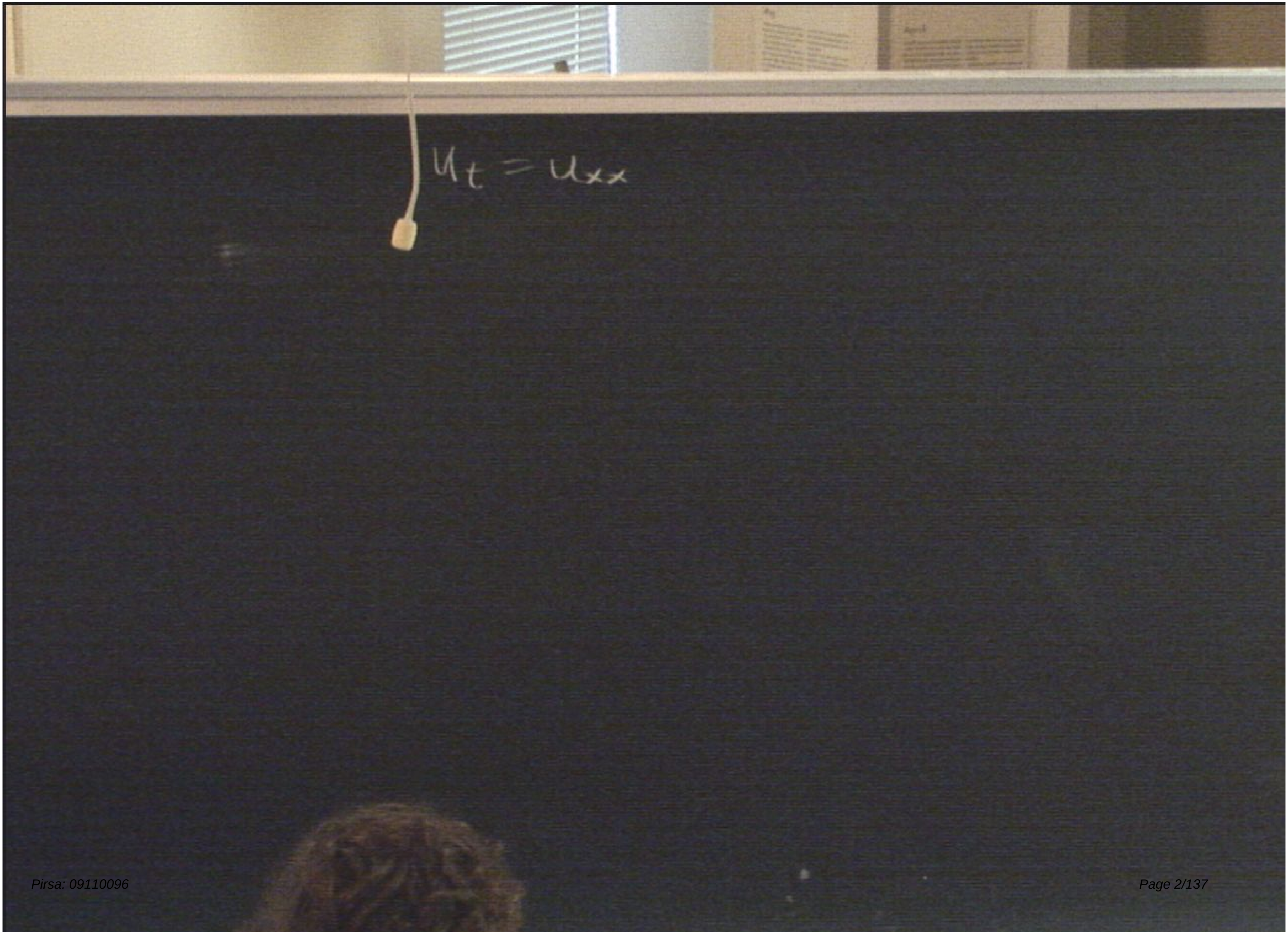
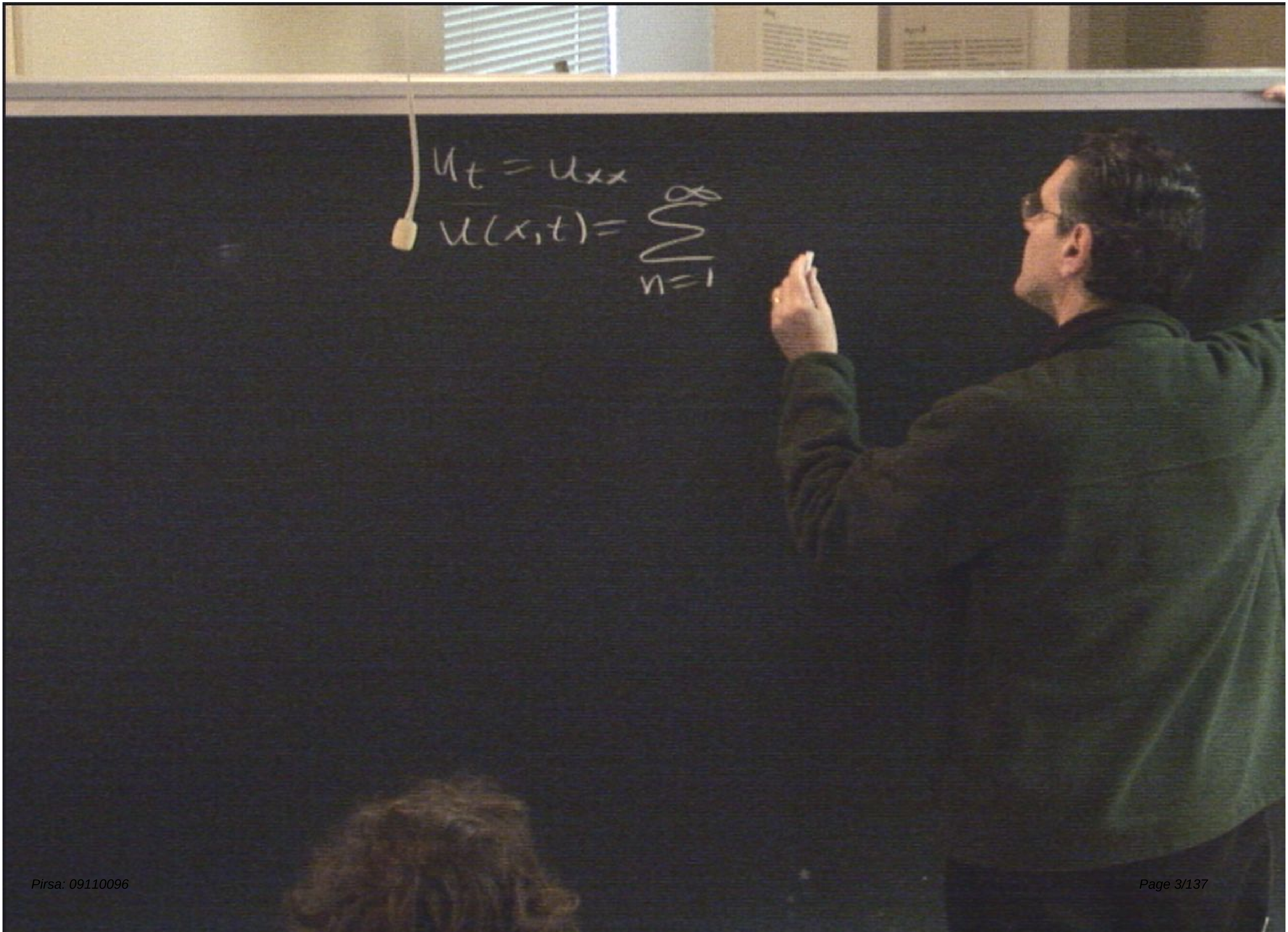
Title: Mathematical Physics (PHYS 624) - Lecture 4

Date: Nov 19, 2009 09:00 AM

URL: http://pirsa.org/09110096

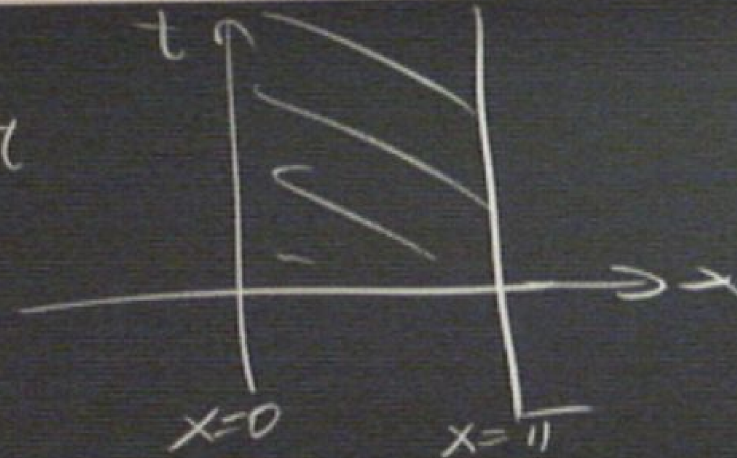
Abstract:

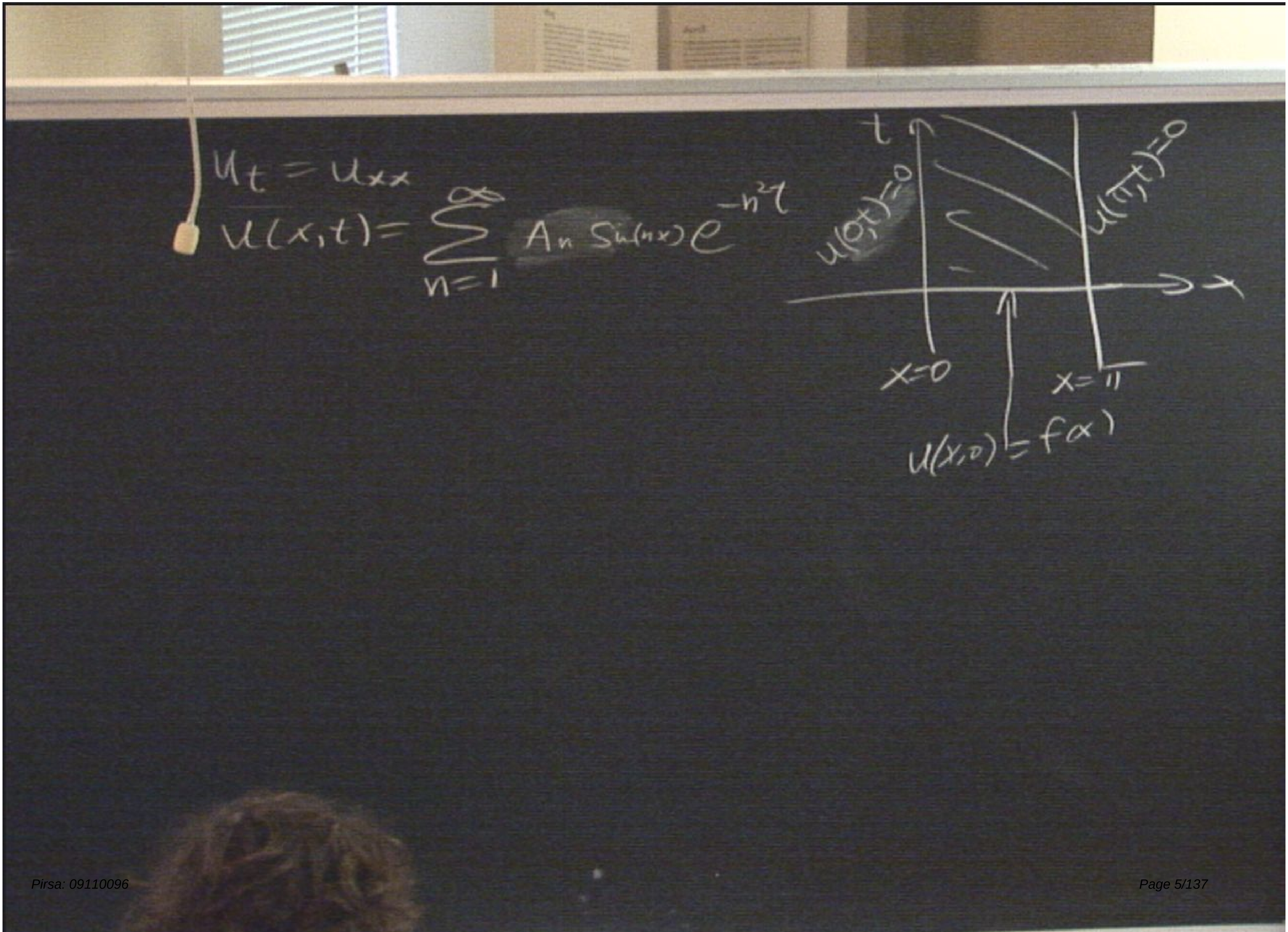

$u_t = u_{xx}$


$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty}$

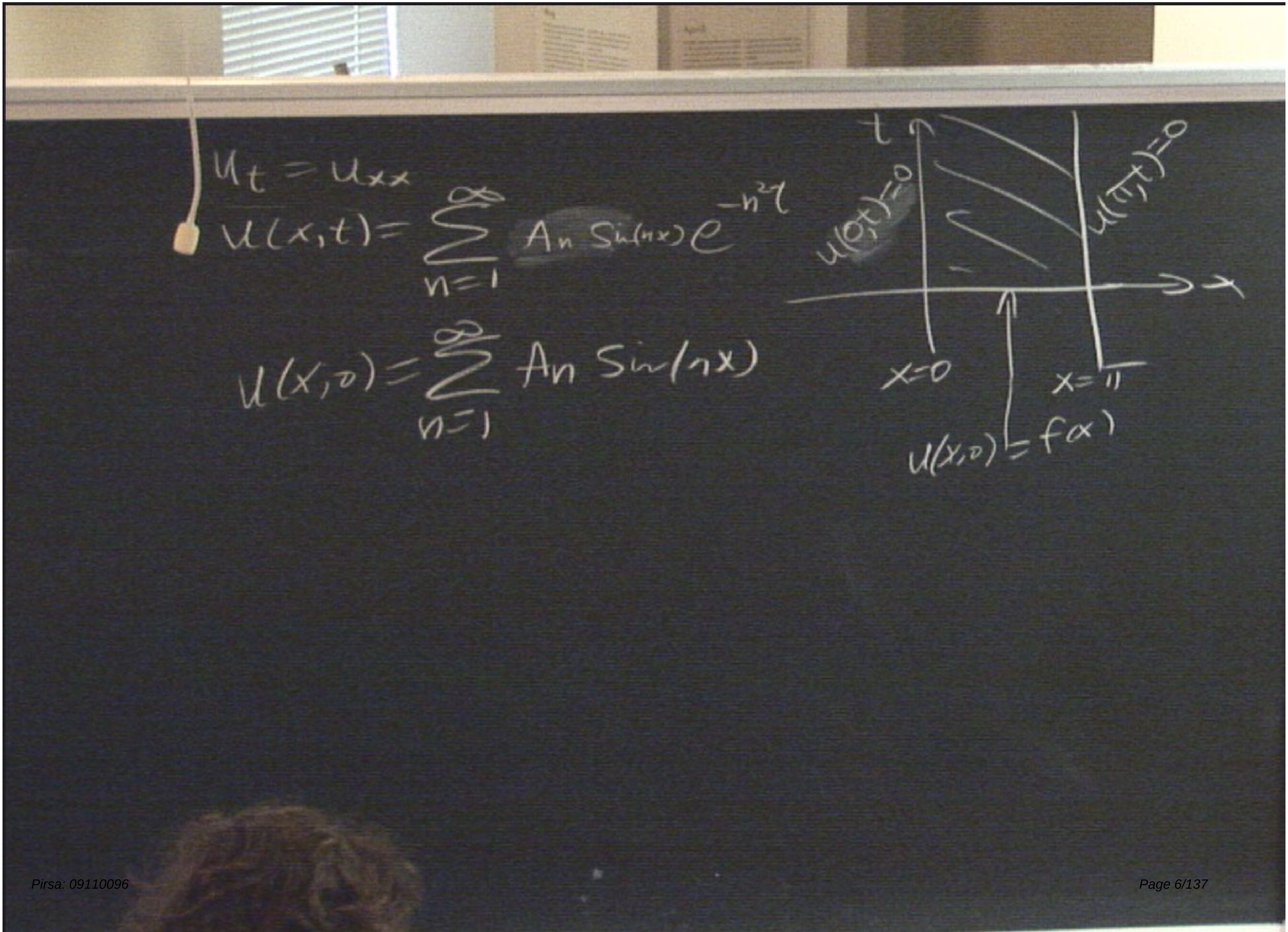
$$u_t = u_{xx}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$





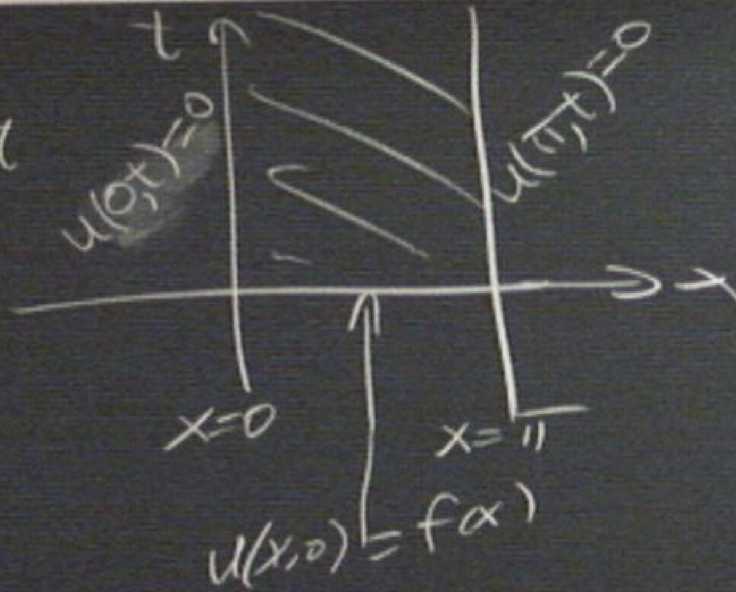
$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$
t
x
u(0,t)=0
u(π,t)=0
x=0
x=π
u(x,0)=f(x)


$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$
$u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$
t
x
$u(0,t)=0$
$u(\pi,t)=0$
$x=0$
$x=\pi$
$u(x,0)=f(x)$

$$u_t = u_{xx}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$

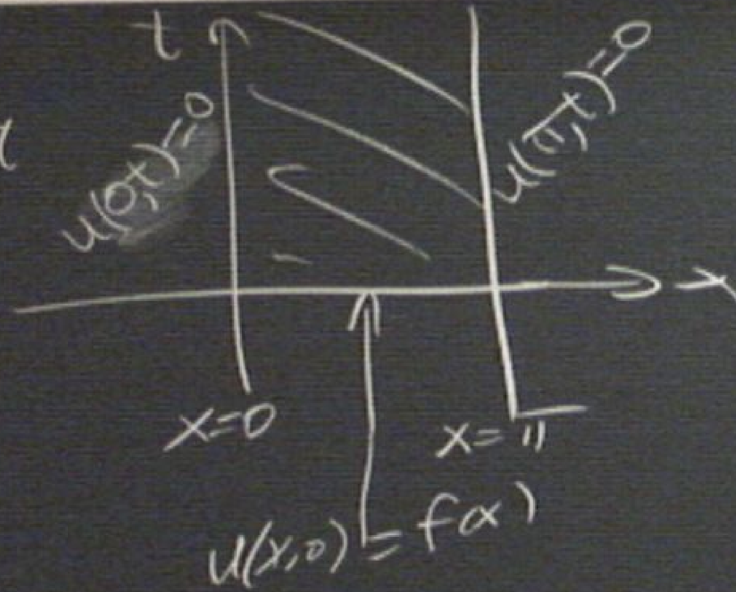
$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$



$$u_t = u_{xx}$$

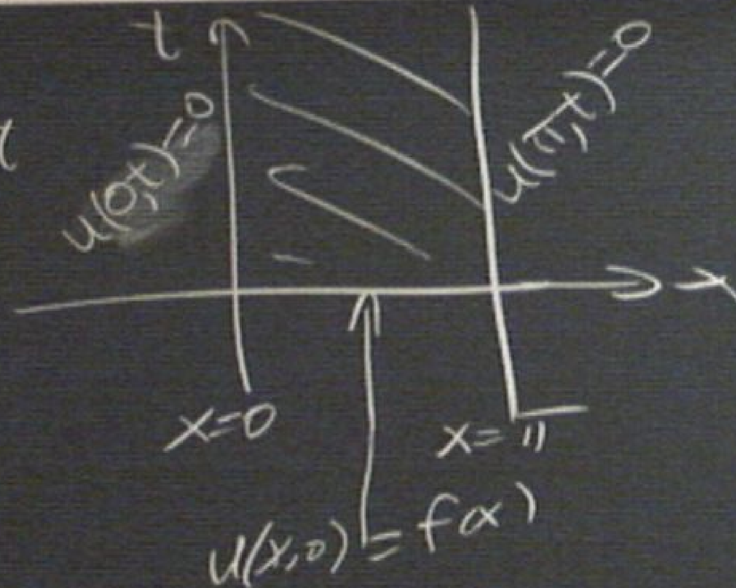
$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$



$$u_t = u_{xx}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$

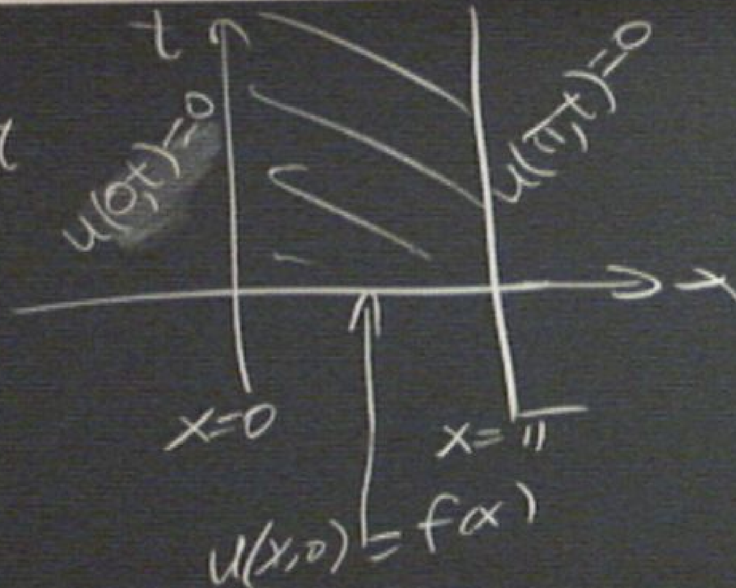


$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx\, \sin(nx)\, f(x)$

$$u_t = u_{xx}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$



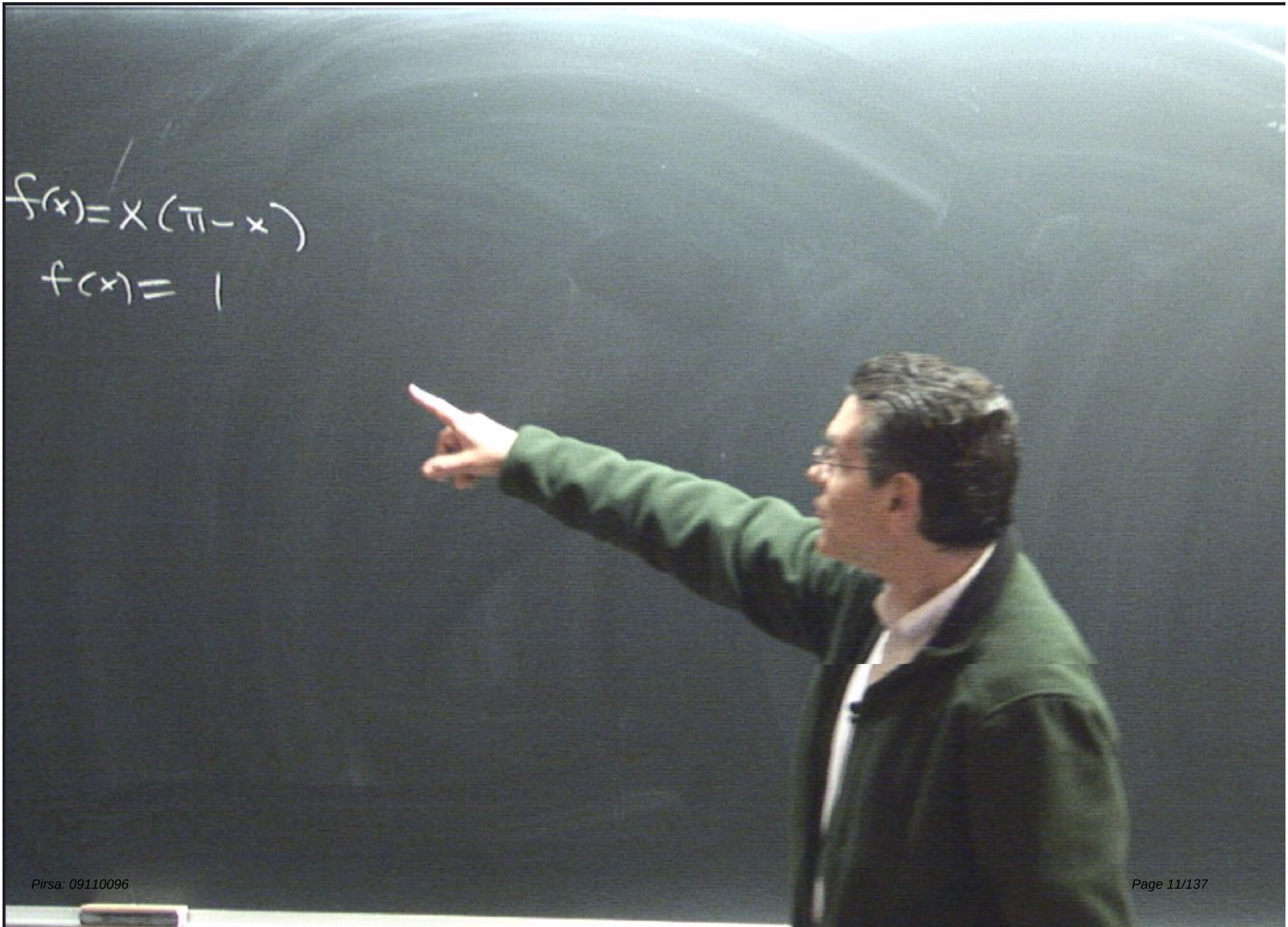
$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx\, \sin(nx)\, f(x)$

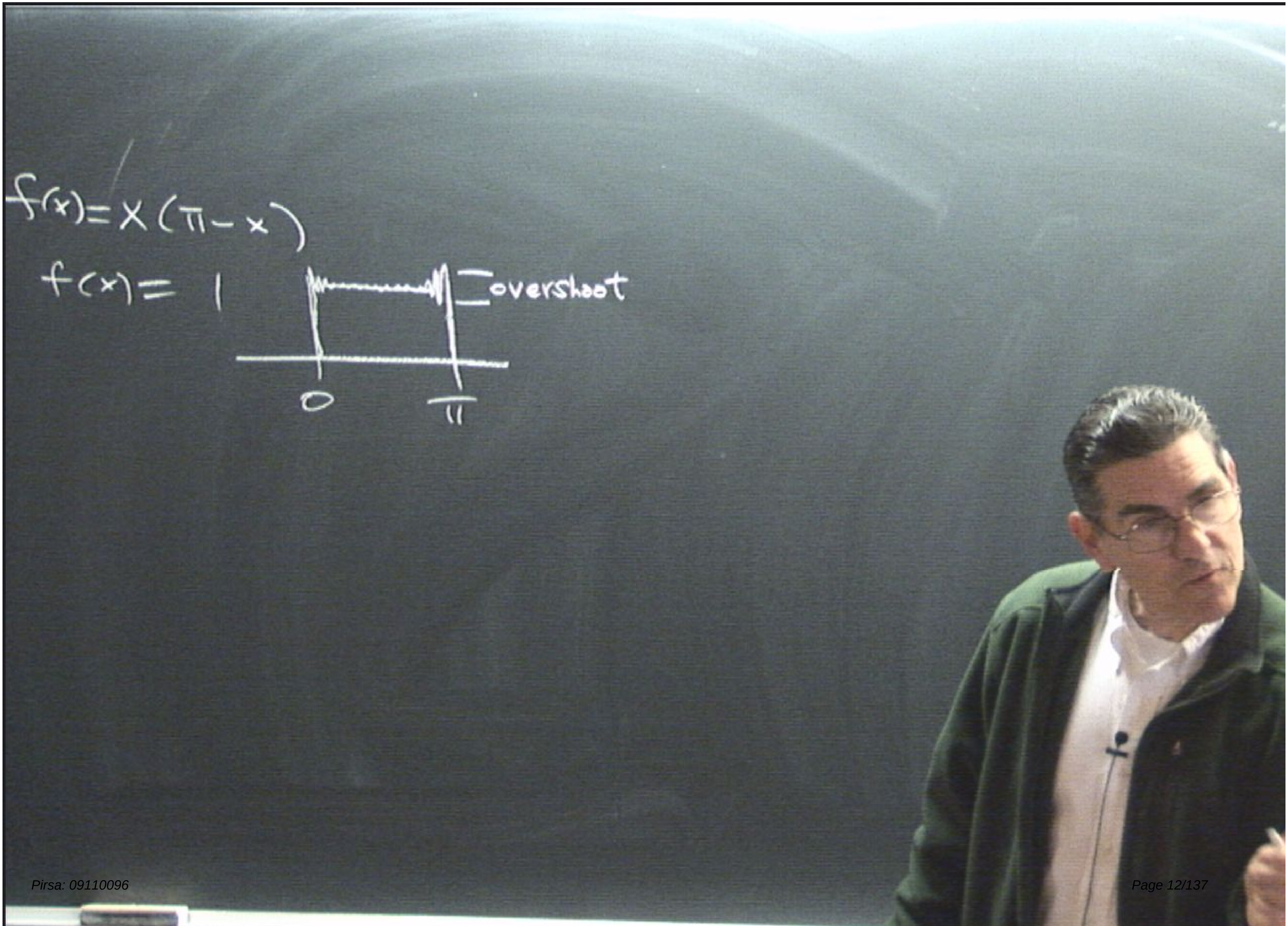
① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?

