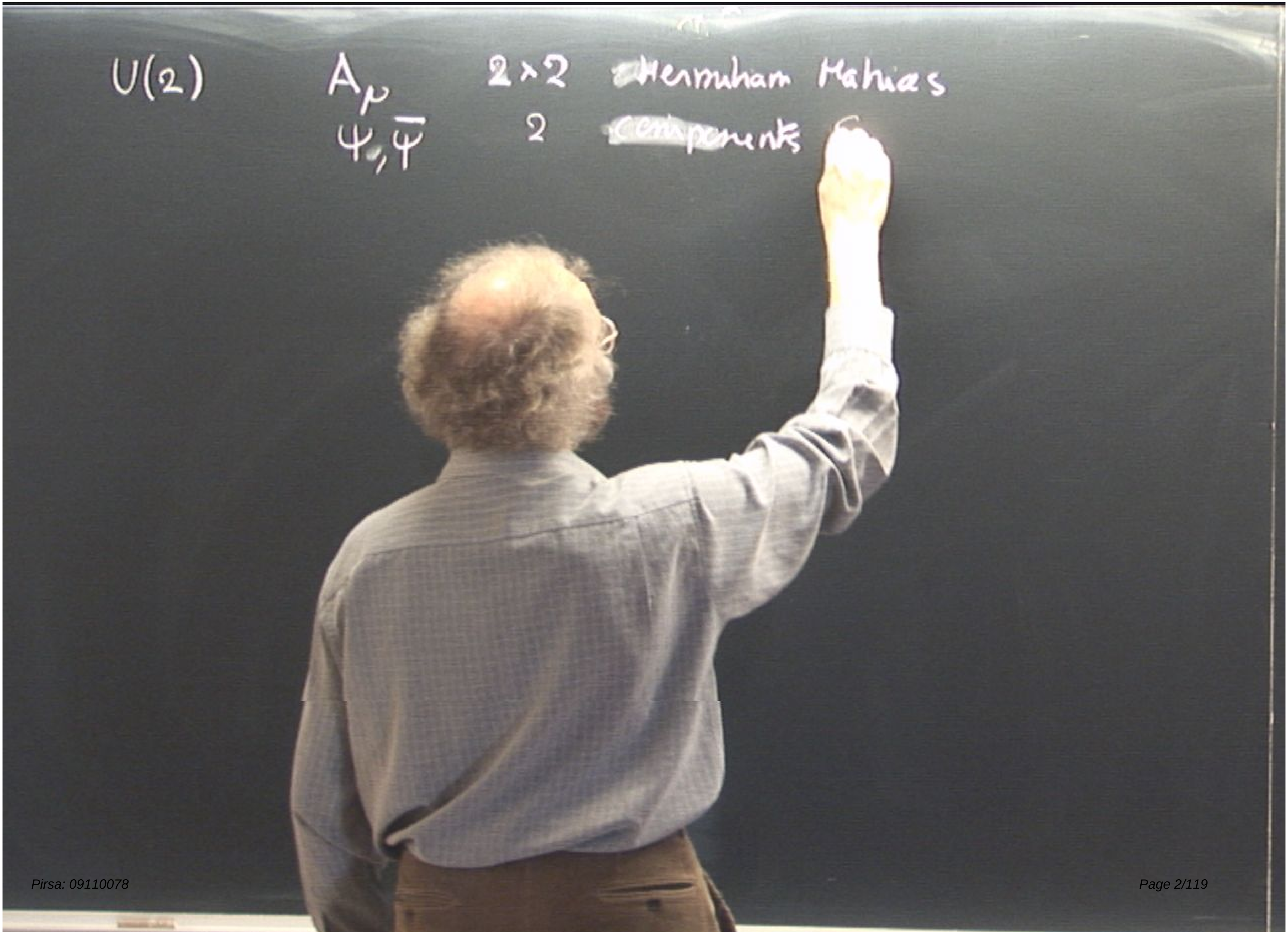
Title: Quantum Field Theory II (PHYS 603) - Lecture 14

Date: Nov 12, 2009 09:00 AM

URL: http://pirsa.org/09110078

Abstract:



$U(2)$ A_μ 2×2 Hermitian Matrices

$\psi, \bar{\psi}$ 2 components

$U(2)$ $A_\mu, c, \bar{c}$: 2×2 Hermitian Matrices

$\psi, \bar{\psi}$ 2 components Spinors

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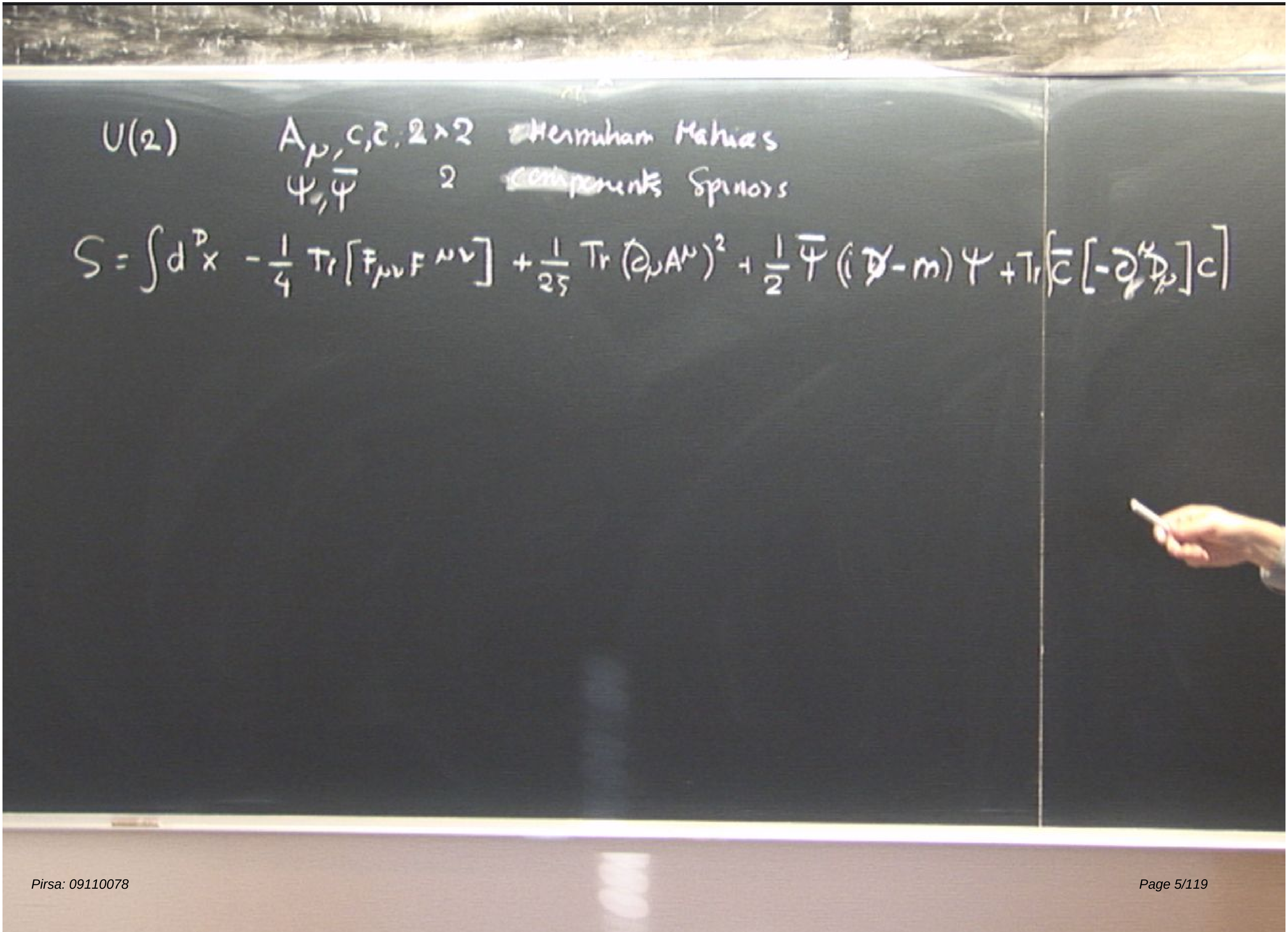
$$S = \int d^D x \; -\frac{1}{4}\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2\xi}\,\mathrm{Tr}\left(\partial_\mu A^\mu\right)^2 + \frac{1}{2}\,\bar{\Psi}\left(i\not{D} - m\right)\Psi +$$



$U(2)$ $\quad A_\mu, c, \bar{c}$: 2×2 Hermitian Matrices

$\psi, \bar{\psi}$ $\quad 2$ components Spinors

$$S = \int d^D x \; -\frac{1}{4}\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{25}\,\mathrm{Tr}\,(\partial_\mu A^\mu)^2 + \frac{1}{2}\,\bar{\Psi}\,(i\not{D} - m)\,\Psi + \mathrm{Tr}\left[\bar{c}\,[-\partial^\mu D_\mu]\,c\right]$$

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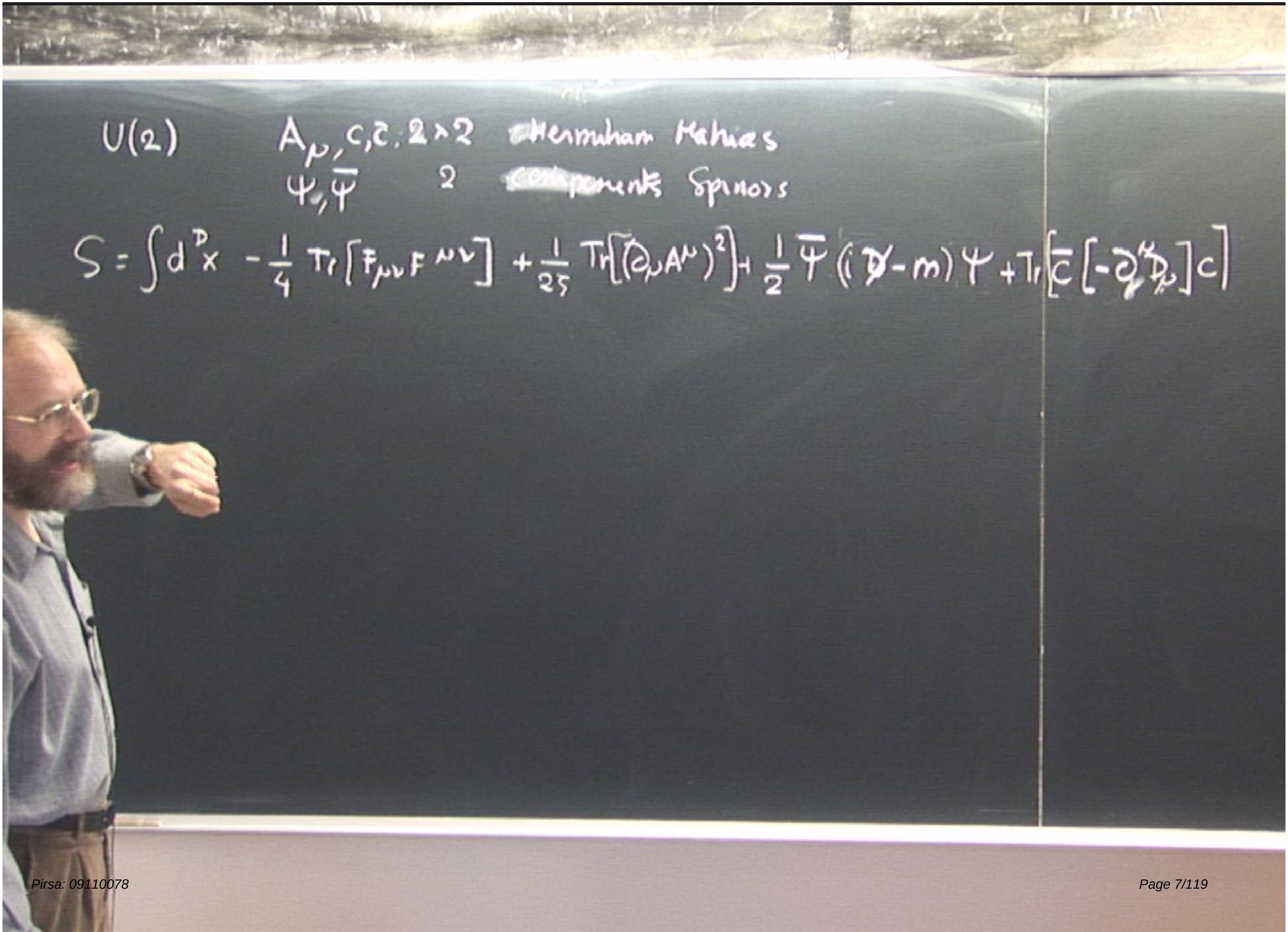

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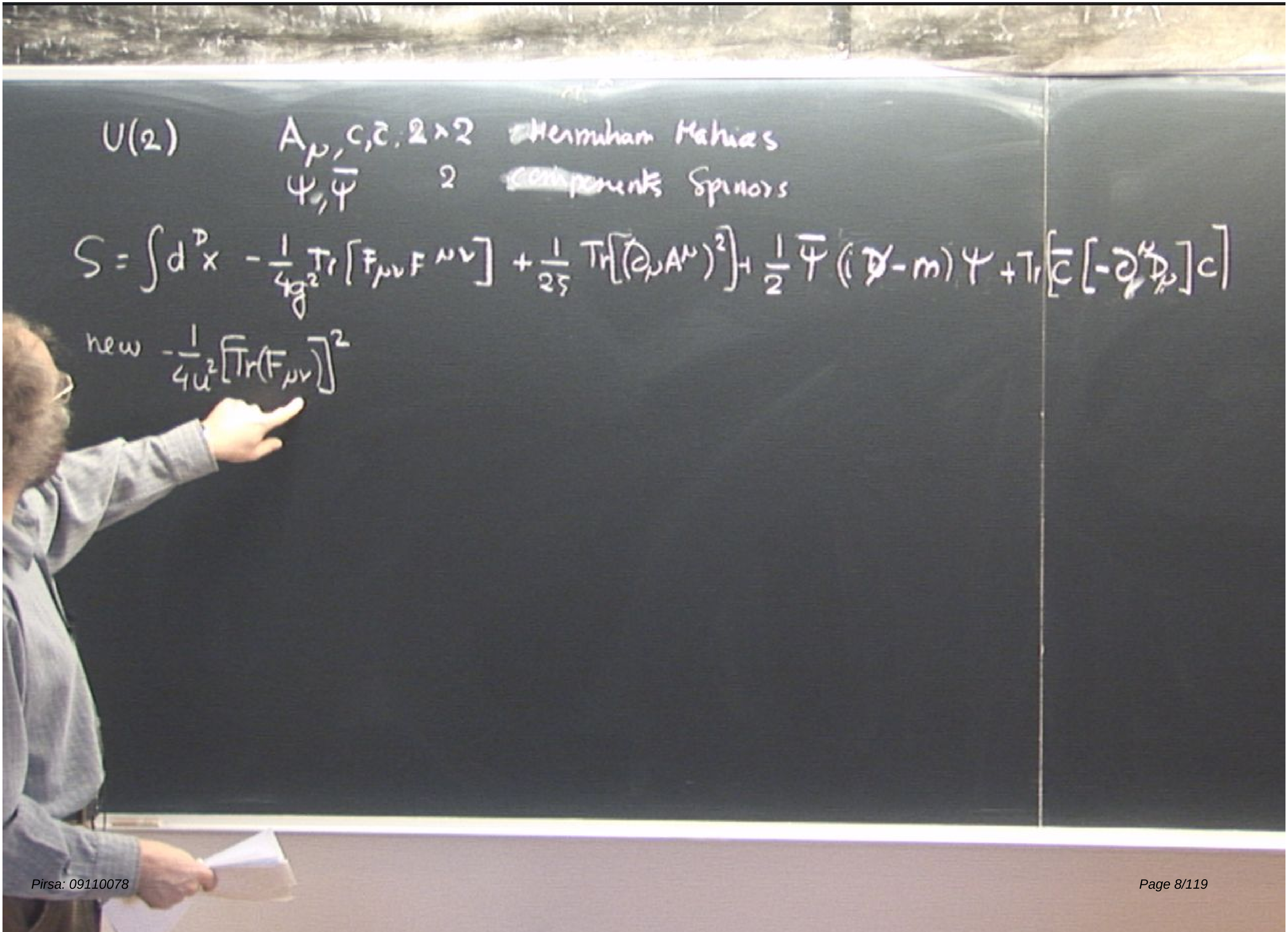

U(2) $A_\mu, c, \bar{c}$: 2×2 Hermitian Matrices

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$$S = \int d^D x \; -\frac{1}{4g^2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2\xi}\mathrm{Tr}\left[(\partial_\mu A^\mu)^2\right] + \frac{1}{2}\bar{\Psi}\left(i\not{D} - m\right)\Psi + \mathrm{Tr}\left[\bar{c}\left[-\partial^\mu D_\mu\right]c\right]$$

new $-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$

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new $-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$ $u \to \infty$ $U(2) \to SU(2)$

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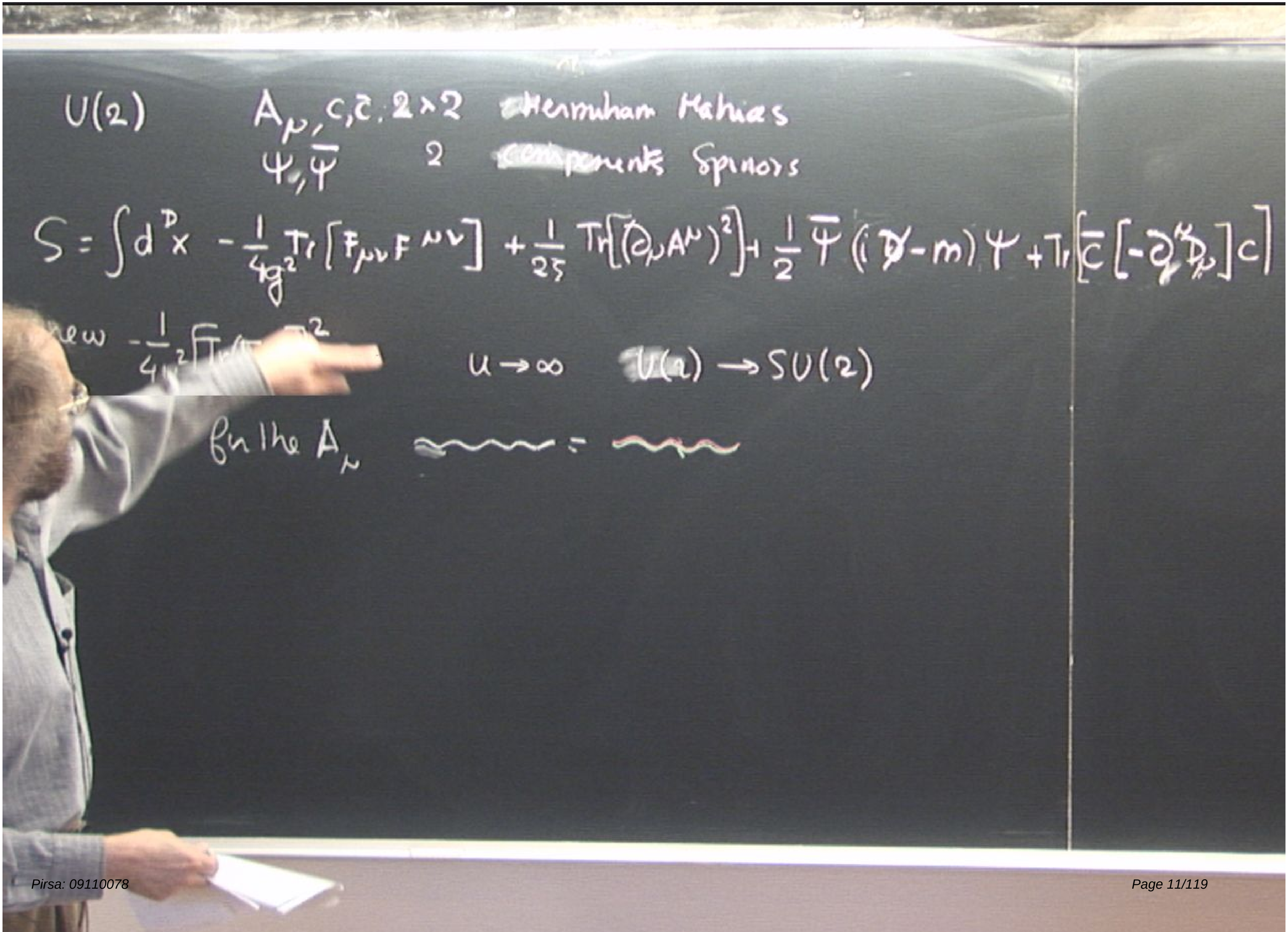
new $-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$ $\qquad u \to \infty \quad U(2) \to SU(2)$

Propagator for the A_μ  =



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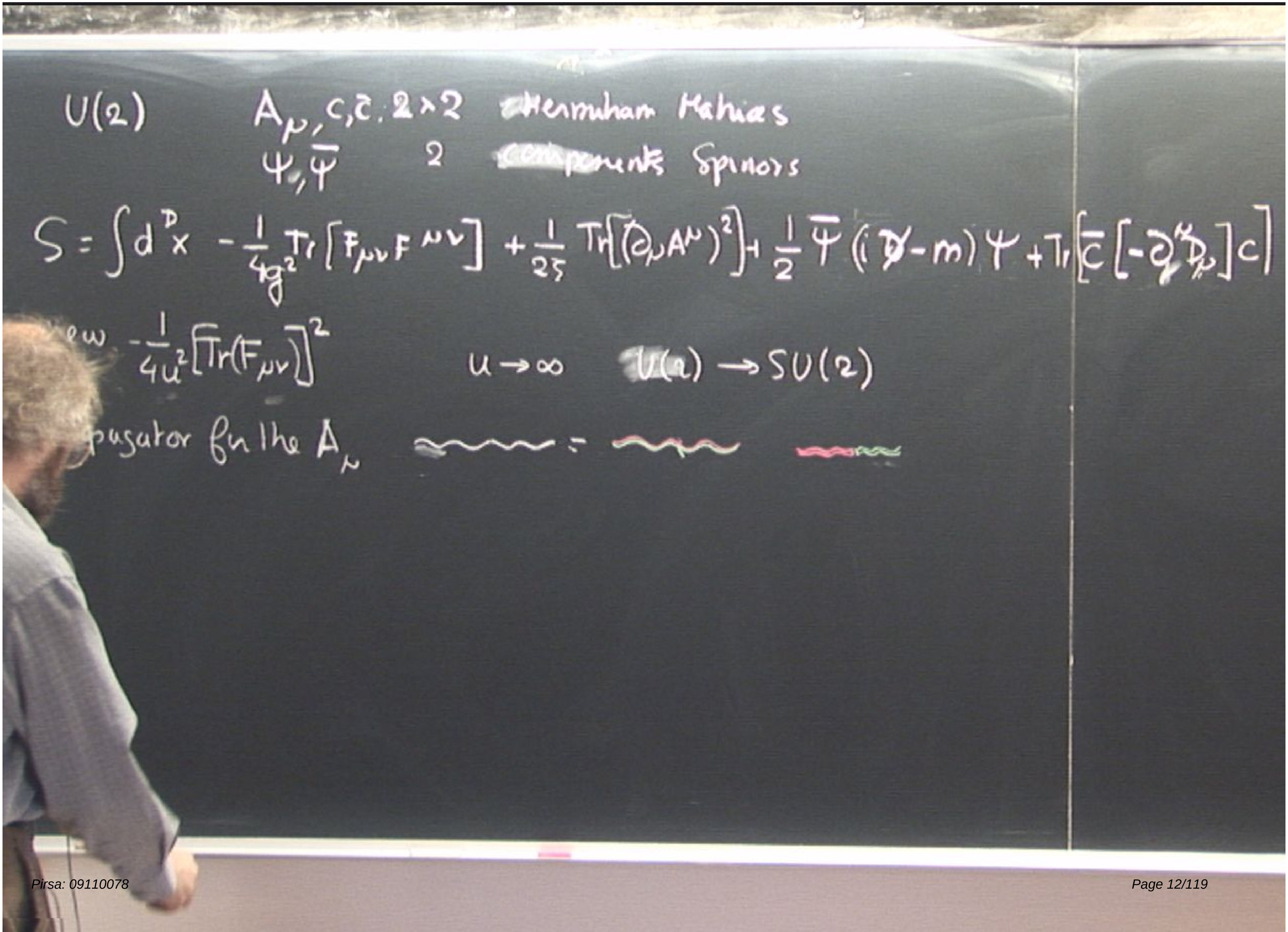
new $-\frac{1}{4u^2}\left[\mathrm{Tr}(\right.$ [illegible] $\left.\right]^2$

$u \to \infty$ $U(2) \to SU(2)$

for the A_μ =



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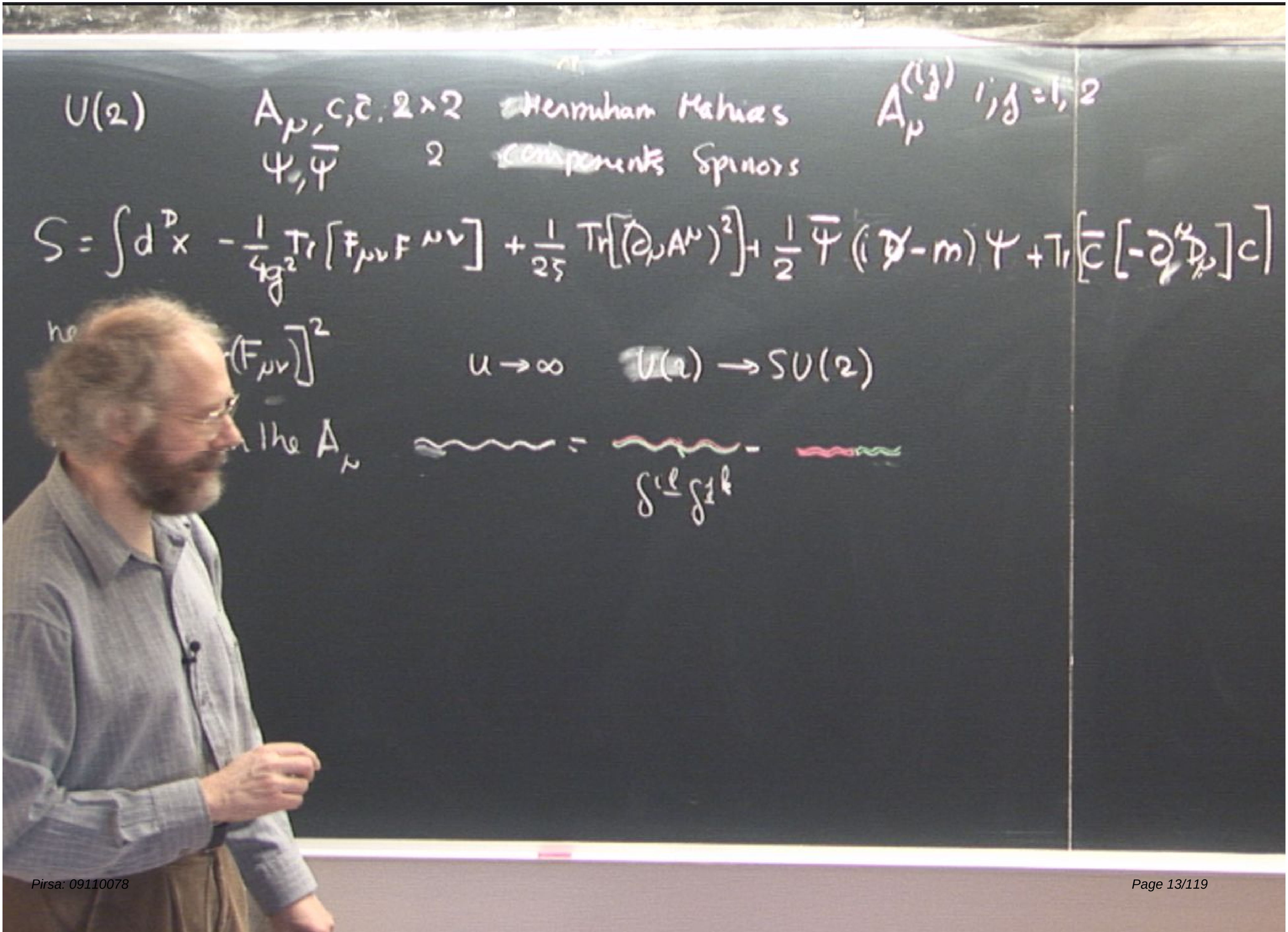
ew $-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$ $\qquad u \to \infty \quad U(2) \to SU(2)$

pagator for the A_μ



U(2) $A_\mu, c, \bar{c}$: 2×2 Hermitian Matrices $A_\mu^{(ij)}$ i, j = 1, 2

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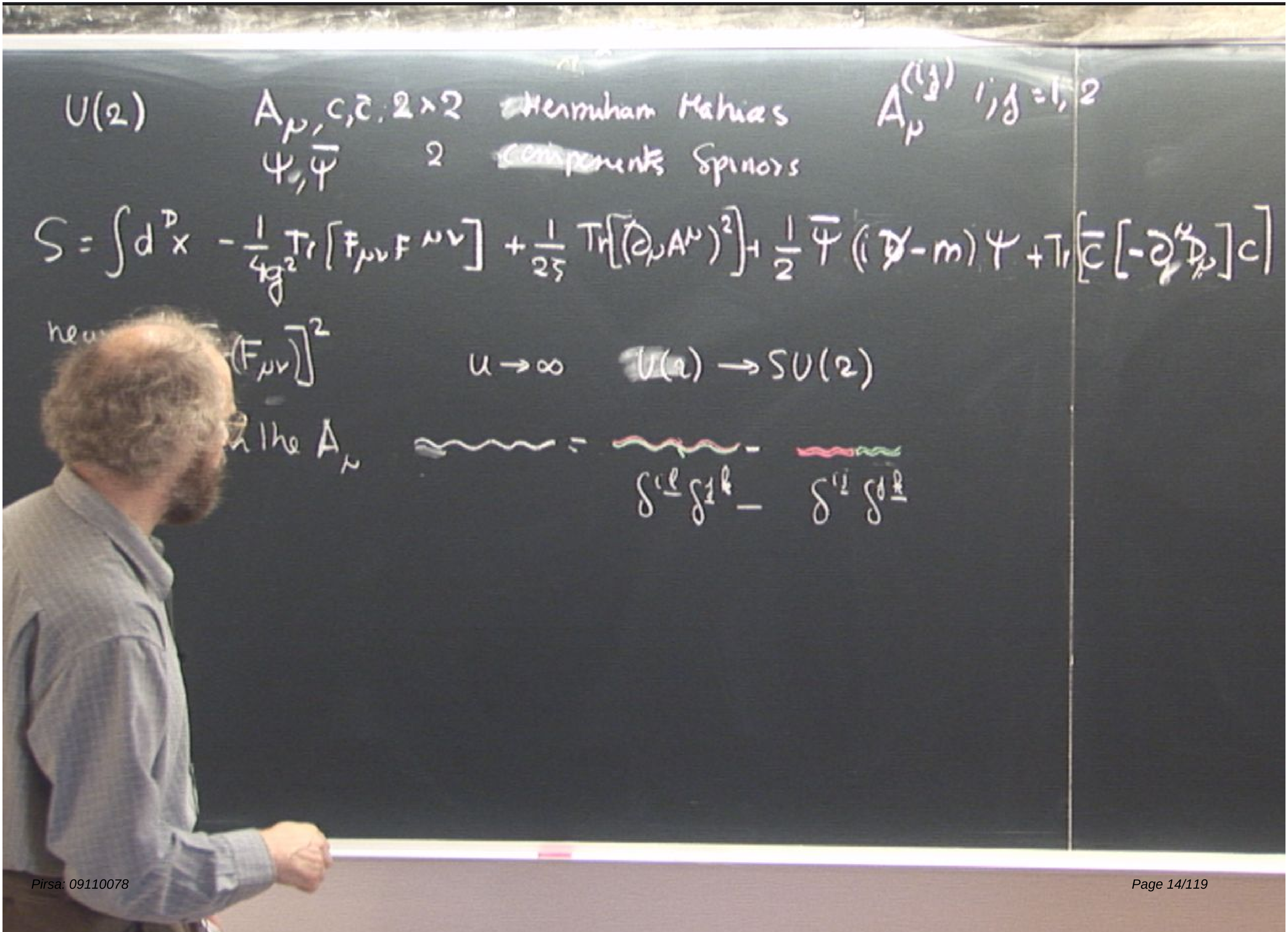
he $(F_{\mu\nu})]^2$

$u \to \infty$ U(2) → SU(2)

the A_μ = −

$\delta^{il}\delta^{jk}$



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heu $(F_{\mu\nu})]^2$ $u \to \infty$ $U(2) \to SU(2)$

