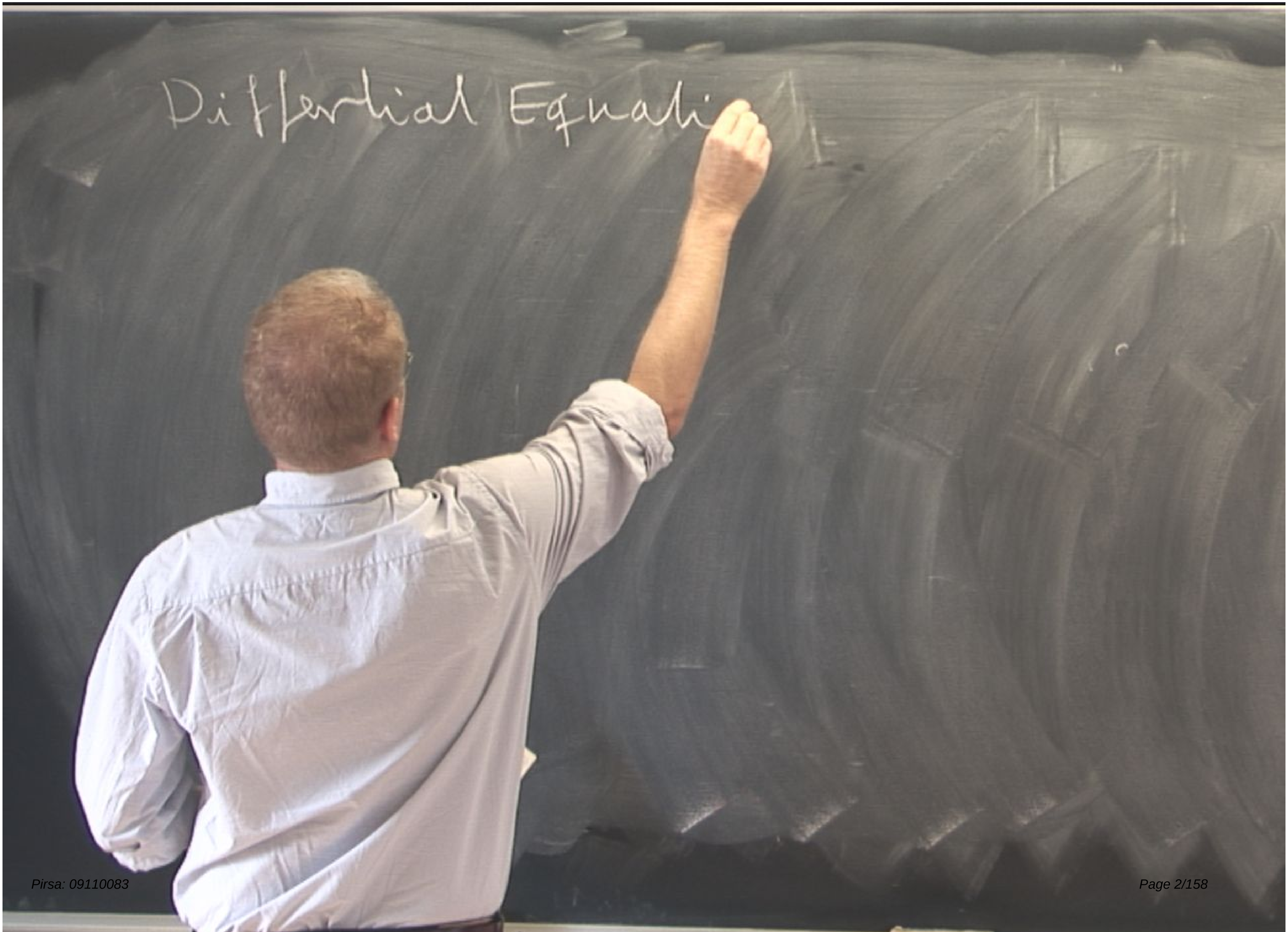
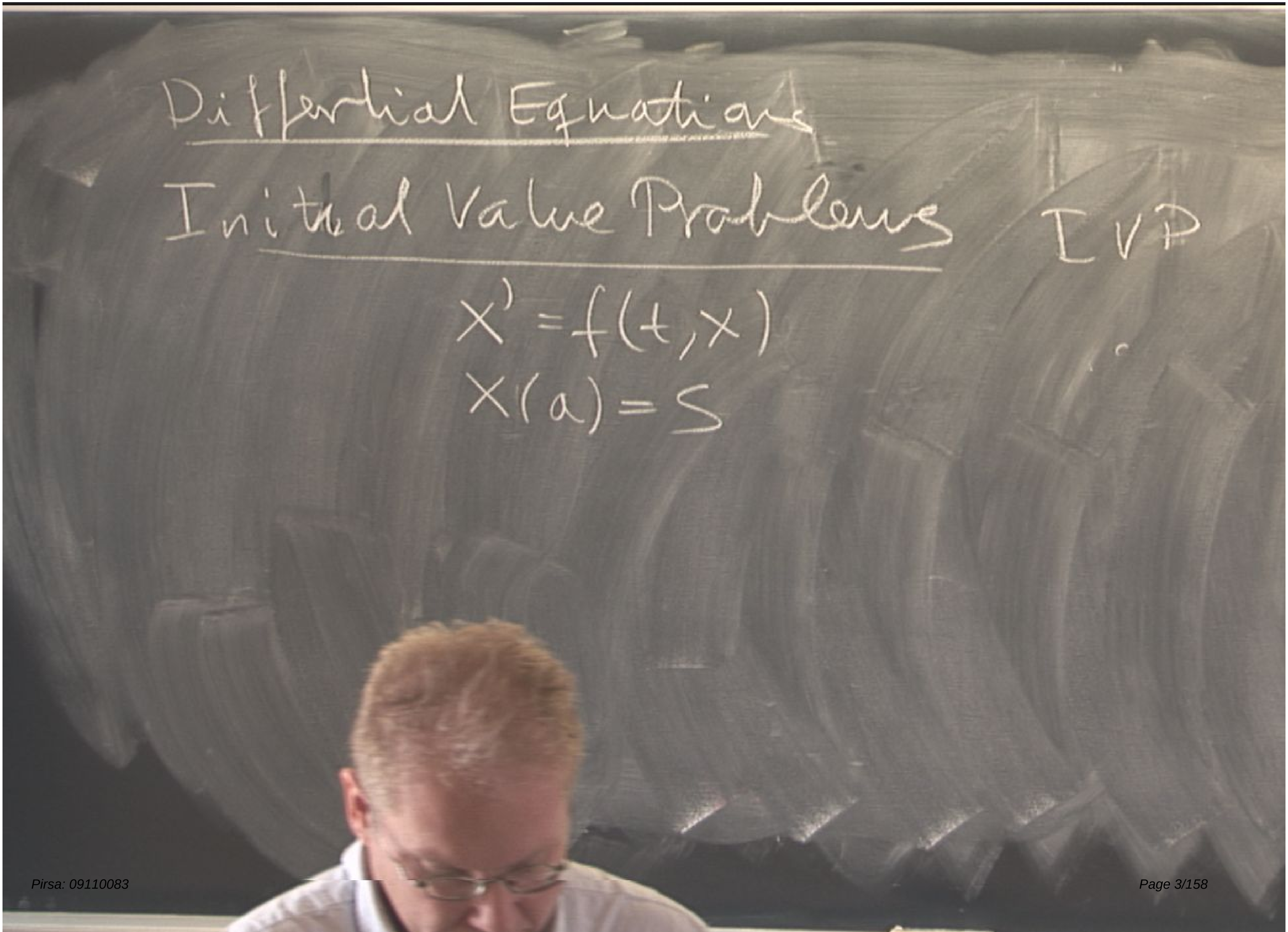
Title: Scientific Computation (PHYS 608) - Lecture 8

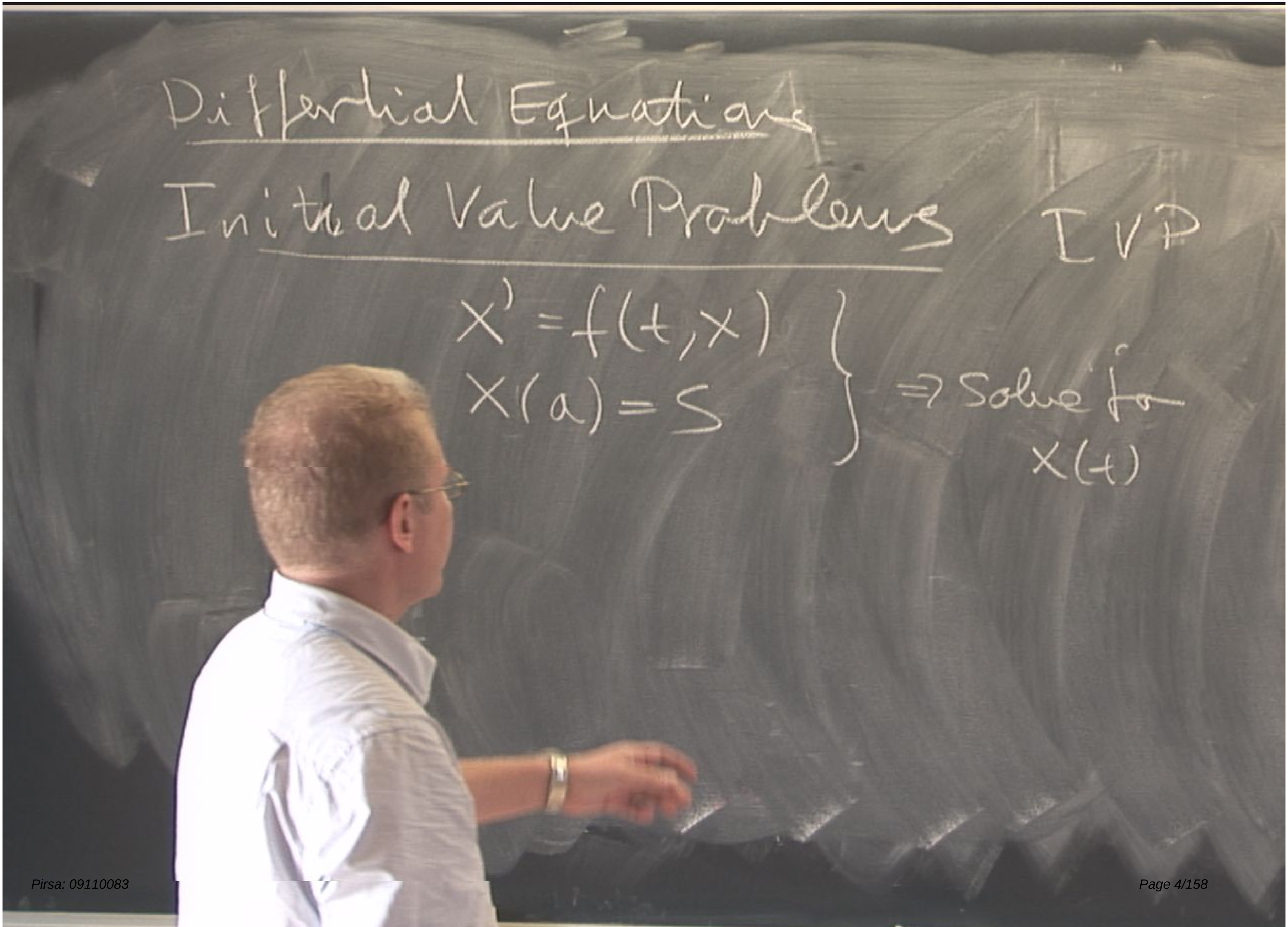
Date: Nov 04, 2009 10:30 AM

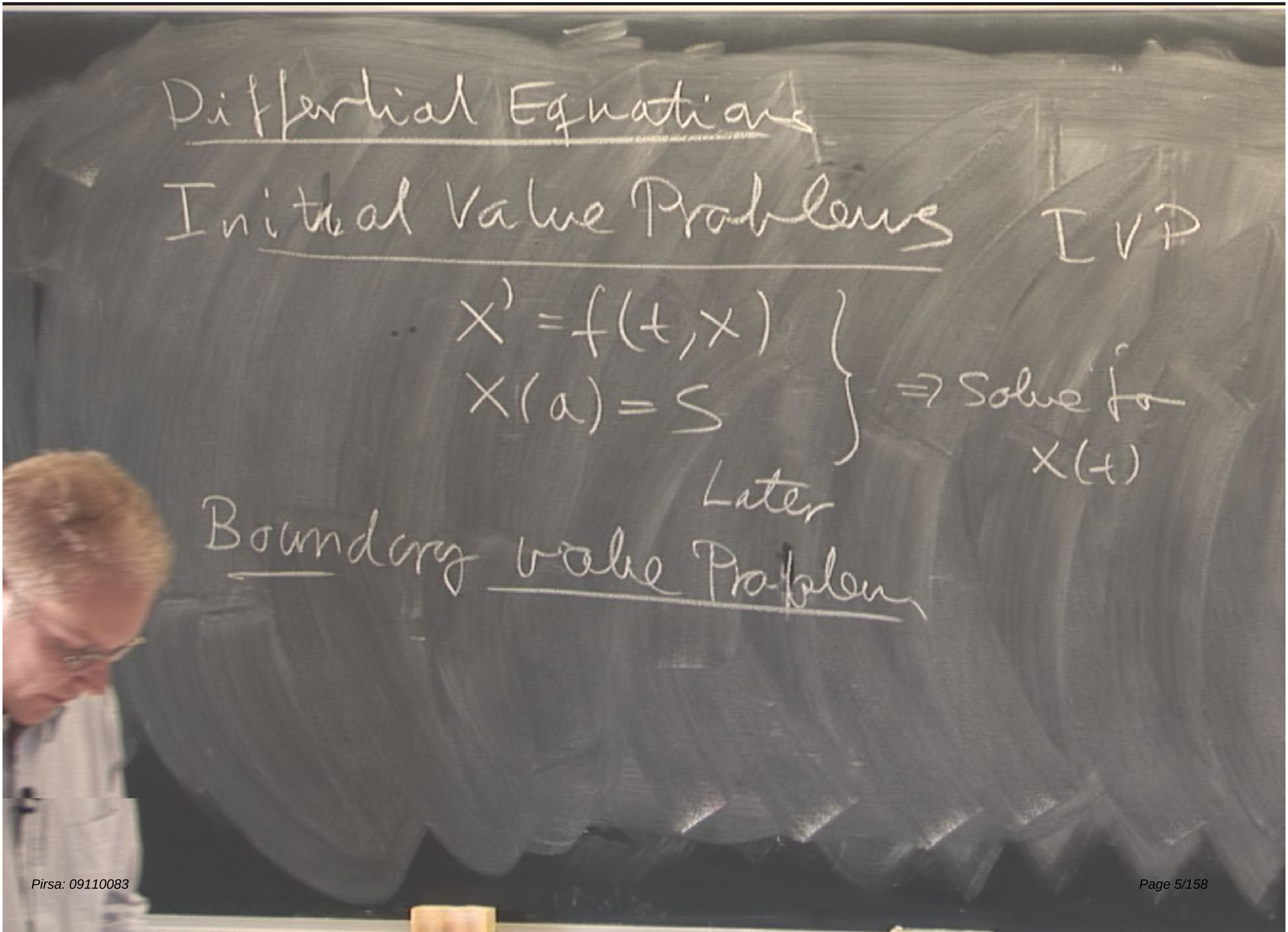
URL: http://pirsa.org/09110083

Abstract:


Differtial Equati


Differtial Equations
Initial Value Problems
IVP
$x' = f(t,x)$
$x(a) = s$


Differential Equations
Initial Value Problems
IVP
x' = f(t, x)
x(a) = s
⇒ Solve for
x(t)


Differential Equations
Initial Value Problems IVP
x' = f(t,x)
x(a) = s
⇒ Solve for x(t)
Later
Boundary value Problem

Differential Equations

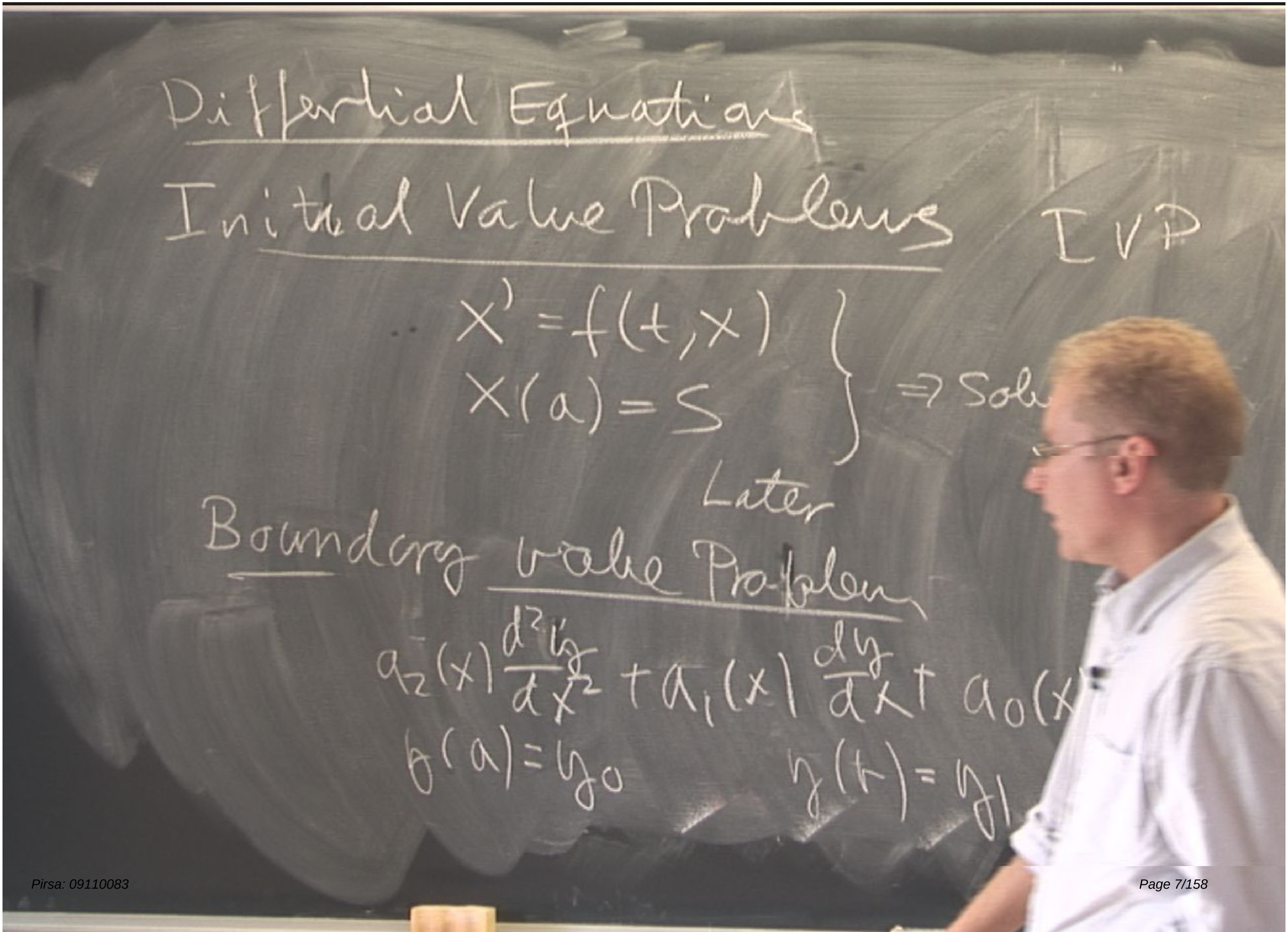
Initial Value Problems   IVP

$$\left.\begin{array}{l} x' = f(t,x) \\ x(a) = s \end{array}\right\} \Rightarrow \text{Solve for } x(t)$$

Later

Boundary value Problem

$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y =$$


Differential Equations
Initial Value Problems IVP
$x' = f(t,x)$
$x(a) = s$
⇒ Sol
Later
Boundary value Problem
$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)$
$y(a) = y_0$
$y(b) = y_1$

Differential Equations

Initial Value Problems IVP

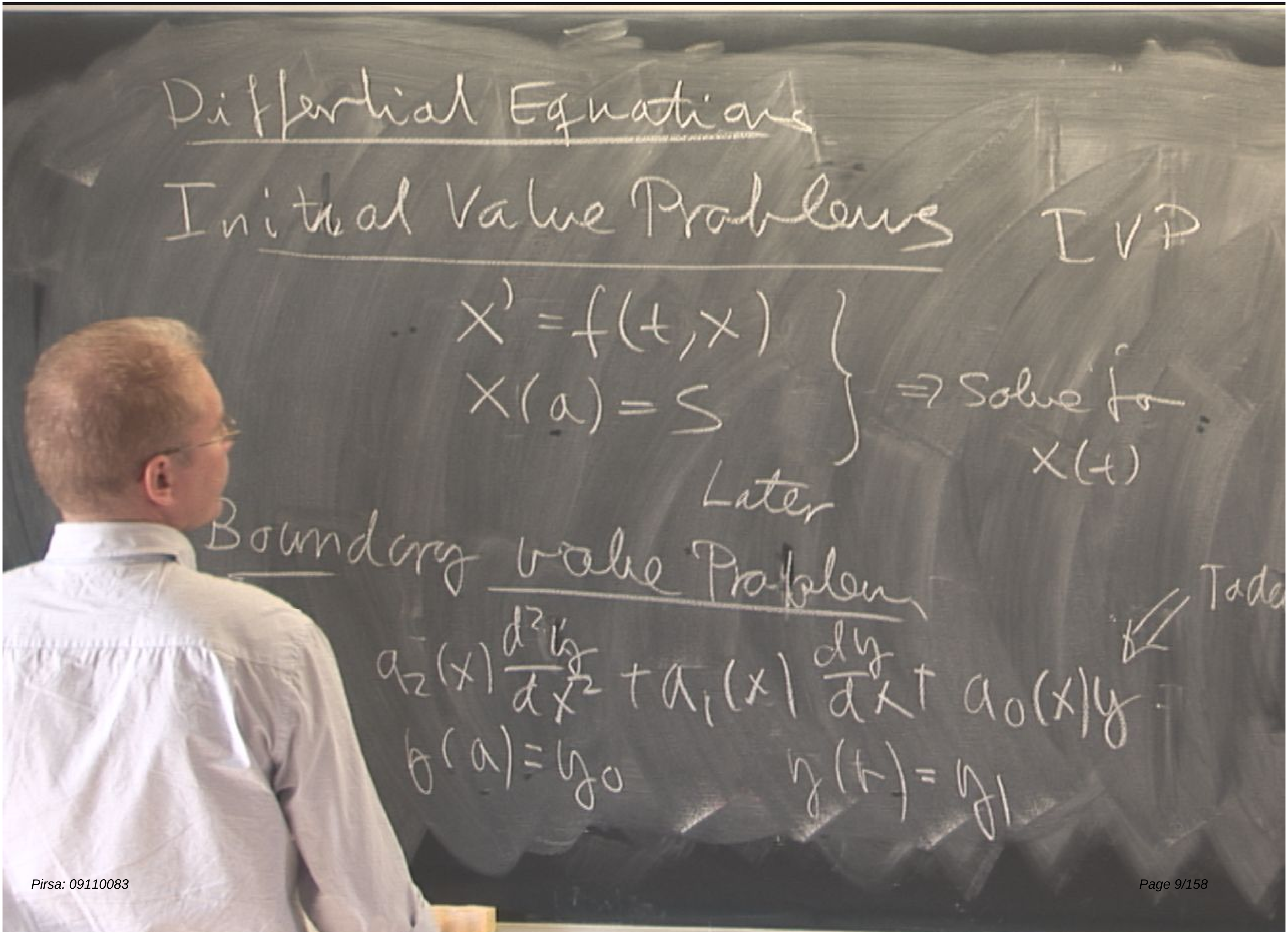
$$\left.\begin{aligned} x' &= f(t,x) \\ x(a) &= s \end{aligned}\right\} \Rightarrow \text{Solve for } x(t)$$

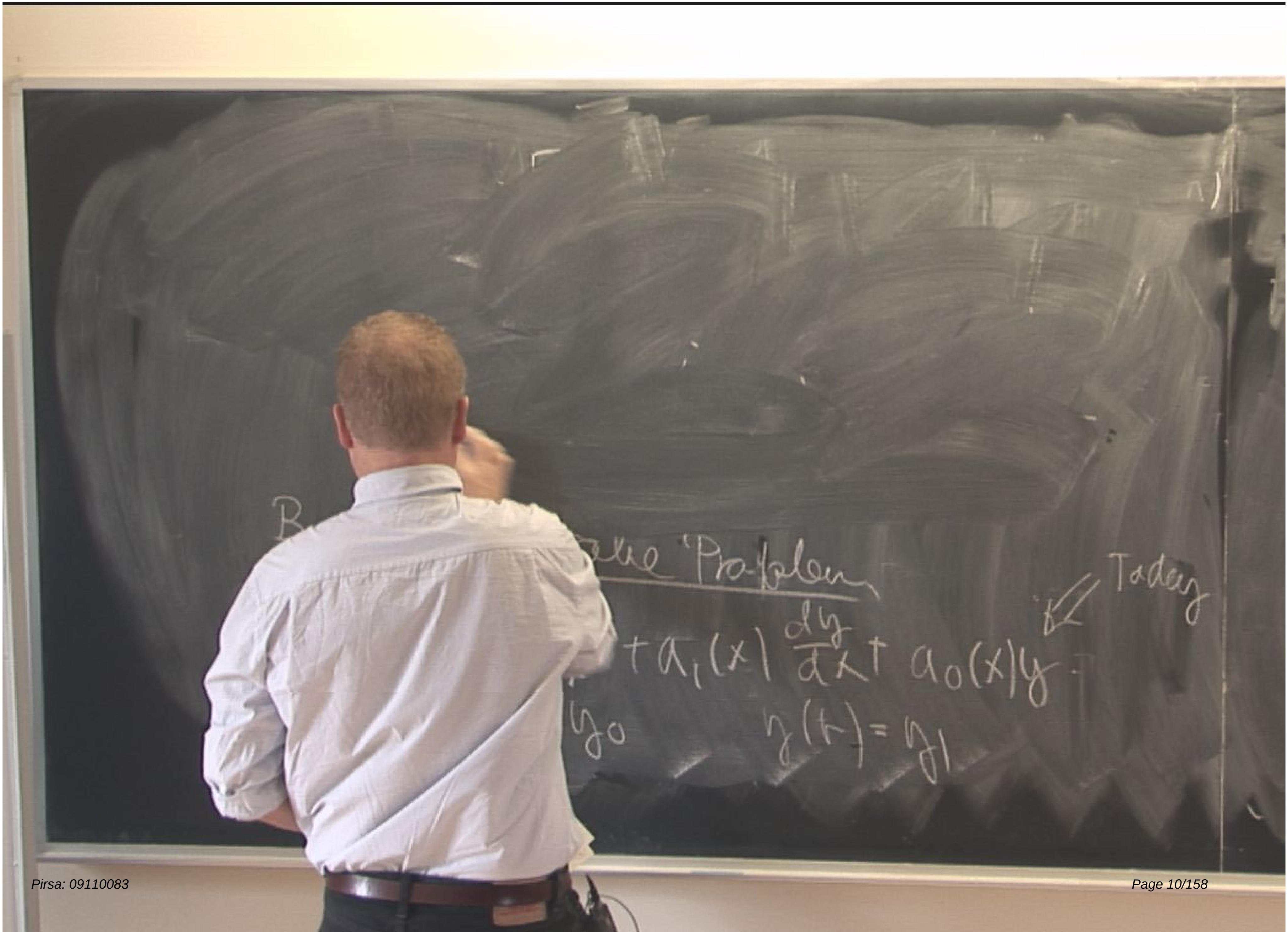
Later

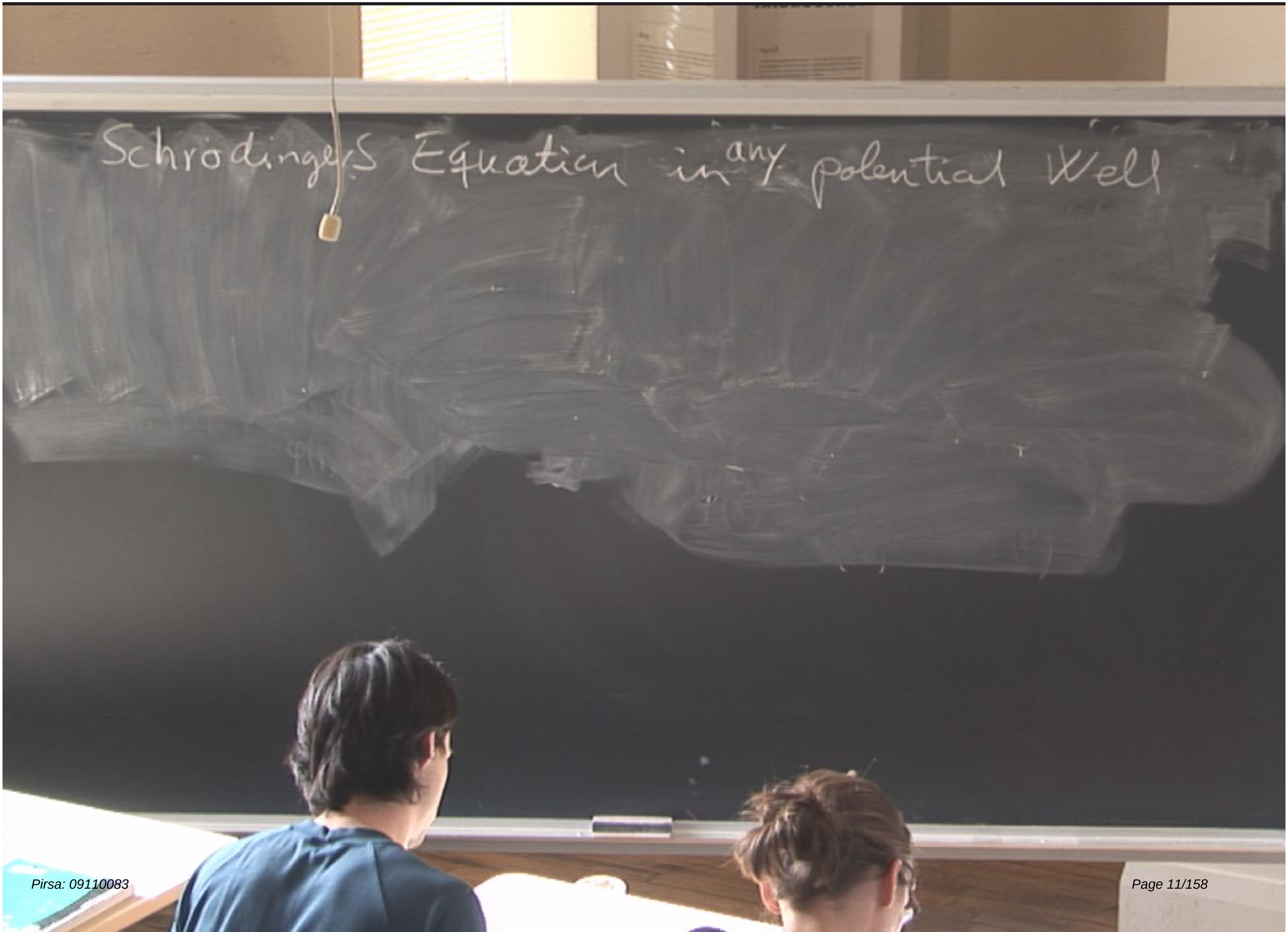
Boundary Value Problem

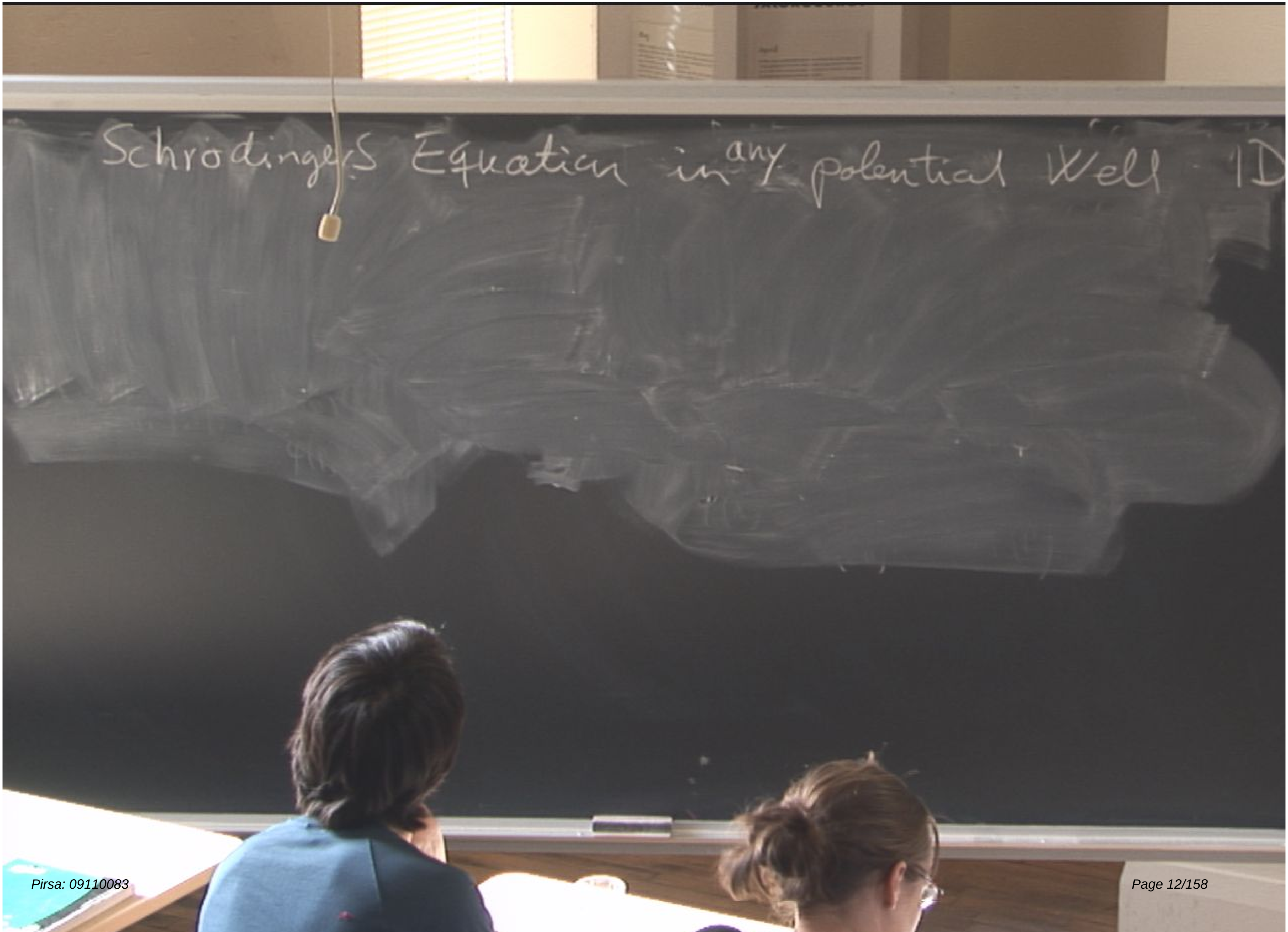
$$a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x)y =$$

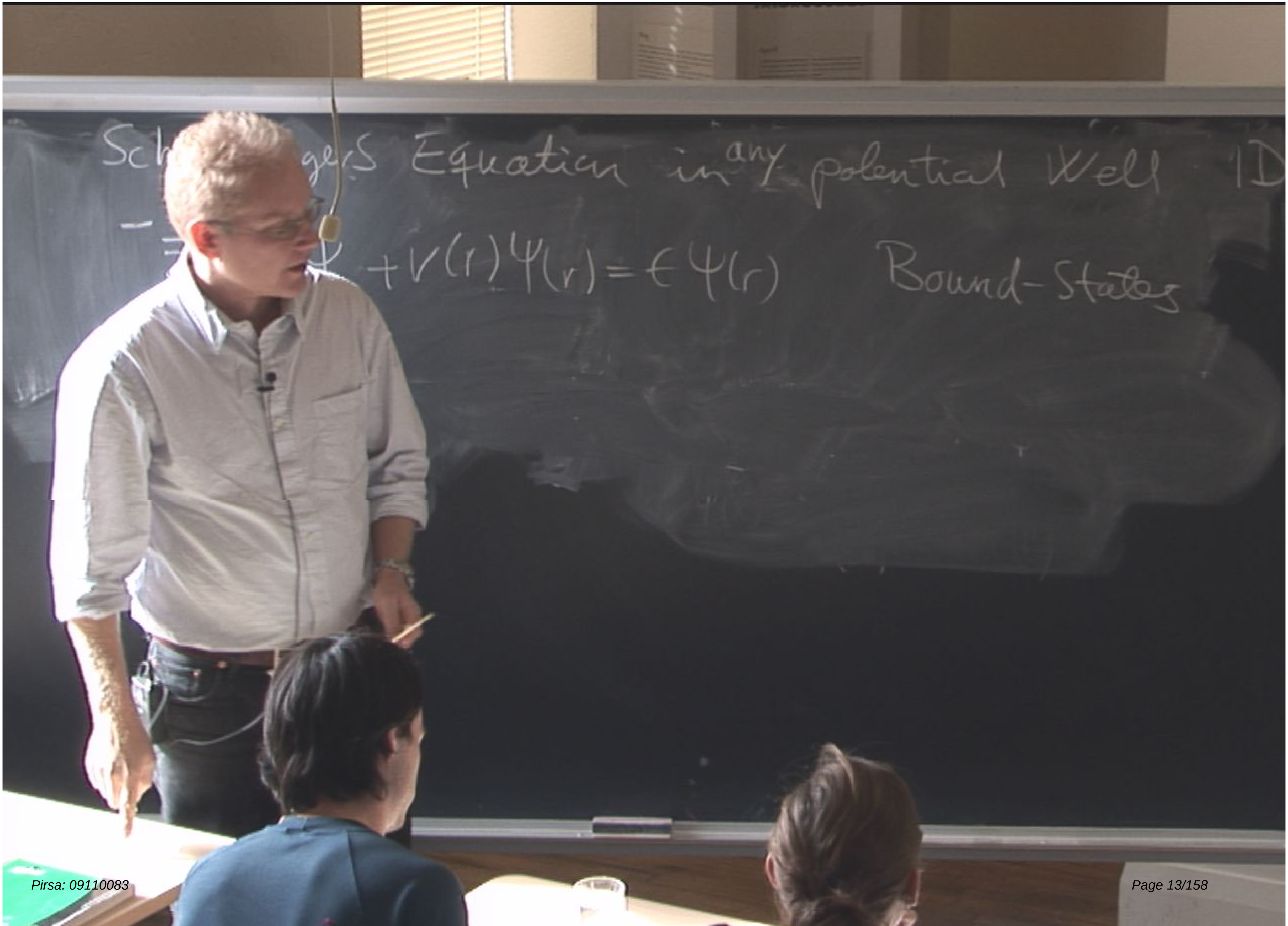
$$y(a) = y_0 \qquad y(b) = y_1$$

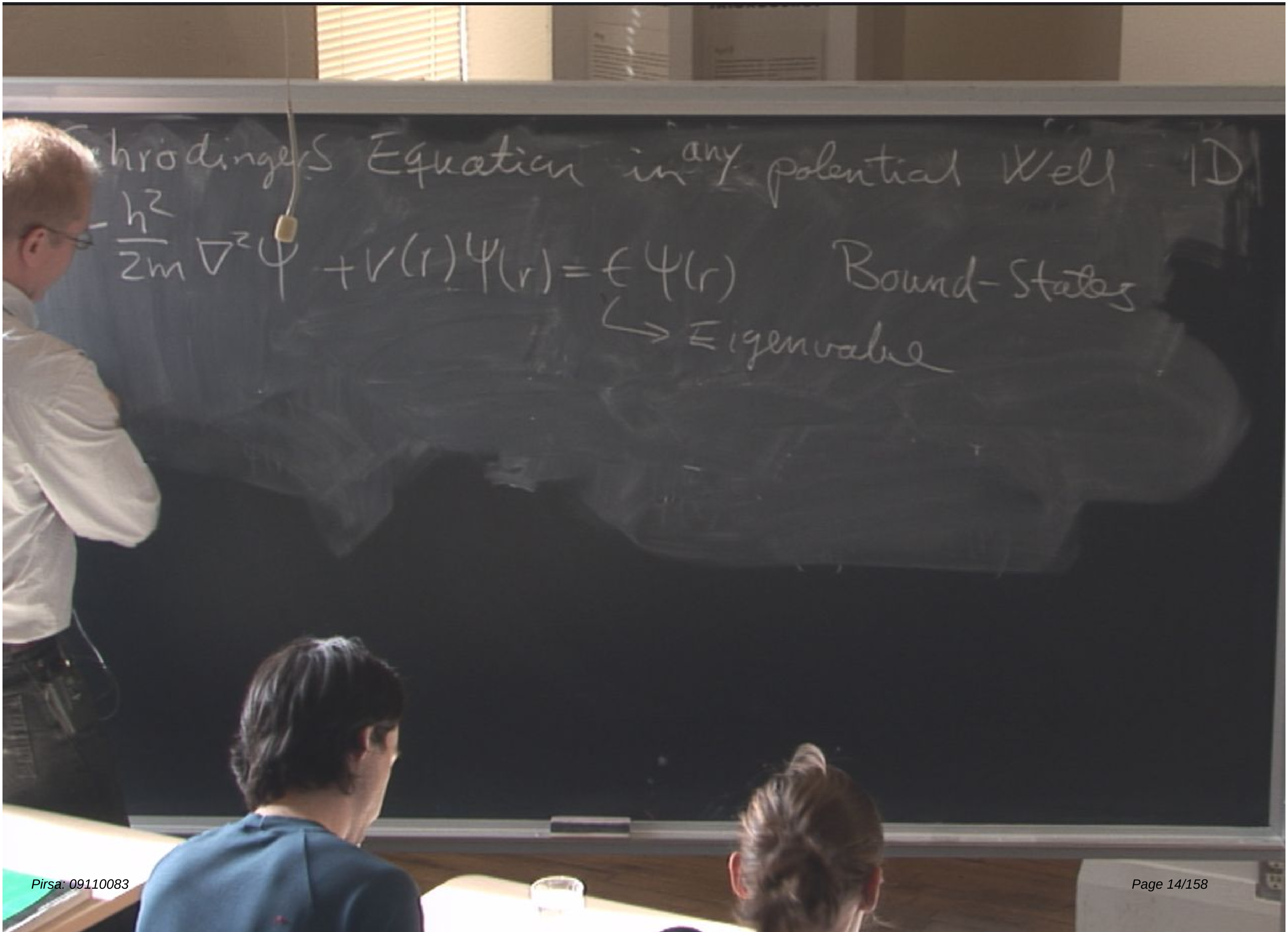

Differential Equations
Initial Value Problems
IVP
x' = f(t, x)
x(a) = s
⇒ Solve for x(t)
Later
Boundary Value Problem
a_2(x) d^2y/dx^2 + a_1(x) dy/dx + a_0(x)y
y(a) = y_0
y(b) = y_1
Today