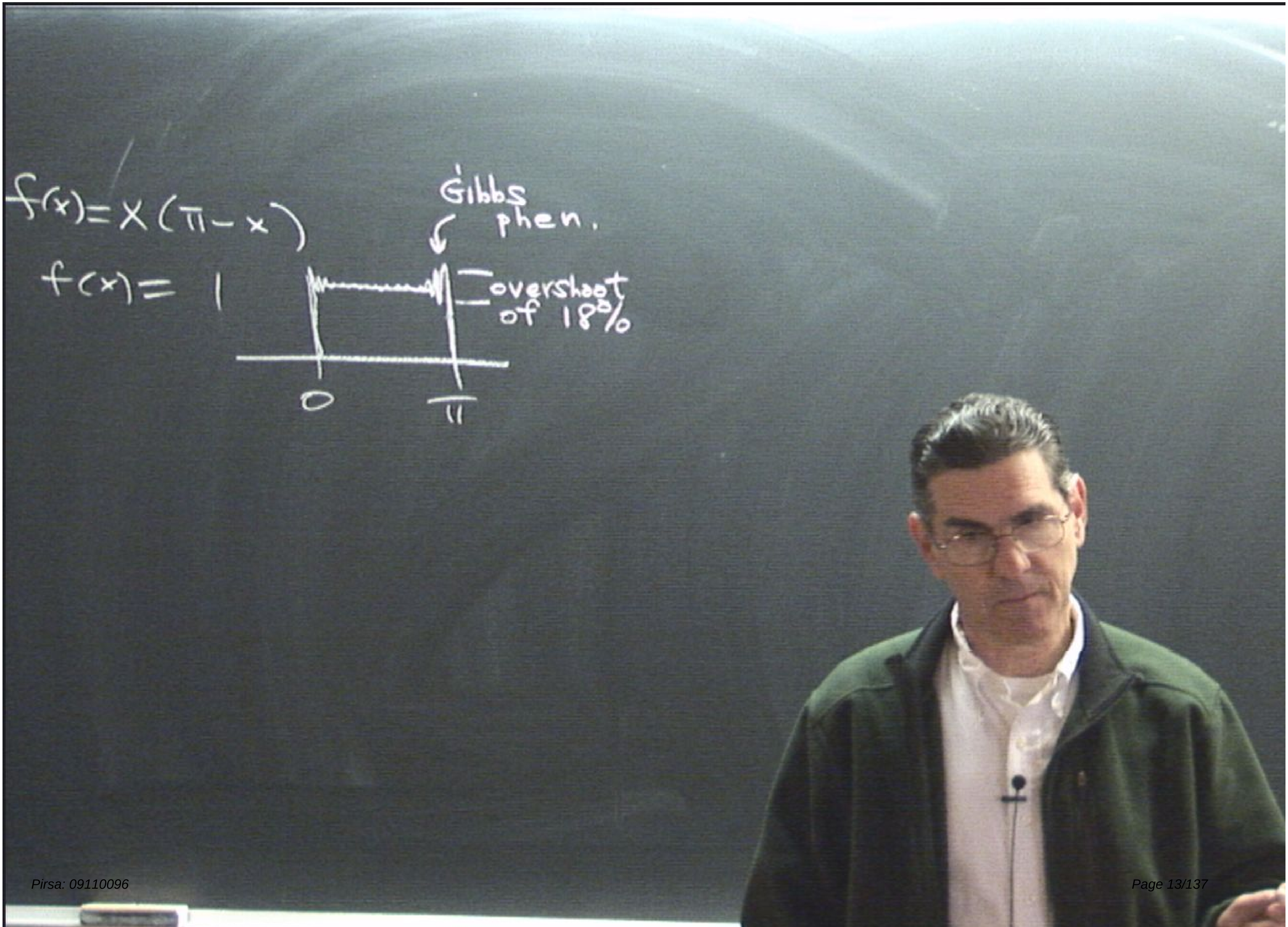
" " to $f(x)$?

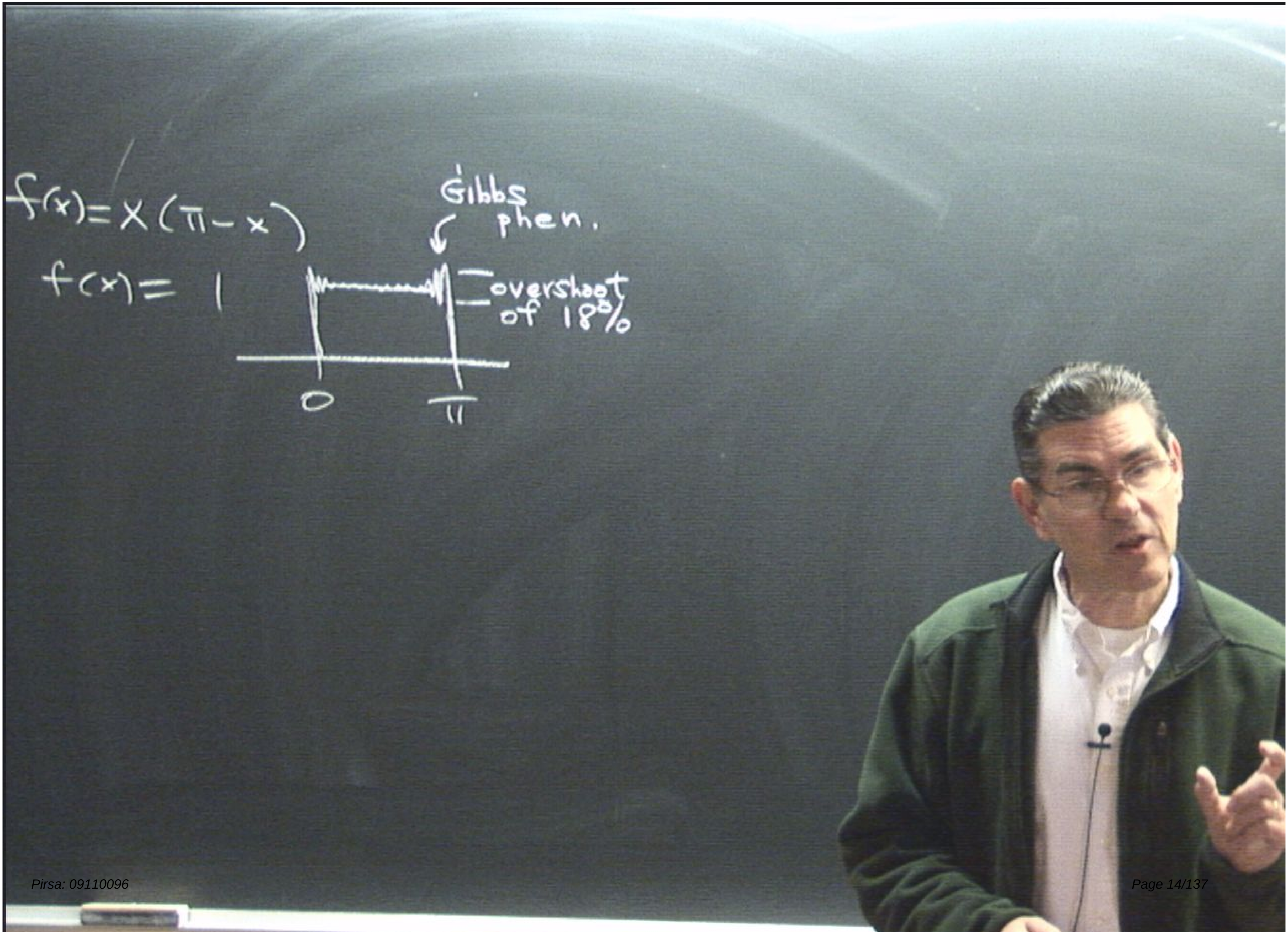
② " "



$f(x) = x(\pi - x)$
$f(x) = 1$


f(x)=x(π−x)
f(x)= 1
overshoot
0
π


f(x)=x(π−x)
f(x)= 1
Gibbs phen.
overshoot of 18%
0
π


f(x)=x(π−x)
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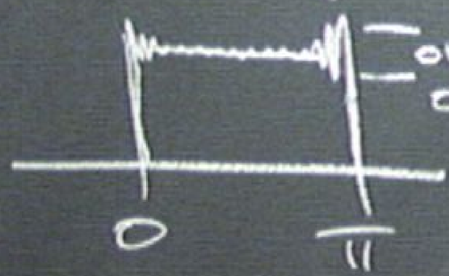
Gibbs phen.
overshoot of 18%
0
π

$f(x)=x(\pi-x)$

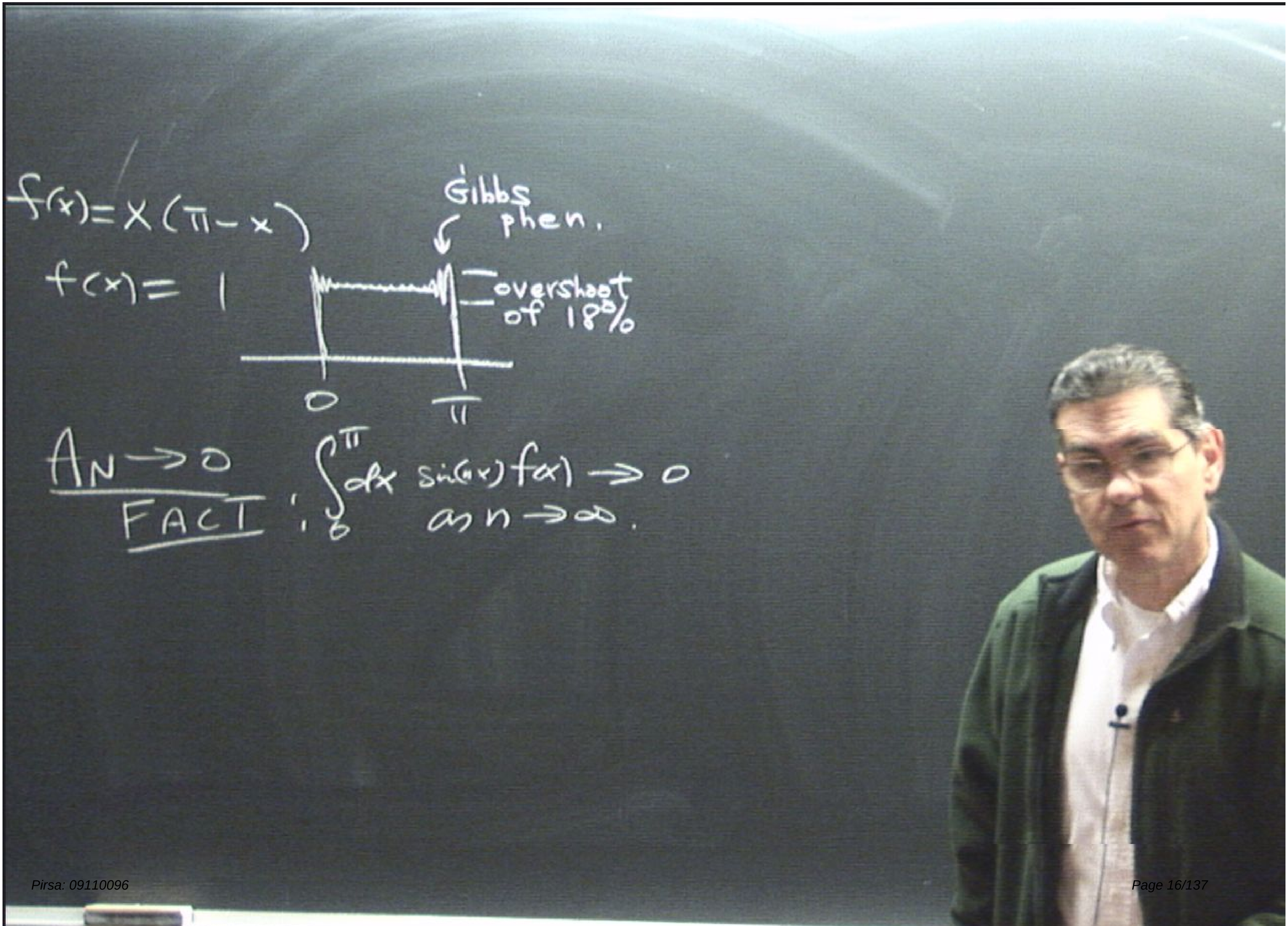
$f(x)=1$

Gibbs phen.



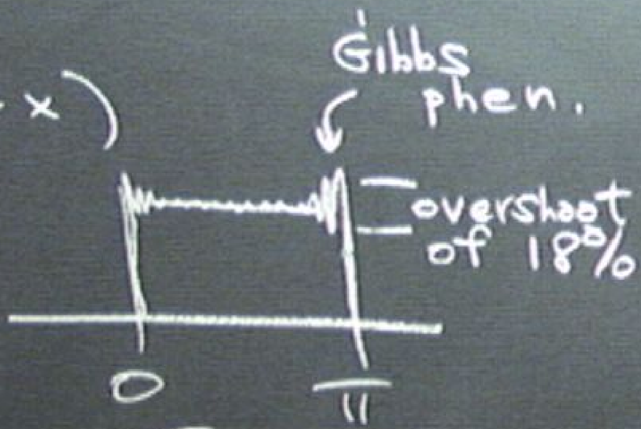
overshoot of 18%

$\underline{A_N \to 0}$


f(x)=x(π−x)
f(x)= 1
Gibbs phen.
overshoot of 18%
0
π
A_N → 0
FACT: ∫_0^π dx sin(nx) f(x) → 0 as n → ∞.

$f(x) = x(\pi - x)$

$f(x) = 1$



$\underline{A_N \to 0}$

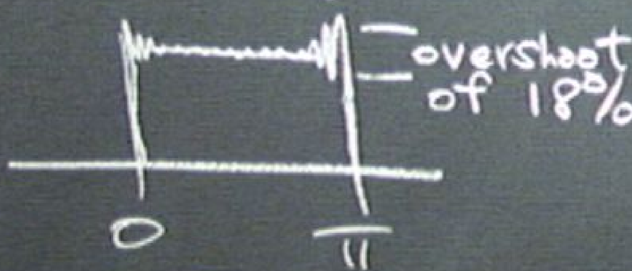
$\underline{\text{FACT}}$: $\int_0^{\pi} dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.

$f(x) = x(\pi - x)$

Gibbs phen.

$f(x) = 1$



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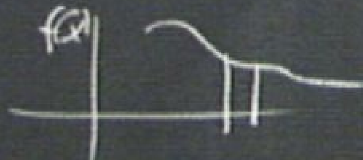

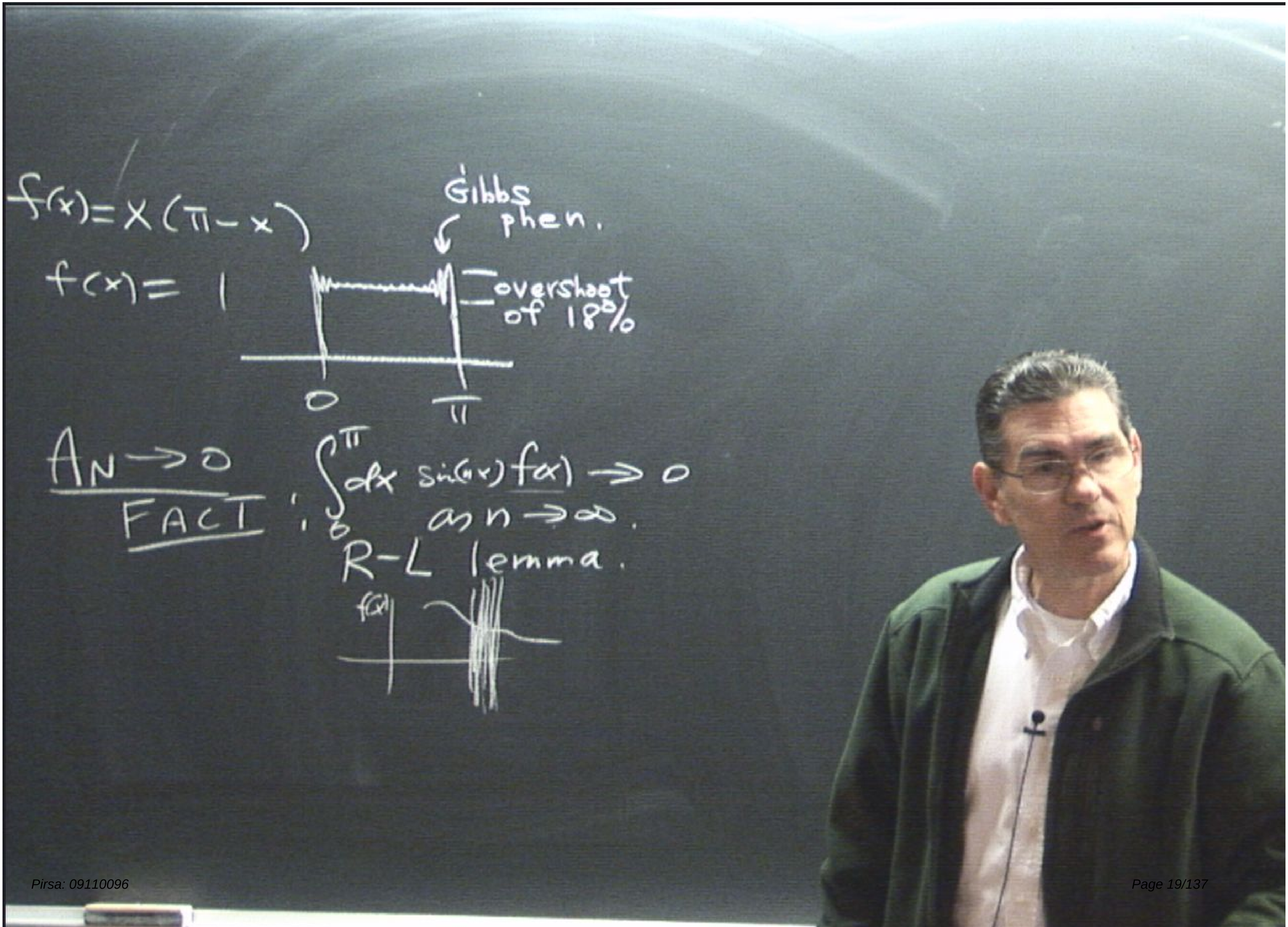

f(x) = x(π − x)
Gibbs phen.
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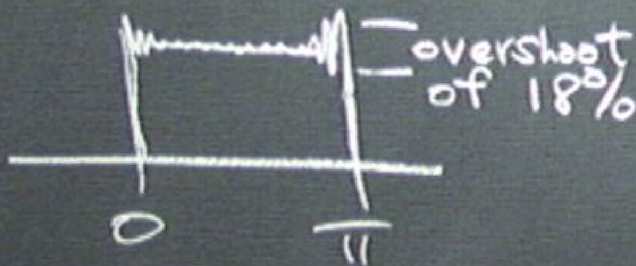
overshoot of 18%
0
π
A_N → 0
FACT: ∫_0^π dx sin(nx) f(x) → 0 as n → ∞.
R-L lemma.
f(x)

$f(x) = x(\pi - x)$

Gibbs phen.

$f(x) = 1$



$\underline{A_N \to 0}$

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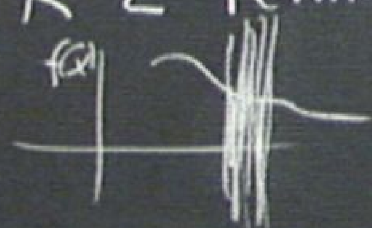

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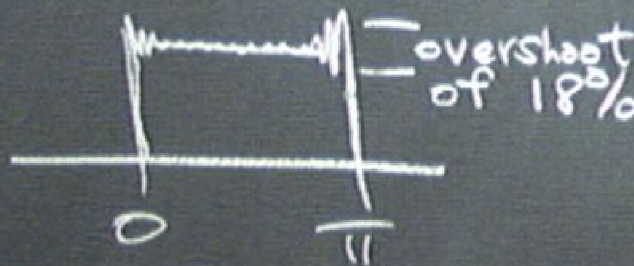

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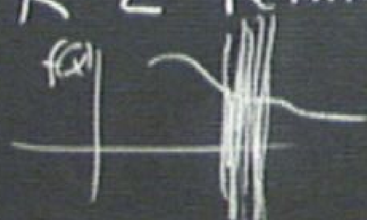


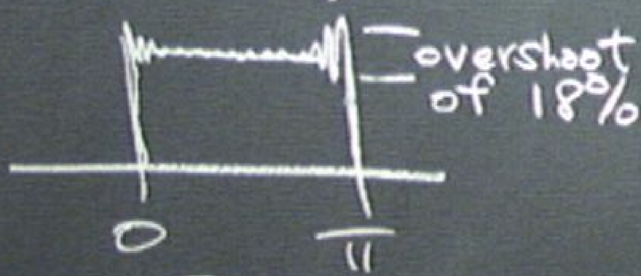
$A_n \sim ?$ as $n \to \infty$

$$A_n = -\frac{\cos(nx)}{n} f(x) \Big|_0^{\pi} + \int_0^{\pi} dx \, \frac{f'(x) \cos(nx)}{n}$$

$x(\pi - x)$

Gibbs phen.

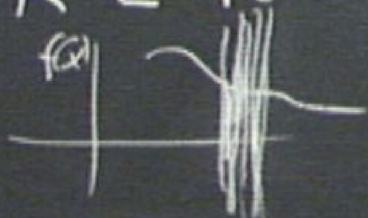
$= 1$



$\to 0$

FACT: $\int_0^{\pi} dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



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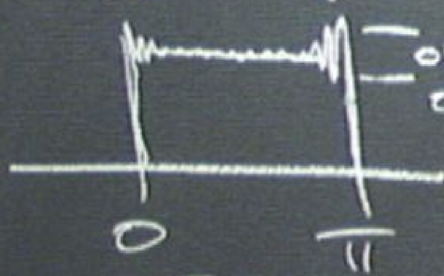
$) = 1$

Gibbs phen.



overshoot of 18%

$\to 0$

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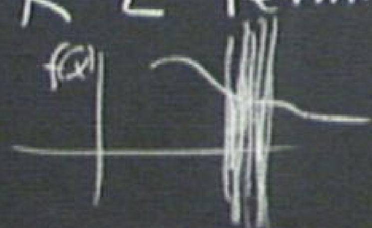


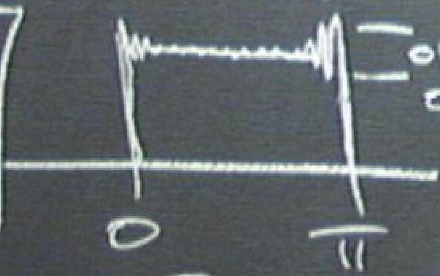
$A_n \sim ?$ as $n \to \infty$

$$A_n = -\frac{\cos(nx)}{n} f(x) \Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi} dx\, \underbrace{f'(x) \cos(nx)}_{\substack{\to 0 \\ \text{as } n\to\infty \\ \text{by R-L}}}$$

vanishes if $f(0) = 0$, $f(\pi) = 0$

$$\boxed{f(x) = x(\pi - x)}$$

$$\boxed{\begin{aligned} f(x) &= 1 \\ A_n &\sim \frac{1}{n} \end{aligned}}$$



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R-L lemma.

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vanishes if $f(0) = 0$, $f(\pi) = 0$

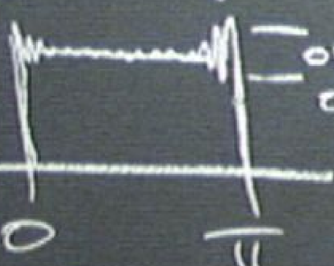
$\to$ 0 as n... by R...

$x(\pi - x)$

$= 1$

$\sim \frac{1}{n}$

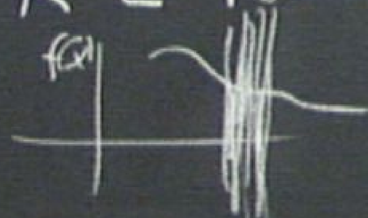
Gibbs phen.



$\to 0$

FACT: $\int_0^\pi dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



$A_n \sim ?$ as $n \to \infty$

$$A_n = \underbrace{-\frac{\cos(nx)}{n} f(x) \Big|_0^\pi}_{\text{vanishes if } f(0)=0,\ f(\pi)=0} + \frac{1}{n} \underbrace{\int_0^\pi dx \, f'(x) \cos(nx)}_{\to 0 \text{ as } n \to \infty \text{ by R-L}}$$

$x(\pi - x)$

$n = 1$

$\sim \frac{1}{n}$

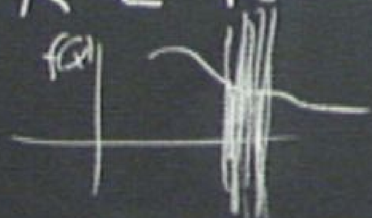
Gibbs phen.

overshoot of 18%

$\rightarrow 0$

FACT: $\int_0^\pi dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



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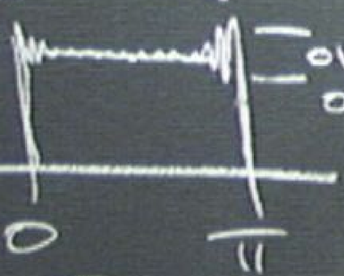
$= 0$

$(\pi - x)$

$= 1$

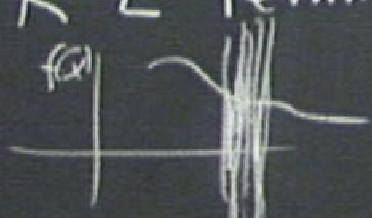
$\frac{1}{n}$



$\to 0$

FACT: $\int_0^\pi dx\, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



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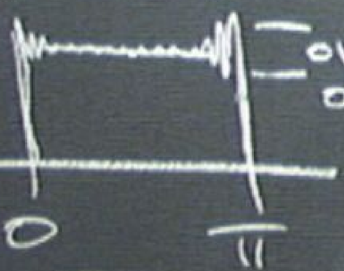
$$= \frac{1}{n^2}\sin(nx) f'(x)\Big|_0^\pi - \frac{1}{n^2}\int_0^\pi f''(x)\sin(nx)\,dx$$

$(\pi - x)$

$= 1$

$\frac{1}{n}$

Gibbs phen.



overshoot of 18%

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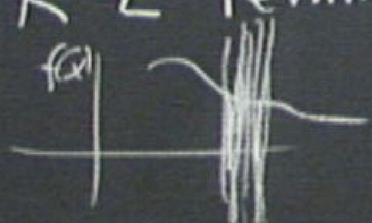

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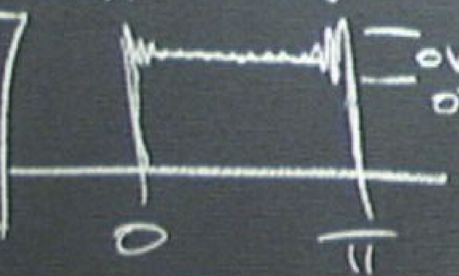
$$= \frac{1}{n^2} \sin(nx) f'(x) \Big|_0^{\pi} - \frac{1}{n^2}\int_0^{\pi} f''(x) \sin(nx)\, dx$$

$(\pi - x)$

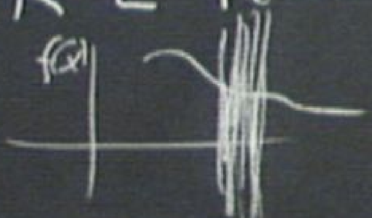
Gibbs phen.



0

CI : $\int_0^\pi dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R–L lemma.



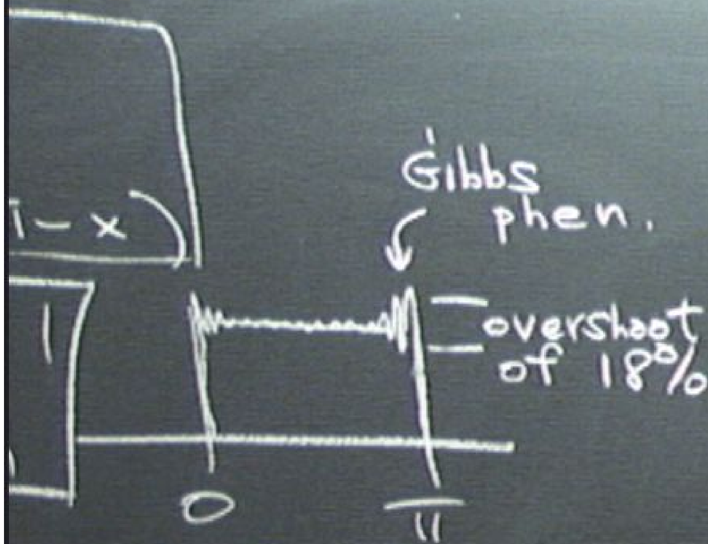
$\sim ?$ as $n \to \infty$

$$A_n = \underbrace{-\frac{\cos(nx)}{n} f(x)\Big|_0^\pi}_{\text{vanishes if } f(0)=0,\ f(\pi)=0} + \frac{1}{n}\int_0^\pi dx \, \underbrace{f'(x) \cos(nx)}_{\to 0 \text{ as } n\to\infty \text{ by R-L}}$$

$$= \cancel{\frac{1}{n^2}\sin(nx) f'(x)\Big|_0^\pi} - \frac{1}{n^2}\int_0^\pi f''(x) \sin(nx)\, dx$$

0

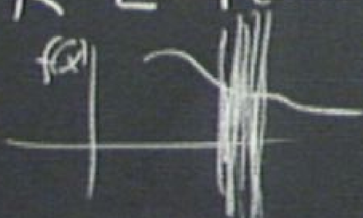
$\sim \frac{1}{n^3}$



0

CI : $\int_0^\pi dx\, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



$\sim ?$ as $n \to \infty$

$$A_n = -\frac{\cos(nx)}{n} f(x)\Big|_0^\pi + \frac{1}{n}\underbrace{\int_0^\pi dx\, f'(x)\cos(nx)}_{\to 0 \text{ as } n\to\infty \text{ by R-L}}$$

vanishes if $f(0)=0$, $f(\pi)=0$

$$= \cancel{\frac{1}{n^2}\sin(nx) f'(x)\Big|_0^\pi} - \frac{1}{n^2}\int_0^\pi f''(x)\sin(nx)\,dx$$

0

$\sim \frac{1}{n^3}$ if $f''(0)$ and/or $f''(\pi) \neq 0$

if $f''(0) = f''(\pi) = 0$

then $A_n \sim$

$$A_n = -\frac{\cos(nx)}{n} f(x)\Big|_0^\pi + \frac{1}{n}\int_0^\pi dx\, f'(x)\cos(nx)$$

vanishes if $f(0)=0$, $f(\pi)=0$

$\to 0$ as $n\to\infty$ by R-L

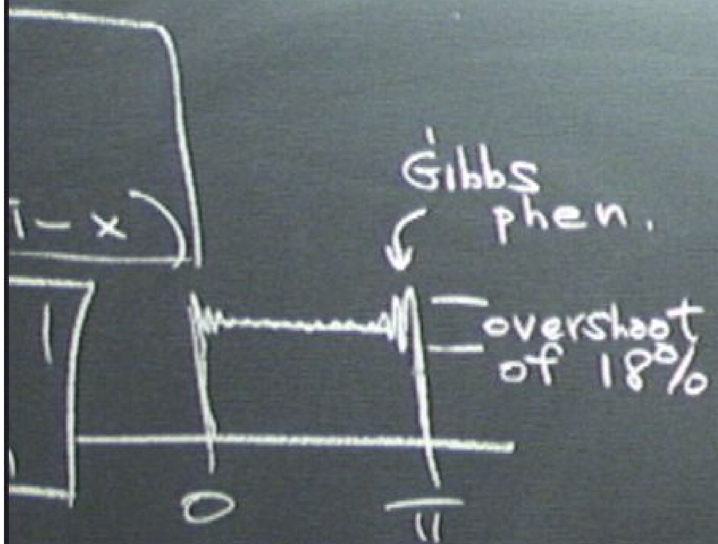
$$= \frac{1}{n^2}\sin(nx) f'(x)\Big|_0^\pi - \frac{1}{n^2}\int_0^\pi f''(x)\sin(nx)$$

0

$\sim \frac{1}{n^3}$ if $f''(0)$ and/or $f''(\pi) \neq 0$

if $f''(0) = f''(\pi) = 0$ then $A_n \sim \frac{1}{n^6}$

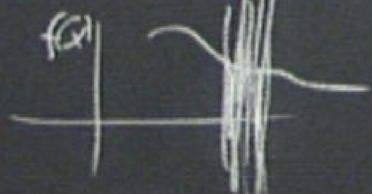
unless $f''''(0) = f''''(\pi) = 0$



0

CI : $\int_0^\pi dx \sin(nx) f(x) \to 0$ as $n\to\infty$.