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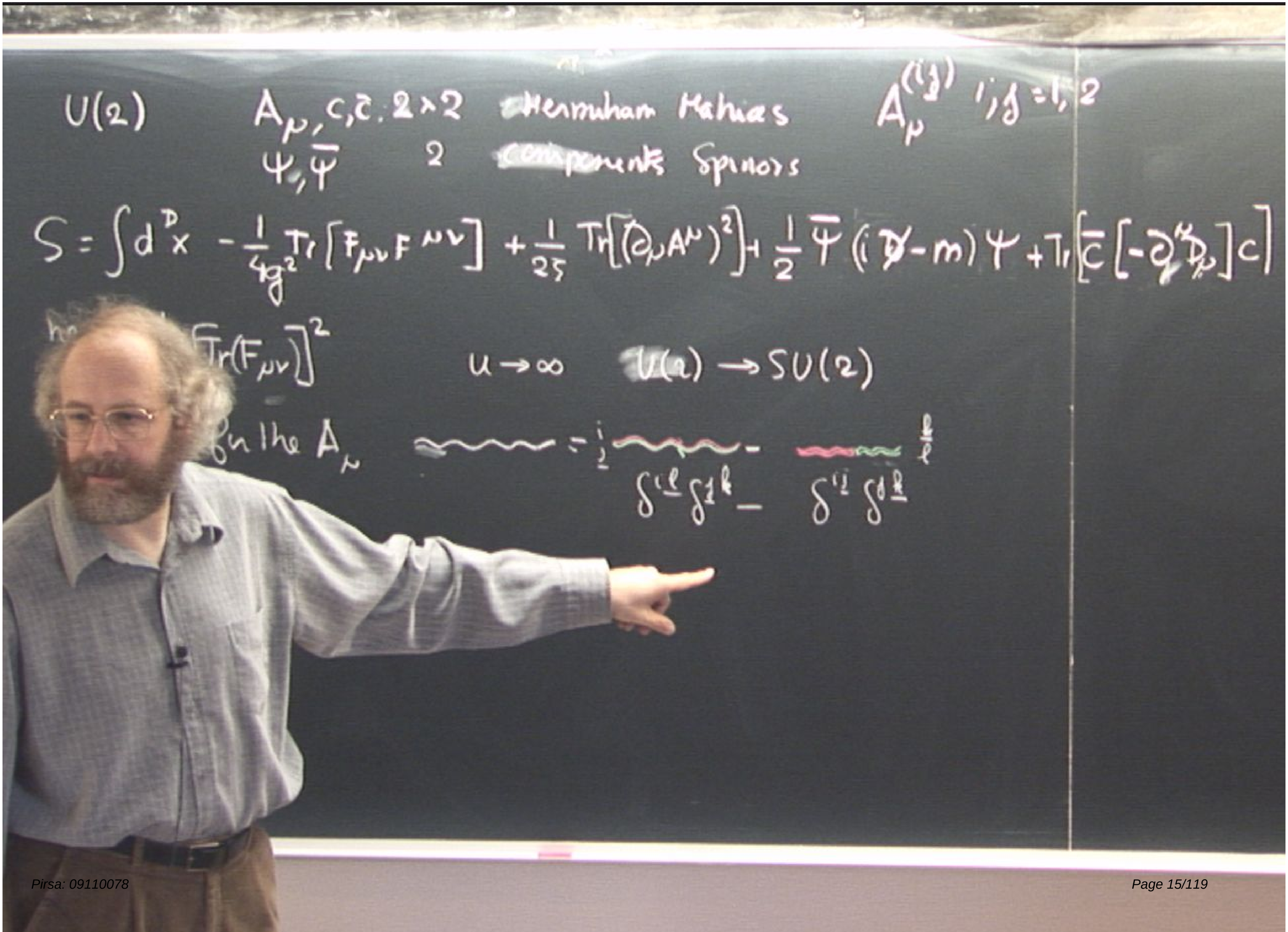
$\delta^{il}\delta^{jk} - \delta^{ij}\delta^{lk}$



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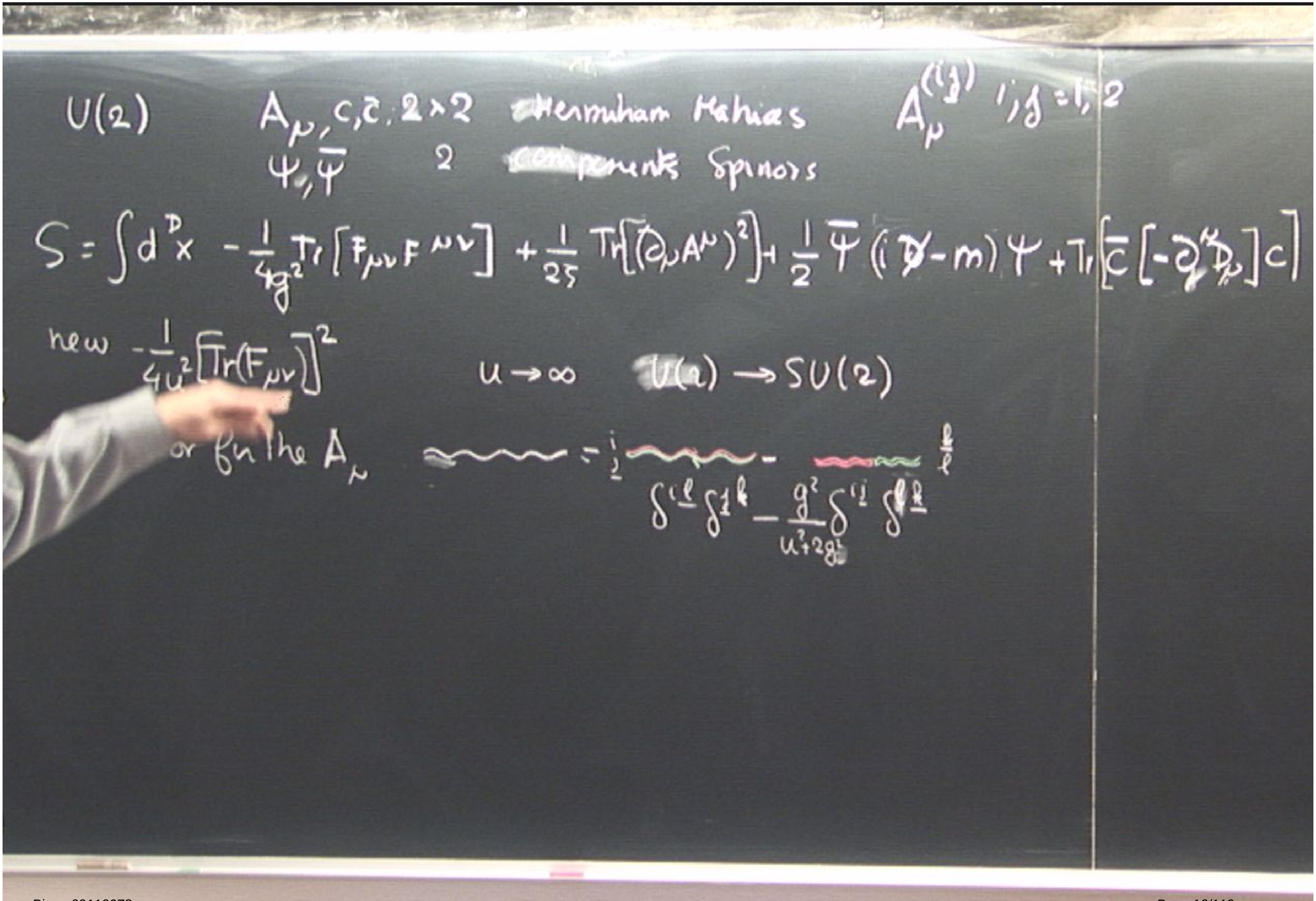
h $\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$

$u \to \infty$ U(2) $\to$ SU(2)

fn the A_μ $= \frac{1}{2}$ $-$

$\delta^{il}\delta^{jk} - \delta^{ij}\delta^{kl}$



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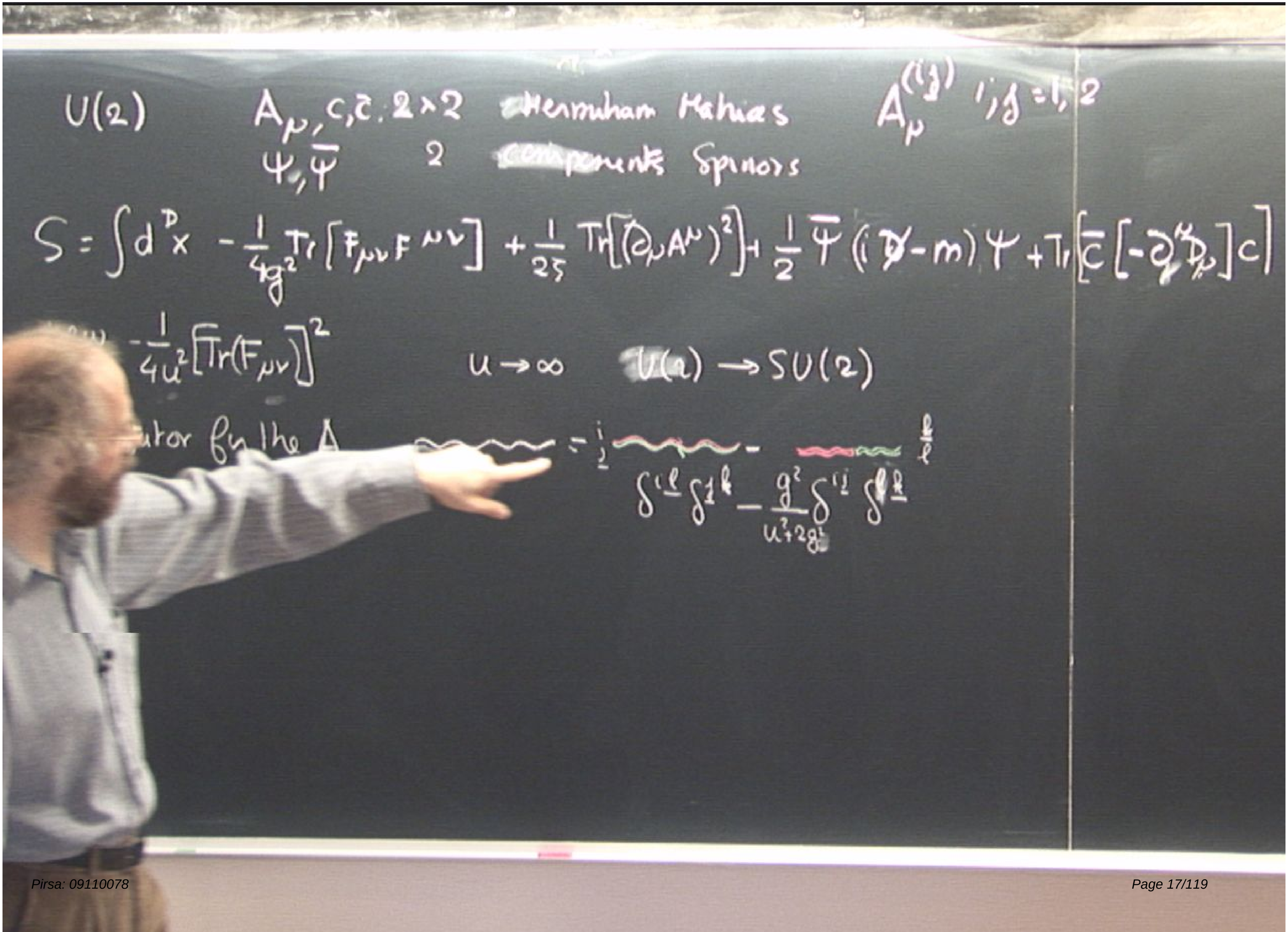
new $-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$ $u \to \infty$ U(2) → SU(2)

or fn the A_μ

$$\delta^{i\ell}\delta^{jk} - \frac{g^2}{u^2+2g^2}\delta^{ij}\delta^{\ell k}$$



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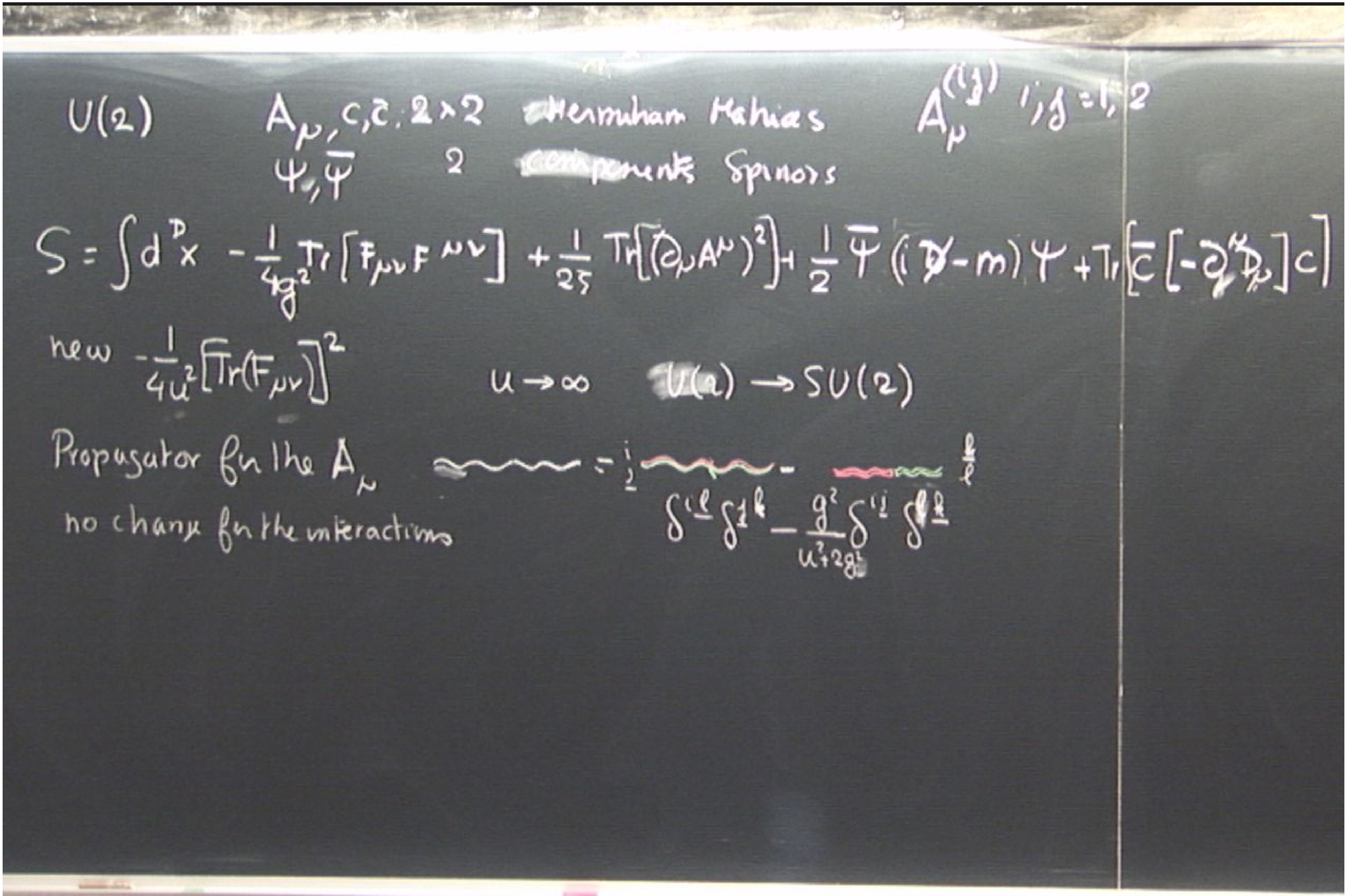
$$-\frac{1}{4u^2}\left[\mathrm{Tr}(F_{\mu\nu})\right]^2$$

$u \to \infty$ $U(2) \to SU(2)$

ator for the A

$$= \frac{i}{2} \quad - \quad {}^{k}_{l}$$

$$\delta^{il}\delta^{jk} - \frac{g^2}{u^2+2g^2}\delta^{ij}\delta^{lk}$$



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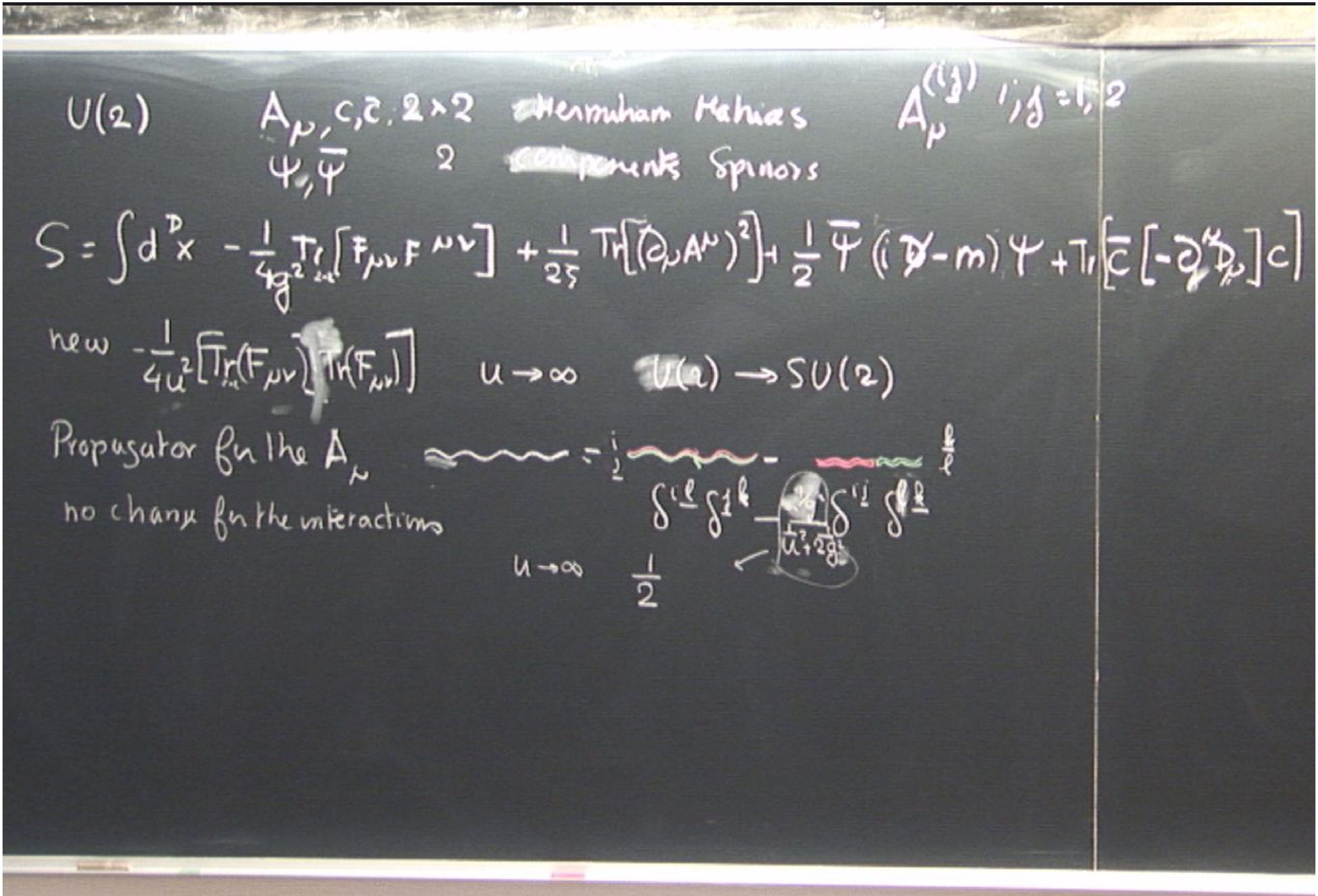
Propagator for the A_μ $= \frac{i}{2}$ $-$ $\begin{smallmatrix} k \\ l \end{smallmatrix}$

$$\delta^{il}\delta^{jk} - \frac{g^2}{u^2+2g^2}\delta^{ij}\delta^{lk}$$

no change for the interactions



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$$S = \int d^D x \; -\frac{1}{4g^2} \mathrm{Tr}_{2\times2}\left[F_{\mu\nu}F^{\mu\nu}\right] + \frac{1}{2\xi}\mathrm{Tr}\left[(\partial_\nu A^\nu)^2\right] + \frac{1}{2}\bar{\psi}(i\not{D} - m)\psi + \mathrm{Tr}\left[\bar{c}\left[-\partial^\mu D_\mu\right]c\right]$$

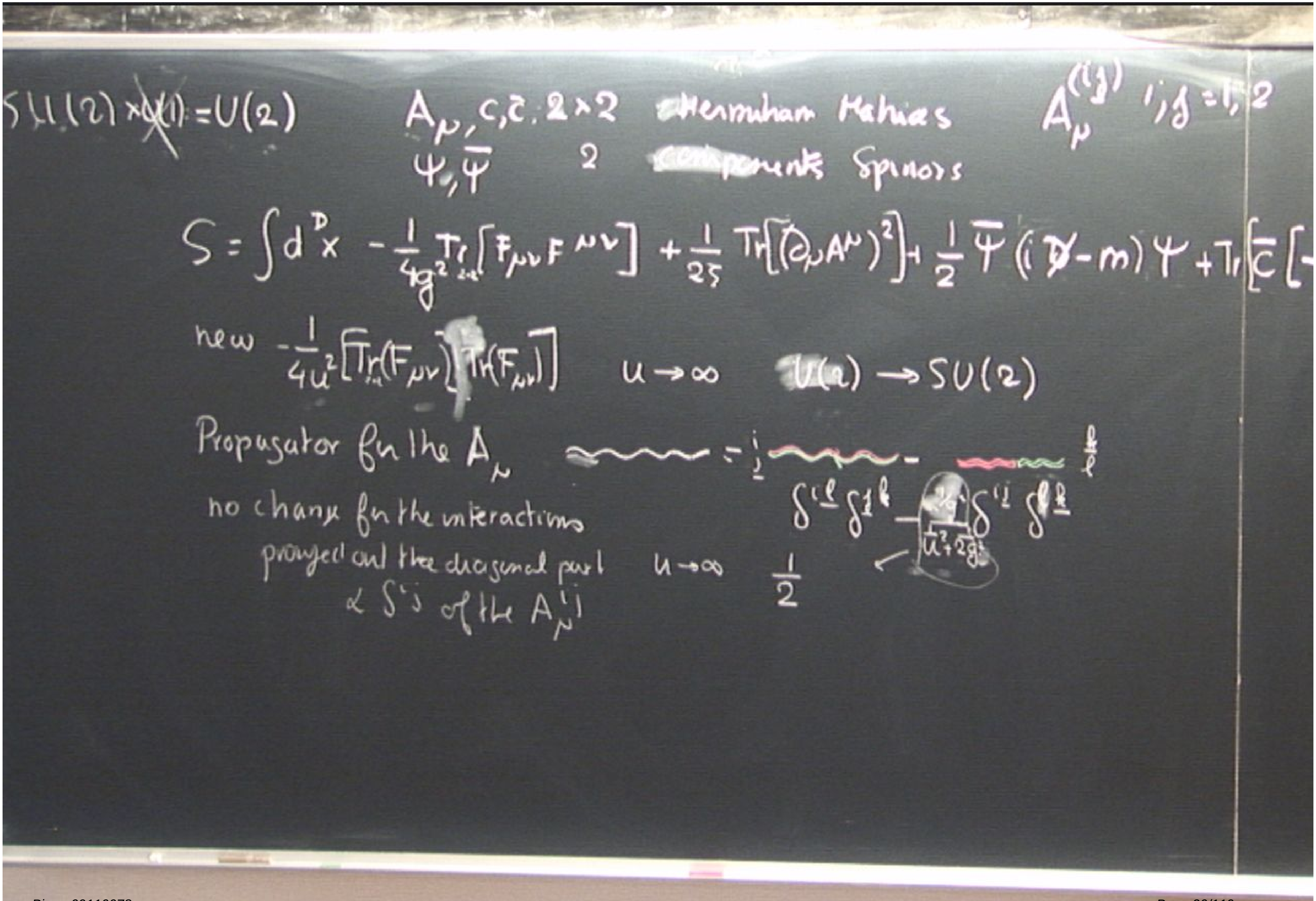
new $-\frac{1}{4u^2}\left[\mathrm{Tr}_{2\times2}(F_{\mu\nu})\right]\left[\mathrm{Tr}(F_{\mu\nu})\right]$ $u \to \infty$ U(2) → SU(2)

Propagator for the A_μ

no change for the interactions

$$= \frac{1}{2}\; \delta^{il}\delta^{jk} - \frac{[illegible]}{u^2+2g^2}\,\delta^{ij}\delta^{lk}$$

$u \to \infty$ $\frac{1}{2}$



$SU(2) \times \cancel{U(1)} = U(2)$ $A_\mu, c, \bar{c}$: 2×2 Hermitian Matrices $A_\mu^{(ij)}$ $i,j = 1, 2$

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new $-\frac{1}{4u^2}\left[\mathrm{Tr}_{2\times 2}(F_{\mu\nu})\, \mathrm{Tr}(F_{\mu\nu})\right]$ $u \to \infty$ $U(2) \to SU(2)$

Propagator for the A_μ $= \frac{i}{2}$ $-$ $\begin{matrix} k \\ \ell \end{matrix}$

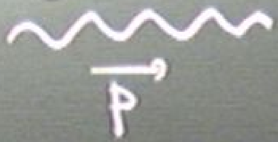
$\delta^{i\ell} \delta^{jk} - \frac{[illegible]}{u^2+2g^2} \delta^{ij} \delta^{\ell k}$

no change for the interactions

projects out the diagonal part $u \to \infty$ $\frac{1}{2}$

$\propto \delta^{ij}$ of the A_μ^{ij}

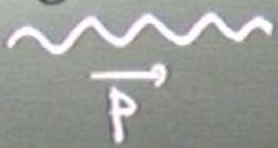
Renormalization



: $\Pi_{\mu\nu}(p)$

$c[-\partial^{\mu} D_{\mu}]c]$

Renormalization



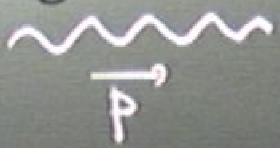
$$\therefore \; \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

$$c\left[-\partial^{\mu} D_{\mu}\right]c\Big]$$

$c\left[-\partial^{\mu} D_{\mu}\right] c]$

Renormalization

A^{μ}



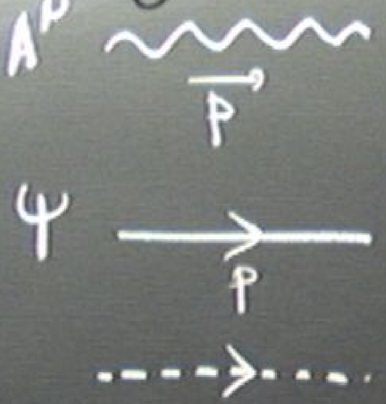
ψ

$$\therefore \ \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

$$\Delta(p) \sim \frac{1}{\not{p}+m}$$

$[-\partial^\mu D_\mu]\,c]$

Renormalization



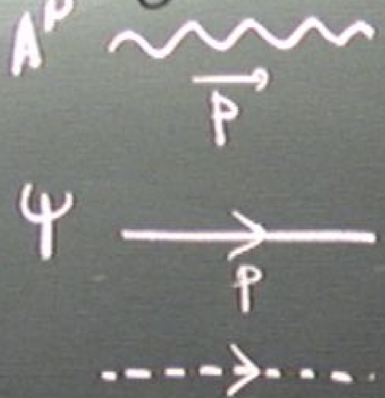
$$\therefore\ \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

$$\Delta(p) \sim \frac{1}{\not{p}+m} \sim \frac{1}{|p|}$$

$$G_{\text{Ghost}} \sim \frac{1}{p^2}$$

$[-\partial^{\mu} D_{\mu}]c]$

Renormalization



$$\therefore \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} + (1-\xi)\frac{p_{\mu}p_{\nu}}{p^2}\right)$$

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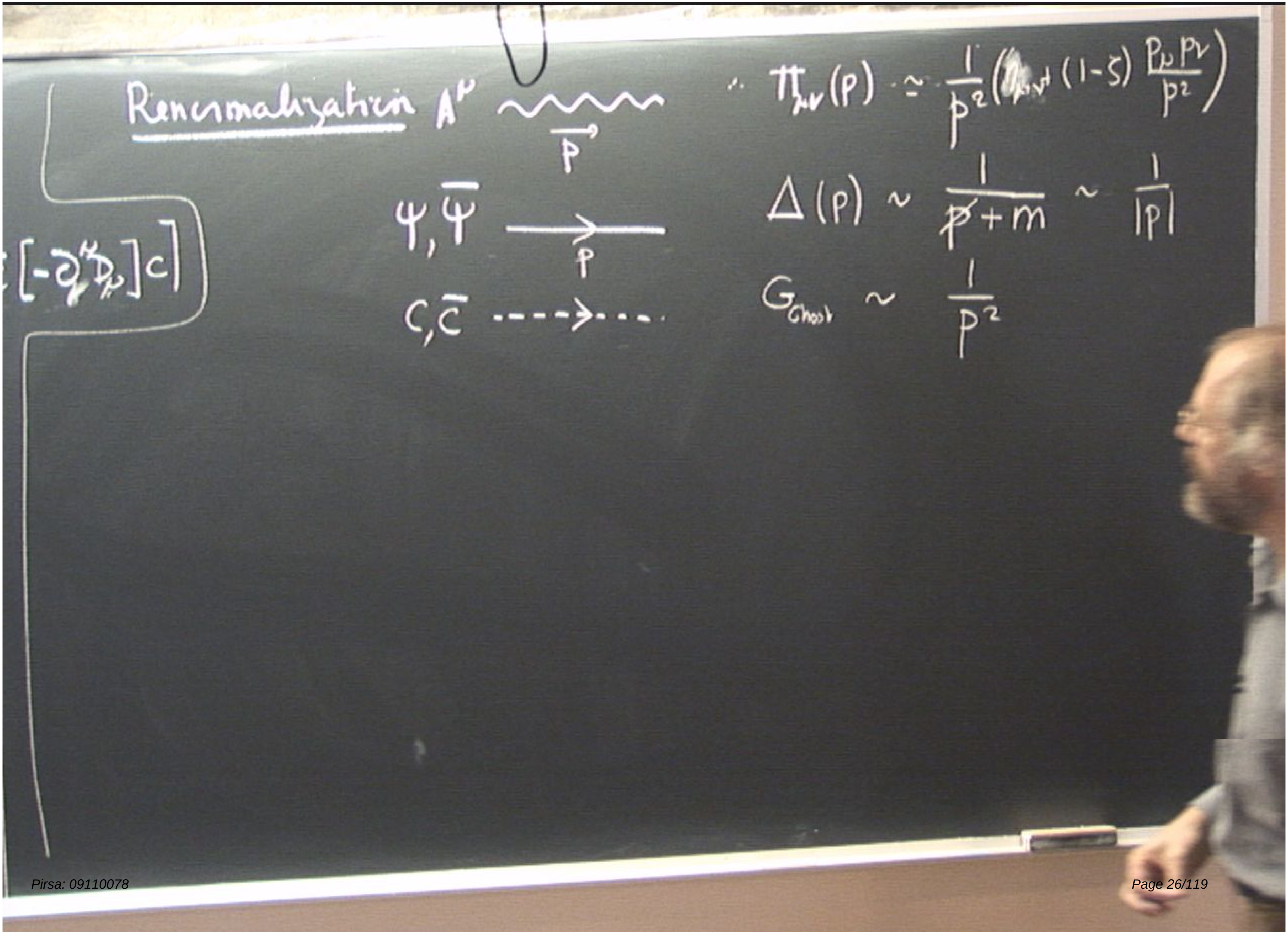

Renormalization A^μ $\underset{\overrightarrow{p}}{\sim\sim\sim}$

$\psi, \bar{\psi}$ $\xrightarrow[p]{}$

$c, \bar{c}$ $\dashrightarrow$

$[-\partial^\mu D_\mu] c]$

$$\therefore \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

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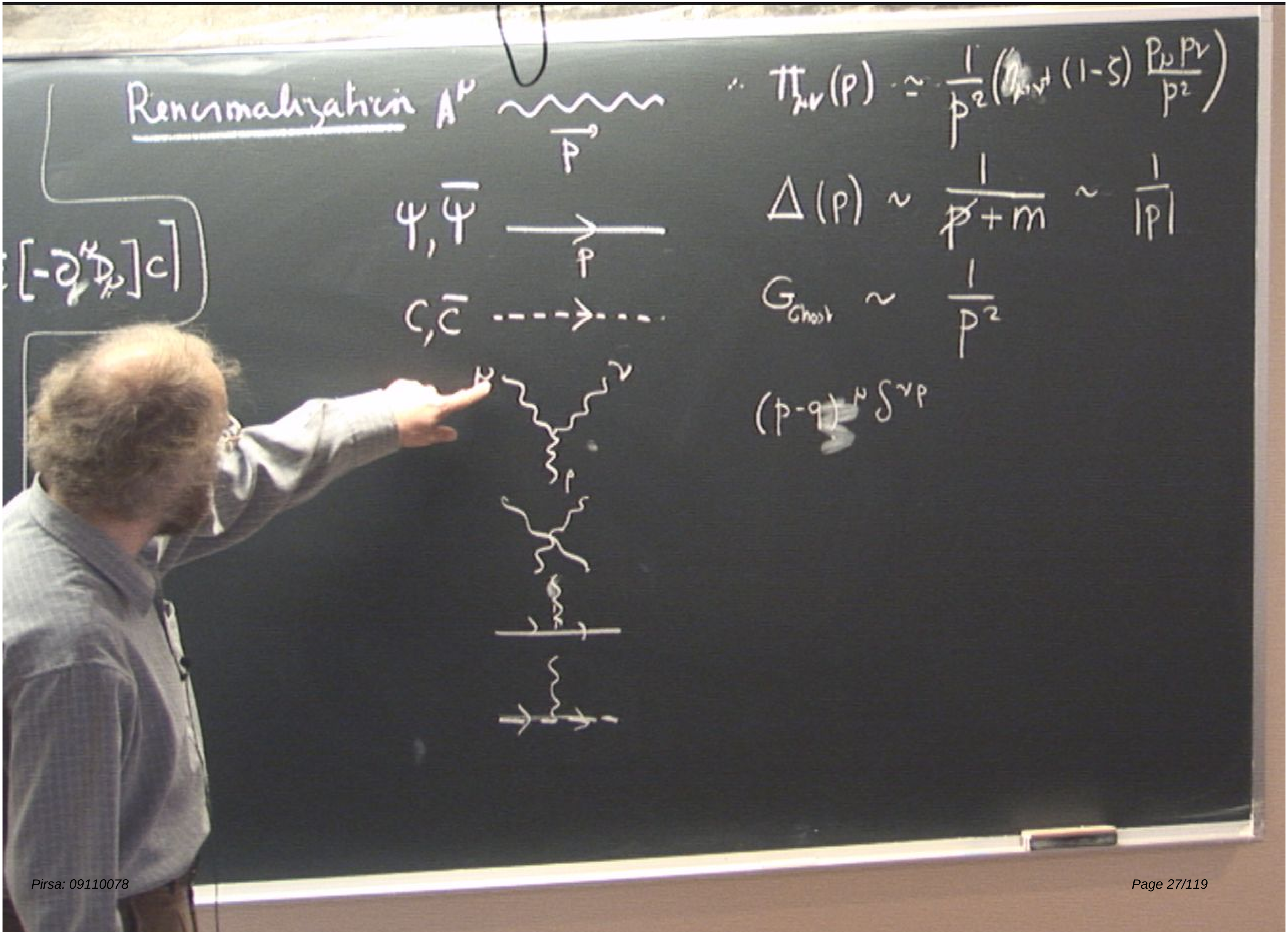


