Title: Mathematical Physics (PHYS 624) - Lecture 10

Date: Nov 27, 2009 09:00 AM

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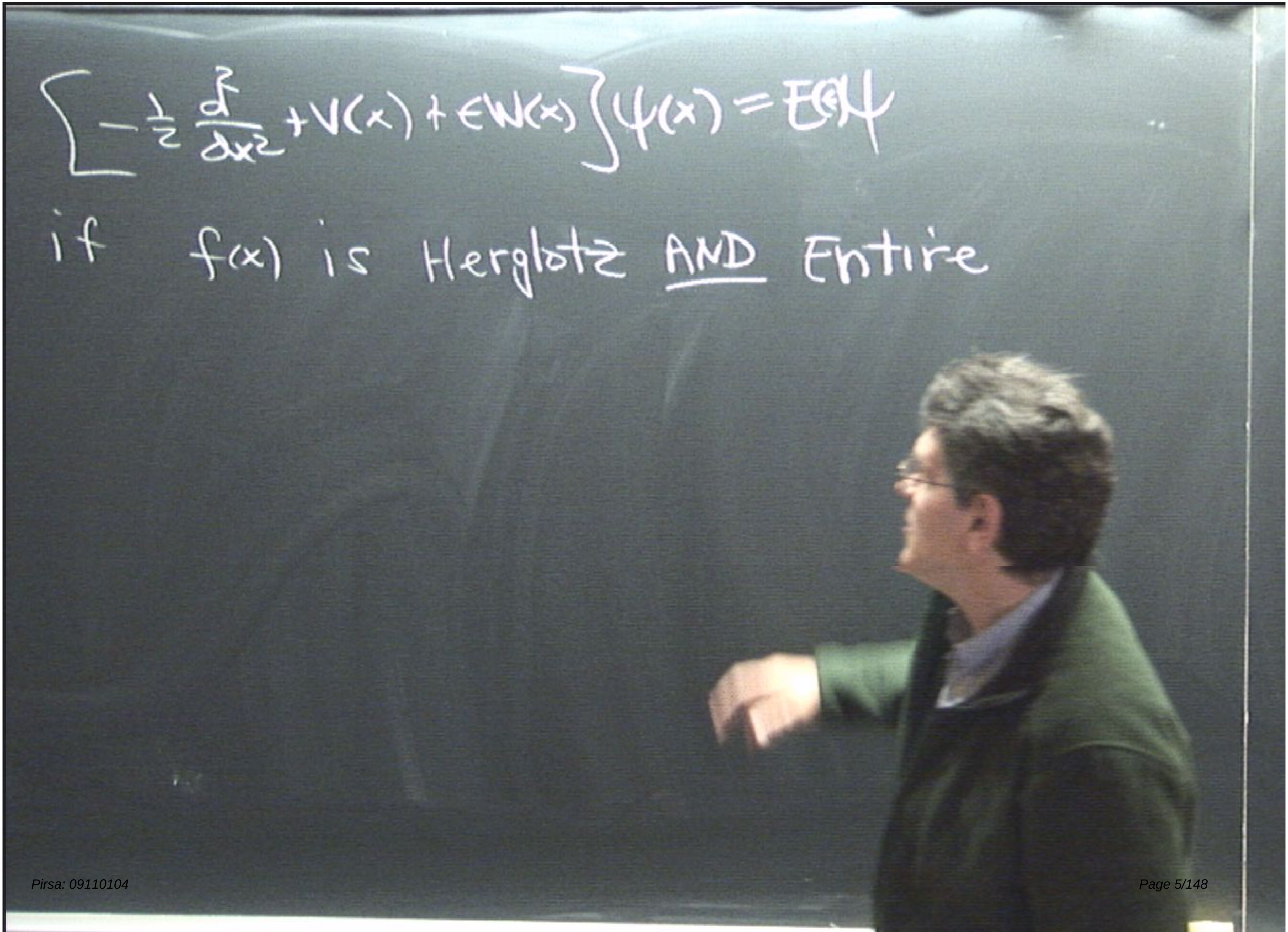
Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

$$-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\Big]\psi(x)=E\psi$$

$$-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\Big]\psi(x)=E(\epsilon)\psi$$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

if f(x) is Herglotz <u>AND</u> Entire

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

if $f(x)$ is Herglotz <u>AND</u> Entire

$f(x)=\sum a_n x^n$

$R \text{ of } C=\infty.$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if f(x) is Herglotz AND Entire

then f(x) is trivial —

i.e. $f(x) = Ax + B$

$f(x) = \sum a_n x^n$

$R \circ C = \infty$.

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

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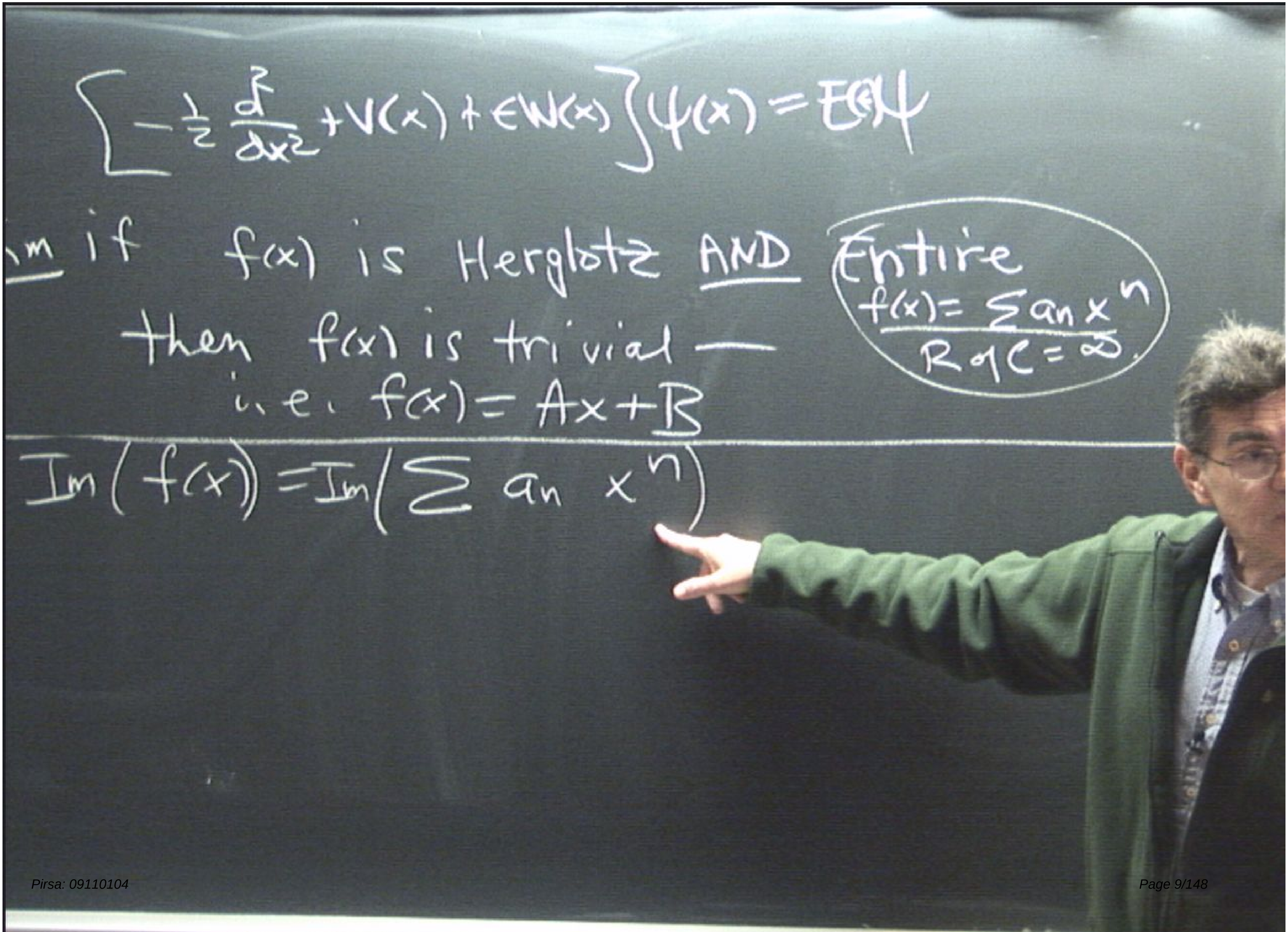

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

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R of C = ∞

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$$\mathrm{Im}(f(x)) = \mathrm{Im}\left(\sum a_n x^n\right)$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

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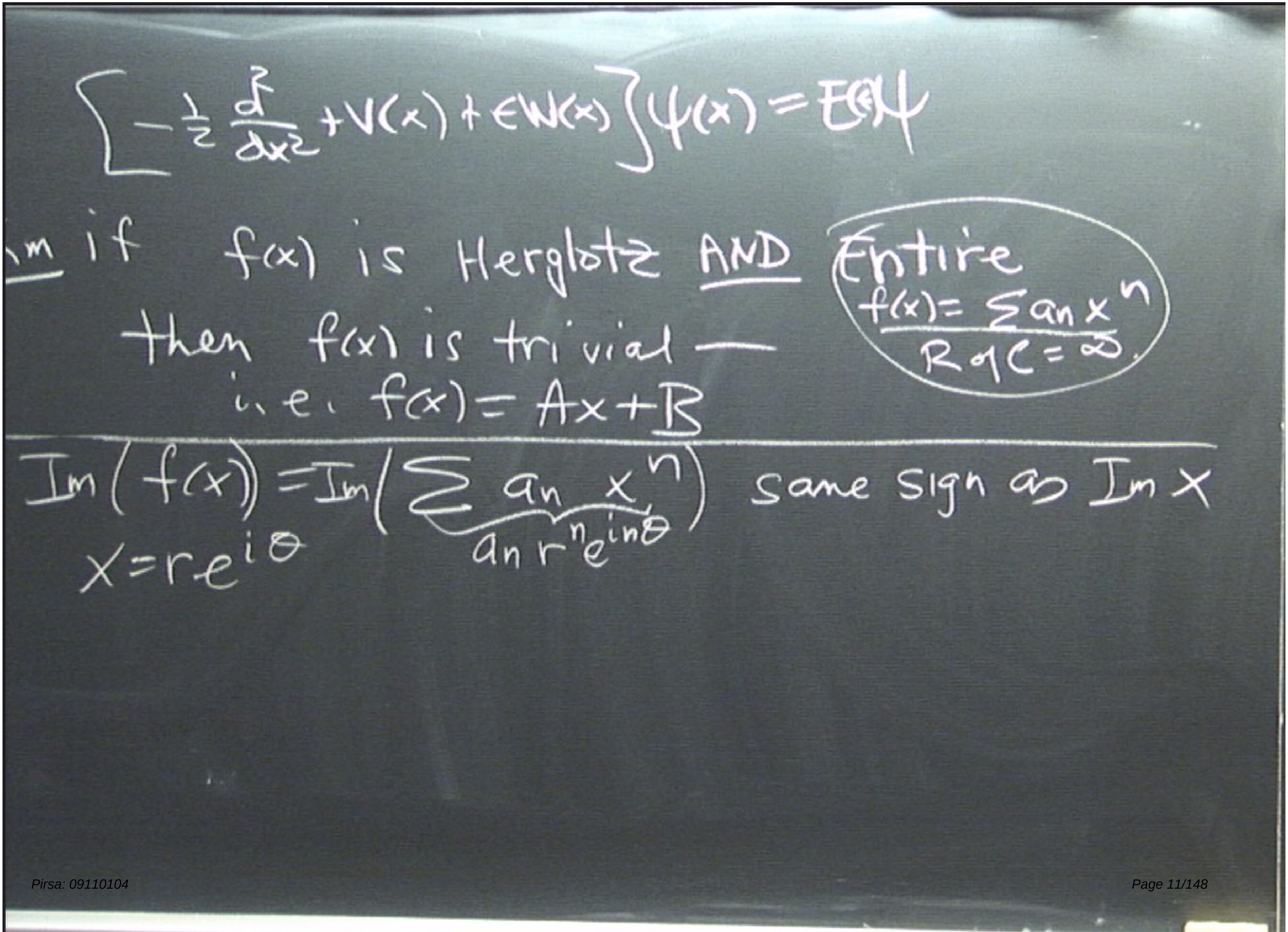

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if $f(x)$ is Herglotz AND Entire

$f(x)=\sum a_n x^n$

R of C $=\infty$

then $f(x)$ is trivial —

i.e. $f(x)=Ax+B$

$$\mathrm{Im}(f(x))=\mathrm{Im}\Big(\sum \underbrace{a_n\ x^n}_{a_n r^n e^{in\theta}}\Big)$$ same sign as $\mathrm{Im}\,x$

$x=re^{i\theta}$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if $f(x)$ is Herglotz AND Entire

$$\boxed{\begin{array}{c}\underline{f(x)=\sum a_n x^n}\\ R\ of\ C=\infty.\end{array}}$$

then $f(x)$ is trivial —

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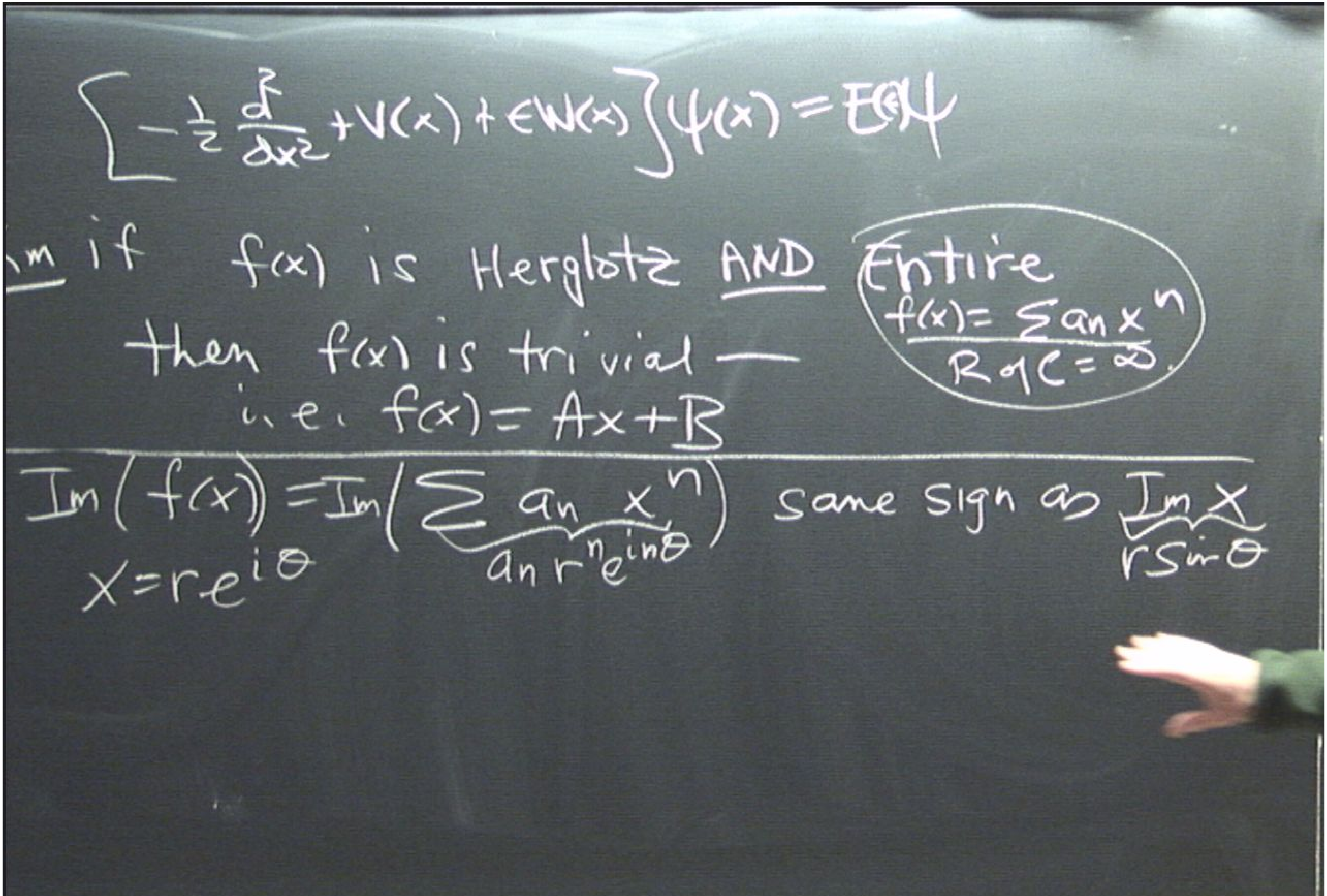
$$\mathrm{Im}(f(x)) = \mathrm{Im}\Big(\underbrace{\sum a_n\ x^n}_{a_n r^n e^{in\theta}}\Big) \quad \text{same sign as } \underbrace{\mathrm{Im}\,x}_{r\sin\theta}$$

$x = re^{i\theta}$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if f(x) is Herglotz AND Entire

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R of C = ∞

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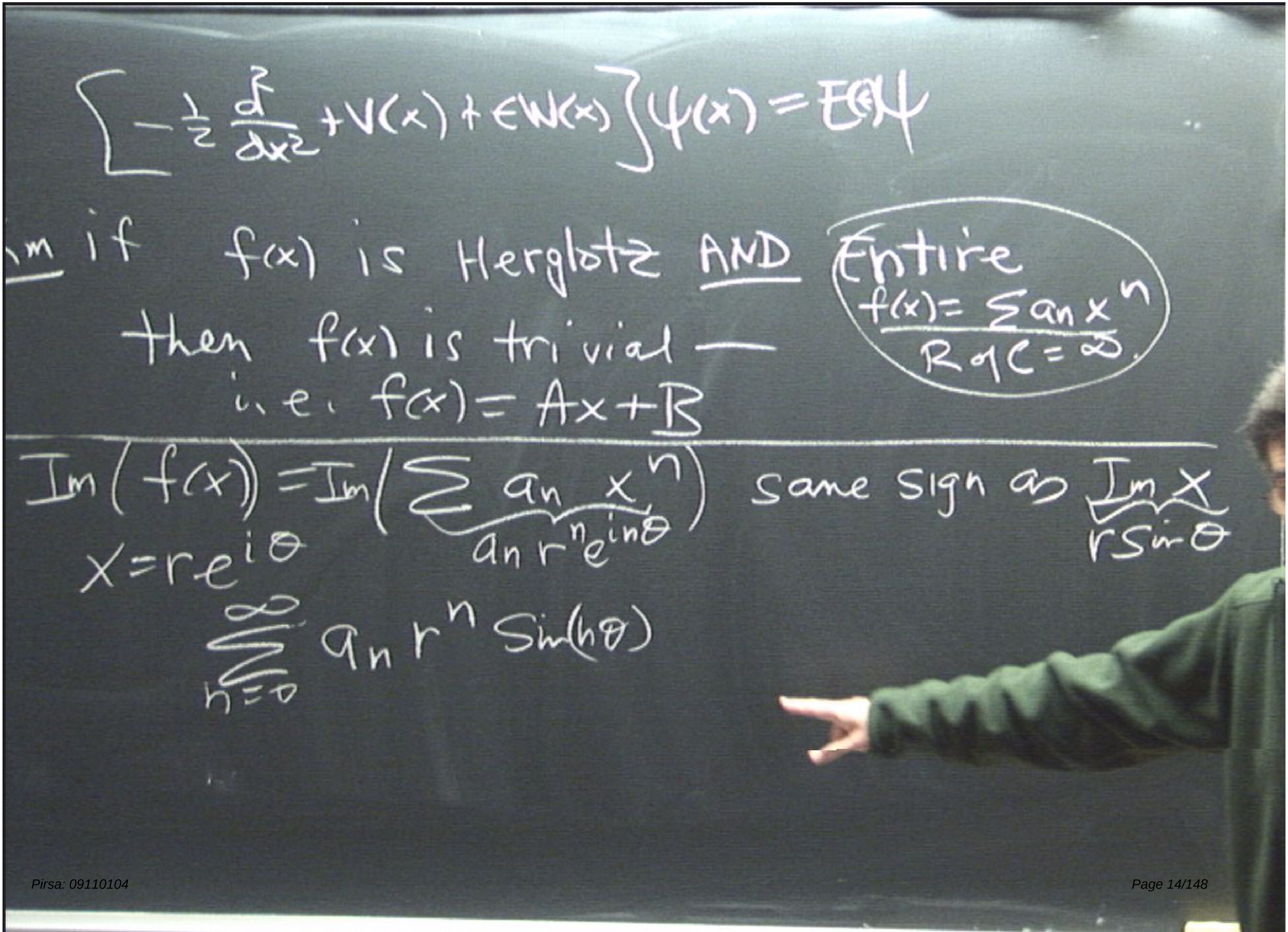

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if f(x) is Herglotz AND Entire

$f(x) = \sum a_n x^n$

RoC $= \infty$

then f(x) is trivial —

i.e. $f(x) = Ax + B$

$$\mathrm{Im}(f(x)) = \mathrm{Im}\Big(\sum \underbrace{a_n\, x^n}_{a_n r^n e^{in\theta}}\Big) \quad \text{same sign as } \underbrace{\mathrm{Im}\, x}_{r\sin\theta}$$

$x = re^{i\theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

<u>m</u> if $f(x)$ is Herglotz <u>AND</u> $\boxed{\text{Entire} \quad \dfrac{f(x)=\sum a_n x^n}{\text{RoC}=\infty}}$

then $f(x)$ is trivial —

i.e. $f(x) = Ax + \underline{B}$

$$\text{Im}(f(x)) = \text{Im}\Big(\sum \underbrace{a_n\, x^n}_{a_n r^n e^{in\theta}}\Big) \quad \text{same sign as } \underbrace{\underline{\text{Im}\, x}}_{r\sin\theta}$$

$x = re^{i\theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta) \quad \text{same sign as } r\sin\theta$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

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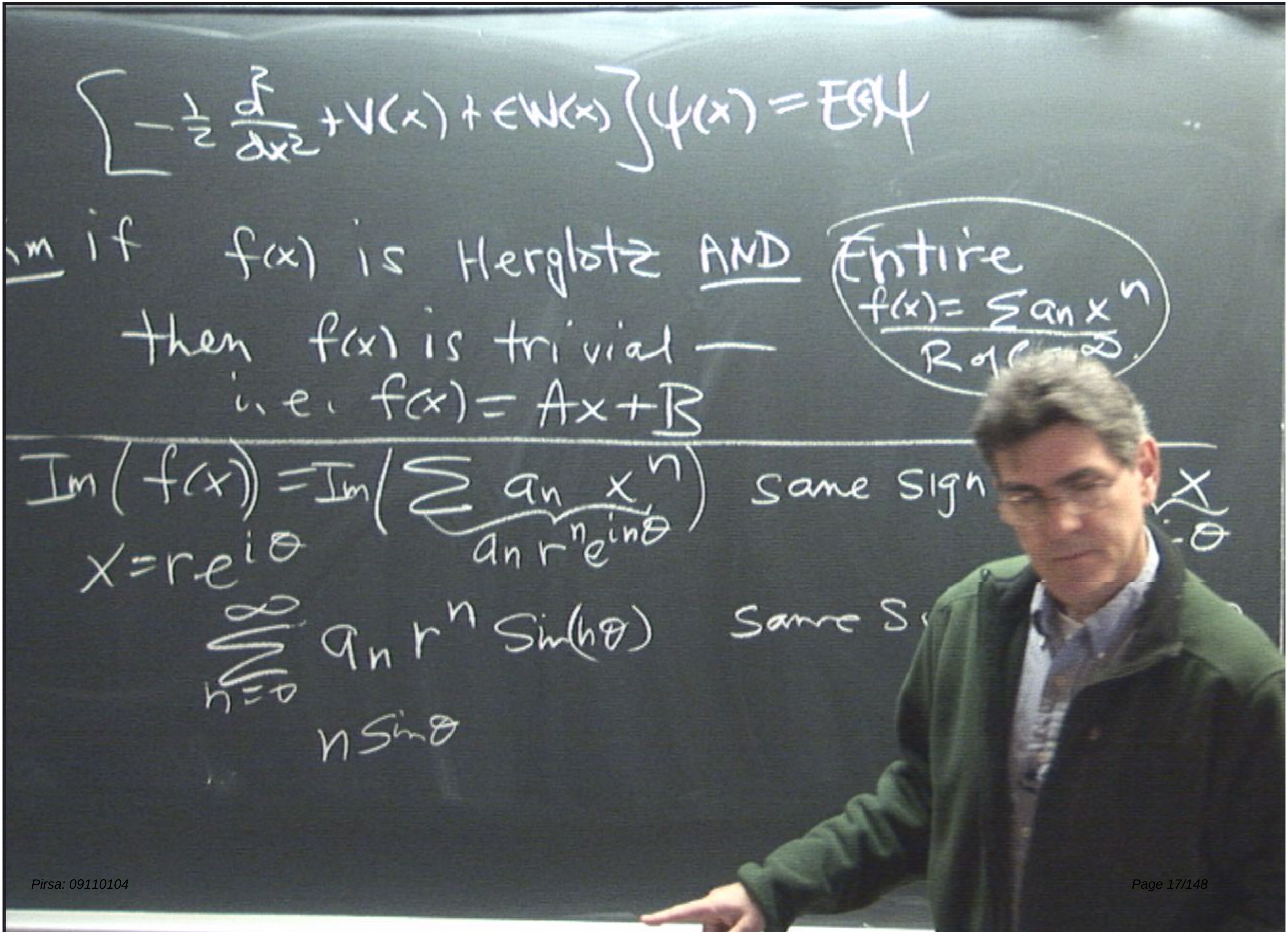
$$\text{Im}(f(x))=\text{Im}\Big(\sum \underbrace{a_n\ x^n}_{a_n r^n e^{in\theta}}\Big)$$ same sign as $\underbrace{\text{Im}\,x}_{r\sin\theta}$

$x=re^{i\theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$ same sign as $r\sin\theta$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

m if f(x) is Herglotz AND Entire

$f(x)=\sum a_n x^n$

R of C [illegible] ∞

then f(x) is trivial —

i.e. $f(x)=Ax+B$

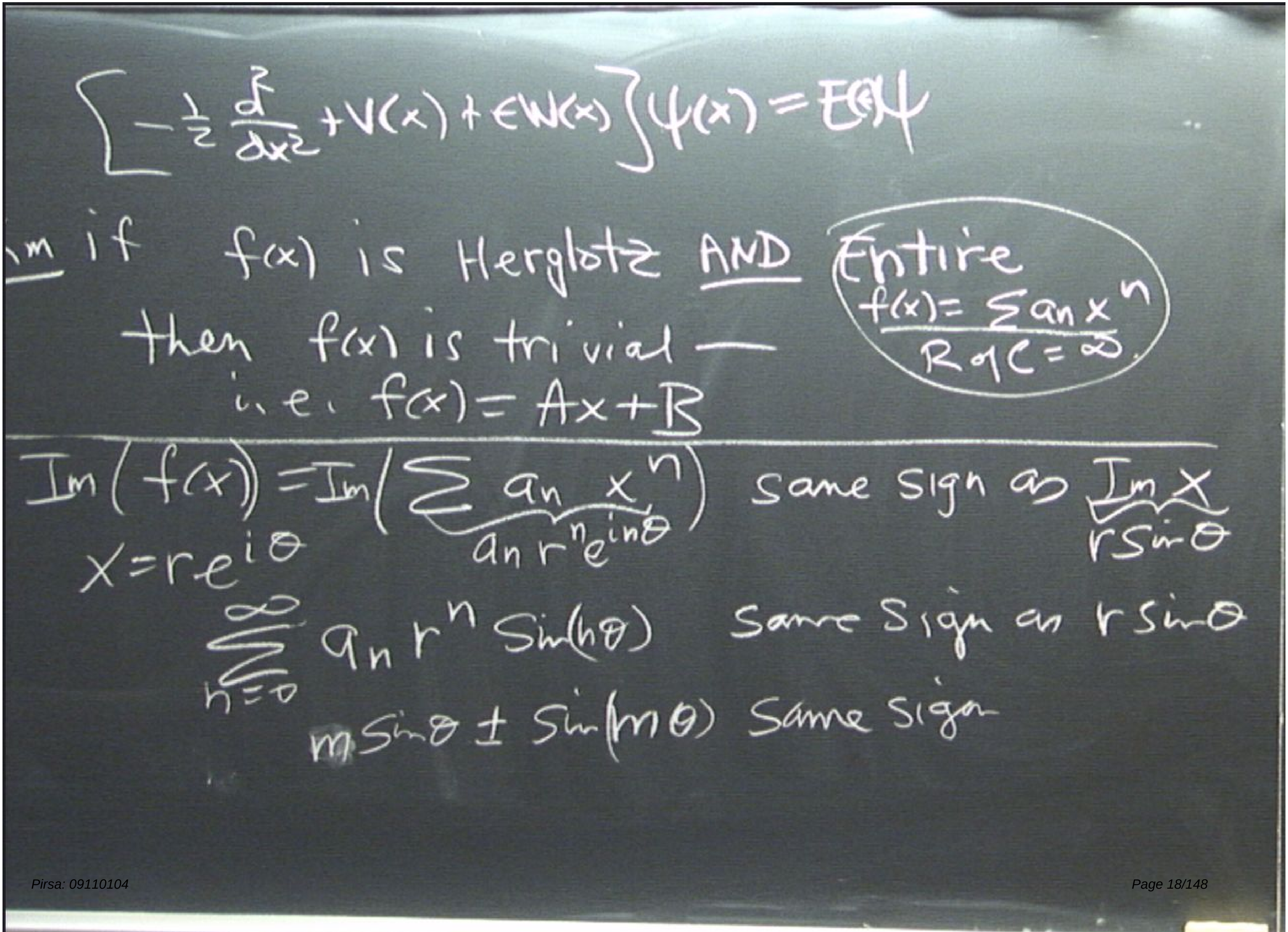
$\mathrm{Im}(f(x))=\mathrm{Im}\left(\sum \underbrace{a_n\ x^n}_{a_n r^n e^{in\theta}}\right)$ same sign [illegible] x

$-\theta$

$x=re^{i\theta}$

$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$ same S[illegible]

$n\sin\theta$



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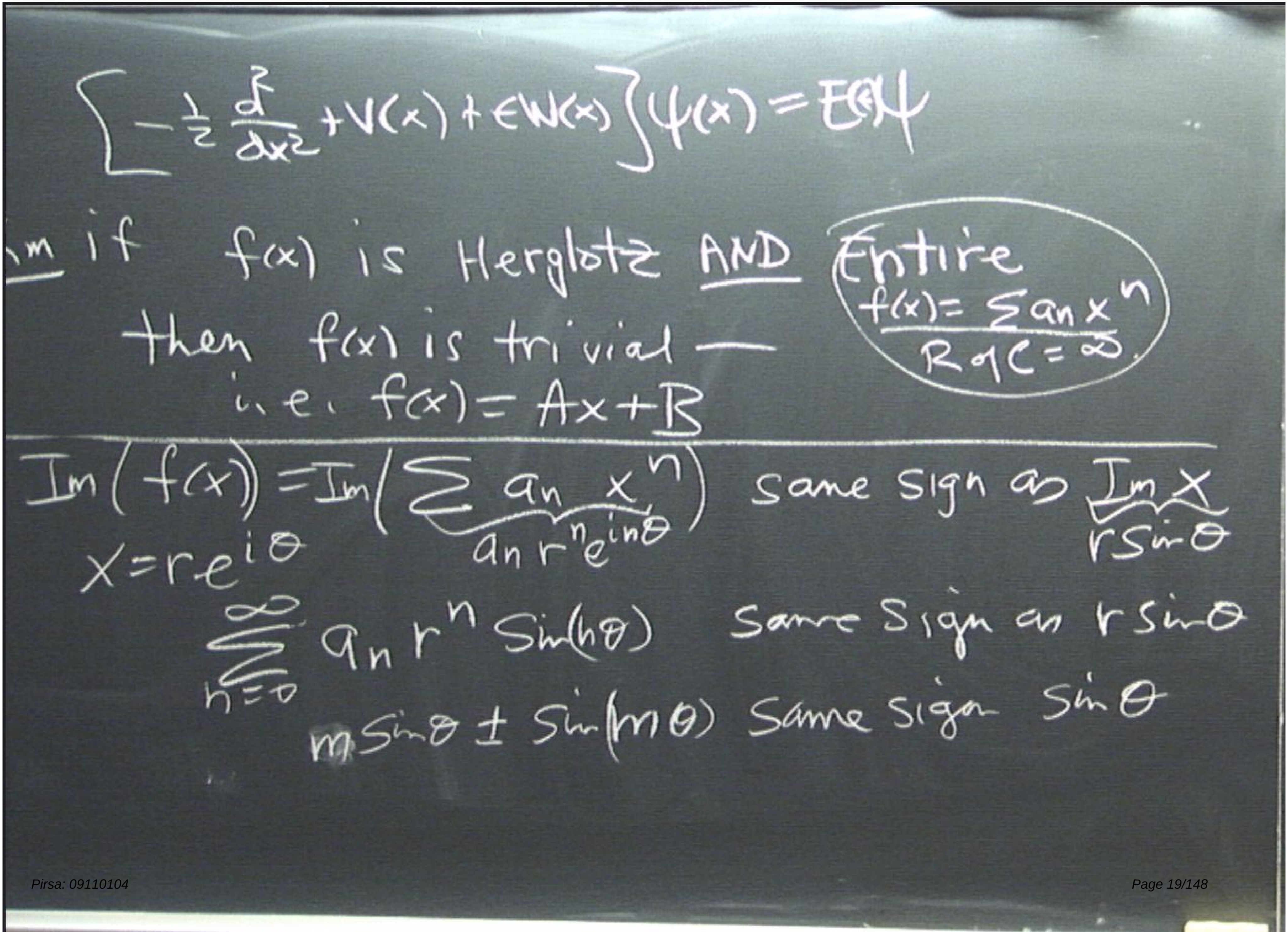
$x=re^{i\theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta) \text{ same sign as } r\sin\theta$$

$$m\sin\theta \pm \sin(m\theta) \text{ same sign}$$



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$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$ Same Sign as $r\sin\theta$

$m\sin\theta \pm \sin(m\theta)$ Same sign $\sin\theta$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

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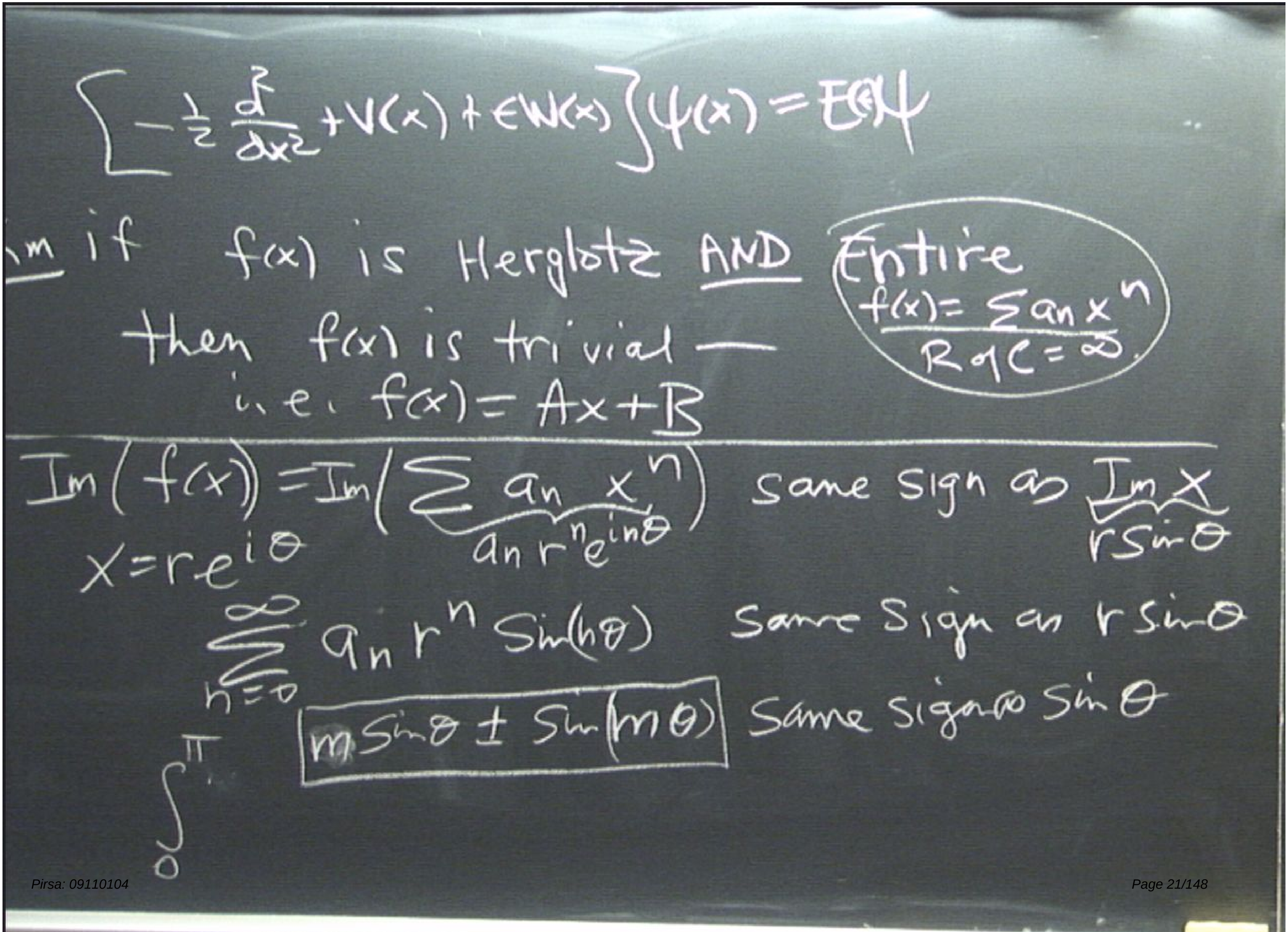
$x=re^{i\theta}$

$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$ same sign as $r\sin\theta$

$\boxed{m\sin\theta \pm \sin(m\theta)}$ same sign $\sin\theta$

$\int_0^{\pi}$



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$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$ same sign as $r\sin\theta$

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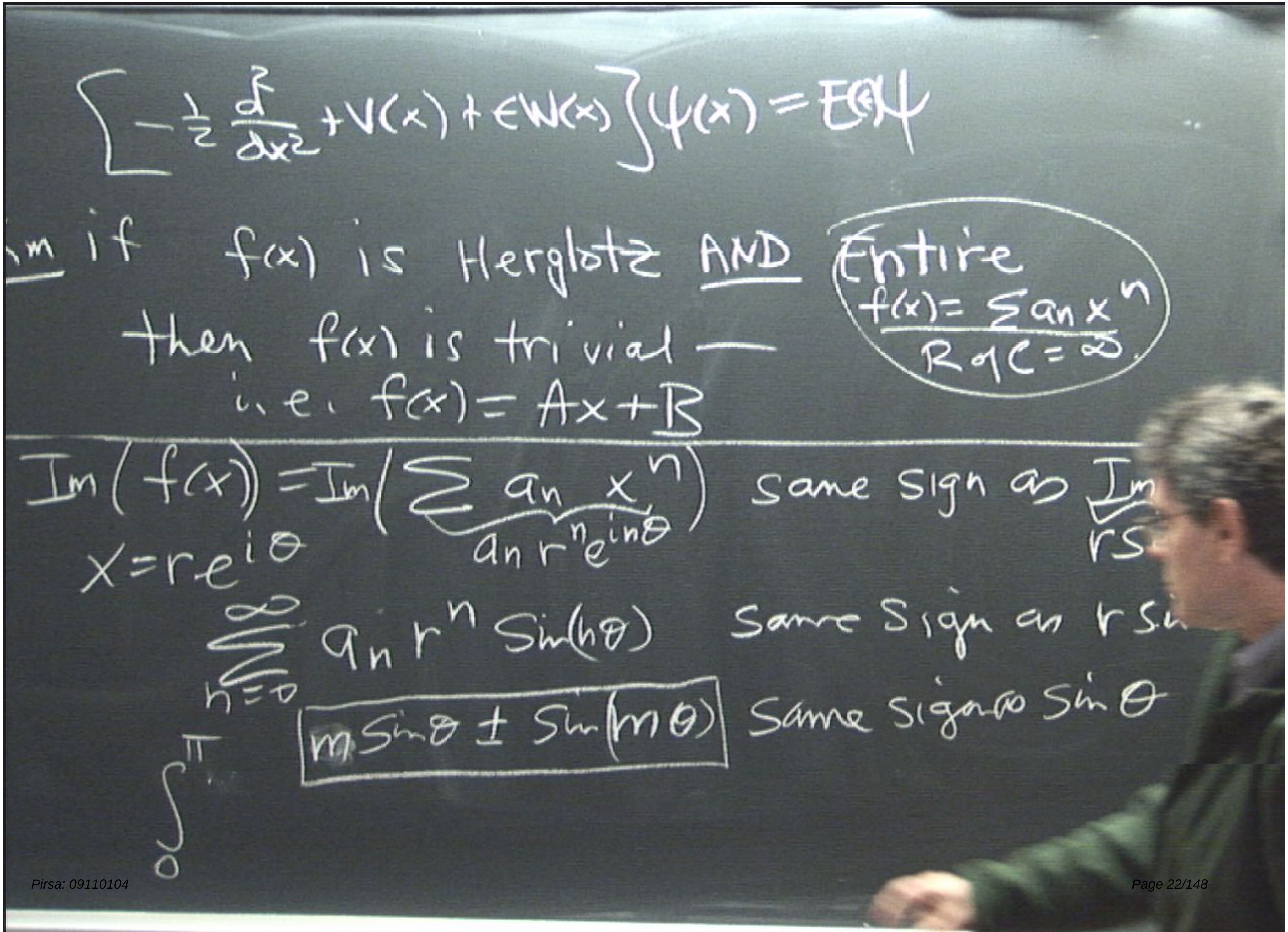