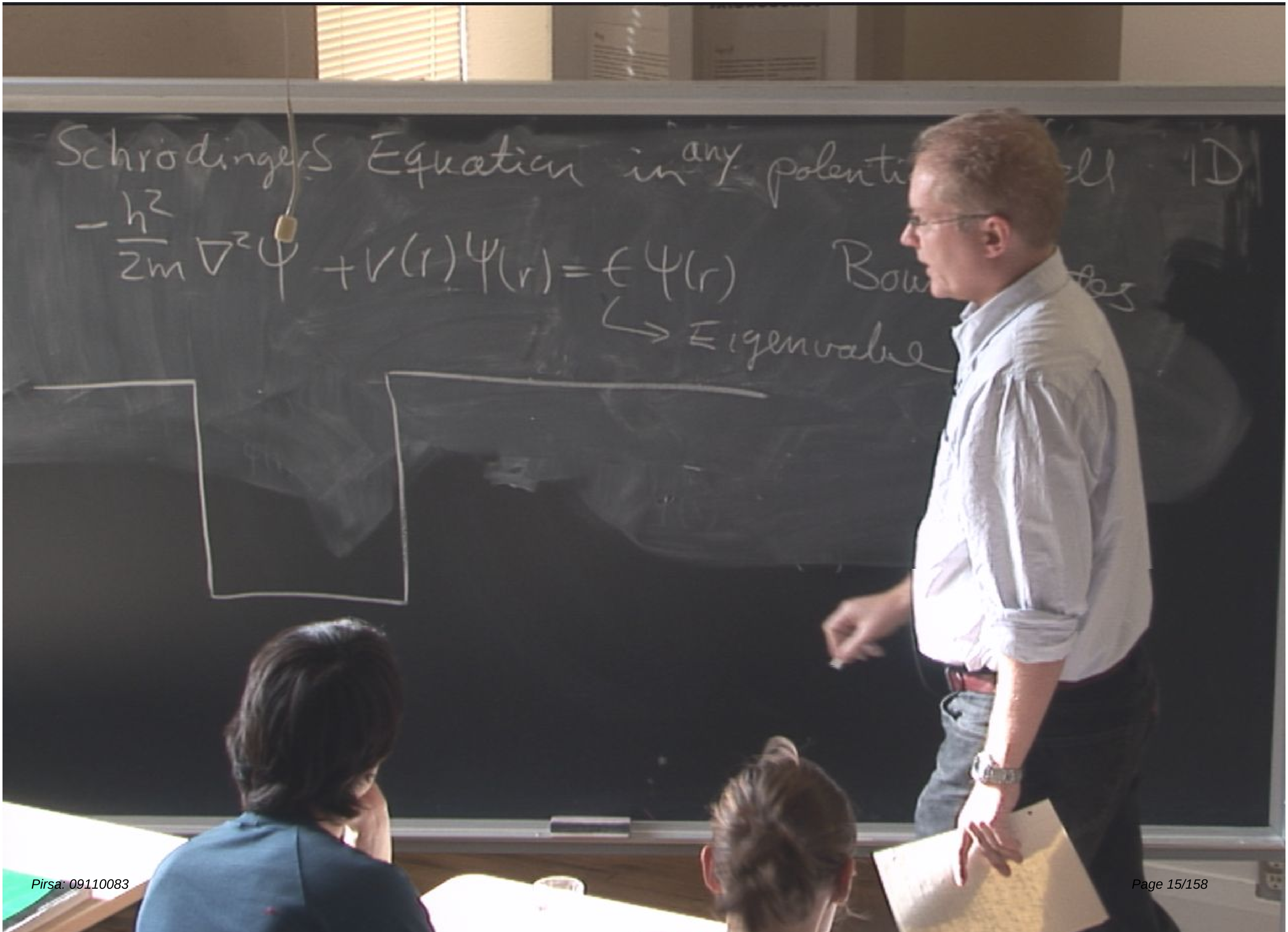

Today

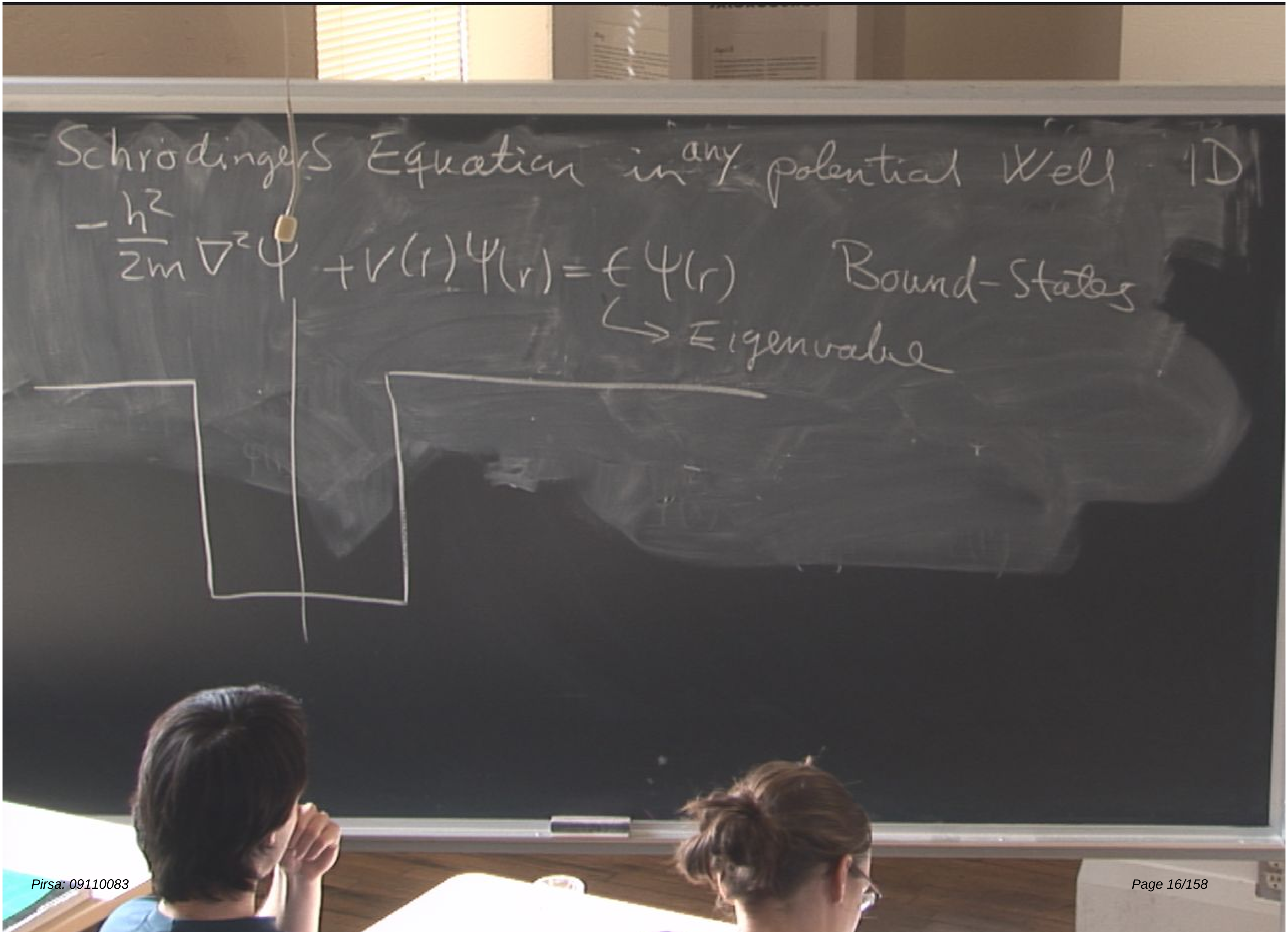

Schrödinger's Equation in any potential Well

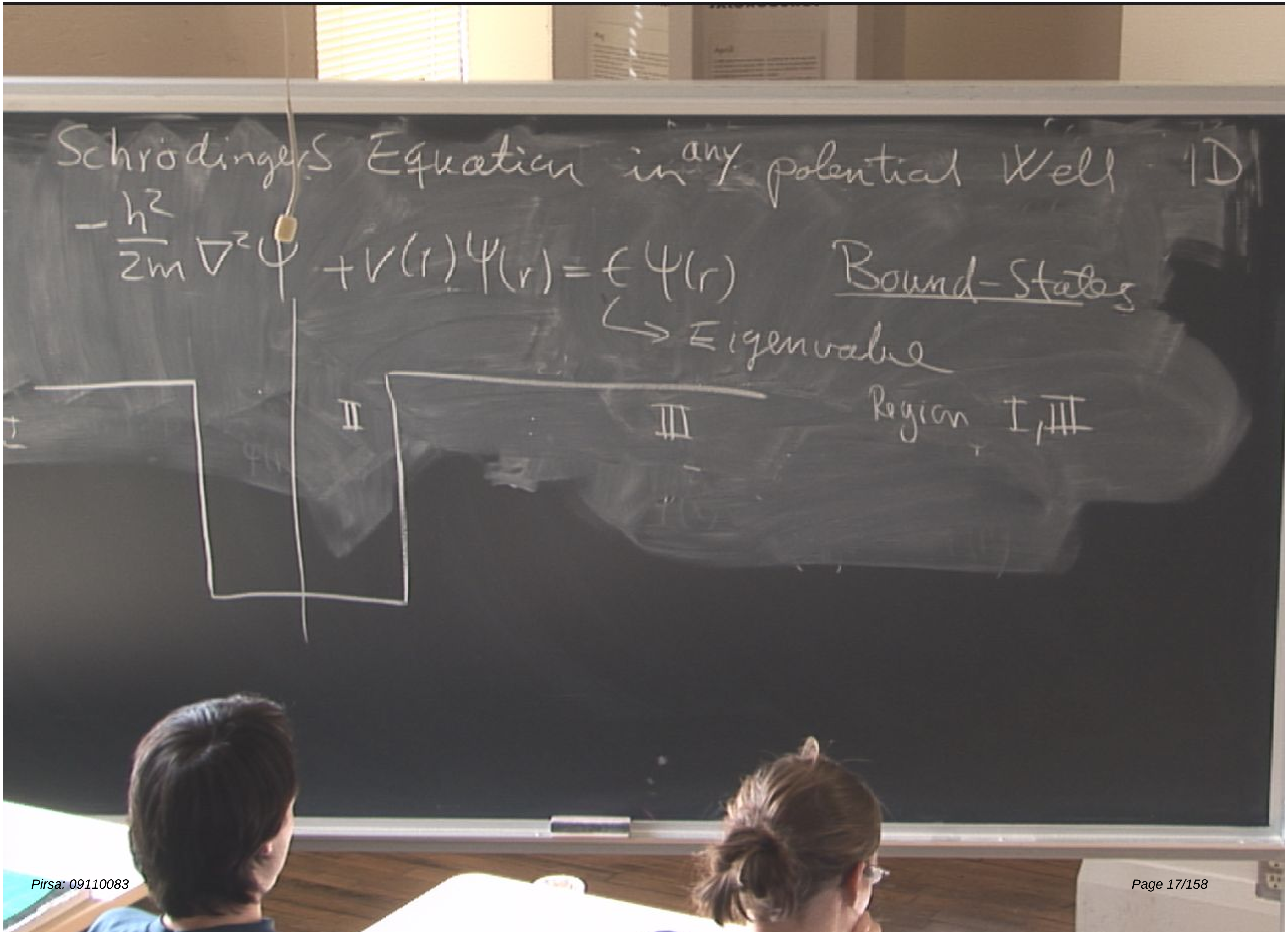

Schrödinger's Equation in any potential Well 1D

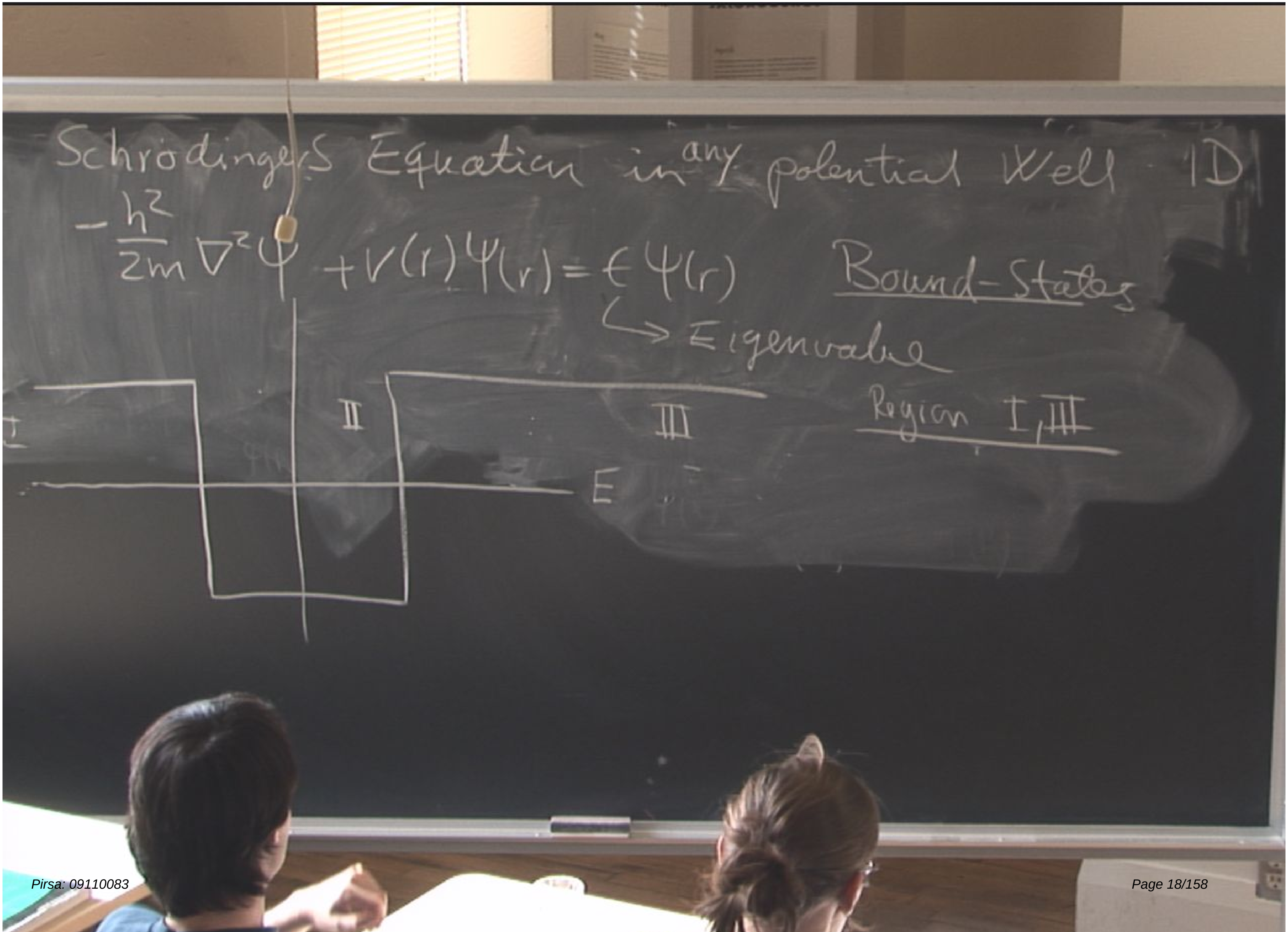

Equation in any potential Well 1D
+V(r)Ψ(r) = ϵΨ(r)
Bound-States

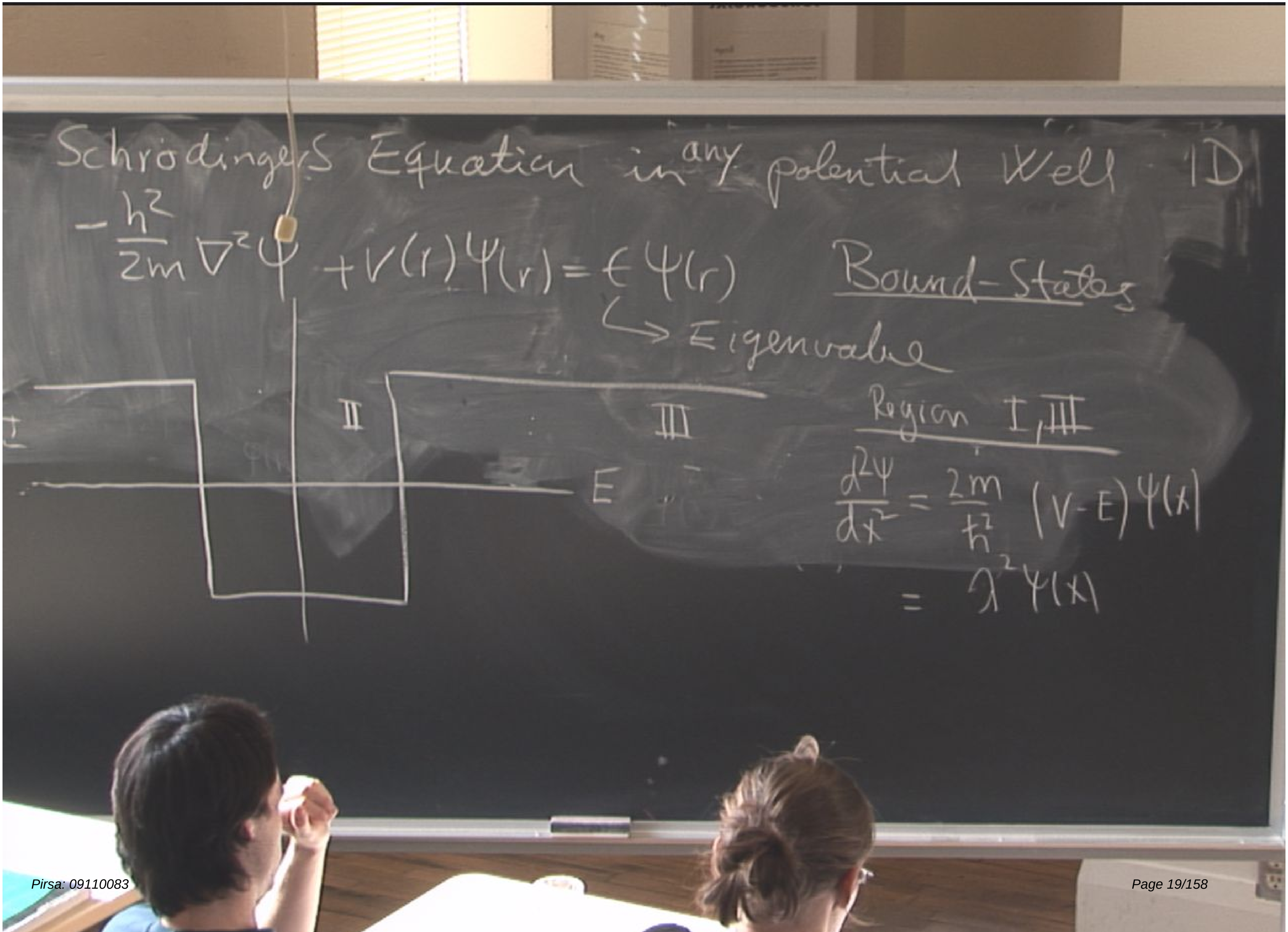

Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Bound-States
Eigenvalue

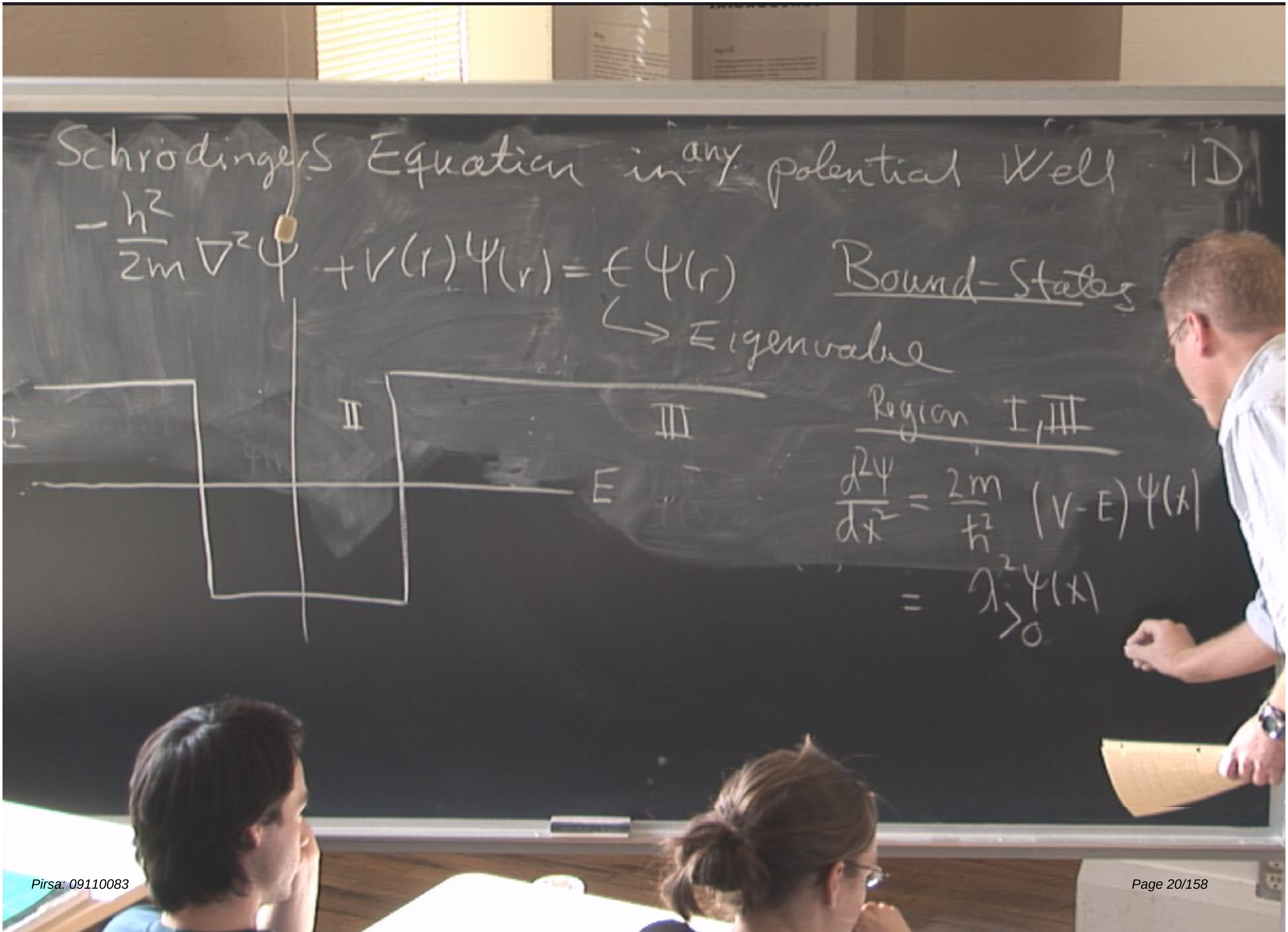

Schrödingers Equation in any potenti ell 1D
$-\frac{h^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Eigenvalue

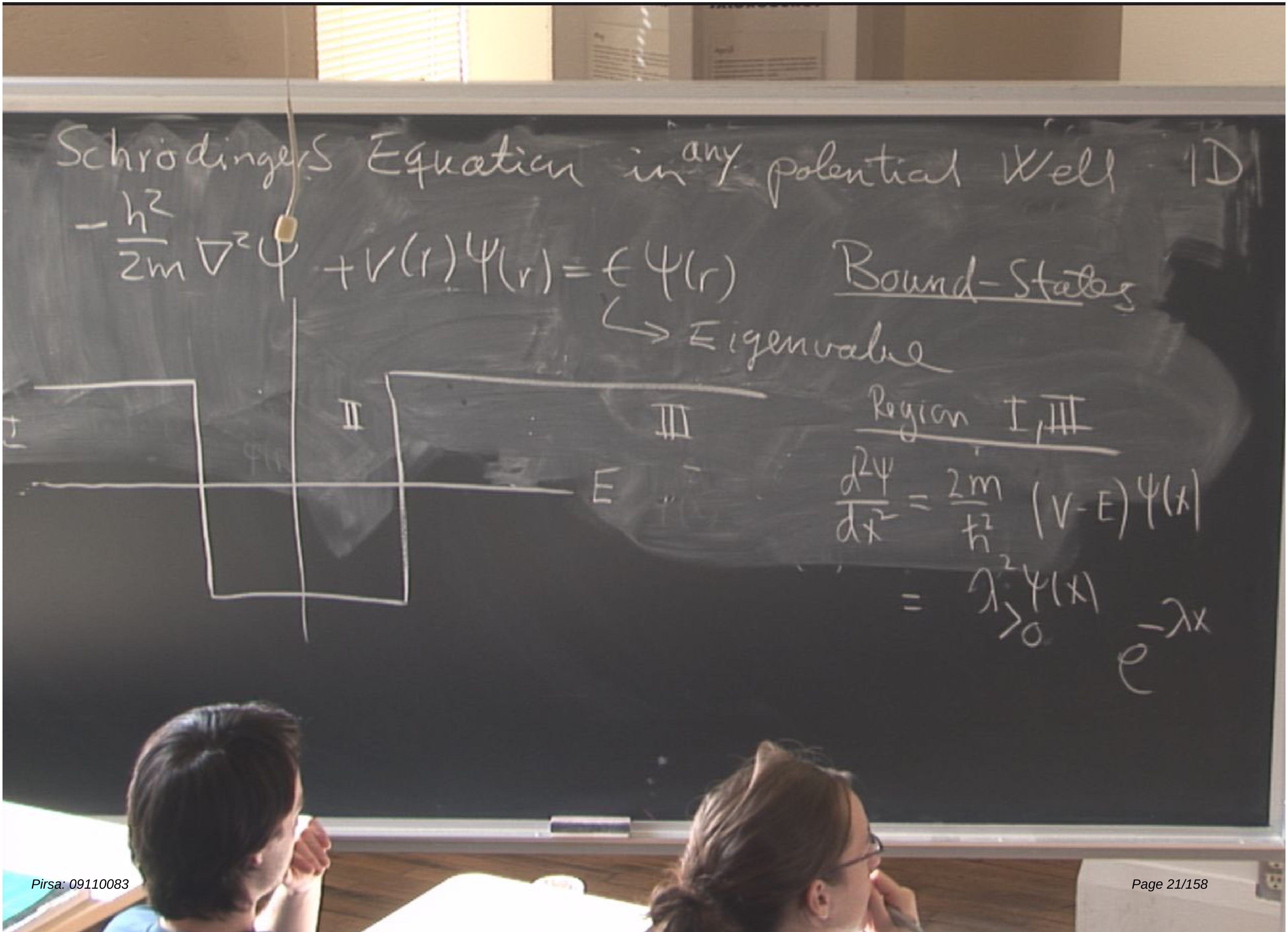

Schrödingers Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi(r) = \epsilon\,\psi(r)$
Bound-States
Eigenvalue

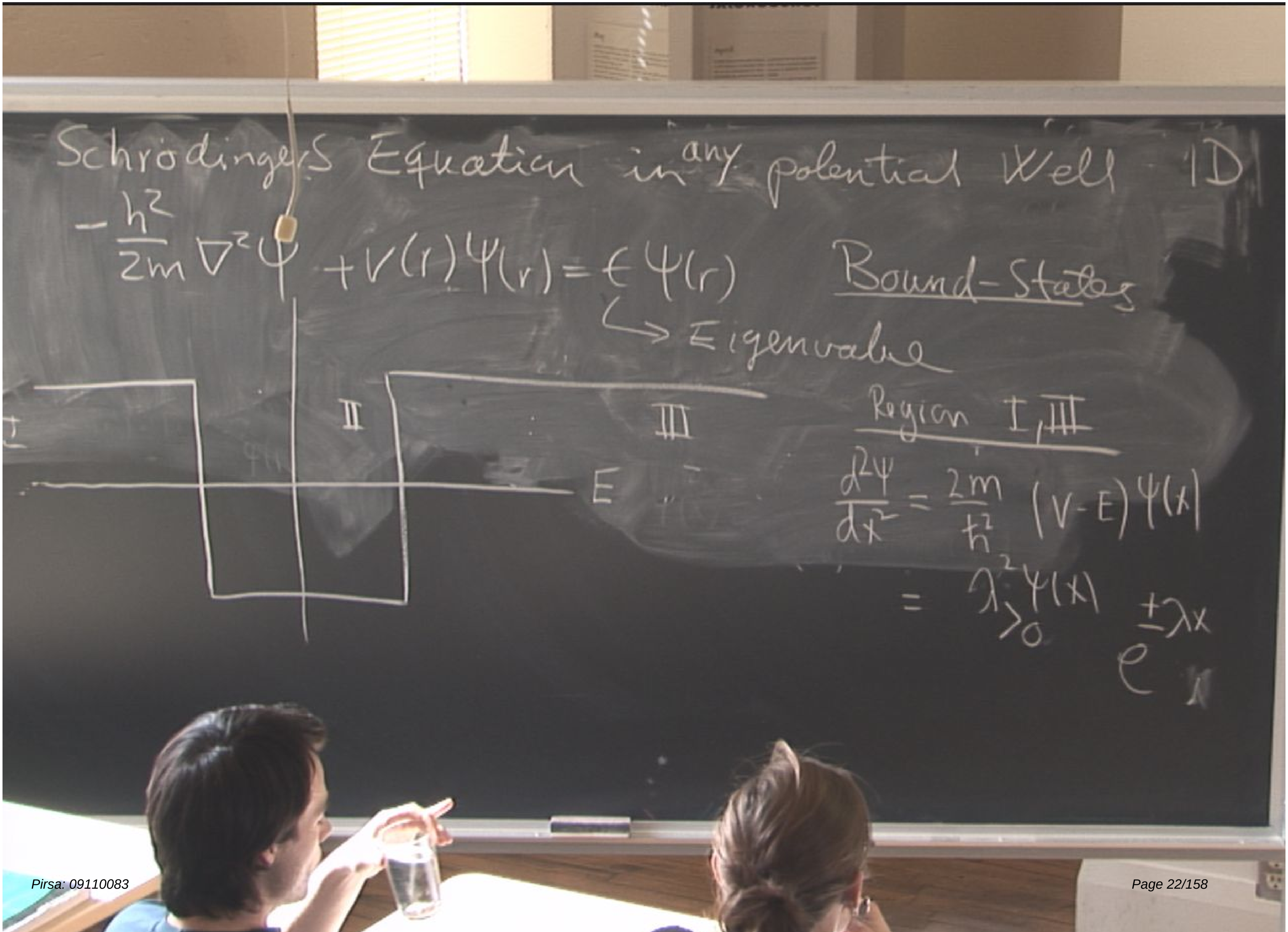

Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Bound-States
Eigenvalue
Region I, III
II
III

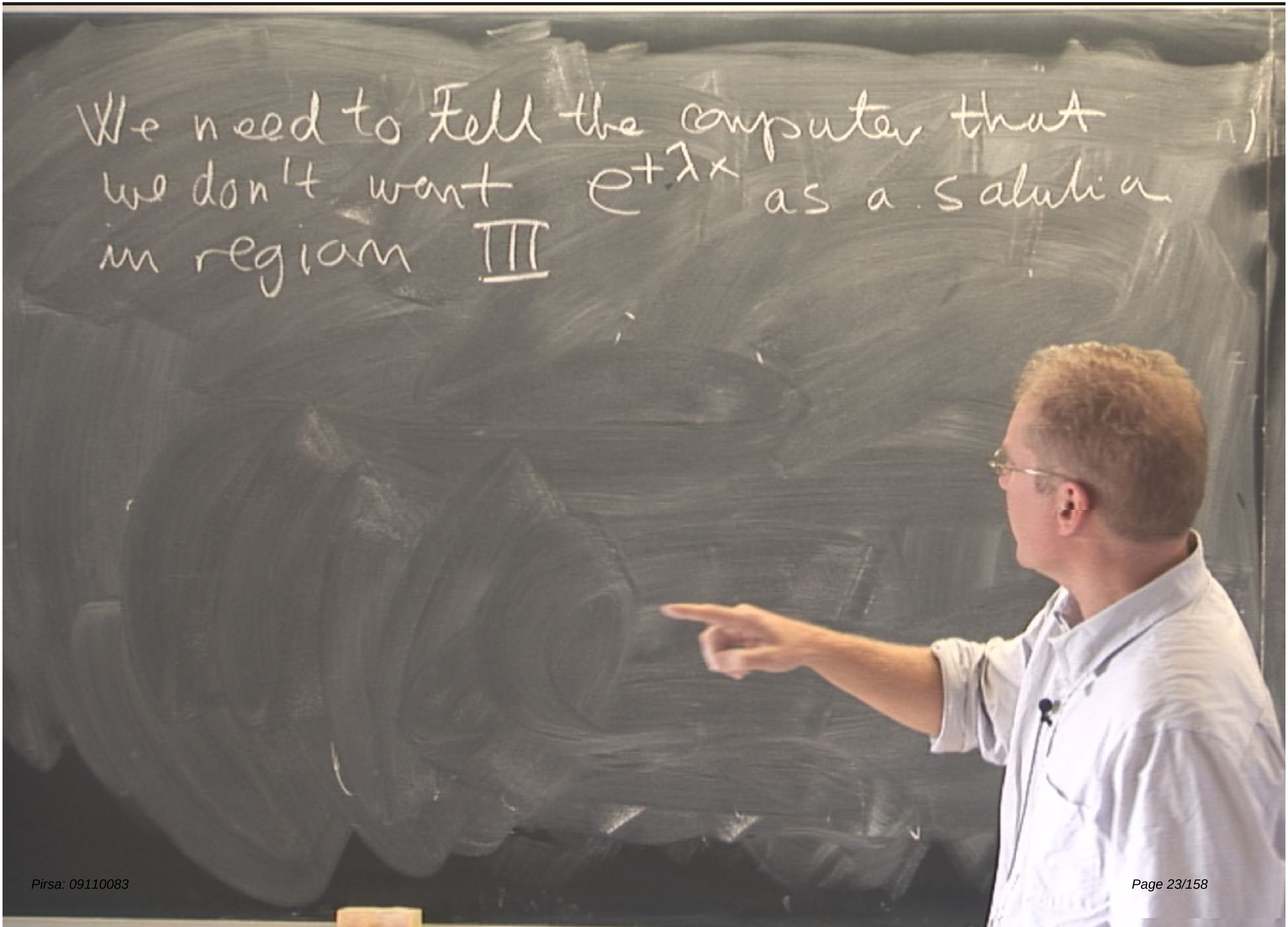

Schrödingers Equation in any potential Well 1D
$-\frac{h^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Bound-States
Eigenvalue
I
II
III
E
Region I, III

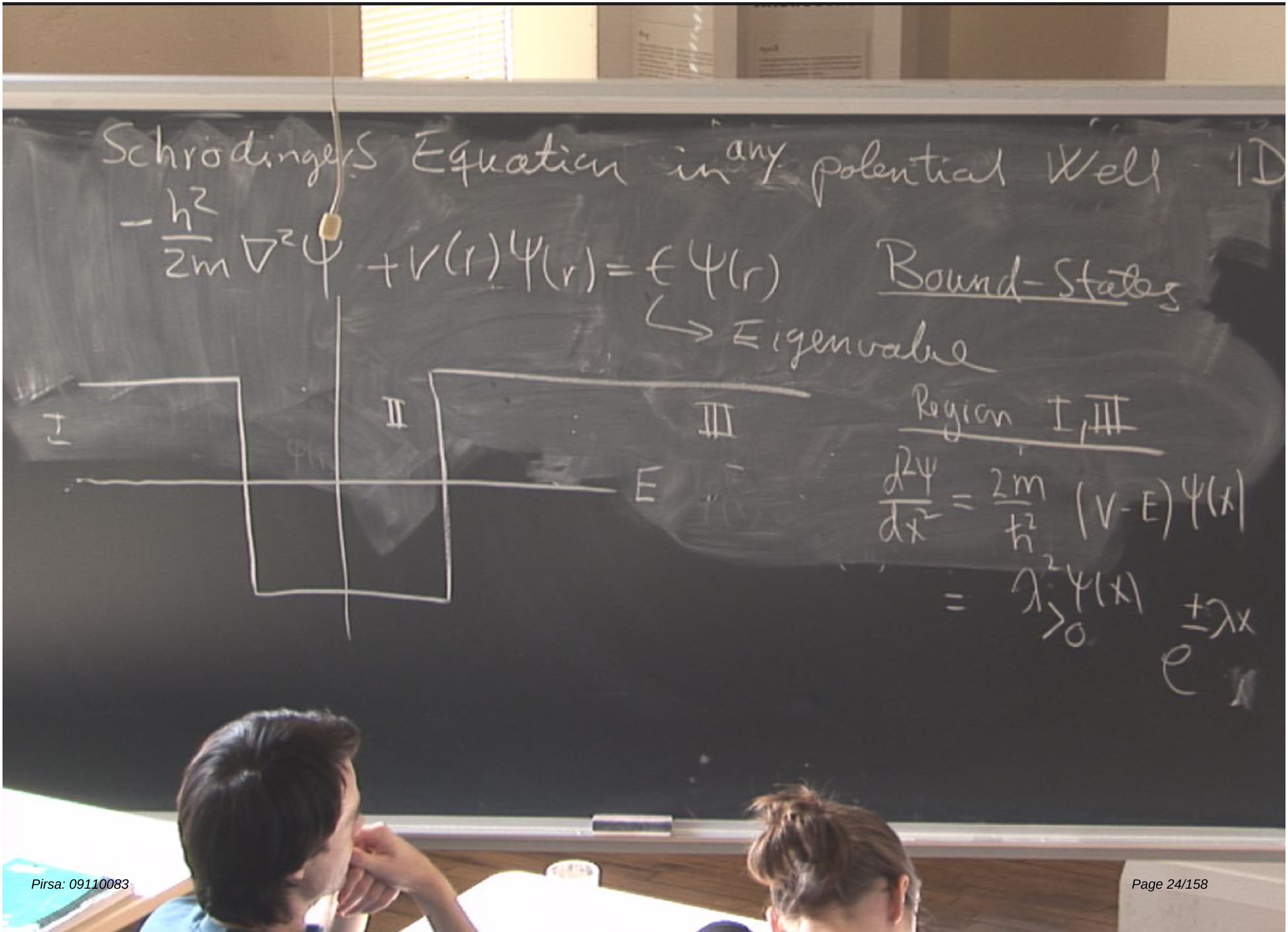

Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\Psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\Psi(x)$
$= \lambda^2\Psi(x)$


