


Title: Mathematical Physics (PHYS 624) - Lecture 10

Date: Nov 27, 2009 09:00 AM

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Abstract:

PERIMETER  INSTITUTE FOR THEORETICAL PHYSICS

$$-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \} \psi(x) = E \psi$$

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if $f(x)$ is Herglotz AND Entire

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

if $f(x)$ is Herglotz AND Entire
 $f(x) = \sum a_n x^n$
 $R \neq C = \infty$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

im if $f(x)$ is Herglotz AND
then $f(x)$ is trivial —
i.e. $f(x) = Ax + B$

$$\text{Entire} \\ f(x) = \sum a_n x^n \\ R \text{ of } C = \infty.$$

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$$\text{Entire } f(x) = \sum a_n x^n$$

$R \text{ or } C = \infty$

$$\text{Im}(f(x)) = \text{Im}\left(\sum a_n x^n\right) \text{ same sign as } \text{Im } x$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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 $f(x) = \sum a_n x^n$
 $R \text{ or } C = \infty$

$\text{Im}(f(x)) = \text{Im}\left(\sum \underbrace{a_n x^n}_{a_n r^n e^{in\theta}}\right)$ same sign as $\text{Im } x$
 $x = r e^{i\theta}$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

im if $f(x)$ is Herglotz AND
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Entire

$$\frac{f(x) = \sum a_n x^n}{R \text{ or } C = \infty}$$

$$\text{Im}(f(x)) = \text{Im}\left(\sum \underbrace{a_n x^n}_{a_n r^n e^{in\theta}}\right)$$

$x = r e^{i\theta}$

same sign as $\frac{\text{Im } x}{r \sin \theta}$

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same sign as $\frac{\text{Im } x}{r \sin \theta}$

$$x = r e^{i\theta}$$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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 $f(x) = \sum a_n x^n$
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$$\text{Im}(f(x)) = \text{Im}\left(\sum a_n x^n\right)$$

same sign

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$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

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$$x = r e^{i\theta}$$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

$$m \sin \theta \pm \sin(m\theta) \text{ Same sign}$$

same sign as $\frac{\text{Im } x}{r \sin \theta}$

same sign as $r \sin \theta$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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$$\frac{f(x) = \sum a_n x^n}{R \text{ or } C = \infty}$$

$$\text{Im}(f(x)) = \text{Im}\left(\sum a_n x^n\right)$$

$x = r e^{i\theta}$

same sign as $\frac{\text{Im } x}{r \sin \theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

same sign as $r \sin \theta$

$$m \sin \theta \pm \sin(m\theta) \text{ same sign } \sin \theta$$

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$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

same sign as $r \sin \theta$

$$\boxed{m \sin \theta \pm \sin(m\theta)}$$

same sign $\sin \theta$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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$x = r e^{i\theta}$

same sign as $\frac{\text{Im } x}{r \sin \theta}$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

same sign as $r \sin \theta$

\int_0^π

$m \sin \theta \pm \sin(m\theta)$

same sign as $\sin \theta$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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 $f(x) = \sum a_n x^n$
 $R \text{ or } C = \infty$

$$\text{Im}(f(x)) = \text{Im}\left(\sum a_n x^n\right)$$

same sign as $\frac{\text{Im}}{r^5}$

$$x = r e^{i\theta}$$

$$\sum_{n=0}^{\infty} a_n r^n \sin(n\theta)$$

same sign as $r \sin$

$$\int_0^\pi m \sin \theta \pm \sin(m\theta)$$

same sign as $\sin \theta$

$mqr \pm amr^m$



$m \leq r \leq n$

positive

$$m \geq r \pm \frac{a_m t^m}{a_m} \\ a_m = 0 \text{ for } m > 1$$

positive

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \right] \psi(x) = E \psi(x)$$

↑
grnd state.

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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grnd state,

you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

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$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n$$

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$$E(E)'' = \sum_{n=0}^{\infty} e^n E_n, \quad \psi'' = \sum_{n=0}^{\infty} e^n \psi_n$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi$$

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$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$-\frac{1}{2} \psi_n'' + V(x) \psi_n$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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$$E(E) = \sum_{n=0}^{\infty} e^{-n} E_n, \quad \psi = \sum_{n=0}^{\infty} e^{-n} \psi_n$$

(n) $-\frac{1}{2} \psi_n'' + V(x) \psi_n +$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \right] \psi(x) = E \psi$$

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$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

(n) $-\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} =$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \right] \psi(x) = E \psi$$

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you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$-\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^n \psi_j E_{n-j}$$

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$$n) \quad -\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=-1}^{n-1} \psi_j E_{n-j}$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \right] \psi(x) = E \psi$$

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$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$-\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi$$

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you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$\textcircled{n} \quad -\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

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$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$-\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

Let $\psi_n = \psi_0^{(n)} Q_n^{(x)}$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

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$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$n) \quad -\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

Let $\psi_n = \psi_0^{(x)} Q_n^{(x)}$

$$-\frac{1}{2} \psi_0'' Q_n - \frac{1}{2} \psi_0 Q_n'' - \psi_0' Q_n'$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi$$

↑
grnd state.

you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n$$

$$n) \quad -\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

Let $\psi_n = \psi_0^{(x)} Q_n^{(x)}$

$$-\frac{1}{2} \psi_0'' Q_n - \frac{1}{2} \psi_0 Q_n'' - \psi_0' Q_n' + V(x) \psi_0 Q_n + W(x) \psi_0 Q_{n-1} =$$

$$\psi(x) = E \psi$$

↑ grnd state,

$$\left(\frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$$

$$\psi = \sum_{n=0}^{\infty} c_n \psi_n(x)$$

$$V(x) \psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$+ V(x) \psi_0 Q_n + W(x) \psi_0 Q_{n-1} = \psi_0$$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n + \psi_0 Q_n E_0$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) + \epsilon W(x) \right] \psi(x) = E \psi(x)$$

↑
grnd state.

solve $\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$\psi = \sum_{n=0}^{\infty} e^{-\beta E_n} \psi_n$$

$$\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$\psi_n = \psi_0 Q_n(x)$$

$$-\frac{\hbar^2}{2m} \psi_0 Q_n'' - \psi_0 Q_n' + V(x) \psi_0 Q_n + W(x) \psi_0 Q_{n-1} = \psi_0 E_n$$

$$\sum_{j=0}^{n-1} Q_j E_{n-j}$$

state,

$$\psi_0$$

E_n

$$n-j + \psi_0 E_n + \psi_n E_0$$

$$\psi_0 \sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n + \cancel{\psi_0 \psi_n E_0} - W \alpha \gamma \psi_0 Q_{n-1}$$

ste,

$$\psi_0$$

n

$$-j + \psi_0 E_n + \psi_n E_0$$

$$-2 \left(\psi_0 \sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_0 E_n \right)$$

$$-W \alpha \gamma \psi_0 Q_{n-1}$$

$$\sum \psi_n + V(x) \psi_n + V(x) \psi_n$$

$$\psi_n = \psi_0 Q_n(x)$$

$$\psi_0 Q_n'' + 2\psi_0 Q_n'$$

$\psi = \psi_0 Q_n(x)$

$\psi_0 Q_n'' + 2\psi_0 Q_n'$

$$V(x) + EW(x) \psi(x) = E\psi$$

↑
grnd state.

solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$\sum_{n=0}^{\infty} e^{E_n} \psi_n, \quad \psi = \sum_{n=0}^{\infty} e^{E_n} \psi_n$$

$$V(x)\psi_n + W(x)\psi_{n-1} = \sum_{j=-1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$= \psi_0^{(1)} Q_n^{(x)}$$

$$\psi_0^2 Q_n'' + 2\psi_0 Q_n'$$

$$= -2 \left(\psi_0 \right)$$

$$\sum_{j=1}^{n-1} Q_j E_{n-j} + \psi_n E_0$$

state,

$$\psi_0$$

$$\psi_n$$

$$\psi_0 E_n + \psi_n E_0$$

$$= -2 \psi_0 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + E_n \right]$$

$$-W(x) Q_n$$

$$\frac{d^2}{dx^2} + V(x) + E W(x) \int \psi(x) = H \psi$$

↑ grnd state,

can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$ $\psi_0 \rightarrow 0$ as $x \rightarrow \pm \infty$

$$\psi'' = \sum_{n=0}^{\infty} e^n E_n, \quad \psi = \sum_{n=0}^{\infty} e^n \psi_n$$

$$\psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$$\psi_n = \psi_0^{(x)} Q_n^{(x)}$$

$$\psi_0^2 Q_n'' + 2\psi_0 Q_n' = \frac{d}{dx} (\psi_0^2 Q_n') = -2 \psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} \right]$$

$\int_{-\infty}^{\infty}$

state,

$$E_0 \psi_0$$

$$\psi_0 \rightarrow 0 \text{ as } x \rightarrow \pm\infty$$

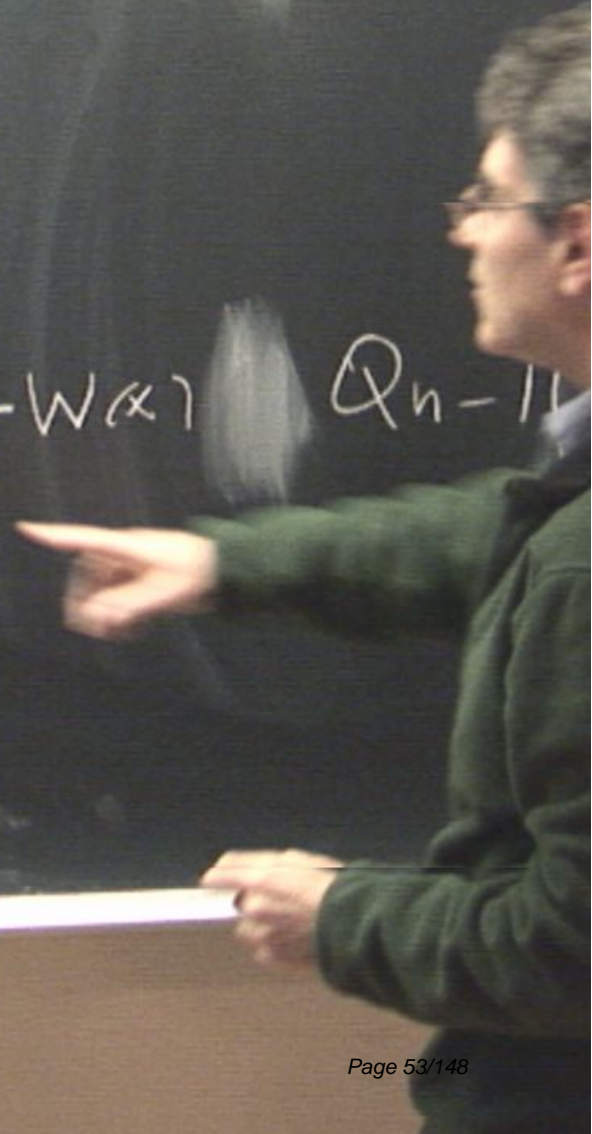
$$E_n =$$

$$\psi_n$$

$$E_{n-1} + \psi_0 E_n + \psi_n E_0$$

$$= -2 \int \psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{\psi_0} \right]$$

$$-W(x) Q_{n-1}$$



state,

$$\psi_0$$

$$\psi_0 \rightarrow 0 \text{ as } x \rightarrow \pm\infty$$

$$E_n =$$

$$\int_{-\infty}^{\infty} \psi_0^2 dx$$

$$\psi_n$$

$$E_{n-1} + \psi_0 E_n + \psi_n E_0$$

$$= -2 \int \psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{-W(x)} Q_{n-1} \right]$$

$\lim_{x \rightarrow \pm\infty} \psi = 0$

$$E_n = \frac{\int_{-\infty}^{\infty} W(x) Q_{n-1} \psi_0^2 - \int_{-\infty}^{\infty} \sum_{j=1}^{n-1} Q_j E_{n-j}}{\int_{-\infty}^{\infty} \psi_0^2 dx}$$

$k_n E_0$

$$\left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{-W(x) Q_{n-1}} \right]$$

$\lim_{m \rightarrow \pm \infty} x \rightarrow \pm \infty$

$$E_n = \frac{\int W(x) Q_{n-1} \psi_0^2 dx - \int \sum_{j=1}^{n-1} Q_j E_{n-j} \psi_0^2 dx}{\int_0^{\infty} \psi_0^2 dx}$$

$k_n E_0$

$$\left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{-W(x) Q_{n-1}} \right]$$

$\lim_{n \rightarrow \pm \infty} \psi \rightarrow 0$

$$E_n = \frac{\int_{-\infty}^{\infty} W(x) Q_{n-1} \psi_0^2 dx - \int_{-\infty}^{\infty} \sum_{j=1}^{n-1} Q_j E_{n-j} \psi_0^2 dx}{\int_{-\infty}^{\infty} \psi_0^2 dx}$$

$E_n E_0$

$$\left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{-W(x) Q_{n-1}} \right]$$

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x) + E W(x) \right] \psi(x) = E \psi(x)$$

↑ grnd state.

you can solve $\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$

$$E(\epsilon) = \sum_{n=0}^{\infty} \epsilon^n E_n, \quad \psi = \sum_{n=0}^{\infty} \epsilon^n \psi_n(x)$$

$$-\frac{1}{2} \psi_n'' + V(x) \psi_n + W(x) \psi_{n-1} = \sum_{j=0}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n$$

Let $\psi_n = \psi_0 Q_n(x)$

$$Q_n(x) = \frac{1}{\psi_0} \int_0^x \int_0^x (-2 + \psi_0^2) dx dx$$

$$E W(x) \psi(x) = E_0 \psi$$

↑ grnd state.

$$\left(-\frac{1}{2} \frac{d^2}{dx^2} + V(x) \right) \psi_0 = E_0 \psi_0$$

$$\psi_0 \rightarrow 0 \text{ as } x \rightarrow \pm \infty$$

$$\psi = \sum_{n=0}^{\infty} c_n \psi_n$$

$$W(x) \psi_{n-1} = \sum_{j=1}^{n-1} \psi_j E_{n-j} + \psi_0 E_n + \psi_n E_0$$

$Q_n(x)$

$$Q_n(x) = \frac{1}{\psi_0^2(x)} \int_0^x \int_{-\infty}^x$$

$$\psi_0^2 \left[\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{\psi_0^2} \right]$$