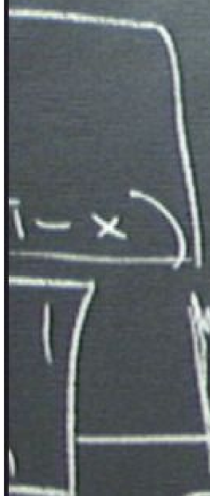
R-L lemma.



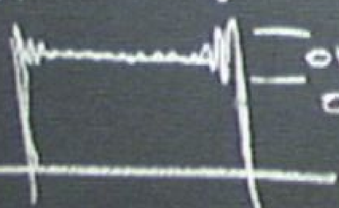
$\sim ?$ as $n\to\infty$

$i-x)$

Gibbs
phen.



overshoot
of 18%

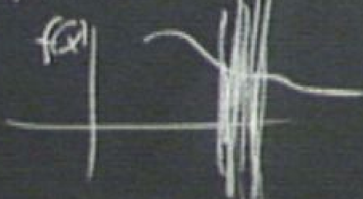


$0 \qquad \pi$

0

cI : $\int_0^\pi dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



$\sim ?$ as $n \to \infty$

$$A_n = -\frac{\cos(nx)}{n} f(x) \Big|_0^\pi + \frac{1}{n} \int_0^\pi dx \, \underbrace{f'(x) \cos(nx)}$$

vanishes if $f(0)=0$, $f(\pi)=0$

$\to 0$ as $n \to \infty$ by R-L

$$= \cancel{\frac{1}{n^2} \sin(nx) f'(x) \Big|_0^\pi} - \frac{1}{n^2} \int_0^\pi f''(x) \sin(nx)\, dx$$

0

$\sim \frac{1}{n^3}$ if $f''(0)$ and/or $f''(\pi) \neq 0$

if $f''(0) = f''(\pi) = 0$

then $A_n \sim \frac{1}{n^5}$

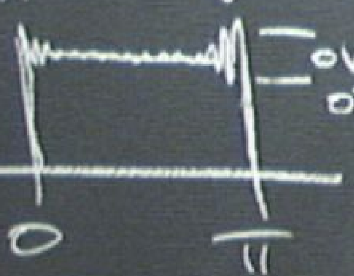
unless $f''''(0) = f''$

$A_n \sim$

$\ldots 1 - x)$

Gibbs phen.

overshoot of 18%



$$A_n = -\frac{\cos(nx)}{n} f(x)\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi} dx\, \underbrace{f'(x)\cos(nx)}_{\to 0 \text{ as } n\to\infty \text{ by R-L}}$$

vanishes if $f(0)=0$, $f(\pi)=0$

$$= \cancel{\frac{1}{n^2}\sin(nx) f'(x)\Big|_0^{\pi}} - \frac{1}{n^3}\int_0^{\pi} f''(x)\sin(nx)\,dx$$

0

$\sim \frac{1}{n^3}$ if $f''(0)$ and/or $f''(\pi) \neq 0$

if $f''(0) = f''(\pi) = 0$

then $A_n \sim \frac{1}{n^5}$

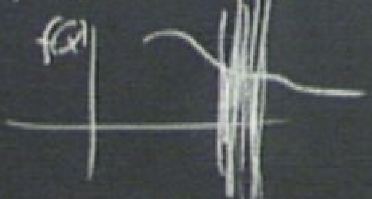
unless $f''''(0) = f''''(\pi) = 0$

$A_n \sim$

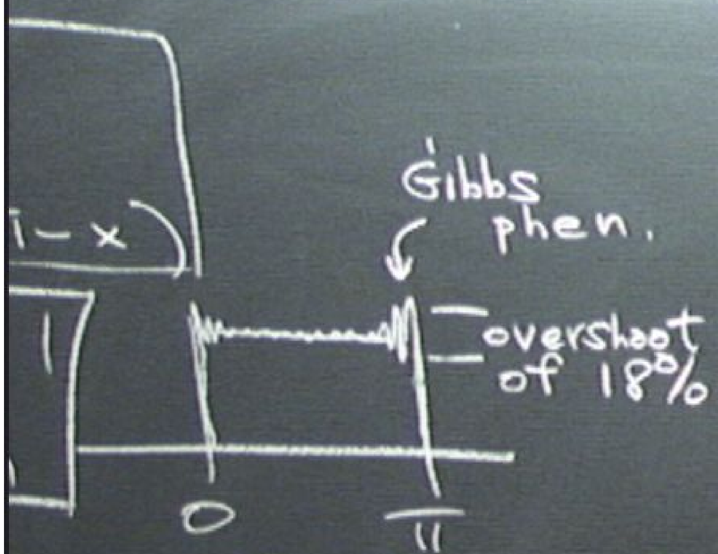
0

CI : $\int_0^{\pi} dx \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



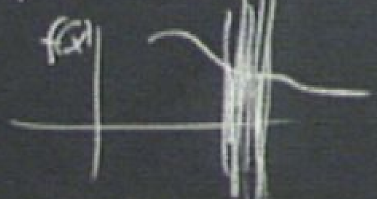
$\sim ?$ as $n \to \infty$



0

$\underline{CI}$: $\displaystyle\int_0^\pi dx\, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



$\sim ?$ as $n \to \infty$

$$A_n = \underbrace{-\frac{\cos(nx)}{n} f(x)\Big|_0^\pi}_{\text{vanishes if } f(0)=0,\ f(\pi)=0} + \underbrace{\frac{1}{n}\int_0^\pi dx\, f'(x)\cos(nx)}_{\to 0 \text{ as } n\to\infty \text{ by R-L}}$$

$$= \underbrace{\cancel{\frac{1}{n^2}\sin(nx) f'(x)\Big|_0^\pi}}_{0} - \frac{1}{n^2}\int_0^\pi f''(x)\sin(nx)\,dx$$

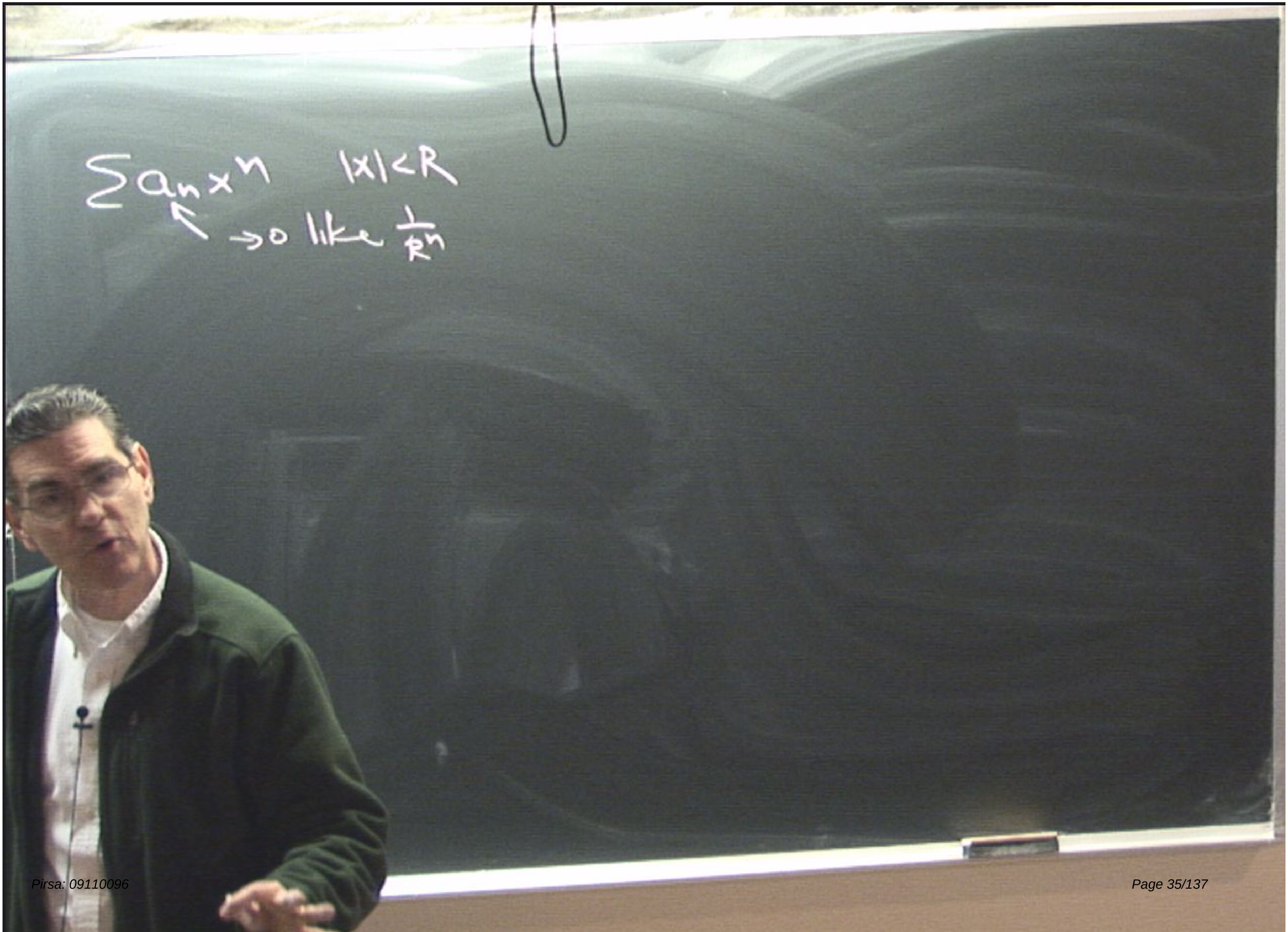
$\sim \frac{1}{n^3}$ if $f''(0)$ and/or $f''(\pi) \neq 0$

if $f''(0) = f''(\pi) = 0$

then $A_n \sim \frac{1}{n^5}$

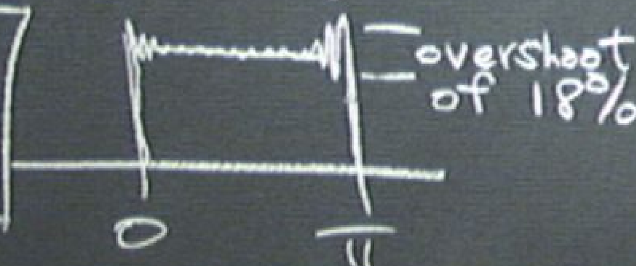
unless $f''''(0) = f''''(\pi) = 0$

$A_n \sim \frac{1}{n^7}$


$\sum a_n x^n$ $|x|<R$
$\rightarrow 0$ like $\frac{1}{R^n}$

$$\boxed{A_n \sim \frac{1}{n^3} \quad f(x) = x(\pi - x)}$$

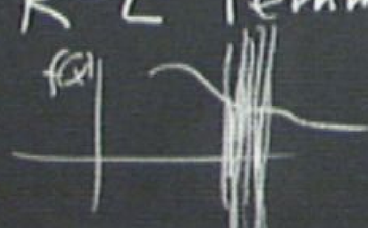
$$\boxed{f(x) = 1 \quad A_n \sim \frac{1}{n}}$$



$\underline{A_N \to 0}$

$\underline{\text{FACT}}$: $\int_0^\pi dx\, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.



$A_n \sim ?$ as $n \to \infty$

$$A_n = \underbrace{-\frac{\cos(nx)}{n} f(x) \Big|_0^\pi}_{0} + \frac{1}{n}\int_0^\pi dx\, f'(x) \cos$$

vanishes if $f(0) = 0$, $f(\pi) = 0$

$\to$ as n by R

$$= \underbrace{\cancel{\frac{1}{n^2} \sin(nx) f'(x) \Big|_0^\pi}}_{0} - \frac{1}{n^2}\int_0^\pi f''$$

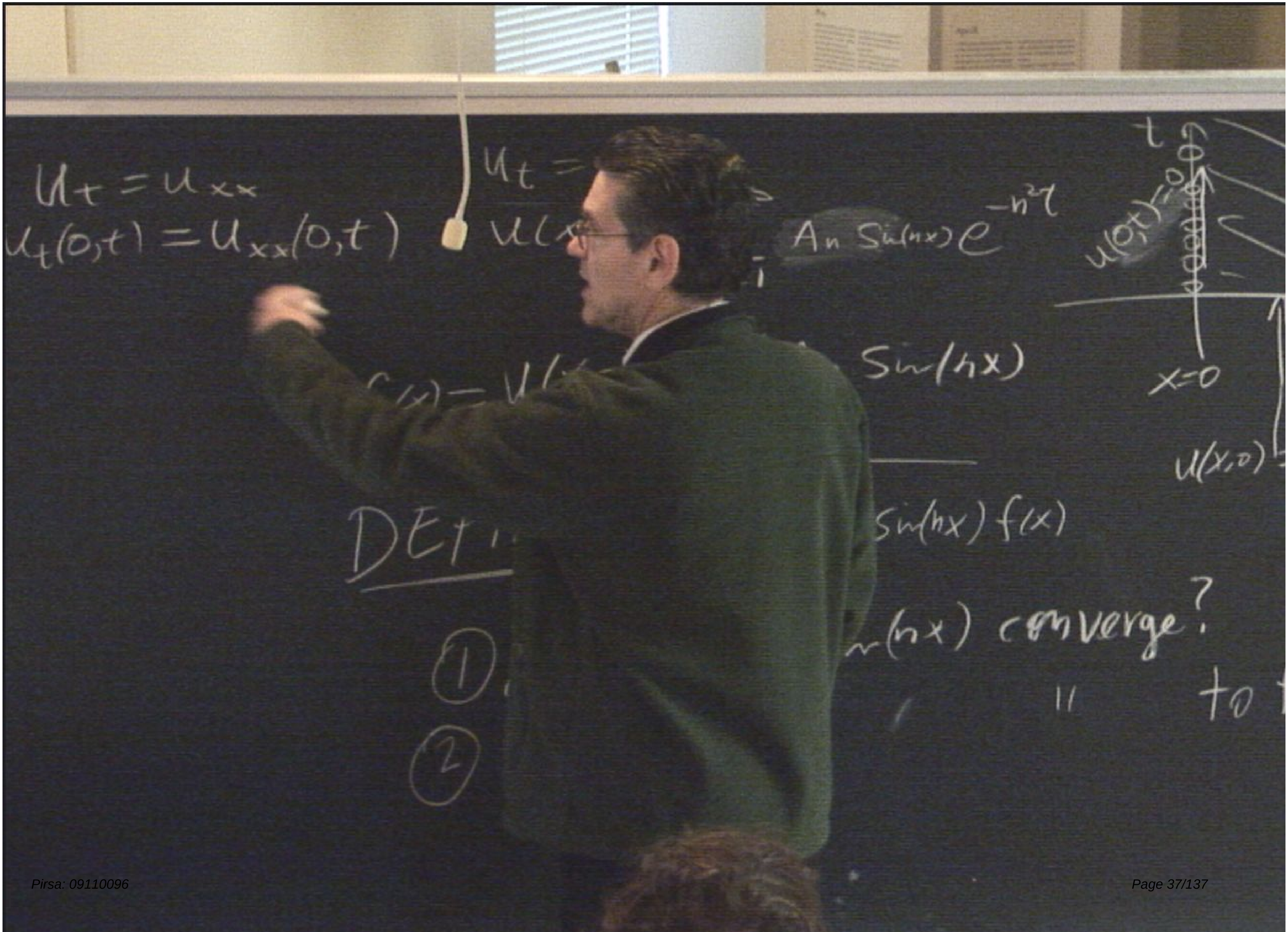
$\sim \frac{1}{n^3}$ if $f''(0)$ and/or f

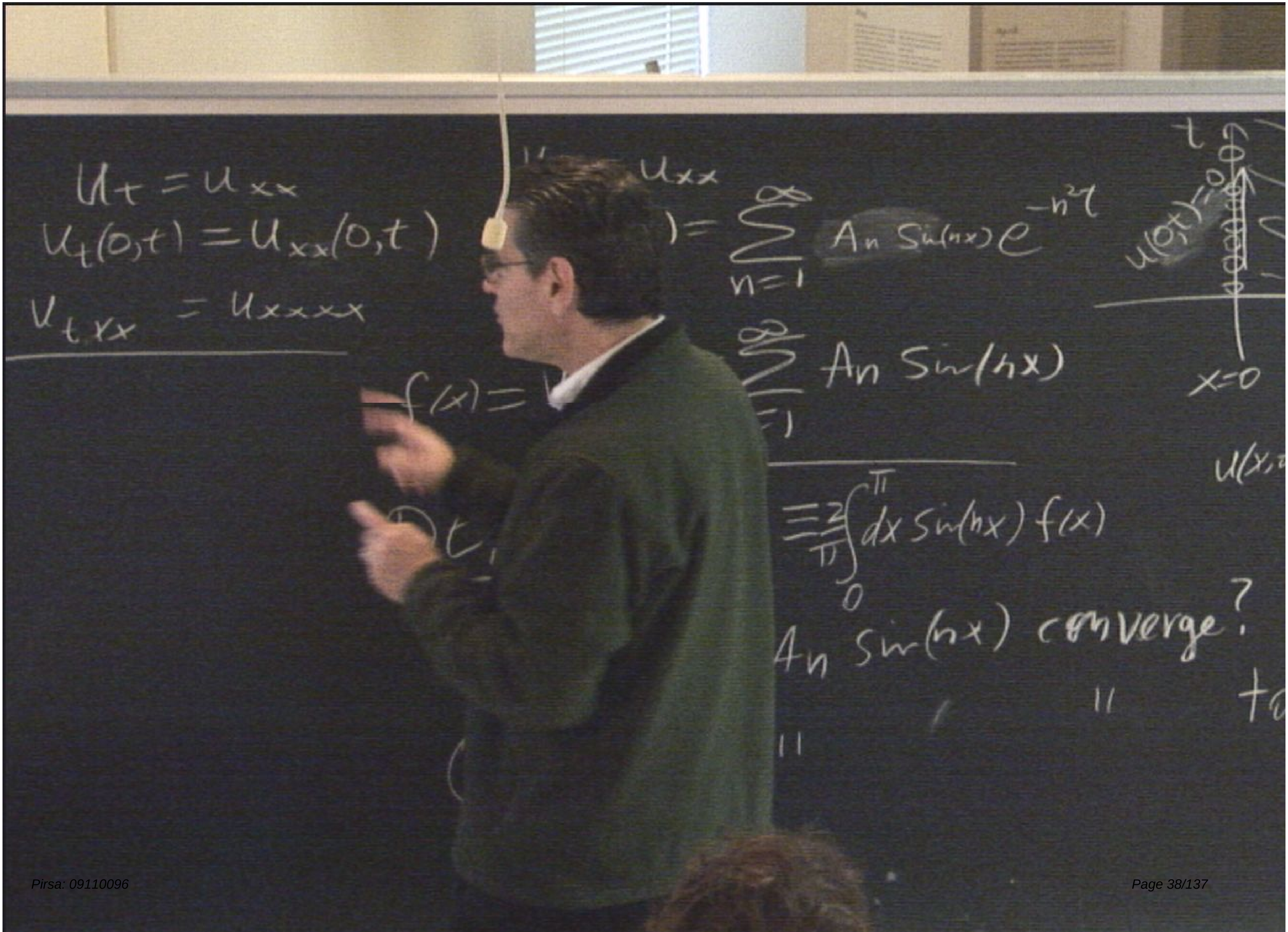
if $f''(0) = f''(\pi) = 0$

then $A_n \sim \frac{1}{n^5}$

unless $f''''(0) = f''''(\pi) =$

$A_n \sim \frac{1}{n^7}$


$u_t = u_{xx}$
$u_t(0,t) = u_{xx}(0,t)$
$u_t =$
$A_n \sin(nx) e^{-n^2 t}$
$u(0,t)=0$
t
$\sin(nx)$
$x=0$
$u(x,0)$
$\sin(nx) f(x)$
converge?
to



Ut = Uxx
Ut(0,t) = Uxx(0,t)
Vtxx = Uxxxx
Uxx
An Sin(nx) e^-n^2 t
n=1
An Sin(nx)
f(x) =
x=0
dx Sin(nx) f(x)
An Sin(nx) converge?

$u_t = u_{xx}$

$u_t(0,t) = u_{xx}(0,t)$

$u_{txx} = u_{xxxx}$

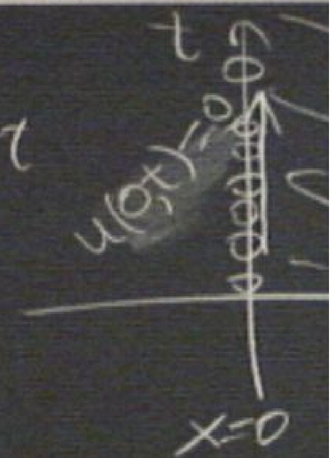
$u_{txxxx} = u_{xxxxxx} = 0$

$u_{xxxx} = 0$

$u_t = u_{xx}$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$$

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$



$u(x,$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$

① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?

② " " " to

$u_t = u_{xx}$

$u_t(0,t) = u_{xx}(0,t)$

$u_{txx} = u_{xxxx}$

$u_{txxxx} = u_{xxxxxx} = 0$

$u_{xxxx} = 0$

$u_t = u_{xx}$

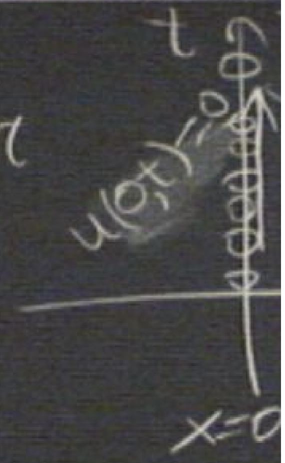
$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx\, \sin(nx)\, f(x)$

① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?

② " " "



$u_t = u_{xx}$

$(0,t) = u_{xx}(0,t)$

$_{xx} = u_{xxxx}$

$_{xx} = u_{xxxxxx} = 0$

$= 0$

$$u_t = u_{xx}$$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$$

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

$u(0,t) = 0$

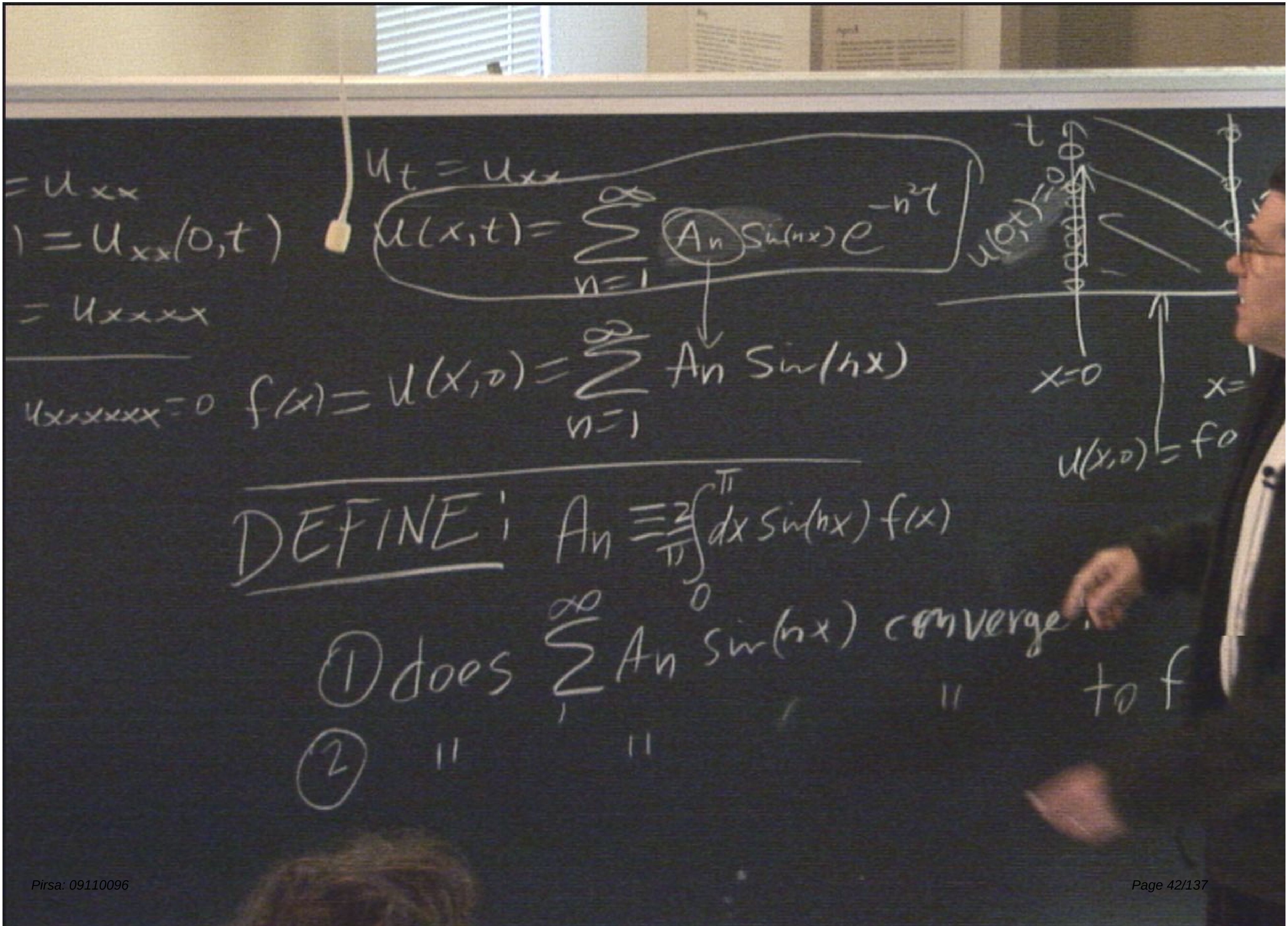
$x = 0$

$u(x,0) = f$

DEFINE: $A_n \equiv \frac{2}{\pi} \int_0^{\pi} dx \sin(nx) f(x)$

① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?

② " " " to


$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$
$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$
DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$
① does $\sum_1^{\infty} A_n \sin(nx)$ converge
to f
②
$u(0,t)=0$
$x=0$
$u(x,0)=f$

$= u_{xx}$

$) = u_{xx}(0,t)$

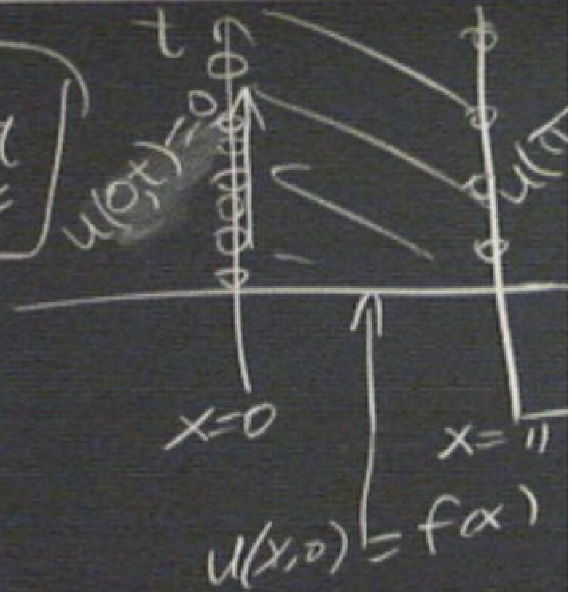
$= u_{xxxx}$

$u_{xxxxxx} = 0$

$u_t = u_{xx}$

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx)\, e^{-n^2 t}$$

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

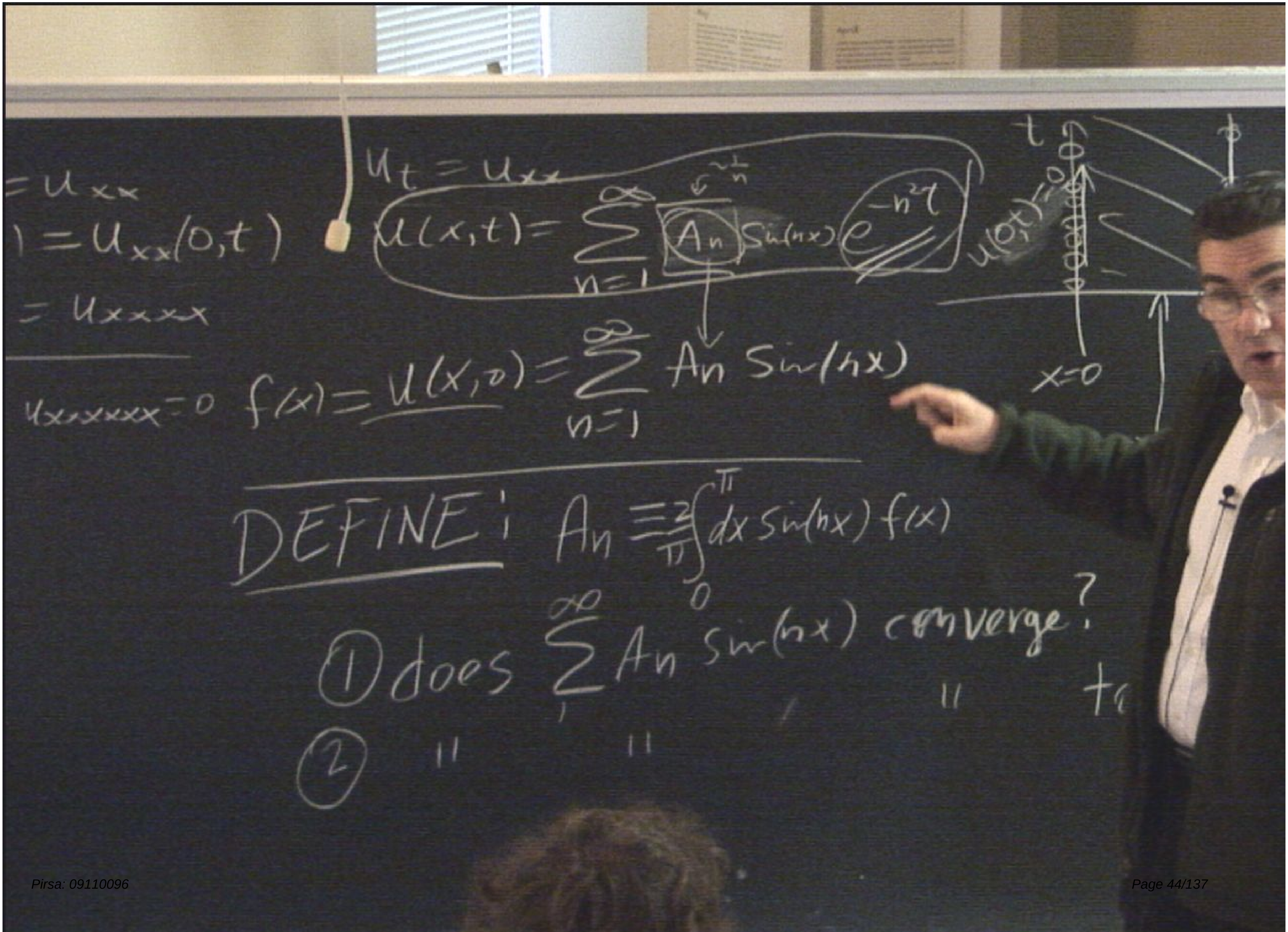


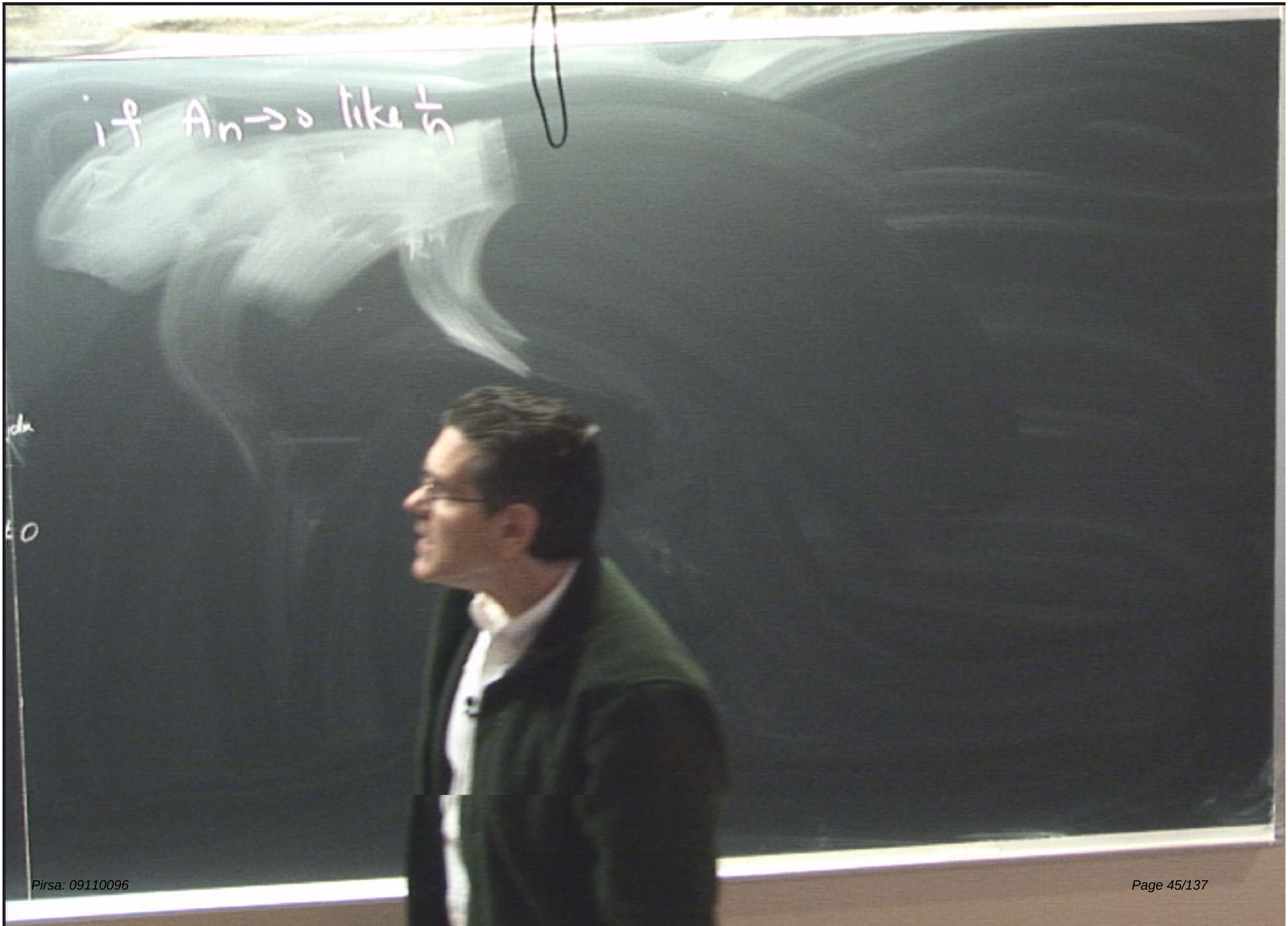
DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx\, \sin(nx)\, f(x)$

① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?