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rS

$x=re^{i\theta}$

$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$ same Sign as r Si

$\boxed{m\sin\theta \pm \sin(m\theta)}$ Same sign as $\sin\theta$

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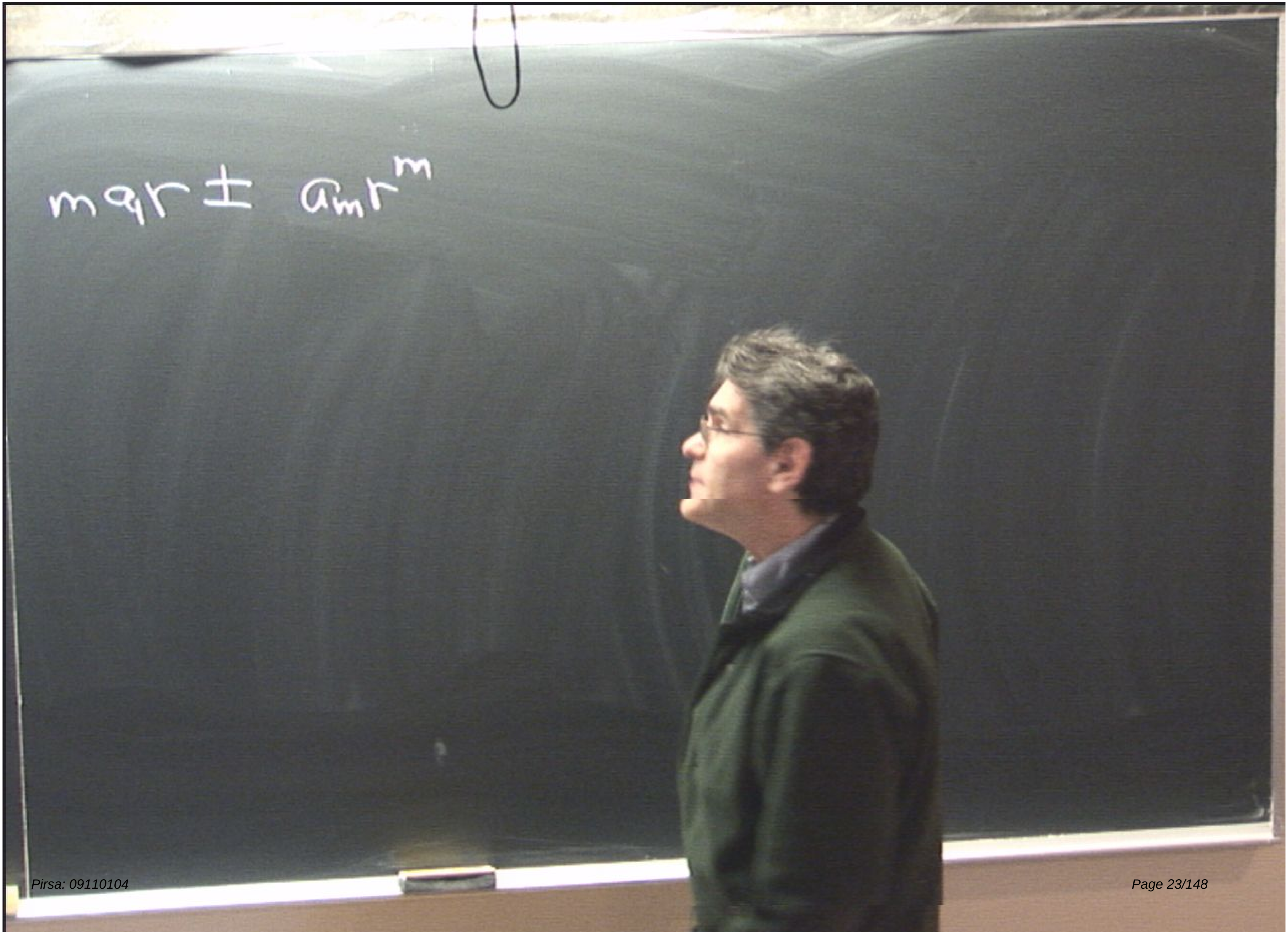
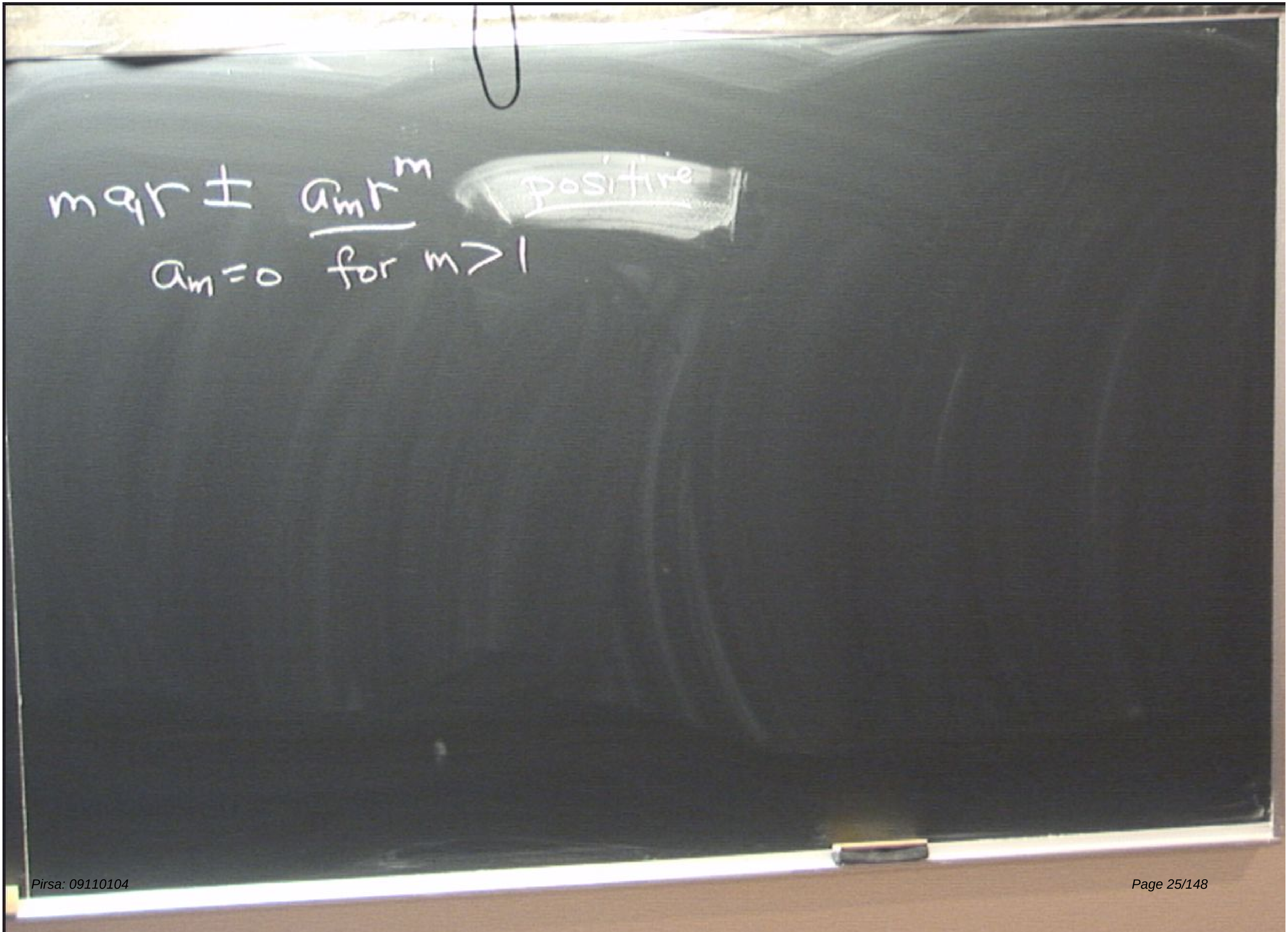


$$mqr \pm a_m r^m$$

$m q r \pm a_m r^m$ positive



$mqr \pm \underline{a_m r^m}$ <u>positive</u>

$a_m = 0$ for $m > 1$

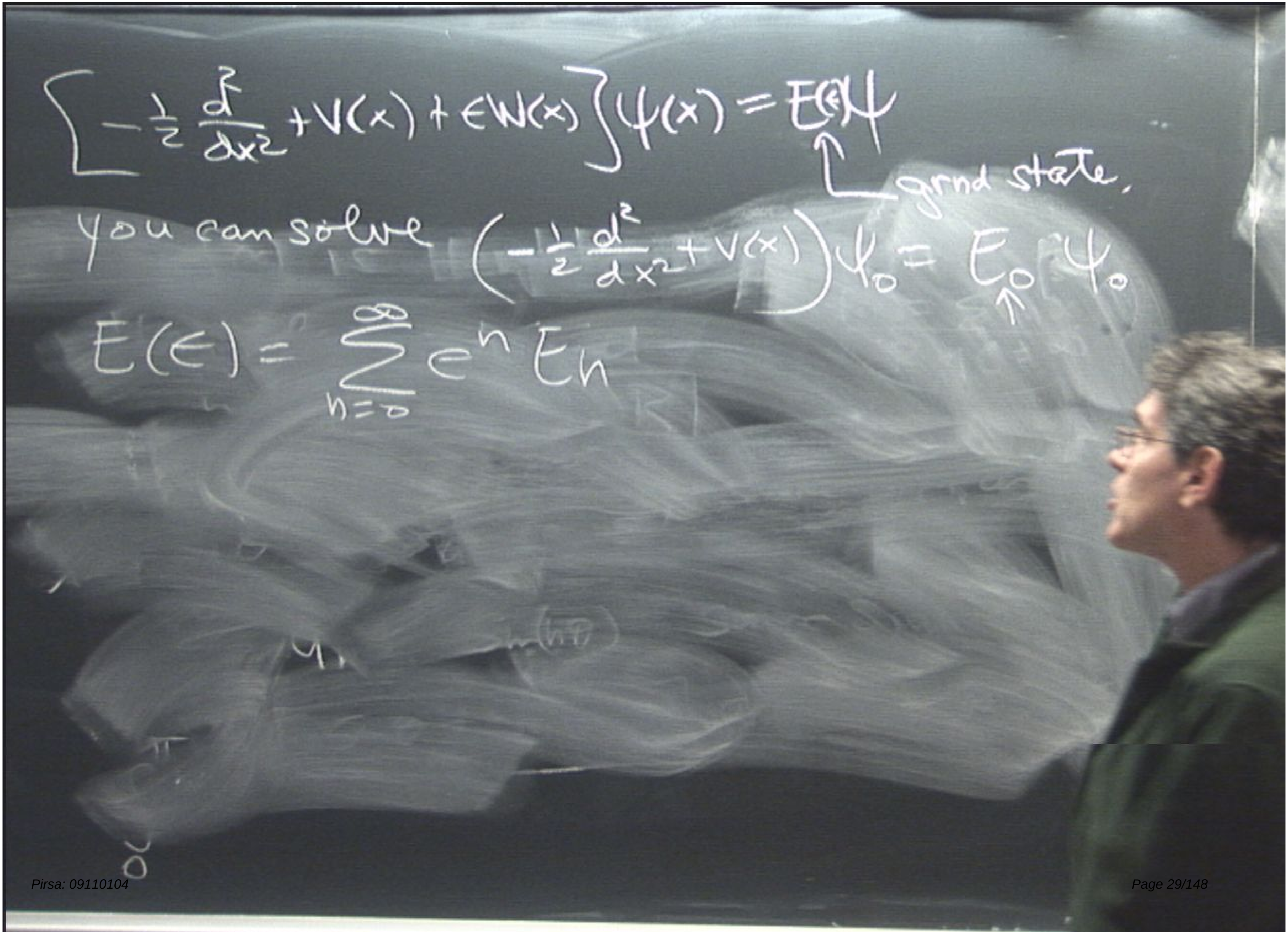
$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

$\uparrow$ grnd state.

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow\ \text{grnd state,}}{E(\epsilon)}\psi$$

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=E_0\psi_0$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0$

$$E(\epsilon)=\sum_{n=0}^{\infty}\epsilon^n E_n$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

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you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=\underset{\uparrow}{E_0}\,\psi_0$

$$E(\epsilon)=\sum_{n=0}^{\infty}\epsilon^n E_n\,,\quad \psi=\sum_{n=0}^{\infty}\epsilon^n\psi_n$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0$

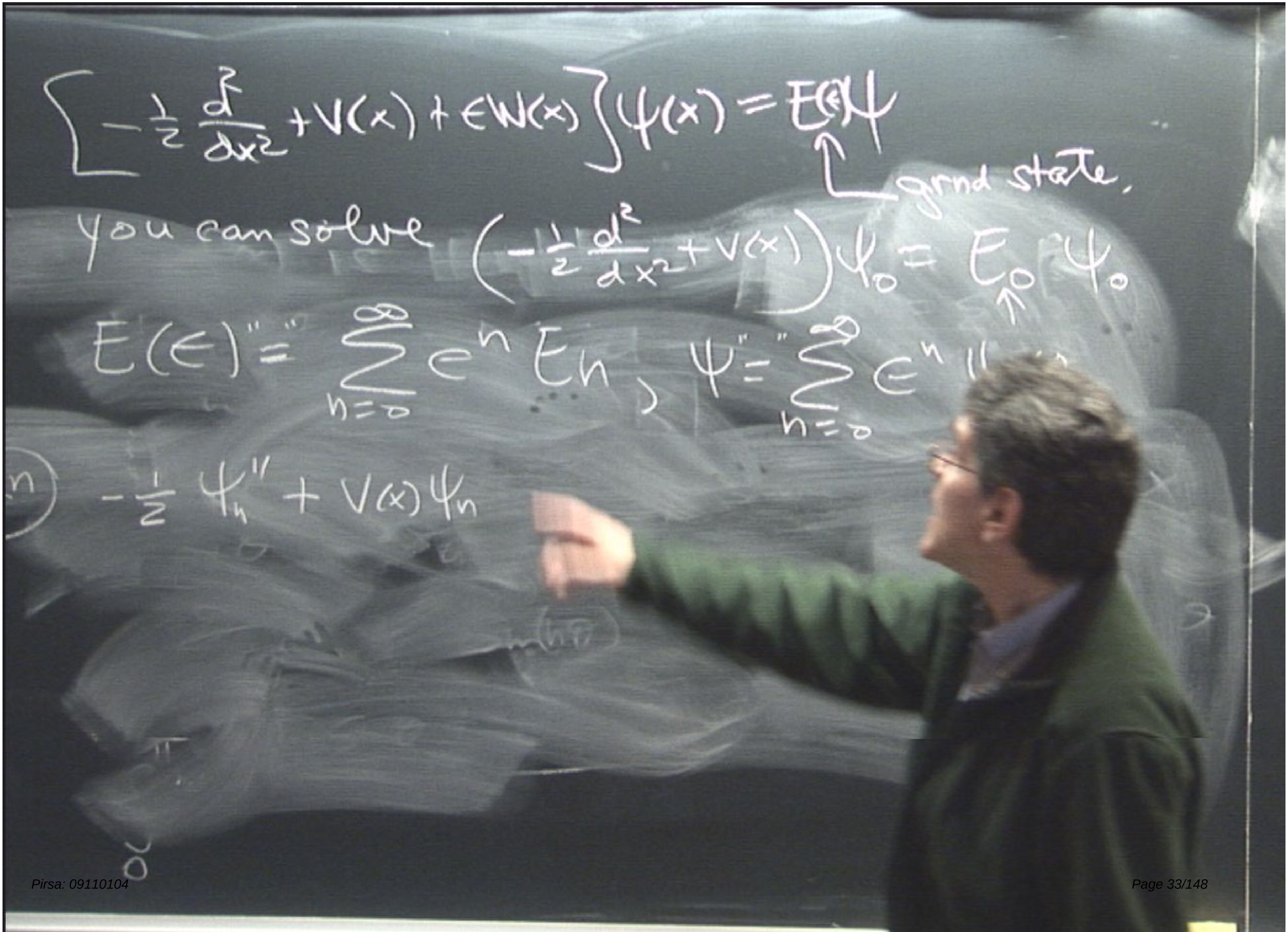
$E(\epsilon)$ "=" $\sum_{n=0}^{\infty}\epsilon^n E_n$, ψ "=" $\sum_{n=0}^{\infty}\epsilon^n \psi_n^{(x)}$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x) = \underset{\uparrow}{E(\epsilon)}\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\psi_0$

$$E(\epsilon) \text{"="} \sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{"="} \sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$



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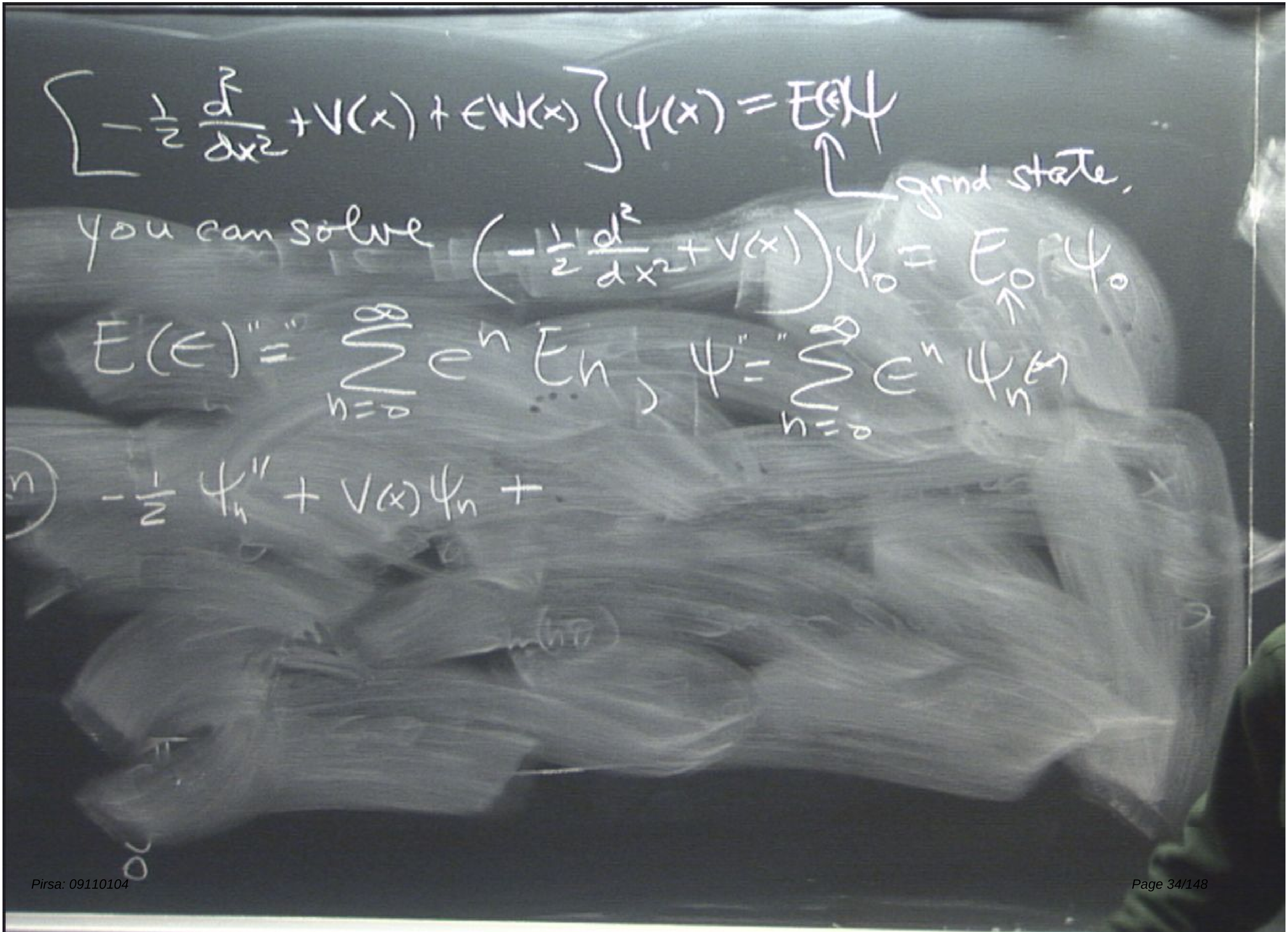
you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\psi_0$

$$E(\epsilon) \text{ "=" } \sum_{n=0}^{\infty} \epsilon^n E_n , \quad \psi \text{ "=" } \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$\textcircled{n}$ $-\frac{1}{2}\psi_n'' + V(x)\psi_n$



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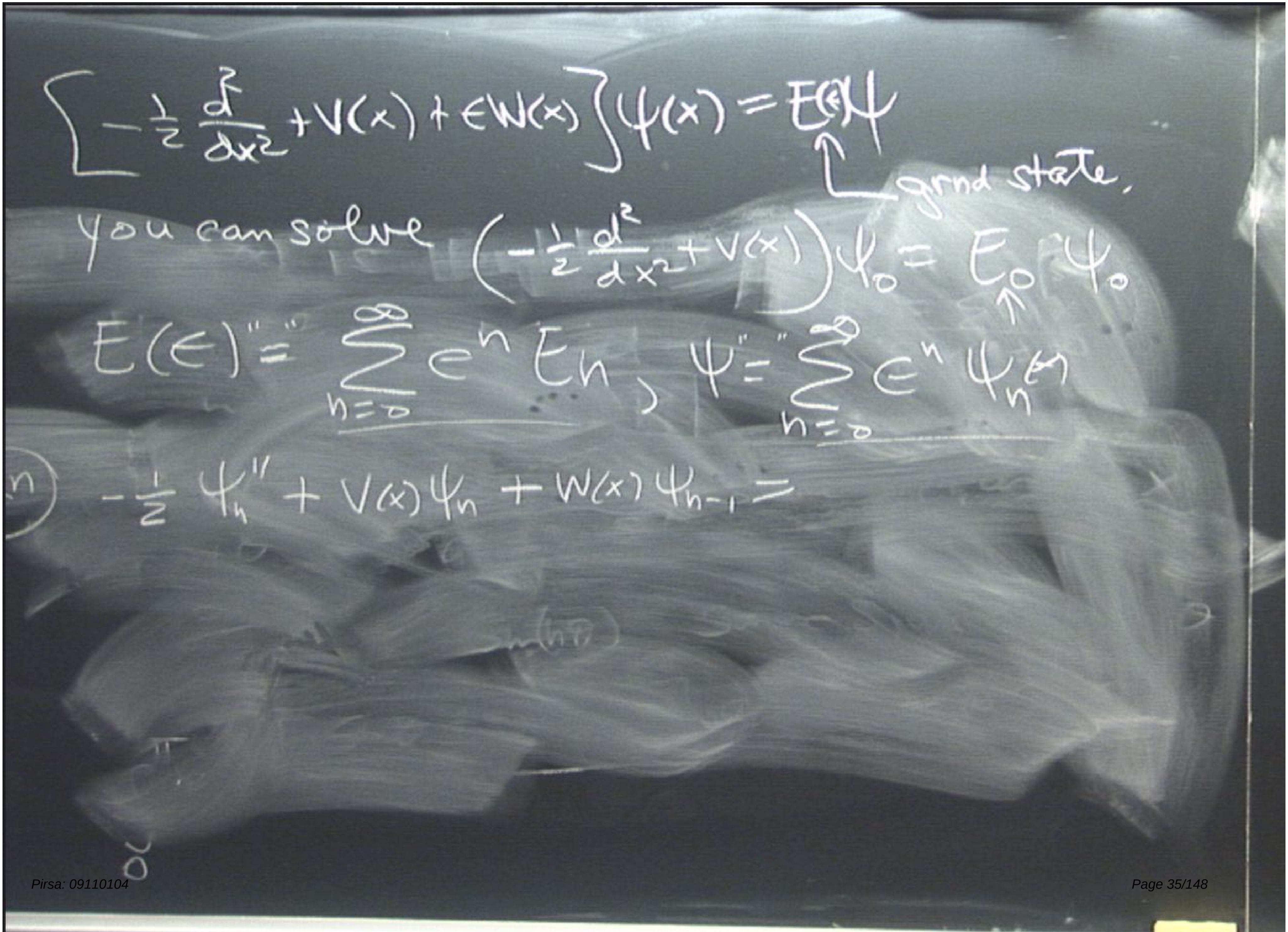
you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\psi_0$

$$E(\epsilon) \text{``}=\text{''} \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi \text{``}=\text{''} \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

(n) $-\frac{1}{2}\psi_n'' + V(x)\psi_n +$



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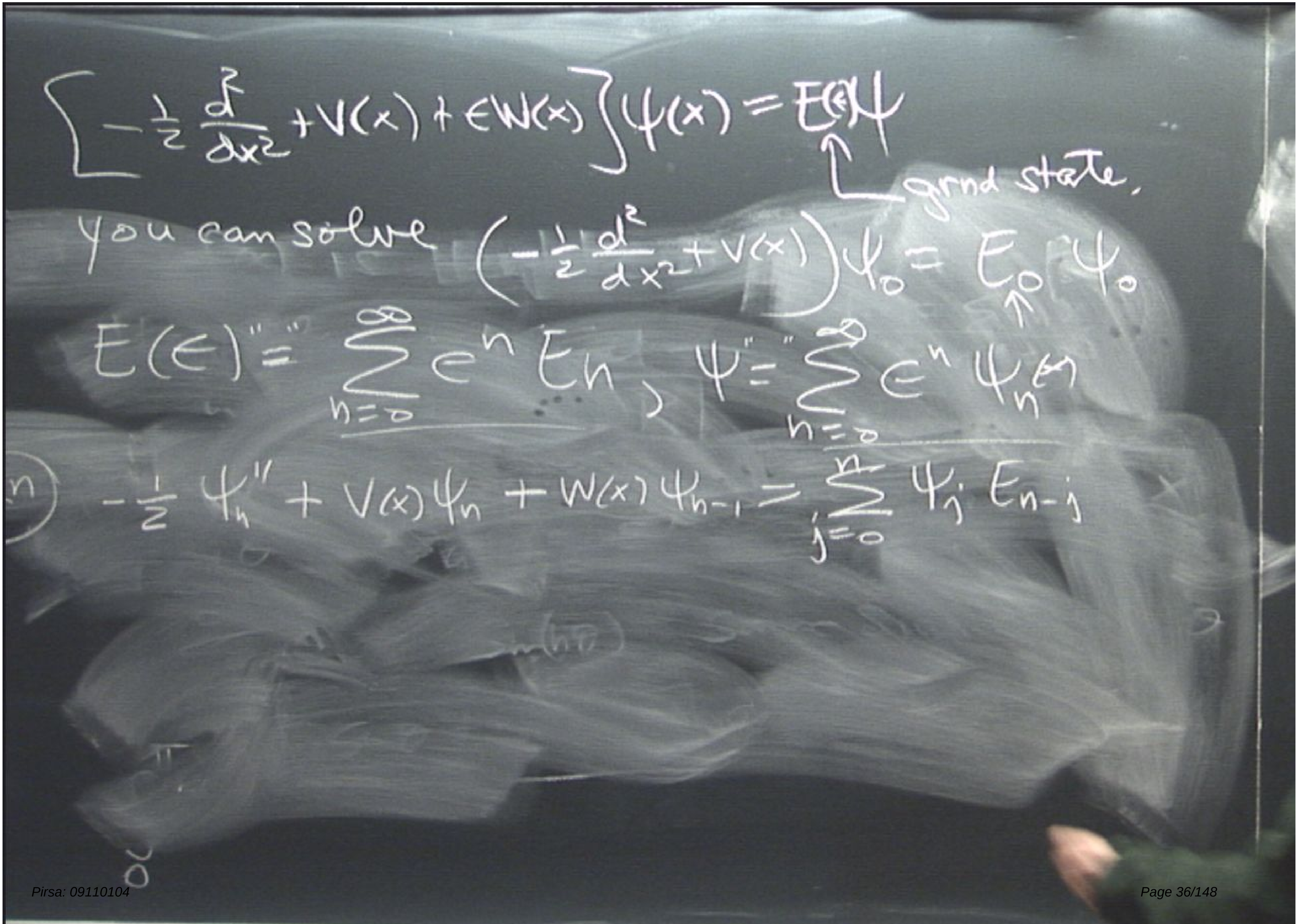
$$E(\epsilon) \text{"="} \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi \text{"="} \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$\bigcirc\!\!\!\!n$ $\quad -\frac{1}{2}\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} =$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

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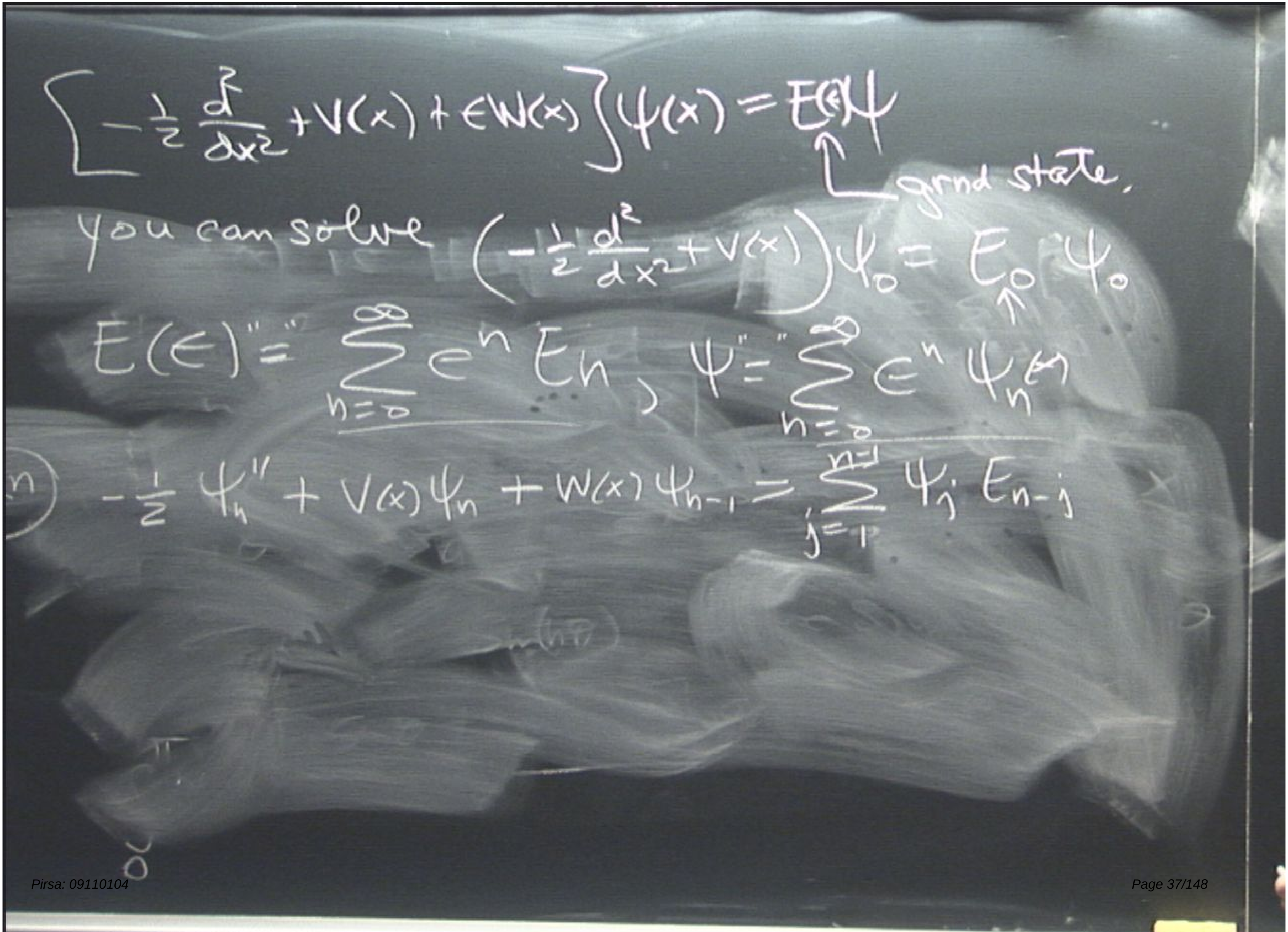
you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0$

$$\underline{E(\epsilon) \text{ "=" } \sum_{n=0}^{\infty} \epsilon^n E_n}\,, \quad \underline{\psi \text{ "=" } \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)}$$

$$\textcircled{n} \quad -\frac{1}{2}\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=0}^{n} \psi_j E_{n-j}$$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=E_0\,\psi_0$

$$E(\epsilon)\text{ "=" }\sum_{n=0}^{\infty}\epsilon^n E_n\,,\quad \psi\text{ "=" }\sum_{n=0}^{\infty}\epsilon^n\,\psi_n(x)$$

(n) $-\frac{1}{2}\psi_n''+V(x)\psi_n+W(x)\psi_{n-1}=\sum_{j=1}^{n-1}\psi_j\,E_{n-j}$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x) = \underset{\uparrow}{E(\epsilon)}\psi$$

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you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\psi_0$

$$E(\epsilon) \text{"="} \sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{"="} \sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$

$$) \quad -\frac{1}{2}\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1}\psi_j E_{n-j} + \psi_0 E_n + \psi_n E$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=\underset{\uparrow}{E_0}\psi_0$

$$E(\epsilon)\text{ ``$=$'' }\sum_{n=0}^{\infty}\epsilon^n E_n,\quad \psi\text{ ``$=$'' }\sum_{n=0}^{\infty}\epsilon^n\psi_n(x)$$

$$n)\quad -\frac{1}{2}\psi_n''+V(x)\psi_n+W(x)\psi_{n-1}=\sum_{j=1}^{n-1}\psi_j E_{n-j}+\psi_0 E_n+\psi_n E_0$$

$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

grnd state,

you can solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=\underset{\uparrow}{E_0}\,\psi_0}$

$$E(\epsilon)\text{"="}\sum_{n=0}^{\infty}\epsilon^n E_n\,,\quad \psi\text{"="}\sum_{n=0}^{\infty}\epsilon^n\psi_n(x)$$

$$(n)\quad -\frac{1}{2}\underline{\psi_n''}+\underline{V(x)\psi_n}+W(x)\psi_{n-1}=\sum_{j=1}^{n-1}\psi_j E_{n-j}+\psi_0E_n+\psi_nE_0$$

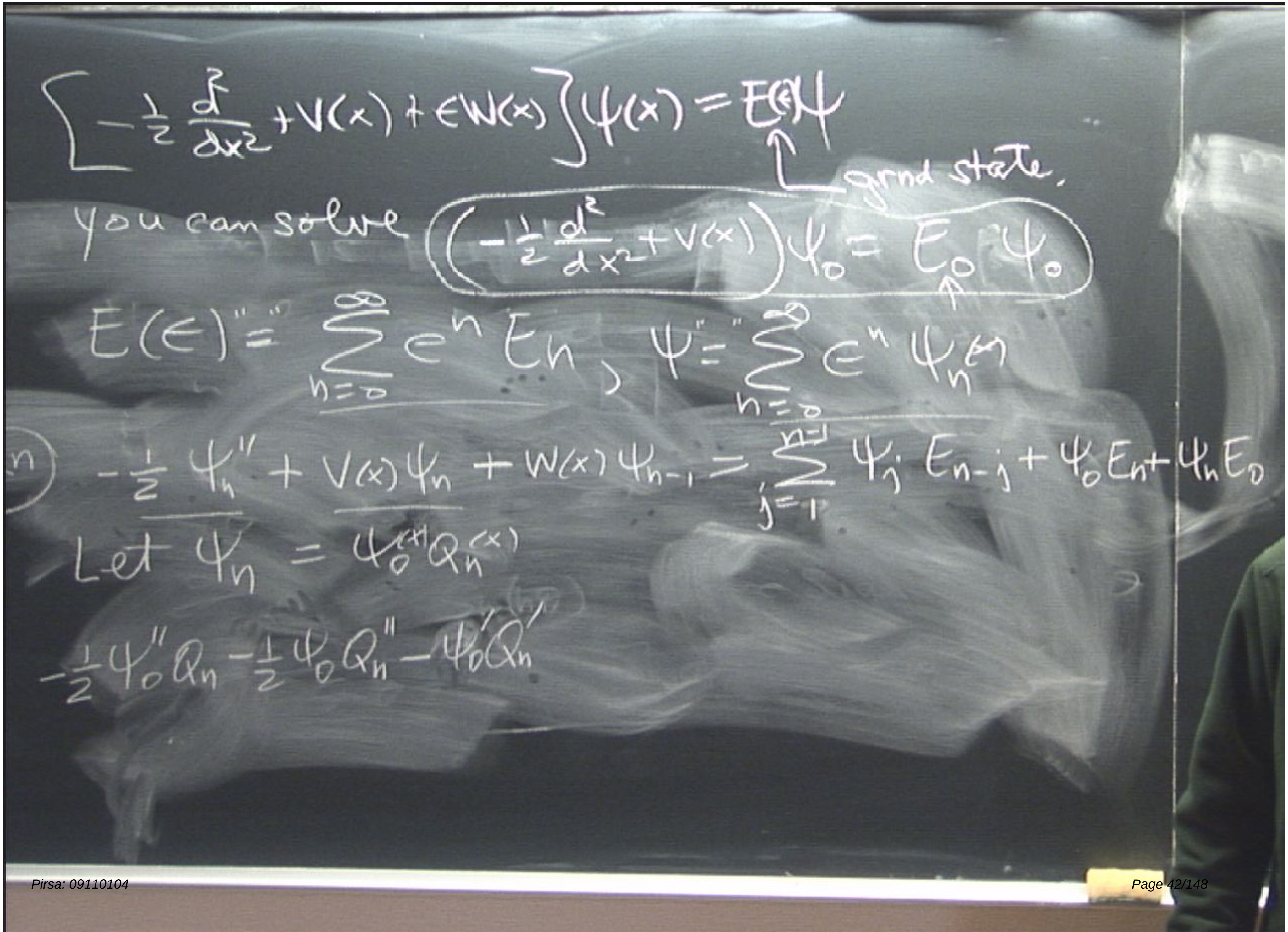
$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow\ \text{grnd state.}}{E(\epsilon)}\psi$$

you can solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0=\underset{\uparrow}{E_0}\psi_0}$

$$E(\epsilon)\text{"="}\sum_{n=0}^{\infty}\epsilon^n E_n,\quad \psi\text{"="}\sum_{n=0}^{\infty}\epsilon^n\psi_n(x)$$

$$(n)\quad -\frac{1}{2}\psi_n''+V(x)\psi_n+W(x)\psi_{n-1}=\sum_{j=1}^{n-1}\psi_j E_{n-j}+\psi_0 E_n+\psi_n E_0$$

Let $\psi_n=\psi_0(x)Q_n(x)$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=\underset{\uparrow}{E(\epsilon)}\psi$$

grnd state.

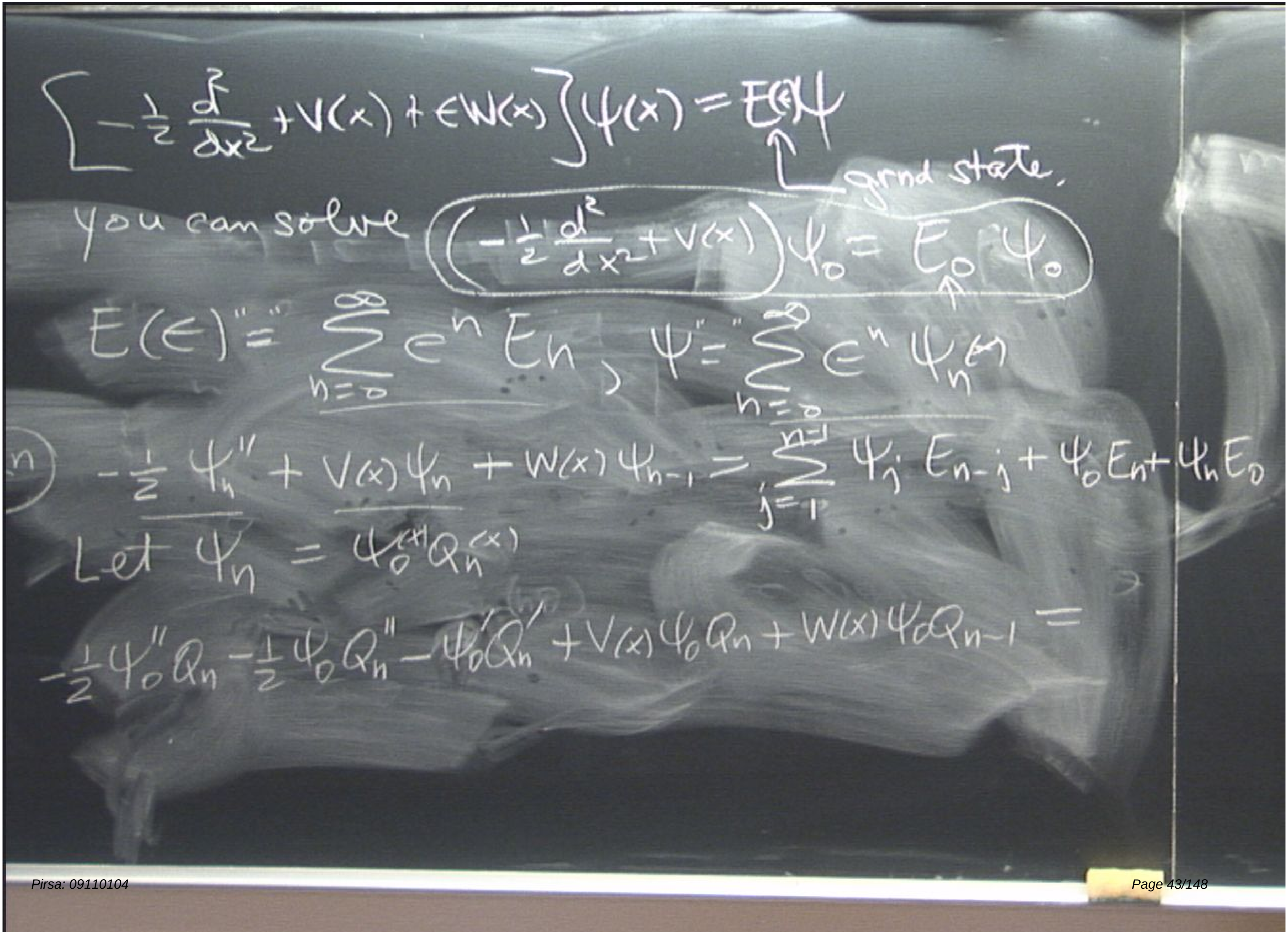
you can solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0}$

$$E(\epsilon) \text{ "=" } \sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{ "=" } \sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$

$$(n) \quad -\frac{1}{2}\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1}\psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

Let $\psi_n = \psi_0(x) Q_n(x)$

$$-\frac{1}{2}\psi_0'' Q_n - \frac{1}{2}\psi_0 Q_n'' - \psi_0' Q_n'$$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

grnd state.

you can solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = E_0\,\psi_0}$

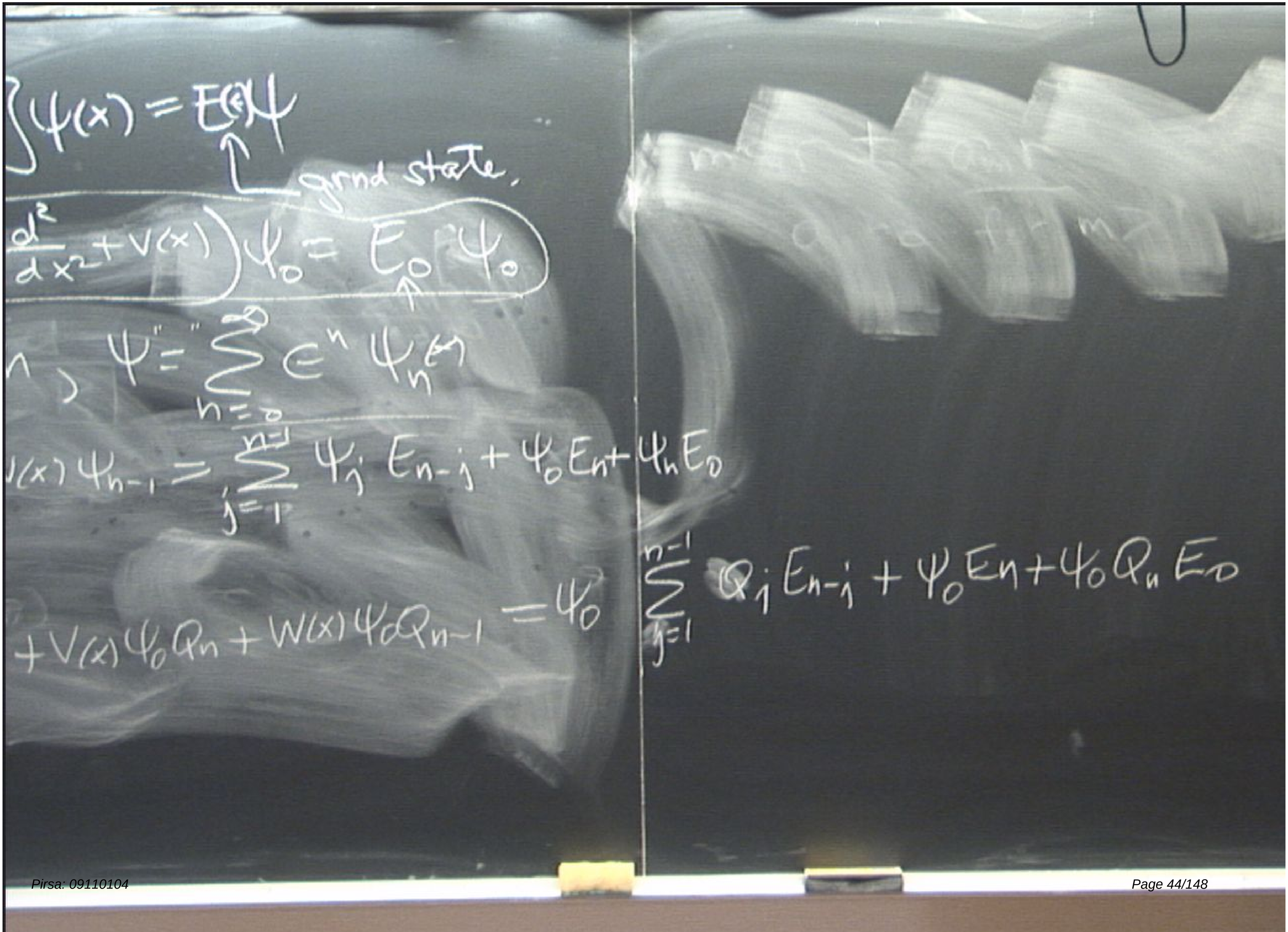
$$E(\epsilon) \text{``=''} \sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{``=''} \sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$

n)

$$-\frac{1}{2}\underline{\psi_n''} + \underline{V(x)\psi_n} + W(x)\psi_{n-1} = \sum_{j=1}^{n-1}\psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

Let $\psi_n = \psi_0(x) Q_n(x)$

$$-\frac{1}{2}\psi_0'' Q_n - \frac{1}{2}\psi_0 Q_n'' - \psi_0' Q_n' + V(x)\psi_0 Q_n + W(x)\psi_0 Q_{n-1} =$$



$$)\psi(x) = E(\epsilon)\psi$$

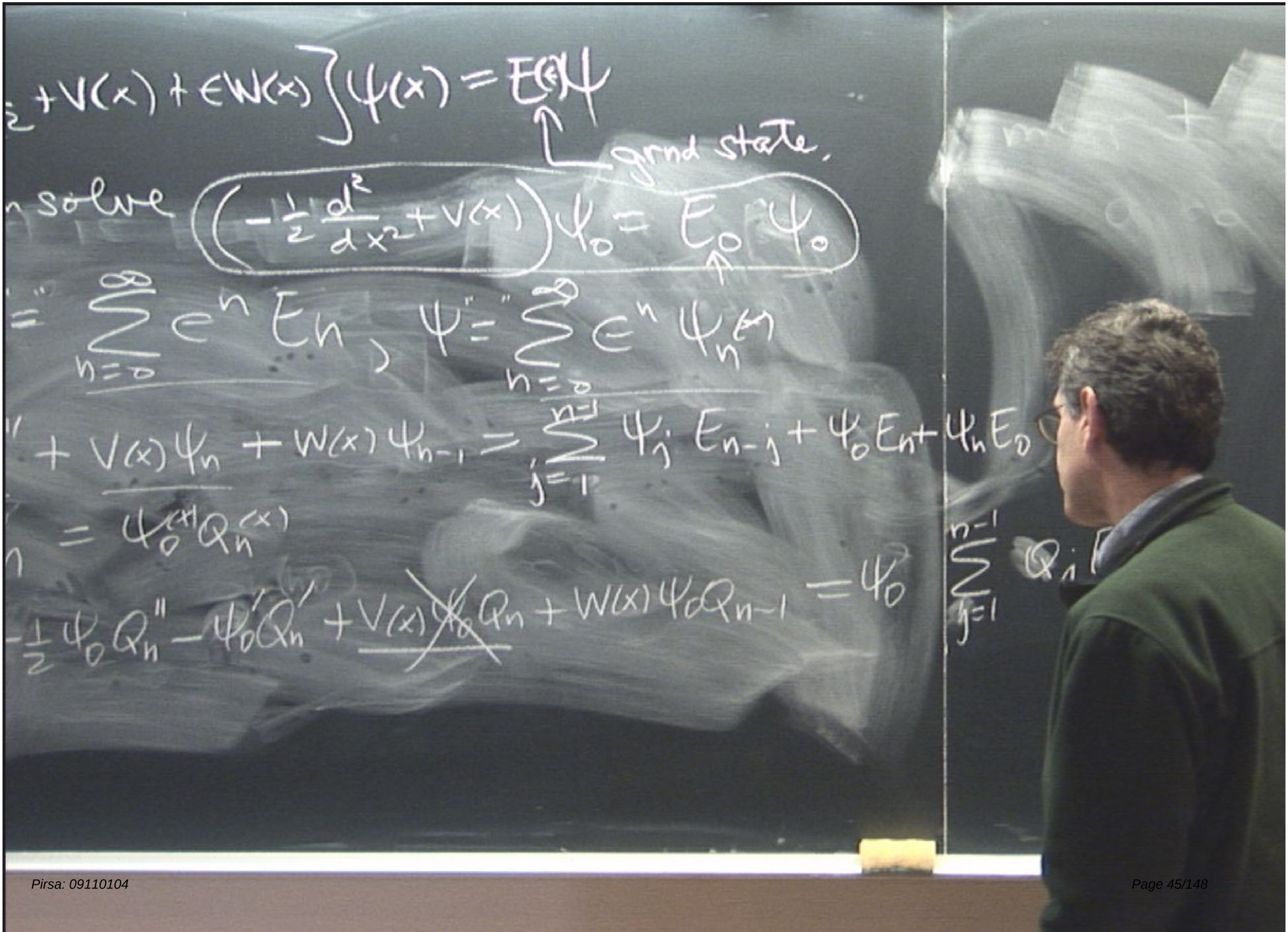
grnd state.

$$\frac{d^2}{dx^2}+V(x))\psi_0 = E_0\psi_0$$

$$n,\ \psi'' = '' \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$$V(x)\psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$+V(x)\psi_0 Q_n + W(x)\psi_0 Q_{n-1} = \psi_0 \sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n + \psi_0 Q_n E_0$$



$$\ldots + V(x) + \epsilon W(x)\Big]\psi(x) = E(\epsilon)\psi$$

grnd state.