Schrödinger's Equation in any potential Well 1D
Bound-States
Eigenvalue
II
III
E
Region I, III


Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
→ Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\Psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\Psi(x)$
$= \lambda^2\Psi(x)$
>0
$e^{-\lambda x}$


Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi(r) = \epsilon\psi(r)$
Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$
$= \lambda^2\psi(x)$
$\lambda > 0$
$e^{\pm\lambda x}$


We need to tell the computer that
we don't want e+λx as a solution
in region III


Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Eigenvalue
Bound-States
I
II
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Region I, III
$\frac{d^2\Psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\Psi(x)$
$= \lambda^2\Psi(x)$
$\lambda > 0$
$e^{\pm\lambda x}$

We need to tell the computer that we don't want $e^{+\lambda x}$ as a solution in region III

Strategy

$\psi(x=-c)$

We need to tell the computer that we don't want $e^{+\lambda x}$ as a solution in region III

Strategy

$$\psi(x=-c)=0$$

We need to tell the computer that we don't want $e^{+\lambda x}$ as a solution in region III

Strategy

$$\psi(x=-c)=0$$

then integrate towards the right

We need to tell the computer ~~that~~ we don't want $e^{+\lambda x}$ as a solution in region III

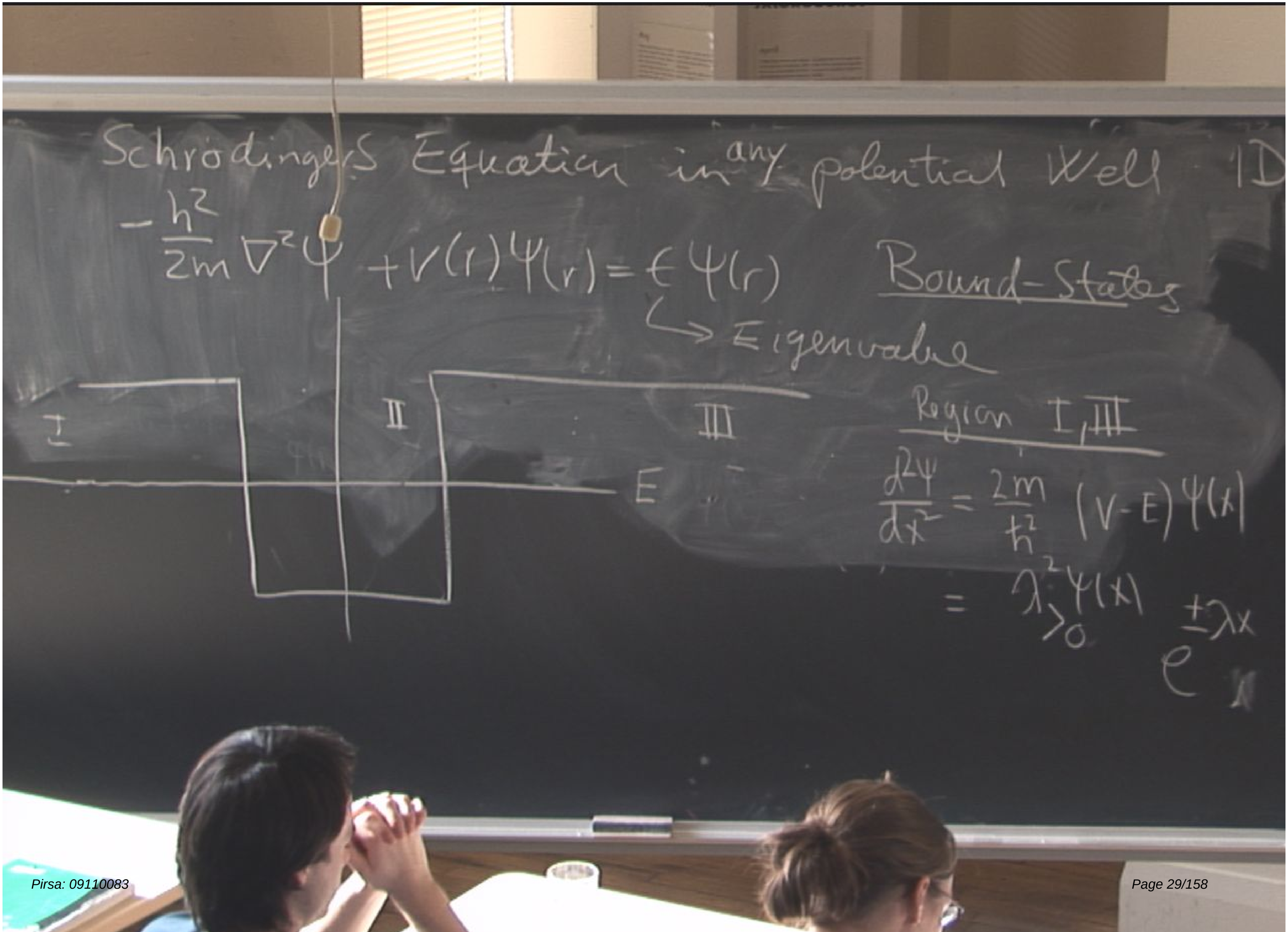
<u>Strategy</u>

$\psi(x=-c) = 0$

then integrate towards the right

Try to get $\psi(x=-c+h)$

↖ step


Schrödinger's Equation in any potential Well 1D
Bound-States
Eigenvalue
I
II
III
E
Region I, III

We need to tell the computer that we don't want $e^{+\lambda x}$ as a solution in region III

Strategy

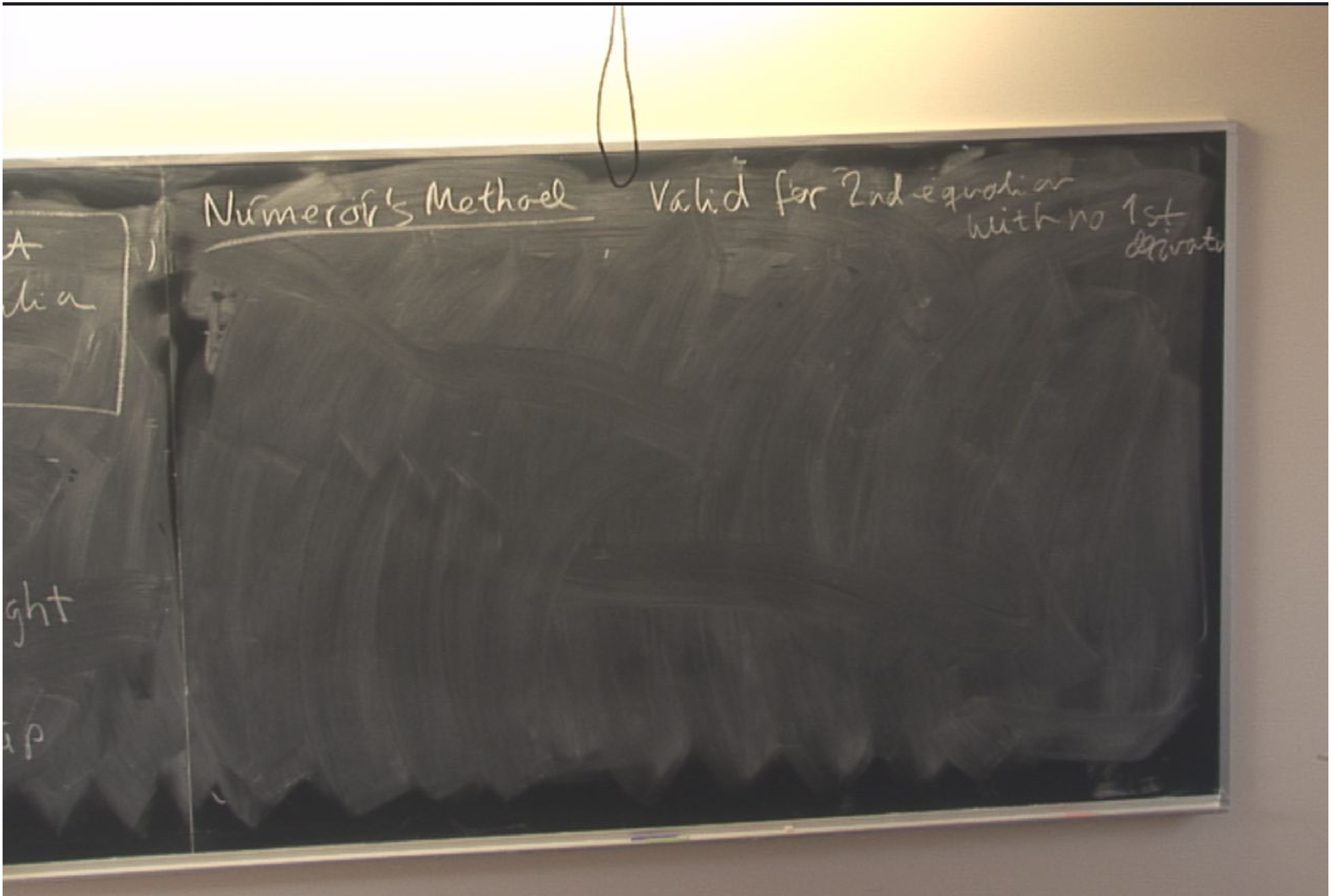
$\psi(x=-c) = 0$

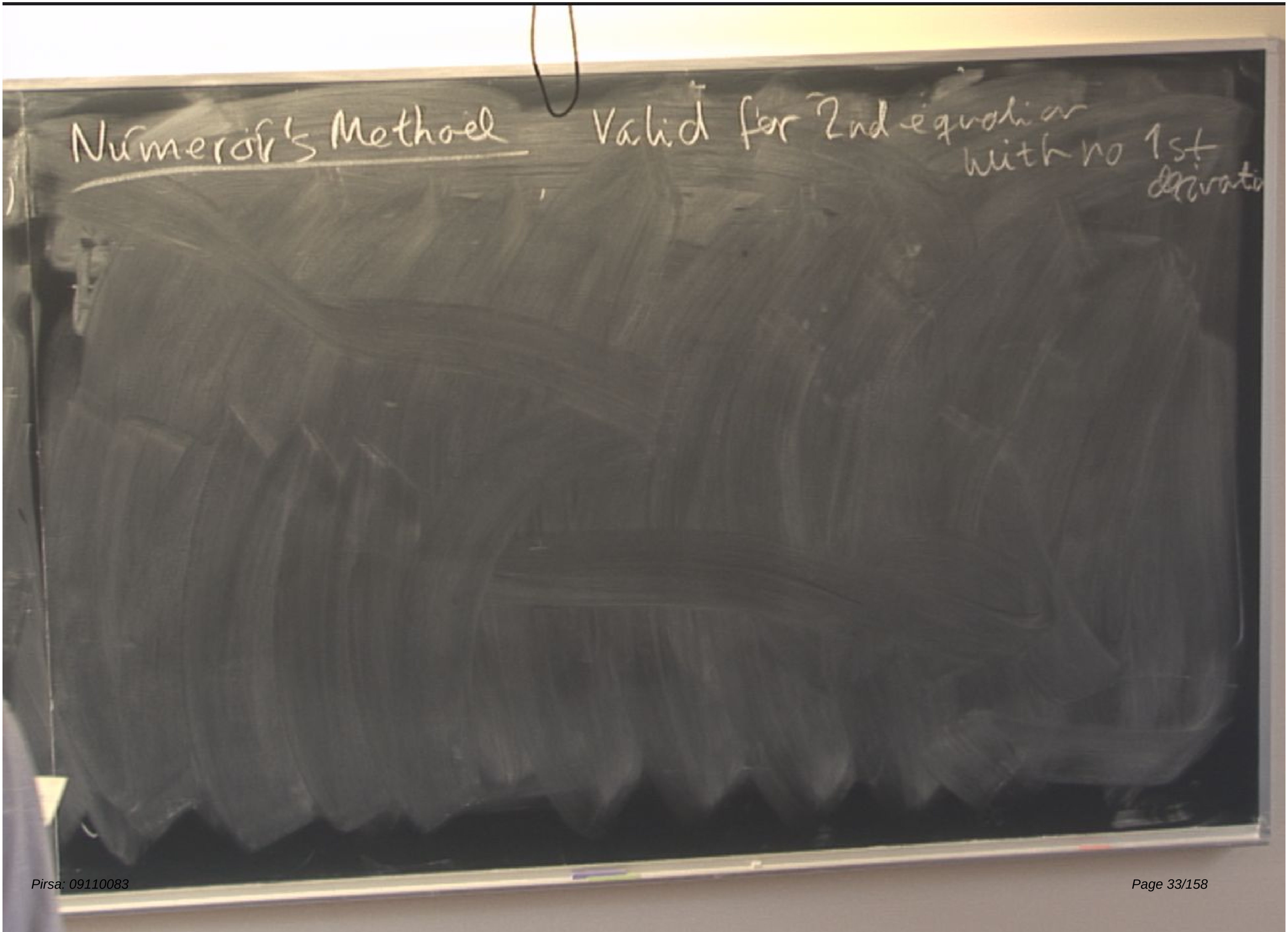
then integrate towards the right

Try to get $\psi(x=-c+h)$

↖ step

Numerov's Method


Numerov's Method
Valid for 2nd equation
with no 1st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Valid for 2nd equation
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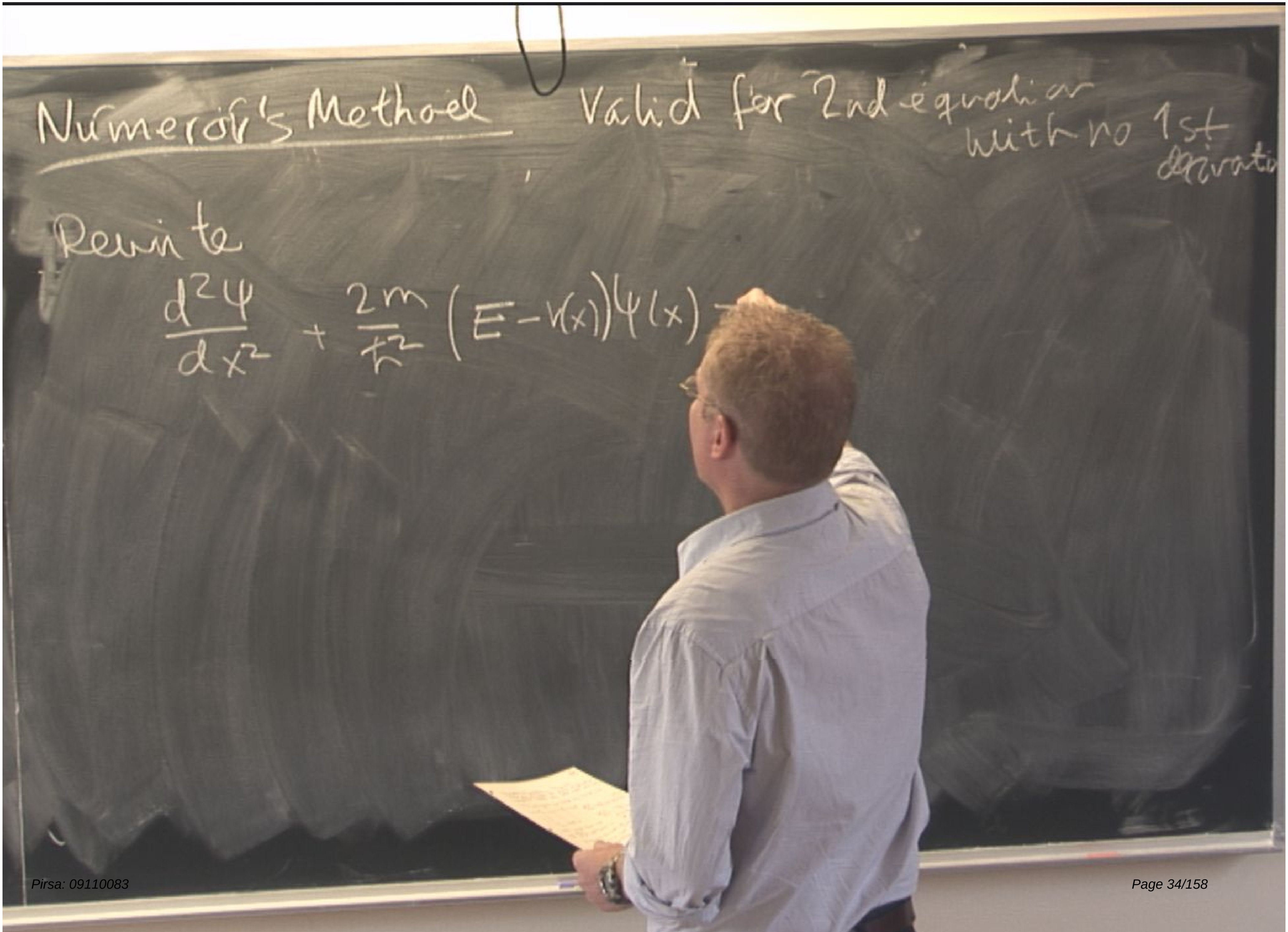

Numerov's Method
Valid for 2nd equation
with no 1st
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) =$

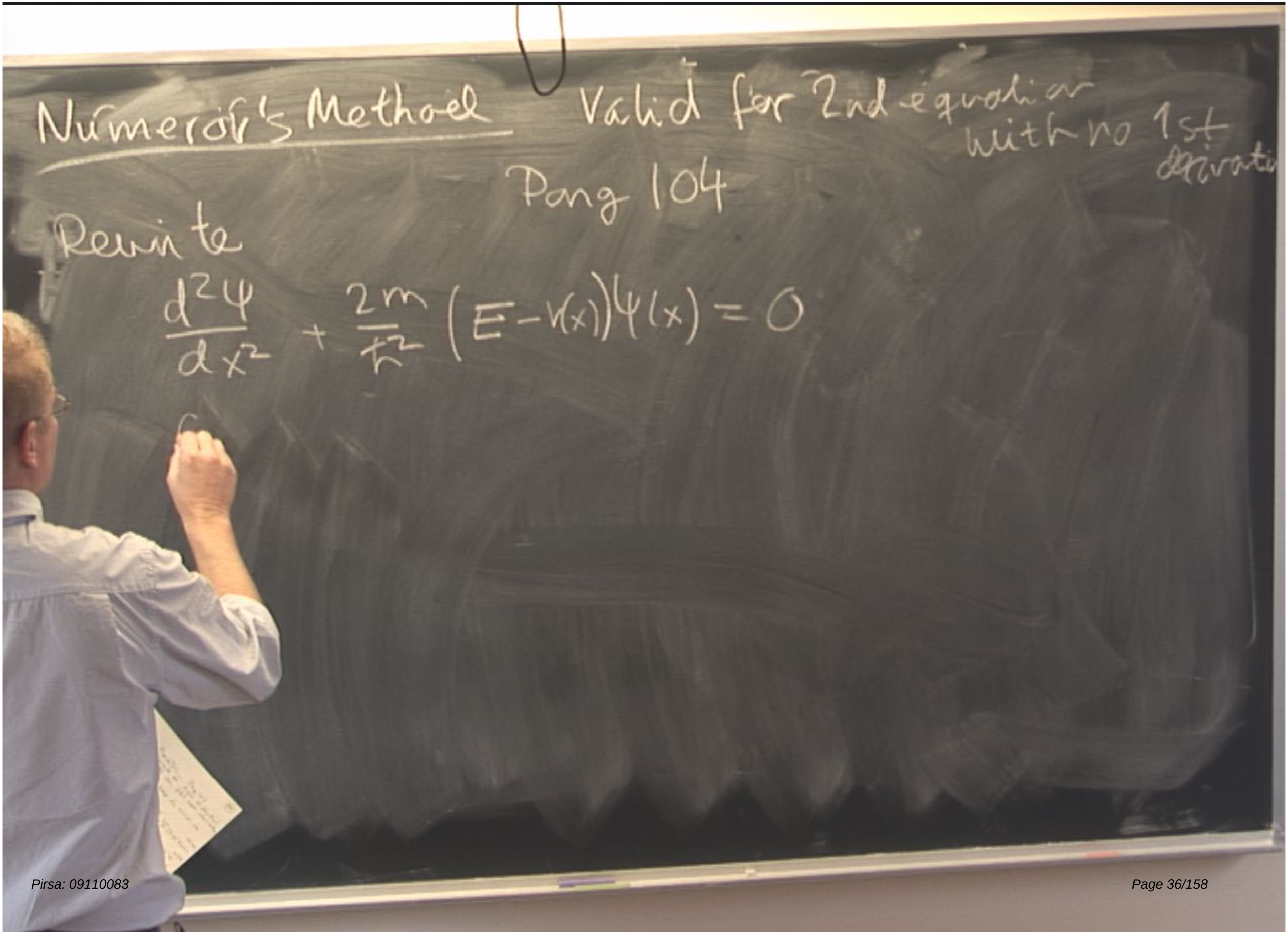
Numerov's Method

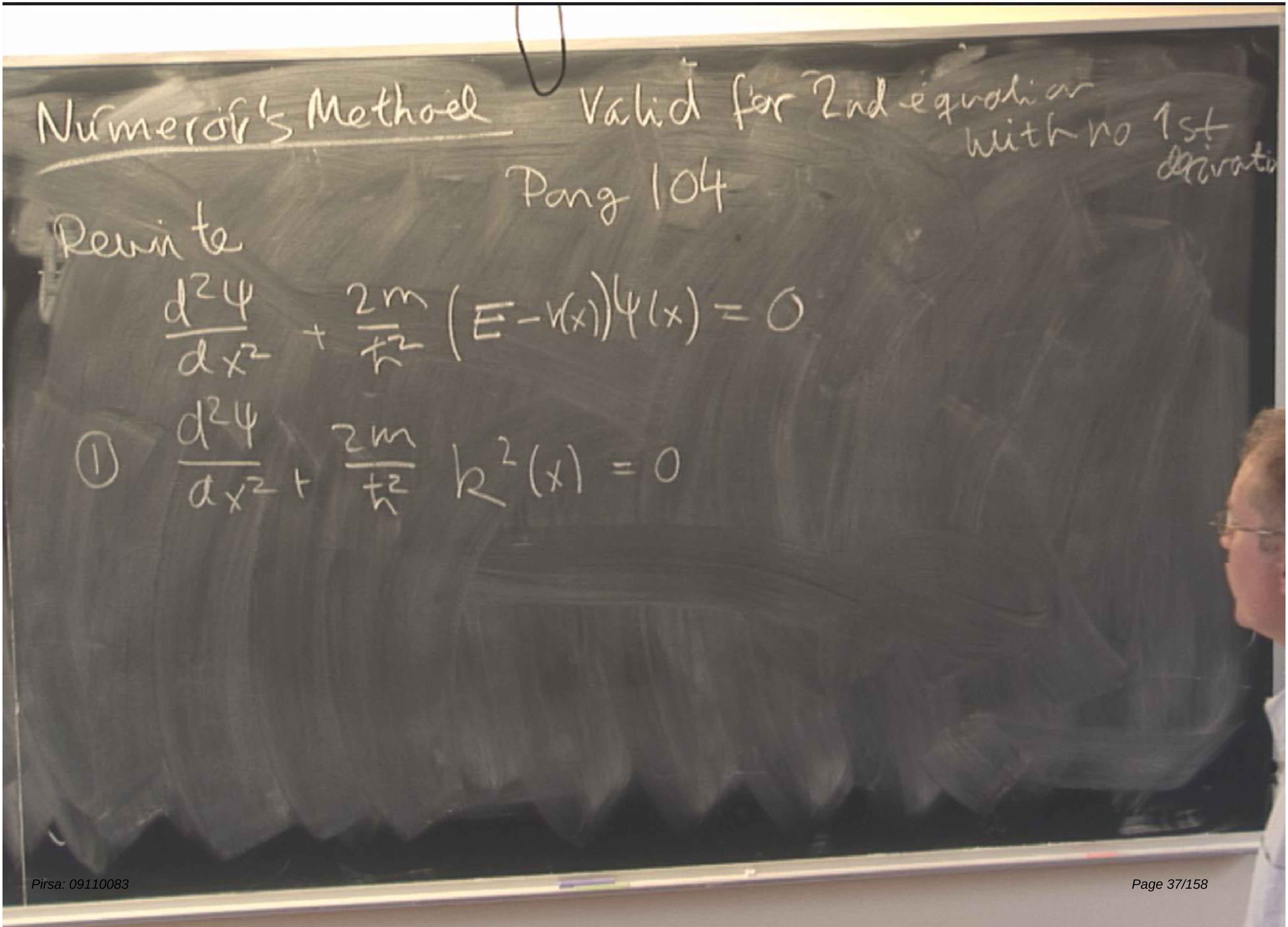
Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$


Numerov's Method
Valid for 2nd equation
with no 1st derivative
Pang 104
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$


Numerov's Method
Valid for 2nd equation with no 1st derivative
Pang 104
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$
① $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$

Numerov's Method Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

(1) $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

Numerov's Method

Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

(1) $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)}$$

Numerov's Method — Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

$$(1) \quad \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2}$$

Numerov's Method — Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

$$① \quad \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2}$$

Numerov's Method — Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

(1) $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)}$$

Numerov's Method

Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

(1) $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)$$

Numerov's Method — Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

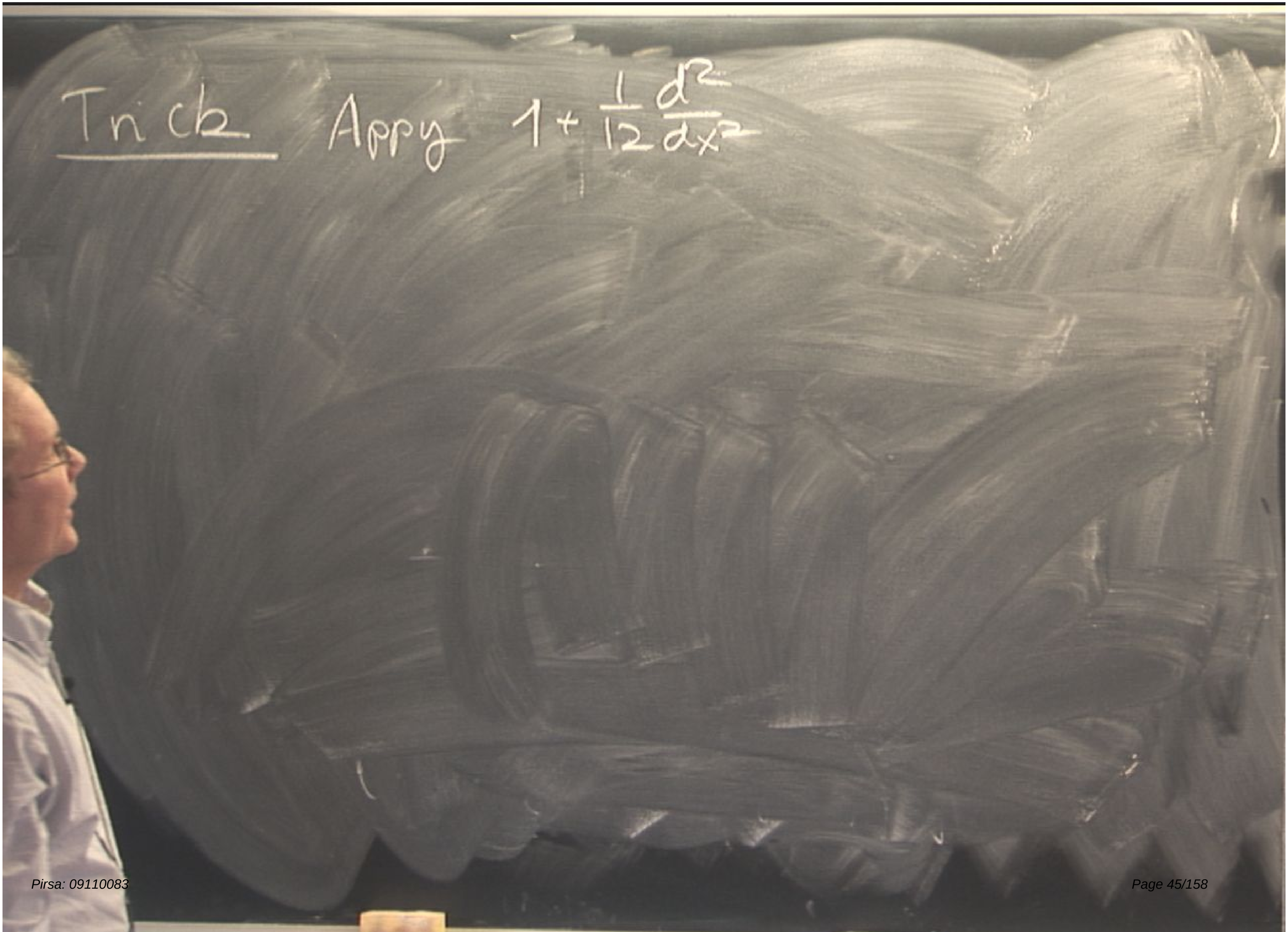
$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

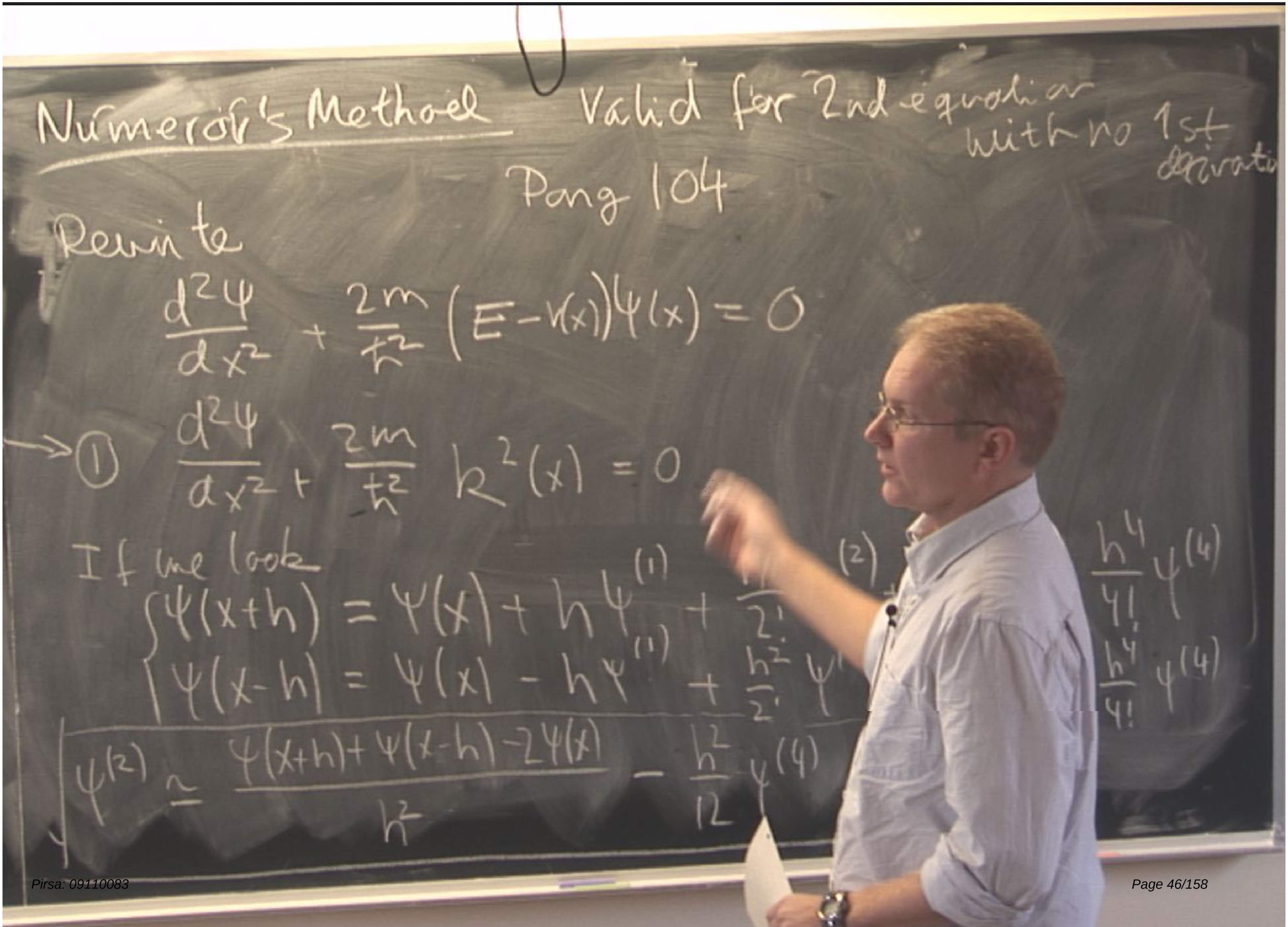
(1) $$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\boxed{\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)}$$


Trick
Appy $1+\frac{1}{12}\frac{d^2}{dx^2}$


Numerov's Method
Valid for 2nd equation
with no 1st
derivati
Pang 104
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$
① $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$
If we look
$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \dots \frac{h^4}{4!}\psi^{(4)}$
$\psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} \dots \frac{h^4}{4!}\psi^{(4)}$
$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)}$

Numerov's Method

Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

$$\textcircled{1} \quad \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} k^2(x) = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\boxed{\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)}$$

Trick Appy $1+\frac{1}{12}\frac{d^2}{dx^2}$ to ①

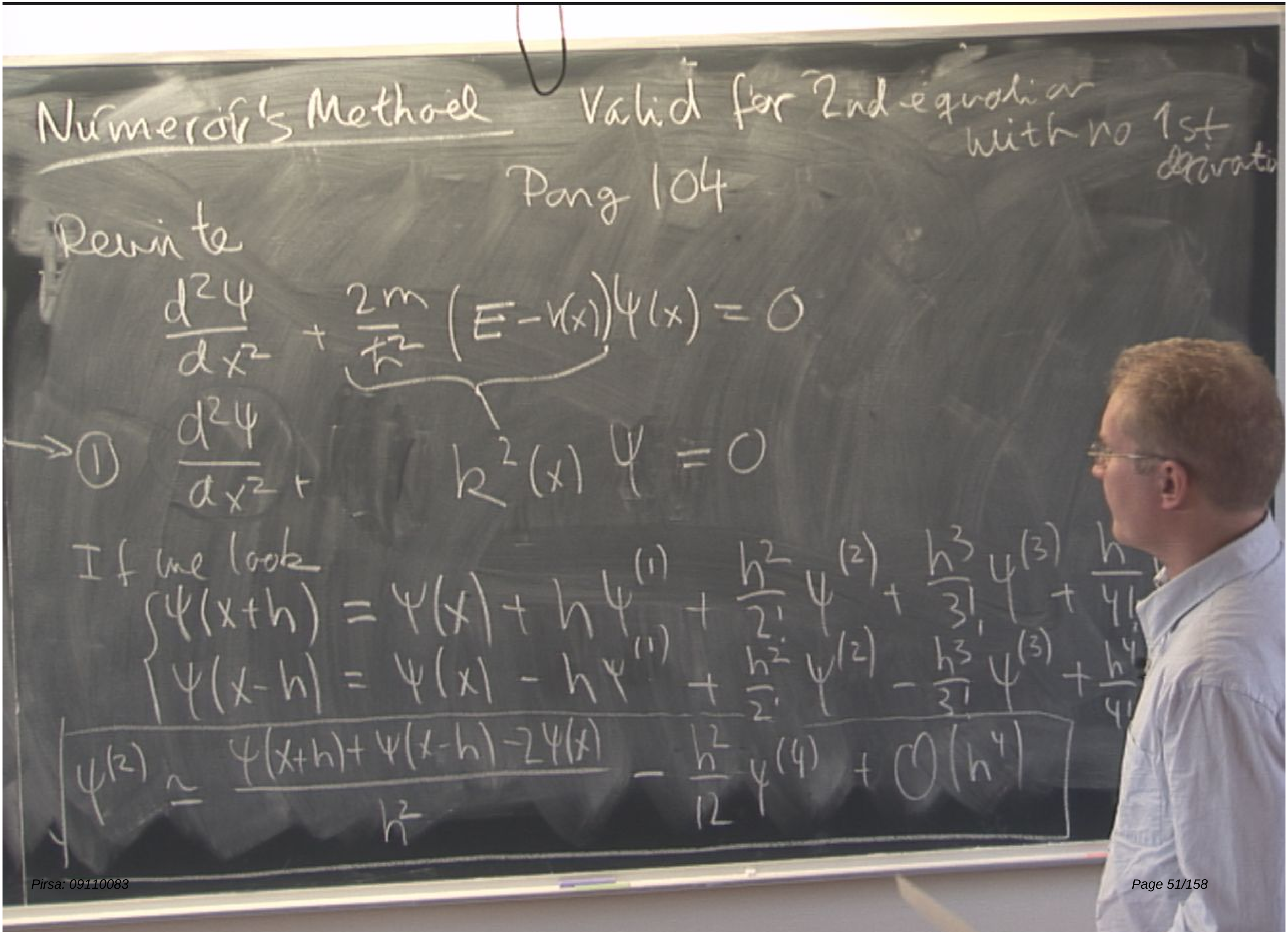
$\psi^{(12)}$

Trick Appy $1 + \frac{h^2}{12}\frac{d^2}{dx^2}$ to (1)

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)}$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2$$


Numerov's Method
Valid for 2nd equation with no 1st derivative
Pang 104
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$
(1) $\frac{d^2\psi}{dx^2} + k^2(x)\psi = 0$
If we look
$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}$
$\psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}$
$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)$