$\lim_{x \rightarrow \pm\infty} \psi = 0$

$$E_n = \frac{\int_{-\infty}^{\infty} W(x) Q_{n-1} \psi^2 dx - \int_{-\infty}^{\infty} \sum_{j=1}^{n-1} Q_j E_{n-j} \psi^2 dx}{\int_{-\infty}^{\infty} \psi^2 dx}$$

$\psi_n E_0$

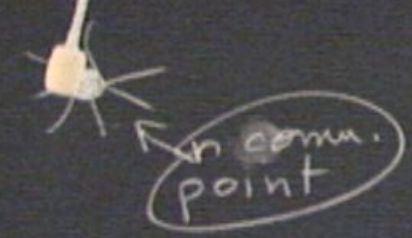
$\sum_{j=1}^{n-1} Q_j E_{n-j} + \frac{E_n}{-W(x) \cdot Q_{n-1}}$

n-Vertex



n comm.
point

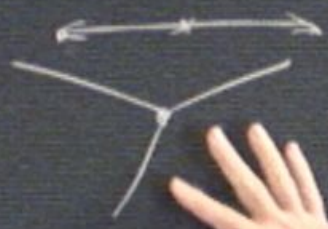
n-Vertex



line



2-line



n-Vertex



n conn. point

line



2-line



2-line
4-vertex



n-Vertex



n conn. point

line



2-line



2-line ———
4-vertex



4-line
4-vertex



n-Vertex



n conn. point

line



2-line



2-line



4-vertex



4-line



4-vertex

1-line



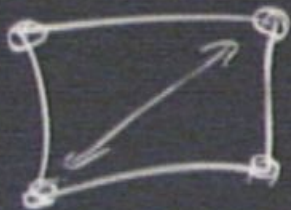
3-vertex



n-Vertex

line

2-line



n comm. point



2-line
4-vertex

4-line
4-vertex

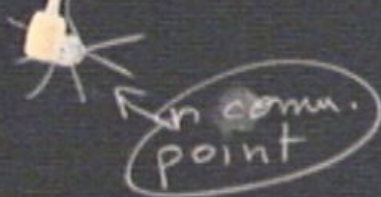
1-line
3-vertex





n-Vertex

line

2-line



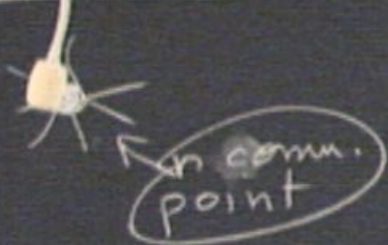
2-line 
 4-vertex 



4-line	
4-vertex	
<hr/>	
1-line	
3-vertex	



n-Vertex



line



2-line



4-line
4-vertex



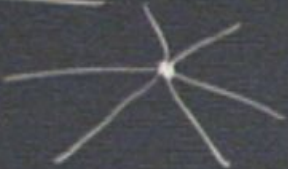
2-line →
4-vertex *



1-line →
3-vertex *



line



2-line



4-v



4-line
4-vertex



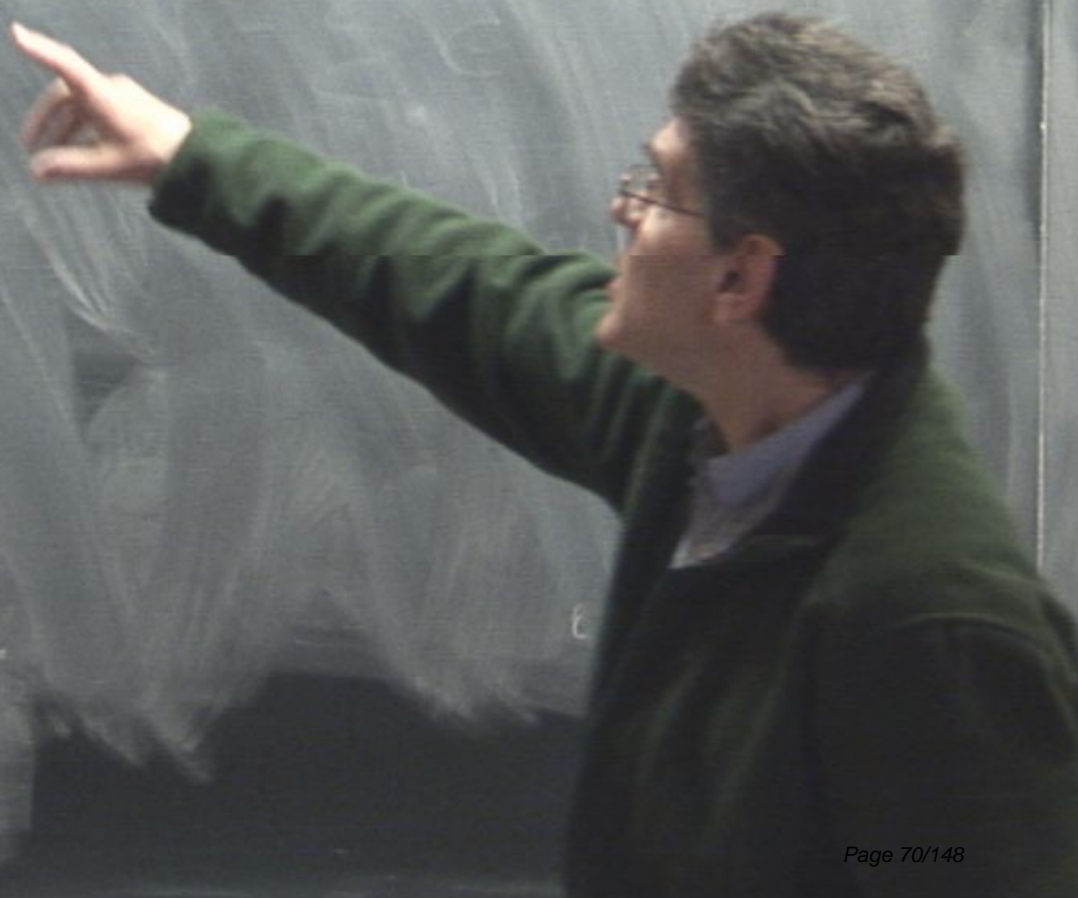
1-line
3-vertex



$E(\mathbb{C})$

\mathbb{C}^n

$$e^{-\lambda x} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \left(\frac{d}{dx}\right)^n$$



$$e^{-\mathcal{M}} \frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-\mathcal{M}} \frac{b_n}{n!} x^n$$

$$\mathcal{P} - \mathcal{M} \frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$\mathcal{P} - \mathcal{M} \frac{b_n}{n!} x^n$$

x=0

$$e^{-M} \sum_{n=0}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-M} \sum_{n=0}^{\infty} \frac{b_n}{n!} x^n$$

$$\Big|_{x=0}$$

$$= 1 +$$

all Feynman
Graphs

$$e^{-\mathcal{M}} \frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-\mathcal{M}} \frac{b_n x^n}{n!}$$

$$\Big|_{x=0}$$

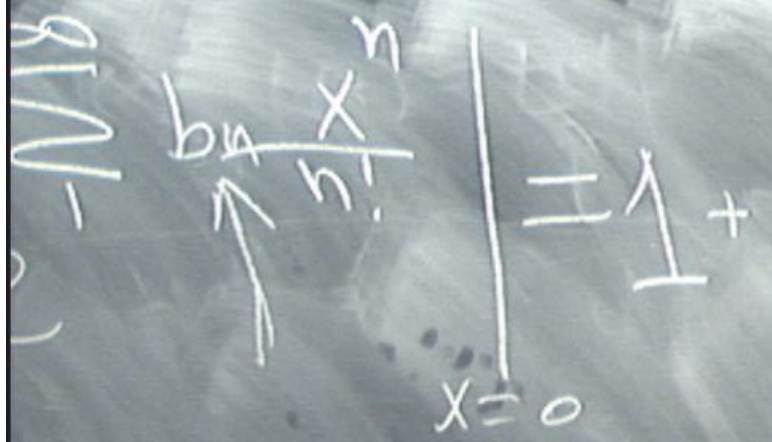
$$= 1 +$$

all Feynman
Graphs (mult by
Symm #'s)

$$\sum_{n=0}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n$$

$$e^{-\sum b_n \frac{x^n}{n!}} \Big|_{x=0} = 1 +$$

all Feynman
graphs (mult by
Symm #'s)
using a_n for every n
and b_n for every



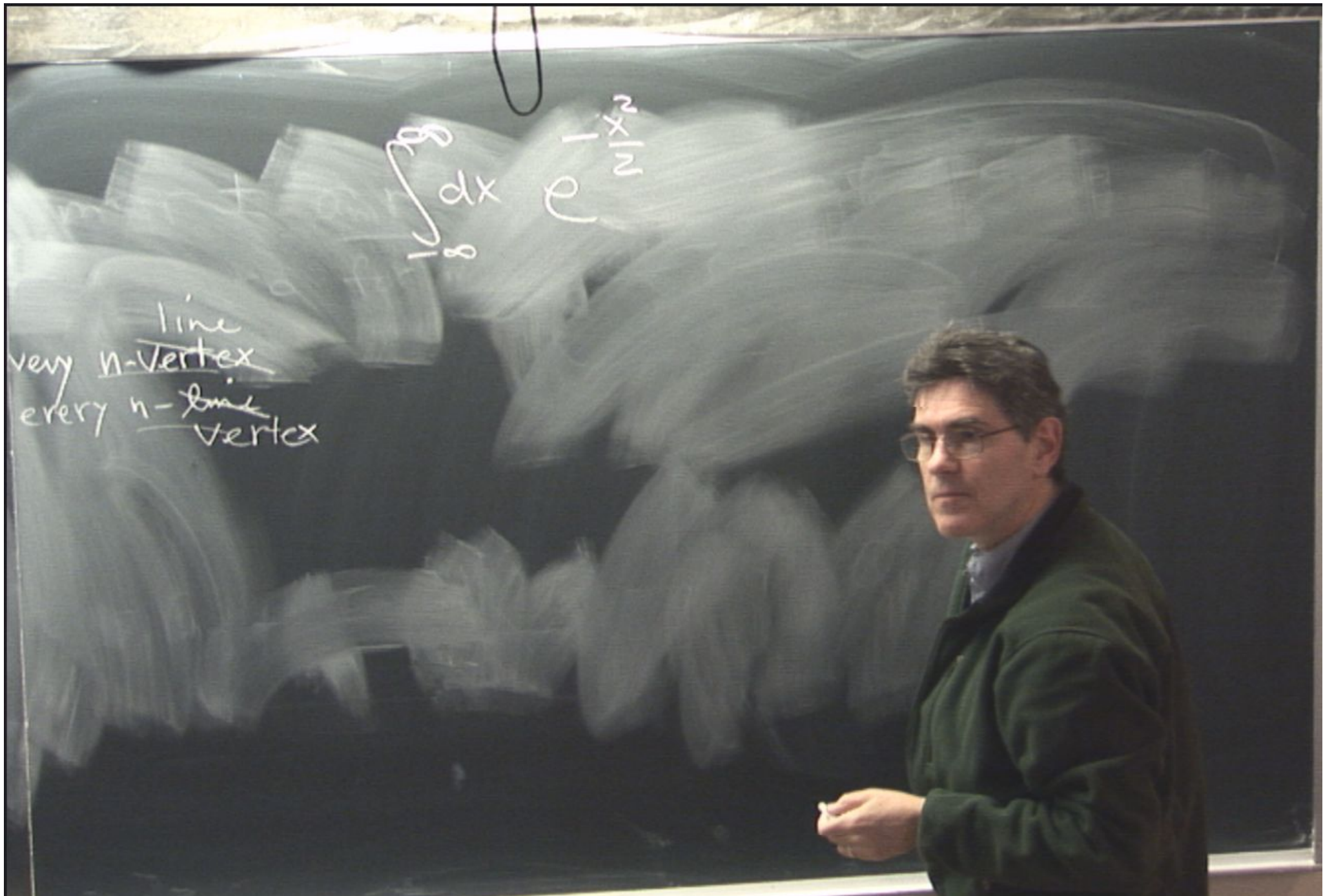
+ all Feynman
 Graphs (mult by
 Symm #'s)

Using A_n for every n-vertex
 and b_n for every n-line
 vertex

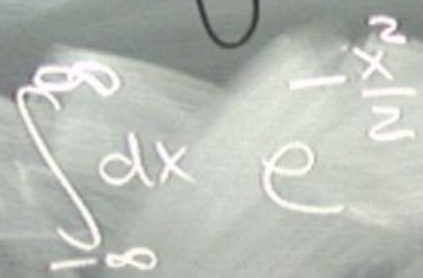
$$\begin{aligned}
 & \frac{b_n}{n!} x^n \\
 & \uparrow \\
 & x=0
 \end{aligned}$$

+ all Feynman
 Graphs (mult by
 Symm #'s)

Using A_n for every n -vertex
 and b_n for every n -line
vertex



line
very n-vertex
every n-~~line~~ vertex



$$\int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - ex^4}$$

line
very n -vertex
every n -~~line~~
vertex

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2} = e^{-x^2}$$

$$N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line
very n-vertex
every n-~~line~~
vertex

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \epsilon x^4}$$

$$N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line
every n -vertex
every n -~~line~~
vertex

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N = \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

line
 every n-vertex
 every n-~~line~~
 vertex

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N = \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

line
 every n-vertex
 every n-~~line~~
 vertex

$$= 1$$



$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N = \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \frac{\epsilon x^4}{1 \cdot x^3} + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

line
very n-vertex
every n-~~line~~
vertex

$$= 1 - 3\epsilon$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4}$$

$$N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \underbrace{\epsilon x^4}_{x \cdot x^3} + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

line
 every n -~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad N \equiv \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line
 every n-vertex
 vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \underbrace{\epsilon x^4}_{x \cdot x^3} + \frac{\epsilon^2}{2!} \underbrace{x^8}_{x \cdot x^7} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \epsilon x^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line
 every n-vertex
 vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