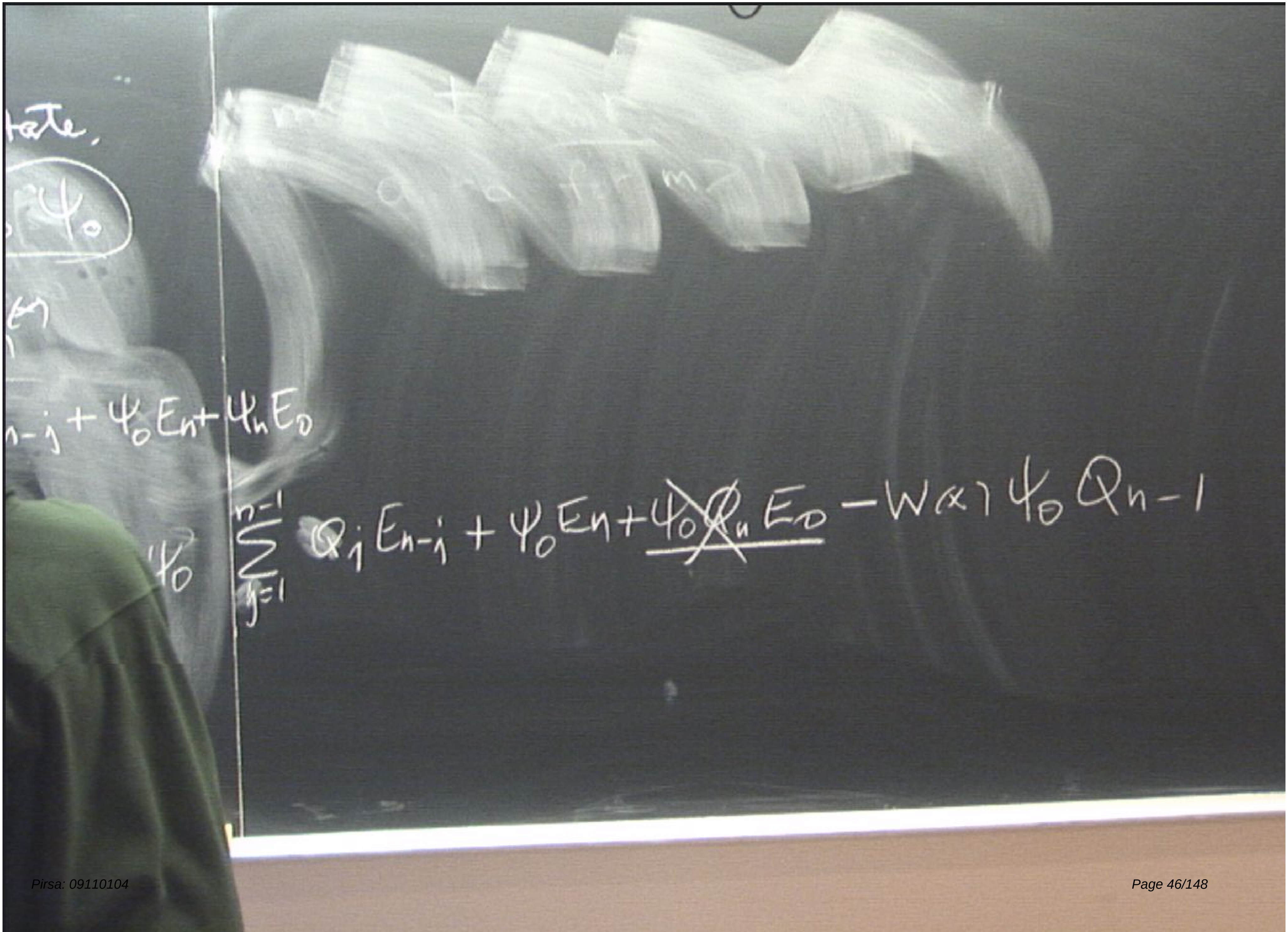
solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = E_0\psi_0$

$$\text{"}=\text{"}\sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{"}=\text{"}\sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$

$$'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1}\psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$n = \psi_0(x) Q_n(x)$$

$$-\frac{1}{2}\psi_0 Q_n'' - \psi_0' Q_n' + V(x)\cancel{\psi_0} Q_n + W(x)\psi_0 Q_{n-1} = \psi_0 \sum_{j=1}^{n-1} Q_j$$



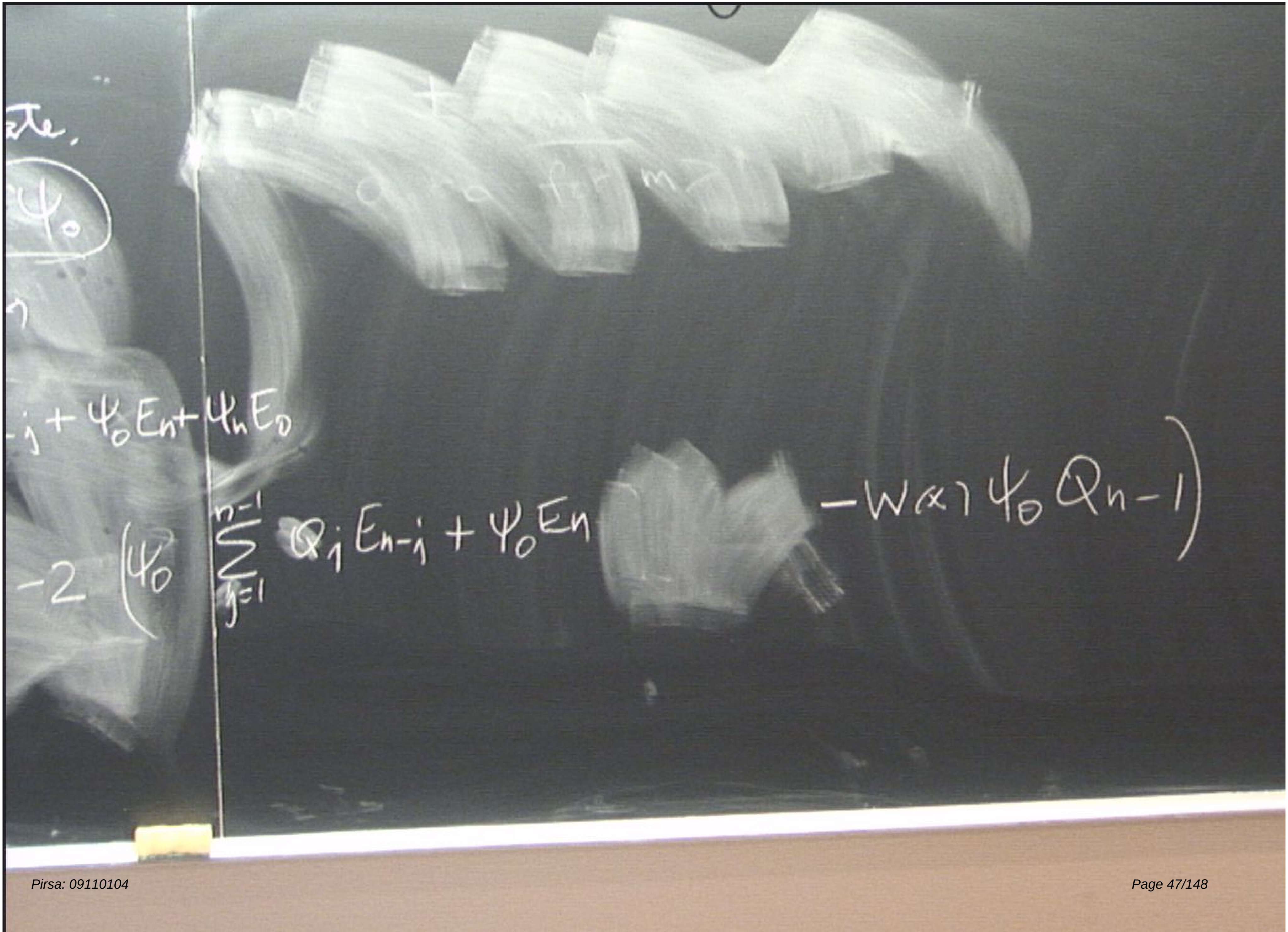
tate.

ψ_0

$_{n-j} + \psi_0 E_n + \psi_n E_0$

ψ_0

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n + \underline{\psi_0 \cancel{Q_n} E_0} - W(x)\, \psi_0 Q_{n-1}$$



te.

ψ_0

n

$_j + \psi_0 E_n + \psi_n E_0$

$$-2 \left(\psi_0 \sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n \quad - W(x)\, \psi_0 Q_{n-1} \right)$$

$$= \psi_n + V(x)\psi_n + W(x)\psi_n$$

$$+ \psi_n = \psi_0(x) Q_n(x)$$

$$\psi_0 Q_n'' + 2\psi_0' Q_n'$$

$$_n = \varphi_0 Q_n(x)$$

$$\varphi_0 Q_n'' + 2\varphi_0' Q_n'$$

$$V(x) + \epsilon W(x) \Big] \psi(x) = E(\epsilon)\psi$$

grnd state.

solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = E_0 \psi_0}$

$$\sum_{n=0}^{\infty} \epsilon^n E_n , \quad \psi " = " \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$$V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$= \psi_0(x) Q_n(x)$$

$$\psi_0^2 Q_n'' + 2\psi_0'\psi_0 Q_n'$$

$$= -2 \Big(\psi_0 \quad \sum_{j=1}^{n-1} Q_j E_{n-j} + \psi$$

state.

Ψ_0

$E_{n-j} + \Psi_0 E_n + \Psi_n E_0$

$= -2 \Psi_0^2 \Big[\sum_{j=1}^{n-1} Q_j E_{n-j} + \quad E_n \quad - W(x) \quad Q_n -$

$$\frac{d}{dx^2} + V(x) + \epsilon W(x) \Big\} \psi(x) = E(\epsilon)\psi$$

$\uparrow$ grnd state.

can solve $\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2} + V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0}$ $\psi_0 \to 0$ as $x \to \pm\infty$

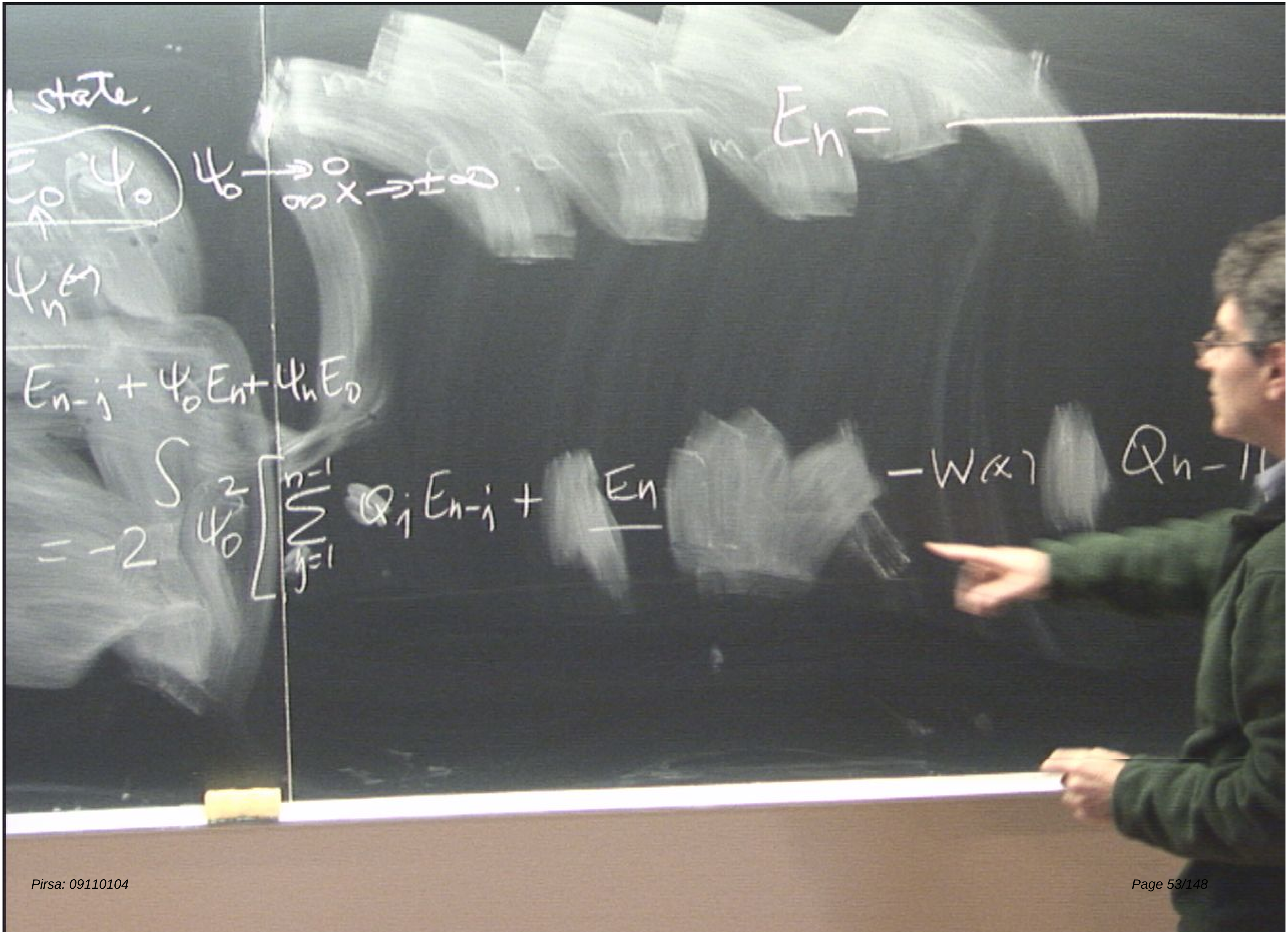
$$) \text{"="} \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi \text{"="} \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$$\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$\psi_n = \psi_0(x) Q_n(x)$$

$$\psi_0^2 Q_n'' + 2\psi_0'\psi_0 Q_n' = \frac{d}{dx}\left(\psi_0^2 Q_n'\right) = -2\psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j}\right.$$

$$\int_{-\infty}^{\infty}$$



state.

$E_0 \psi_0$ $\quad \psi_0 \to 0$ as $x \to \pm\infty$.

$\psi_n^{(n)}$

$E_n =$

$E_{n-j} + \psi_0 E_n + \psi_n E_0$

$$= -2 \int \psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{\;} \quad - W(x) \quad Q_{n-1} \right.$$

state.

$E_0 \Psi_0$ $\Psi_0 \to 0$ as $x \to \pm\infty$

$$E_n = \frac{\qquad}{\int_{-\infty}^{\infty} \Psi_0^2 \, dx}$$

$\Psi_n(x)$

$E_{n-j} + \Psi_0 E_n + \Psi_n E_0$

$$= -2 \int \Psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \underline{E_n} \quad - W(x) \quad Q_{n-1} \right]$$

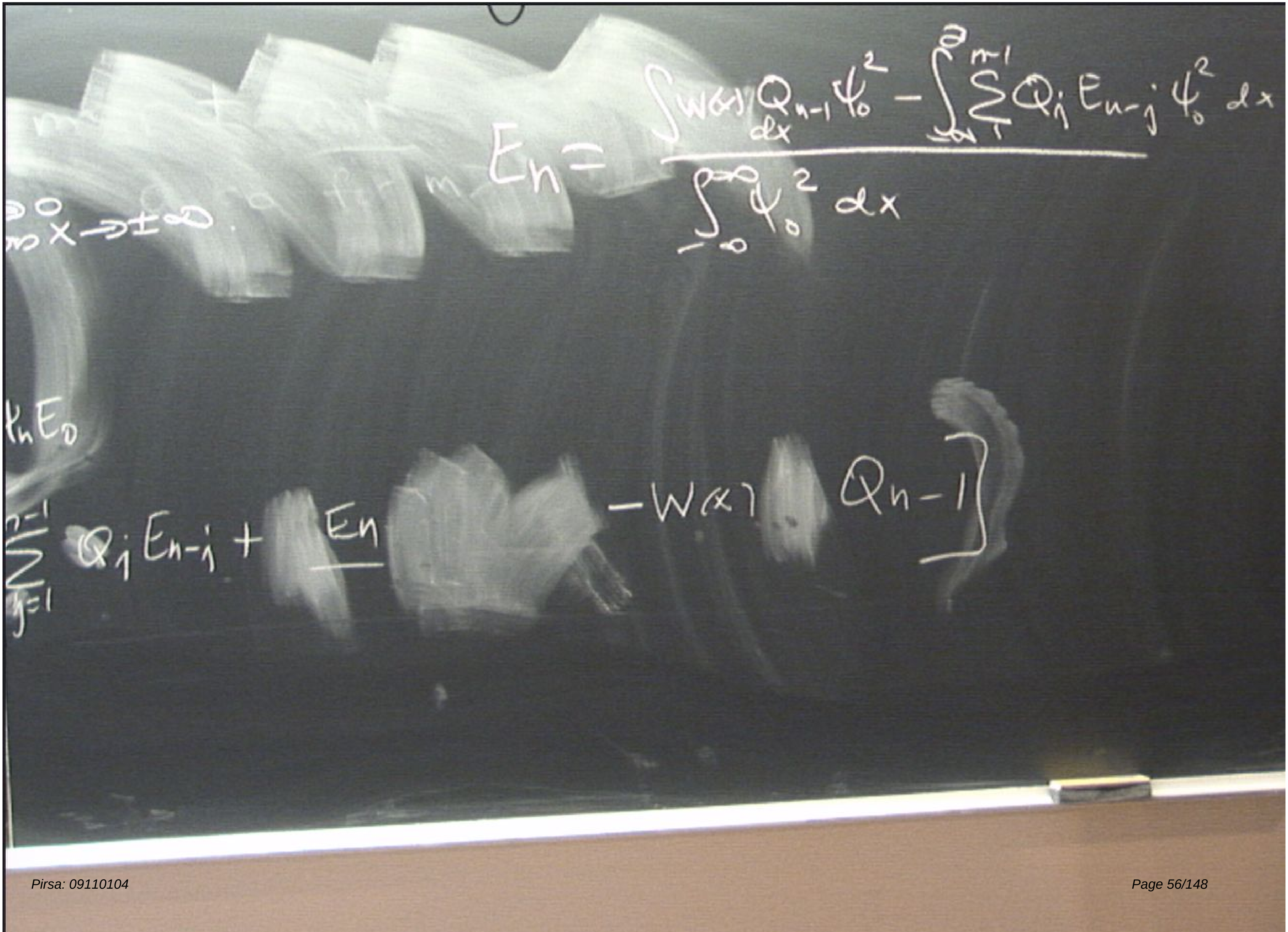
$$E_n = \frac{\int W(x)\, Q_{n-1}\psi_0^2 - \int_{-\infty}^{\infty}\sum_{j=1}^{n-1} Q_j E_{n-j}}{\int_{-\infty}^{\infty}\psi_0^2\, dx}$$

$\to 0$

$x \to \pm\infty$

$\psi_n E_0$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \underline{E_n} \quad - W(x) \quad Q_{n-1} \Big]$$



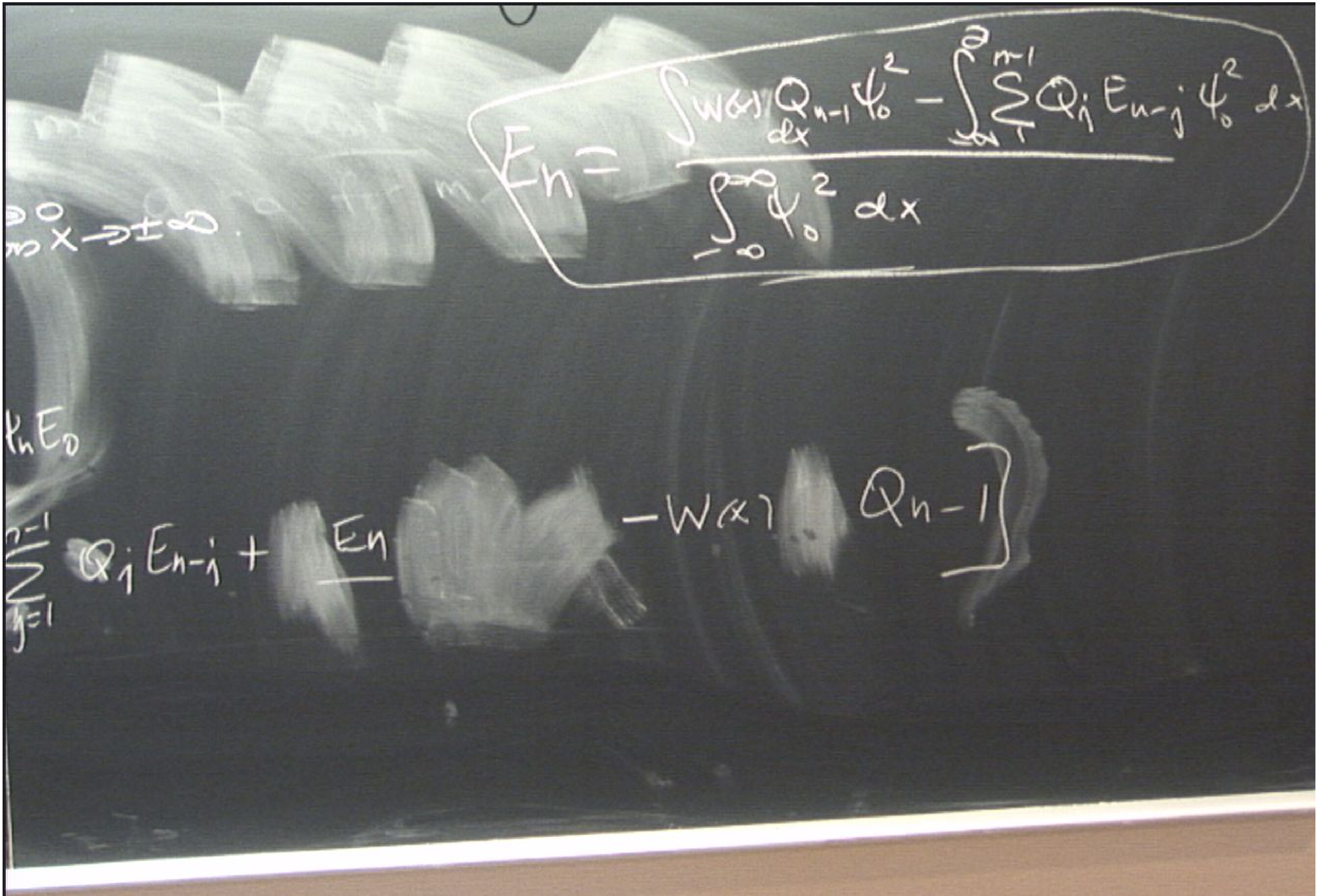
$= 0$

$x \to \pm\infty$

$$E_n = \frac{\int W(x)\, \underset{dx}{Q_{n-1}}\,\psi_0^2 - \int_{-\infty}^{\infty}\sum_{1}^{n-1} Q_j E_{n-j}\,\psi_0^2\, dx}{\int_{-\infty}^{\infty}\psi_0^2\, dx}$$

$\psi_n E_0$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \underline{E_n} \quad -W(x) \quad Q_{n-1}\Big]$$



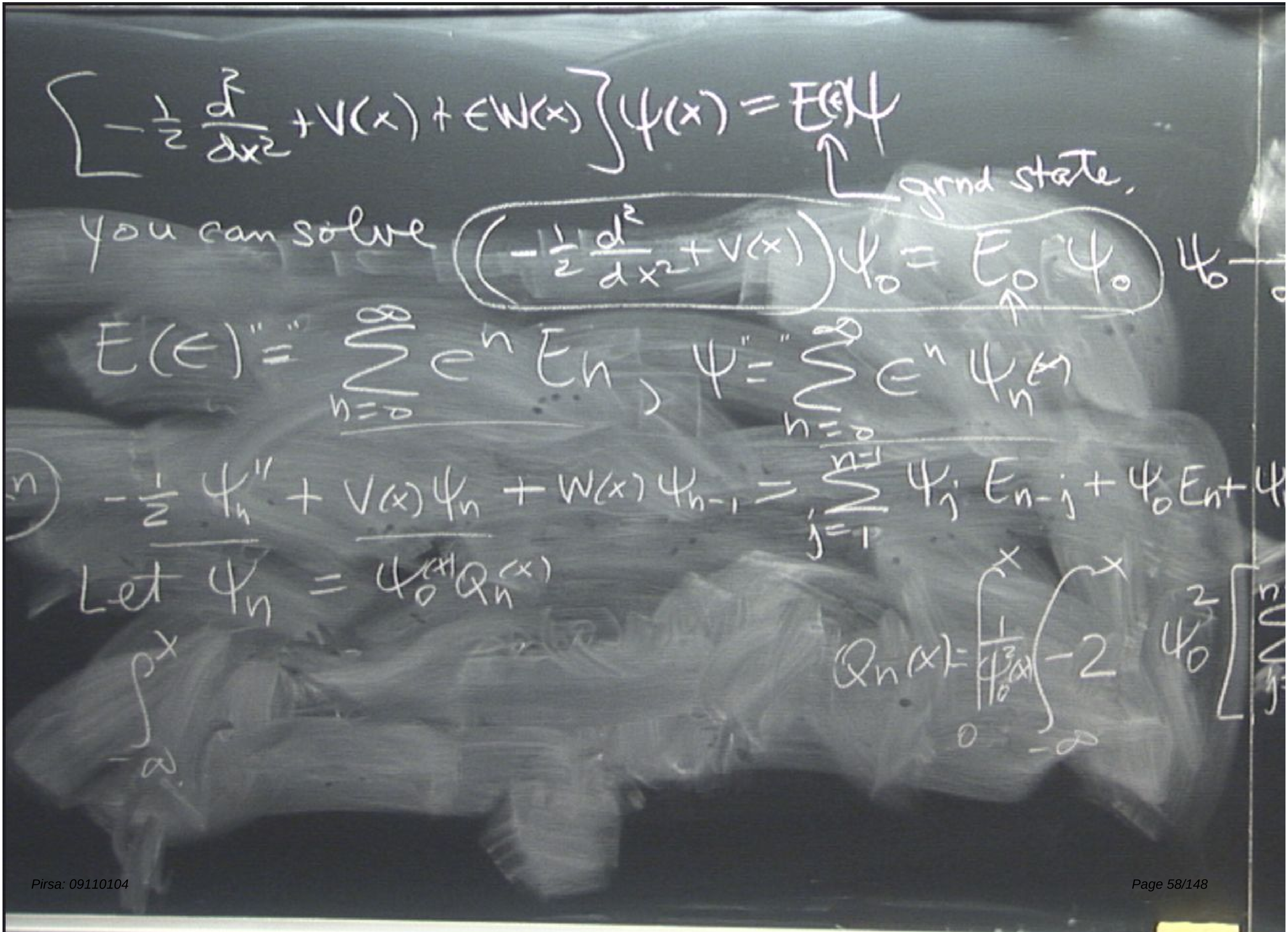
= 0

x → ±∞

$$E_n = \frac{\int W(x) \underset{dx}{Q_{n-1}} \psi_0^2 - \int_{-\infty}^{\infty} \sum_{j=1}^{n-1} Q_j E_{n-j} \psi_0^2 \, dx}{\int_{-\infty}^{\infty} \psi_0^2 \, dx}$$

$\psi_n E_0$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \underline{E_n} \quad - W(x) \quad Q_{n-1} \Big]$$



$$\left[-\frac{1}{2}\frac{d^2}{dx^2}+V(x)+\epsilon W(x)\right]\psi(x)=E(\epsilon)\psi$$

grnd state.

you can solve $\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = E_0\psi_0$ $\quad \psi_0 -$

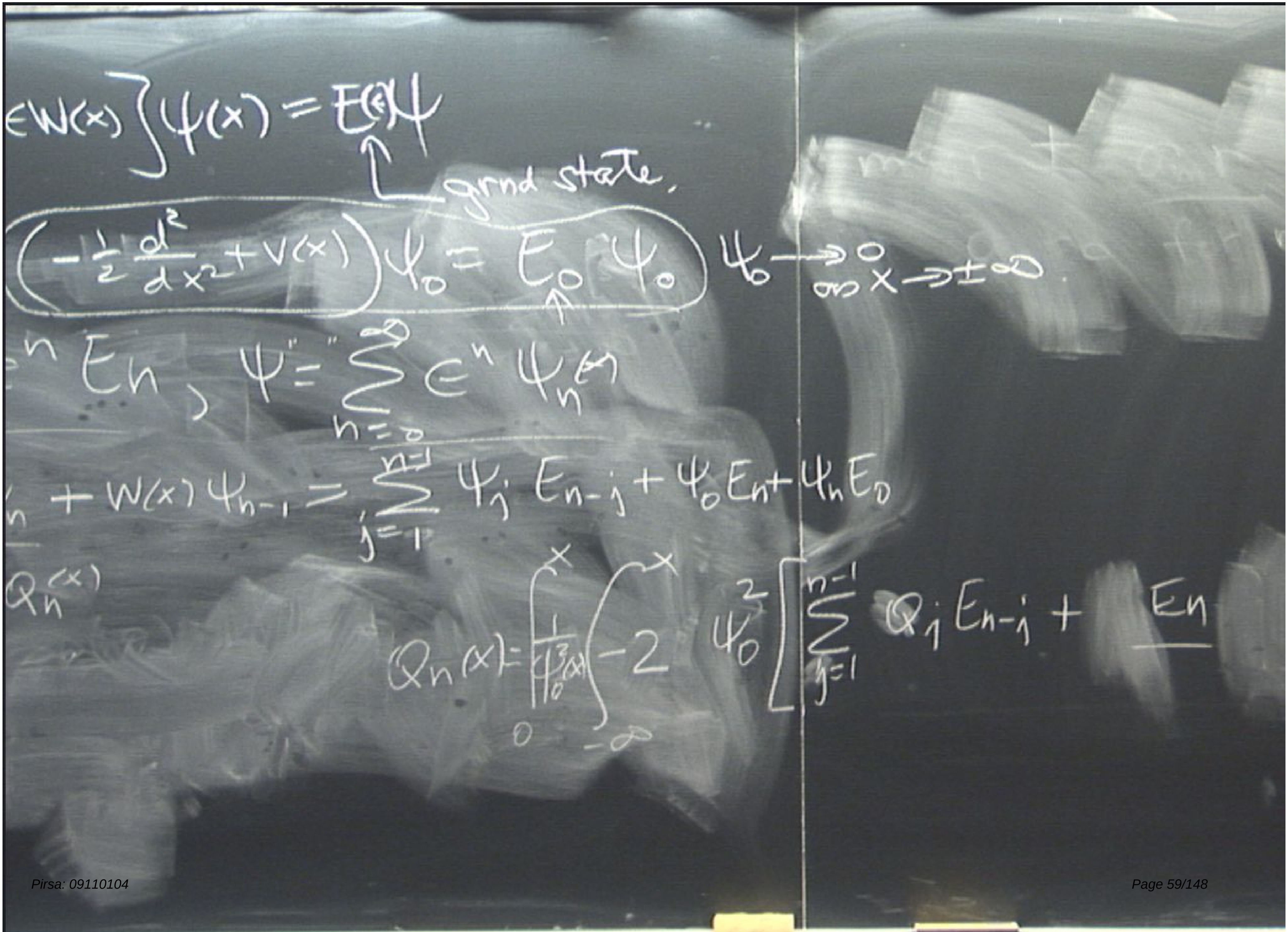
$$E(\epsilon) \text{"="} \sum_{n=0}^{\infty}\epsilon^n E_n, \quad \psi \text{"="} \sum_{n=0}^{\infty}\epsilon^n \psi_n(x)$$

$$(n) \quad -\frac{1}{2}\psi_n'' + V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1}\psi_j E_{n-j} + \psi_0 E_n + \psi$$

Let $\psi_n = \psi_0(x) Q_n(x)$

$$\int_{-\infty}^{x}$$

$$Q_n(x) = \int_0^x \frac{1}{\psi_0^2(x)}\left(-2\int_{-\infty}^{x}\psi_0^2\left[\sum_{j}^{n}\right.\right.$$



$\epsilon W(x) \Big] \psi(x) = E(\epsilon)\psi$

$\uparrow$ grnd state.

$$\boxed{\left(-\frac{1}{2}\frac{d^2}{dx^2}+V(x)\right)\psi_0 = \underset{\uparrow}{E_0}\,\psi_0} \quad \psi_0 \to 0 \text{ as } x \to \pm\infty.$$

$$\epsilon^n E_n, \quad \psi \text{ "=" } \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$$_n + W(x)\psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$Q_n(x)$

$$Q_n(x) = \int_0^x \frac{1}{\psi_0^3(x)} \int_{-\infty}^x -2 \quad \psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \quad \underline{E_n}\right.$$

$\to 0$

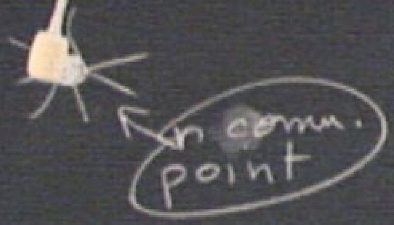
as $x \to \pm\infty$

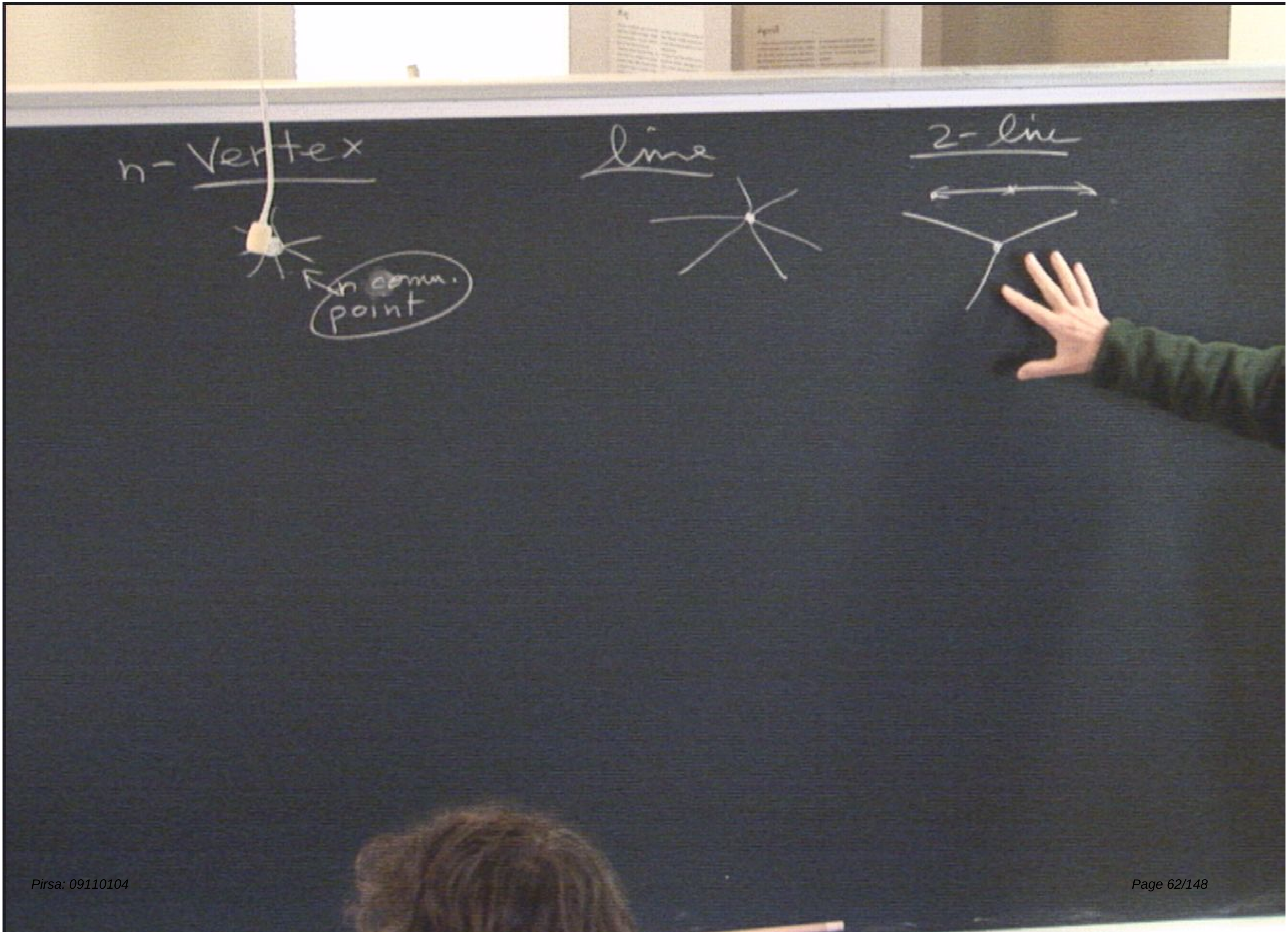
$$\boxed{E_n = \frac{\int W(x)\, \underset{dx}{Q_{n-1}}\, \psi_0^2 - \int_{-\infty}^{\infty} \sum_{1}^{n-1} Q_j E_{n-j}\, \psi_0^2 \, dx}{\int_{-\infty}^{\infty} \psi_0^2 \, dx}}$$

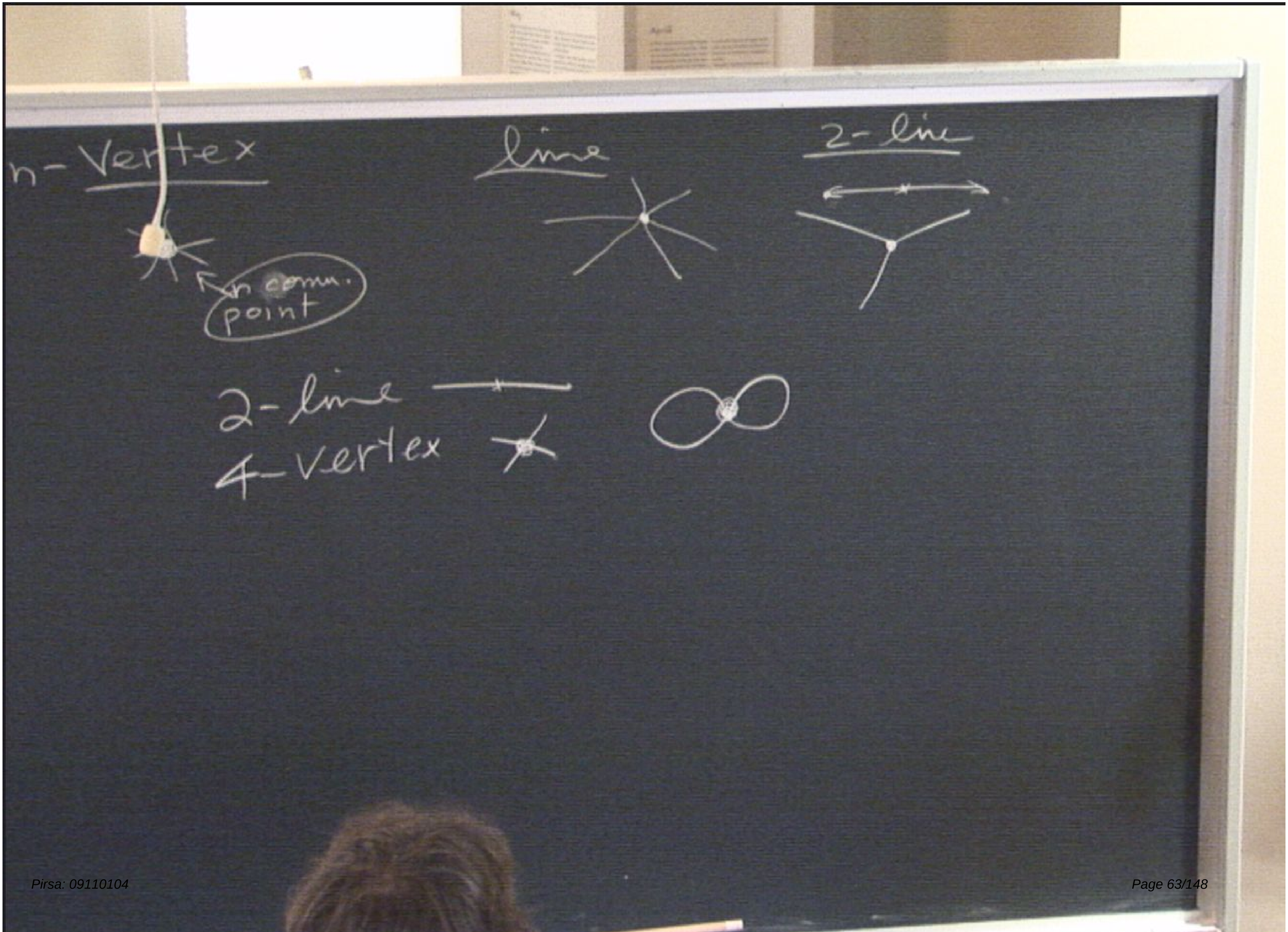
$\psi_n E_0$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \underline{E_n} \qquad - W(x) \qquad Q_{n-1} \Big]$$