Numerov's Method

Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

$$\text{(1)} \quad \frac{d^2\psi}{dx^2} + k^2(x)\,\psi = 0$$

If we look

$$\begin{cases} \psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \\ \psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)} \end{cases}$$

$$\boxed{\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)}$$

Numerov's Method Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

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$$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)}$$

$$\psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)}$$

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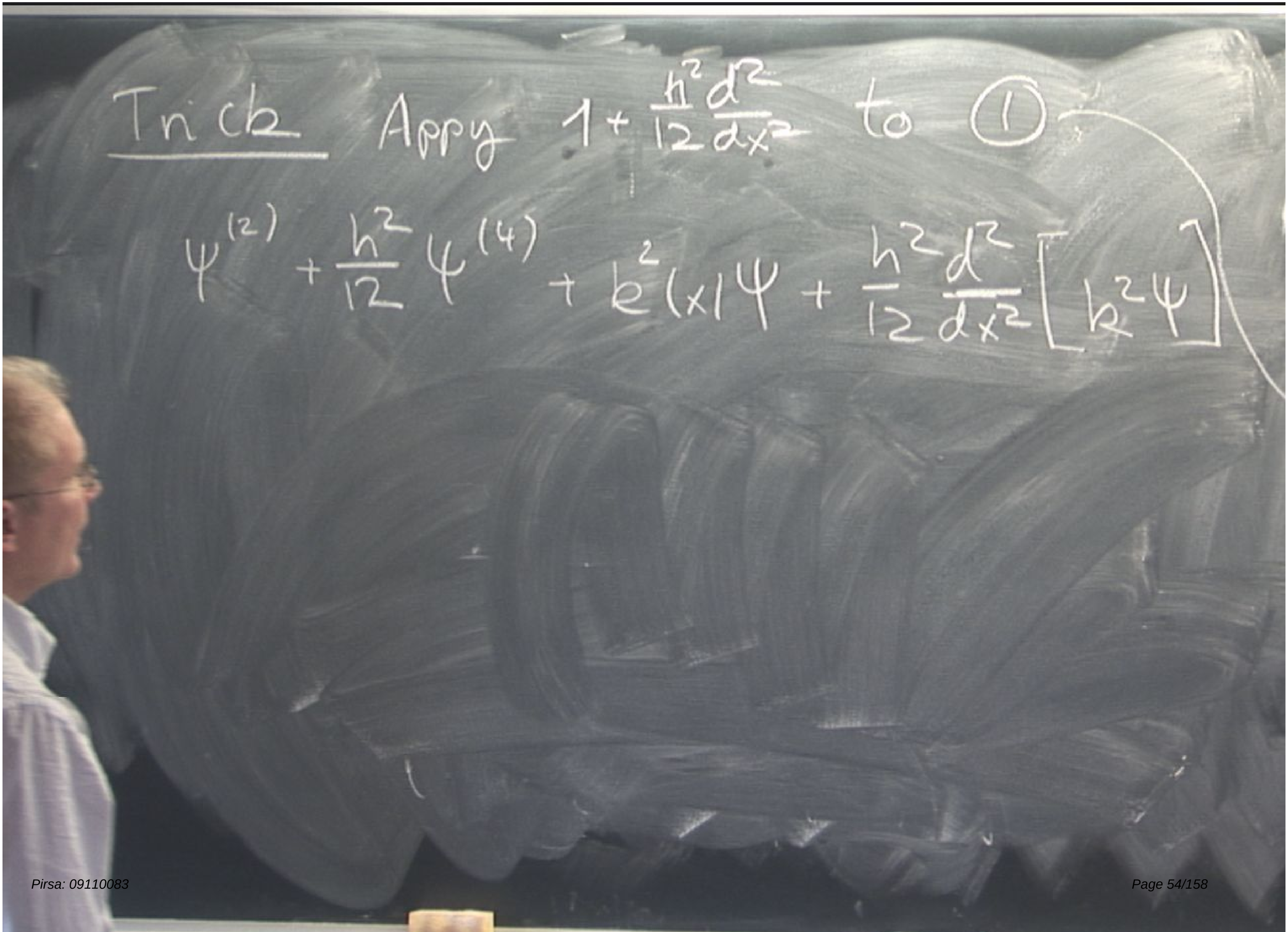
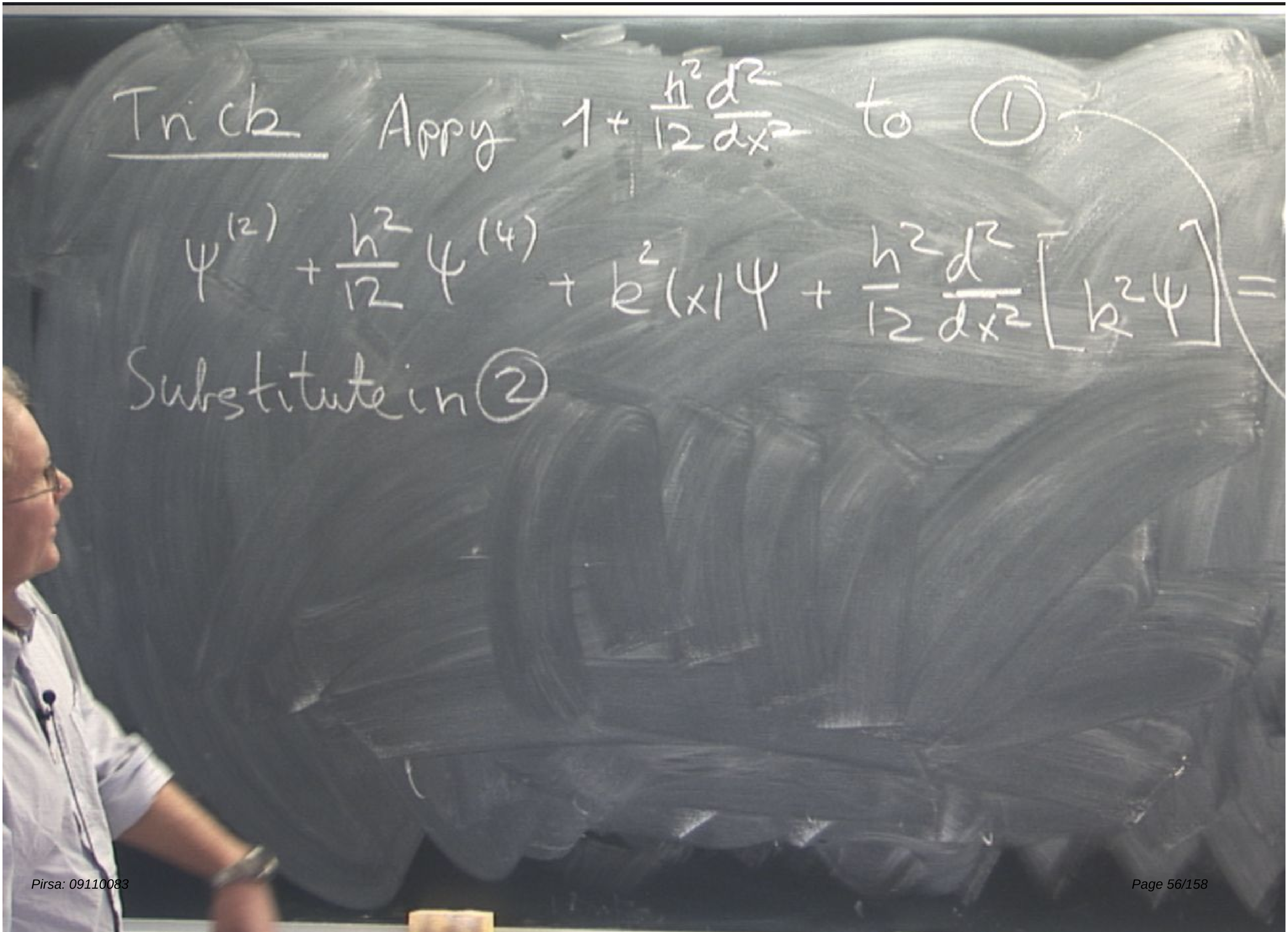
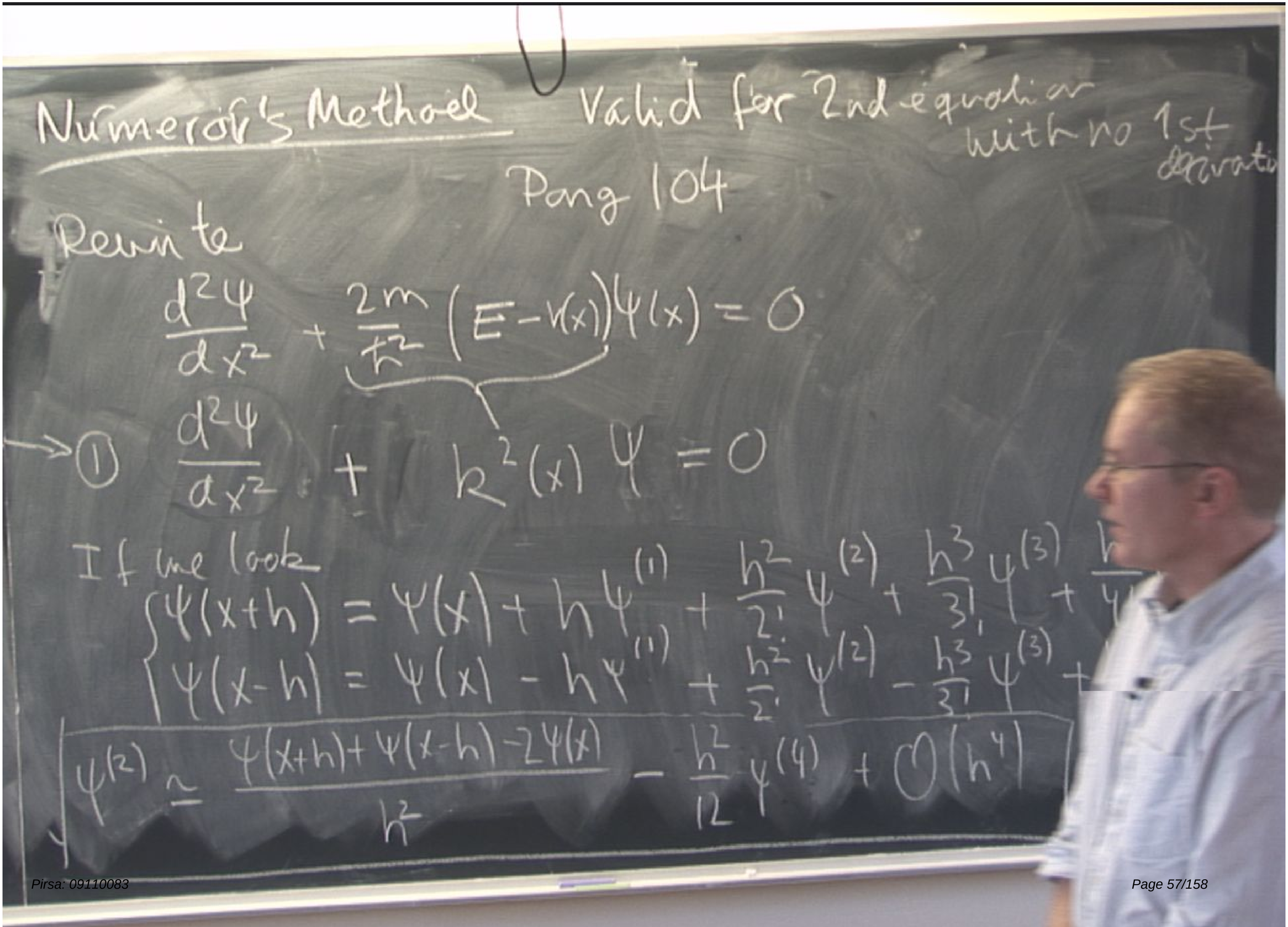

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①
$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right]$

Trick Apply $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$$

Substitute in ②


Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①
$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$
Substitute in ②


Numerov's Method
Valid for 2nd equation with no 1st derivati
Pang 104
Rewrite
$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi(x) = 0$
(1) $\frac{d^2\psi}{dx^2} + k^2(x)\psi = 0$
If we look
$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} +$
$\psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} +$
$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4)$

Numerov's Method

Valid for 2nd equation with no 1st derivative

Pang 104

Rewrite

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$

$$\text{(1)} \quad \frac{d^2\psi}{dx^2} + k^2(x)\,\psi = 0$$

If we look

$$\psi(x+h) = \psi(x) + h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} + \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)}$$

$$\psi(x-h) = \psi(x) - h\psi^{(1)} + \frac{h^2}{2!}\psi^{(2)} - \frac{h^3}{3!}\psi^{(3)} + \frac{h^4}{4!}\psi^{(4)}$$

$$\psi^{(2)} \simeq \frac{\psi(x+h) + \psi(x-h) - 2\psi(x)}{h^2} - \frac{h^2}{12}\psi^{(4)} + O(h^4) \quad \text{(2)}$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$$

Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2}$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to (1)

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$$

Substitute in (2)

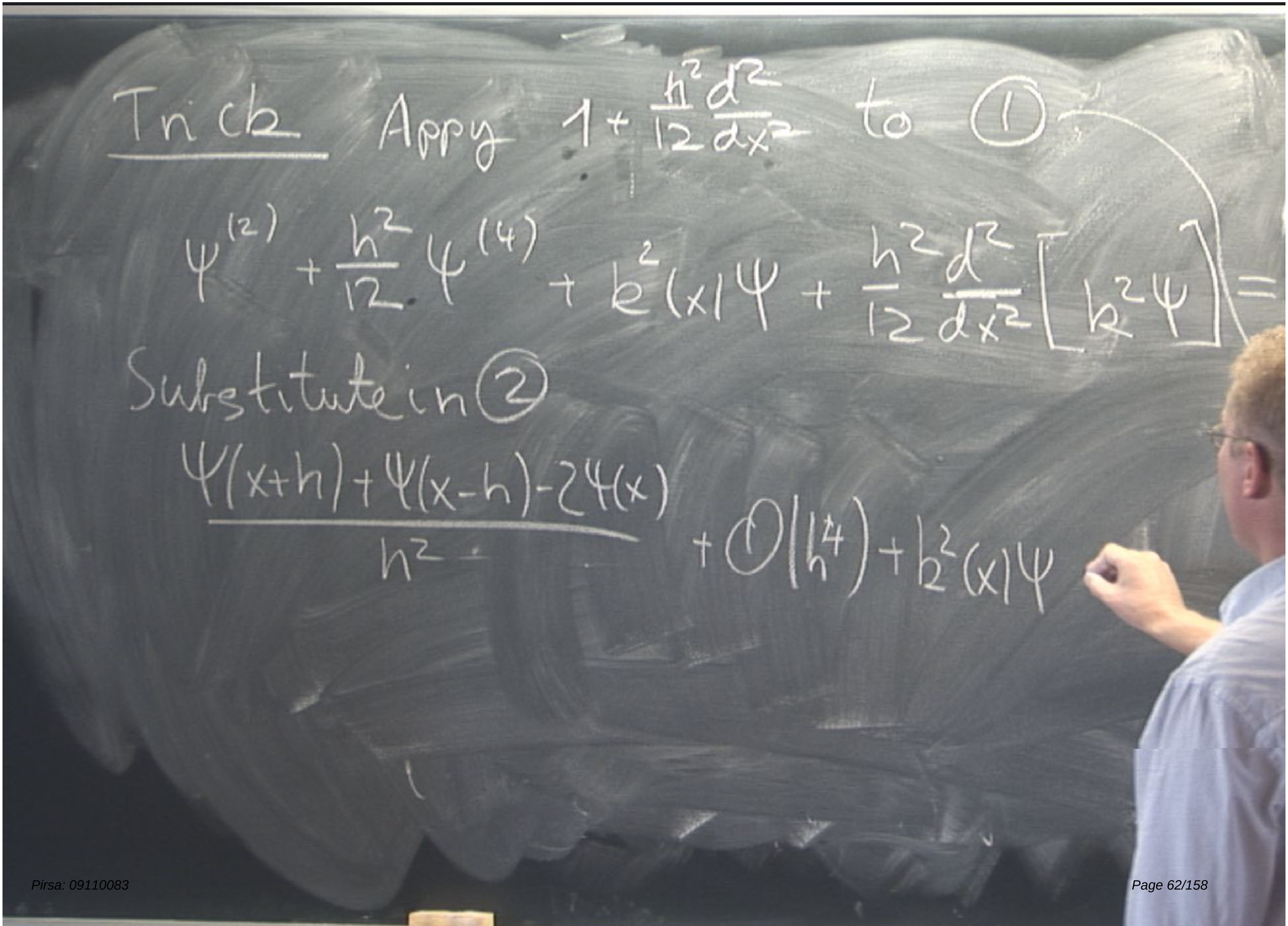
$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2}$$

Trick Appy $1 + \frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$$

Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + \mathcal{O}(h^4)$$


Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①
$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] =$
Substitute in ②
$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi$

Trick: Apply $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to (1)

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Substitute in (2)

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Trick Apply $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

→ Rewrite

$$\psi(x+h) = \ldots \psi(x)$$

Trick Apply $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to (1)

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Substitute in (2)

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

→ Rewrite

$$\psi(x+h) = \ldots\psi(x)\ldots\psi(x-h)$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

→ Rewrite

$$\psi(x+h) = \ldots\ldots\psi(x)\ldots\ldots\psi(x-h)$$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①

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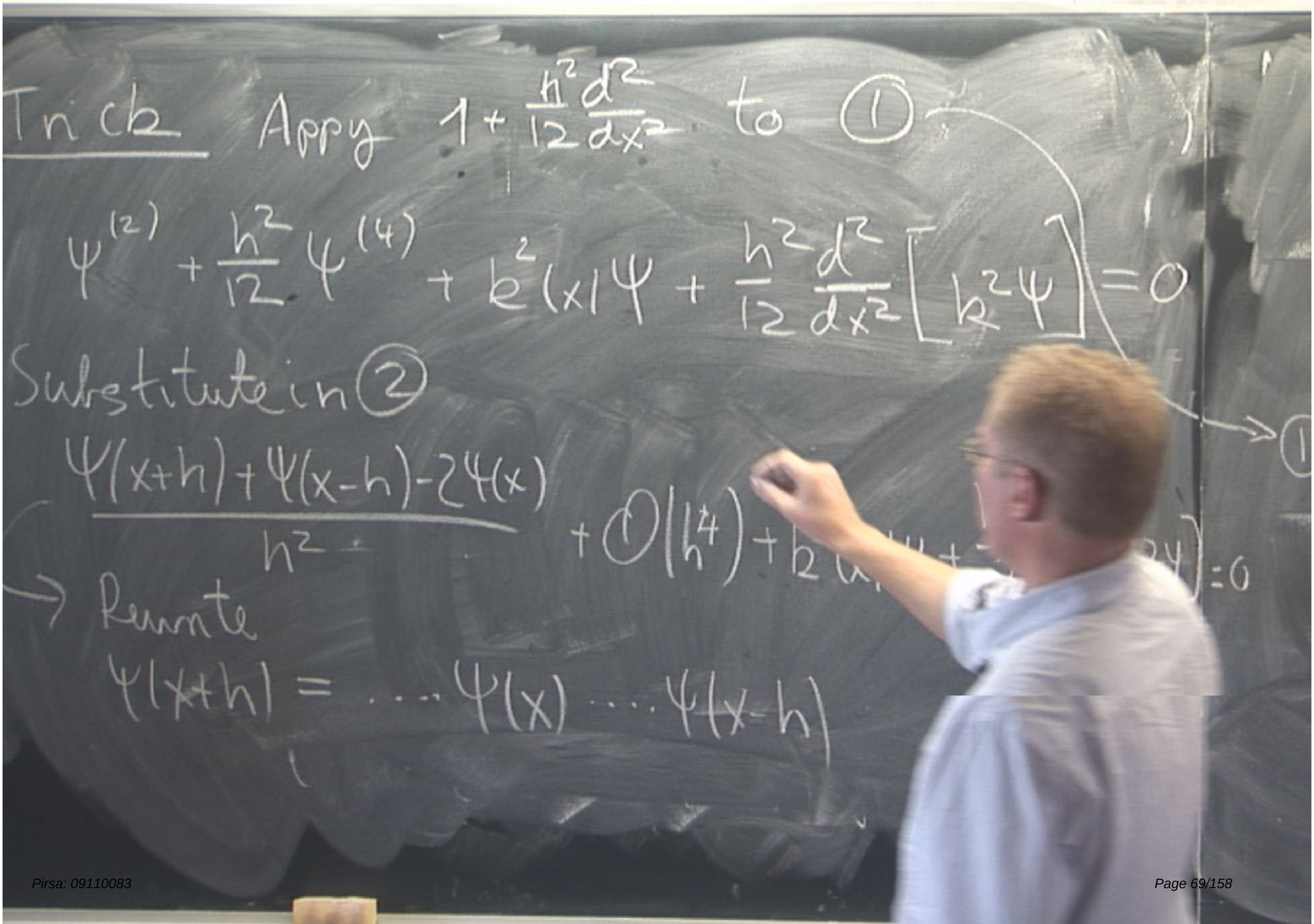
Substitute in ②

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

→ Rewrite

$$\psi(x+h) = \ldots\psi(x)\ldots\psi(x-h)$$

→ ①


Trick
Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to ①
$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$
Substitute in ②
$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k$
$=0$
Rewrite
$\psi(x+h) = \ldots\psi(x)\ldots\psi(x-h)$

Trick Appy $1+\frac{h^2}{12}\frac{d^2}{dx^2}$ to (1)

$$\psi^{(2)} + \frac{h^2}{12}\psi^{(4)} + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

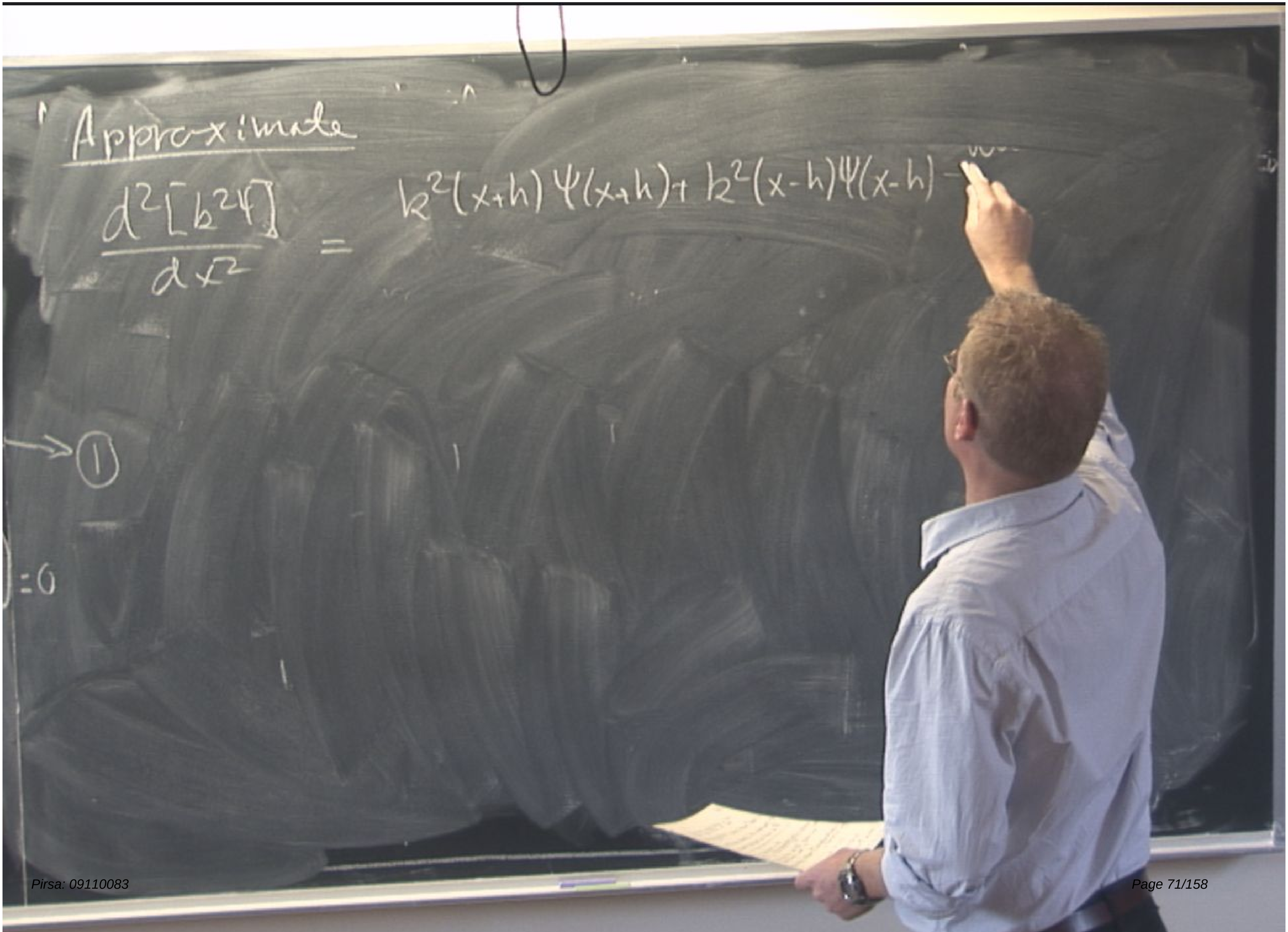
Substitute in (2)

Works because no 1st derivative

$$\frac{\psi(x+h)+\psi(x-h)-2\psi(x)}{h^2} + O(h^4) + k^2(x)\psi + \frac{h^2}{12}\frac{d^2}{dx^2}\left[k^2\psi\right] = 0$$

→ Rewrite

$$\psi(x+h) = \ldots\psi(x)\ldots\psi(x-h)$$


Approximate
d²[k²Ψ] / dx² = k²(x+h)Ψ(x+h) + k²(x-h)Ψ(x-h) -
①
) = 0

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\,\psi(x+h) + k^2(x-h)\,\psi(x-h) - 2k^2(x)\,\psi(x)}{}$$

(1)

:0

Approximate

$$\frac{d^2[b^2\psi]}{dx^2} = \frac{k^2(x+h)\,\psi(x+h) + k^2(x-h)\,\psi(x-h) - 2k^2(x)\,\psi(x)}{h^2} + O$$

→ (1)

$) = 0$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\,\psi(x+h) + k^2(x-h)\,\psi(x-h) - 2k^2(x)\,\psi(x)}{h^2} + O(h^2)$$

→ ①

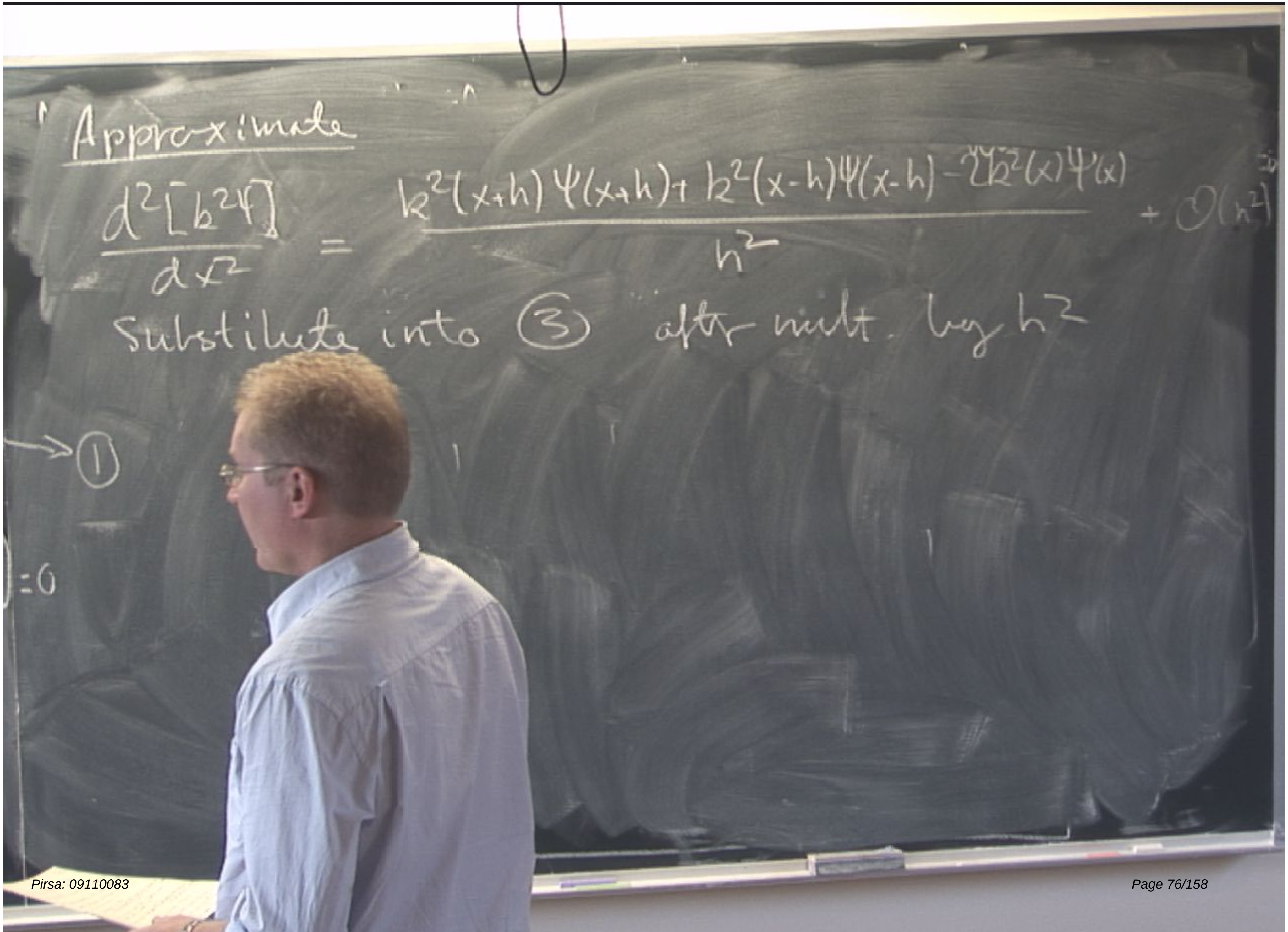
$) = 0$

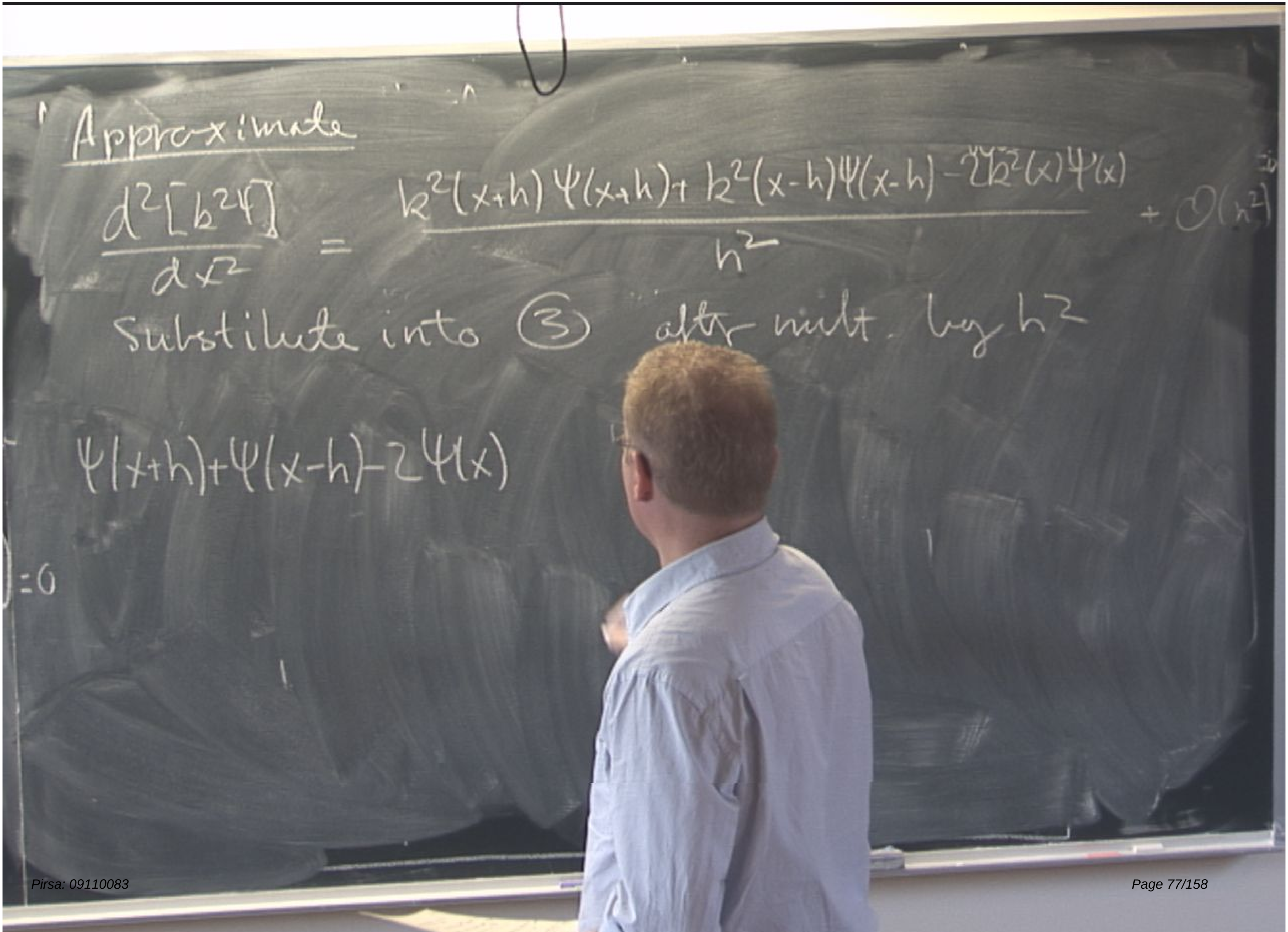
Approximate

$$\frac{d^2[b^2\psi]}{dx^2} = \frac{k^2(x+h)\,\psi(x+h) + k^2(x-h)\,\psi(x-h) - 2k^2(x)\,\psi(x)}{h^2} + O(h^2)$$

Substitute into (3)

→ (1)


Approximate
$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$
Substitute into (3) after mult. by h^2
(1)
=0


Approximate
d²[k²ψ]/dx² = [k²(x+h)ψ(x+h) + k²(x-h)ψ(x-h) - 2k²(x)ψ(x)] / h² + O(h²)
Substitute into ③ after mult. by h²
ψ(x+h)+ψ(x-h)-2ψ(x)
)=0

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

$$\psi(x+h) + \psi(x-h) - 2\psi(x) + h^2 k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)\right] = 0$$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

$$\psi(x+h)+\psi(x-h)-2\psi(x) + h^2 k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h) + k^2(x- \ldots \; 2k^2(x)\psi(x)\right] = 0$$

Isolate

$$\psi(x+h) = \frac{1}{1+\frac{h^2}{12}k^2(x+h)}\Big[$$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

$$\psi(x+h) + \psi(x-h) - 2\psi(x) + h^2 k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)\right] = 0$$

Isolate

$$\psi(x+h) = \frac{1}{1 + \frac{h^2}{12}k^2(x+h)}\Big[$$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

$$\psi(x+h)+\psi(x-h)-2\psi(x)+h^2k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h)+k^2(x-h)\psi(x-h)-2k^2(x)\psi(x)\right] = 0$$

Isolate

$$\psi(x+h) = \frac{1}{1+\frac{h^2}{12}k^2(x+h)}\Big[2\psi(x)$$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

$$\psi(x+h) + \psi(x-h) - 2\psi(x) + h^2 k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi\ldots\right.$$

$= 0$

Isolate

$$\psi(x+h) = \frac{1}{1+\frac{h^2}{12}k^2(x+h)}\Big[2\psi(x) - \psi(x-h) - h^2 k^2(x)\psi(x) - \frac{h^2}{12}k^2(x-h)\psi(x-h)$$

$$\psi(x+h) = \frac{\qquad\qquad}{1 + \frac{h^2}{12} k^2(x+h)}$$

Approximate

$$\frac{d^2[k^2\psi]}{dx^2} = \frac{k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)}{h^2} + O(h^2)$$

Substitute into ③ after mult. by h^2

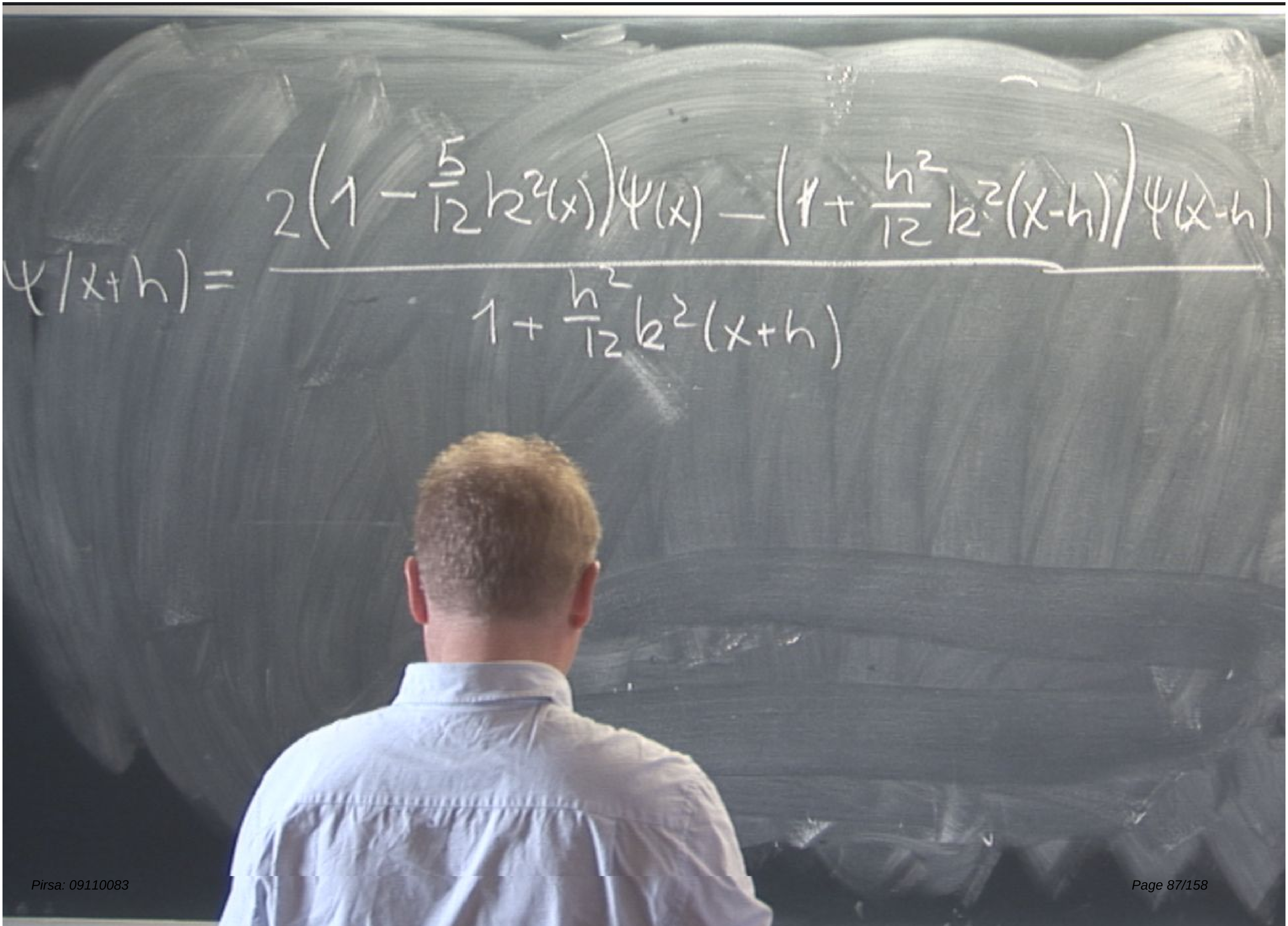
$$\psi(x+h) + \psi(x-h) - 2\psi(x) + h^2 k^2(x)\psi + \frac{h^2}{12}\left[k^2(x+h)\psi(x+h) + k^2(x-h)\psi(x-h) - 2k^2(x)\psi(x)\right] = 0$$