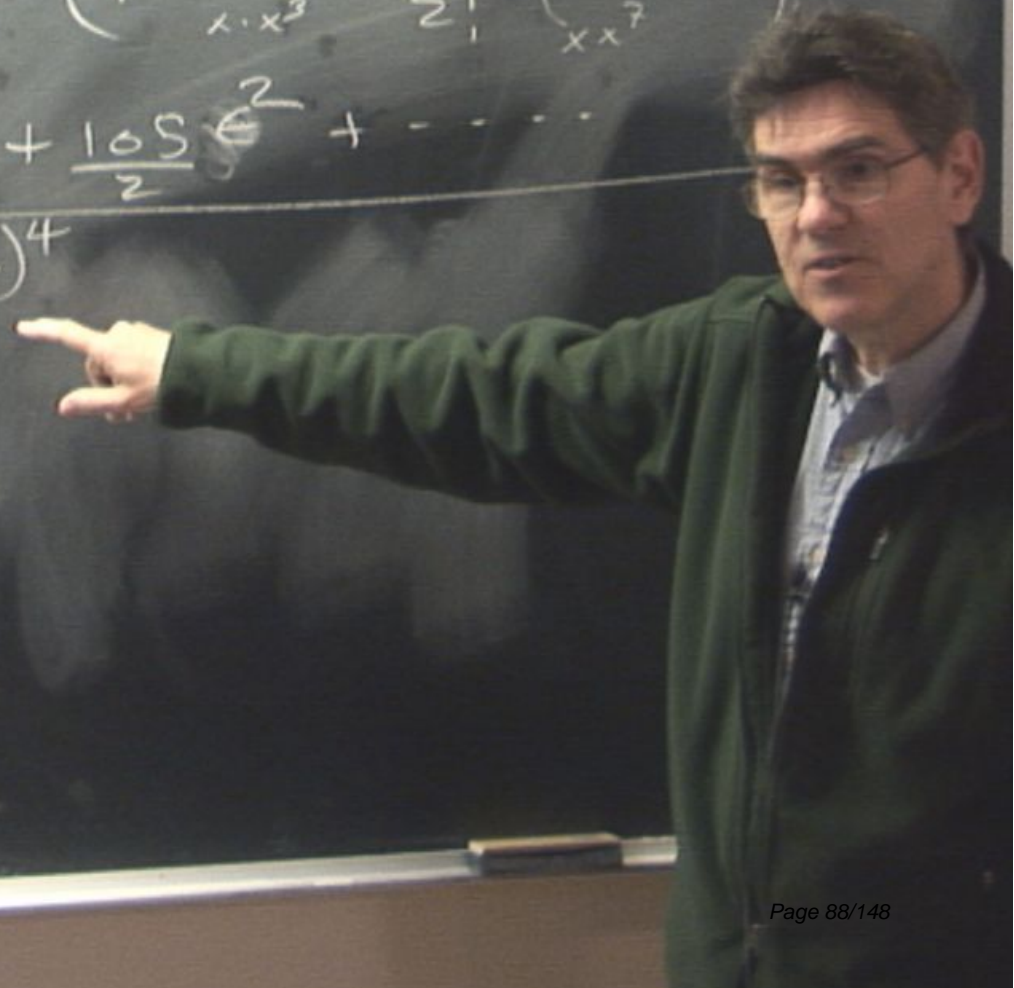
$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} J=0 \\ N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \end{array} \right\}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

very line
n-vertex
every n-~~line~~
vertex

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4}$$



$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} J=0 \\ N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \end{array} \right\}$$

very line n-vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

every line vertex

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \\ J=0 \end{array} \right\}$$

very line n-vertex
every line n-vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \\ J=0 \end{array} \right\}$$

very line n-vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}} \quad \left. \begin{array}{l} \\ J=0 \end{array} \right\}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \Big|_{J=0} \quad N \equiv \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \frac{\epsilon x^4}{x x^3} + \frac{\epsilon^2 x^8}{2! x x^7} + \dots \right)$$

line
n-vertex
every n-~~line~~
vertex

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^4}{2}} \quad \Big|_{J=0}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2}x^2 - \epsilon x^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

line
n-vertex
or n-~~line~~
vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}} \left(\frac{J}{2}\right)$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2}J^2}$$



$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{1}{2}x^2 - \epsilon x^4 + Jx} \quad \Big|_{J=0} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line
n-vertex
or n-~~line~~
vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}} \quad \Big|_{J=0}$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2}J^2} \quad \Big|_{J=0}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2} x^2 - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} J=0 \\ N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \end{array} \right\}$$

line
n-vertex
or n-~~line~~
vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2} x^2} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}} \left(\frac{J^2}{2}\right)$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2} J^2} \quad \left. \begin{array}{l} \uparrow \text{vertex} \\ \uparrow \text{line} \\ J=0 \end{array} \right\}$$

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \\ J=0 \end{array} \right\}$$

line
n-vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

or n-~~line~~
vertex

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2}} \quad \left. \begin{array}{l} J=0 \\ \left(\frac{J^2}{2}\right) \end{array} \right\}$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2} J^2} \quad \left. \begin{array}{l} \uparrow \\ J=0 \end{array} \right\}$$

4-vertex

2-line

$$e^{-\sum \frac{1}{n!} a_n (d/dx)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

$$x=0$$

$$= 1 +$$

all Feynman
Graphs (mult by
Symm #'s)

Using a_n for ev
and b_n for e

$$e^{-\int V dx} \sum_{n=0}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n$$

$$e^{-\int V dx} \sum_{n=0}^{\infty} \frac{b_n}{n!} x^n \Big|_{x=0}$$

= 1 + all Feynman
 Graphs (mult by
 Symm #'s)
 Using a_n for ev
 and b_n for e

$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2}x^2 - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \\ J=0 \end{array} \right\}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

line
n-vertex
every n-~~line~~
vertex

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{4} + \frac{J^2}{2}} \quad \left. \begin{array}{l} J=0 \end{array} \right\}$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2}J^2} \quad \left. \begin{array}{l} \uparrow \\ \text{4-vertex} \end{array} \right\} \quad \left. \begin{array}{l} \uparrow \\ \text{2-line} \\ J=0 \end{array} \right\}$$

$$e^{-\sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

$$x=0$$

$$= 1 +$$

all Feynman
graphs (mult by
Symm #'s)

using a_n for ev
and b_n for e

$$\frac{F-V}{-24E}$$

$$e^{-\sum \frac{1}{n!} a_n (d/dx)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

$$x=0$$

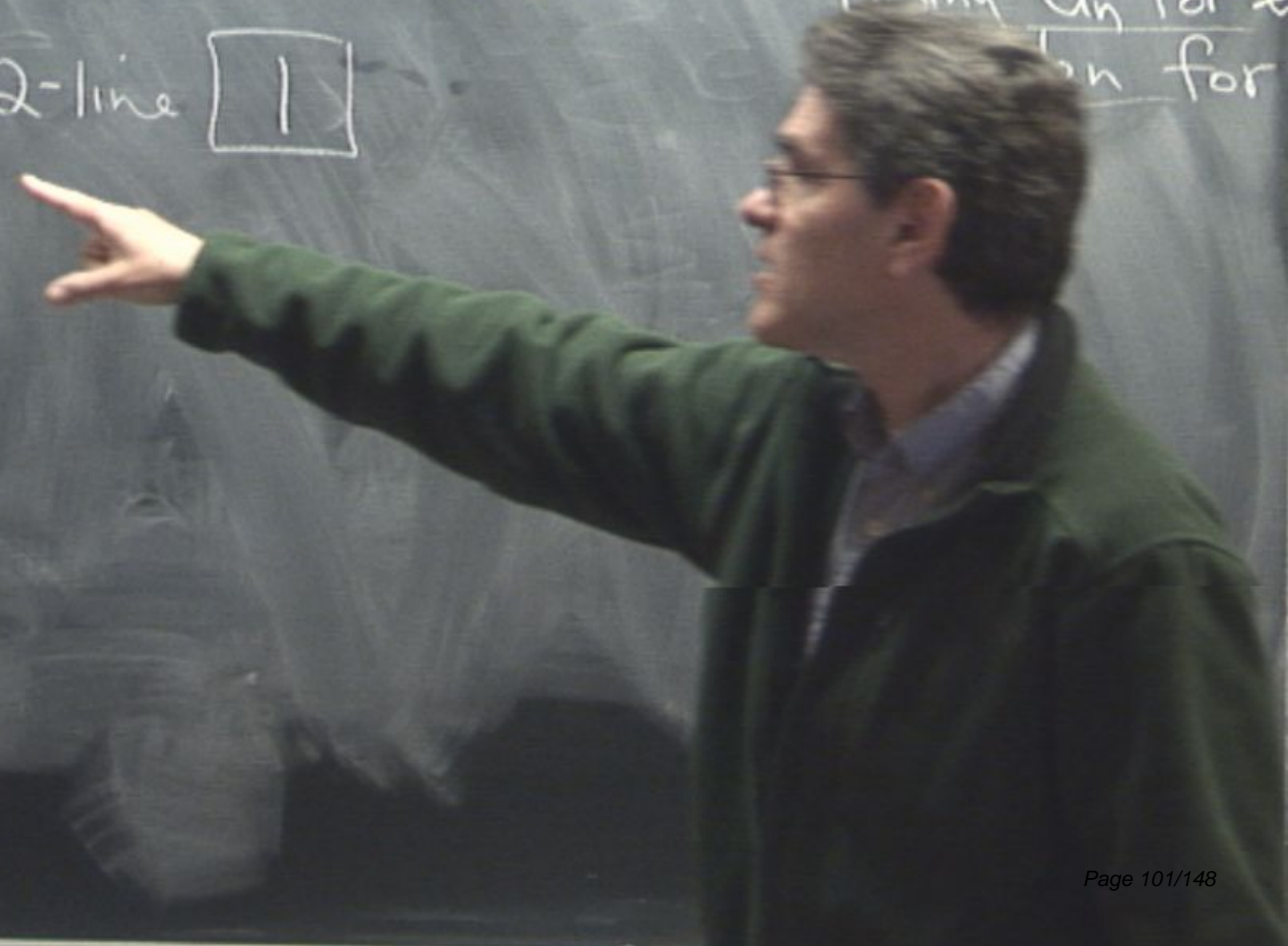
$$= 1 +$$

all Feynman
Graphs (mult by
Symm #'s)

using a_n for ev
for e

$$\frac{4-v}{-24\epsilon}$$

$$2\text{-line } 1$$



$$e^{-\sum \frac{1}{n!} a_n (d/dx)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n} \Big|_{x=0} = 1 +$$

all Feynman
 Graphs (mult by
 Symm #'s)
 Using a_n for ev
 and b_n for e

$$4-v \quad \boxed{-24E}$$

$$2\text{-line} \quad \boxed{1}$$

$$e^{-\int \sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\int \sum \frac{b_n}{n!} x^n} \Big|_{x=0} = 1 +$$

all Feynman
 Graphs (mult by
 Symm #'s)
 using a_n for ev
 and b_n for e

4-v $\boxed{-24E}$

2-line $\boxed{1}$



$$e^{-\sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

$x=0$

$$= 1 +$$

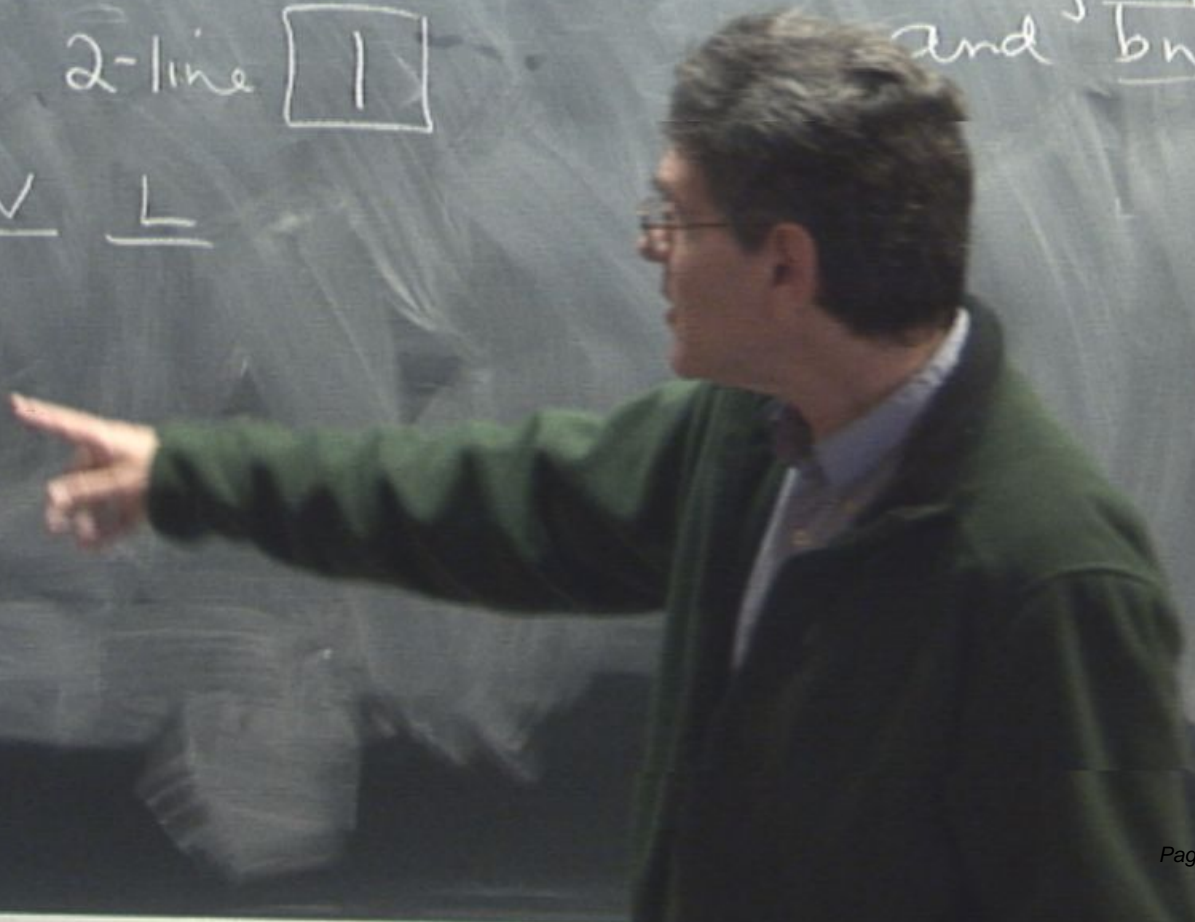
all Feynman
Graphs (mult by
Symm #'s)

using a_n for even
and b_n for odd

$\frac{4-v}{2} \boxed{-24E}$

2-line $\boxed{1}$

$\frac{3N}{2} \quad \frac{V}{2} \quad \frac{L}{2}$



$$e^{-\int \sum \frac{1}{n!} a_n (d^n)} \uparrow$$

$$e^{-\int \sum \frac{b_n}{n!} x^n}$$

$$= 1 + \text{all Feynman Graphs (mult by Symm #'s)}$$

$x=0$

Using a_n for even
and b_n for odd

$$4-v \quad \boxed{-24E}$$

$$2\text{-line} \quad \boxed{1}$$

$$\frac{1}{8} \frac{3N}{-24E} \frac{V}{1} \frac{L}{1}$$



$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2}x^2 - \epsilon x^4 + Jx} \quad \left. \begin{array}{l} N = \int_{-\infty}^{\infty} dx e^{-x^2/2} \\ J=0 \end{array} \right\}$$

very line
n-vertex
every n-~~line~~
vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2}x^2} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{2}\epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2}x^2 + Jx - \frac{J^2}{2\epsilon} + \frac{J^2}{2}} \quad \left. \begin{array}{l} J=0 \end{array} \right\}$$

$$= e^{-\epsilon \left(\frac{d}{dJ}\right)^4} e^{\frac{1}{2}J^2} \quad \left. \begin{array}{l} \uparrow \\ 4\text{-vertex} \end{array} \right\} \quad \left. \begin{array}{l} \uparrow \\ 2\text{-line} \\ J=0 \end{array} \right\}$$

$$e^{-\int \sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\int \sum \frac{b_n}{n!} x^n}$$

$$x=0$$

$$= 1 +$$

all Feynman
Graphs (mult by
Symm #'s)

