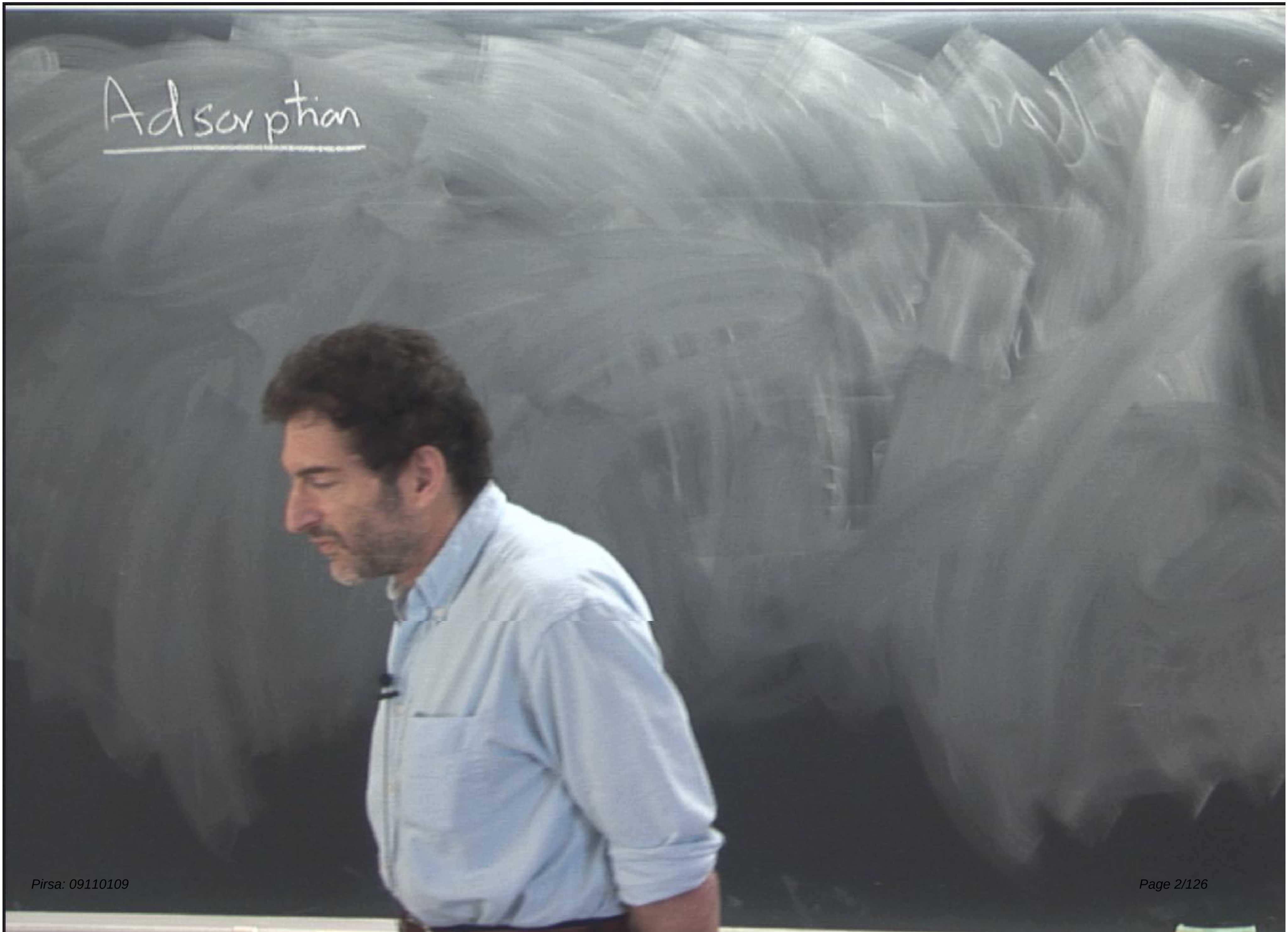
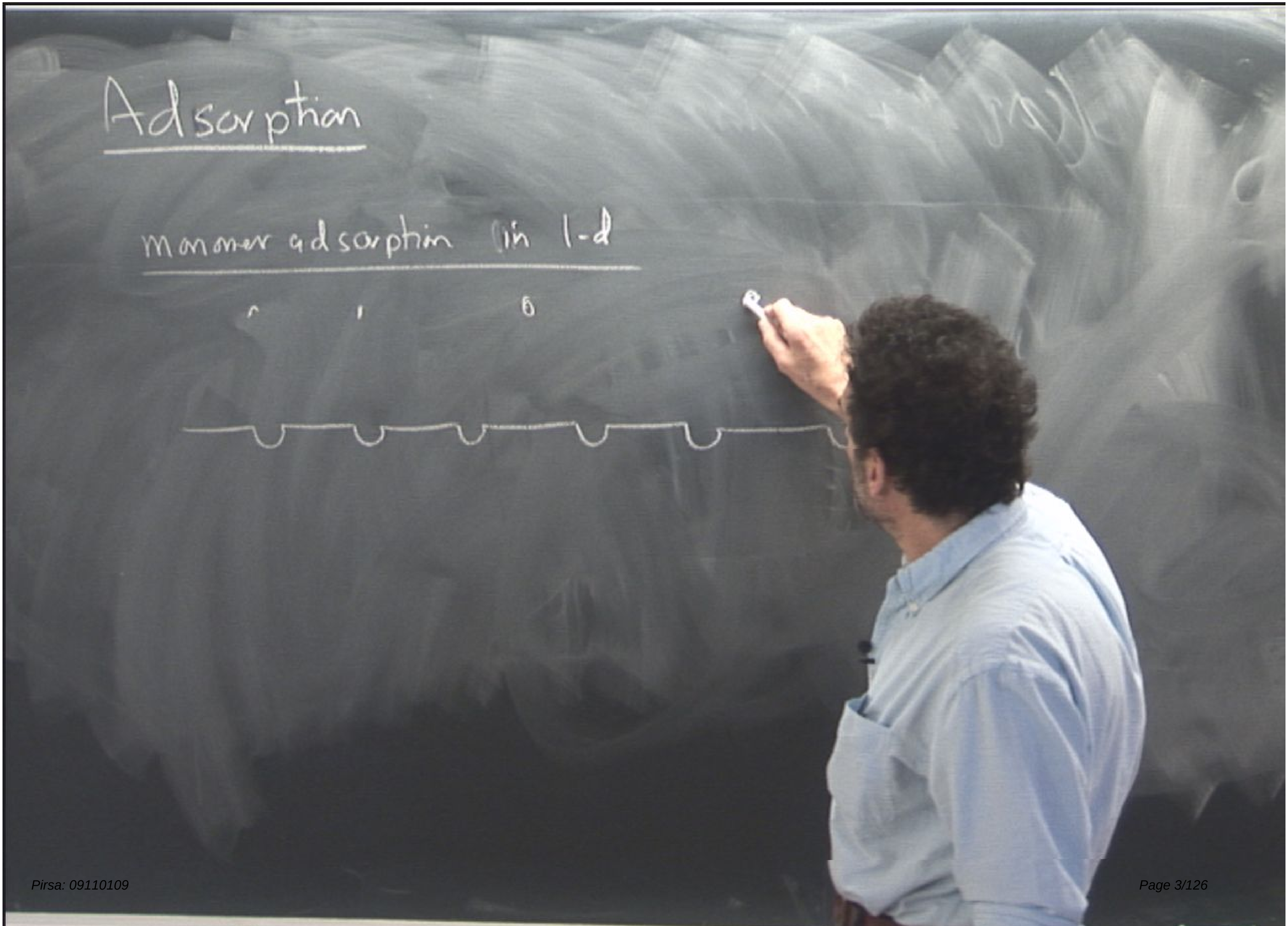
Title: Non-Equilibrium Systems (PHYS 606) - Lecture 5

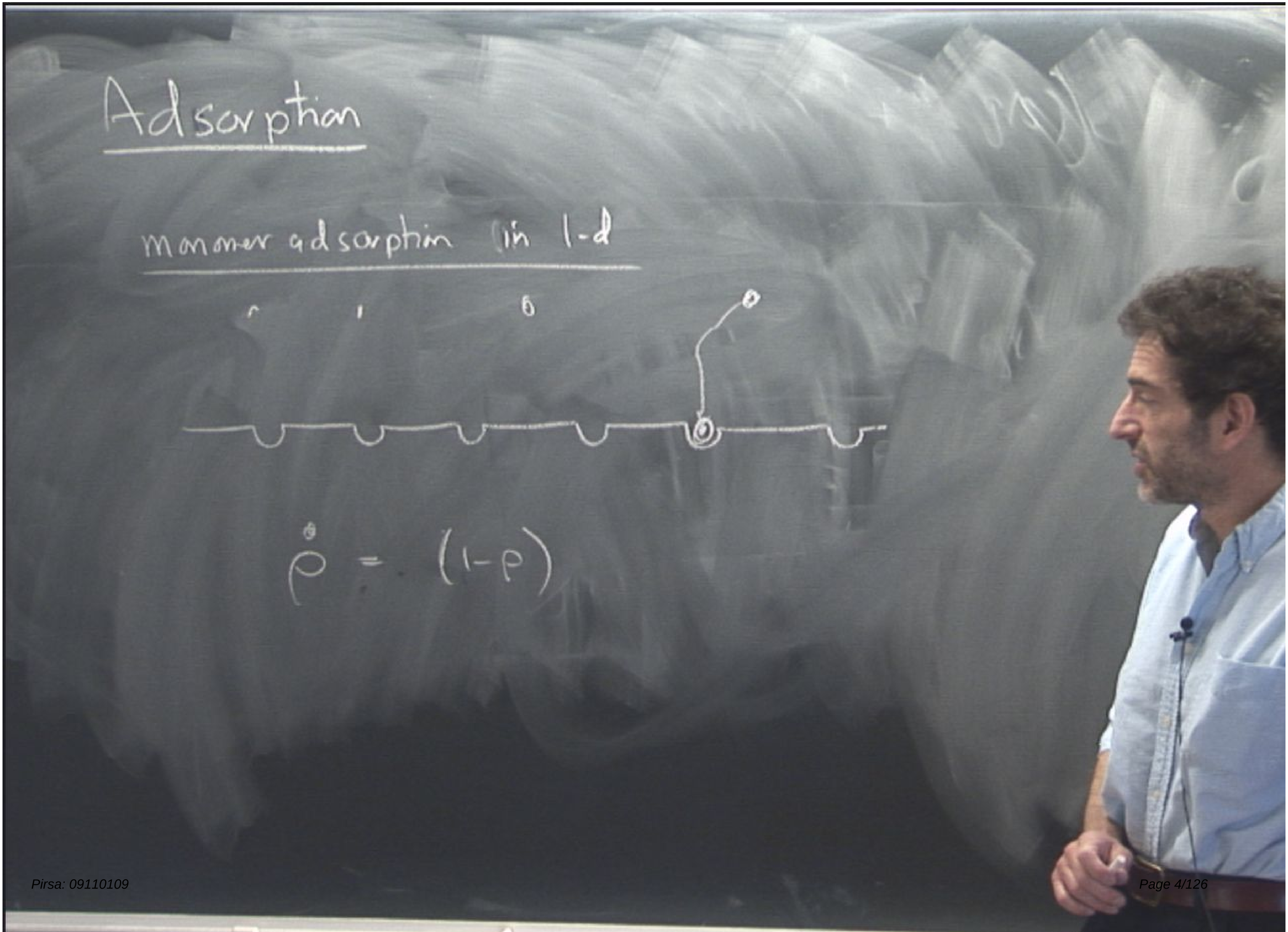
Date: Nov 20, 2009 10:30 AM

URL: http://pirsa.org/09110109

Abstract:

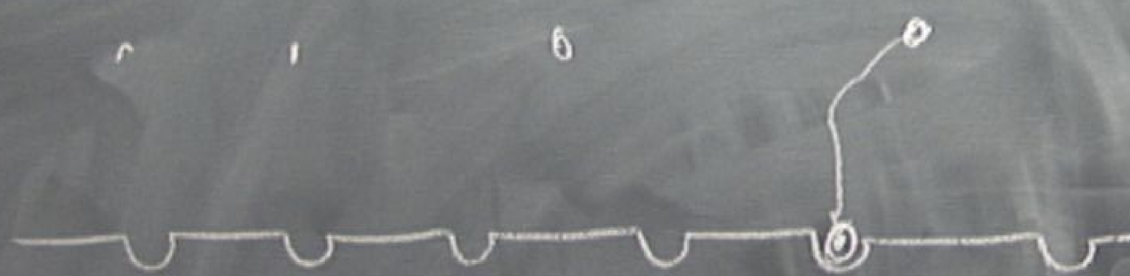

Adsorption


Adsorption
monomer adsorption in 1-d


Adsorption
monomer adsorption in 1-d
$\dot{\rho} = (1-\rho)$

Adsorption

monomer adsorption in 1-d

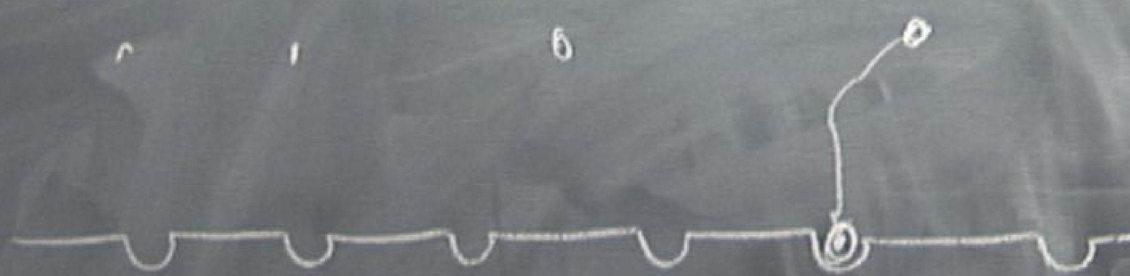


$$\dot{\rho} = (1-\rho) \quad \text{rate eq}^{\underline{n}}$$

$$\rho(t) = 1 - e^{-t}$$

Adsorption

monomer adsorption in 1-d

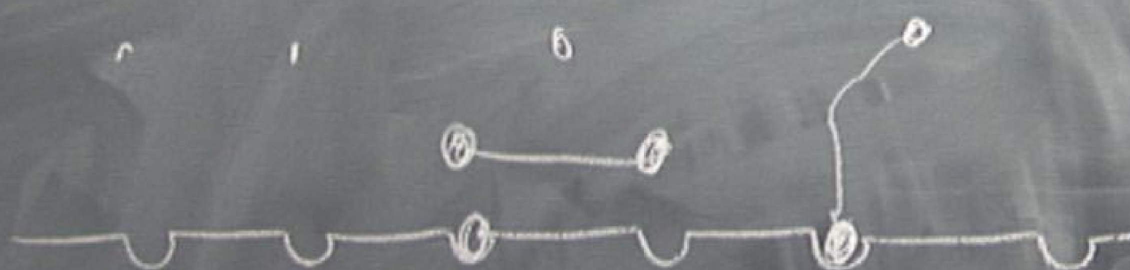


$$\dot{\rho} = (1-\rho) \quad \text{rate eqn}$$

$$\rho(t) = 1 - e^{-t}$$

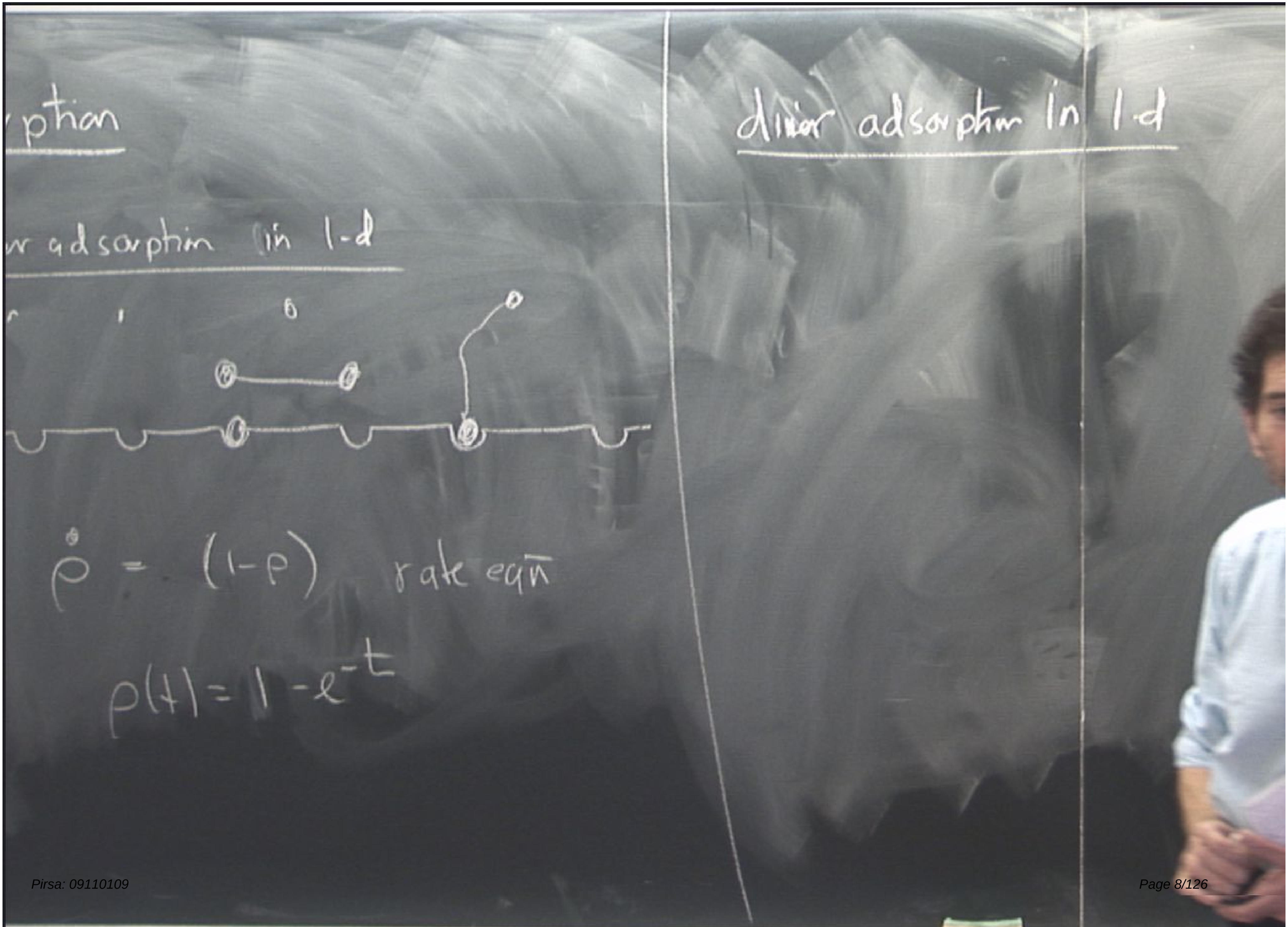
Adsorption

monomer adsorption in 1-d



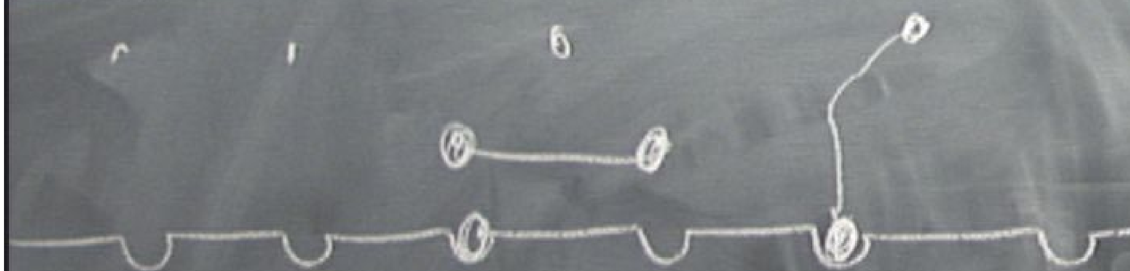
$$\dot{\rho} = (1-\rho) \quad \text{rate eqn}$$

$$\rho(t) = 1 - e^{-t}$$


ption
r adsorption in 1-d
$\dot{\rho} = (1-\rho)$ rate eqn
$\rho(t) = 1 - e^{-t}$
dimer adsorption in 1-d

sorption

monomer adsorption in 1-d



$$\dot{\rho} = (1-\rho) \quad \text{rate eq}^{\underline{n}}$$

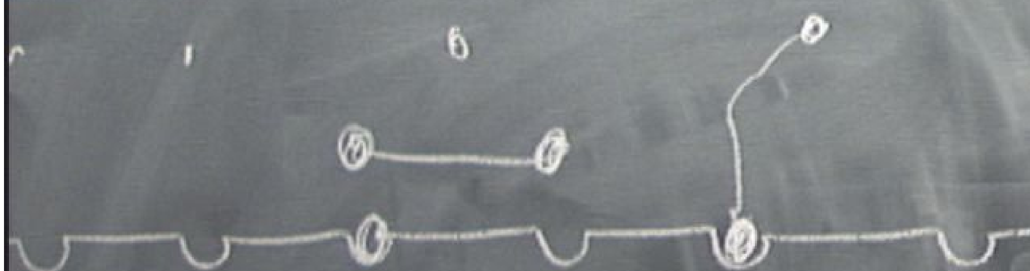
$$\rho(t) = 1 - e^{-t}$$

dimer adsorption in 1-d

$$\rho_\infty = 1 - e^{-2}$$

ption

r adsorption in 1-d



$\dot{\rho} = (1-\rho)$ rate eqn

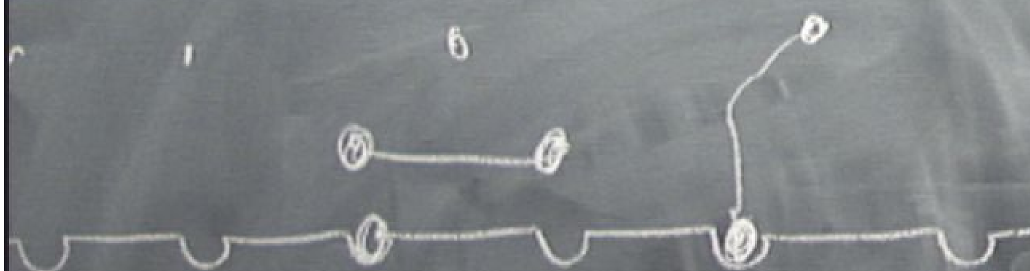
$\rho(t) = 1 - e^{-t}$

dimer adsorption in 1-d

$\rho_\infty = 1 - e^{-2} \cong .864$

ption

r adsorption in 1-d



$$\dot{\rho} = (1-\rho) \quad \text{rate eqn}$$

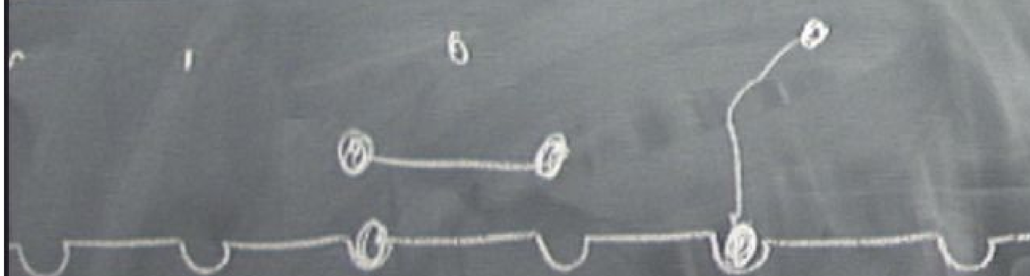
$$\rho(t) = 1 - e^{-t}$$

dimer adsorption in 1-d

$$\rho_\infty = 1 - e^{-2} \cong .864$$

ption

r adsorption in 1-d

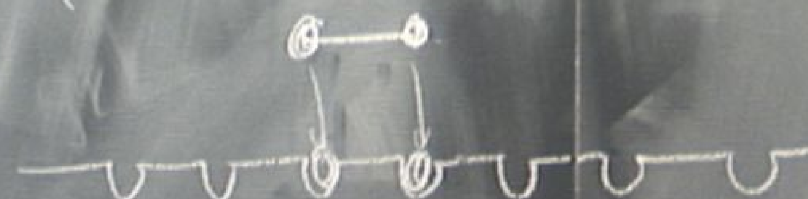


$\dot{\rho} = (1-\rho)$ rate eqn

$\rho(t) = 1 - e^{-t}$

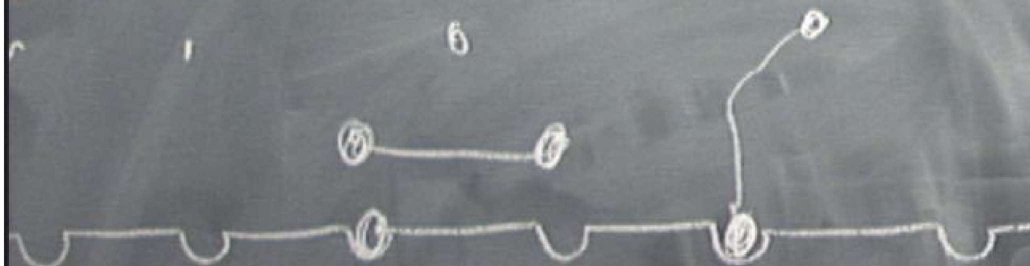
dimer adsorption in 1-d

$\rho_\infty = 1 - e^{-2} \cong .864.$



ption

r adsorption in 1-d

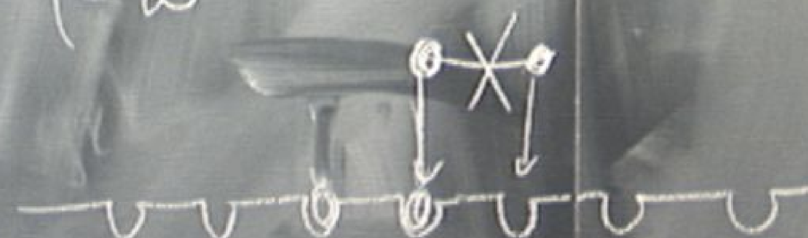


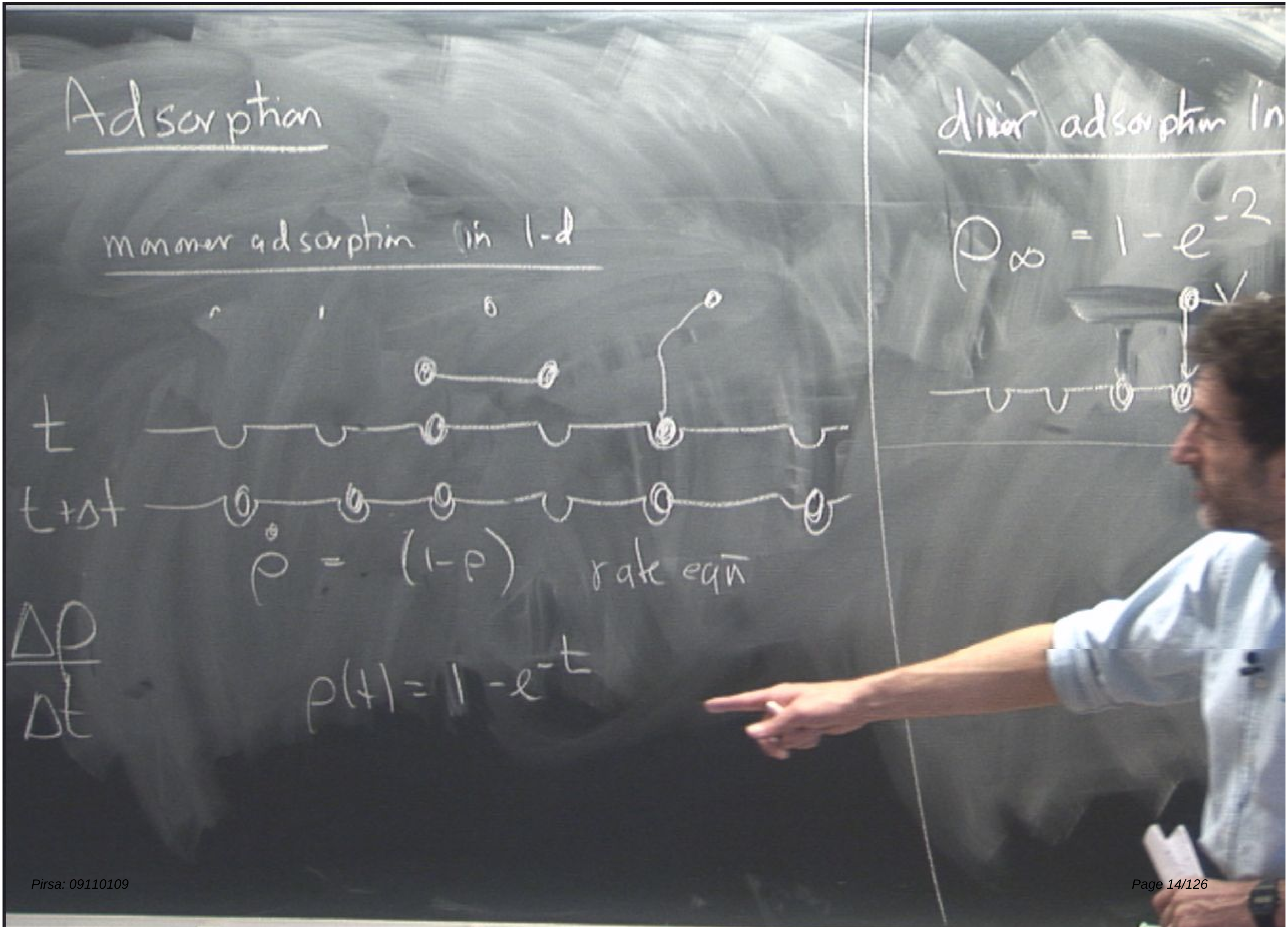
$$\dot{\rho} = (1-\rho) \quad \text{rate eq}\overline{\text{n}}$$

