Title: Quantum Field Theory II (PHYS 603) - Lecture 15

Date: Nov 13, 2009 09:00 AM

URL: http://pirsa.org/09110079

Abstract:

$$-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}(i\not{D} - m)\Psi + \text{gauge fixing ter}$$

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Renormalisable (at 1 loop) : true at all orders

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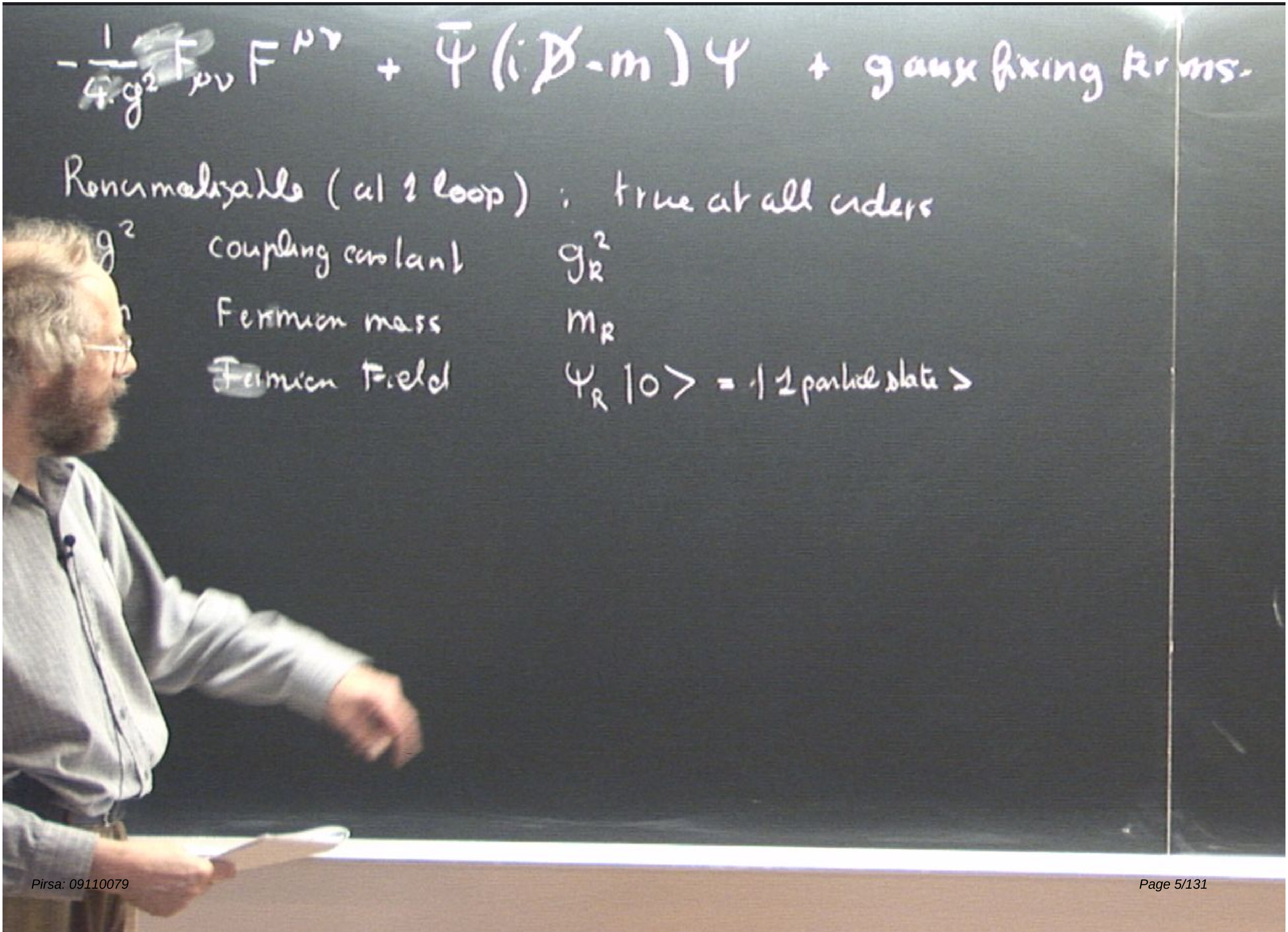
Renormalizable (at 1 loop) : true at all orders

g^2 coupling constant

m Fermion mass

Ψ Fermion field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Fermion mass m_R
Fermion Field $\Psi_R |0\rangle = |1\text{ particle state}\rangle$

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Running c.c. g_R depends on the energy scale at which it is "measured" or "defined"

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$$\mu \frac{d}{d\mu} g(\mu) = \beta(g(\mu))$$

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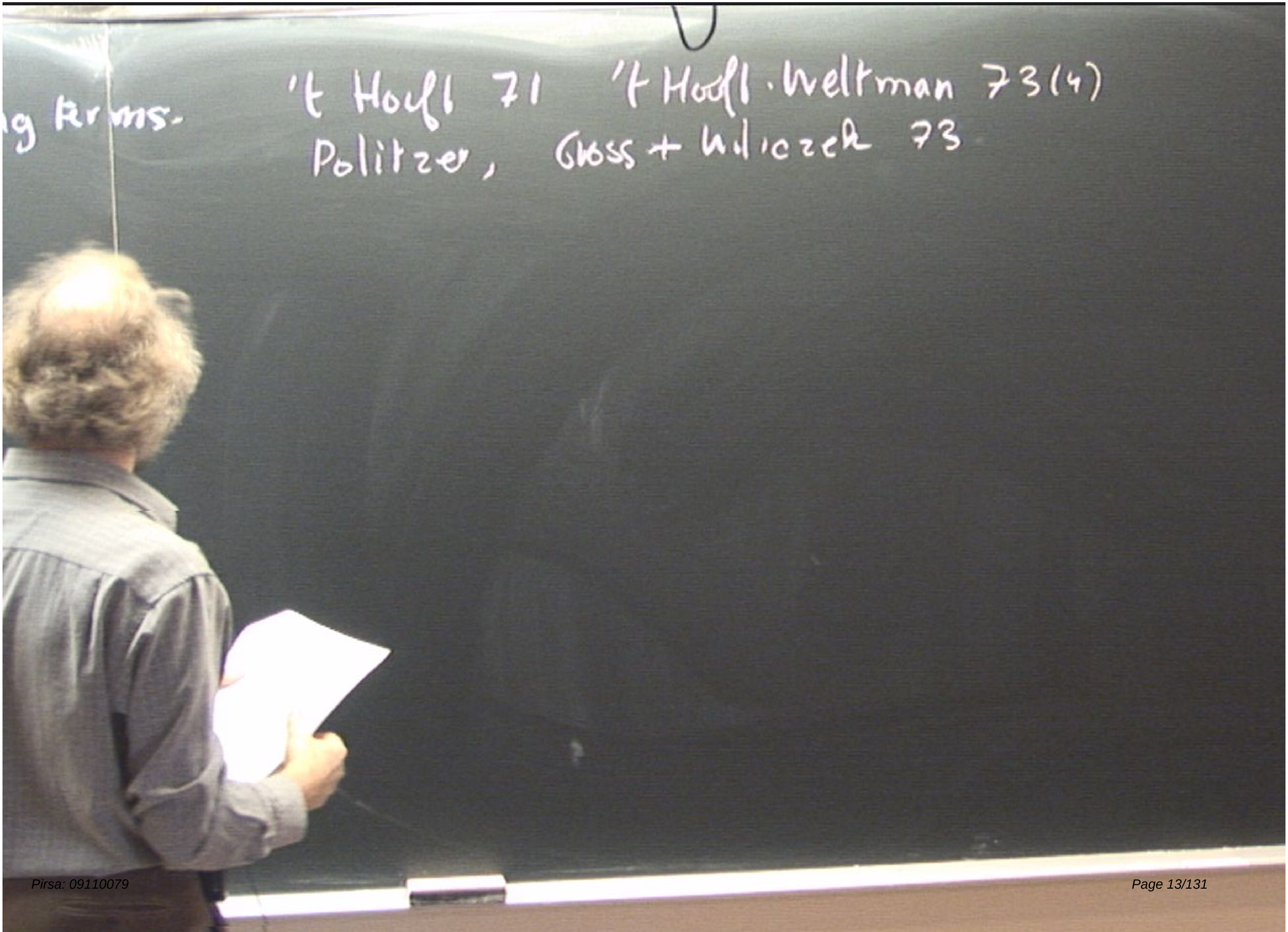
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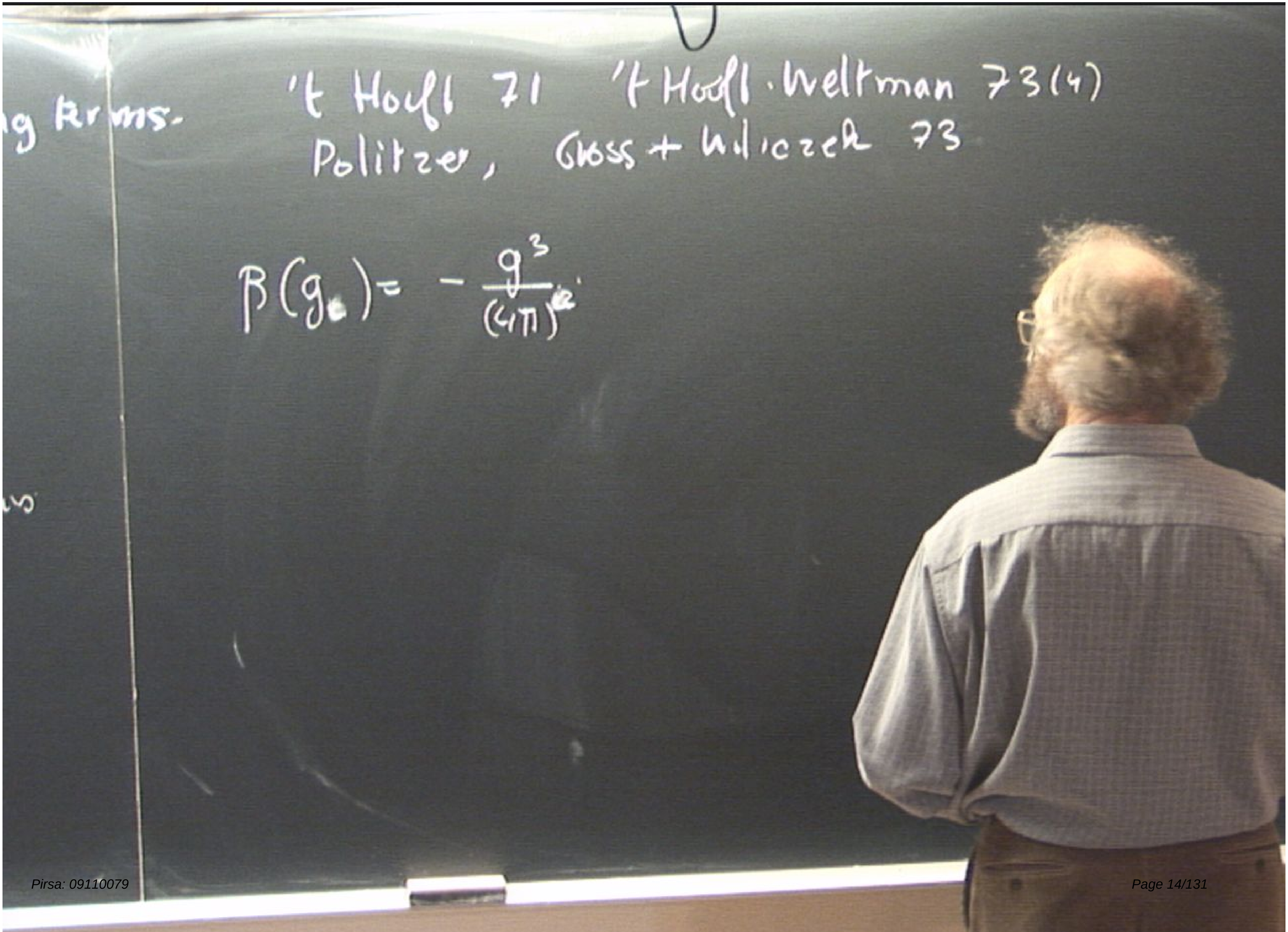
Politzer, Gross + Wilczek 73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$\beta(g_0) = -\frac{g^3}{(4\pi)^2}$

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Politzer, Gross + Wilczek 73

$N = 2 \quad SU(2)$

$SU(N)$

$$\beta(g_0) = -\frac{g^3}{(4\pi)^2} \cdot \frac{11}{3}\left(\frac{N^2-1}{2N}\right)$$

gauge fields
+ g.f terms

ns.

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$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}\left(\frac{N^2-1}{2N}\right) - \frac{4}{3}N_F \cdot \frac{1}{2}\right]$$

gauge fields + g.f terms

Fermions if they are in the fundamental representation

N-component Dirac Fermion

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at one loop

gauge fields + g.f. terms

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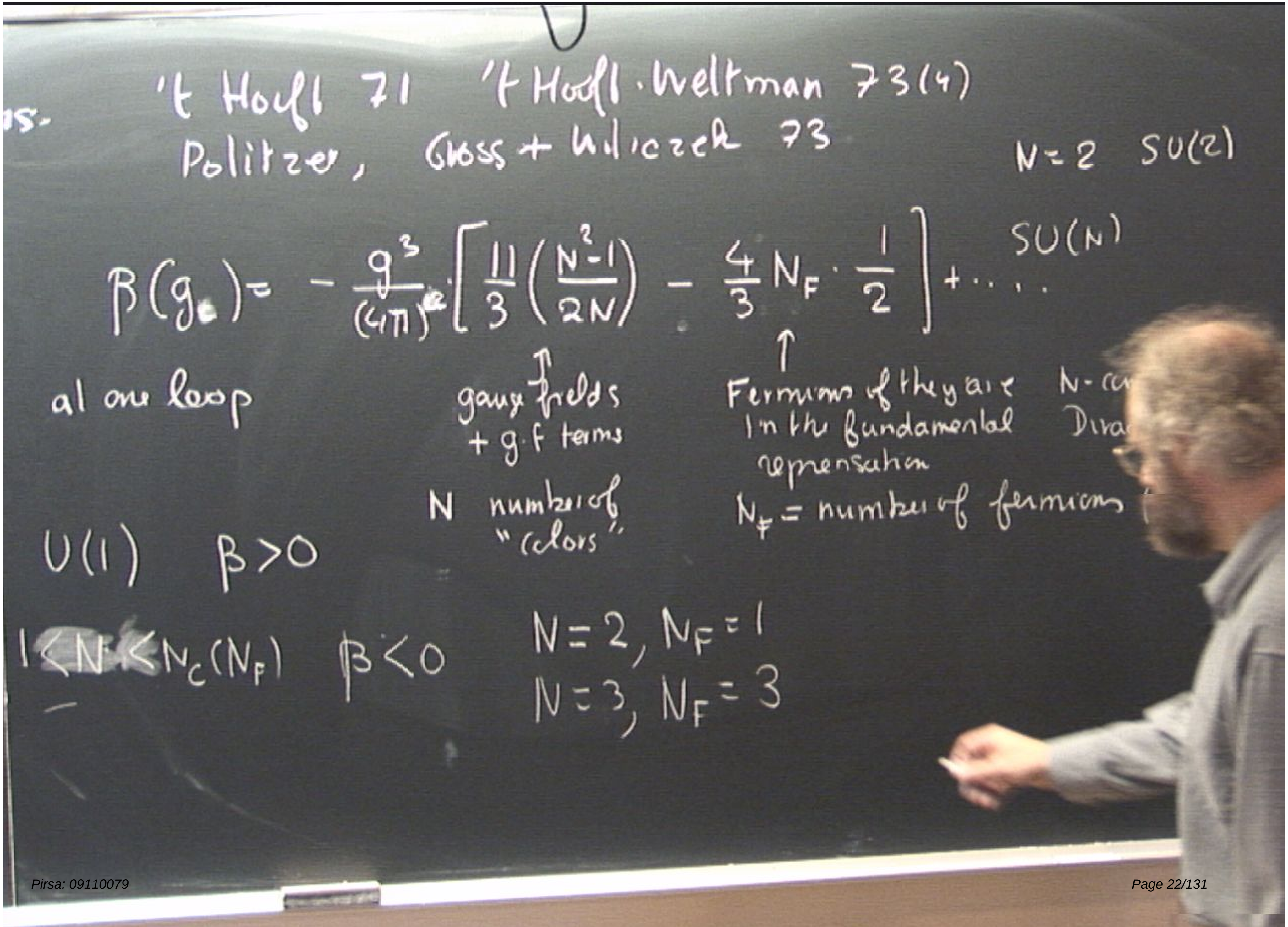
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at one loop
gauge fields + g.f terms
Fermions if they are in the fundamental representation
N number of "colors"
N_F = number of fermions
U(1) β > 0
1 < N < $N_c(N_F)$ β < 0
N = 2, N_F = 1
N = 3, N_F = 3

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gauge fields + g.f terms

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$U(1)$ $\beta > 0$

Asymptotic Freedom

$1 < N < N_c(N_F)$ $\beta < 0$

$N=2, N_F=1$

$N=3, N_F=3$ QCD $g(\mu) \searrow 0$ when $\mu \nearrow$

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Higher Loops

at one loop

gauge fields + g.f terms

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Higher Loops

...oop

gauge fields + g.f terms

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Asymptotic Freedom

$g(\mu) \searrow 0$ when $\mu \nearrow \ldots$

$\ldots(N) = SU(N) \times U(1)$

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at one loop

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Asymptotic Freedom

$g(\mu) \searrow 0$ when $\mu \nearrow \infty$

$U(N) = SU(N) \times U(1)$ decouples

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$N=2$ $SU(2)$

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$$\boxed{\begin{array}{l}\text{Asymptotic Freedom}\\ g(\mu)\searrow 0 \text{ when } \mu\nearrow\end{array}}$$

$U(N)=SU(N)\times \cancel{U(1)}$ decouples

Higgs Mechanism and "Gauge symmetry breaking"

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Symmetry breaking (spontaneous)

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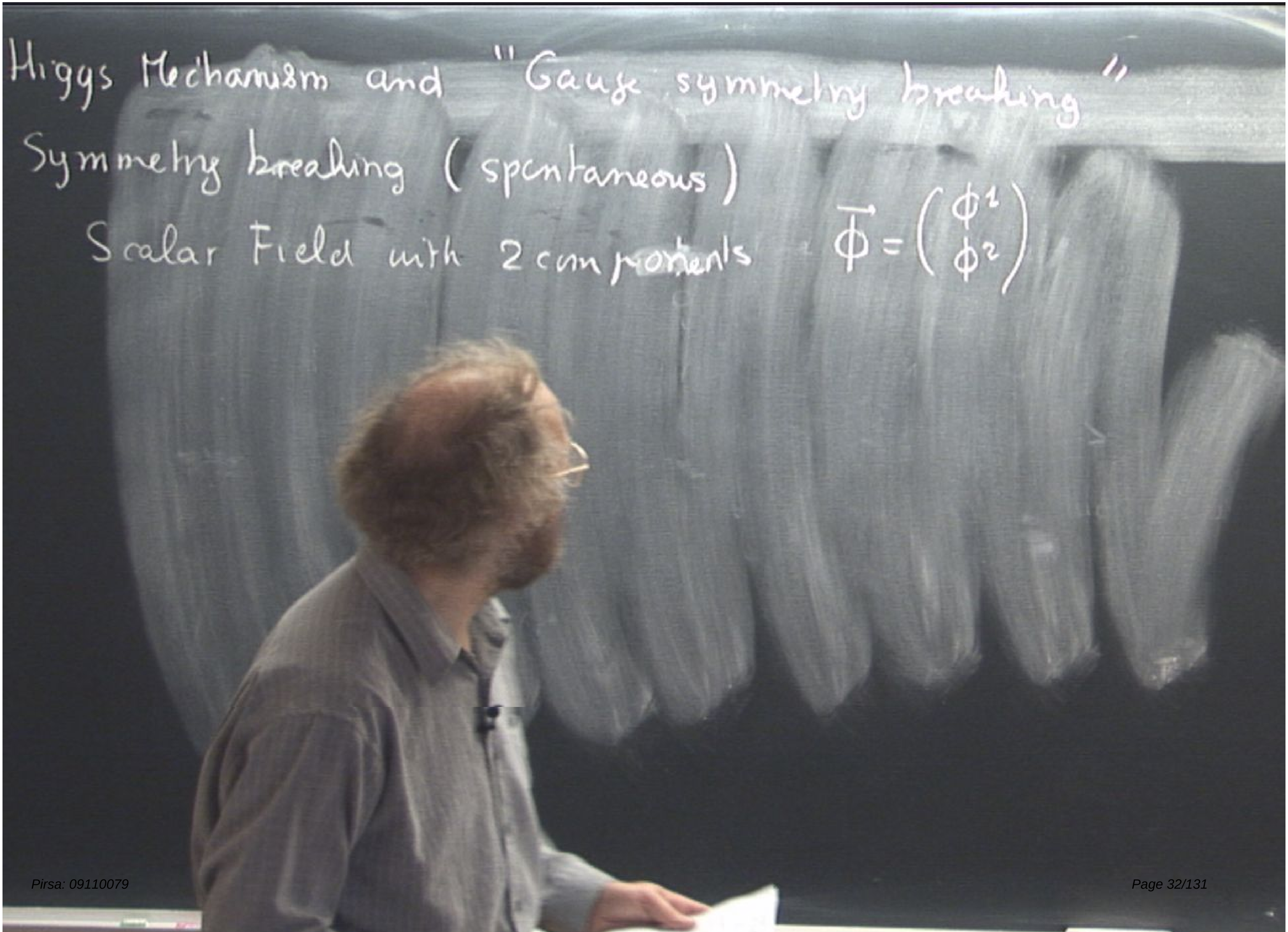
Symmetry breaking (spontaneous)