Using a_n for ev
and b_n for e

$$4-v \quad \boxed{-24E}$$

$$2\text{-line} \quad \boxed{1}$$



$$\frac{1}{8} \frac{3N}{-24E} \frac{V}{1} L = -3E$$

$$e^{-\int \sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\int \sum \frac{b_n}{n!} x^n}$$

$$x=0$$

$$= 1 +$$

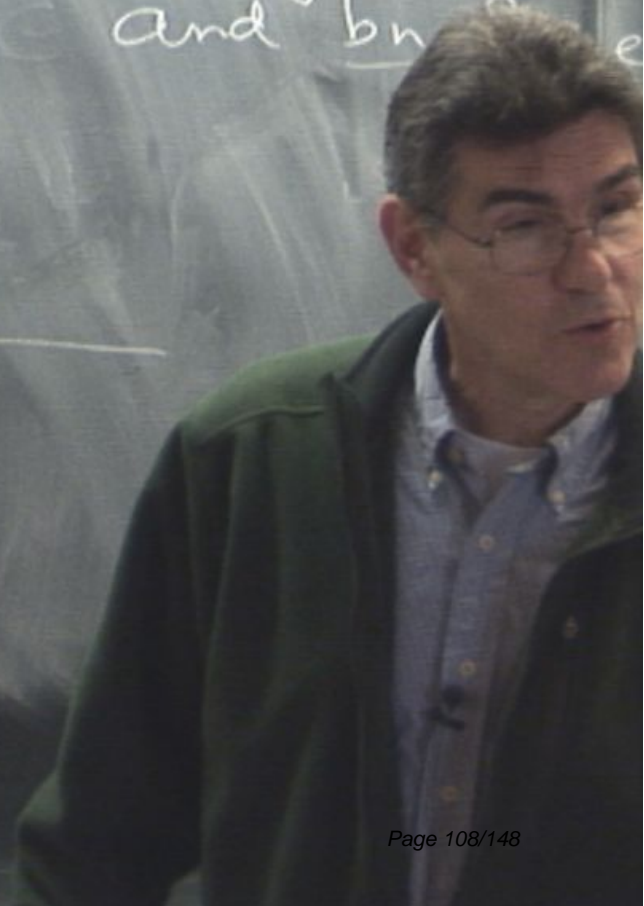
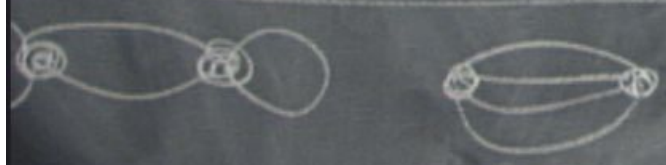
all Feynman
graphs (mult by
Symm #'s)

using a_n for ev
and b_n for o

4-v -24E

2-line 1

$$\frac{1}{8} \frac{3N}{-24E} \frac{V}{1} \frac{L}{1} = -3E$$



$$e^{-\sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

$$\Big|_{x=0} = 1 +$$

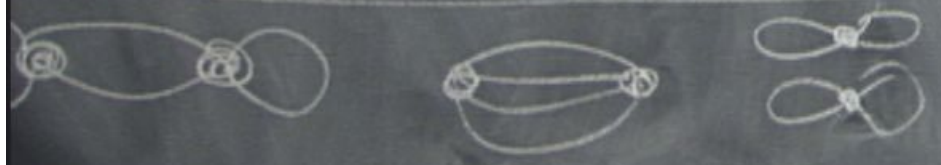
all Feynman
Graphs (mult by
Symm #'s)

Using a_n for ev
and b_n for e

$\frac{4-v}{-24E}$

2-line $\boxed{1}$

$$\frac{1}{8} \frac{3N}{-24E} \frac{V}{1} \frac{L}{1} = -3E$$



$$e^{-\int \sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\int \sum \frac{b_n}{n!} x^n}$$

$$x=0$$

$$= 1 +$$

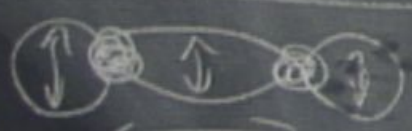
all Feynman
Graphs (mul
Symm #'s)

Using a_n for
and b_n for

$$\frac{4-v}{-24\epsilon}$$

$$2\text{-line } 1$$

$$\frac{1}{8} \frac{3N}{-24\epsilon} \frac{V}{1} \frac{L}{1} = -3\epsilon$$



$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{28}\right) (24\epsilon)^2 = \epsilon^2 \left[\right]$$

$$e^{-\sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

$$e^{-\sum \frac{b_n}{n!} x^n}$$

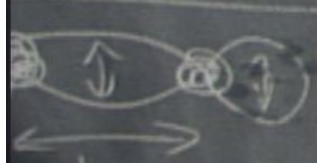
$$\left. \frac{d}{dx} \right|_{x=0} = 1 +$$

all Feynman
 Graphs (mult by
 Symm #'s)
 using a_n for even
 and b_n for odd

$$F-V \quad \boxed{-24E}$$

$$2\text{-line} \quad \boxed{1}$$

$$\frac{1}{8} \frac{SN}{-24E} \frac{V}{L} = -3E$$



$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128} \right) (24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$


$$e^{-\sum \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

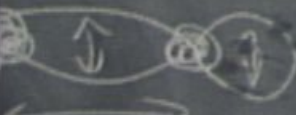


$$e^{-\sum \frac{b_n}{n!} x^n} \Big|_{x=0} = 1 +$$

all Feynman
 Graphs (mult by
 Symm #'s)
 using a_n for even
 and b_n for odd

F-V -24E

2-line 1

 $\frac{1}{8} \frac{SN}{-24E} \frac{V}{1} \frac{L}{1} = -3E$

 $\frac{1}{16}$ +  $\frac{1}{48}$ +  $\frac{1}{128}$ $(24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$

$\frac{24}{24}$


$$e^{-\sum_{n=0}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx}\right)^n}$$

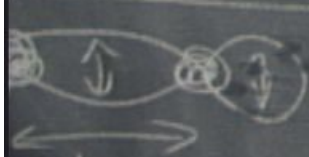
$$e^{-\sum_{n=0}^{\infty} \frac{b_n}{n!} x^n} \Big|_{x=0} = 1 +$$

all Feynman
 Graphs (mult by
 Symm #'s)
 using a_n for even
 and b_n for odd

F-V -24E

2-line 1

 $\frac{1}{8} \frac{SN}{-24E} \frac{V}{1} \frac{L}{1} = -3E$



$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{128} \right) (24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$

$$\frac{24+8+3}{24}$$

$$\frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-\sqrt{g}x} \sum_{n=0}^{\infty} \frac{b_n}{n!} x^n \Big|_{x=0} = 1 +$$

all Feynman
graphs (mult by
Symm #'s)

using a_n for every n -leg
and b_n for every n -

$$\boxed{-24\epsilon}$$

2-line $\boxed{1}$

$$\frac{1}{8} \frac{SN}{-24\epsilon} \frac{V}{1} = -3\epsilon$$



$$+ \left(\frac{1}{48} + \frac{1}{128} \right) (24\epsilon)^2 = 36\epsilon^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$

$$\frac{24+8+3}{24} = \frac{35}{24} 36\epsilon^2 = \frac{105}{2} \epsilon^2$$

$$\frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-\mathcal{M}} \sum_{n=0}^{\infty} \frac{b_n}{n!} x^n \Big|_{x=0} = 1 +$$

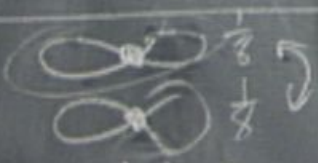
all Feynman
Graphs (mult by
Symm #'s)

Using a_n for every n -leg
and b_n for every n -

$$\boxed{-24\epsilon}$$

2-line $\boxed{1}$

$$\frac{1}{8} \frac{SN}{-24\epsilon} \frac{V}{1} = -3\epsilon$$


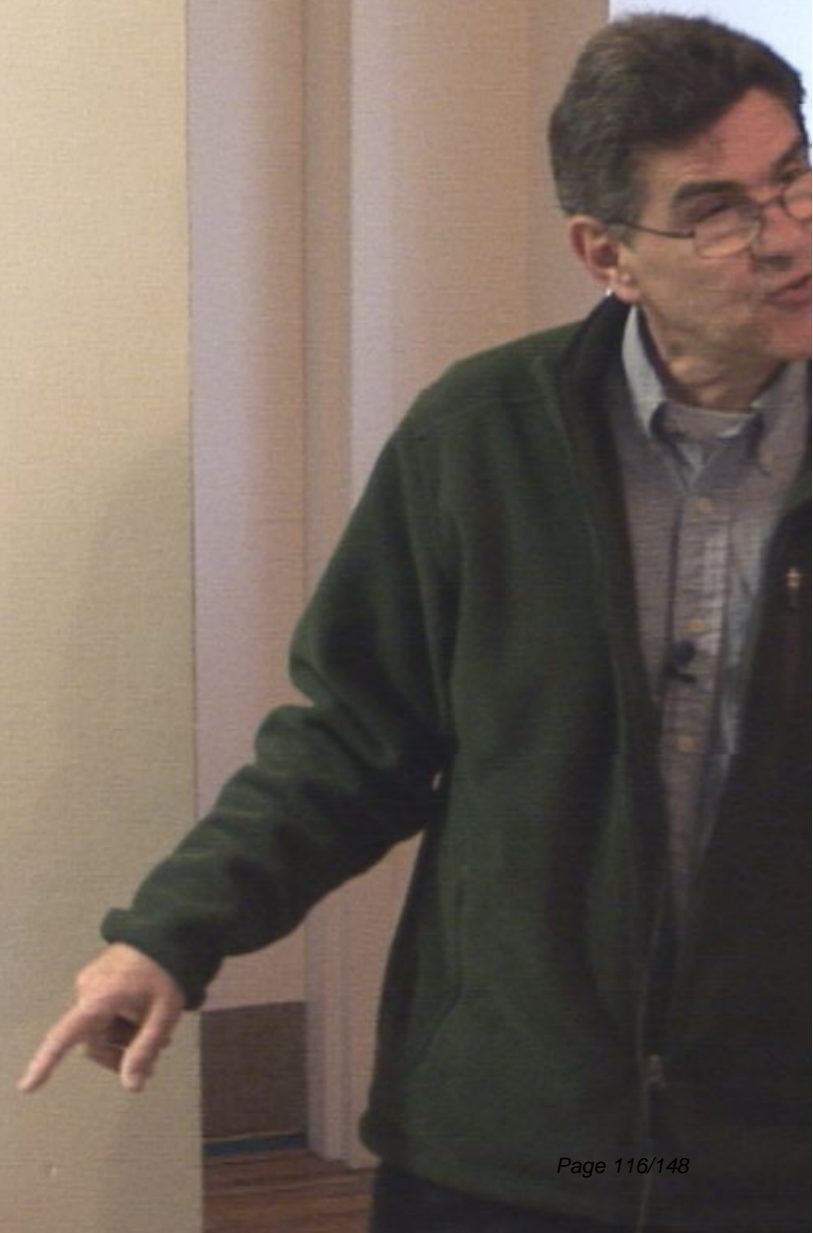


$$+ \left(\frac{1}{48} + \frac{1}{128} \right) (24\epsilon)^2 = 36\epsilon^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$

