

Title: Mathematical Physics (PHYS 624) - Lecture 8

Date: Nov 25, 2009 09:00 AM

URL: <http://pirsa.org/09110102>

Abstract:



perimeter scholars  
INTERNATIONAL

$a_n: (a_0=1) a_1 a_2 a_3 \dots$

$$Q_n: \quad (a_0=1) \quad a_1 \quad a_2 \quad a_3 \quad \dots \quad (2n)!$$

1315  
1 5 61 ? (24)  
 $a_n: (a_0=1) a_1 a_2 a_3 \dots$

1315

1   5   61   ?   (2n)!

$a_n: (a_0=1) \quad a_1 \quad a_2 \quad a_3 \quad \dots$

$b_n: (b_0=1) \quad b_1 \quad b_2 \quad b_3 \quad \dots \quad n^2$

$$\begin{array}{ccccccc}
 & & & & 1 & 3 & 5 & \dots & (2n)! \\
 & & & & 1 & 5 & 61 & \underline{?} & \\
 a_n: & (a_0=1) & a_1 & a_2 & a_3 & \dots & & & \\
 \downarrow & & & & & & & & \\
 b_n: & (b_0=1) & b_1 & b_2 & b_3 & \dots & & & n^2
 \end{array}$$

---


$$\text{I. } a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$\begin{array}{ccccccc}
 & & & & 1 & 5 & 61 & \frac{1315}{?} & & (2n)! \\
 & & & & & & & & & \\
 a_n: & (a_0=1) & a_1 & a_2 & a_3 & \dots & & & & \\
 \downarrow & & & & & & & & & \\
 b_n: & (b_0=1) & b_1 & b_2 & b_3 & \dots & & & & n^2
 \end{array}$$

$$\text{I, } a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$\text{II, } P_0(x) = 1 \quad P_1(x) = x$$



$$\begin{array}{ccccccc}
 & & & & 1 & 3 & 5 & \dots & (2n)! \\
 & & & & 1 & 5 & 61 & \underline{?} & \\
 a_n: & (a_0=1) & a_1 & a_2 & a_3 & \dots & & & \\
 \downarrow & & & & & & & & \\
 b_n: & (b_0=1) & b_1 & b_2 & b_3 & \dots & & & n^2
 \end{array}$$

$$\text{I. } a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$\begin{aligned}
 \text{II. } P_0(x) &= 1 & P_1(x) &= x \\
 P_{n+1}(x) &= x P_n(x) - b_n P_{n-1}(x)
 \end{aligned}$$

$$\begin{array}{ccccccc}
 & & & & 1 & 3 & 5 \\
 & & & & 1 & 5 & 61 \\
 & & & & & \underline{?} & \\
 & & & & & & (2n)! \\
 a_n: & (a_0=1) & a_1 & a_2 & a_3 & \dots & \\
 \downarrow & & & & & & \\
 b_n: & (b_0=1) & b_1 & b_2 & b_3 & \dots & n^2
 \end{array}$$

$$\text{I. } a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$\begin{array}{l}
 \text{II. } P_0(x) \equiv 1 \quad P_1(x) = x \\
 P_{n+1}(x) = x P_n(x) - b_n P_{n-1}(x) \\
 \text{(monic poly.)}
 \end{array}$$

$$\begin{array}{ccccccc}
 & & & & 1 & 5 & 61 & \underline{?} & (2n)! \\
 & & & & 1 & 5 & 61 & \underline{?} & (2n)! \\
 a_n: & (a_0=1) & a_1 & a_2 & a_3 & \dots & & & \\
 \downarrow & & & & & & & & \\
 \underline{b_n}: & (b_0=1) & b_1 & b_2 & b_3 & \dots & & & n^2
 \end{array}$$

$$\text{I. } a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$\begin{array}{l}
 \text{II. } P_0(x) = 1 \quad P_1(x) = x \\
 P_{n+1}(x) = x P_n(x) - b_n P_{n-1}(x) \\
 \text{(monic poly.)} \\
 P_2(x) = x^2 - b_1 \\
 P_3(x) = x^3 - b_1 x - b_2 x
 \end{array}$$

$$a_n: (a_0=1) \quad 1 \quad 5 \quad 61 \quad \underline{?} \quad (2n)! \\
b_n: (b_0=1) \quad b_1 \quad b_2 \quad b_3 \quad \dots \quad n^2$$

$$I. \quad a_n = \int_{-L}^L W(x) x^{2n} dx \quad \left( \int_{-L}^L W(x) dx = 1 \right)$$

$$II. \quad P_0(x) = 1 \quad P_1(x) = x$$

$$P_{n+1}(x) = x P_n(x) - b_n P_{n-1}(x)$$

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$$P_2(x) = x^2 - b_1$$

$$P_3(x) = x^3 - (b_1 + b_2)x$$

1 3 5 ... (2n)!

1   5   61   ?

$$a_n: (a_0=1) \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$b_n: (b_0=1) \quad b_1 \quad b_2 \quad b_3 \quad \dots \quad n^2$$

I.  $a_n = \int_{-L}^L W(x) x^{2n} dx$     $\left( \int_{-L}^L W(x) dx = 1 \right)$

even

II.  $P_0(x) = 1$     $P_1(x) = x$

$$P_{n+1}(x) = x P_n(x) - b_n P_{n-1}(x)$$

(monic poly.)

$$P_2(x) = x^2 - b_1$$

$$P_3(x) = x^3 - (b_1 + b_2)x$$

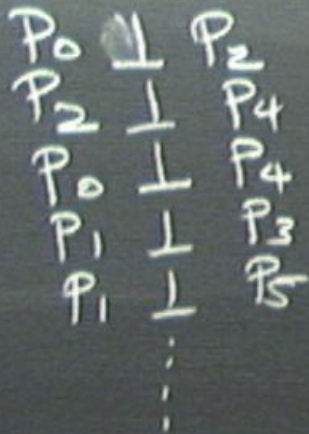
III. Demand  $\{P_n\}$  is orthog w.r.t.  $W(x)$

$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \quad \text{if } n \neq m.$$

obvious!  $\int P_{2n}^{\text{even}} P_{2m+1}^{\text{odd}} W(x) dx = 0 \quad \checkmark$

$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n}^{\text{even}} P_{2m+1}^{\text{odd}} W(x) dx = 0 \checkmark$