Renormalization A^μ

$\psi, \bar{\psi}$

$c, \bar{c}$

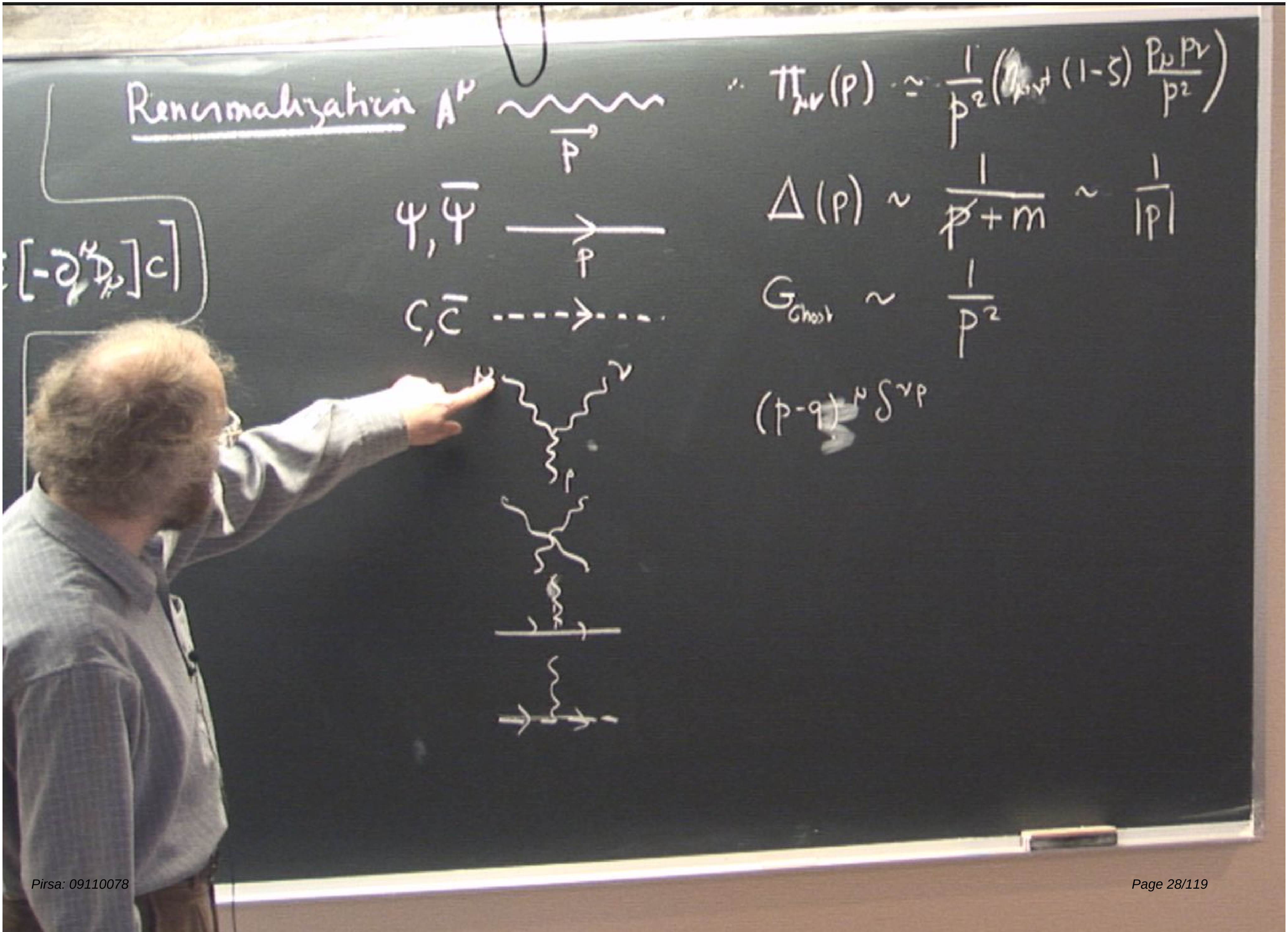
$[-\partial^\mu D_\mu]c]$

$$\therefore \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

$$\Delta(p) \sim \frac{1}{\not{p}+m} \sim \frac{1}{|p|}$$

$$G_{Ghost} \sim \frac{1}{p^2}$$

$$(p-q)^\mu \delta^{\nu\rho}$$



Renormalization A^μ

$\psi, \bar{\psi}$

$c, \bar{c}$

$[-\partial^\mu D_\mu]c]$

$\therefore\ \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} + (1-\zeta)\frac{p_\mu p_\nu}{p^2}\right)$

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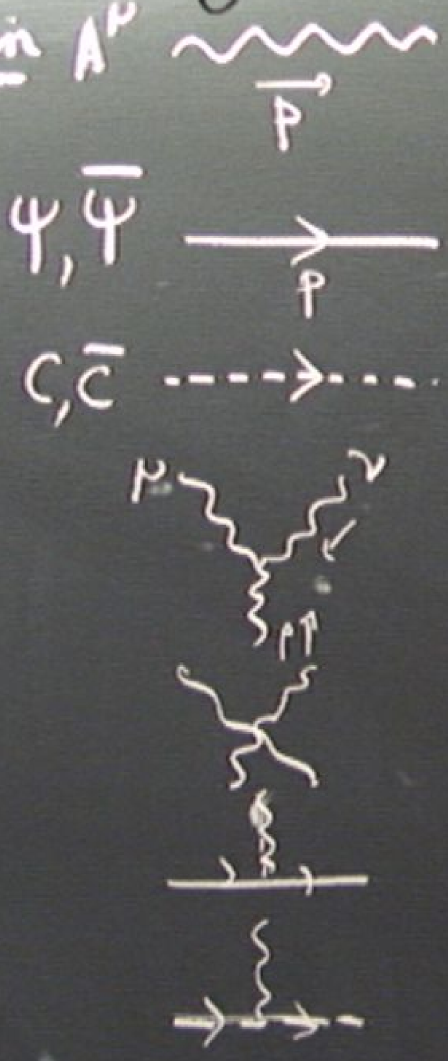
$G_{Ghost} \sim \frac{1}{p^2}$

$(p-q)^\mu \delta^{\nu\rho}$

$[-\partial^\mu D_\mu]c]$

Renormalization



$$\therefore \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu} (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

$$\Delta(p) \sim \frac{1}{\not{p}+m} \sim \frac{1}{|p|}$$

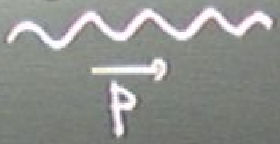
$$G_{\text{Ghost}} \sim \frac{1}{p^2}$$

$$(p-q)^\mu \delta^{\nu\rho} \sim |p|$$

$[-\partial^\mu D_\mu]c]$

Renormalization

A^μ



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$\psi, \bar{\psi}$

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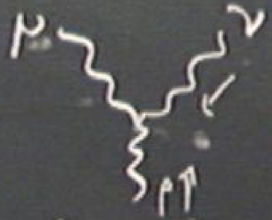
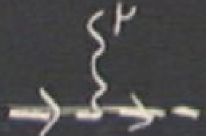


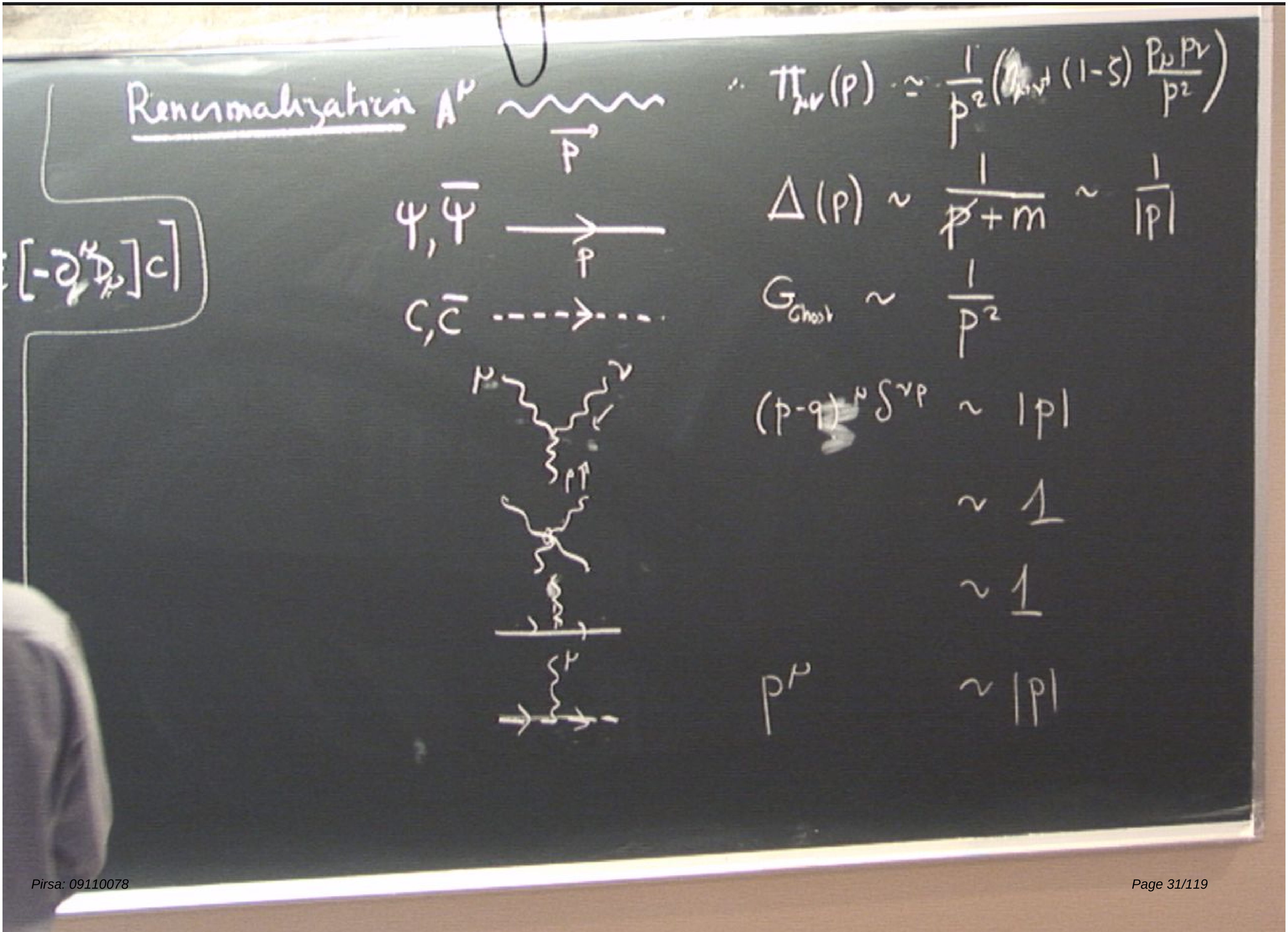
$$(p-q)^\nu \delta^{\nu\rho} \sim |p|$$

$$\sim 1$$

$$\sim 1$$



$$p^\mu \sim |p|$$



$[-\partial^\mu D_\mu]c]$

Renormalization

A^μ, $\vec{p}$ $\quad \therefore \; \Pi_{\mu\nu}(p) \simeq \frac{1}{p^2}\left(\eta_{\mu\nu}(1-\zeta)\frac{p_\mu p_\nu}{p^2}\right)$

$\psi, \bar{\psi}$, p $\quad \Delta(p) \sim \frac{1}{\not{p}+m} \sim \frac{1}{|p|}$

$c, \bar{c}$ $\quad G_{Ghost} \sim \frac{1}{p^2}$

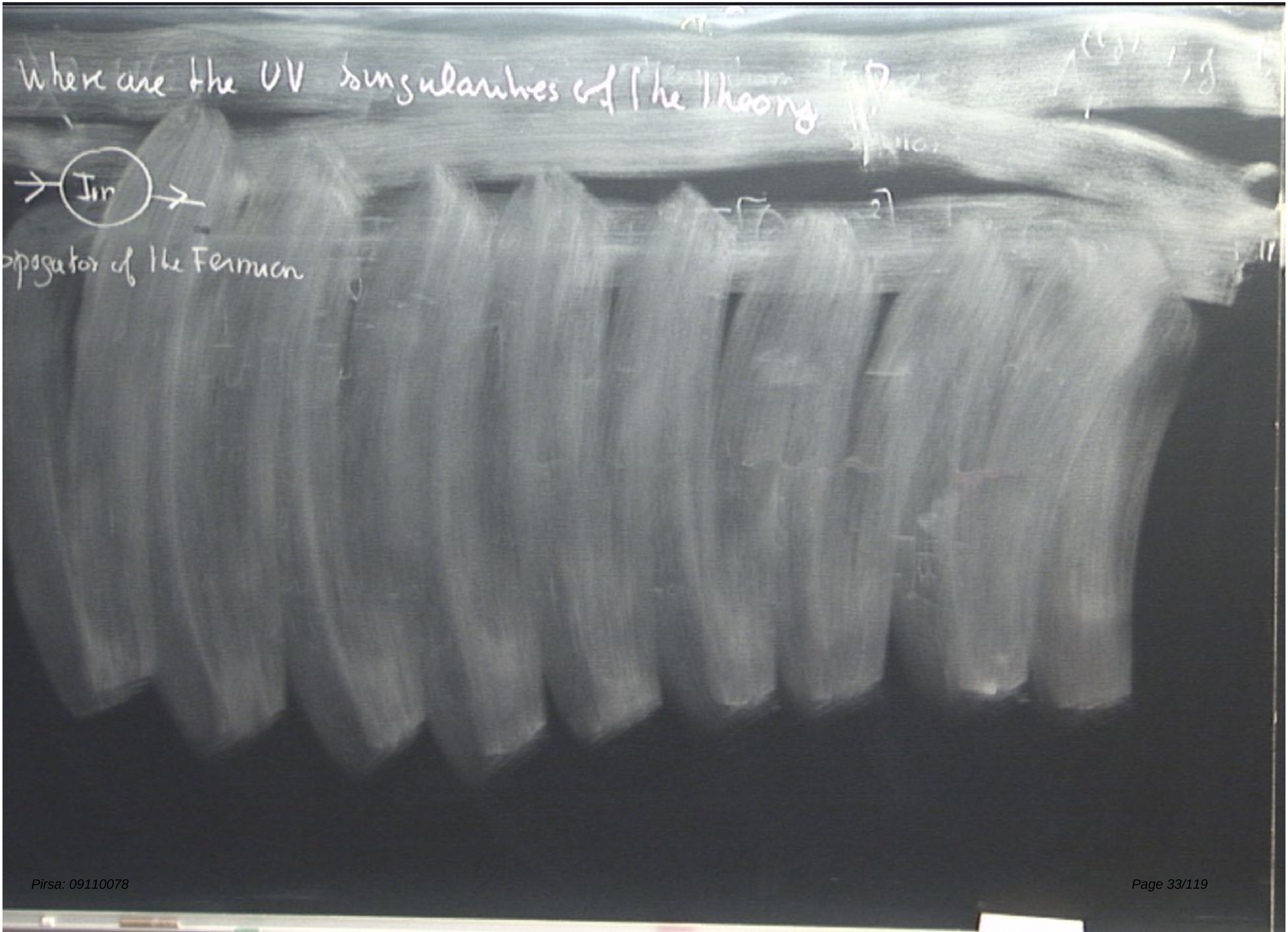
μ, ν, ρ $\quad (p-q)^\mu \delta^{\nu\rho} \sim |p|$

~ 1

~ 1

μ $\quad p^\mu \sim |p|$

Where are the UV singularities of the theory ?



Where are the UV singularities of the theory

opagator of the Fermion

Where are the UV singularities of the theory?

opagator of the Fermion

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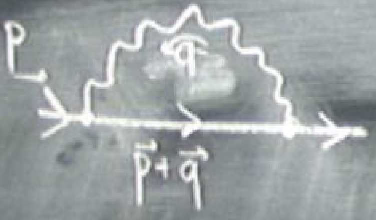


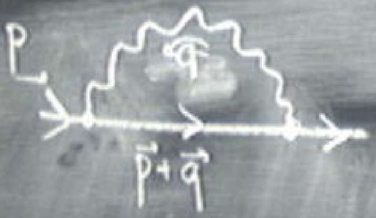
$$\simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not{p}+\not{q})^2+m} \quad \ldots$$

opagator of the Fermion

Where are the UV singularities of the theory ?

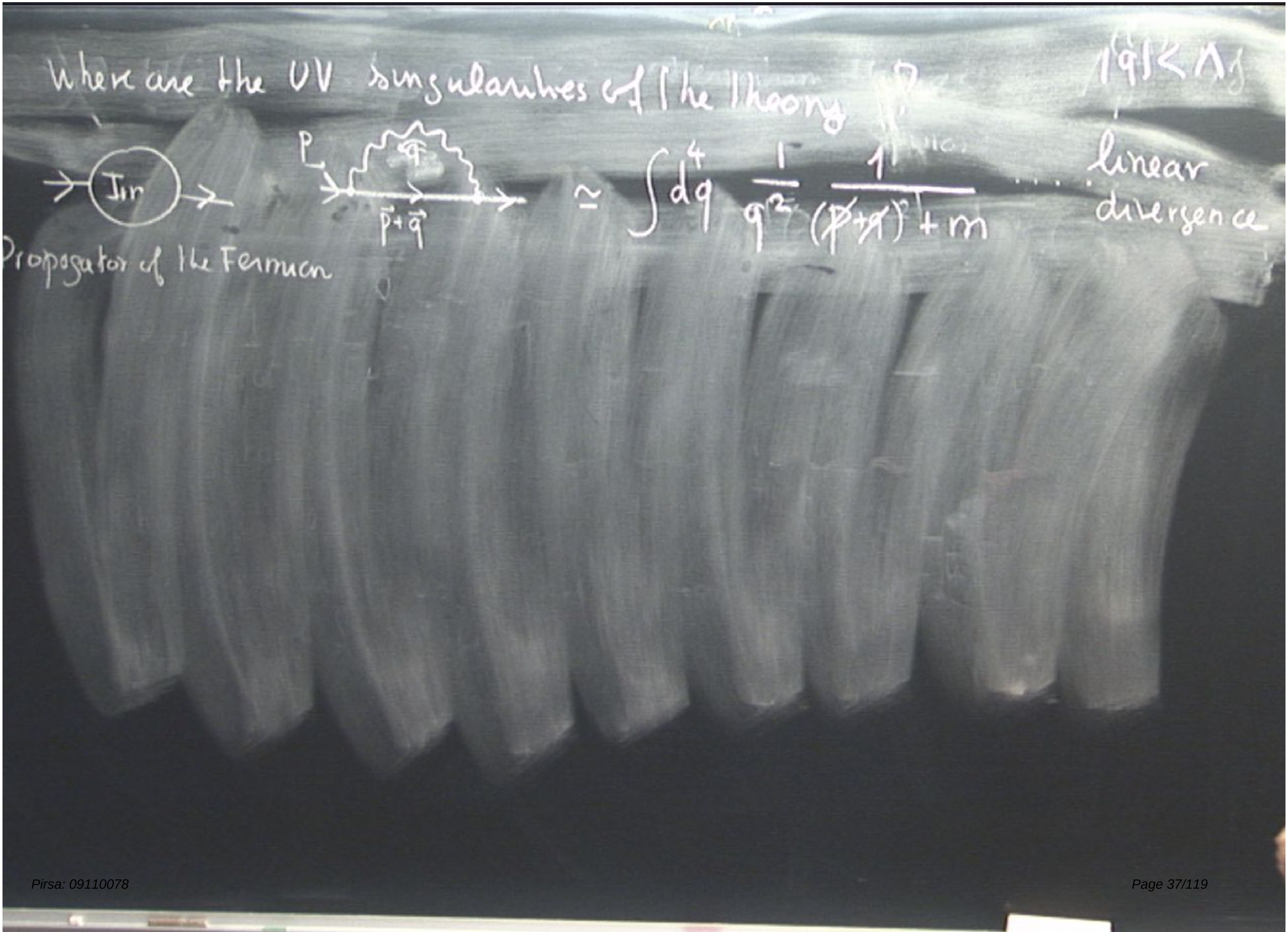
$\Lambda^{(i,j)}$, i,j

Propagator of the Fermion



$$\simeq \int d^4q \; \frac{1}{q^2} \; \frac{1}{(\not{p}+\not{q})^2 + m} \; \ldots$$

linear divergence



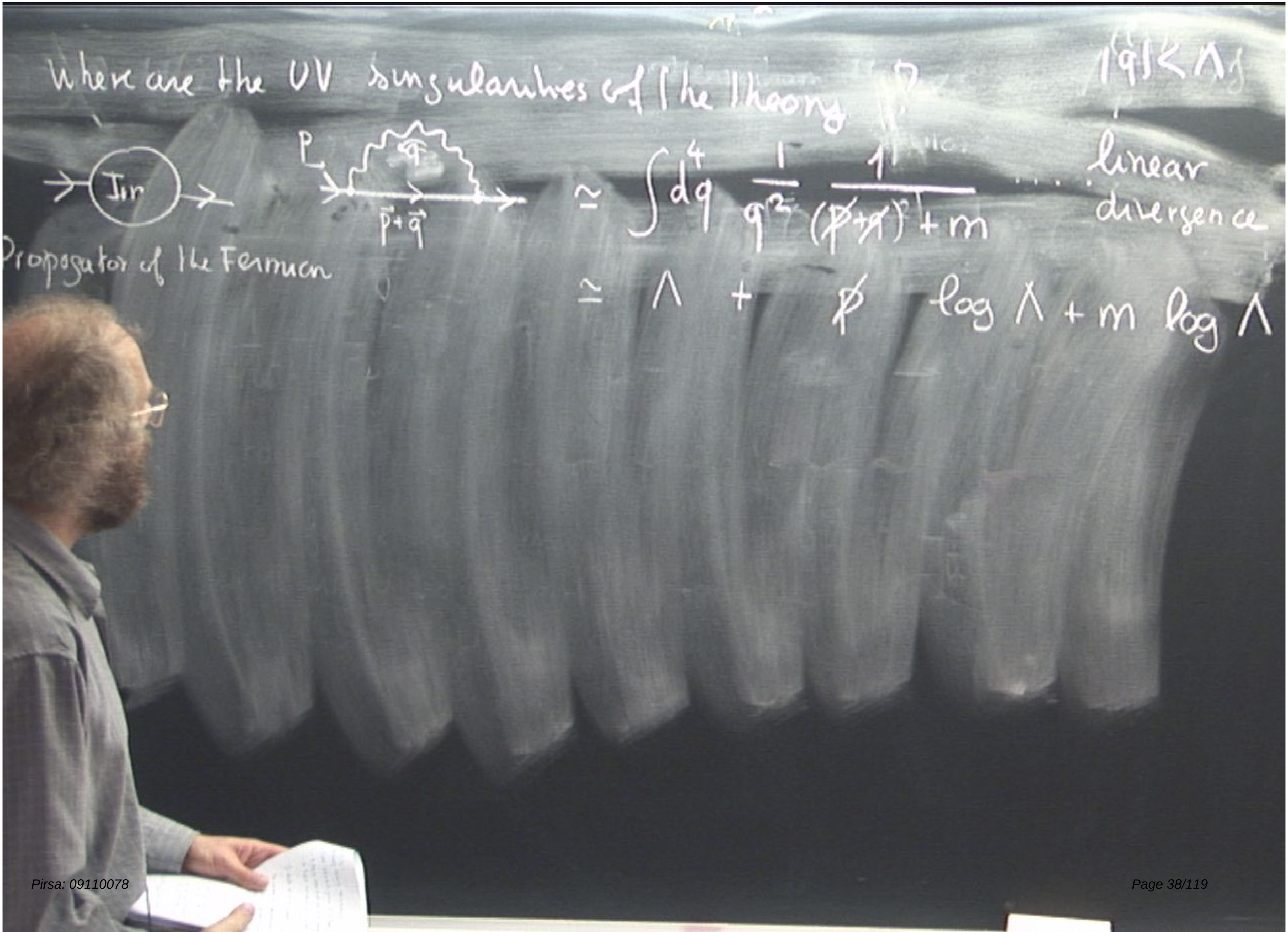
Where are the UV singularities of the theory ?

$|q| < \Lambda$;

Propagator of the Fermion

$$\simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not p + \not q) + m} \cdots$$

linear divergence



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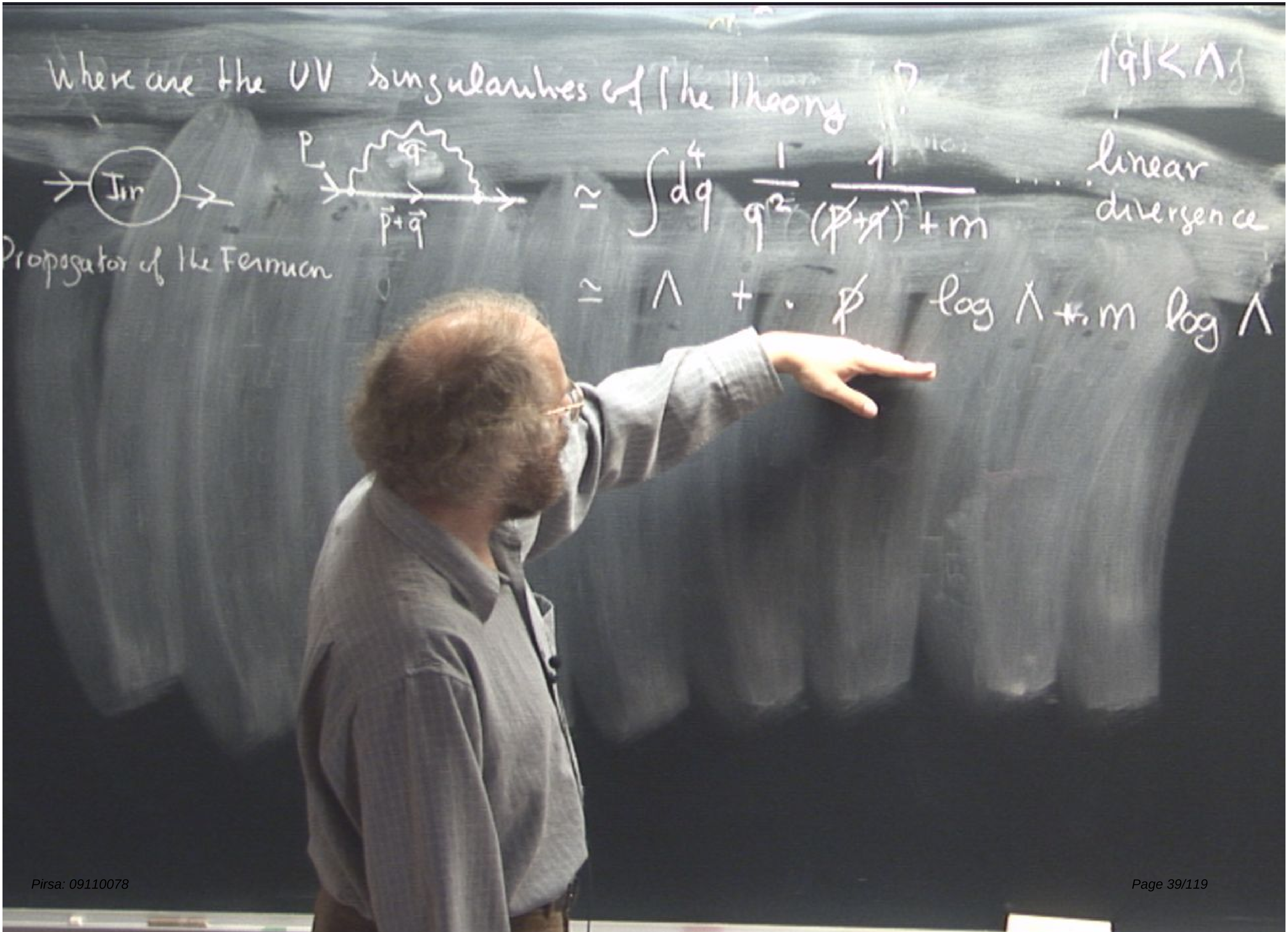

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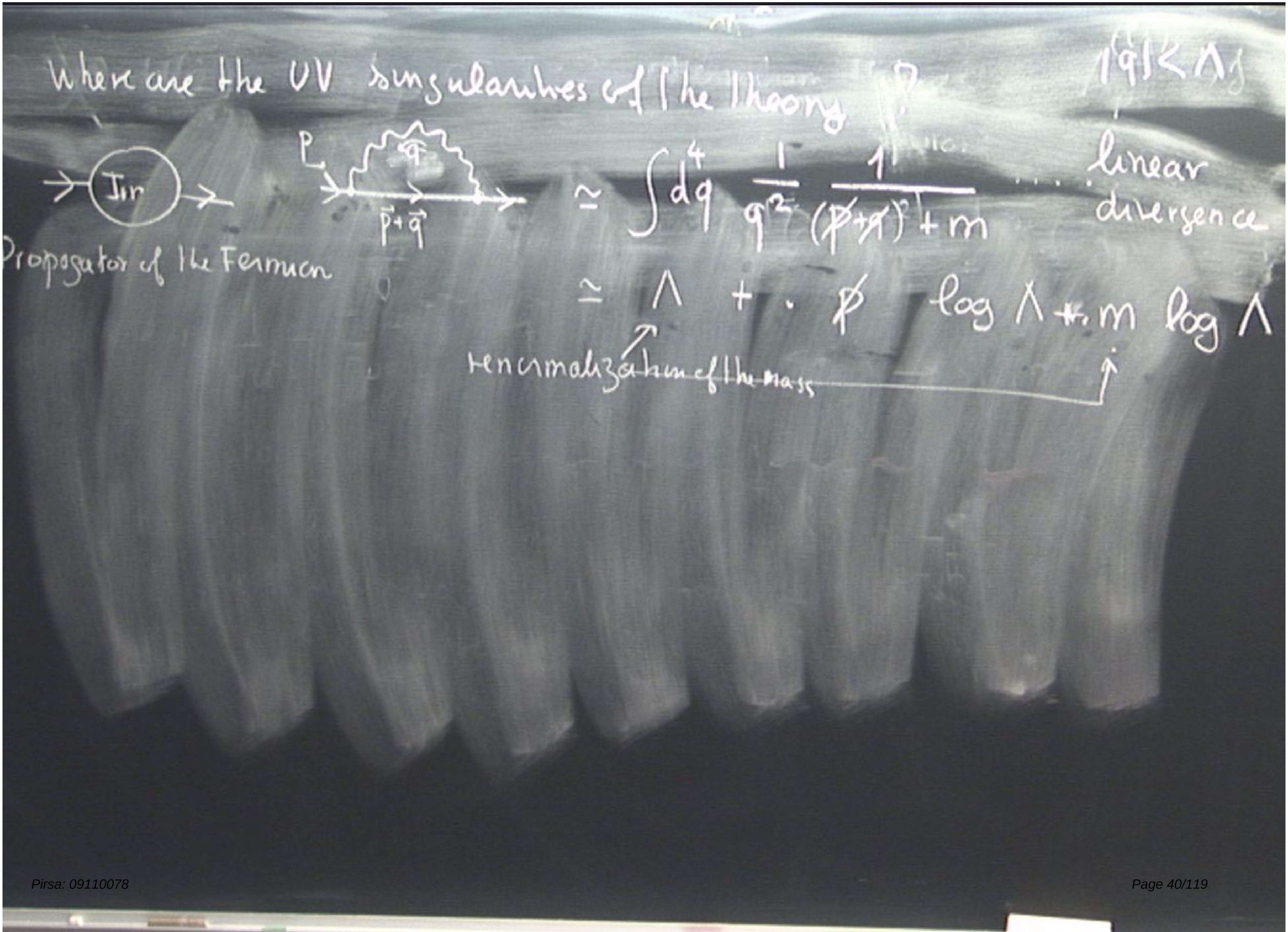