n-Vertex




n-Vertex
line
2-line
n comm. point



n-Vertex

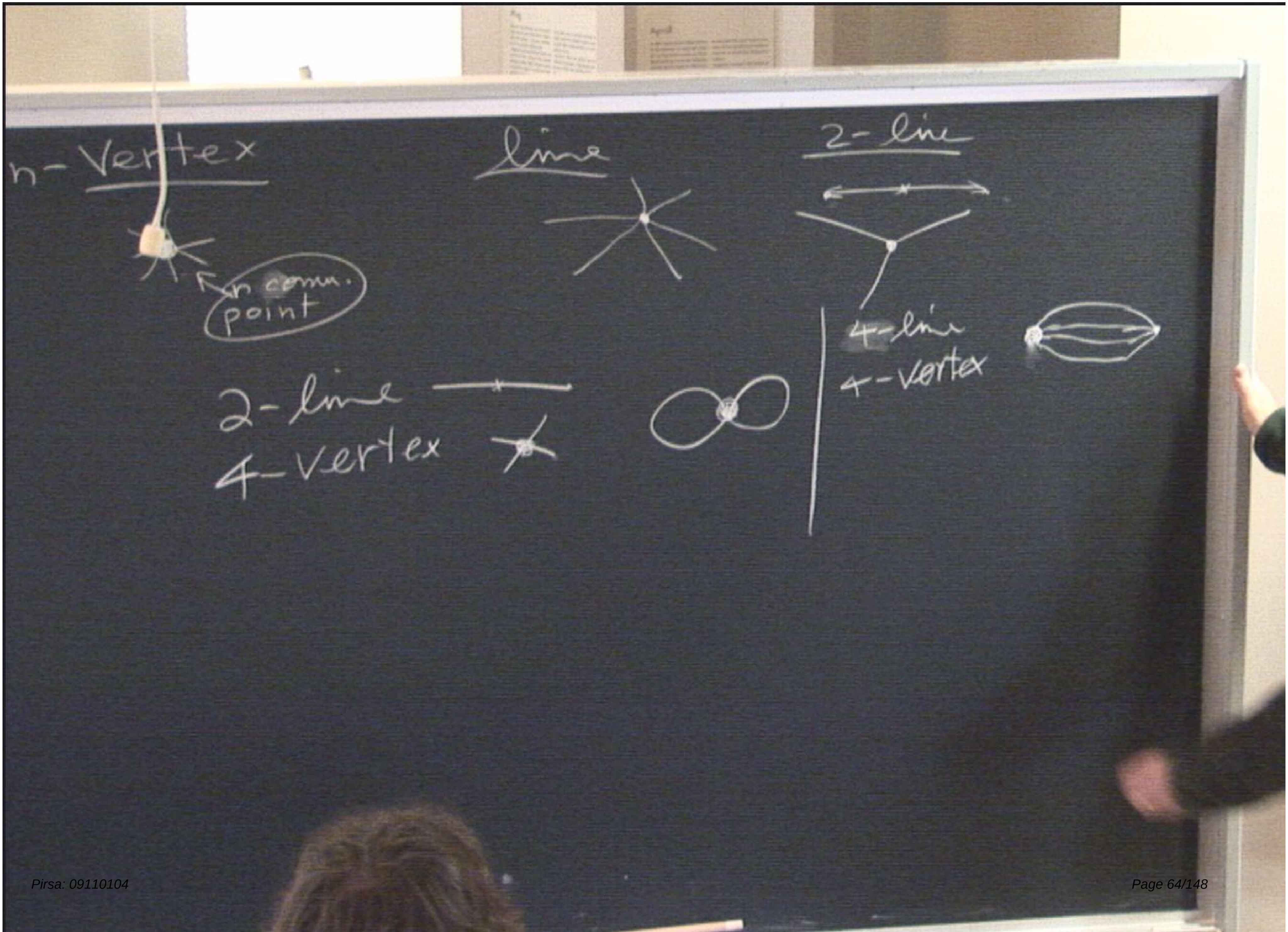
n comm. point

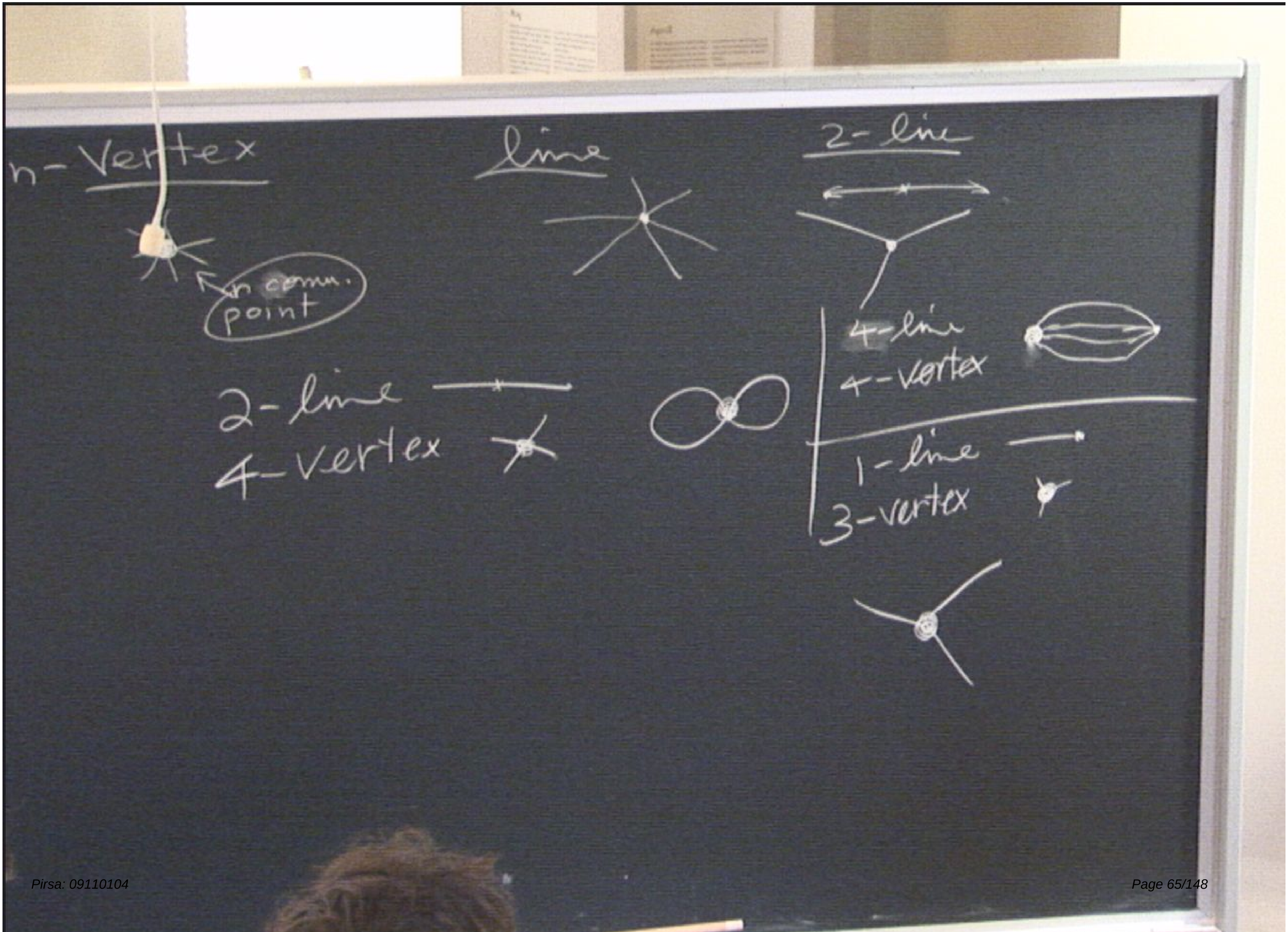
line

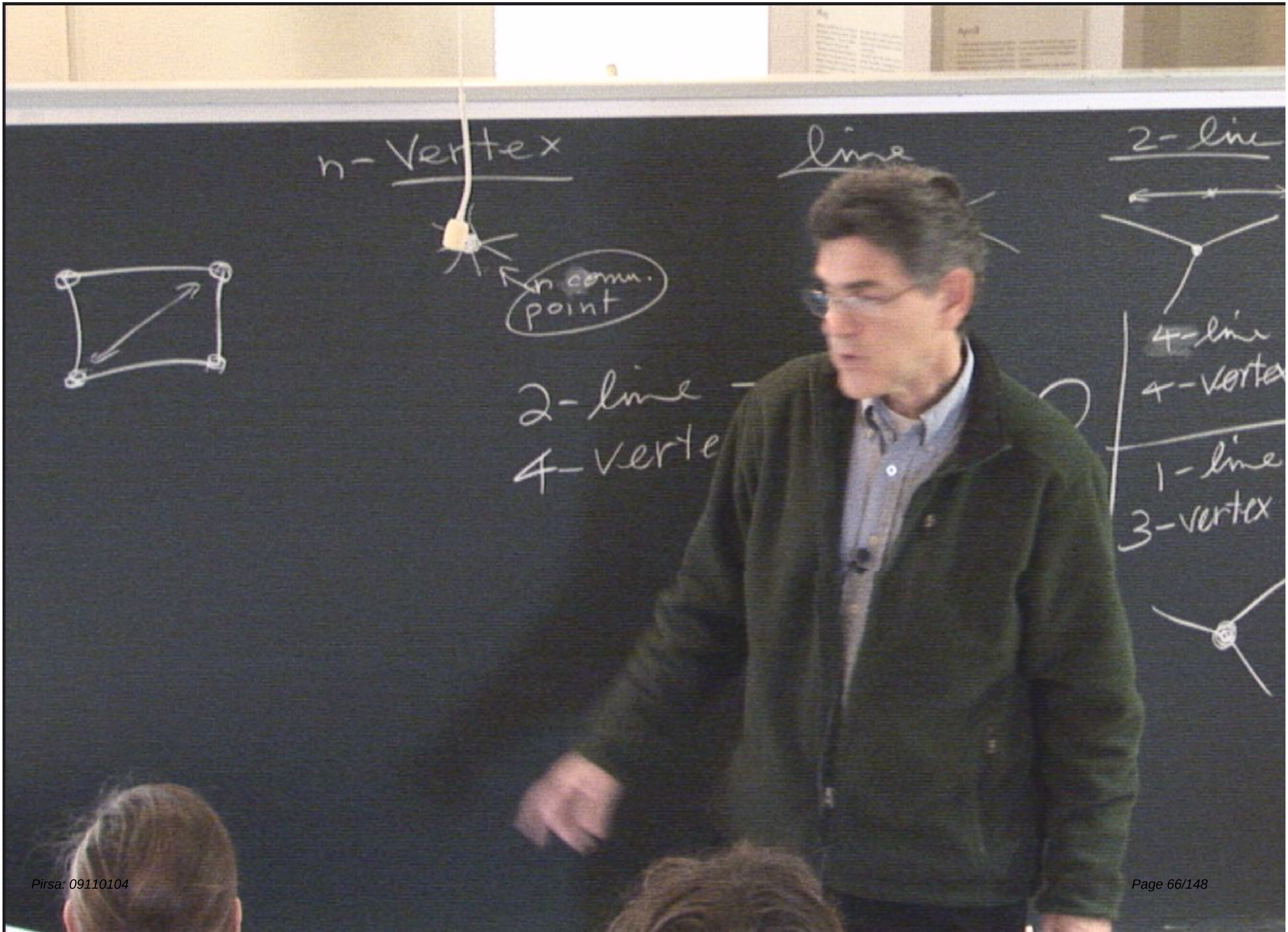
2-line

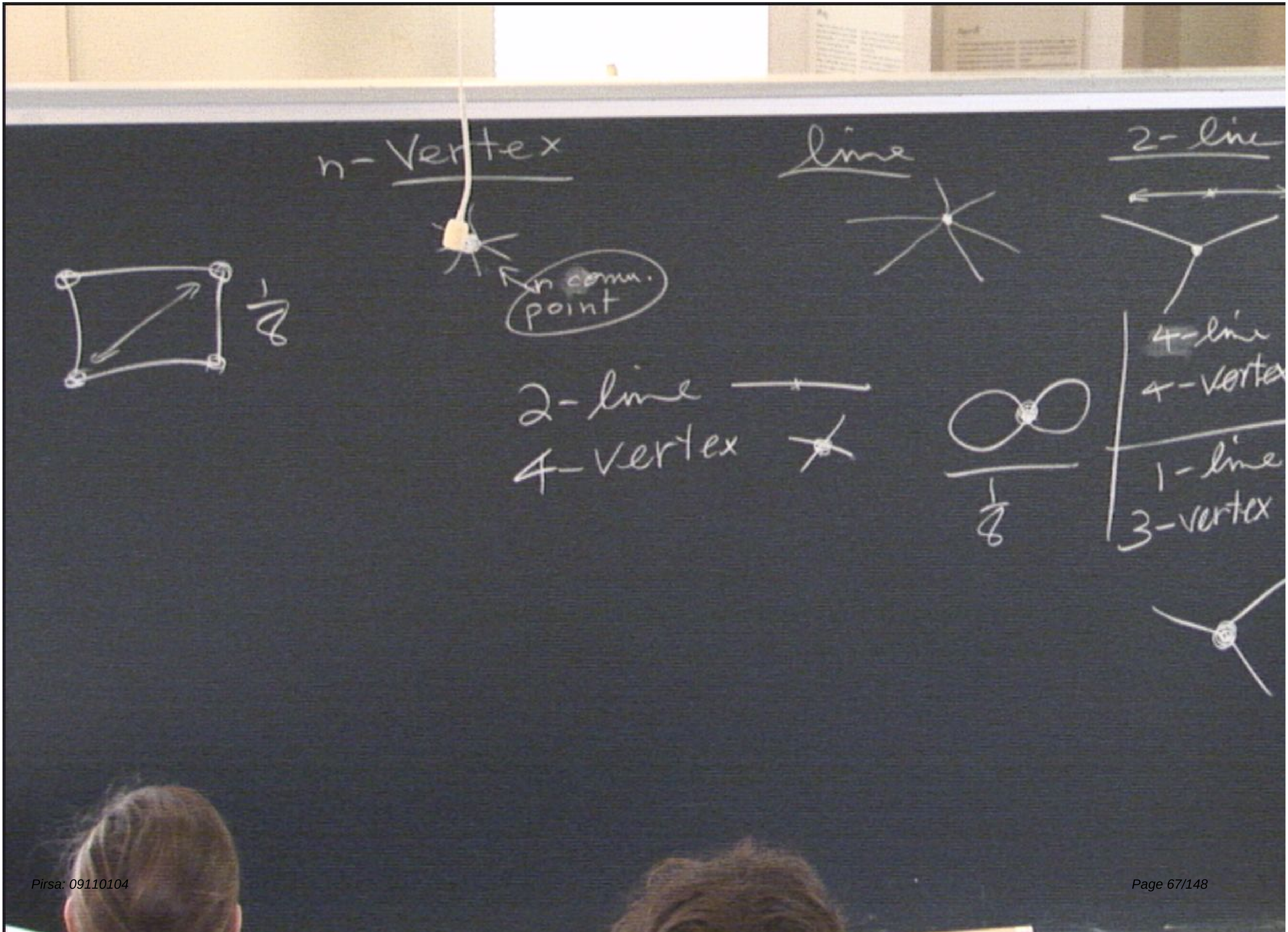
2-line

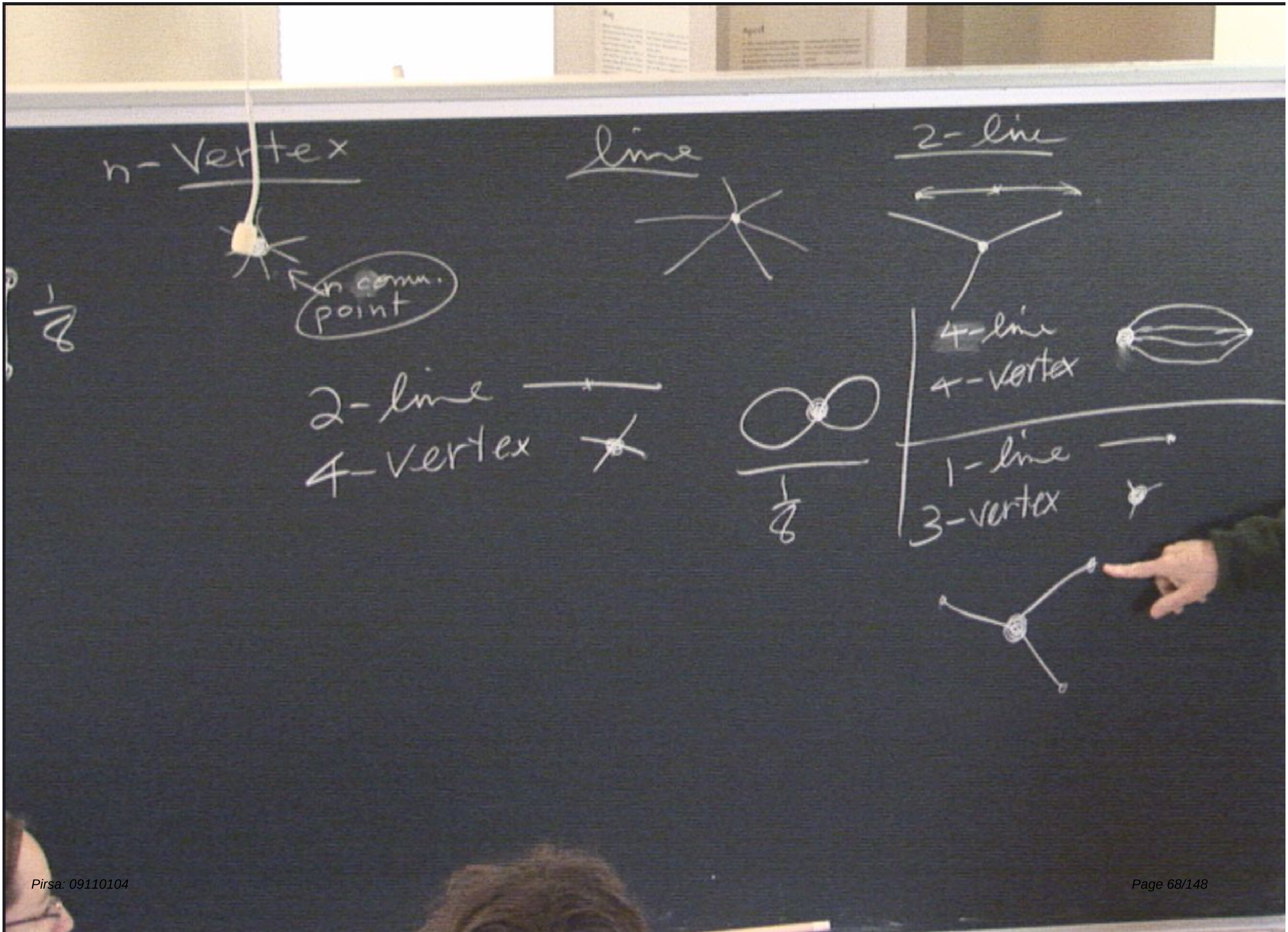
4-vertex

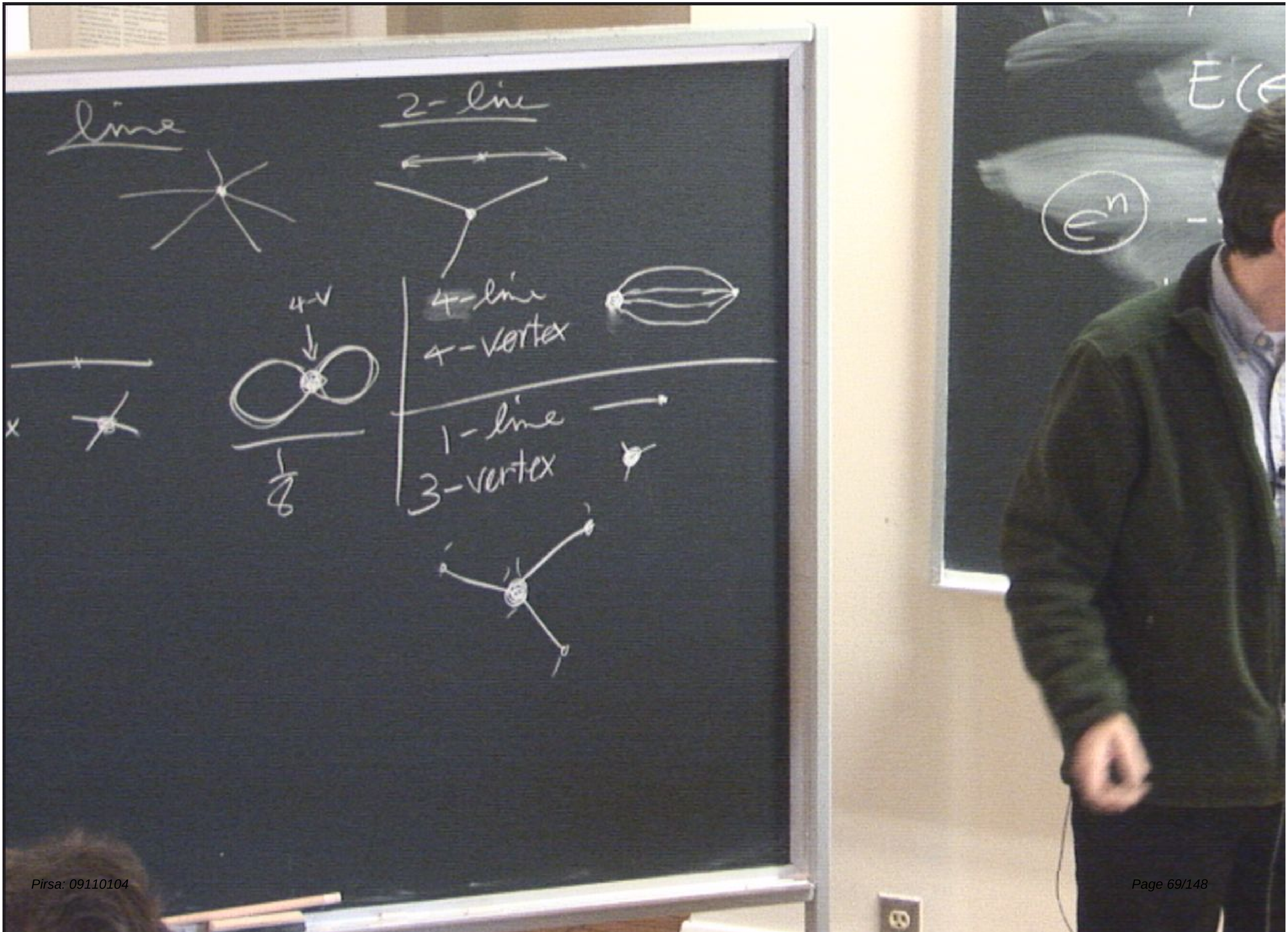

n-Vertex
line
2-line
n comm. point
2-line
4-vertex
4-line
4-vertex

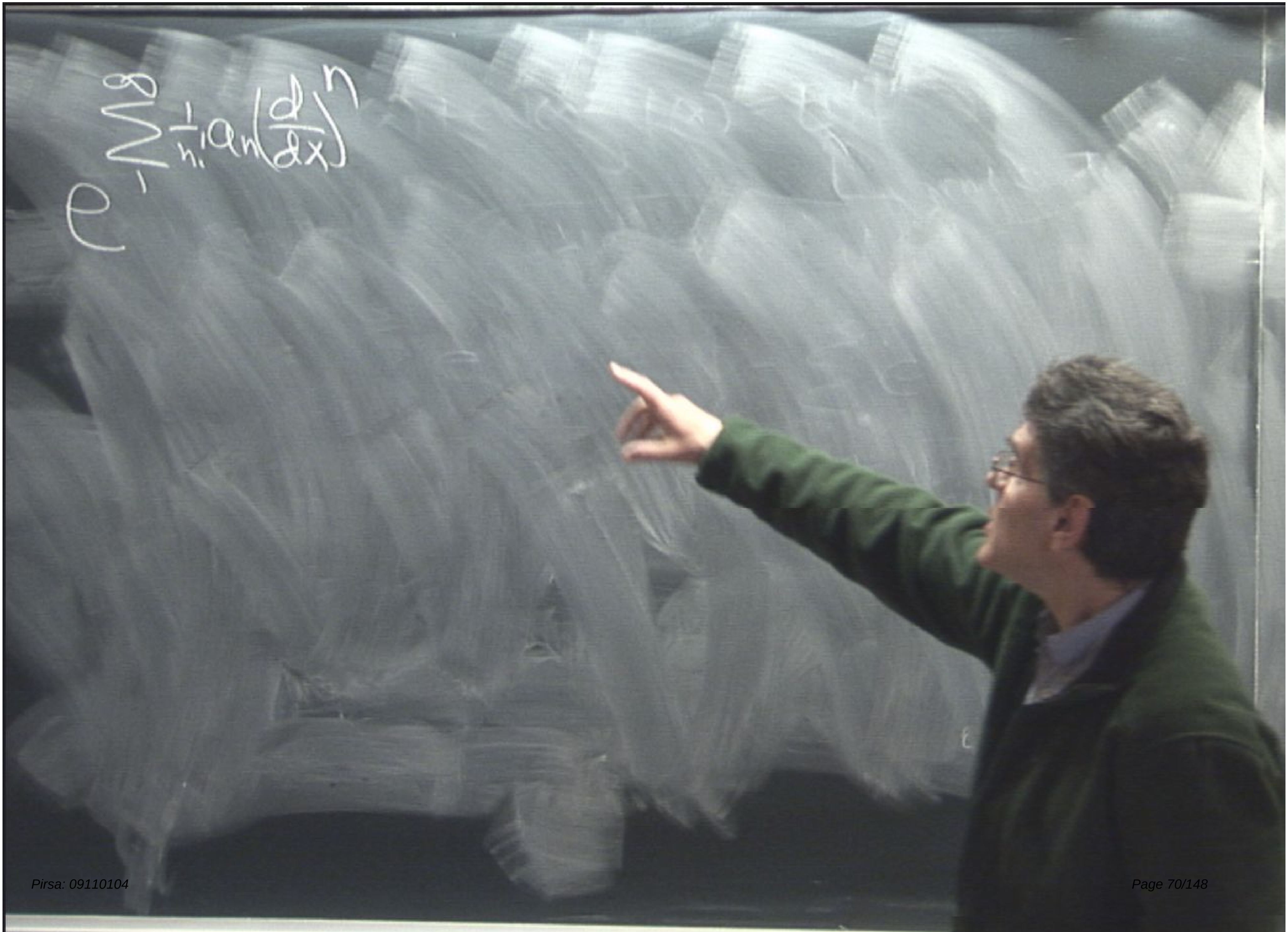

n-Vertex
n comm. point
line
2-line
2-line
4-vertex
4-line
4-vertex
1-line
3-vertex


n-Vertex
n comm. point
2-line
4-verte
line
2-line
4-line
4-vertex
1-line
3-vertex


n-Vertex
n comm. point
1/8
line
2-line
2-line
4-vertex
1/8
4-line
4-vertex
1-line
3-vertex


n-Vertex
line
2-line
n comm. point
$\frac{1}{8}$
2-line
4-vertex
$\frac{1}{8}$
4-line
4-vertex
1-line
3-vertex


line
2-line
4-v
1/8
4-line
4-vertex
1-line
3-vertex
E(ε
ε^n



$$e^{\sum_{1}^{\infty}\frac{1}{n!}a_n\left(\frac{d}{dx}\right)^n}$$

$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \qquad e^{\sum_1^\infty b_n \frac{x^n}{n!}}$$

$$e^{\sum_{1}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \qquad e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}}\Bigg|_{x=0}$$

$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 + \text{all Feynman Graphs}$$

$$e^{\sum_{1}^{\infty}\frac{1}{n!}a_n\left(\frac{d}{dx}\right)^n}\; e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}}\Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

$$\sum_{1}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n \; e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 +$$

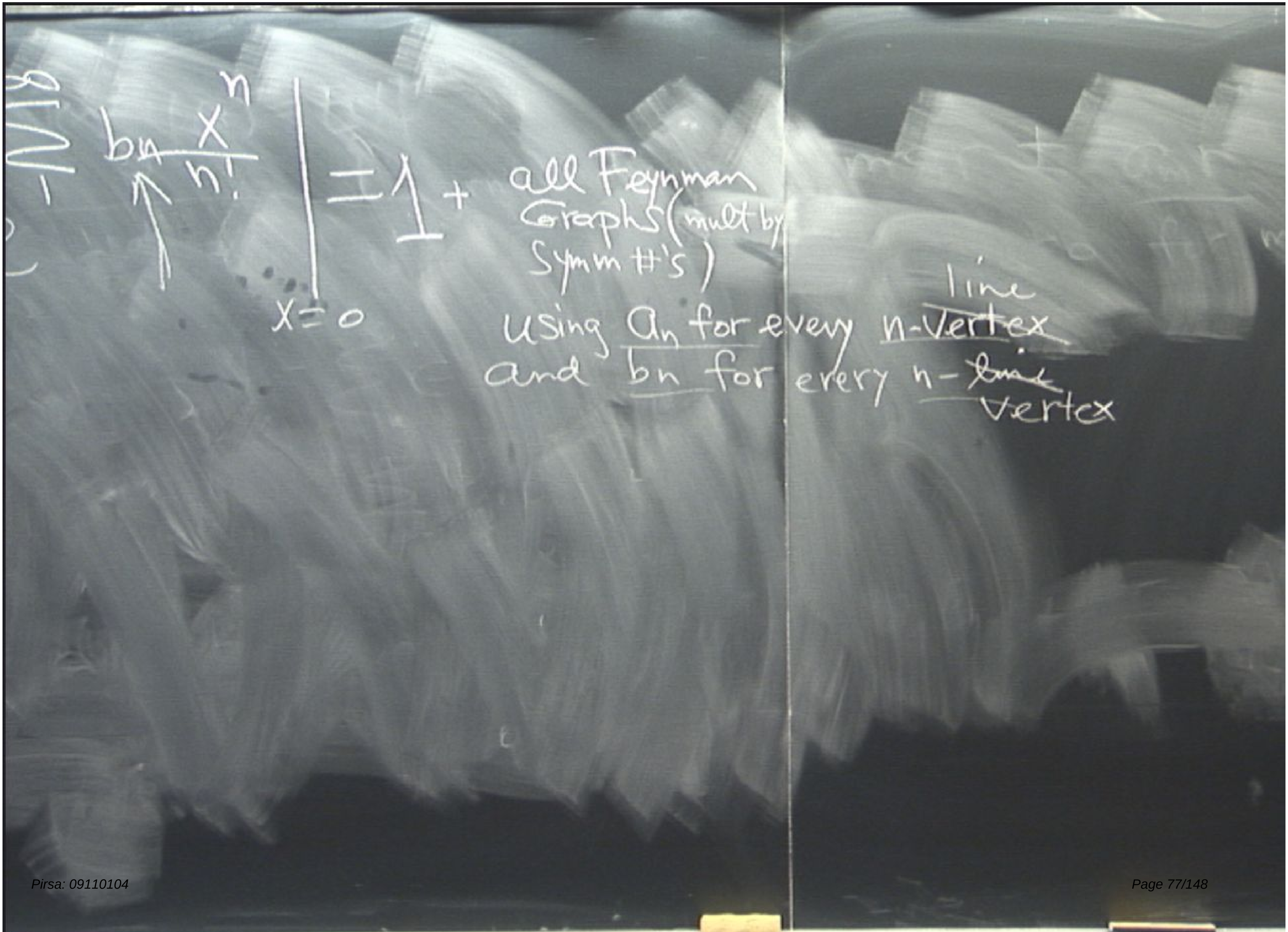
all Feynman
Graphs (mult by
Symm #'s)

using a_n for every n-
and b_n for every

$$\sum_{1}^{\infty} \underset{\uparrow}{b_n} \frac{x^n}{n!} \Bigg|_{x=0} = 1 +$$ all Feynman Graphs (mult by Symm #'s)

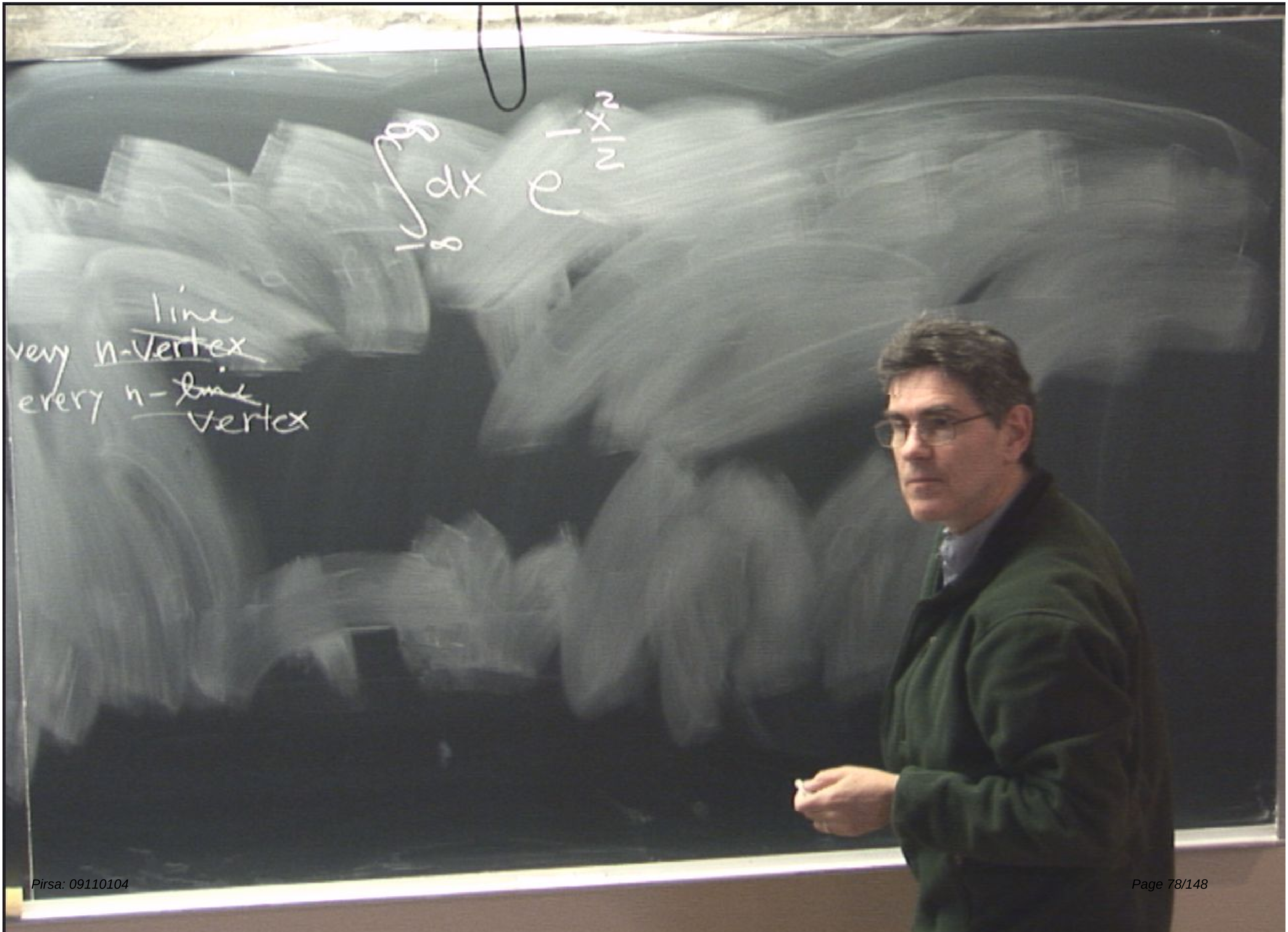
using $\underline{a_n}$ for every $\underline{\text{n-}}$~~vertex~~ line

and $\underline{b_n}$ for every $\underline{\text{n-}}$~~line~~ vertex



$$\sum_{1} b_n \frac{x^n}{n!}\Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

using a_n for every n-~~vertex~~ line
and b_n for every n-~~line~~ vertex


$\int_{-\infty}^{\infty} dx \, e^{-\frac{x^2}{2}}$
line
very n-~~Vertex~~
erery n-~~line~~ vertex

$$\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4}$$

very n-~~vertex~~ line

every n-~~line~~ vertex

$$\frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

line
very n-~~Vertex~~
every n-~~line~~
vertex

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

very n-~~vertex~~ line

every n-~~line~~ vertex

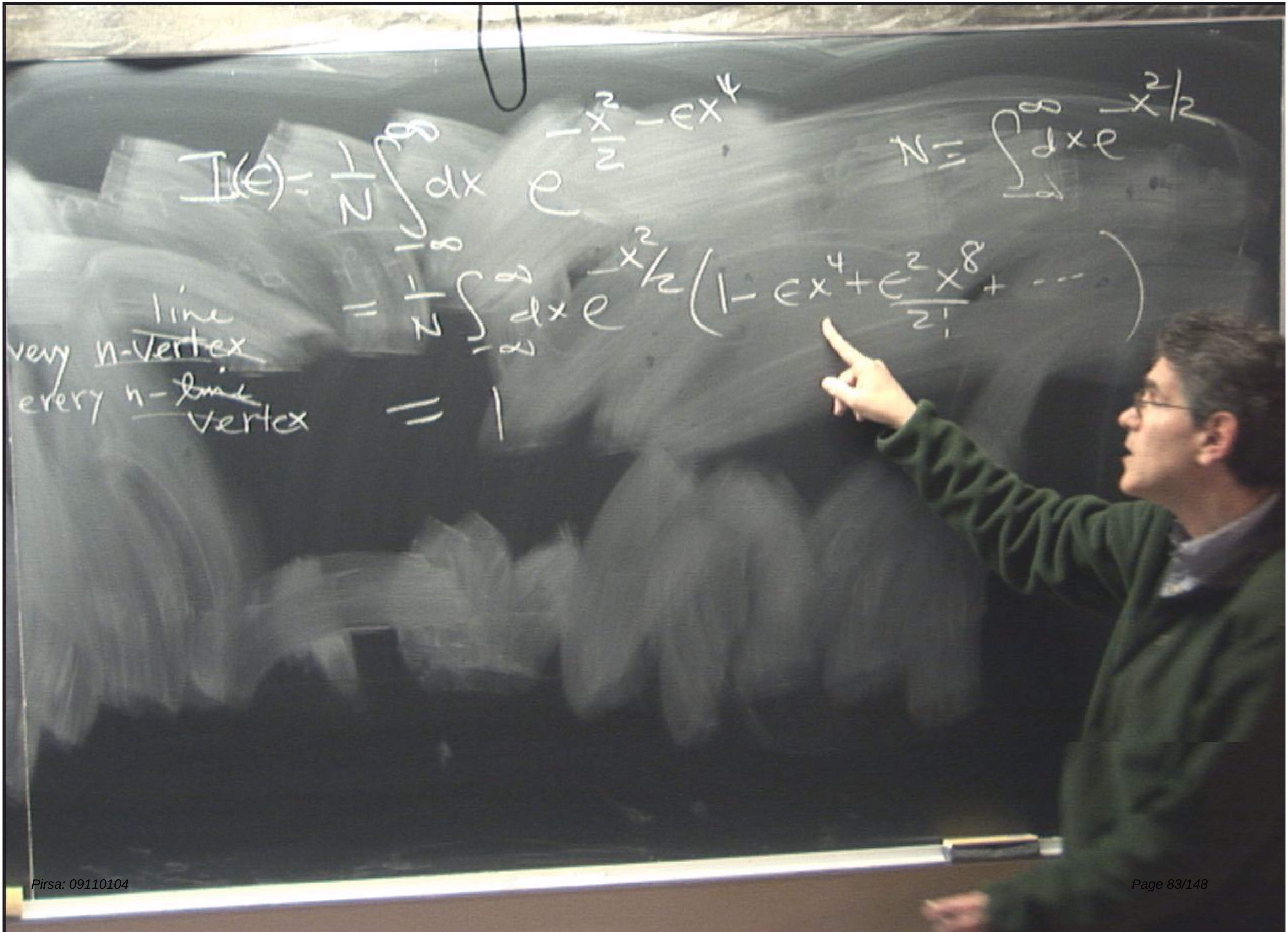
$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \cdots\right)$$

line
very n-Vertex

every n-~~line~~
vertex

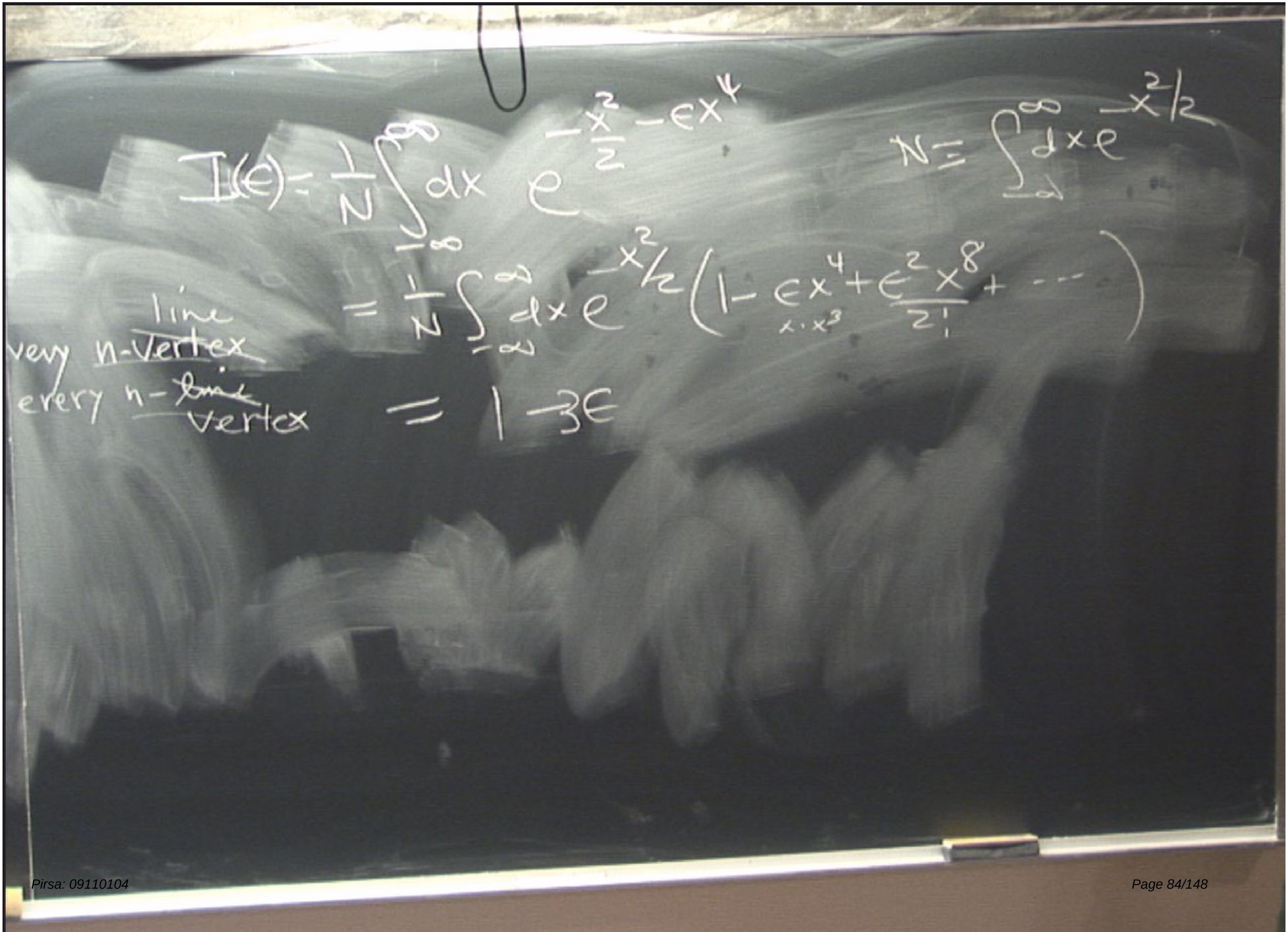


$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \cdots\right)$$

line
very n-vertex
every n-~~line~~ vertex

$$= 1$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} - \epsilon x^4} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x \cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 x^8}{2!} + \cdots\right)$$

$$= 1 - 3\epsilon$$

line

very n-~~Vertex~~

every n-~~line~~ vertex

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x \cdot x^3}{\epsilon x^4} + \underset{x x^7}{\frac{\epsilon^2 x^8}{2!}} + \cdots\right)$$

line

very n-vertex

every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

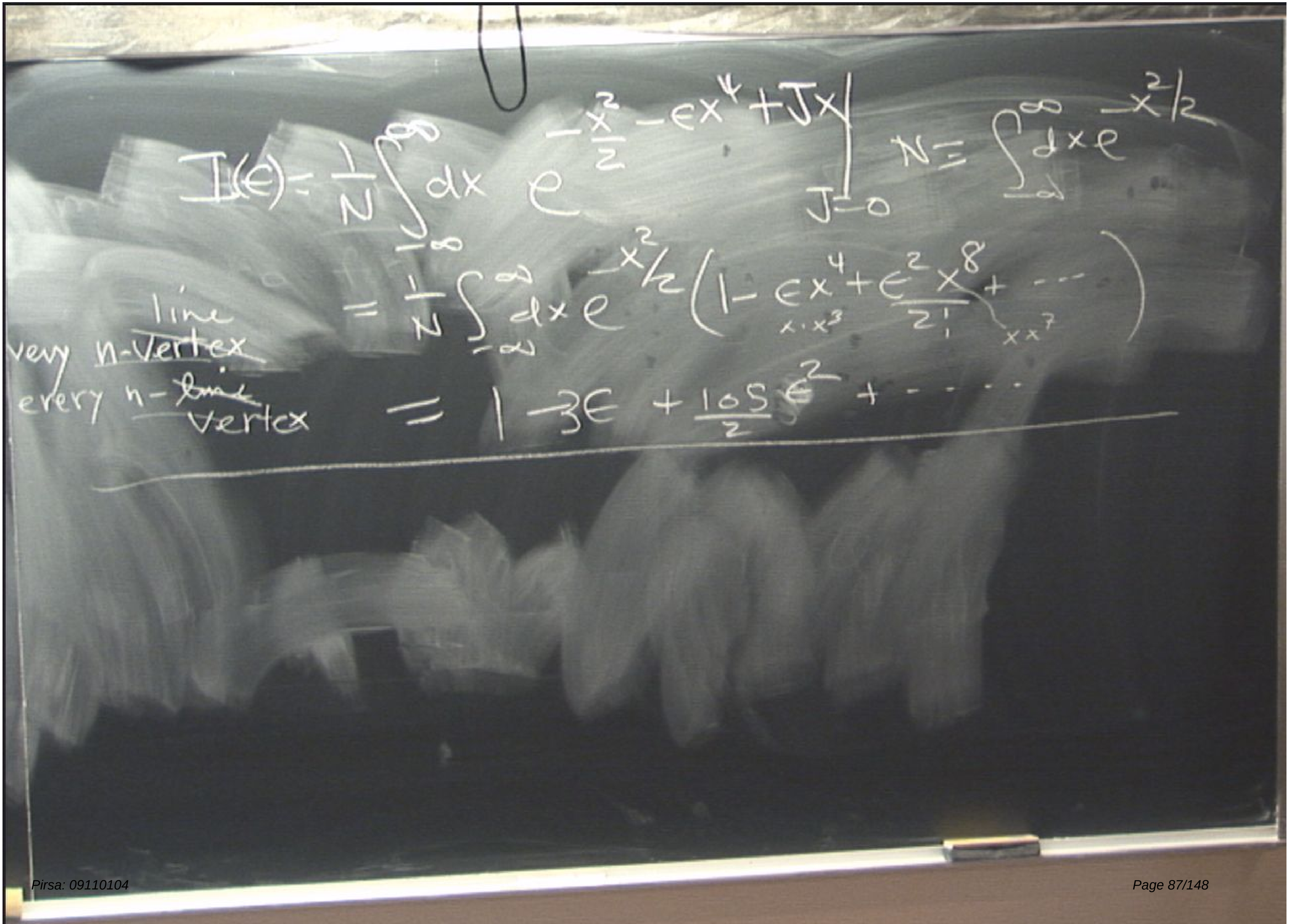
$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x \cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\,x^7}{x^8}}{2!} + \cdots\right)$$

very n-~~Vertex~~ line

every n-~~line~~ Vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$



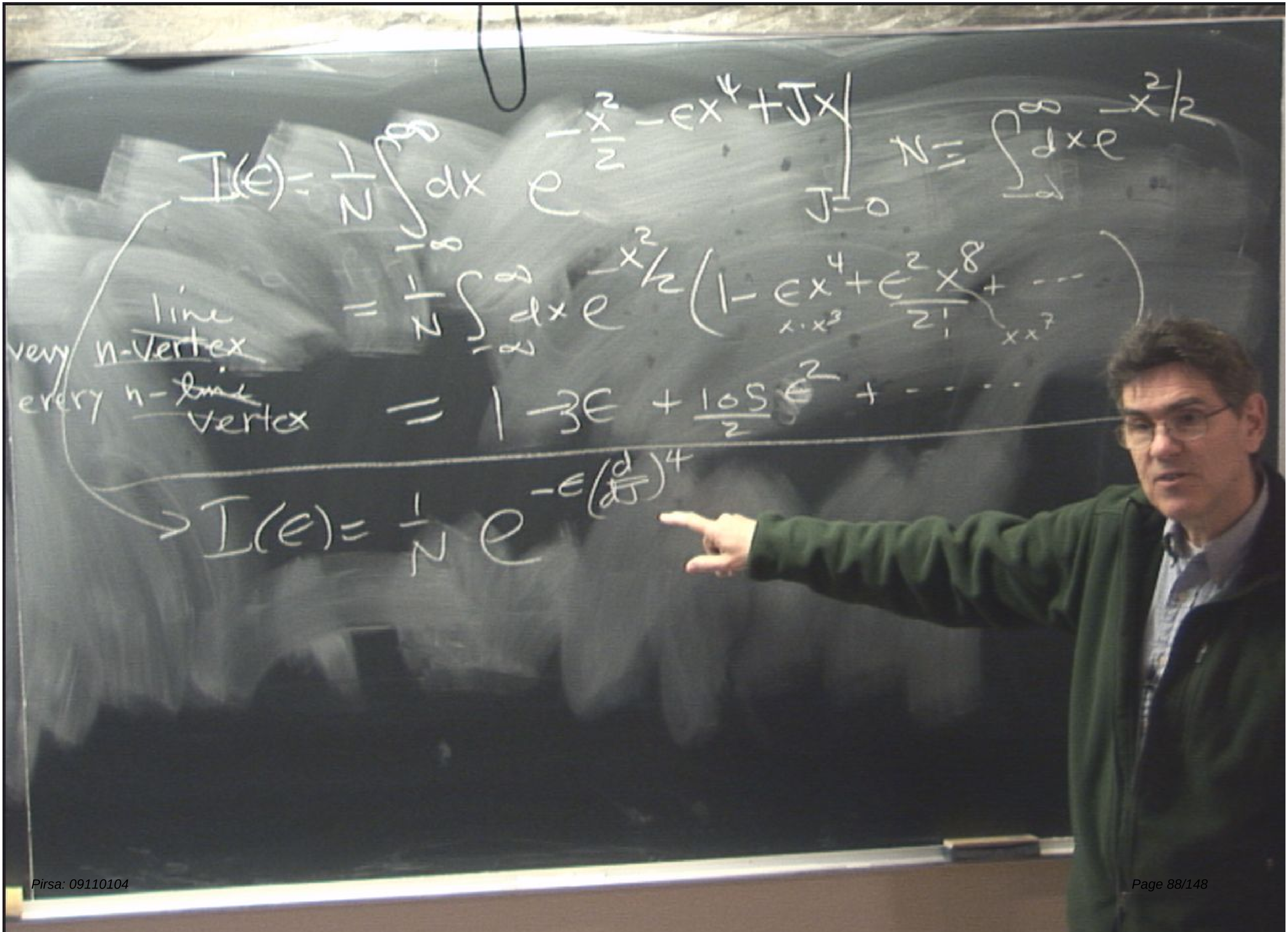
$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x\cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\right)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

very n-~~vertex~~ line

every n-~~line~~ vertex



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

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$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

line

very n-vertex

every n-~~line~~ vertex

$$I(\epsilon) = \frac{1}{N} e^{-\epsilon\left(\frac{d}{dJ}\right)^4}$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx \, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx \, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx \, e^{-x^2/2}\left(1 - \epsilon \underset{x \cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\right)$$

very n-line vertex

every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

$$I(\epsilon) = \frac{1}{N} e^{-\epsilon\left(\frac{d}{dJ}\right)^4}$$

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$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x \cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\right)$$

line

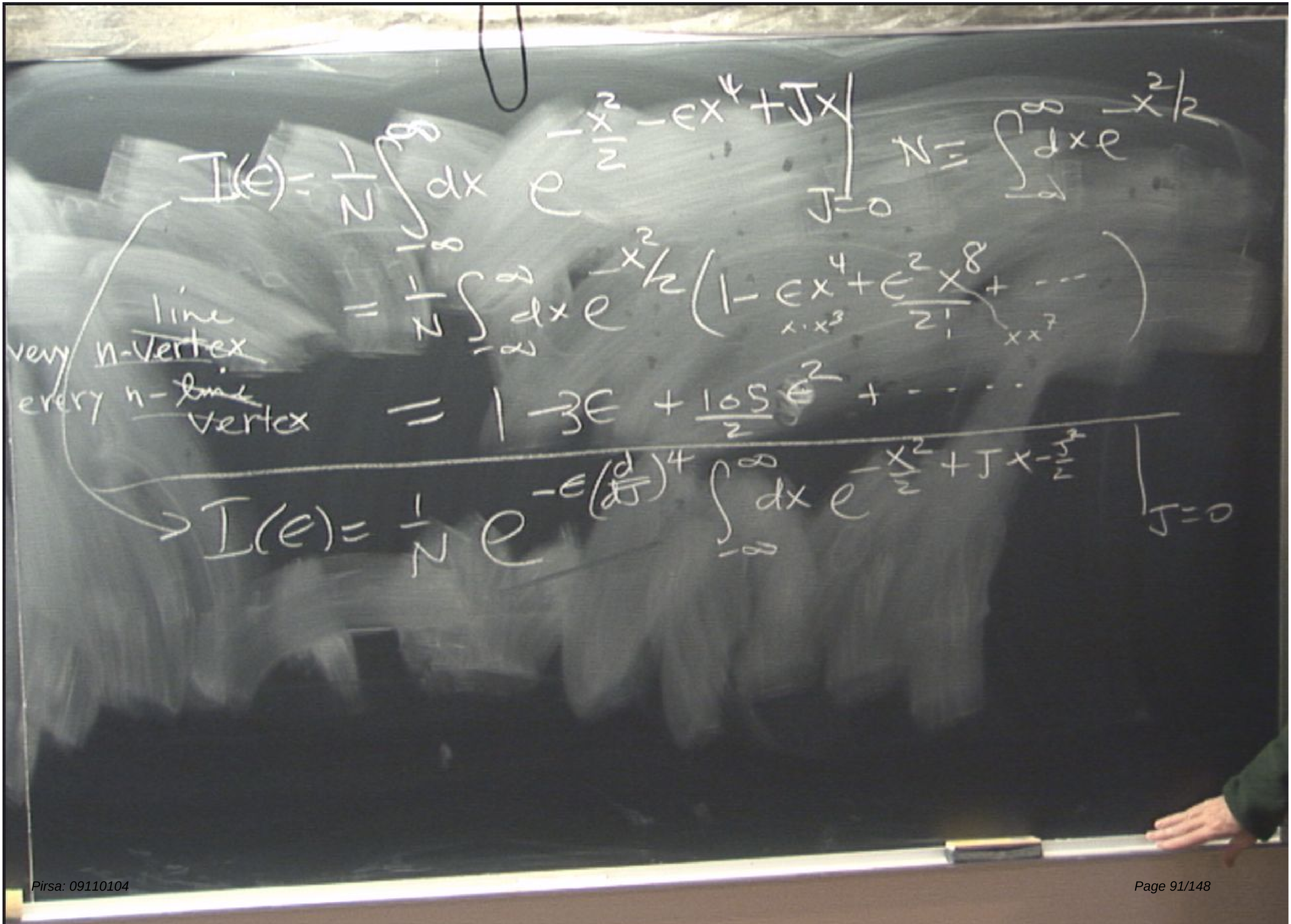
very n-Vertex

every n-~~line~~

vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

$$I(\epsilon) = \frac{1}{N} e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} + Jx}$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