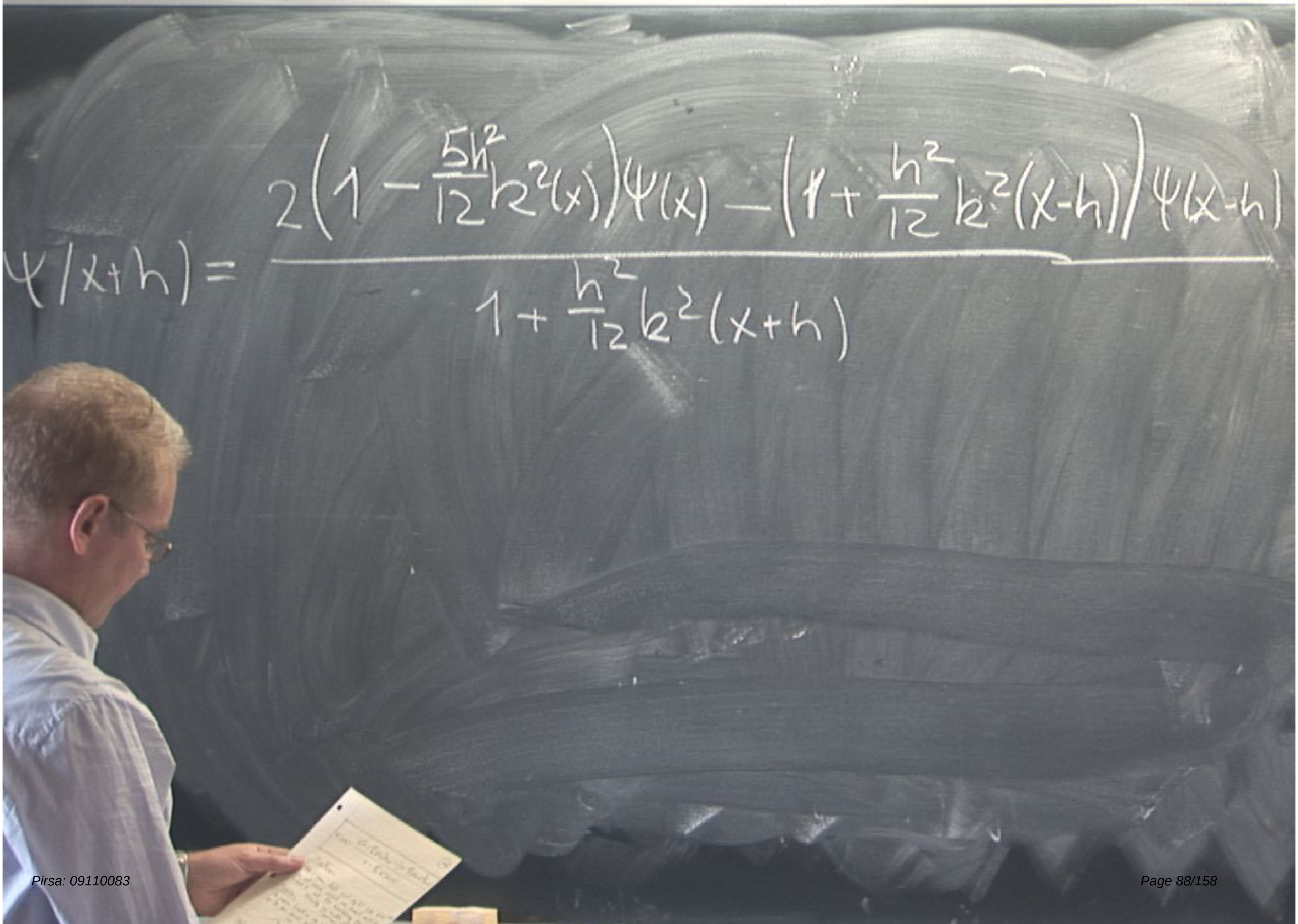
Isolate

$$\psi(x+h) = \frac{1}{1 + \frac{h^2}{12}k^2(x+h)}\left[2\psi(x) - \psi(x-h) - h^2 k^2(x)\psi(x) - \frac{h^2}{12}k^2(x-h)\psi(x-h) + \frac{2h^2}{12}k^2(x)\psi(x)\right]$$

$$\psi(x+h) = \frac{2\left(1 - \frac{5}{12}k^2(x)\right)\psi(x) - }{1 + \frac{h^2}{12}k^2(x+h)}$$

$$\psi(x+h) = \frac{2\left(1 - \frac{5}{12}k^2(x)\right)\psi(x) - \left(1 + \frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1 + \frac{h^2}{12}k^2(x+h)}$$


$\psi(x+h) = \dfrac{2\left(1-\frac{5}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$


$\psi(x+h) = \frac{2\left(1-\frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$

$$\psi(x+h) = \frac{2\left(1-\frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$$

Strategy II

$$\psi(x=-c)=0$$

$$\psi(x+h) = \frac{2\left(1 - \frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1 + \frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1 + \frac{h^2}{12}k^2(x+h)}$$

Strategy II

$\psi(x=-c)=0$

Should c depend on E

$$\psi(x+h) = \frac{2\left(1 - \frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1 + \frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1 + \frac{h^2}{12}k^2(x+h)}$$

Strategy II

$\psi(x=-c)=0 \qquad \psi(x=-c+h)=\varepsilon$

Should c depend on E

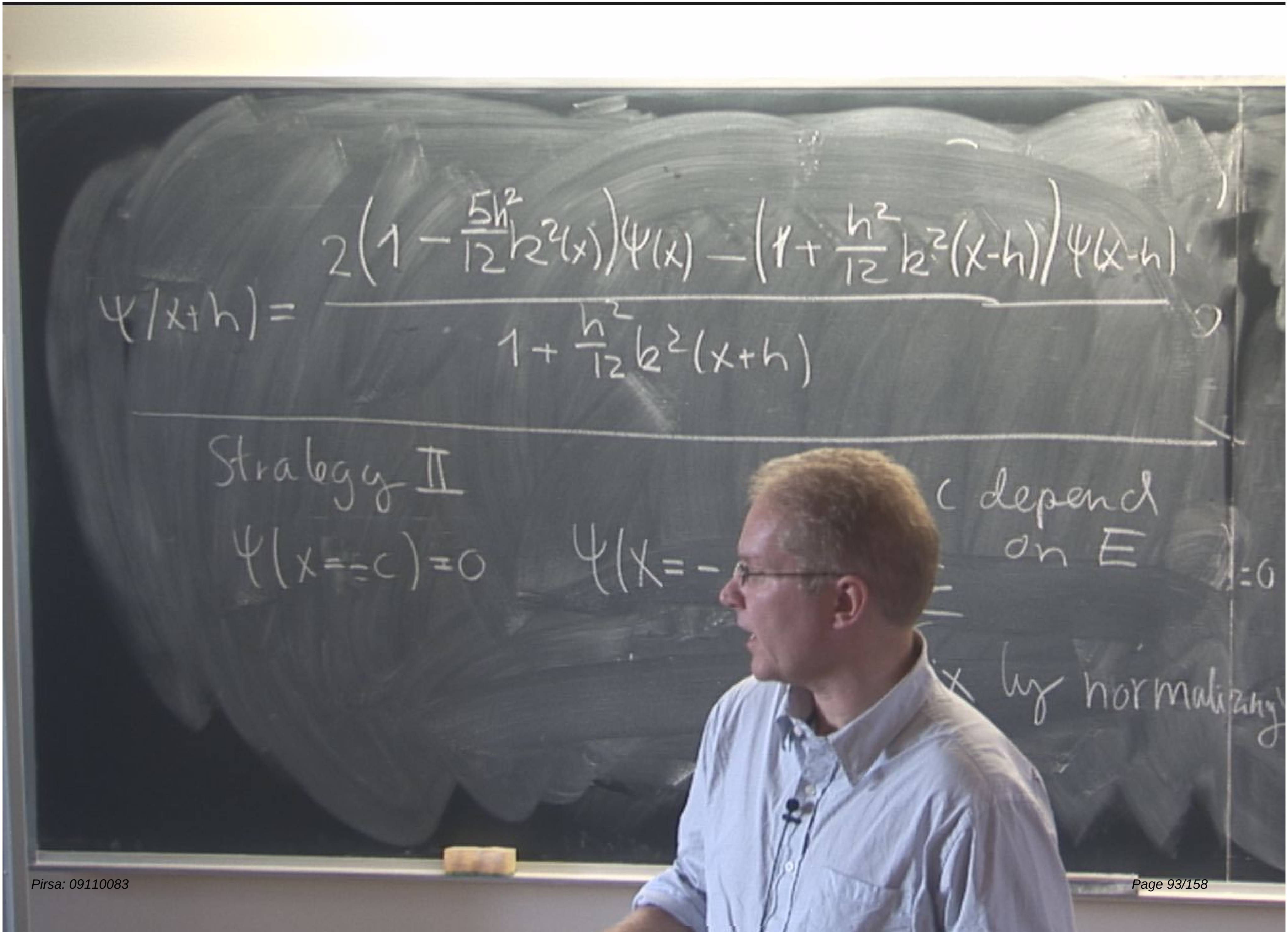
$$\psi(x+h) = \frac{2\left(1 - \frac{5h^2}{12} k^2(x)\right)\psi(x) - \left(1 + \frac{h^2}{12} k^2(x-h)\right)\psi(x-h)}{1 + \frac{h^2}{12} k^2(x+h)}$$

Strategy II

Should c depend on E

$\psi(x=-c)=0 \qquad \psi(x=-c+h)=\varepsilon$

(Fix by norm


$\psi(x+h) = \frac{2\left(1-\frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$
Strategy II
$\psi(x=-c)=0$
c depend
on E

$$\psi(x+h) = \frac{2\left(1-\frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$$

Strategy II

$\psi(x=-c)=0 \qquad \psi(x=-c+h)=\varepsilon$

Should c depend on E

(Fix by normalizing)

$$\psi(x+h) = \frac{2\left(1-\frac{5h^2}{12}k^2(x)\right)\psi(x) - \left(1+\frac{h^2}{12}k^2(x-h)\right)\psi(x-h)}{1+\frac{h^2}{12}k^2(x+h)}$$