Higgs Mechanism and "Gauge symmetry breaking"
Symmetry breaking (spontaneous)
Scalar Field with 2 components
$\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

Higgs Mechanism and "Gauge symmetry breaking"

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Higgs Mechanism and "Gauge symmetry breaking"

Symmetry breaking (spontaneous)

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2 \right) \right] \qquad |\vec{\Phi}|^2 = \phi^1 \phi^1 + \phi^2 \phi^2$$

$$V(|\Phi|^2) = \frac{r}{2} |\vec{\Phi}|^2 + \frac{g}{4} |\vec{\Phi}|^4$$

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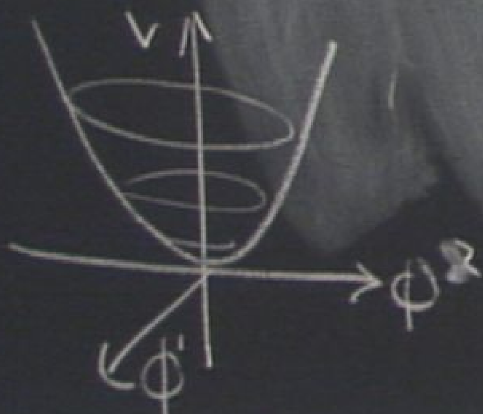
$$V(|\phi|^2) = \frac{r}{2} |\vec{\Phi}|^2 + \frac{g}{4} |\vec{\Phi}|^4 \qquad g > 0$$

$t > 0, \; t = m^2$



Mechanism and "Gauge symmetry breaking" ms

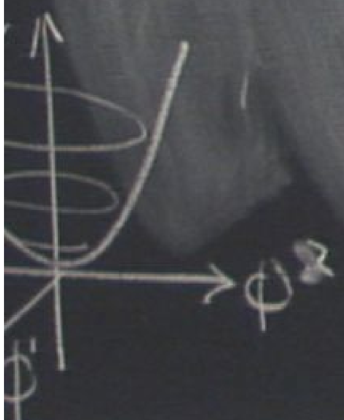
metry breaking (spontaneous) global

calar Field with 2 components $\vec{\Phi} = \begin{pmatrix}\phi^1 \\ \phi^2\end{pmatrix}$ $U(1)$ symmetry

$$\int d^4x \left[-\frac{1}{2}\left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi}\right) - V\left(|\vec{\Phi}|^2\right) \right] \qquad |\vec{\Phi}|^2 = \phi^1\phi^1 + \phi^2\phi^2$$

$$(|\Phi|^2) = \frac{t}{2}|\vec{\Phi}|^2 + \frac{g}{4}|\vec{\Phi}|^4 \qquad g > 0$$

$> 0,\ t = m^2$



iggs Mechanism and "Gauge symmetry breaking"

ymmetry breaking (spontaneous) glob

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ symm

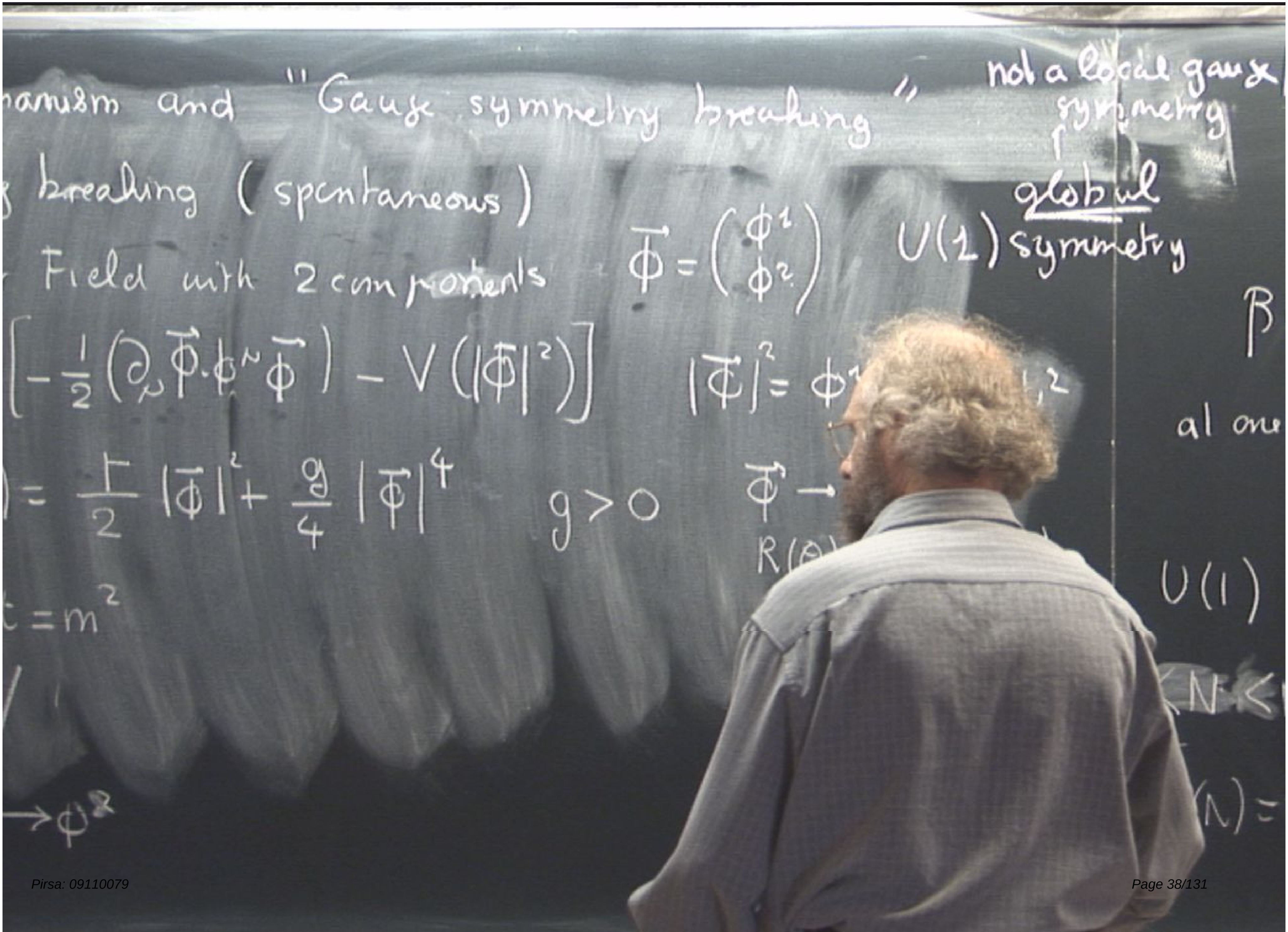
$^4x\left[-\frac{1}{2}\left(\partial_\mu \vec{\Phi}\cdot\partial^\mu \vec{\Phi}\right) - V\left(|\vec{\Phi}|^2\right)\right]$ $|\vec{\Phi}|^2 = \phi^1\phi^1 + \phi^2\phi^2$

$) = \frac{r}{2}|\vec{\Phi}|^2 + \frac{g}{4}|\vec{\Phi}|^4$ $g > 0$ $\vec{\Phi} \to R(\theta)\vec{\Phi}$

$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$= m^2$

$\to \phi^2$



nanism and "Gauge symmetry breaking"
not a local gauge symmetry
global
breaking (spontaneous)
Field with 2 components
$\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$
U(1) symmetry
$\left[-\frac{1}{2}(\partial_\nu \vec{\Phi} \cdot \partial^\nu \vec{\Phi}) - V(|\vec{\Phi}|^2)\right]$
$|\vec{\Phi}|^2 = \phi^1$
$= \frac{t}{2} |\vec{\Phi}|^2 + \frac{g}{4} |\vec{\Phi}|^4$
$g > 0$
$\vec{\Phi} \rightarrow$
$t = m^2$
β
al one
U(1)

Higgs Mechanism and "Gauge symmetry breaking" not a

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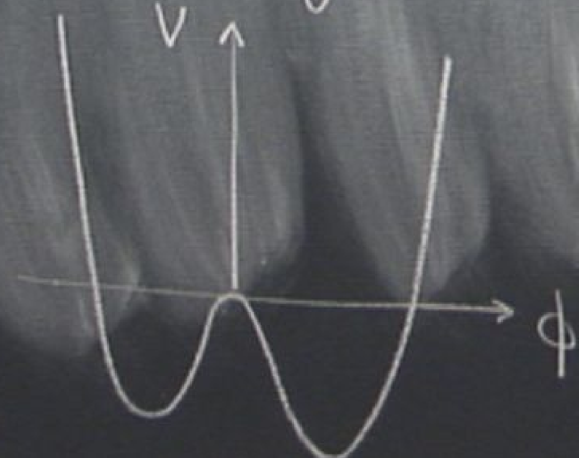
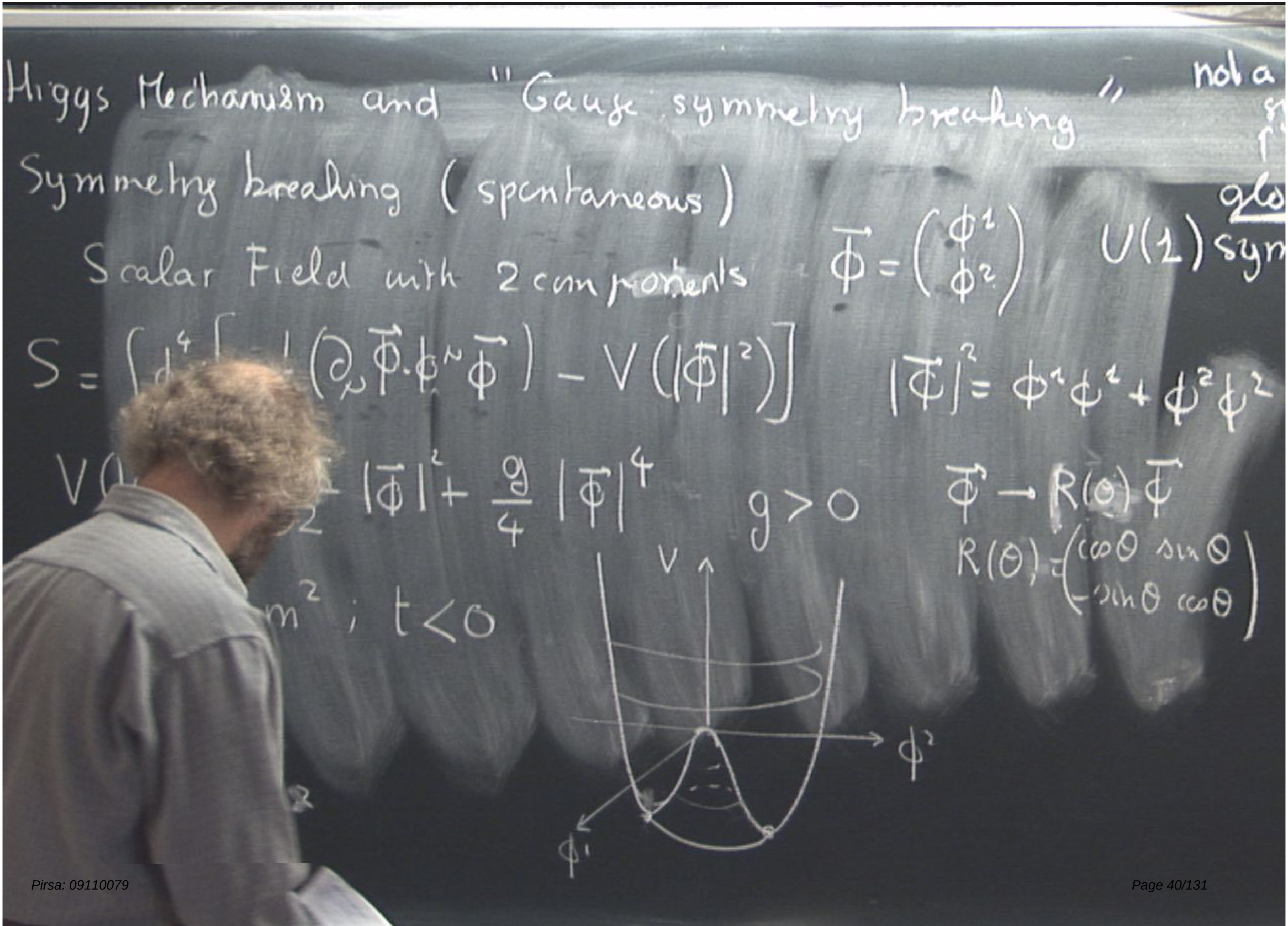
$t > 0,\ t = m^2 \ ; \ t < 0$




Higgs Mechanism and "Gauge symmetry breaking"
Symmetry breaking (spontaneous)
Scalar Field with 2 components
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U(1)
$V(|\vec{\Phi}|^2)$
$|\vec{\Phi}|^2 = \phi^1\phi^1 + \phi^2\phi^2$
$|\vec{\Phi}|^2 + \frac{g}{4}|\vec{\Phi}|^4$
$g > 0$
$\vec{\Phi} \to R(\theta)\vec{\Phi}$
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m^2 ; $t < 0$
V
ϕ^2
ϕ^1

Higgs Mechanism and "Gauge symmetry breaking" not a

Symmetry breaking (spontaneous) glo

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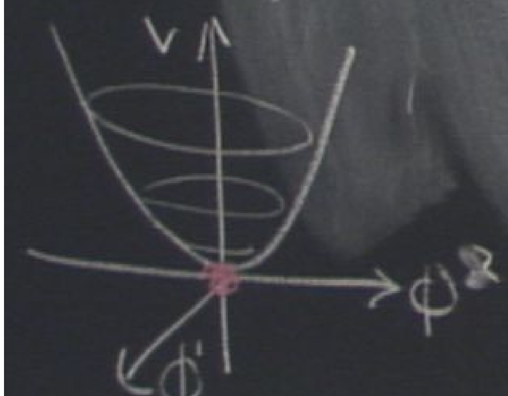

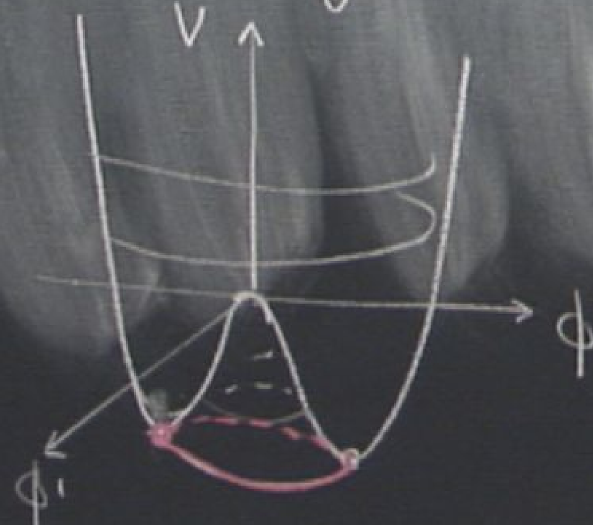

Higgs Mechanism and "Gauge symmetry breaking" not a

Symmetry breaking (spontaneous)

glo

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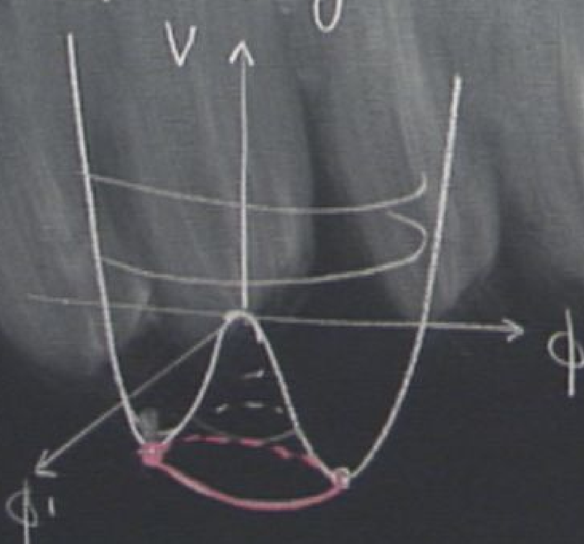
$\langle \vec{\Phi} \rangle = 0$



classical minima (ground state)

$$\langle \vec{\Phi} \rangle = \vec{u}\, \phi_0$$

Higgs Mechanism and "Gauge symmetry breaking" not a ...

Symmetry breaking (spontaneous) glo...

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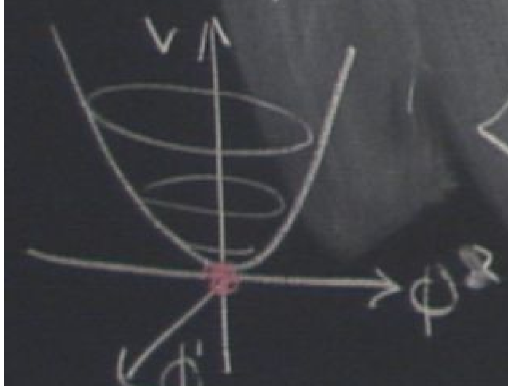
$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2\right) \right] \qquad |\vec{\Phi}|^2 = \phi^1\phi^1 + \phi^2\phi^2$$

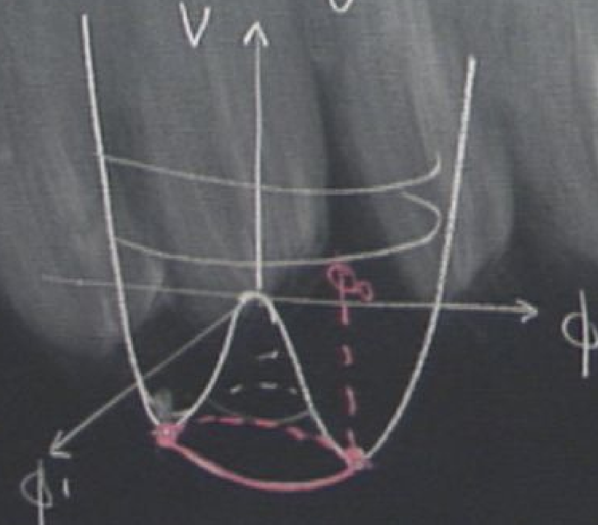
$$V(|\Phi|^2) = \frac{t}{2} |\vec{\Phi}|^2 + \frac{g}{4} |\vec{\Phi}|^4 \qquad g > 0 \qquad \vec{\Phi} \to R(\theta)\vec{\Phi}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$t > 0$, $t = m^2$; $t < 0$

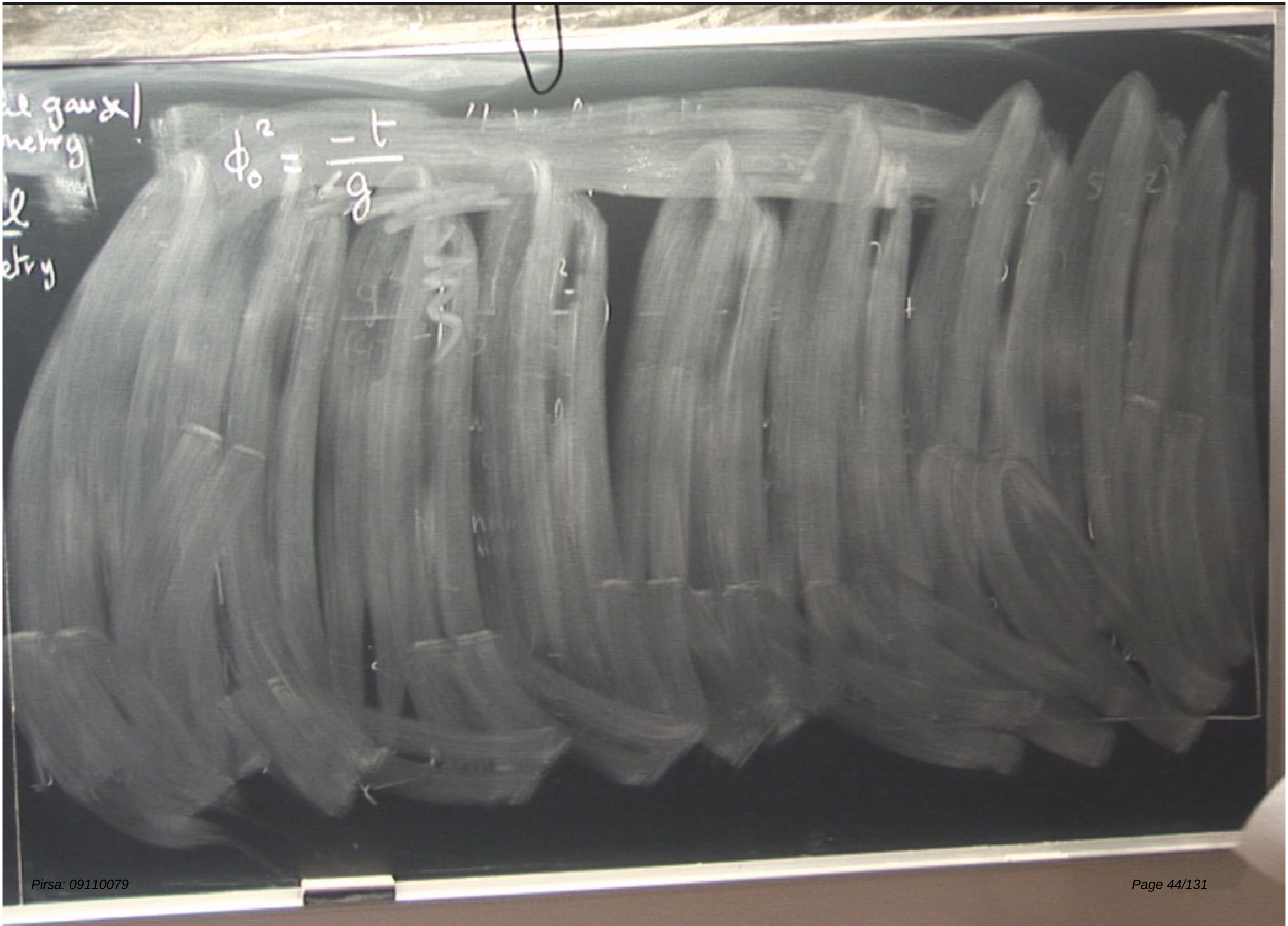


$\langle \vec{\Phi} \rangle = 0$



classical minima (groundstate)

$$\langle \vec{\Phi} \rangle = \vec{u}\, \phi_0$$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$\phi_0^2 = \frac{-t}{g}$
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$$\phi_0^2 = \frac{-t}{g}$$

U(1) symmetry is spontaneously broken

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$$\phi_0^2 = \frac{-t}{g}$$

U(1) symmetry is spontaneously broken

the vaccum state $|0\rangle$ is not invariant !