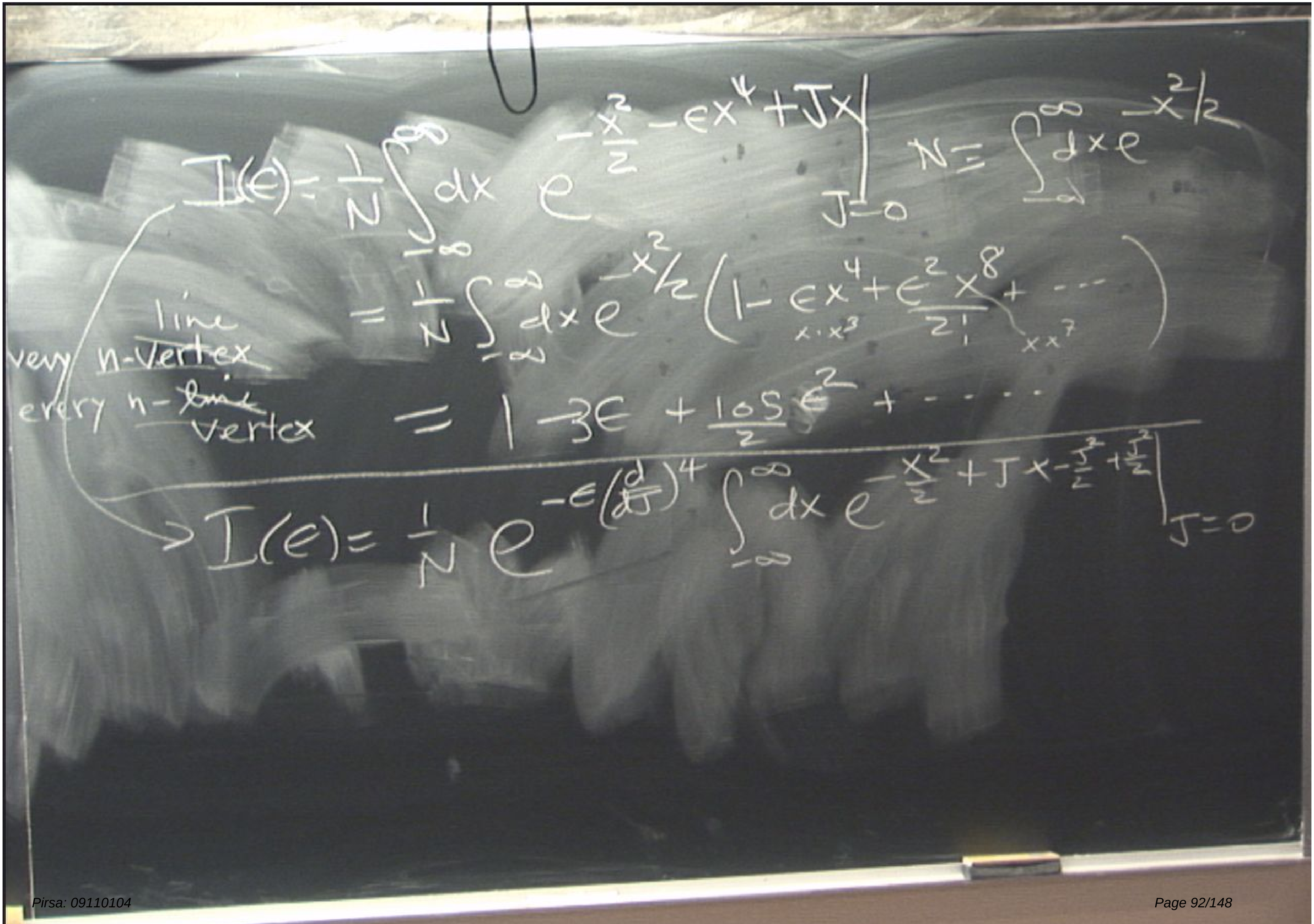
$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \overset{}{\underset{x\,x^7}{x^8}}}{2!} + \cdots\right)$$

line
very n-Vertex

every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

$$I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}}\Big|_{J=0}$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\Big)$$

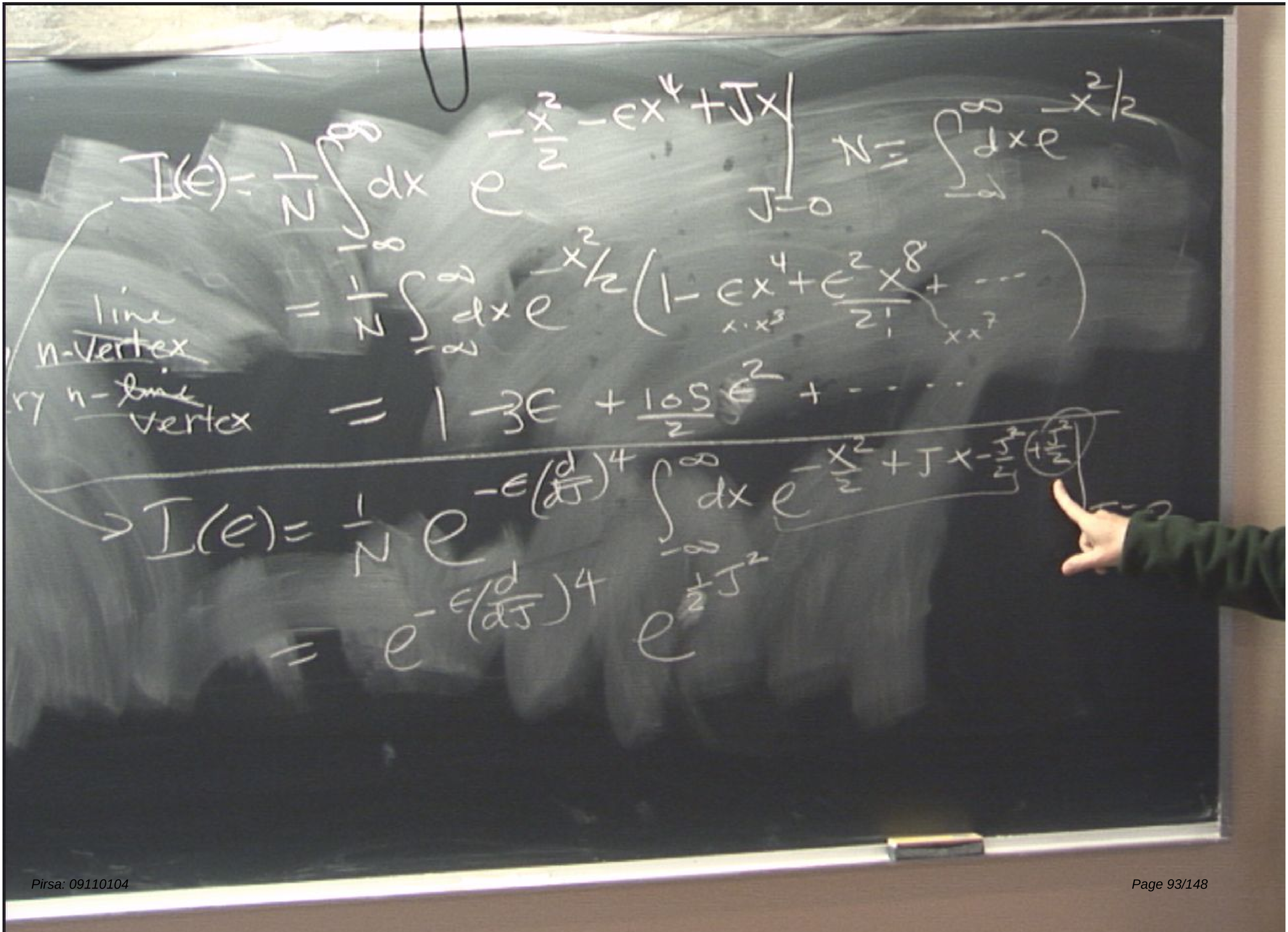
$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

line

very n-vertex

every n-~~line~~ vertex

$$I(\epsilon) = \frac{1}{N} e^{-\epsilon\left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}\Big|_{J=0}$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

line

n-vertex

ry n-~~line~~ vertex

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-x^2/2}\Big(1 - \underset{x \cdot x^3}{\epsilon x^4} + \underset{x x^7}{\frac{\epsilon^2 x^8}{2!}} + \cdots\Big)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

$$I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}\Big|_{J=0}$$

$$= e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\; e^{\frac{1}{2}J^2}$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x\cdot x^3}{\epsilon x^4} + \underset{x\, x^7}{\frac{\epsilon^2 x^8}{2!}} + \cdots\right)$$

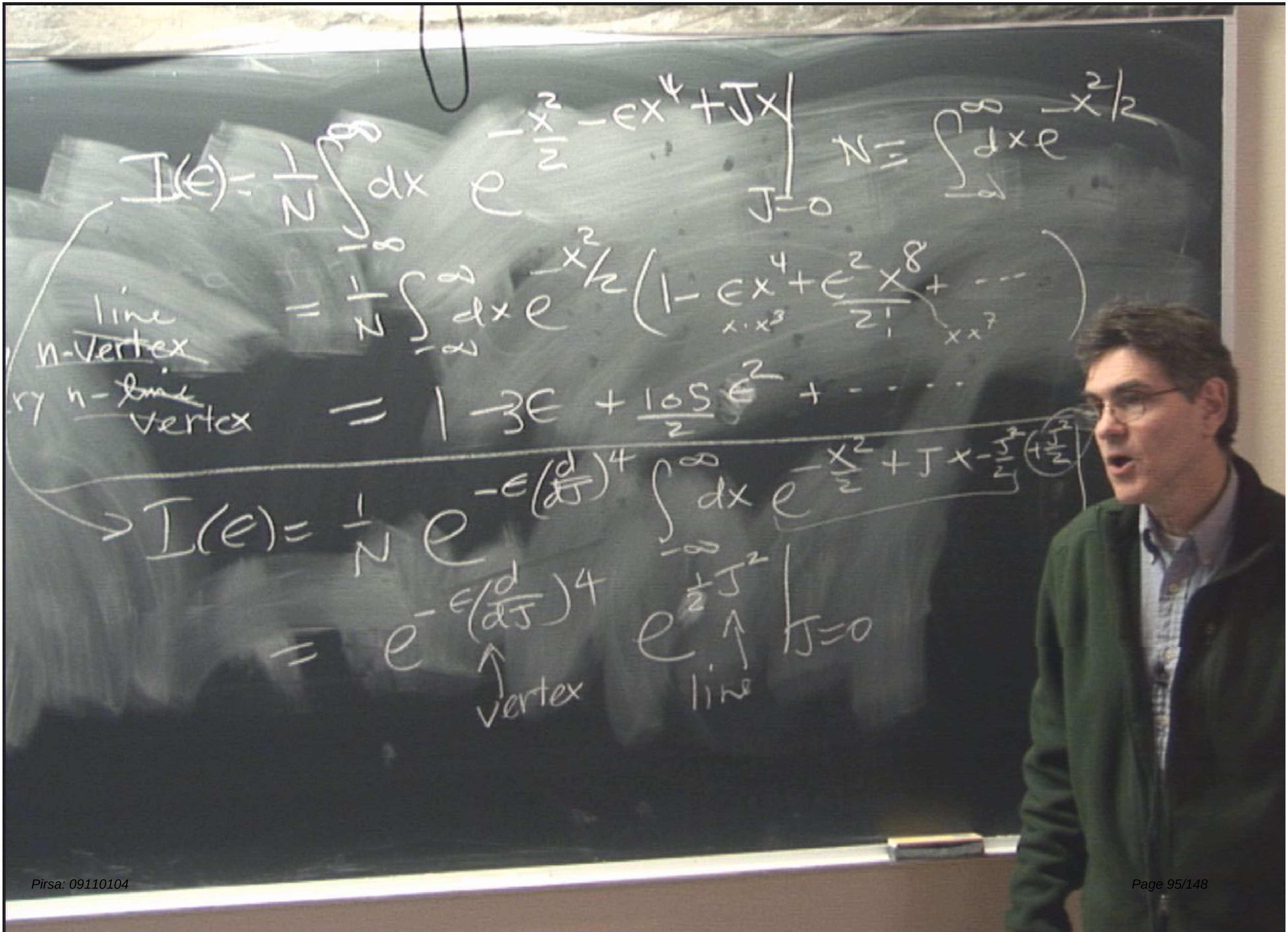
line
n-Vertex

ry n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

$$I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\; e^{\underbrace{-\frac{x^2}{2} + Jx - \frac{J^2}{2}}\boxed{+\frac{J^2}{2}}}\Big|_{J=0}$$

$$= e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\, e^{\frac{1}{2}J^2}\Big|_{J=0}$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\,x^7}{x^8}}{2!} + \cdots\Big)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

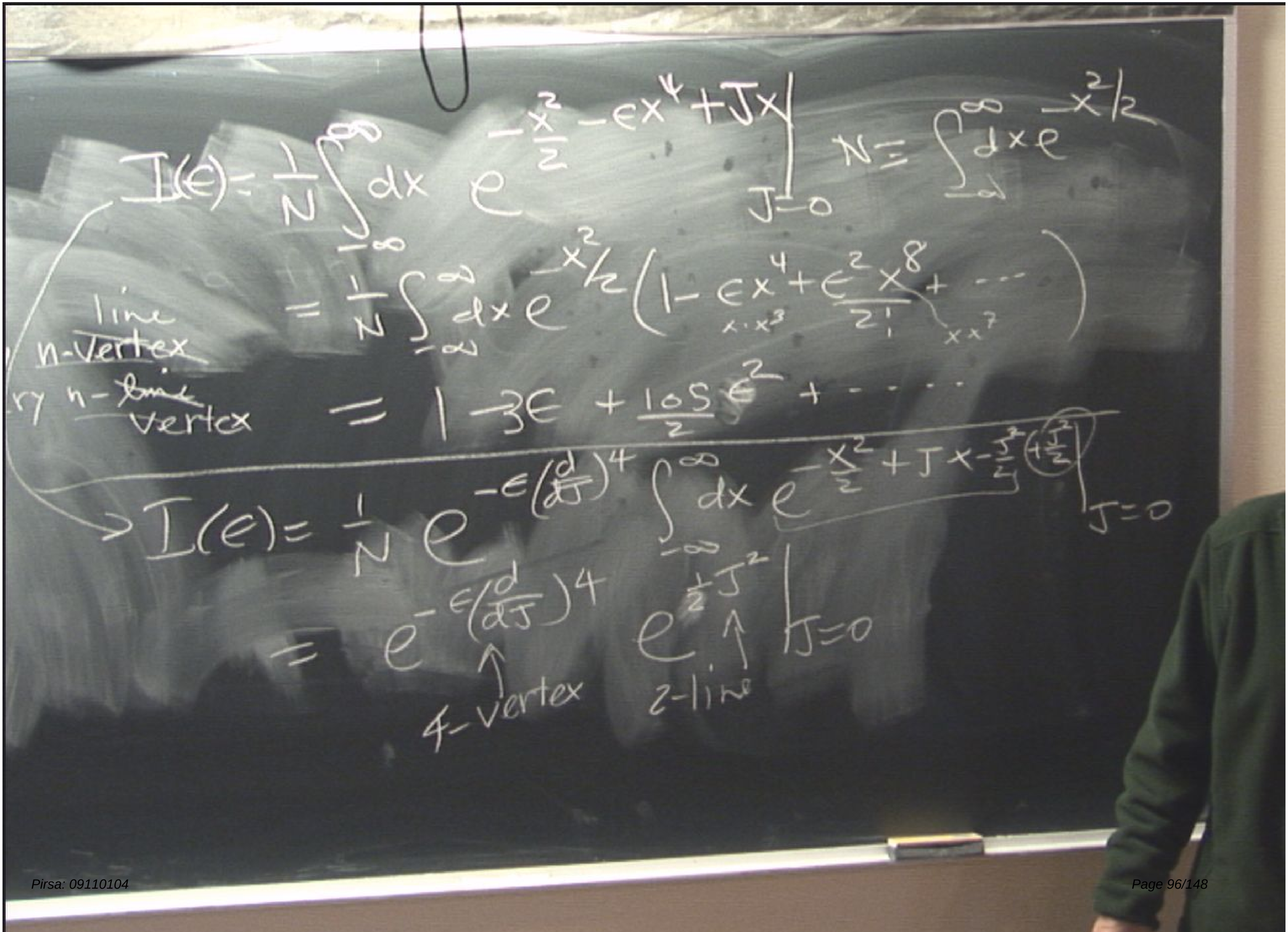
line

n-vertex

ry n-~~line~~ vertex

$$\to I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}\left(+\frac{J^2}{2}\right)}$$

$$= \underset{\substack{\uparrow\\ \text{vertex}}}{e^{-\epsilon\left(\frac{d}{dJ}\right)^4}}\;\underset{\substack{\uparrow\\ \text{line}}}{e^{\frac{1}{2}J^2}}\Big|_{J=0}$$



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\,x^7}{x^8}}{2!} + \cdots\right)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$$

line

n-vertex

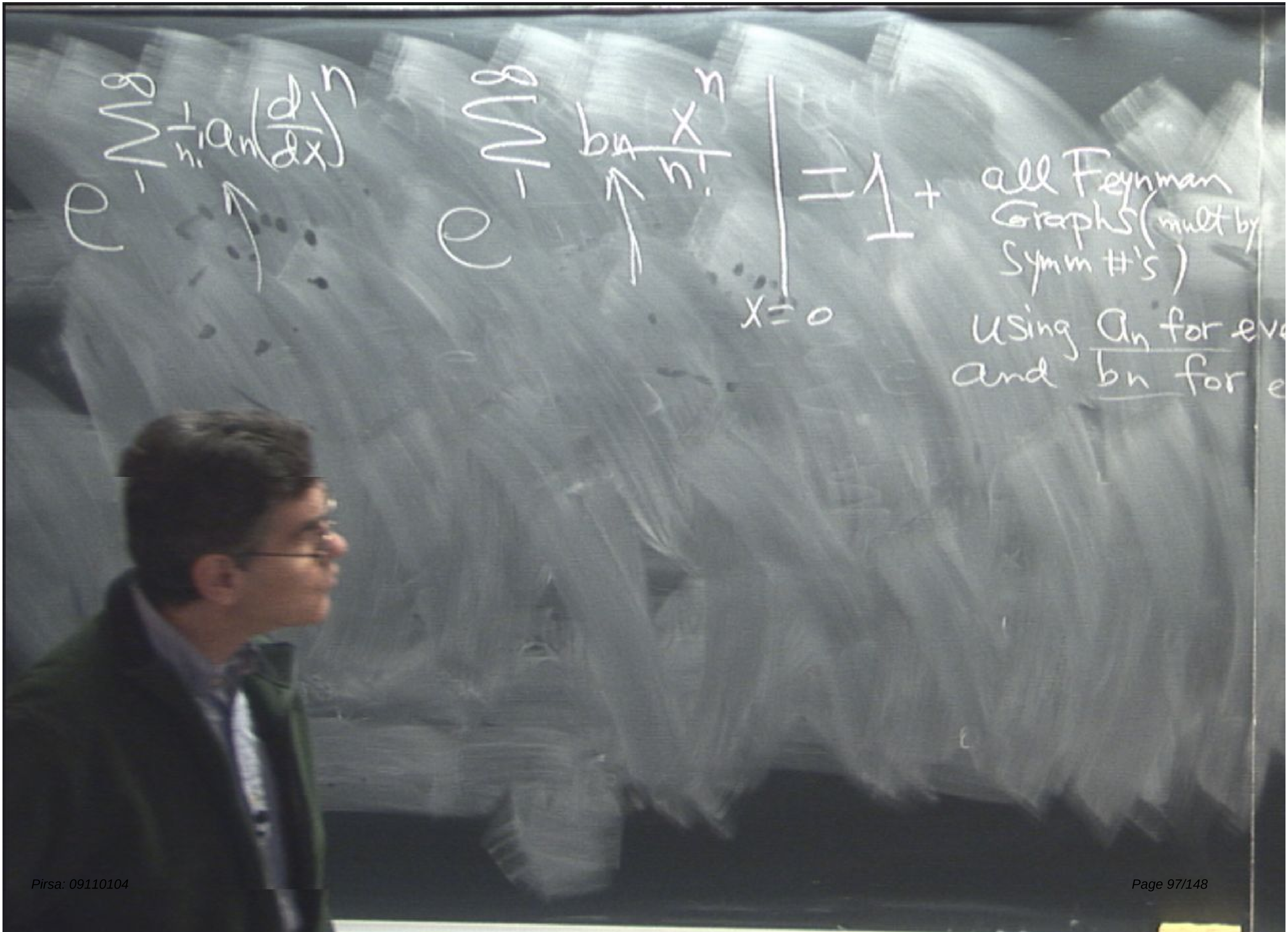
ry n- ~~line~~ vertex

$$\to I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}\left(+\frac{J^2}{2}\right)}\Big|_{J=0}$$

$$= e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\, e^{\frac{1}{2}J^2}\Big|_{J=0}$$

4-vertex

2-line



$$e^{\sum_{1}^{\infty}\frac{1}{n!}a_n\left(\frac{d}{dx}\right)^n} \quad e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}}\Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

using $\underline{a_n \text{ for ev}}$

and $\underline{b_n}$ for e

$$e^{\underset{\uparrow}{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}} \; e^{\underset{\uparrow}{\sum_1^\infty b_n \frac{x^n}{n!}}} \Bigg|_{x=0} = 1 +$$

all Feynman Graphs (mult by Symm #'s)

using $\underline{a_n}$ for ev

and $\underline{b_n}$ for e

$\underline{F-V}$

$$I(\epsilon)=\frac{1}{N}\int_{-\infty}^{\infty}dx\, e^{-\frac{x^2}{2}-\epsilon x^4+Jx}\Big|_{J=0} \qquad N=\int_{-\infty}^{\infty}dx\, e^{-x^2/2}$$

$$=\frac{1}{N}\int_{-\infty}^{\infty}dx\, e^{-x^2/2}\Big(1-\epsilon \underbrace{x^4}_{x\cdot x^3}+\frac{\epsilon^2 \underbrace{x^8}_{x x^7}}{2!}+\cdots\Big)$$

line

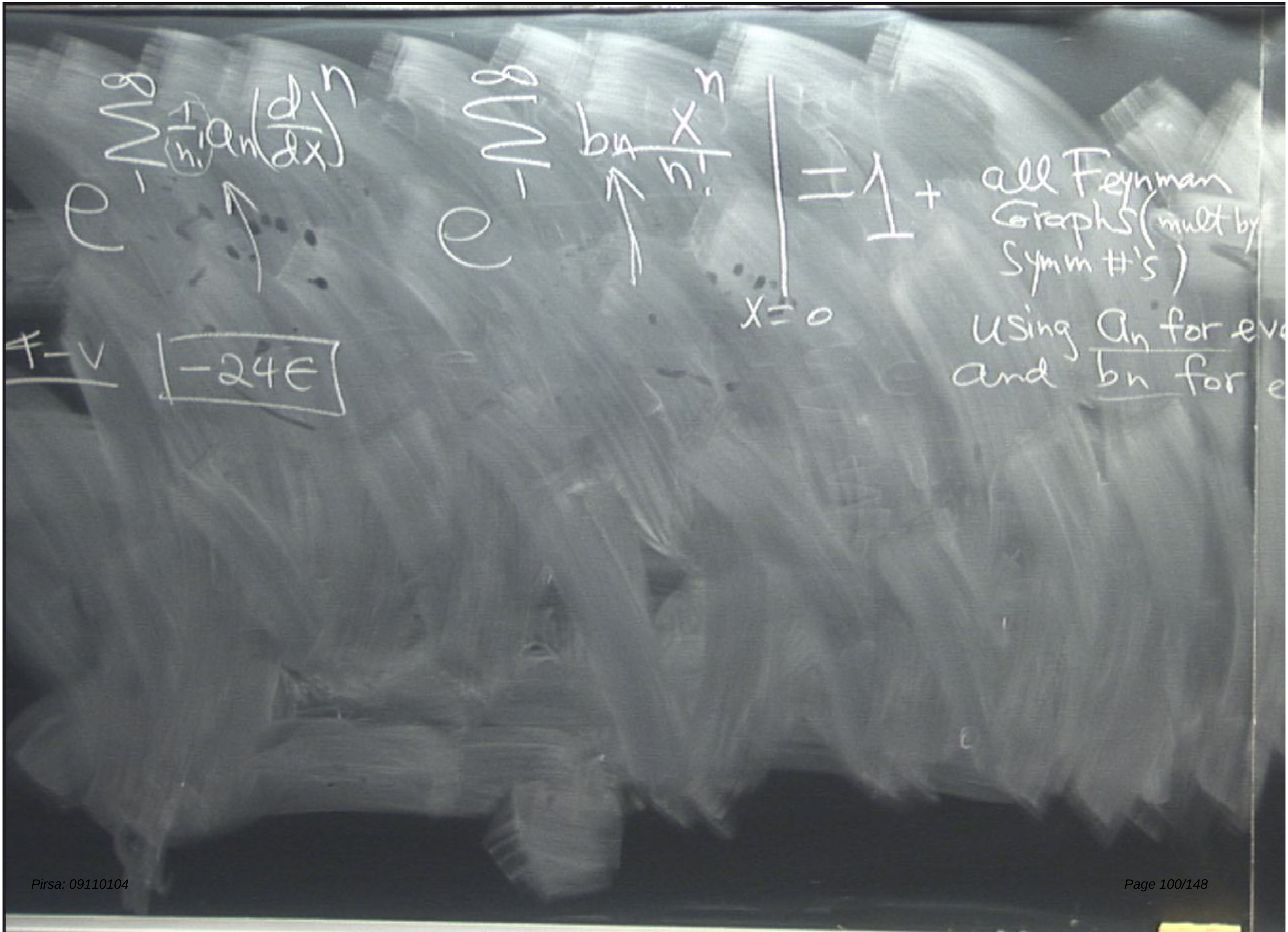
very n-Vertex

every n-~~line~~ vertex

$$=1-3\epsilon+\frac{105}{2}\epsilon^2+\cdots$$

$$I(\epsilon)=\frac{1}{N}e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty}dx\, e^{-\frac{x^2}{2}+Jx-\frac{J^2}{2}\left(+\frac{J^2}{2}\right)}\Big|_{J=0}$$

$$=\underset{\text{4-vertex}}{e^{-\epsilon\left(\frac{d}{dJ}\right)^4}}\ \underset{\text{2-line}}{e^{\frac{1}{2}J^2}}\Big|_{J=0}$$

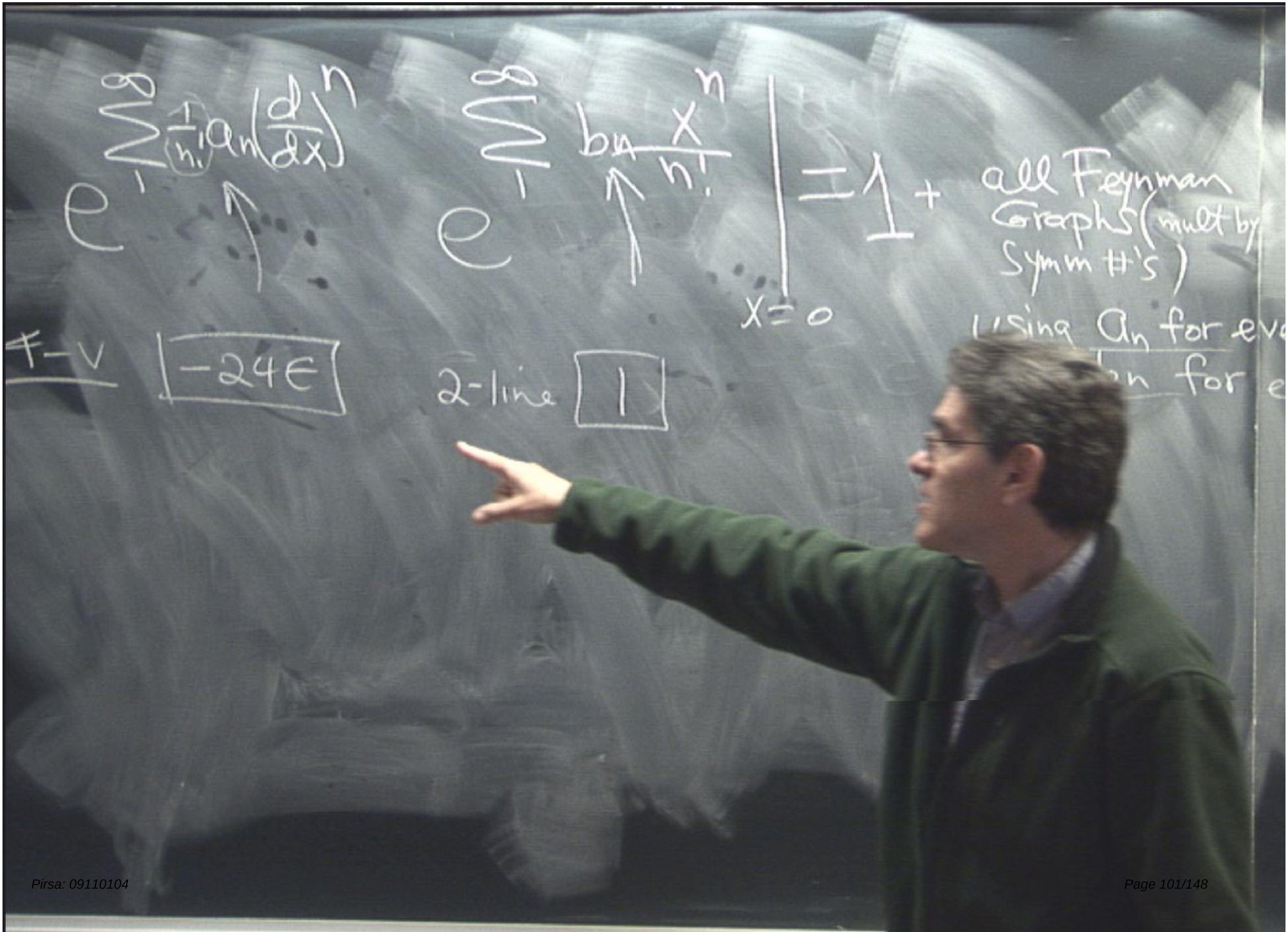


$$e^{\sum_{1}^{\infty}\frac{1}{n!}a_n\left(\frac{d}{dx}\right)^n}\;e^{\sum_{1}^{\infty}b_n\frac{x^n}{n!}}\Bigg|_{x=0}=1+$$ all Feynman Graphs (mult by Symm #'s)

using $\underline{a_n}$ for ev

and $\underline{b_n}$ for e

$\underline{F-V}$ $\boxed{-24\epsilon}$



$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \qquad e^{\sum_1^\infty b_n \frac{x^n}{n!}}\Bigg|_{x=0} = 1 +$$

all Feynman Graphs (mult by Symm #'s)

using a_n for ev

[illegible]n for e

4–V $\boxed{-24\epsilon}$

2-line $\boxed{1}$

$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Bigg|_{x=0} = 1 +$$

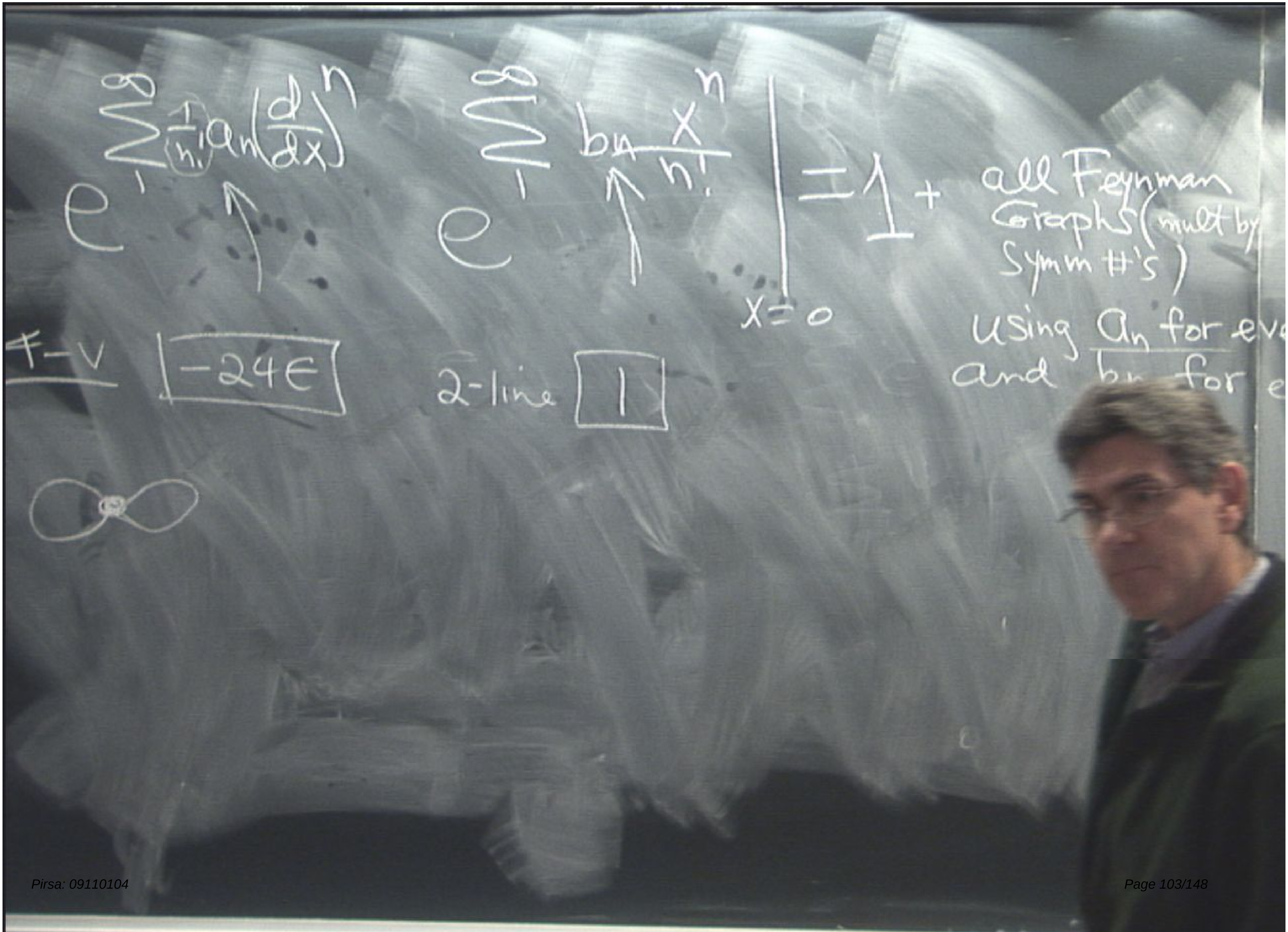
all Feynman Graphs (mult by Symm #'s)

using a_n for ev

and b_n for e

4-V $\boxed{-24\epsilon}$

2-line $\boxed{1}$



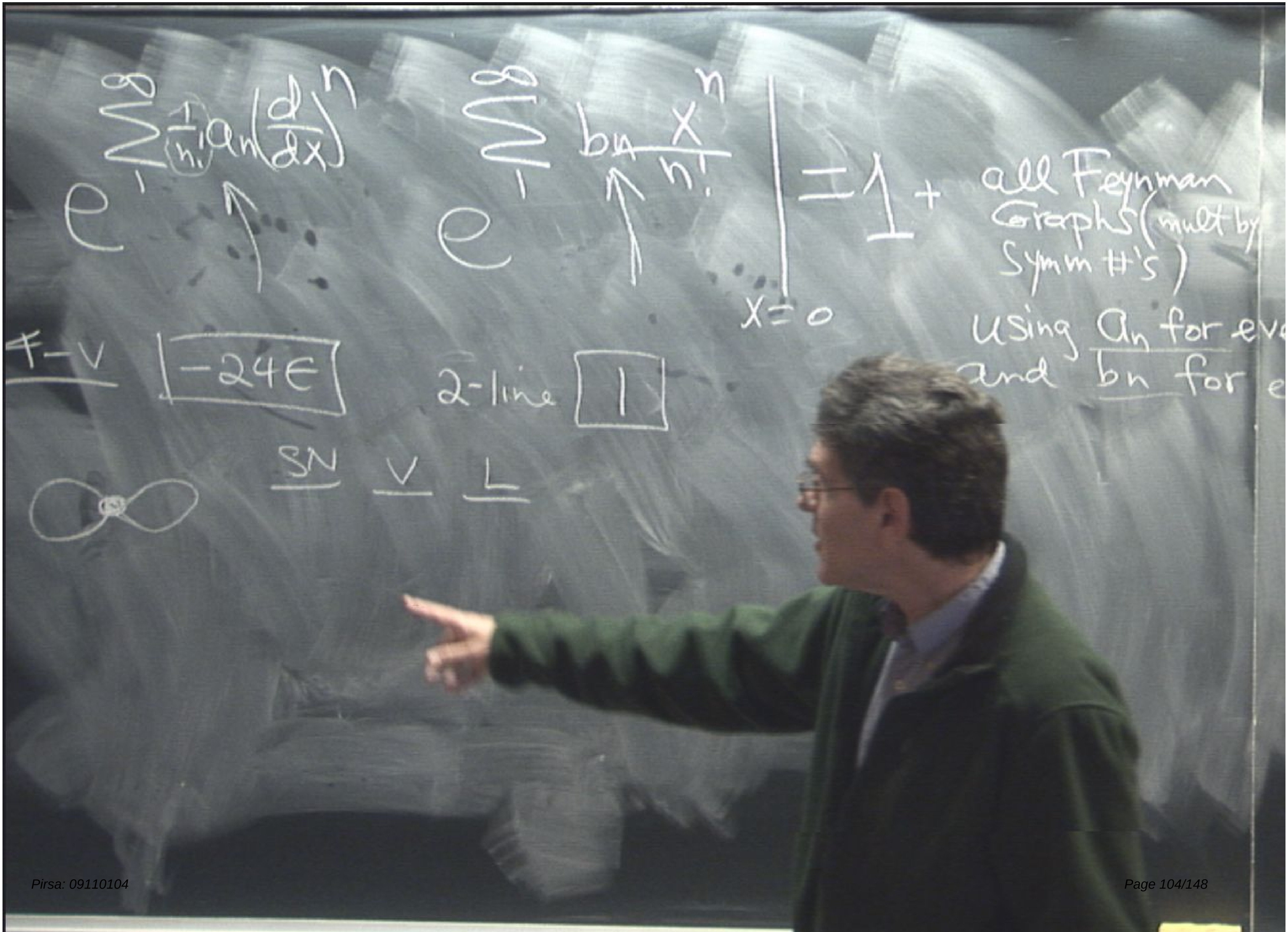
$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \uparrow \quad e^{\sum_1^\infty b_n \frac{x^n}{n!}} \uparrow \Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