Strategy II

$\psi(x=-c)=0$ $\quad\psi(x=-c+h)=+\varepsilon$

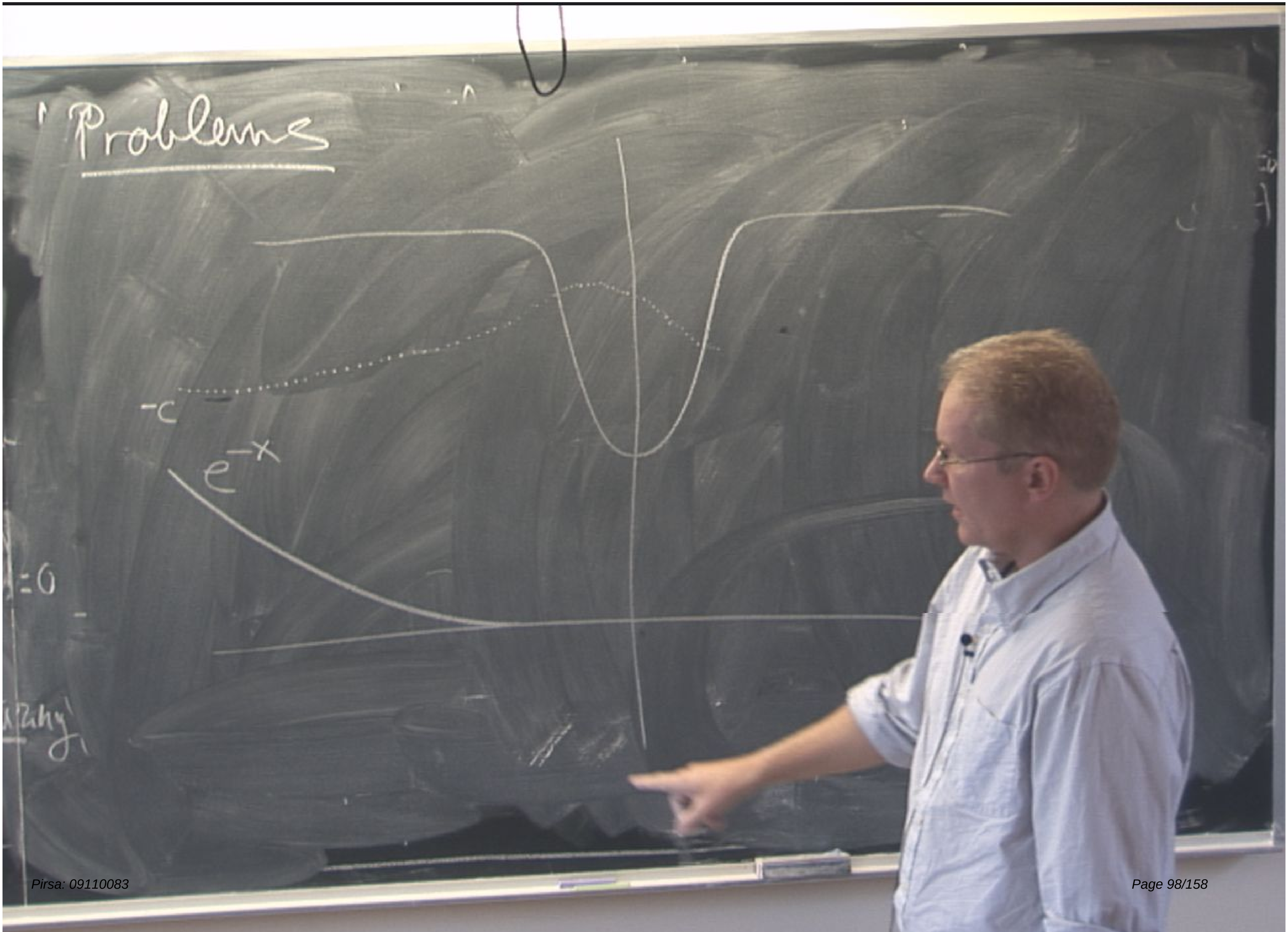
Should c depend on E

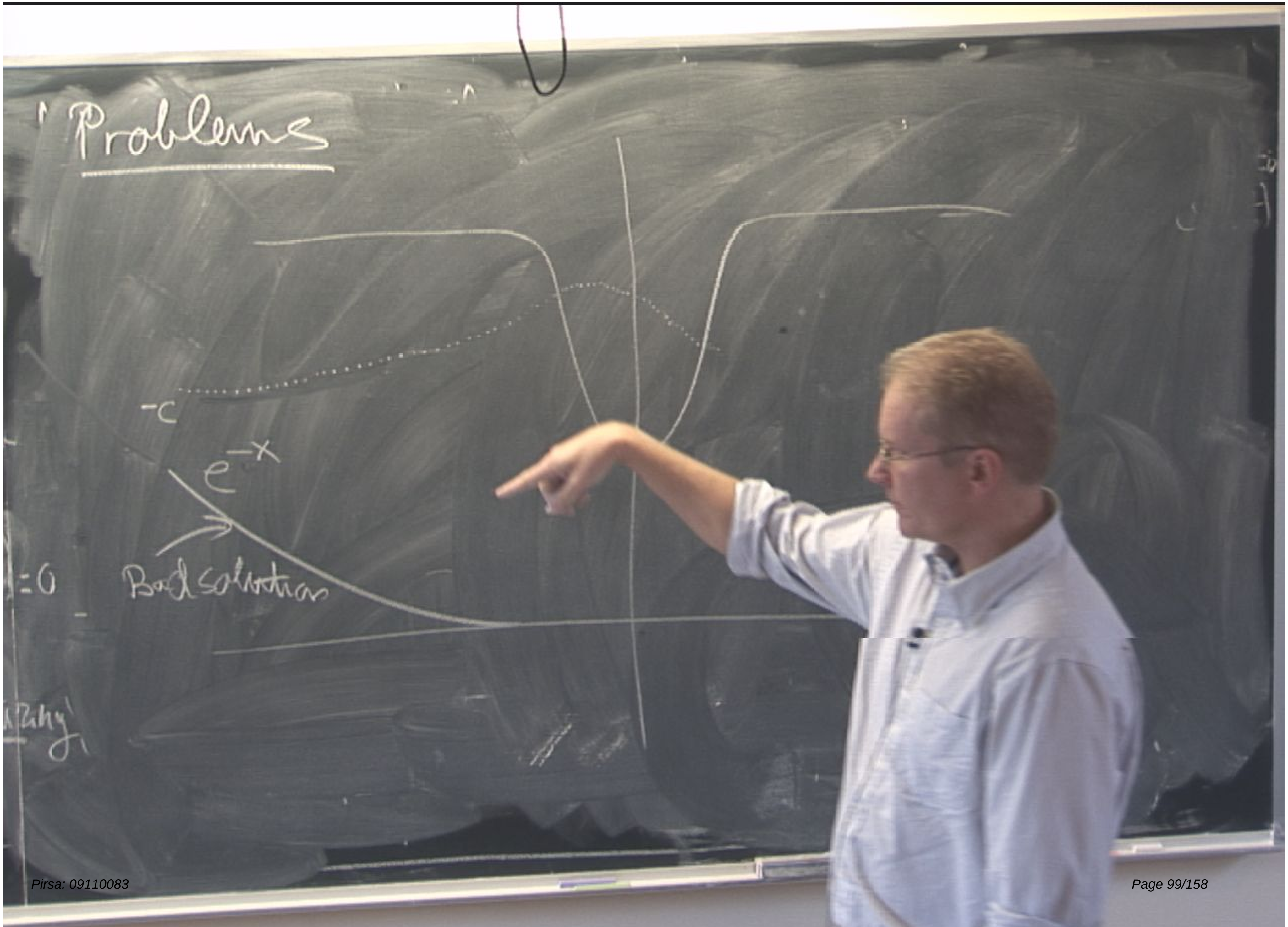
(Fix by normalizing

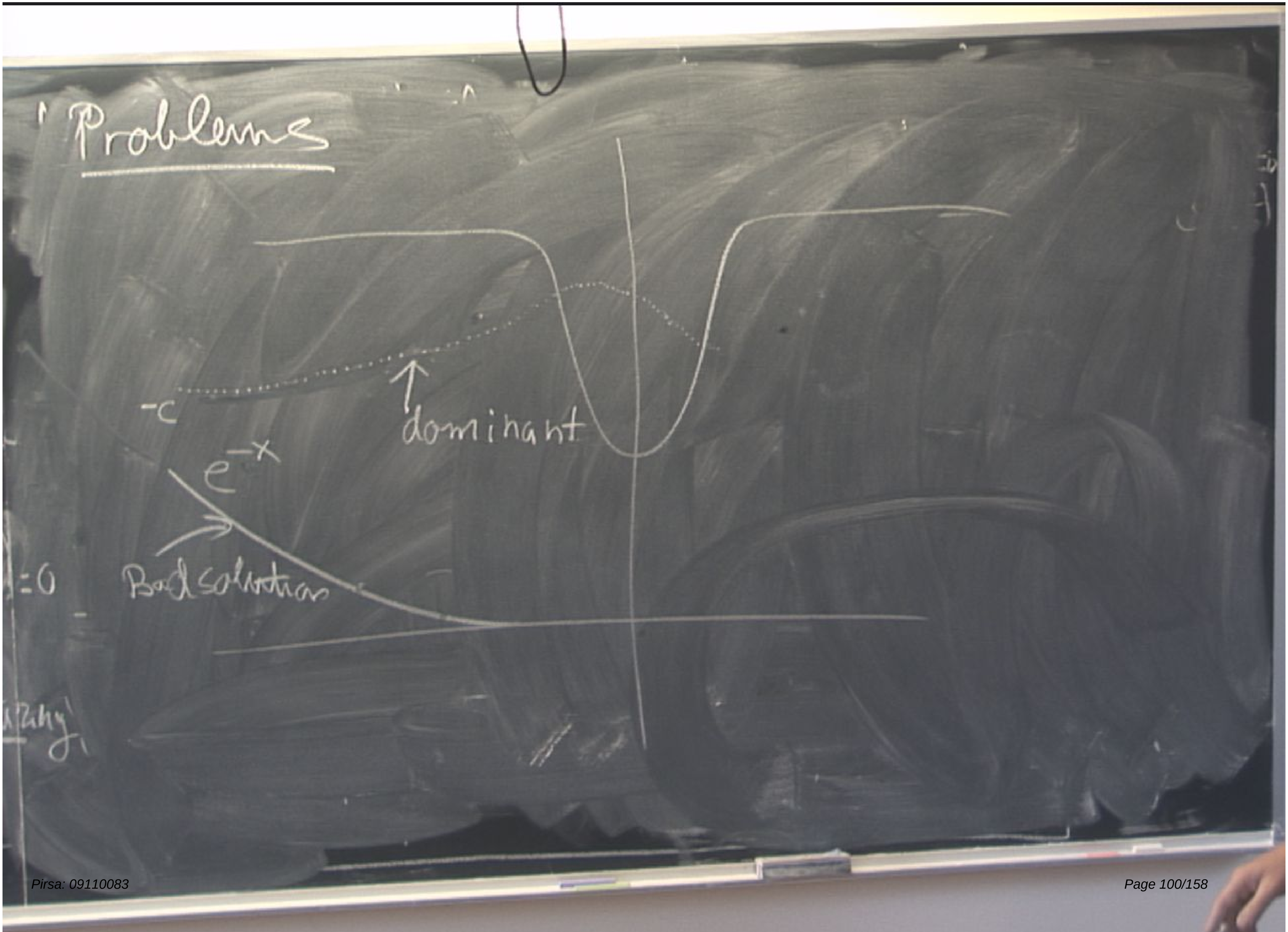
Fixed them all

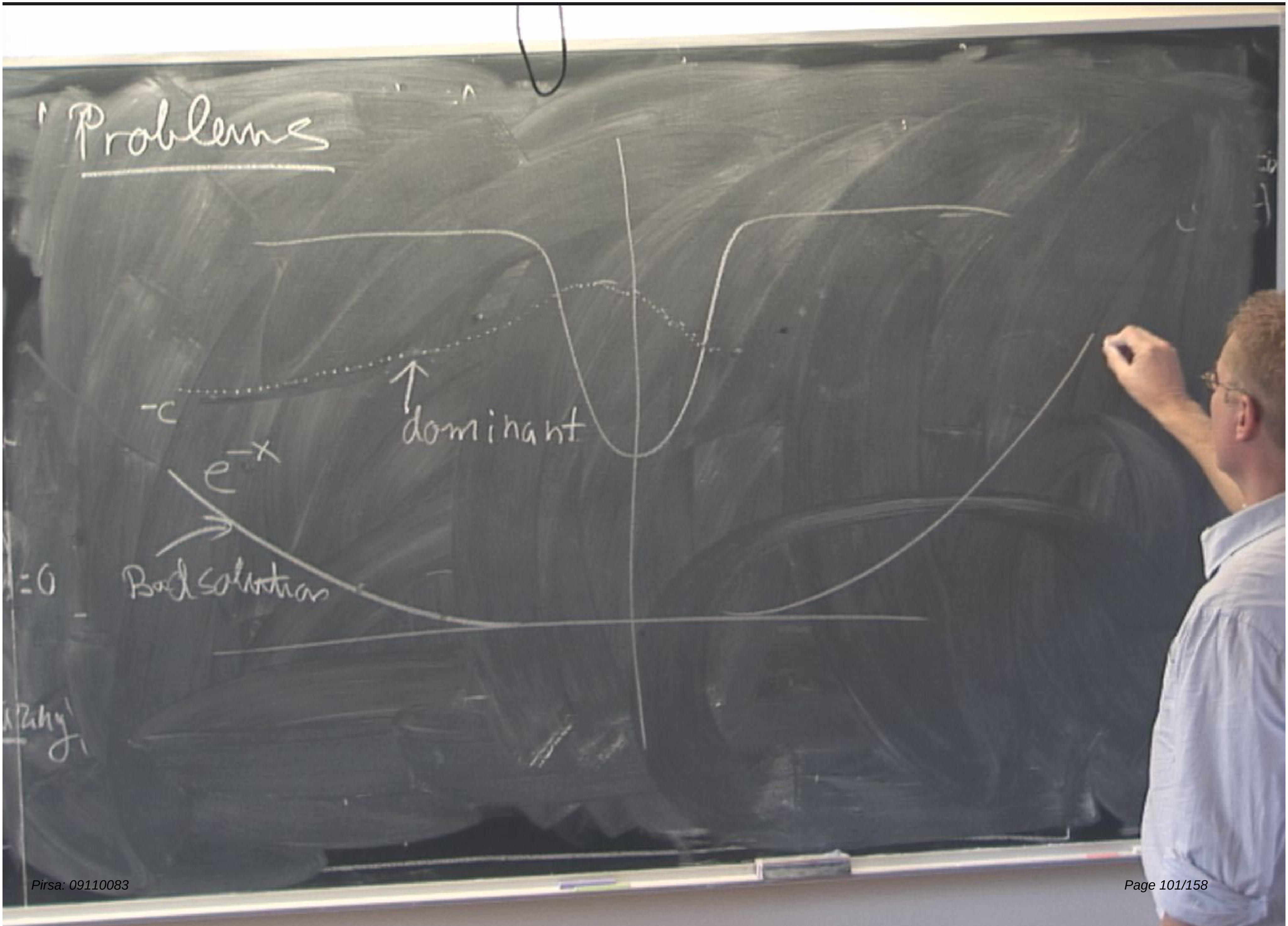

Problems
-c

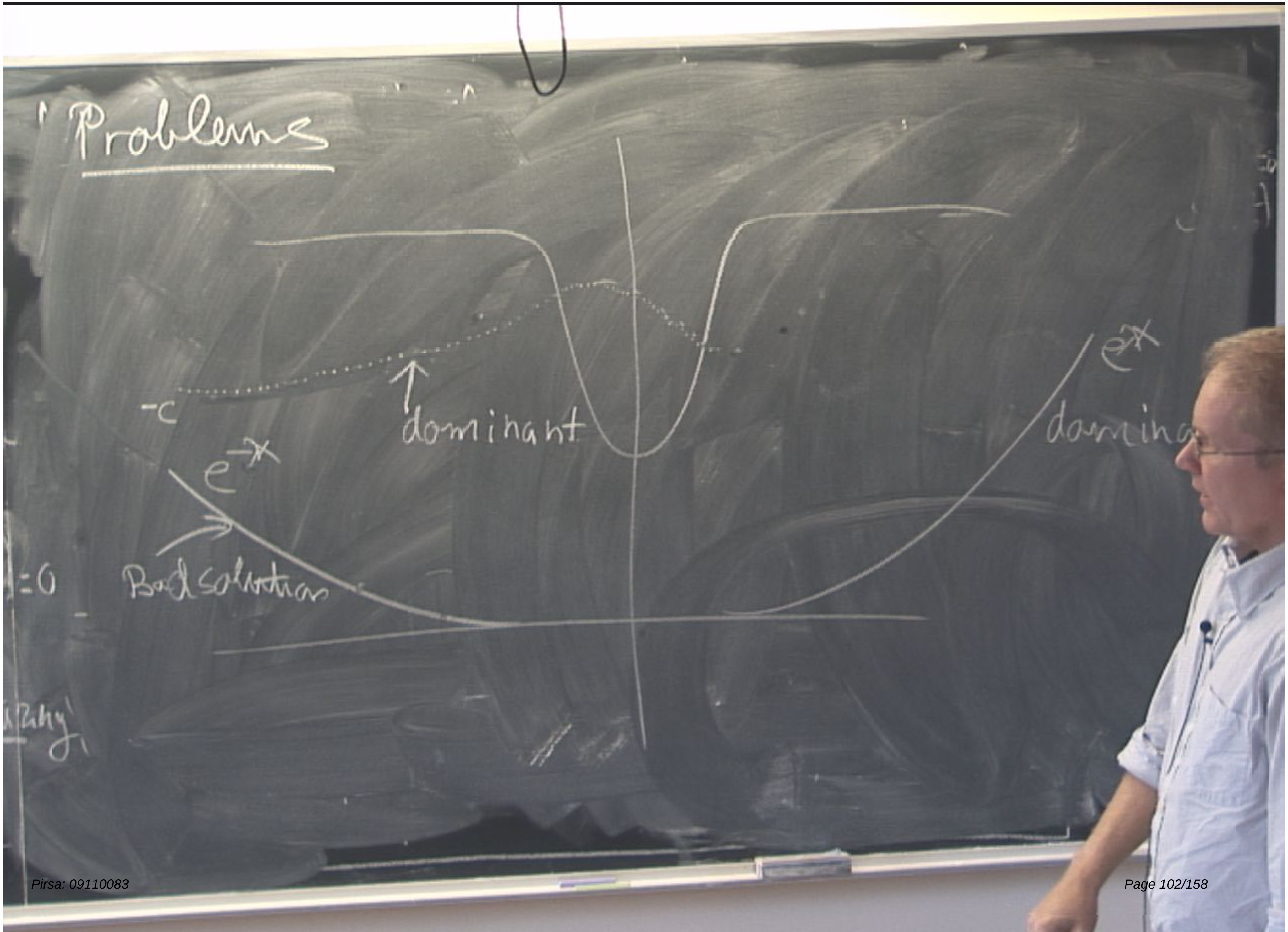

Problems
−c
=0


Problems
-c
e-x
=0

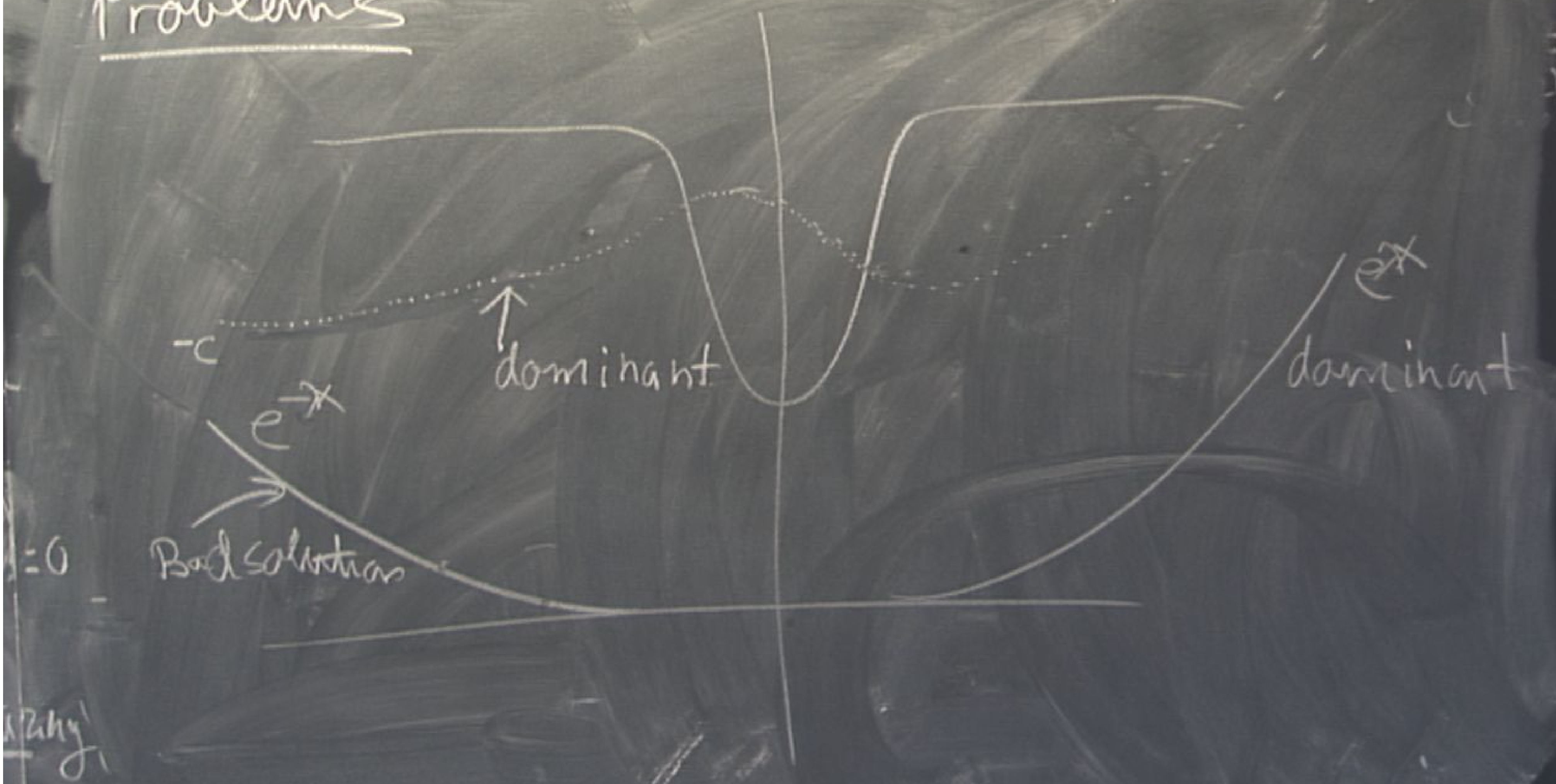

Problems
-c
e^{-x}
Bad solution

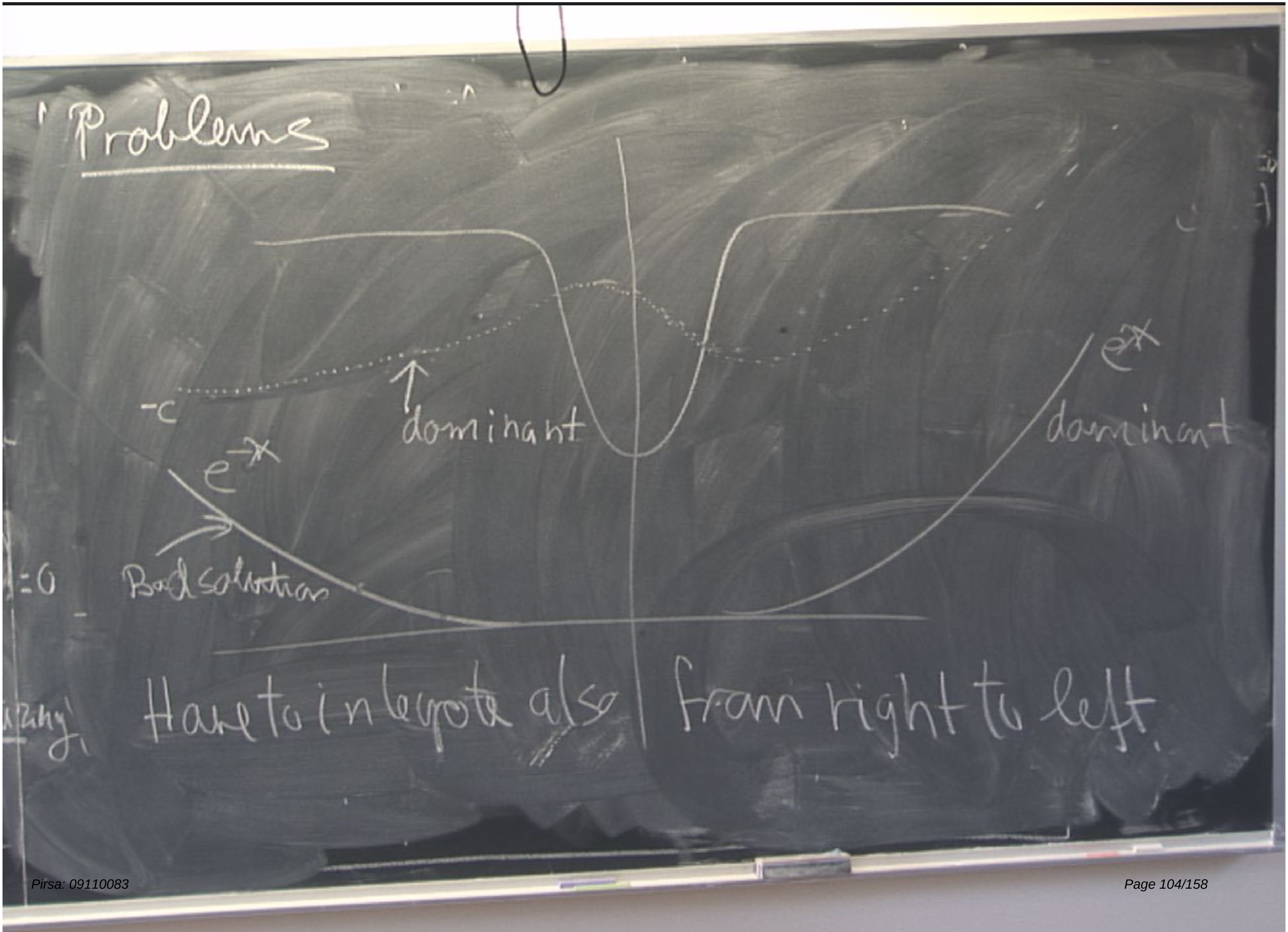

Problems
-c
dominant
e^{-x}
Bad solutions

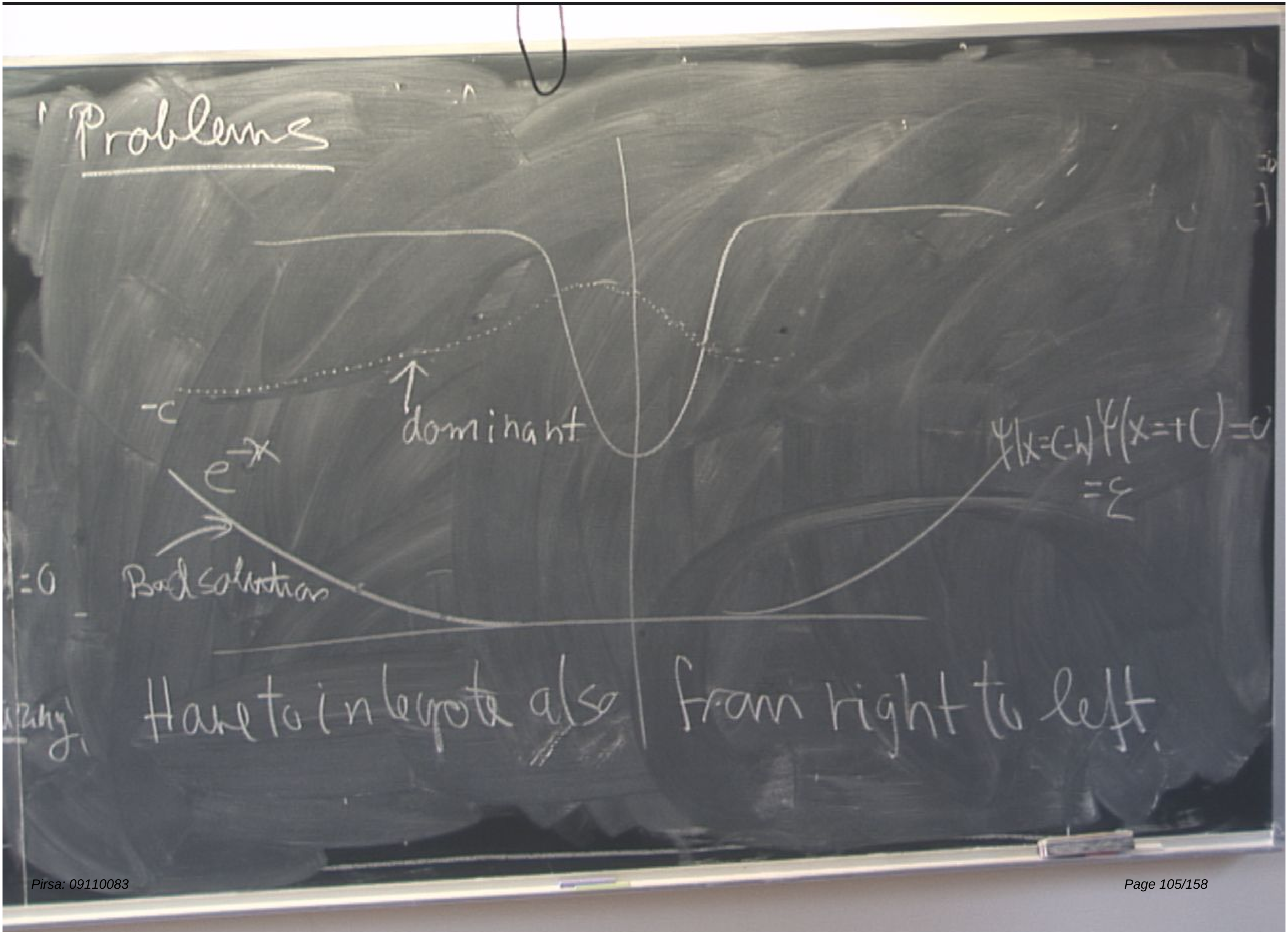

Problems
-c
dominant
e^{-x}
Bad solution

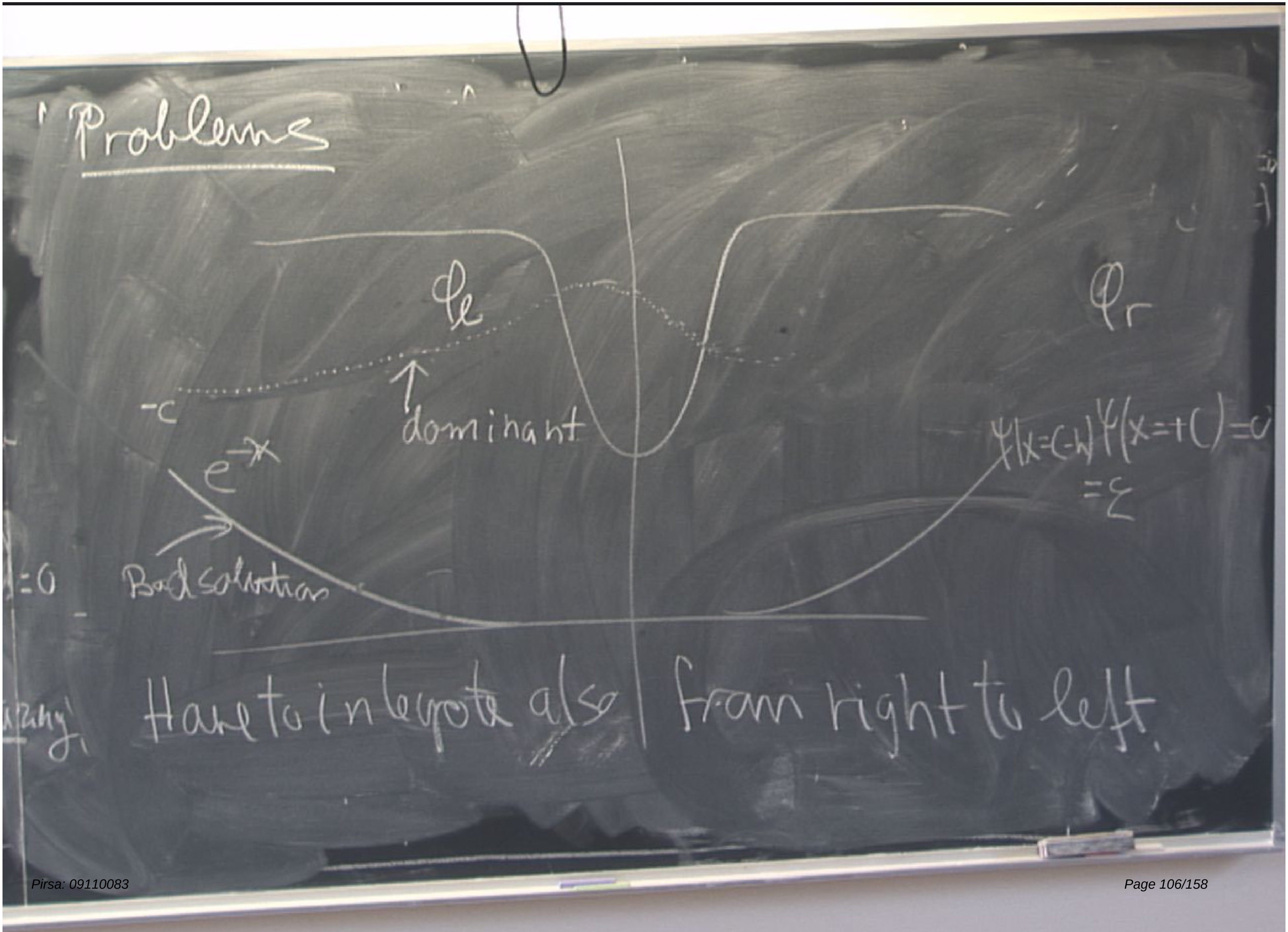

Problems
dominant
-c
Bad solution

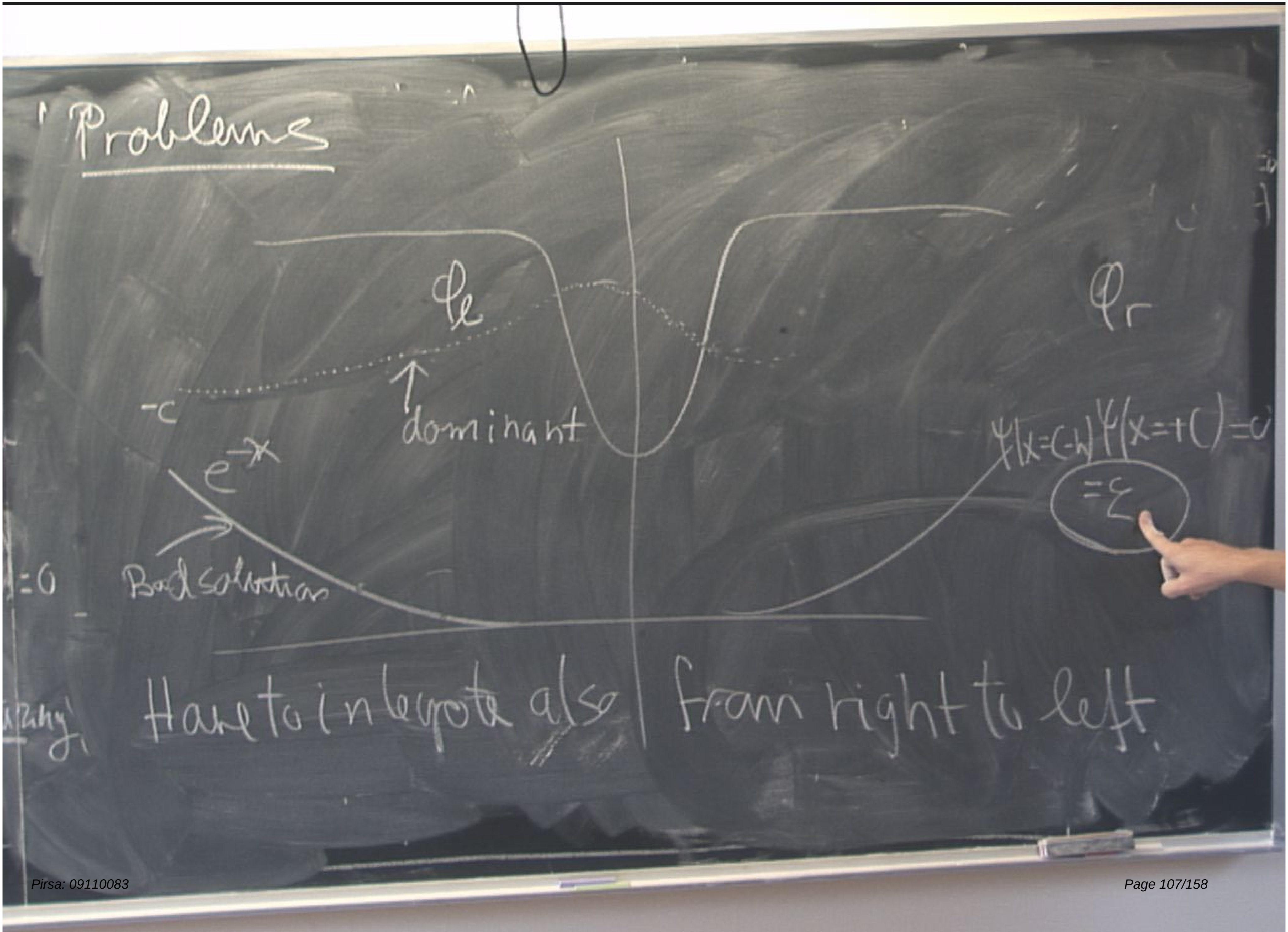
Problems

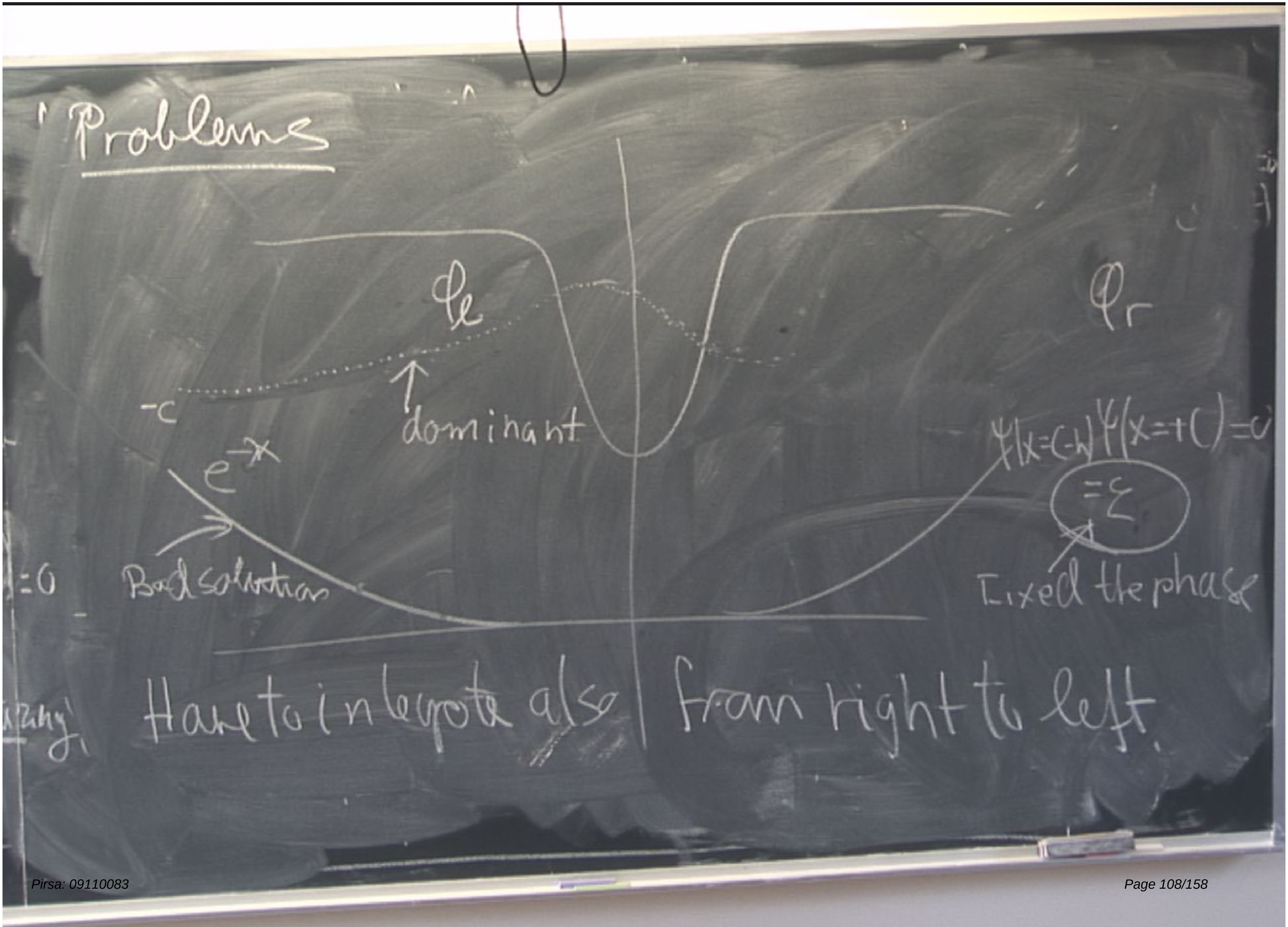


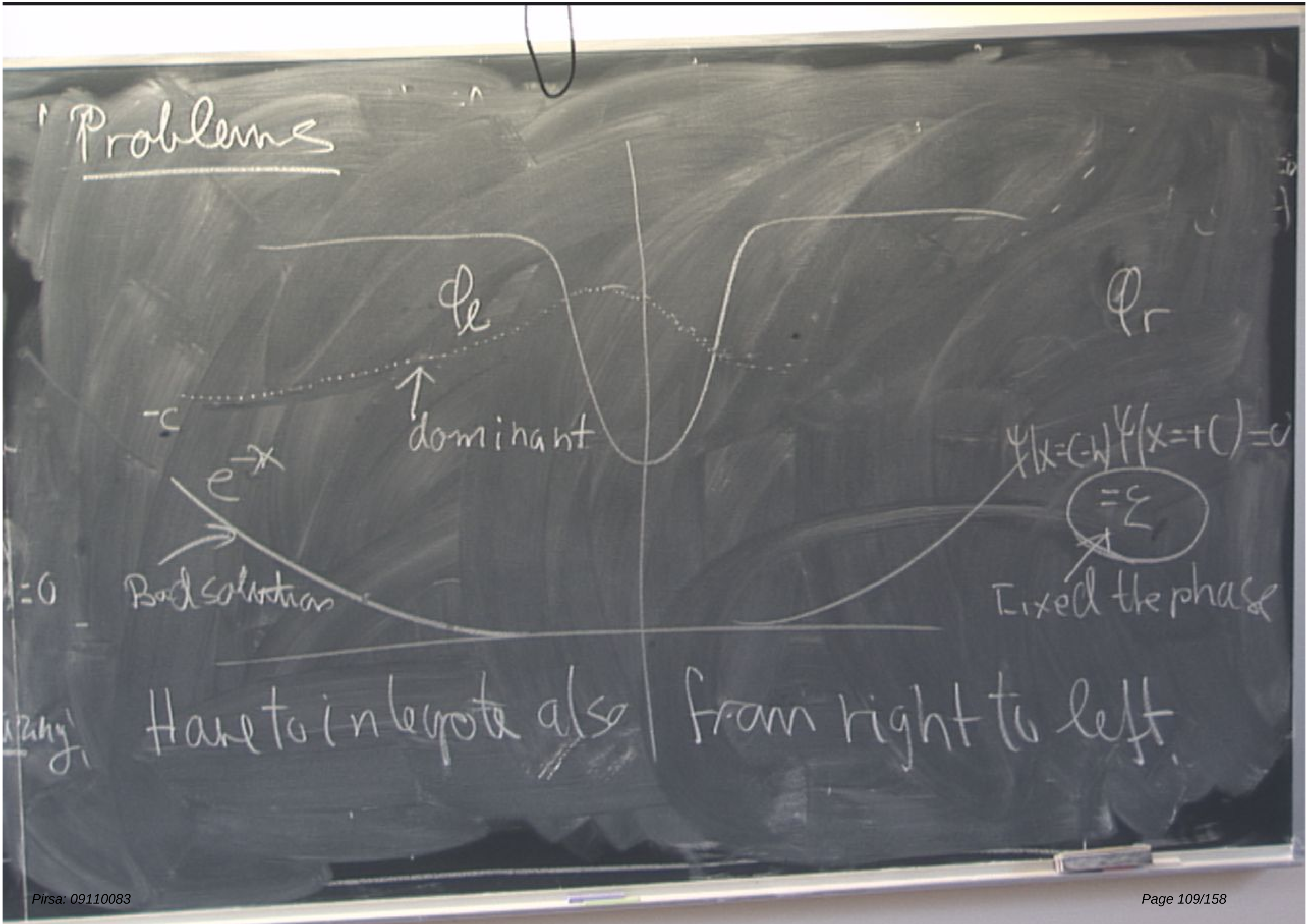

Problems
-c
dominant
e^{-x}
Bad solution
e^{x}
dominant
Have to integrate also from right to left.

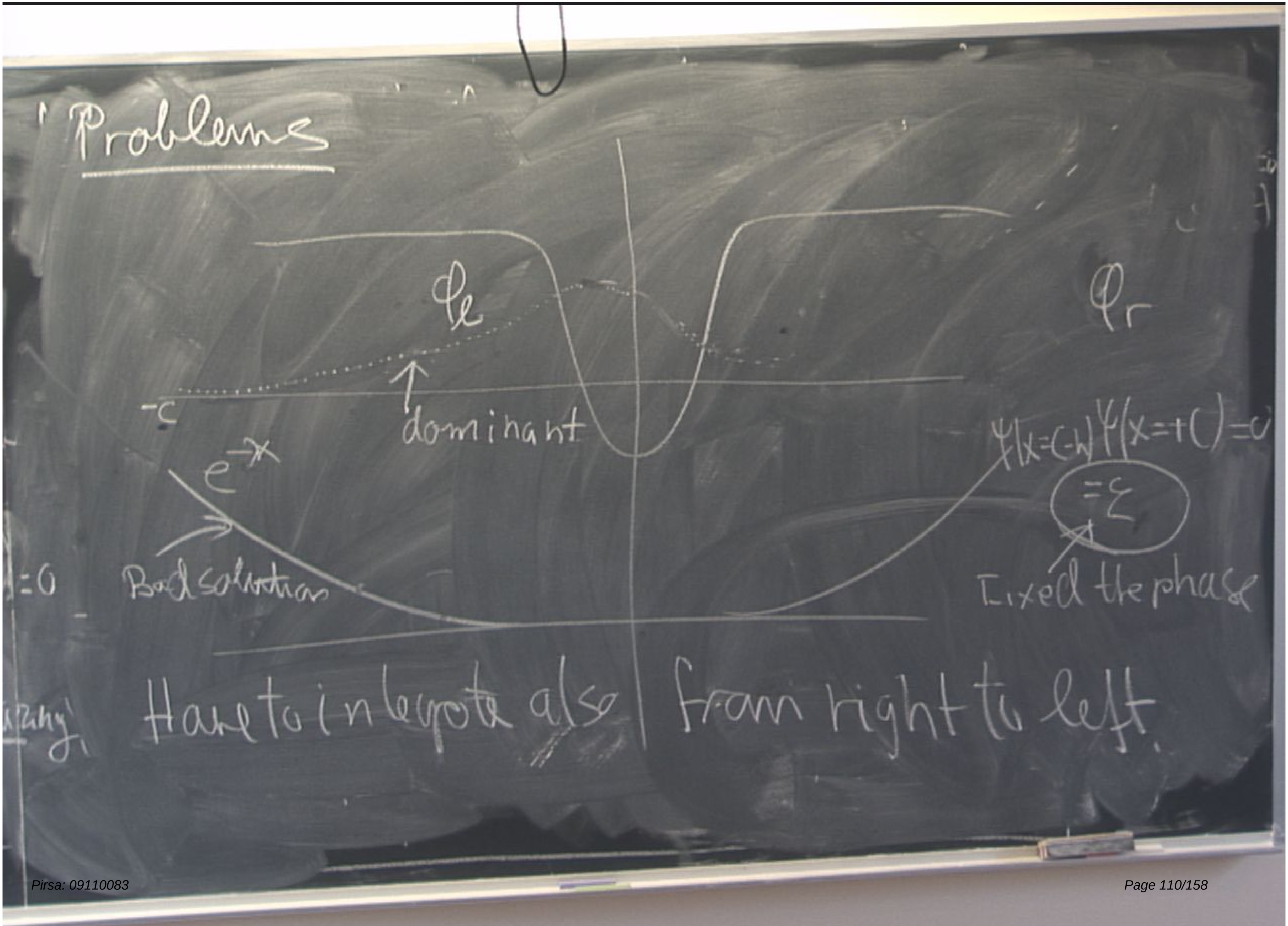

Problems
-c
dominant
e^{-x}
Bad solutions
=ε
Have to integrate also from right to left.

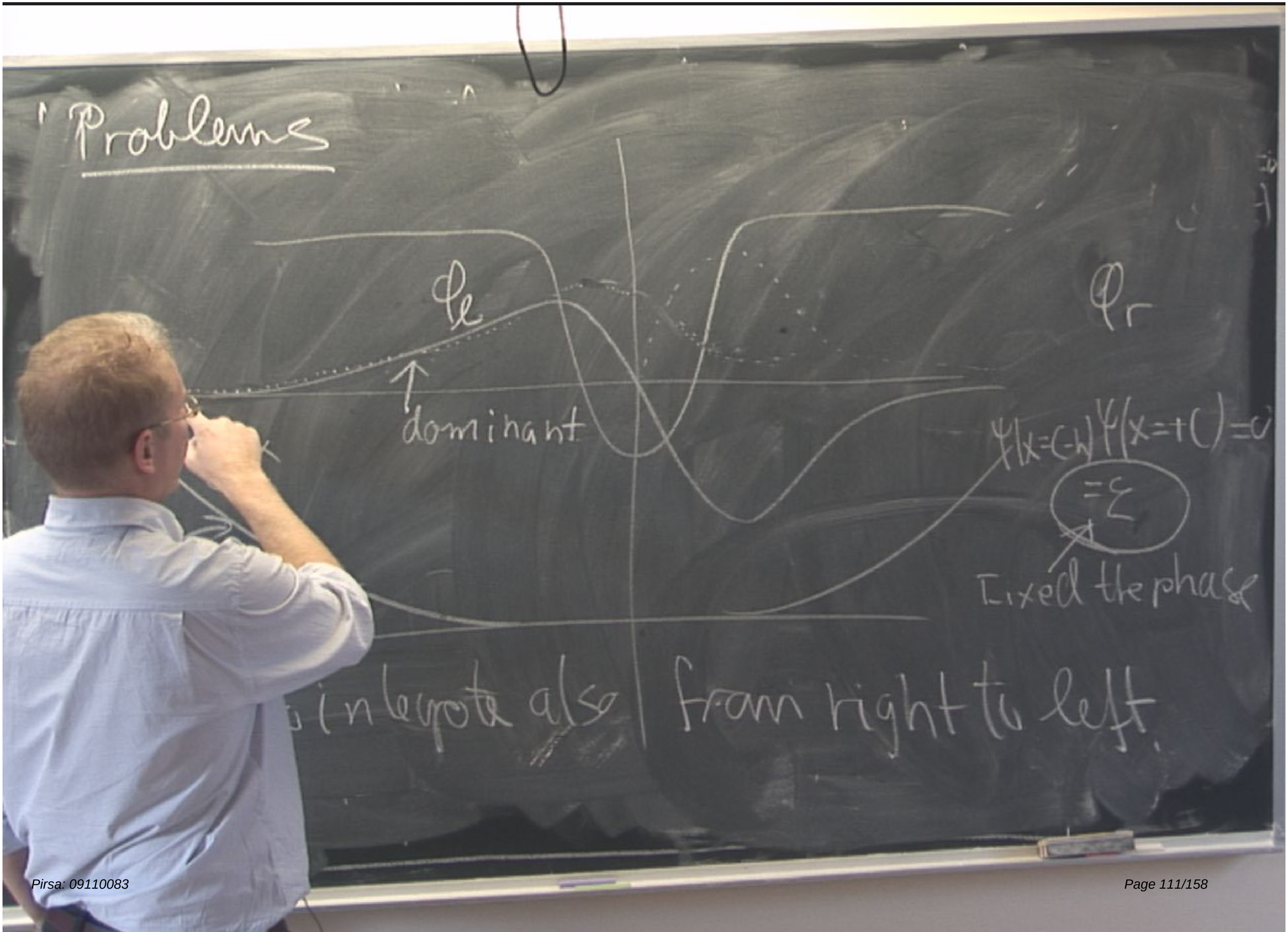

Problems
φ_ℓ
φ_r
dominant
$-c$
e^{-x}
Bad solution
$=\varepsilon$
Have to integrate also from right to left.

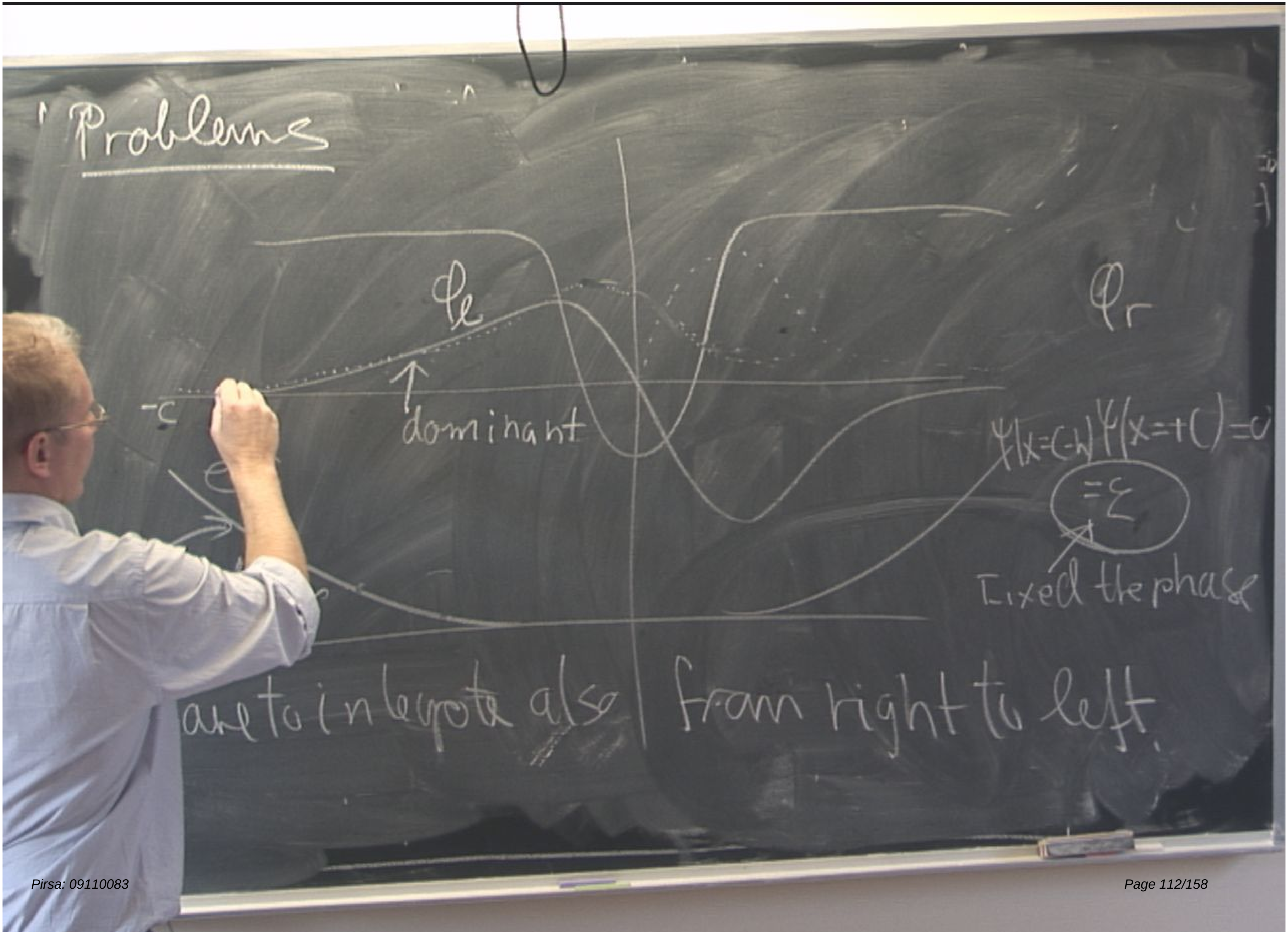

Problems
φl
φr
-c
dominant
e^-x
Bad solutions
=ε
Have to integrate also from right to left.

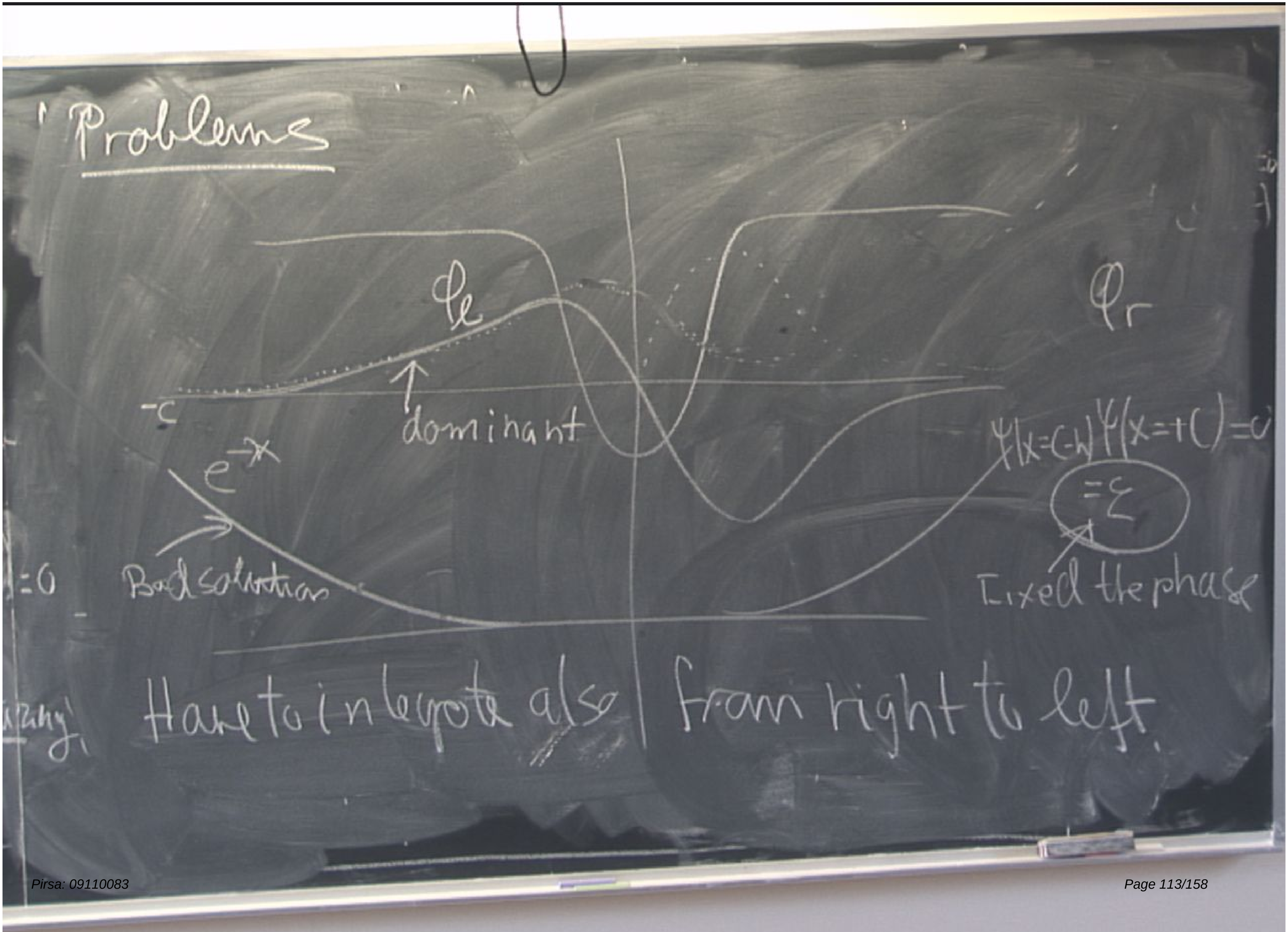

Problems
dominant
-c
Bad solution
Fixed the phase
=ε
Have to integrate also from right to left.

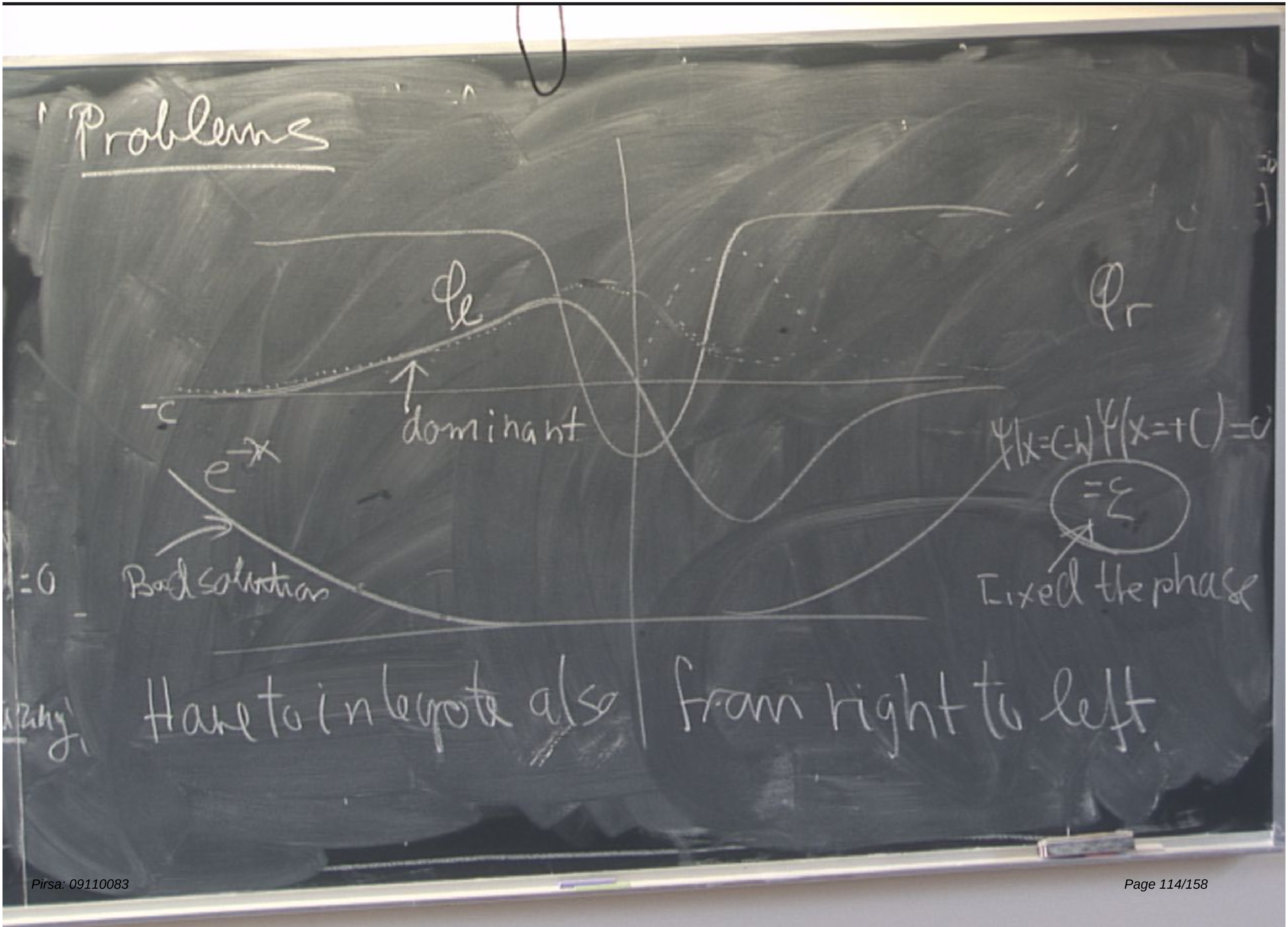

Problems
φ_l
φ_r
dominant
$-c$
e^{-x}
Bad solution
$\psi(x=-c)\psi(x=+c)=0$
$=\varepsilon$
Fixed the phase
Have to integrate also from right to left.


Problems
dominant
e^{-x}
Bad solution
Fixed the phase
Have to integrate also from right to left.

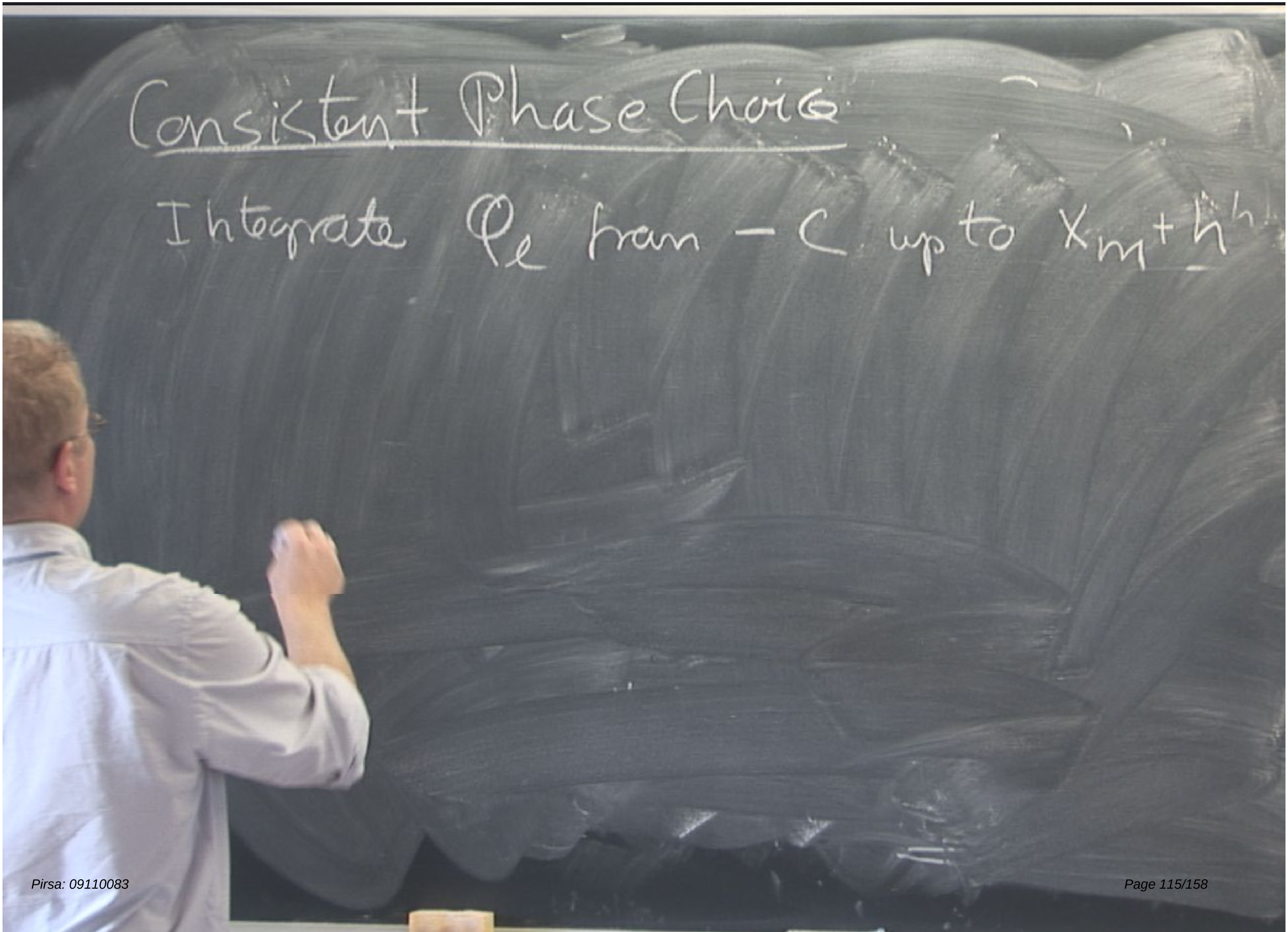

Problems
dominant
=ε
fixed the phase
from right to left.


Problems
φl
φr
dominant
-c
=ε
fixed the phase
are to integrate also from right to left.


Problems
dominant
Bad solutions
Fixed the phase
Have to integrate also from right to left.



Problems
φ_l
φ_r
dominant
$-c$
e^{-x}
Bad solutions
$=\varepsilon$
Fixed the phase
Have to integrate also from right to left.