$$\frac{24+8+3}{24} = \frac{35}{24} 36\epsilon^2 = \frac{105}{2} \epsilon^2$$


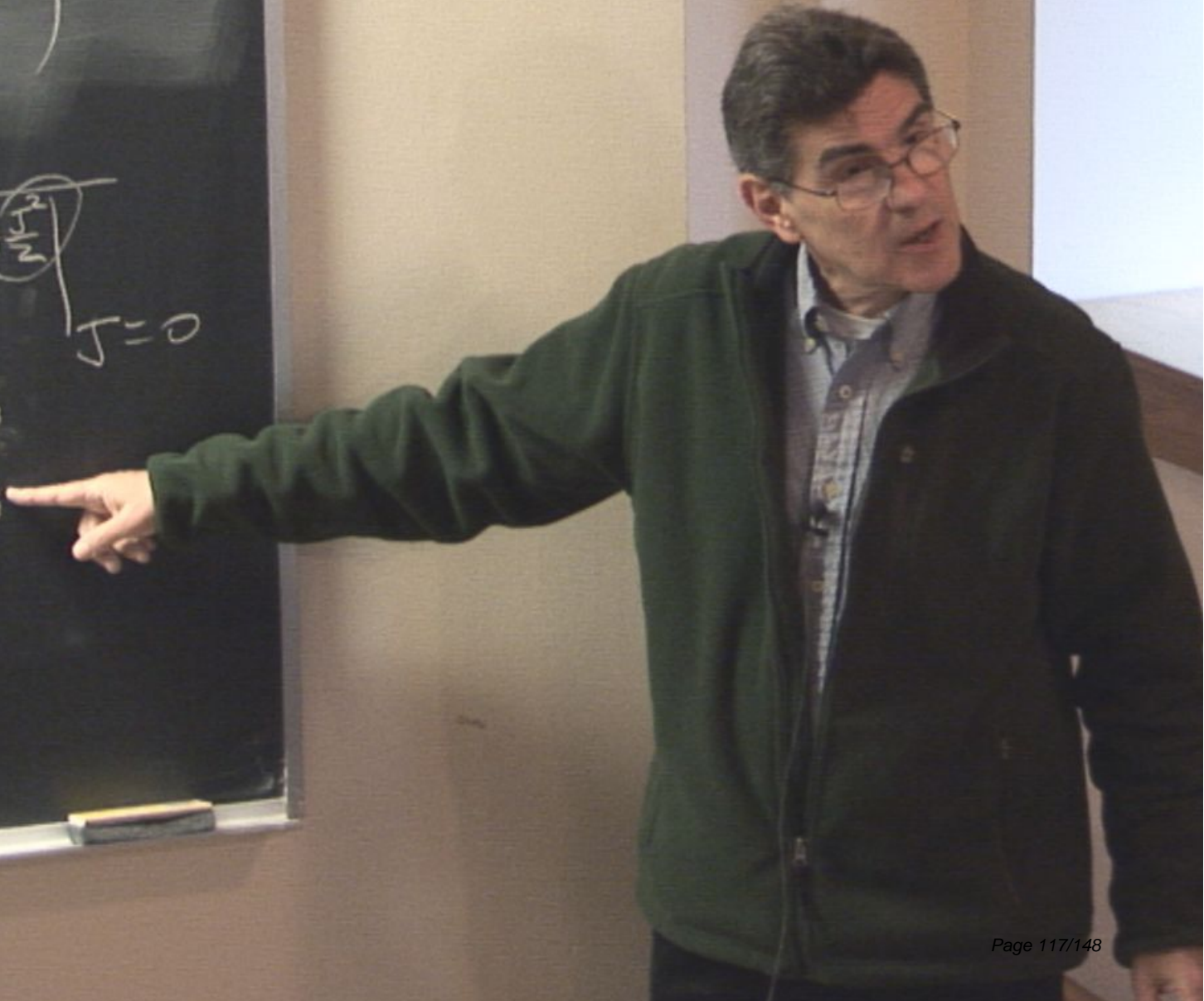
$J=0$
 $x^4 + \frac{e^2 x^8}{2!} + \dots$
 x^3 $2!$ $x x^7$
 $e^2 + \dots$


 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 $J=0$
 J^2
 \uparrow
 \hbar_0 $J=0$

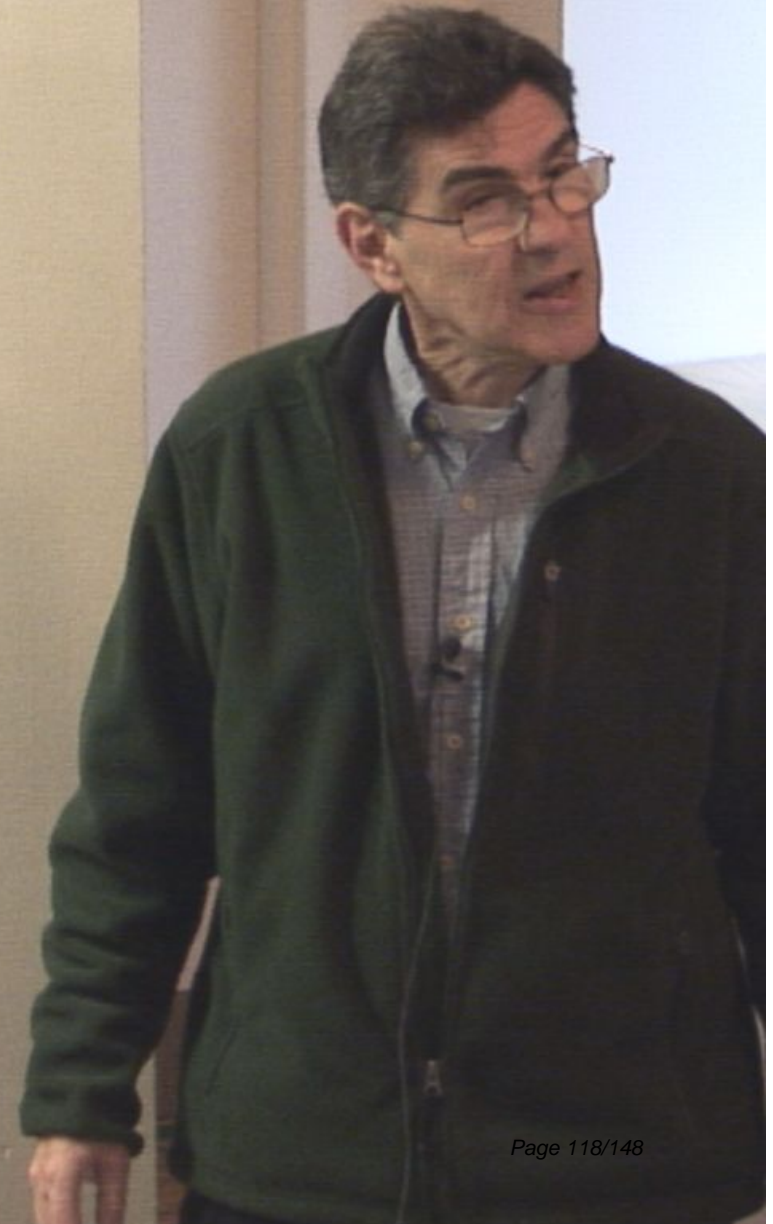



$J=0$
 $x^4 + \frac{\epsilon^2 x^8}{2!} + \dots$
 $\epsilon^2 + \dots$

 $x \left(e^{-\frac{x^2}{2}} + Jx - \frac{J^2}{2} + \frac{J^2}{2} \right)$
 $J=0$
 J^2
 \uparrow
 $J=0$
 z_0





$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 x^3
 $\epsilon^2 + \dots$
 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 $J=0$
 J^2
 \uparrow
 $J=0$




$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 x^3
 x^7
 $\epsilon^2 + \dots$

 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 $J=0$
 J^2
 \uparrow
 $J=0$
 z_0




The diagram shows a square with its four vertices and two diagonals, forming a complete graph K_4 . To its right is a triangle with a central point connected to each of its three vertices, forming a wheel graph W_3 .

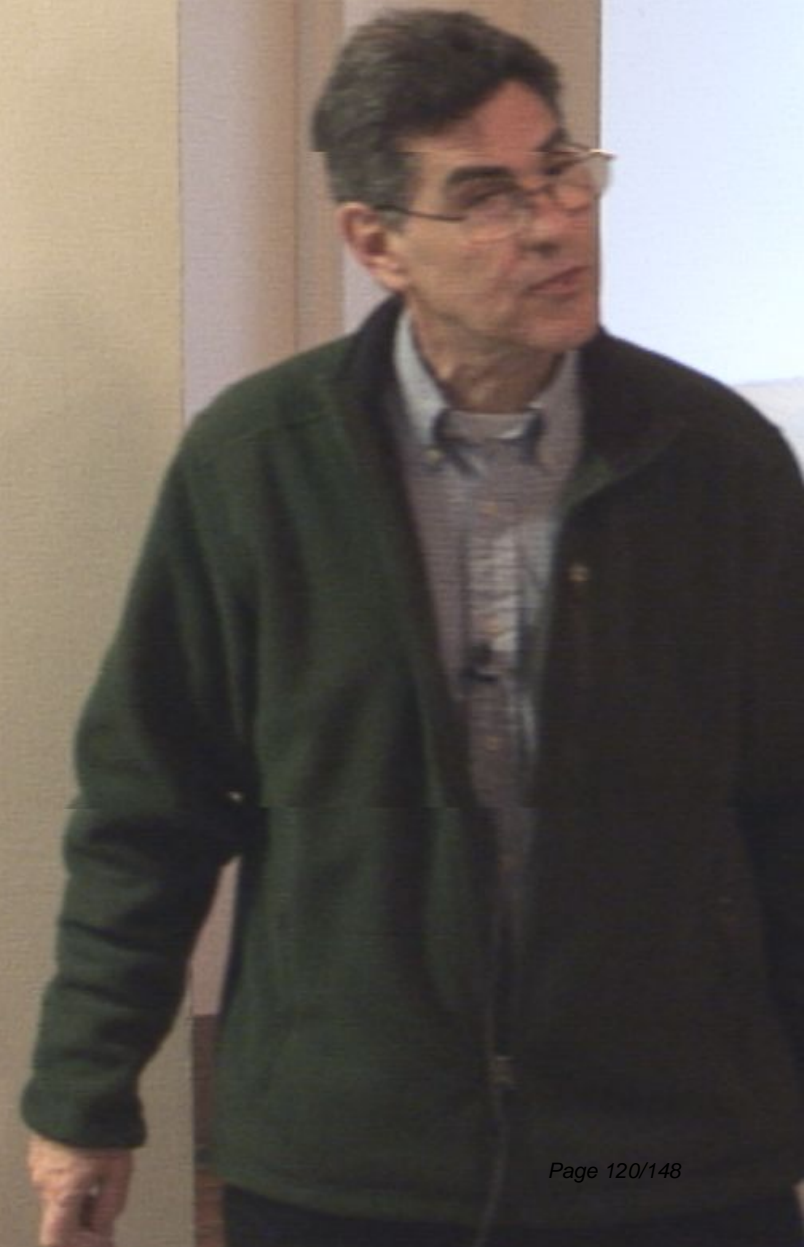


$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 x^3
 $\epsilon^2 + \dots$

 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 J^2
 \uparrow
 $J=0$
 $\hbar e$




$J=0$ 4




$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 x^3 2^1 $x x^7$
 $\epsilon^2 + \dots$



$x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$

J^2 | $J=0$
 \uparrow
 \hbar_0






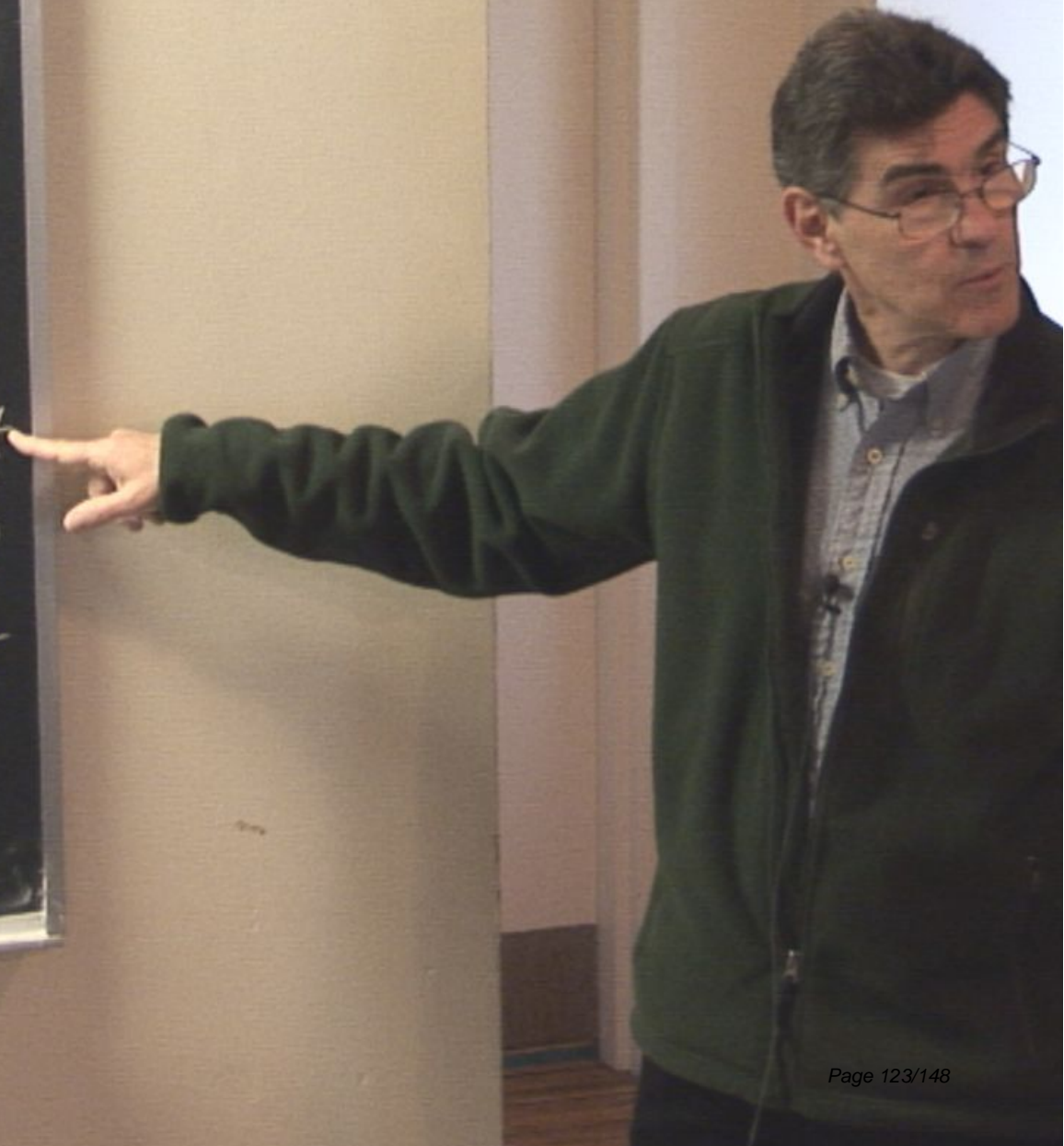
$J=0$
 4
 x
 3

$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 $\epsilon^2 + \dots$
 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 J^2
 $J=0$