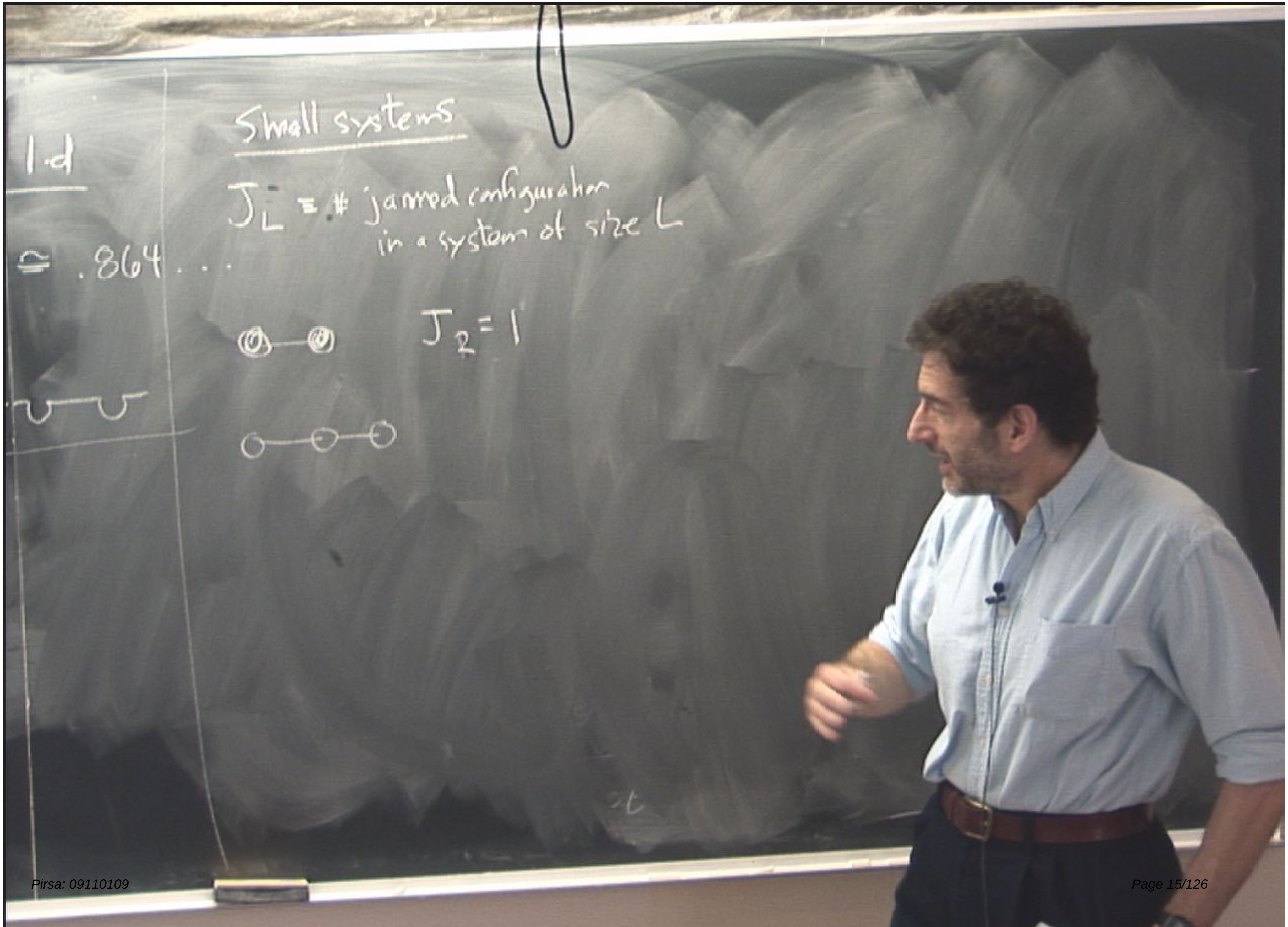
$$\rho(t) = 1 - e^{-t}$$

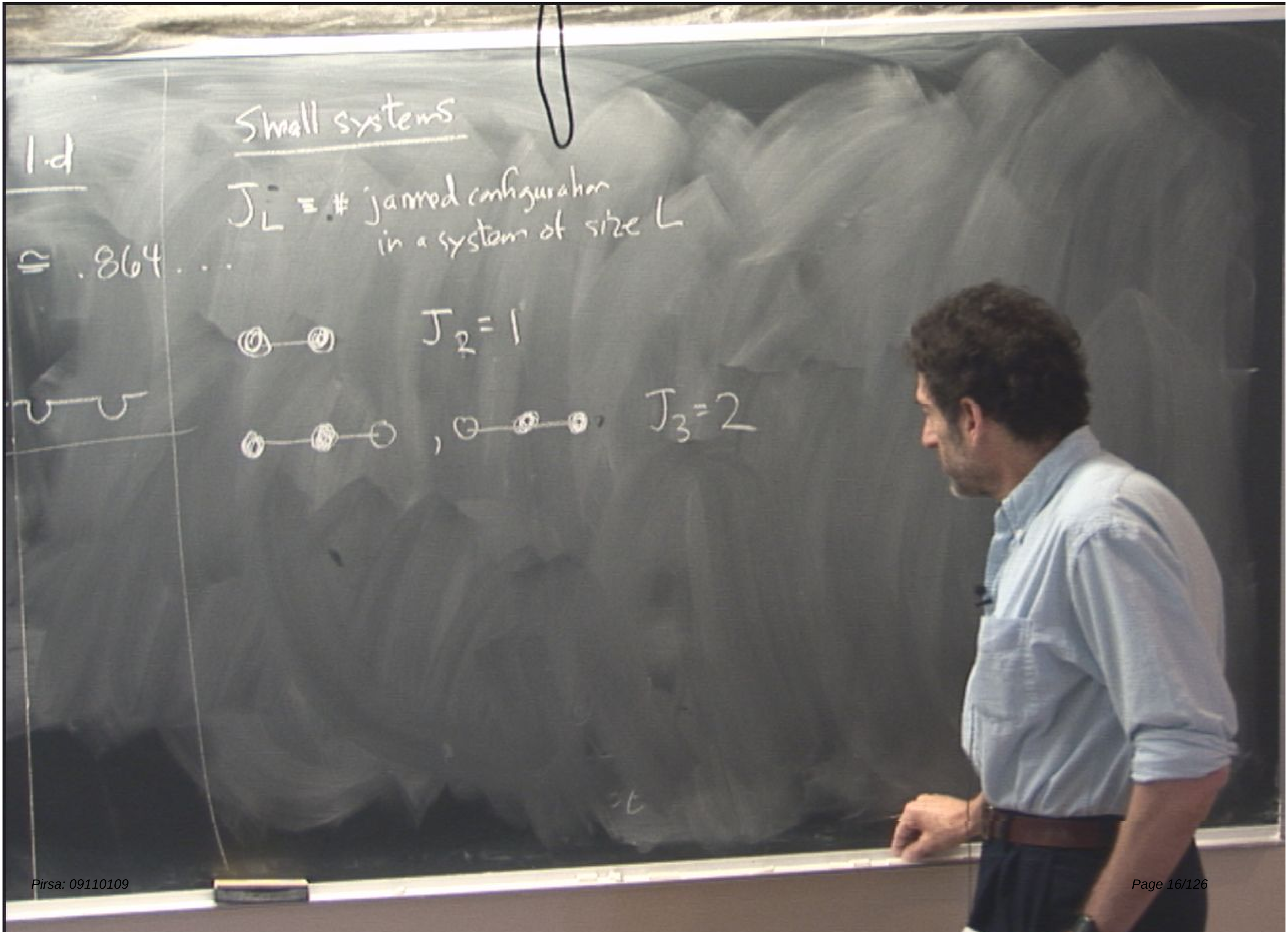
dimer adsorption in 1-d

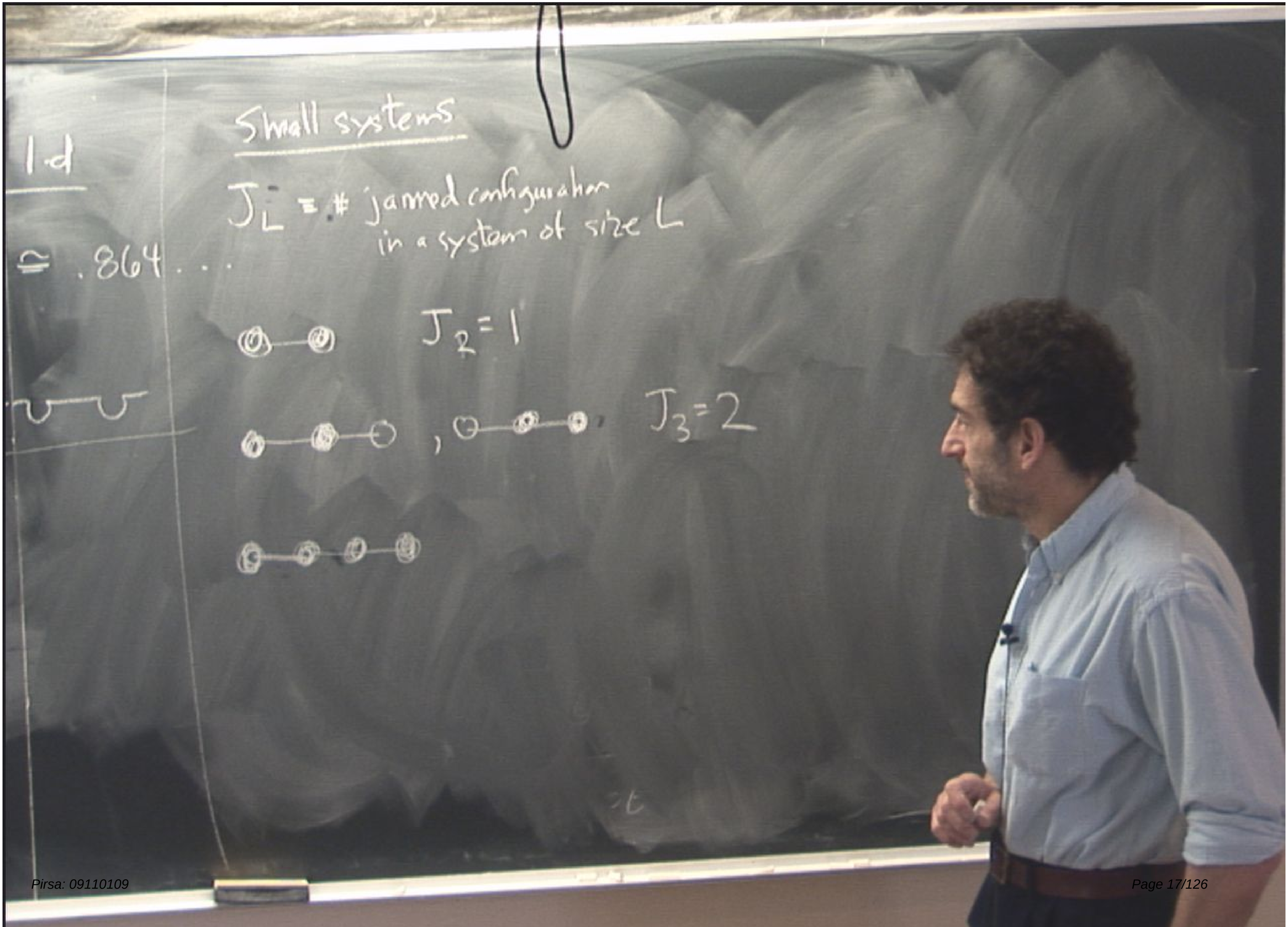
$$\rho_\infty = 1 - e^{-2} \cong .864 .$$

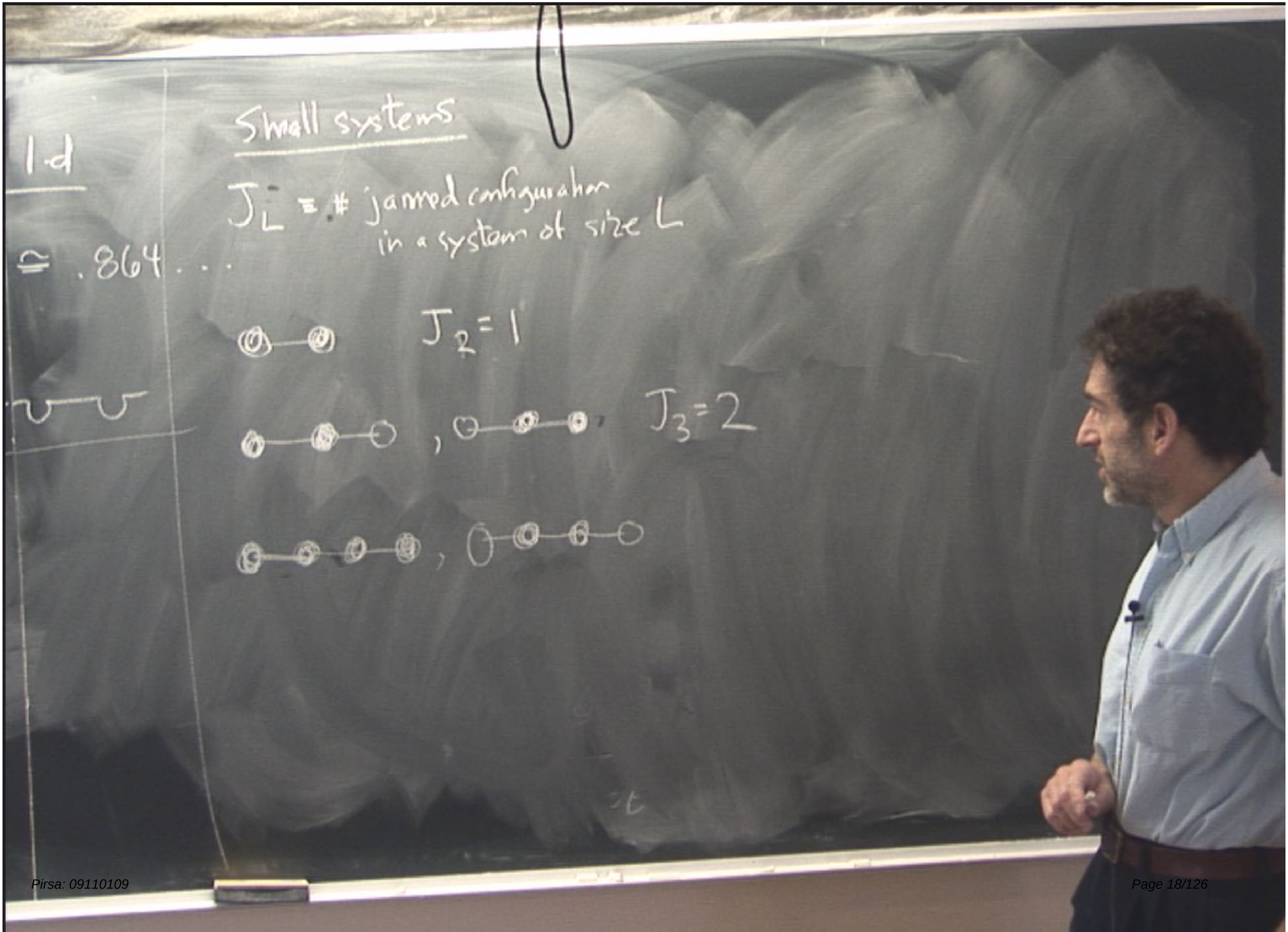



Adsorption
monomer adsorption in 1-d
t
t+Δt
ρ̇ = (1-ρ) rate eqn
Δρ/Δt
ρ(t) = 1 - e^{-t}
dimer adsorption in
ρ_∞ = 1 - e^{-2}


1.d
≅ .864...
Small systems
J_L ≡ # jammed configuration
in a system of size L
J_2 = 1


1.d
≅ .864...
Small systems
J_L ≡ # jammed configuration in a system of size L
J_2 = 1
J_3 = 2


1.d
≅ .864...
Small systems
J_L ≡ # jammed configuration
in a system of size L
J_2 = 1
J_3 = 2



Small systems
1.d
J_L ≡ # jammed configuration in a system of size L
≅ .864...
J_2 = 1
J_3 = 2

1.d

$\cong .864\ldots$

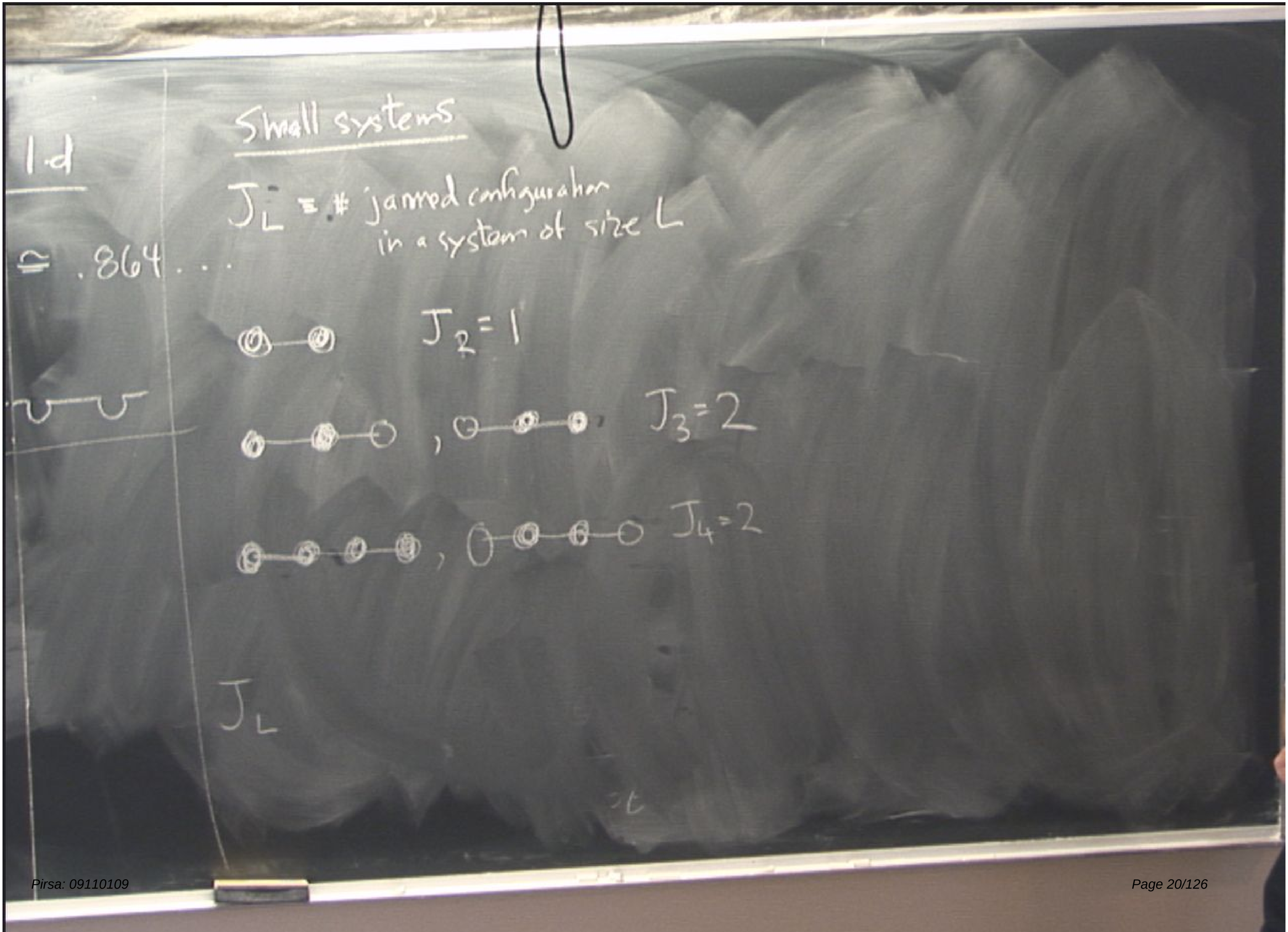
Small systems

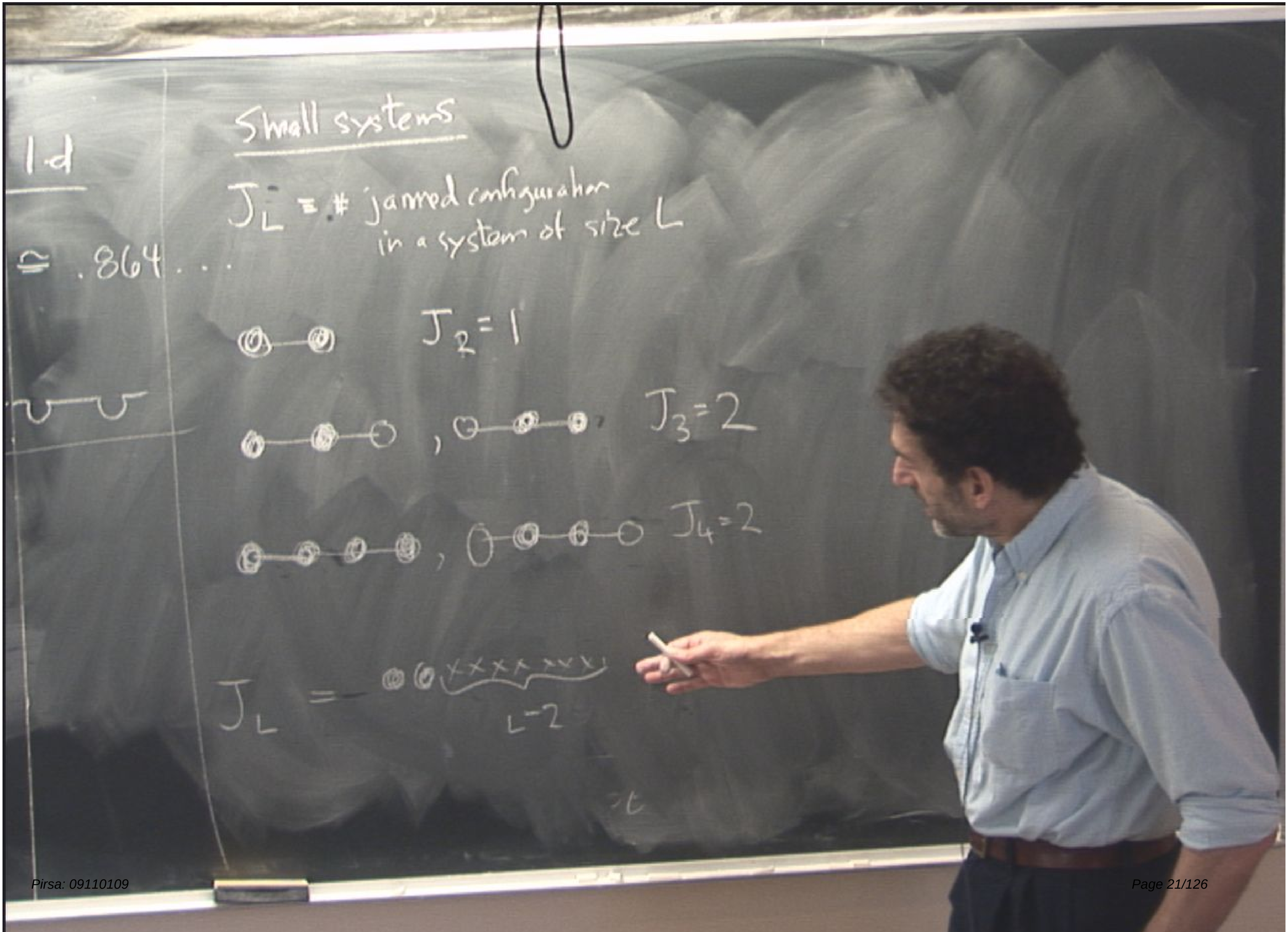
$J_L \equiv$ # jammed configuration in a system of size L