Consistent Phase Choice
Integrate φℓ from − C up to xm + h

Consistent Phase Choice

Integrate φ_ℓ from $-c$ up to $x_m + h$

Integrate φ_r from $+c$ down to $x_m - h$

Consistent Phase Choice

Integrate φ_ℓ from $-c$ up to $x_m + h$

Integrate φ_r from $+c$ down to $x_m - h$

Consistent Phase Choice

Integrate φ_ℓ from $-c$ up to $x_m + h$

Integrate φ_r from $+c$ down to $x_m - h$

$$\varphi_r \to \varphi_r / \varphi_\ell(x_m)$$

Consistent Phase Choice

Integrate φ_ℓ from $-C$ up to $x_m + h$

Integrate φ_r from $+C$ down to $x_m - h$

$$\varphi_r \to \varphi_r / \varphi_\ell(x_m)$$

Consistent Phase Choice

Integrate φ_ℓ from $-c$ up to $x_m + h$

Integrate φ_r from $+c$ down to $x_m - h$

$$\varphi_r \to \varphi_r / \varphi_\ell(x_m)$$

Automatically $\varphi_r(x_m) = \varphi_\ell(x_m)$

Consistent Phase Choice

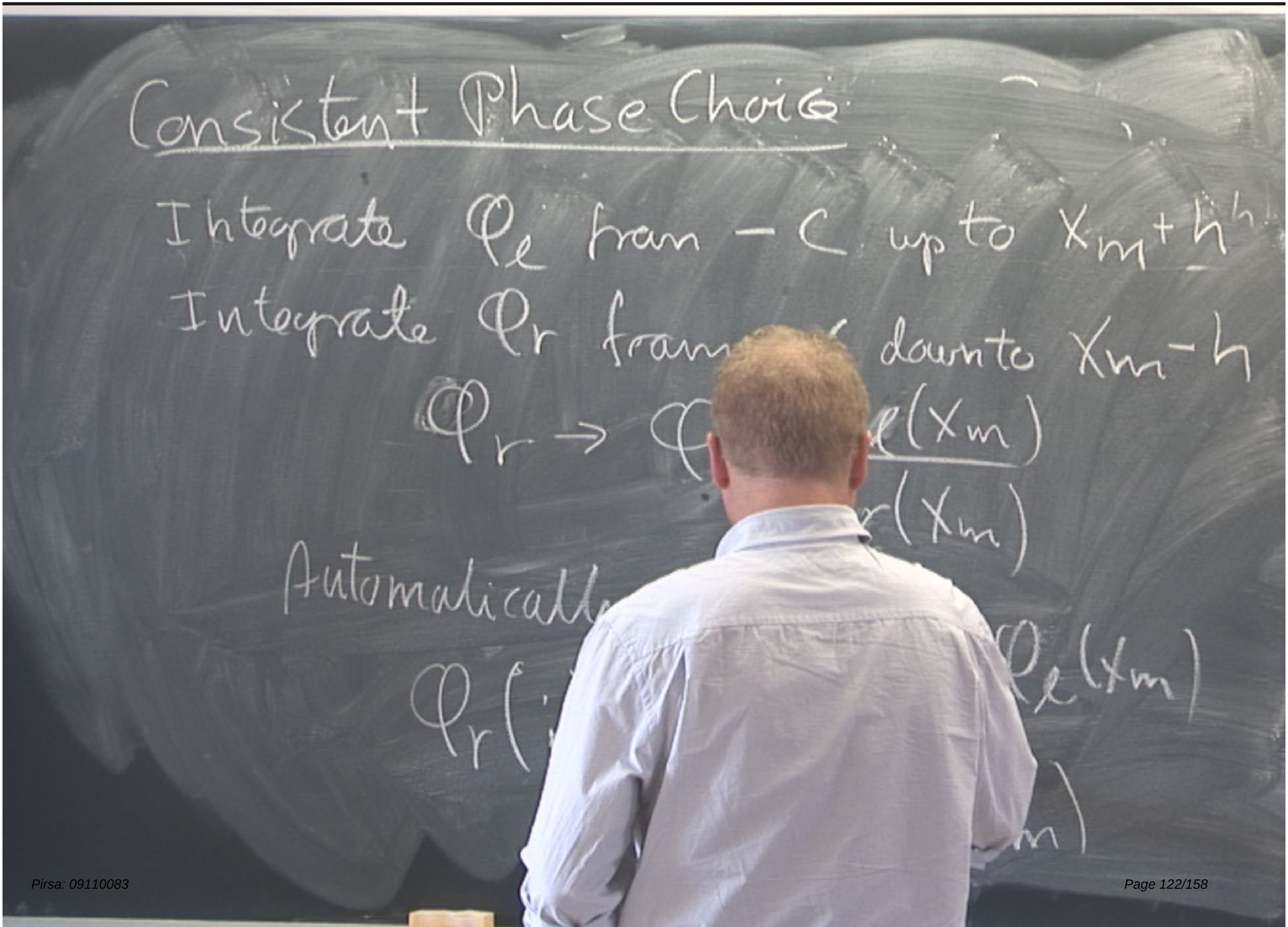
Integrate φ_ℓ from $-C$ up to $x_m + h$

Integrate φ_r from $+C$ down to $x_m - h$

$$\varphi_r \to \varphi_r / \varphi_\ell(x_m)$$

Automatically $\varphi_r(x_m) = \varphi_\ell(x_m)$

$$\varphi_r(\cdot) = \varphi_r(\cdot)/\varphi_\ell(x_m)$$


Consistent Phase Choice
Integrate φl from −C up to xm+h
Integrate φr from C down to xm−h
φr → φ
l(xm)
r(xm)
Automatically
φr(
φl(xm)

Consistent Phase Choice

Integrate φ_ℓ from $-C$ up to $x_m + h$

Integrate φ_r from $+C$ down to $x_m - h$

$$\varphi_r \to \varphi_r \frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$

Automatically $\varphi_r(x_m) = \varphi_\ell(x_m)$

$$\varphi_r(\cdot) = \varphi_r(\cdot)\frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$

Consistent Phase Choice

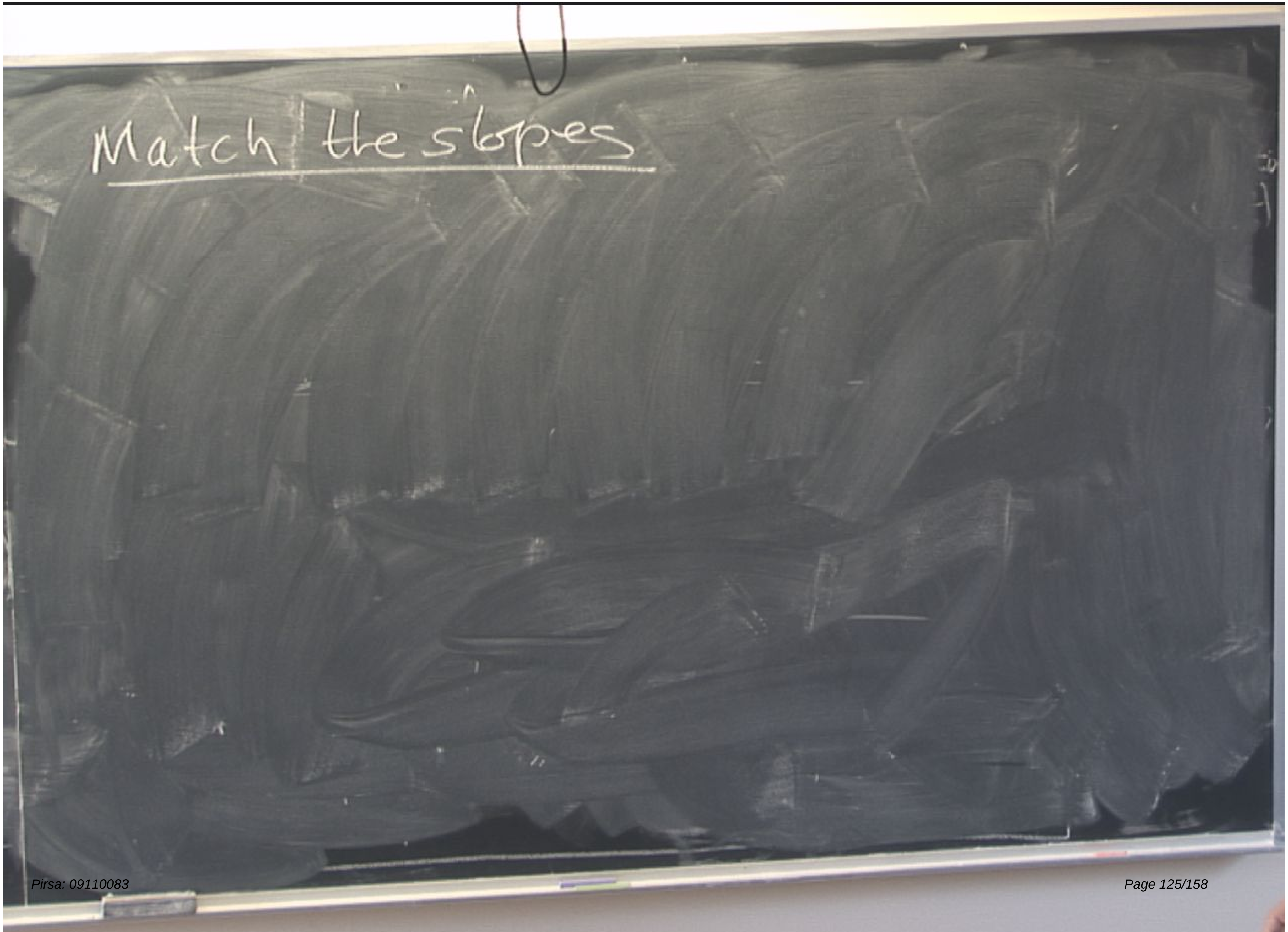
Integrate φ_ℓ from $-C$ up to $x_m + h$

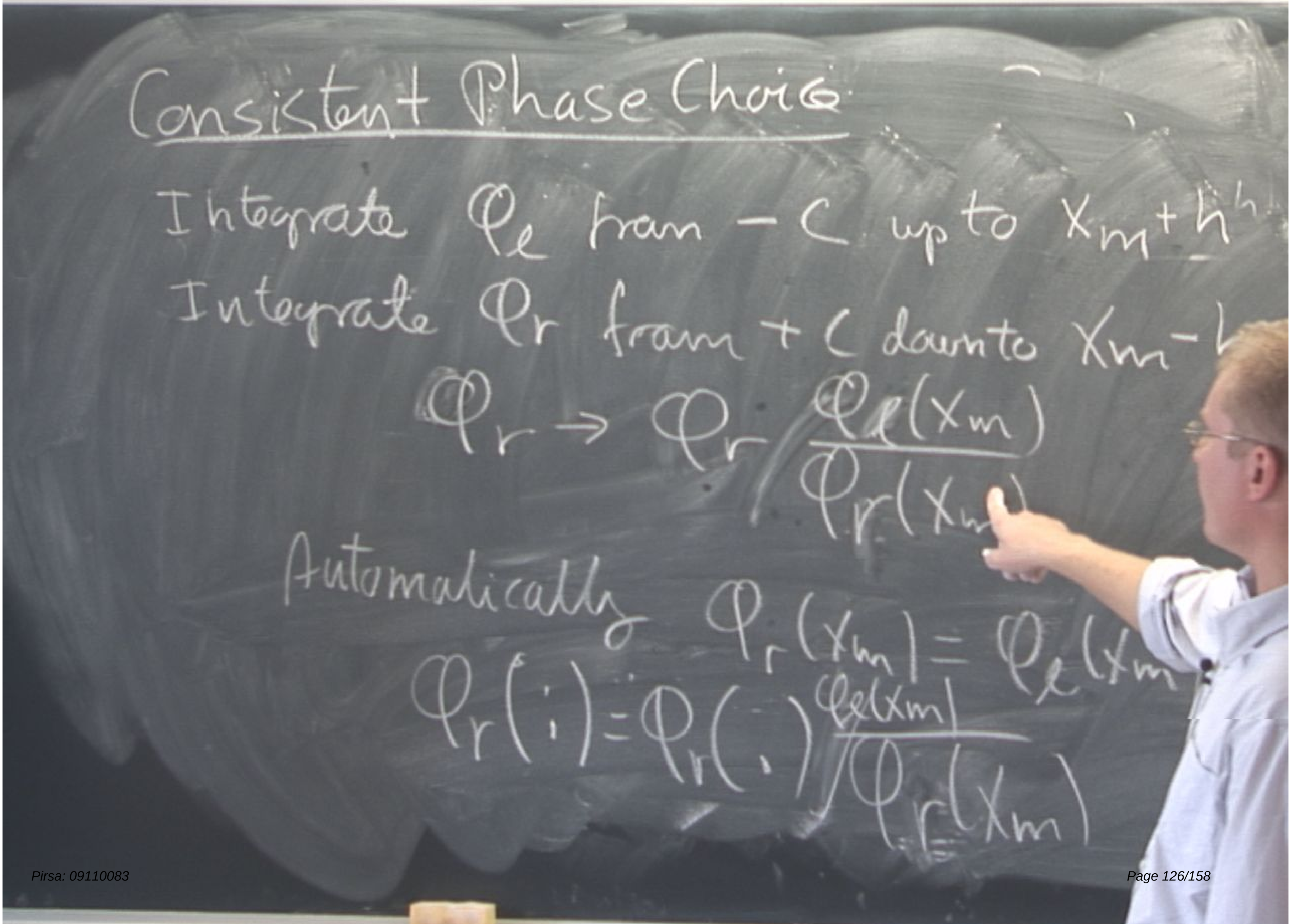
Integrate φ_r from $+C$ down to $x_m - h$

$$\varphi_r \to \varphi_r \frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$

Automatically $\varphi_r(x_m) = \varphi_\ell(x_m)$

$$\varphi_r(\cdot) = \varphi_r(\cdot)\,\frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$


Match the slopes


Consistent Phase Choice
Integrate φl from −C up to xm+h
Integrate φr from +C down to xm−h
φr → φr φl(xm)/φr(xm)
Automatically φr(xm) = φl(xm)
φr(:) = φr(:) φl(xm)/φr(xm)

Consistent Phase Choice

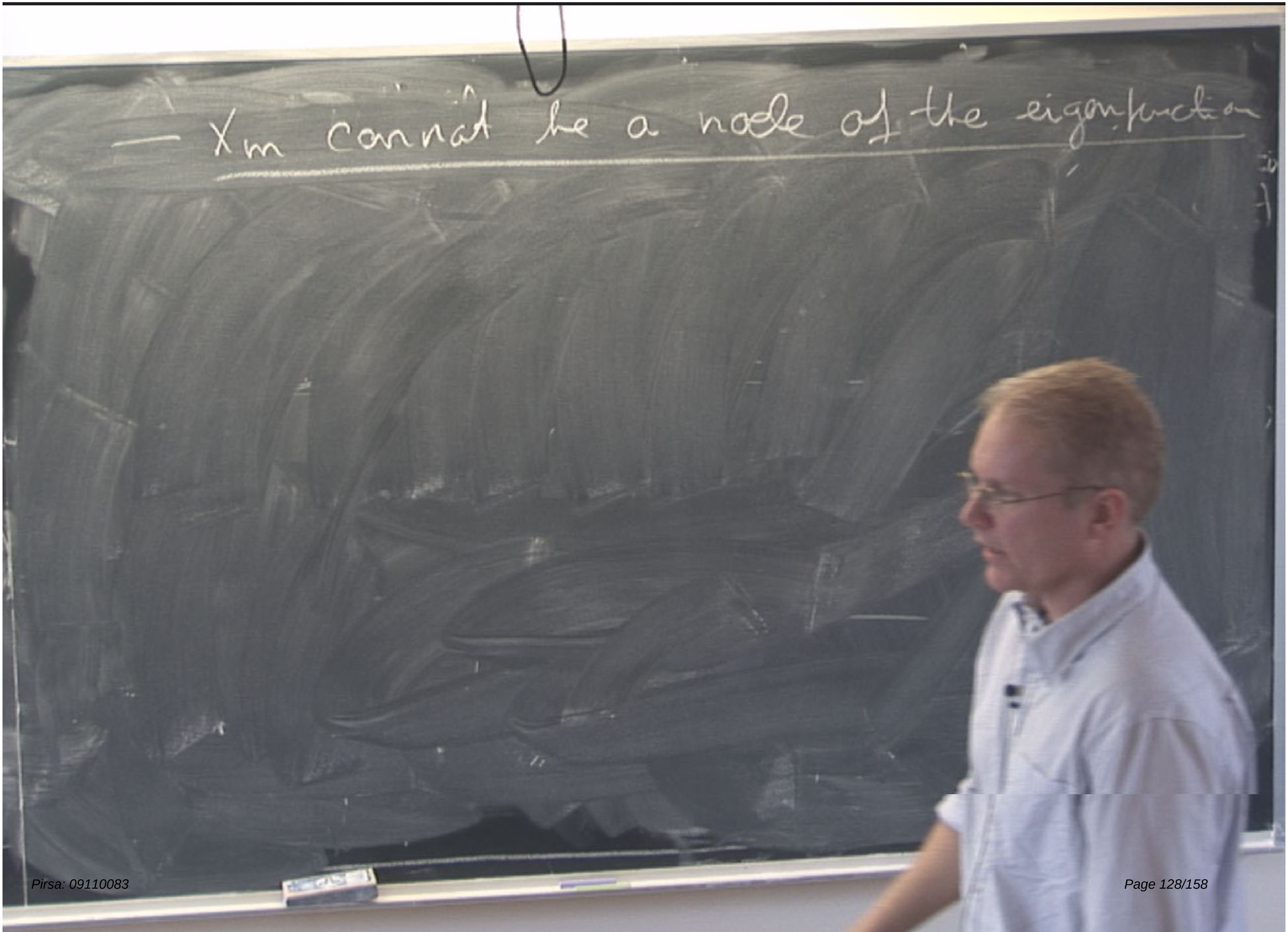
Integrate φ_ℓ from $-c$ up to $x_m + h$

Integrate φ_r from $+c$ down to $x_m - h$

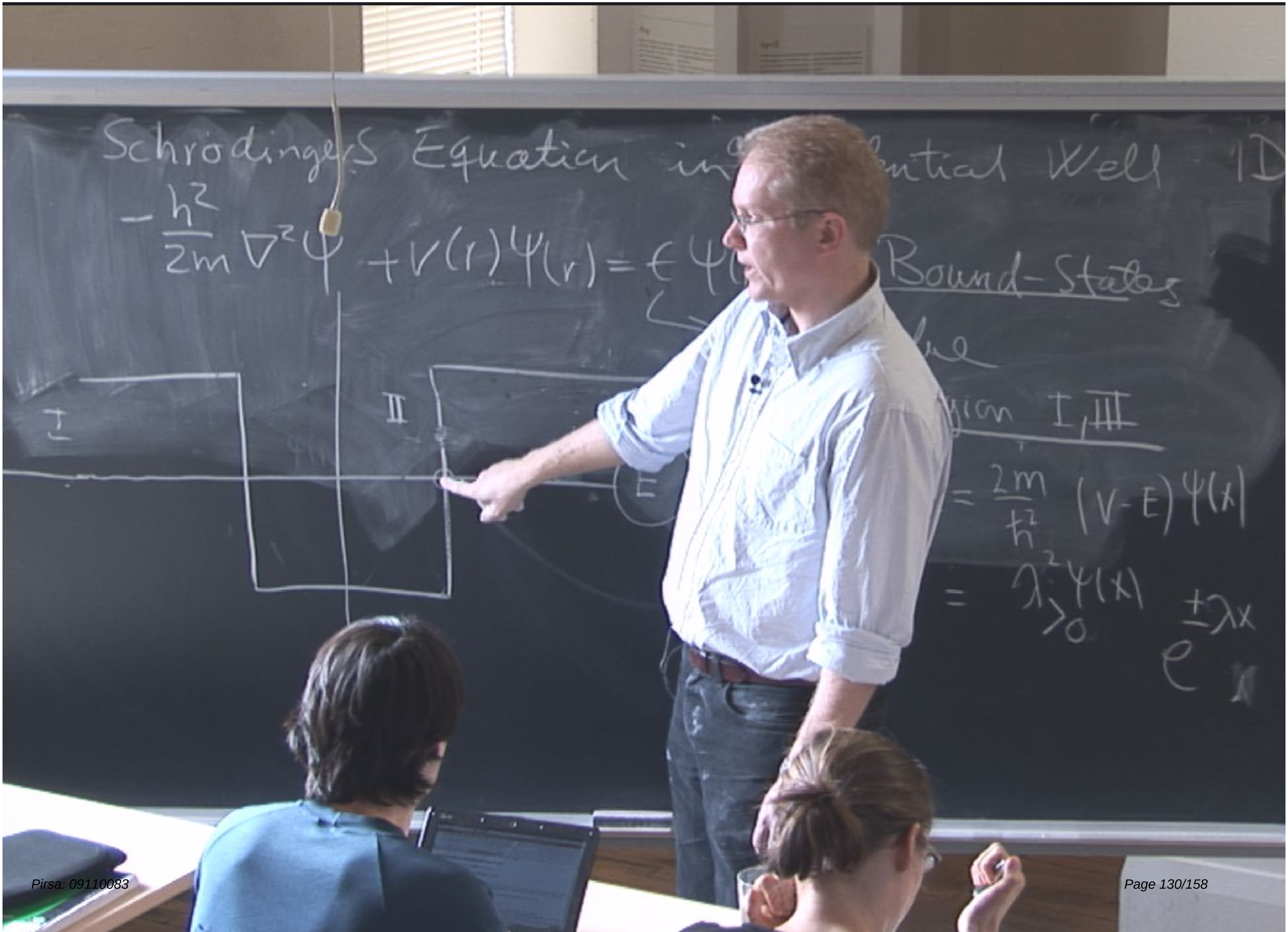
$$\varphi_r \to \varphi_r \frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$

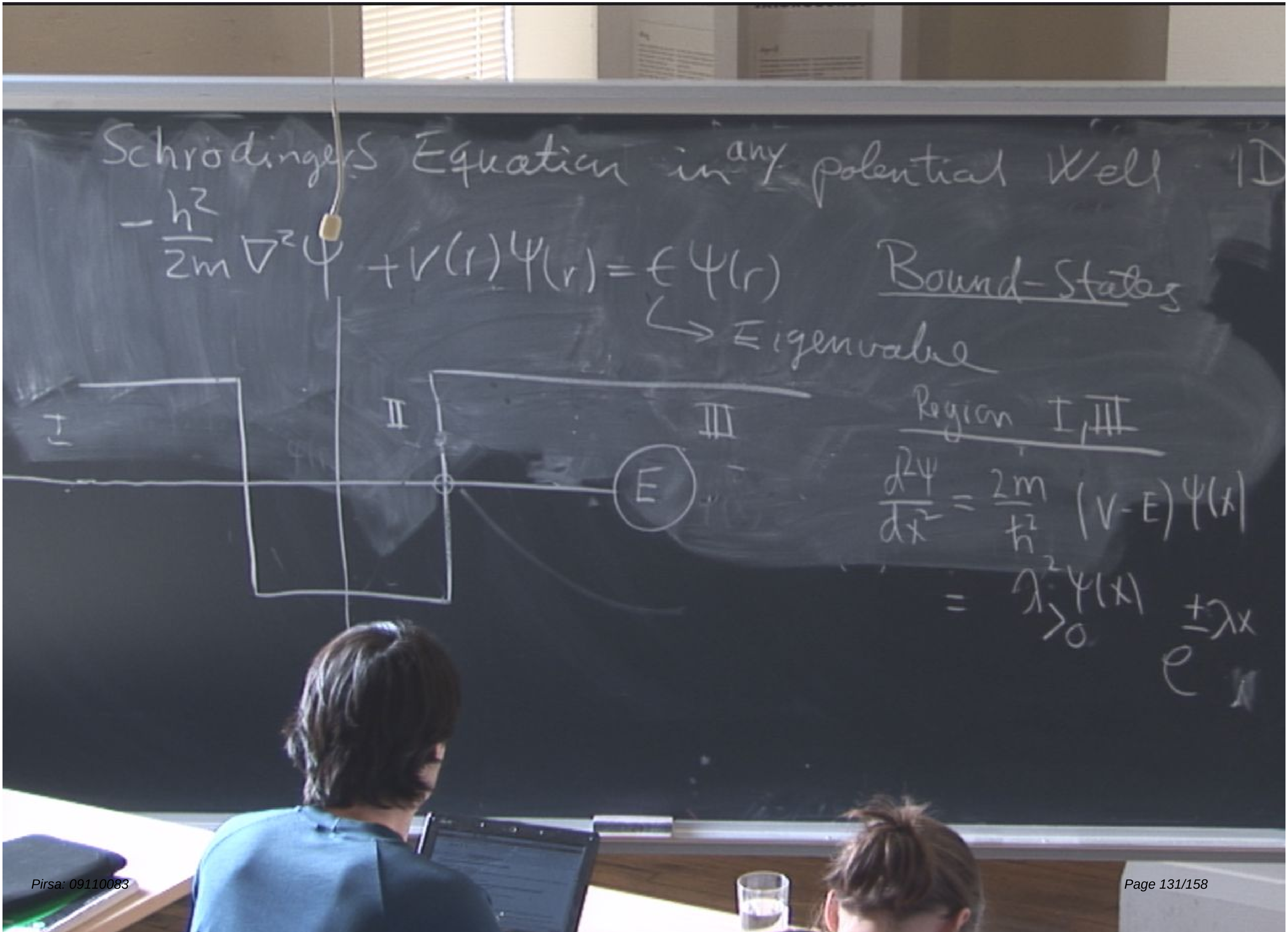
Automatically $\varphi_r(x_m) = \varphi_\ell(x_m)$

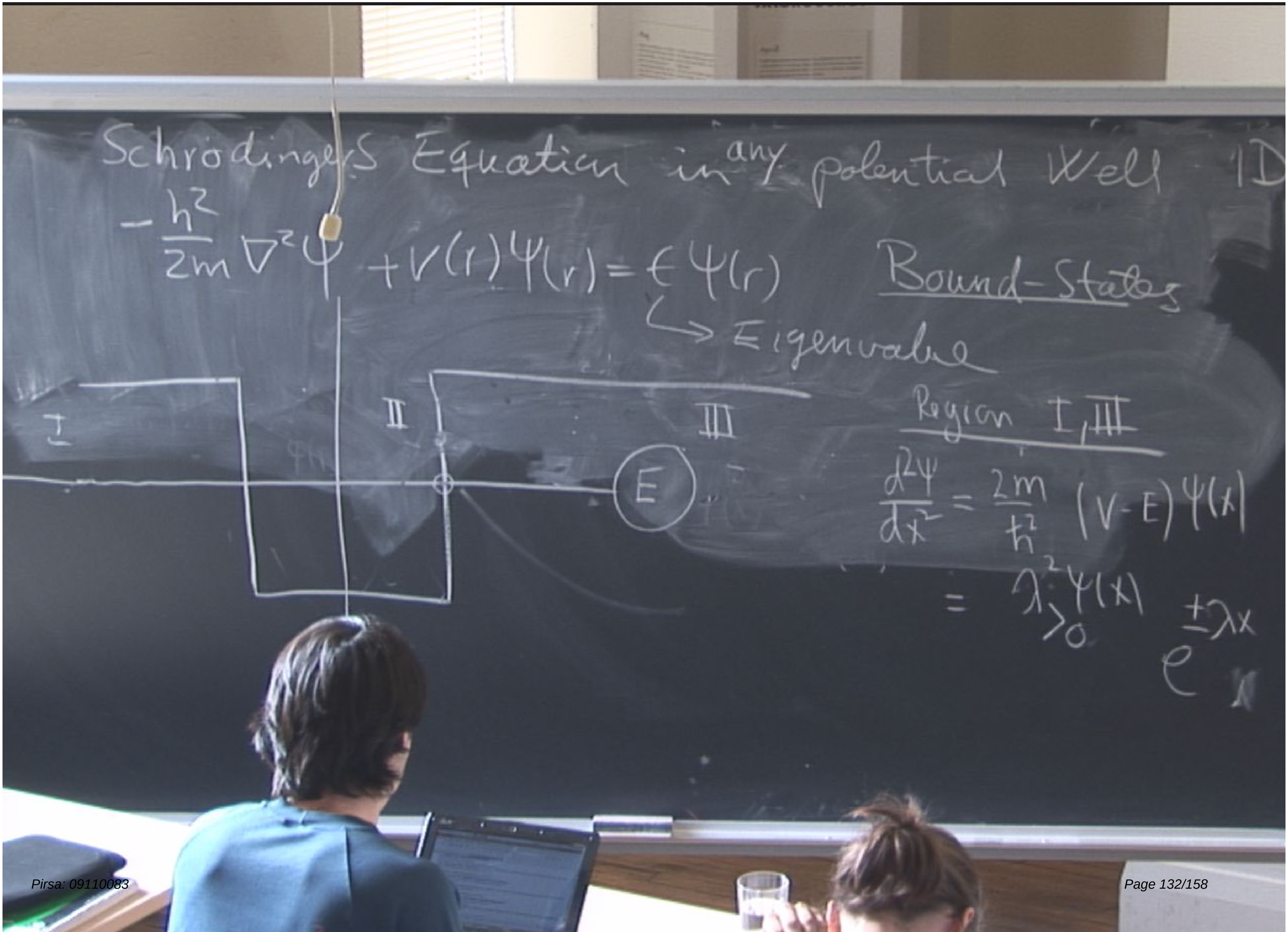
$$\varphi_r(\cdot) = \varphi_r(\cdot)\frac{\varphi_\ell(x_m)}{\varphi_r(x_m)}$$


— X_m cannot be a node of the eigenfunction

— x_m cannot be a node of the eigenfunction


Schrödinger's Equation
Bound-States
I
II
E


Schrodinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$
$= \lambda^2\psi(x)$
$\lambda > 0$
$e^{\pm\lambda x}$


Schrodinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi(r) = \epsilon\,\psi(r)$
Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$
$= \lambda^2\psi(x)$
$\lambda > 0$
$e^{\pm\lambda x}$

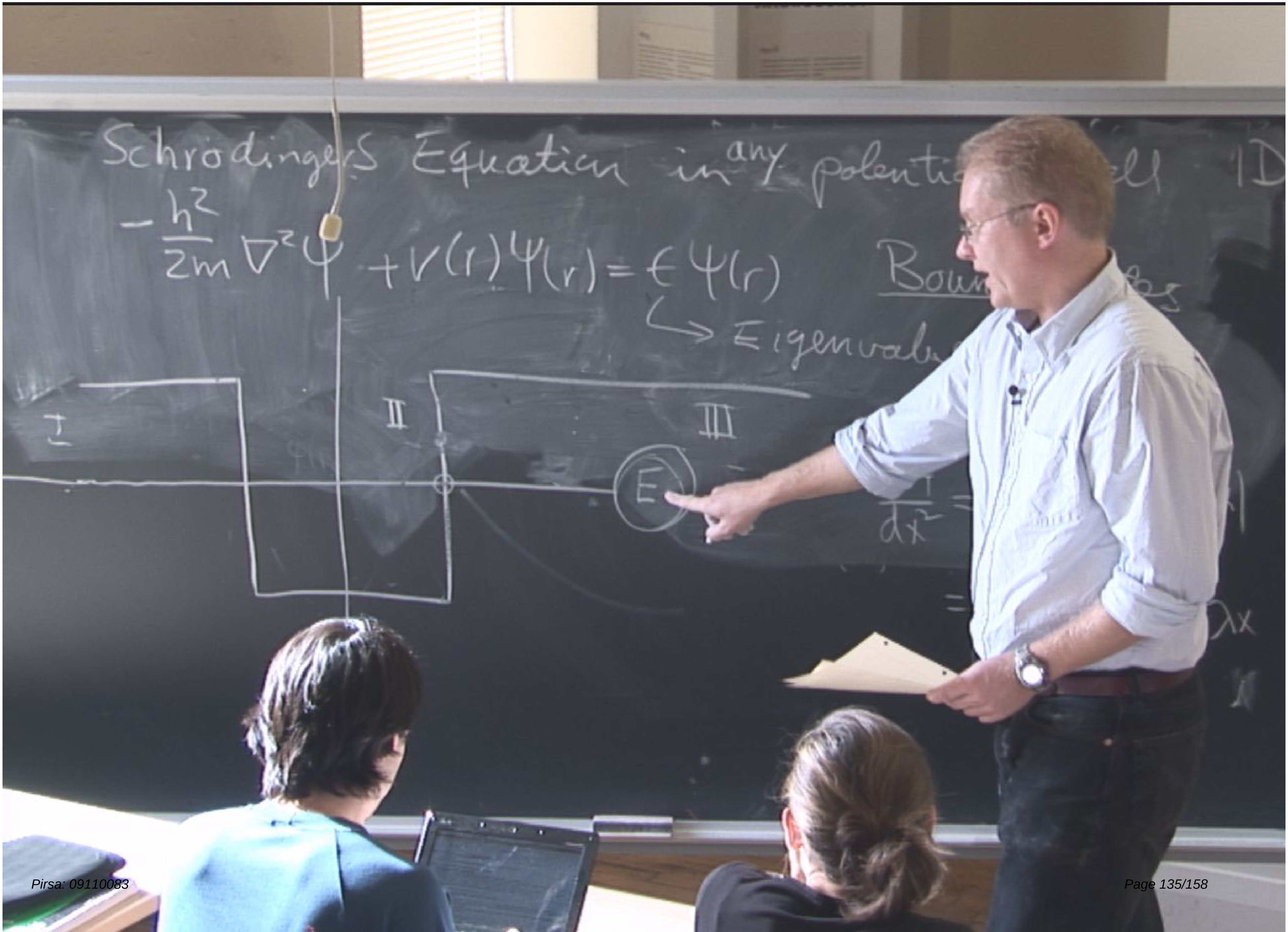
— x_m cannot be a node of the eigenfunction

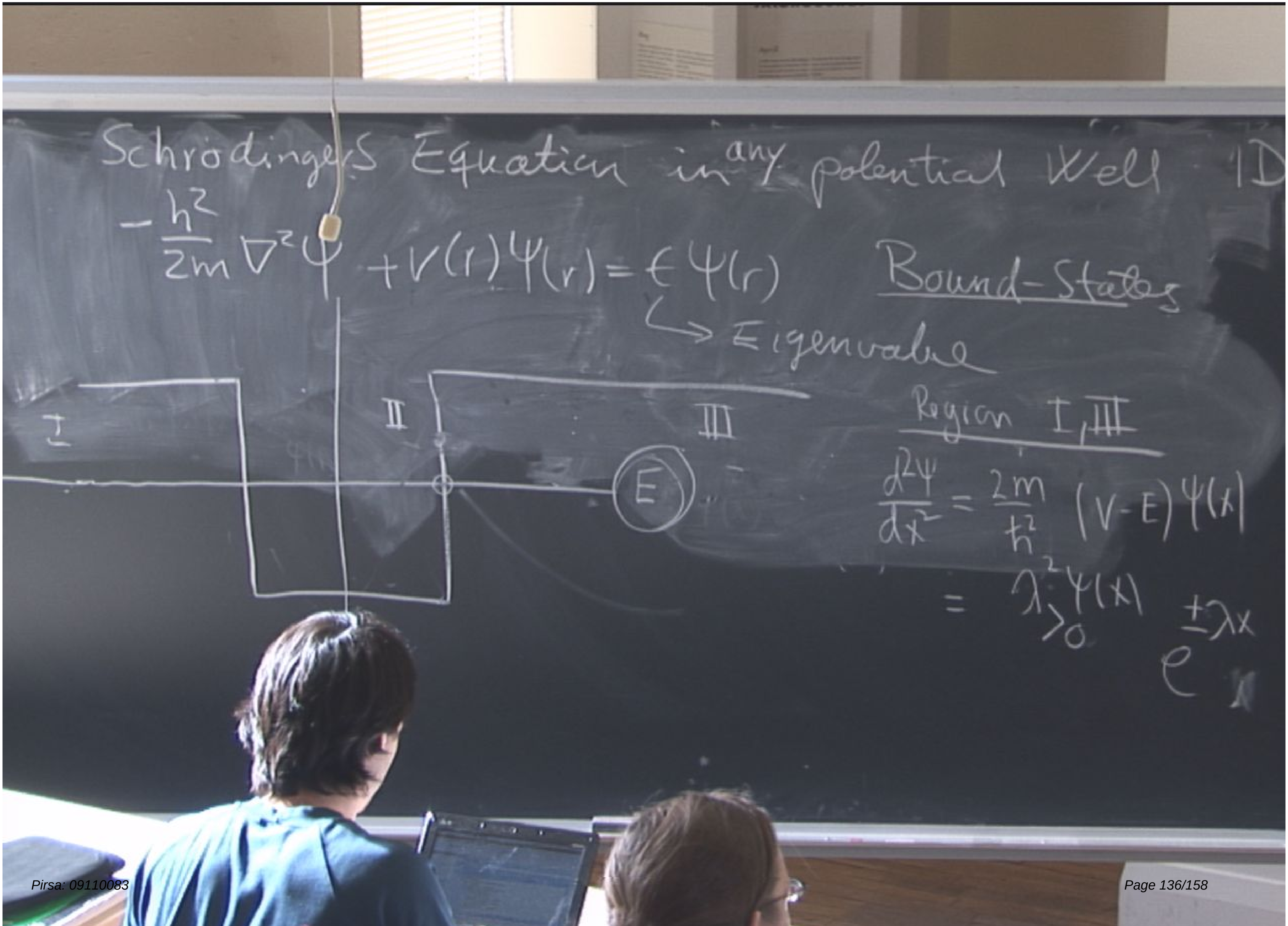
Match the Slopes

— x_m cannot be a node of the eigenfunction

Match the Slopes

$F(E)$

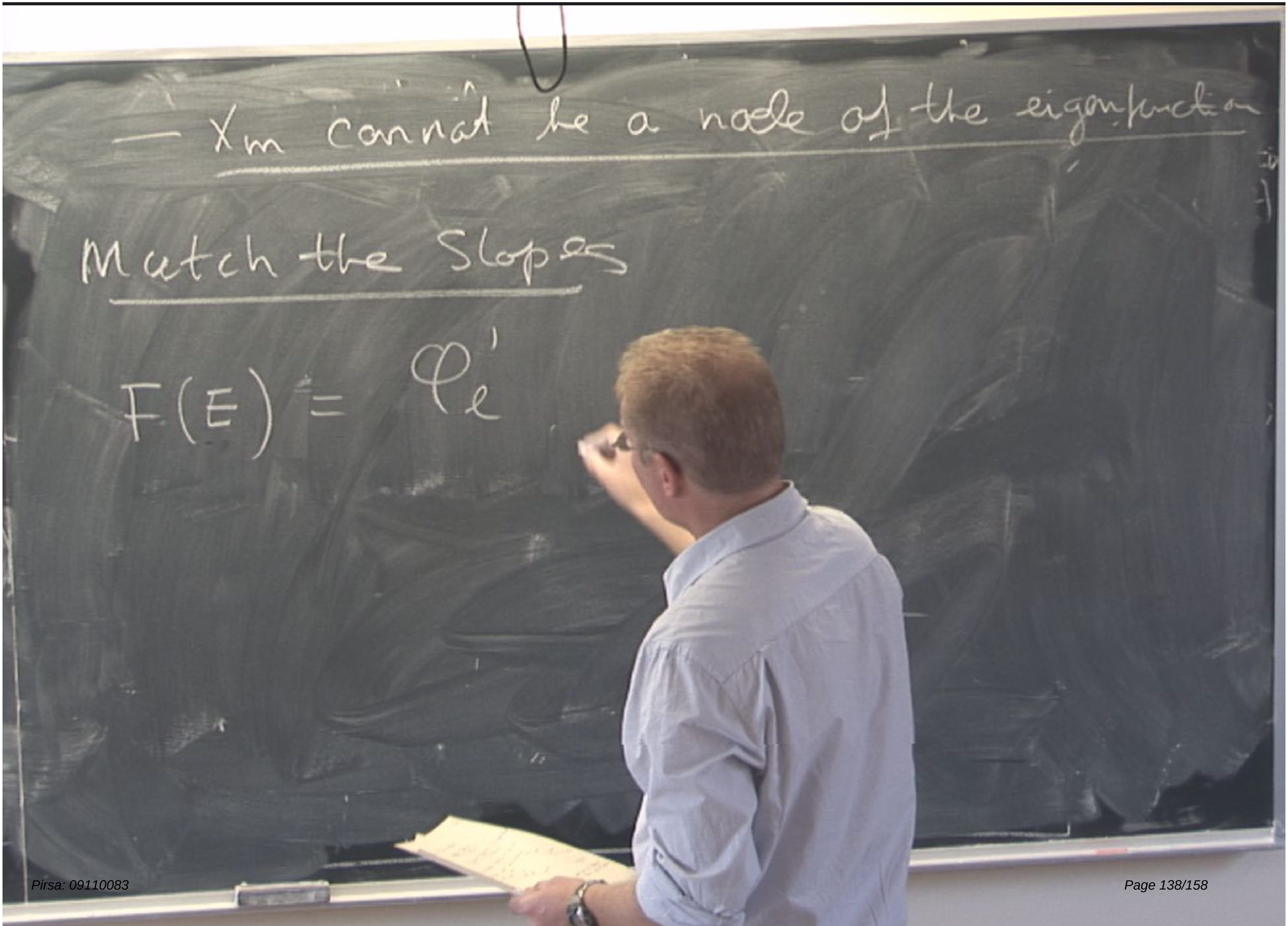

Schrodinger's Equation in any potential well 1D
$-\frac{\hbar^2}{2m}\nabla^2\Psi + V(r)\Psi(r) = \epsilon\Psi(r)$
Eigenval
I
II
III
E


Schrödinger's Equation in any potential Well 1D
$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi(r) = \epsilon\psi(r)$
Eigenvalue
Bound-States
I
II
III
E
Region I, III
$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V-E)\psi(x)$
$= \lambda^2\psi(x)$
$\lambda > 0$
$e^{\pm\lambda x}$