$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$

$P_{2n}$   
even
 $P_{2m+1}$   
odd
 $W(x)$   
even

$\left. \begin{array}{l} P_0 \quad | \quad P_2 \\ P_2 \quad | \quad P_4 \\ P_0 \quad | \quad P_4 \\ P_1 \quad | \quad P_3 \\ P_1 \quad | \quad P_5 \\ \vdots \end{array} \right\}$  uniquely determine reln between a's + b's.

$$a_1 = b_1$$

$$a_2 = b_1 (b_1 + b_2)$$

$$a_3 = b_1 b_2 b_3 + b_1 (b_1 + b_2)^2$$



$$\sum_{n=0}^{\infty} a_n x^n$$

$$\frac{b_0}{1 - b_1 x} \frac{1}{1 - b_2 x} \frac{1}{1 - b_3 x} \frac{1}{1 - b_4 x} \dots$$

$$\sum_{n=0}^{\infty} a_n x^n$$

$$\frac{b_0}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \cdot \frac{1}{1 - b_4 x} \cdots$$

$b_0$

$\sum_{n=0}^{\infty} a_n x^n$

$$\frac{\frac{b_0}{1-b_1x}}{1-b_2x} \frac{1}{1-b_3x} \frac{1}{1-b_4x} \dots$$

$$b_0, \frac{b_0}{1-b_1x}, \frac{b_0}{1-b_1x} \frac{1}{1-b_2x}, \dots$$

$$\sum_{n=0}^{\infty} a_n x^n$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$(s_0, s_1, s_2, \dots) \rightarrow L$

$b_0, \frac{b_0}{1-b_1x}, \frac{b_0}{1-b_1x} \frac{1}{1-b_2x}, \dots \rightarrow L$

$$\frac{\frac{b_0}{1-b_1x}}{1-b_2x} \frac{1}{1-b_3x} \frac{1}{1-b_4x} \dots$$

$$\sum_{n=0}^{\infty} a_n x^n$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$(s_0, s_1, s_2, \dots) \rightarrow L$

$b_0, \frac{b_0}{1-b_1x}, \frac{b_0}{1-\frac{b_1x}{1-b_2x}}, \dots \rightarrow L$

$$\frac{\frac{b_0}{1-b_1x}}{1-b_2x} \frac{1}{1-b_3x} \frac{1}{1-b_4x} \dots$$

Wall

$$\sum_{n=0}^{\infty} a_n x^n$$

$$1 + a_1 x + a_2 x^2 + \dots$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$(s_0, s_1, s_2, \dots) \rightarrow L$

$$a_0 = b_0 = 1$$

$$a_1 = b_1$$

$$\frac{b_0}{1 - b_1 x}$$

$$\frac{1 - b_2 x}{1 - b_3 x}$$

$$\frac{1 - b_4 x}{\dots}$$

Wall

$$b_0, \frac{b_0}{1 - b_1 x}, \frac{b_0}{1 - b_1 x} \frac{1 - b_2 x}{1 - b_2 x}, \dots \rightarrow L$$

$$\frac{b_0(1 + b_1 x)}{1 + b_1 x}$$

$$\sum_{n=0}^{\infty} a_n x^n$$

$$1 + a_1 x + a_2 x^2 + \dots$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$$\frac{b_0}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \cdot \frac{1}{1 - b_4 x} \dots$$

Wall

$(S_0, S_1, S_2, \dots) \rightarrow L$

$$a_0 = b_0 = 1$$

$$a_1 = b_1$$

$$b_0 \left( \frac{1}{1 - b_1 x} \right) \left( \frac{1}{1 - b_2 x} \right) \dots \rightarrow L$$

$$\frac{b_0 (1 + b_1 x + \dots)}{(1 + b_1 x + \dots)}$$

$$\frac{1}{1 - b_1 x (1 + b_2 x)}$$

$$1 + b_1 x (1 + b_2 x) + b_1^2 x^2 + \dots$$

$$1 + b_1 x + b_1 (b_1 + b_2) x^2 + \dots$$

$$\sum_{n=0}^{\infty} \frac{a_n x^n}{1 + a_1 x + a_2 x^2 + \dots}$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$S_0, S_1, S_2, \dots \rightarrow L$

$$a_0 = b_0 = 1$$

$$a_1 = b_1$$

$$a_2 = b_1(b_1 + b_2)$$

$$\frac{b_0}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \cdot \frac{1}{1 - b_4 x} \dots$$

Wall

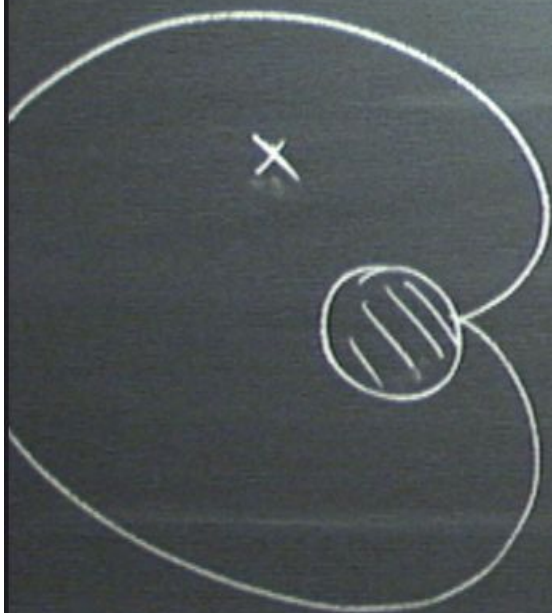
$$b_0 \left( \frac{1}{1 - b_1 x} \right) \left( \frac{1}{1 - b_2 x} \right) \dots \rightarrow L$$

$$b_0 (1 + b_1 x + \dots)$$

$$\frac{1}{1 - b_1 x (1 + b_2 x)} \left( 1 + b_1 x (1 + b_2 x) + b_1^2 x^2 + \dots \right)$$

$$1 + b_1 x + b_1 (b_1 + b_2) x^2 + \dots$$





1315  
 1 5 61 ?  
 (21)

$$a_n: (a_0=1) \quad a_1 \quad a_2 \quad a_3 \quad \dots$$

$$b_n: (b_0=1) \quad b_1 \quad b_2 \quad b_3 \quad \dots$$

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(monic poly.)