" to $f(x)$?

② " "


$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$
$= u_{xx}(0,t)$
$= u_{xxxx}$
$u_{xxxxxx} = 0$
$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$
$x=0$
DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$
① does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?
②


if $A_n \to 0$ like $\frac{1}{n}$

$u_t = u_{xx}$

$u(0,t) = u_{xx}(0,t)$

$u_{xx} = u_{xxxx}$

$u_{xx} = u_{xxxxxx} = 0$

$= 0$

$u_t = u_{xx}$

$u(x,t) = \sum_{n=1}^{\infty} \boxed{A_n} \sin(nx) e^{-n^2 t}$

$u(0,t) = 0$

$f(x) = \underline{u(x,0)} = \sum_{n=1}^{\infty} A_n \sin(nx)$

$x=0$

$u(x,0) = f$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$

① does $\boxed{\sum_{1}^{\infty} A_n \sin(nx)}$ converge?

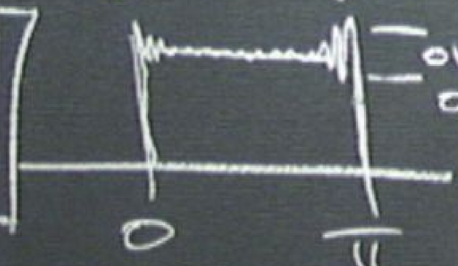
② " " " to $f($

Prove that the F S for $f(x)=1$ converges to 1

$$\boxed{\begin{array}{c} A_n \sim \frac{1}{n^3} \\ f(x) = x(\pi - x) \end{array}}$$

$$\boxed{\begin{array}{l} f(x) = 1 \\ A_n \sim \frac{1}{n} \end{array}}$$

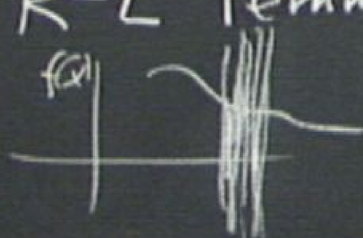
Gibbs phen.



$\underline{A_N \to 0}$

$\underline{\text{FACT}}$: $\int_0^{\pi} dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R–L lemma.



$A_n \sim ?$ as $n \to \infty$

$$\int_0^{\infty} dx \, \frac{\sin x}{x}$$

$$A_n = -\frac{\cos(nx)}{n} f(x) \Big|_0^{\pi} + \frac{1}{n} \int_0^{\pi} dx \, f'(x) \cos$$

$\underline{0}$

vanishes if $f(0) = 0$, $f(\pi) = 0$

$\to$ as n by R

$$= \cancel{\frac{1}{n^2} \sin(nx) f'(x) \Big|_0^{\pi}} - \frac{1}{n^2} \int_0^{\pi} f''(x)$$

0

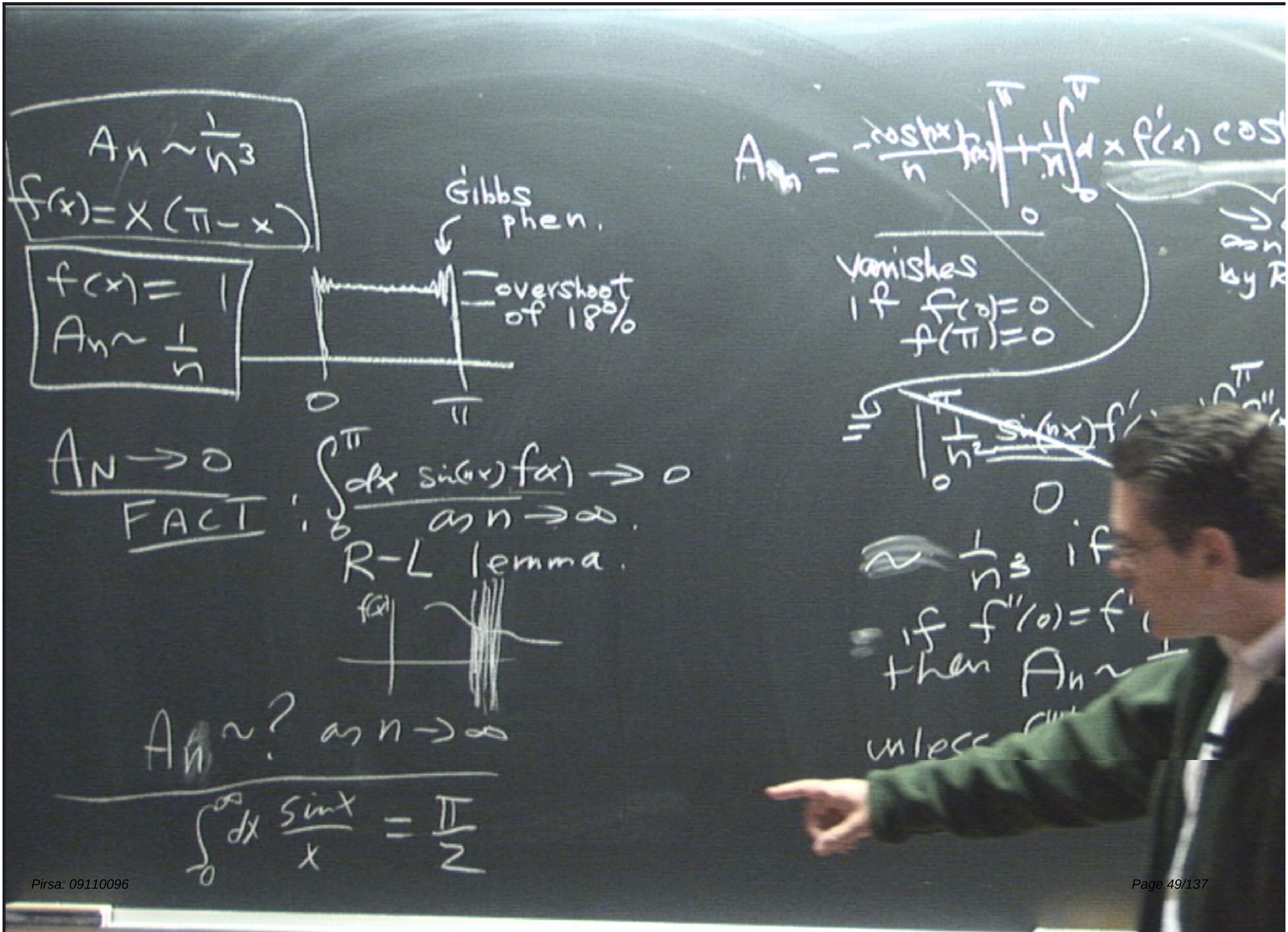
$\sim \frac{1}{n^3}$ if $f''(0)$ or and

if $f''(0) = f''(\pi) = 0$

then $A_n \sim \frac{1}{n^5}$

unless $f''''(0) = f''''(\pi)$

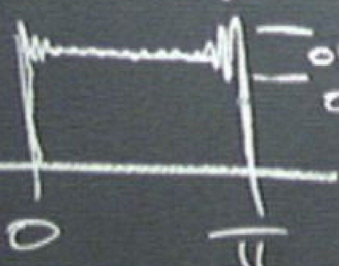
$A_n \sim \frac{1}{n^7}$


An ~ 1/n^3
f(x) = x(π − x)
f(x) = 1
An ~ 1/n
Gibbs phen.
overshoot of 18%
0
π
AN → 0
FACT: ∫_0^π dx sin(nx) f(x) → 0 as n → ∞.
R-L lemma.
f(x)
An ~ ? as n → ∞
∫_0^∞ dx sin x / x = π/2
An = −cos(nx)/n f(x) |_0^π + 1/n ∫_0^π dx f′(x) cos
vanishes if f(0) = 0, f(π) = 0
0
~ 1/n^3 if
if f″(0) = f′
then An ~
unless

$$A_n \sim \frac{1}{n^3}$$
$$f(x) = x(\pi - x)$$

$$f(x) = 1$$
$$A_n \sim \frac{1}{n}$$

Gibbs phen.



overshoot of 18%

$A_N \to 0$

FACT: $\int_0^{\pi} dx \, \sin(nx) f(x) \to 0$ as $n \to \infty$.

R-L lemma.

$A_n \sim ?$ as $n \to \infty$

$$\int_0^{\infty} dx \, \frac{\sin x}{x} = \frac{\pi}{2}$$

$$A_n = -\frac{\cos(nx)}{n} f(x)\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi} dx \, f'(x) \cos$$

vanishes if $f(0)=0$, $f(\pi)=0$

$\to$ as n by R

$$= \cancel{\frac{1}{n^2}\sin(nx) f'(x)\Big|_0^{\pi}} - \frac{1}{n^2}\int_0^{\pi} f''$$

0

$\sim \frac{1}{n^3}$ if $f''(0)$ and/or

if $f''(0) = f''(\pi) = 0$ then $A_n \sim \frac{1}{n^5}$

unless $f''''(0) = f''''(\pi) =$

$A_n \sim \frac{1}{n^7}$

Prove that the FS for $f(x)=1$ converges to 1

$$A_n = \frac{2}{\pi}\int_0^{\pi} dx \; 1 \; \sin(nx) = \frac{-2}{\pi n} \underbrace{\cos(nx)\Big|_0^{\pi}}_{(-1)^n}$$

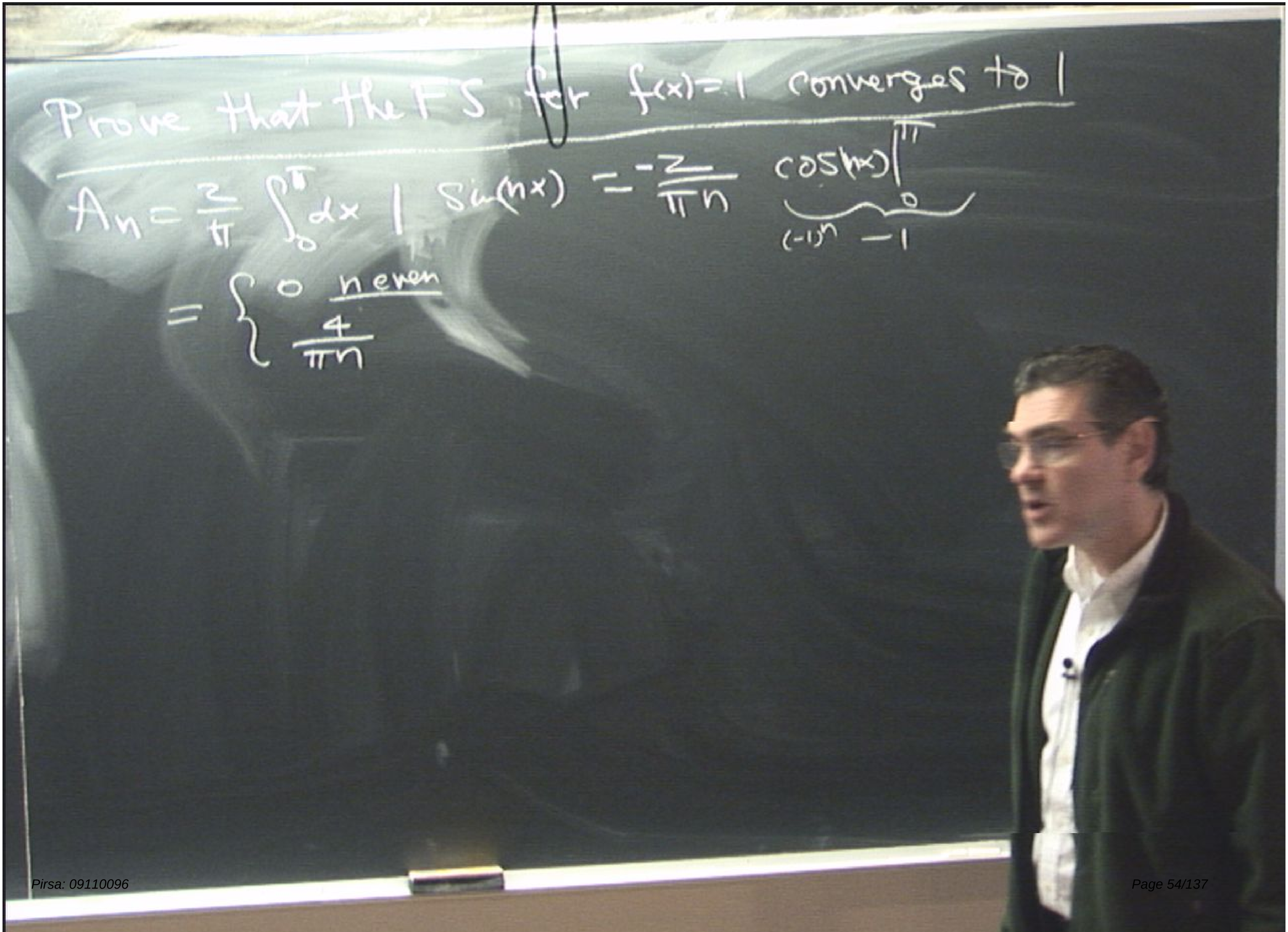
Prove that the FS for $f(x)=1$ converges to 1

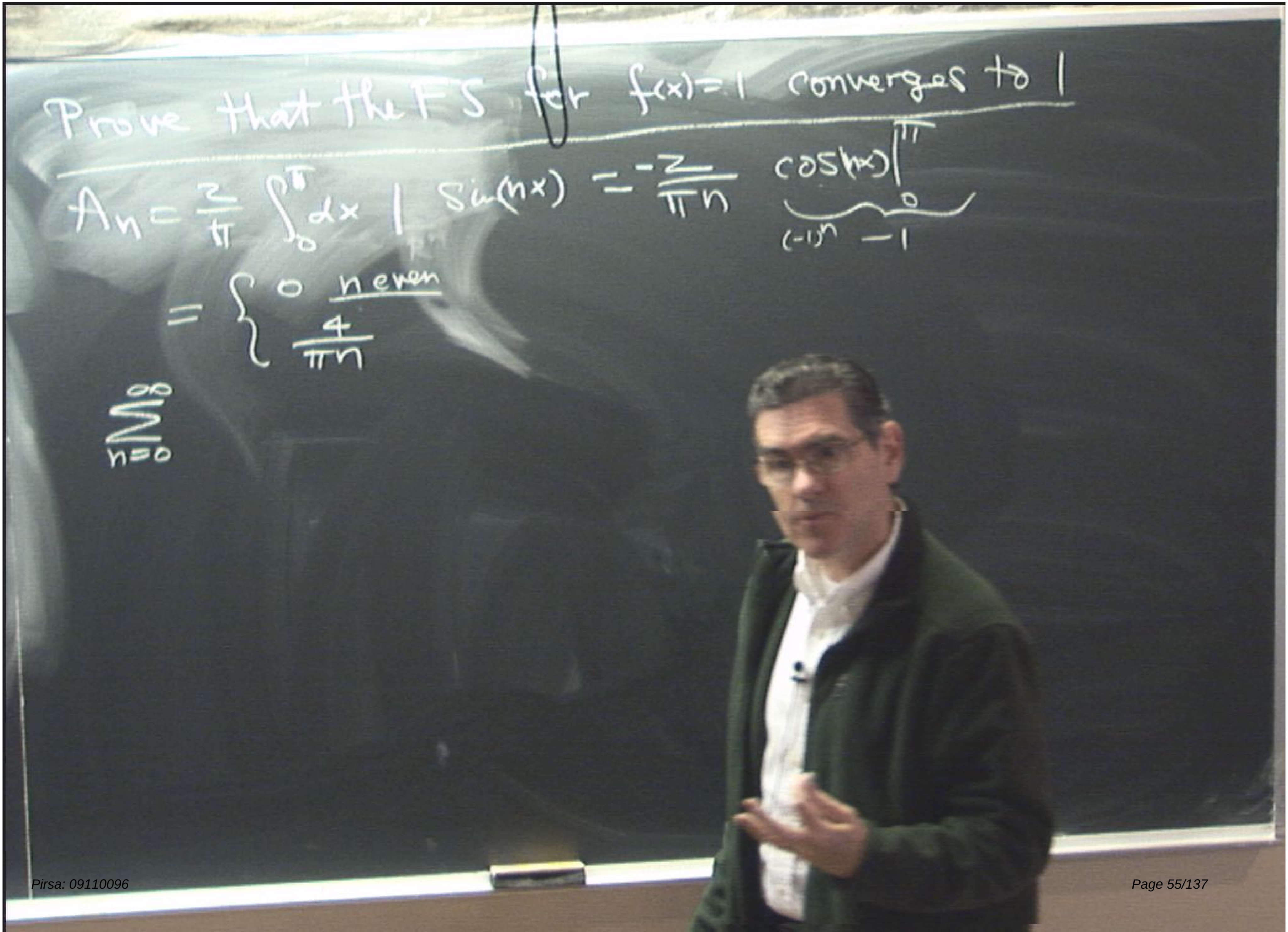
$$A_n = \frac{2}{\pi}\int_0^\pi dx\; 1\; \sin(nx) = \frac{-2}{\pi n}\underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$$

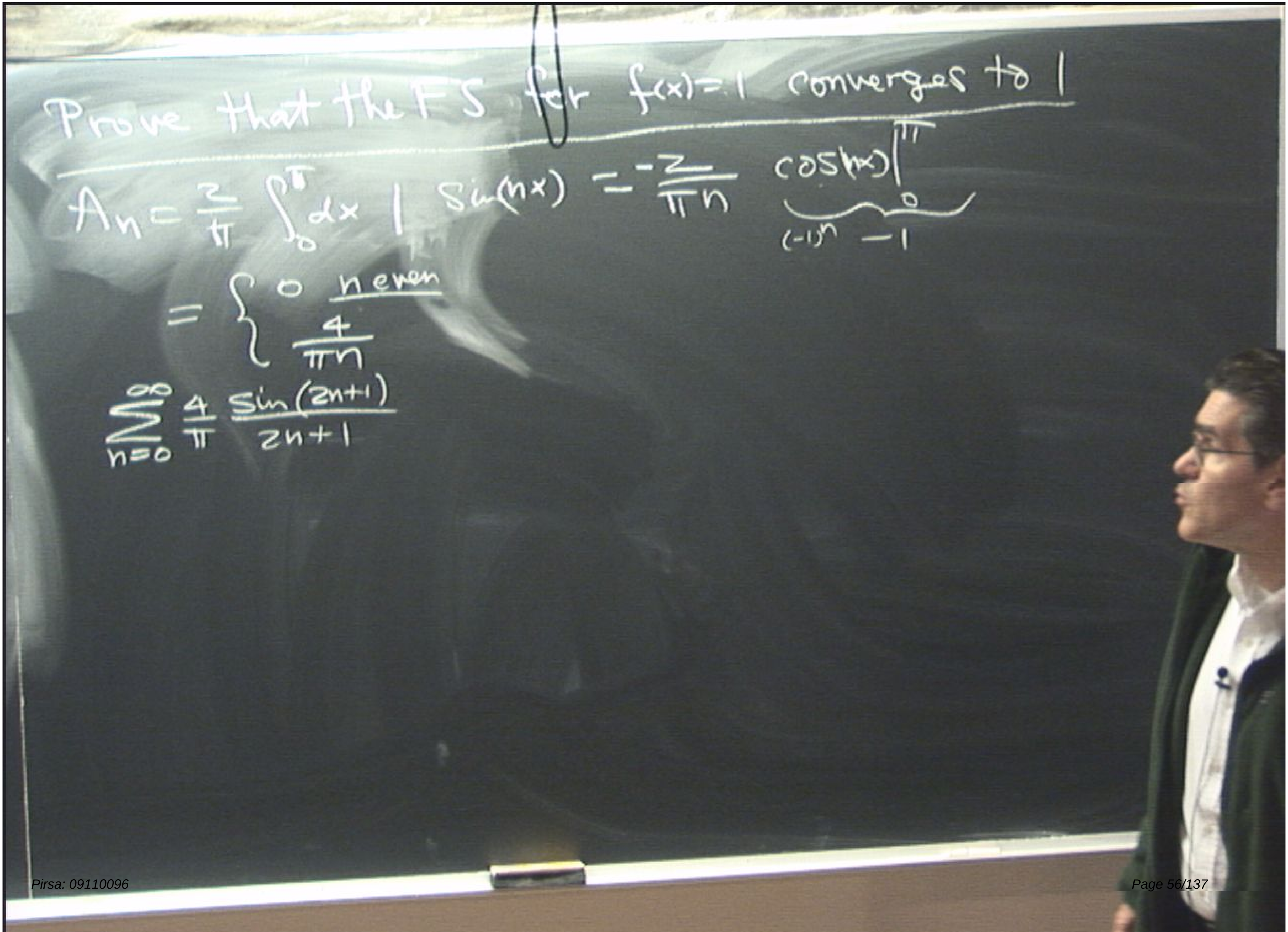
Prove that the FS for $f(x)=1$ converges to 1

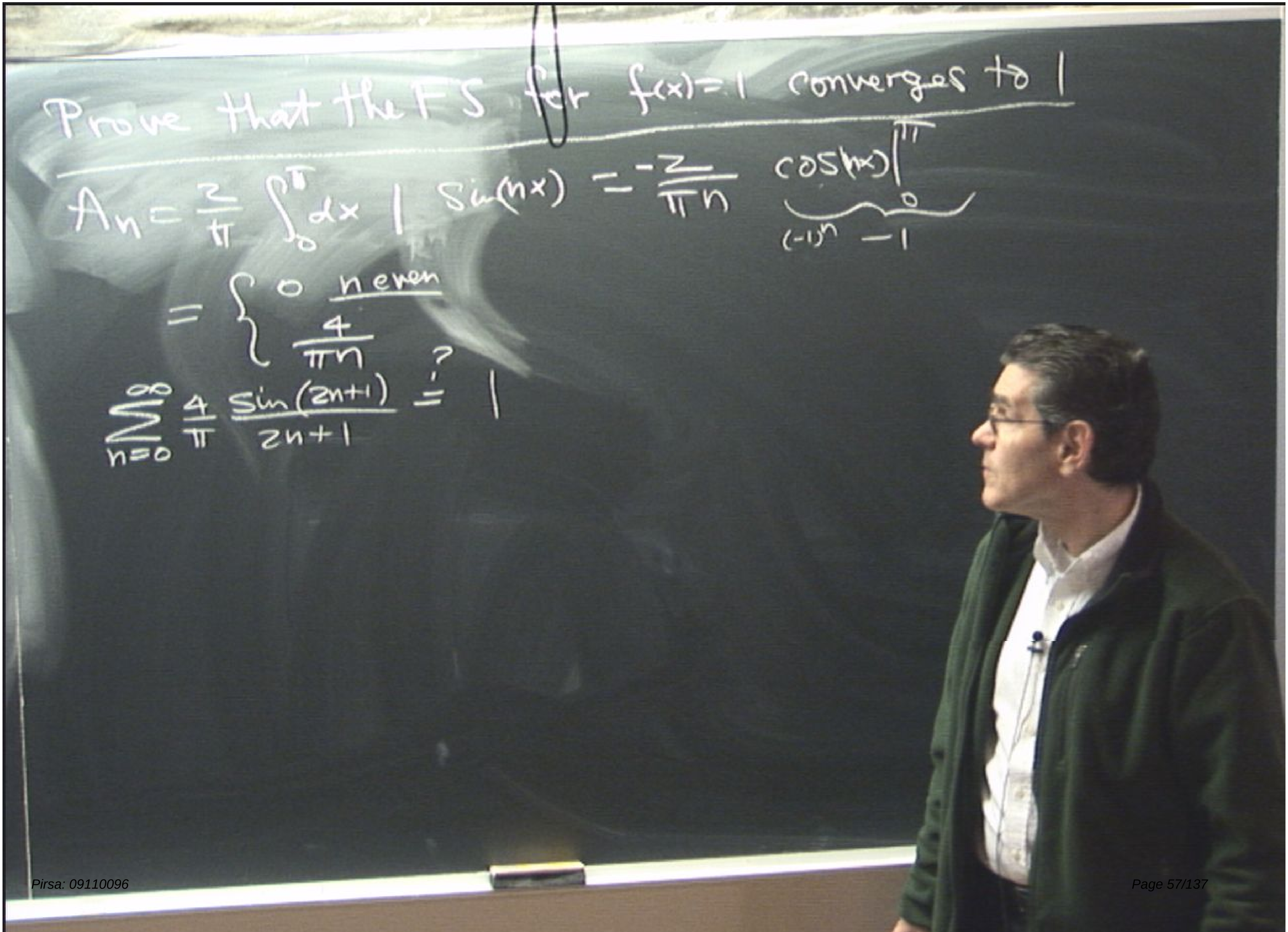
$$A_n = \frac{2}{\pi}\int_0^\pi dx \; 1 \; \sin(nx) = \frac{-2}{\pi n} \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n \; - 1}$$

$$= \begin{cases} 0 & \underline{n \text{ even}} \\ \end{cases}$$


Prove that the FS for $f(x)=1$ converges to 1
$A_n = \frac{2}{\pi}\int_0^\pi dx \; 1 \; \sin(nx) = -\frac{2}{\pi n} \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} & \end{cases}$


Prove that the FS for $f(x)=1$ converges to 1
$A_n = \frac{2}{\pi}\int_0^\pi dx \, 1 \, \sin(nx) = \frac{-2}{\pi n} \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} \end{cases}$
$\sum_{n=0}^{\infty}$


Prove that the FS for f(x)=1 converges to 1
$A_n = \frac{2}{\pi}\int_0^{\pi} dx\, 1\, \sin(nx) = -\frac{2}{\pi n} \underbrace{\cos(nx)\Big|_0^{\pi}}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} & \end{cases}$
$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin(2n+1)}{2n+1}$


Prove that the FS for f(x)=1 converges to 1
$A_n = \frac{2}{\pi}\int_0^\pi dx\, 1\, \sin(nx) = -\frac{2}{\pi n}\underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} \end{cases}$
$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin(2n+1)}{2n+1} \overset{?}{=} 1$

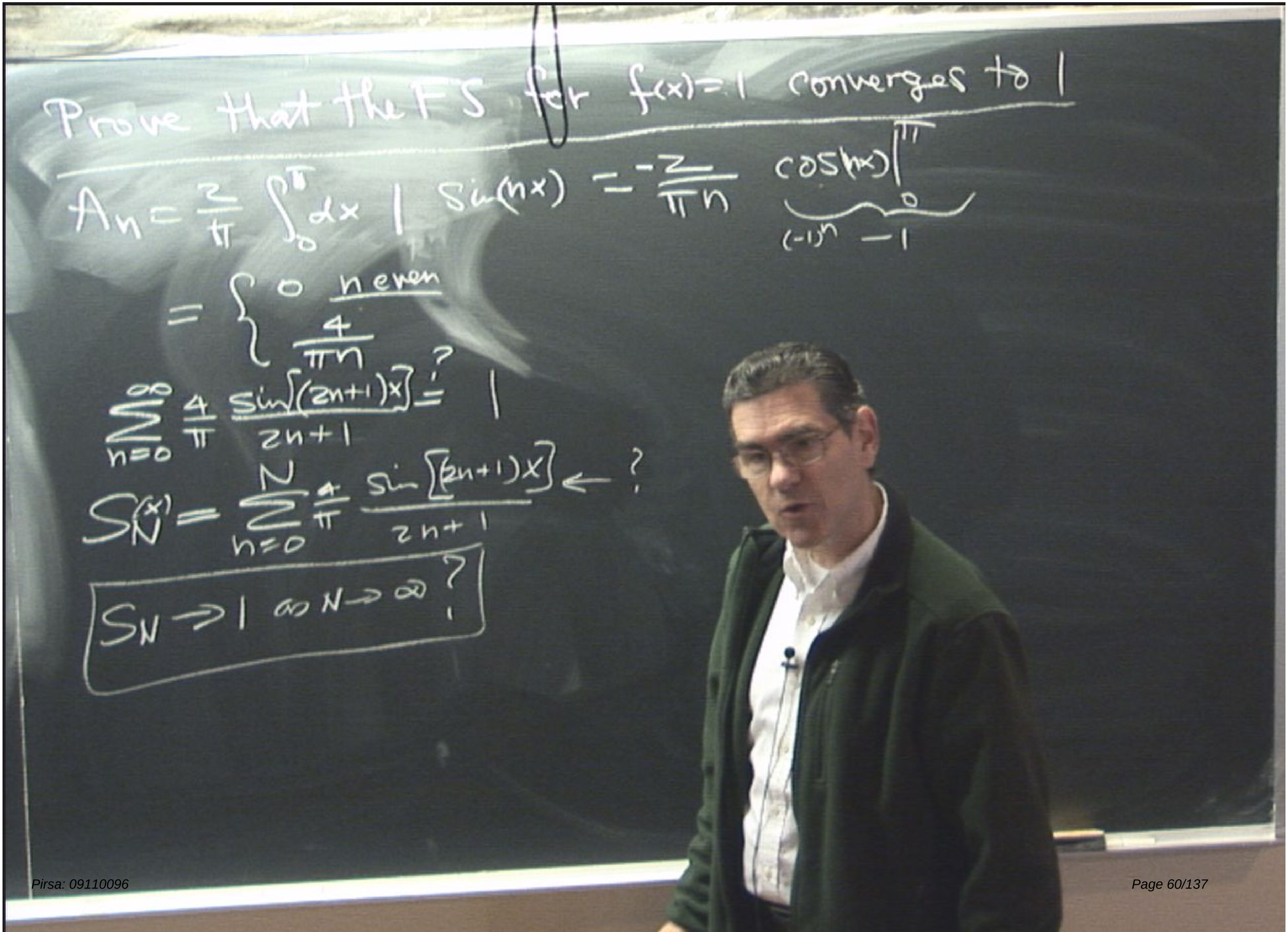
Prove that the FS for $f(x)=1$ converges to 1

$$A_n = \frac{2}{\pi}\int_0^\pi dx\ 1\ \sin(nx) = \frac{-2}{\pi n}\ \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$$

$$= \begin{cases} 0 & \underline{n\ \text{even}} \\ \dfrac{4}{\pi n} & \end{cases}$$

$$\sum_{n=0}^{\infty} \frac{4}{\pi}\frac{\sin[(2n+1)x]}{2n+1} \stackrel{?}{=} 1$$


Prove that the F S for f(x)=1 converges to 1
$A_n = \frac{2}{\pi}\int_0^\pi dx \, 1 \, \sin(nx) = -\frac{2}{\pi n} \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} \end{cases}$
$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1} \stackrel{?}{=} 1$
$S_N(x) = \sum_{n=0}^{N} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1}$
$S_N \to 1$ as $N \to \infty$?