using a_n for ev

and b_n for e

4-V $\boxed{-24\epsilon}$

2-line $\boxed{1}$



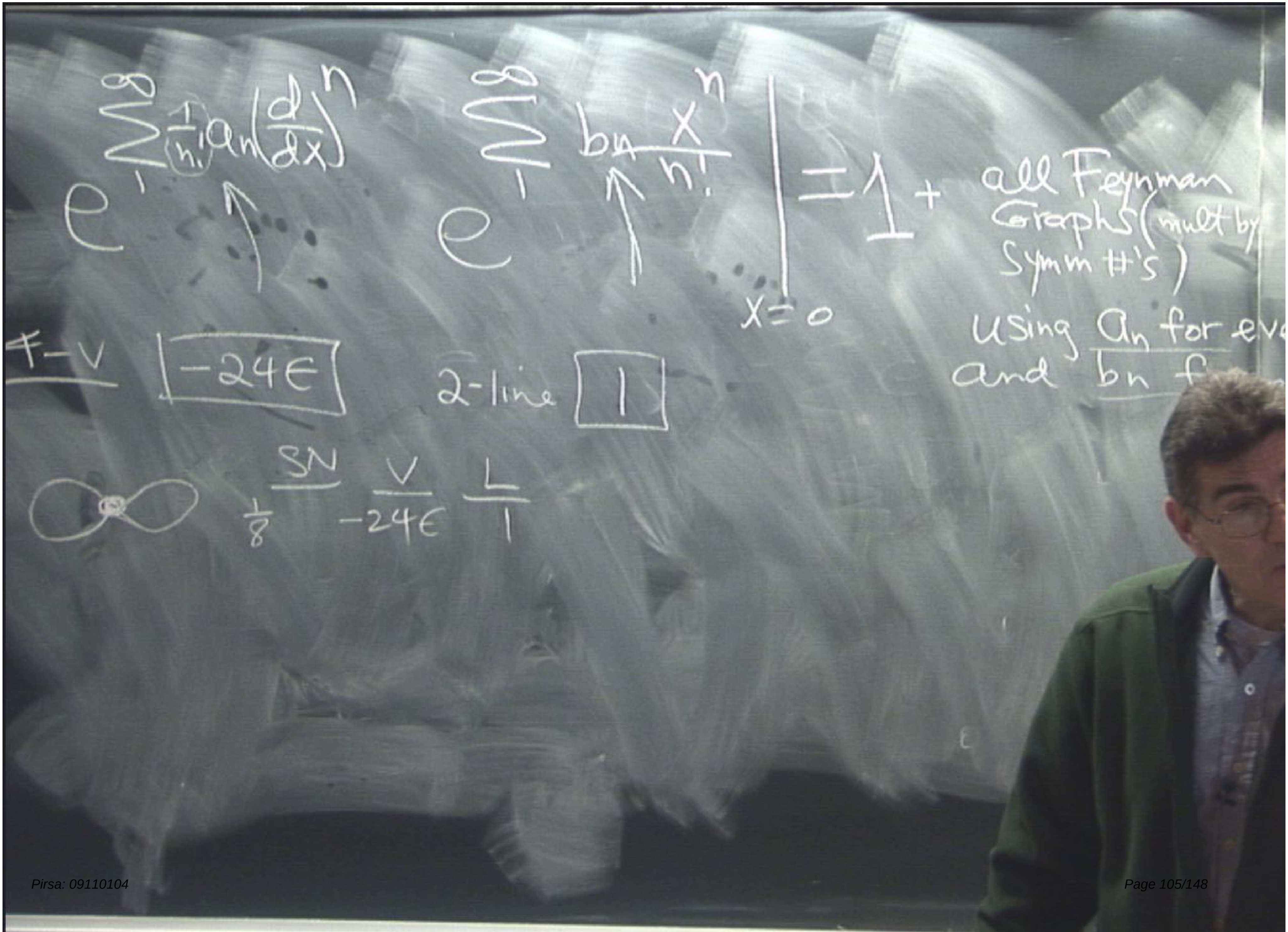
$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

using a_n for eve

and b_n for e

4–V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

SN V L



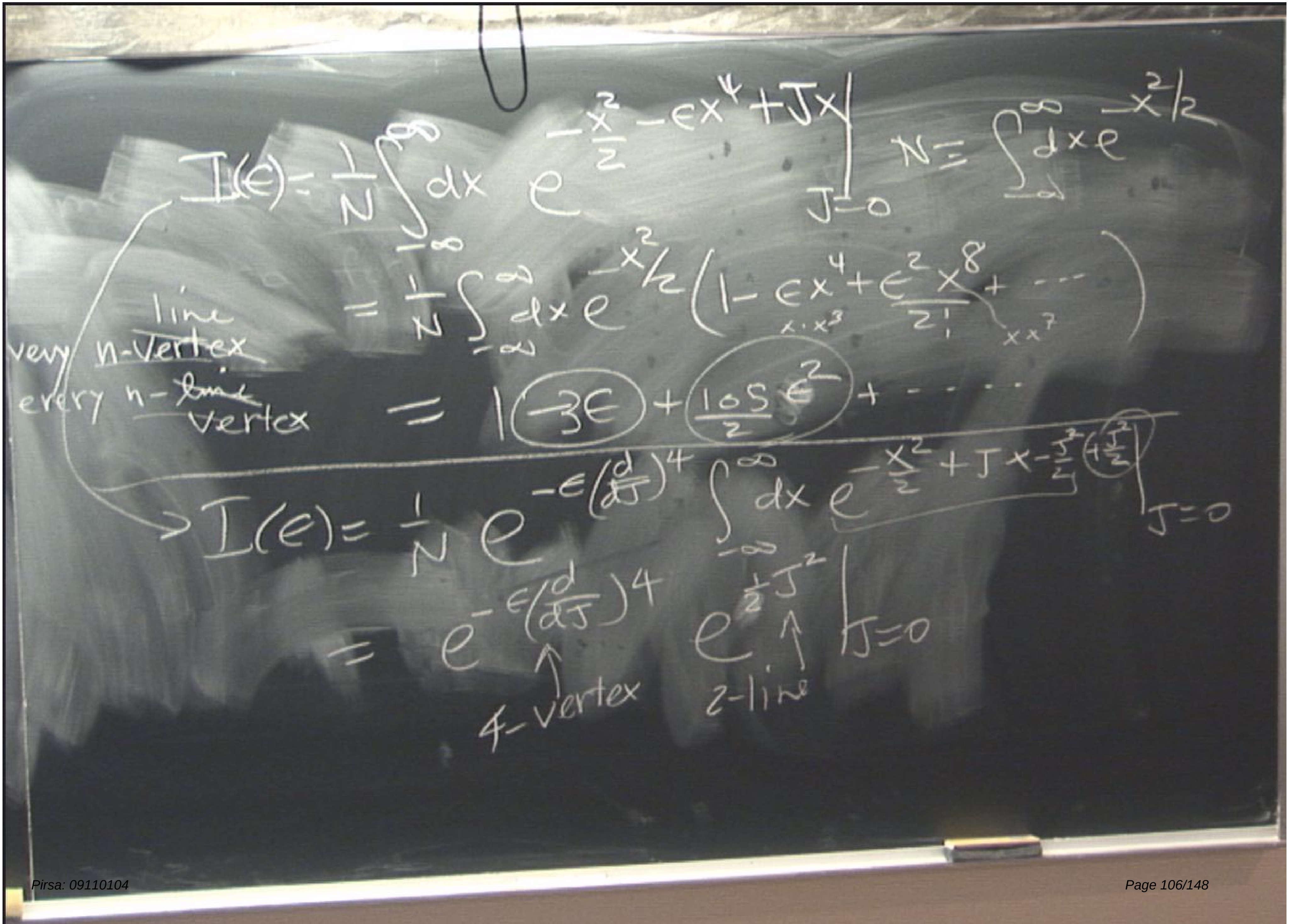
$$e^{\sum_{1}^{\infty}\frac{1}{n!}a_n\left(\frac{d}{dx}\right)^n} \quad e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}}\Bigg|_{x=0} = 1 +$$

all Feynman Graphs (mult by Symm #'s)

using a_n for ev
and b_n f

F–V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

	SN	V	L
	$\frac{1}{8}$	-24ϵ	1



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} dx\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\, x^7}{x^8}}{2!} + \cdots\right)$$

very n-~~vertex~~ line

every n-~~line~~ vertex

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

$$I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx\; e^{\underbrace{-\frac{x^2}{2} + Jx - \frac{J^2}{2}}\boxed{+\frac{J^2}{2}}}\Big|_{J=0}$$

$$= e^{-\epsilon\overset{\uparrow}{\left(\frac{d}{dJ}\right)^4}}\; e^{\overset{\uparrow}{\frac{1}{2}J^2}}\Big|_{J=0}$$

4-vertex 2-line

$$e^{\sum_{1}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_{1}^{\infty} b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 +$$

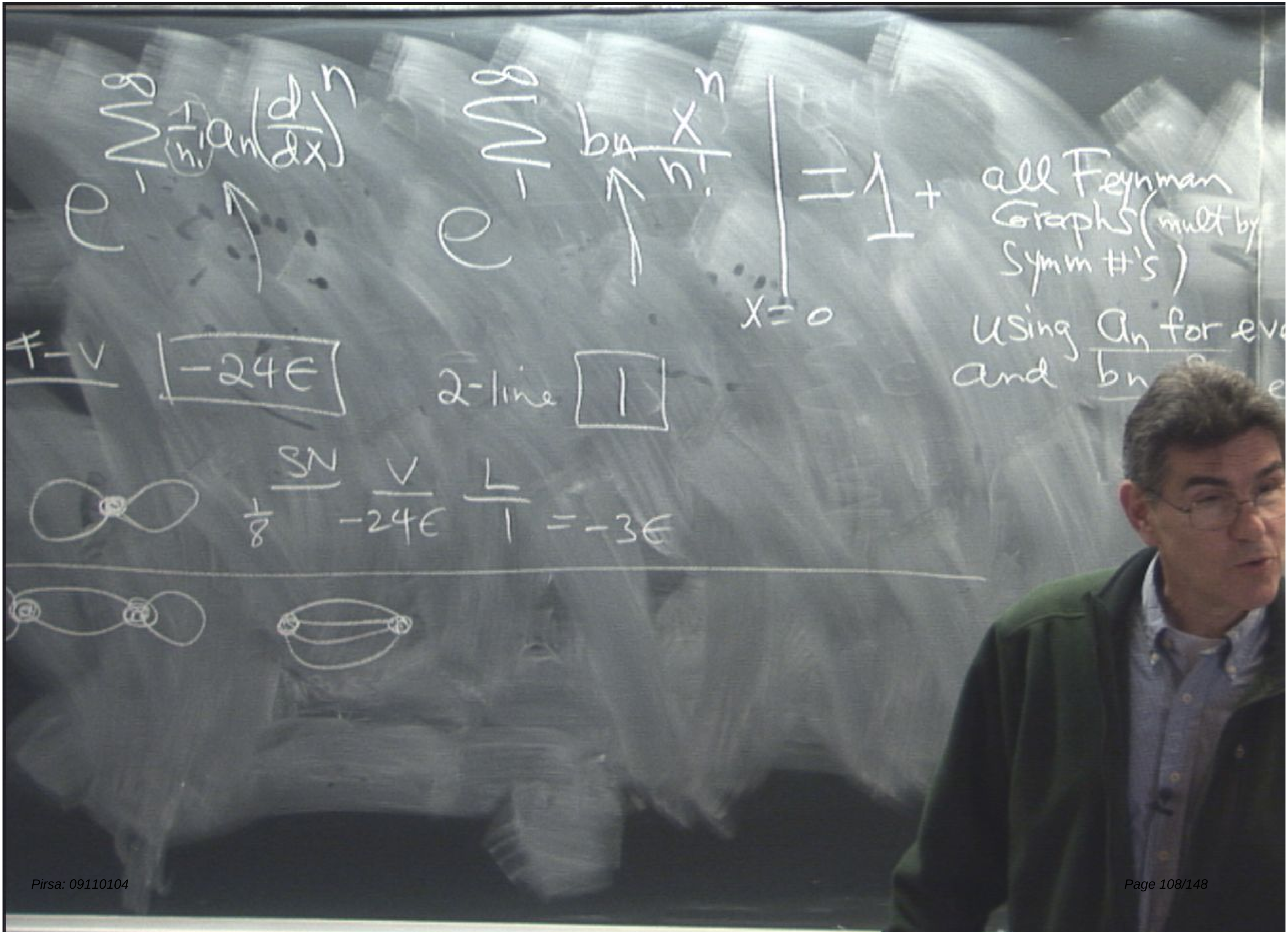
all Feynman Graphs (mult by Symm #'s)

using $\underline{a_n}$ for ve

and $\underline{b_n}$ for e

$\underline{\text{F}-\text{V}}$ $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$$\overset{\text{SN}}{\frac{1}{8}} \quad \overset{\text{V}}{-24\epsilon} \quad \overset{\text{L}}{1} = -3\epsilon$$



$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 +$$

all Feynman Graphs (mult by Symm #'s)

using a_n for eve

and b_n

$\underline{F-V}$ $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$$\underset{\frac{1}{8}}{\underline{SN}} \quad \underset{-24\epsilon}{\underline{V}} \quad \underset{1}{\underline{L}} = -3\epsilon$$

$$e^{\sum_1^\infty \frac{1}{n!} \underset{\uparrow}{a_n} \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty \underset{\uparrow}{b_n} \frac{x^n}{n!}} \Bigg|_{x=0} = 1 +$$

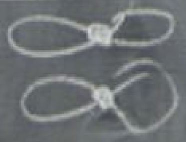
all Feynman Graphs (mult by Symm #'s)

using a_n for ev

and b_n for e

4-V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

	SN	V	L	
	$\frac{1}{8}$	-24ϵ	1	$= -3\epsilon$



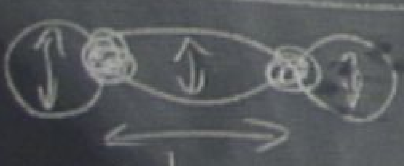
$$\left. e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \, e^{\sum_1^\infty b_n \frac{x^n}{n!}} \right|_{x=0} = 1 +$$

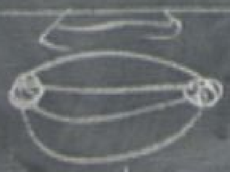
all Feynma
Graphs (mul
Symm #'s)

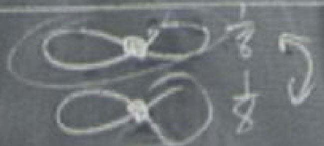
using a_n fo
and b_n f

4-V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$$\begin{array}{ccc} \underline{SN} & \underline{V} & \underline{L} \\ \frac{1}{8} & -24\epsilon & 1 \end{array} = -3\epsilon$$







$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128}\right)(24\epsilon)^2 = \epsilon^2 \Big[\quad \Big]$$

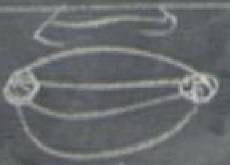
$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 + $$

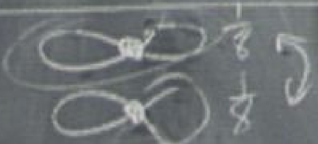
all Feynman Graphs (mult by Symm #'s)

using $\underline{a_n}$ for eve
and $\underline{b_n}$ for e

$\underline{F\text{–}V}$ $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$$\frac{1}{8} \quad \frac{\underline{SN}}{} \quad \frac{\underline{V}}{-24\epsilon} \quad \frac{\underline{L}}{1} = -3\epsilon$$





$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128}\right)(24\epsilon)^2 = 36\epsilon^2\left[1 + \frac{1}{3} + \frac{1}{8}\right]$$

$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}}\Bigg|_{x=0} = 1 +$$

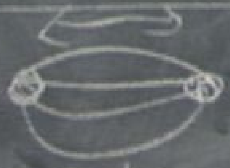
all Feynman Graphs (mult by Symm #'s)

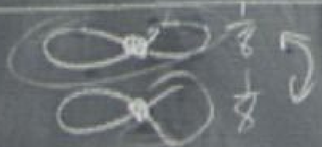
using $\underline{a_n}$ for eve

and $\underline{b_n}$ for e

F–V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$$\frac{SN}{\frac{1}{8}} \quad \frac{V}{-24\epsilon} \quad \frac{L}{1} = -3\epsilon$$





$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128}\right)(24\epsilon)^2 = 36\epsilon^2\left[\underbrace{1+\frac{1}{3}+\frac{1}{8}}\right]$$

$$\overline{24}$$

$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n} \; e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 +$$

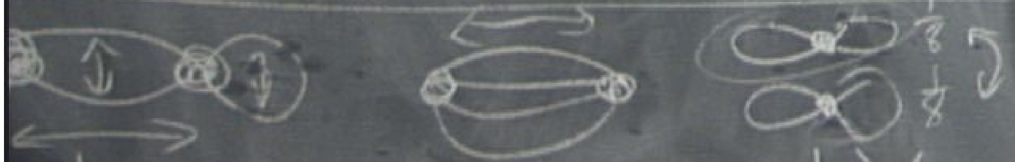
all Feynman Graphs (mult by Symm #'s)

using a_n for eve

and b_n for e

F–V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$\frac{SN}{\frac{1}{8}}$ $\frac{V}{-24\epsilon}$ $\frac{L}{1} = -3\epsilon$



$$\frac{1}{16} + \frac{1}{48} + \frac{1}{128})(24\epsilon)^2 = 36\epsilon^2\left[\underbrace{1+\frac{1}{3}+\frac{1}{8}}_{\frac{24+8+3}{24}}\right]$$

$$\frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n \qquad e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

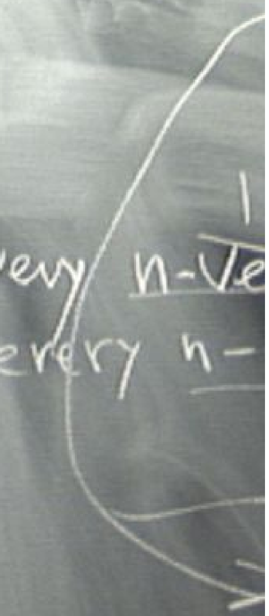
using a_n for every n-Ve

and b_n for every n-

$$\boxed{-24\epsilon} \qquad \text{2-line } \boxed{1}$$

$$\frac{1}{8} \quad \frac{SN}{} \quad \frac{V}{-24\epsilon} \quad \frac{L}{1} = -3\epsilon$$

$$+ \quad \frac{1}{48} + \frac{1}{128}\Big)(24\epsilon)^2 = 36\epsilon^2 \left[\underbrace{1 + \frac{1}{3} + \frac{1}{8}}_{\frac{24+8+3}{24}}\right] = \frac{35}{24} 36\epsilon^2 = \underline{\frac{105}{2}\epsilon^2}$$



$$\frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n \qquad e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Bigg|_{x=0} = 1 +$$

all Feynman Graphs (mult by Symm #'s)

using $\underline{a_n}$ for every n-ve

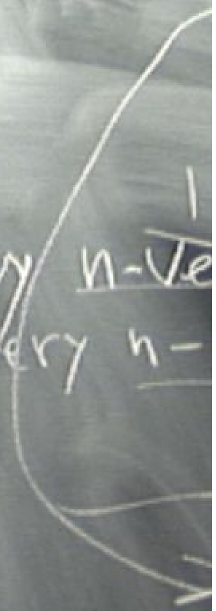
and $\underline{b_n}$ for every n-

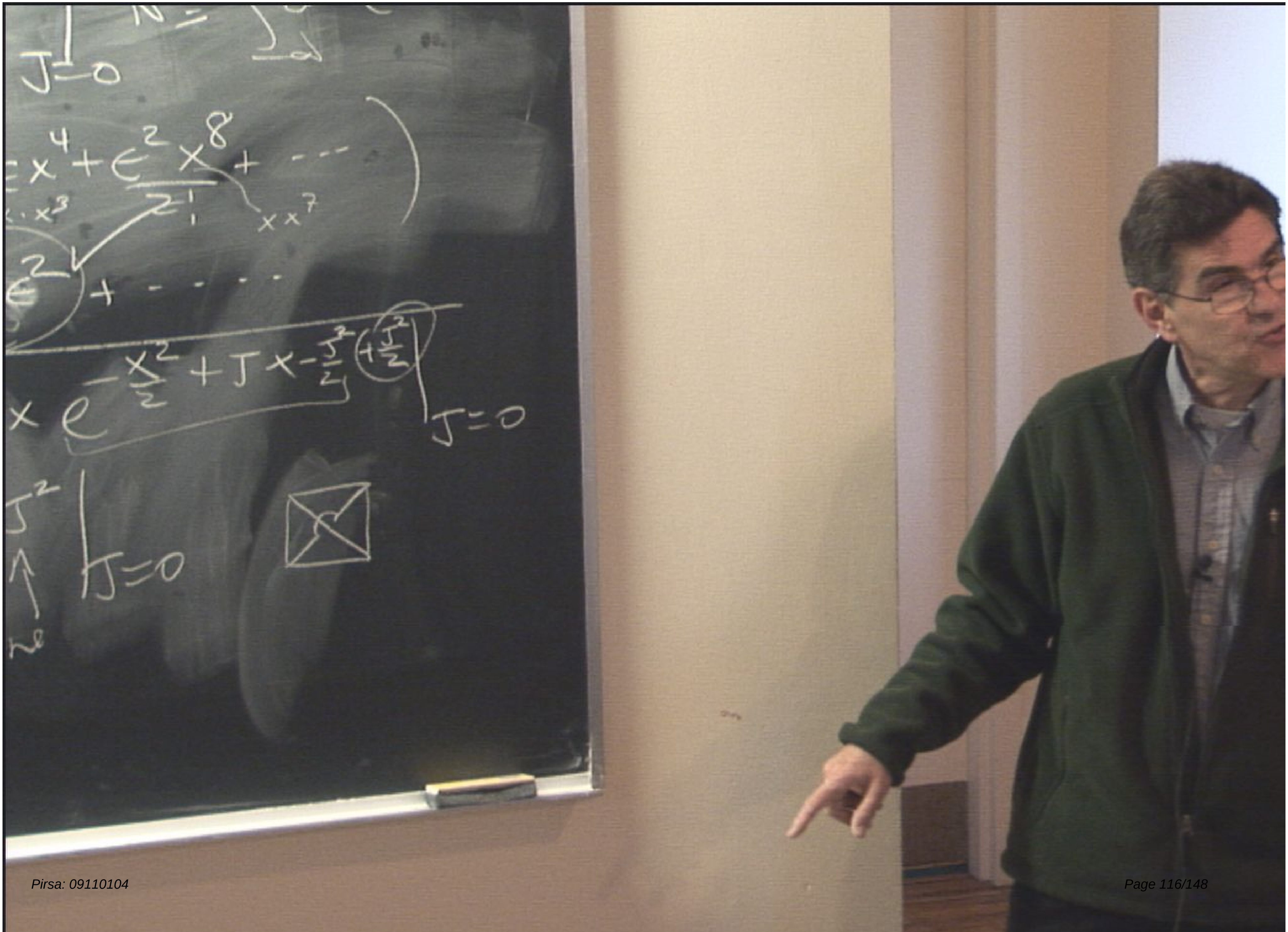
$\boxed{-24\epsilon}$ 2-line $\boxed{1}$

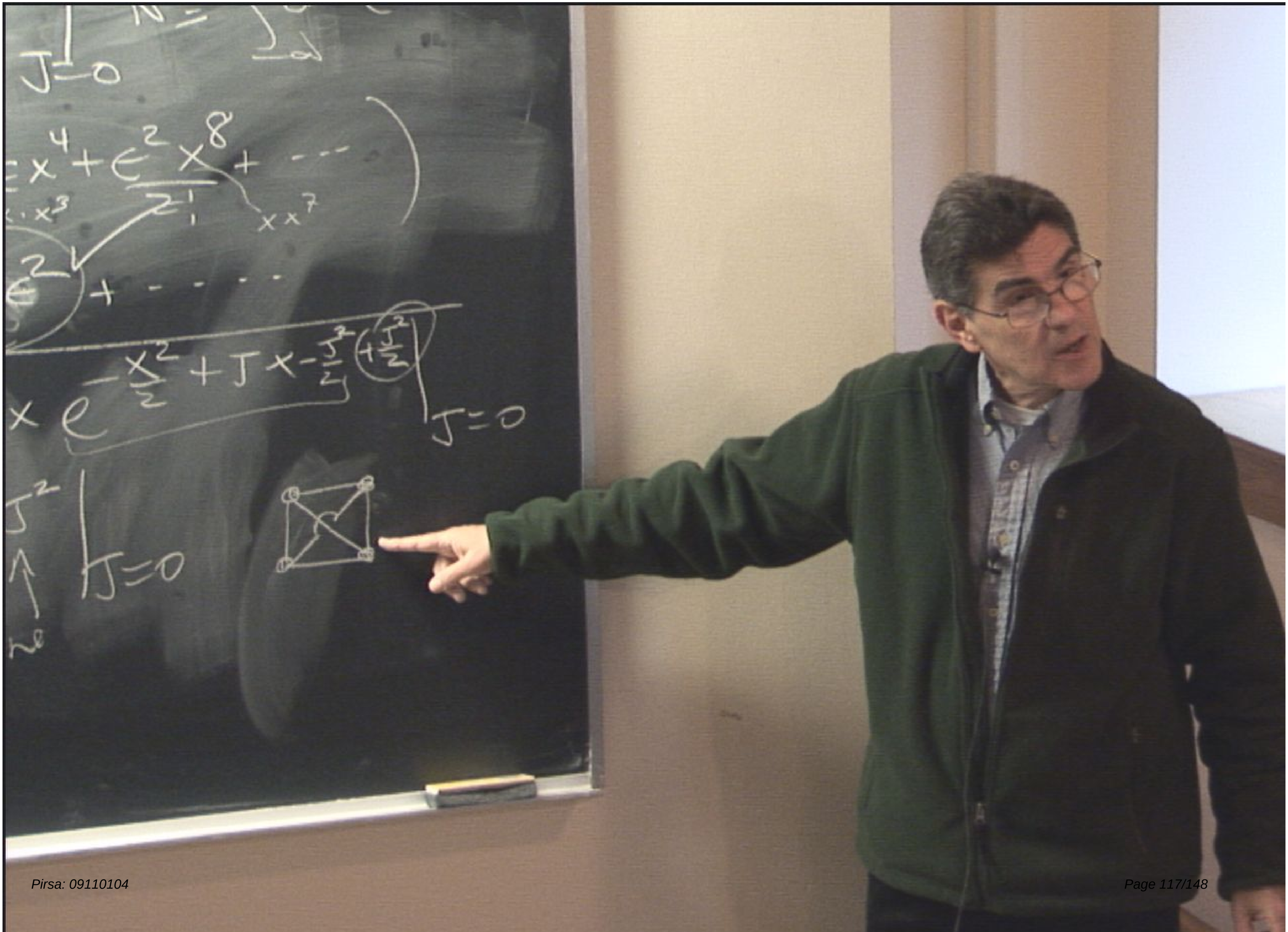
$$\frac{SN}{\frac{1}{8}} \quad \frac{V}{-24\epsilon} \quad \frac{L}{1} = -3\epsilon$$

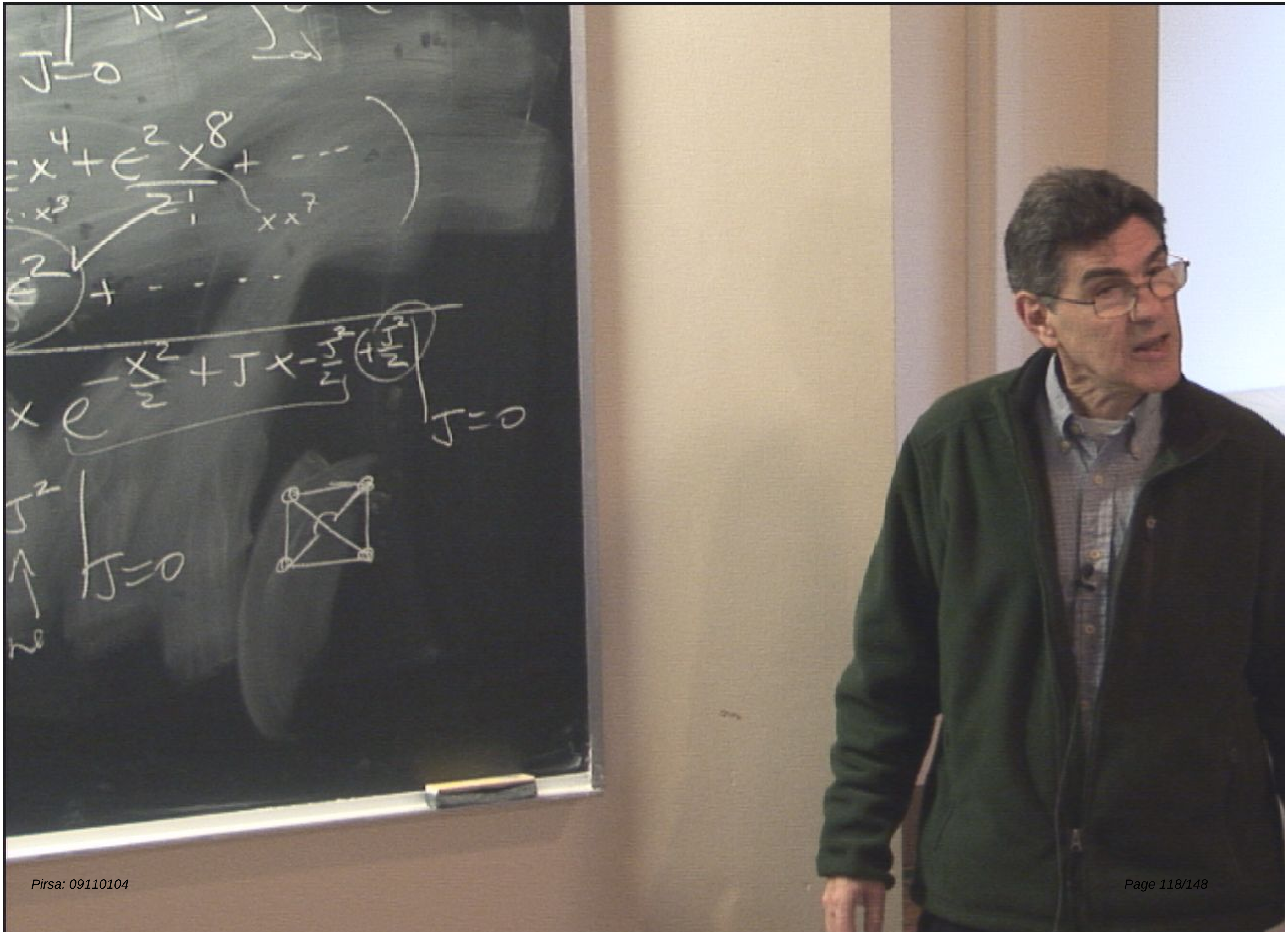
$$+ \frac{1}{48} + \frac{1}{128}\Big)(24\epsilon)^2 = 36\epsilon^2 \left[\underbrace{1 + \frac{1}{3} + \frac{1}{8}}\right]$$

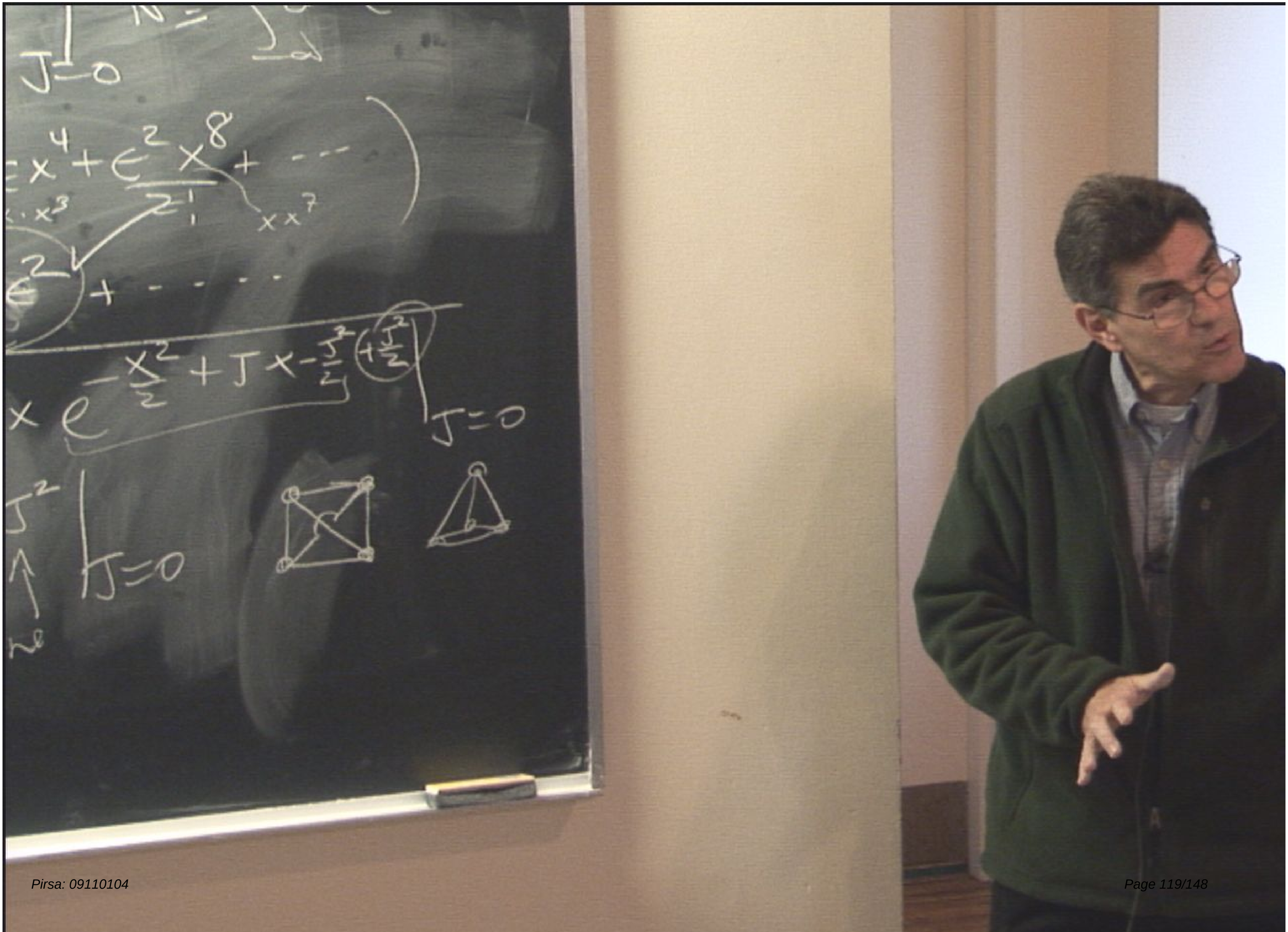
$$\frac{24+8+3}{24} = \frac{35}{24} 36\epsilon^2 = \frac{105}{2}\epsilon^2$$

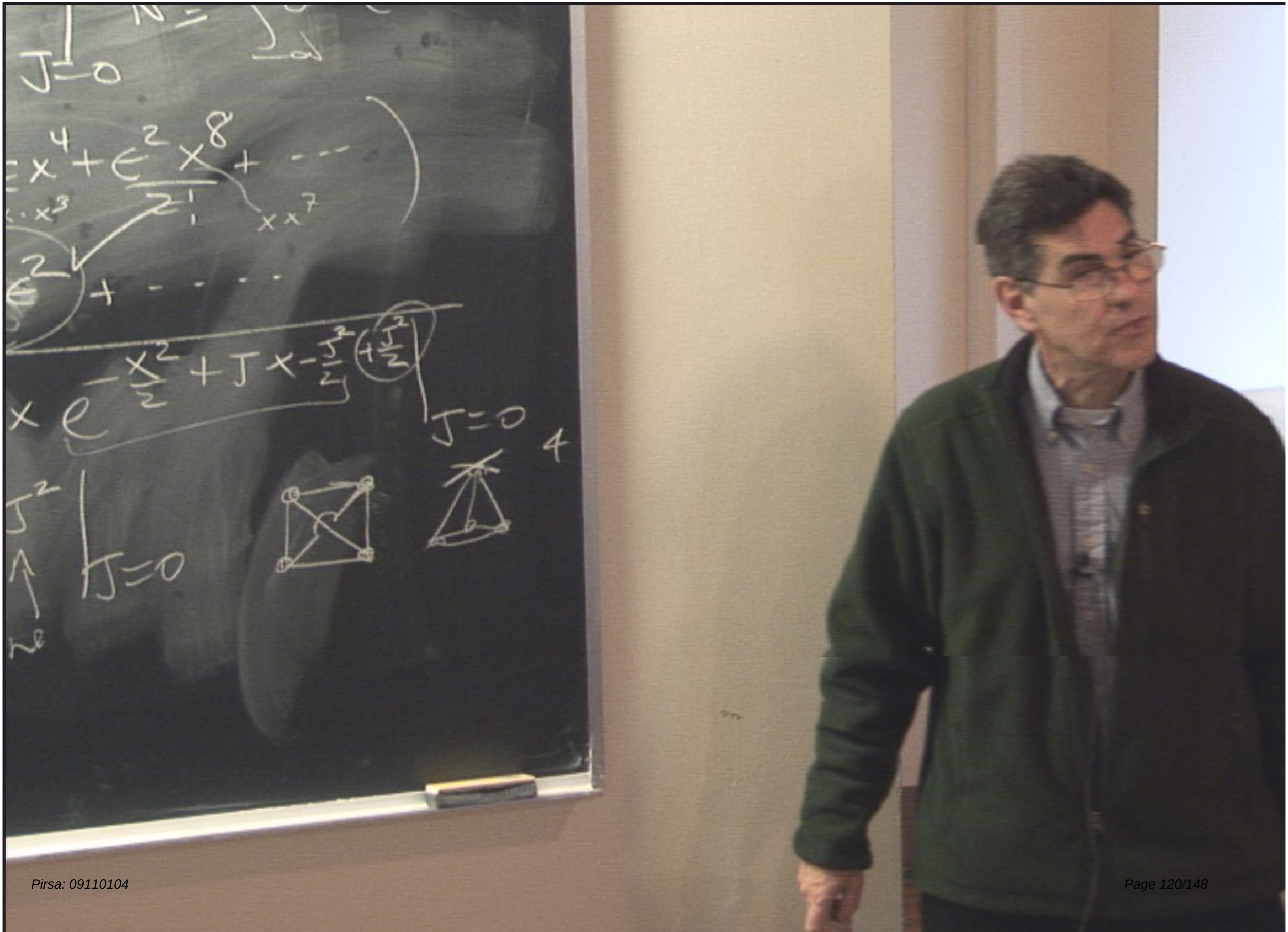


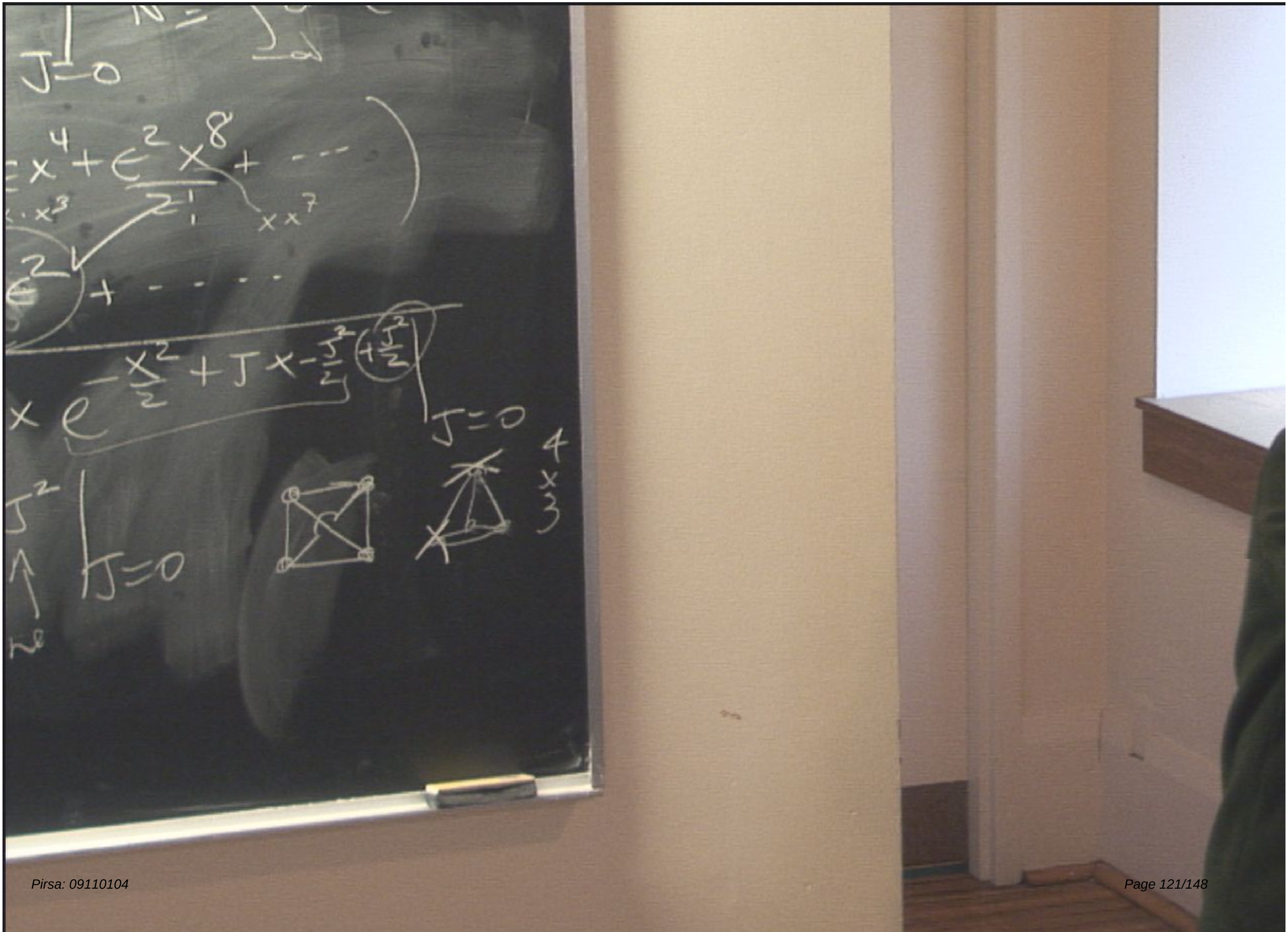

J=0
x⁴ + ε² x⁸ + ----
2!
x·x³
x x⁷
ε² + - - - - -
x e^{-x²/2 + Jx - J²/2} (+J²/2)
J=0
J²
J=0

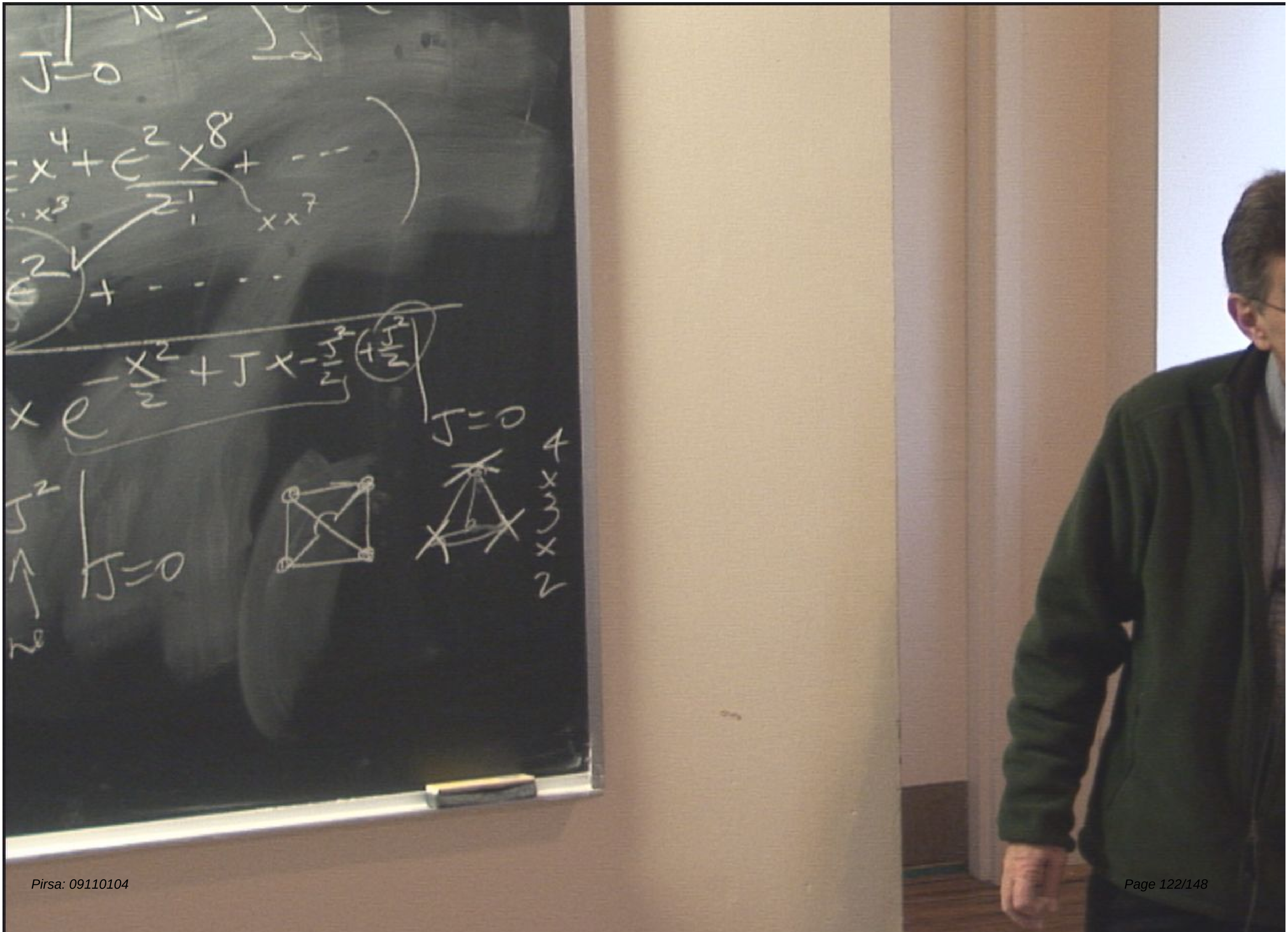

J=0
x^4 + ε^2 x^8/2! + ----
x x^7
ε^2 + - - - - -
e^{-x^2/2 + Jx - J^2/2 + J^2/2}
J=0
J^2
J=0