$$P_2(x) = x^2 - b_1$$

$$P_3(x) = x^3 - (b_1 + b_2)x$$

III. Demand  $\{P_n\}$  is orthog w.r.t.  $W(x)$

$$C_0 \ln(1-xC_1) \ln(1-xC_2) \ln \dots$$

$$C_0 > C_0 \ln(1-xC_1)$$

$$C_0 \ln(1-xC_1) \ln(1-xC_2)$$

$$\sum_{n=0}^{\infty} a_n x^n$$

$$1 + a_1 x + a_2 x^2 + \dots$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$$\frac{b_0}{1-b_1x} \cdot \frac{1}{1-b_2x} \cdot \frac{1}{1-b_3x} \cdot \frac{1}{1-b_4x} \dots$$

$$S_0, S_1, S_2, \dots \rightarrow L$$

$$a_0 = b_0 = 1$$

$$a_1 = b_1$$

$$a_2 = b_1(b_1 + b_2)$$

$$b_0 \left( \frac{1}{1-b_1x} \cdot \frac{1}{1-b_2x} \right) \rightarrow L$$

$$\frac{b_0(1+b_1x)}{1+b_1x} \cdot \frac{1}{1-b_2x}$$

$$\frac{1}{1-b_1x(1+b_2x)} \cdot (1+b_1x + b_1^2x^2 + \dots)$$

$$1 + b_1x + b_1^2x^2 + \dots + b_1(b_1 + b_2)x + \dots$$

$C_0 \ln(1-xC_1) \ln(1-xC_2) \ln \dots$   
 $C_0 \ln(1-xC_1)$   
 $C_0 \ln(1-xC_1 \ln(1-xC_2))$

$$\sum_{n=0}^{\infty} \frac{a_n x^n}{1+a_1 x+a_2 x^2+\dots}$$

$$S_N = \sum_{n=0}^N a_n x^n$$

$$\frac{b_0}{1-b_1 x} \frac{1}{1-b_2 x} \frac{1}{1-b_3 x} \dots \frac{1}{1-b_n x}$$

$$\text{Ans} = \sum a_n x^n$$

$S_0, S_1, S_2, \dots \rightarrow L$

$$a_0 = b_0 = 1$$

$$a_1 = b_1$$

$$a_2 = b_1(b_1 + b_2)$$

$$b_0 \left( \frac{b_0}{1-b_1 x} \right) \left( \frac{1}{1-b_2 x} \right) \dots \rightarrow L$$

$$\frac{b_0(1+b_1 x)}{(1+b_1 x)}$$

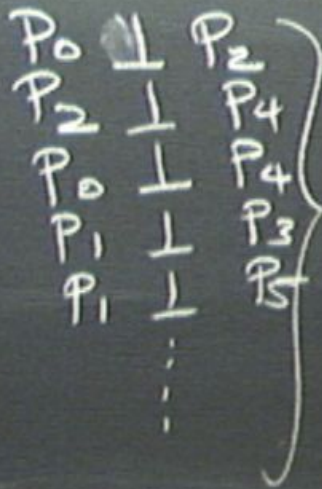
$$\frac{1}{1-b_1 x(1+b_2 x)}$$

$$1 + b_1 x(1+b_2 x) + b_1^2 x^2 + b_1 x(b_1 + b_2 x) + \dots$$

$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$

$\begin{matrix} P_{2n} & P_{2m+1} \\ \text{even} & \text{odd} \end{matrix}$

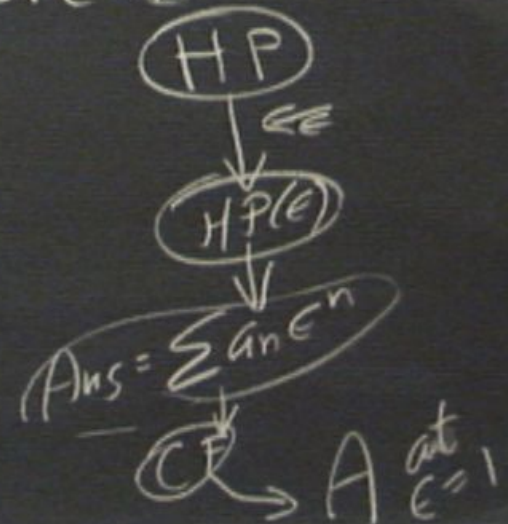


uniquely determine reln between a's + b's.

$$a_1 = b_1$$

$$a_2 = b_1(b_1 + b_2)$$

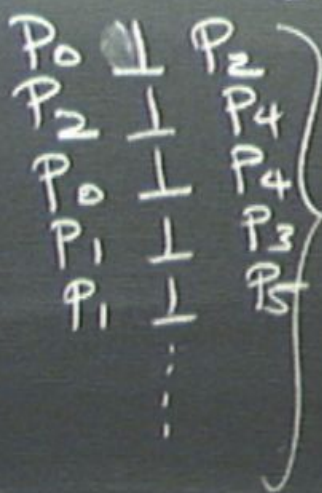
$$a_3 = b_1 b_2 b_3 + b_1(b_1 + b_2)^2$$



$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$

$\begin{matrix} P_{2n} & P_{2m+1} \\ \text{even} & \text{odd} \end{matrix}$

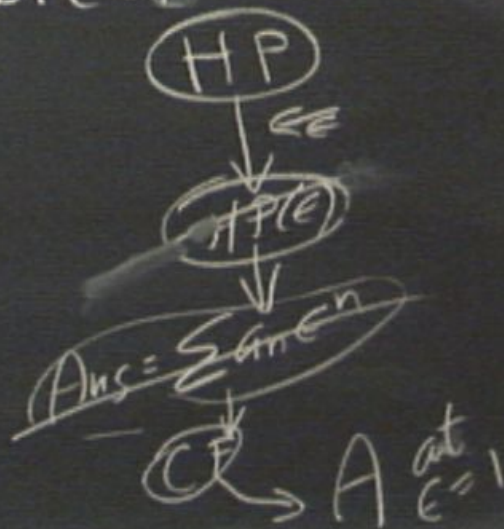


uniquely determine reln between a's + b's.

$$a_1 = b_1$$

$$a_2 = b_1(b_1 + b_2)$$

$$a_3 = b_1 b_2 b_3 + b_1(b_1 + b_2)^2$$



From  
Grains of  
Pollen to  
Evidence  
for Atom.

How  
Big Is A  
Molecule?