...ggys Mechanism and "Gauge symmetry breaking" not a lo... sym...

...ymmetry breaking (spontaneous) glob...

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ U(1) symm...

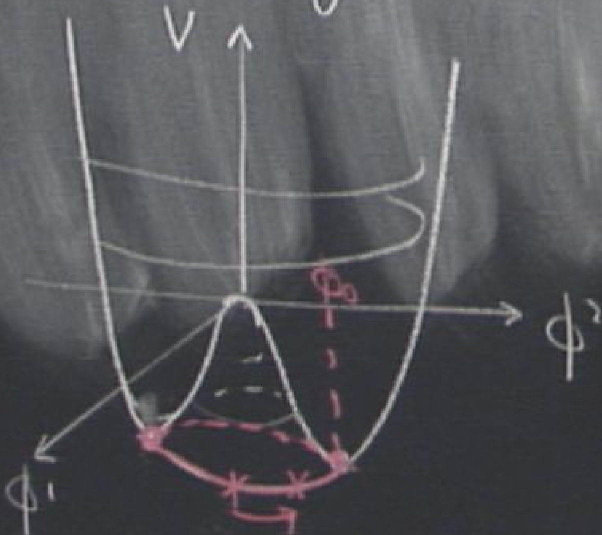
$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2 \right) \right] \qquad |\vec{\Phi}|^2 = \phi^1 \phi^1 + \phi^2 \phi \ldots$$

$$V(|\Phi|^2) = \frac{t}{2} |\vec{\Phi}|^2 + \frac{g}{4} |\vec{\Phi}|^4 \qquad g > 0 \qquad \vec{\Phi} \to R(\theta) \vec{\Phi}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin \ldots \\ -\sin\theta & \ldots \end{pmatrix}$$

$t > 0,\ t = m^2\ ;\ t < 0$

$\langle \vec{\Phi} \rangle = 0$



classical m...
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$\langle \vec{\Phi} \rangle = v \ldots$

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$$\phi_0^2 = \frac{-t}{g}$$

U(1) symmetry is spontaneously broken
the vaccum state $|0\rangle$ is not invariant !

Q charge $\quad Q|0\rangle \neq 0$

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$$\phi_0^2 = \frac{-t}{g}$$

$$\Phi = \phi_1 + i\,\phi_2 \qquad \text{complex field}$$

$$\bar{\Phi} = \phi_1 - i\,\phi_2$$

U(1) symmetry is spontaneously broken

the vacuum state $|0\rangle$ is not invariant!

Q charge $\qquad Q|0\rangle \neq 0$

... gauge | ...metry

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$$\phi_0^2 = \frac{-t}{g}$$

$$\phi = \phi_1 + i\,\phi_2$$ complex field

$$\bar{\phi} = \phi_1 - i\,\phi_2$$

U(1) symmetry is spontaneously broken

the vaccum state $|0\rangle$ is not invariant !

Q charge $\quad Q|0\rangle \neq 0$ $\quad$ current

Noether theorem $\longrightarrow$ $J_\mu(x) = i\left[\partial_\mu\bar{\phi}\cdot\phi - \bar{\phi}\,\partial_\mu\phi\right]$

$$Q = \int d^3x\, J_0(x)$$

$$\phi_0^2 = \frac{-t}{g}$$

$\Phi = \phi_1 + i\phi_2$ complex field

$\bar{\Phi} = \phi_1 - i\phi_2$

U(1) symmetry is spontaneously broken

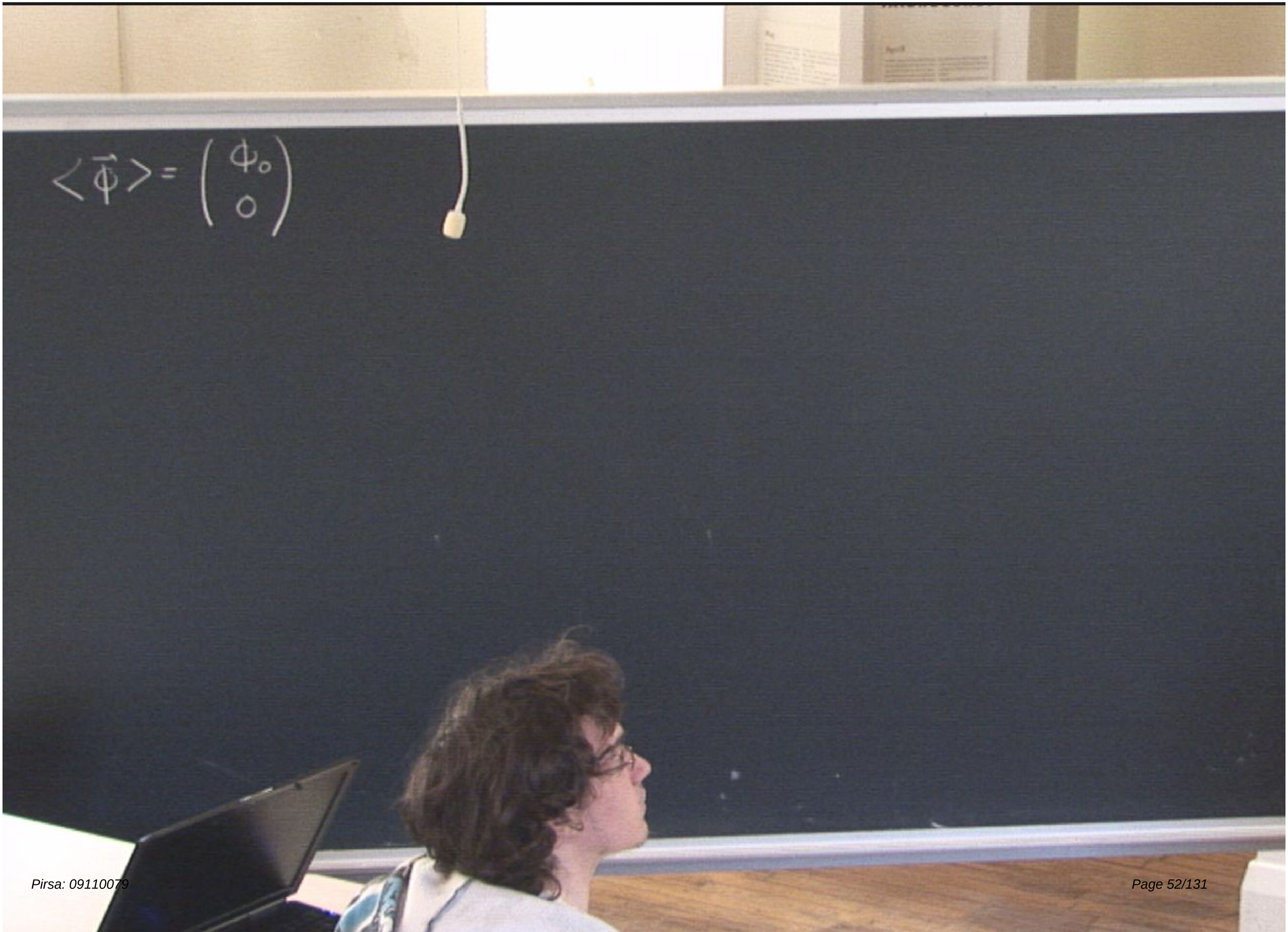
The vacuum state $|0\rangle$ is not invariant!

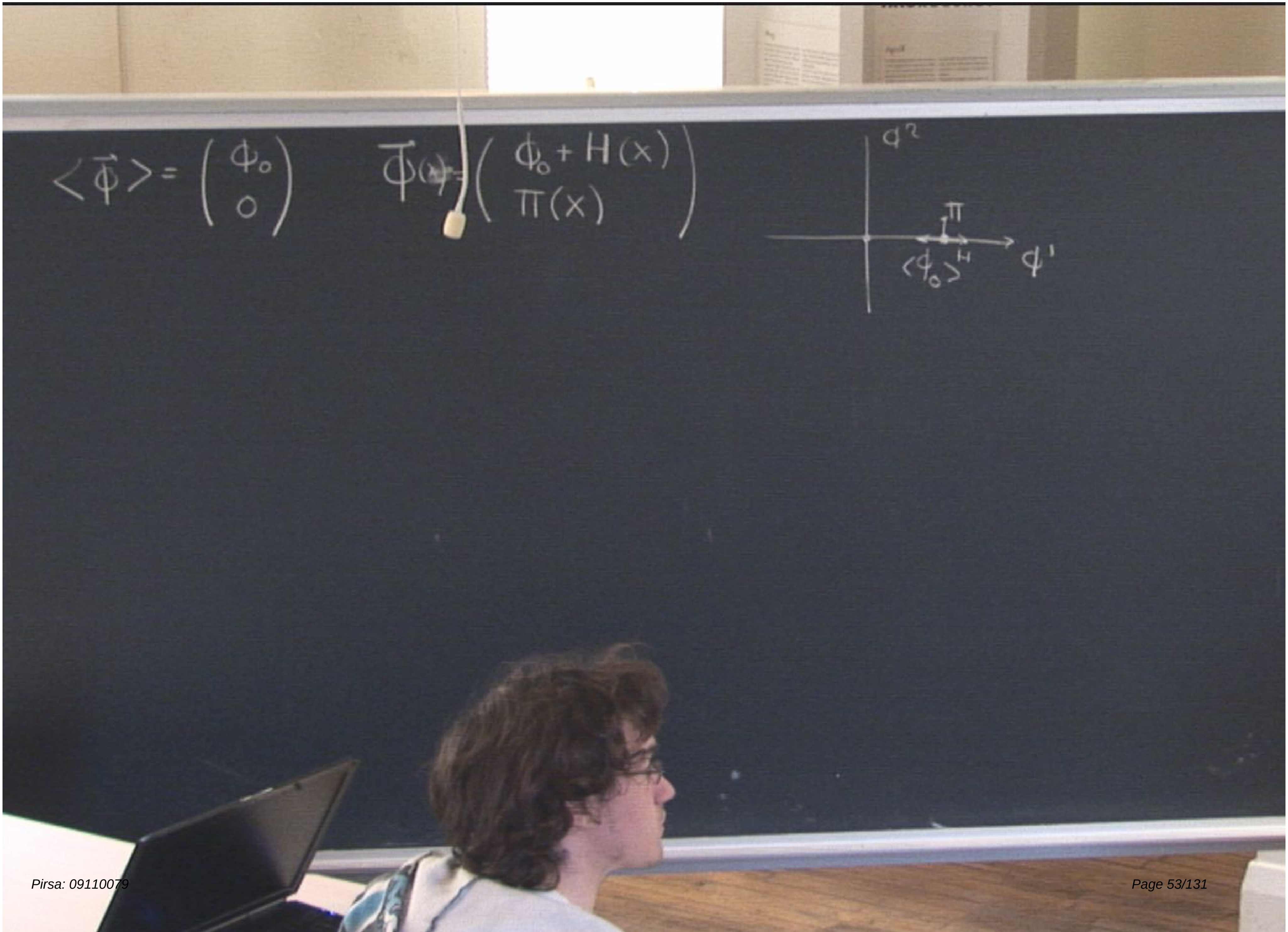
Q charge $\quad Q|0\rangle \neq 0$

current

Noether theorem $\longrightarrow$ $J_\mu(x) = i\left[\partial_\mu \bar{\Phi} \cdot \Phi - \bar{\Phi}\,\partial_\mu \Phi\right]$

$Q = \int d^3x\, J_0(x)$ $\quad [Q, H] = 0$ symmetry


$\langle \vec{\Phi} \rangle = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$



$\langle \vec{\phi} \rangle = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$
$\vec{\Phi}(x) = \begin{pmatrix} \phi_0 + H(x) \\ \pi(x) \end{pmatrix}$
ϕ^2
π
$\langle \phi_0 \rangle$
H
ϕ^1

iggs Mechanism and "Gauge symmetry breaking" not a lo sym

ymmetry breaking (spontaneous) glob

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ U(1) symm

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2 \right) \right]$$

$$V(|\phi|^2) = -\frac{t}{2} |\phi|^2 + \frac{g}{4} |\phi|^4$$

t

0

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iggs Mechanism and "Gauge symmetry breaking" not a lo... sym...

glob...

ymmetry breaking (spontaneous)

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ symm...

$$S = \int d^4x \left[-\frac{1}{2}\left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi}\right) - V\left(|\vec{\Phi}|^2\right)\right]$$

$$V(|\phi|^2) = -\frac{t}{2}|\phi|^2 + \frac{g}{4}|\phi|^4$$

$$V = -\frac{M^2}{8}\phi_0^2 + \frac{M^2}{2}H^2 + \frac{M^2}{2\phi_0}\left(H^3 + H\pi^2\right) + \frac{M^2}{8\phi_0^2}\left(\pi^4 + H^4 + 2\pi^2 H^2\right)$$

iggs Mechanism and "Gauge symmetry breaking" not a lo sym

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symmetry breaking (spontaneous) glob

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ symm

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\nu \vec{\Phi} \cdot \partial^\nu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2 \right) \right]$$

$$V(|\phi|^2) = -\frac{t}{2} |\phi|^2 + \frac{g}{4} |\phi|^4$$

$$V = -\frac{M^2}{8} \phi_0^2 + \frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} (H^3 + H\pi^2) + \frac{M^2}{8\phi_0^2} (\pi^4 + H^4 + 2\pi^2 H^2)$$

$$M^2 = 2g\,\phi_0^2$$

$-\phi$

0

Higgs Mechanism and "Gauge symmetry breaking"   not a ...

Symmetry breaking (spontaneous)

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ sym...

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi} \right) - V\left(|\vec{\Phi}|^2 \right) \right]$$

$$V(|\phi|^2) = -\frac{t}{2} |\phi|^2 + \frac{g}{4} |\phi \ldots$$

$$U(H,\pi) = \left(-\frac{M^2}{8} \phi_0^2 \right) + \frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} \left(H^3 + H\pi^2 \right) + \frac{M^2}{8\phi_0^2} \left(\pi^4 + H^4 + 2\pi^2 H^2 \right)$$

$$M^2 = 2g\phi_0^2$$

$$S = \int d^4x \left[-\frac{1}{2} \left[(\partial\pi)^2 + (\partial H)^2 \right] - U(H,\pi) \right]$$

Higgs Mechanism and "Gauge symmetry breaking" not a ...

Symmetry breaking (spontaneous)

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ sym...

$$S = \int d^4x \left[-\frac{1}{2} \left(\partial_\nu \vec{\Phi} \cdot \partial^\nu \vec{\Phi} \right) - V(|\vec{\Phi}|^2) \right] \qquad V(|\phi|^2) = -\frac{t}{2} |\phi|^2 + \frac{g}{4} |\phi ...$$

$$U(H,\pi) = \left(-\frac{M^2}{8} \phi_0^2 \right) + \frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} (H^3 + H\pi^2) + \frac{M^2}{8\phi_0^2} (\pi^4 + H^4 + 2\pi^2 H^2)$$

$M^2 = 2 g \phi_0^2$

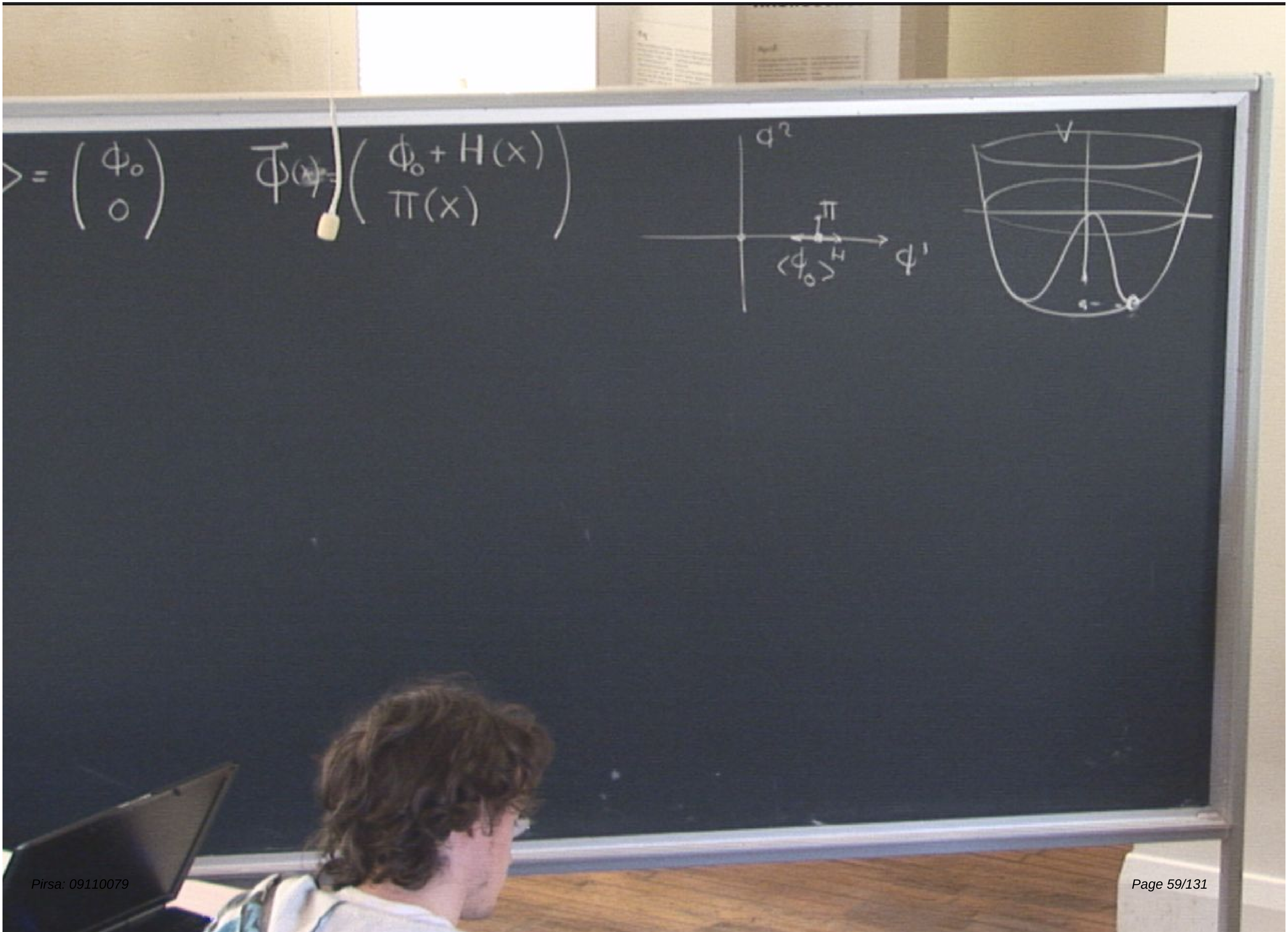
$$S = \int d^4x \left[-\frac{1}{2} \left[(\partial\pi)^2 + (\partial H)^2 \right] - U(H,\pi) \right]$$

mass terms

H is a massive field with mass M

π is a massless field

$-\phi$


> = (φ₀ 0)
Φ(x) = (φ₀ + H(x) π(x))
φ²
π
⟨φ₀⟩
H
φ¹
V

iggs Mechanism and "Gauge symmetry breaking" not a lo[cal] sym[metry]

[s]ymmetry breaking (spontaneous) glob[al]