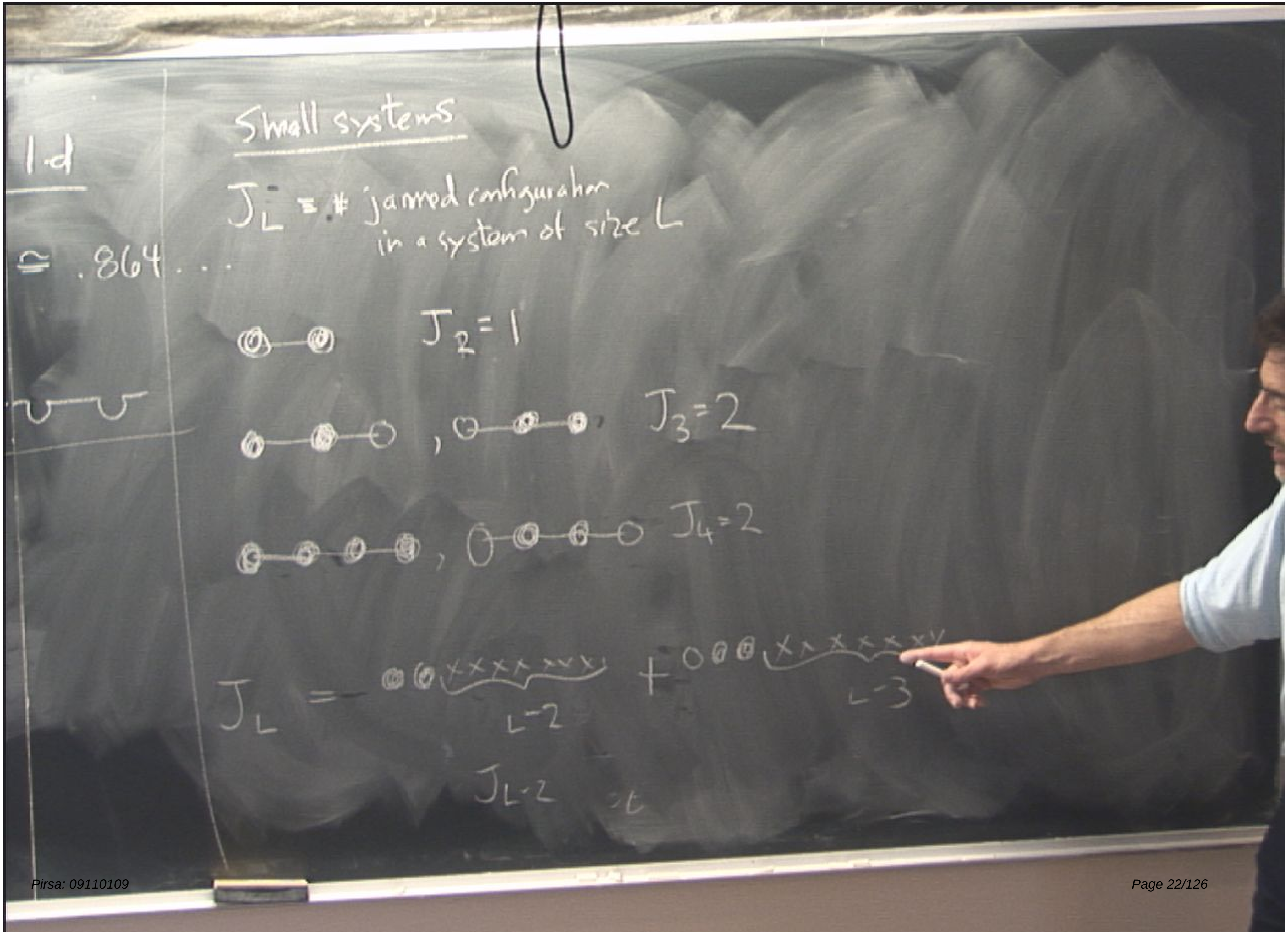
$J_2 = 1$

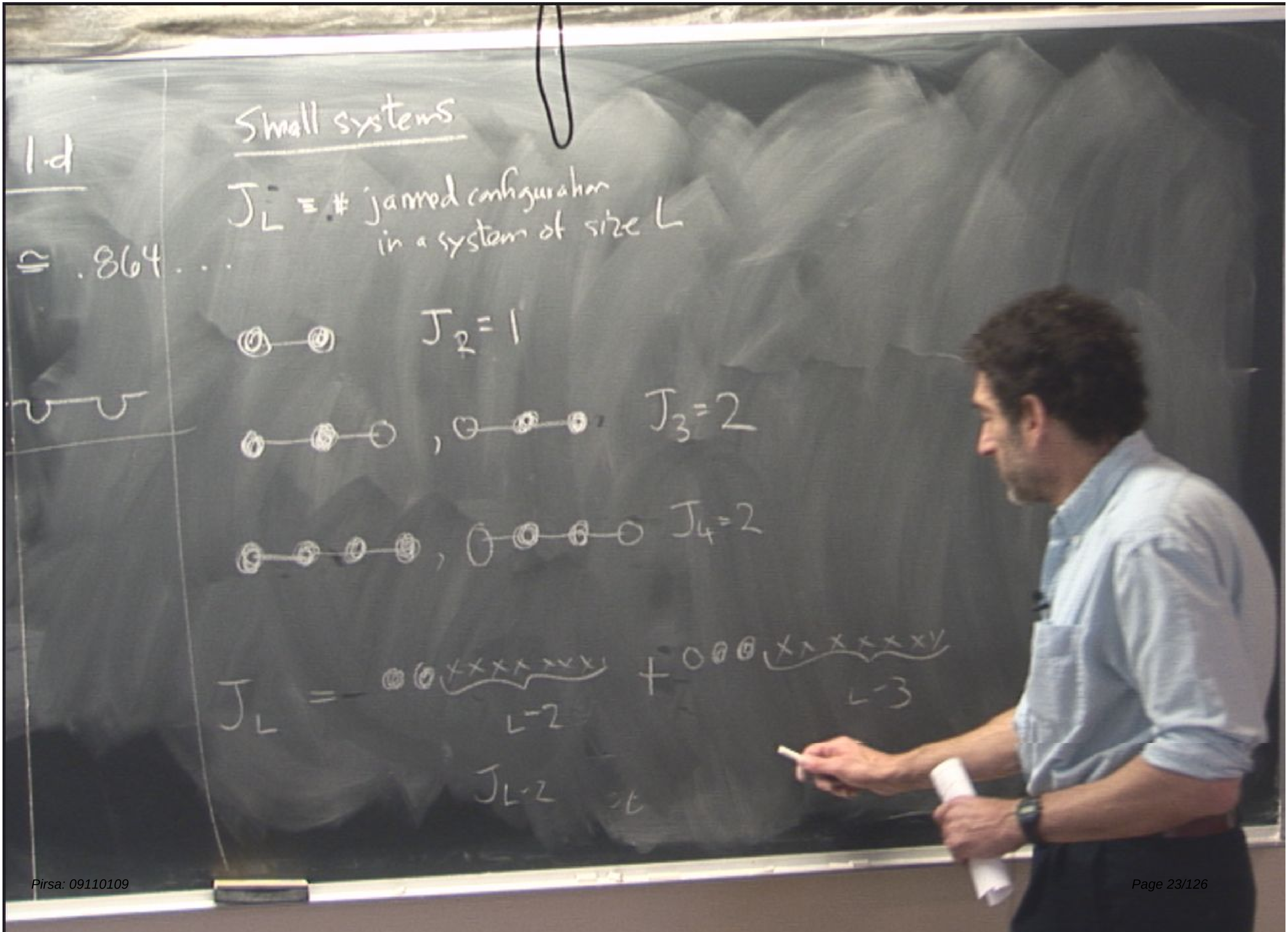
$J_3 = 2$

$J_4 = 2$


Small systems
1.d
≅ .864...
J_L ≡ # jammed configuration
in a system of size L
J_2 = 1
J_3 = 2
J_4 = 2
J_L


Small systems
1.d
$J_L \equiv$ # jammed configuration in a system of size L
≅ .864 . . .
$J_2 = 1$
$J_3 = 2$
$J_4 = 2$
$J_L =$
L−2


1.d
≅ .864...
Small systems
J_L ≡ # jammed configuration in a system of size L
J_2 = 1
J_3 = 2
J_4 = 2
J_L =
L-2
L-3
J_L-2


Small systems
1.d
≅ .864...
J_L ≡ # jammed configuration
in a system of size L
J_2 = 1
J_3 = 2
J_4 = 2
J_L =
L-2
L-3
J_L-2

$$J_L = J_{L-2} + J_{L-3}$$

1-d

$\cong .864\ldots$

Small systems

$J_L \equiv$ # jammed configuration in a system of size L

$J_2 = 1$

$J_3 = 2$

$J_4 = 2$

$$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$$

$$J_{L-2} \qquad J_{L-3}$$

1.d

$\cong .864\ldots$

Small systems

$J_L \equiv$ # jammed configuration in a system of size L

$J_2 = 1$

$J_3 = 2$

$J_4 = 2$

$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$

J_{L-2} J_{L-3}

$J_L = J_{L-2} + J_{L-3}$

$J_L \underset{L\to\infty}{\sim} 1.342$

1.d

$\cong .864\ldots$

Small systems

$J_L \equiv$ # jammed configuration in a system of size L

$J_2 = 1$

$J_3 = 2$

$J_4 = 2$

$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$

$J_{L-2} + J_{L-3}$

$J_L = J_{L-2} + J_{L-3}$

$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$

1-d

$\cong .864\ldots$

Small systems

$J_L \equiv$ # jammed configuration in a system of size L

$J_2 = 1$

$J_3 = 2$

$J_4 = 2$

$J_L = \underbrace{\times\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$

$J_{L-2} \qquad J_{L-3}$

$J_L = J_{L-2} + J_{L-3}$

$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$

1-d

$\cong .864\ldots$

Small systems

$J_L \equiv$ # jammed configuration in a system of size L

$J_2 = 1$

$J_3 = 2$

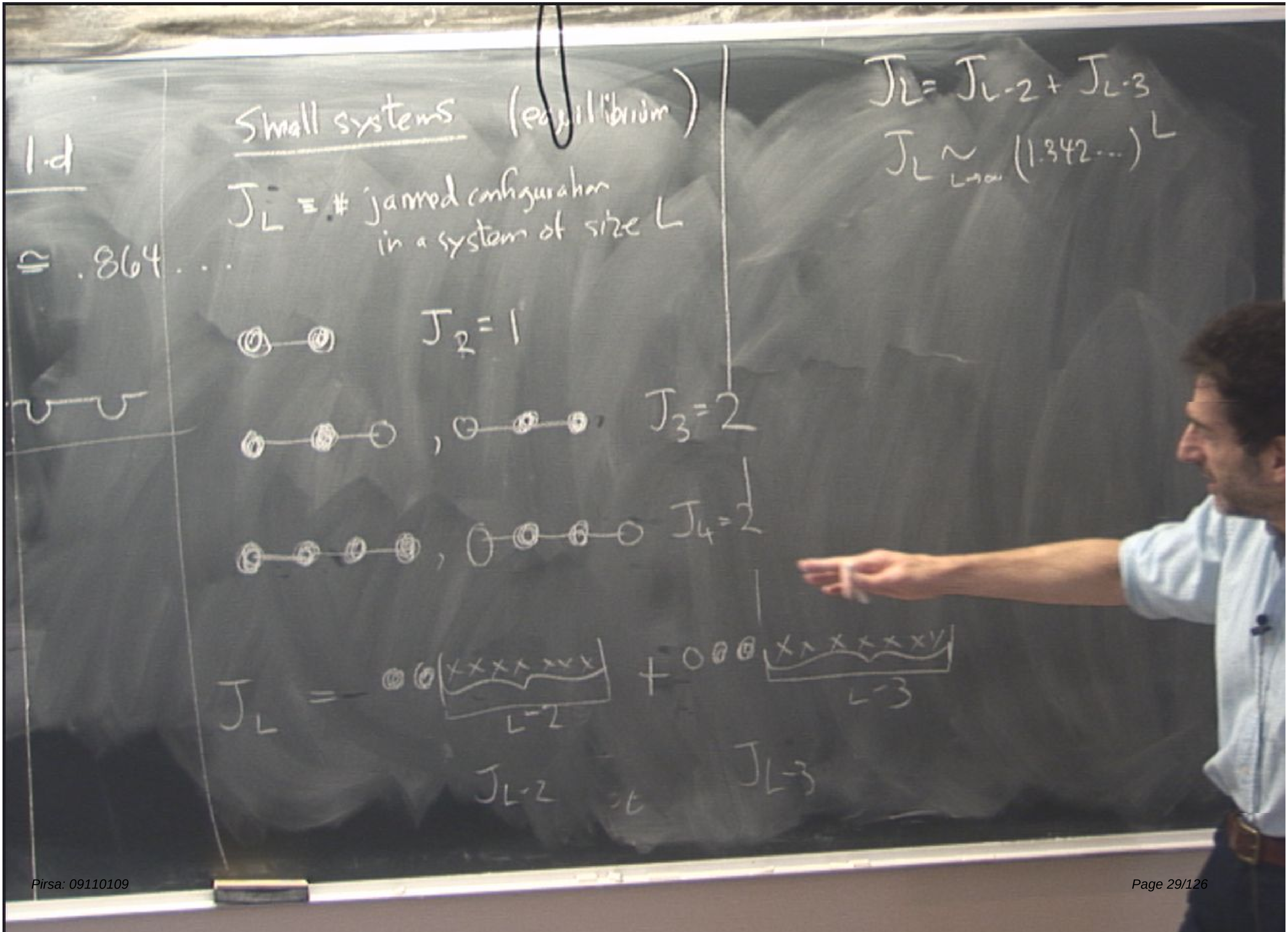
$J_4 = 2$

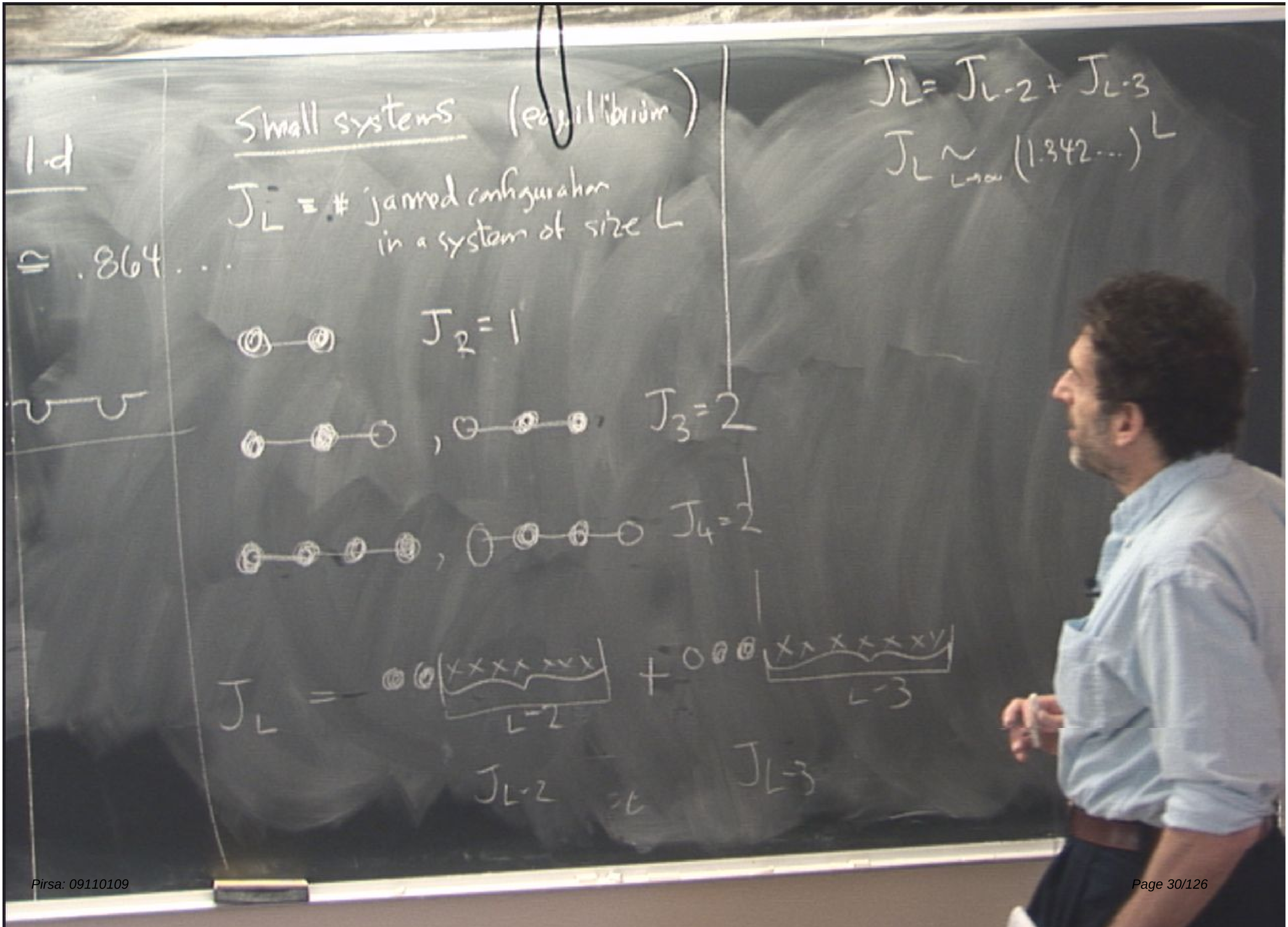
$$J_L = \bullet\bullet\underbrace{\times\times\times\times\times\times\times}_{L-2} + \circ\bullet\bullet\underbrace{\times\times\times\times\times\times\times}_{L-3}$$

$$J_{L-2} \qquad\qquad J_{L-3}$$

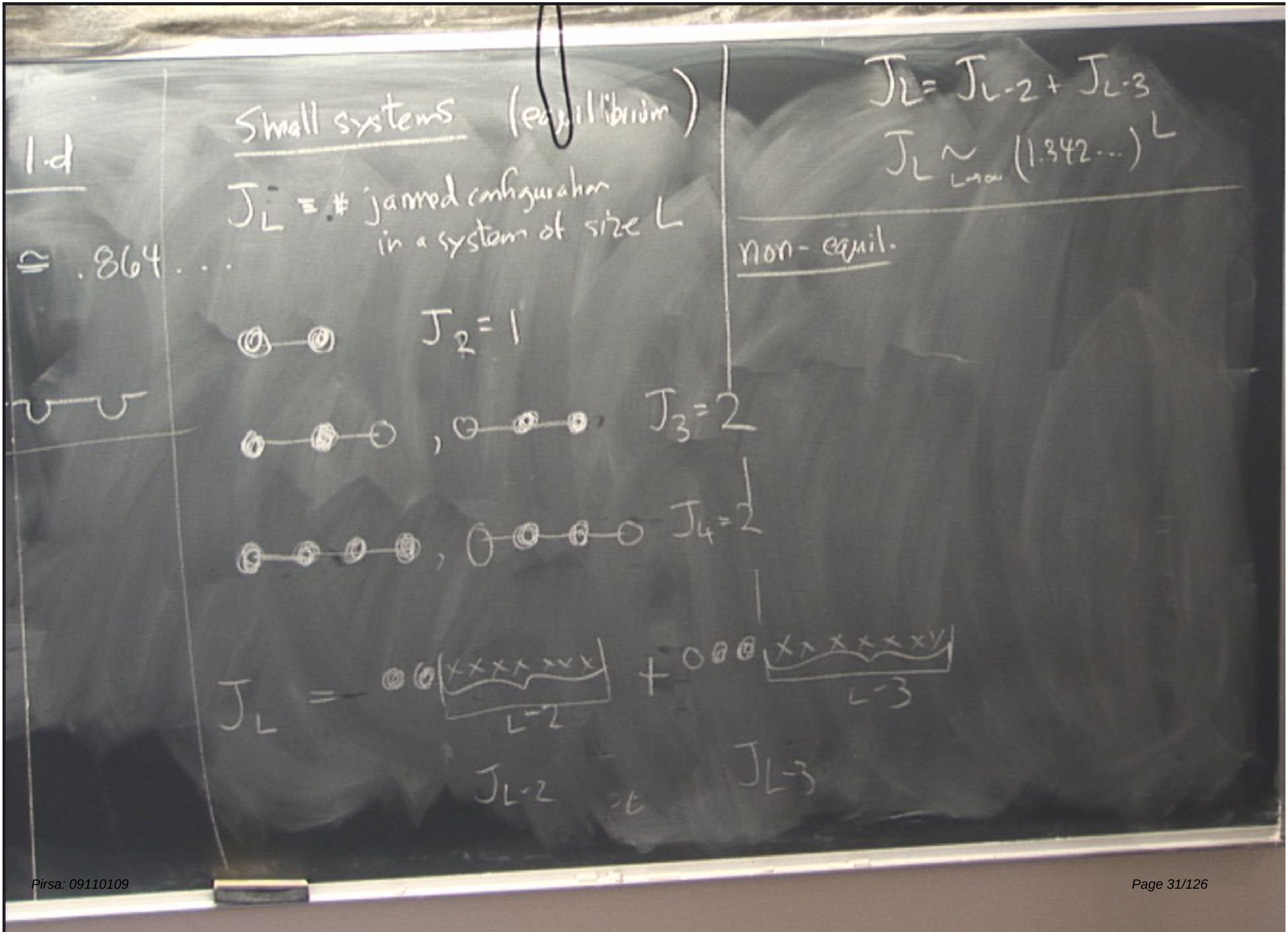
$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$


1.d
≅ .864...
Small systems (equilibrium)
$J_L \equiv$ # jammed configuration in a system of size L
$J_2 = 1$
$J_3 = 2$
$J_4 = 2$
$J_L =$
L-2
+
L-3
J_{L-2}
J_{L-3}
$J_L = J_{L-2} + J_{L-3}$
$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$



Small systems (equilibrium)
1.d
≅ .864...
J_L ≡ # jammed configuration in a system of size L
$J_2 = 1$
$J_3 = 2$
$J_4 = 2$
J_L = 00 xxxxxxx + 000 xxxxxxx
L-2
L-3
J_{L-2}
J_{L-3}
$J_L = J_{L-2} + J_{L-3}$
$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$



1.d
≅ .864...
Small systems (equilibrium)
J_L ≡ # jammed configuration in a system of size L
J_2 = 1
J_3 = 2
J_4 = 2
J_L =
L-2
L-3
J_{L-2}
J_{L-3}
J_L = J_{L-2} + J_{L-3}
J_L ~ (1.342...)^L
L→∞
non-equil.

1-d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$

$J_3 = 2$

$J_4 = 2$

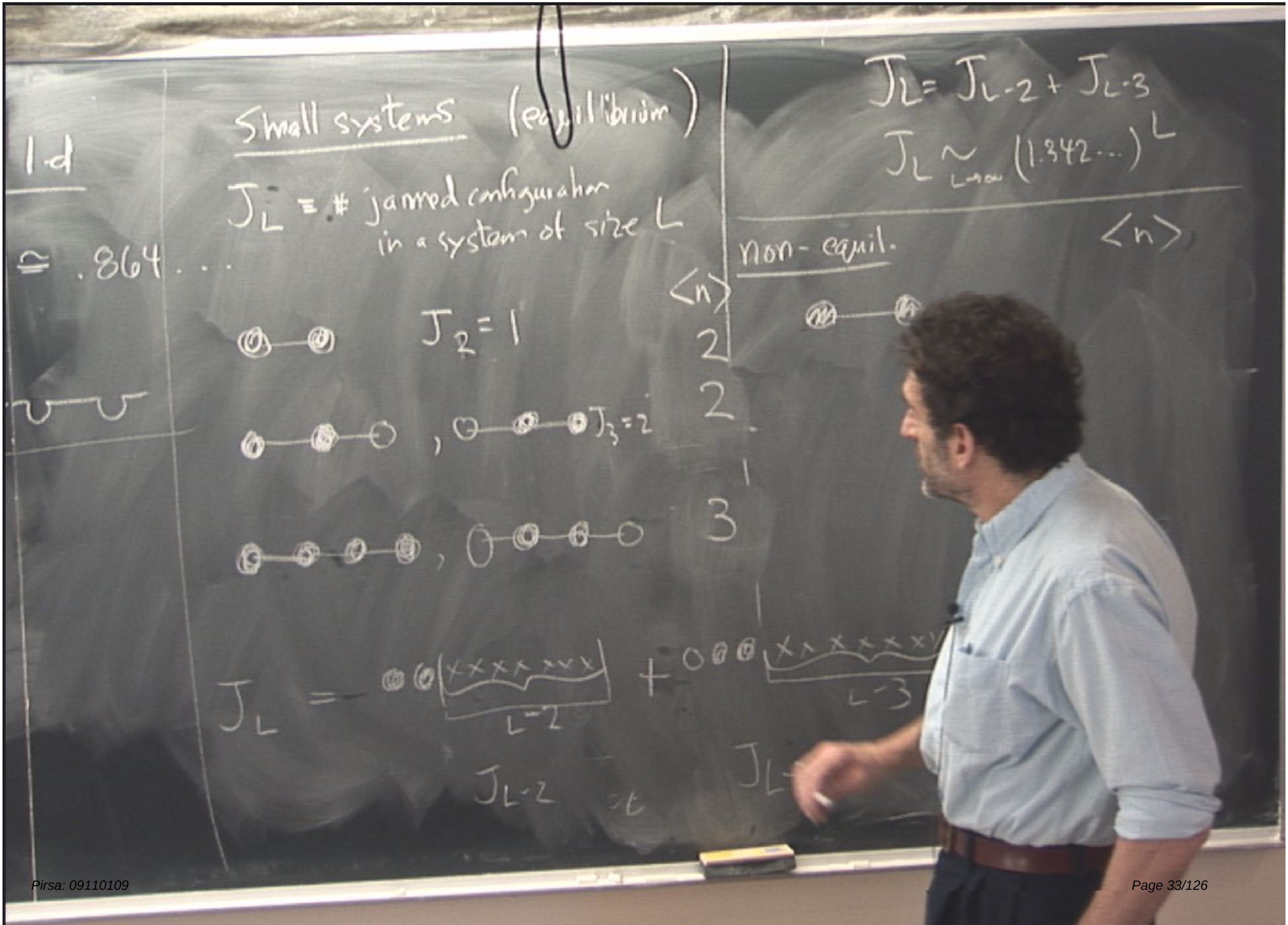
$$J_L = \underbrace{\circ\circ\, xxxxxxx}_{L-2} + \underbrace{\circ\circ\circ\, xxxxxxx}_{L-3}$$

$$J_{L-2} \qquad J_{L-3}$$

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil.