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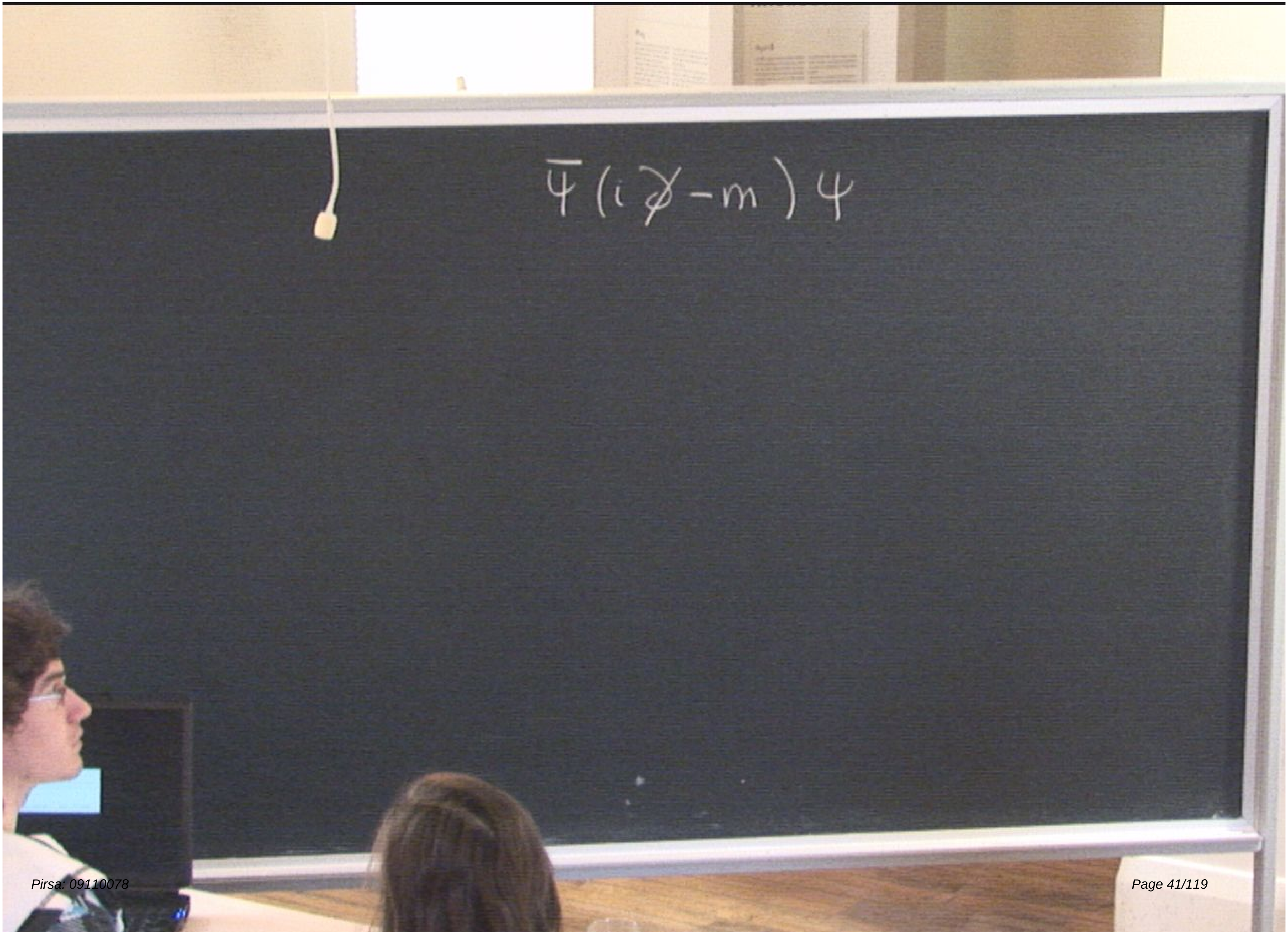
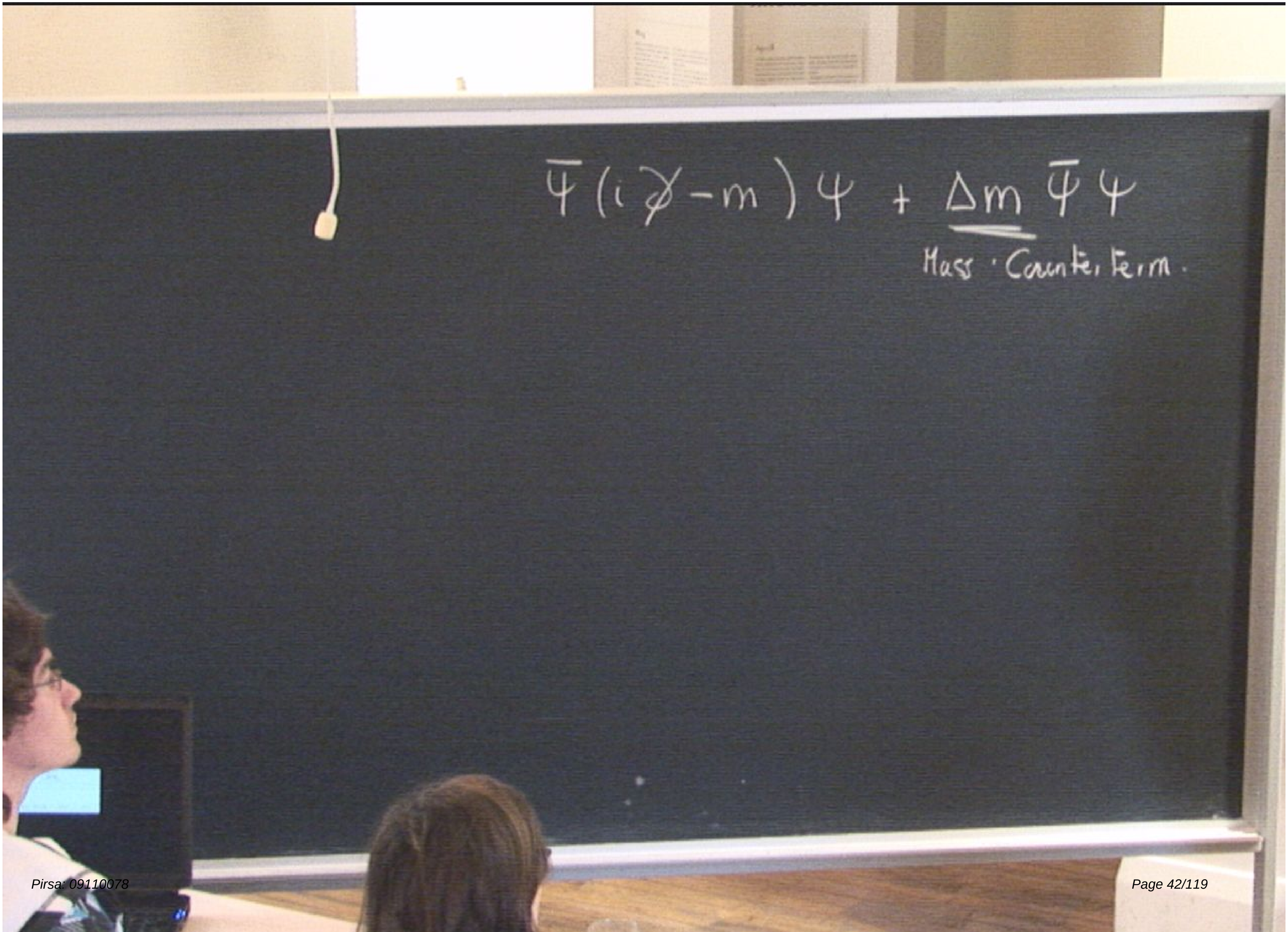
renormalization of the mass

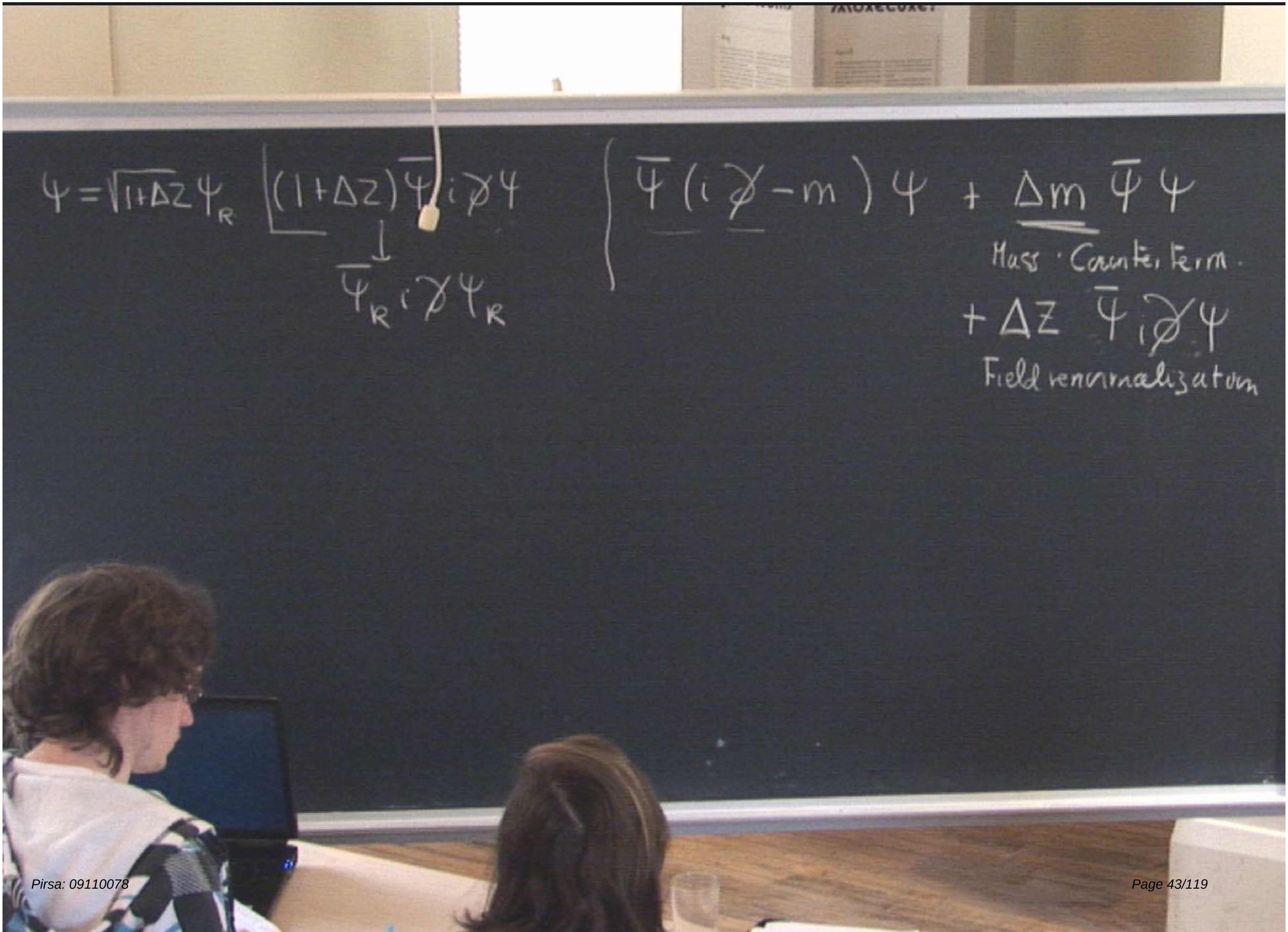


$$\bar{\Psi}(i\not{\partial}-m)\Psi$$



$$\bar{\Psi}(i\not{\partial}-m)\Psi+\underline{\Delta m}\,\bar{\Psi}\Psi$$

Mass · Counter term.



$$\psi = \sqrt{1+\Delta Z}\,\psi_R \qquad (1+\Delta Z)\bar{\psi}\, i\not{\partial}\psi$$

$$\downarrow$$

$$\bar{\psi}_R\, i\not{\partial}\psi_R$$

$$\int \bar{\psi}(i\not{\partial}-m)\psi + \underline{\Delta m}\,\bar{\psi}\psi$$

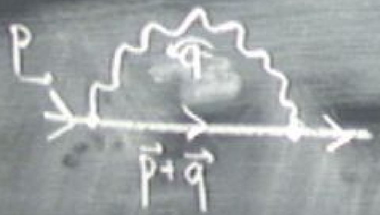
Mass · Counter term.

$$+\Delta Z\,\bar{\psi}\, i\not{\partial}\psi$$

Field renormalization

Where are the UV singularities of the theory ? $|q| < \Lambda$

opagator of the Fermion



$$\simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not{p}+\not{q})+m} \; \ldots$$

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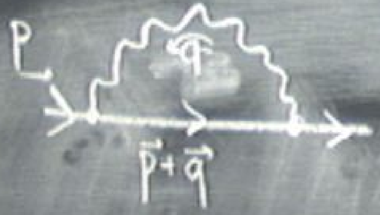

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Λ divergence is absent ! renormalization of the mass

If $m = 0$, it is not renormalized (≠ case of a scalar field)

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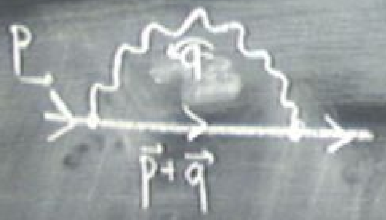

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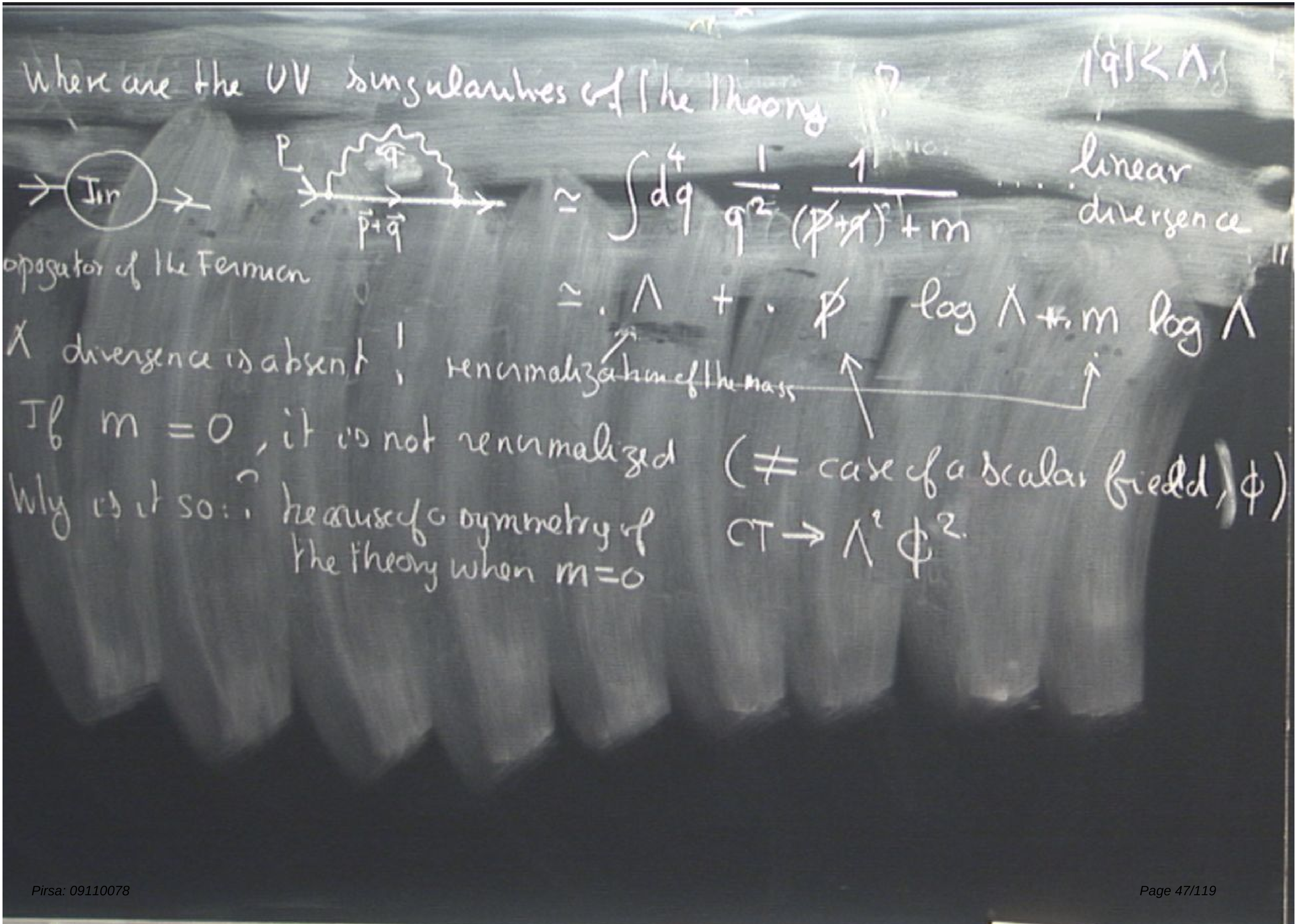

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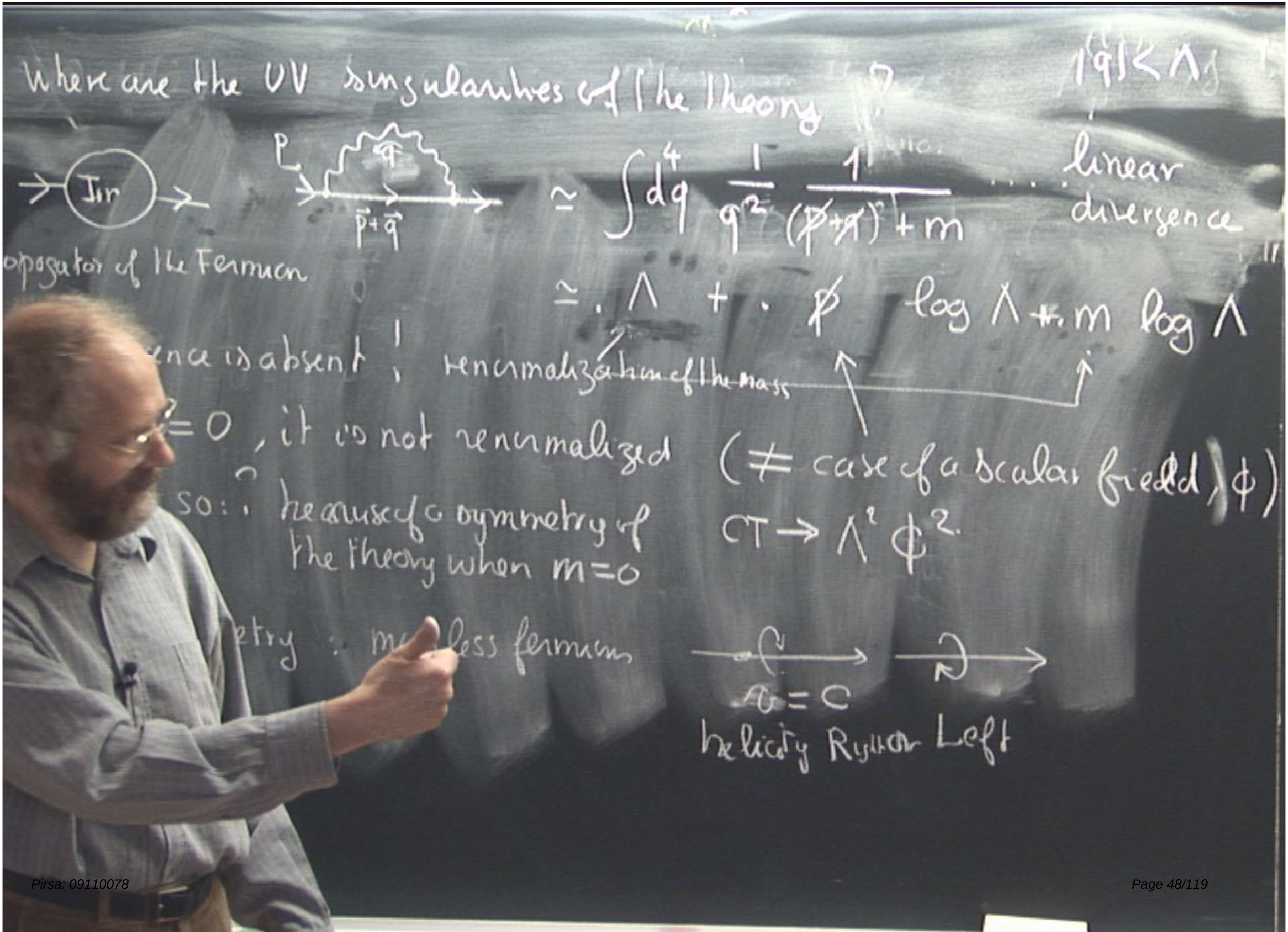

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$\sigma = c$

helicity Right Left

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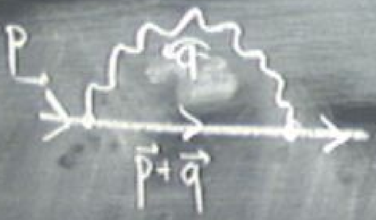

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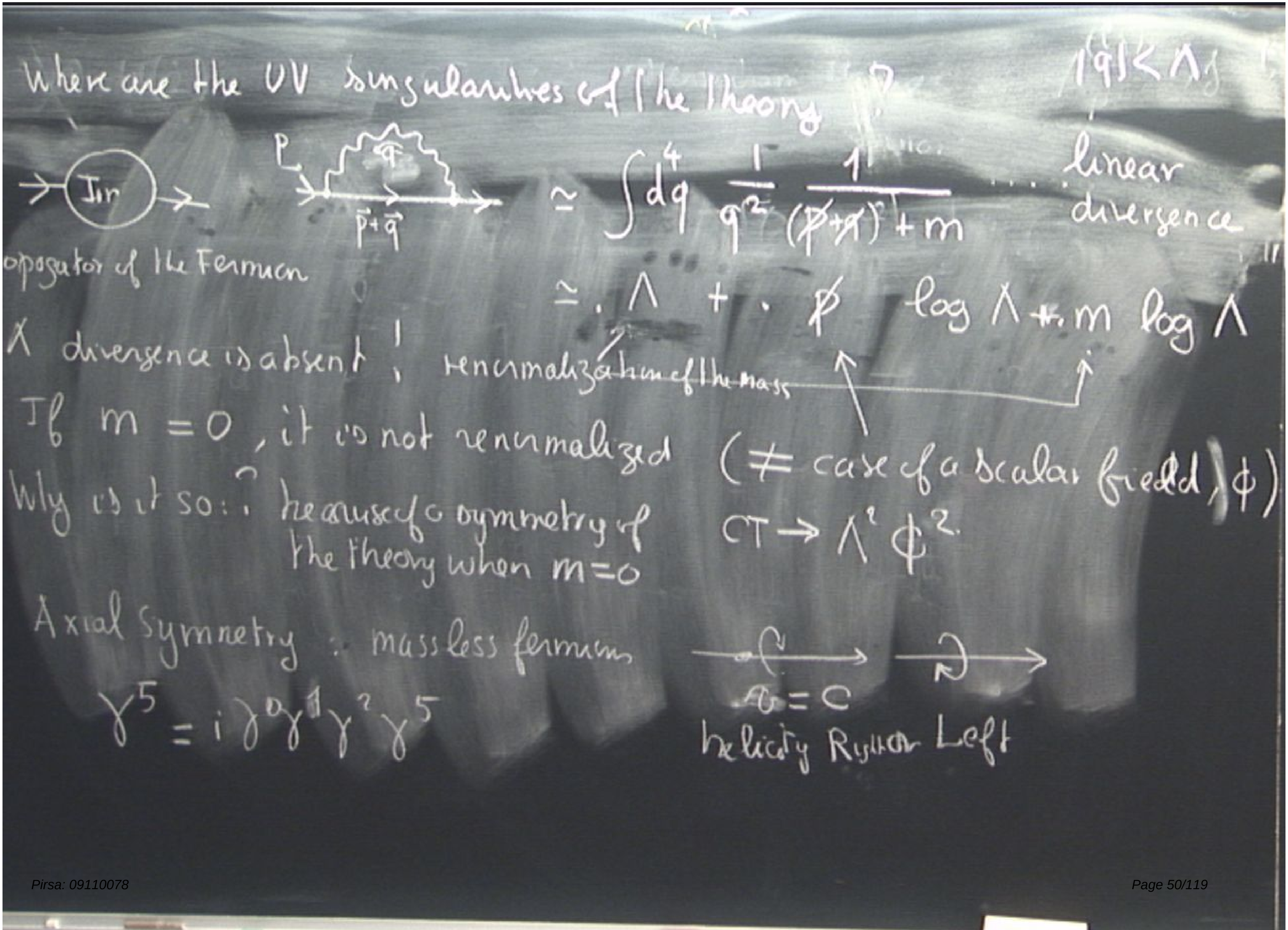
Axial Symmetry : massless fermions

$v = c$

helicity Right or Left



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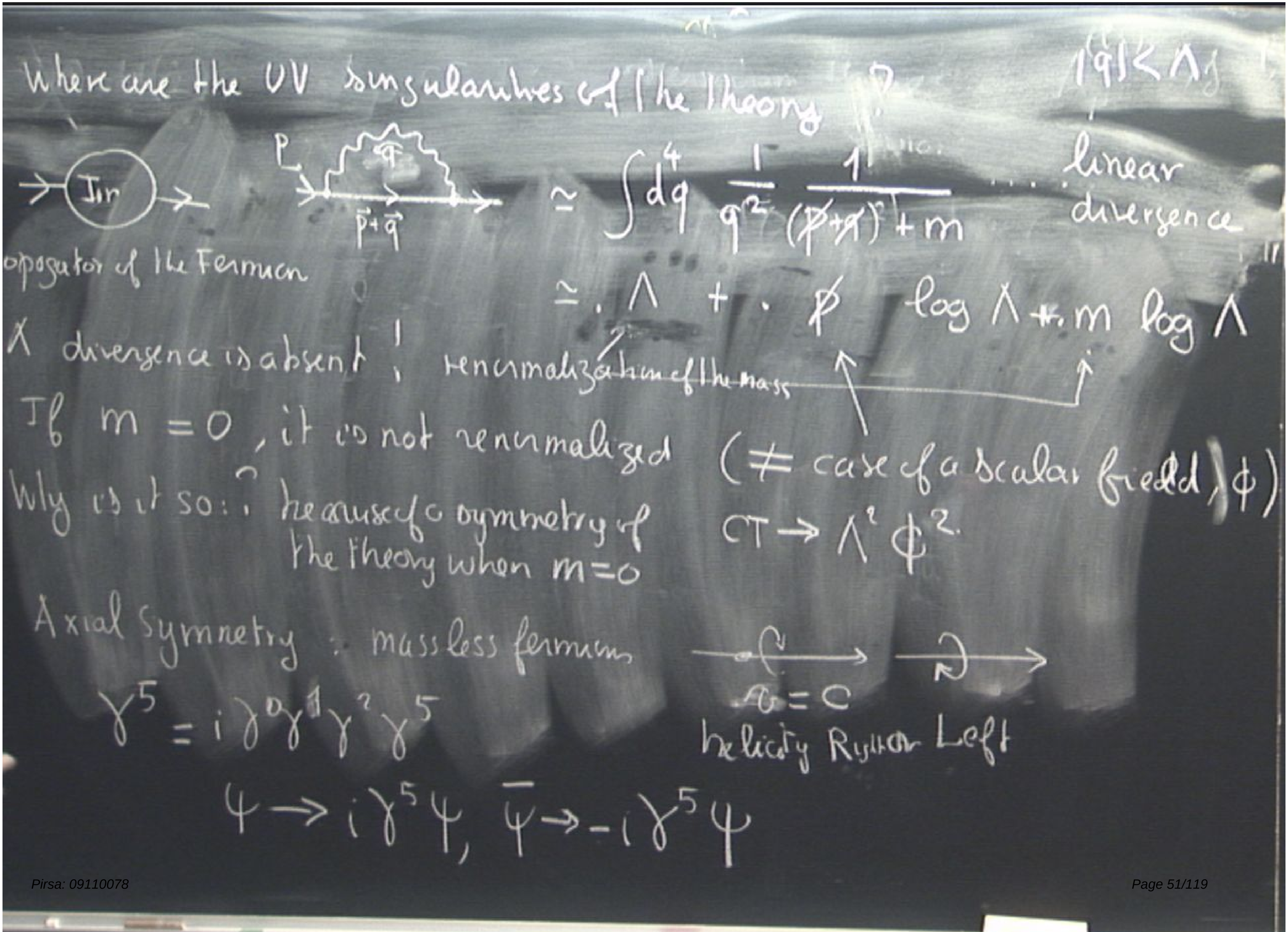

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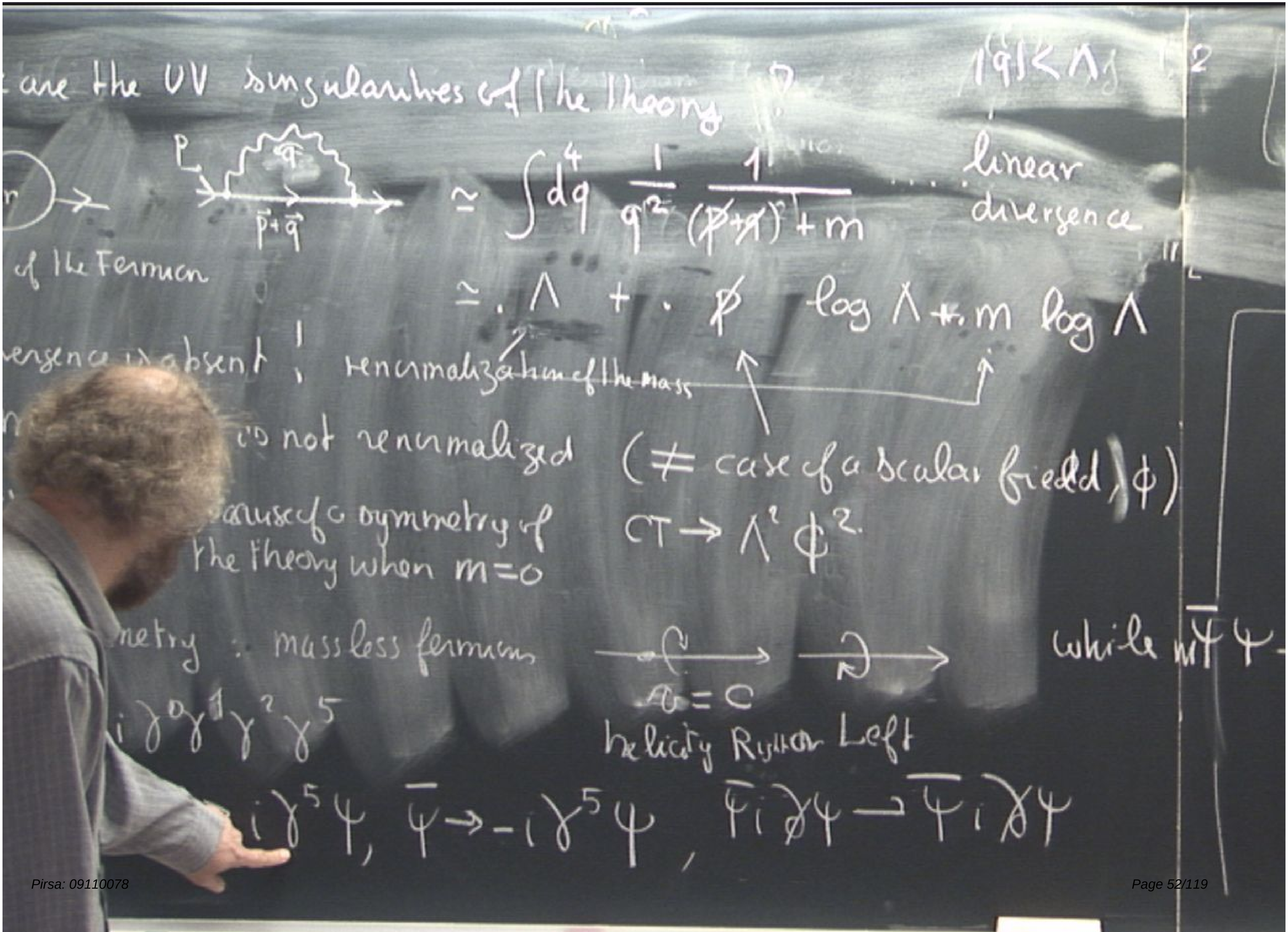

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$$\simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not p + \not q) + m} \quad \ldots \text{ linear divergence}$$

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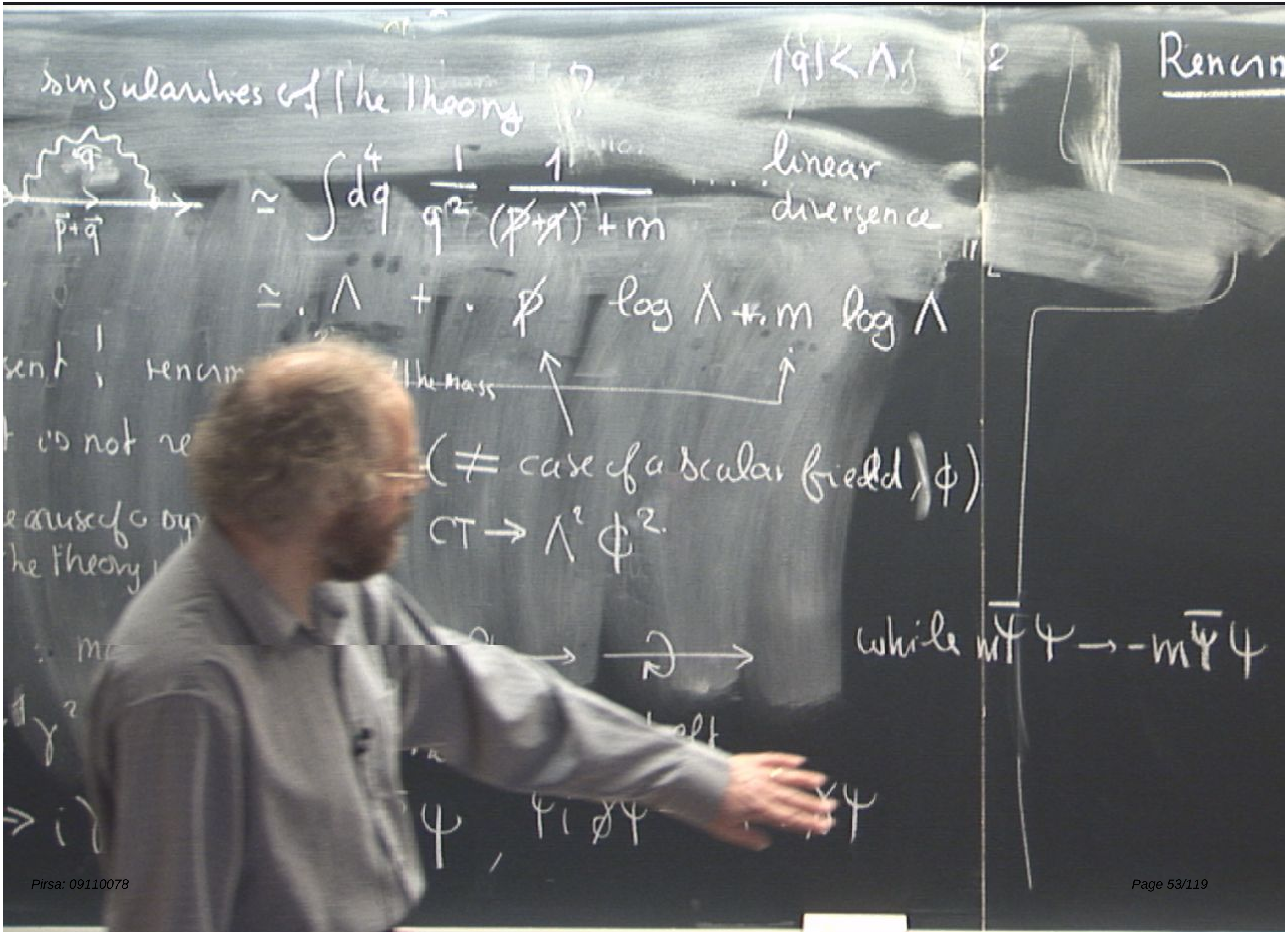
while $m\bar{\Psi}\Psi$ -

$i\gamma^0\gamma^1\gamma^2\gamma^5$

$i\gamma^5\Psi, \; \bar{\Psi} \to -i\gamma^5\Psi, \; \bar{\Psi} i \not\partial \Psi \to \bar{\Psi} i \not\partial \Psi$



singularities of the theory ?