Scalar Field with 2 components $\vec{\Phi} = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$ $U(1)$ symm[etry]

$$S = \int d^4x \left[-\frac{1}{2}\left(\partial_\mu \vec{\Phi} \cdot \partial^\mu \vec{\Phi}\right) - V\left(|\vec{\Phi}|^2\right) \right] \qquad V(|\phi|^2) = -\frac{t}{2}|\phi|^2 + \frac{g}{4}|\phi|^4$$

$$[U](H, \Pi) = \left(-\frac{M^2}{8}\phi_0^2\right) + \frac{M^2}{2}H^2 + \frac{M^2}{2\phi_0}\left(H^3 + H\Pi^2\right) + \frac{M^2}{8\phi_0^2}\left(\Pi^4 + H^4 + 2\Pi^2 H^2\right)$$

$M^2 = 2g\,\phi_0^2$

$$S = \int d^4x \left[-\frac{1}{2}\left[(\partial\Pi)^2 + (\partial H)^2\right] - U(H, \Pi) \right]$$

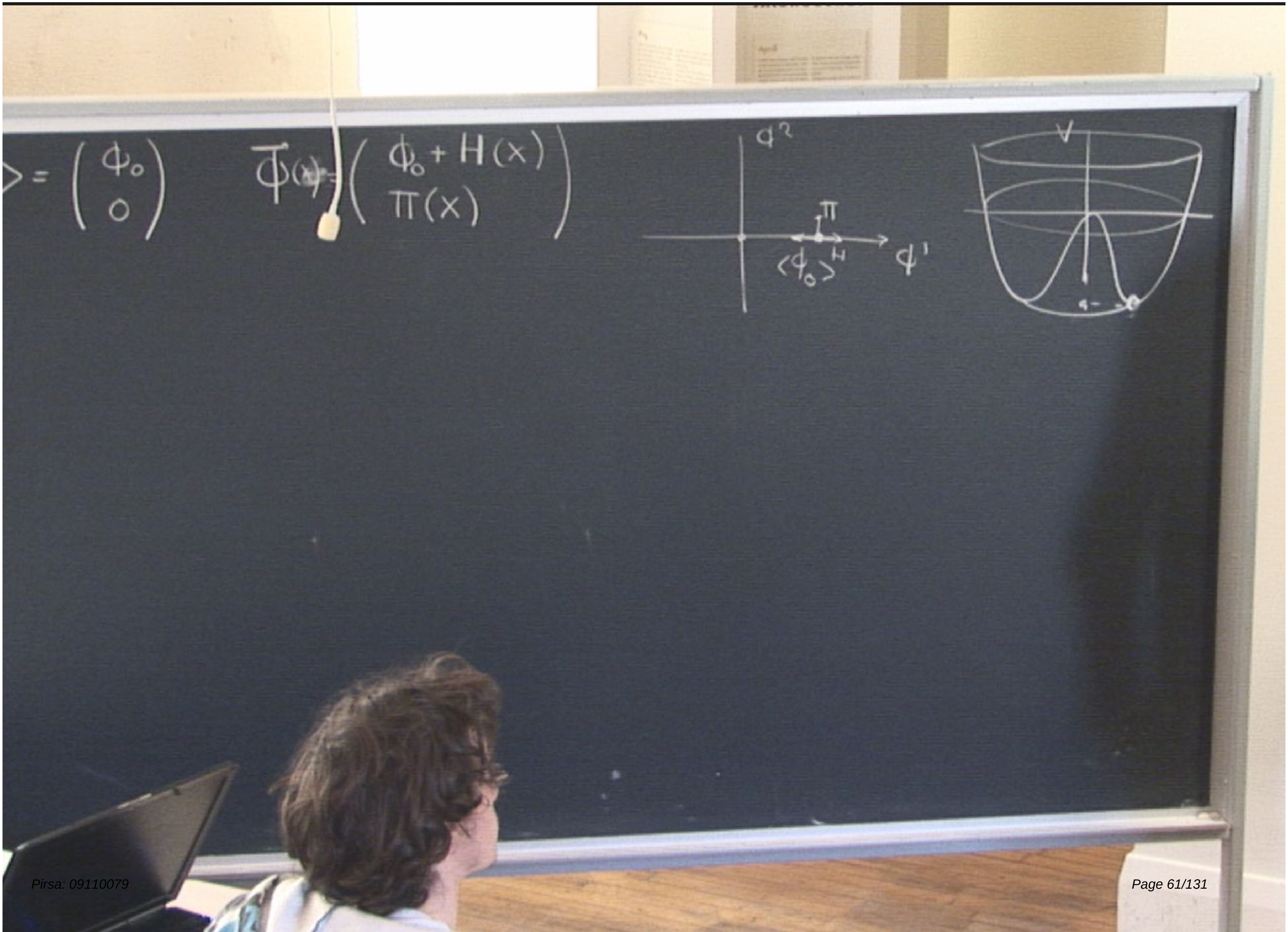
$\phi_0^2 = -t/g$

mass terms

H is a massive field with mass M

Π is a massless field

$-\phi$


$\rangle = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$
$\Phi(x) = \begin{pmatrix} \phi_0 + H(x) \\ \pi(x) \end{pmatrix}$
ϕ^2
π
$\langle \phi_0 \rangle$
H
ϕ^1
V

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π is a Goldstone Boson:

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π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

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π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ———— $M_H = M > 0$

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π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ———— $M_H = M > 0$

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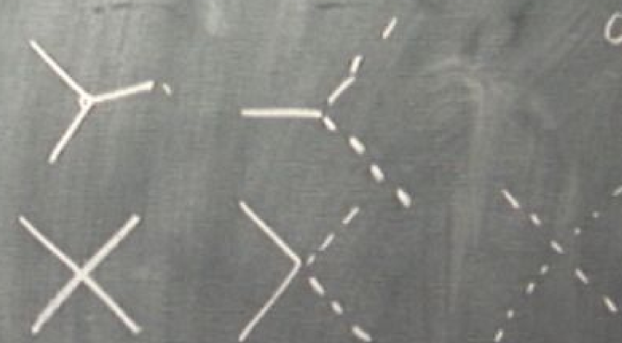
π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ———— $M_H = M > 0$



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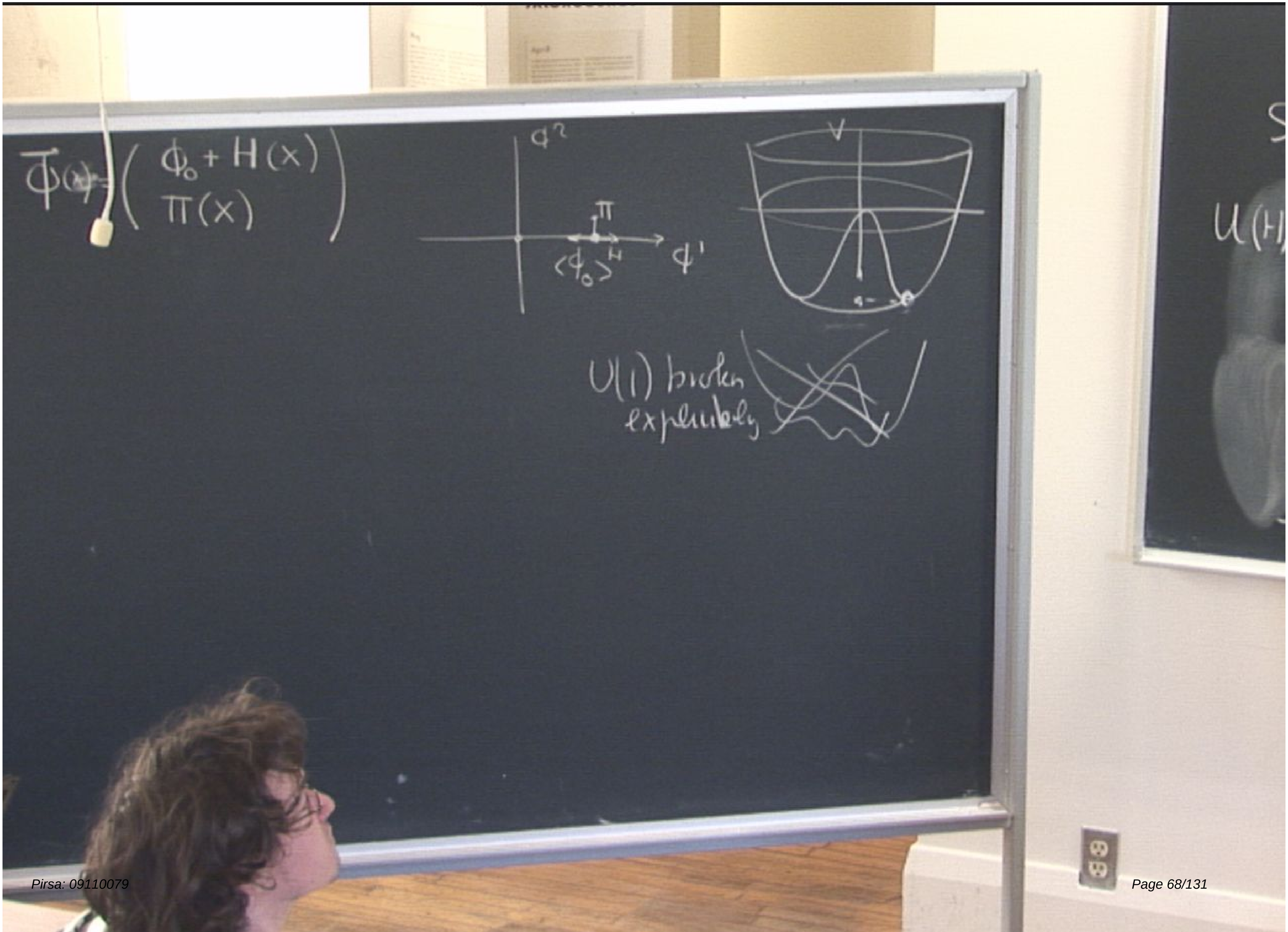
π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$

U(1) symmetry


Φ(x) = (φ₀ + H(x)
π(x))
φ2
π
⟨φ0⟩
H
φ1
V
U(1) broken
explicitly

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π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$

U(1) symmetry

What is happening if ϕ is now coupled to a gauge theory

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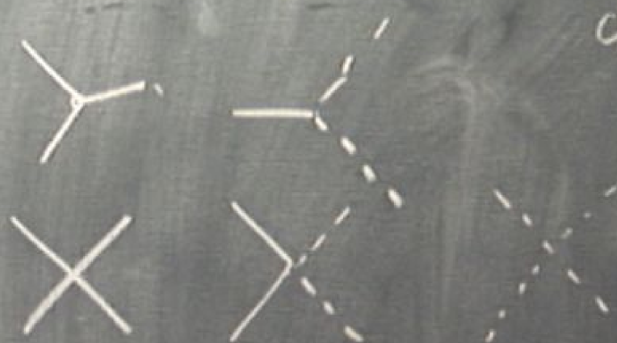
π is a Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$



U(1) symmetry

What is happening if ϕ is now coupled to a gauge theory

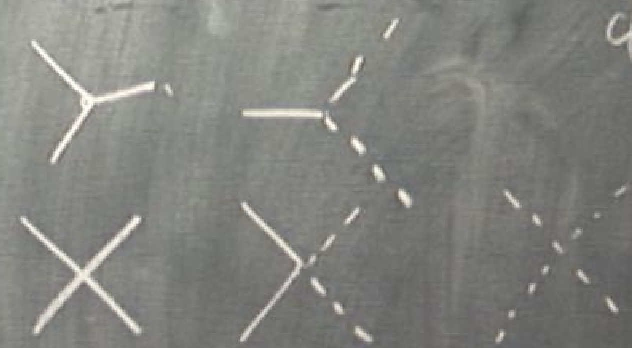
$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field.

$t > 0$ scalar QED

π is a Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$

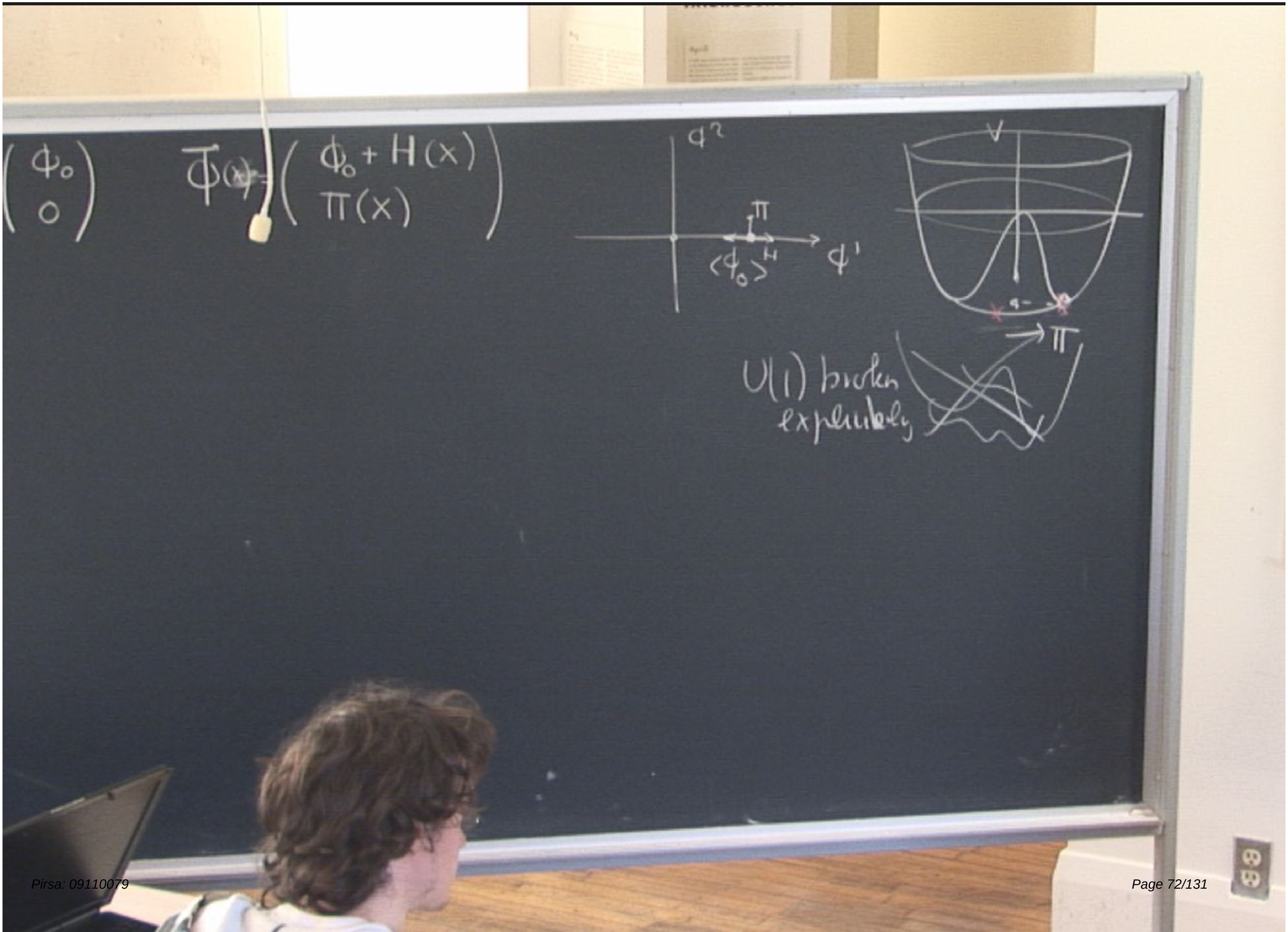


U(1) symmetry

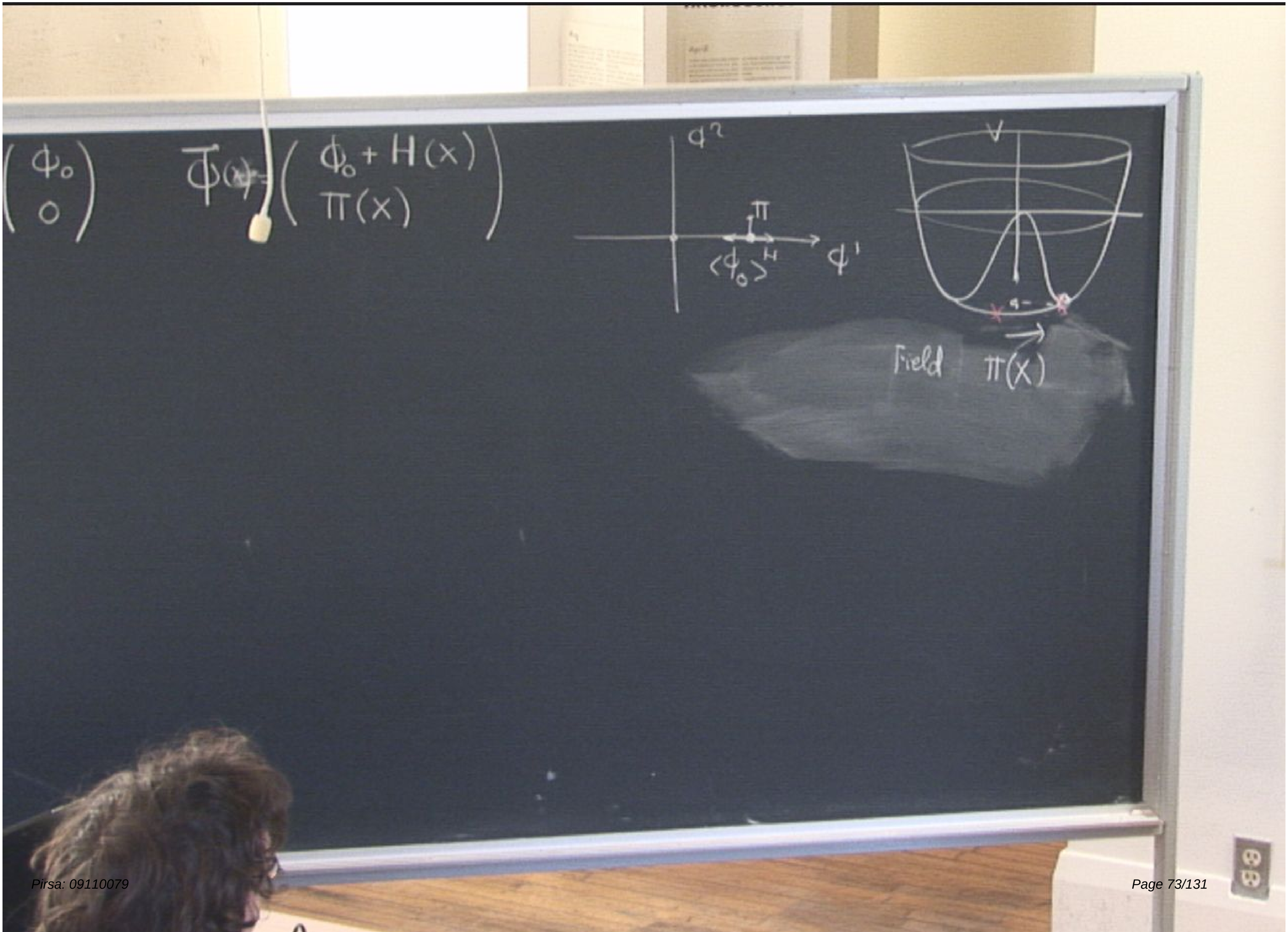
What is happening if ϕ is now coupled to a gauge theory

$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field.