Prove that the F S for $f(x)=1$ converges to 1
$A_n = \frac{2}{\pi}\int_0^\pi dx \; 1 \; \sin(nx) = -\frac{2}{\pi n} \underbrace{\cos(nx)\Big|_0^\pi}_{(-1)^n - 1}$
$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} \end{cases}$
$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1} \stackrel{?}{=} 1$
$S_N(x) = \sum_{n=0}^{N} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1} \leftarrow ?$
$\boxed{S_N \to 1 \text{ as } N \to \infty \,?}$

Prove that the FS for $f(x)=1$ converges to 1

$$A_n = \frac{2}{\pi}\int_0^{\pi} dx \; 1 \; \sin(nx) = -\frac{2}{\pi n} \underbrace{\cos(nx)\Big|_0^{\pi}}_{(-1)^n - 1}$$

$$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} & \end{cases}$$

$$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1} \overset{?}{=} 1$$

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$$\boxed{S_N \to 1 \text{ as } N \to \infty ?}$$

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Prove that the F S for $f(x)=1$ converges to 1

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....

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$S_N^i(x) = \frac{2}{\pi} e^{-i(2N+1)x}$

$\supset 0$

Prove that the F S

$A_n = \frac{2}{\pi} \int_0^{\pi} dx \mid \sin$

$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} & ? \end{cases}$

$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1} \stackrel{?}{=}$

$S_N(x) = \sum_{n=0}^{N} \frac{4}{\pi} \frac{\sin[(2n+1)x]}{2n+1}$

$\boxed{S_N \to 1 \text{ as } N \to \infty \, ?}$

$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1-e^{2ix}}{1-e^{2ix}}$$

Prove that the F S

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Prove that the FS for f(x)

$$A_n = \frac{2}{\pi}\int_0^\pi dx \; 1 \; \sin(nx) = -\frac{2}{\pi}$$

$$= \begin{cases} 0 & n \text{ even} \\ \frac{4}{\pi n} & \end{cases}$$

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Prove that the FS for $f(x)=1$ converges to 1

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$$\frac{\sin[(2n+1)x]}{2n+1} \overset{?}{=} 1$$

$$\sum_{n=0}^{N} \frac{4}{\pi}\frac{\sin[(2n+1)x]}{2n+1} \leftarrow ?$$

$$\boxed{\text{as } N \to \infty \; ?}$$

$$S_N'(x) = \sum_{n=0}^{N} \frac{4}{\pi}\cos[(2n+1)x]$$

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$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1 - e^{\overbrace{2ix(2N+2)}^{4N+4}}}{1 - e^{2ix}} = \frac{2}{\pi} \frac{e^{-i(2N+1)x} - e^{2N+3}}{1 - e^{2ix}}$$

$$S'_N(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1 - e^{\overbrace{2ix(2N+2)}^{4N+4}}}{1 - e^{2ix}}$$

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$$e^{ix}$$

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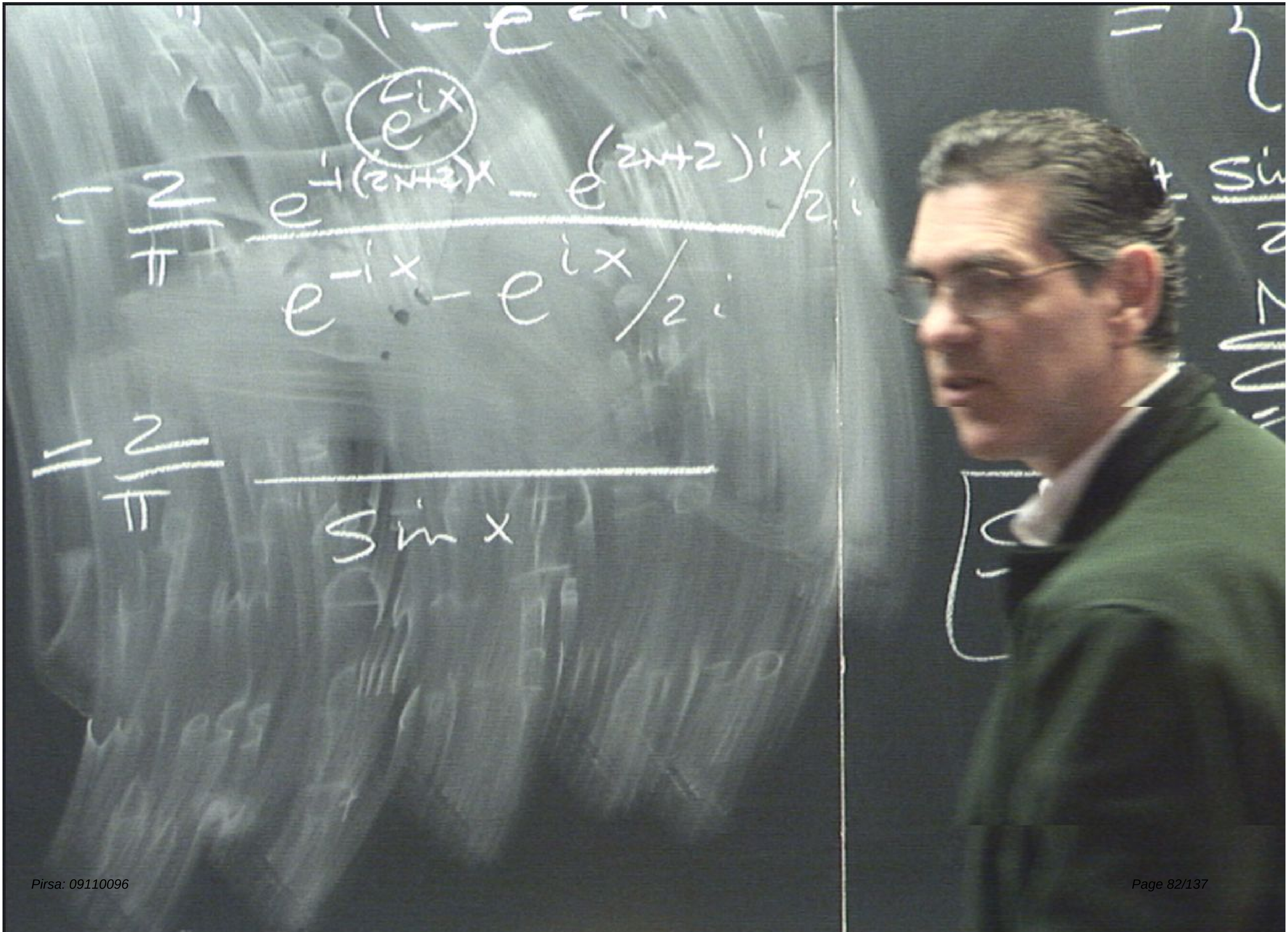
e^{ix}
$= \frac{2}{\pi} \frac{e^{-i(2N+2)x} - e^{(2N+2)ix}/2i}{e^{-ix} - e^{ix}/2i}$
$= \frac{2}{\pi} \frac{}{\sin x}$

$$= \frac{2}{\pi}\,\frac{e^{-i(2N+2)x} - e^{(2N+2)ix}/2i}{e^{-ix} - e^{ix}/2i}$$

$$e^{ix}$$

$$= \frac{2}{\pi}\,\frac{\sin((2N+2)x)}{\sin x}$$

$$= \{$$

$$\sum_{n=0}^{\infty} \frac{4}{\pi} \frac{\sin}{2}$$

$$S_N^{(x)} = \sum_{n=}^{N}$$

$$\boxed{S_N \to 1}$$

Prove that the FS for $f(x)=1$ converges to 1

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$$1 + r + r^2 + \cdots + r^N = \frac{1 - r^{N+1}}{1 - r} \qquad e^{2ix(2N+1)}$$

$$x \neq 0, \pi \qquad r = 1 \qquad e$$

$$\boxed{S_N'(x) = \frac{2}{\pi}\,\frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int^x dt\,\frac{2}{\pi}\,\frac{\sin(2N+2)t}{\sin t}$$

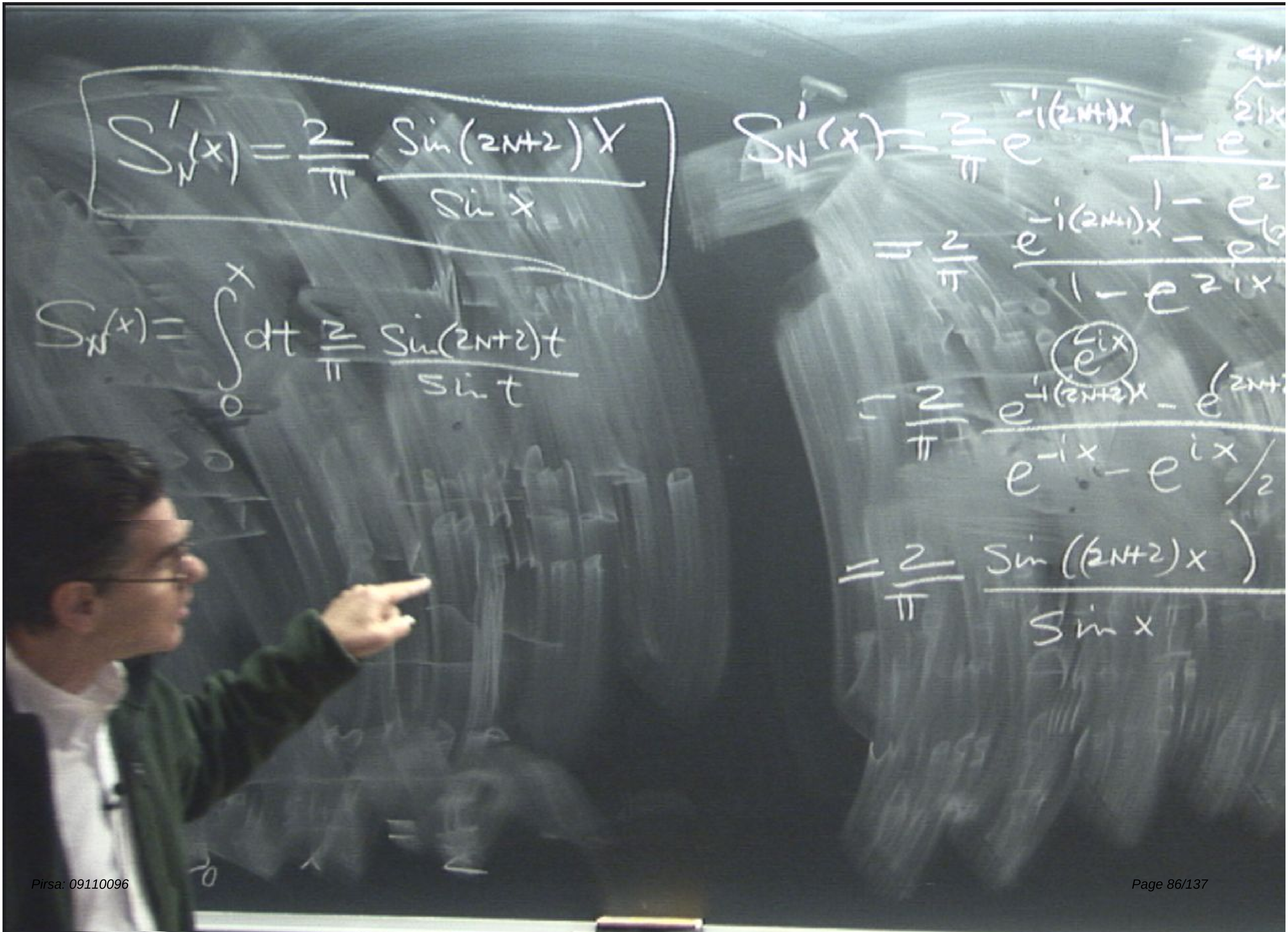
$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x}\,\frac{1-e^{\ldots}}{1-e^{2\ldots}}$$

$$= \frac{2}{\pi}\,\frac{e^{-i(2N+1)x} - e^{\ldots}}{1-e^{2ix}}$$

$$e^{ix}$$

$$= \frac{2}{\pi}\,\frac{e^{-i(2N+2)x} - e^{(2N+\ldots}}{e^{-ix} - e^{ix}/2\ldots}$$

$$= \frac{2}{\pi}\,\frac{\sin((2N+2)x)}{\sin x}$$



$S'_N(x) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$
$S_N(x) = \int_0^x dt \frac{2}{\pi} \frac{\sin(2N+2)t}{\sin t}$
$S'_N(x) = \frac{2}{\pi} e^{-i(2N+1)x}$
$= \frac{2}{\pi} \frac{e^{-i(2N+1)x} - e}{1 - e^{2ix}}$
e^{ix}
$= \frac{2}{\pi} \frac{e^{-i(2N+2)x} - e^{(2N+2}}{e^{-ix} - e^{ix} / 2}$
$= \frac{2}{\pi} \frac{\sin((2N+2)x)}{\sin x}$

$$\boxed{S_N'(x) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int_0^x dt \, \frac{2}{\pi} \sin(2N+2)t \left[\frac{1}{\sin t} - \frac{1}{t} + \frac{1}{t}\right]$$

$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1 - e^{2ix \ldots}}{1 - e^{2 \ldots}}$$

$$= \frac{2}{\pi} \frac{e^{-i(2N+1)x} - e^{\ldots}}{1 - e^{2ix}}$$

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$$= \frac{2}{\pi} \frac{e^{-i(2N+2)x} - e^{(2N+2)\ldots}}{e^{-ix} - e^{ix}} \Big/ 2$$

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$$\boxed{S_N'(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int_0^x dt\, \frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t}-\frac{1}{t}\right)+\frac{1}{t}\right]$$

$$I_1 = \int \cdots \sin(2N+2)t\left(\frac{1}{\sin t}-\frac{1}{t}\right) \qquad I_1 \quad I_2$$

$$S_N'(x) = \frac{2}{\pi}e^{-i(2N+1)x}\frac{1-e^{\ldots}}{1-e^{2\ldots}}$$

$$= \frac{2}{\pi}\frac{e^{-i(2N+1)x} - e^{\ldots}}{1-e^{2ix}}$$

$$e^{ix}$$

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$$\boxed{S_N'(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

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$$I_1 = \int \cdots\cdots \sin(2N+2)t\left(\frac{1}{\sin t} - \frac{1}{t}\right) \quad I_2$$

$$(N \to \infty)$$

$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1-e \cdots}{1-e \cdots}$$

$$= \frac{2}{\pi} \frac{e^{-i(2N+1)x} - e \cdots}{1-e^{2ix}}$$

$$e^{ix}$$

$$= \frac{2}{\pi} \frac{e^{-i(2N+2)x} - e^{(2N+2 \cdots}}{e^{-ix} - e^{ix}/2}$$

$$= \frac{2}{\pi} \frac{\sin((2N+2)x)}{\sin x}$$

$$\boxed{S_N'(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int_0^x dt\, \frac{2}{\pi} \sin(2N+2)t \left[\underbrace{\left(\frac{1}{\sin t} - \frac{1}{t}\right)}_{I_1} + \underbrace{\frac{1}{t}}_{I_2}\right]$$

$$I_1 = \int_0^x \cdots\cdots \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$(N \to \infty)$ $\longrightarrow 0$ as $N \to \infty$.

$$S_N'(x) = \frac{2}{\pi} e^{-i(2N+1)x} \frac{1 - e^{2ix \ldots}}{1 - e^{2 \ldots}}$$

$$= \frac{2}{\pi} \frac{e^{-i(2N+1)x} - e^{\ldots}}{1 - e^{2ix}}$$

e^{ix}

$$= \frac{2}{\pi} \frac{e^{-i(2N+2)x} - e^{(2N+2)\ldots}}{e^{-ix} - e^{ix}} /2$$

$$= \frac{2}{\pi} \frac{\sin((2N+2)x)}{\sin x}$$

$$\boxed{S'_N(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int_0^x dt\, \frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$$I_1 \qquad\qquad I_2$$

$$I_1 = \int_0^x \sin(2N+2)t\left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$(N \to \infty)$ $\longrightarrow 0$ as $N \to \infty$.

$$I_2 = \frac{2}{\pi}\int_0^x dt\, \frac{\sin(2N+2)t}{t}$$

$$S'_N(x) = \frac{2}{\pi} e^{-i(2N+1)x}\frac{1-e^{2ix\ldots}}{1-e^{2\ldots}}$$

$$= \frac{2}{\pi}\frac{e^{-i(2N+1)x} - e^{\ldots}}{1-e^{2ix}}$$

$$e^{ix}$$

$$= \frac{2}{\pi}\frac{e^{-i(2N+2)x} - e^{(2N+2)\ldots}}{e^{-ix} - e^{ix}/2}$$

$$= \frac{2}{\pi}\frac{\sin((2N+2)x)}{\sin x}$$

$$\boxed{S_N'(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

$$S_N(x) = \int_0^x dt\, \frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$$I_1 \qquad\qquad I_2$$

$$I_1 = \int_0^x \cdots\cdots \sin(2N+2)t\left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$(N \to \infty)$

$\longrightarrow 0$ as $N \to \infty$.

$$I_2 = \frac{2}{\pi}\int_0^{x(2N+2)} dt\, \frac{\sin(2N+2)t}{t(2N+2)} \quad s$$

$$= \frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\, \frac{\sin s}{s}\right) \to 1$$

$\to 0$

$$S_N'(x) = \frac{2}{\pi}e^{-i(2N+1)x}\frac{1-e^{2ix\cdots}}{1-e^{2\cdots}}$$

$$= \frac{2}{\pi}\frac{e^{-i(2N+1)x} - e^{\cdots}}{1-e^{2ix}}$$

e^{ix}

$$= \frac{2}{\pi}\frac{e^{-i(2N+2)x} - e^{(2N+\cdots}}{e^{-ix} - e^{ix}/2}$$

$$= \frac{2}{\pi}\frac{\sin((2N+2)x)}{\sin x}$$

$$\boxed{S_N'(x) = \frac{2}{\pi}\frac{\sin(2N+2)x}{\sin x}}$$

Near $x \neq 0$?

$$S_N(x) = \int_0^x dt \, \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

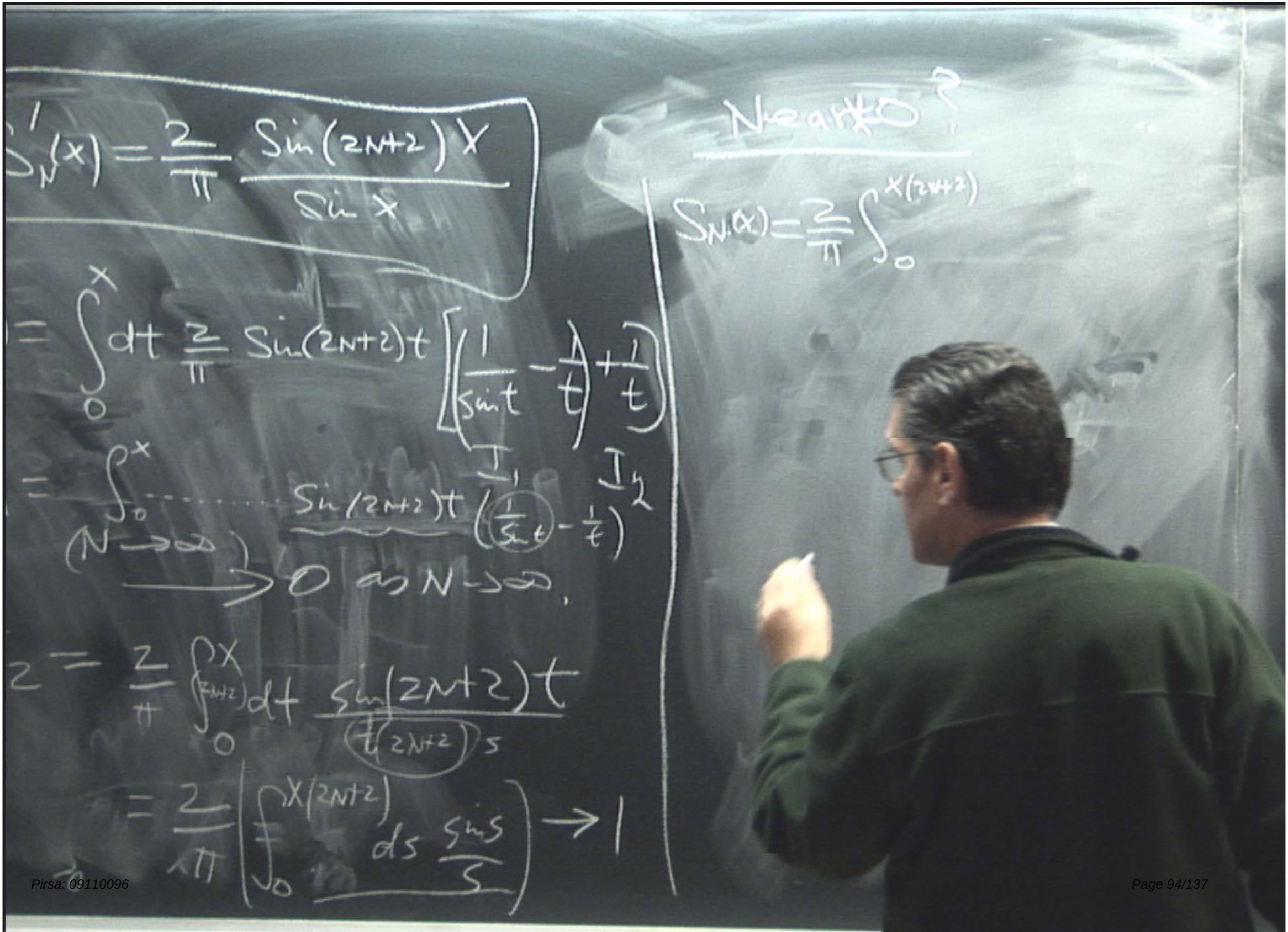
$$I_1 = \int_0^x \cdots \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$(N \to \infty)$

$\longrightarrow 0$ as $N \to \infty$.

$$I_2 = \frac{2}{\pi}\int_0^x (2N+2)\, dt \, \frac{\sin(2N+2)t}{t(2N+2)} \quad s$$

$$= \frac{2}{\pi}\left(\int_0^{x(2N+2)} ds \, \frac{\sin s}{s}\right) \to 1$$


$S_N'(x) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$
$= \int_0^x dt \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t} \right) + \frac{1}{t} \right]$
I_1 I_2
$= \int_0^x \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t} \right)$
$(N \to \infty)$
$\to 0$ as $N \to \infty$.
$I_2 = \frac{2}{\pi} \int_0^{x(2N+2)} dt \frac{\sin(2N+2)t}{t(2N+2)}$ s
$= \frac{2}{\pi} \left(\int_0^{x(2N+2)} ds \frac{\sin s}{s} \right) \to 1$
Near x≈0 ?
$S_N(x) = \frac{2}{\pi} \int_0^{x(2N+2)}$

$$S_N'(x) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$$

$$= \int_0^x dt \, \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$I_1 \qquad I_2$

$$I_1 = \int_0^x \cdots \underbrace{\sin(2N+2)t}_{(N\to\infty)} \left(\frac{1}{\sin t} - \frac{1}{t}\right) \longrightarrow 0 \text{ as } N\to\infty.$$

$$I_2 = \frac{2}{\pi} \int_0^{x(2N+2)} dt \, \frac{\sin(2N+2)t}{t(2N+2)} \; s$$

$$= \frac{2}{\pi} \left(\int_0^{x(2N+2)} ds \, \frac{\sin s}{s} \right) \to 1$$

Near $x=0$?

$$S_N(x) = \frac{2}{\pi} \int_0^{x(2N+2)} ds \, \frac{\sin s}{s}$$

$$) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$$

$$\int dt \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$I_1 \quad I_2$

$$\int_0^x \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$N \to \infty$

$\to 0$ as $N \to \infty$.

$$= \frac{2}{\pi} \int_0^{x(2N+2)} dt \frac{\sin(2N+2)t}{t(2N+2)} s$$

$$= \frac{2}{\pi} \left(\int_0^{x(2N+2)} ds \frac{\sin s}{s} \right) \to 1$$

Near $x = 0$?

$$S_N(x) = \frac{2}{\pi} \int_0^{x(2N+2)} ds \frac{\sin s}{s}$$

$$x \sim \frac{1}{2N+2}$$

Pr

A

S

S

$$) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$$

$$\int dt \, \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$$\int_0^x \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$N \to \infty$

$\to 0$ as $N \to \infty$.

$$= \frac{2}{\pi} \int_0^{x(2N+2)} dt \, \frac{\sin(2N+2)t}{t(2N+2)}$$

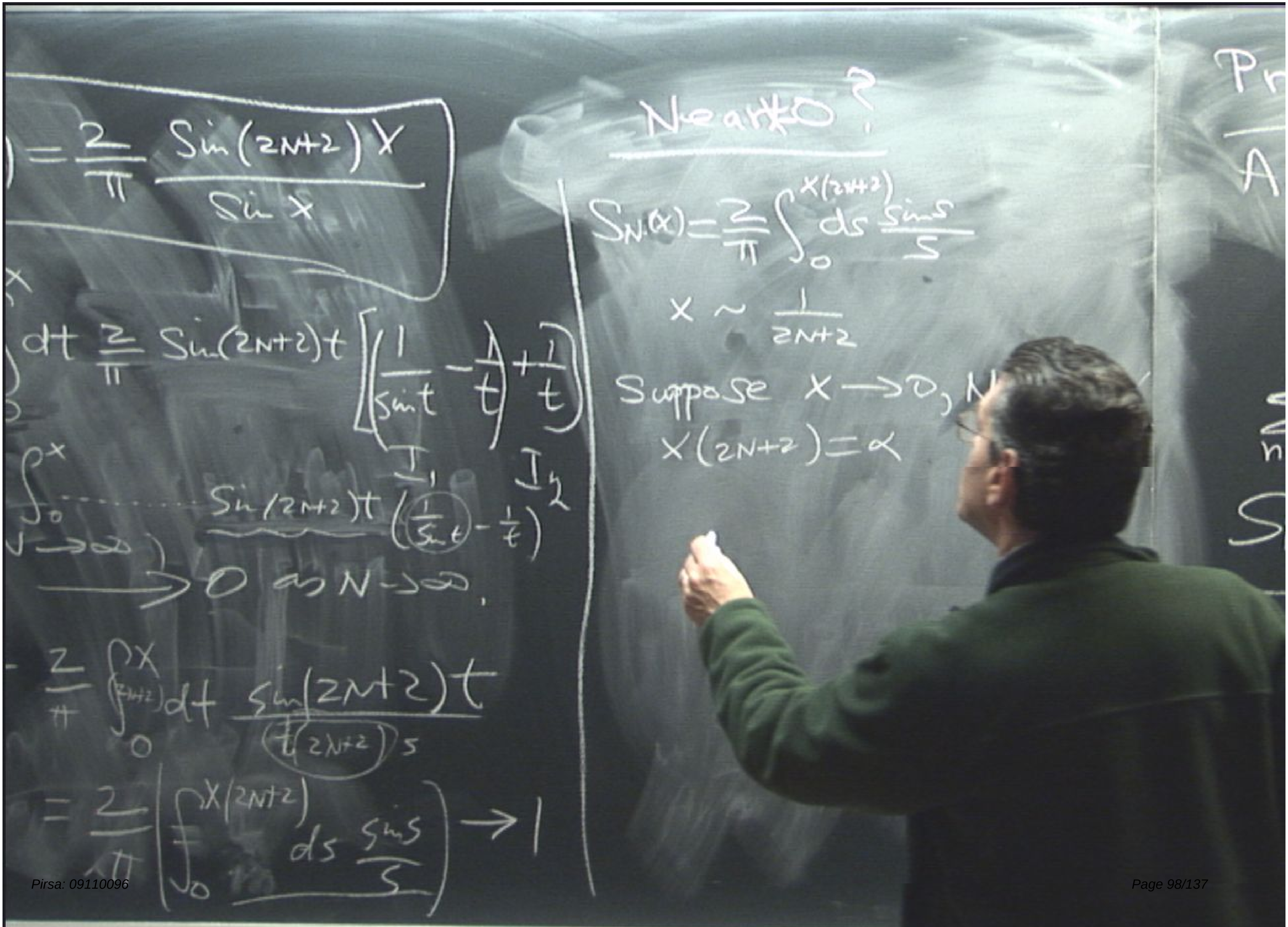
$$= \frac{2}{\pi} \left(\int_0^{x(2N+2)} ds \, \frac{\sin s}{s} \right) \to 1$$

Near $x = 0$?

$$S_N(x) = \frac{2}{\pi} \int_0^{x(2N+2)} ds \, \frac{\sin s}{s}$$

$$x \sim \frac{1}{2N+2}$$

Suppose $x \to 0$, $N \to \infty$.


$) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$
$dt\ \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$
I_1 I_2
$\int_0^x \sin(2N+2)t \left(\frac{1}{\sin t} - \frac{1}{t}\right)$
$\to 0$ as $N \to \infty$.
$\frac{2}{\pi} \int_0^{x(2N+2)} dt\ \frac{\sin(2N+2)t}{t(2N+2)}$
$= \frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\ \frac{\sin s}{s}\right) \to 1$
Near x=0 ?
$S_N(\alpha) = \frac{2}{\pi} \int_0^{x(2N+2)} ds\ \frac{\sin s}{s}$
$x \sim \frac{1}{2N+2}$
Suppose $x \to 0$, N
$x(2N+2) = \alpha$

$$) = \frac{2}{\pi}\,\frac{\sin(2N+2)x}{\sin x}$$

x

$$\int dt\,\frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t}-\frac{1}{t}\right)+\frac{1}{t}\right]$$

$I_1 \qquad I_2$

$$\int_0^x \sin(2N+2)t\left(\frac{1}{\sin t}-\frac{1}{t}\right)$$

$N\to\infty)$

$\to 0$ as $N\to\infty$.

$$=\frac{2}{\pi}\int_0^{x(2N+2)} dt\,\frac{\sin(2N+2)t}{t(2N+2)}\;s$$

$$=\frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}\right)\to 1$$

Near $x=0$?

$$S_N(x)=\frac{2}{\pi}\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}$$

$$x\sim\frac{1}{2N+2}$$

Suppose $x\to 0,\ N\to\infty$.

$$x(2N+2)=\alpha$$

$$=\frac{2}{\pi}\int_0^{\alpha} ds\,\frac{\sin s}{s}$$

Pr

A

n

S

$$) = \frac{2}{\pi} \frac{\sin(2N+2)x}{\sin x}$$

$$\int dt \, \frac{2}{\pi} \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$$I_1 \qquad I_2$$

$$\int_0^x \frac{\sin(2N+2)t}{} \left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$$N \to \infty$$

$$\to 0 \text{ as } N \to \infty.$$

$$\frac{2}{\pi} \int_0^{x(2N+2)} dt \, \frac{\sin(2N+2)t}{t(2N+2)} \; s$$

$$= \frac{2}{\pi} \left(\int_0^{x(2N+2)} ds \, \frac{\sin s}{s} \right) \to 1$$

Near $x=0$?

$$S_N(x) = \frac{2}{\pi} \int_0^{x(2N+2)} ds \, \frac{\sin s}{s}$$

$$x \sim \frac{1}{2N+2}$$

Suppose $x \to 0, N \to \infty$

$$x(2N+2) = \alpha$$

$$= \frac{2}{\pi} \int_0^{\alpha} ds \, \frac{\sin s}{s}$$

$$= \frac{2}{\pi} \, Si(\alpha)$$

Pr

A

$$) = \frac{2}{\pi}\,\frac{\sin(2N+2)x}{\sin x}$$

x

$$\int dt\,\frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t}-\frac{1}{t}\right)+\frac{1}{t}\right]$$

$$\qquad\qquad I_1 \qquad I_2$$

$$\int_0^x \underbrace{\sin(2N+2)t}\left(\frac{1}{\sin t}-\frac{1}{t}\right)$$

$N\to\infty$)

$\to 0$ as $N\to\infty$.

$$= \frac{2}{\pi}\int_0^{x(2N+2)} dt\,\frac{\sin(2N+2)t}{t(2N+2)}\;s$$

$$= \frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}\right)\to 1$$

Near $x = 0$?

$$S_N(x) = \frac{2}{\pi}\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}$$

$$x \sim \frac{1}{2N+2}$$

Suppose $x\to 0$, $N\to\infty$.

$$x(2N+2) = \alpha$$

$$= \frac{2}{\pi}\underbrace{\int_0^{\alpha} ds\,\frac{\sin s}{s}}$$

$$= \frac{2}{\pi}\,Si(\alpha)$$

Pr

A

S

S

$$) = \frac{2}{\pi}\,\boxed{\frac{\sin(2N+2)x}{\sin x}}$$

x

$$\int dt\,\frac{2}{\pi}\sin(2N+2)t\left[\left(\frac{1}{\sin t}-\frac{1}{t}\right)+\frac{1}{t}\right]$$

$$I_1 \qquad I_2$$

$$\int_0^x \sin(2N+2)t\left(\frac{1}{\sin t}-\frac{1}{t}\right)$$

$N\to\infty$

$\to 0$ as $N\to\infty$.

$$=\frac{2}{\pi}\int_0^{x(2N+2)} dt\,\frac{\sin(2N+2)t}{t(2N+2)}\ s$$

$$=\frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}\right)\to 1$$

Near $x=0$?

$$S_N(\alpha)=\frac{2}{\pi}\int_0^{x(2N+2)} ds\,\frac{\sin s}{s}$$

$$x\sim\frac{1}{2N+2}$$

Suppose $x\to 0,\ N\to\infty$.

$$x(2N+2)=\alpha$$

$$=\frac{2}{\pi}\int_0^{\alpha} ds\,\frac{\sin s}{s}$$

$$=\frac{2}{\pi}\,\boxed{Si(\alpha)}$$

$$Si(0)=0$$

Pr

A

n

S

S

$$) = \frac{2}{\pi}\ \frac{\sin(2N+2)x}{\sin x}$$

x

$$\int dt\ \frac{2}{\pi}\ \sin(2N+2)t \left[\left(\frac{1}{\sin t} - \frac{1}{t}\right) + \frac{1}{t}\right]$$

$I_1 \qquad I_2$

$$\int_0^x \underbrace{\sin(2N+2)t}\left(\frac{1}{\sin t} - \frac{1}{t}\right)$$

$N \to \infty$

$\to 0$ as $N \to \infty$.

$$= \frac{2}{\pi}\int_0^{x(2N+2)} dt\ \frac{\sin(2N+2)t}{t(2N+2)}\ s$$

$$= \frac{2}{\pi}\left(\int_0^{x(2N+2)} ds\ \frac{\sin s}{s}\right) \to 1$$

Near ~~x~~ 0 ?

$$S_N(\alpha) = \frac{2}{\pi}\int_0^{x(2N+2)} ds\ \frac{\sin s}{s}$$

$$x \sim \frac{1}{2N+2}$$

Suppose $x \to 0$, $N \to \infty$.

$$x(2N+2) = \alpha$$

$$= \frac{2}{\pi}\underbrace{\int_0^{\alpha} ds\ \frac{\sin s}{s}}$$

$$= \frac{2}{\pi}\boxed{Si(\alpha)}$$

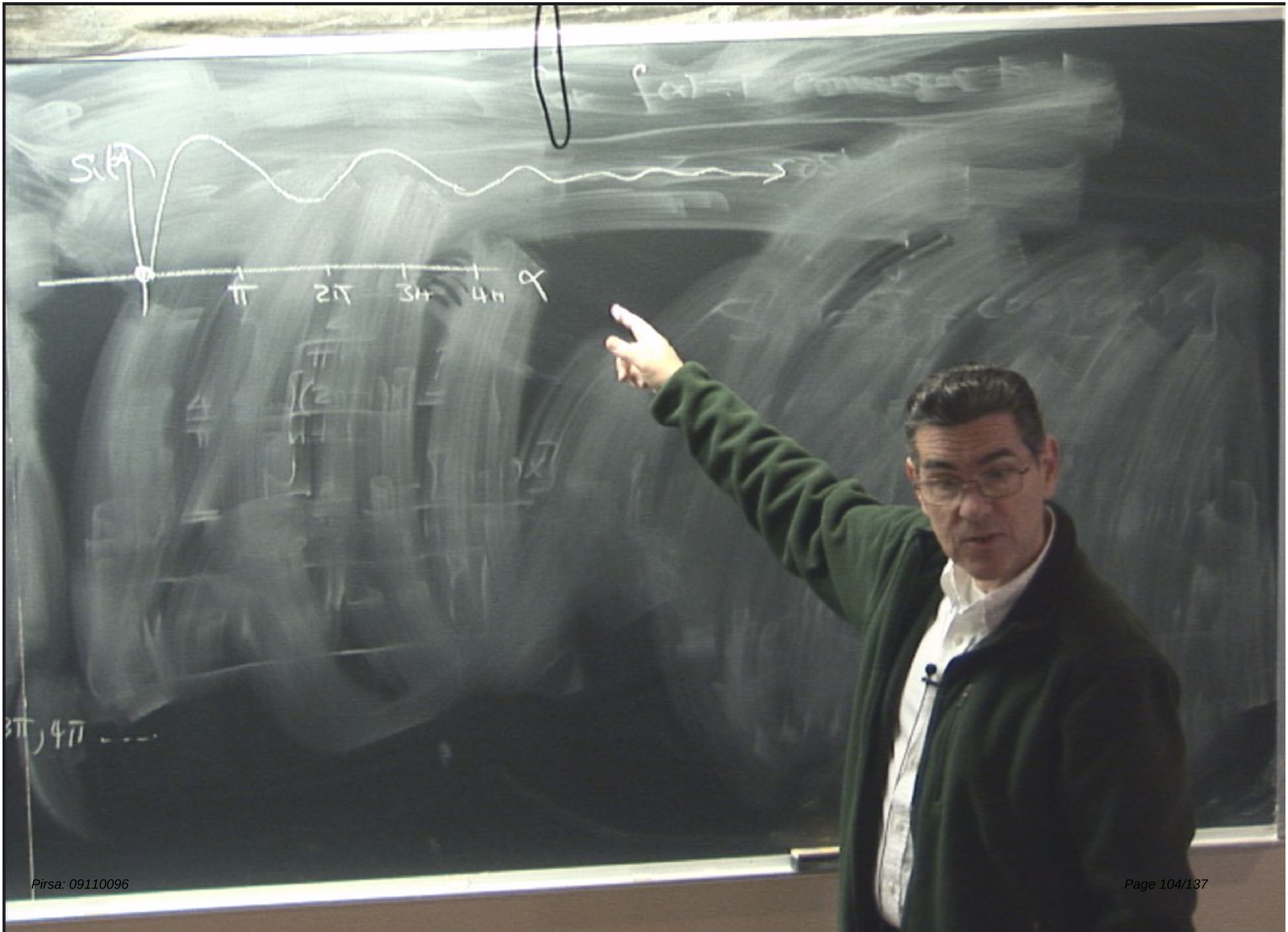
$$Si(0) = 0$$

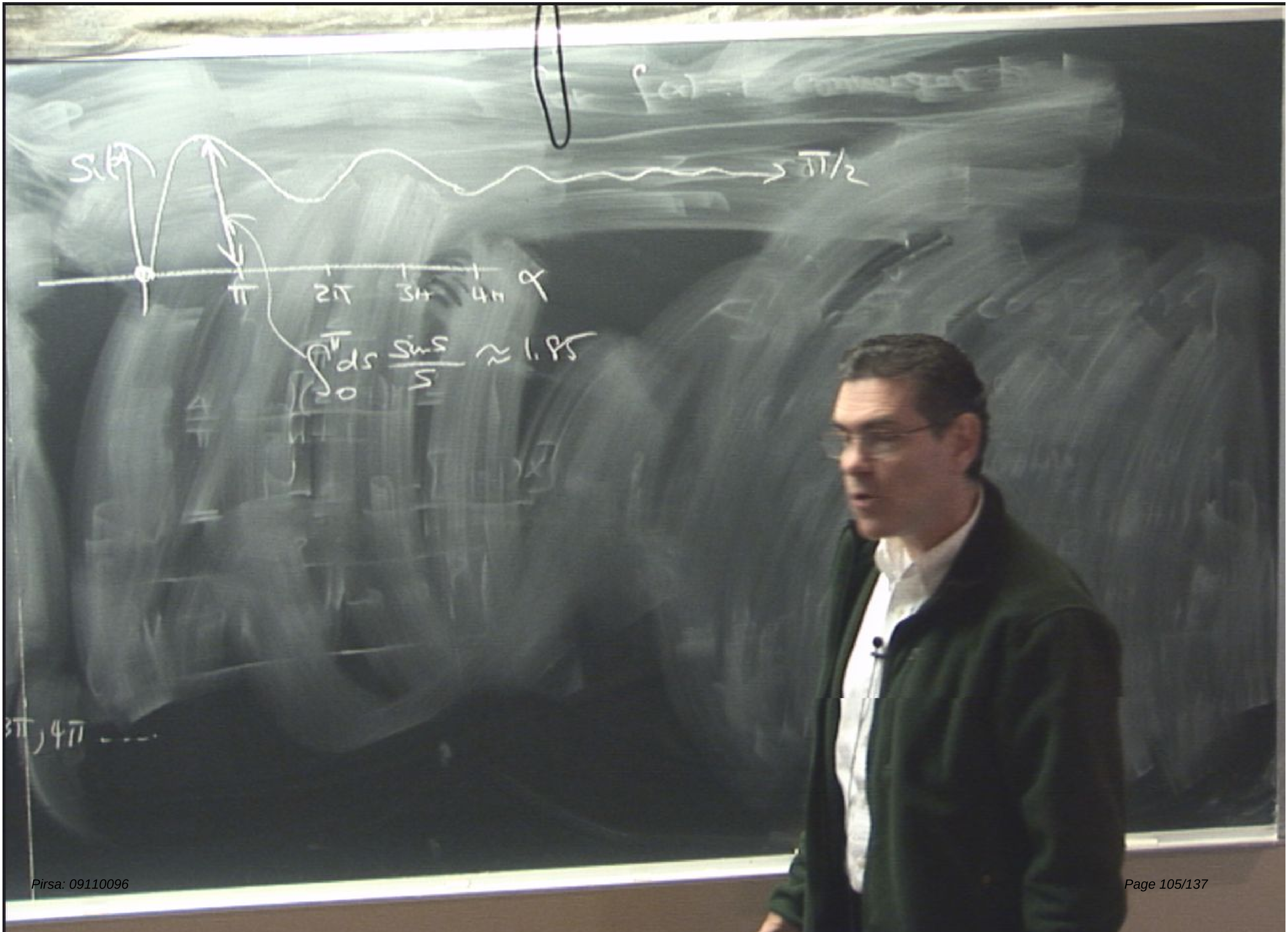
$$Si'(\alpha) = \frac{\sin s}{s} \qquad s = \pi, 2\pi, 3\pi, 4\pi \ldots$$

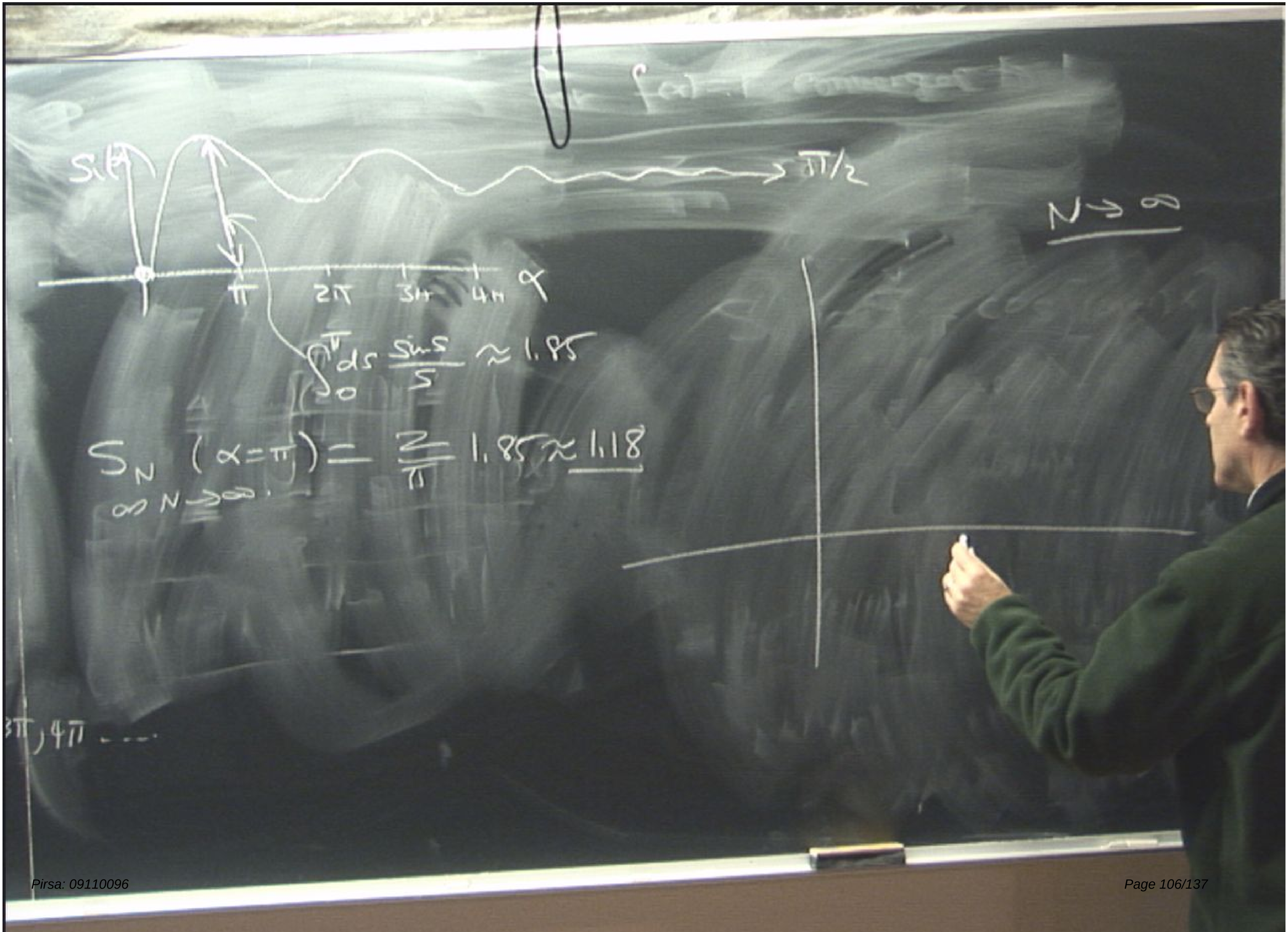
n

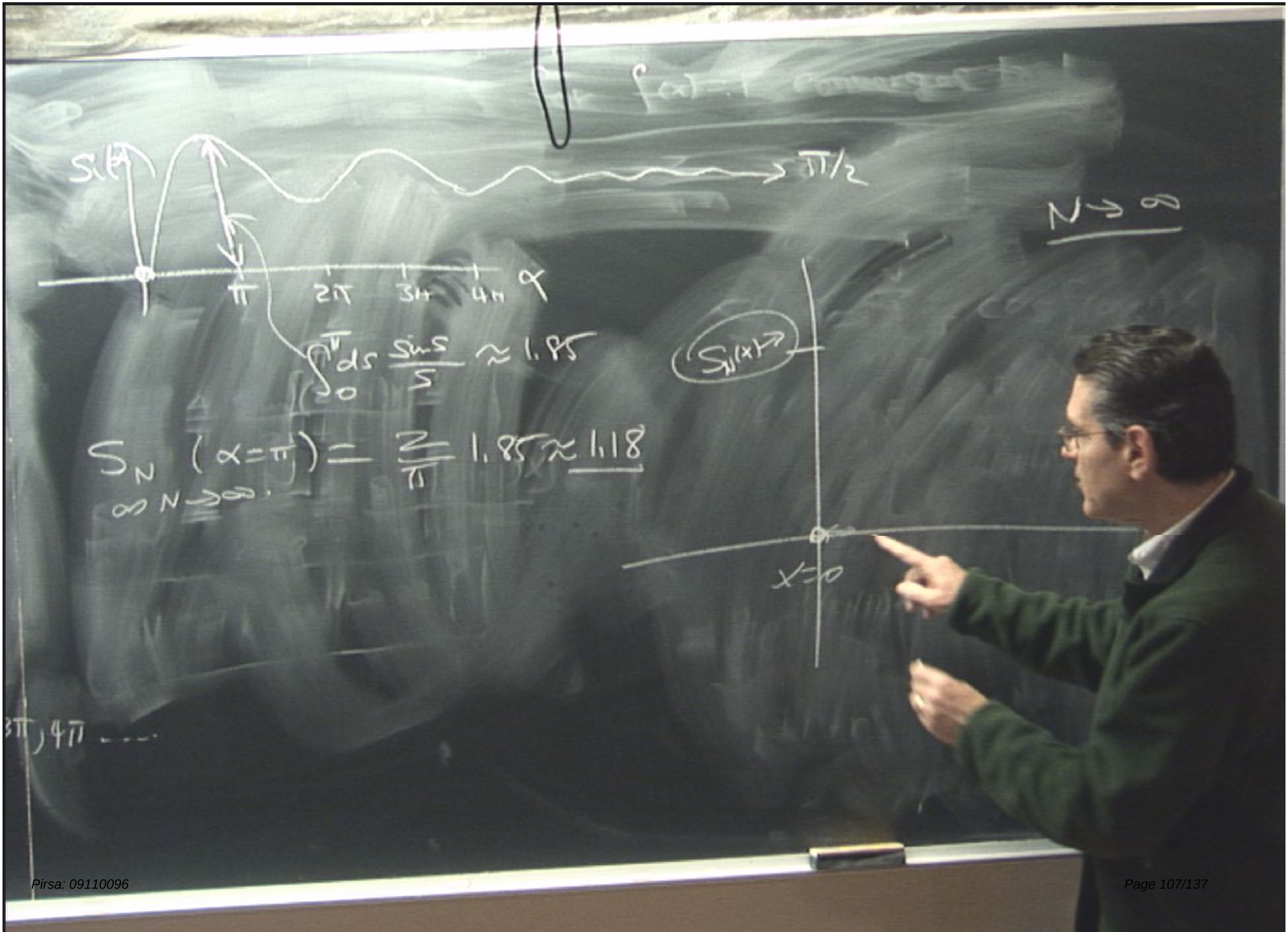
S

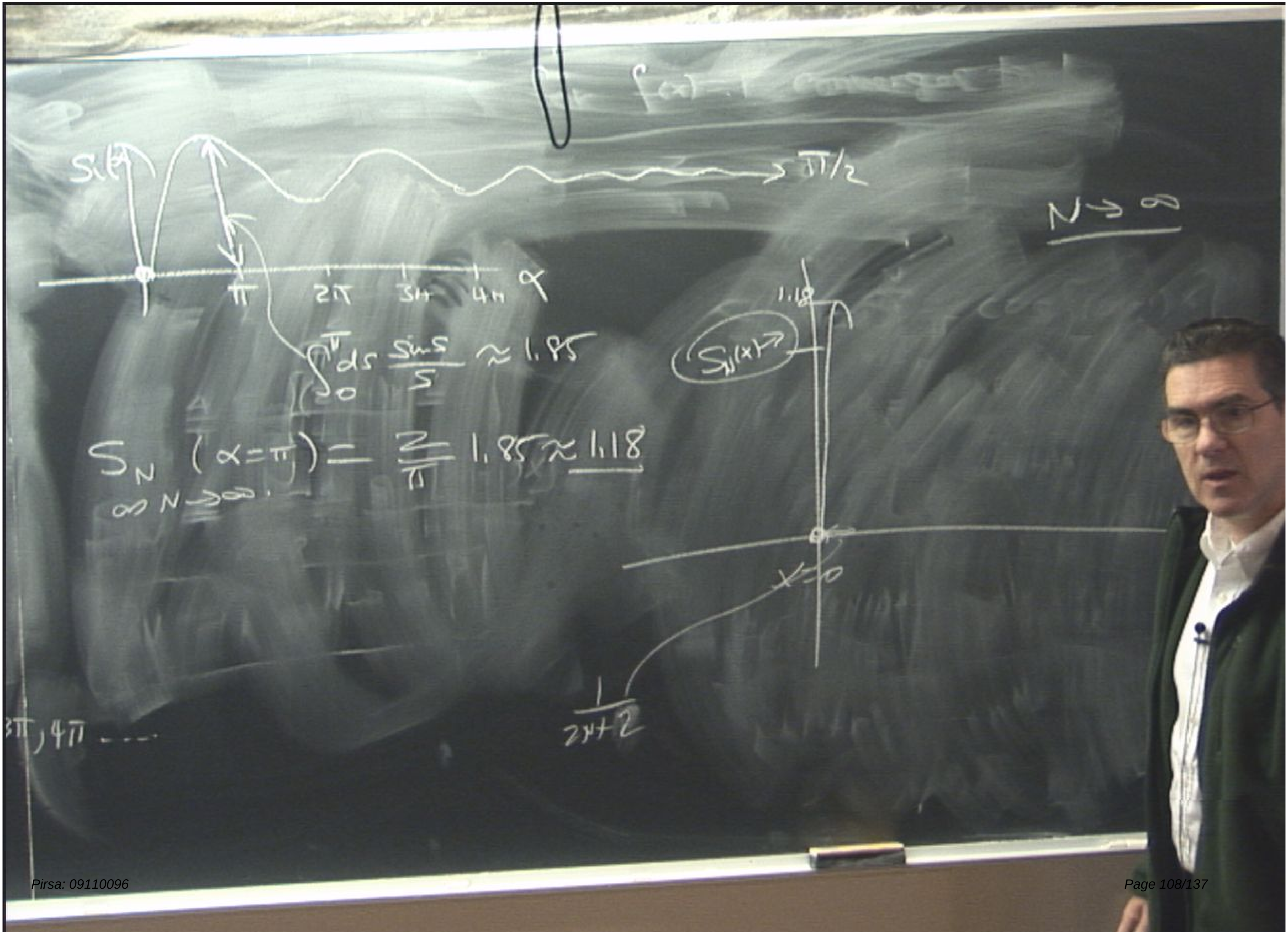
S

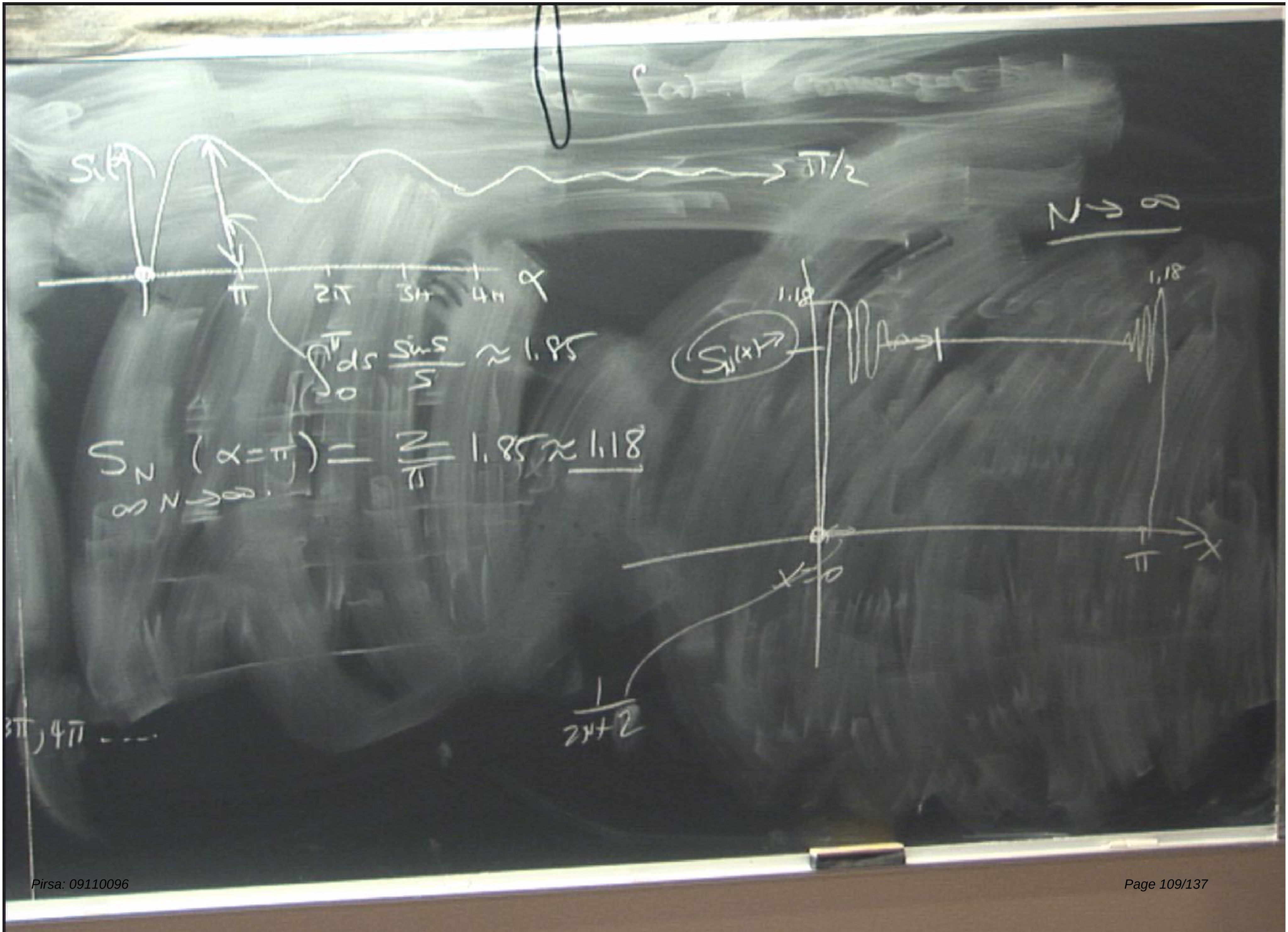

Si(α)
π
2π
3π
4π
α
3π, 4π ....

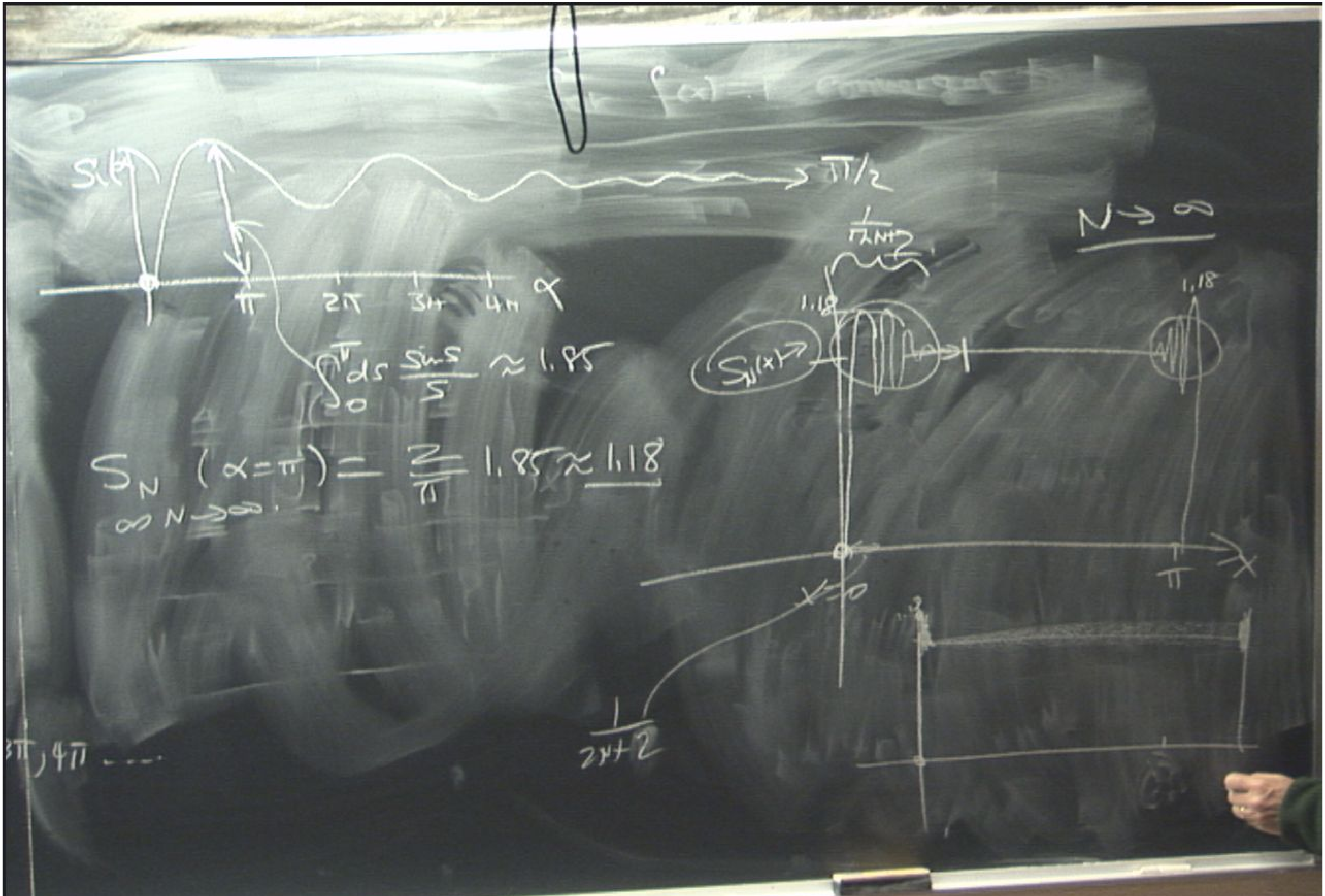

Si(α)
π 2π 3π 4π α
∫₀^π ds sin s / s ≈ 1.85
3π, 4π ....