$|q| < \Lambda$

2

Renorm

$$\simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not p+\not q)+m}$$

linear divergence

$$\simeq \cdot \Lambda + \cdot \not p \log \Lambda + \cdot m \log \Lambda$$

sent ! renorm ... the mass

t is not re ... ($\neq$ case of a scalar field ϕ)

e cause of a sym ... $CT \to \Lambda^2 \phi^2$

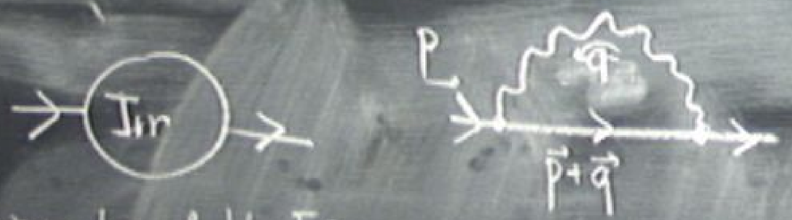
he theory

m ... $\longrightarrow$ ... while $m\bar\Psi\Psi \to -m\bar\Psi\Psi$

γ^2 ... left

$\to i\gamma$ ... ψ, $\bar\psi i \not\partial \psi$ ... $\bar\psi\psi$

Where are the UV singularities of the theory ? $|q|<$



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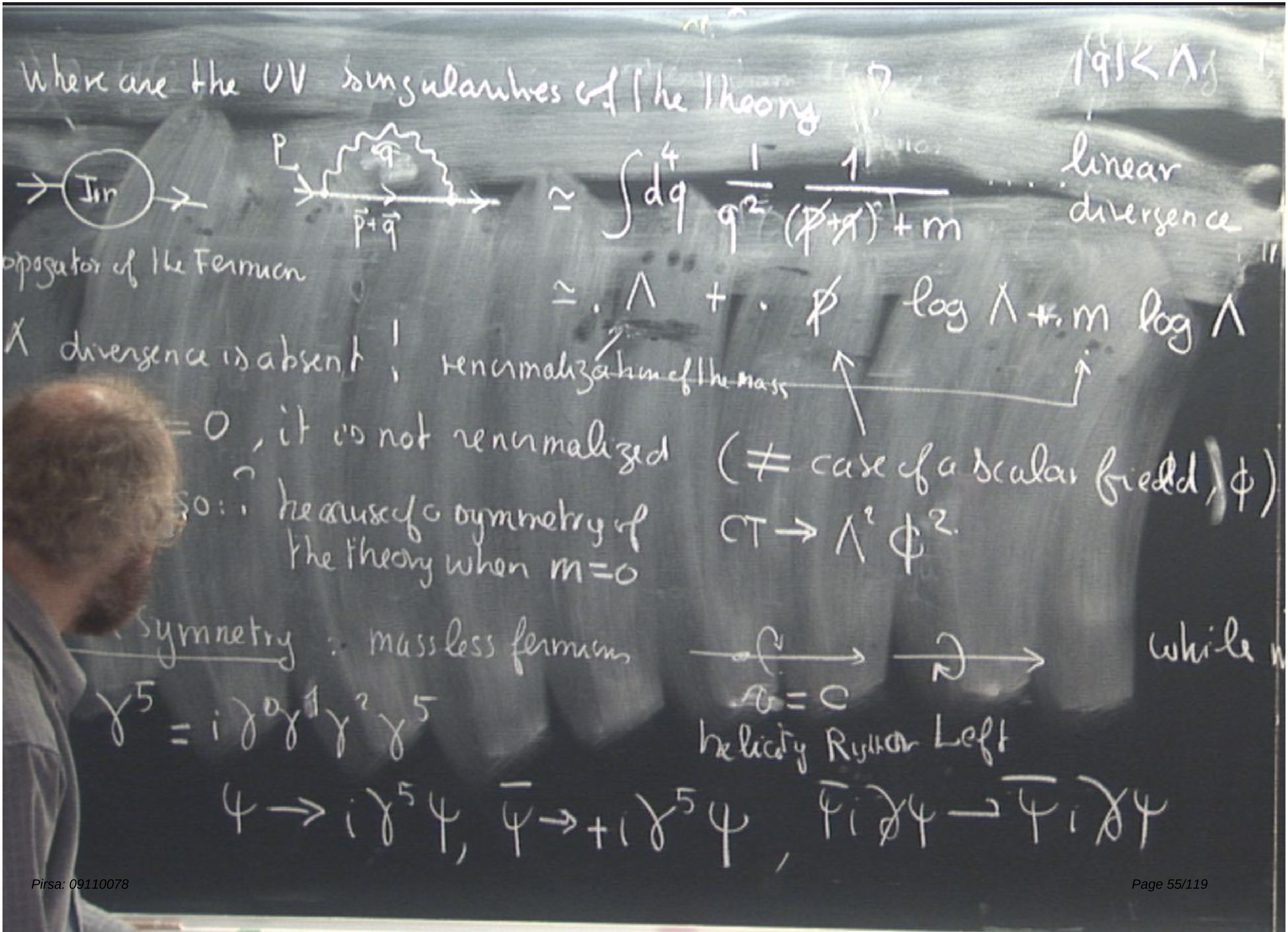

Where are the UV singularities of the theory ? $|q| < \Lambda$

$$\rightarrow \boxed{\Sigma_{Irr}} \rightarrow \qquad p, \; q, \; \vec{p}+\vec{q} \qquad \simeq \int d^4q \, \frac{1}{q^2} \, \frac{1}{(\not{p}+\not{q}) + m} \; \ldots$$

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$$\psi \to e^{i\alpha\gamma_5}\psi$$

$$\bar{\psi} \to \bar{\psi}\, e^{i\alpha\gamma_5}$$

α a real parameter

U(1) symmetry

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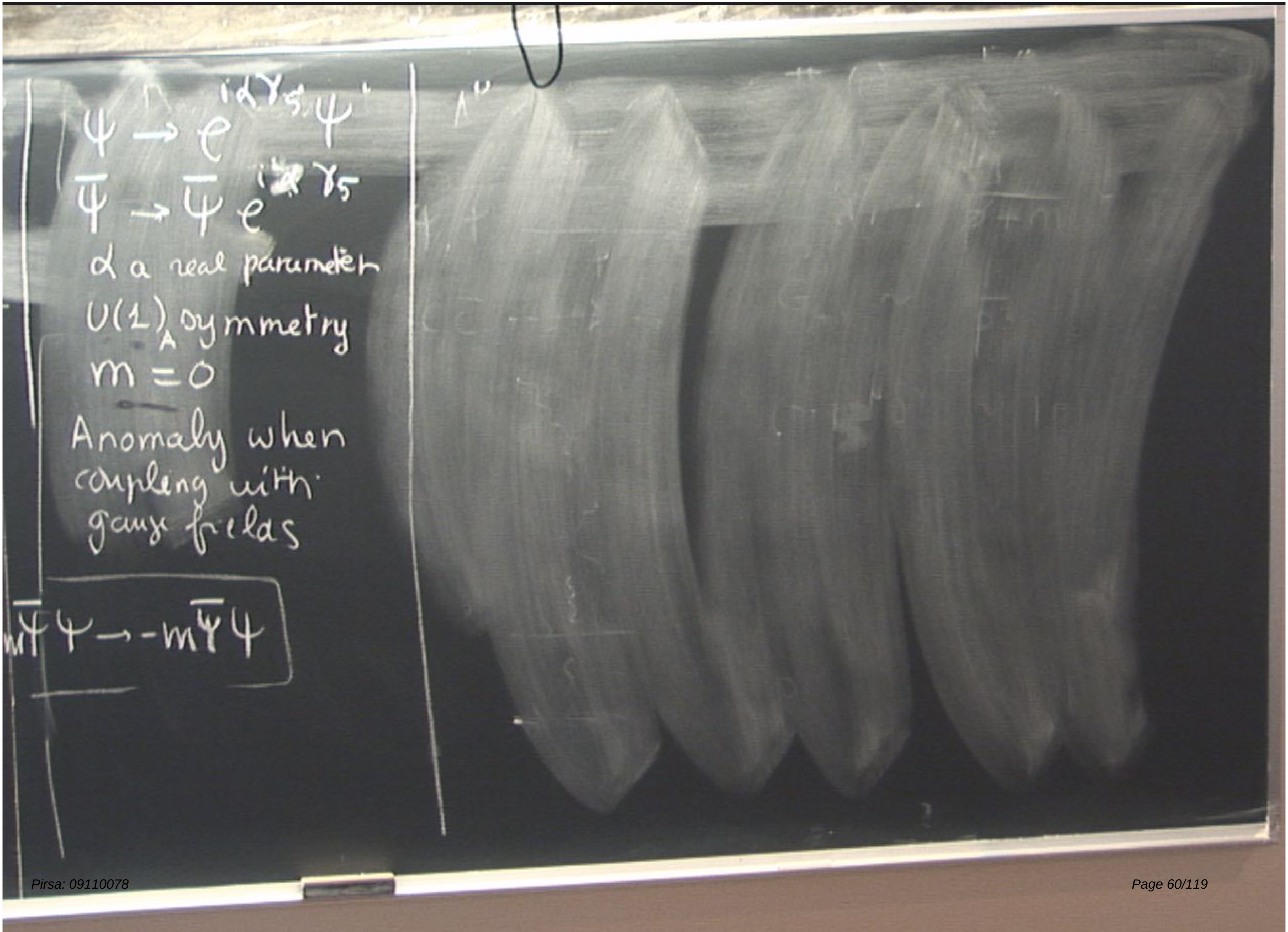

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Anomaly when coupling with gauge fields

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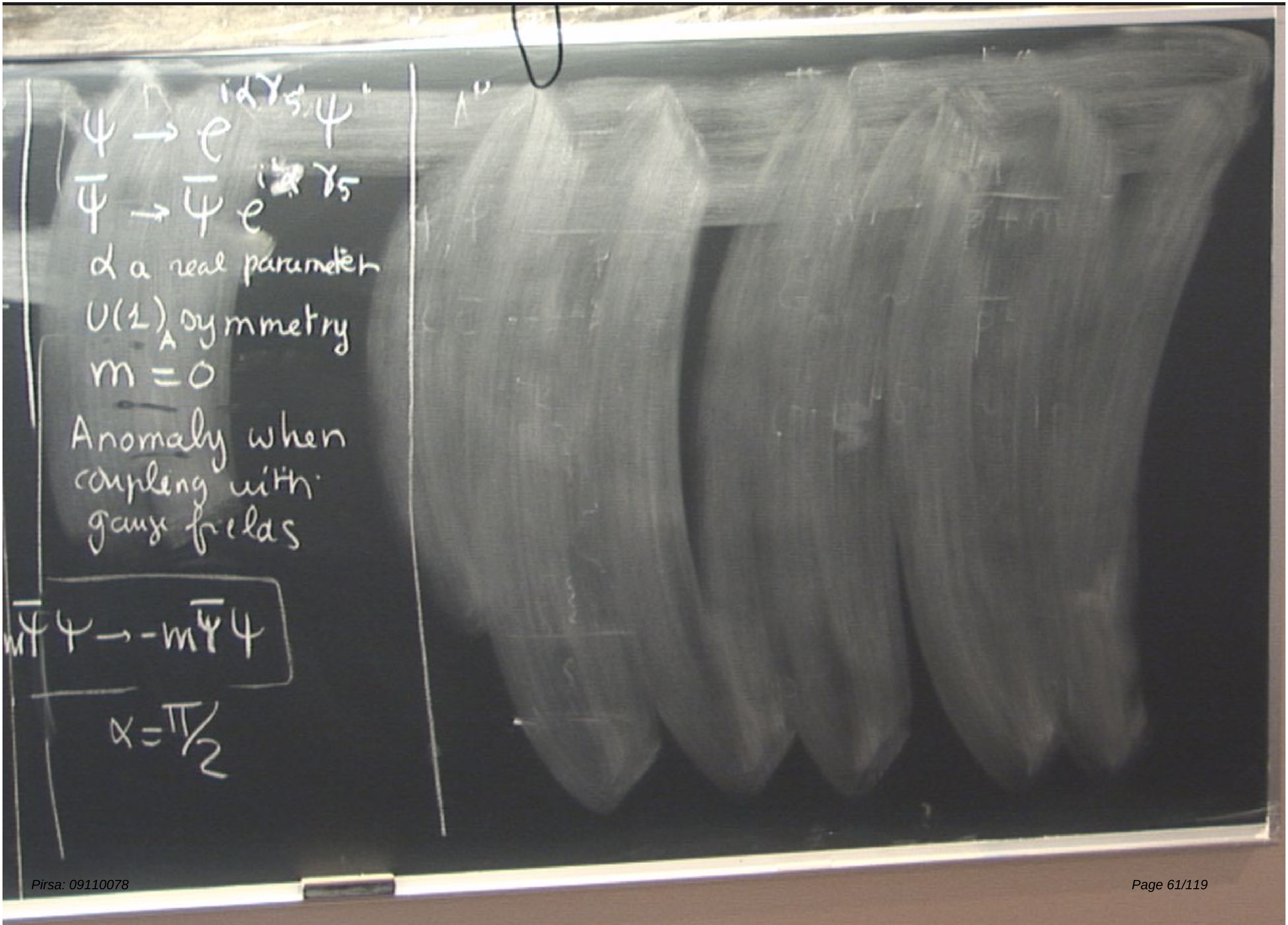

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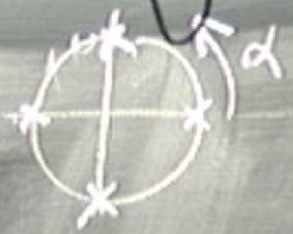

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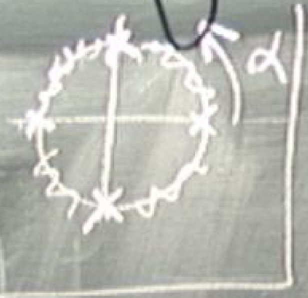

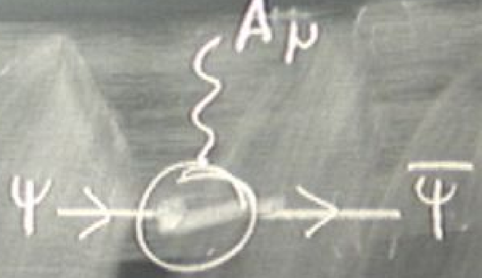

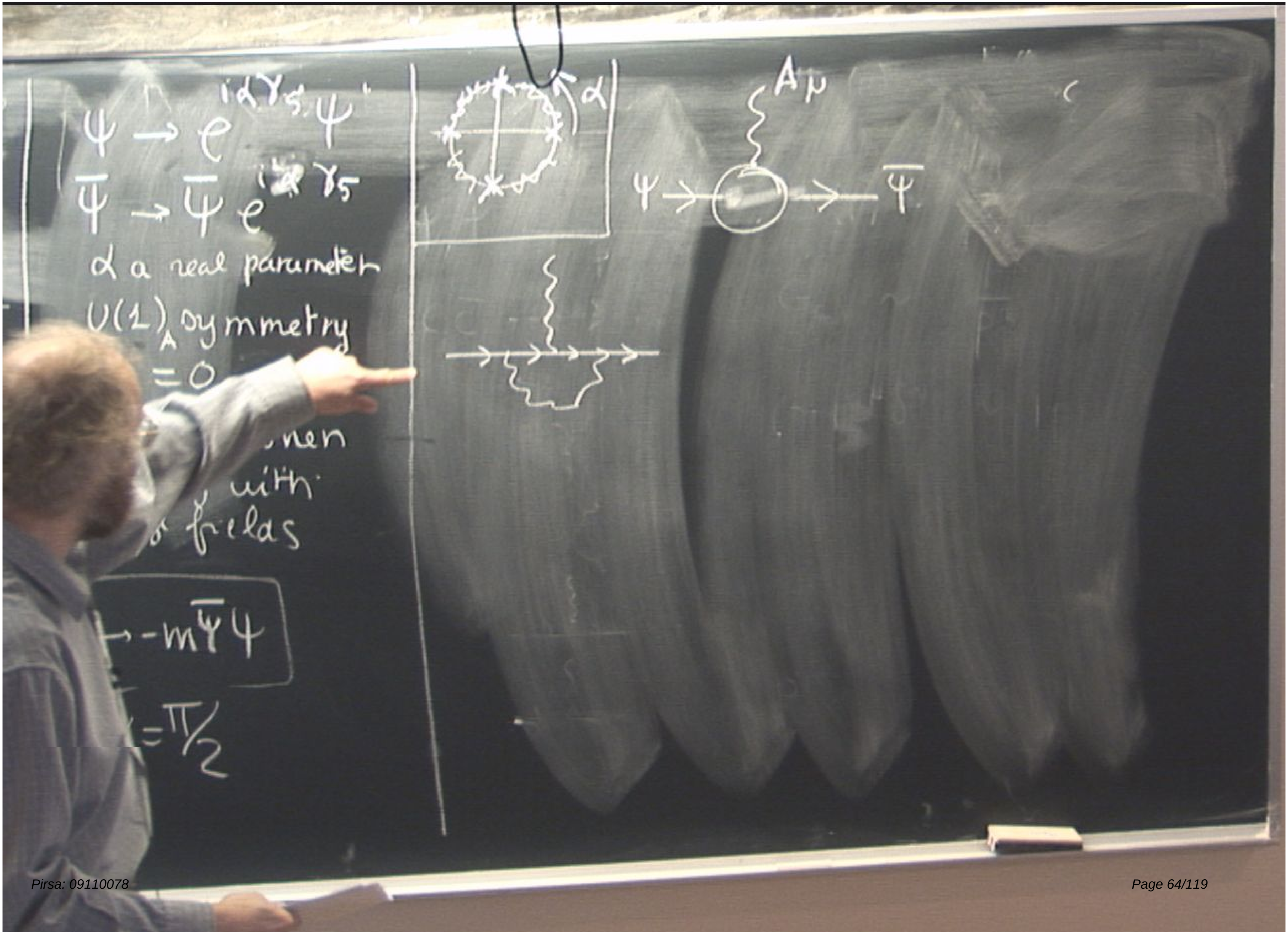

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$= 0$

hen

with

fields

$\boxed{\to -m\bar{\psi}\psi}$

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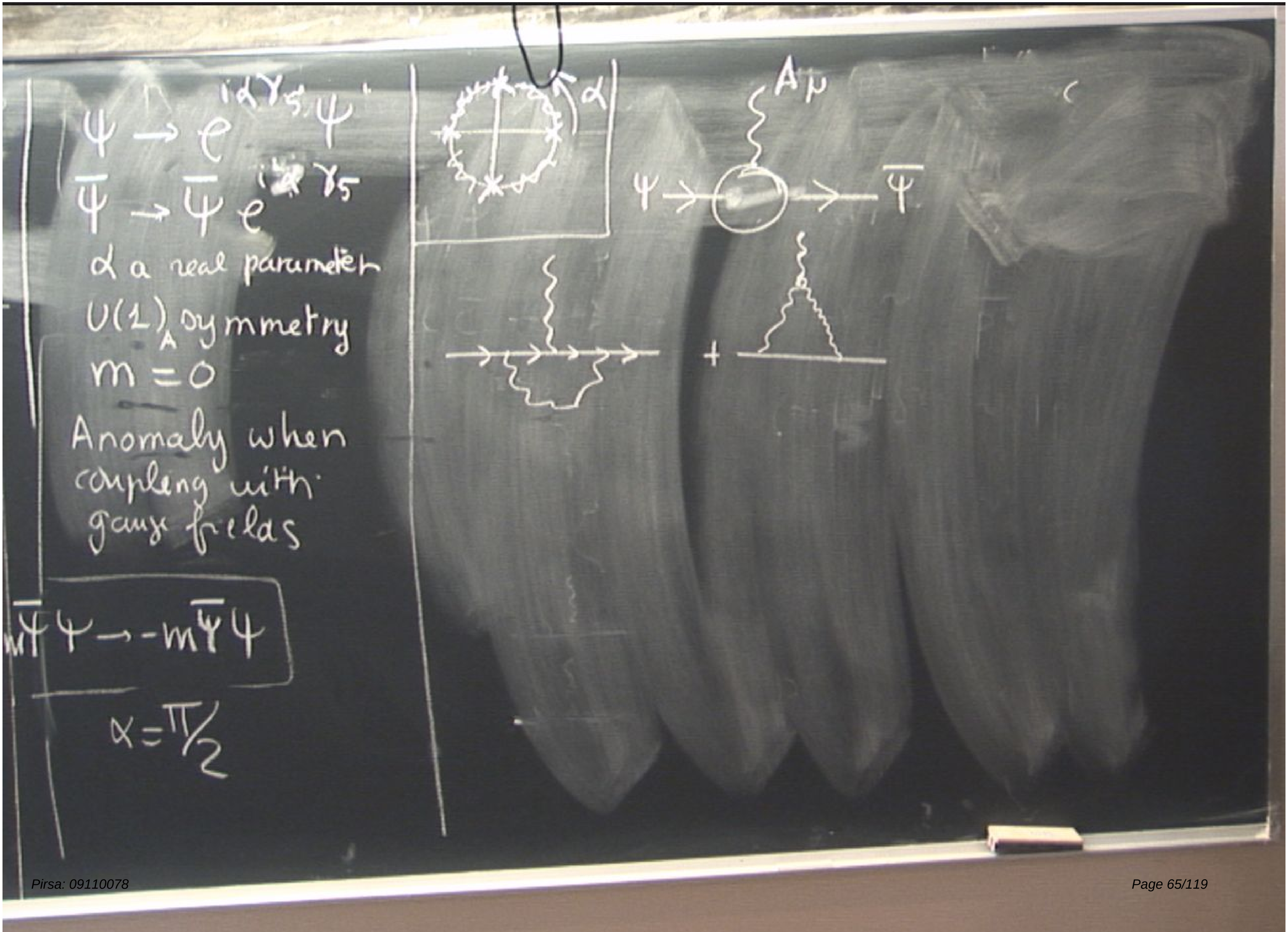

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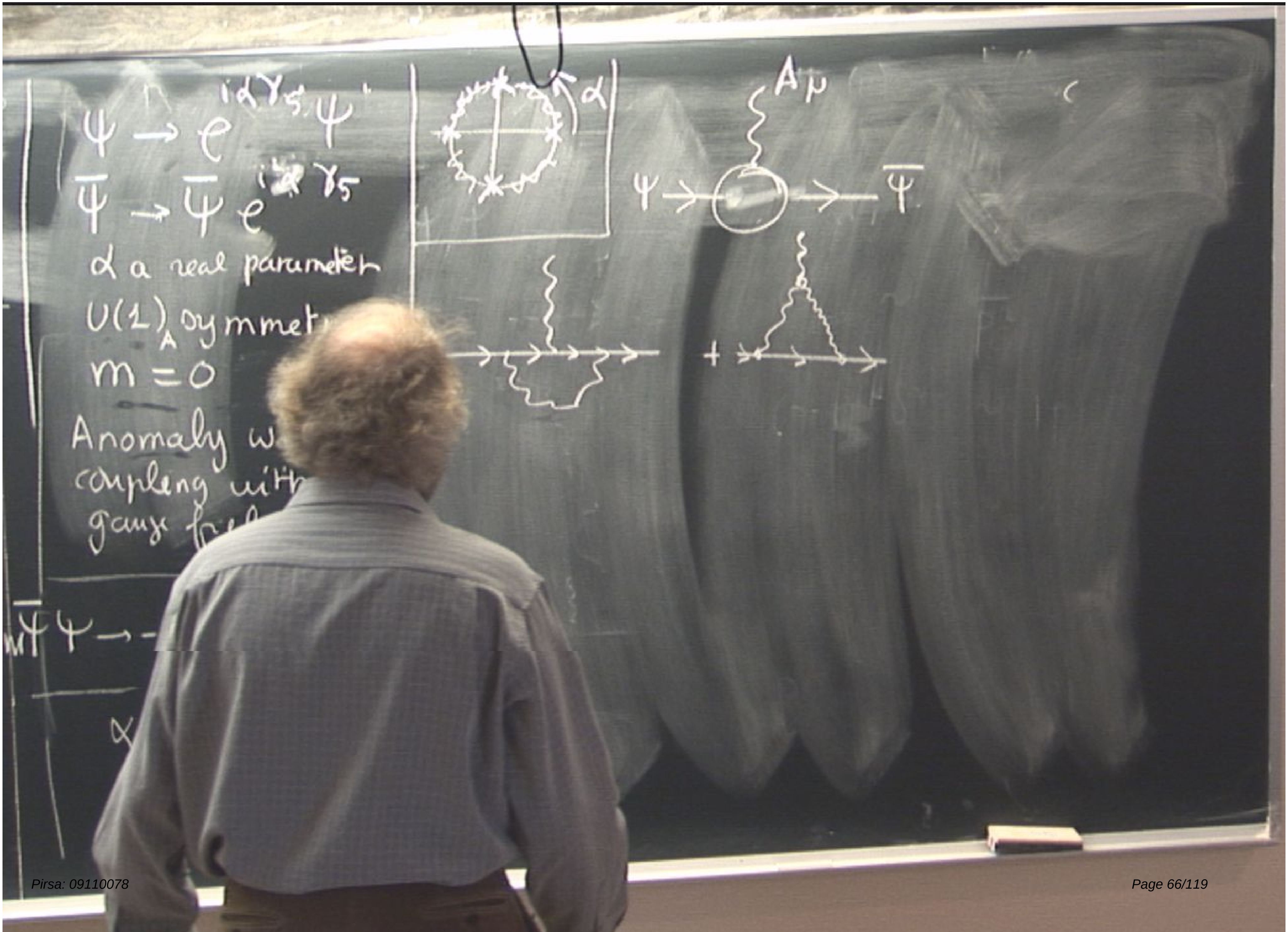

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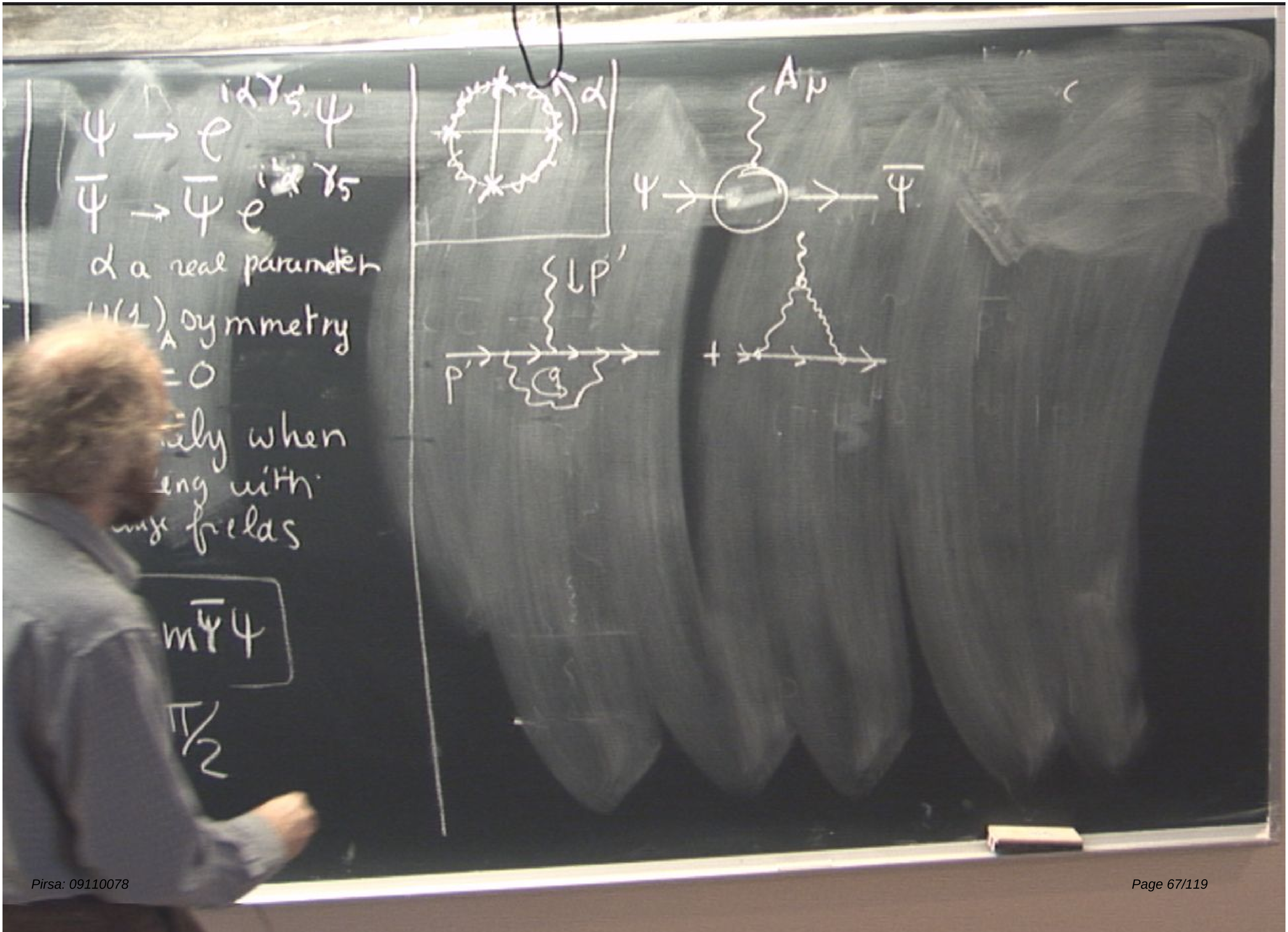