$\ln(1-x) \ln(1-x) \ln(1-x)$   
 $\ln(1-x) \ln(1-x)$   
 $\ln(1-x) \ln(1-x)$

$\sum_{n=0}^{\infty} a_n x^n$   
 $1 + a_1 x + a_2 x^2 + \dots$

$S_N = \sum_{n=0}^N a_n x^n$

$S_0, S_1, S_2, \dots \rightarrow L$

$a_0 = b_0 = 1$   
 $a_1 = b_1$   
 $a_2 = b_1(b_1 + b_2)$

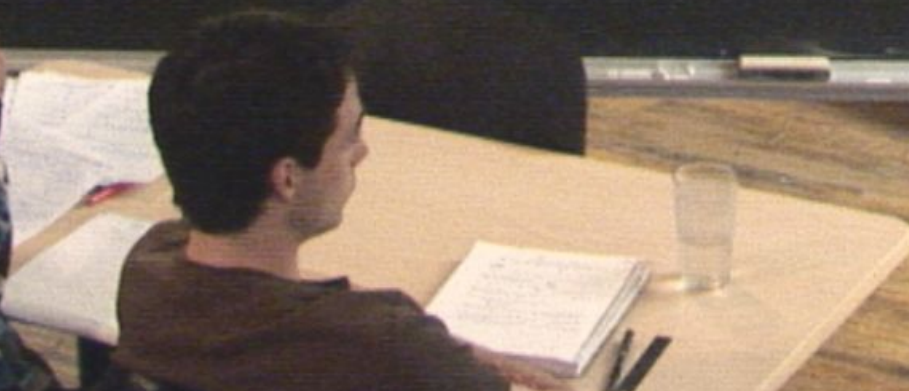
$\frac{b_0}{1 - b_1 x}$   
 $\frac{1}{1 - b_1 x}$   
 $\frac{1}{1 - b_1 x (1 + b_2 x)}$   
 $\frac{1 + b_1 x (1 + b_2 x)}{1 + b_1 x + b_1 b_2 x^2 + \dots}$

$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$   
 $\ln(1-x)^2 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{x^{n+m}}{nm}$   
 $\ln(1-x)^3 = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{x^{n+m+k}}{nmk}$

$\ln(1-x) \ln(1-x) \ln(1-x) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{x^{n+m+k}}{nmk}$

$\ln(1-x) \ln(1-x) \ln(1-x) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{x^{n+m+k}}{nmk}$

$\ln(1-x) \ln(1-x) \ln(1-x) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \frac{x^{n+m+k}}{nmk}$



$$\frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \rightarrow \frac{1}{1 - b_1 x}, \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x}, \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

$$\frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \rightarrow \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

$$\rightarrow \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

$$\rightarrow \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

$$\rightarrow \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

III. Partial Fractions (Partialbruchzerlegung)

$$\frac{1}{(1 - b_1 x)(1 - b_2 x)(1 - b_3 x)} = \frac{A}{1 - b_1 x} + \frac{B}{1 - b_2 x} + \frac{C}{1 - b_3 x}$$

$$1 = A(1 - b_2 x)(1 - b_3 x) + B(1 - b_1 x)(1 - b_3 x) + C(1 - b_1 x)(1 - b_2 x)$$

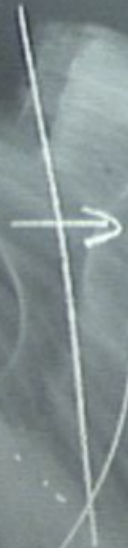
$$1 = A(1 - b_2 x - b_3 x + b_2 b_3 x^2) + B(1 - b_1 x - b_3 x + b_1 b_3 x^2) + C(1 - b_1 x - b_2 x + b_1 b_2 x^2)$$

$$1 = (A + B + C) + (-Ab_2 - Bb_1 - Cb_1)x + (Ab_2 b_3 + Bb_1 b_3 + Cb_1 b_2)x^2$$

$$\begin{cases} A + B + C = 1 \\ -Ab_2 - Bb_1 - Cb_1 = 0 \\ Ab_2 b_3 + Bb_1 b_3 + Cb_1 b_2 = 0 \end{cases}$$



$$\frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$



$$\left( \frac{1}{1 - b_1 x}, \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x}, \frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x} \right)$$

$$\left( \frac{1}{1 - b_1 x}, \frac{1 - b_2 x}{1 - b_1 x - b_2 x} \right)$$

$$\rightarrow \frac{1}{1 - \frac{b_1 x (1 - b_3 x)}{1 - b_3 x - b_2 x}}$$

$$\rightarrow \frac{1 - b_3 x - b_2 x}{1 - b_3 x - b_2 x - b_1 x (1 - b_3 x)}$$

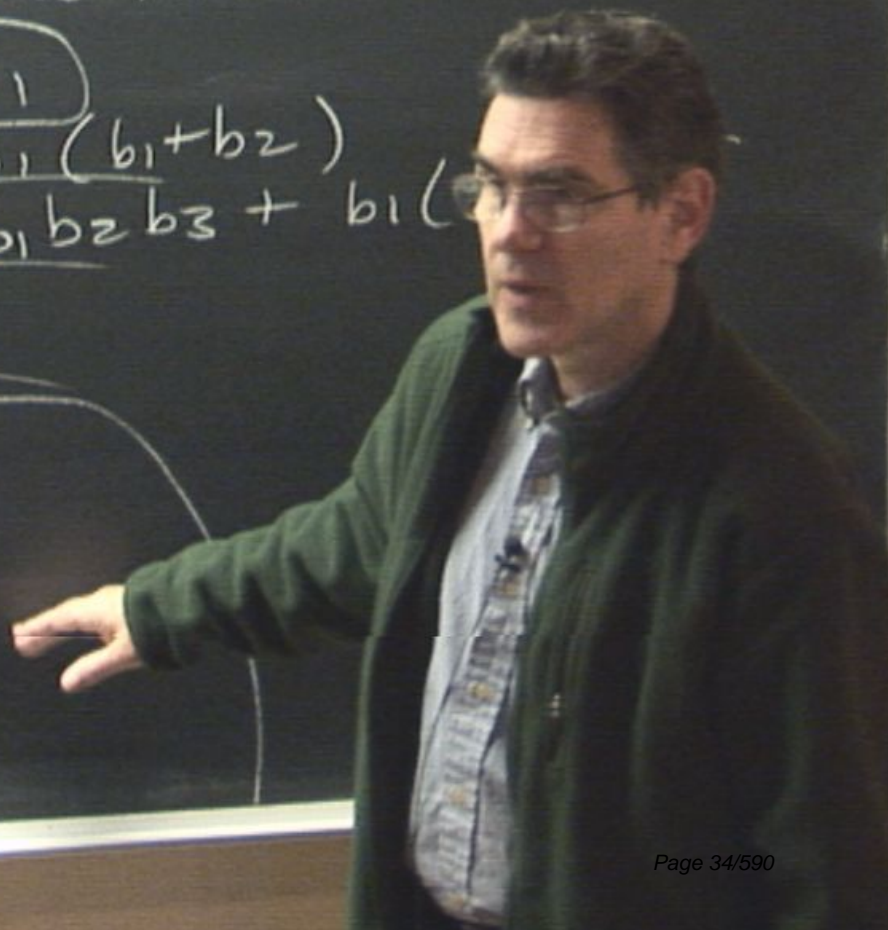
$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$   
even      odd      even