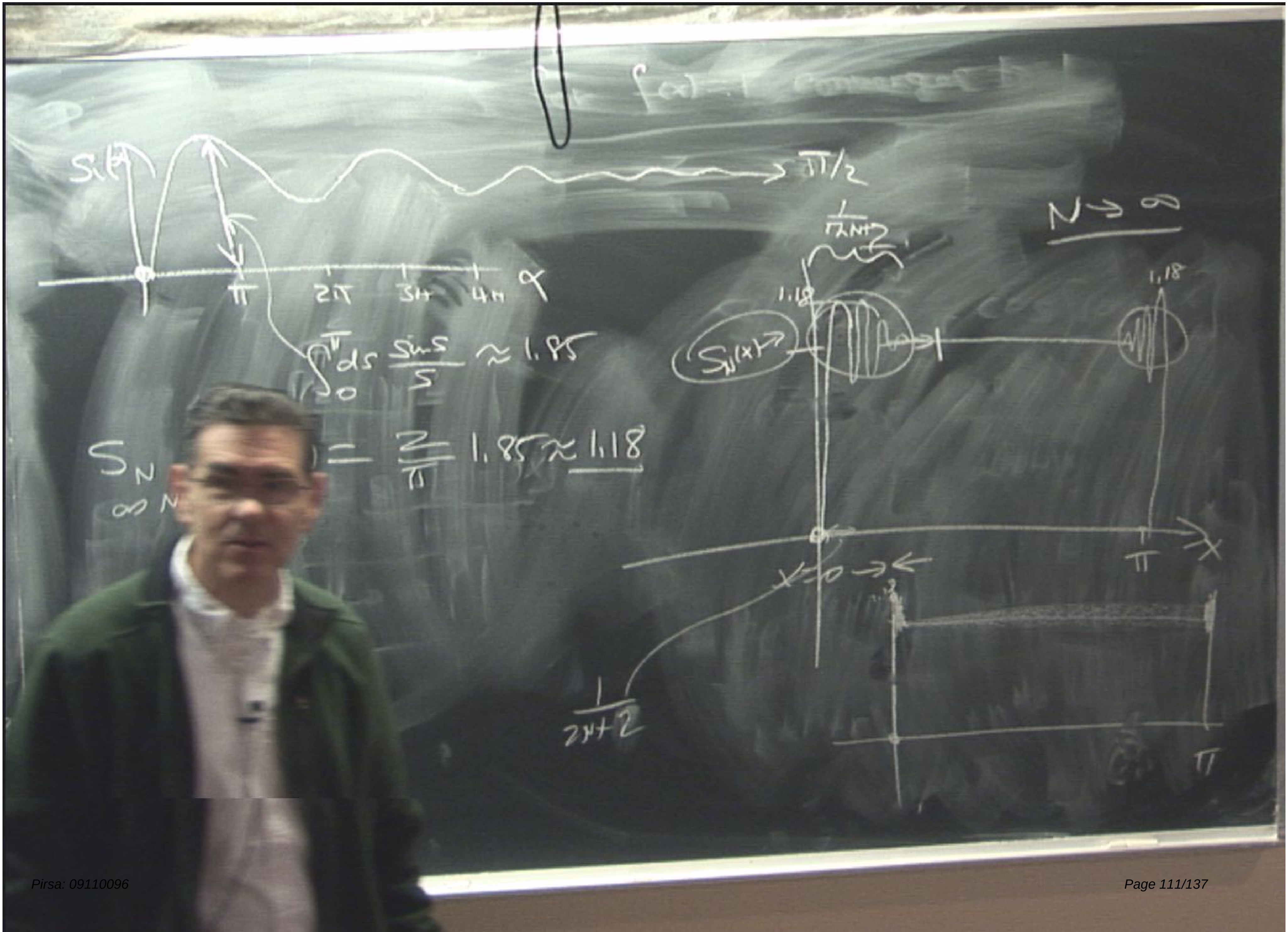

Si(α)
π/2
π 2π 3π 4π α
$\int_0^\pi ds \frac{\sin s}{s} \approx 1.85$
$S_N (\alpha=\pi) = \frac{2}{\pi} 1.85 \approx 1.18$
$N \to \infty$
$N \to \infty$
3π, 4π ....

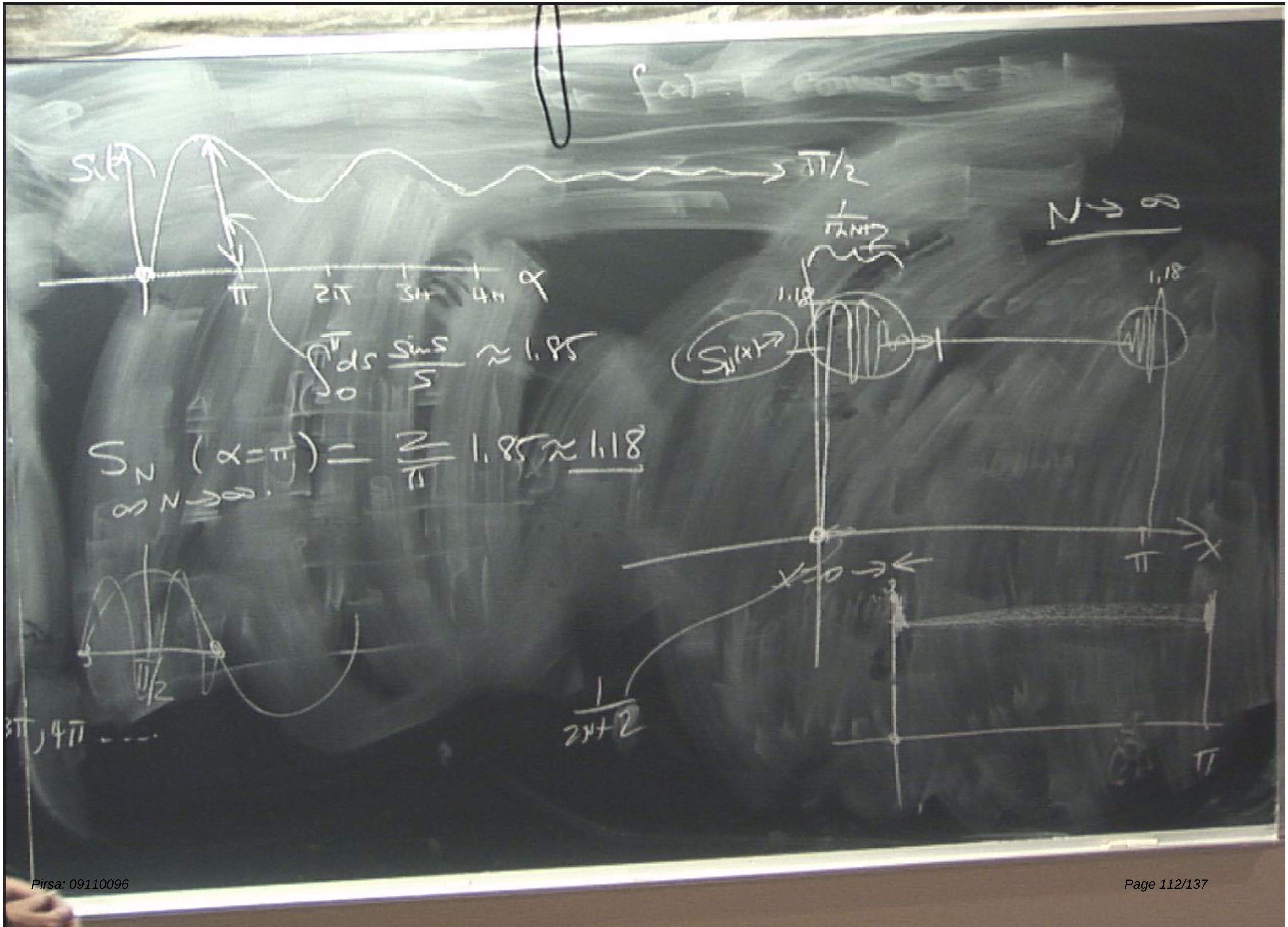

Si(α)
π/2
N → ∞
π 2π 3π 4π α
∫₀^π ds sin s / s ≈ 1.85
S_N(x)
S_N (α=π) = 2/π 1.85 ≈ 1.18
∞ N → ∞.
x=0
3π, 4π ....


Si(α)
→ π/2
N → ∞
π 2π 3π 4π α
∫_0^π ds sin s / s ≈ 1.85
S_N (α=π) = 2/π 1.85 ≈ 1.18
N → ∞
1.18
S_N(x)
x=0
1/(2N+2)
3π, 4π ....

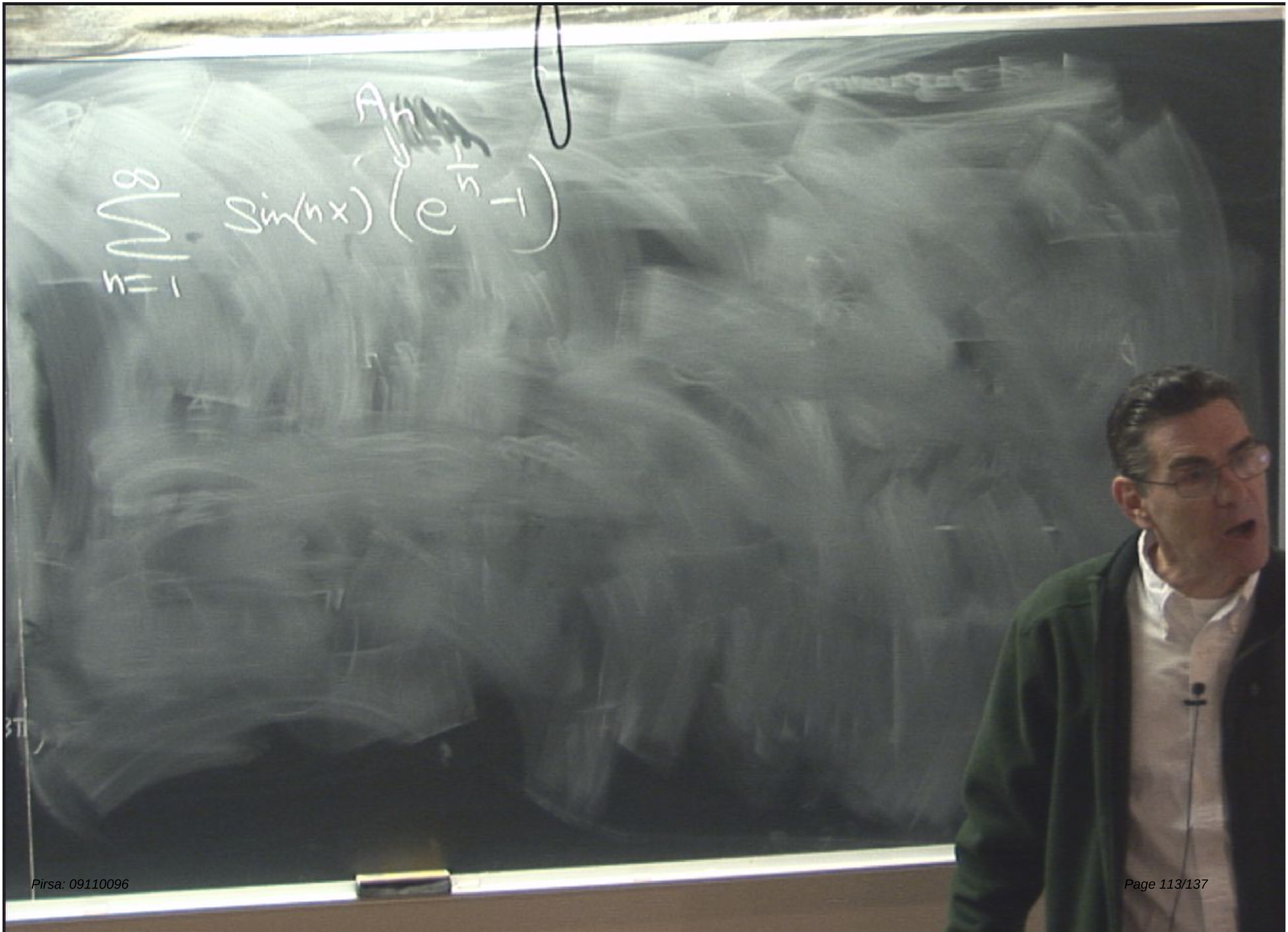

Si(α)
→ π/2
N → ∞
π 2π 3π 4π α
1.18
1.18
S_N(x)
$\int_0^\pi ds \frac{\sin s}{s} \approx 1.85$
$S_N(\alpha=\pi) = \frac{2}{\pi} 1.85 \approx 1.18$
as N → ∞.
x=0
π
x
$\frac{1}{2N+2}$
3π, 4π ....

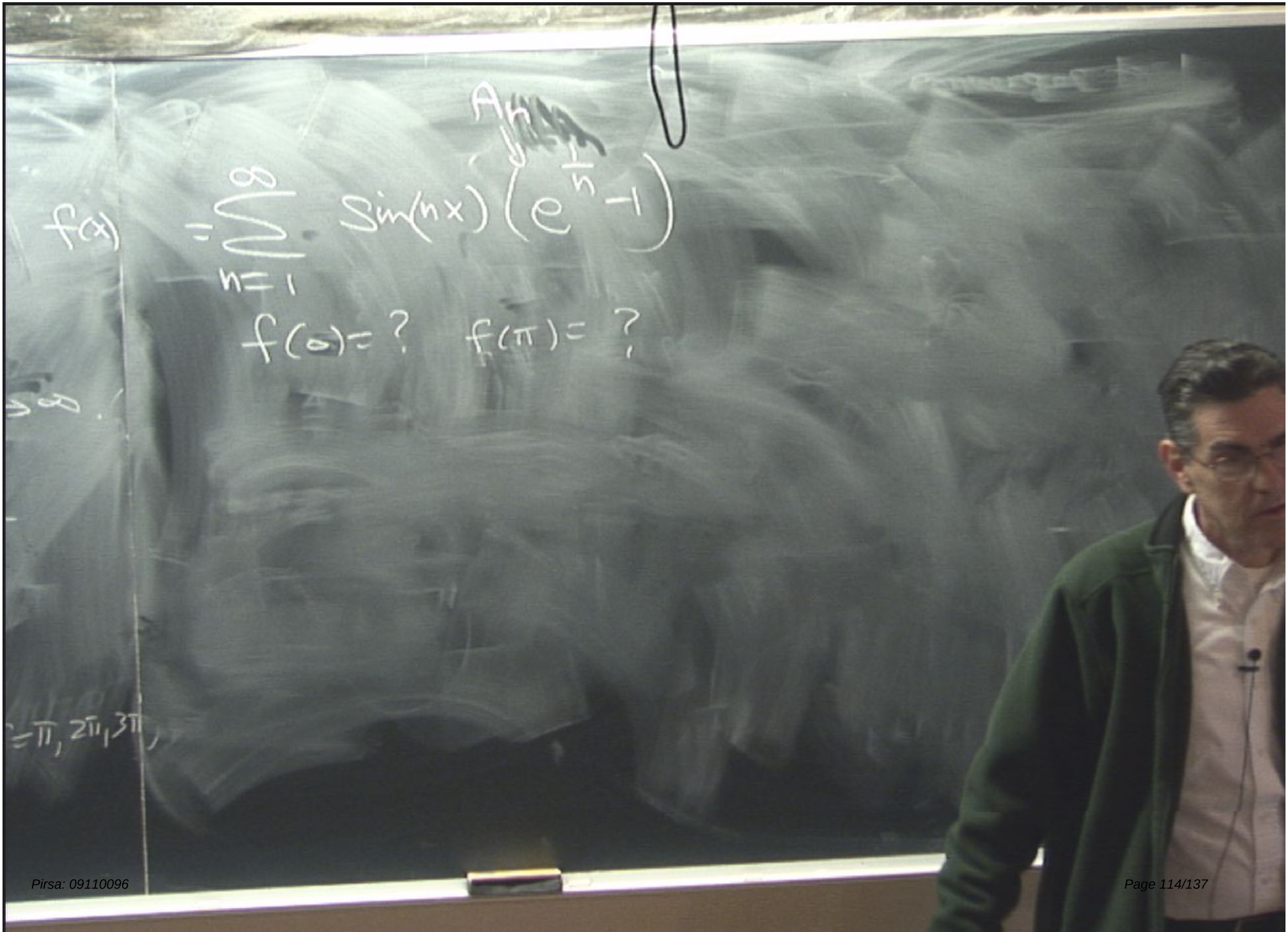

S(α)
π/2
π 2π 3π 4π α
$\int_0^\pi ds \frac{\sin s}{s} \approx 1.85$
$S_N(\alpha=\pi) = \frac{2}{\pi} 1.85 \approx 1.18$
N → ∞
1.18
$S_N(x)$
x = 0
π
$\frac{1}{2N+2}$
3π, 4π ....


π/2
N → ∞
1.18
≈ 1.85
2/π 1.85 ≈ 1.18

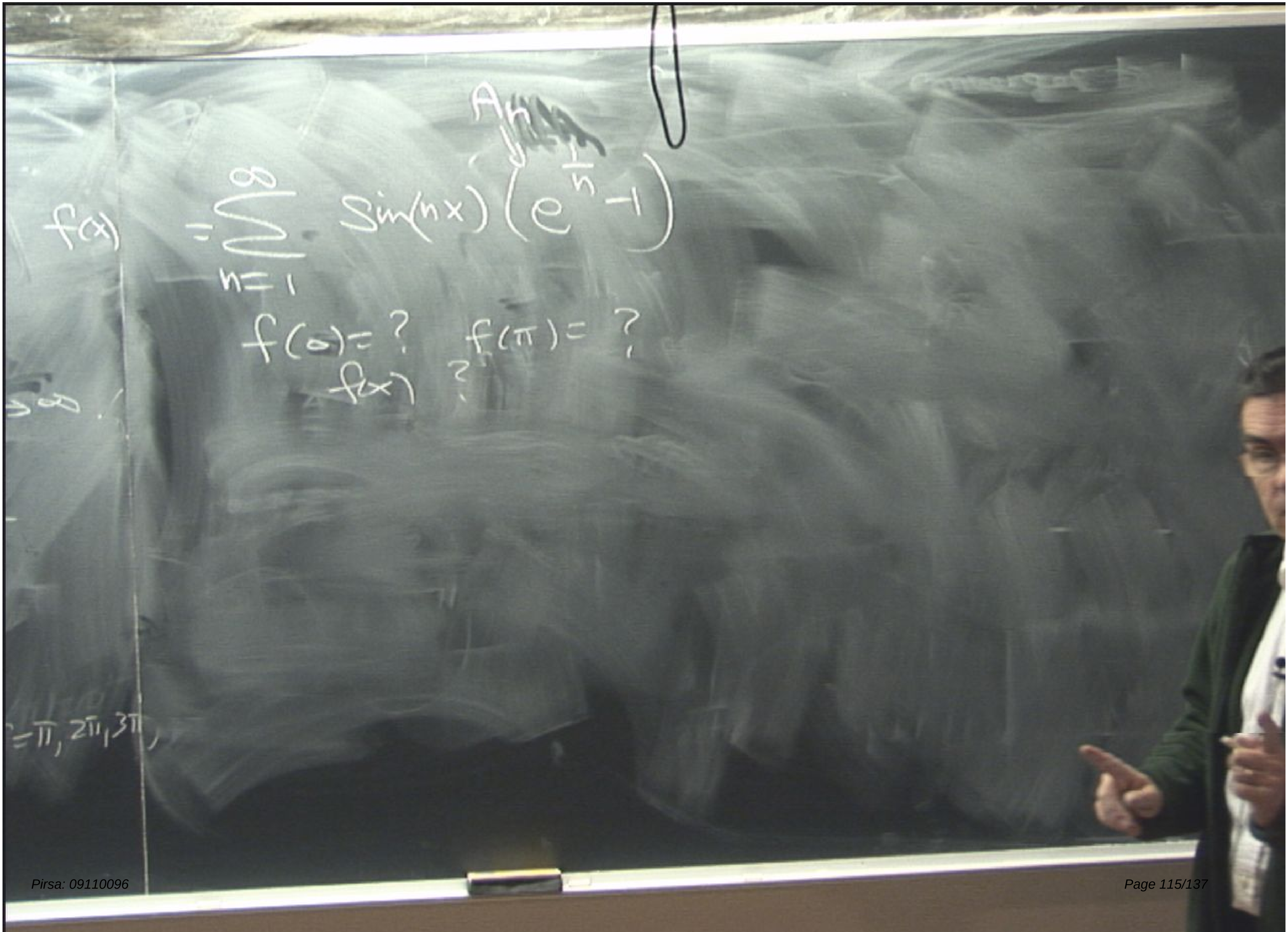


$Si(\alpha)$
$\pi/2$
π 2π 3π 4π α
$\int_0^\pi ds \frac{\sin s}{s} \approx 1.85$
$S_N(\alpha=\pi) = \frac{2}{\pi} 1.85 \approx 1.18$
$N \to \infty$
$\pi/2$
$3\pi, 4\pi \ldots$
$N \to \infty$
$\frac{1}{2N+2}$
1.18
$S_N(x)$
1.18
π
x
$x=0$
$\frac{1}{2N+2}$
π


A_n
$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$



$f(x) = \sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}} - 1\right)$
$f(0) = ?$ $f(\pi) = ?$
$= \pi, 2\pi, 3\pi,$


A_n
f(x) = Σ_{n=1}^{∞} sin(nx) (e^{1/n} − 1)
f(0) = ?   f(π) = ?
f(x) ?
= π, 2π, 3π

A_n

$$f(x) = \sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx)$$

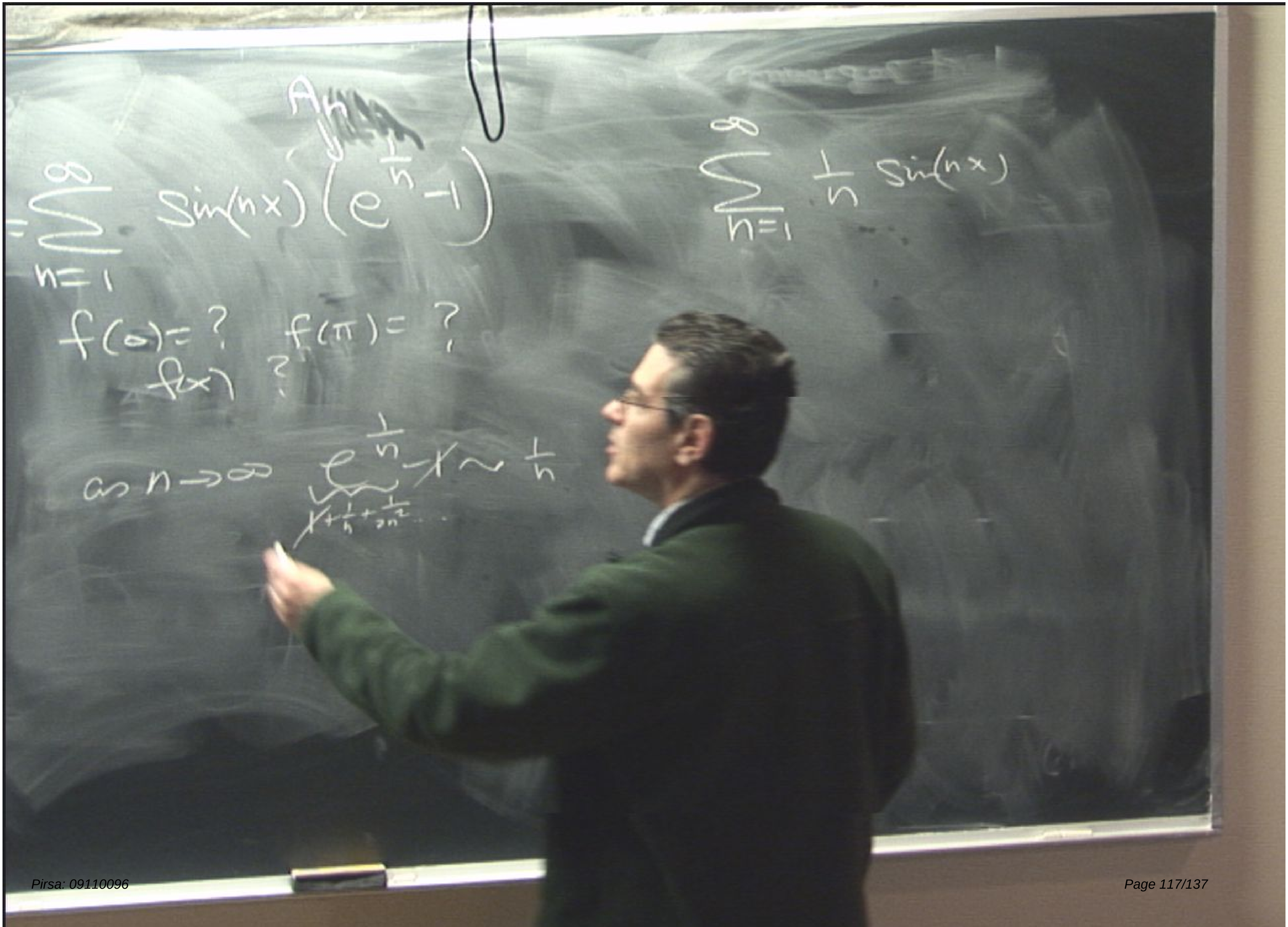
$f(0)=?$ $\quad f(\pi)=?$

$f(x)$?

as $n\to\infty$ $\quad \underbrace{e^{\frac{1}{n}}}_{\cancel{1}+\frac{1}{n}+\frac{1}{2n^2}\ldots} - \cancel{1} \sim \frac{1}{n}$

$\to\infty$

$=\pi, 2\pi, 3\pi$


∑ n=1 ∞ sin(nx) (e^(1/n) − 1)
∑ n=1 ∞ 1/n sin(nx)
f(0) = ? f(π) = ?
f(x) ?
as n → ∞ e^(1/n) − 1 ~ 1/n
1 + 1/n + 1/2n² ...