$\psi \rightarrow e^{i\alpha\gamma_5}\psi$

$\overline{\psi} \rightarrow \overline{\psi}\, e^{i\alpha\gamma_5}$

α a real parameter

$U(1)_A$ symmetry

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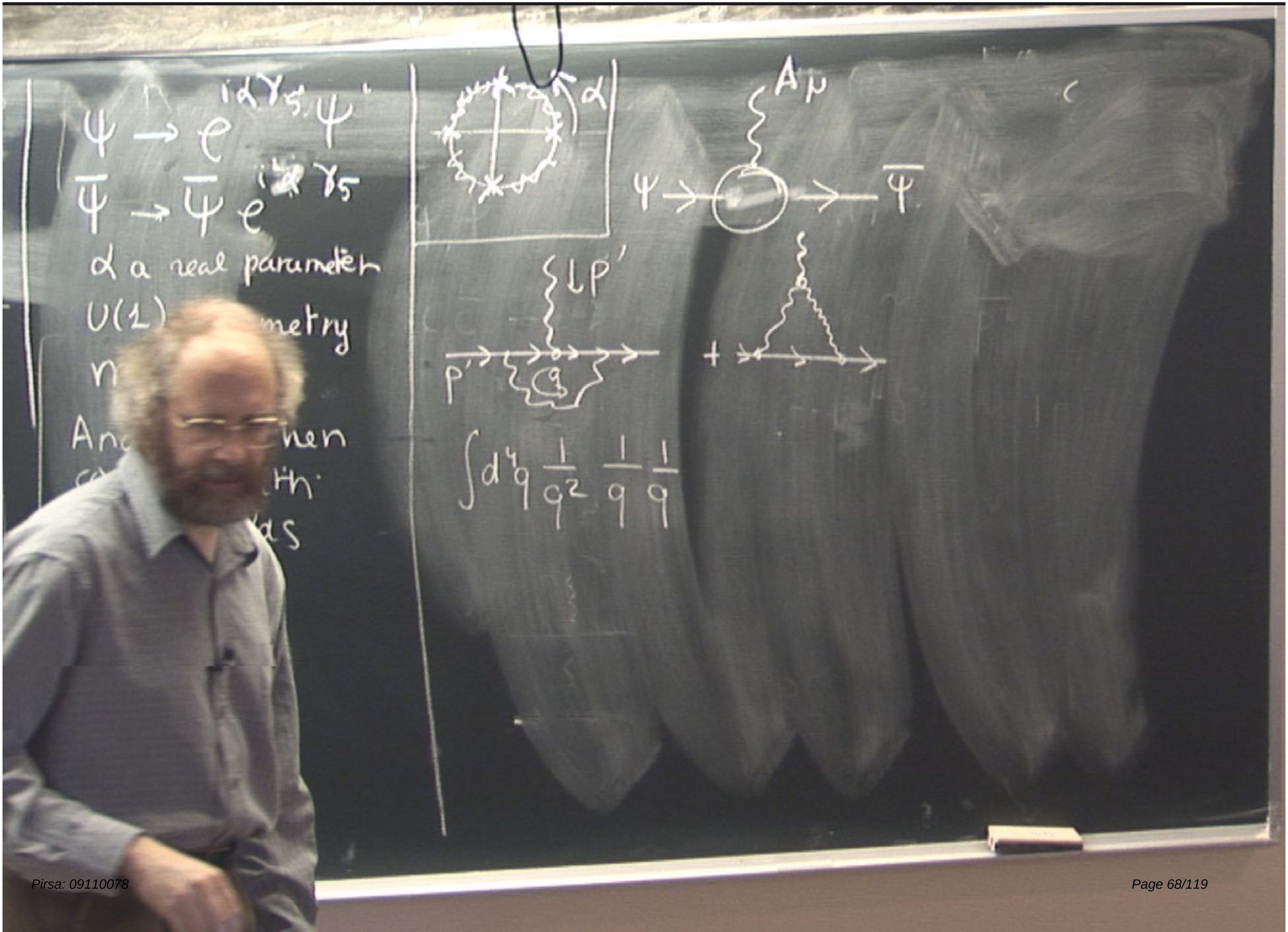
$\boxed{m\overline{\psi}\psi}$

$\pi/2$


$\psi \to e^{i\alpha\gamma_5}\psi$
$\bar{\psi} \to \bar{\psi}\, e^{i\alpha\gamma_5}$
α a real parameter
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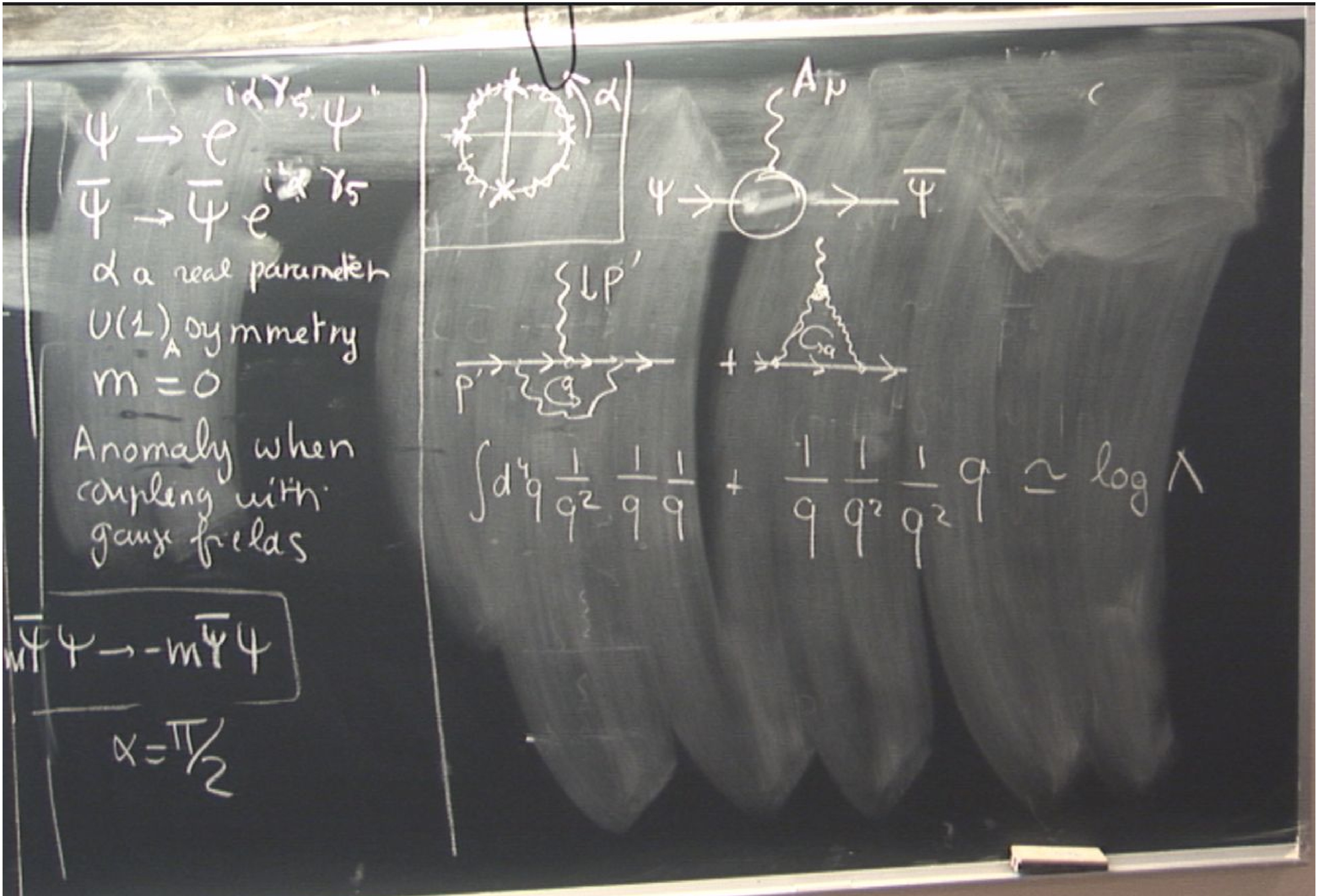
$\int d^4q \frac{1}{q^2}\frac{1}{q}\frac{1}{q}$



$$\psi \to e^{i\alpha\gamma_5}\psi$$

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α a real parameter

$U(1)_A$ symmetry

$m = 0$

Anomaly when coupling with gauge fields

$$\boxed{m\bar{\psi}\psi \to -m\bar{\psi}\psi}$$

$$\alpha = \pi/2$$

$$\int d^4q \,\frac{1}{q^2}\,\frac{1}{q}\,\frac{1}{q} + \frac{1}{q}\,\frac{1}{q^2}\,\frac{1}{q^2}\, q \simeq \log \Lambda$$

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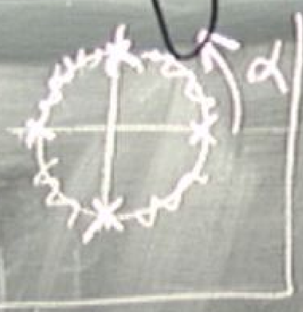

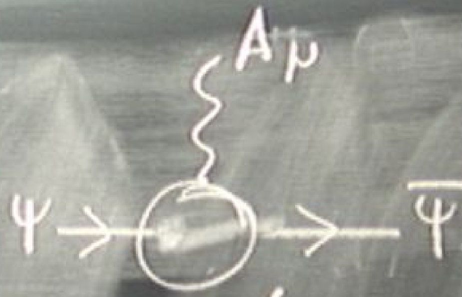

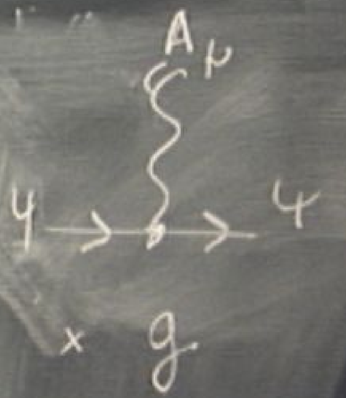

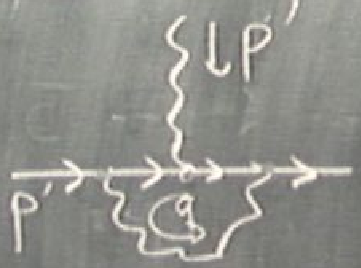

+

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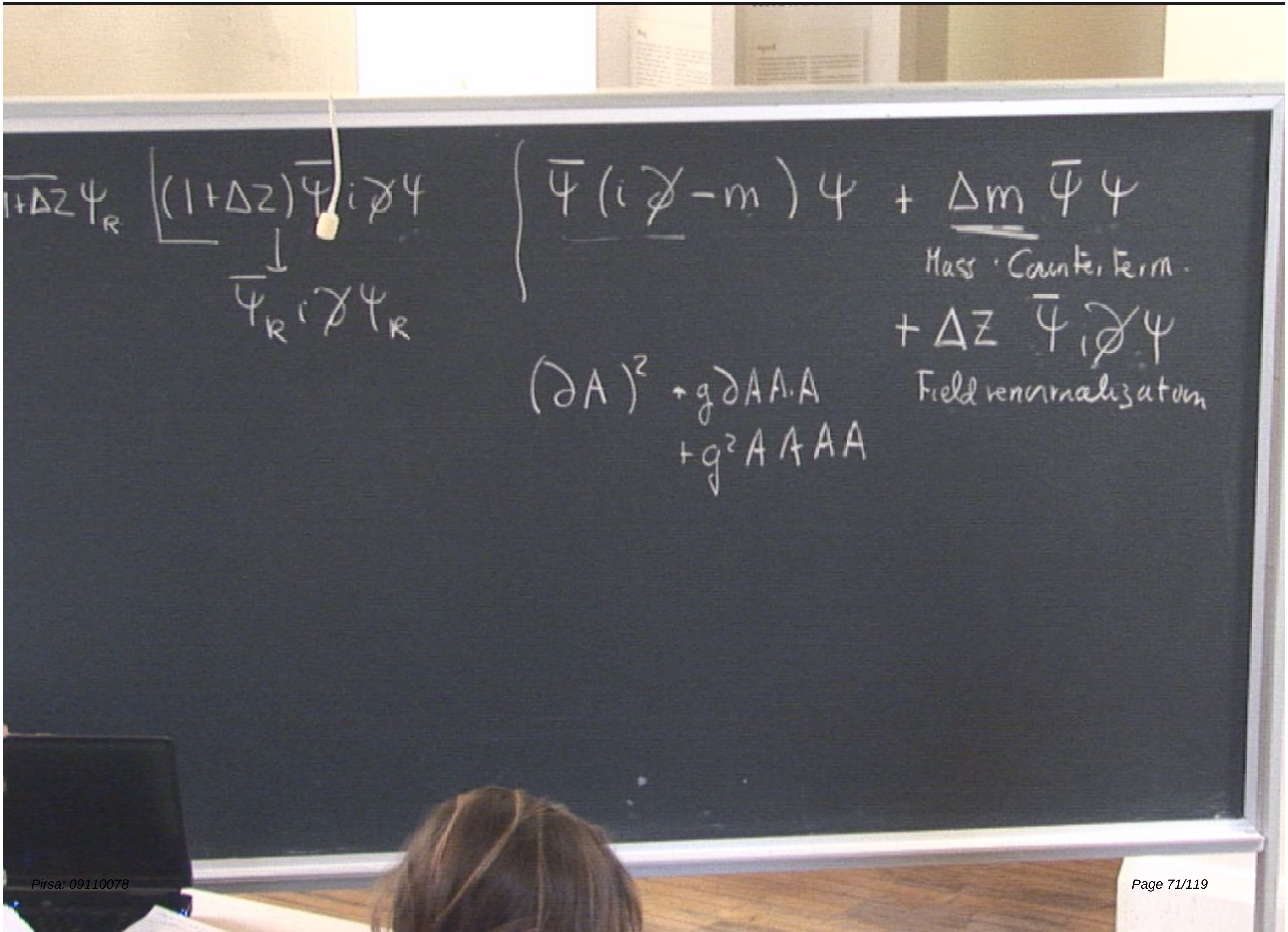


$$\overline{1+\Delta Z}\,\psi_R \qquad \underline{\left|(1+\Delta Z)\right.}\,\bar{\psi}\, i\partial\!\!\!/\,\psi \xrightarrow{} \bar{\psi}_R\, i\partial\!\!\!/\,\psi_R$$

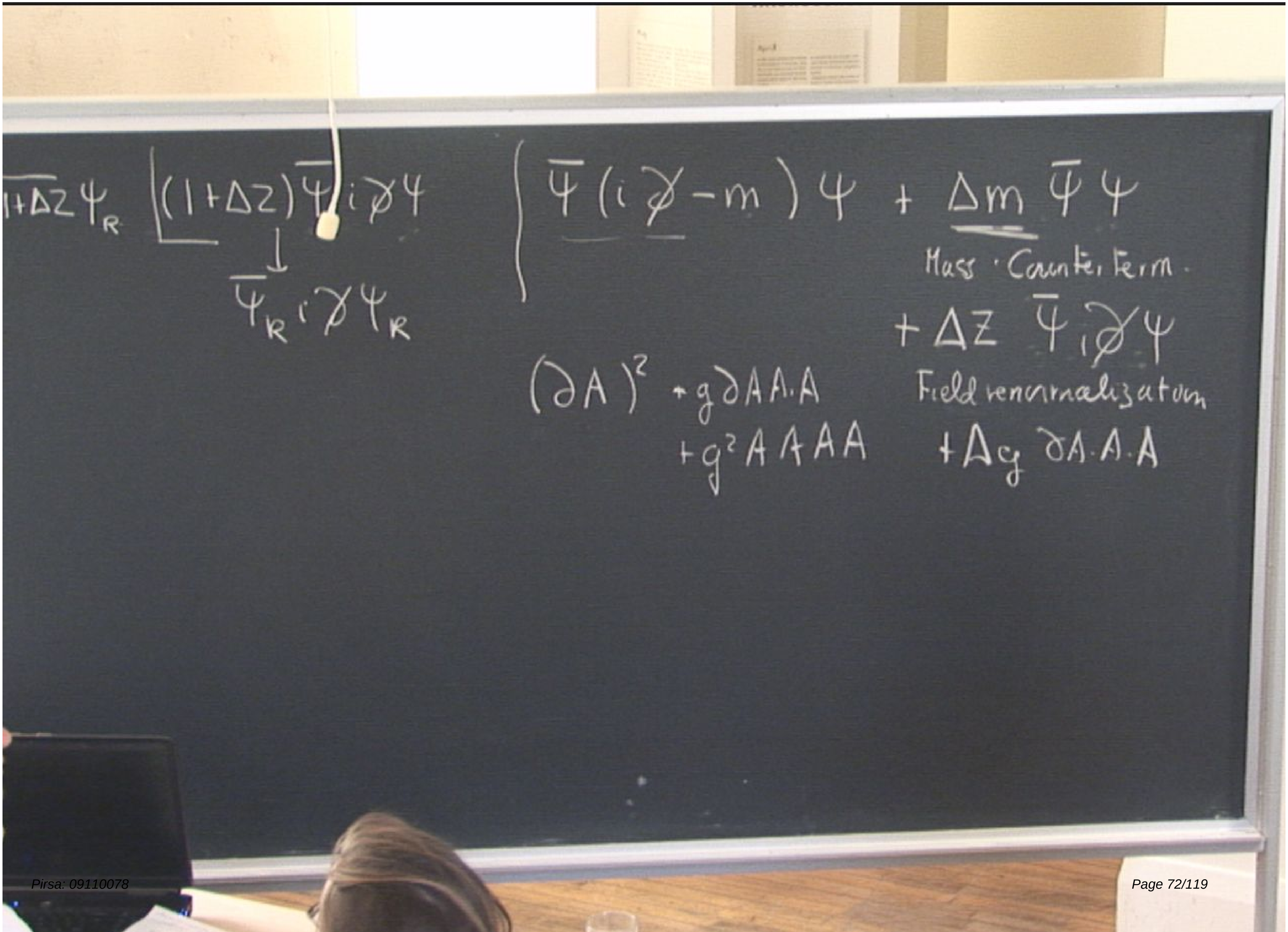
$$\int \underline{\bar{\psi}\,(i\partial\!\!\!/} - m)\,\psi + \underline{\Delta m}\,\bar{\psi}\psi$$

Mass · Counter term.

$$+\Delta Z\,\bar{\psi}\, i\partial\!\!\!/\,\psi$$

Field renormalization

$$(\partial A)^2 + g\,\partial A A\cdot A + g^2 AAAA$$



$\overline{1+\Delta Z}\,\Psi_R$

$$(1+\Delta Z)\bar{\Psi}\, i\not{\partial}\Psi \;\Big\downarrow\; \bar{\Psi}_R\, i\not{\partial}\,\Psi_R$$

$$\int \underline{\bar{\Psi}\,(i\underline{\not{\partial}}-m)\,\Psi} + \underline{\Delta m}\,\bar{\Psi}\Psi$$

Mass · Counter term.

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Field renormalization

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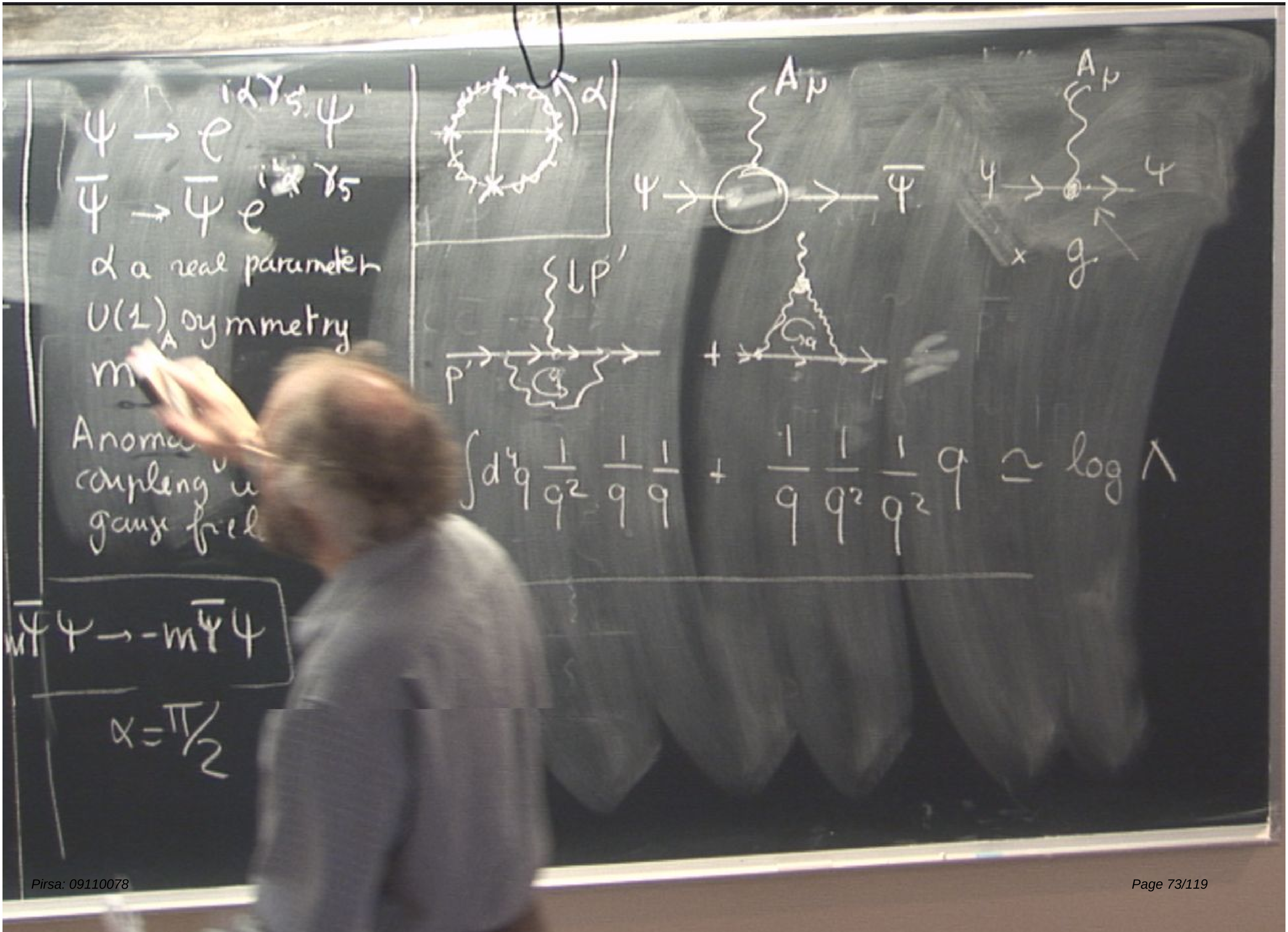

$\psi \to e^{i\alpha\gamma_5}\psi$

$\bar\psi \to \bar\psi e^{i\alpha\gamma_5}$

α a real parameter

$U(1)_A$ symmetry

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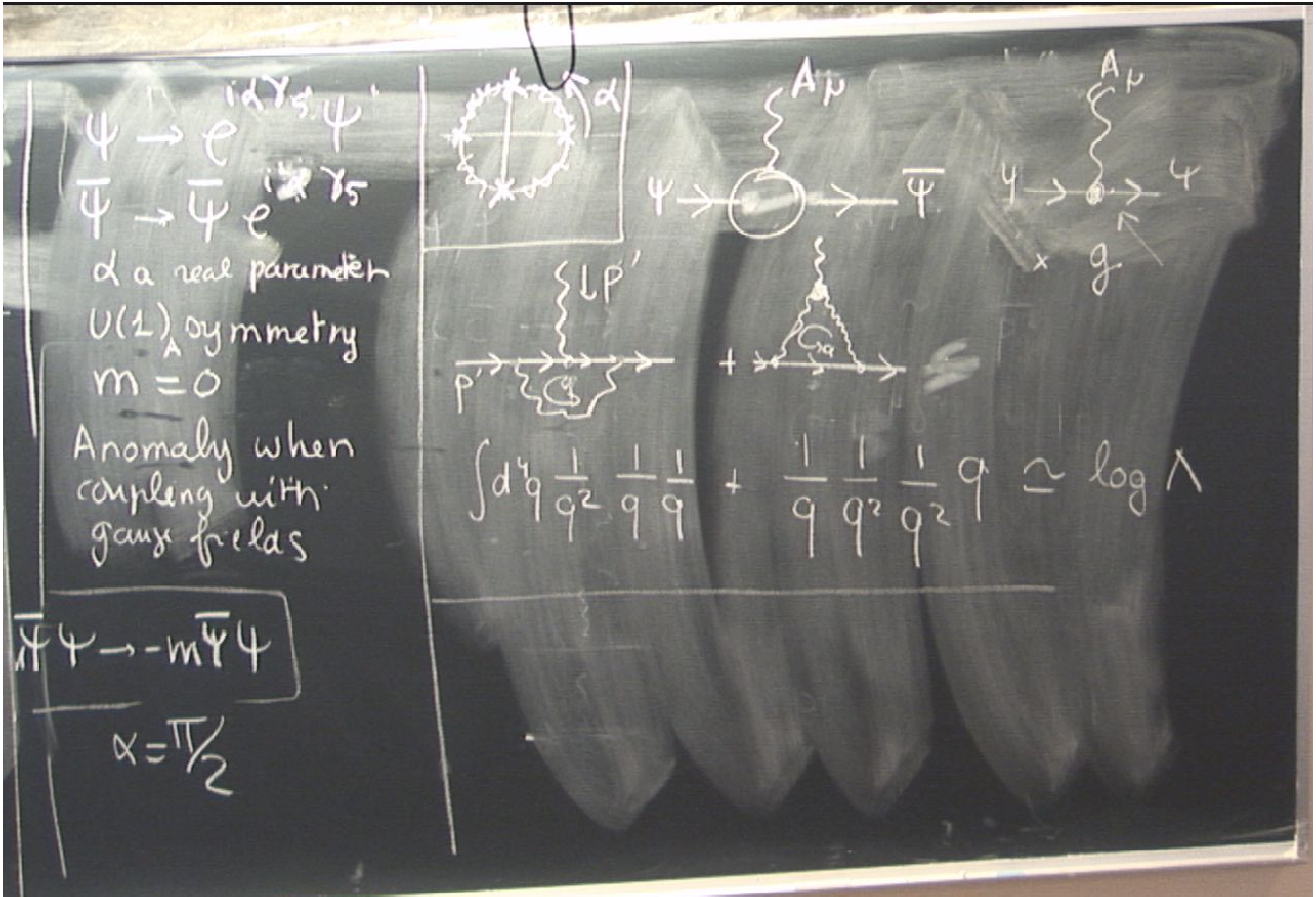

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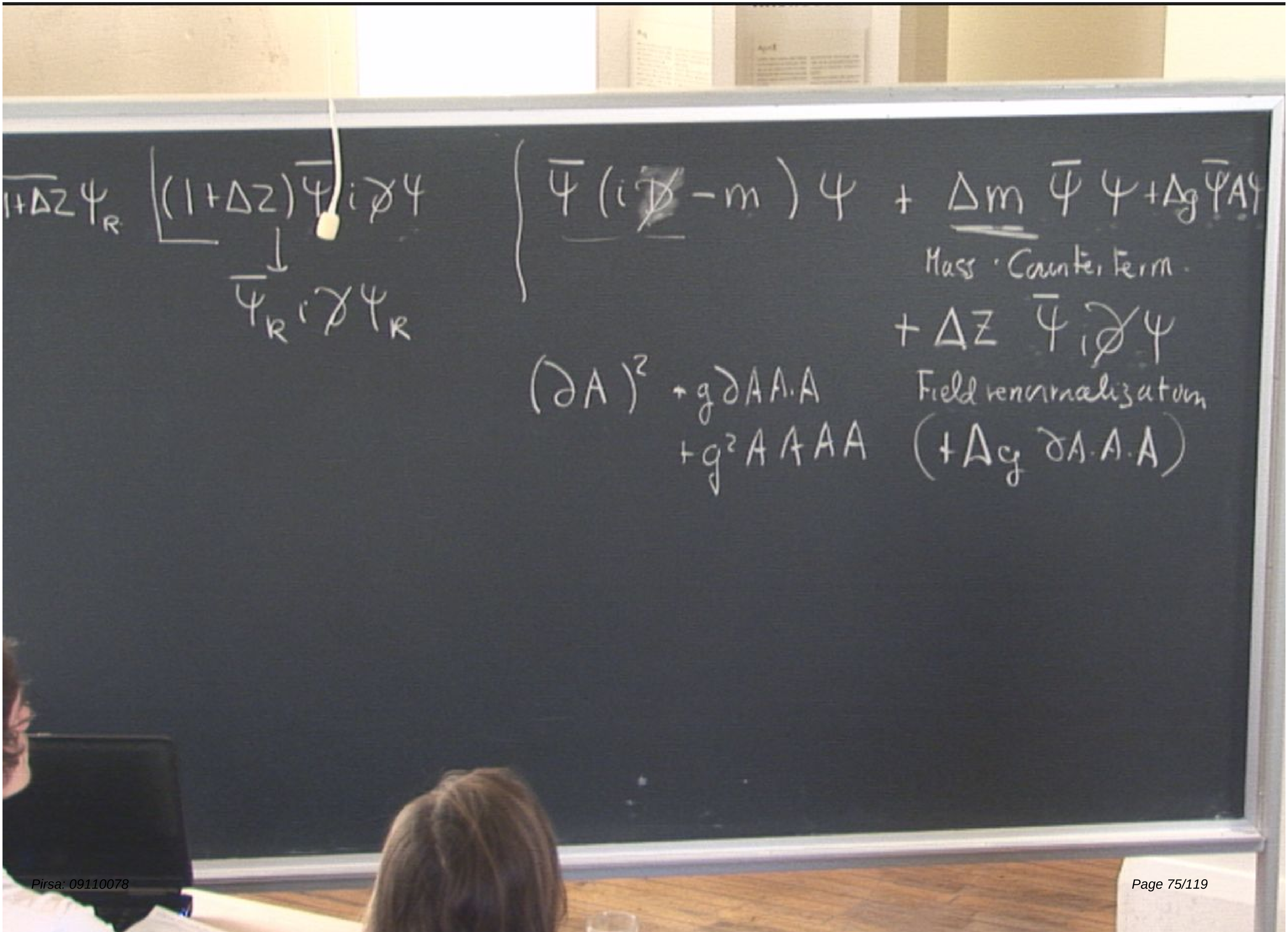


$\overline{1+\Delta Z}\,\psi_R$

$(1+\Delta Z)\,\bar\psi\, i\not\partial\psi$

$\downarrow$

$\bar\psi_R\, i\not\partial\,\psi_R$

$\int \bar\psi\,(i\not D - m)\,\psi + \underline{\Delta m}\,\bar\psi\psi + \Delta g\,\bar\psi A\psi$

Mass · Counter term ·

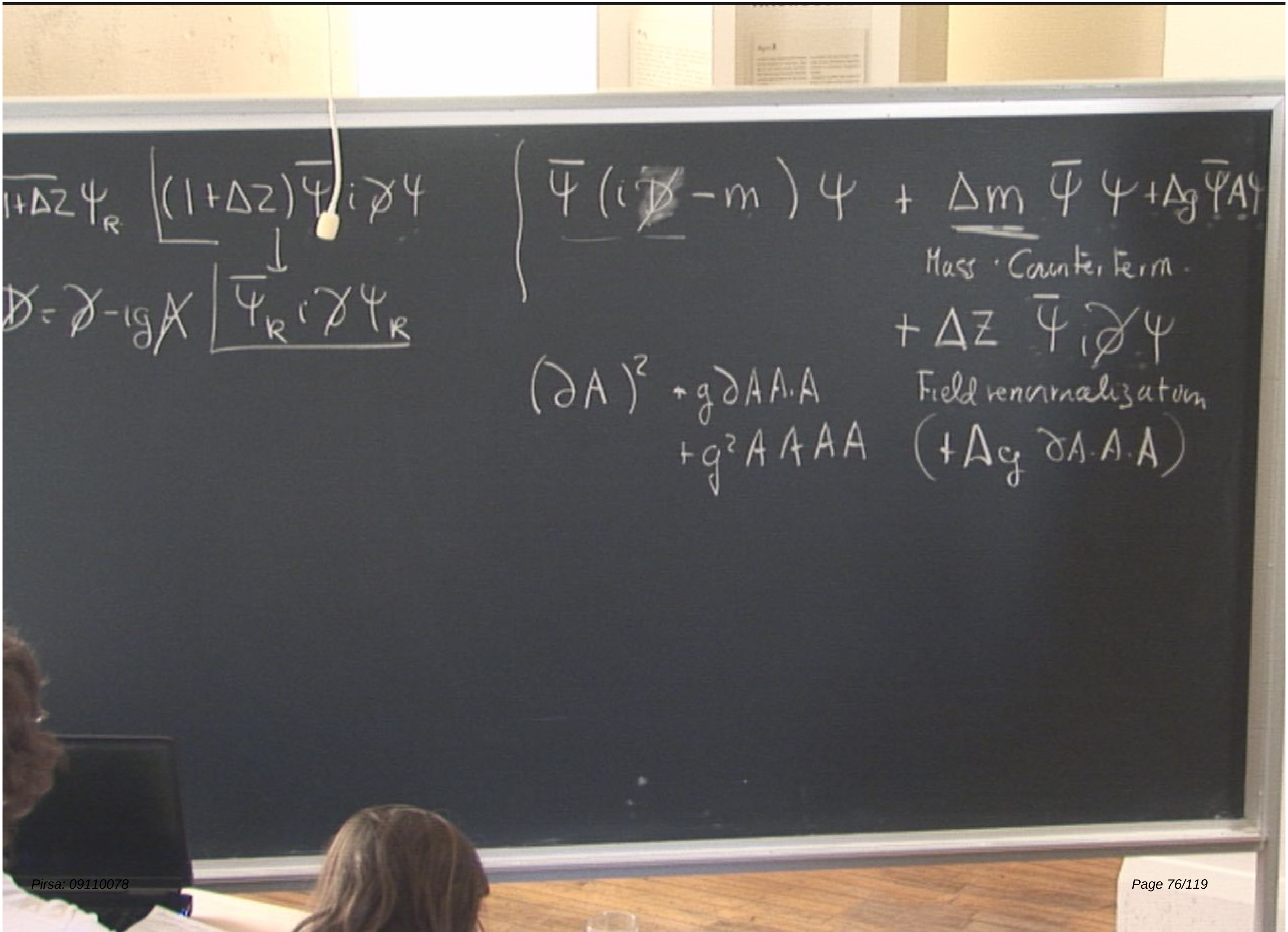
$+\Delta Z\,\bar\psi\, i\not\partial\,\psi$

Field renormalization

$(\partial A)^2 + g\,\partial A A\cdot A$

$+ g^2 AAAA$

$(+\Delta g\,\partial A\cdot A\cdot A)$



$\overline{1+\Delta Z}\,\psi_R$

$$\boxed{(1+\Delta Z)\bar{\psi}\, i\not{\partial}\psi} \;\downarrow\; \boxed{\bar{\psi}_R\, i\not{\partial}\psi_R}$$

$\not{D} = \not{\partial} - ig\not{A}$

$$\int \bar{\psi}(i\not{D} - m)\psi + \underline{\Delta m}\,\bar{\psi}\psi + \Delta g\,\bar{\psi}A\psi$$

Mass Counter term.

$$+\Delta Z\,\bar{\psi}\, i\not{\partial}\psi$$

Field renormalization

$$(\partial A)^2 + g\,\partial A A\cdot A + g^2 AAAA \quad (+\Delta g\,\partial A\cdot A\cdot A)$$

$\overline{1+\Delta Z}\,\psi_R$

$(1+\Delta Z)\,\bar{\psi}\, i\,\not{\partial}\,\psi$

$\downarrow$

$\not{D} = \not{\partial} - ig\not{A}$

$\bar{\psi}_R\, i\,\not{\partial}\,\psi_R$

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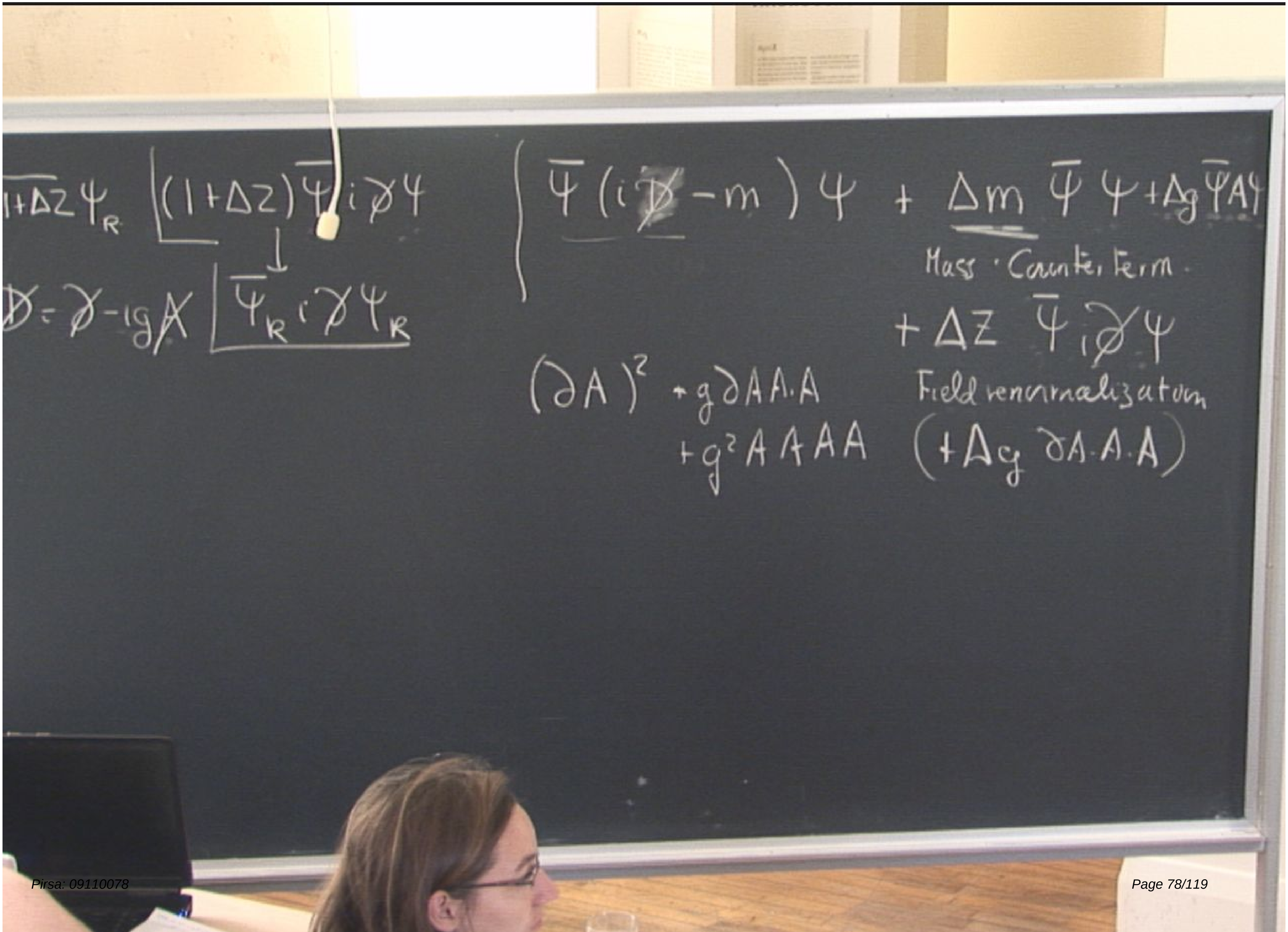

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$\downarrow$

$\bar{\psi}_R\, i\partial\!\!\!/\psi_R$

$D\!\!\!/ = \partial\!\!\!/ - igA\!\!\!/$

$\bar{\psi}(iD\!\!\!/ - m)\psi + \underline{\Delta m}\,\bar{\psi}\psi + \Delta g\,\bar{\psi}A\psi$

Mass Counter term.