1.d
≃ .864...
Small systems (equilibrium)
$J_L \equiv$ # jammed configuration in a system of size L
⟨n⟩
$J_2 = 1$
2
$J_3 = 2$
2
3
$J_L = J_{L-2} + J_{L-3}$
$J_L \underset{L\to\infty}{\sim} (1.342...)^L$
non-equil.
⟨n⟩
L-2
L-3
J_{L-2}

1.d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ 2

$J_3 = 2$ 2

3

$$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$$

$$J_{L-2} \qquad J_{L-3}$$

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil. $\langle n \rangle$

1 d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ 2

$J_3 = 2$ 2

3

$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$

J_{L-2} J_{L-3}

$J_L = J_{L-2} + J_{L-3}$

$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$

non-equil. $\langle n \rangle$

or

1.d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

●—● $\quad J_2 = 1 \quad$ 2

●—●—○ , ○—●—● $\quad J_3 = 2 \quad$ 2

●—●—●—● , ○—●—●—○ $\quad$ 3

$$J_L = \bullet\bullet\underbrace{\times\times\times\times\times\times\times}_{L-2} + \circ\bullet\bullet\underbrace{\times\times\times\times\times\times\times}_{L-3}$$

$J_{L-2} \qquad J_{L-3}$

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil.

$\langle n \rangle$

●—● $\quad$ 2

●—●—○ or ○—●—● $\quad$ 2

●—●—○—○ or ○—○—●—● $\quad$ 1/3

1 d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ — 2

$J_3 = 2$ — 2

3

$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$

$J_{L-2} \qquad J_{L-3}$

$J_L = J_{L-2} + J_{L-3}$

$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$

non-equil.

$\langle n \rangle$

2

or — 2

or — 1/3

1/3

or — 1/3

1.d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ 2

$J_3 = 2$ 2

3

$$J_L = \underbrace{xxxxxxx}_{L-2} + \underbrace{xxxxxxx}_{L-3}$$

J_{L-2} J_{L-3}

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil. $\langle n \rangle$

2

or 2

or 1/3

1/3

or 1/3

$$\langle n \rangle = \frac{2}{3}4 + \frac{1}{3}2 = \frac{10}{3}$$

1.d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ — 2

$J_3 = 2$ — 2

3

$$J_L = \underbrace{\bullet\bullet\,xxxxxxxx}_{L-2} + \underbrace{\circ\bullet\bullet\,xxxxxxx}_{L-3}$$

$$J_{L-2} + J_{L-3}$$

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil.

$\langle n \rangle$

2

2

or

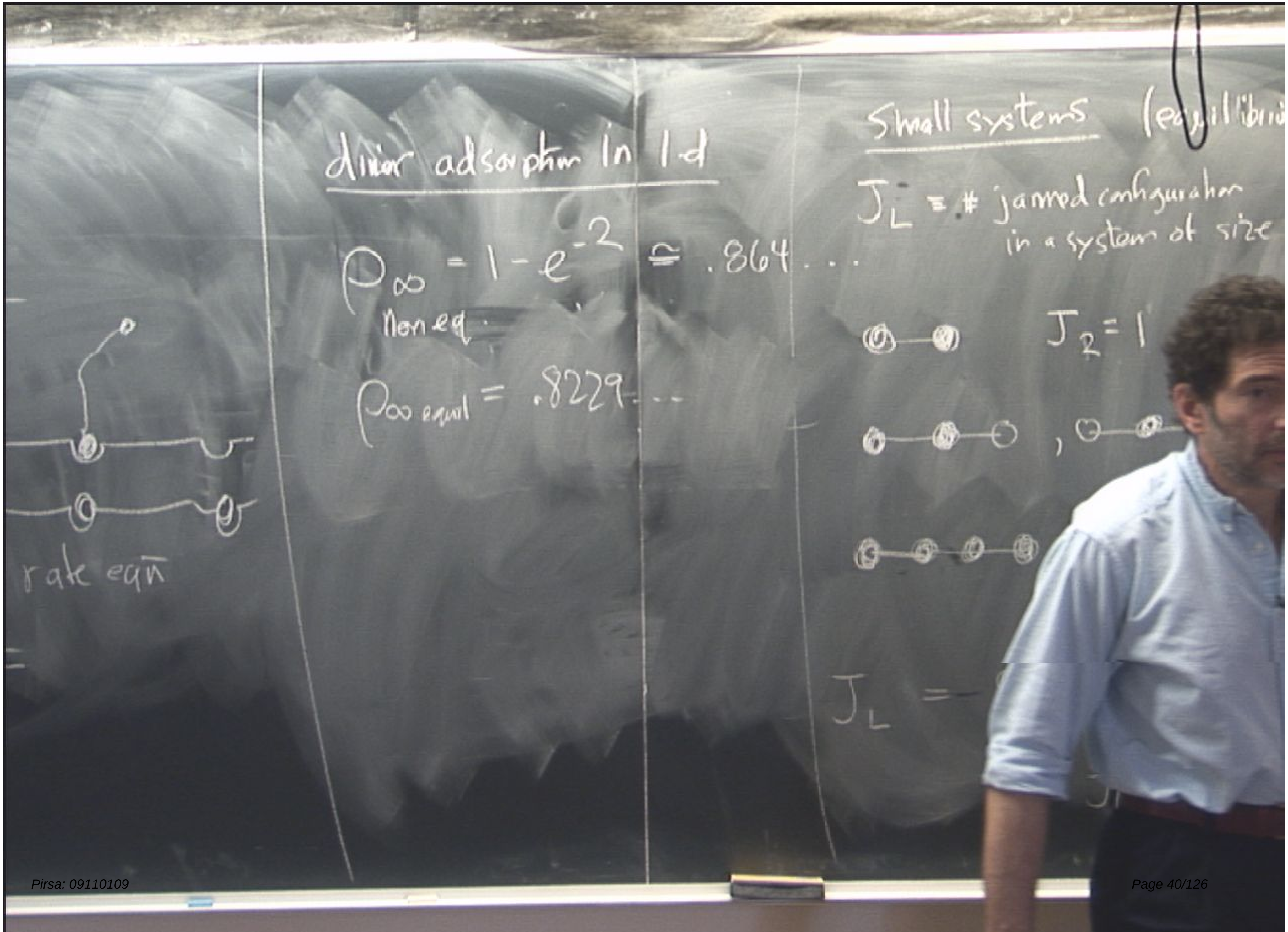
or $1/3$

$1/3$

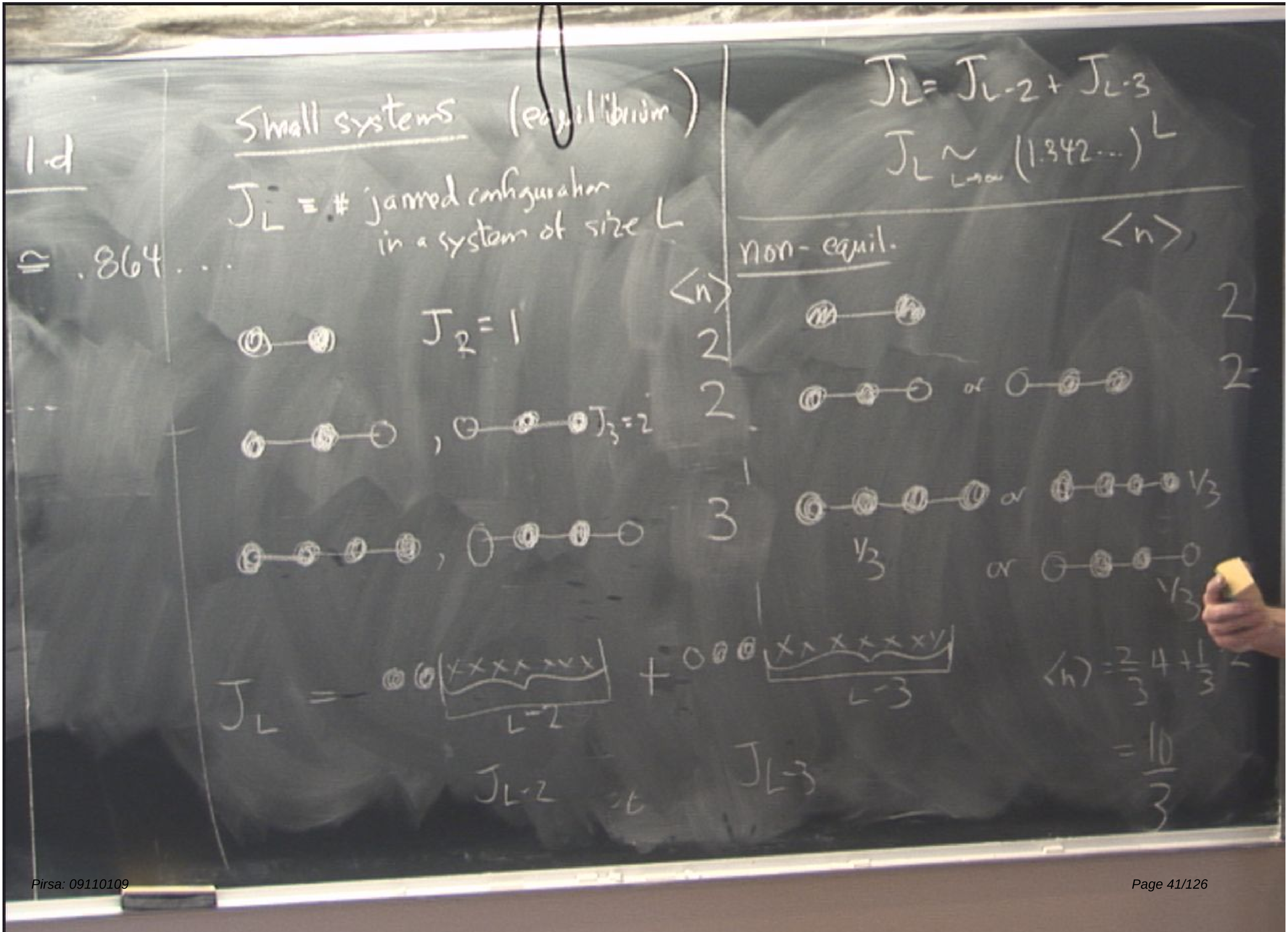
or

$1/3$

$$\langle n \rangle = \frac{2}{3}4 + \frac{1}{3}2 = \frac{10}{3}$$



dimer adsorption in 1-d
$\rho_{\infty} = 1 - e^{-2} \cong .864 \ldots$
non eq
$\rho_{\infty\,\mathrm{equil}} = .8229 \ldots$
rate eqn
Small systems
$J_L \equiv$ # jammed configuration
in a system of size
$J_2 = 1$
$J_L =$


1.d
≅ .864...
Small systems (equilibrium)
J_L ≡ # jammed configuration in a system of size L
⟨n⟩
J_2 = 1
2
J_3 = 2
2
3
J_L = J_{L-2} + J_{L-3}
L-2
L-3
J_{L-2}
J_{L-3}
J_L = J_{L-2} + J_{L-3}
J_L ~ (1.342...)^L
L→∞
non-equil.
⟨n⟩
2
or
2
or
1/3
1/3
or
1/3
⟨n⟩ = 2/3 4 + 1/3 2
= 10/3

1.d

$\cong .864\ldots$

Small systems (equilibrium)

$J_L \equiv$ # jammed configuration in a system of size L

$\langle n \rangle$

$J_2 = 1$ — 2

$J_3 = 2$ — 2

3

$$J_L = \underbrace{\times\times\times\times\times\times\times}_{L-2} + \underbrace{\times\times\times\times\times\times\times}_{L-3}$$

J_{L-2} J_{L-3}

$$J_L = J_{L-2} + J_{L-3}$$

$$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$$

non-equil.

$\langle n \rangle$

2

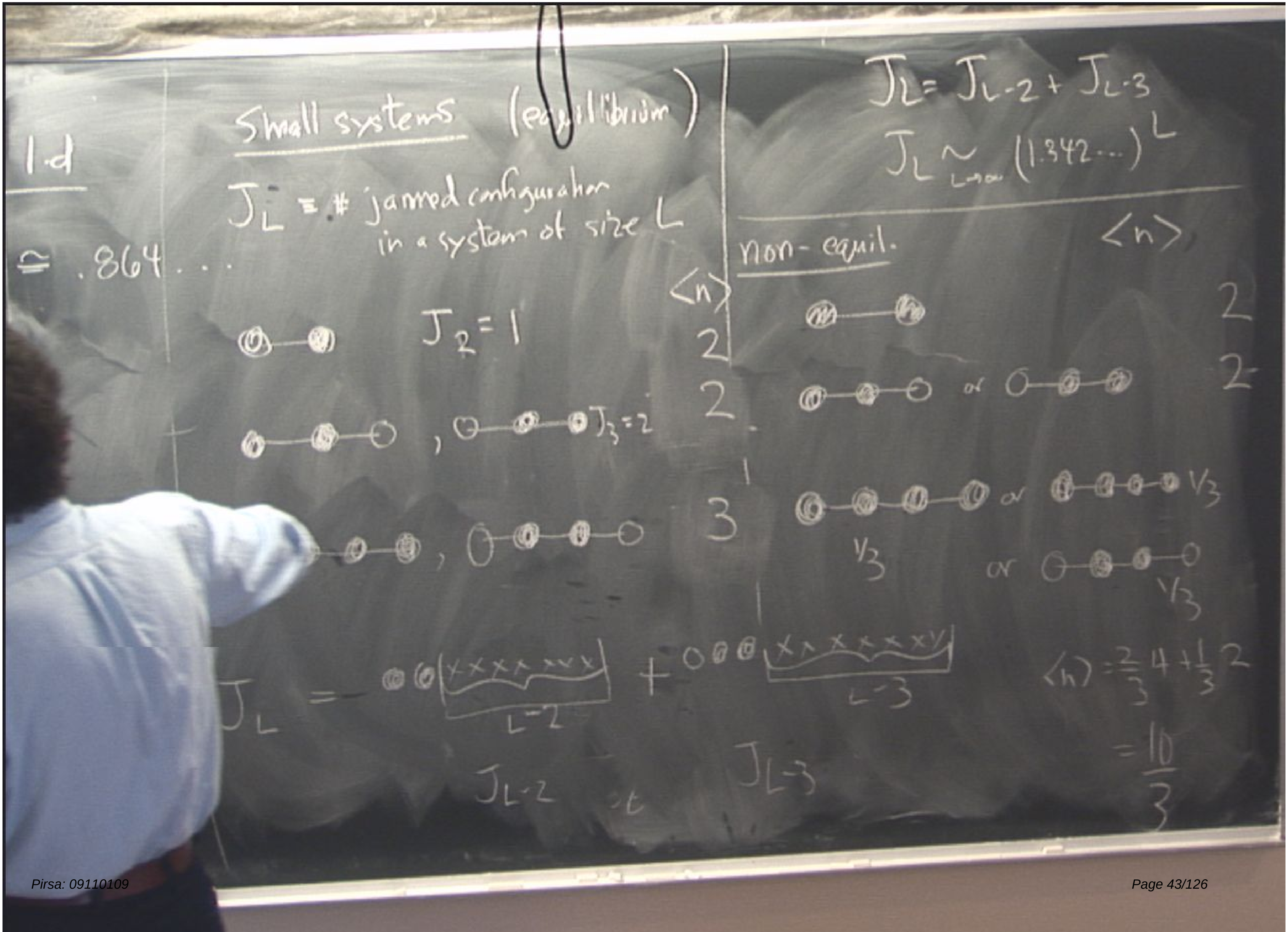
2

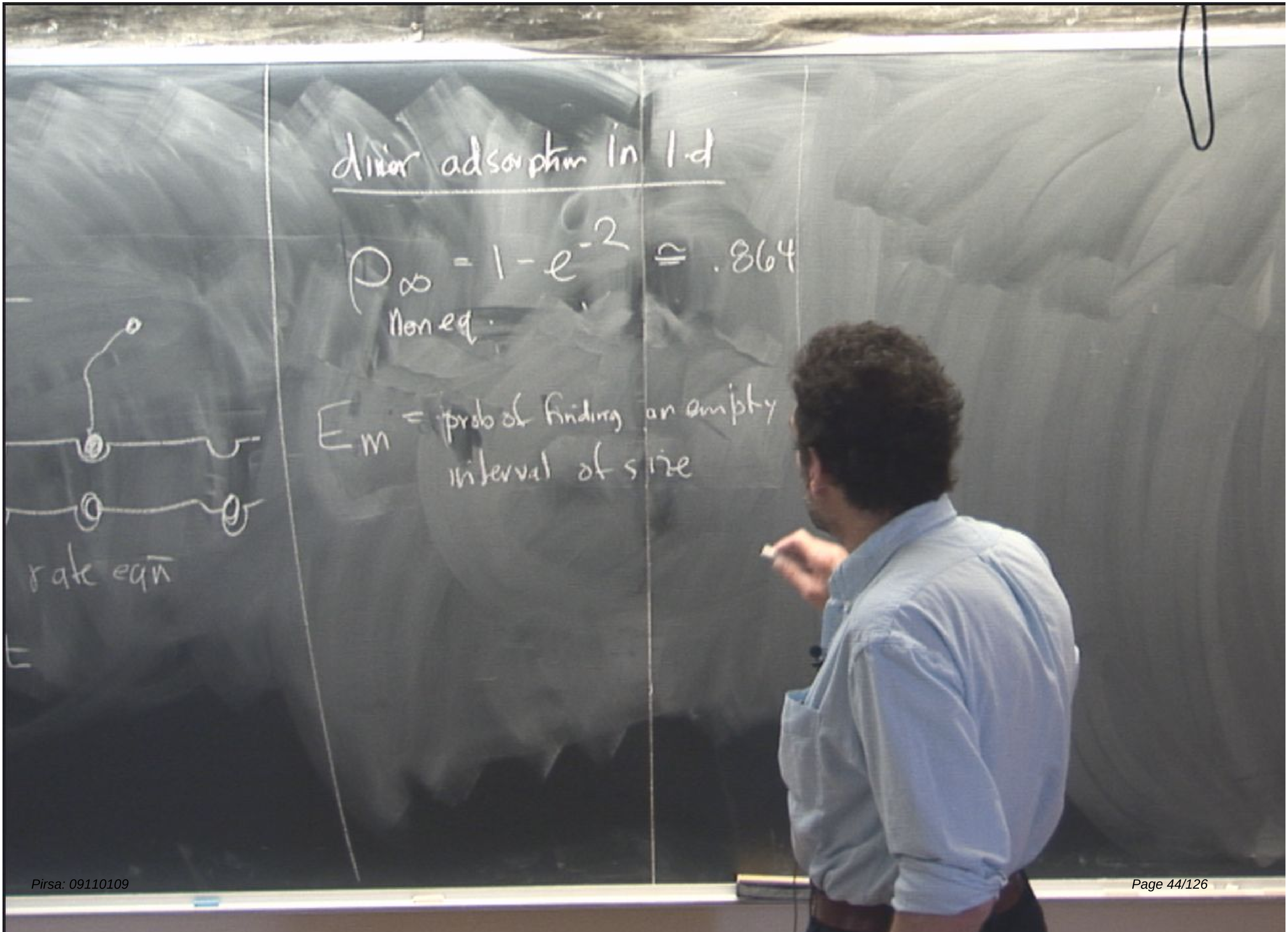
or — $1/3$

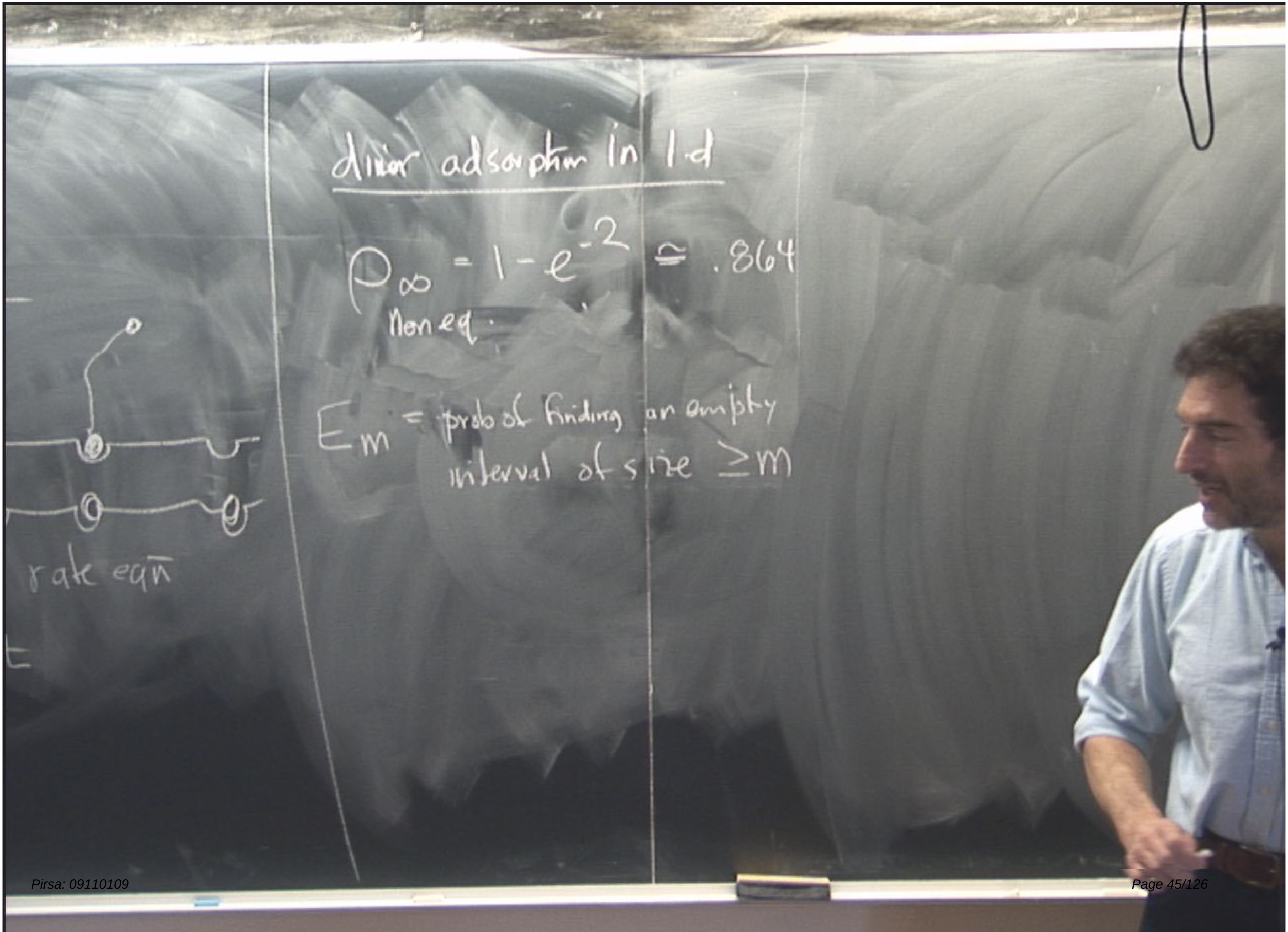
$1/3$

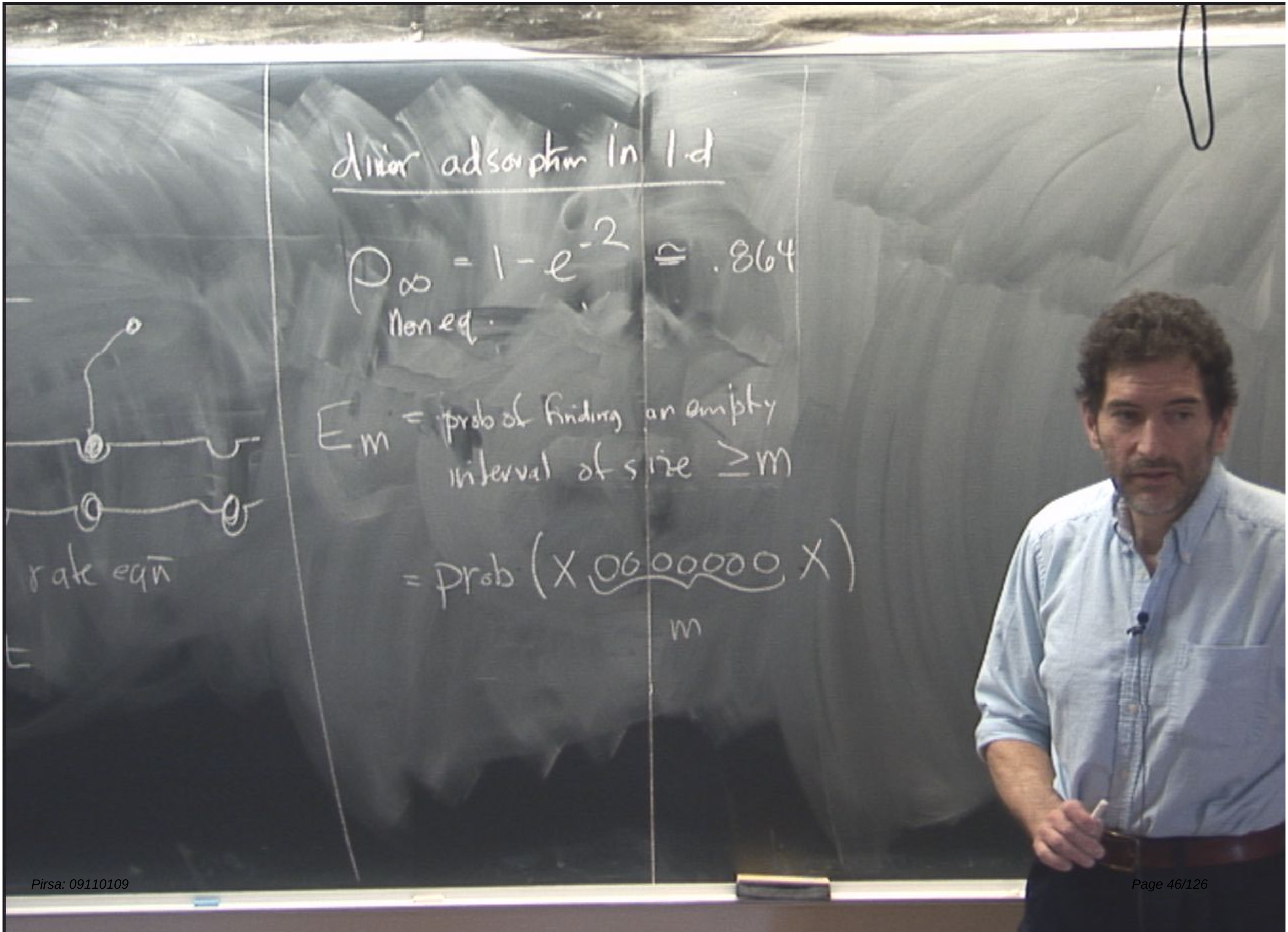
or — $1/3$

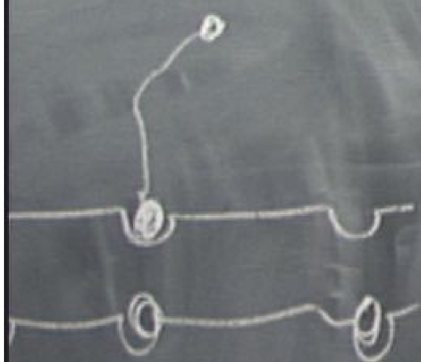
$$\langle n \rangle = \frac{2}{3}4 + \frac{1}{3}2 = \frac{10}{3}$$


Small systems (equilibrium)
1.d
$\simeq .864\ldots$
$J_L \equiv$ # jammed configuration in a system of size L
$\langle n \rangle$
$J_2 = 1$
2
$J_3 = 2$
2
3
$J_L = J_{L-2} + J_{L-3}$
$J_L \underset{L\to\infty}{\sim} (1.342\ldots)^L$
non-equil.
$\langle n \rangle$
2
2
1/3
1/3
1/3
$J_L = \underbrace{xxxxxxx}_{L-2} + \underbrace{xxxxxxx}_{L-3}$
J_{L-2}
J_{L-3}
$\langle n \rangle = \frac{2}{3} 4 + \frac{1}{3} 2$
$= \frac{10}{3}$


dimer adsorption in 1-d
$\rho_\infty = 1 - e^{-2} \cong .864$
non eq.
E_m = prob of finding an empty interval of size
rate eqn


dimer adsorption in 1-d
$\rho_{\infty} = 1 - e^{-2} \cong .864$
non eq.
E_m = prob of finding an empty interval of size $\geq m$
rate eqn


dimer adsorption in 1-d
$\rho_{\infty} = 1 - e^{-2} \cong .864$
noneq.
E_m = prob of finding an empty interval of size $\geq m$
= prob $(X \underbrace{0000000}_{m} X)$
rate eqn



rate eqn

dimer adsorption in 1 d

$$\rho_{\infty} = 1 - e^{-2} \cong .864$$

non eq.