$t > 0$ scalar QED , $t < 0$



$\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$
$\Phi(x) = \begin{pmatrix} \phi_0 + H(x) \\ \pi(x) \end{pmatrix}$
ϕ^2
π
H
$\langle \phi_0 \rangle$
ϕ^1
V
π
U(1) broken
explicitly


Φ(x) = (φ₀ + H(x) / π(x))
φ²
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⟨φ₀⟩
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φ¹
V
Field π(x)

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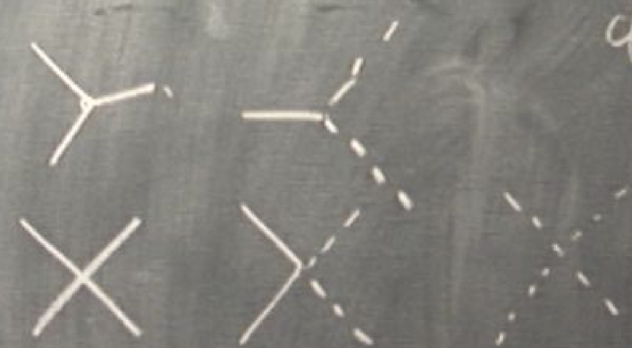
π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$



U(1) symmetry

What is happening if ϕ is now coupled to a gauge theory

$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field. $\phi = \phi_1 + i\,\phi_2$

$t > 0$ scalar QED , $t < 0$ $\phi \to e^{i\alpha}\phi$ U(1)

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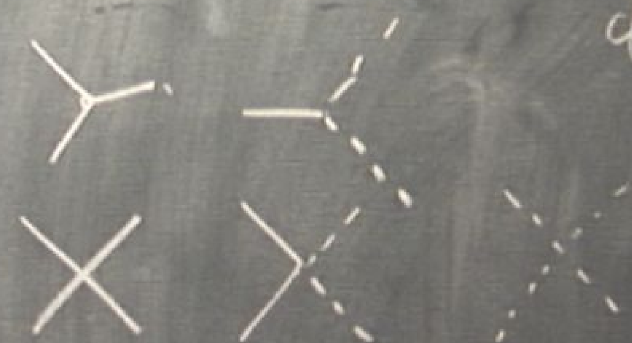
π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$

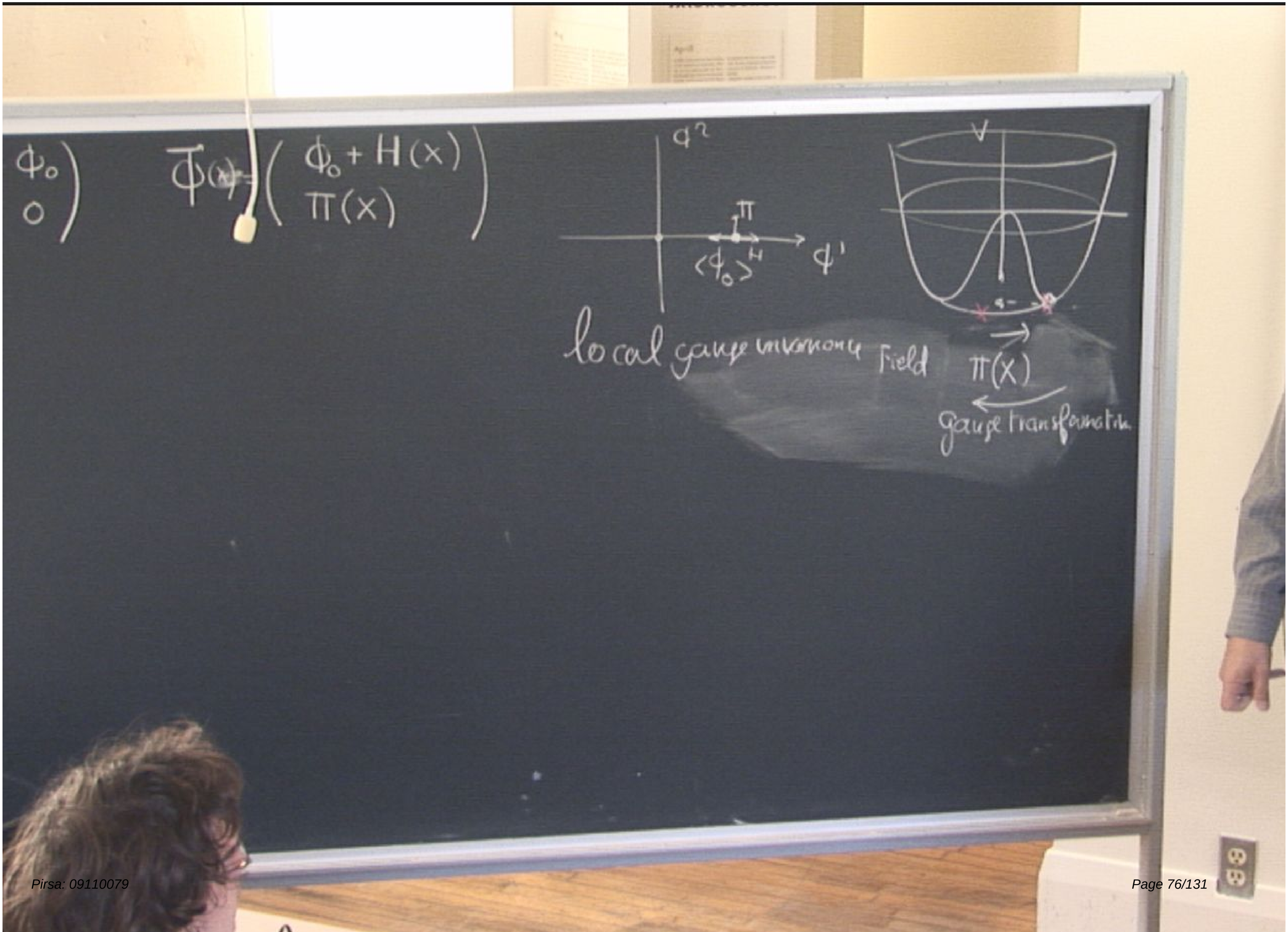


U(1) symmetry

What is happening if ϕ is now coupled to a gauge theory

$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field. $\quad \phi = \phi_1 + i\,\phi_2$

$t > 0$ scalar: ϕED , $\quad t < 0 \quad \phi \to e^{i\alpha}\phi \quad U(1)$


$\Phi(x) = \begin{pmatrix} \phi_0 + H(x) \\ \pi(x) \end{pmatrix}$
ϕ^2
π
H
$\langle\phi_0\rangle$
ϕ^1
V
local gauge invariance
Field
$\pi(X)$
gauge transformation

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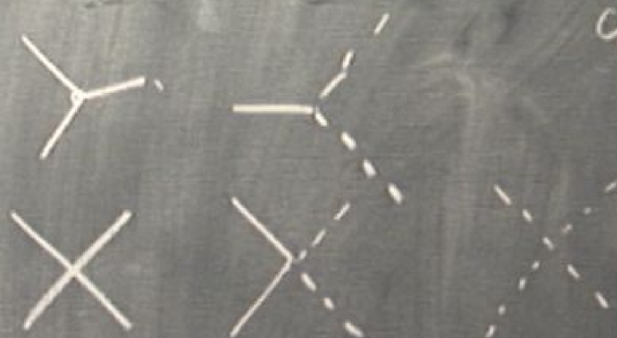
π is Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$



$U(1)$ symmetry

What is happening if ϕ is now coupled to a gauge theory

$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field. $\quad \phi = \phi_1 + i\,\phi_2$

$t > 0$ scalar QED , $\quad t < 0 \qquad \phi \to e^{i\alpha}\phi \quad U(1)$

$U(1)$ symmetry ?

ϕ complex scalar Field + A_μ gauge potential

massless field

$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

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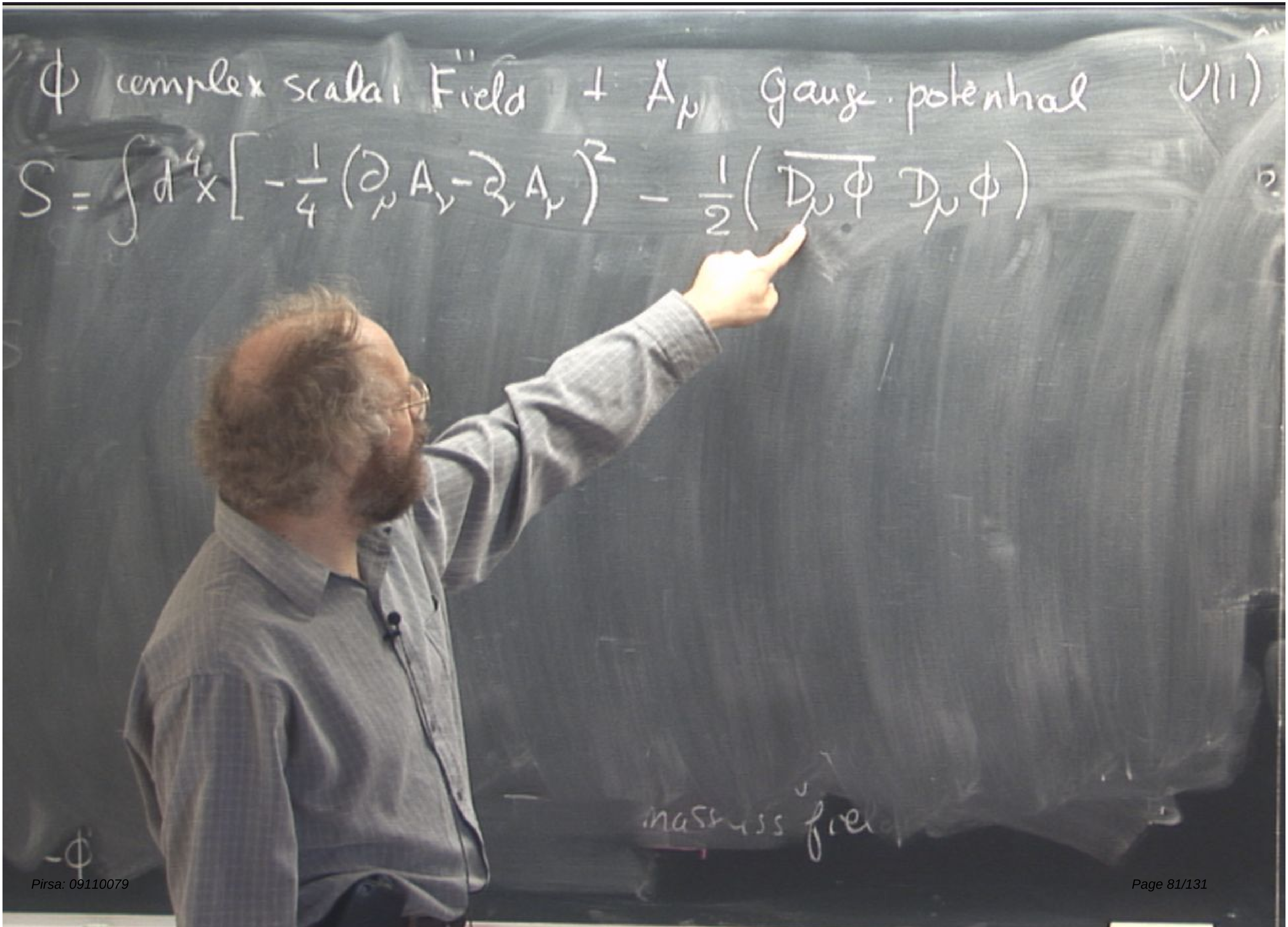
ϕ complex scalar "Field" + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right.$$

massless field


ϕ complex scalar Field + A_μ gauge potential U(1)
$S = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi \right)\right.$
massless

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi)\right]$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$$V = \frac{t}{2}\bar{\phi}\phi + \frac{g}{4}(\bar{\phi}\phi)^2$$

massless field

ϕ

π is a Goldstone Boson: massless $\Leftarrow$ spontaneous breaking of symmetry

π - - - - - - $m_\pi = 0$

H ——— $M_H = M > 0$

U(1) symmetry

What is happening if ϕ is now coupled to a gauge theory

$\Phi = (\phi^1, \phi^2) \longleftrightarrow U(1)$ gauge field. $\phi = \phi_1 + i\,\phi_2$ $\bar{\phi} = \phi_1 - i\,\phi_2$

$t > 0$ scalar QED, $t < 0$ $\phi \to e^{i\alpha}\phi$ U(1)

U(1) symmetry ?

ϕ complex scalar Field + A_μ gauge potential U(1)

$$S = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right]$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$$V = \frac{t}{2}\bar{\phi}\phi + \frac{g}{4}(\bar{\phi}\phi)^2$$

$$\phi \to e^{i\alpha e}\phi$$
$$\bar{\phi} \to e^{-i\alpha e}\phi$$

massless fiel

$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi)\right]$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$$V = \frac{t}{2}\bar{\phi}\phi + \frac{g}{4}(\bar{\phi}\phi)^2$$

$$\phi \to e^{i\alpha(x)e}\phi$$

$$\bar{\phi} \to e^{-i\alpha(x)e}\bar{\phi}$$

$t < 0$

massless fie

$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\overline{\phi}\phi)\right]$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$$V = \frac{t}{2}\overline{\phi}\phi + \frac{g}{4}(\overline{\phi}\phi)^2$$

$$\phi \to e^{i\alpha(x)e}\phi$$
$$\overline{\phi} \to e^{-i\alpha(x)e}\overline{\phi}$$
$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

Gauge Transformations

massless field

$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\nu \phi}\, D_\mu \phi\right) - V(\overline{\phi}\phi)\right]$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$$V = \frac{t}{2}\overline{\phi}\phi + \frac{g}{4}(\overline{\phi}\phi)^2$$

$$\phi \to e^{i\alpha(x)e}\phi$$
$$\overline{\phi} \to e^{-i\alpha(x)e}\overline{\phi}$$
$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

Gauge Transformations

Gauge Fixing condition on the complex field ϕ

$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\nu \phi}\, D_\mu \phi\right) - V(\overline{\phi}\phi)\right.$$

$D_\mu = \partial_\mu - ieA_\mu$ e charge of the field ϕ

$V = \frac{t}{2}\overline{\phi}\phi + \frac{g}{4}(\overline{\phi}\phi)^2$

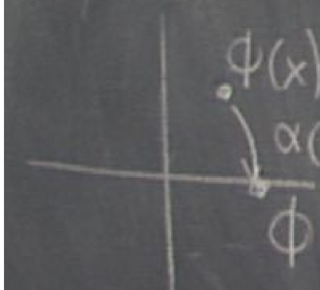
$\phi \to e^{i\alpha(x)e}\phi$

$\overline{\phi} \to e^{-i\alpha(x)e}\overline{\phi}$

$A_\mu \to A_\mu + \partial_\mu \alpha(x)$

Gauge Transformations

Gauge Fixing condition on the complex field ϕ :



$\mathrm{Im}\,\phi(x) = 0$

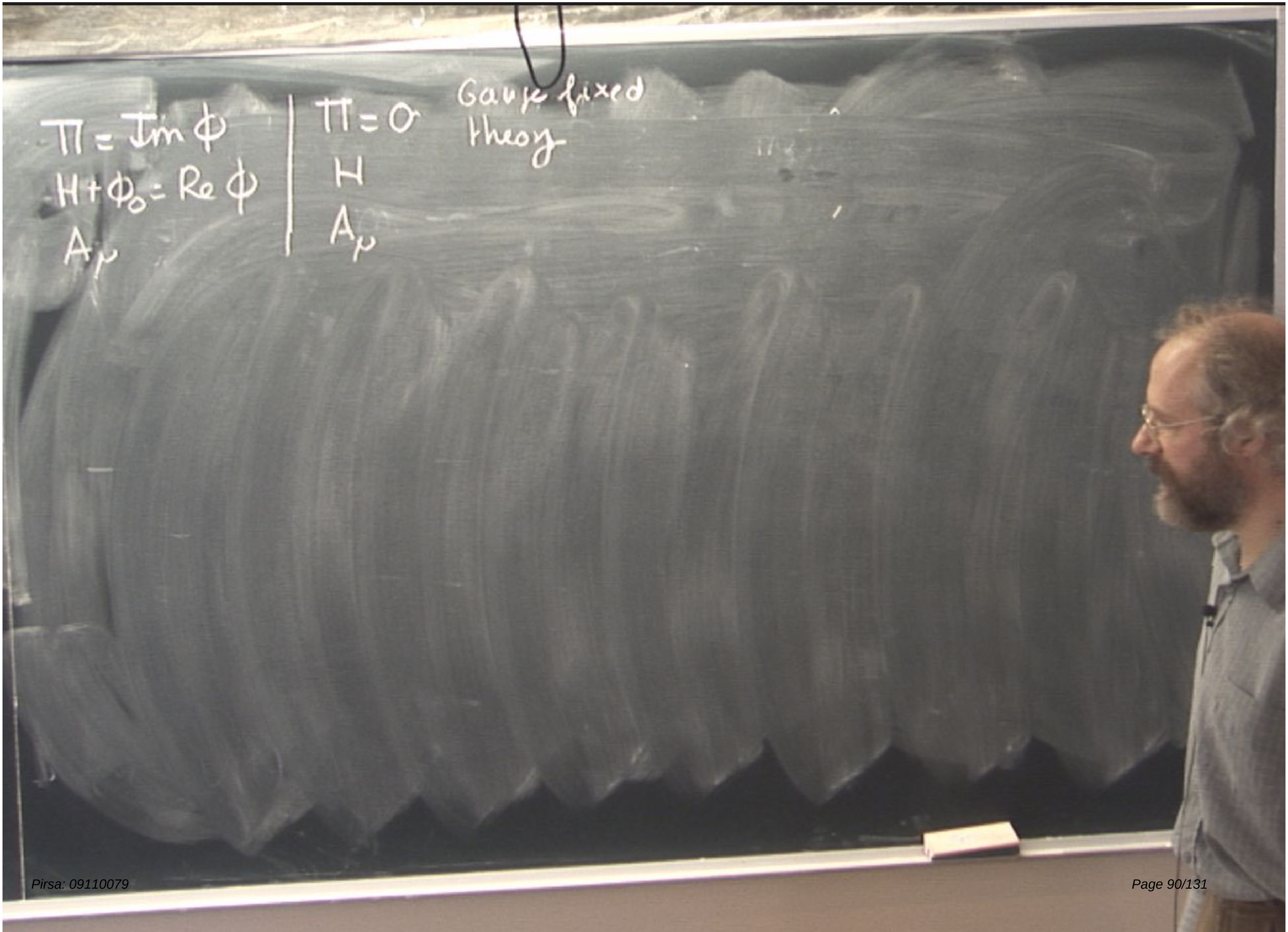
$\Longleftrightarrow \phi(x)$ real

unitarity gauge

$-\phi$

$$\Pi = \mathrm{Im}\,\phi$$

$$H + \phi_0 = \mathrm{Re}\,\phi$$


$\Pi = \mathrm{Im}\,\phi$
$H + \phi_0 = \mathrm{Re}\,\phi$
A_μ
$\Pi = 0$
H
A_μ
Gauge fixed theory

$\Pi = \mathrm{Im}\,\phi$

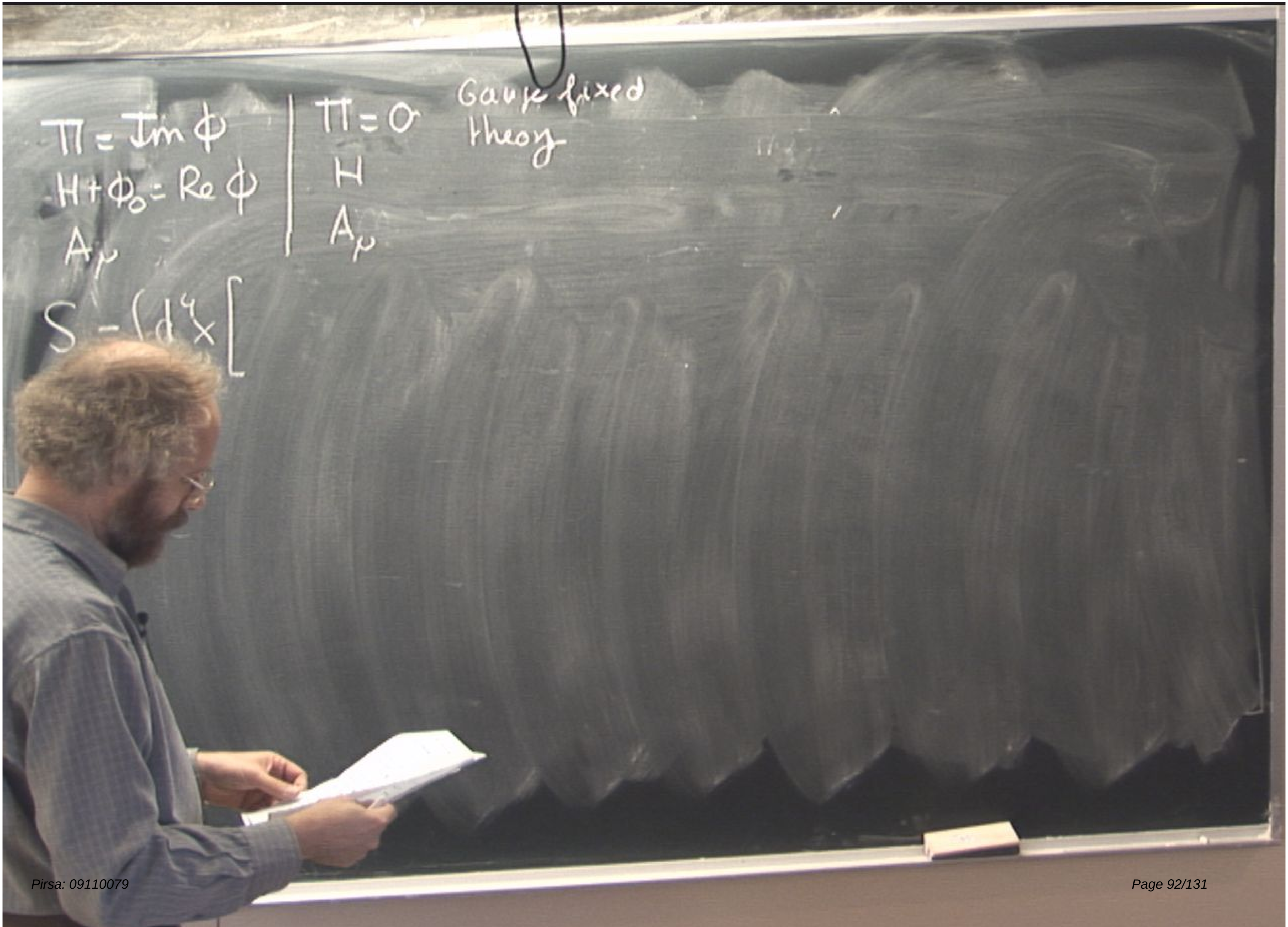
$H + \Phi_0 = \mathrm{Re}\,\phi$

A_μ

$\Pi = 0$ Gauge fixed theory

H

A_μ


Π = Im φ
H + φ0 = Re φ
Aμ
Π = 0
H
Aμ
Gauge fixed
theory
S = ∫d4x [

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$S_{GF} = \int d^4x \left[-\frac{1}{2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 \right.$$