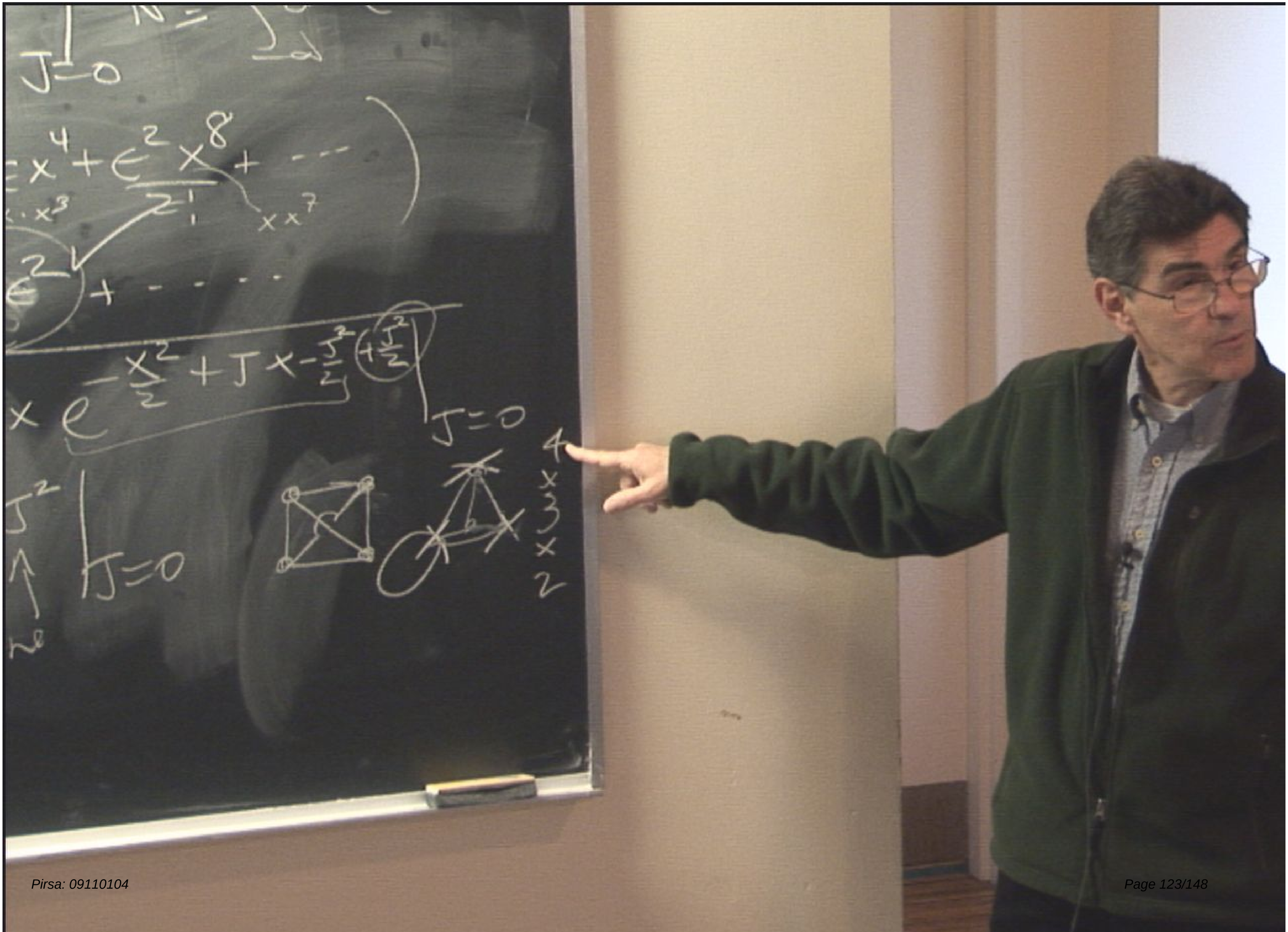

J=0
x^4 + ε^2 x^8 + ----
x·x^3
2!
x x^7
ε^2 + ----
x e^{-x^2/2 + Jx - J^2/2} +J^2/2
J=0
J^2
J=0

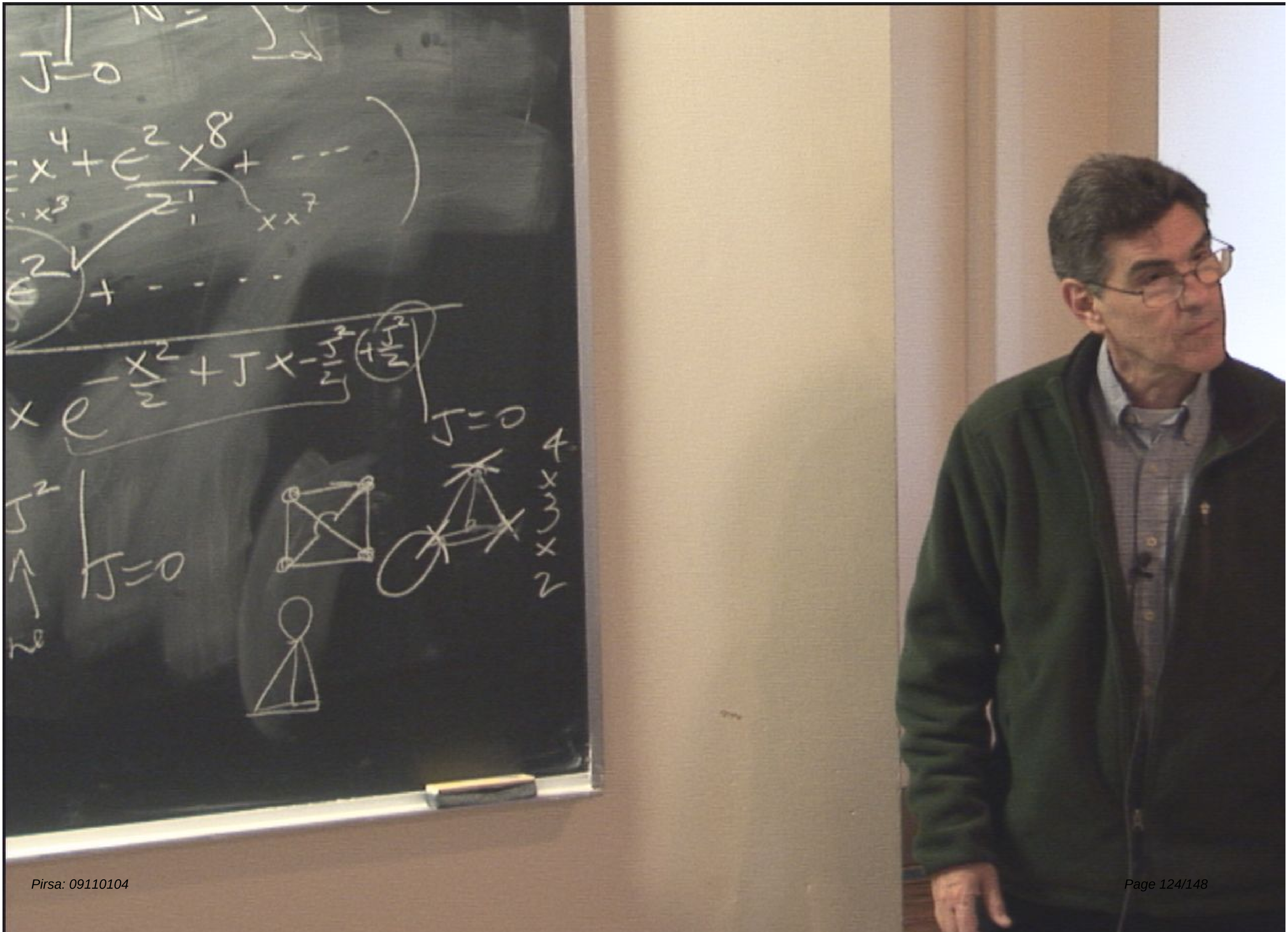

J=0
x⁴ + ε² x⁸ + - - -
2!
x · x³
x x⁷
ε² + - - - -
x e^{-x²/2 + Jx - J²/2} (+J²/2)
J=0
J²
J=0


J=0
x^4 + ε^2 x^8 / 2! + ---
x · x^3
x x^7
ε^2 + - - - -
x e^{-x^2/2 + Jx - J^2/2} (+ J^2/2)
J=0
4
J^2
J=0

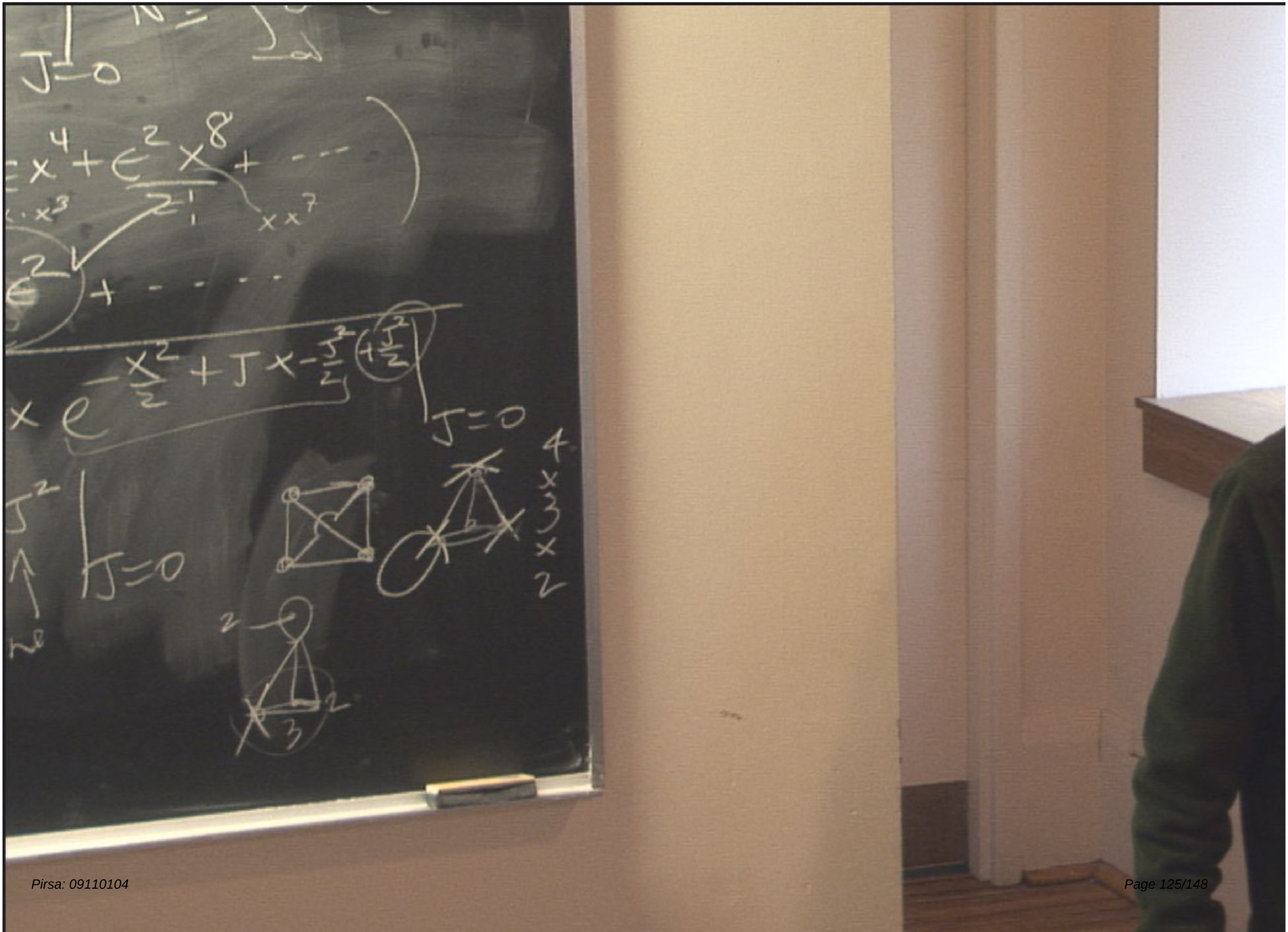

J = 0
x^4 + ε^2 x^8 + - - -
2!
x · x^3
x x^7
ε^2 + - - - -
x e^{-x^2/2 + Jx - J^2/2} + J^2/2
J = 0
4
x
3
J^2
J = 0

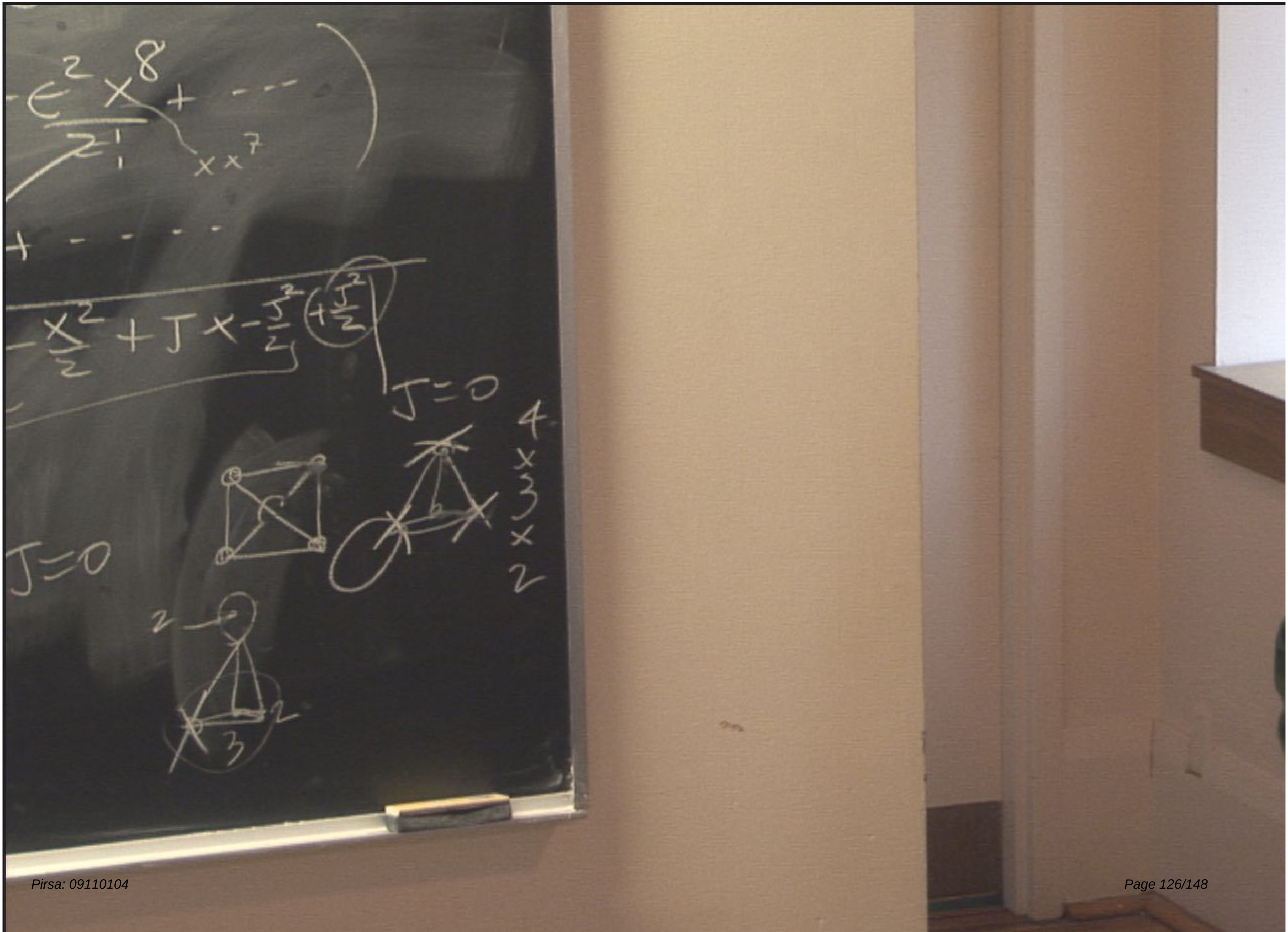

J = 0
x^4 + ε^2 x^8 + ---
2!
x · x^3
x x^7
ε^2 + - - - - -
e^{-x^2/2 + Jx - J^2/2 + J^2/2}
J = 0
J^2
J = 0
4 × 3 × 2


J=0
x^4 + ε^2 x^8/2! + ---
x·x^3
x x^7
ε^2 + - - - - -
e^{-x^2/2 + Jx - J^2/2} (+J^2/2)
J=0
4 × 3 × 2
J^2
J=0



J=0
x^4 + ε^2 x^8 + ----
2!
x · x^3
x x^7
ε^2 + - - - - -
e^{-x^2/2 + Jx - J^2/2} + J^2/2
J=0
J^2
J=0
4 × 3 × 2


J=0
x^4 + ε^2 x^8/2! + ----
x·x^3
x x^7
ε^2 + - - - - -
x e^{-x^2/2 + Jx - J^2/2} (+ J^2/2)
J=0
J^2
J=0
4 × 3 × 2
2
2
3



$\epsilon^2 \frac{x^8}{2!} + \cdots$

$x x^7$

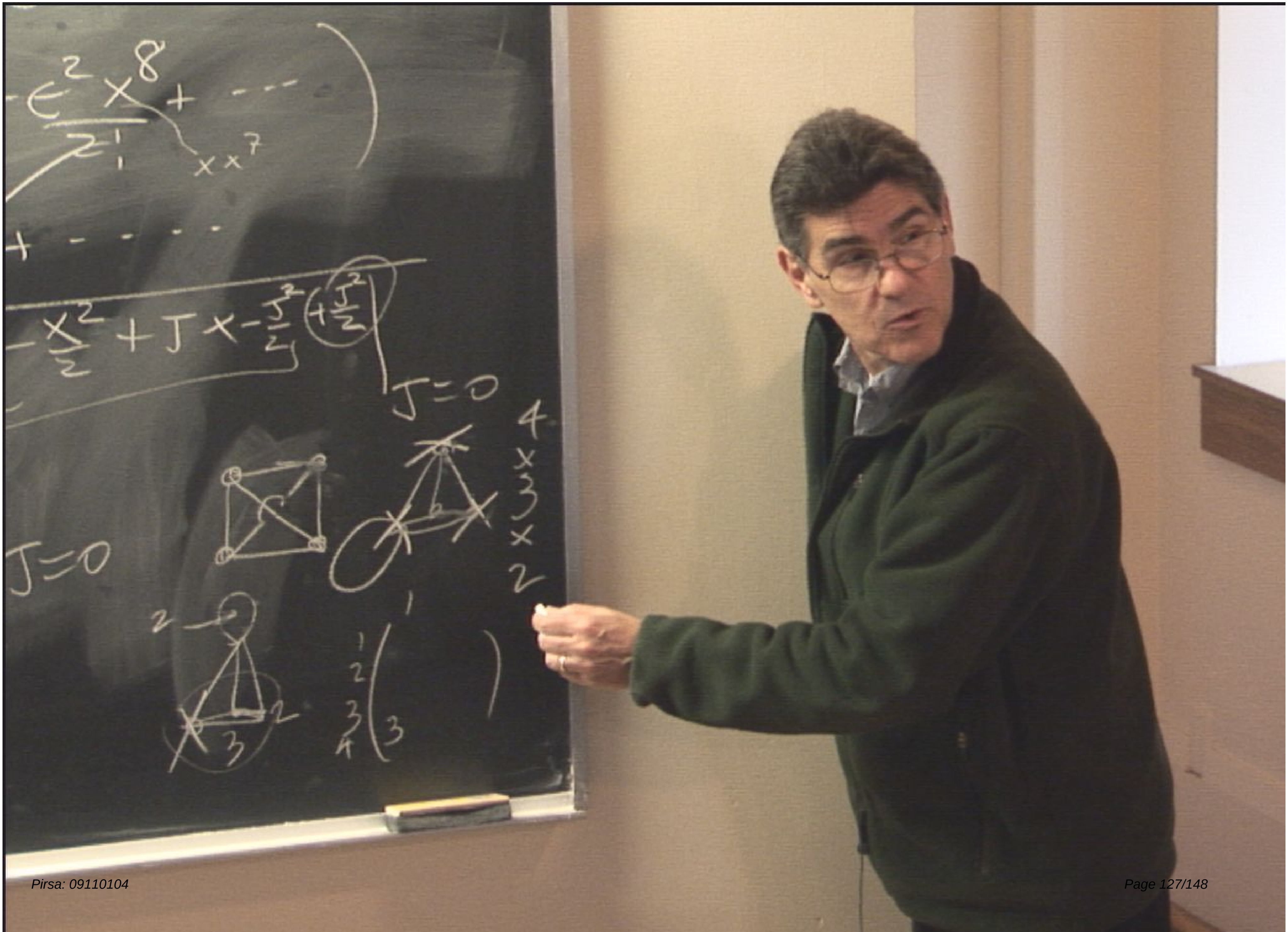
$+ \cdots$

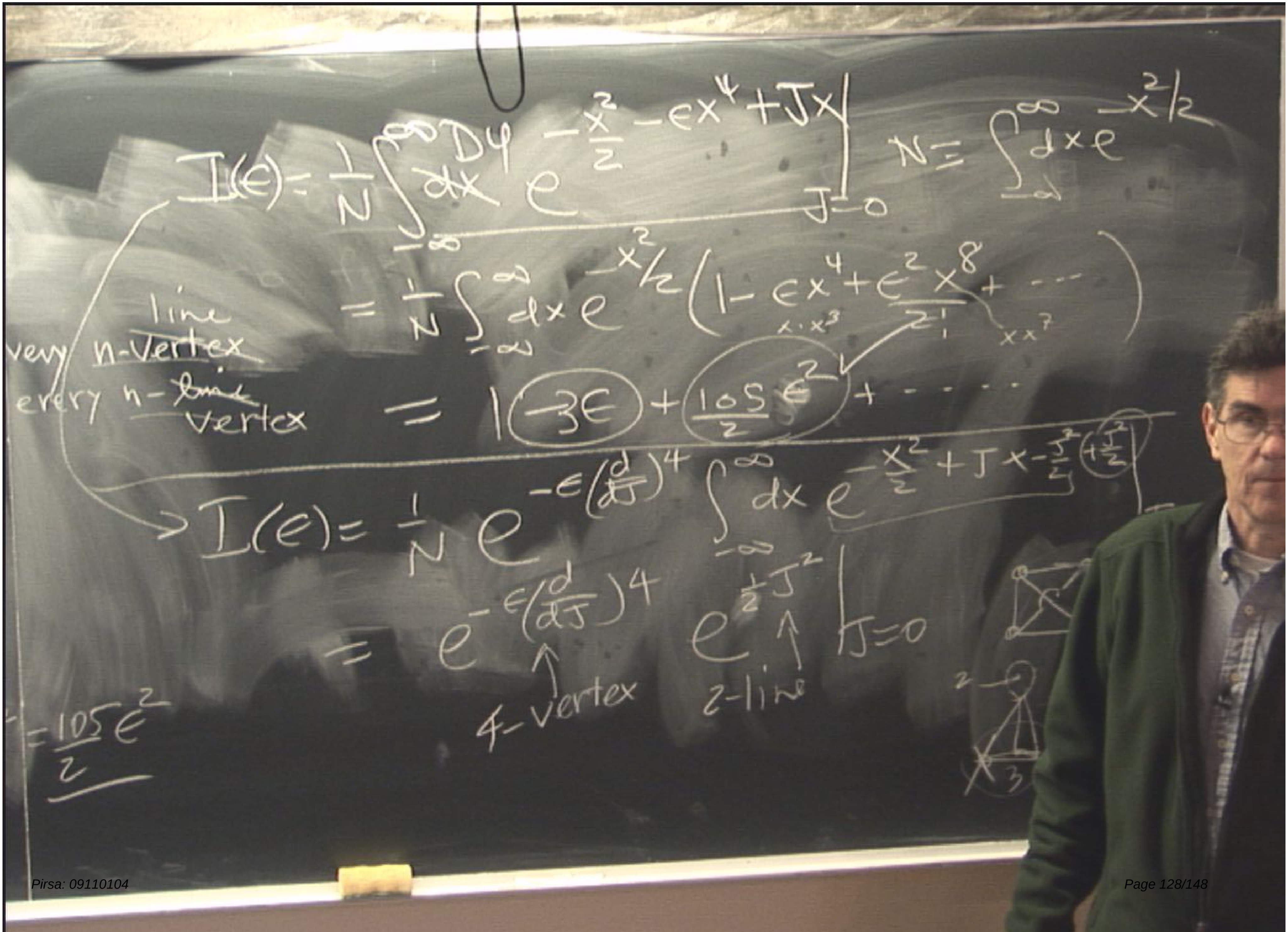
$-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}$

$J=0$

$4 \times 3 \times 2$

$J=0$


J=0
J=0
4
x
3
x
2



$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \mathcal{D}\varphi \, \cancel{dx}\; e^{-\frac{x^2}{2}-\epsilon x^4+Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1-\epsilon \underset{x\cdot x^3}{x^4}+\frac{\epsilon^2 \underset{xx^7}{x^8}}{2!}+\cdots\Big)$$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

line

very n-Vertex

every n- ~~line~~ vertex

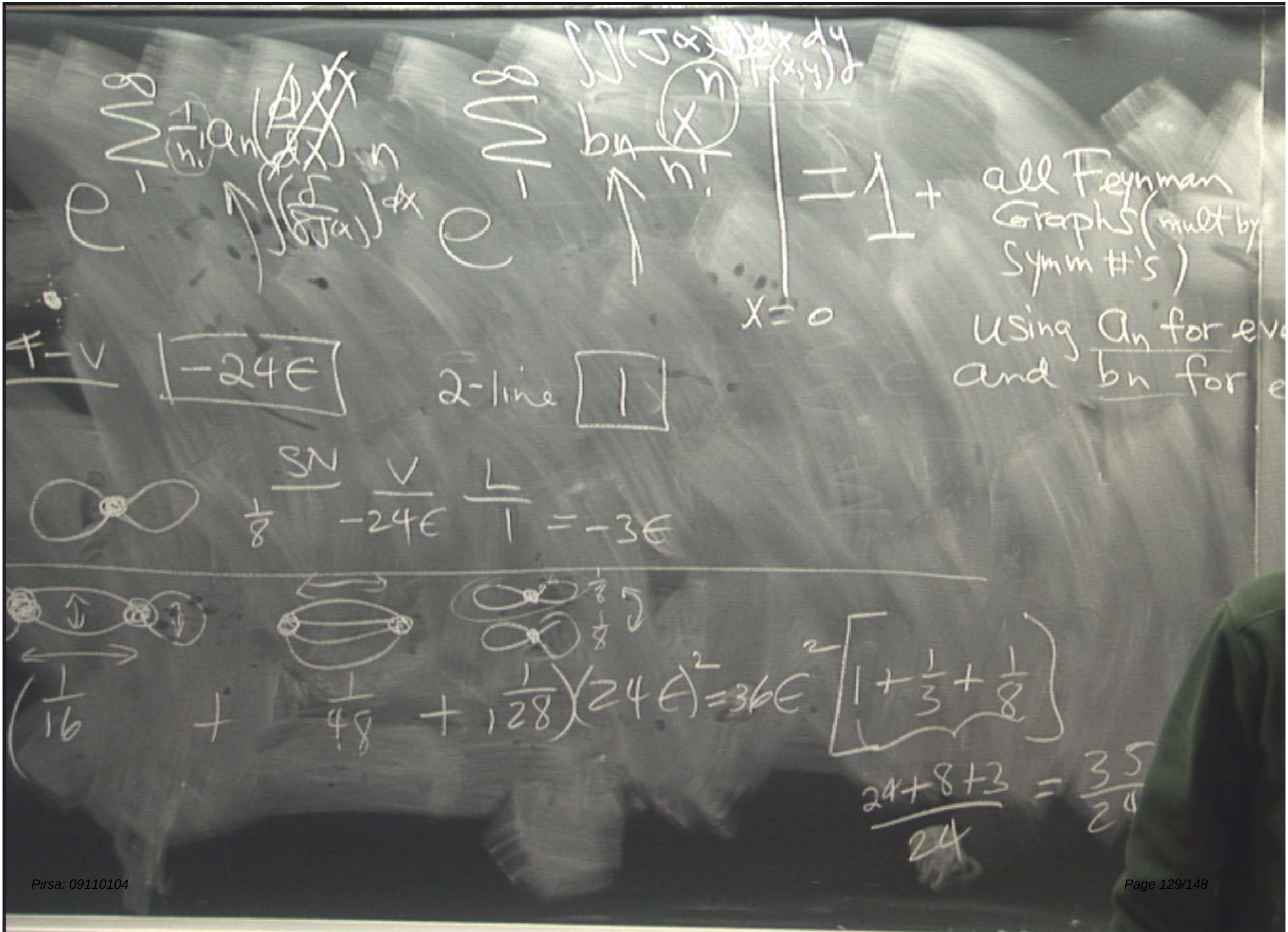
$$I(\epsilon) = \frac{1}{N}\, e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2}+Jx-\frac{J^2}{2}\boxed{+\frac{J^2}{2}}}\Big|$$

$$= e^{-\epsilon\left(\frac{d}{dJ}\right)^4}\; e^{\frac{1}{2}J^2}\Big|_{J=0}$$

4-vertex

2-line

$$= \frac{105}{2}\epsilon^2$$



$$e^{\sum_1^\infty \frac{1}{n!} a_n \left(\cancel{\frac{d}{dx}}\right)^n} \quad e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Bigg|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

$$\uparrow \int\!\!\int\left(\frac{\delta}{\delta J(\alpha)}\right) dx \qquad \uparrow \int\!\!\int (J(\alpha)) \, dx\, dy \, F(x,y)$$

using $\underline{a_n}$ for ev

and $\underline{b_n}$ for e

$\underline{F-V}$ $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

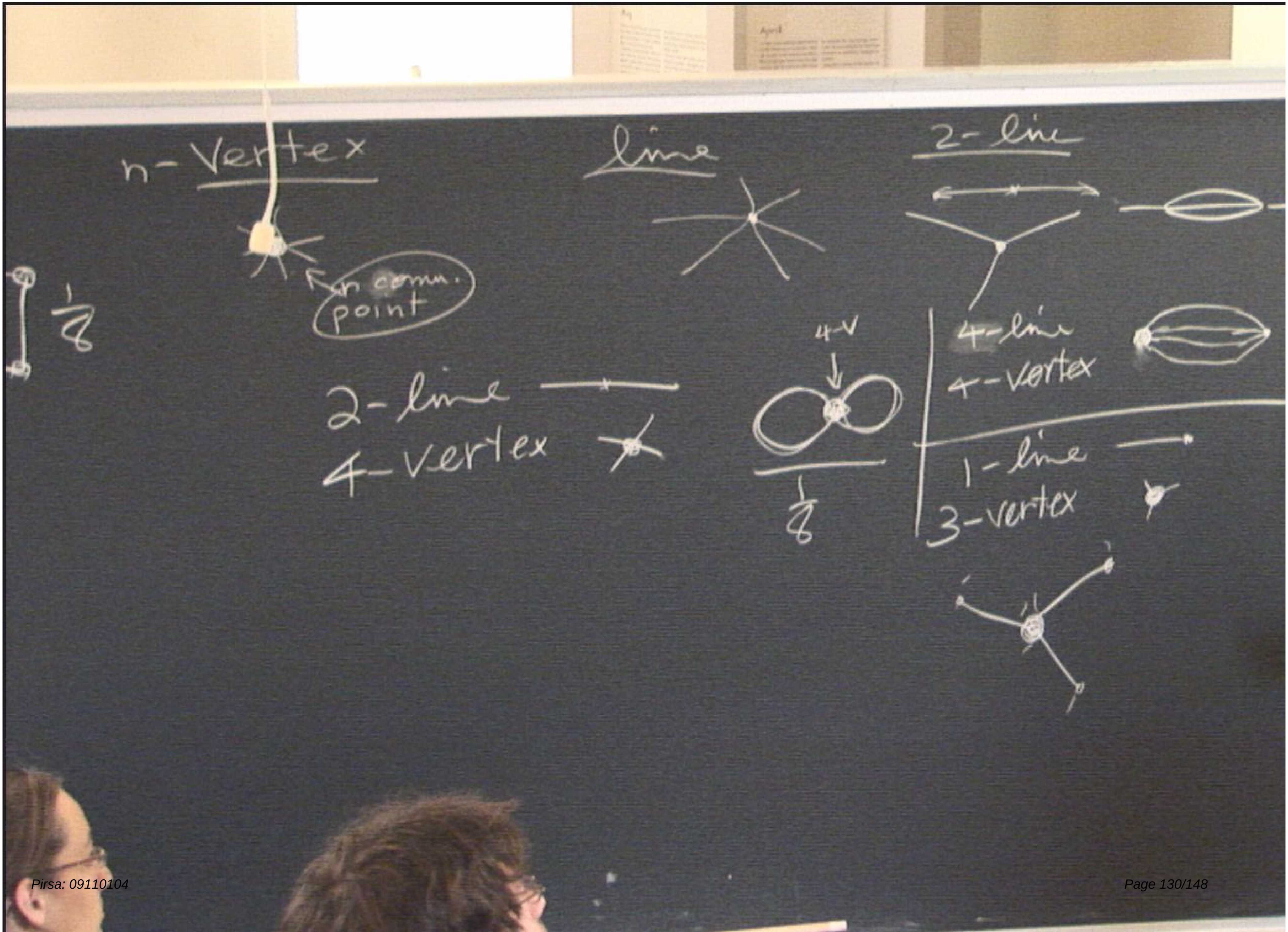
$$\frac{1}{8} \quad \frac{\underline{SN}}{} \quad \frac{\underline{V}}{-24\epsilon} \quad \frac{\underline{L}}{1} = -3\epsilon$$

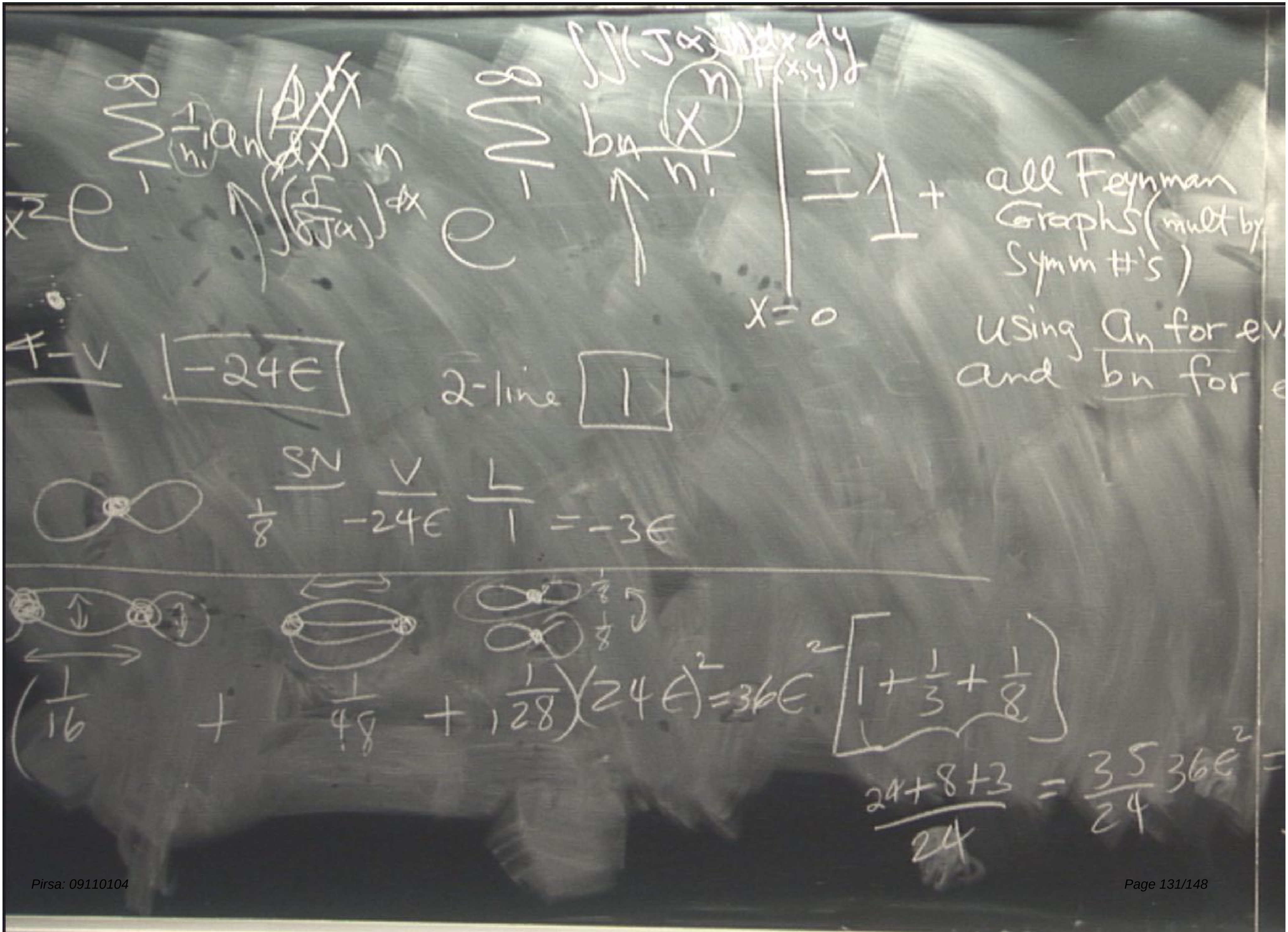
$\frac{1}{8}$

$\frac{1}{8}$

$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128}\right)(24\epsilon)^2 = 36\epsilon^2\left[\underbrace{1 + \frac{1}{3} + \frac{1}{8}}\right]$$

$$\frac{24+8+3}{24} = \frac{35}{24}$$


n-Vertex
n comm. point
line
2-line
1/8
2-line
4-vertex
4-V
1/8
4-line
4-vertex
1-line
3-vertex



$$x^2 \quad e^{\sum_1^\infty \frac{1}{n!} a_n (\cancel{dx})^n}$$

$$\uparrow \int \left(\frac{\delta}{\delta J(x)}\right)^4 dx$$

$$\iint (J(x) \ldots) dx\, dy\; F(x,y)$$

$$e^{\sum_1^\infty b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 + \text{all Feynman Graphs (mult by Symm \#'s)}$$

using a_n for ev

and b_n for e

F-V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

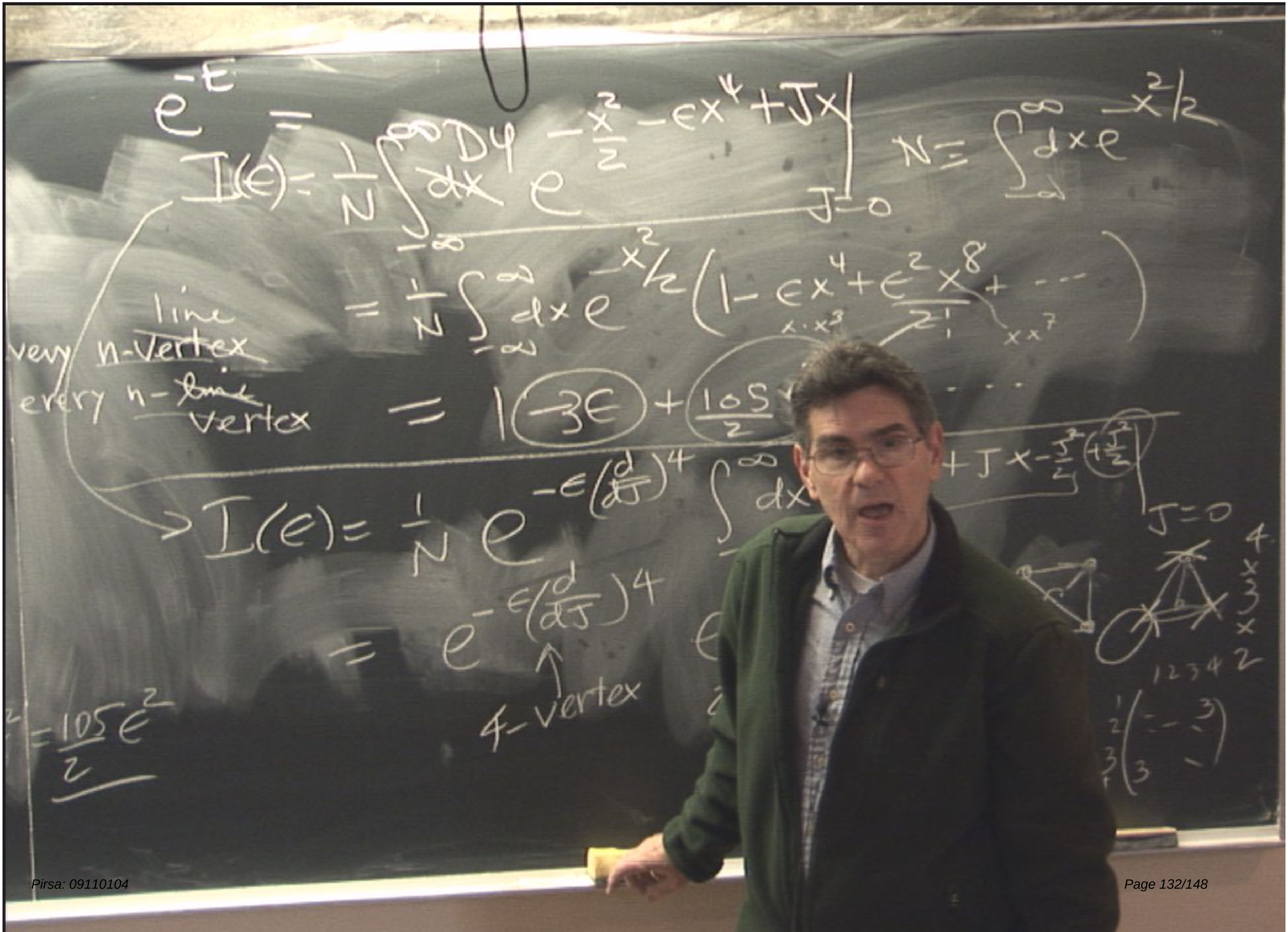
$$\frac{1}{8} \quad \frac{SN}{} \quad \frac{V}{-24\epsilon} \quad \frac{L}{1} = -3\epsilon$$

$\frac{1}{8}$

$\frac{1}{8}$

$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128}\right)(24\epsilon)^2 = 36\epsilon^2 \left[1 + \frac{1}{3} + \frac{1}{8}\right]$$

$$\frac{24+8+3}{24} = \frac{35}{24}\, 36\epsilon^2 =$$



$$e^{-E} =$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{\cancel{dx}}\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \underset{x\cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 x^8}{2!} + \underset{x x^7}{\cdots}\Big)$$

very n-~~Vertex~~ line

every n-~~line~~ vertex

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}} \cdots$$

$$I(\epsilon) = \frac{1}{N} e^{-\epsilon\left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx \quad + Jx - \frac{J^2}{2}\left(+\frac{J^2}{2}\right)\Big|_{J=0}$$

$$= e^{-\epsilon\left(\frac{d}{dJ}\right)^4} \quad e$$

4-vertex

$$= \frac{105}{2}\epsilon^2$$

4 × 3 × 2

1 2 3 4 2

x^2 $e^{\sum_1^\infty \frac{1}{n!} a_n \left(\cancel{\frac{d}{dx}}\right)^n}$ $\uparrow \int \left(\frac{\delta}{\delta J(x)}\right) dx$

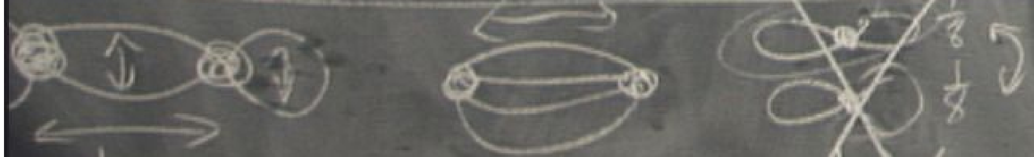
$e^{\sum_1^\infty b_n \frac{x^n}{n!}}$ $\uparrow \iint (J(x)J(y)) dx\,dy\, F(x,y)$

$\Big|_{x=0} = 1 +$ all Feynman Graphs (mult by Symm #'s)

using a_n for ev
and b_n for e

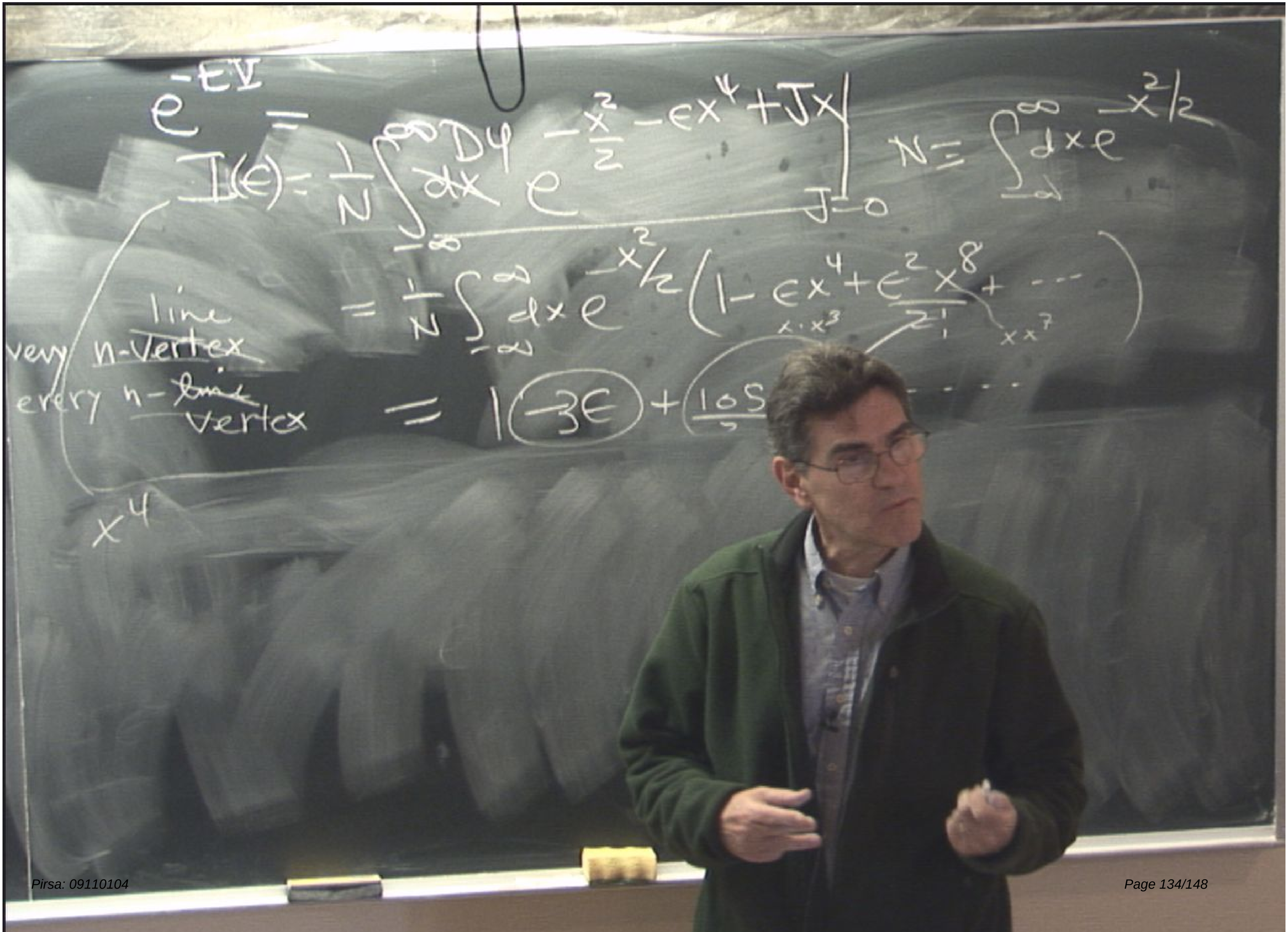
F−V $\boxed{-24\epsilon}$ 2-line $\boxed{1}$