E_m = prob of finding an empty interval of size $\geq m$

$$= \text{prob}\,(X\,\underbrace{0000000}_{m}\,X)$$

dimer adsorption in 1-d

$$\rho_{\infty \text{ non eq}} = 1 - e^{-2} \cong .864$$

E_m = prob of finding an empty interval of size $\geq m$

$$= \text{prob}\,(X\,\underbrace{oo\,ooooo}_{m}\,X)$$

$$\frac{\partial E_m}{\partial t}$$

dimer adsorption in 1-d

$$\rho_{\infty}^{\text{non eq}} = 1 - e^{-2} \cong .864$$

E_m = prob of finding an empty interval of size $\geq m$

$$= \text{prob}\,(X\,\underbrace{00\,0000}_{m}\,X)$$

$$\frac{\partial E_m}{\partial t} = -(m-1)\,E_m - 2E_{m+1}$$

dimer adsorption in 1-d

$$\rho_\infty = 1 - e^{-2} \cong .864$$

non eq.

E_m = prob of finding an empty interval of size $\geq m$

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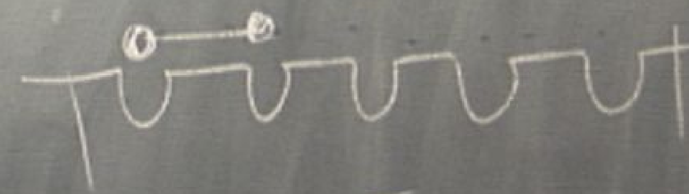


5

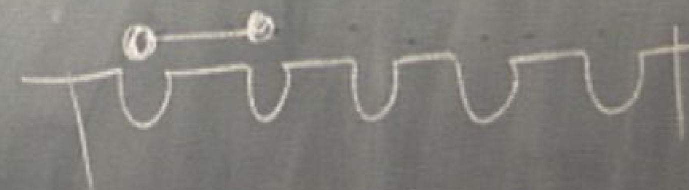
dimer adsorption in 1-d

$$\rho_{\infty}^{\text{non eq}} = 1 - e^{-2} \cong .864$$

E_m = prob of finding an empty interval of size $\geq m$

$$= \text{prob}\,(X\,\underbrace{000000}_{m}\,X)$$

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$



5

in 1d

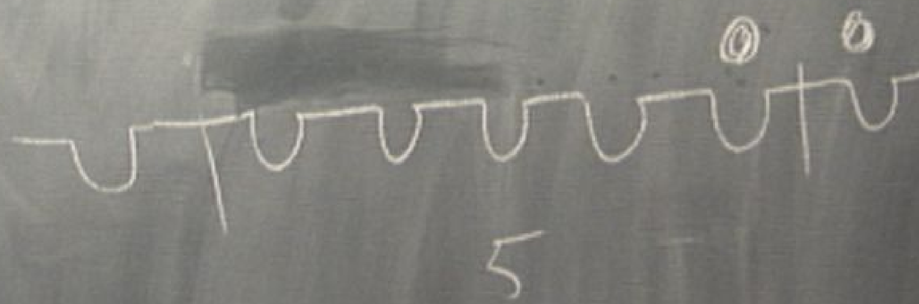
$2 \cong .864$

ding an empty

of size $\geq m$

$(\underbrace{00|00000}_{m}\ X)$

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$



in 1d

$2 \cong .864$

ding an empty

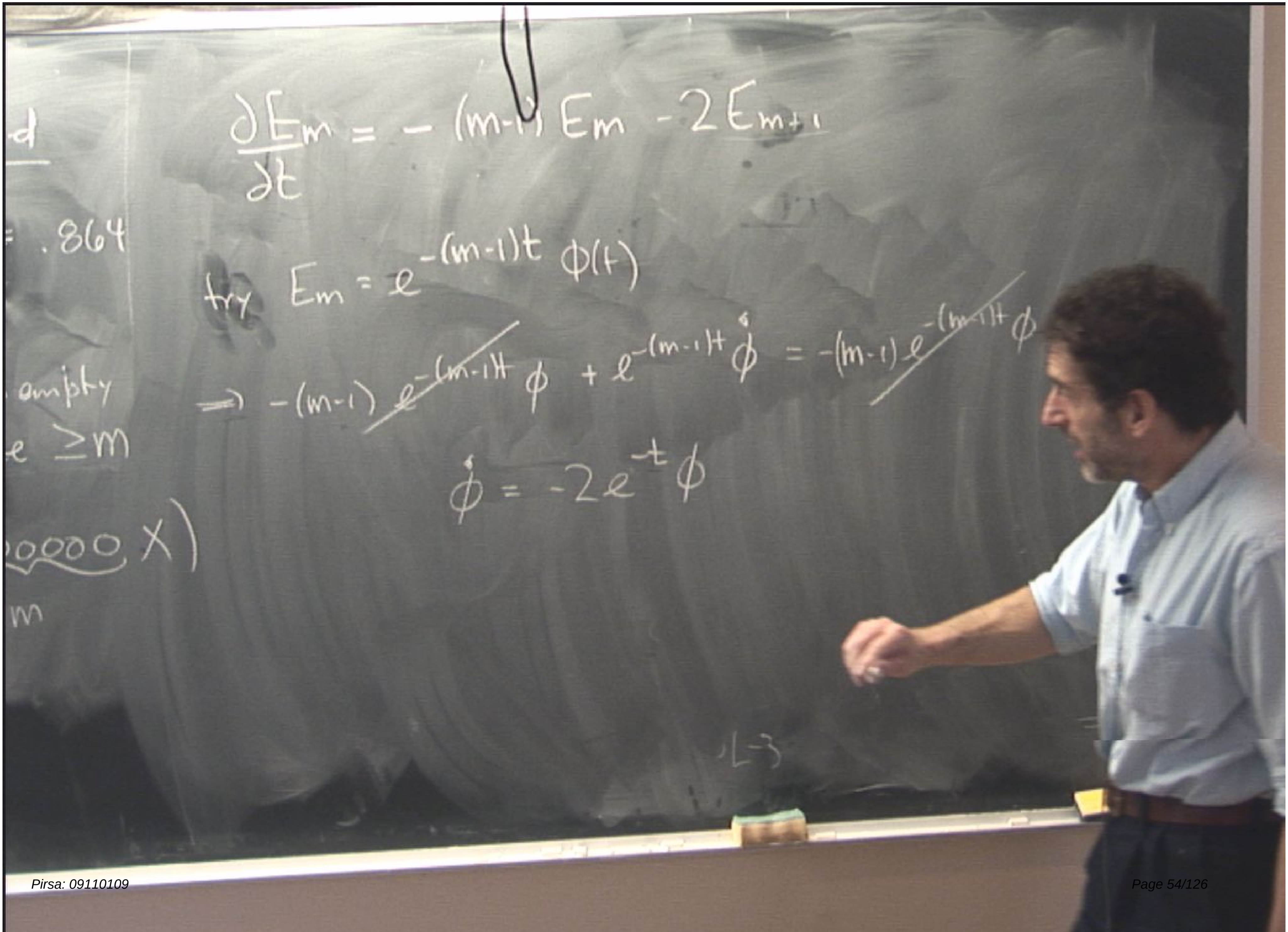
of size $\geq m$

$(\underbrace{00\,00000}_{m} X)$

$$\frac{\partial E_m}{\partial t} = -(m-1) E_m - 2 E_{m+1}$$

try $E_m = e^{-(m-1)t} \phi(t)$

$$\Rightarrow -(m-1) e^{-(m-1)t} \phi + e^{-(m-1)t} \dot{\phi} = -(m-1) e^{-(m-1)t} \phi - 2$$


d
.864
empty
e ≥ m
0000 X)
m
$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$
try $E_m = e^{-(m-1)t}\,\phi(t)$
$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}}\phi + e^{-(m-1)t}\dot{\phi} = -(m-1)\cancel{e^{-(m-1)t}}\phi$
$\dot{\phi} = -2e^{-t}\phi$

d

= .864

empty

e $\geq m$

$0000\,X)$

m

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$

try $E_m = e^{-(m-1)t}\phi(t)$

$$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}}\phi + e^{-(m-1)t}\dot{\phi} = -(m-1)\cancel{e^{-(m-1)t}\phi} - 2e^{-mt}\phi$$

$$\boxed{\dot{\phi} = -2e^{-t}\phi}$$

d

$= .864$

empty

e $\geq m$

$0000\,X)$

m

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$

try $E_m = e^{-(m-1)t}\phi(t)$

$$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}\phi} + e^{-(m-1)t}\dot{\phi} = \cancel{-(m-1)e^{-(m-1)t}\phi} - 2e^{-mt}\phi$$

$$\boxed{\dot{\phi} = -2e^{-t}\phi}$$

d

$= .864$

empty

$e \geq m$

$0000\,X)$

m

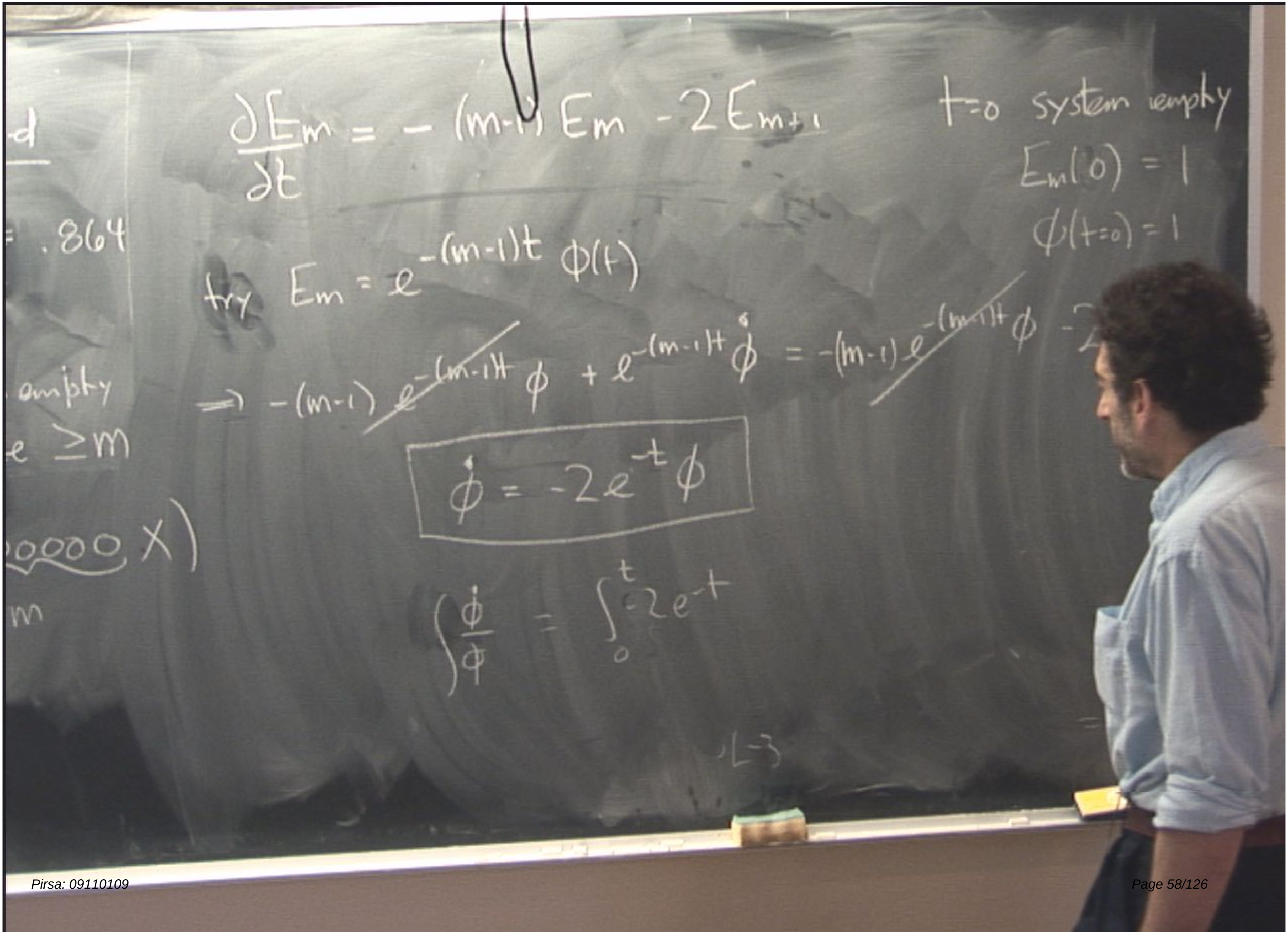
$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$

$t=0$ system empty

try $E_m = e^{-(m-1)t}\,\phi(t)$

$$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}}\,\phi + e^{-(m-1)t}\,\dot{\phi} = -(m-1)\cancel{e^{-(m-1)t}}\,\phi - 2e^{-mt}\,\phi$$

$$\boxed{\dot{\phi} = -2e^{-t}\,\phi}$$



$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$
$t=0$ system empty
$E_m(0) = 1$
$\phi(t=0) = 1$
.864
try $E_m = e^{-(m-1)t}\phi(t)$
$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}}\phi + e^{-(m-1)t}\dot\phi = -(m-1)\cancel{e^{-(m-1)t}}\phi$
empty
$e \geq m$
$\boxed{\dot\phi = -2e^{-t}\phi}$
$\int\frac{\dot\phi}{\phi} = \int_0^t -2e^{-t}$

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$

$t=0$ system empty

$E_m(0) = 1$

$\phi(t=0) = 1$

try $E_m = e^{-(m-1)t}\phi(t)$

$$\Rightarrow -(m-1)\cancel{e^{-(m-1)t}}\phi + e^{-(m-1)t}\dot{\phi} = -(m-1)\cancel{e^{-(m-1)t}}\phi - 2e^{-mt}\phi$$

$$\boxed{\dot{\phi} = -2e^{-t}\phi}$$

$\ln\phi(t) = -2(1-e^{-t})$

$$\int\frac{\dot{\phi}}{\phi} = \int_0^t -2e^{-t}$$

-d

.864

empty

$\geq m$

0000 x)

m

-d

$= .864$

empty

$e \geq m$

$\underline{0000}\, X)$

m

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1}$$

$t=0$ system empty

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-d

= .864

empty

e $\geq m$

0000 X)

m

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$t=0$ system empty

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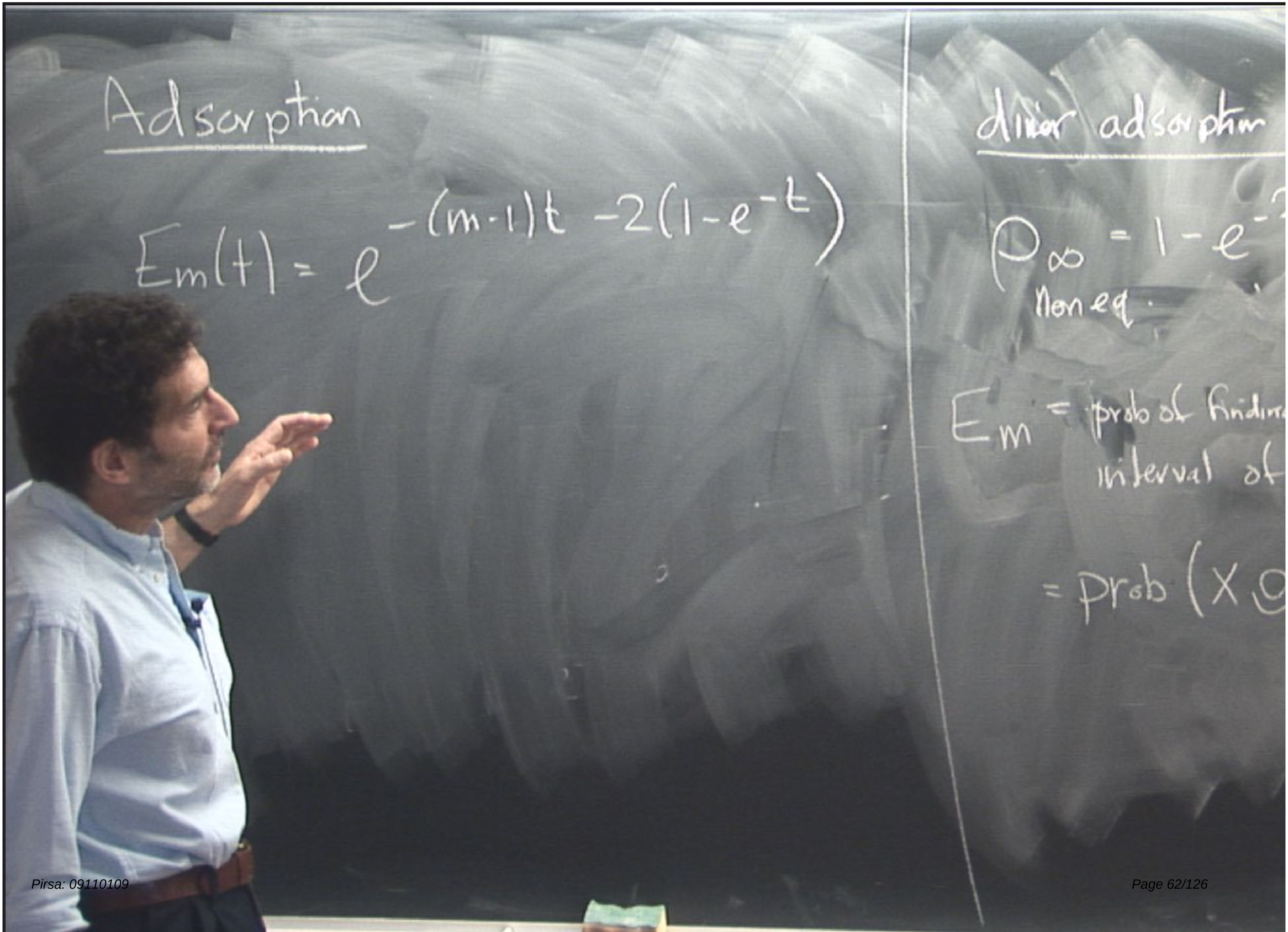
$$\boxed{\dot{\phi} = -2e^{-t}\phi}$$

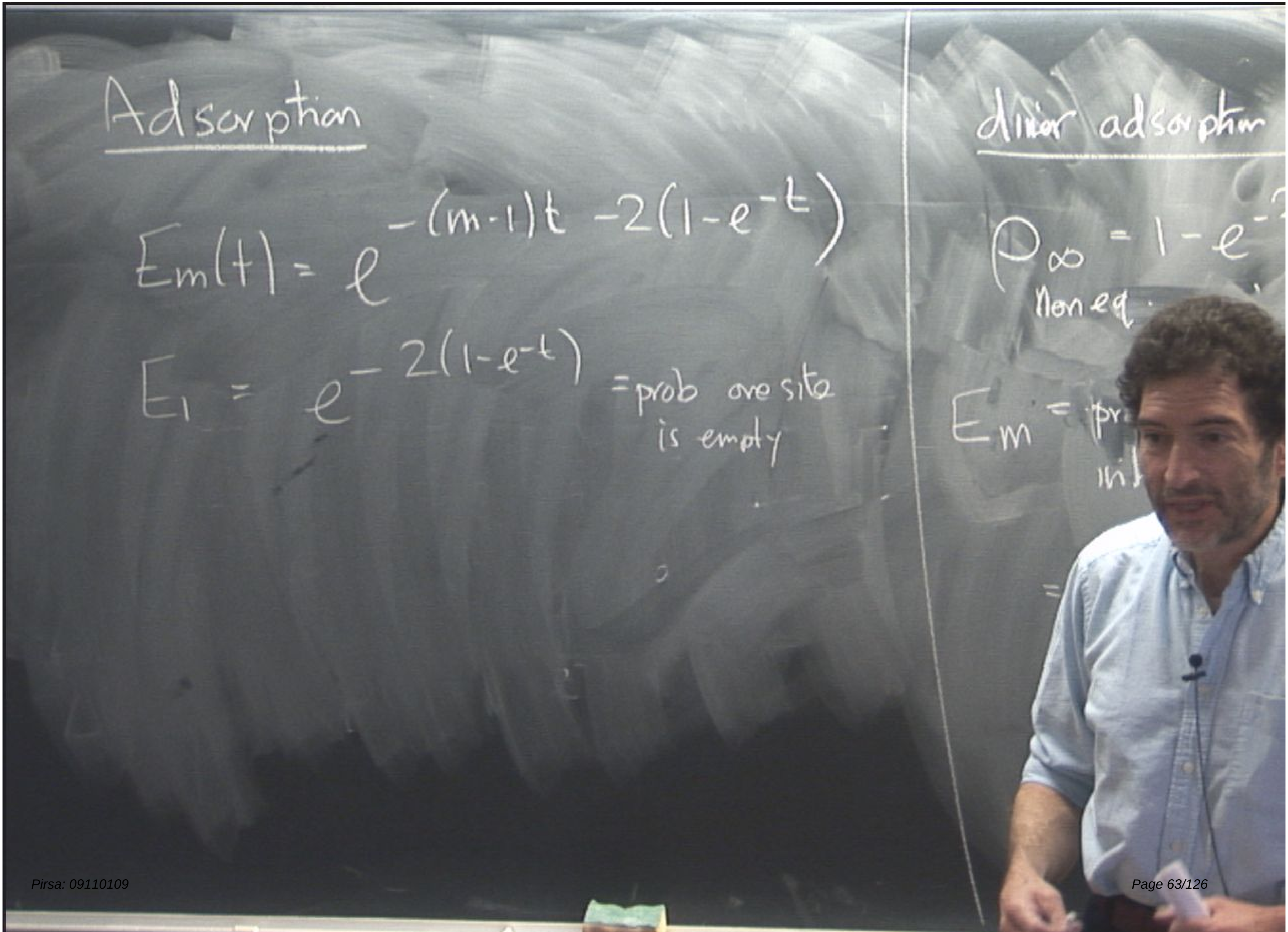
$$\ln\phi(t) = -2(1-e^{-t})$$

$$\phi(t) = \exp(-2(1-e^{-t}))$$

$$\int\frac{\dot{\phi}}{\phi} = \int_0^t -2e^{-t}$$



Adsorption
$E_m(t) = e^{-(m-1)t - 2(1-e^{-t})}$
dimer adsorption
$\rho_\infty = 1 - e$
Non eq.
E_m = prob of
interval of
= prob (X


Adsorption
$E_m(t) = e^{-(m-1)t - 2(1-e^{-t})}$
$E_1 = e^{-2(1-e^{-t})}$
= prob one site is empty
dimer adsorption
Non eq.

Adsorption

$$E_m(t) = e^{-(m-1)t - 2(1-e^{-t})}$$

$$E_1 = e^{-2(1-e^{-t})}$$ = prob one site is empty

$$\boxed{\rho = 1 - E_1 = 1 - e^{-2(1-e^{-t})}}$$

dimer adsorption

$\rho_\infty = 1 - e^{-2}$

Non eq.

E_m = prob of finding interval of

= prob (X

Adsorption

$$E_m(t) = e^{-(m-1)t - 2(1-e^{-t})}$$

$$E_1 = e^{-2(1-e^{-t})}$$ = prob one site is empty

$$\boxed{\rho = 1 - E_1 = 1 - e^{-2(1-e^{-t})}}$$

$$\xrightarrow[t\to\infty]{} 1 - e^{-2}$$

dimer adsorption

$\rho_\infty = 1 - e^{-2}$

Non eq.