$\left. \begin{array}{l} P_0 \perp P_2 \\ P_2 \perp P_4 \\ P_0 \perp P_4 \\ P_1 \perp P_3 \\ P_1 \perp P_5 \\ \vdots \end{array} \right\}$  uniquely determine reln between a's + b's.

$$\begin{aligned} a_1 &= b_1 \\ a_2 &= b_1(b_1 + b_2) \\ a_3 &= b_1 b_2 b_3 + b_1 \end{aligned}$$

$$\frac{P_0}{Q_0}, \frac{P_0}{Q_1}, \frac{P_1}{Q_1}, \frac{P_1}{Q_2}$$



$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$

$\begin{matrix} P_{2n} & P_{2m+1} \\ \text{even} & \text{odd} \end{matrix}$

$\left. \begin{matrix} P_0 & \perp & P_2 \\ P_2 & \perp & P_4 \\ P_0 & \perp & P_4 \\ P_1 & \perp & P_3 \\ P_1 & \perp & P_5 \\ & \vdots & \end{matrix} \right\}$  uniquely determine reln between a's + b's.

$$\begin{aligned} a_1 &= b_1 \\ a_2 &= b_1(b_1 + b_2) \\ a_3 &= b_1 b_2 b_3 + b_1(b_1 + b_2)^2 \\ &\vdots \end{aligned}$$

$$\boxed{P_n^m(x)} = \frac{\text{Poly of deg } m}{\text{Poly of deg } n}$$

$$\frac{P_0}{Q_0}, \frac{P_0}{Q_1}, \frac{P_1}{Q_1}, \frac{P_1}{Q_2}, \frac{P_2}{Q_2}, \frac{P_2}{Q_3}, \frac{P_3}{Q_3}$$

Padé

$$P_0^0, P_1^0, P_1^1, P_2^1, P_2^2, P_3^2, P_3^3, \dots$$

$$\int_{-L}^L dx P_n(x) P_m(x) W(x) = 0 \text{ if } n \neq m.$$

obvious!  $\int_{-L}^L P_{2n} P_{2m+1} W(x) dx = 0 \checkmark$

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$P_0^0, P_1^0, P_1^1, P_2^1, P_2^2, P_3^2, P_3^3, \dots$   
 "main" or "diagonal"

$$\frac{1}{1 - b_1 x} \cdot \frac{1}{1 - b_2 x} \cdot \frac{1}{1 - b_3 x}$$

ANSWER  $\sum_{n=0}^{\infty} a_n x^n$

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$$\sum_{n=0}^N a_n x^n = P_m(x)$$

$N = n+m$

$N+1$   
Coeff(s)  
 $a_0, a_1, \dots, a_N$

Poly  $m$   $(m+1)$   
Poly  $n$   $(n+1)$

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Poly  $m$   $(m+1)$   
Poly  $n$   $(n)$   
 $x^n + \dots + x^{n-1}$

$$N+1 = \frac{n+m+1}{1}$$

$$N = n+m \quad \checkmark$$

$$x^5 + x = 1$$

$$x_0 = 0.75487767$$

$$\boxed{\epsilon x^5 + x = 1}$$

$$x = 1 - \epsilon + 5\epsilon^2 - 35\epsilon^3 + 285\epsilon^4 - 2530\epsilon^5 + 23751\epsilon^6 + \dots$$

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$$R = \frac{4^4}{5^5} = 0.08192$$

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$$[3,3]\text{-Padé} \rightarrow 0.76369 \text{ [1.2\% error]}$$

$$\boxed{x^5 + \epsilon x = 1}$$

$$x = 1 - \frac{1}{5}\epsilon - \frac{1}{25}\epsilon^2 - \frac{1}{125}\epsilon^3 + \frac{21}{15625}\epsilon^5 + \frac{78}{78125}\epsilon^6 + \dots$$

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$$x(1) = \sum_{n=0}^6 a_n = 21476$$

$$[3,3]\text{-Padé} \rightarrow 0.76369 \text{ [1.2\% error]}$$

$$\underline{x^5 + \epsilon x = 1}$$

$$x = 1 - \frac{1}{5}\epsilon - \frac{1}{25}\epsilon^2 - \frac{1}{125}\epsilon^3 + \frac{21}{15625}\epsilon^5 + \frac{78}{78125}\epsilon^6 + \dots$$

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$$[6,6 \text{ Padé}] \quad 0.75487654 \quad (0.00015\% \text{ error})$$

[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

$$\boxed{x^5 + \epsilon x = 1}$$

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[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

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$$[6,6 \text{ Padé}] \quad 0.75487654 \quad (0.00015\% \text{ error})$$

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[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

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$$[6,6 \text{ Padé}] \quad 0.75487654 \quad (0.00015\% \text{ error})$$

[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

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[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

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$$[6,6 \text{ Padé}] \quad 0.75487654 \quad (0.00015\% \text{ error})$$

[3,3]-Padé  $\rightarrow$  0.75434 (0.07% error)

$$\boxed{x^5 + \epsilon x = 1}$$

$$x = 1 - \frac{1}{5}\epsilon - \frac{1}{25}\epsilon^2 - \frac{1}{125}\epsilon^3 + \frac{21}{15625}\epsilon^5 + \frac{78}{78125}\epsilon^6 + \dots$$

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**Table 8.9** A comparison of the convergence rates of the Taylor and Padé approximants to  $e^x$  at  $x = 1$

The actual values of the approximants and their relative errors [relative error = (approximant - exact value)/exact value] are listed. Observe that (a) the Taylor approximants are monotone increasing while the Padé approximants have errors with the sign pattern  $++--++--\dots$ , which will be explained in Sec. 8.5; and (b) each Padé approximant is constructed from and contains the same information as the Taylor approximant listed on the same line, but its relative error is noticeably smaller. It will be shown in Sec. 8.5 that the error in the Padé approximants is smaller than the error in the Taylor approximants by a factor proportional to  $2^n$  as  $n \rightarrow +\infty$