— x_m cannot be a node of the eigenfunction

Match the Slopes

$F(E)$


— Xm cannot be a node of the eigenfunction
Match the Slopes
F(E) = φ'e

— x_m cannot be a node of the eigenfunction

Match the Slopes

$$F(E) = \frac{\varphi_l'(x_m) - \varphi_r'(x_m)}{2h\,\varphi_r(x_m)}$$

— x_m cannot be a node of the eigenfunction

Match the Slopes

$$F(E) = \frac{\varphi_\ell'(x_m) - \varphi_r'(x_m)}{2h\,\varphi_r(x_m)}$$

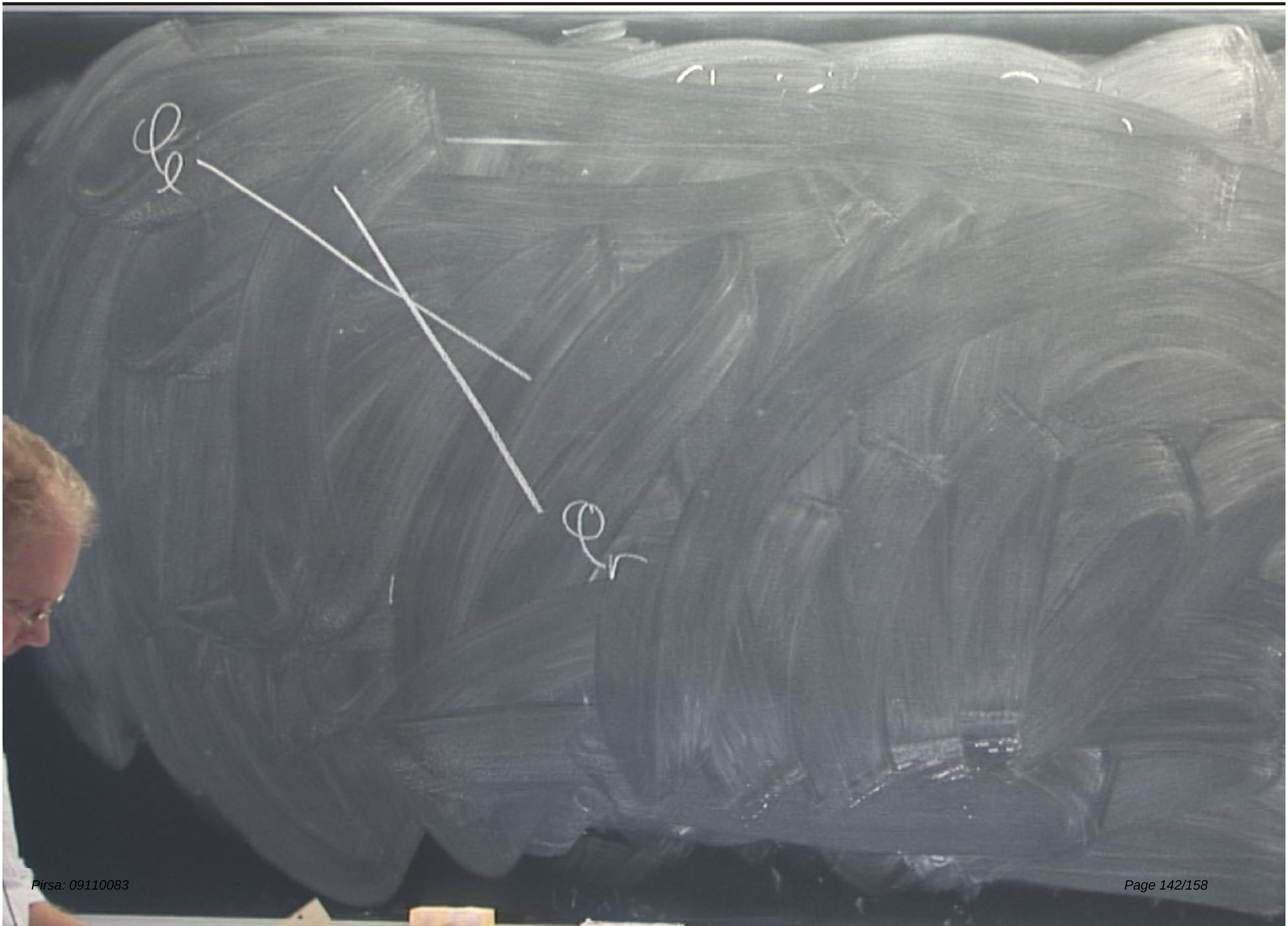
$$= \frac{\varphi_\ell(x_m+h) - \varphi_\ell(x_m-h) - (\varphi_r(x_m+h) - \varphi_r(x_m \ldots}{2h\,\varphi_r(x_m)}$$

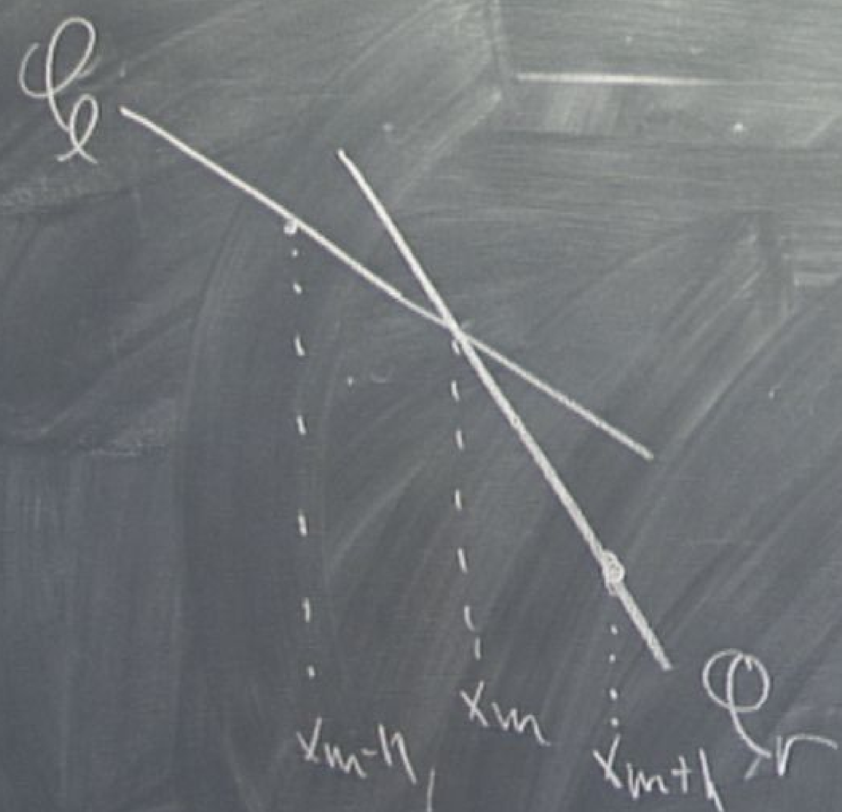
— x_m cannot be a node of the eigenfunction

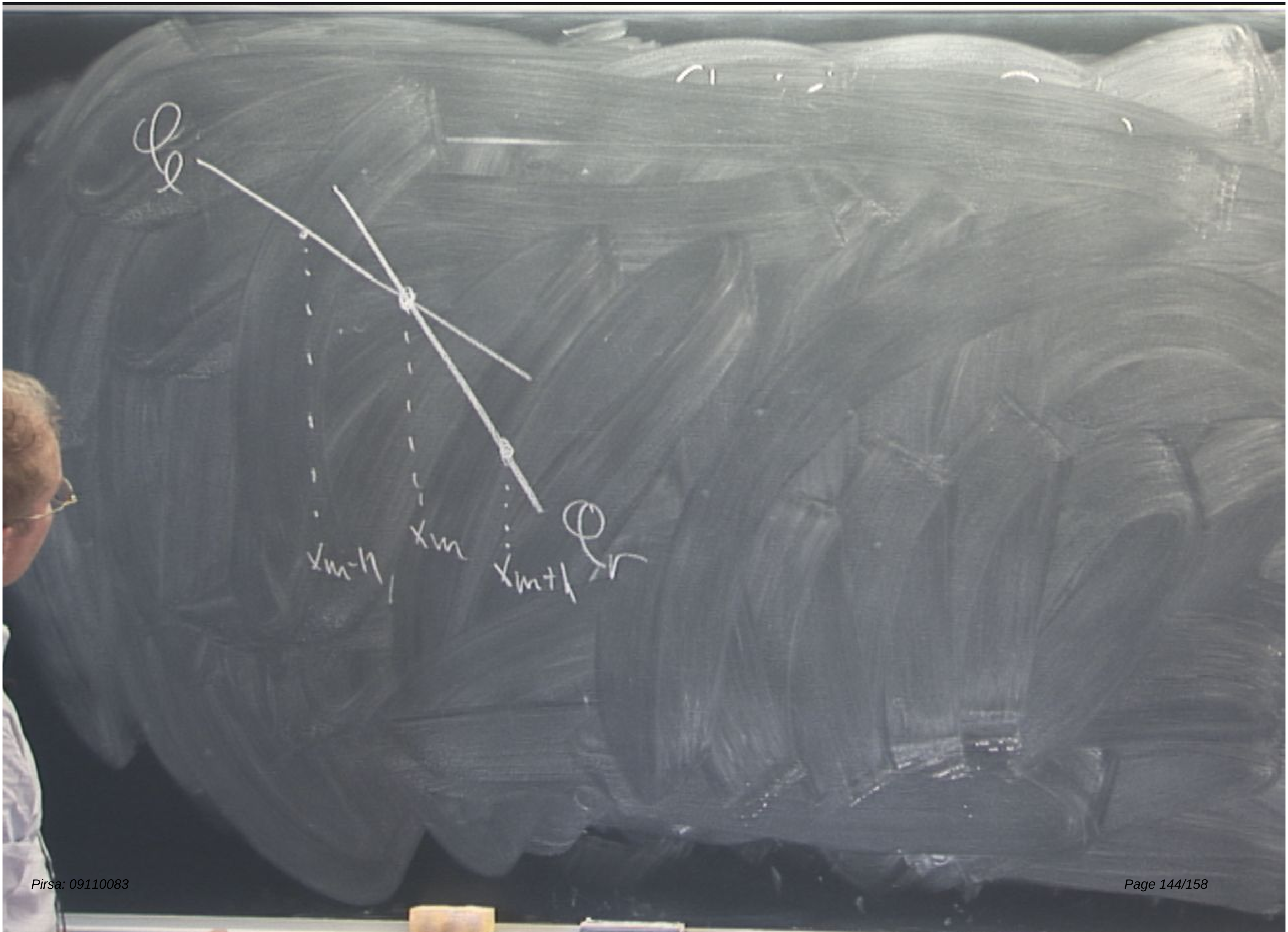
Match the Slopes

$$F(E) = \frac{\varphi_l'(x_m) - \varphi_r'(x_m)}{2h\,\varphi_r(x_m)}$$

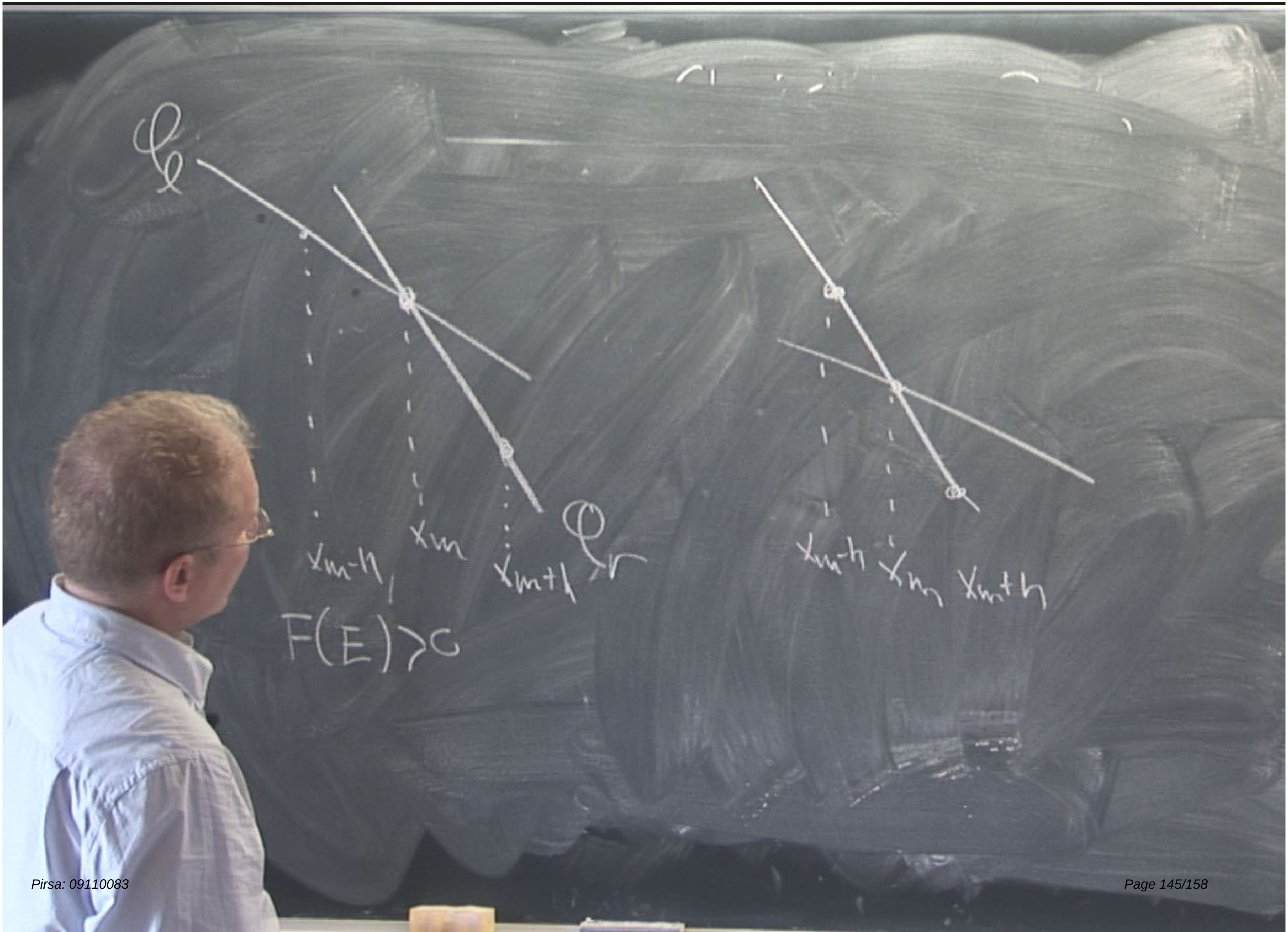
$$= \frac{\varphi_l(x_m+h) - \varphi_l(x_m-h) - \left(\varphi_r(x_m+h) - \varphi_r(x_m-h)\right)}{2h\,\varphi_r(x_m)}$$

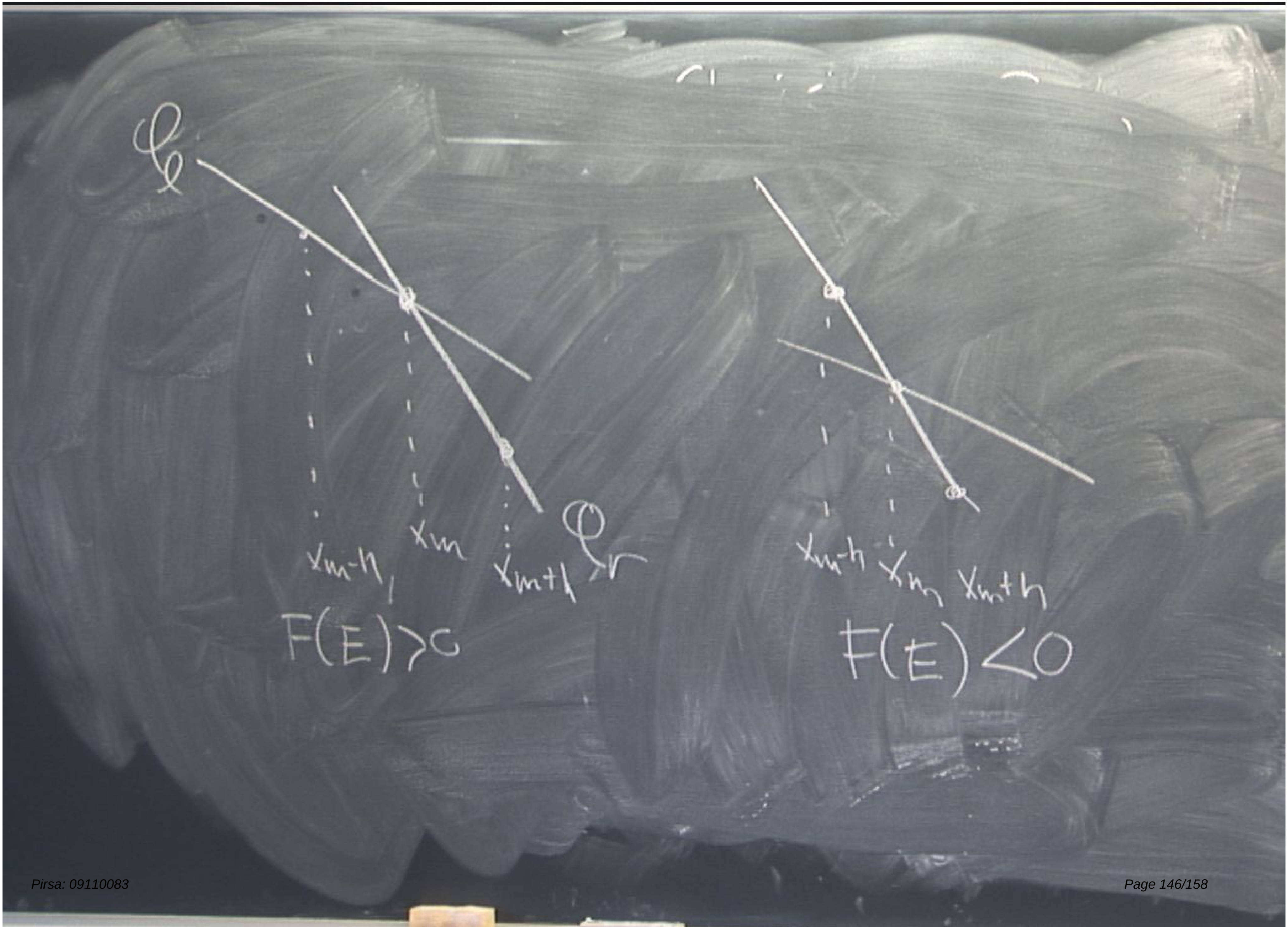

φ_ℓ
φ_r


φ_ℓ
x_{m-1},
x_m
x_{m+1}
φ_r



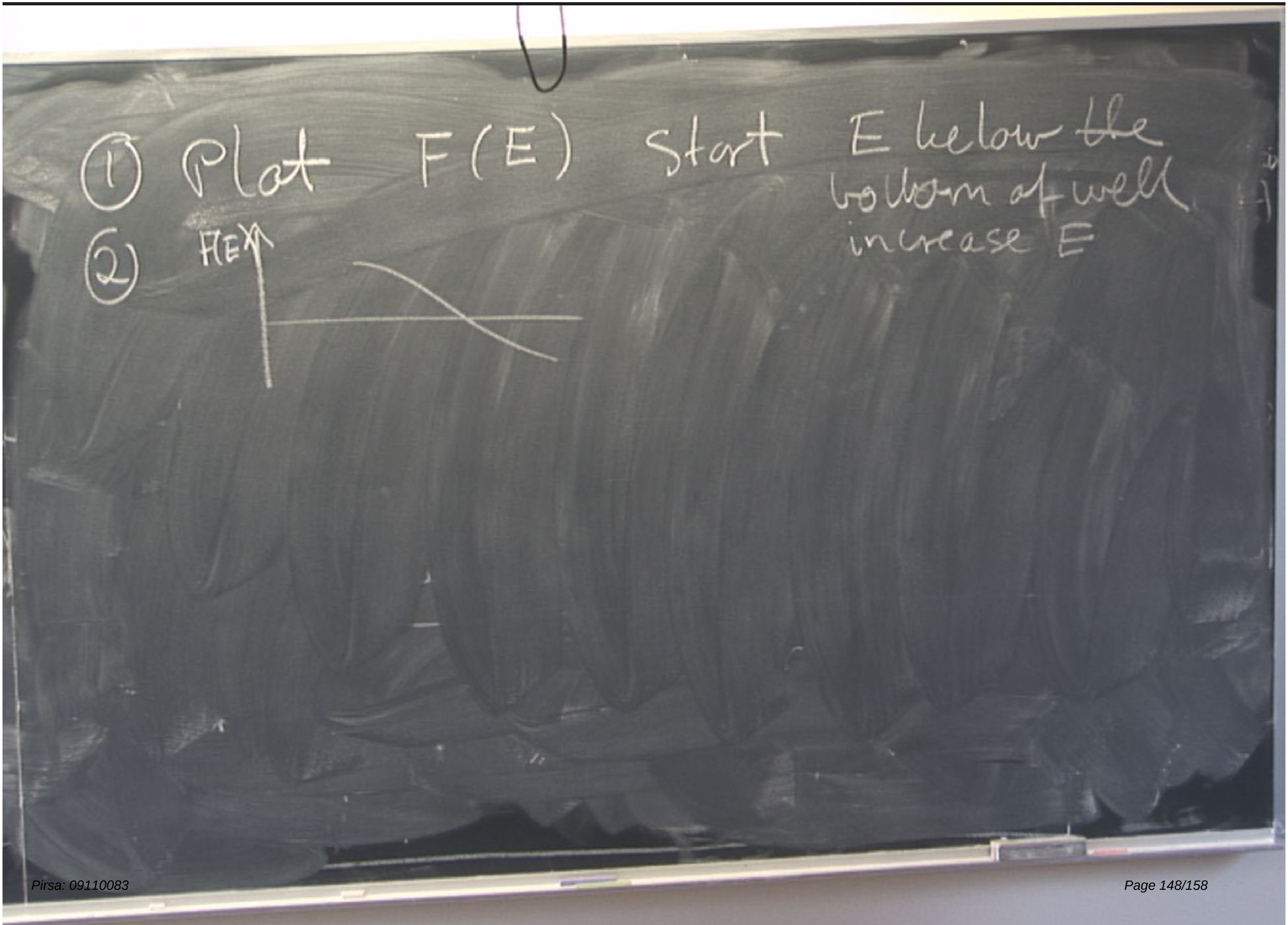
φ_l
x_{m-1},
x_m
x_{m+1}
φ_r


φ_l
x_{m-1}
x_m
x_{m+1}
φ_r
$F(E) > 0$
x_{m-1}
x_m
x_{m+1}



φ_l
$x_m - h$
x_m
$x_m + h$
φ_r
F(E)>0
$x_m - h$
x_m
$x_m + h$
F(E)<0

① Plot F(E) start E below the bottom of well
increase E
②


① Plot F(E)
Start E below the bottom of well
increase E
② F(E)

① Plot F(E) Start E below the bottom of well increase E

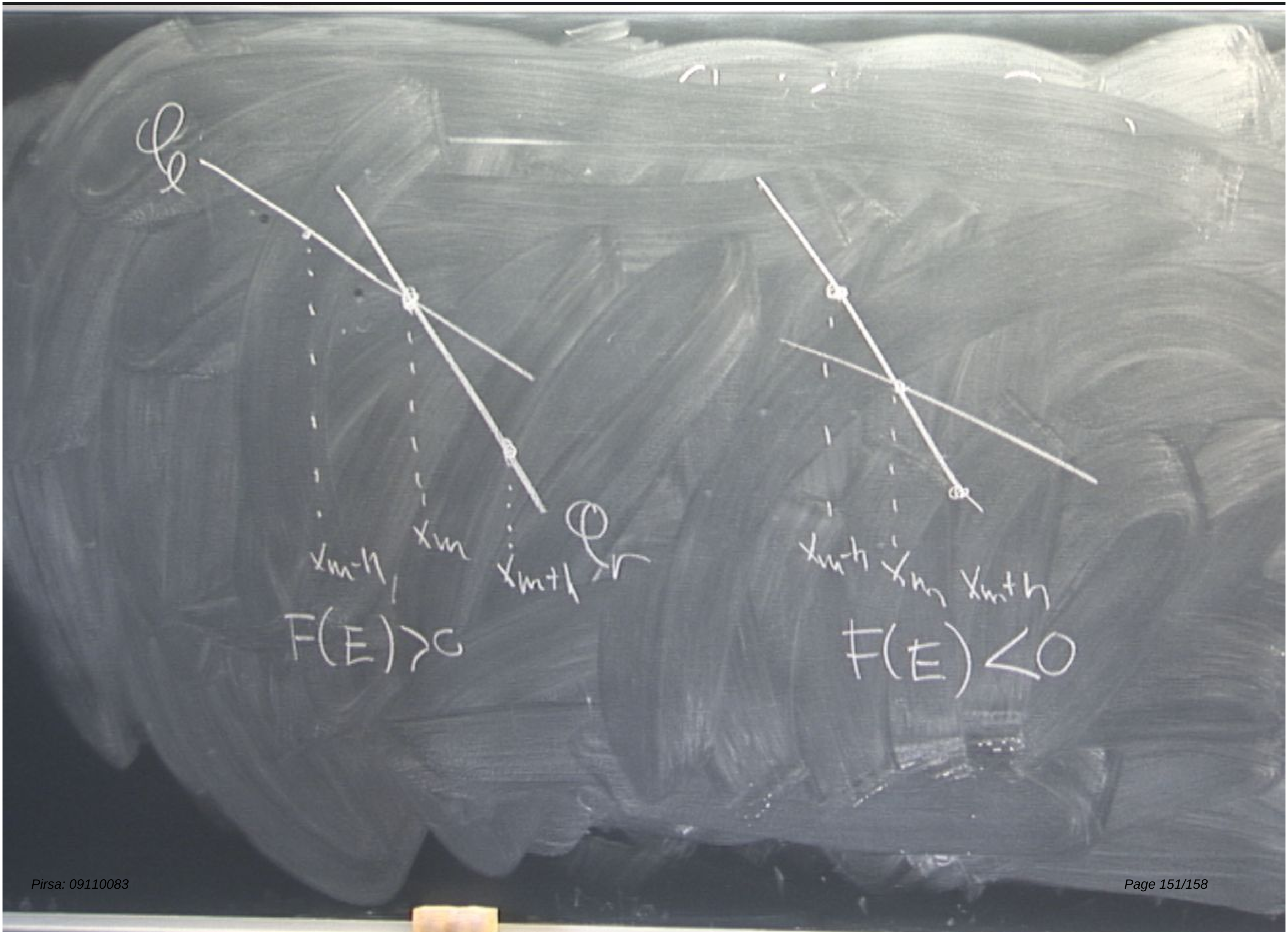
②

1. Plot $F(E)$ Start E below the bottom of well, increase E

2. 

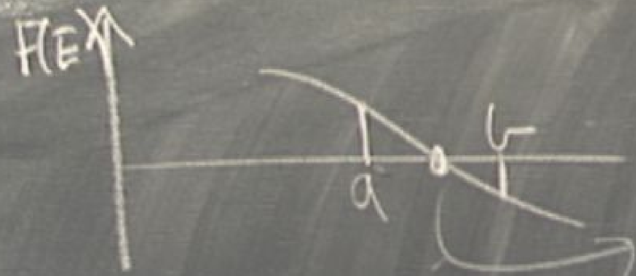
determine $[a,b]$

3. Run bisection on $f(E)$ on the $[a,b)$


φ_l
$x_m - h$
x_m
$x_m + h$
φ_r
$F(E) > 0$
$x_m - h$
x_m
$x_m + h$
$F(E) < 0$

① Plot F(E) Start E below the bottom of well increase E

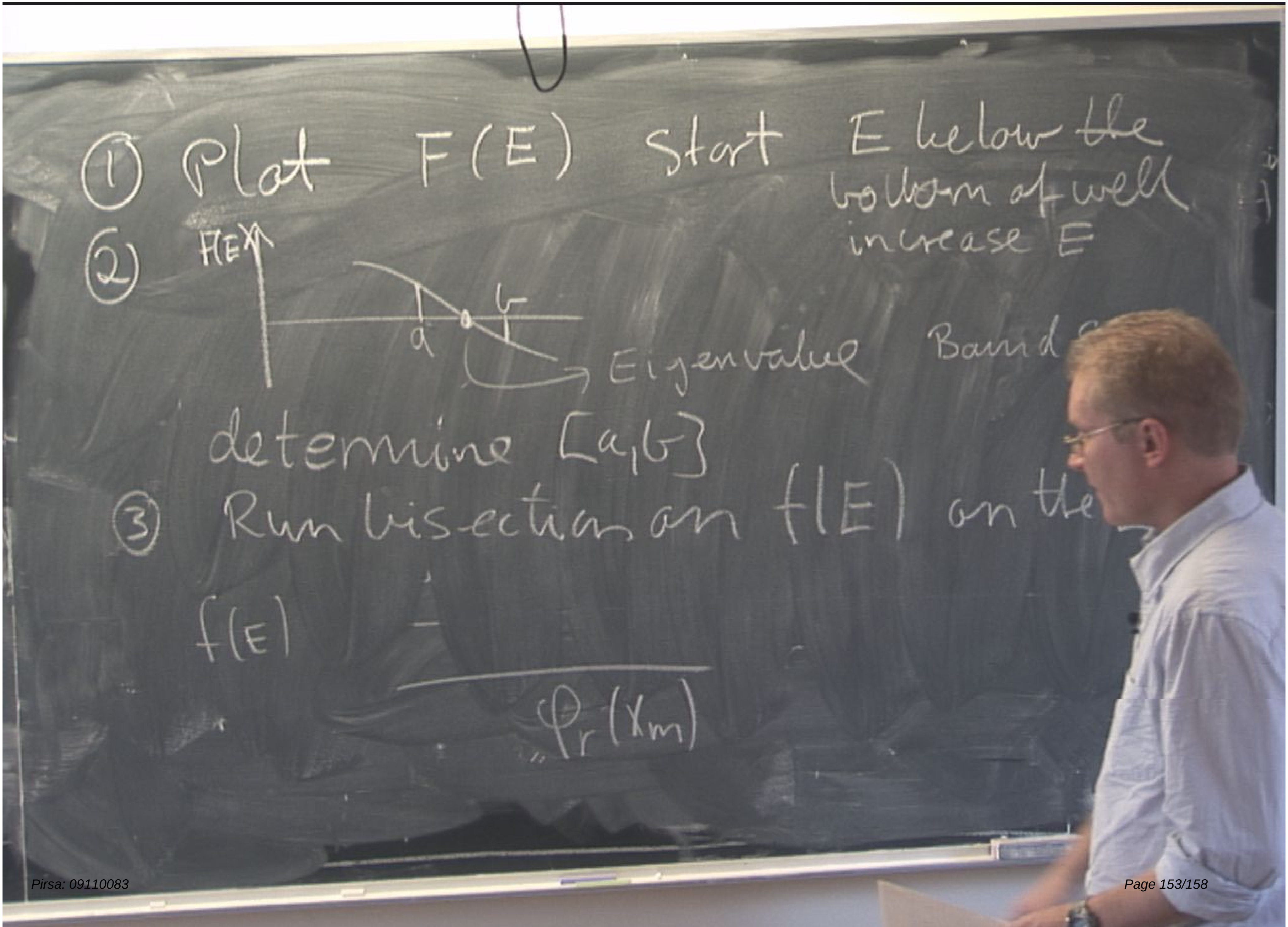
②



Eigenvalue Bound State

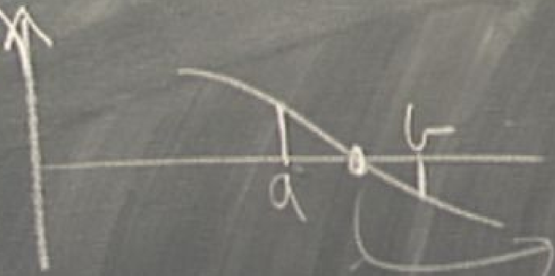
determine [a, b]

③ Run bis-ection on f(E) on the [a, b)


① Plot F(E) Start E below the bottom of well increase E
② F(E)
a
b
Eigenvalue
Band
determine [a,b]
③ Run bisection on f(E) on the
f(E)
φr(xm)

① Plot F(E) Start E below the bottom of well increase E

② $F(E)$



Eigenvalue Bound State

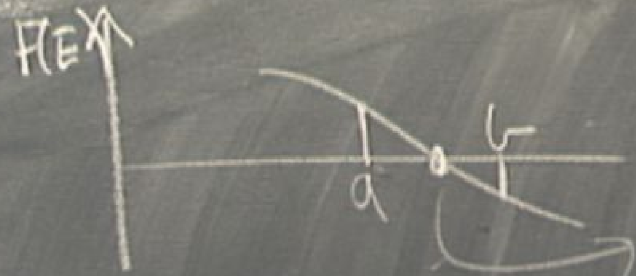
determine $[a, b]$

③ Run bisection on $f(E)$ on the $[a, b)$

$$f(E) \quad \frac{\quad}{\varphi_r(x_m)}$$

① Plot F(E) Start E below the bottom of well increase E

②



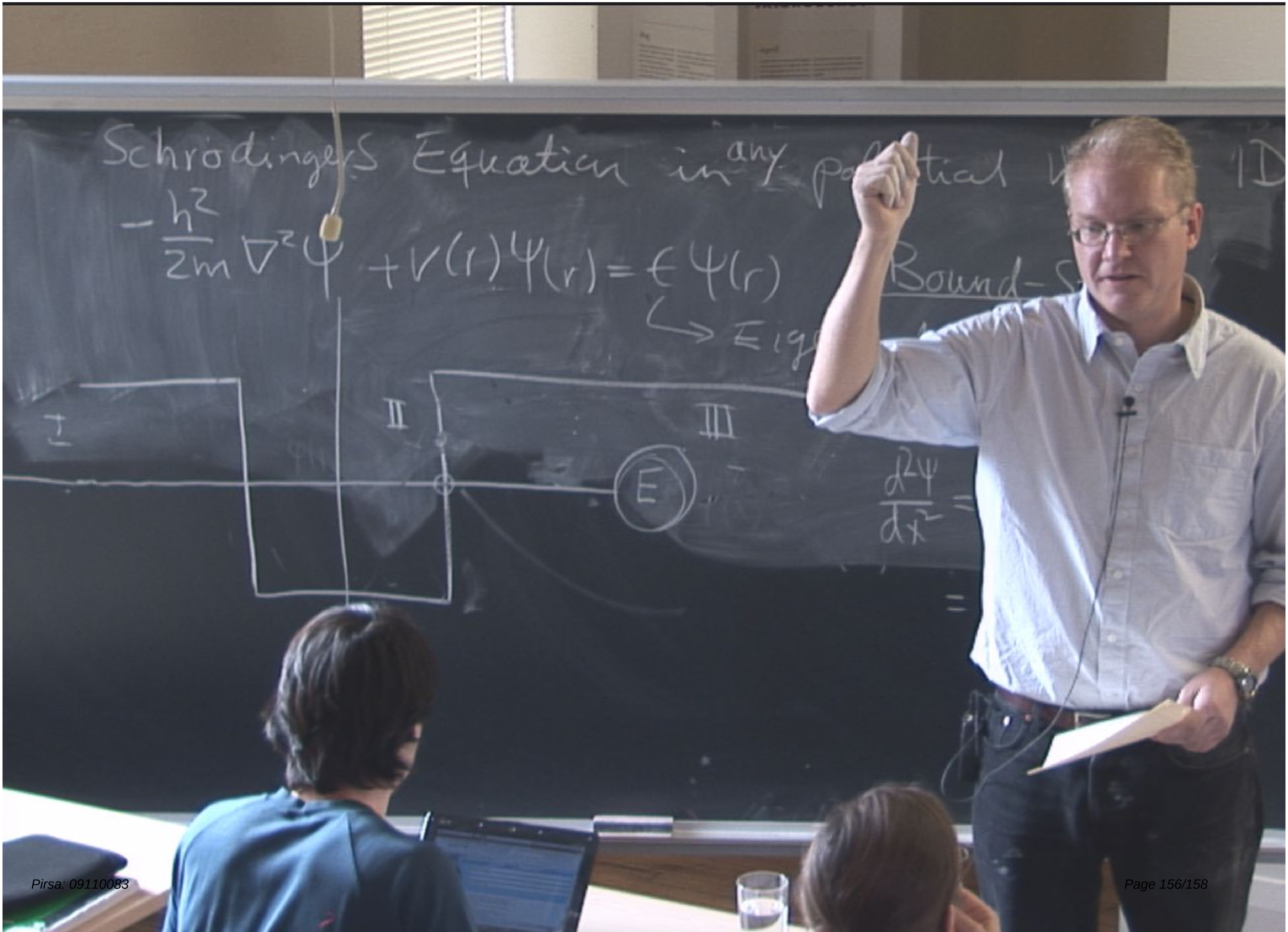
Bound State

determine [a,b]

③ Run bisection on f(E) on the [a,b)

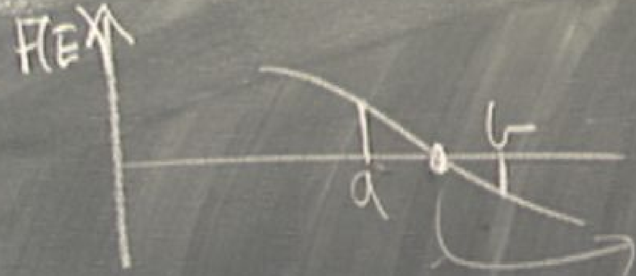
f(E)

$\varphi_r(x_m)$



Schrodinger's Equation in any potential
Bound-S
Eig
I
II
III
E

① Plot F(E) Start E below the bottom of well increase E

② 

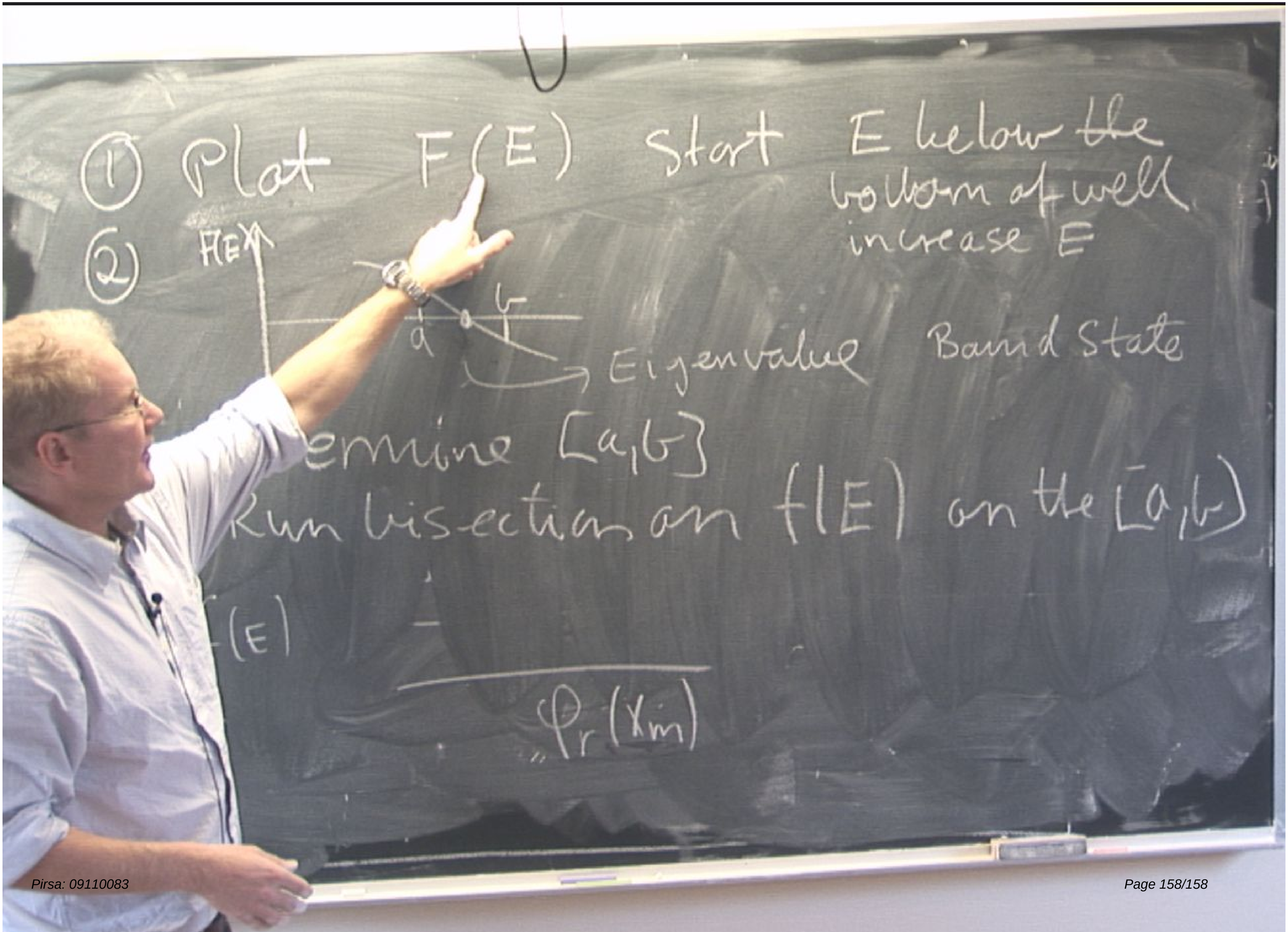
Bound State

determine [a, b]

③ Run bisection on f(E) on the [a, b)

f(E)

$\varphi_r(x_m)$


① Plot F(E) Start E below the
bottom of well
increase E
② F(E)
a
b
Eigenvalue
Bound State
ermine [a,b]
Run bis-ection on f(E) on the [a,b)
(E)
φr(Xm)