E_m = prob of finding interval of

= prob (X

Adsorption

$$E_m(t) = e^{-(m-1)t - 2(1-e^{-t})}$$

$$E_1 = e^{-2(1-e^{-t})}$$ = prob one site is empty

$$\boxed{\rho = 1 - E_1 = 1 - e^{-2(1-e^{-t})}} \xrightarrow[t\to\infty]{} 1 - e^{-2}$$

dimer adsorption

$\rho_\infty = 1 - e^{-2}$

Non eq.

E_m = prob of finding interval of

= prob (X ...

1.d

$\cong .864$

an empty

ize $\geq m$

00000 X)

m

$$\frac{\partial E_m}{\partial t} = -(m-1) E_m - 2 E_{m+1} \quad m \geq 1 \quad t=0 \text{ system empty}$$

$$E_m(0) = 1$$

$$\phi(t=0) = 1$$

try $E_m = e^{-(m-1)t} \phi(t)$

$$\Rightarrow -(m-1) \cancel{e^{-(m-1)t}} \phi + e^{-(m-1)t} \dot{\phi} = -(m-1) \cancel{e^{-(m-1)t}} \phi - 2 e^{-mt} \phi$$

$$\boxed{\dot{\phi} = -2 e^{-t} \phi}$$

$$\ln \phi(t) = -2(1-e^{-t})$$

$$\phi(t) = \exp(-2(1-e^{-t}))$$

$$\int \frac{\dot{\phi}}{\phi} = \int_0^t -2e^{-t}$$

1.d

$\cong .864$

an empty

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$00000\,X)$

m

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$$\boxed{\dot{\phi} = -2e^{-t}\phi}$$

$$\ln\phi(t) = -2(1-e^{-t})$$

$$\phi(t) = \exp(-2(1-e^{-t}))$$

$$\int\frac{\dot{\phi}}{\phi} = \int_0^t -2e^{-t}$$

$m-1)t - 2(1-e^{-t})$

$(1-e^{-t})$

(O) = prob one site is empty

$$\boxed{1-e^{-2(1-e^{-t})}}$$

∞ $1-e^{-2}$

Irreversible car adsorption in 1-d

$\rho_\infty \cong 0.747\ldots\ldots$

$\frac{\partial E_m}{\partial t}$

try E_m

$\Rightarrow -(m-$

$1)t - 2(1-e^{-t})$

(0)

$\overleftrightarrow{}$

$e^{-t})$ = prob one site is empty

$$\boxed{e^{-2(1-e^{-t})}}$$

$1-e^{-2}$

Irreversible car adsorption in 1d

$\rho_\infty \cong 0.747\ldots\ldots$

$E(x) \equiv$ prob of finding an empty interval of length $\geq X$

$\frac{\partial E_m}{\partial t} =$

try $E_m =$

$\Rightarrow -(m-1)$

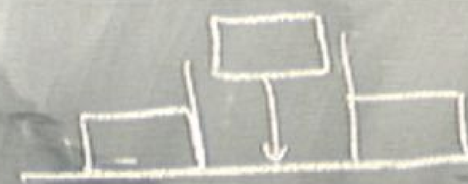
$e^{-t})$

ove site

mpty

$t)$

Irreversible car adsorption in 1-d



$\rho_\infty \cong 0.747\ldots\ldots$

$E(x) \equiv$ prob of finding an empty interval of length $\geq x$

$e^{-t})$

ove site

mpty

$t)$

Irreversible car adsorption in 1-d

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Irreversible car adsorption in 1 d

$\rho_\infty \cong 0.747\ldots\ldots$

$E(x) \equiv$ prob of finding an empty interval of length $\geq X$

$$\frac{\partial E(x,t)}{\partial t} = -$$

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E$$

$)$

$e^{-(m-1)}$

$2e$

reversible

car. adsorption in 1 d

$\rho_\infty \cong 0.747\ldots\ldots$

$E(x) \equiv$ prob of finding an empty interval of length $\geq X$

$$\frac{\partial E(x,t)}{\partial t} = -$$

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_m$$

)

$e^{-(m-1)t}\,\phi$

$X > 1$

$\boxed{e^{-t}\,\phi}$

$2e^{-t}$

reversible

car. adsorption in 1 d

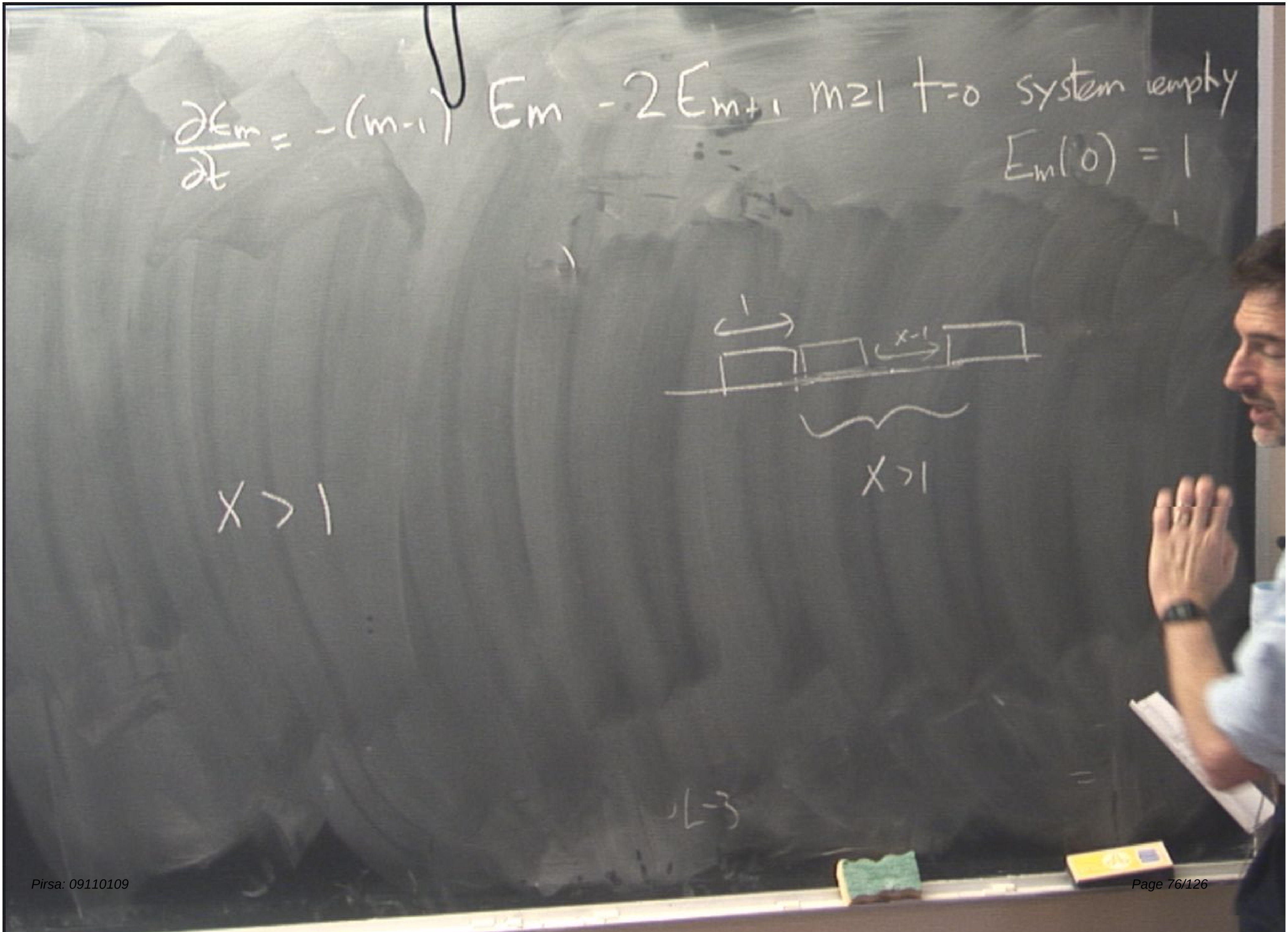
$\rho_\infty \cong 0.747\ldots\ldots$

$E(x) \equiv$ prob of finding an empty interval of length $\geq x$

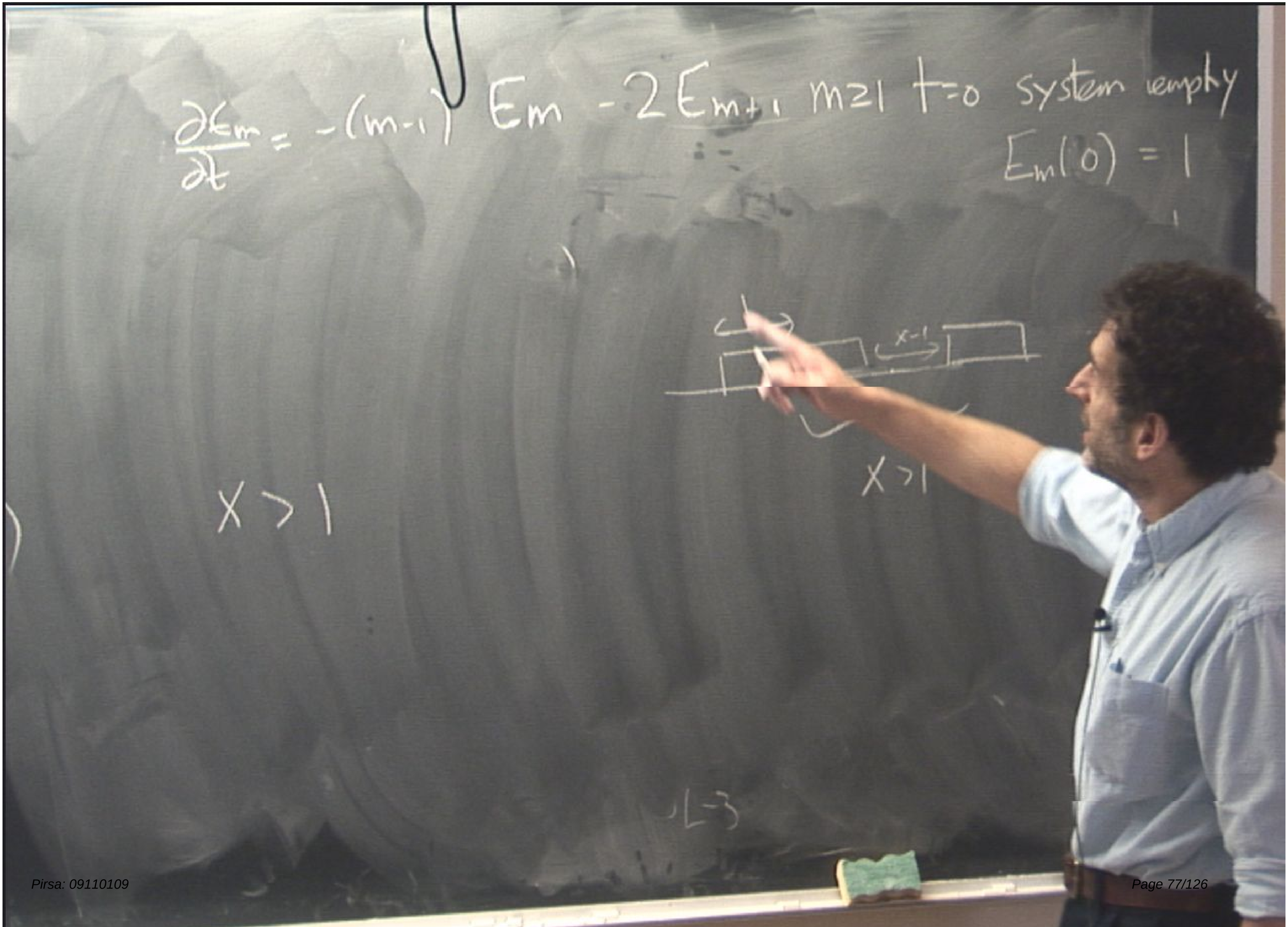
$$\frac{\partial E(x,t)}{\partial t} = -(x-1)$$

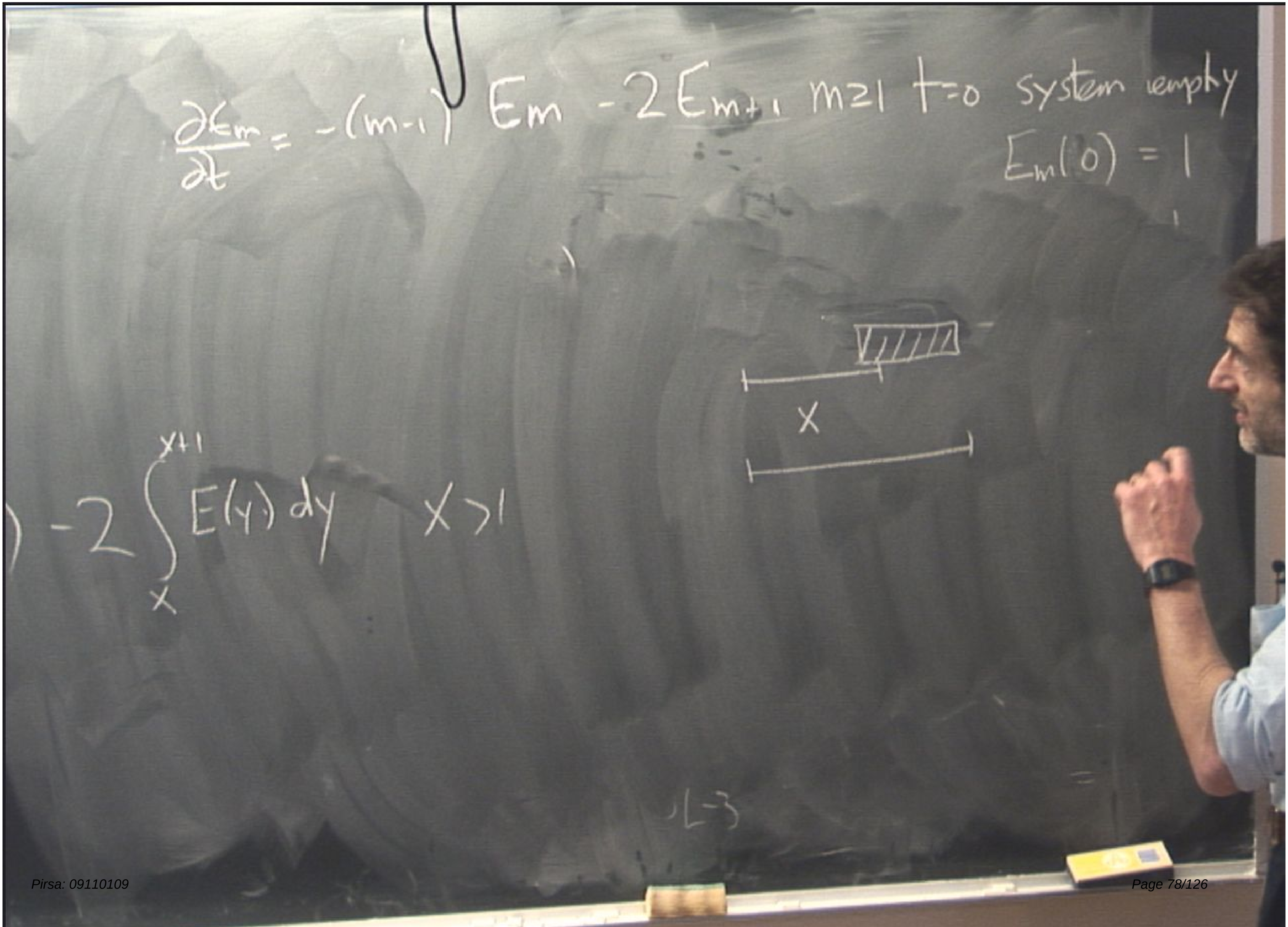
$x > 1$

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2 E_m$$



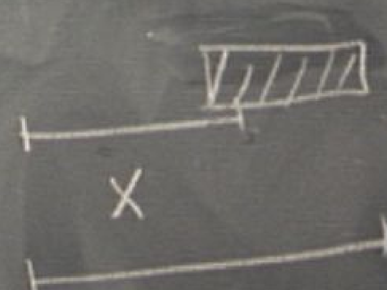
∂Em/∂t = -(m-1) Em - 2 Em+1 m≥1 t=0 system empty
Em(0) = 1
1
x-1
x>1
x>1
L-3


∂E_m/∂t = -(m-1) E_m - 2 E_{m+1} m≥1 t=0 system empty
E_m(0) = 1
x-1
X > 1
X > 1
L-3


∂Em/∂t = -(m-1) Em - 2 Em+1   m≥1   t=0 system empty
Em(0) = 1
X
-2 ∫ from x to x+1 E(y) dy   x>1

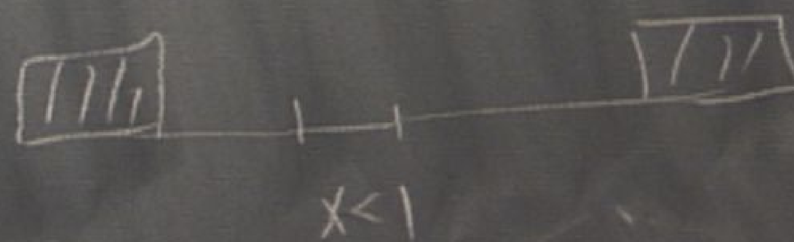
$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system em}$$

$$E_m(0) =$$



$$2\int_x^{x+1} E(y)\, dy \qquad x>1$$

$$x<1$$



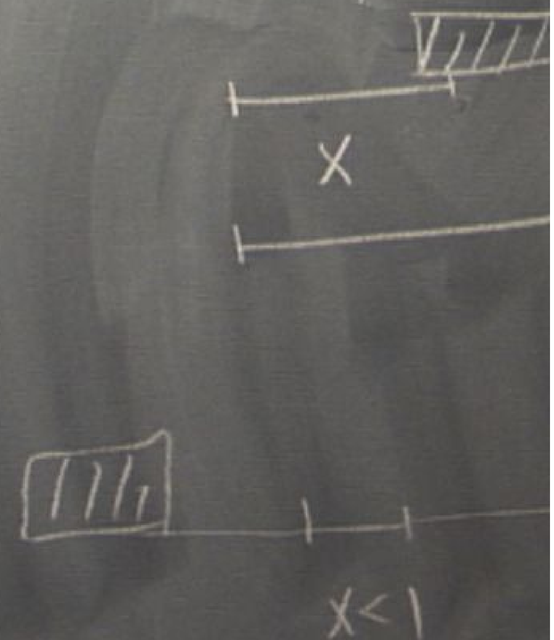
...sorption in 1-d

...747......

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1} \quad m \geq 1 \quad t$$

...of finding an empty
...rval of length $\geq X$

$$\begin{cases} = -(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy & x > 1 \\ = -(1-x)E(1) & x < 1 \end{cases}$$



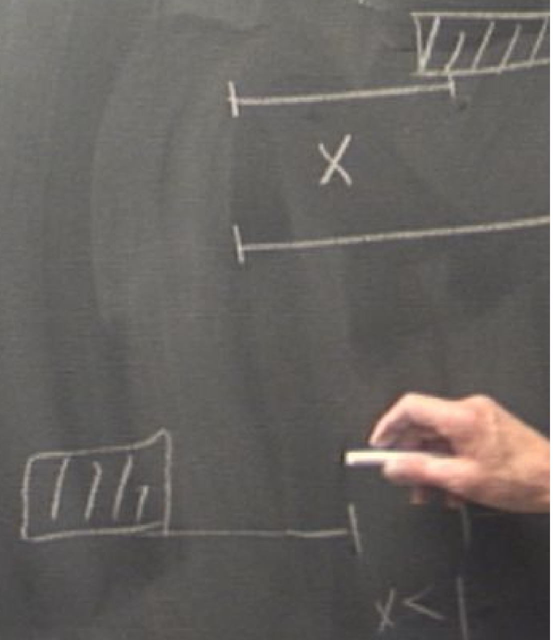
sorption in 1-d

.747......

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2 E_{m+1} \quad m \geq 1$$

of finding an empty
rval of length $\geq X$

$$\begin{cases} = -(x-1)E(x) - 2\displaystyle\int_x^{x+1} E(y)\,dy & x>1 \\ = -(1-x)E(1) & x<1 \end{cases}$$



ld

...

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system}$$

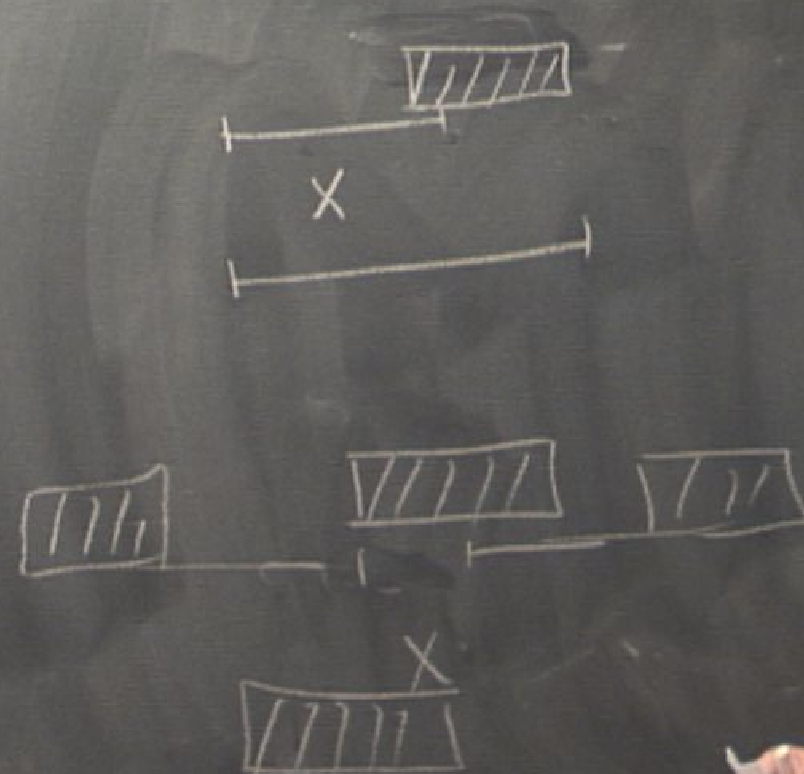
$E_m(0)$

empty

$\geq X$

$$(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy \qquad X>1$$

$$-x)E(1) \qquad X<1$$



L-3

l.d

$$\frac{\partial E_m}{\partial t} = -(m-1)E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system}$$

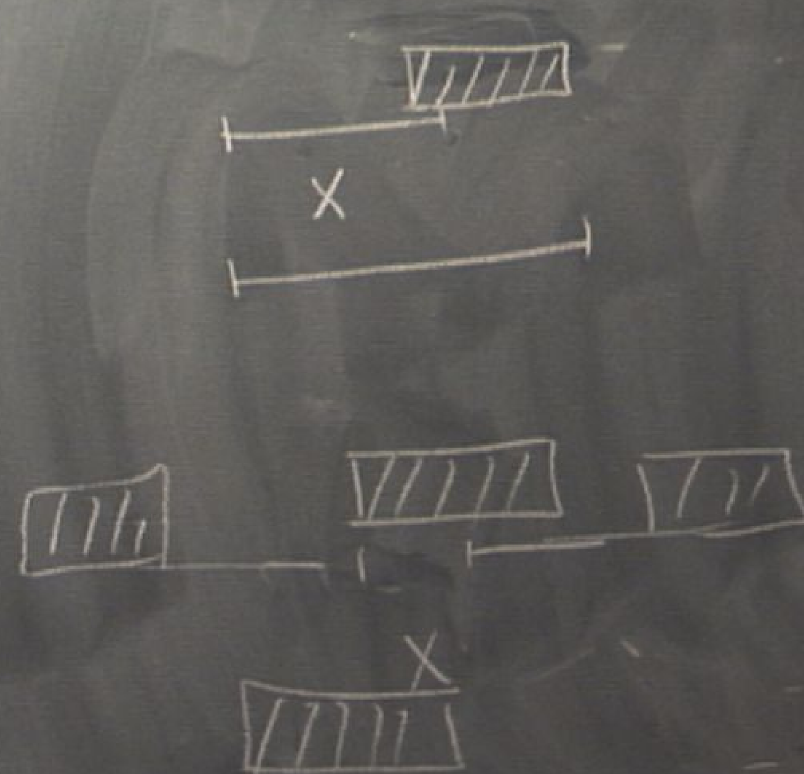
$E_m(0)$

empty

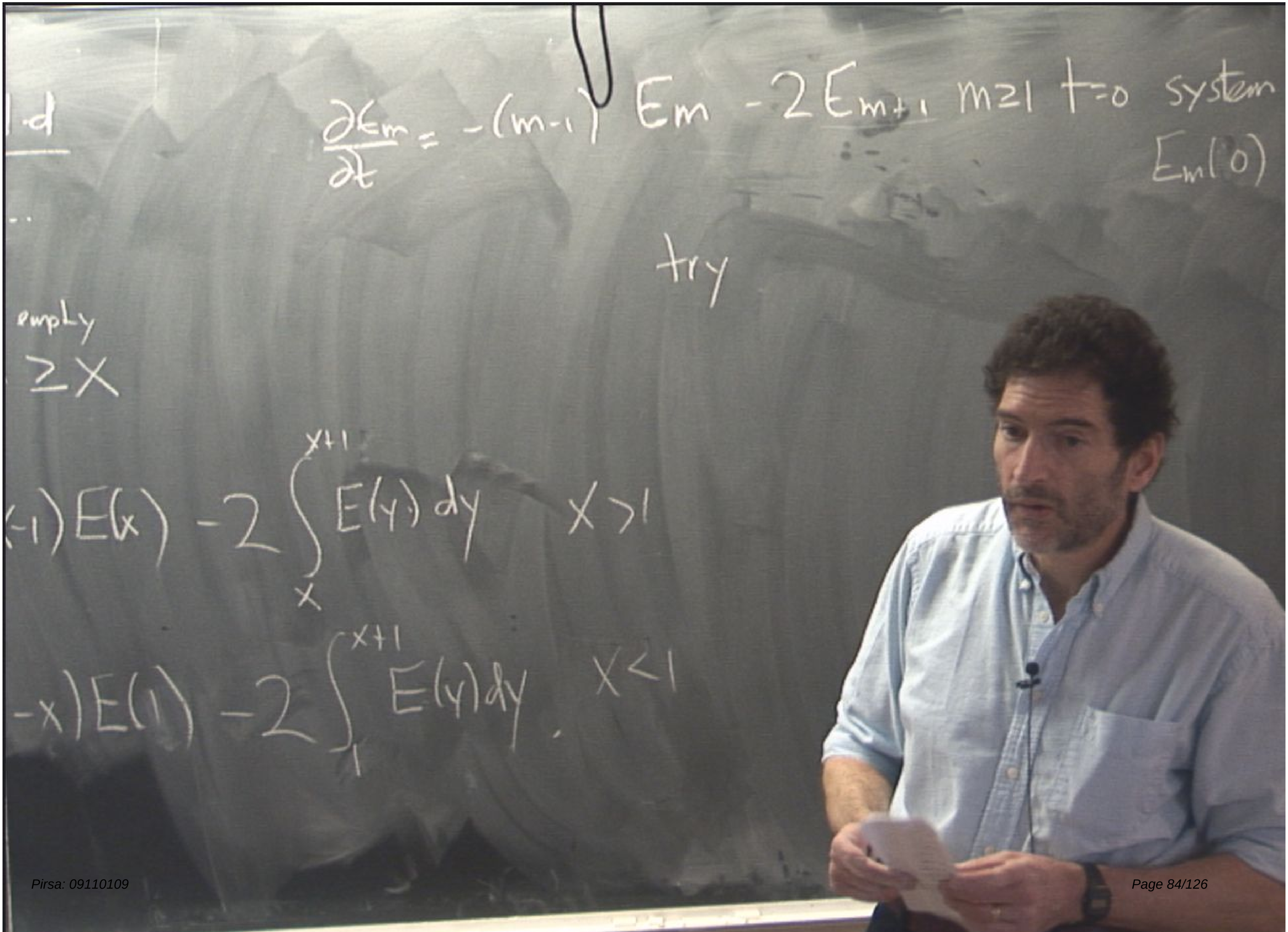
$\geq x$

$$(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy \qquad x>1$$

$$(\ldots-x)E(1) - 2\int_1^{x+1} E(y)\,dy \qquad x<1$$



L-3



$\frac{\partial E_m}{\partial t} = -(m-1) E_m - 2 E_{m+1}$ $m \geq 1$ $t=0$ system
$E_m(0)$
try
empty
$\geq X$
$(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy$ $x > 1$
$(1-x)E(1) - 2\int_1^{x+1} E(y)\,dy$ $x < 1$