A_n

$$= \sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx)$$

$f(0)=?$ $f(\pi)=?$

$f(x)?$?

as $n \to \infty$ $\underbrace{e^{\frac{1}{n}}}_{\cancel{1}+\frac{1}{n}+\frac{1}{2n^2}\cdots} - \cancel{1} \sim \frac{1}{n}$

$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

A_n

$$\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx) = \frac{\pi - x}{2}$$

$f(0) = ?$ $f(\pi) = ?$

$f(x)$?

as $n \to \infty$ $\underbrace{e^{\frac{1}{n}}}_{\cancel{1}+\frac{1}{n}+\frac{1}{2n^2}\cdots} - \cancel{1} \sim \frac{1}{n}$

$$A_n$$

$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

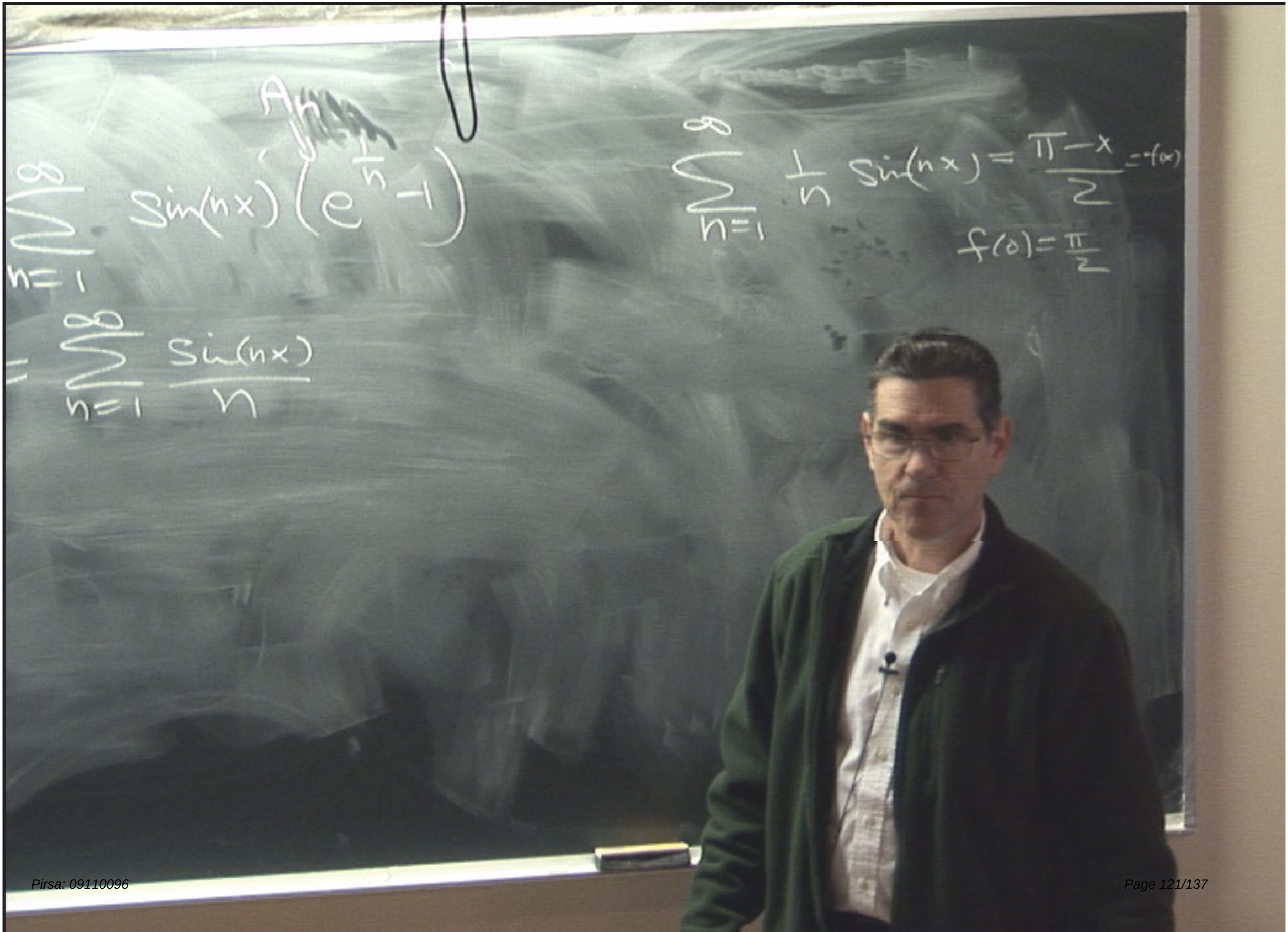
$$\underline{f(0)=?} \quad \underline{f(\pi)=?}$$

$$f(x)\ ?$$

$$\text{as } n\to\infty \quad \underbrace{e^{\frac{1}{n}}}_{\cancel{1}+\frac{1}{n}+\frac{1}{2n^2}\cdots}-\cancel{1}\sim\frac{1}{n}$$

$$\sum_{n=1}^{\infty}\frac{1}{n}\sin(nx)=\frac{\pi-x}{2}=f(x)$$

$$f(0)=\frac{\pi}{2}$$



A_n
∑_{n=1}^∞ sin(nx)(e^{-1/n} − 1)
= ∑_{n=1}^∞ sin(nx)/n
∑_{n=1}^∞ (1/n) sin(nx) = (π − x)/2 = f(x)
f(0) = π/2

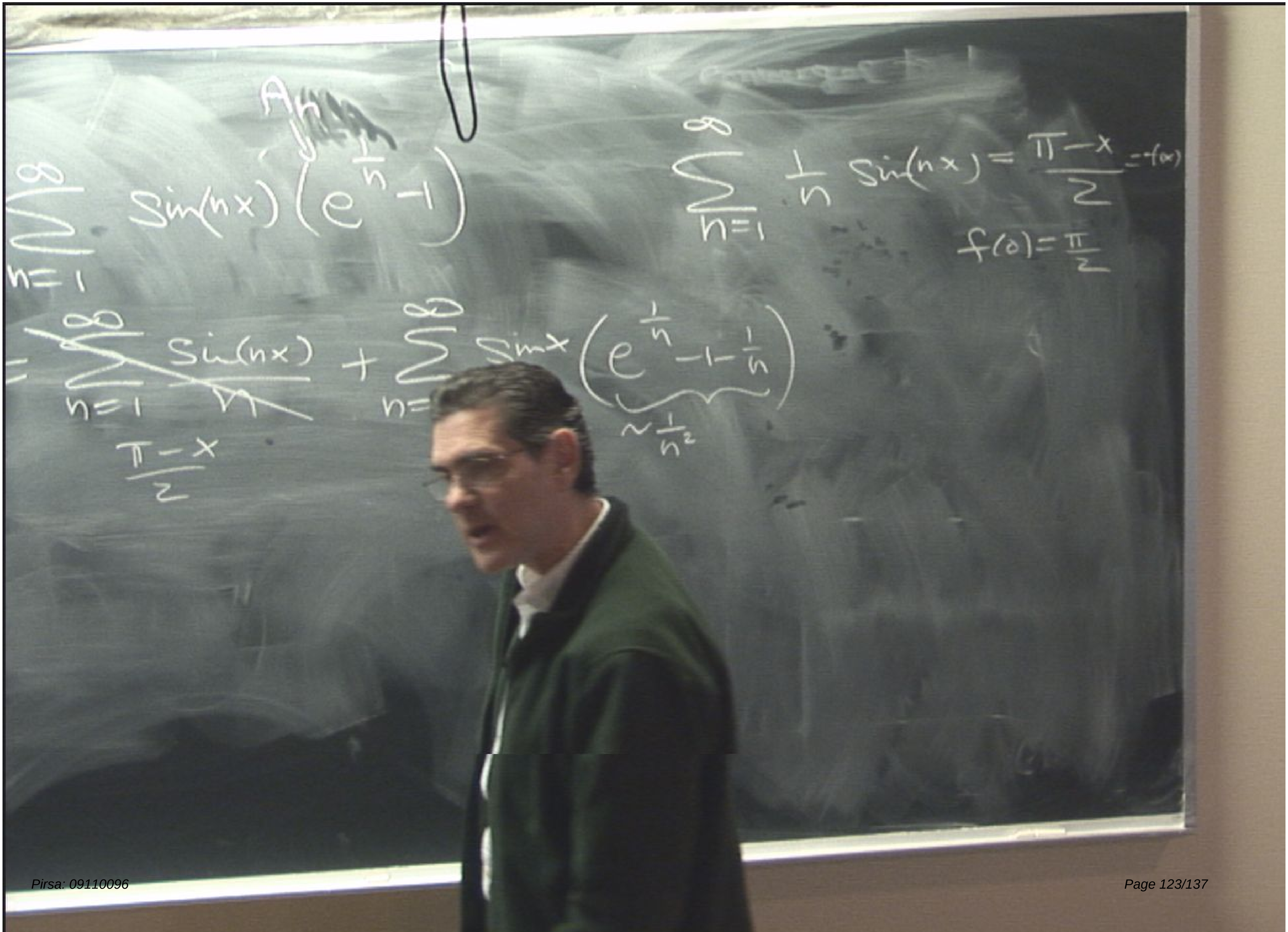
A_n

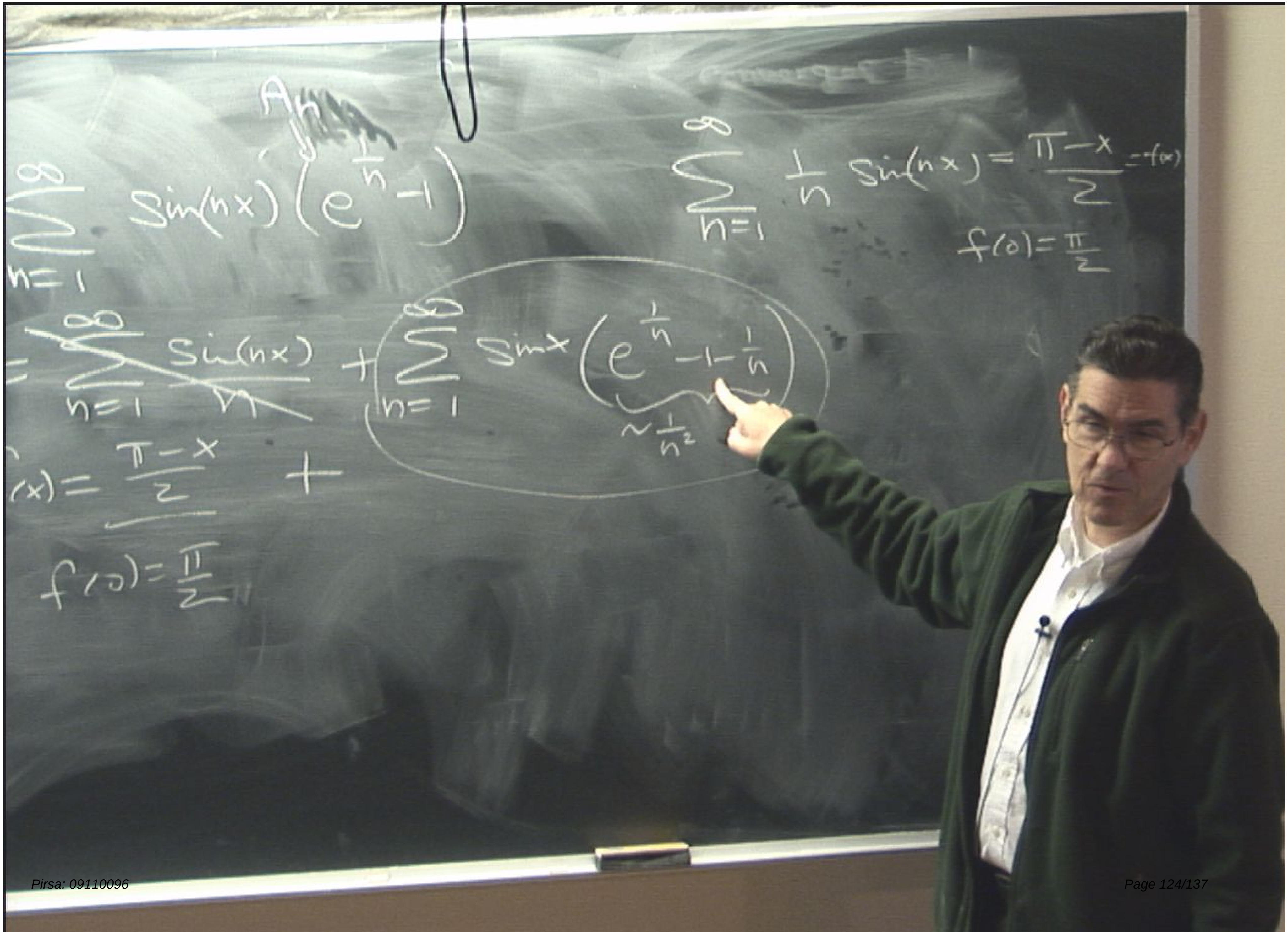
$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

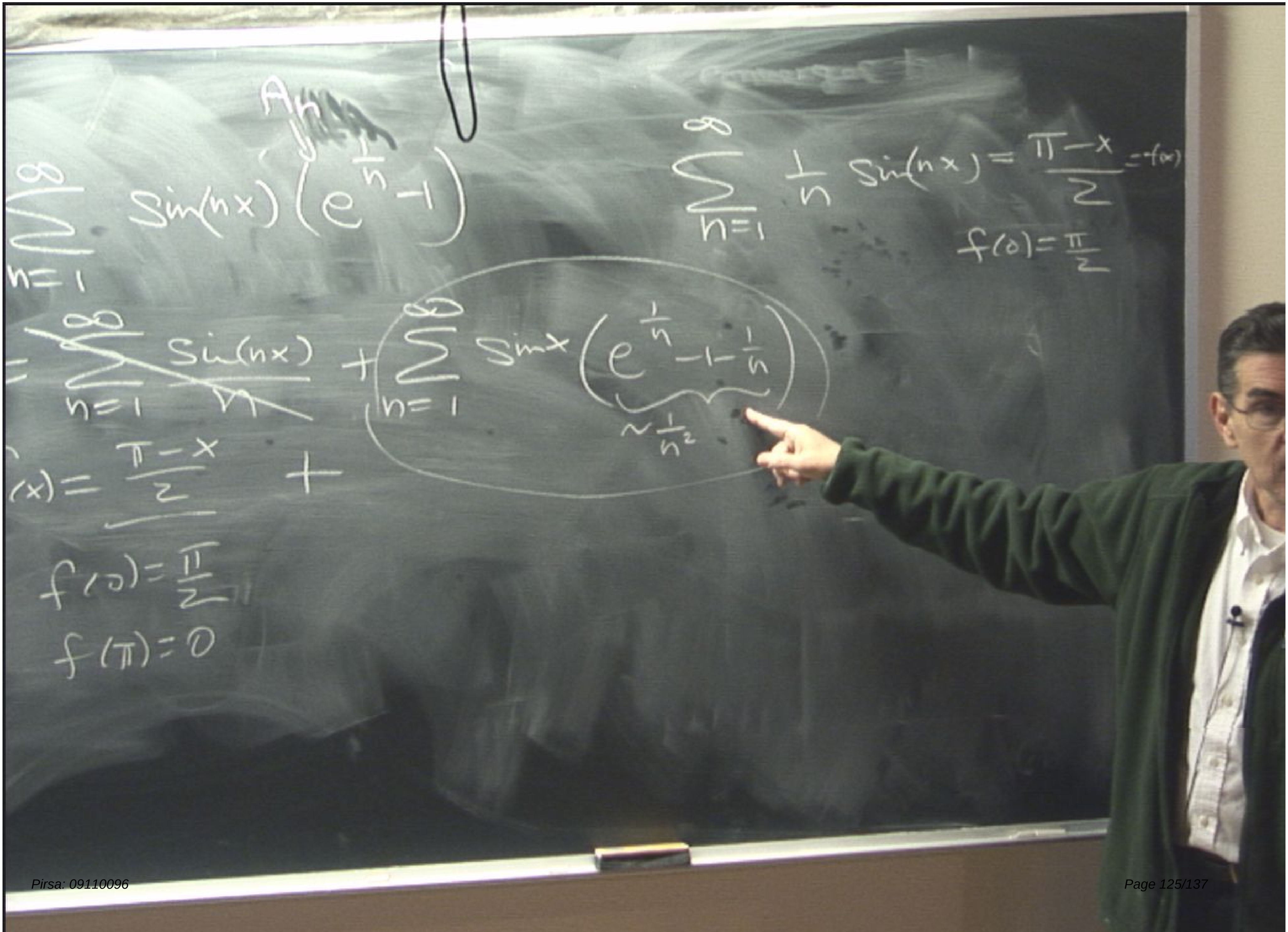
$$\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$= \sum_{n=1}^{\infty} \frac{\sin(nx)}{n} + \sum_{n=1}^{\infty} \sin nx\left(e^{\frac{1}{n}}-1-\frac{1}{n}\right)$$


$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$
$= \sum_{n=1}^{\infty} \frac{\sin(nx)}{n} + \sum_{n=1}^{\infty} \sin nx\left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim\frac{1}{n^2}}\right)$
$\frac{\pi-x}{2}$
$\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx) = \frac{\pi-x}{2} = f(x)$
$f(0)=\frac{\pi}{2}$


∑ sin(nx)(e^{1/n} − 1)
n=1
∑ (1/n) sin(nx) = (π − x)/2 = f(x)
n=1
f(0) = π/2
∑ sin(nx)/n + ∑ sinx (e^{1/n} − 1 − 1/n)
n=1
~ 1/n²
f(x) = (π − x)/2 +
f(0) = π/2


A_n
$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$
$\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx) = \frac{\pi - x}{2} = f(x)$
$f(0)=\frac{\pi}{2}$
$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin nx \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$
$f(x) = \frac{\pi - x}{2}$ +
$f(0)=\frac{\pi}{2}$
$f(\pi)=0$

A_n

$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin nx \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$f(x) = \frac{\pi - x}{2} \quad +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$A_n$$

$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin x \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$f(x) = \frac{\pi - x}{2} \quad +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

A_n

$$\sum_{n=1}^{\infty} \sin(nx)\left(e^{\frac{1}{n}}-1\right)$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin nx \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$f(x) = \frac{\pi - x}{2} \quad +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$g(x) = \sum \frac{\sin(nx)}{2n^2}$$

$$\sum_{n=1}^{\infty} \underline{\sin(nx)\left(e^{\frac{1}{n}}-1\right)}$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin x \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$f(x) = \frac{\pi - x}{2} +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$g(x) = \sum \frac{\sin(nx)}{2n^2}$$

$$f(x) = \frac{\pi - x}{2} + g(x) + \sum_{n=1}^{\infty} \sin x \left(e^{\frac{1}{n}} - 1 - \frac{1}{n} - \frac{1}{2n^2}\right)$$

$$\sum_{n=1}^{\infty} \frac{\sin(nx)\left(e^{\frac{1}{n}}-1\right)}{}$$

A_n

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin nx \left(\underbrace{e^{\frac{1}{n}} - 1 - \frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$g(x) = \sum \frac{\sin(nx)}{2n^2}$$

$$f(x) = \frac{\pi - x}{2} +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$f(x) = \frac{\pi - x}{2} + g(x) + \sum_{n=1}^{\infty} \sin x \left(\underbrace{e^{\frac{1}{n}} - 1 - \frac{1}{n} - \frac{1}{2n^2}}_{\frac{1}{n^3}}\right)$$

A_n

$$= \sum_{n=1}^{\infty} \underline{\sin(nx)\left(e^{\frac{1}{n}}-1\right)}$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin x \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$f(x) = \underline{\frac{\pi - x}{2}} +$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$f(x) = \frac{\pi - x}{2} + g(x) + \sum_{n=1}^{\infty} \sin x \left(\underbrace{e^{\frac{1}{n}}-1-\frac{1}{n}-\frac{1}{2n^2}}_{\frac{1}{n^3}}\right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$g(x) = \sum \frac{\sin(nx)}{2n^2}$$

A_n

$$= \sum_{n=1}^{\infty} \underline{\sin(nx)\left(e^{\frac{1}{n}}-1\right)}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) = \frac{\pi - x}{2} = f(x)$$

$$f(0) = \frac{\pi}{2}$$

$$= \cancel{\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}} + \sum_{n=1}^{\infty} \sin nx \left(\underbrace{e^{\frac{1}{n}} - 1 - \frac{1}{n}}_{\sim \frac{1}{n^2}}\right)$$

$$g(x) = \sum \frac{\sin(nx)}{2n \ldots}$$

$$f(x) = \frac{\pi - x}{2} +$$

$$\sum_{n=0} (\alpha a_n + \beta b_n) = \alpha \sum a_n + \beta \sum b_n$$

$$f(0) = \frac{\pi}{2}$$

$$f(\pi) = 0$$

$$f(x) = \frac{\pi - x}{2} + g(x) + \sum_{n=1}^{\infty}$$

$u_t = u_{xx}$

$u(0,t) = u_{xx}(0,t)$

$u_{xx} = u_{xxxx}$

$u_{xx} = u_{xxxxxx} = 0$

$= 0$

$u_t = u_{xx}$

$$u(x,t) = \sum_{n=1}^{\infty} \boxed{A_n} \sin(nx)\, e^{-n^2 t}$$

$u(0,t) = 0$

t

$$f(x) = u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

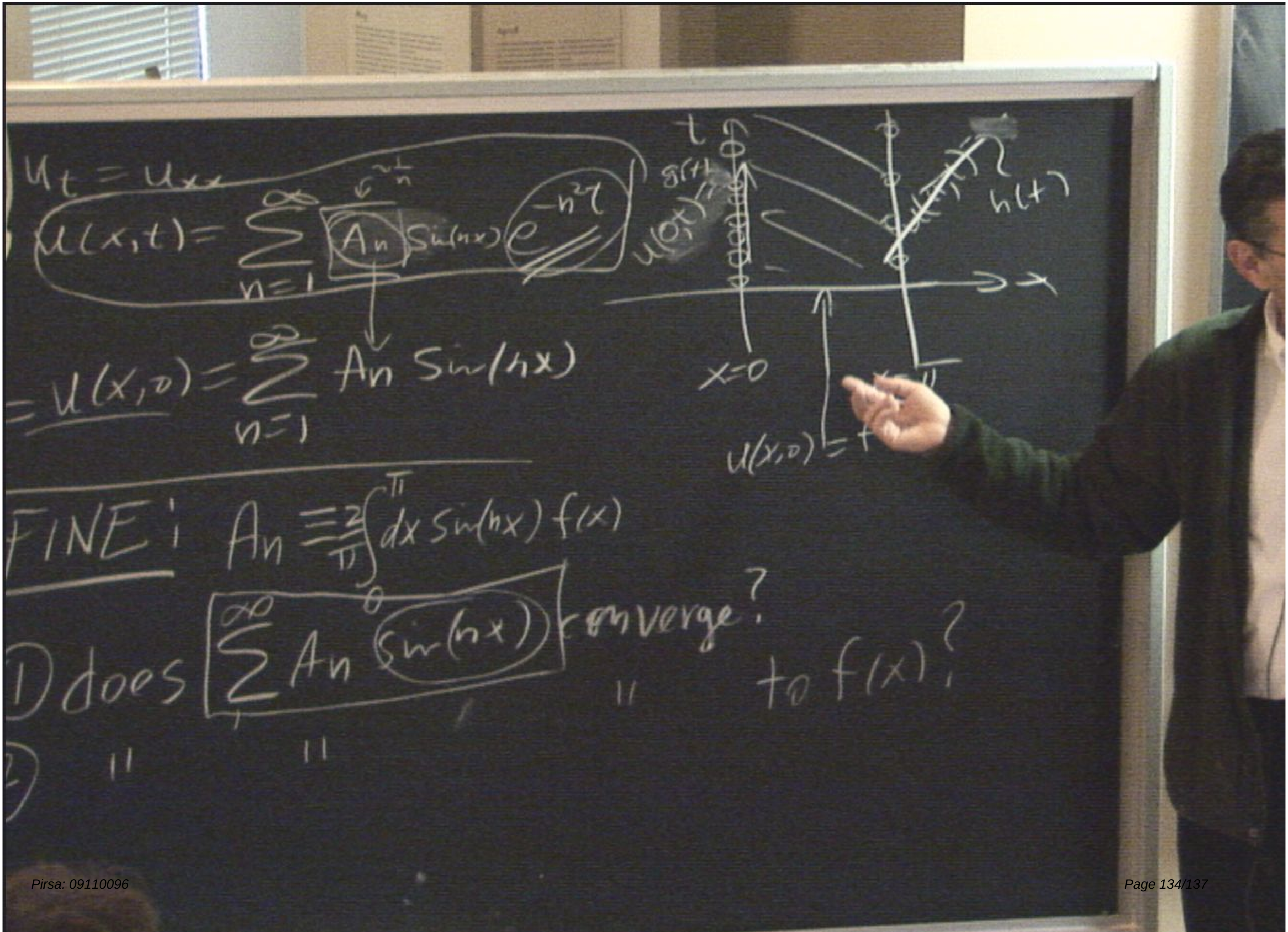
$x = 0$

$u(x,0) = f$

DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$

① does $\boxed{\sum_{n=1}^{\infty} A_n \sin(nx)}$ converge? " to $f($

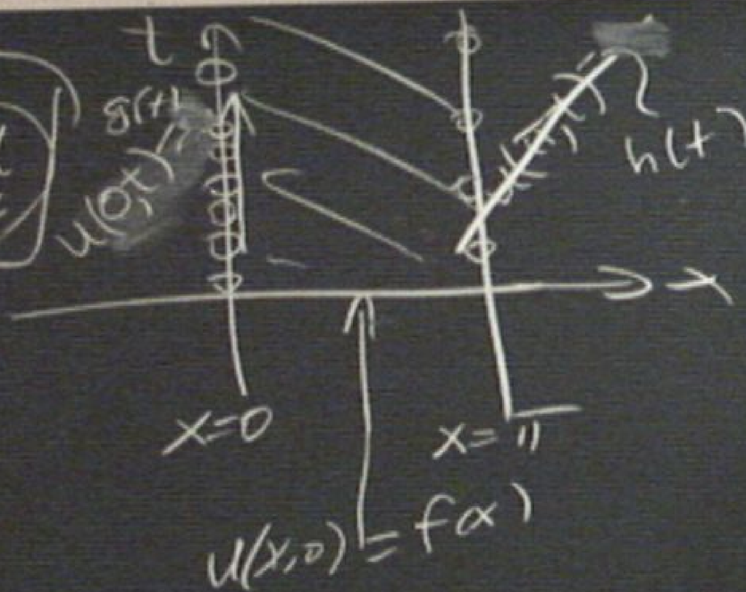
② " "


u_t = u_xx
u(x,t) = Σ_{n=1}^{∞} A_n Sin(nx) e^{-n²t}
u(x,0) = Σ_{n=1}^{∞} An Sin(nx)
FINE: An ≡ 2/π ∫_0^π dx Sin(nx) f(x)
1) does Σ An Sin(nx) converge?
to f(x)?
2)
g(t)
u(0,t)
h(t)
x=0
u(x,0) = f

$u_t = u_{xx}$

$$u(x,t) = \sum_{n=1}^{\infty} \boxed{\overset{\sim \frac{1}{n}}{A_n}} \sin(nx)\, e^{-n^2 t}$$

$$= u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$$

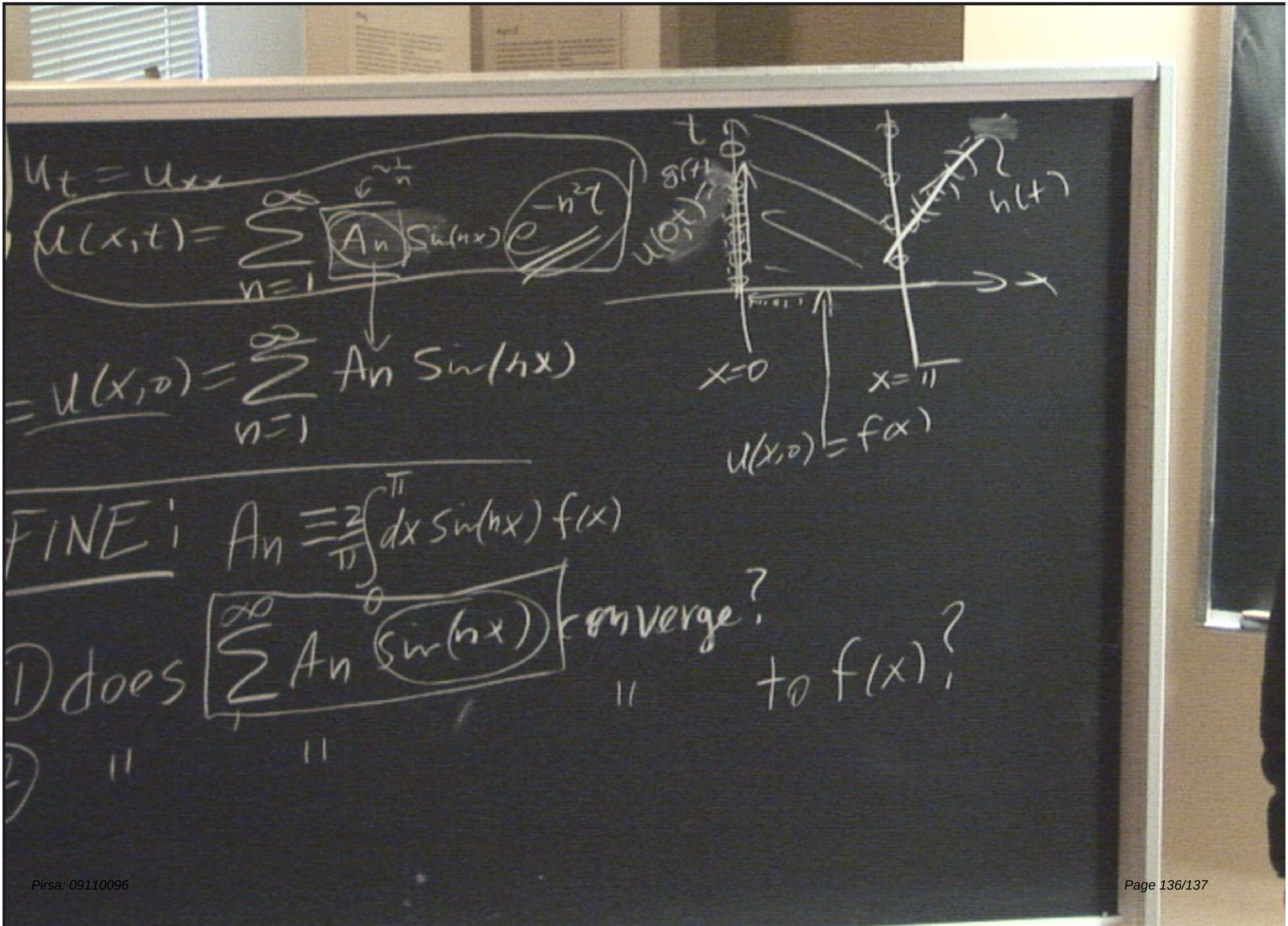


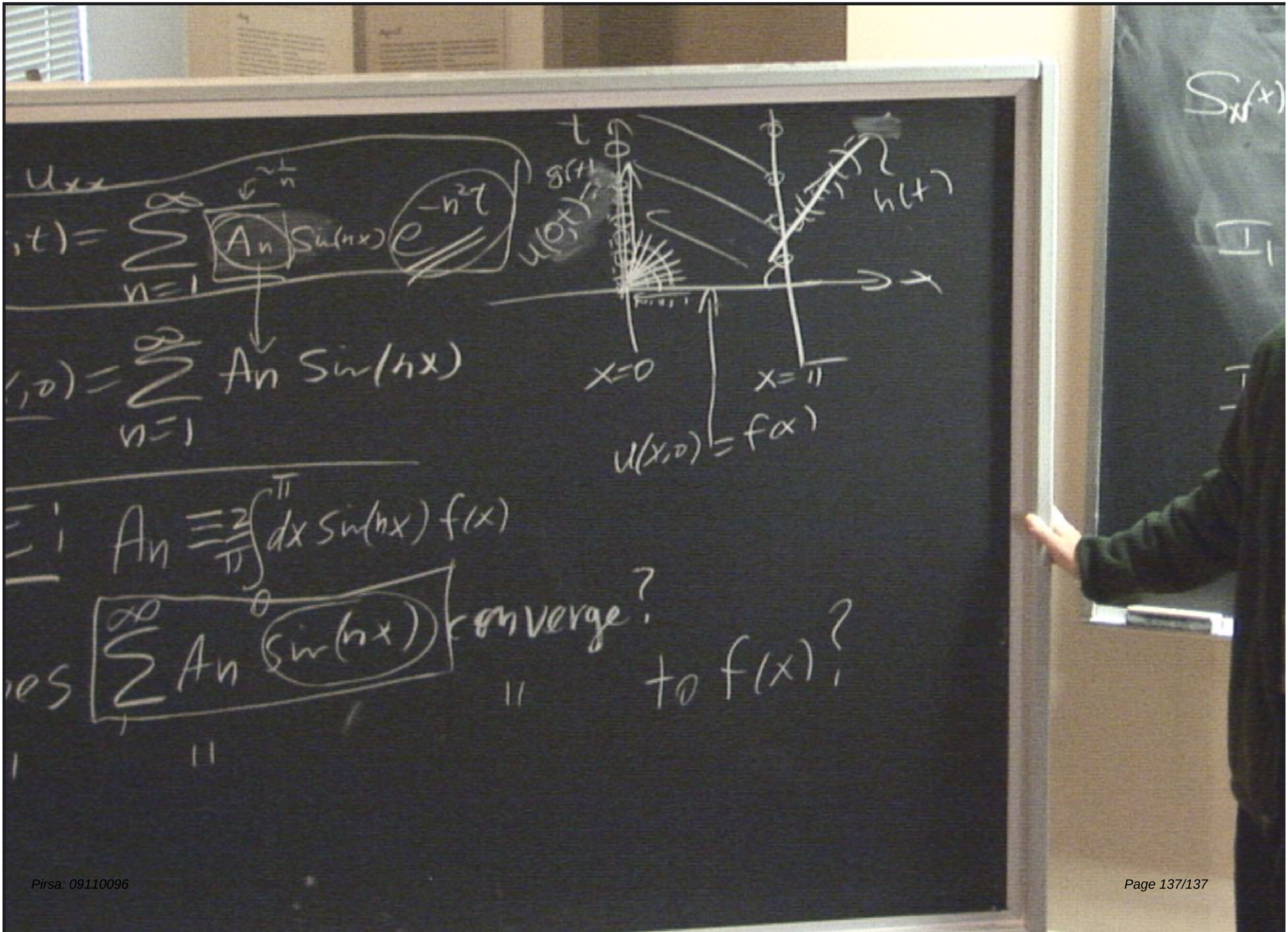
DEFINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx\, \sin(nx)\, f(x)$

(1) does $\boxed{\sum_{n=1}^{\infty} A_n \sin(nx)}$ converge?

" " to $f(x)$?

(2) " "


$u_t = u_{xx}$
$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(nx) e^{-n^2 t}$
$= u(x,0) = \sum_{n=1}^{\infty} A_n \sin(nx)$
FINE: $A_n \equiv \frac{2}{\pi}\int_0^{\pi} dx \sin(nx) f(x)$
1) does $\sum_{1}^{\infty} A_n \sin(nx)$ converge?
to f(x)?
g(t)
u(0,t)
h(t)
x=0
x=π
u(x,0) = f(x)


u_xx
,t) = Σ An Sin(nx) e^{-n²t}
n=1
(,0) = Σ An Sin(nx)
n=1
An ≡ 2/π ∫ dx Sin(nx) f(x)
es Σ An Sin(nx) converge?
to f(x)?
t
g(t)
u(0,t)
h(t)
x
x=0
x=π
u(x,0) = f(x)
S_N(x)