 $J=0$ 4
 x 3
 x 2
 x 2



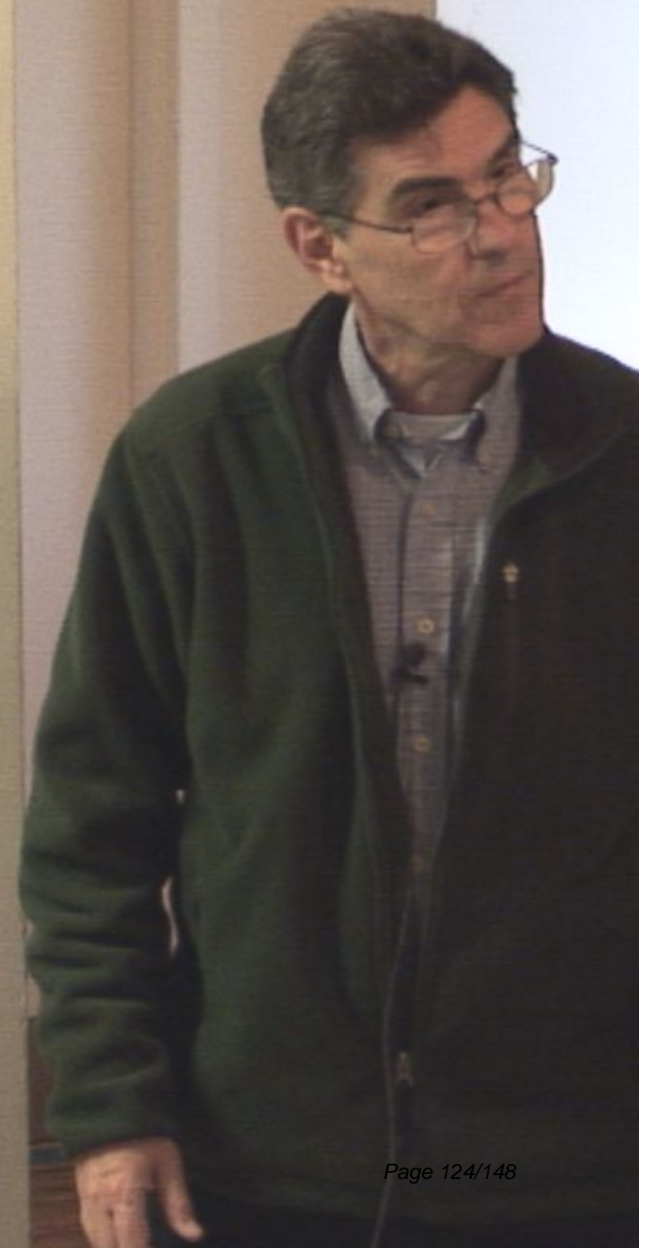
$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 $\epsilon^2 + \dots$

 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 J^2
 $J=0$

 4
 x
 3
 x
 2



$J=0$
 $x^4 + \epsilon^2 x^8 + \dots$
 $\epsilon^2 + \dots$
 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 J^2
 $J=0$
 h_0

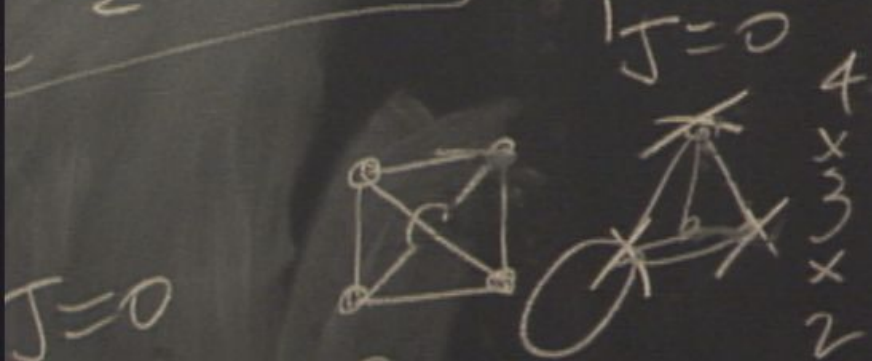
$J=0$
 4
 x
 3
 x
 2



$J=0$
 $x^4 + \frac{2}{x^3} + \frac{x^8}{2!} + \dots$
 $x^2 + \dots$
 $x e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$
 J^2
 $J=0$
 2
 3
 $J=0$
 4
 x
 3
 x
 2

$$\left(\frac{x^2}{2} + \dots \right)$$

$$x^2 + Jx - \frac{J^2}{2} + \frac{J^2}{2}$$



$$\left(\frac{x^2}{2} + Jx - \frac{J^2}{2} \left(\frac{J^2}{2} \right) \right)$$

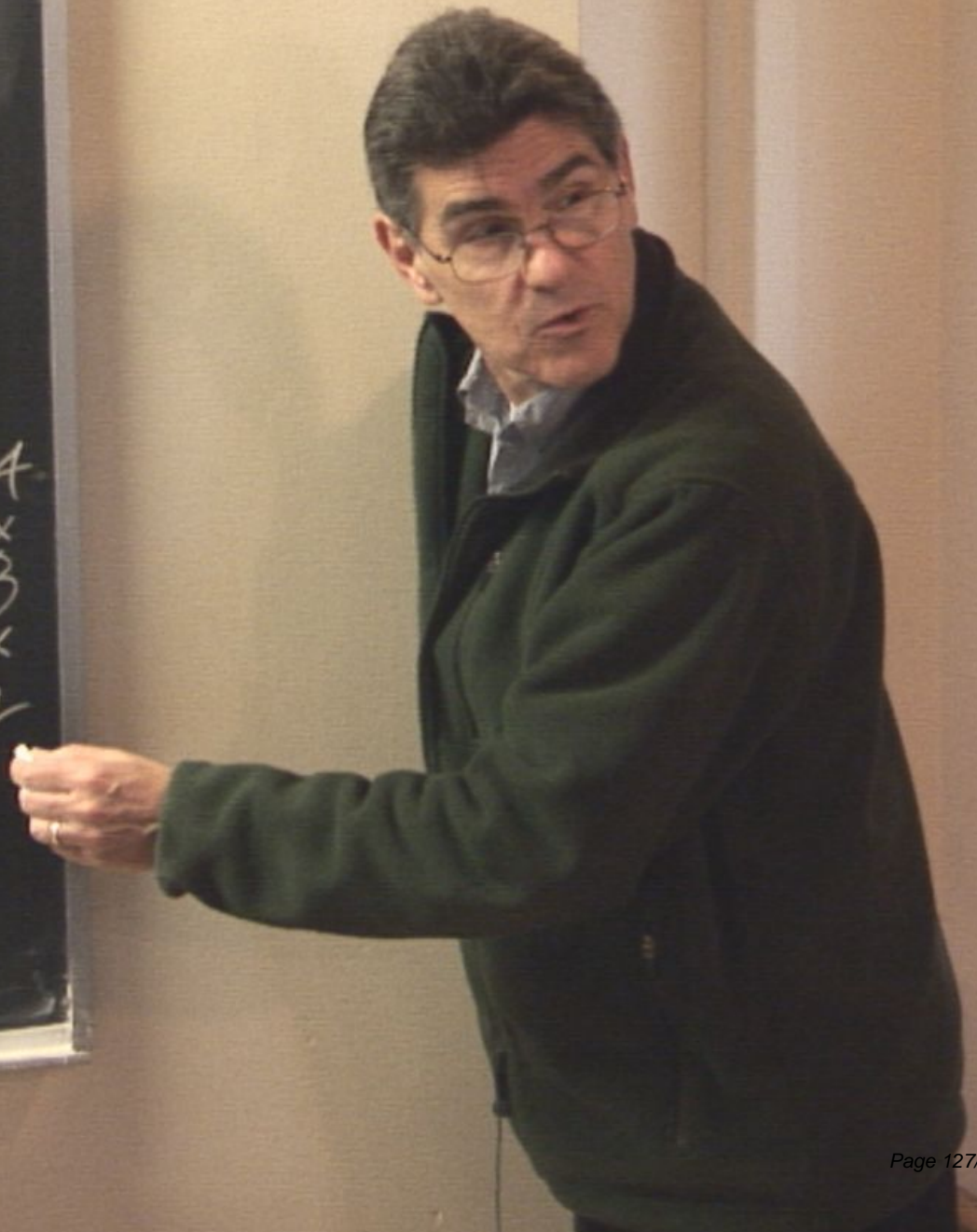
$$J=0$$

$$J=0$$

$$2 \rightarrow$$

$$1 \quad 2 \quad 3 \quad 4$$

$$\left(\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \right) 3$$



$$I(\epsilon) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{\epsilon}{2} x^2 - \epsilon x^4 + Jx} \quad J=0 \quad N \equiv \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

line
n-vertex
every n-~~line~~
vertex

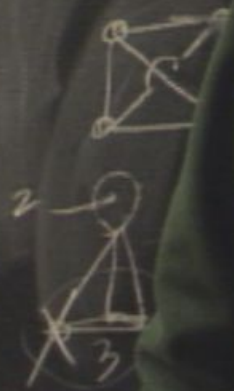
$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} (1 - \epsilon x^4 + \frac{\epsilon^2}{2!} x^8 + \dots)$$

$$= 1 - 3\epsilon + \frac{105}{2} \epsilon^2 + \dots$$

$$\rightarrow I(\epsilon) = \frac{1}{N} e^{-\epsilon (\frac{d}{dJ})^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} + Jx - \frac{J^2}{2} + \frac{J^2}{2}}$$

$$= e^{-\epsilon (\frac{d}{dJ})^4} e^{\frac{1}{2} J^2} \quad J=0$$

4-vertex
2-line



$$= \frac{105}{2} \epsilon^2$$


$$e^{-\sum \frac{1}{n!} a_n \left(\frac{\partial}{\partial x} \right)^n}$$

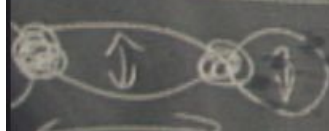
$$e^{-\sum \frac{1}{n!} b_n \left(\frac{\partial}{\partial x} \right)^n} = 1 + \int_{x=0}^{\infty} \int_{y=0}^{\infty} \mathcal{P}(J(x,y)) \mathcal{H}(x,y) dx dy$$

all Feynman
Graphs (mult by
Symm #'s)
Using a_n for ev
and b_n for e

4-v $\boxed{-24E}$

2-line $\boxed{1}$

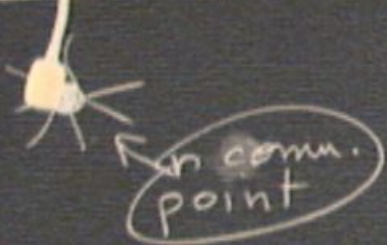
 $\frac{1}{8} \frac{SN}{-24E} \frac{V}{1} \frac{L}{1} = -3E$

 $\frac{1}{16}$ +  $\frac{1}{48}$ +  $\frac{1}{128}$

$$(24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$

$$\frac{24+8+3}{24} = \frac{35}{24}$$

n-Vertex



line



2-line



4-line
4-vertex



2-line →
4-vertex →



1-line
3-vertex




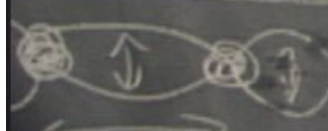


$$e^{-\sum \frac{1}{n!} a_n \left(\frac{\partial}{\partial x} \right)^n}$$

$$e^{-\sum \frac{1}{n!} b_n \left(\frac{\partial}{\partial x} \right)^n} = 1 + \int_{x=0}^{\infty} \dots$$

all Feynman Graphs (mult by Symm #'s)
Using a_n for ev and b_n for e

4-v $\boxed{-24E}$ 2-line $\boxed{1}$

 $\frac{1}{8} \frac{SN}{-24E} \frac{L}{1} = -3E$

 $\frac{1}{16}$ +  $\frac{1}{48}$ +  $\frac{1}{128}$ $(24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$

$$\frac{24+8+3}{24} = \frac{35}{24} 36E^2 =$$

$$I(e) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line every n-vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} (1 - ex^4 + \frac{e^2 x^8}{2!} + \dots)}$$