$\frac{\partial E_m}{\partial t} = -(m-1) E_m - 2 E_{m+1} \quad m \geq 1 \quad t=0$ system

$E_m(0)$

try

empty

$\geq X$

$(x-1) E(x) - 2 \int_x^{x+1} E(y)\,dy \quad x > 1$

$(1-x) E(1) - 2 \int_1^{x+1} E(y)\,dy \quad x < 1$

$$\frac{\partial E_m}{\partial t} = -(m-1)\,E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system}$$

$E_m(0)$

try $E(x,t) = e^{-(x-1)t}$

empty

$\geq X$

$$(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy \quad x > 1$$

$$-x)E(1) - 2\int_1^{x+1} E(y)\,dy \quad x < 1$$

$|d$

$\frac{\partial E_m}{\partial t} = -(m-1) E_m - 2 E_{m+1}$ $m \geq 1$ $t=0$ system

$E_m(0)$

...

try $E(x,t) = e^{-(x-1)t} E(1,t)$

empty

$\geq X$

$(x-1)E(x) - 2\int_x^{x+1} E(y)\,dy$ $x>1$

$(1-x)E(1) - 2\int_1^{x+1} E(y)\,dy$, $x<1$

=

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system empty}$$

$$E_m(0) = 1$$

try $E(x,t) = e^{-(x-1)t}\, E(1,t)$

$$\Rightarrow E(x,t) = \exp\left[-\int_0^t \frac{2}{t'}\left(1-e^{-t'}\right) dt'\right]$$

$$2\int_x^{x+1} E(y)\, dy \qquad x>1$$

$$\int_1^{x+1} E(y)\, dy \qquad x<1$$

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system empty}$$

$$E_m(0) = 1$$

try $E(x,t) = e^{-(x-1)t}\, E(1,t)$

$$\Rightarrow E(x,t) = \exp\left[- \int_0^t \frac{2}{t'} \left(1 - e^{-t'}\right) dt' \right]$$

$$x) - 2 \int_x^{x+1} E(y)\, dy \qquad x > 1$$

$x = 1$ $\quad \frac{\partial E(0,t)}{\partial t}$

$$E(1) - 2 \int_1^{x+1} E(y)\, dy \qquad x < 1$$

$$\frac{\partial E_m}{\partial t} = -(m-1)\, E_m - 2E_{m+1} \quad m \geq 1 \quad t=0 \text{ system empty}$$

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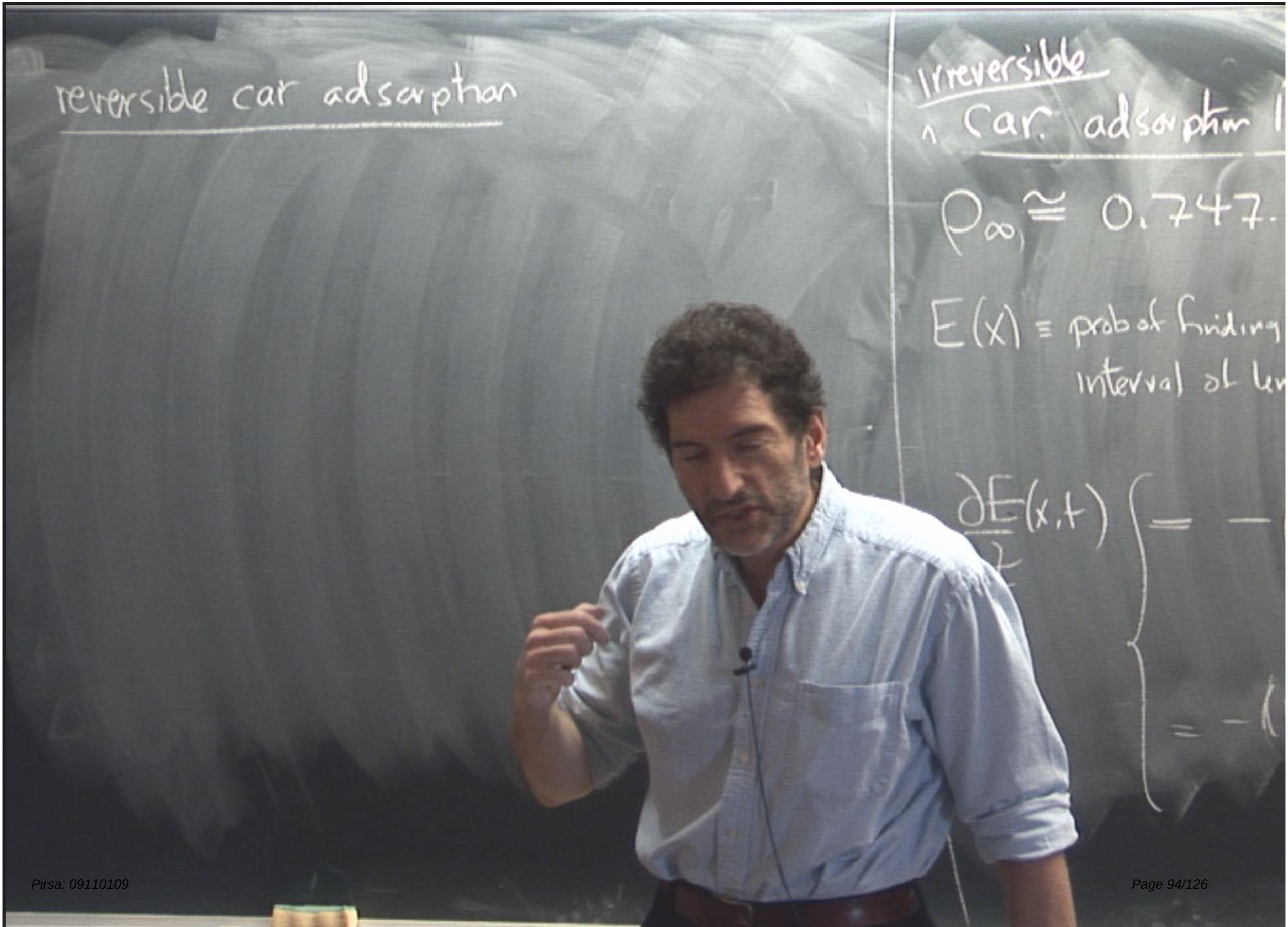
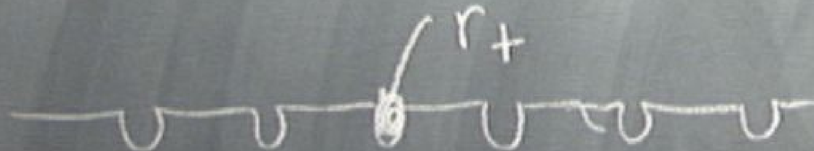

reversible car adsorption
irreversible
car adsorption
$\rho_\infty \cong 0.747$
$E(x) \equiv$ prob of finding
interval of
$\frac{\partial E(x,t)}{\partial t}$

reversible car adsorption
prelim: reversible monomer adsorption
irreversible
car adsorption
$\rho_\infty \cong 0.747$
$E(x) \equiv$ prob of finding
interval of len
$\frac{\partial E}{\partial t}(x,t)$

reversible car adsorption

prelim: reversible monomer adsorption



irreversible car adsorption

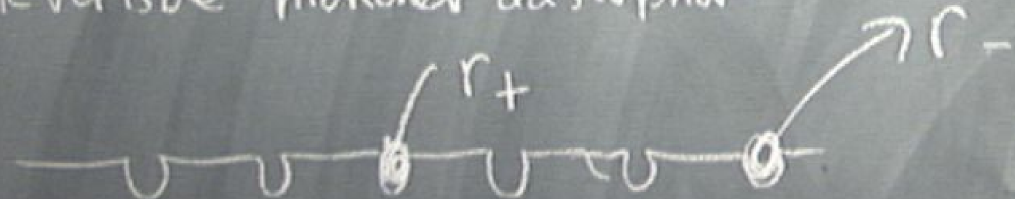
$\rho_\infty \cong 0.747\ldots$

$E(x) \equiv$ prob of finding
interval of l

$\dfrac{\partial E}{\partial t}(x,t) \Big\{ =$

reversible car adsorption

prelim: reversible monomer adsorption



irreversible car adsorption

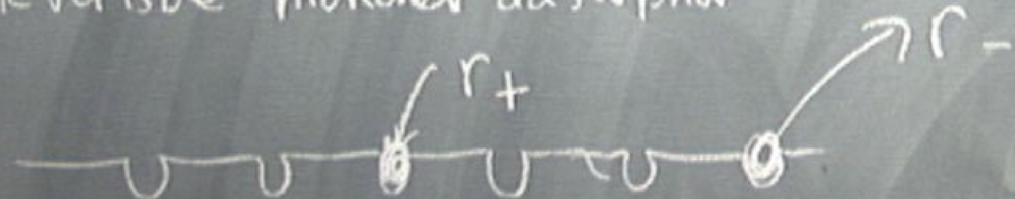
$\rho_\infty \cong 0.747\ldots$

$E(x) \equiv$ prob of f... interval ...

$\frac{\partial E(x,t)}{\partial t}$ ...

reversible car adsorption

prelim: reversible monomer adsorption



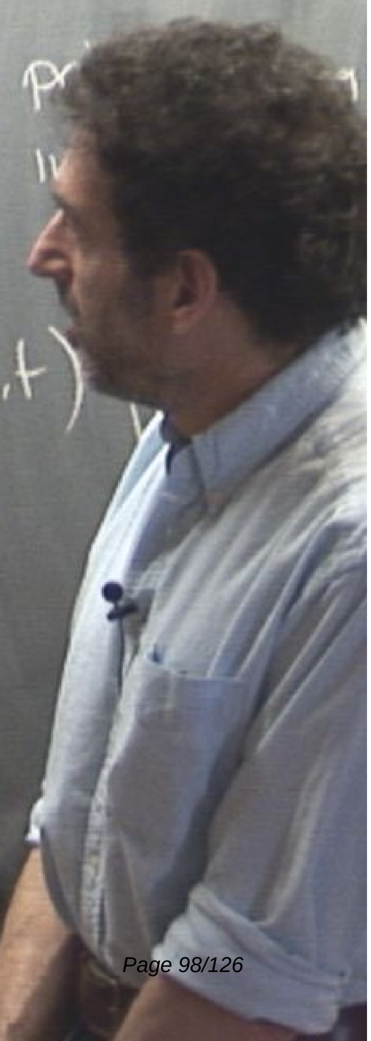
$\frac{\partial \rho}{\partial t}$

irreversible car adsorption

$\rho_\infty \cong 0.747\ldots$

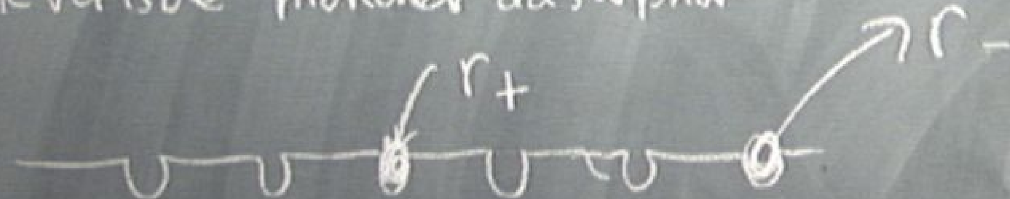
$E(x) \equiv p$

$\frac{\partial E}{\partial t}(x,t)$



reversible car adsorption

prelim: reversible monomer adsorption



$$\frac{\partial \rho}{\partial t} = r_+(1-\rho) - r_-\rho$$

irreversible car adsorption

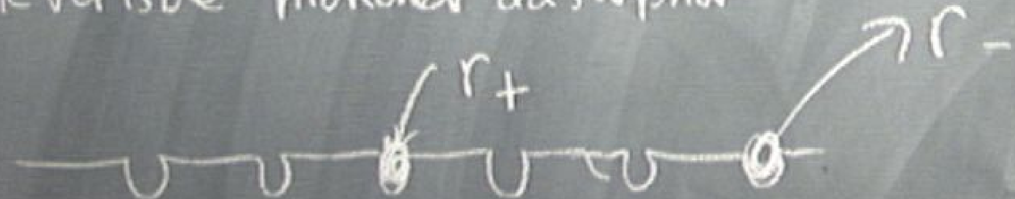
$\rho_\infty \cong 0.747\ldots$

$E(x) \equiv$ prob of finding
interval of len

$$\frac{\partial E(x,t)}{\partial t} \begin{cases} = - \\ = -(\end{cases}$$

reversible car adsorption

prelim: reversible monomer adsorption



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irreversible car adsorption

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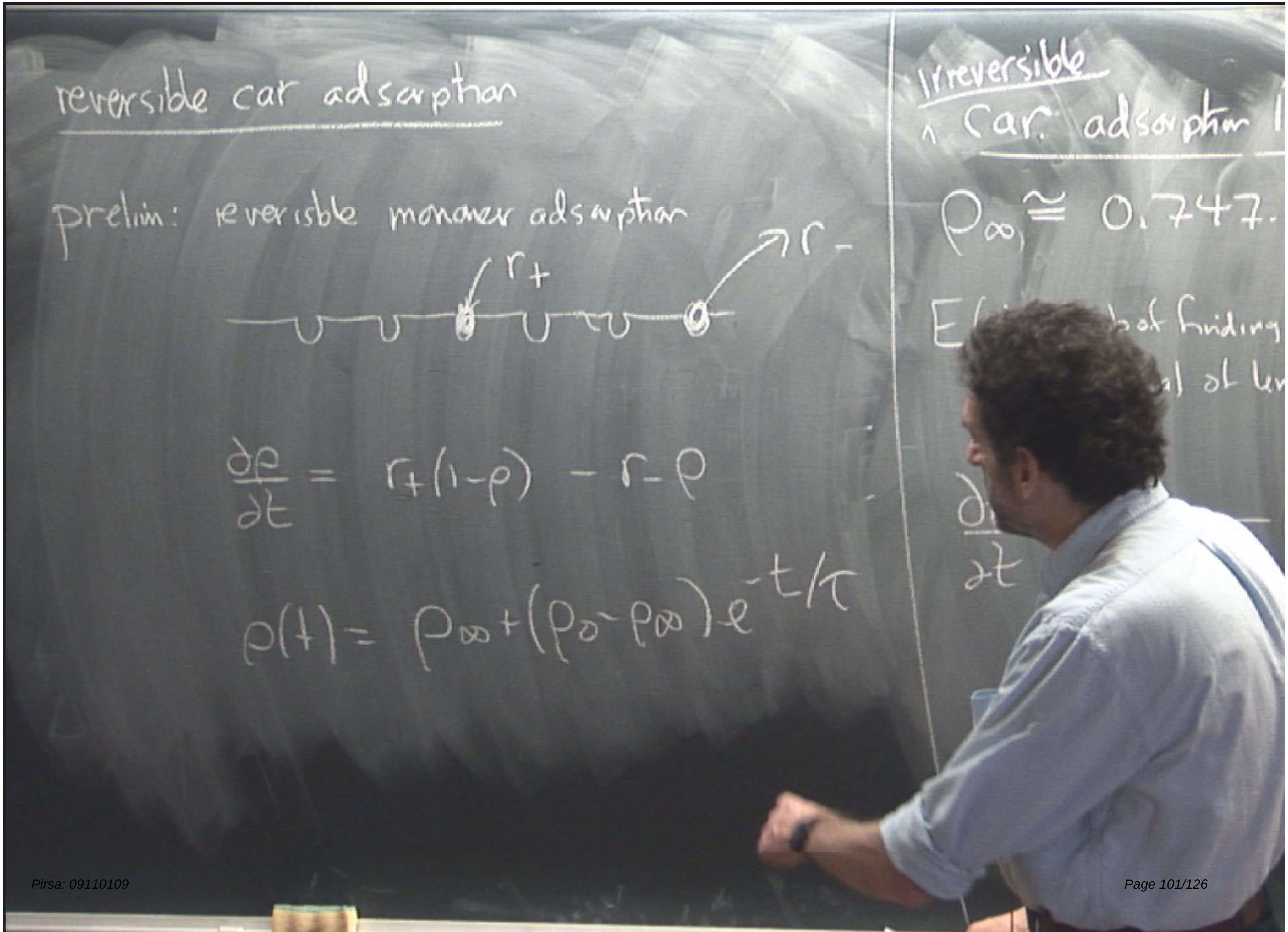
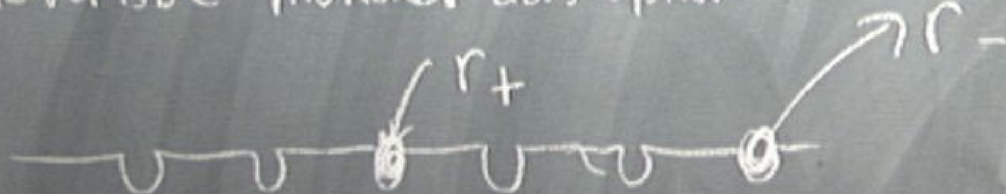
interval of len

$$\frac{\partial E}{\partial t}(x,t) \begin{cases} = - \\ = -(\end{cases}$$


reversible car adsorption
prelim: reversible monomer adsorption
r_+
r_-
$\frac{\partial \rho}{\partial t} = r_+(1-\rho) - r_-\rho$
$\rho(t) = \rho_\infty + (\rho_0 - \rho_\infty) e^{-t/\tau}$
irreversible
car adsorption
$\rho_\infty \cong 0.747$
of finding

reversible car adsorption

prelim: reversible monomer adsorption



$$\frac{\partial \rho}{\partial t} = r_+(1-\rho) - r_- \rho$$

$$\rho(t) = \rho_\infty + (\rho_0 - \rho_\infty) e^{-t/\tau}$$

$$\tau = (r_+ + r_-)^{-1} \;,\; \rho_\infty = \frac{r_+}{r_+ + r_-}$$

irreversible car adsorption

$\rho_\infty \cong 0.747\ldots$

$E(x) \equiv$ prob of finding ... interval of len...

$$\frac{\partial E}{\partial t}(x,t) \begin{cases} = - \ldots \\ = - (\ldots \end{cases}$$

adsorption

r_+ $\rightarrow r_-$

$) - r_- \rho$

$+(\rho_0 - \rho_\infty) e^{-t/\tau}$

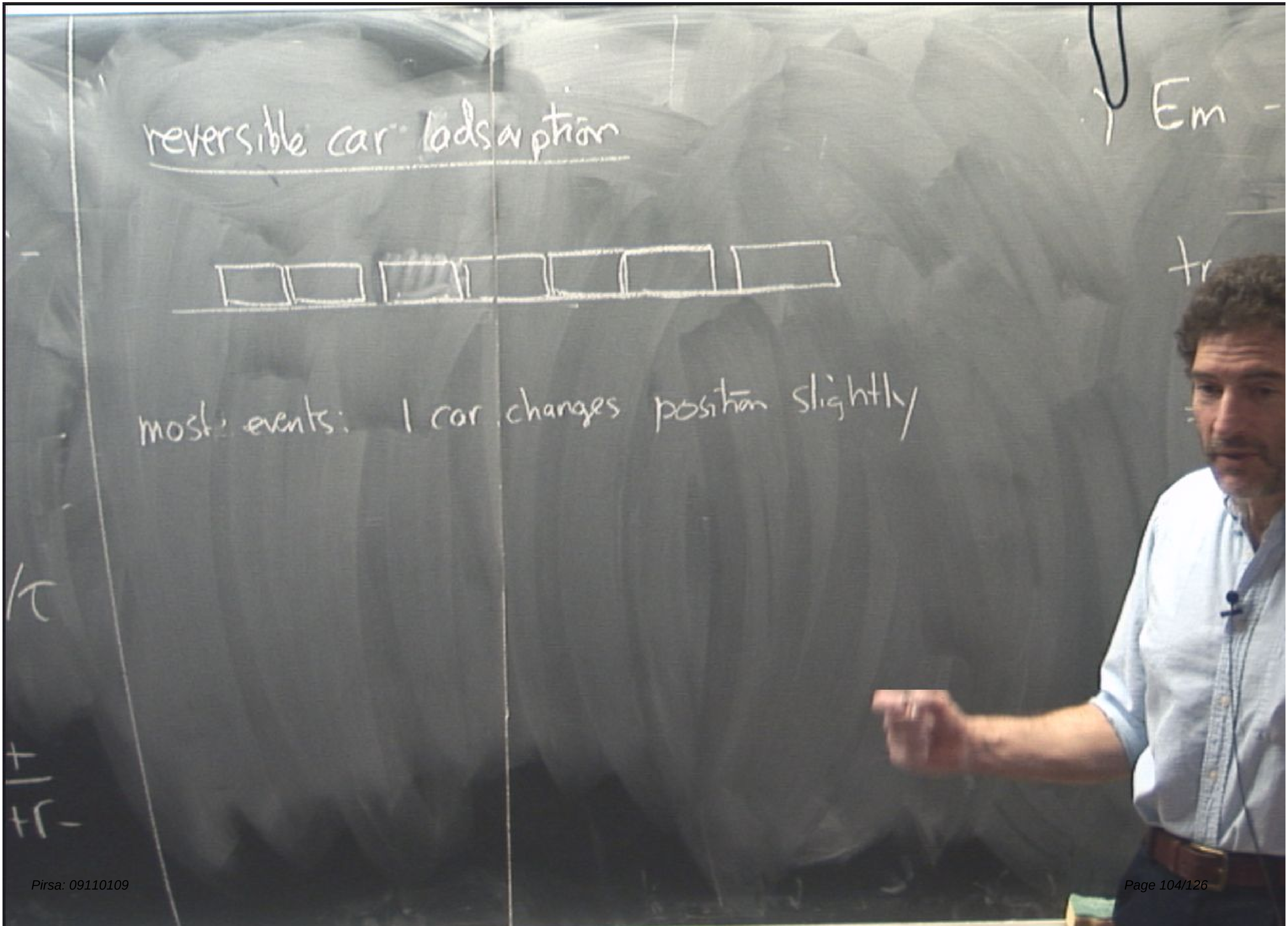
$+r_-)^{-1}$, $\rho_\infty = \dfrac{r_+}{r_+ + r_-}$

reversible car /adsorption

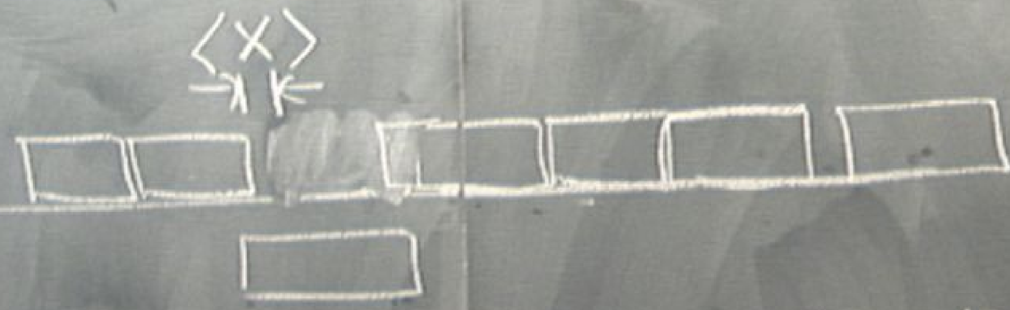
$$\frac{\partial \mathcal{E}_m}{\partial t} =$$

$$-2 \int_{x}^{x+1} E(y)\, dy$$

$$-2 \int_{-1}^{x+1} E(y)$$


reversible car adsorption
most events: 1 car changes position slightly
Em

reversible car adsorption



most events: 1 car changes position slightly

Em

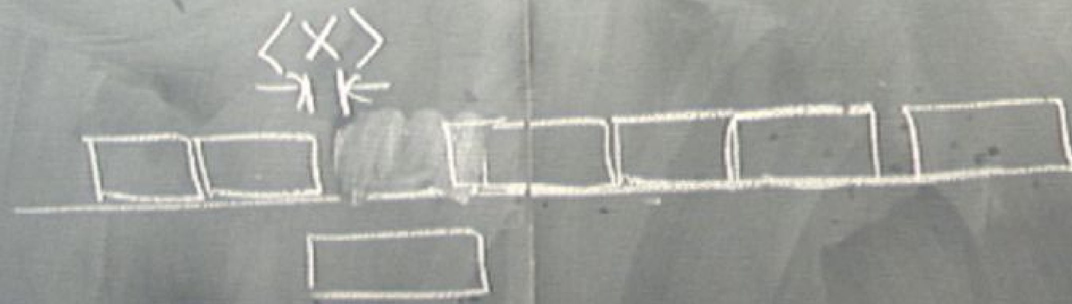
try

⇒

/τ

+

+r-reversible car (ad)sorption

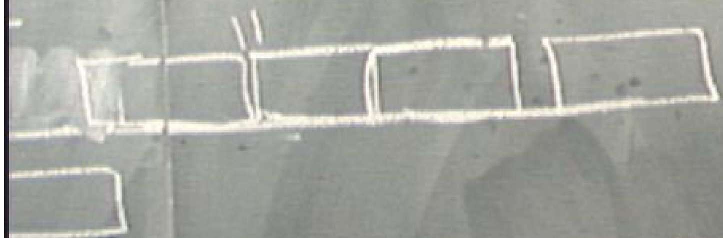


most events: 1 car changes position slightly

rare cooperative events: 1 more car can park

adsorption

$$\rho = \frac{1}{1+\langle x\rangle}$$

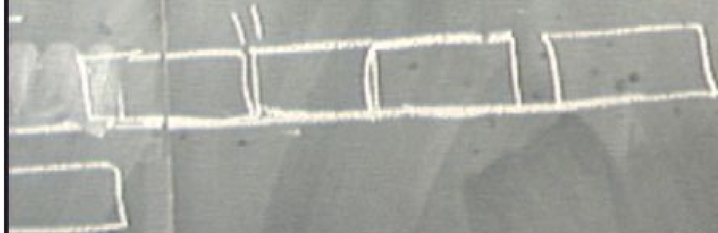


1 car changes position slightly

e: 1 more car can park

adsorption

$$\rho = \frac{1}{1+\langle x\rangle} \qquad \langle x\rangle = \frac{1}{\rho} - 1$$