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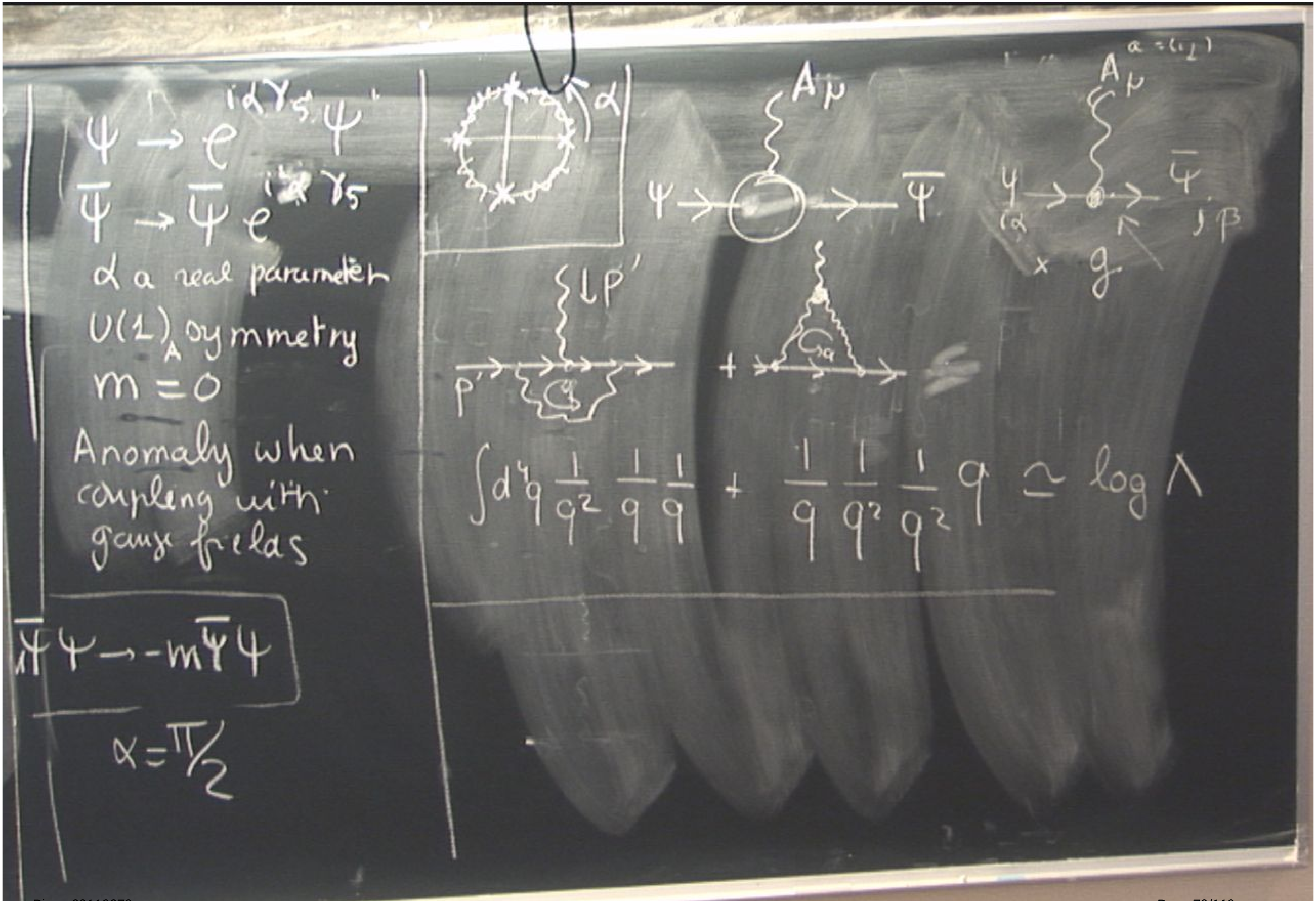
Field renormalization

$(\partial A)^2 + g\partial A A\cdot A$

$+g^2 AAAA$ $(+\Delta g\ \partial A\cdot A\cdot A)$



$\psi \to e^{i\alpha\gamma_5}\psi$

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α a real parameter

$U(1)_A$ symmetry

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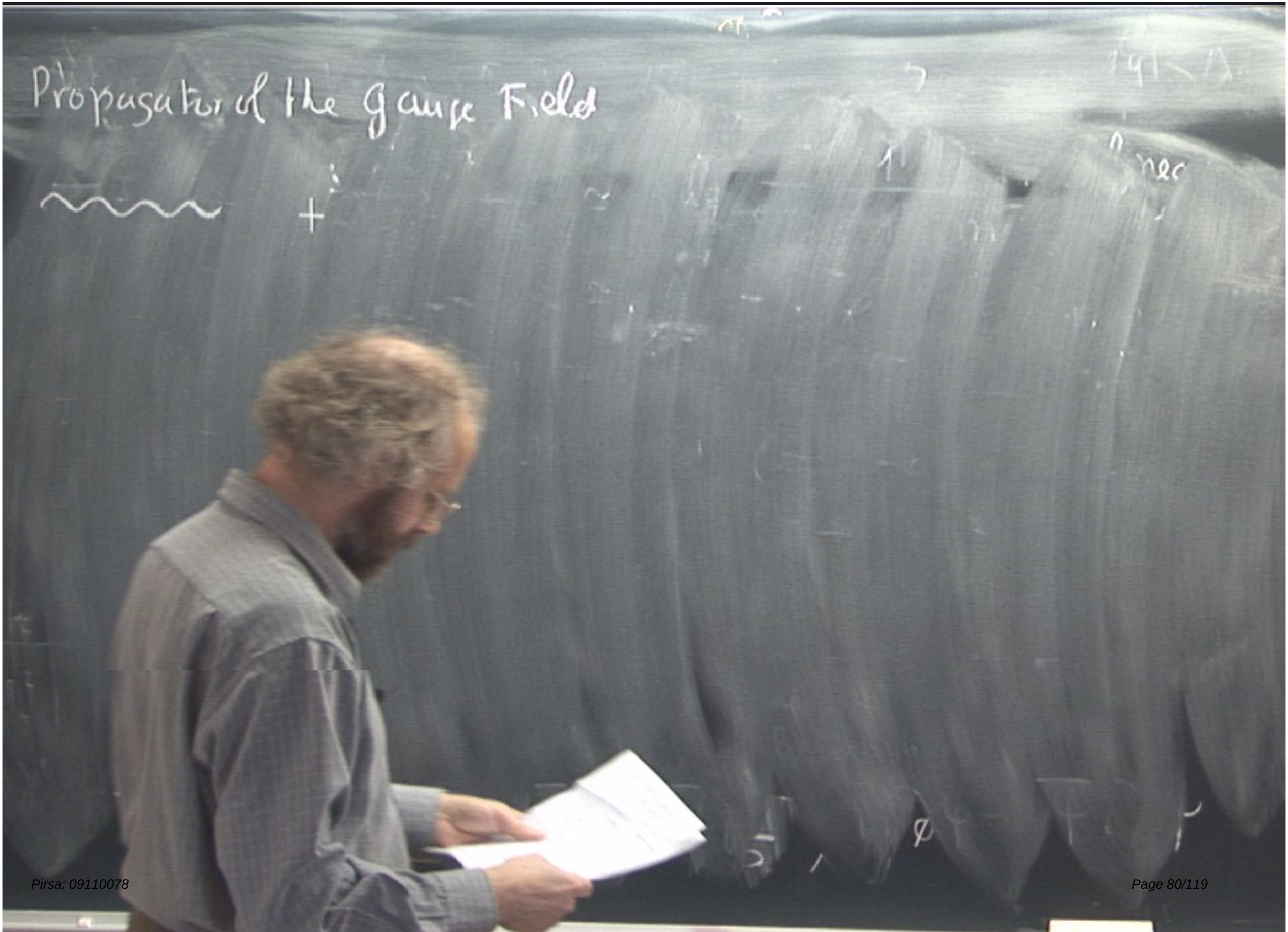
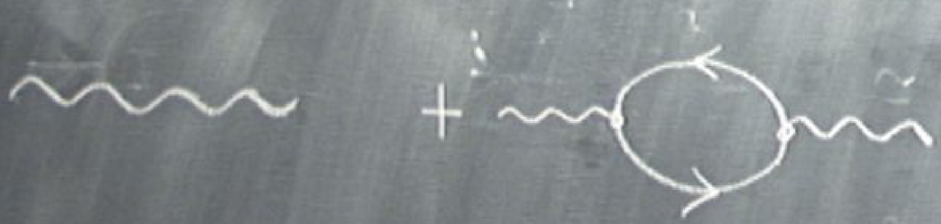


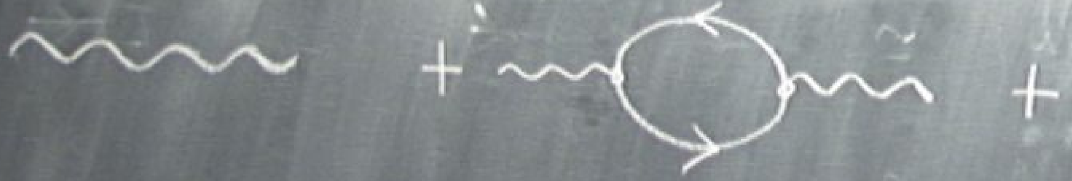
Propagator of the Gauge Field

Propagator of the gauge Field



Vaccum polarization diagram in QED

Propagator of the gauge Field



Vaccum polarization diagram in QED

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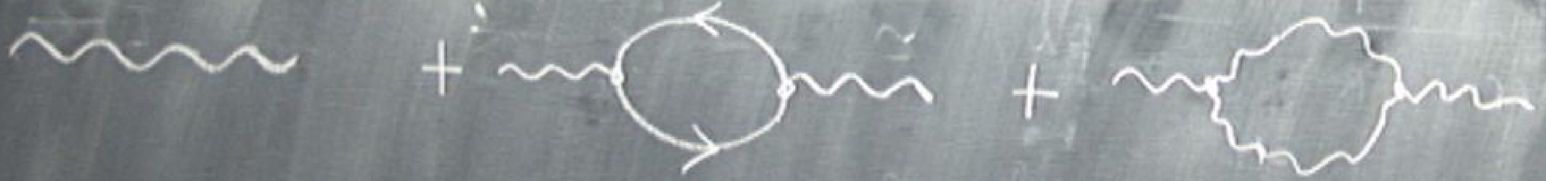
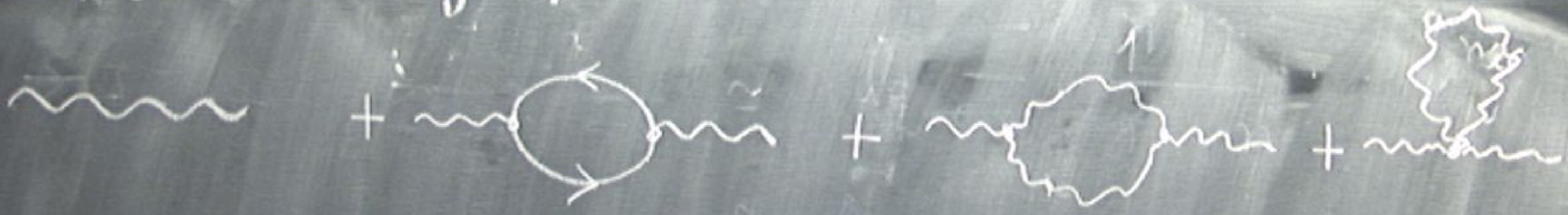


Vaccum polarization
diagram in QED

Propagator of the Gauge Field



Vaccum polarization
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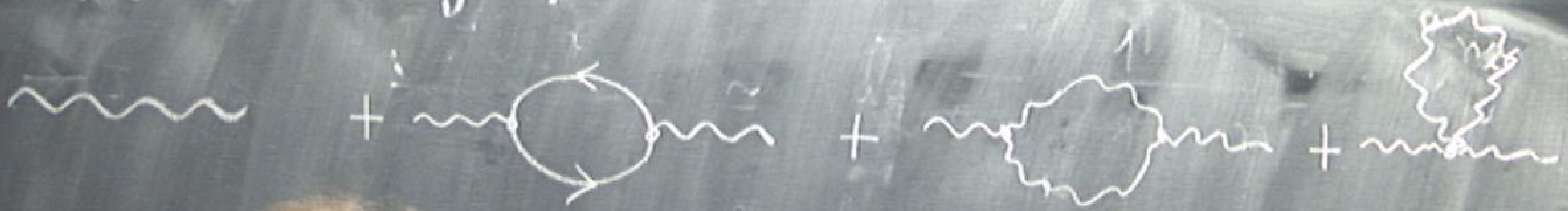
Propagator of the Gauge Field



Vaccum polarization
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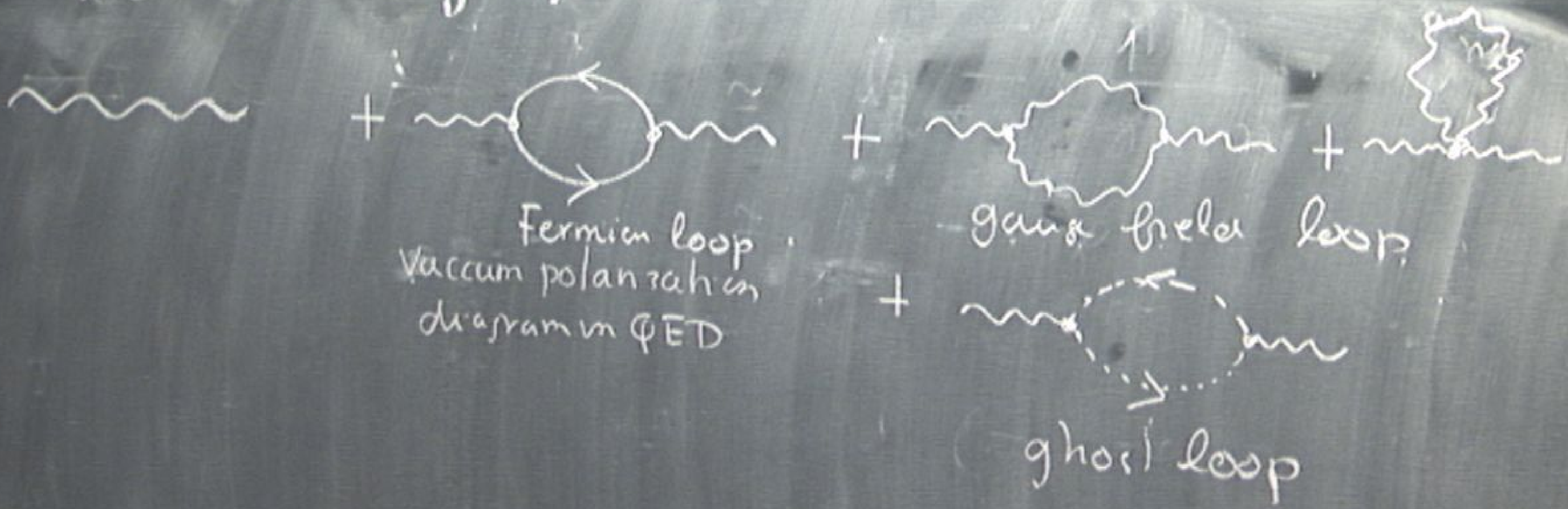
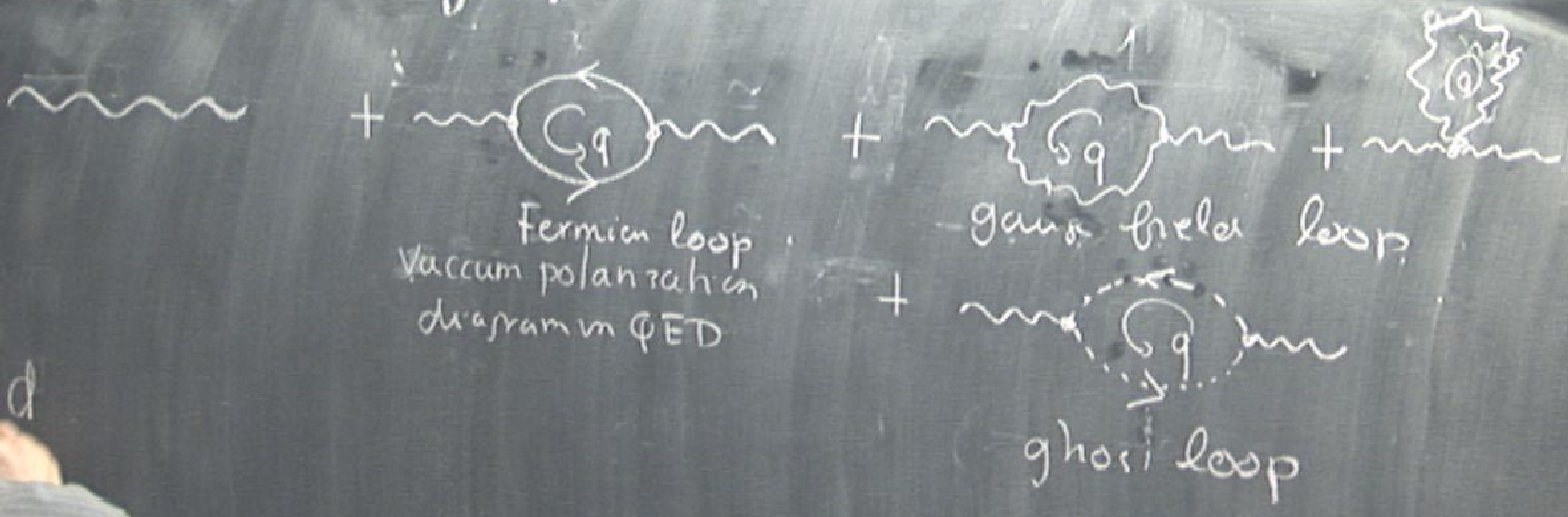


Propagator of the gauge Field

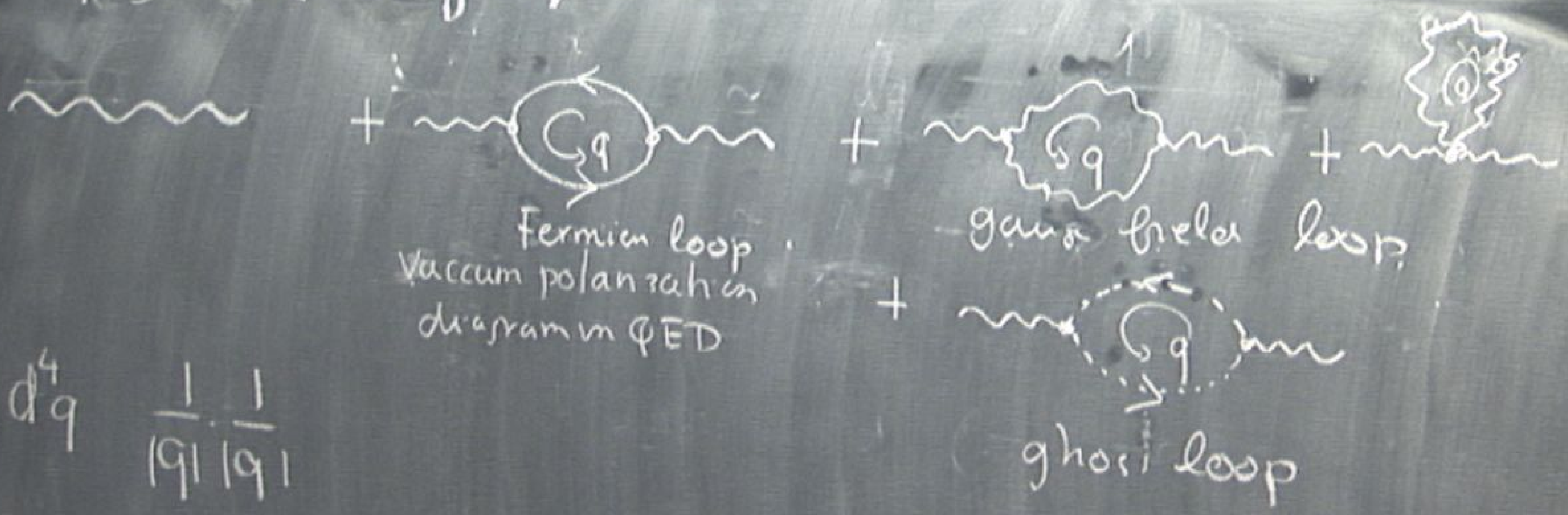


Fermion loop.
Vaccum polarization diagram in QED

gauge field loop

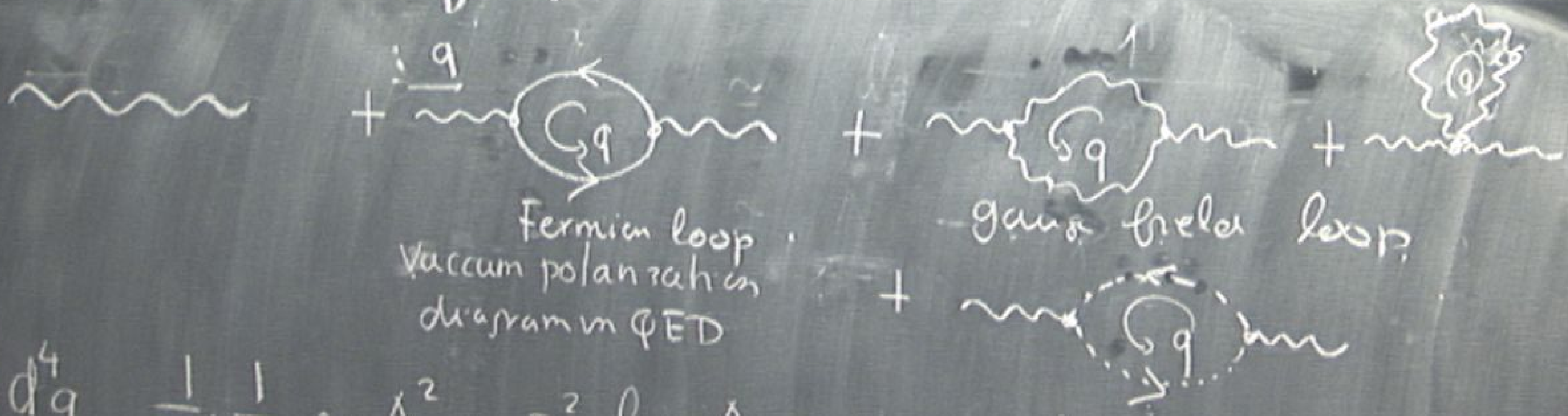
ghost loop

Propagator of the gauge Field



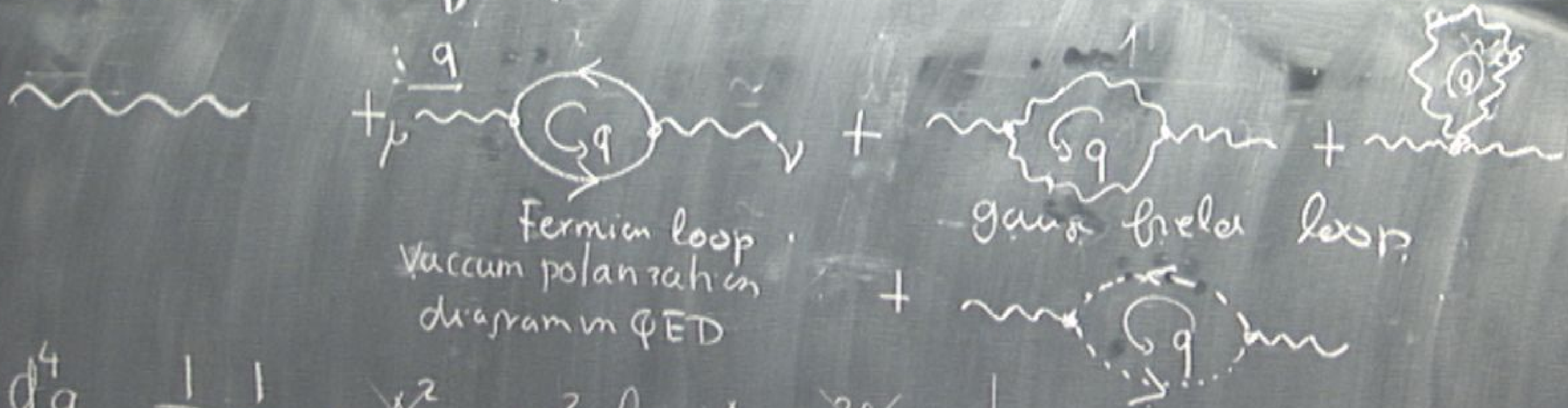
$$\int d^4q \quad \frac{1}{|q|}\cdot\frac{1}{|q|}$$

Propagator of the gauge Field



$$d^4q \quad \frac{1}{|q|}\cdot\frac{1}{|q|} \sim \Lambda^2 + p^2 \log \Lambda$$

Propagator of the gauge Field



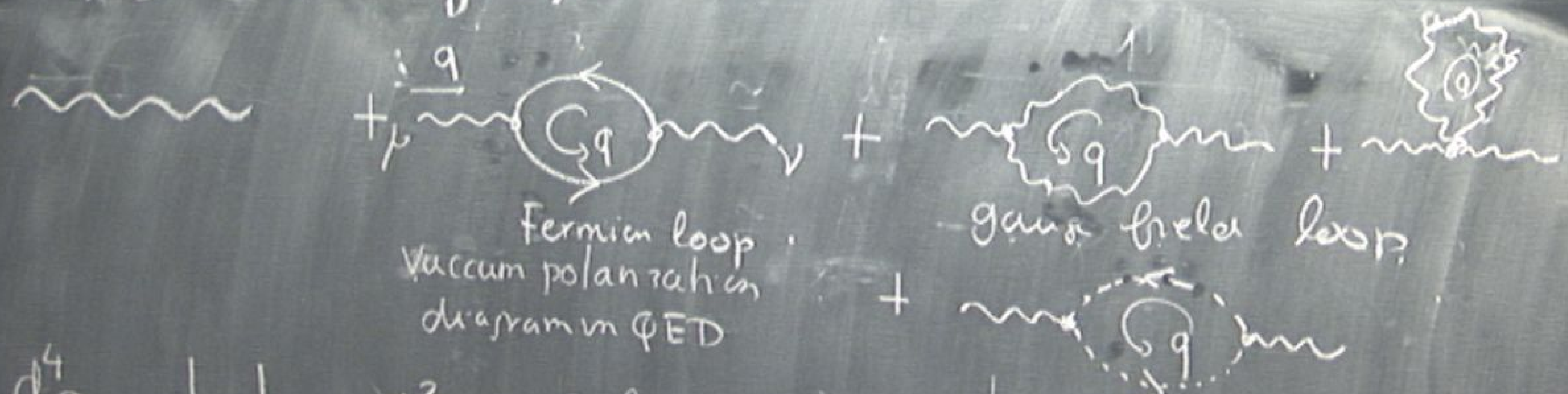
$$\int d^4q \, \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda^2} + \underbrace{p^2 \log\Lambda}_{} + \cancel{m^2\log\Lambda}$$

ghost loop

gauge symmetry U(1)