$n$	Taylor series $\sum_{k=0}^n \frac{1}{k!}$	Padé approximant $P_M^N(1)$	Relative errors	
			Taylor series	Padé approximant
0	1.0	$P_0^0 = 1$		
1	2.0	$P_1^0 = \infty$	-2.64 (-1)	$\infty$
2	2.5	$P_1^1 = 3$	-8.03 (-2)	1.04 (-1)
3	2.666 67	$P_2^1 = 2.666 67$	-1.90 (-2)	-1.90 (-2)
4	2.708 33	$P_2^2 = 2.714 29$	-3.66 (-3)	-1.47 (-3)
5	2.716 67	$P_3^2 = 2.718 75$	-5.94 (-4)	1.72 (-4)
6	2.718 06	$P_3^3 = 2.718 31$	-8.32 (-5)	1.03 (-5)
7	2.718 25	$P_4^3 = 2.718 28$	-1.02 (-5)	-8.31 (-7)
8	2.718 28	$P_4^4 = 2.718 28$	-1.13 (-6)	-4.05 (-8)
9	2.718 28	$P_5^4 = 2.718 28$	-1.11 (-7)	2.48 (-9)
10	2.718 28	$P_5^5 = 2.718 28$	-1.00 (-8)	1.02 (-10)
11	2.718 28	$P_6^5 = 2.718 28$	-8.32 (-10)	-5.02 (-12)
12	2.718 28	$P_6^6 = 2.718 28$	-6.36 (-11)	-1.77 (-13)
13	2.718 28	$P_7^6 = 2.718 28$	-4.52 (-12)	7.31 (-15)
14	2.718 28	$P_7^7 = 2.718 28$	-3.00 (-13)	2.27 (-16)
15	2.718 28	$P_8^7 = 2.718 28$	-1.87 (-14)	-8.03 (-18)

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3	2.666 67	$P_2^1 = 2.666 67$	-1.90 (-2)	-1.90 (-2)
4	2.708 33	$P_2^2 = 2.714 29$	-3.66 (-3)	-1.47 (-3)
5	2.716 67	$P_3^2 = 2.718 75$	-5.94 (-4)	1.72 (-4)
6	2.718 06	$P_3^3 = 2.718 31$	-8.32 (-5)	1.03 (-5)
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14	2.718 28	$P_7^7 = 2.718 28$	-3.00 (-13)	2.27 (-16)
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2	2.5	$P_1^1 = 3$	-8.03 (-2)	1.04 (-1)
3	2.666 67	$P_2^1 = 2.666 67$	-1.90 (-2)	-1.90 (-2)
4	2.708 33	$P_2^2 = 2.714 29$	-3.66 (-3)	-1.47 (-3)
5	2.716 67	$P_3^2 = 2.718 75$	-5.94 (-4)	1.72 (-4)
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11	2.718 28	$P_6^5 = 2.718 28$	-8.32 (-10)	-5.02 (-12)
12	2.718 28	$P_6^6 = 2.718 28$	-6.36 (-11)	-1.77 (-13)
13	2.718 28	$P_7^6 = 2.718 28$	-4.52 (-12)	7.31 (-15)
14	2.718 28	$P_7^7 = 2.718 28$	-3.00 (-13)	2.27 (-16)
15	2.718 28	$P_8^7 = 2.718 28$	-1.87 (-14)	-8.03 (-18)
16	2.718 28	$P_8^8 = 2.718 28$	-1.09 (-15)	-2.22 (-19)
17	2.718 28	$P_9^8 = 2.718 28$	-6.06 (-17)	6.89 (-21)
18	2.718 28	$P_9^9 = 2.718 28$	-3.18 (-18)	1.71 (-22)
19	2.718 28	$P_{10}^9 = 2.718 28$	-1.59 (-19)	-4.74 (-24)
20	2.718 28	$P_{10}^{10} = 2.718 28$	-7.54 (-21)	-1.07 (-25)

Table 8.10 Same as in Table 8.9 except that the approximants are evaluated at  $x = 5$

$n$	Taylor series $\sum_{k=0}^n \frac{5^k}{k!}$	Padé approximants $P_{M}^N(5)$	Relative errors	
			Taylor series	Padé approximants
7	128.619	$P_4^3 = 71.385$	-1.33 (-1)	-5.19 (-1)
8	138.307	$P_4^4 = 128.619$	-6.81 (-2)	-1.33 (-1)
9	143.689	$P_5^4 = 158.621$	-3.18 (-2)	6.88 (-2)
10	146.381	$P_5^5 = 149.697$	-1.37 (-2)	8.65 (-3)
11	147.604	$P_6^5 = 148.001$	-5.45 (-3)	-2.78 (-3)
12	148.114	$P_6^6 = 148.362$	-2.02 (-3)	-3.43 (-4)
13	148.310	$P_7^6 = 148.427$	-6.98 (-4)	9.05 (-5)
14	148.380	$P_7^7 = 148.415$	-2.26 (-4)	1.03 (-5)
15	148.403	$P_8^7 = 148.413$	-6.90 (-5)	-2.28 (-6)
16	148.410	$P_8^8 = 148.413$	-1.99 (-5)	-2.41 (-7)
17	148.412	$P_9^8 = 148.143$	-5.42 (-6)	4.56 (-8)
18	148.413	$P_9^9 = 148.413$	-1.40 (-6)	4.48 (-9)
19	148.413	$P_{10}^9 = 148.413$	-3.45 (-7)	-7.43 (-10)
20	148.413	$P_{10}^{10} = 148.413$	-8.11 (-8)	-6.81 (-11)
21	148.413	$P_{11}^{10} = 148.413$	-1.82 (-8)	1.00 (-11)
22	148.413	$P_{11}^{11} = 148.413$	-3.91 (-9)	8.58 (-13)
23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
25	148.413	$P_{13}^{12} = 148.413$	-3.05 (-11)	1.09 (-15)
26	148.413	$P_{13}^{13} = 148.413$	-5.60 (-12)	8.29 (-17)

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23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
25	148.413	$P_{13}^{12} = 148.413$	-3.05 (-11)	1.09 (-15)
26	148.413	$P_{13}^{13} = 148.413$	-5.60 (-12)	8.29 (-17)



Table 8.10 Same as in Table 8.9 except that the approximants are evaluated at  $x = 5$