$\frac{1}{8}$ $\frac{SN}{}$ $\frac{V}{-24\epsilon}$ $\frac{L}{1} = -3\epsilon$



$\left(\frac{1}{16} + \frac{1}{48} + \cancel{\frac{1}{128}}\right)(24\epsilon)^2 = 36\epsilon^2 \left[\underbrace{1 + \frac{1}{3} + \frac{1}{8}}\right]$

$\frac{24+8+3}{24} = \frac{35}{24} 36\epsilon^2 =$



$$e^{-tV} =$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{\cancel{dx}}\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x \cdot x^3}{x^4} + \epsilon^2 \frac{\overset{}{\underset{x x^7}{x^8}}}{2!} + \cdots\right)$$

$$= 1 \boxed{-3\epsilon} + 105 \cdots$$

line

every ~~n-Vertex~~

every n- ~~line~~ vertex

x^4

e^{-tV} =

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \cancel{dx}\, D\varphi\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N \equiv \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \underset{x \cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 x^8}{2!} + \cdots\Big) \quad x\,x^7$$

$$= 1 \boxed{-3\epsilon} + \boxed{\underline{105}\,\epsilon^2} + \cdots$$

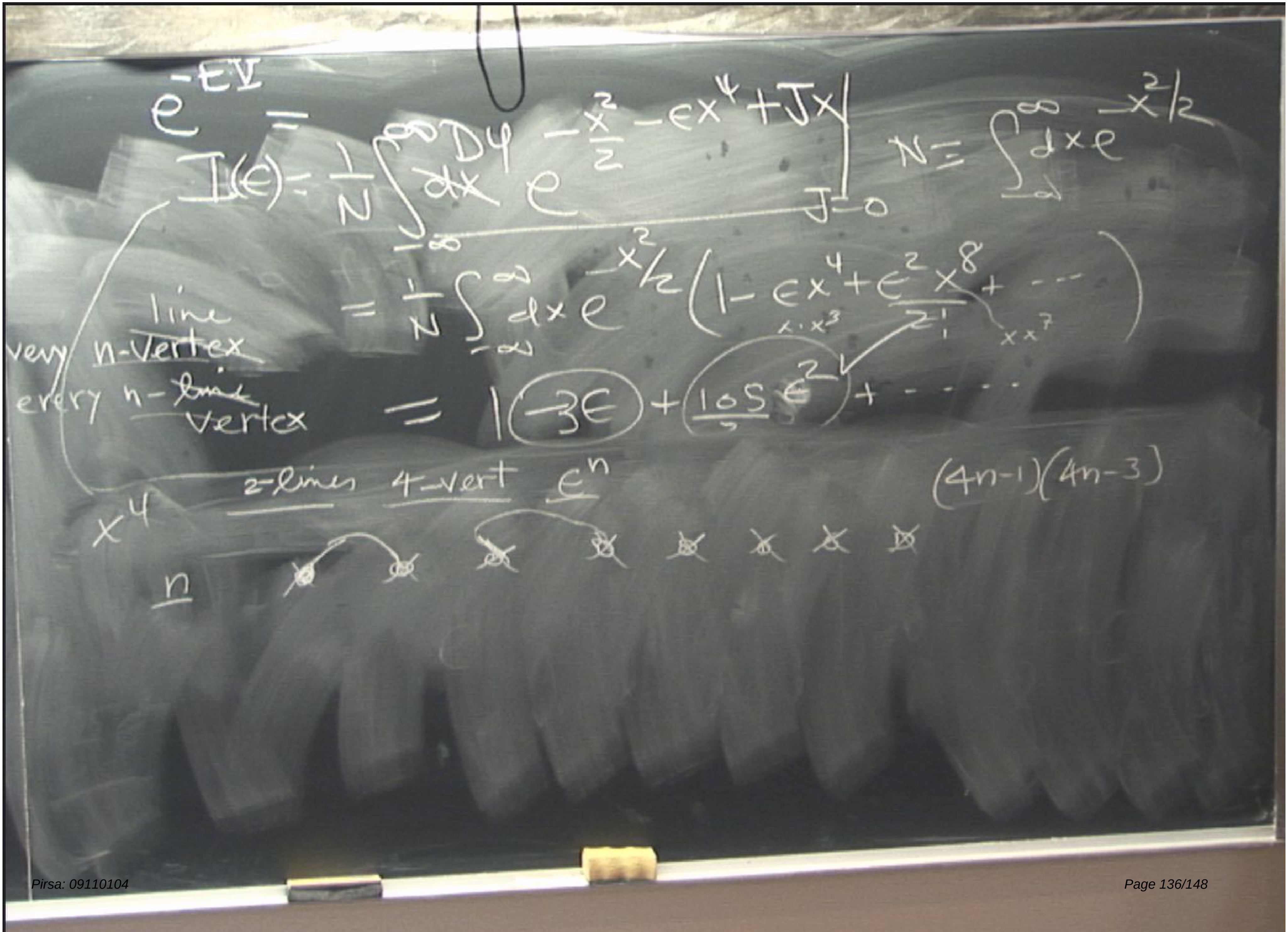
every n-vertex
line

every n-~~line~~
vertex

2-lines 4-vert ϵ^n

x^4

n



$e^{-\epsilon V} =$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{\cancel{dx}}\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \underset{x\cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 \underset{x\,x^7}{x^8}}{2!} + \cdots\Big)$$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

line

every n-vertex

every n-~~line~~ vertex

2-lines 4-vert ϵ^n $(4n-1)(4n-3)$

x^4

n

$$e^{-\epsilon V} =$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \cancel{dx}\, D\varphi\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\right)$$

line

~~very~~ n-vertex

every n-~~line~~ vertex

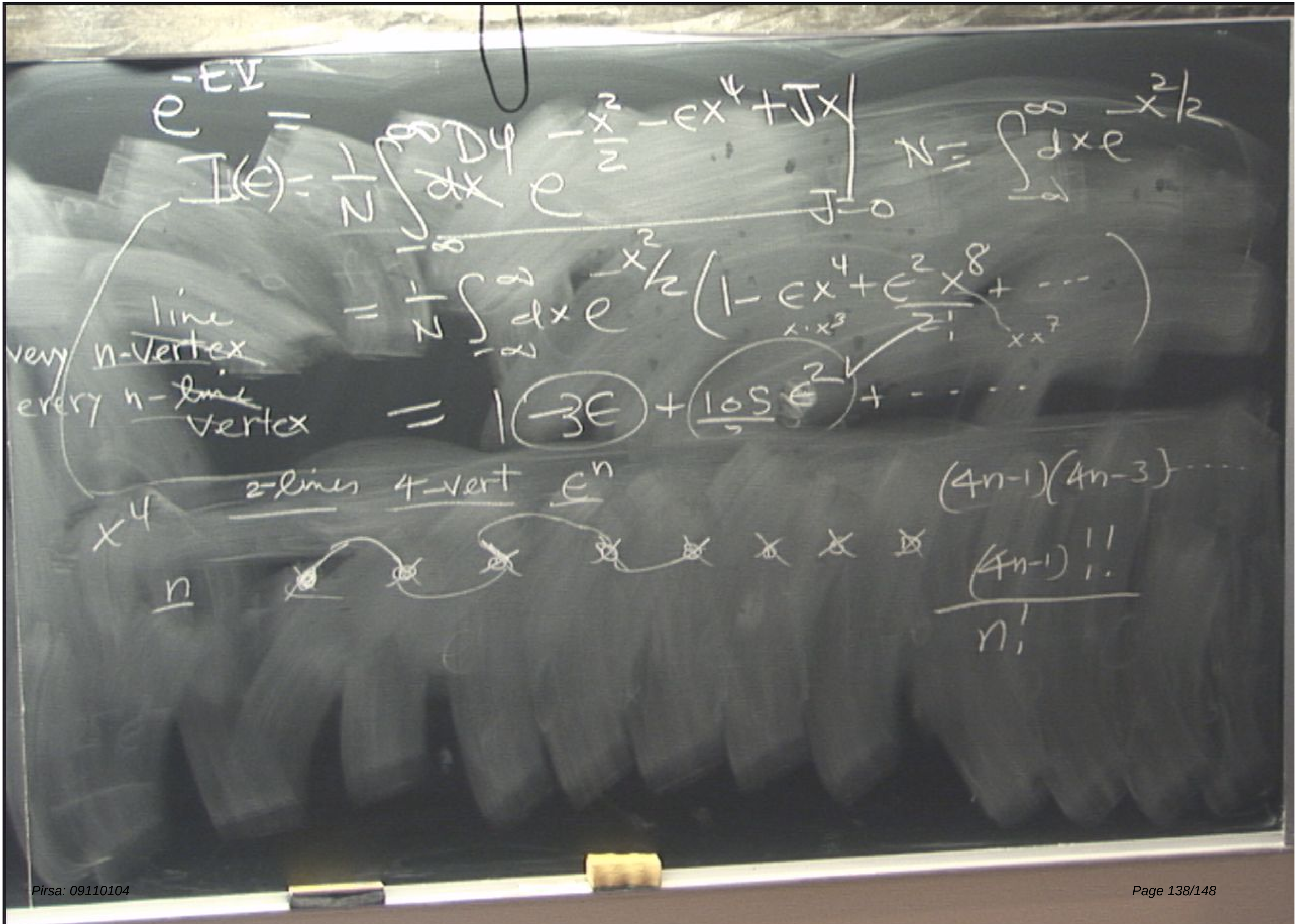
$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

x^4 2-lines 4-vert ϵ^n

$(4n-1)(4n-3)\cdots$

n

$(4n-1)!!$



e^{-tV} =

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{dx}\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\, x^7}{x^8}}{2!} + \cdots\right)$$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

line

every n-vertex

every n-~~line~~ vertex

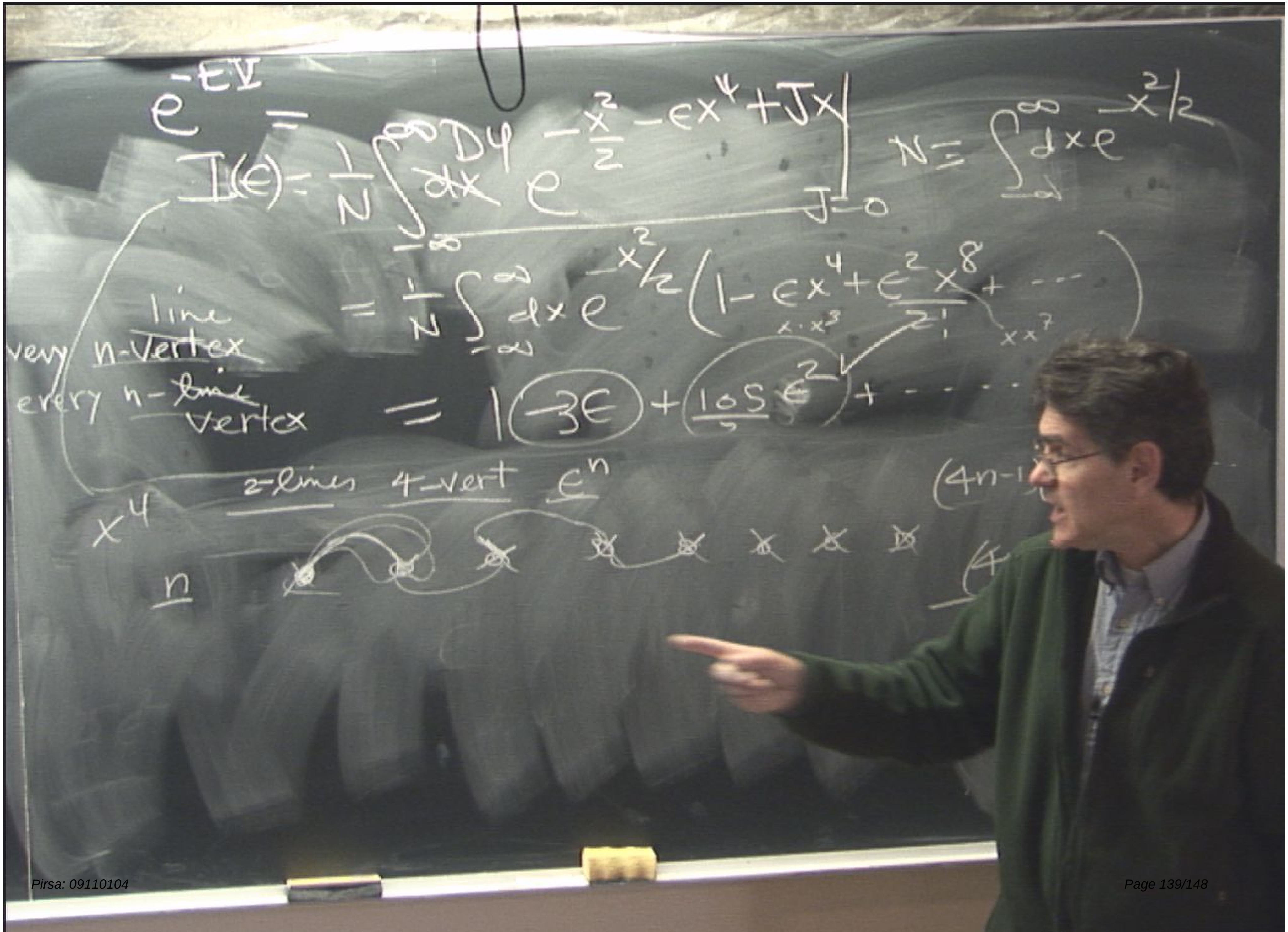
2-lines 4-vert ϵ^n

x^4

n

$(4n-1)(4n-3)\cdots\cdots$

$$\frac{(4n-1)!!}{n!}$$



$$e^{-tV} = $$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \cancel{dx}\, D\varphi\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 x^8}{2!} + \cdots\Big) \quad x\,x^7$$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

line

every n-vertex

every n-~~line~~ vertex

x^4 2-lines 4-vert ϵ^n $(4n-1)$

n $(4$

e^{-tV} =

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \cancel{dx}\, D\varphi \, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \epsilon \underset{x \cdot x^3}{x^4} + \frac{\epsilon^2 \underset{x\, x^7}{x^8}}{2!} + \cdots\Big)$$

line

every n-Vertex

every n-~~line~~ vertex

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

2-lines 4-vert ϵ^n $(4n-1)(4n-3)\cdots$

x^4

n

$$\frac{(4n-1)!!}{n!\, 24^n}$$

$$e^{-\epsilon V}$$

$$=$$

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \cancel{dx}\, D\varphi\; e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \underset{x \cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 \underset{x x^7}{x^8}}{2!} + \cdots\Big)$$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

every n-vertex / line

every n-~~line~~ vertex

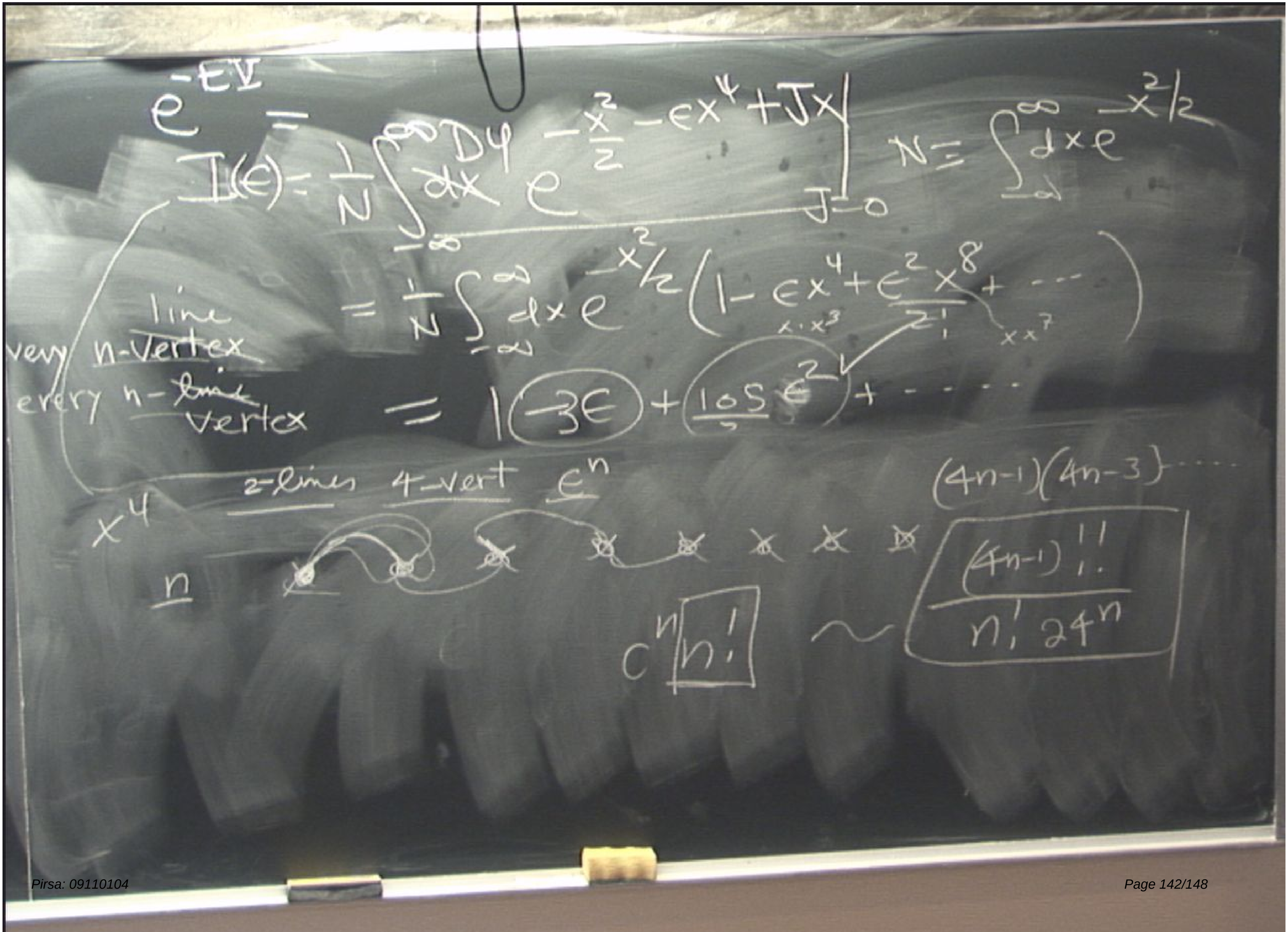
2-lines 4-vert ϵ^n

x^4

n

$(4n-1)(4n-3)\cdots\cdots$

$$\boxed{\frac{(4n-1)!!}{n!\,24^n}}$$



e^{-tV} =

$$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{\cancel{dx}}\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0} \qquad N = \int_{-\infty}^{\infty} dx\, e^{-x^2/2}$$

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1 - \epsilon \underset{x\cdot x^3}{x^4} + \frac{\epsilon^2 \overset{x x^7}{x^8}}{2!} + \cdots\Big)$$

line

very n-Vertex

every n-~~line~~ Vertex

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

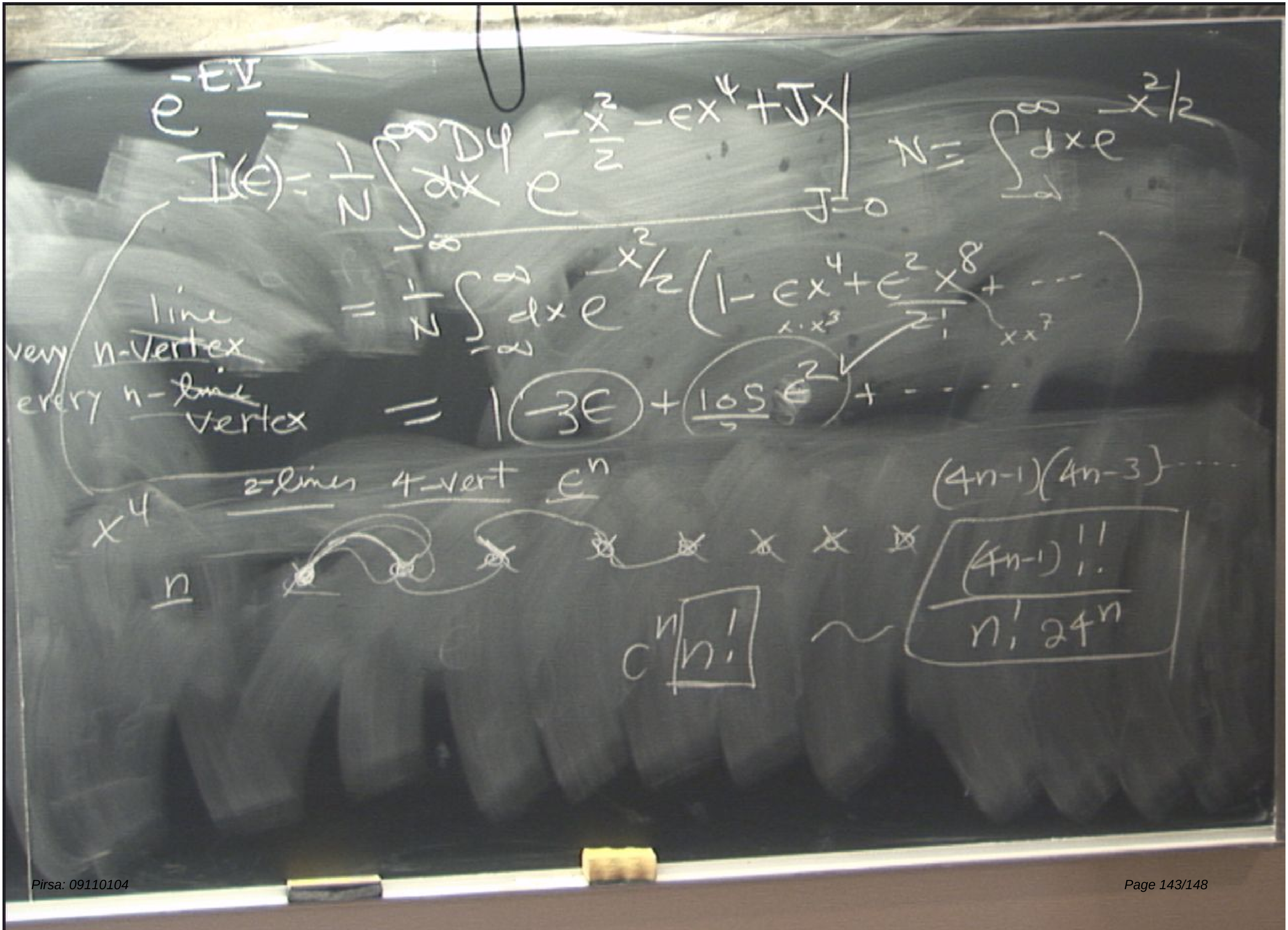
2-lines 4-vert ϵ^n

x^4

n

$(4n-1)(4n-3)\cdots$

$$c^n \boxed{n!} \sim \boxed{\frac{(4n-1)!!}{n!\, 24^n}}$$



$e^{-\epsilon V} =$

$I(\epsilon) = \frac{1}{N}\int_{-\infty}^{\infty} \overset{D\varphi}{\cancel{dx}}\, e^{-\frac{x^2}{2} - \epsilon x^4 + Jx}\Big|_{J=0}$ $N = \int dx\, e^{-x^2/2}$

$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x \cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 x^8}{2!} + \cdots\right)$ $x\,x^7$

$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \cdots$

$\frac{\text{line}}{\text{every n-vertex}}$

every n-~~line~~ vertex

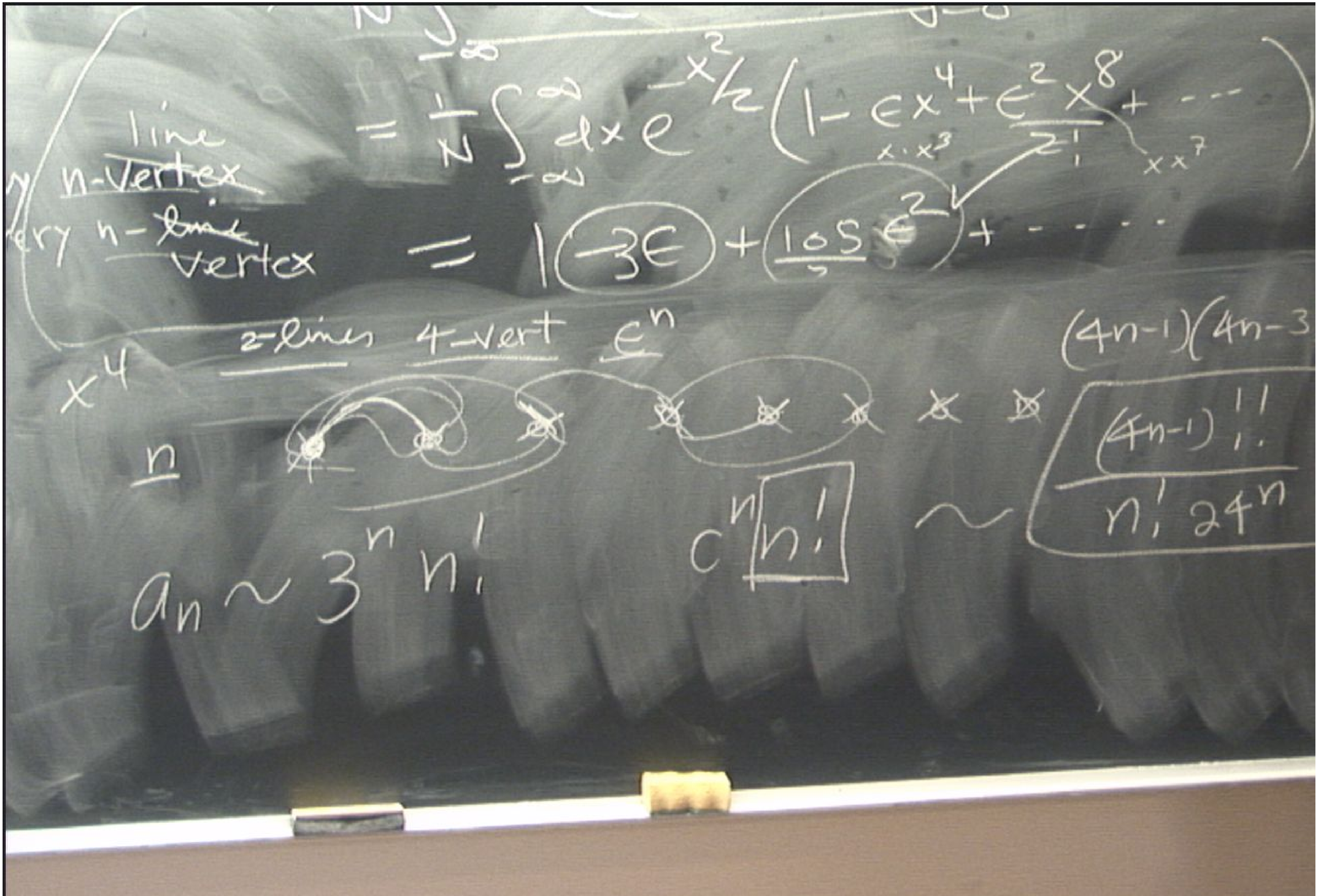
2-lines 4-vert ϵ^n

x^4

n

$(4n-1)(4n-3)\cdots$

$c^n \boxed{n!} \sim \boxed{\frac{(4n-1)!!}{n!\,24^n}}$



line

n-Vertex

n- ~~line~~ vertex

$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1-\underset{x\cdot x^3}{\epsilon x^4}+\frac{\epsilon^2 \underset{x x^7}{x^8}}{2!}+\cdots\right)$$

$$= 1\boxed{-3\epsilon}+\boxed{\underline{105}\,\epsilon^2}+\cdots$$

2-lines 4-vert ϵ^n

$(4n-1)(4n-3$

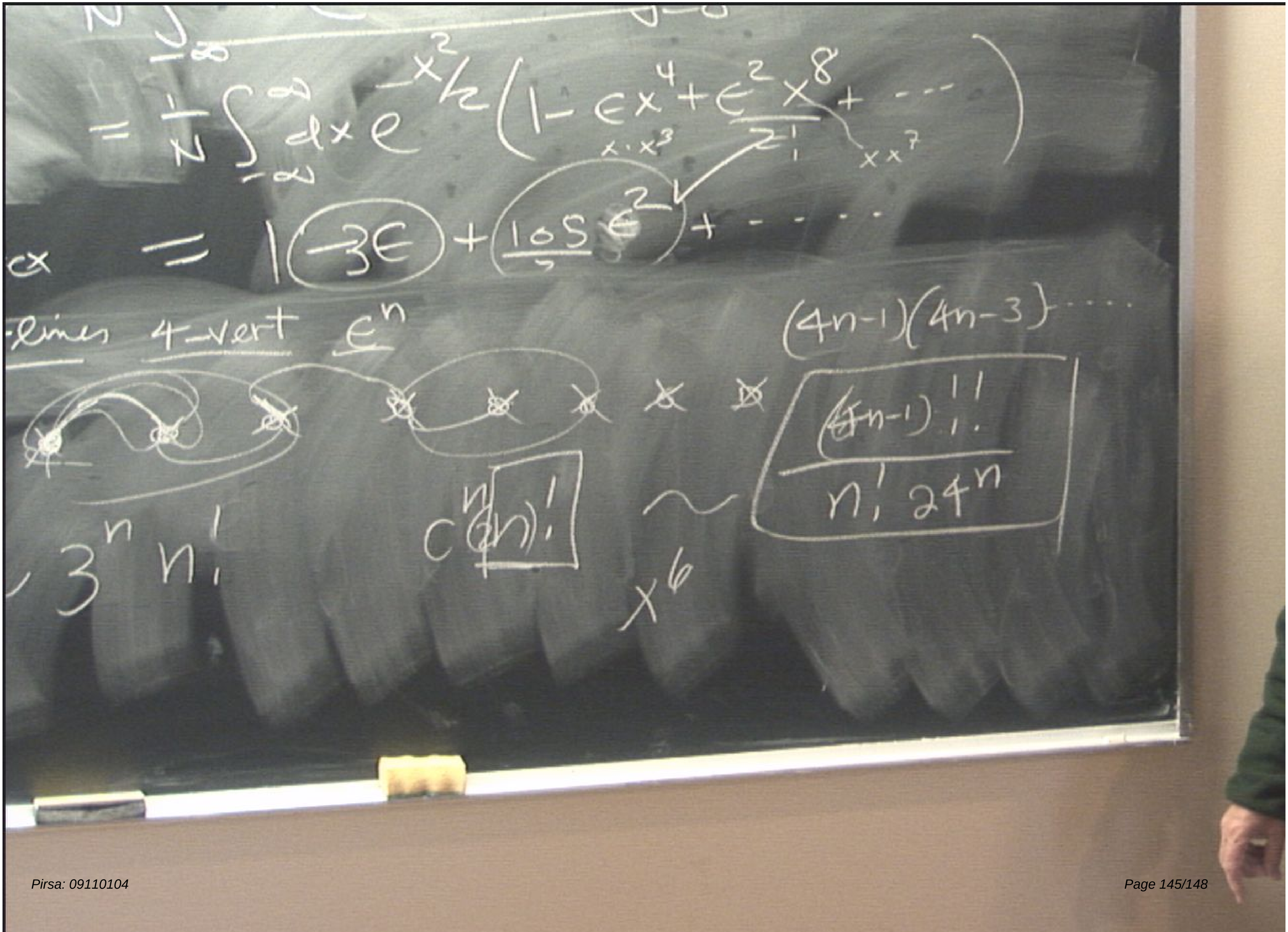
x^4

$\underline{n}$

$$\sim \boxed{\frac{(4n-1)!!}{n!\,24^n}}$$

$c^n\boxed{n!}$

$a_n \sim 3^n n!$



$$= \frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1 - \underset{x\cdot x^3}{\epsilon x^4} + \frac{\epsilon^2 x^8}{2!} + \cdots\right)$$

$x\,x^7$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

lines 4-vert ϵ^n

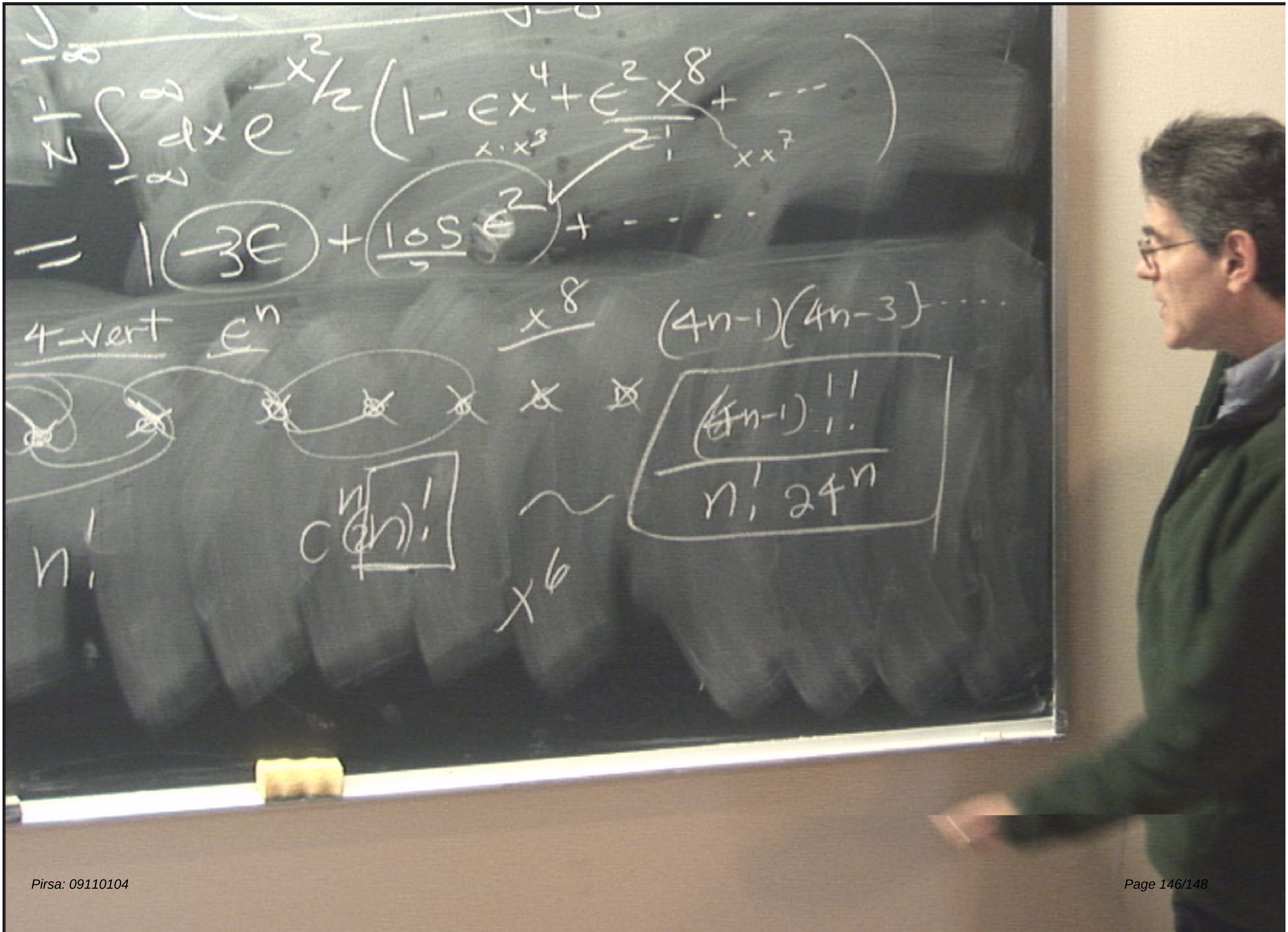
$(4n-1)(4n-3)\cdots\cdots$

$3^n\, n!$

$c^n \boxed{(2n)!}$

$\sim \boxed{\frac{(4n-1)!!}{n!\, 24^n}}$

x^6



$$\frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1-\underset{x\cdot x^3}{\epsilon x^4}+\frac{\epsilon^2 x^8}{2!}+\cdots\right)$$

$x x^7$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

4-vert $\quad \epsilon^n \quad x^8 \quad (4n-1)(4n-3)\cdots$

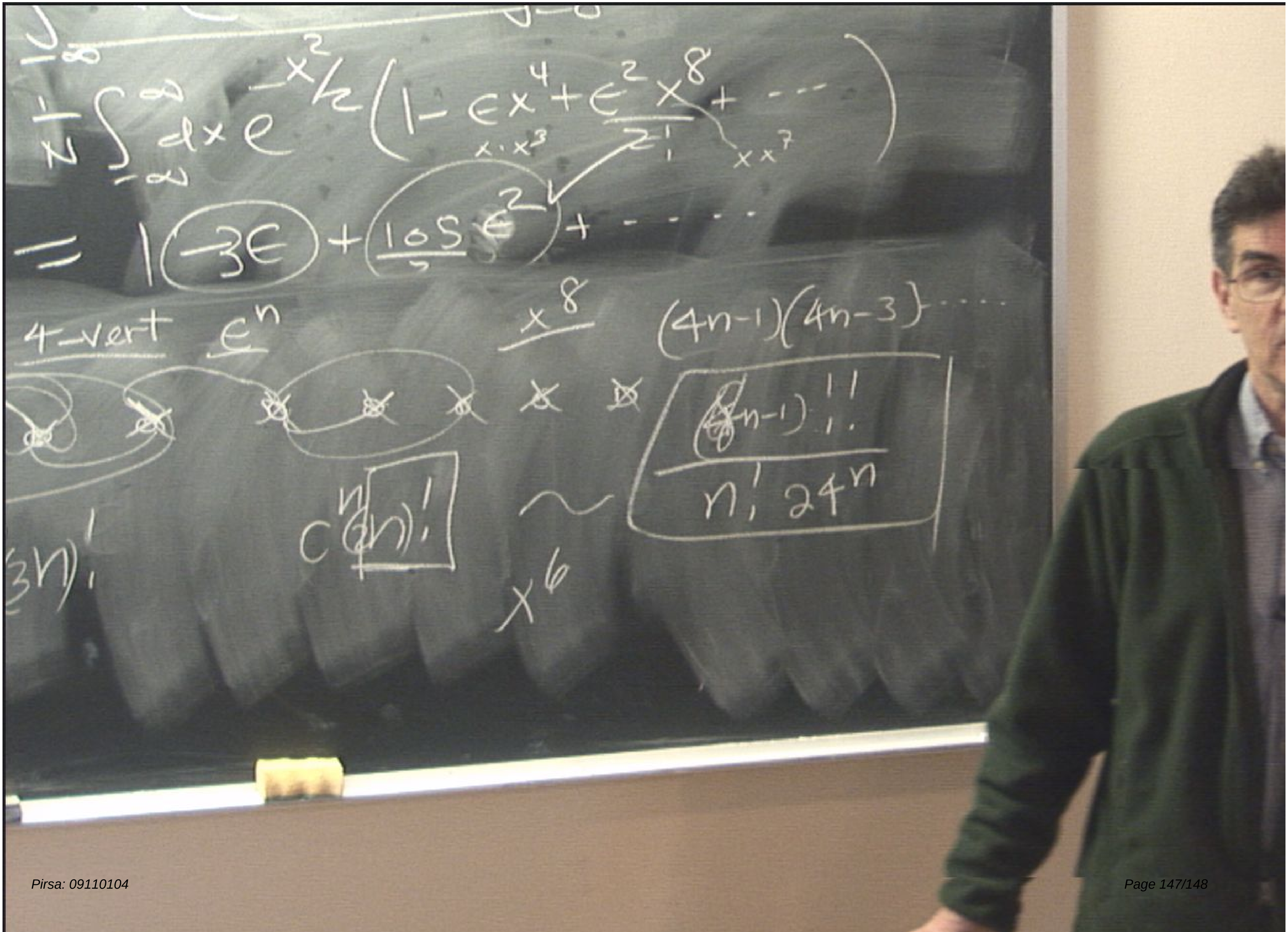
$$\boxed{\frac{(4n-1)!!}{n!\,24^n}}$$

$n!$

$c^n \boxed{(2n)!}$

$\sim$

x^6



$$\frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\Big(1-\underset{x\cdot x^3}{\epsilon x^4}+\frac{\epsilon^2 \overset{}{x^8}}{2!}+\cdots\Big)$$

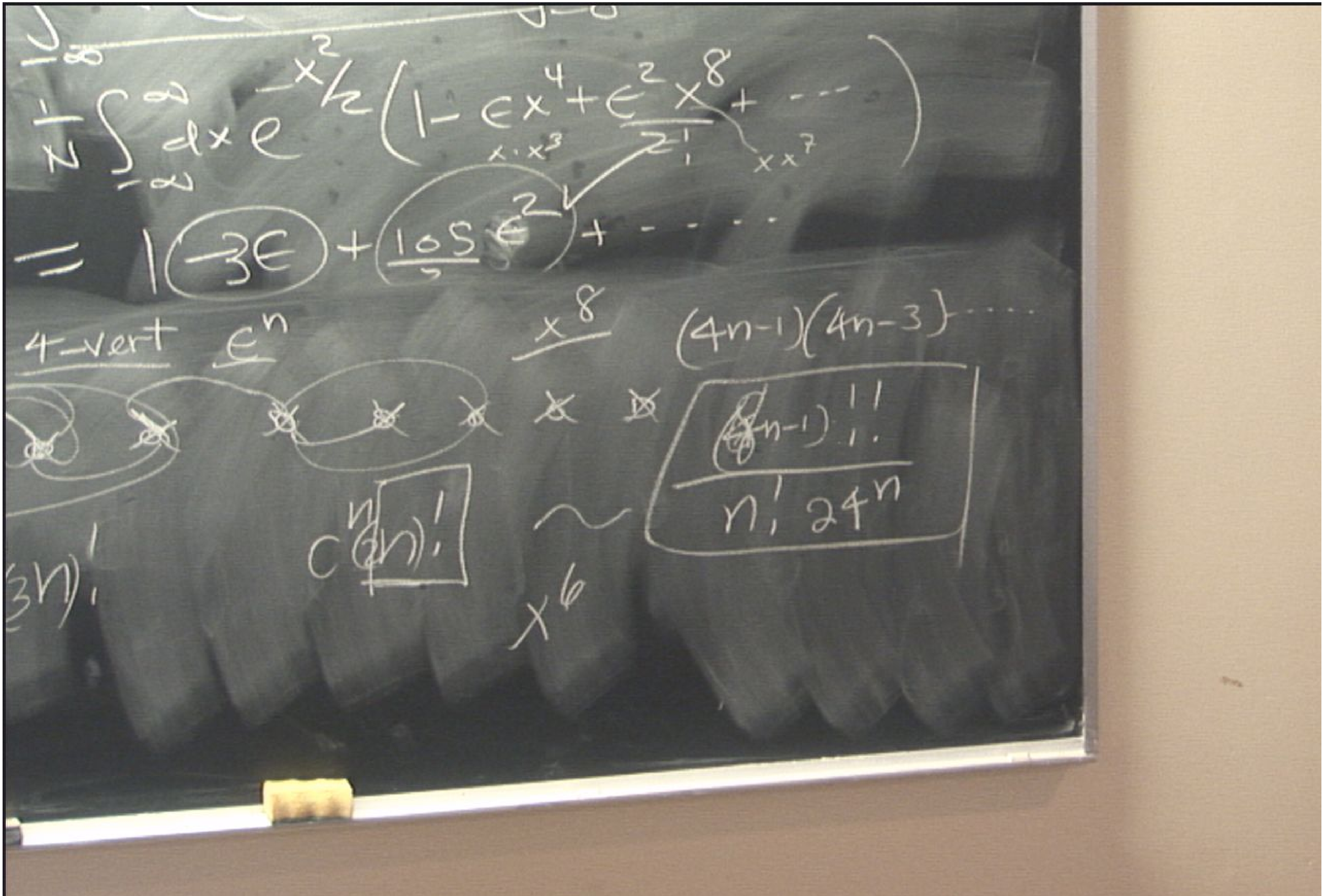
$x\,x^7$

$$= 1 \boxed{-3\epsilon} + \boxed{\frac{105}{2}\epsilon^2} + \cdots$$

4-vert $\quad \epsilon^n \quad x^8 \quad (4n-1)(4n-3)\cdots$

$$\boxed{\frac{(4n-1)!!}{n!\,24^n}}$$

$$c^n\,\boxed{(2n)!} \sim \qquad (3n)! \qquad x^6$$



$$\frac{1}{N}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\left(1-\underset{x\cdot x^3}{\epsilon x^4}+\epsilon^2 \frac{\underset{x x^7}{x^8}}{2!}+\cdots\right)$$

$$= 1 \boxed{-3\epsilon} + \frac{105}{2}\epsilon^2 + \cdots$$

4-vert ϵ^n x^8 $(4n-1)(4n-3)\cdots$

$(3n)!$ $c^n\boxed{(2n)!}$ $\sim \boxed{\frac{(4n-1)!!}{n!\,24^n}}$

x^6