$\Pi = \mathrm{Im}\,\phi$	$\Pi = 0$	Gauge fixed theory
$H + \phi_0 = \mathrm{Re}\,\phi$	H	
A_μ	A_μ	

$$S_{GF} = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left[\left(\partial_\mu H\right)^2 + e^2 \left(A_\mu \phi_0 + A_\mu H\right)^2\right]\right]$$

$\Pi = \mathrm{Im}\,\phi$	$\Pi = 0$	Gauge fixed theory
$H + \phi_0 = \mathrm{Re}\,\phi$	H	
A_μ	A_μ	

$$S_{GF} = \int d^4x \Big[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\Big[\left(\partial_\mu H\right)^2 + e^2\left(A_\mu \phi_0 + A_\mu H\right)^2\Big]$$

$$- \frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4$$

$\Pi = \mathrm{Im}\,\phi$	$\Pi = 0$	Gauge fixed theory
$H + \phi_0 = \mathrm{Re}\,\phi$	H	
A_μ	A_μ	

$$S_{GF} = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left[\left(\partial_\mu H\right)^2 + e^2\left(A_\mu \phi_0 + A_\mu H\right)^2\right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right)\right.$$

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$S_{GF} = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left[\left(\partial_\mu H\right)^2 + e^2\left(A_\mu \phi_0 + A_\mu H\right)^2\right] - \left(\frac{M^2}{2}H^2 + \frac{M^2}{2\phi_0}H^3 + \frac{M^4}{8\phi_0^2}H^4\right)\right]$$

H - still a masse boson with mass M

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$S_{GF} = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left[\left(\partial_\mu H\right)^2 + e^2 \left(A_\mu \phi_0 + A_\mu H\right)^2 \right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4 \right) \right.$$

H still a masse boson with mass M

$$-\frac{1}{4}\left(\partial_\nu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{e^2}{2} \phi_0^2 A_\mu^2$$

$\Pi = \mathrm{Im}\,\phi$	$\Pi = 0$
$H + \phi_0 = \mathrm{Re}\,\phi$	H
A_μ	A_μ

Gauge fixed theory

$$S_{GF} = \int d^4x \Big[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\Big[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2\Big] - \Big(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\Big)$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\nu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2}\phi_0^2 A_\mu^2$$

mass term for the gauge Field

masse $\simeq e\,\phi_0 = m_A$

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$S_{GF} = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2 \right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4 \right) \right.$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2}\phi_0^2 A_\mu^2$$

mass term for the gauge Field

masse $\simeq e\,\phi_0 = m_A \neq 0$

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$S_{GF} = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2 \right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4 \right) \right.$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2} \phi_0^2 A_\mu^2$$

mass term for the gauge Field

masse $\simeq e\,\phi_0 = M_A \neq 0$

$$\Pi = \mathrm{Im}\,\phi \qquad \Pi = 0$$
$$H + \phi_0 = \mathrm{Re}\,\phi \qquad H$$
$$A_\mu \qquad A_\mu$$

Gauge fixed theory

$$e^2 A_\mu A_\mu H^2 + 2e^2 A_\mu^2 H \phi_0$$

$$S_{GF} = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2\right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right)\right.$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2}\phi_0^2 A_\mu^2$$

mass term for the gauge Field

masse $\simeq e\,\phi_0 = M_A \neq 0$

$$-\frac{1}{2}\left((\partial_\mu H)^2 + M^2 H^2\right)$$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2} \left(\overline{D_\mu \phi}\, D_\mu \phi \right) - V(\bar{\phi}\phi) \right]$$

A_μ massive vector boson

H massive scalar boson

ration

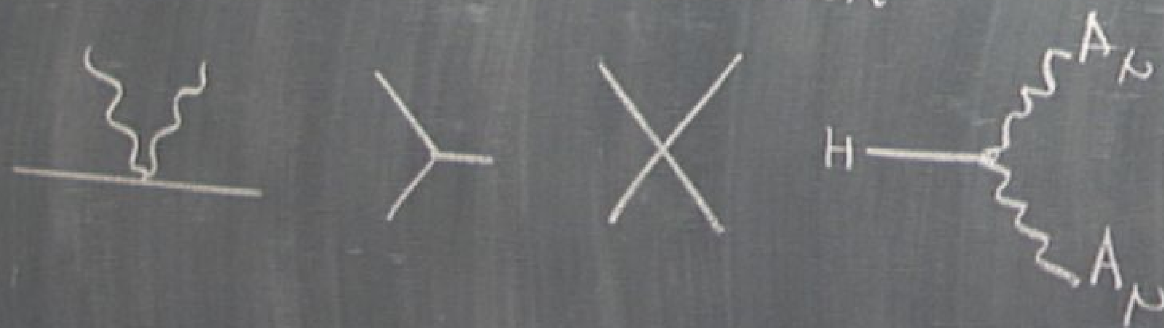
$-\phi$

ϕ complex scalar Field + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right.$$

A_μ massive vector boson

H massive scalar boson



ratios

$-\phi$

ϕ complex scalar "Field" + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right]$$

A_μ massive vector boson ← $U(1)$ symmetry is "broken"

H massive scalar boson

A_μ becomes massive with a mass $M_A = e\phi_0$

new coupling

$-\phi$

ϕ complex scalar "Field" + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi)\right]$$

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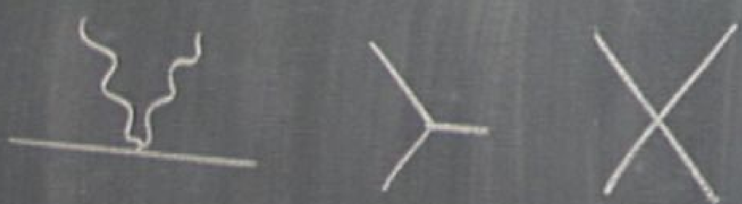
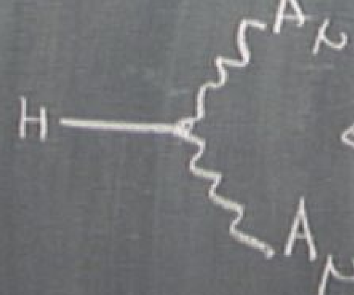




← new coupling $e^2\phi_0$

$-\phi$

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$$\boxed{e^2 A_\mu A_\mu H^2 + 2e^2 A_\mu^2 H \phi_0}$$

$$S_{GF} = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2\right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right)\right]$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2}\phi_0^2 A_\mu^2$$

$$-\frac{1}{2}\left((\partial_\mu H)^2 + M^2 H^2\right)$$

$$-\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^2 + \frac{M^4}{8\phi_0^2} H^4$$

$+$

mass term for the gauge Field

masse $\simeq e\,\phi_0 = M_A \neq 0$

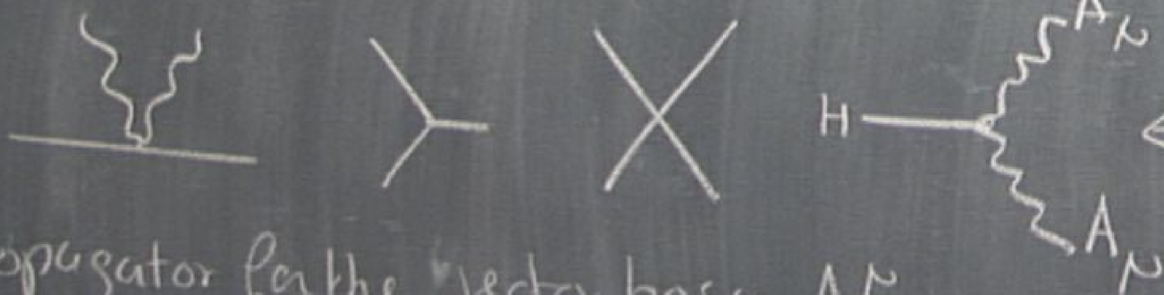
ϕ complex scalar "Field" + "A_μ" gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right]$$

A_μ ~~~ massive vector boson ← $U(1)$ symmetry is "broken"

H —— massive scalar boson

A^μ becomes massive with a mass $M_A = e\phi_0$



← new coupling $e^2\phi_0$

propagator for the vector boson A^μ

$$\Pi_{\mu\nu}(p) = \frac{-i}{p^2+M^2}\left(\eta^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}\right)$$

$-\phi$

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$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi)\right]$$

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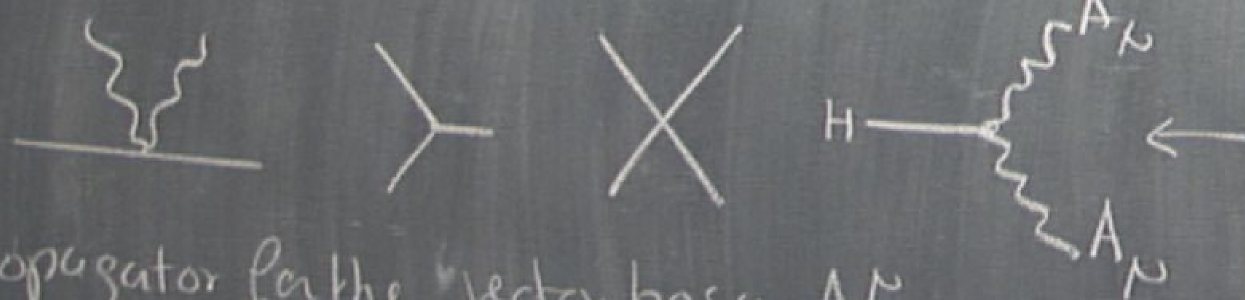
new coupling $e^2\phi_0$



propagator for the vector boson A^μ

$$\Pi_{\mu\nu}(p) = \frac{-i}{p^2+M^2}\left(\eta^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}\right)$$

$$p^2 = -p_0^2 + \vec{p}^{\,2}$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix}$$

$-\phi$

ϕ complex scalar "Field" + A_μ gauge potential $U(1)$

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H —— massive scalar boson

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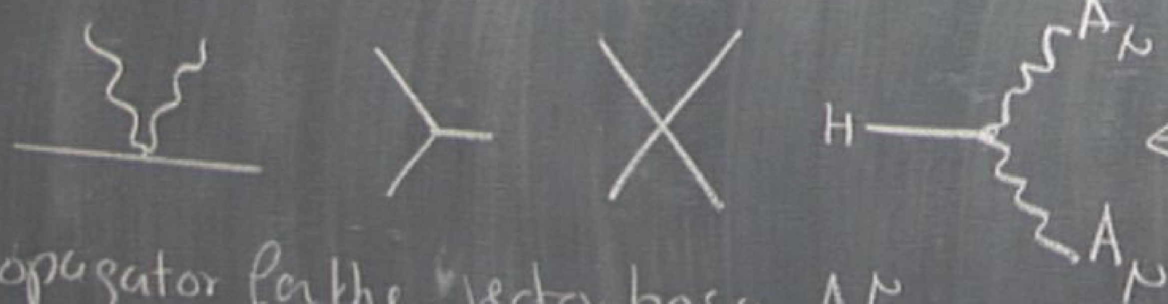


← new coupling

$e^2 \phi_0$

propagator for the vector boson A^μ

$$\Pi_{\mu\nu}(p) = \frac{-i}{p^2 + M^2}\left(\eta^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}\right) \qquad p^2 = -p_0^2 + \vec{p}^{\,2}$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix}$$

$-\phi$

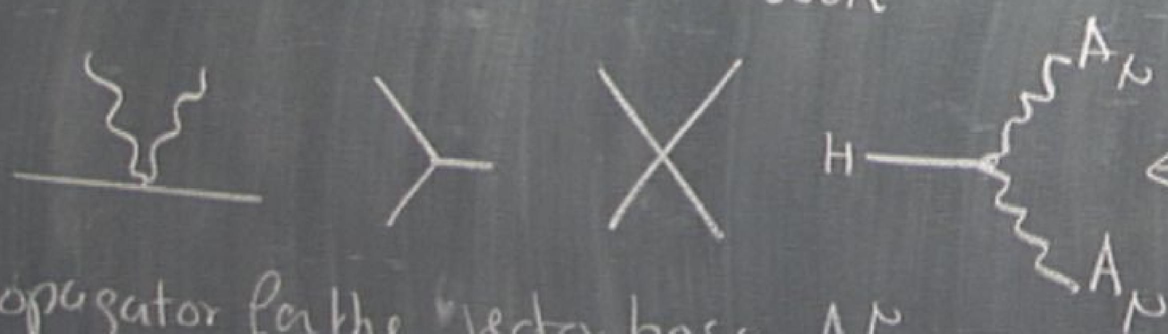
ϕ complex scalar "Field" + A_μ gauge potential $U(1)$

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right]$$

A_μ ~~~~ massive vector boson ← $U(1)$ symmetry is "broken"

H —— massiv scalar boson

A^μ becomes massive with a mass $M_A = e\phi_0$



← new coupling $e^2 \phi_0$

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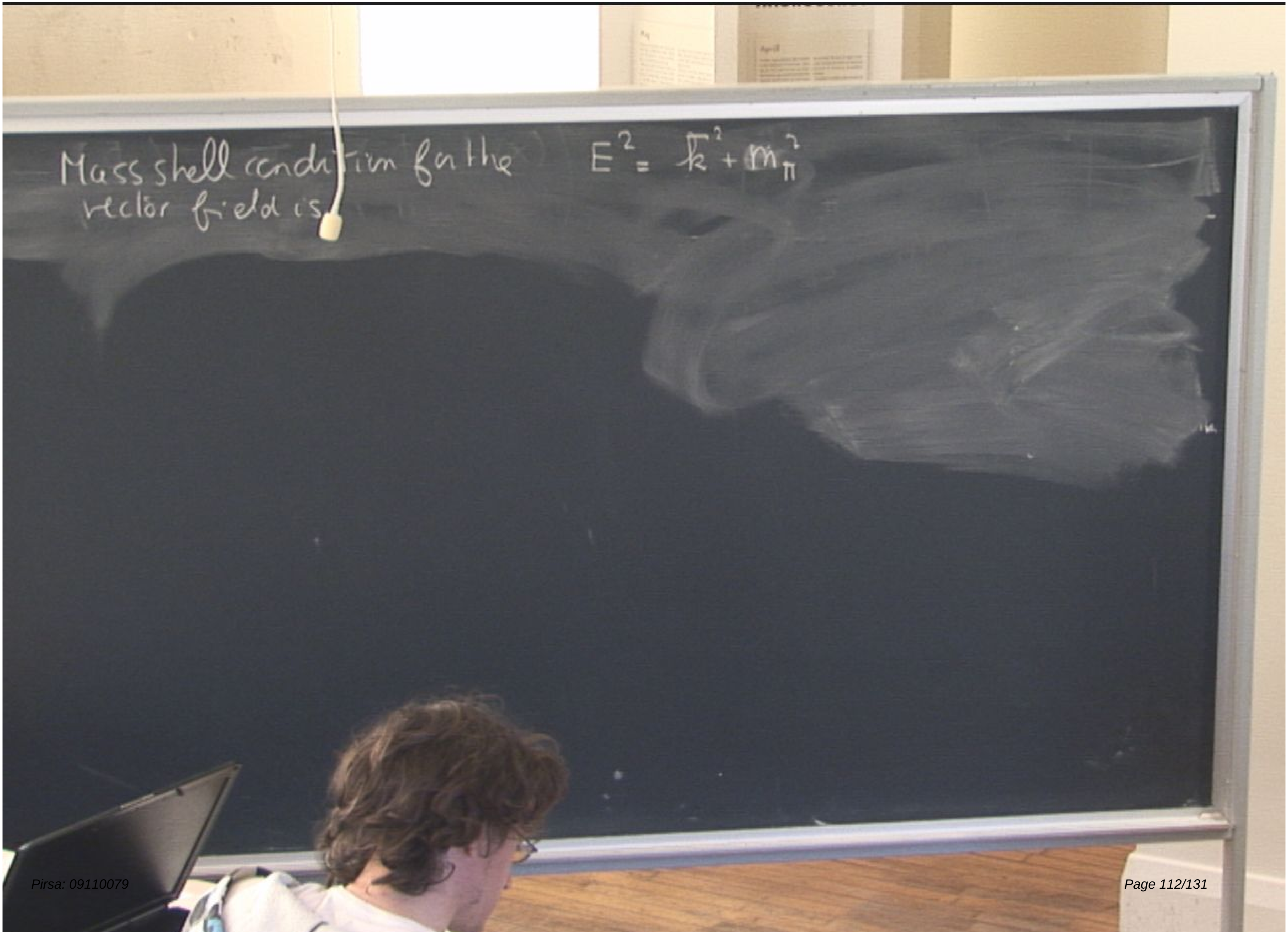
$$p^2 = -p_0^2 + \vec{p}^{\,2}$$

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix}$$

$-\phi$


Mass shell condition for the vector field is
$E^2 = \bar{k}^2 + m_\pi^2$

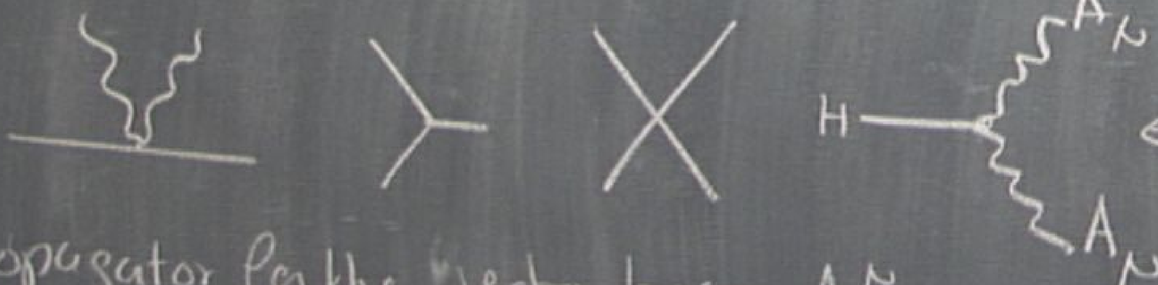
ϕ complex scalar Field + A_μ gauge potential U(1)

$$S = \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 - \frac{1}{2}\left(\overline{D_\mu \phi}\, D_\mu \phi\right) - V(\bar{\phi}\phi) \right]$$

A_μ ~~~~ massive vector boson ← U(1) symmetry is "broken"