$n$	Taylor series $\sum_{k=0}^n \frac{5^k}{k!}$	Padé approximants $P_{M}^N(5)$	Relative errors	
			Taylor series	Padé approximants
7	128.619	$P_4^3 = 71.385$	-1.33 (-1)	-5.19 (-1)
8	138.307	$P_4^4 = 128.619$	-6.81 (-2)	-1.33 (-1)
9	143.689	$P_5^4 = 158.621$	-3.18 (-2)	6.88 (-2)
10	146.381	$P_5^5 = 149.697$	-1.37 (-2)	8.65 (-3)
11	147.604	$P_6^5 = 148.001$	-5.45 (-3)	-2.78 (-3)
12	148.114	$P_6^6 = 148.362$	-2.02 (-3)	-3.43 (-4)
13	148.310	$P_7^6 = 148.427$	-6.98 (-4)	9.05 (-5)
14	148.380	$P_7^7 = 148.415$	-2.26 (-4)	1.03 (-5)
15	148.403	$P_8^7 = 148.413$	-6.90 (-5)	-2.28 (-6)
16	148.410	$P_8^8 = 148.413$	-1.99 (-5)	-2.41 (-7)
17	148.412	$P_9^8 = 148.143$	-5.42 (-6)	4.56 (-8)
18	148.413	$P_9^9 = 148.413$	-1.40 (-6)	4.48 (-9)
19	148.413	$P_{10}^9 = 148.413$	-3.45 (-7)	-7.43 (-10)
20	148.413	$P_{10}^{10} = 148.413$	-8.11 (-8)	-6.81 (-11)
21	148.413	$P_{11}^{10} = 148.413$	-1.82 (-8)	1.00 (-11)
22	148.413	$P_{11}^{11} = 148.413$	-3.91 (-9)	8.58 (-13)
23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
25	148.413	$P_{13}^{12} = 148.413$	-3.05 (-11)	1.09 (-15)
26	148.413	$P_{13}^{13} = 148.413$	-5.60 (-12)	8.29 (-17)

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23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
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22	148.413	$P_{11}^{11} = 148.413$	-3.91 (-9)	8.58 (-13)
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23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
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9	143.689	$P_5^4 = 158.621$	-3.18 (-2)	6.88 (-2)
10	146.381	$P_5^5 = 149.697$	-1.37 (-2)	8.65 (-3)
11	147.604	$P_6^5 = 148.001$	-5.45 (-3)	-2.78 (-3)
12	148.114	$P_6^6 = 148.362$	-2.02 (-3)	-3.43 (-4)
13	148.310	$P_7^6 = 148.427$	-6.98 (-4)	9.05 (-5)
14	148.380	$P_7^7 = 148.415$	-2.26 (-4)	1.03 (-5)
15	148.403	$P_8^7 = 148.413$	-6.90 (-5)	-2.28 (-6)
16	148.410	$P_8^8 = 148.413$	-1.99 (-5)	-2.41 (-7)
17	148.412	$P_9^8 = 148.143$	-5.42 (-6)	4.56 (-8)
18	148.413	$P_9^9 = 148.413$	-1.40 (-6)	4.48 (-9)
19	148.413	$P_{10}^9 = 148.413$	-3.45 (-7)	-7.43 (-10)
20	148.413	$P_{10}^{10} = 148.413$	-8.11 (-8)	-6.81 (-11)
21	148.413	$P_{11}^{10} = 148.413$	-1.82 (-8)	1.00 (-11)
22	148.413	$P_{11}^{11} = 148.413$	-3.91 (-9)	8.58 (-13)
23	148.413	$P_{12}^{11} = 148.413$	-8.07 (-10)	-1.13 (-13)
24	148.413	$P_{12}^{12} = 148.413$	-1.60 (-10)	-9.13 (-15)
25	148.413	$P_{13}^{12} = 148.413$	-3.05 (-11)	1.09 (-15)
26	148.413	$P_{13}^{13} = 148.413$	-5.60 (-12)	8.29 (-17)

Table 8.10 Same as in Table 8.9 except that the approximants are evaluated at  $x = 5$

n	Taylor series $\sum_{k=0}^n \frac{5^k}{k!}$	Padé approximants $P_{M}^N(5)$	Relative errors	
			Taylor series	Padé approximants
7	128.619	$P_4^3 = 71.385$	-1.33 (-1)	-5.19 (-1)
8	138.307	$P_4^4 = 128.619$	-6.81 (-2)	-1.33 (-1)
9	143.689	$P_5^4 = 158.621$	-3.18 (-2)	6.88 (-2)
10	146.381	$P_5^5 = 149.697$	-1.37 (-2)	8.65 (-3)
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z=0$**

In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

$P_N^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
6	0.810 930 216 2	0.693 147 180 7	0.549 306 177 9
7	0.810 930 216 2	0.693 147 180 6	0.549 306 146 7
8	0.810 930 216 2	0.693 147 180 6	0.549 306 144 5

$P_{N+1}^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
5	0.810 930 216 2	0.693 147 179 9	0.549 306 034 1
6	0.810 930 216 2	0.693 147 180 5	0.549 306 136 4
7	0.810 930 216 2	0.693 147 180 6	0.549 306 143 8
8	0.810 930 216 2	0.693 147 180 6	0.549 306 144 3

**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

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3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
6	0.810 930 216 2	0.693 147 180 7	0.549 306 177 9
7	0.810 930 216 2	0.693 147 180 6	0.549 306 146 7
8	0.810 930 216 2	0.693 147 180 6	0.549 306 144 5

$P_{N+1}^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
5	0.810 930 216 2	0.693 147 179 9	0.549 306 034 1
6	0.810 930 216 2	0.693 147 180 5	0.549 306 136 4
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$

$P_N^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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$P_{N+1}^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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$P_{N+1}^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
5	0.810 930 216 2	0.693 147 179 9	0.549 306 034 1
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

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$N$	$x = 0.5$	$x = 1$	$x = 2$
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3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
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4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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$-xG_1 \ln(1-xG_2 \ln \dots$   
 $-xG_1),$   
 $G \ln(1-xG_2))$