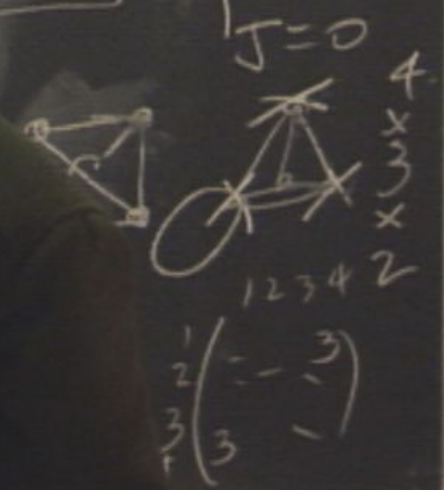
every n-~~line~~ vertex

$$= 1 - 3e + \frac{105e^2}{2} - \dots$$

$$\rightarrow I(e) = \frac{1}{N} e^{-e \left(\frac{d}{dJ}\right)^4} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - Jx - \frac{J^2}{2} \left(\frac{J^2}{2}\right)} \quad J=0$$

$$= e^{-e \left(\frac{d}{dJ}\right)^4}$$

↑
4-vertex



$$= \frac{105e^2}{2}$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} a_n \left(\frac{d}{dx} \right)^n$$

$$e^{-\int_0^x J(x) dx} = 1 + \text{all Feynman Graphs (mult by Symm #'s)}$$

F-V -24E

2-line 1

Using a_n for even
and b_n for odd

$$\frac{1}{8} \frac{SN}{-24E} \frac{V}{1} = -3E$$



$$\left(\frac{1}{16} + \frac{1}{48} + \frac{1}{28} \right) (24E)^2 = 36E^2 \left[1 + \frac{1}{3} + \frac{1}{8} \right]$$

$$\frac{24+8+3}{24} = \frac{35}{24} 36E^2 =$$

$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}}$$

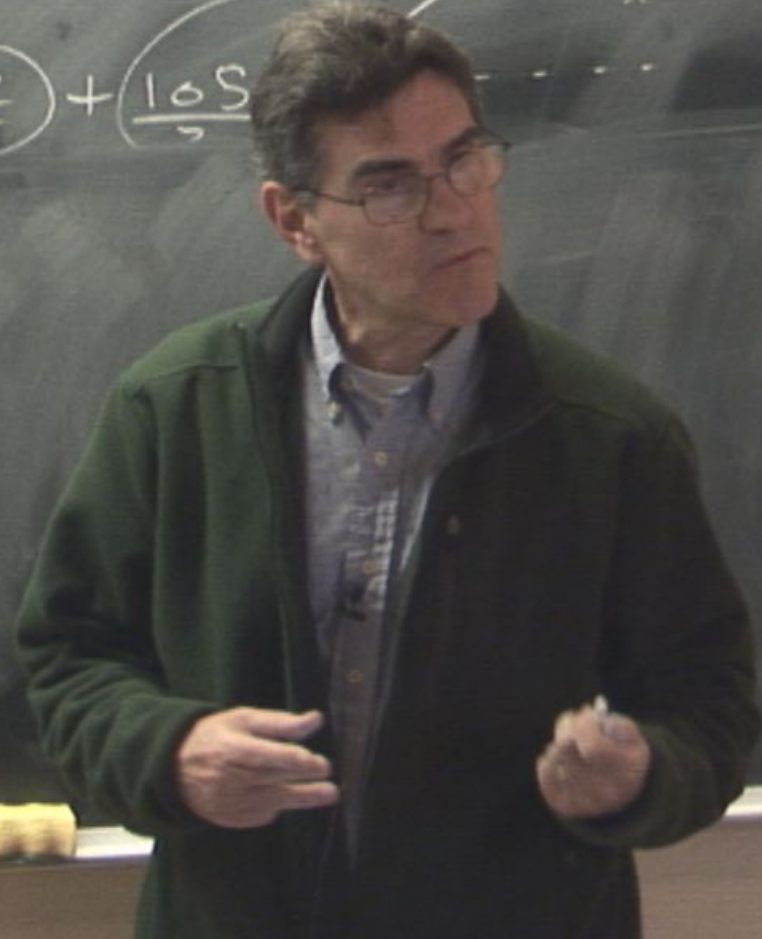
$$I(e) = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad J=0$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - ex^4 + \frac{e^2 x^8}{2!} + \dots \right)$$

line
 every n-vertex
 vertex

$$= 1 - 3e + \frac{10e^2}{5} - \dots$$

x^4

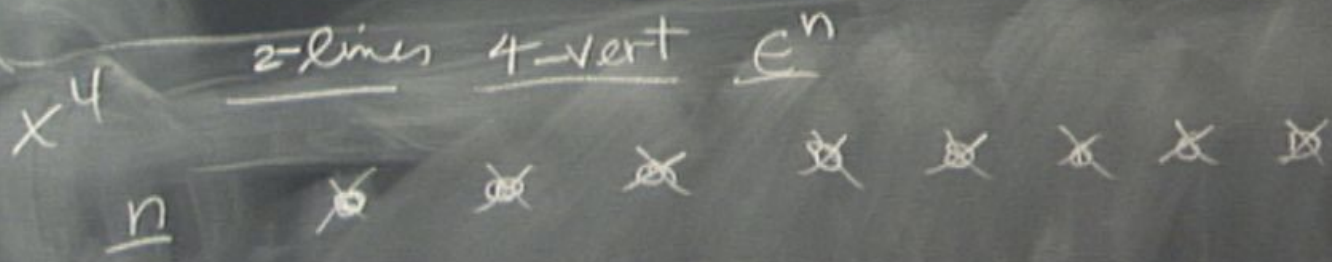


$$e^{-EV} = \frac{1}{Z} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N \equiv \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

very line n-vertex
every n-~~line~~ vertex

$$= \frac{1}{Z} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - ex^4 + \frac{e^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3E + \frac{105}{9} E^2 + \dots$$

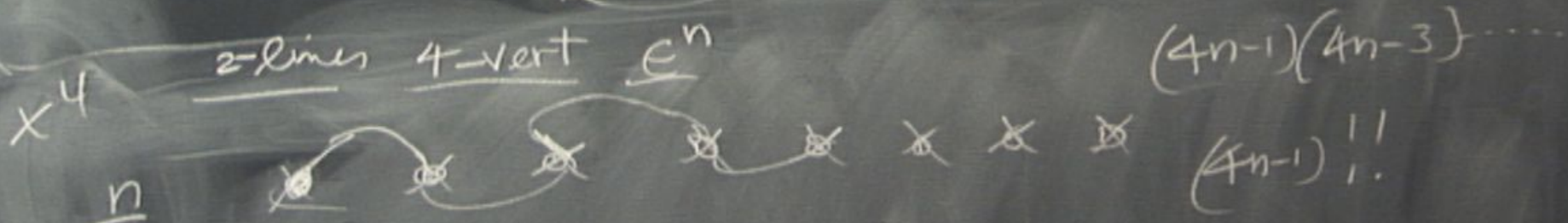


$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

very line n-vertex every n-~~line~~ vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - ex^4 + \frac{e^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3E + \frac{105}{5} E^2 + \dots$$

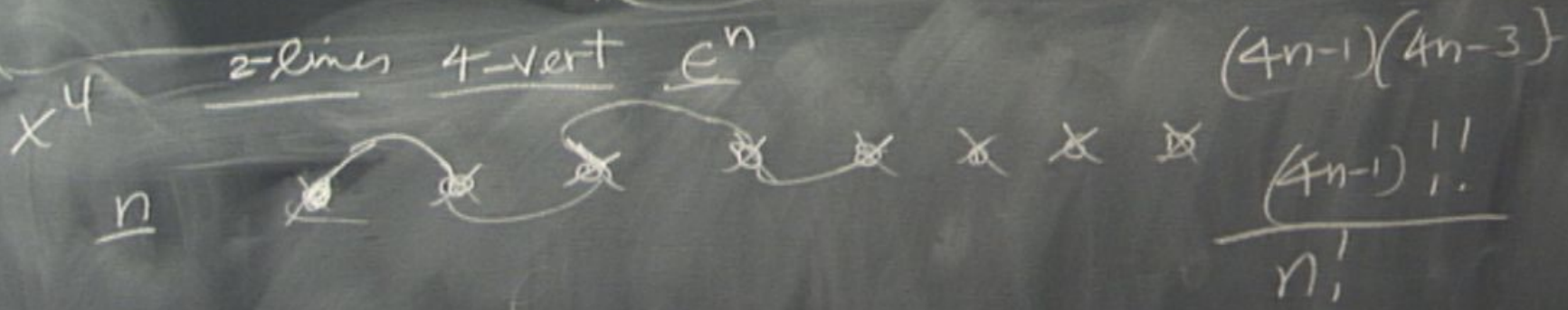


$$e^{-EV} = \frac{1}{Z} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N \equiv \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

$$I(\epsilon) = \frac{1}{Z} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + \frac{\epsilon^2}{2!} x^8 + \dots}$$

very n-vertex
every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$

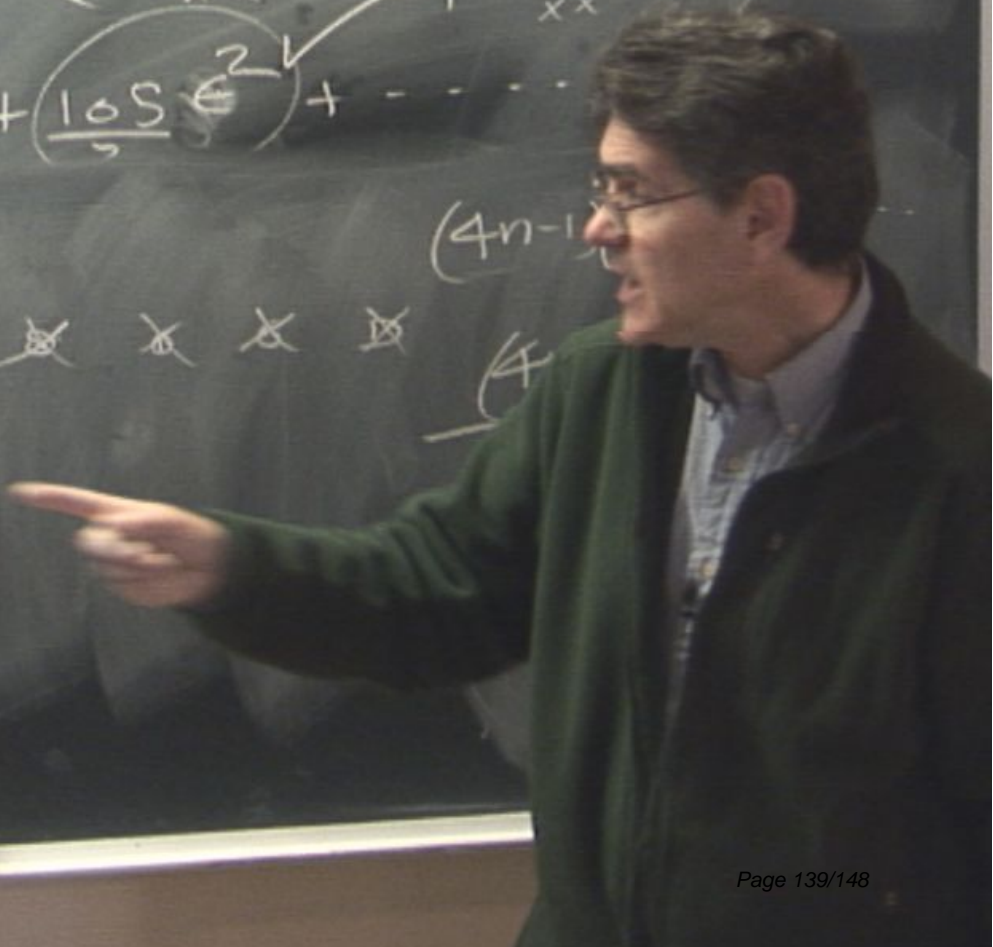
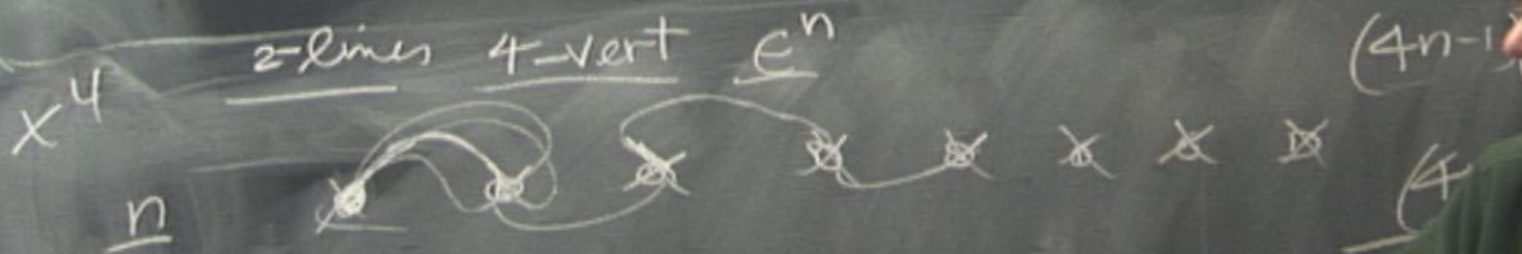


$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

very line n-vertex every n-~~line~~ vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} (1 - ex^4 + \frac{e^2 x^8}{2!} + \dots)$$

$$= 1 - 3E + \frac{105}{5} E^2 + \dots$$

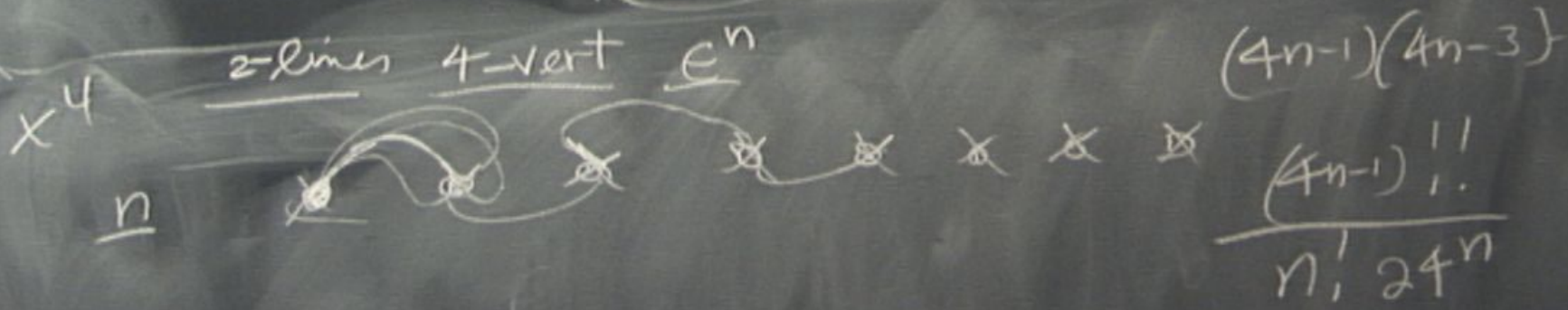


$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad N \equiv \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2}} \left(1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots \right)$$

very line n-vertex
every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$

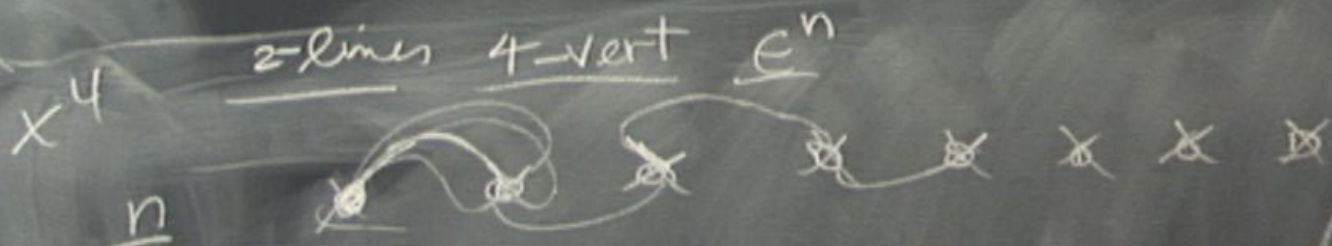


$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - ex^4 + \frac{e^2 x^8}{2!} + \dots \right)$$