H —— massiv scalar boson

A^μ becomes massive with a mass $m_A = e\phi_0$



← new coupling $e^2 \phi_0$

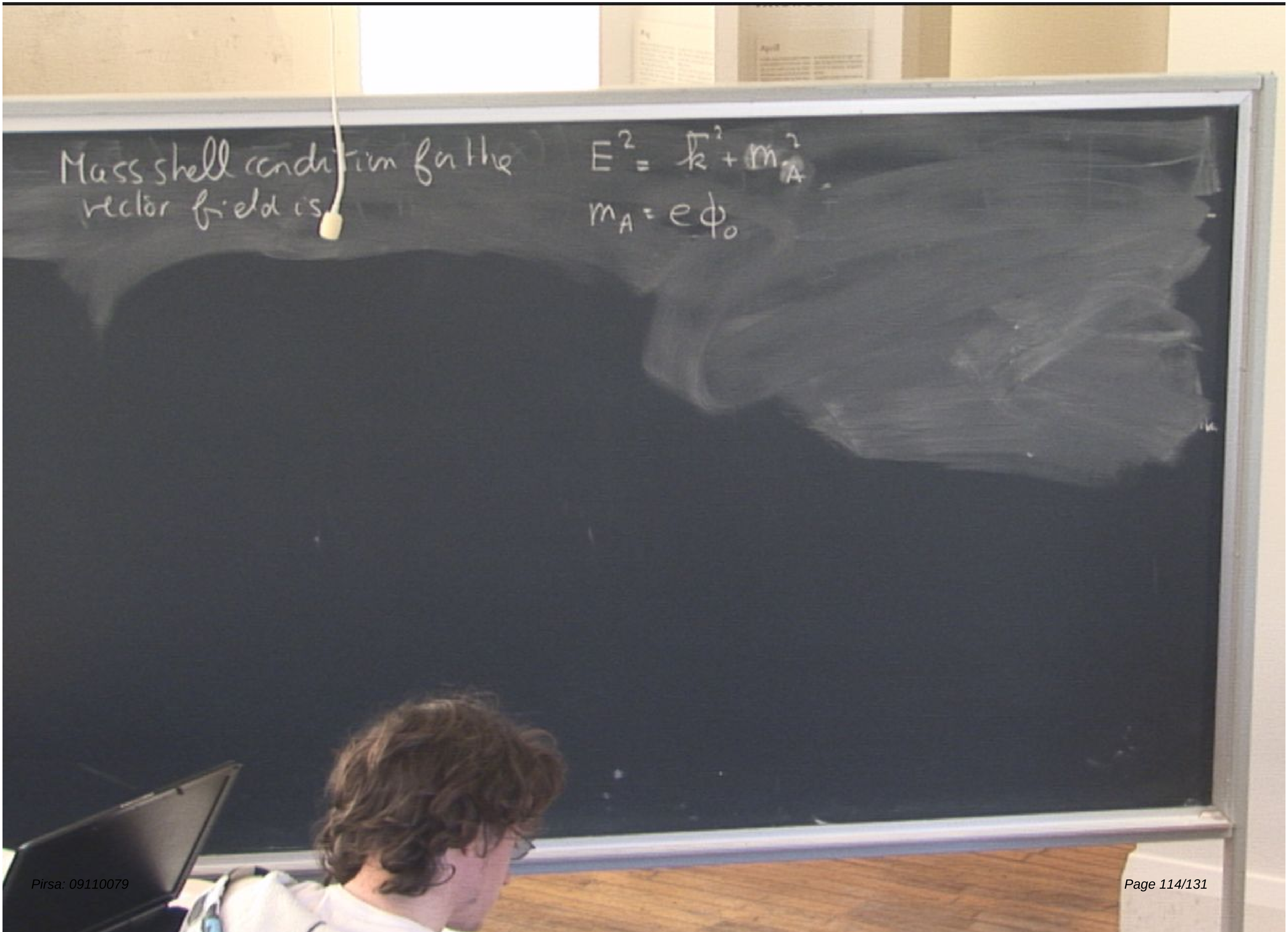
propagator for the vector boson A^μ

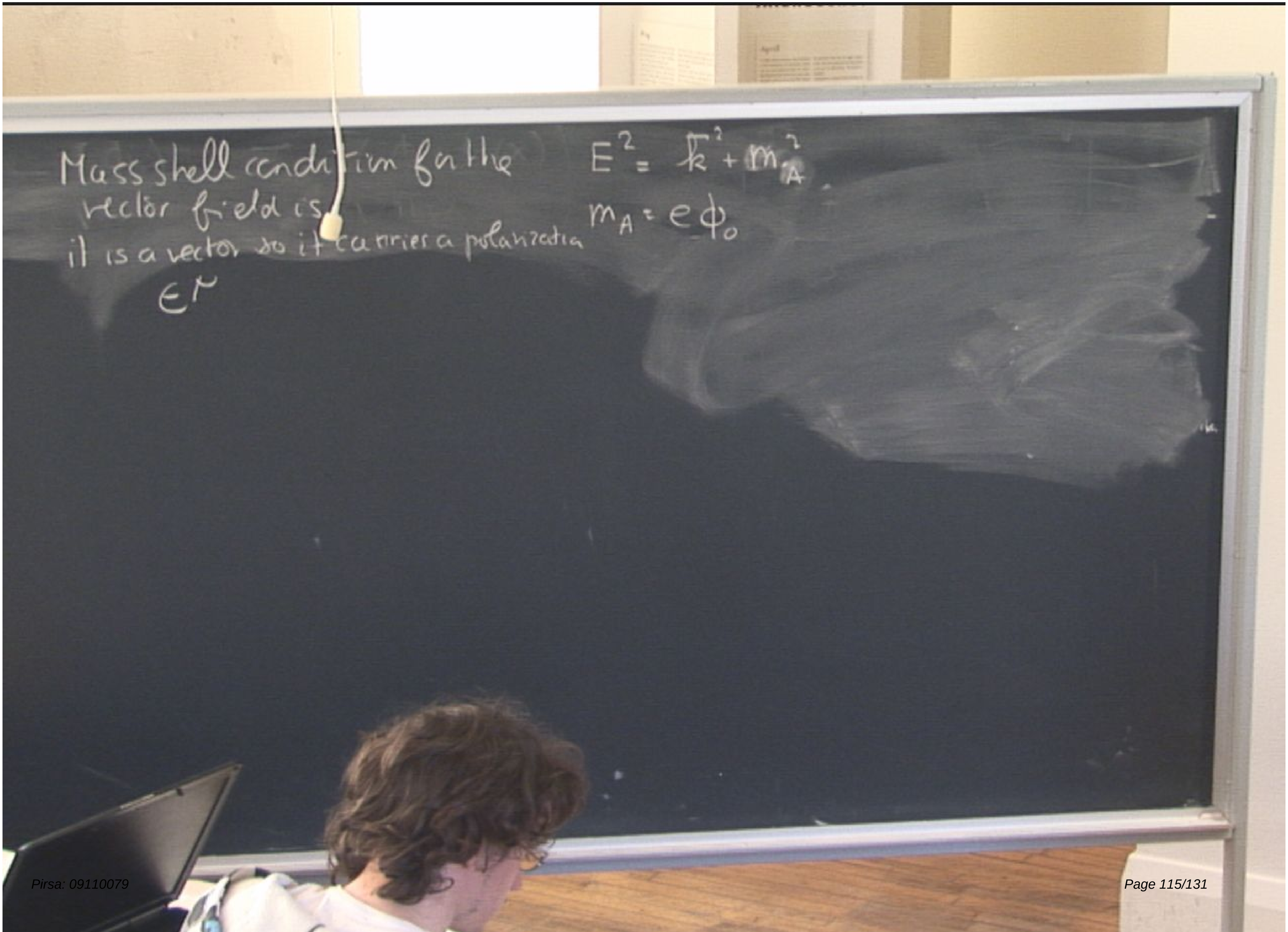
$$\Pi_{\mu\nu}(p) = \frac{-i}{p^2 + m_A^2}\left(\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m_A^2}\right)$$

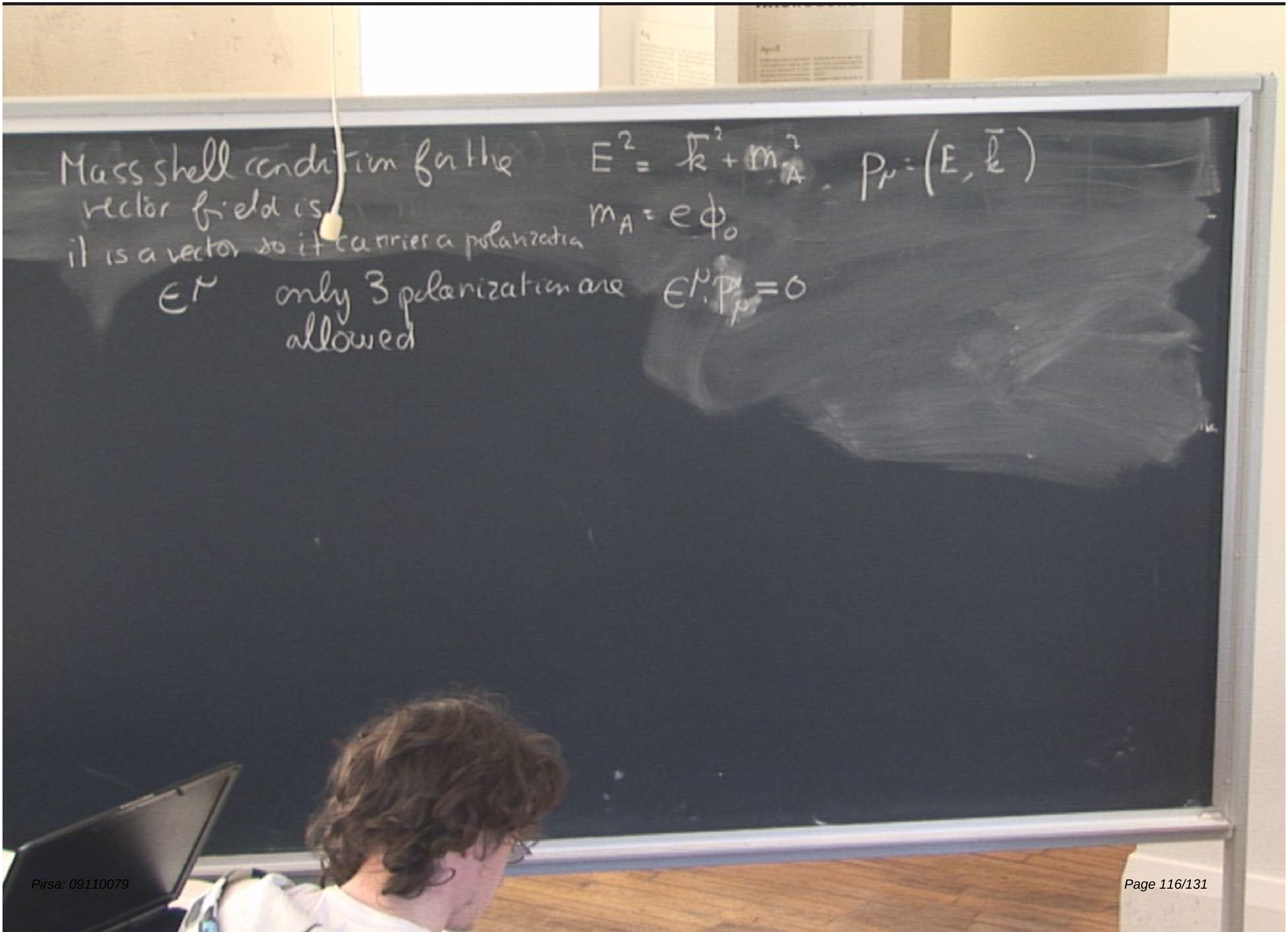
$$p^2 = -p_0^2 + \vec{p}^{\,2}$$

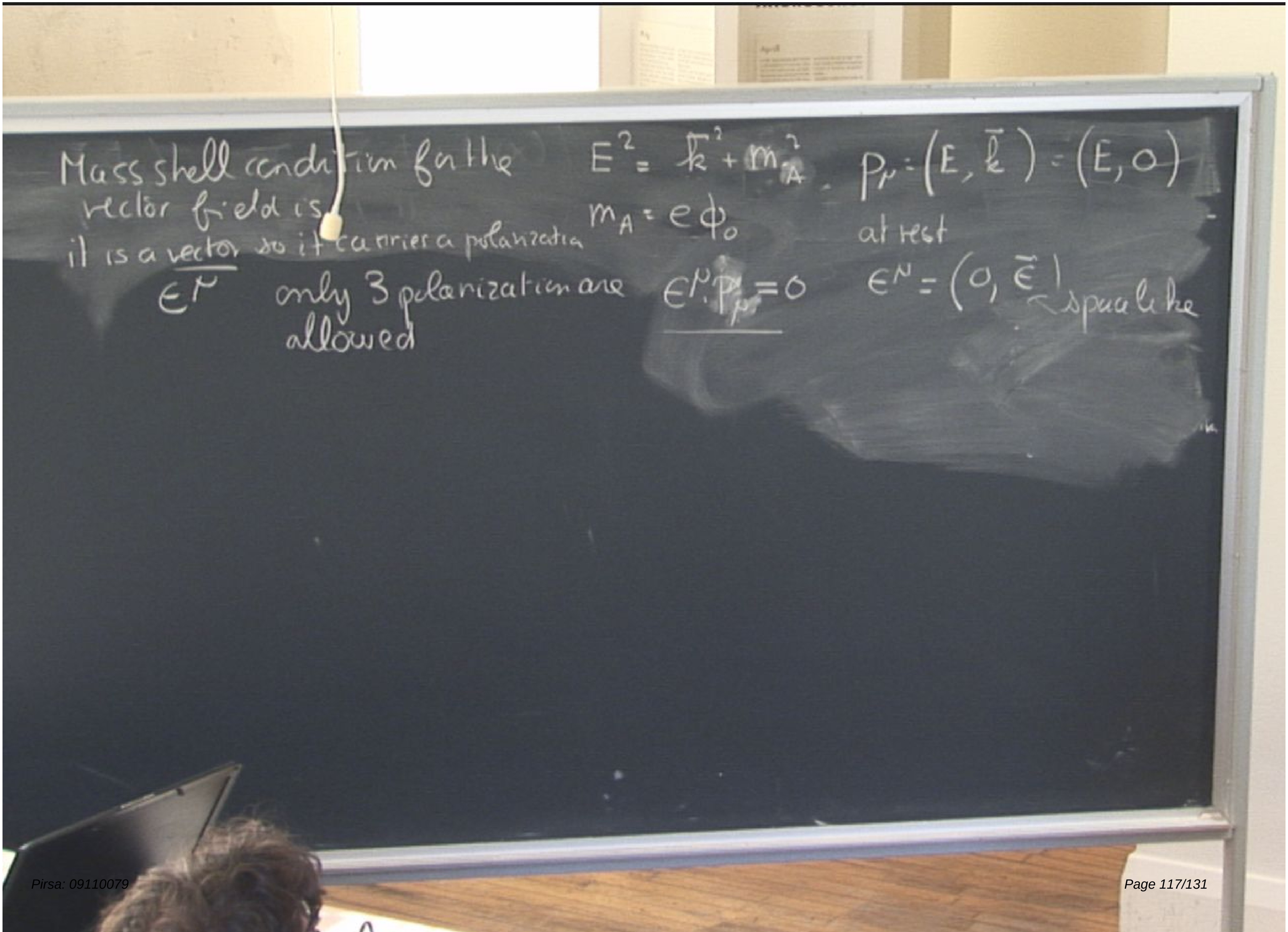
$$\eta^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix}$$

$-\phi$


Mass shell condition for the vector field is
$E^2 = \vec{k}^2 + m_A^2$
$m_A = e\phi_0$


Mass shell condition for the vector field is
$E^2 = k^2 + m_A^2$
$m_A = e\phi_0$
it is a vector so it carries a polarization
ϵ^μ


Mass shell condition for the vector field is
$E^2 = \bar{k}^2 + m_A^2$
$p_\mu = (E, \bar{k})$
$m_A = e\phi_0$
it is a vector so it carries a polarization
ϵ^μ only 3 polarization are allowed
$\epsilon^\mu p_\mu = 0$


Mass shell condition for the vector field is
$E^2 = \vec{k}^2 + m_A^2$
$p_\mu = (E, \vec{k}) = (E, 0)$
$m_A = e\phi_0$
at rest
it is a vector so it carries a polarization
ϵ^μ
only 3 polarization are allowed
$\epsilon^\mu p_\mu = 0$
$\epsilon^\mu = (0, \vec{\epsilon})$
spacelike

Mass shell condition for the vector field is: $E^2 = \vec{k}^2 + m_A^2$ $\quad p_\mu = (E, \vec{k}) = (E, 0)$

$m_A = e\phi_0$ $\quad$ at rest

it is a vector so it carries a polarization

ϵ^μ $\quad$ only 3 polarization are allowed $\quad \epsilon^\mu p_\mu = 0$ $\quad \epsilon^\mu = (0, \vec{\epsilon})$ spacelike

$\Pi = \mathrm{Im}\,\phi$

$H + \phi_0 = \mathrm{Re}\,\phi$

A_μ

$\Pi = 0$

H

A_μ

Gauge fixed theory

$$\boxed{e^2 A_\mu A_\mu H^2 + 2e^2 A_\mu^2 H \phi_0}$$

$$S_{GF} = \int d^4x \left[-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{2}\left[(\partial_\mu H)^2 + e^2 (A_\mu \phi_0 + A_\mu H)^2\right] - \left(\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right)\right.$$

H still a masse boson with mass M

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{e^2}{2}\phi_0^2 A_\mu^2$$

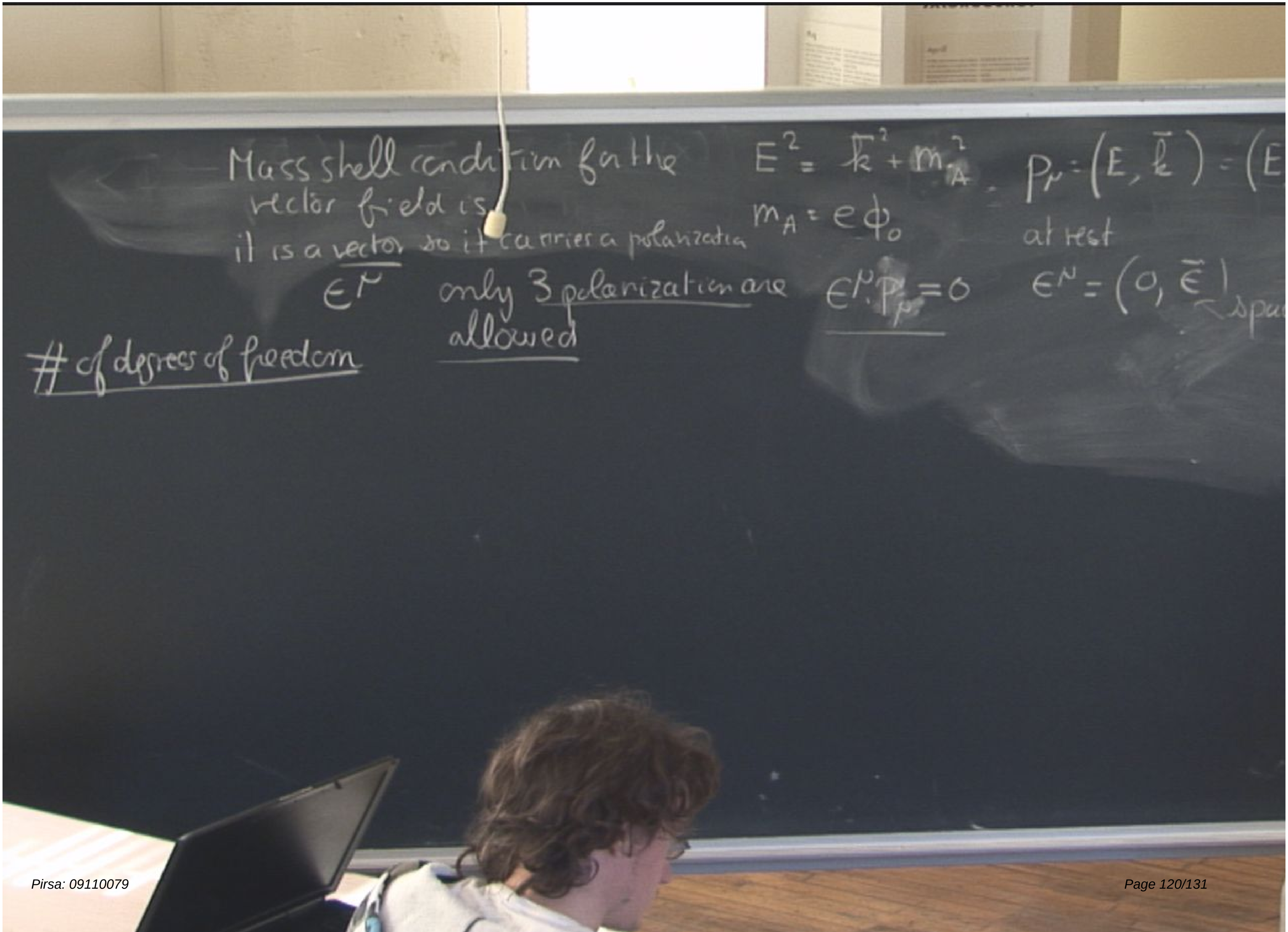
$$-\frac{1}{2}\left((\partial_\mu H)^2 + M^2 H^2\right)$$

$$-\frac{M^2}{2} H^2 + \frac{M^2}{2\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4$$

mass term for the gauge field

masse $\simeq e\,\phi_0 = M_A \neq 0$

$+$



Mass shell condition for the vector field is
$E^2 = \vec{k}^2 + m_A^2$
$p_\mu = (E, \vec{k}) = (E$
$m_A = e\phi_0$
at rest
it is a vector so it carries a polarization
ϵ^μ
only 3 polarization are allowed
$\epsilon^\mu p_\mu = 0$
$\epsilon^\mu = (0, \vec{\epsilon})$
# of degrees of freedom