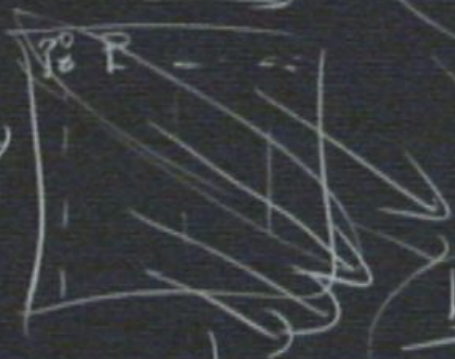
$$\sum_0^{\infty} \frac{a_n x^n}{1+a_1 x+a_2 x^2+\dots}$$

$$S_N = \sum_0^N a_n x^n$$

$S_0, S_1, S_2, \dots \rightarrow L$

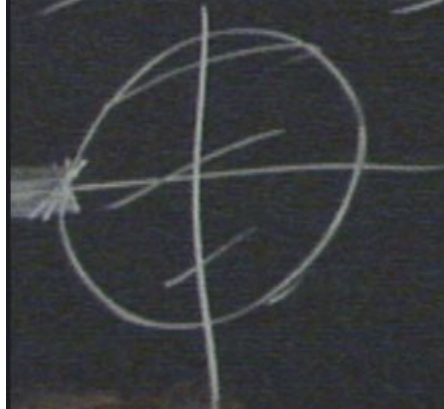
$$\begin{aligned}
 a_0 &= b_0 = 1 \\
 a_1 &= b_1 \\
 a_2 &= b_1(b_1 + b_2)
 \end{aligned}$$

$$\frac{\frac{b_0}{1-b_1 x}}{1-b_2 x} \frac{1}{1-b_3 x} \dots$$



$$\begin{aligned}
 & \frac{b_0}{1-b_1 x} \\
 & \frac{b_0(1+b_1 x+\dots)}{1+b_1 x+\dots} \\
 & \frac{1}{1-b_1 x} \frac{1}{1-b_2 x} \\
 & 1-b_1 x(1+b_2 x)
 \end{aligned}$$

$n \in \mathbb{N}$





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3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
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In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

$P_N^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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$N$	$x = 0.5$	$x = 1$	$x = 2$
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2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
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In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

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$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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5	0.810 930 216 2	0.693 147 185 0	0.549 306 618 4
6	0.810 930 216 2	0.693 147 180 7	0.549 306 177 9
7	0.810 930 216 2	0.693 147 180 6	0.549 306 146 7
8	0.810 930 216 2	0.693 147 180 6	0.549 306 144 5

$P_{N+1}^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.810 810 810 8	0.692 307 692 3	0.545 454 545 5
2	0.810 928 961 7	0.693 121 693 1	0.549 019 607 8
3	0.810 930 203 2	0.693 146 417 4	0.549 285 176 8
4	0.810 930 216 1	0.693 147 157 9	0.549 304 620 9
5	0.810 930 216 2	0.693 147 179 9	0.549 306 034 1
6	0.810 930 216 2	0.693 147 180 5	0.549 306 136 4
7	0.810 930 216 2	0.693 147 180 6	0.549 306 143 8
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**Table 8.13 Padé summation of the Taylor series for  $z^{-1} \ln(1+z)$  about  $z = 0$**

In Sec. 8.5, it will be shown that the sequences of Padé approximants  $P_N^N(z)$  and  $P_{N+1}^N(z)$  converge rapidly, even beyond the circle of convergence of the Taylor series  $|z| < 1$ . Observe that for real positive  $x$  the Padé approximants  $P_N^N(x)$  monotonically decrease and  $P_{N+1}^N(x)$  monotonically increase with  $N$  to the common limit  $\ln(1+x)/x$ . Thus, for any  $N$ , these Padé approximants supply upper and lower bounds on  $\ln(1+x)/x$ .

$P_N^N(x)$			
$N$	$x = 0.5$	$x = 1$	$x = 2$
1	0.812 500 000 0	0.700 000 000 0	0.571 428 571 4
2	0.810 945 273 6	0.693 333 333 3	0.550 724 637 7
3	0.810 930 365 3	0.693 152 454 8	0.549 402 823 0
4	0.810 930 217 7	0.693 147 332 4	0.549 312 879 5
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$C_1 \ln(1-x) C_2 \ln(\dots)$   
 $\ln(1-x_2)$

$$\sum_0^{\infty} \frac{a_n x^n}{1 + a_1 x + a_2 x^2 + \dots}$$

$$S_N = \sum_0^N a_n x^n$$

$S_0, S_1, S_2, \dots \rightarrow L$

$$\begin{aligned}
 a_0 &= b_0 = 1 \\
 a_1 &= b_1 \\
 a_2 &= b_1(b_1 + b_2)
 \end{aligned}$$

$$\frac{\frac{b_0}{1-b_1x}}{1-b_2x} \dots \frac{1}{1-b_3x} \dots \frac{1}{1-b_4x} \dots$$



$$\begin{aligned}
 & \frac{b_0}{1-b_1x} \\
 & \frac{b_0(1+b_1x)}{1+b_1x} \\
 & \frac{1}{1-b_1x} \frac{1}{1-b_2x} \\
 & \frac{1}{1-b_1x(1+b_2x)}
 \end{aligned}$$

$C_1 \ln(1-x) C_2 \ln(\dots)$   
 $\ln(1-xe_2)$

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$$\frac{b_0}{1-b_1x} = \frac{b_0(1+b_1x)}{1+b_1x}$$

$$\frac{1}{1-b_1x} = \frac{1}{1-b_2x}$$

$$\frac{1}{1-b_1x(1+b_2x)}$$

$(1-x\alpha_1) \ln(1-x\alpha_2) \ln \dots$   
 $(1-x\alpha_1)$   
 $-x\alpha_2 \ln(1-x\alpha_2)$

$\sum_{n=0}^{\infty} a_n x^n$   
 $1 + a_1 x + a_2 x^2 + \dots$

$S_N = \sum_{n=0}^N a_n x^n$

$a_n \in \mathbb{R}^n$

$S_0, S_1, S_2, \dots \rightarrow L$

$a_0 = b_0 = 1$   
 $a_1 = b_1$   
 $a_2 = b_1(b_1 + \dots)$

$\frac{b_0}{1-b_1x}$   
 $\frac{1}{1-b_1x}$   
 $\frac{1}{1-b_1x(1+b_2x)}$   
 $1 + b_1x + b_1^2x^2 + \dots$

$\frac{b_0}{1-b_1x}$   
 $\frac{1}{1-b_1x}$   
 $\frac{1}{1-b_1x(1+b_2x)}$   
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 $\frac{1}{1-b_1x}$   
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 $1 + b_1x + b_1^2x^2 + \dots$

$(1-xc_1 \ln(1-xc_2 \ln \dots$   
 $(1-xc_1)$   
 $-xc_1 \ln(1-xc_2))$

$$\sum_{n=0}^{\infty} \frac{a_n x^n}{1+a_1 x+a_2 x^2+\dots}$$

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