$$= 1 - 3E + \frac{105}{5} E^2 + \dots$$

line
n-vertex
every n-~~line~~
vertex



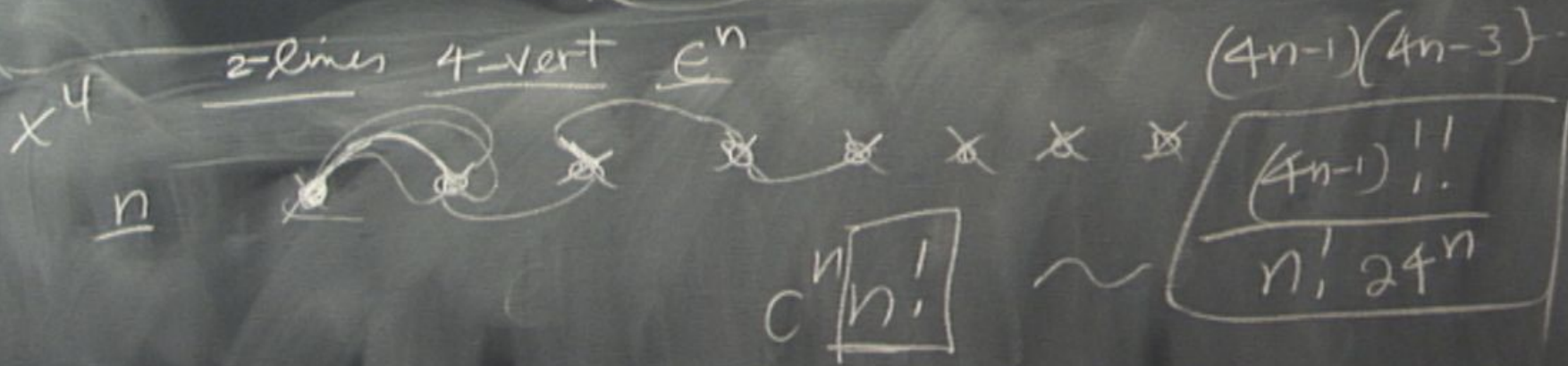
$$\frac{(4n-1)!!}{n! 2^n}$$

$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - \epsilon x^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

very line n-vertex every n-~~line~~ vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-x^2/2} (1 - \epsilon x^4 + \frac{\epsilon^2 x^8}{2!} + \dots)$$

$$= 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$

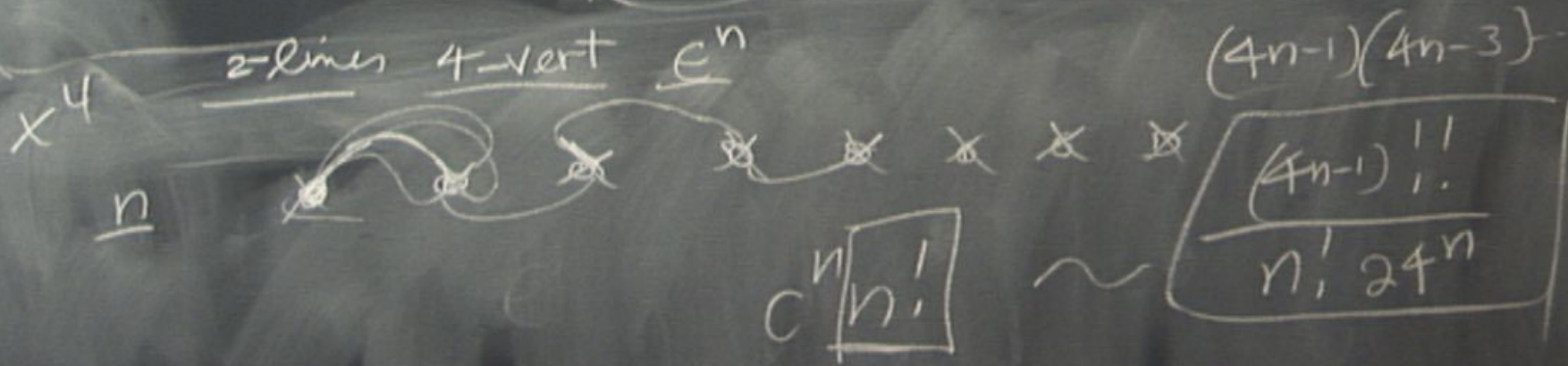


$$e^{-EV} = \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} - ex^4 + Jx} \quad N = \int_{-\infty}^{\infty} dx e^{-x^2/2}$$

line n-vertex
every n-~~line~~ vertex

$$= \frac{1}{N} \int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2} (1 - ex^4 + \frac{e^2 x^8}{2!} + \dots)}$$

$$= 1 - 3E + \frac{105}{5} E^2 + \dots$$

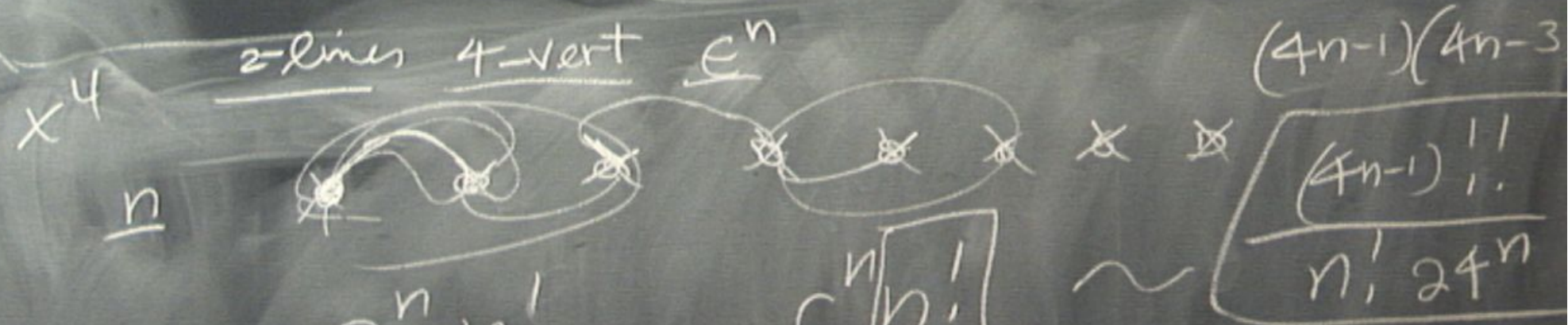


line n-vertex

$$= \frac{1}{2} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \frac{e^{-x^4}}{x \cdot x^3} + \frac{e^{-2x^8}}{2! \cdot x \cdot x^7} + \dots \right)$$

every n-~~line~~ vertex

$$= 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$



$$a_n \sim 3^n n!$$

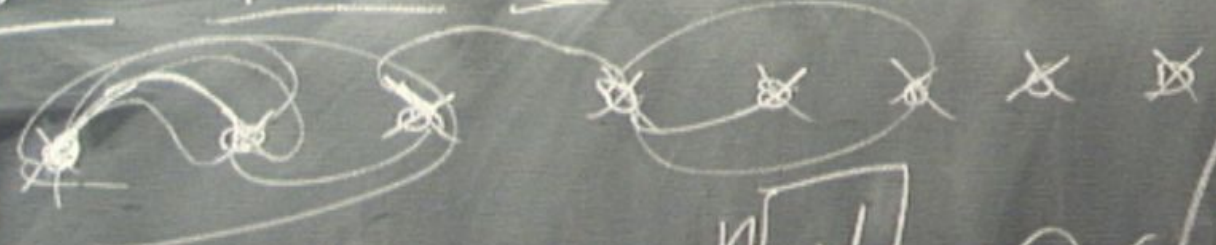
$$c^n \boxed{n!}$$

$$\frac{(4n-1)(4n-3) \dots (4n-1)!!}{n! 24^n}$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \underbrace{e^{-x^4}}_{x \cdot x^3} + \underbrace{e^{-2x^8}}_{2! \cdot x \cdot x^7} + \dots \right)$$

$$\approx 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$

lines 4-vert e^n



$$(4n-1)(4n-3) \dots$$

$$\frac{(4n-1)!!}{n! 24^n}$$

$$3^n n!$$

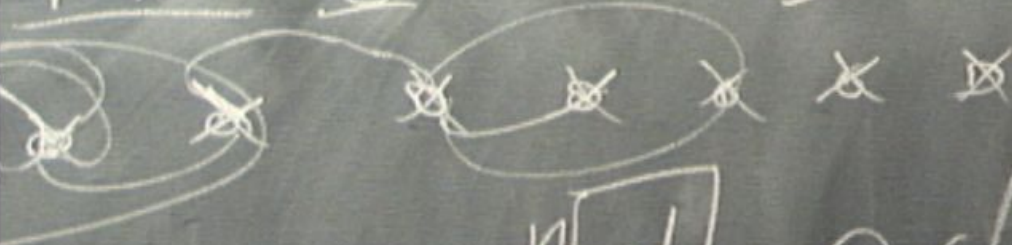
$$c_n \frac{n!}{(n!)!}$$

$$x^6$$

$$\int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \frac{e^{-x^4}}{x \cdot x^3} + \frac{e^{-2x^8}}{2! \cdot x \cdot x^7} + \dots \right)$$

$$= 1 - 3\epsilon + \frac{105}{5} \epsilon^2 + \dots$$

4-vert e^n $\frac{x^8}{(4n-1)(4n-3)} \dots$

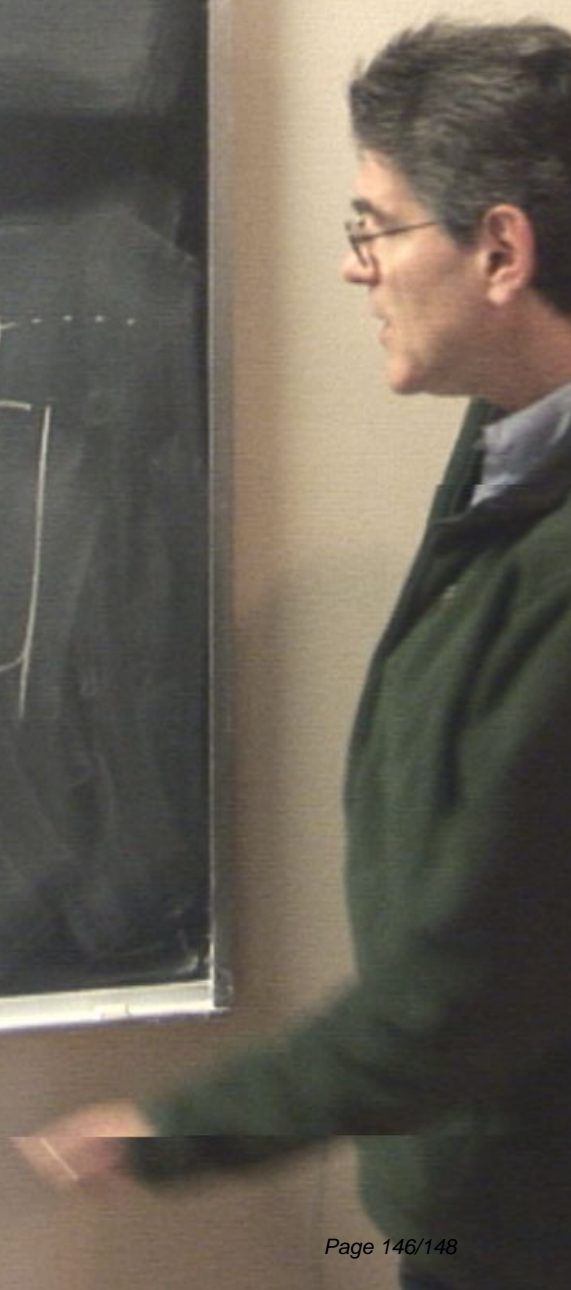


$n!$

$$\frac{n!}{(n!)!}$$

x^6

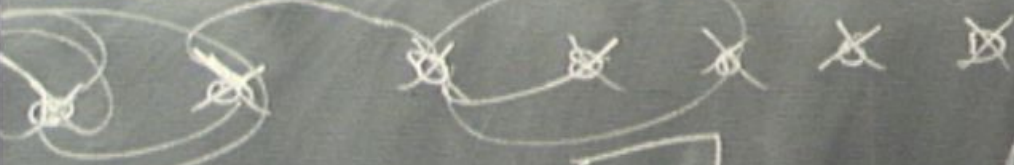
$$\frac{(4n-1)!!}{n! 24^n}$$



$$\int_{-\infty}^{\infty} dx e^{-x^2/k} (1 - e^{-x^4} + e^{-2x^8} + \dots)$$

$$= 1 - \frac{3\epsilon}{5} + \frac{105\epsilon^2}{5} + \dots$$

4-vert e^n



$$(3n)!$$

$$\frac{n!}{(n!)!}$$

$$\frac{x^8}{x^6}$$

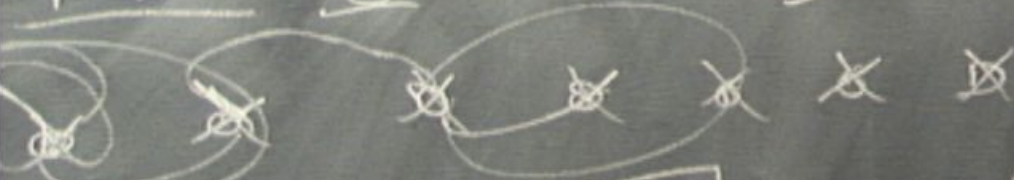
$$(4n-1)(4n-3)\dots$$

$$\frac{(n-1)!!}{n! 24^n}$$

$$\int_{-\infty}^{\infty} dx e^{-x^2/2} \left(1 - \frac{x^4}{4!} + \frac{x^8}{8!} - \dots \right)$$

$$= 1 - \frac{3\epsilon}{5} + \frac{105\epsilon^2}{5} + \dots$$

4-vert e^n



$$\frac{x^8}{(4n-1)(4n-3) \dots}$$

$$\frac{(n-1)!!}{n! 24^n}$$

$$C(n)!$$

$$x^6$$