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_ν

$t > 0$

A_μ 2 polarizations

ϕ 2 states

$\Pi = \mathrm{Im}\,\phi$ | $\Pi = 0$ Gauge fixed theory

$H + \phi_0 = \mathrm{Re}\,\phi$ | H

A_μ | A_μ

$t > 0$

A_μ 2 polarizations

ϕ 2 states

$2+2$

$\Rightarrow$

$=$

$t < 0$

A_μ 3 polarizations

H 1

$3+1$

$\Pi = \mathrm{Im}\,\phi$

$H + \phi_0 = \mathrm{Re}\,\phi$

A_μ

$\Pi = 0$ Gauge fixed theory

H

A_μ

"The gauge field has "eaten" the goldstone boson. and became massive

$t > 0$

A_μ 2 polarizations $\Rightarrow$

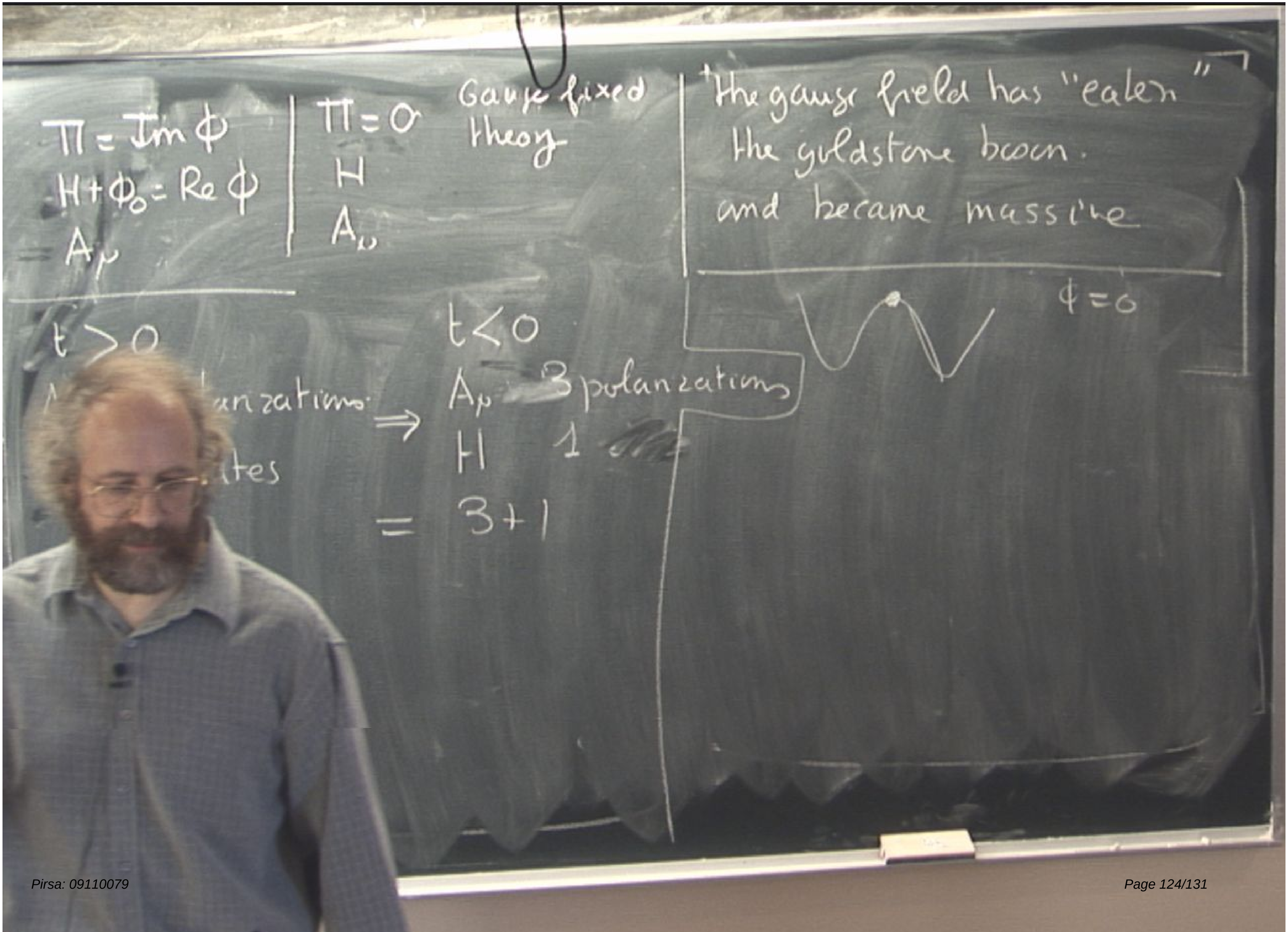
ϕ 2 states

$2+2$ $=$ $3+1$

$t < 0$

A_μ 3 polarizations

H 1


Π = Im φ
H + φ0 = Re φ
Aμ
Π = 0
H
Aν
Gauge fixed theory
"The gauge field has "eaten" the goldstone boson. and became massive
φ = 0
t > 0
t < 0
Aμ 3 polarizations
H 1
= 3+1

$\Pi = \mathrm{Im}\,\phi$

$H + \phi_0 = \mathrm{Re}\,\phi$

A_μ

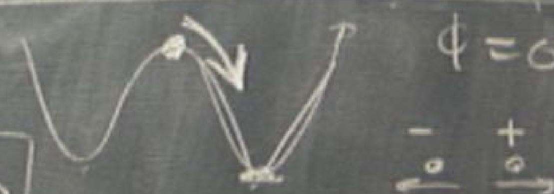
$\Pi = 0$

H

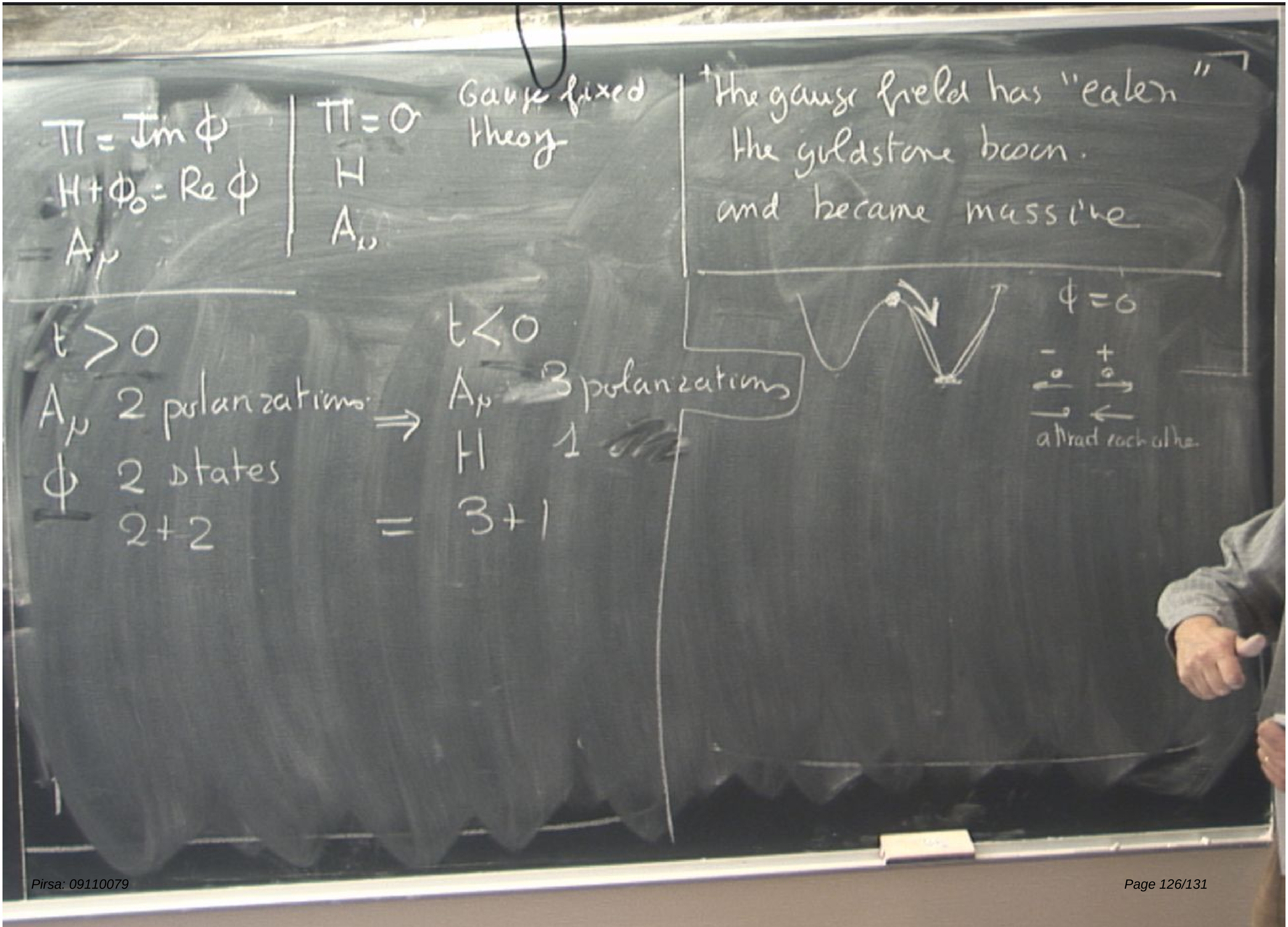
A_μ

Gauge fixed theory

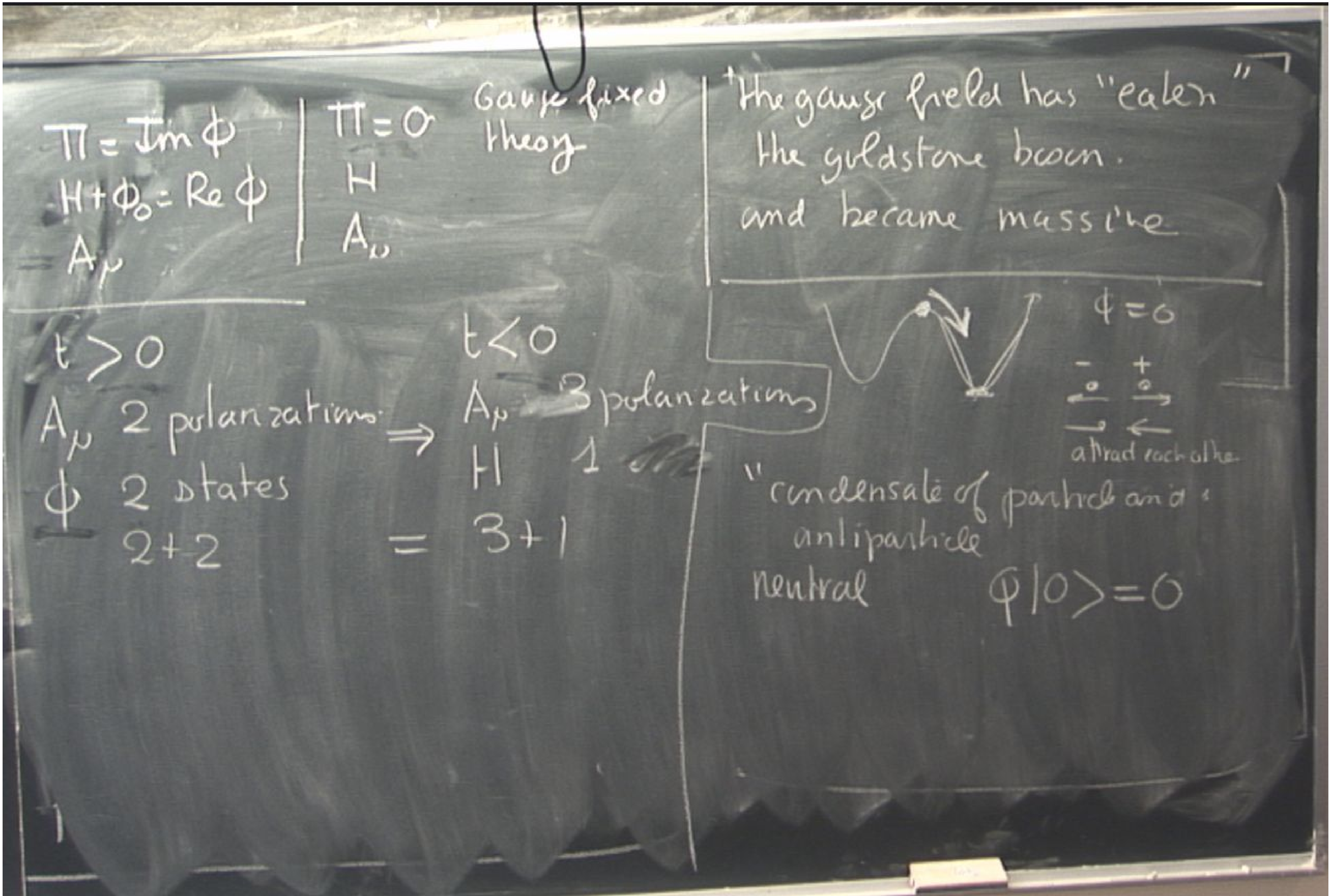
"The gauge field has "eaten" the goldstone boson. and became massive"



$t > 0$		$t < 0$
A_μ 2 polarizations	$\Rightarrow$	A_μ 3 polarizations
		H 1
ϕ 2 states		
$2+2$	$=$	$3+1$



$\Pi = \mathrm{Im}\,\phi$
$H + \phi_0 = \mathrm{Re}\,\phi$
A_μ
$\Pi = 0$
H
A_μ
Gauge fixed theory
"The gauge field has "eaten" the goldstone boson. and became massive
$\phi = 0$
$-$ $+$
attract each other
$t > 0$
A_μ 2 polarizations
ϕ 2 states
$2+2$
$\Rightarrow$
$=$
$t < 0$
A_μ 3 polarizations
H 1
$3+1$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H
A_ν
Gauge fixed
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2+2
⇒
t<0
A_μ 3 polarizations
H 1
= 3+1
"condensate of particle and
antiparticle
neutral
φ|0> = 0

$\Pi = \mathrm{Im}\,\phi$

$H + \phi_0 = \mathrm{Re}\,\phi$

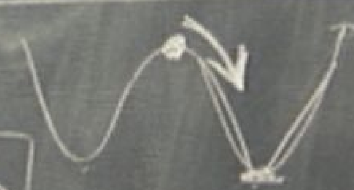
A_μ

$\Pi = 0$ Gauge fixed theory

H

A_ν

"The gauge field has "eaten" the goldstone boson. and became massive



$\phi = 0$

$-$ $+$

attract each other

$t > 0$

A_μ 2 polarizations $\Rightarrow$

ϕ 2 states

$2 + 2$ $=$

$t < 0$

A_μ 3 polarizations

H 1

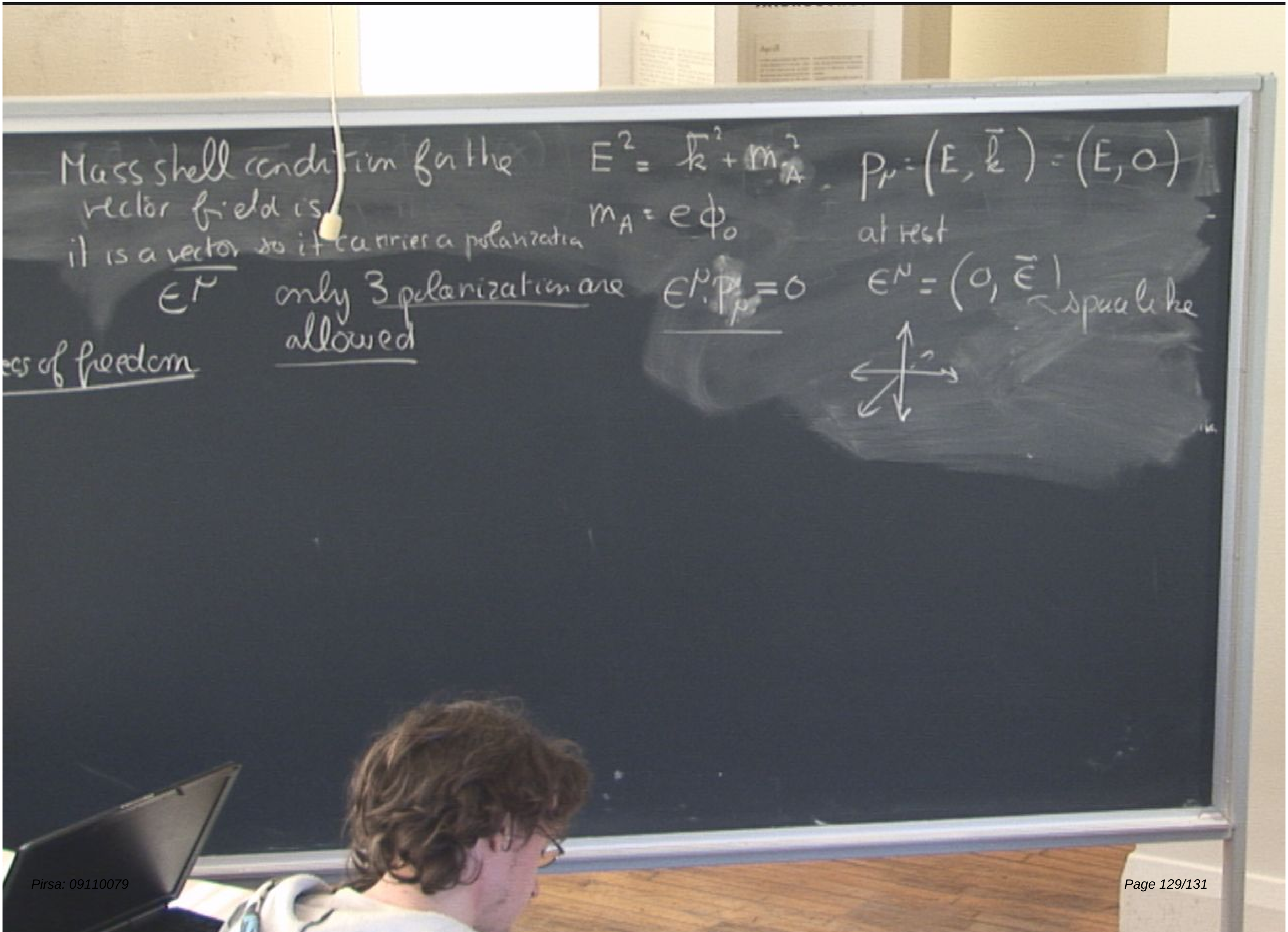
$3 + 1$

"condensate of particle and antiparticle

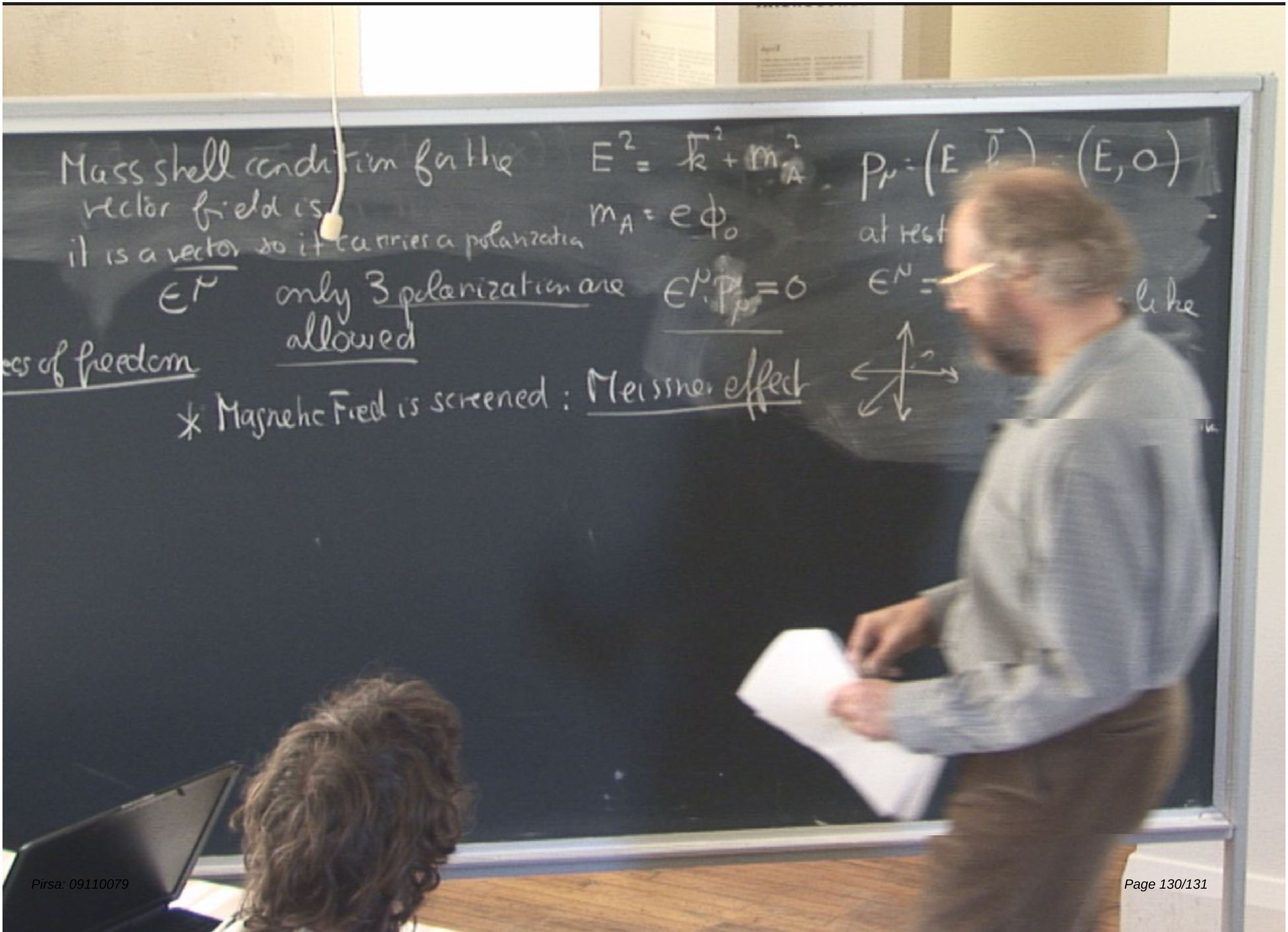
neutral $\quad \phi|0\rangle = 0$

still

e



Mass shell condition for the
vector field is
it is a vector so it carries a polarization
ϵ^μ
only 3 polarization are allowed
of freedom
$E^2 = \vec{k}^2 + m_A^2$
$m_A = e\phi_0$
$\epsilon^\mu p_\mu = 0$
$p_\mu = (E, \vec{k}) = (E, 0)$
at rest
$\epsilon^\mu = (0, \vec{\epsilon})$
spacelike


Mass shell condition for the vector field is
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at rest
it is a vector so it carries a polarization
ϵ^μ
only 3 polarization are allowed
$\epsilon^\mu p_\mu = 0$
ees of freedom
* Magnetic Field is screened : Meissner effect

$\Pi = \mathrm{Im}\,\phi$

$H + \phi_0 = \mathrm{Re}\,\phi$

A_μ

$\Pi = 0$

H

A_μ

Gauge fixed theory

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attract each other

$t > 0$

A_μ 2 polarizations $\Rightarrow$

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$2 + 2 \quad = \quad 3 + 1$

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A_μ 3 polarizations

H 1

Topological Effects $\leftarrow$

"condensate of particle and antiparticle"

neutral $\qquad \phi|0\rangle = 0$

still