1 car changes position slightly

e: 1 more car can park

$$\rho = \frac{1}{1+\langle x\rangle}$$

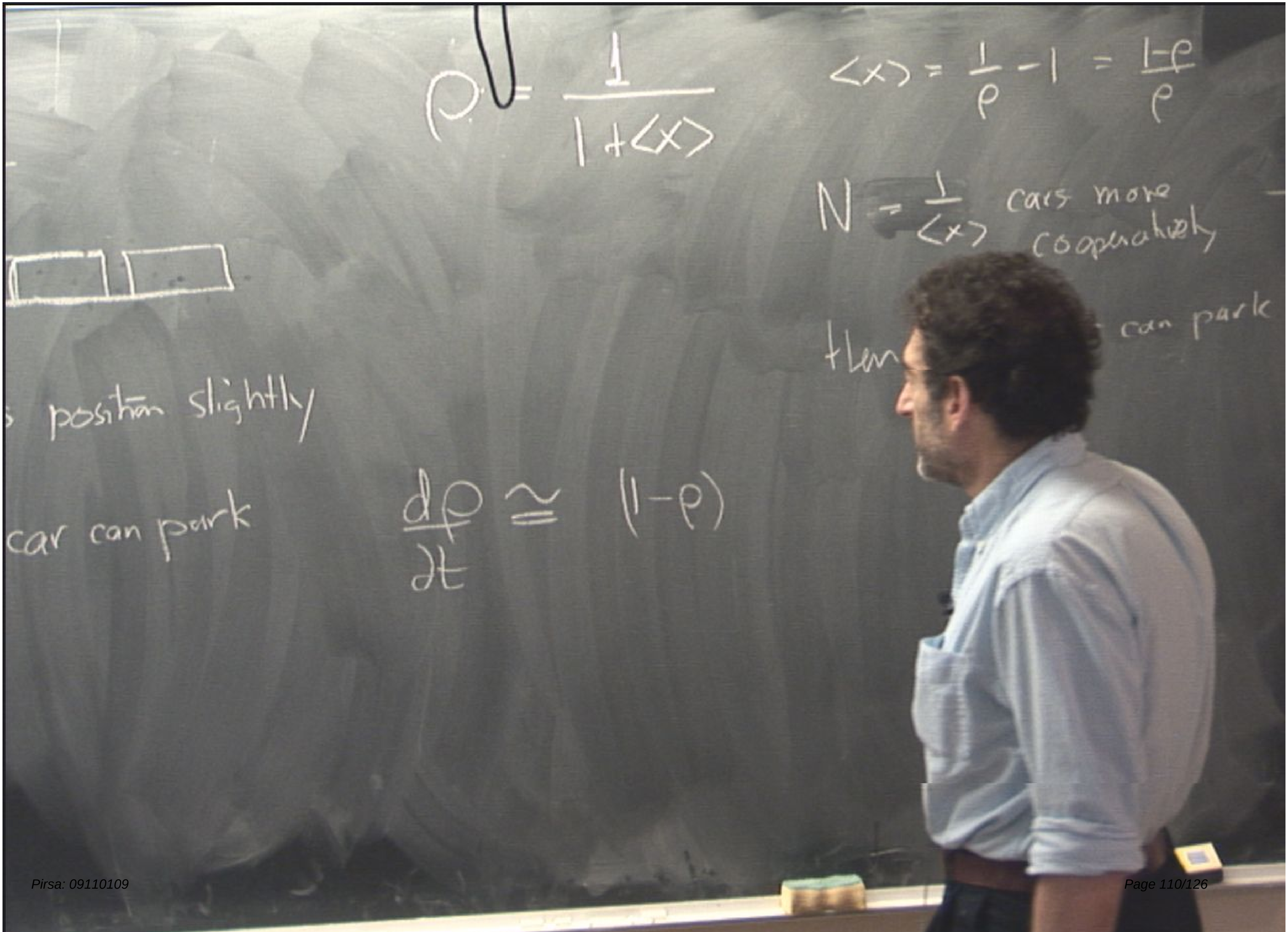
$$\langle x\rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$$

$N = \frac{1}{\langle x\rangle}$ cars more cooperatively

then 1 more car can park

s position slightly

car can park


ρ = 1 / (1 + <x>)
<x> = 1/ρ − 1 = (1−ρ)/ρ
N = 1/<x>
cars more cooperatively
can park
then
position slightly
car can park
dρ/∂t ≅ (1−ρ)

$$\rho = \frac{1}{1+\langle x \rangle}$$

$$\langle x \rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$$

$N = \frac{1}{\langle x \rangle}$ cars more cooperatively

then 1 more car can park

s position slightly

car can park

$$\frac{d\rho}{\partial t} \cong (1-\rho) r_+$$

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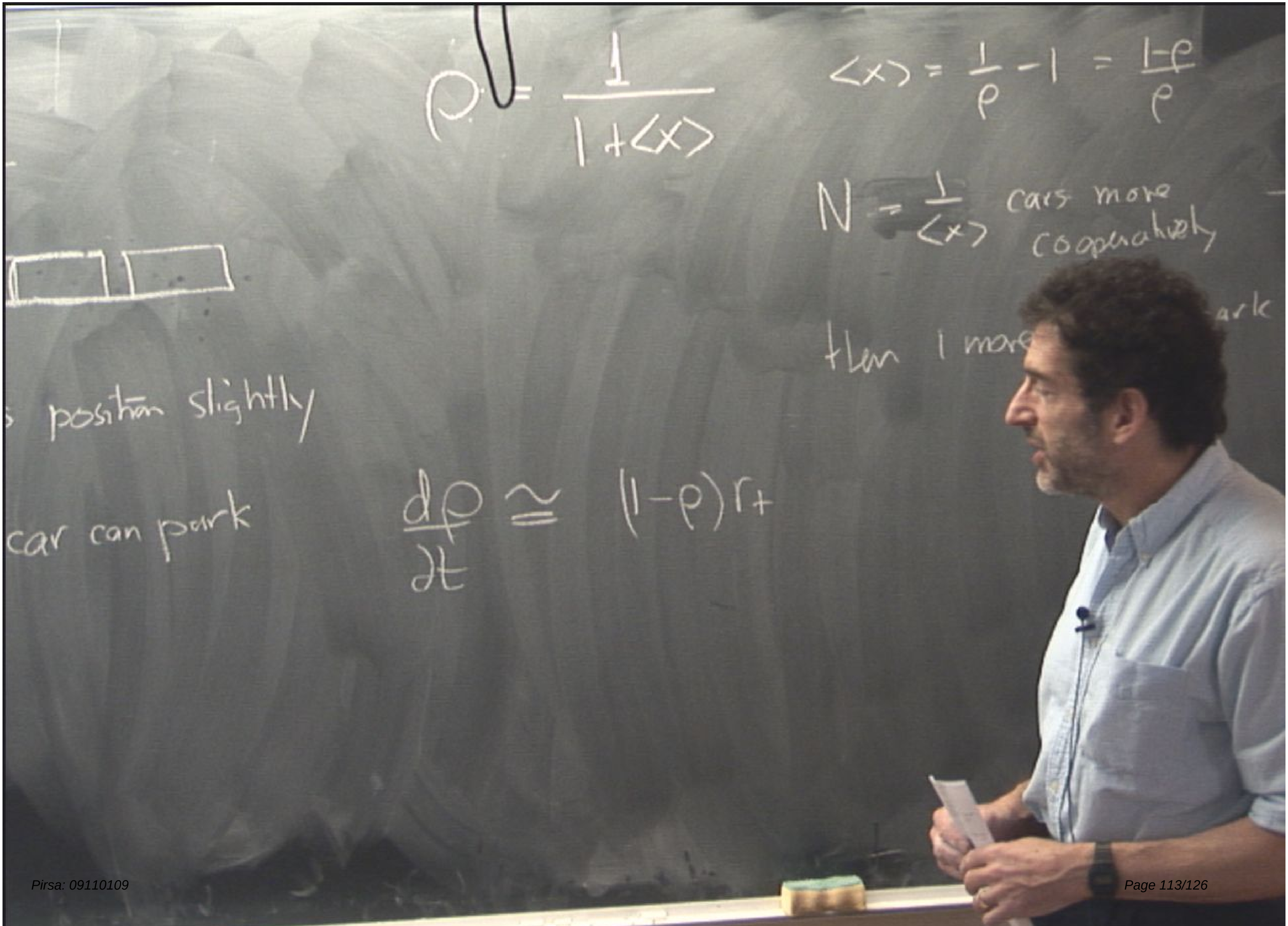
$\rho = \frac{1}{1+\langle x\rangle}$
$\langle x\rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$
$N = \frac{1}{\langle x\rangle}$ cars more cooperatively
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position slightly
car can park
$\frac{d\rho}{\partial t} \cong (1-\rho)r_+$

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s position slightly

car can park

$$\frac{d\rho}{\partial t} \cong (1-\rho) r_+ e^{-N}$$

$$\rho = \frac{1}{1+\langle x\rangle}$$

$$\langle x\rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$$

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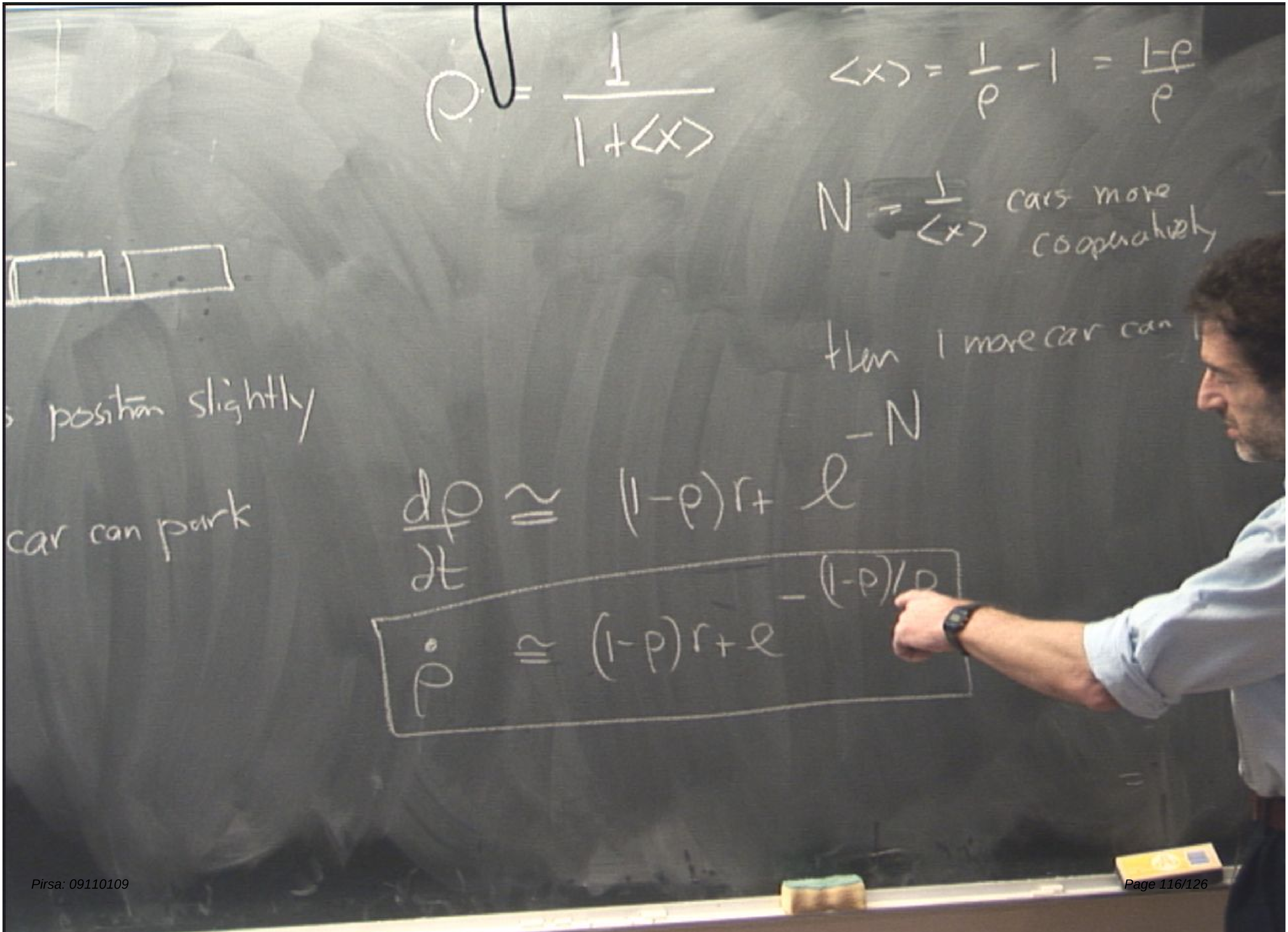
then 1 more car can park

s position slightly

car can park

$$\frac{d\rho}{\partial t} \cong (1-\rho)r_+ e^{-N}$$

$$\cong (1-\rho)r_+ e^{-(1-\rho)/\rho}$$



$\rho = \frac{1}{1+\langle x \rangle}$
$\langle x \rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$
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position slightly
car can park
$\frac{d\rho}{\partial t} \cong (1-\rho) r_+ e^{-N}$
$\dot{\rho} \cong (1-\rho) r_+ e^{-(1-\rho)/\rho}$

$$\rho = \frac{1}{1+\langle x \rangle}$$

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s position slightly

car can park

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$$\rho = \frac{1}{1+\langle x\rangle}$$

$$\langle x\rangle = \frac{1}{\rho} - 1 = \frac{1-\rho}{\rho}$$

$N = \frac{1}{\langle x\rangle}$ cars more cooperatively

then 1 more car can park

s position slightly

car can park

$$\frac{d\rho}{\partial t} \cong (1-\rho) r_+ e^{-N}$$

$$\boxed{\dot{\rho} \cong (1-\rho) r_+ e^{-\rho/1-\rho}}$$

$\underline{t \to \infty}$

$g \equiv$

$\rho = \frac{1}{1+\ldots}$

$\frac{d\rho}{dt} \cong (1-\ldots$

$\boxed{\dot{\rho} \cong (1-\rho)\ldots}$

$\underline{t \to \infty}$

$$g \equiv \frac{1}{1-\rho}$$

$$\rho = 1 - \frac{1}{g}$$

$$\rho = \frac{1}{1+\ldots}$$

$$\frac{d\rho}{dt} \cong (1-\ldots$$

$$\boxed{\dot{\rho} \cong (1-\rho)\ldots}$$

$\underline{t \to \infty}$

$$g \equiv \frac{1}{1-\rho}$$

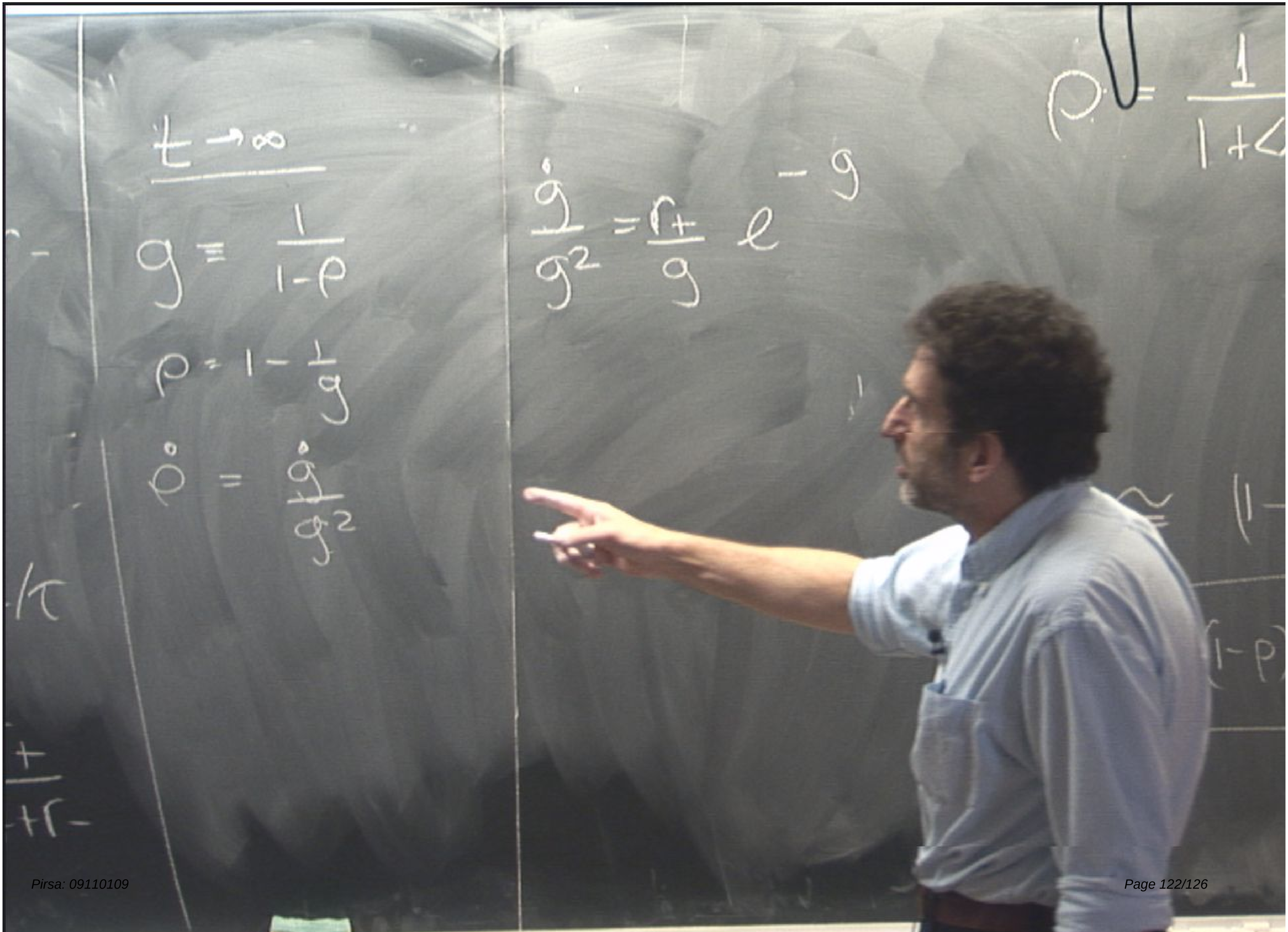
$$\rho = 1 - \frac{1}{g}$$

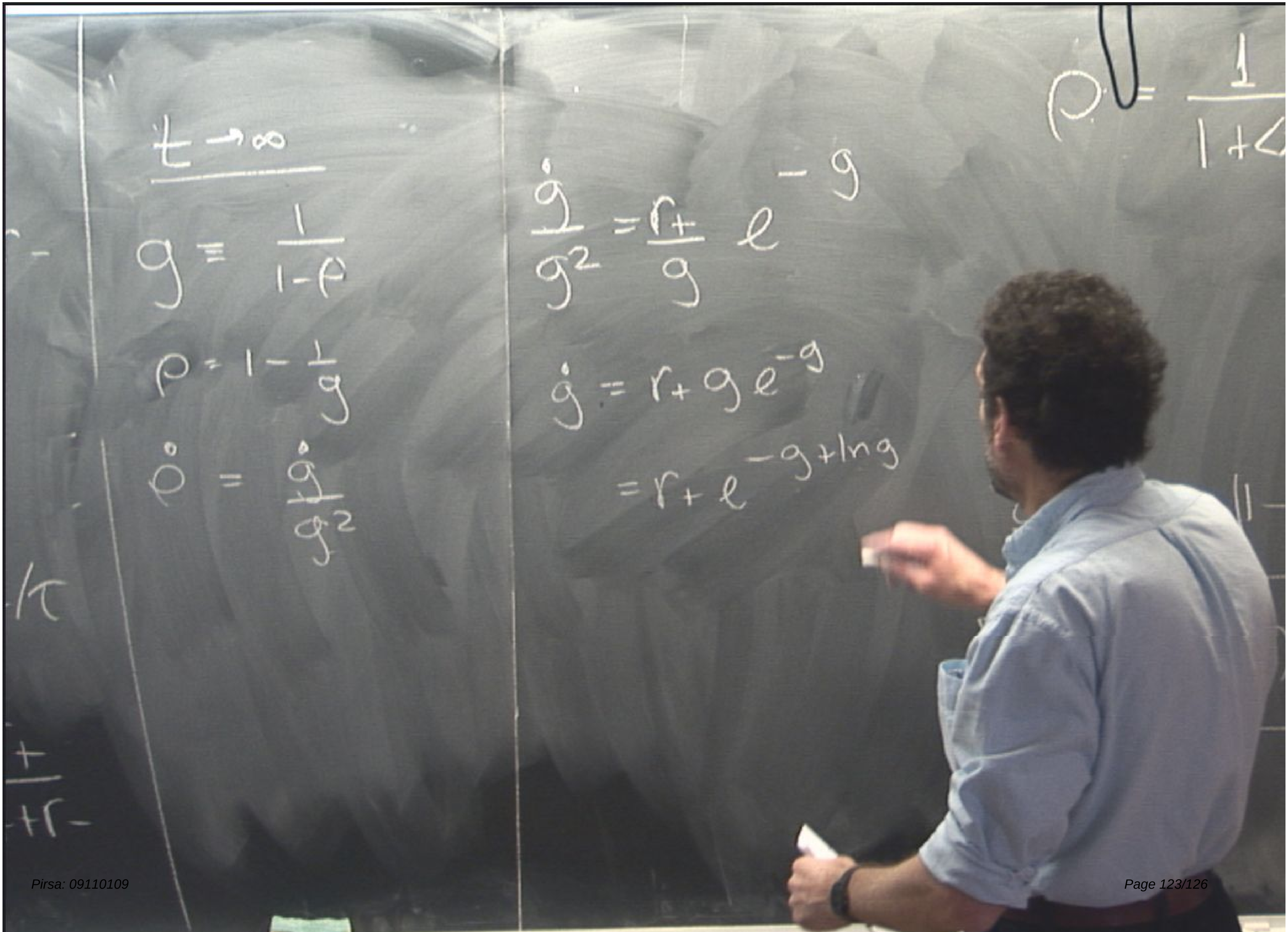
$$\dot{\rho} = \frac{\dot{g}}{g^2}$$

$$\rho = \frac{1}{1+\langle}$$

$$\frac{d\rho}{dt} \cong (1-$$

$$\boxed{\dot{\rho} \cong (1-\rho)}$$


t → ∞
g ≡ 1/(1-ρ)
ρ = 1 - 1/g
ρ̇ = ġ/g²
ġ/g² = r+/g e^-g


$t \to \infty$
$g \equiv \frac{1}{1-\rho}$
$\rho = 1 - \frac{1}{g}$
$\dot{\rho} = \frac{\dot{g}}{g^2}$
$\frac{\dot{g}}{g^2} = \frac{r_+}{g} e^{-g}$
$\dot{g} = r_+ g e^{-g}$
$= r_+ e^{-g+\ln g}$

$$\underline{t \to \infty}$$

$$g \equiv \frac{1}{1-\rho}$$

$$\rho = 1 - \frac{1}{g}$$

$$\dot{\rho} = \frac{\dot{g}}{g^2}$$

$$\frac{\dot{g}}{g^2} = \frac{r_+}{g} e^{-g}$$

$$\dot{g} = r_+ g e^{-g}$$

$$= r_+ e^{-g + \ln g}$$

$$g \propto \ln t$$

$$\rho = \frac{1}{1+\langle}$$

$$\frac{d\rho}{dt} \cong ($$

$$\boxed{\dot{\rho} \cong (1-}$$

$\underline{t \to \infty}$

$$g \equiv \frac{1}{1-\rho}$$

$$\rho = 1 - \frac{1}{g}$$

$$\dot{\rho} = \frac{\dot{g}}{g^2}$$

$$\frac{\dot{g}}{g^2} = \frac{r_+}{g} e^{-g}$$

$$\dot{g} = r_+ g e^{-g}$$

$$= r_+ e^{-g+\ln g}$$

$$g \propto \ln t$$

$$\boxed{\rho \cong 1 - \frac{1}{\ln t}}$$

$\underline{t \to \infty}$

$$g \equiv \frac{1}{1-\rho}$$

$$\rho = 1 - \frac{1}{g}$$

$$\dot{\rho} = \frac{\dot{g}}{g^2}$$

$$\frac{\dot{g}}{g^2} = \frac{r_+}{g} e^{-g}$$

$$\dot{g} = r_+ g e^{-g}$$

$$= r_+ e^{-g + \ln g}$$

$$g \propto \ln t$$

$$\boxed{\rho \cong 1 - \frac{1}{\ln t}}$$

$$\rho = \frac{1}{1+ \ldots}$$

$$\frac{d\rho}{dt} \cong (1- \ldots$$

$$\boxed{\dot{\rho} \cong (1-\rho) \ldots}$$

$/\tau$

$\frac{+}{+r_-}$