$|q| < \Lambda$

Propagator of the gauge Field



$$\int d^4q \quad \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log \Lambda}_{p^\mu p^\nu - g^{\mu\nu} p^2} + m^2 \cancel{\log \Lambda}$$

ghost loop

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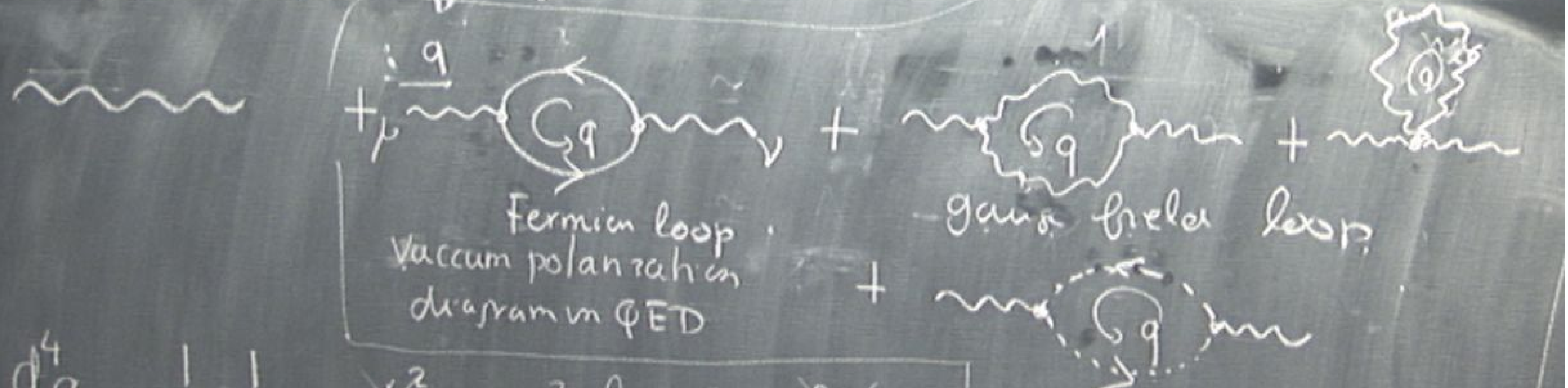
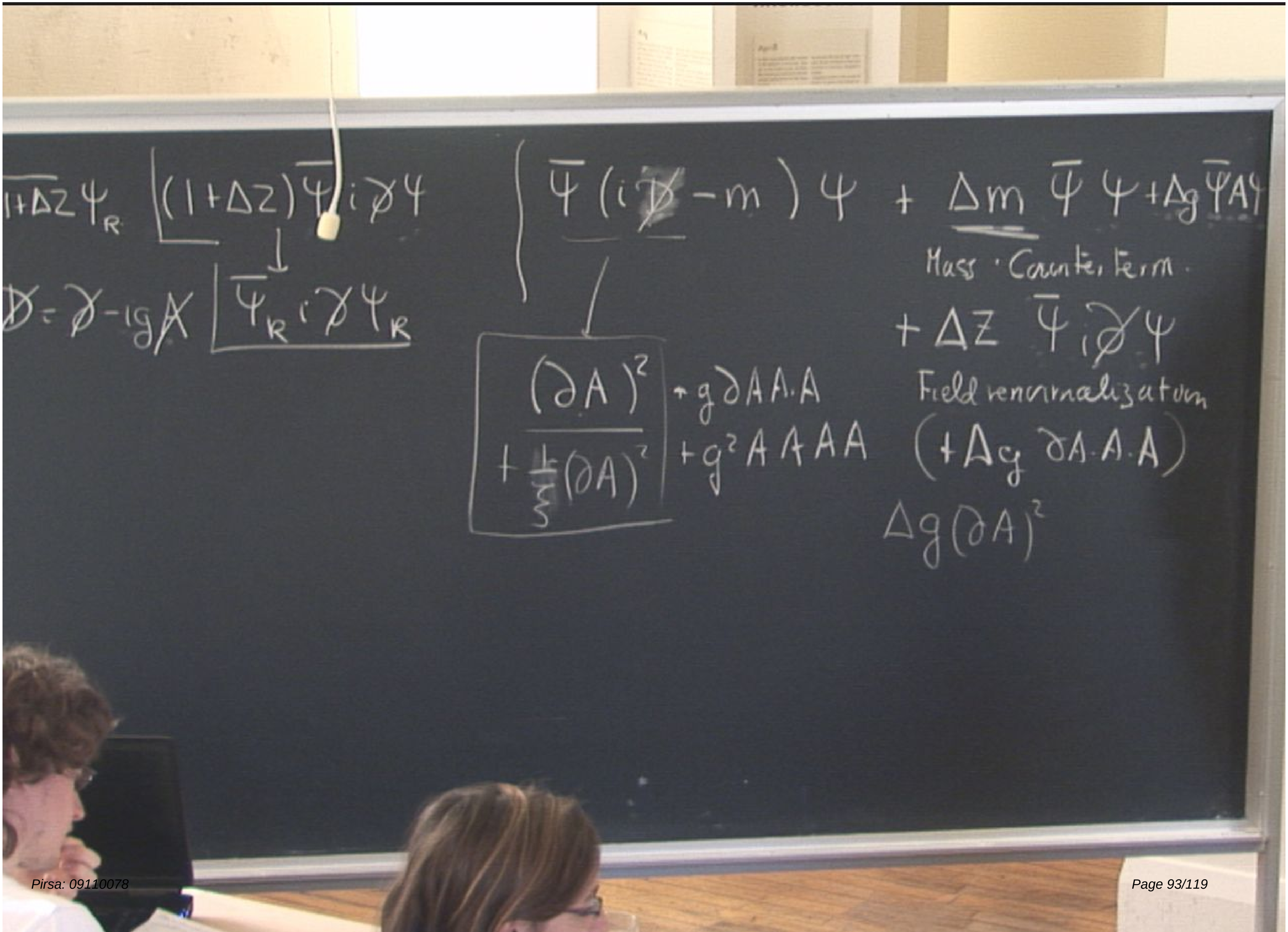
Propagator of the gauge Field



$$\int d^4q \; \frac{1}{|q|}\,\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log \Lambda}_{} + \cancel{m^2 \log\Lambda}$$

gauge symmetry U(1) $\quad p^\mu p^\nu - g^{\mu\nu} p^2$



$\overline{1+\Delta Z}\,\psi_R$

$\boxed{(1+\Delta Z)\bar\psi\, i\not\partial\psi}$

$\downarrow$

$\not D = \not\partial - ig\not A \quad \boxed{\bar\psi_R\, i\not\partial\,\psi_R}$

$$\bar\psi\,(i\not D - m)\,\psi + \underline{\Delta m}\,\bar\psi\,\psi + \Delta g\,\bar\psi \not A\psi$$

Mass · Counter term ·

$+\Delta Z\;\bar\psi\, i\not\partial\,\psi$

Field renormalization

$(+\Delta g\;\partial A\cdot A\cdot A)$

$\Delta g(\partial A)^2$

$$\boxed{(\partial A)^2 + \frac{1}{\xi}(\partial A)^2} \begin{array}{l} + g\,\partial A\,A\cdot A \\ + g^2 AAAA \end{array}$$

$$\psi \to e^{i\alpha\gamma_5}\psi$$

$$\bar\psi \to \bar\psi e^{i\alpha\gamma_5}$$

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Anomaly when coupling with gauge fields

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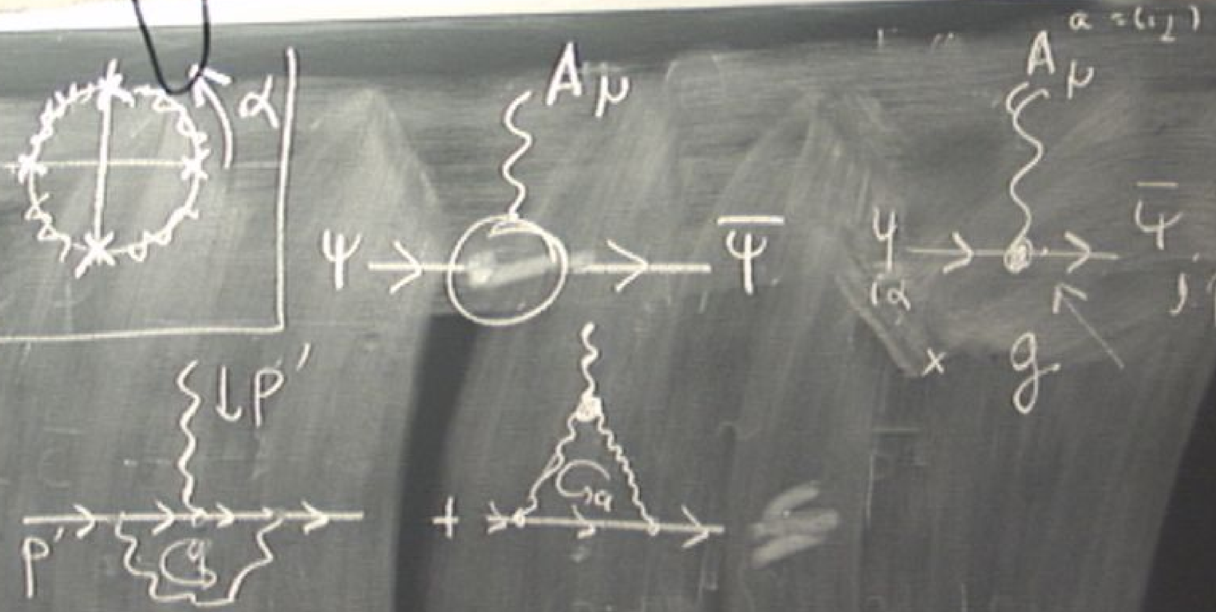
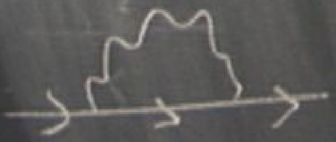
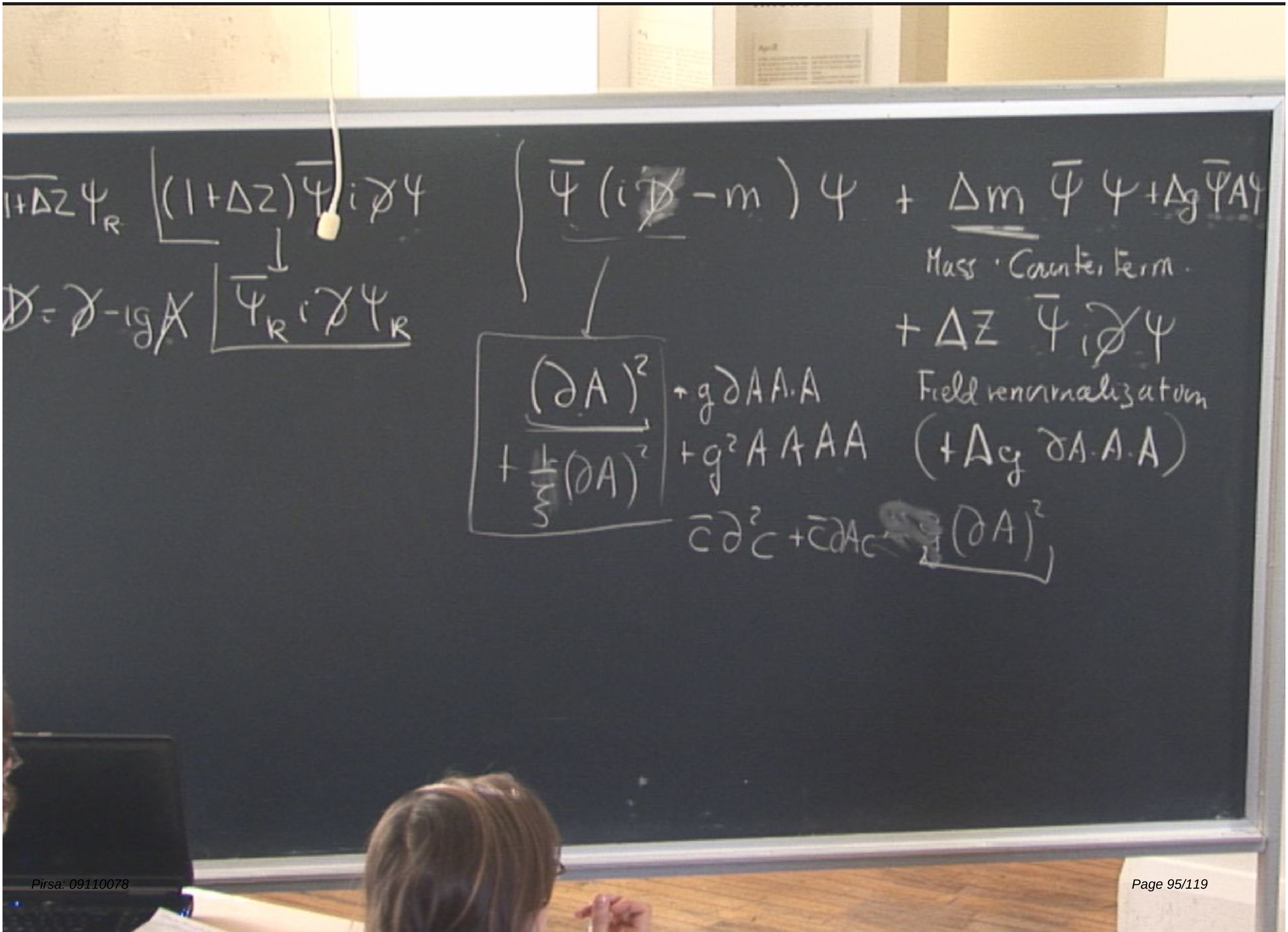


$$\int d^4q \frac{1}{q^2}\frac{1}{q}\frac{1}{q} + \frac{1}{q}\frac{1}{q^2}\frac{1}{q^2}q \simeq \log\Lambda$$





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$\boxed{\underline{(\partial A)^2} + \frac{1}{\xi}(\partial A)^2}$

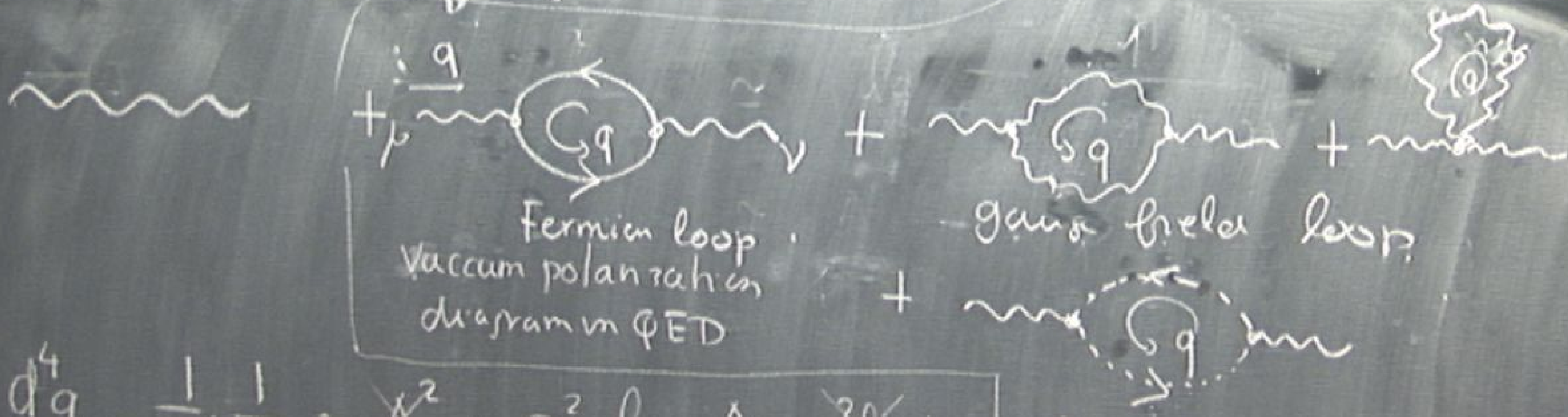
$+ g\,\partial A A\cdot A$

$+ g^2 AAAA$

$(+\Delta g\,\partial A\cdot A\cdot A)$

$\bar{c}\partial^2 c + \bar{c}\partial A c$ [illegible] $(\partial A)^2$

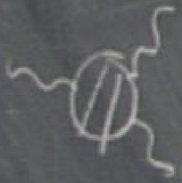
Propagator of the gauge Field



$$\int d^4q \; \frac{1}{|q|}\,\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log\Lambda} + m^2 \cancel{\log\Lambda}$$

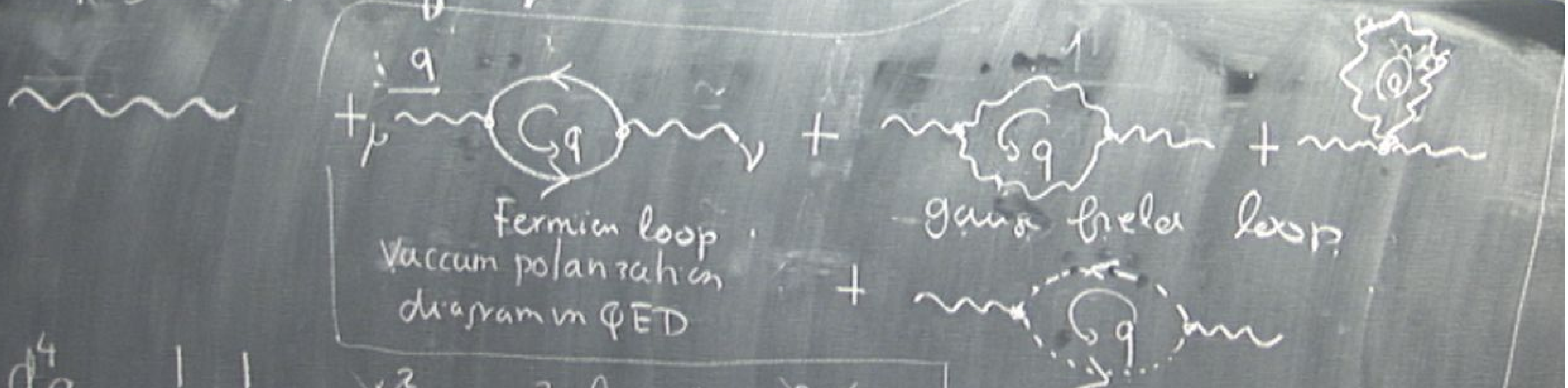
ghost loop

gauge symmetry U(1) $p^\mu p^\nu - g^{\mu\nu} p^2$



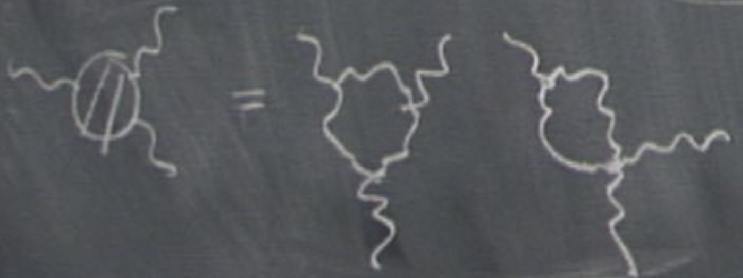
$|q| < \Lambda$

Propagator of the gauge Field



$$\int d^4q \; \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log \Lambda}_{} + m^2 \cancel{\log}\Lambda$$

gauge symmetry U(1) $\quad p^\mu p^\nu - g^{\mu\nu} p^2$



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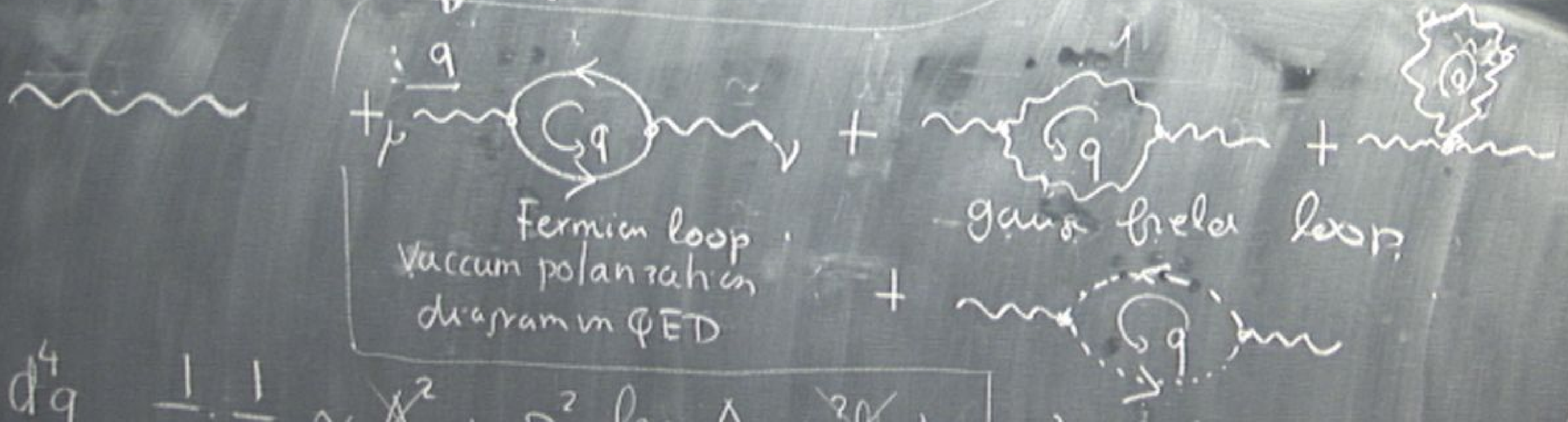

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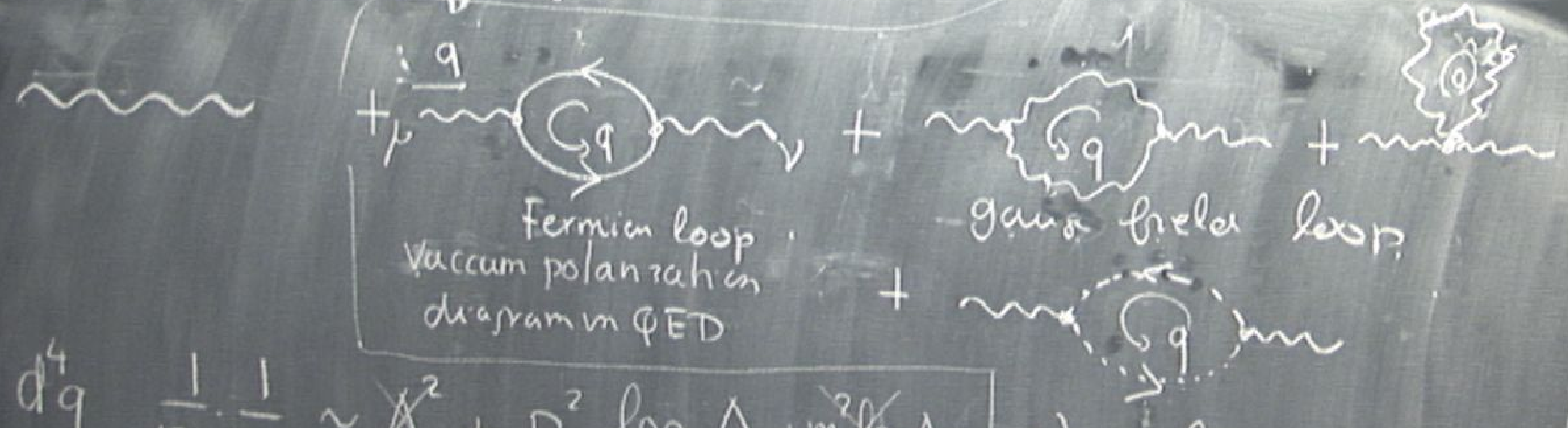
gauge symmetry U(1) $\quad p^\mu p^\nu - g^{\mu\nu} p^2$

Propagator of the gauge Field



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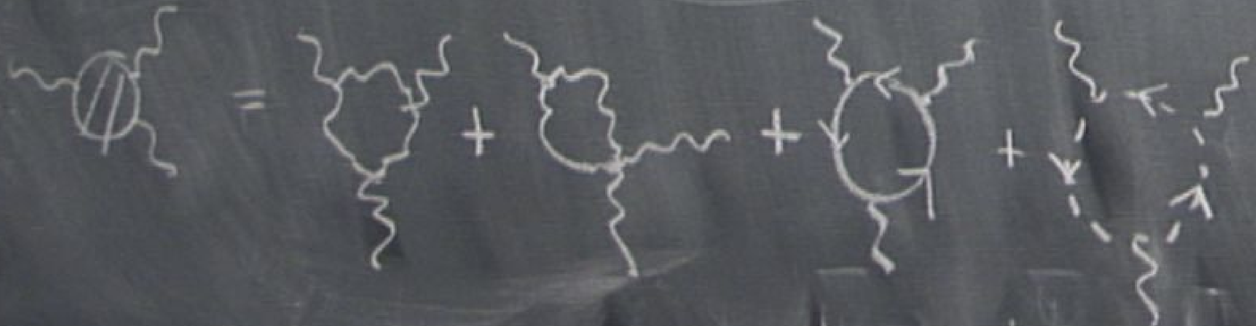

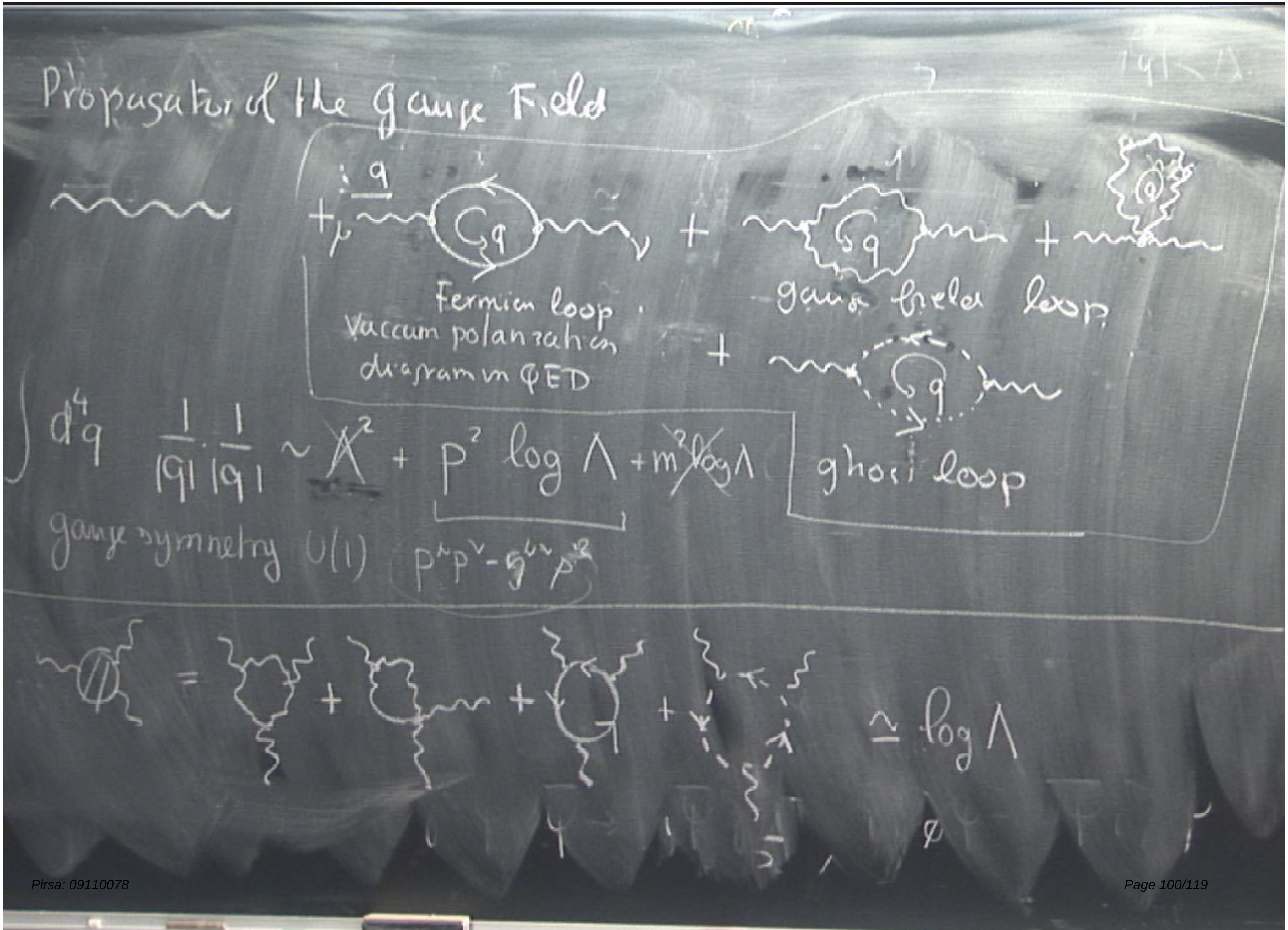

Propagator of the gauge Field

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Fermion loop

Vaccum polarization diagram in QED

gauge field loop

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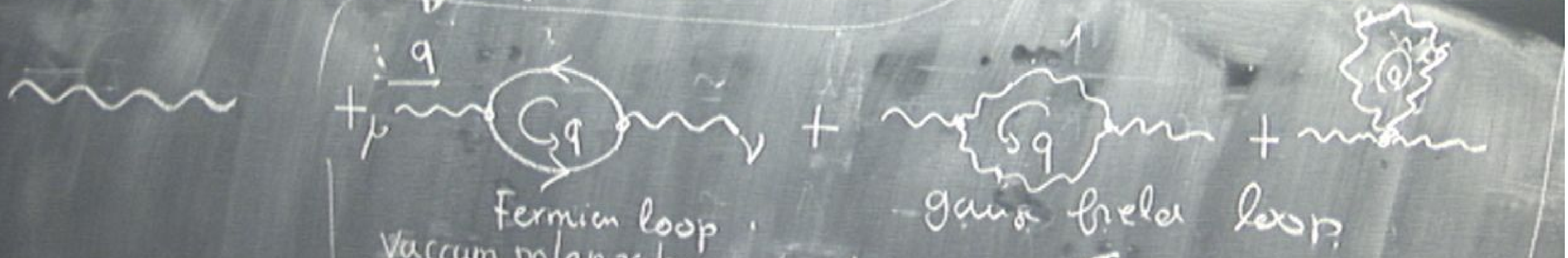
$$= \quad + \quad + \quad + \quad \simeq \log\Lambda$$

Propagator of the gauge Field

$|q| < \Lambda$

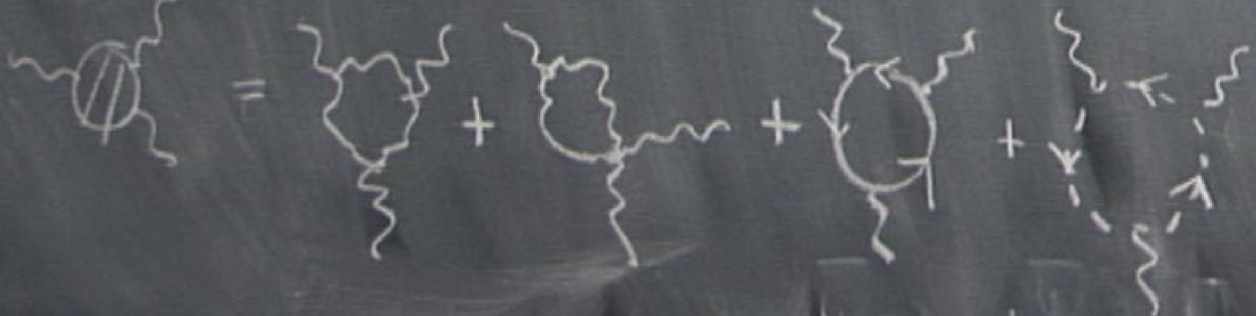


Vaccum polarization diagram in QED

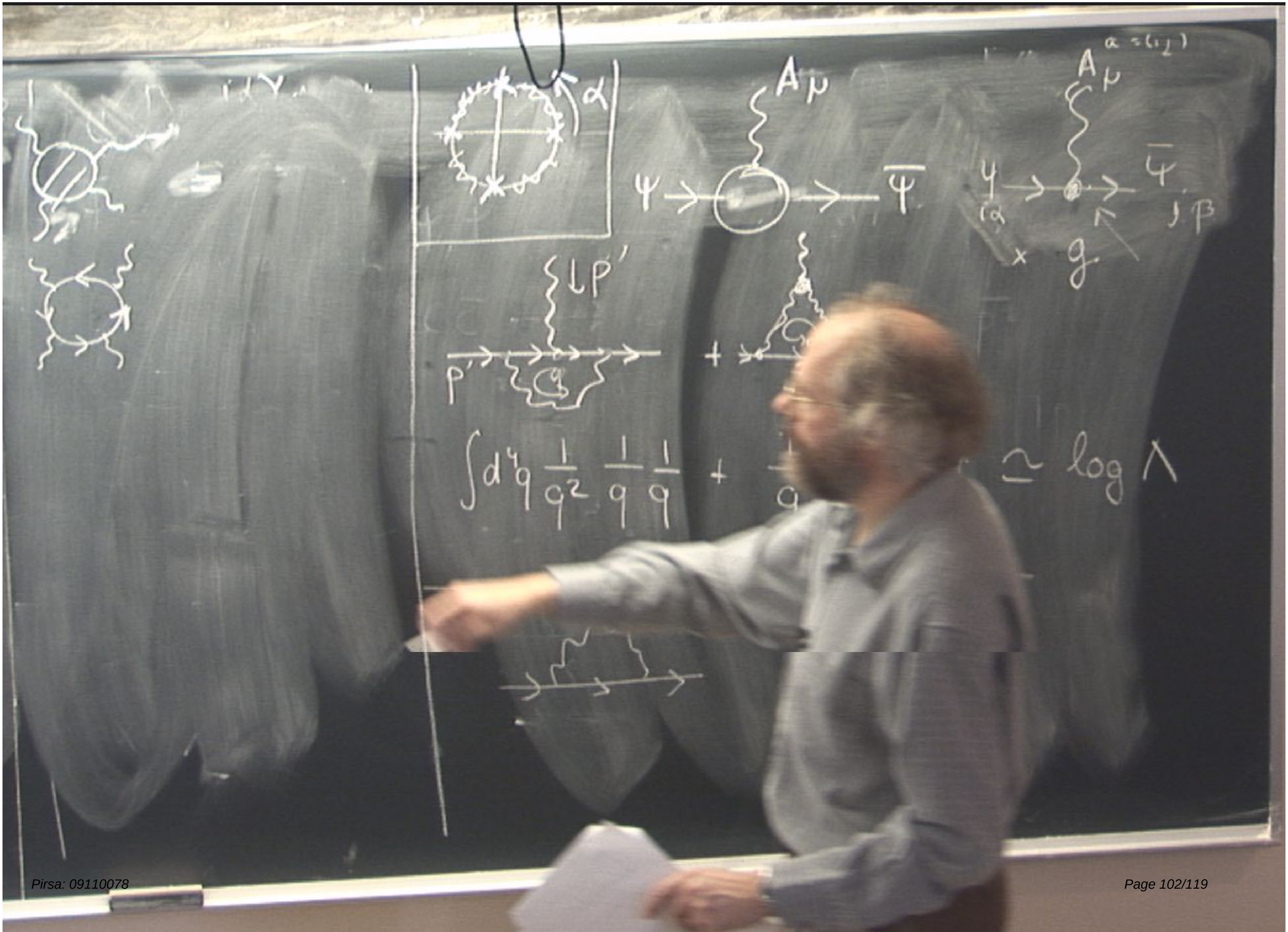
ghost loop

$$\int d^4q \, \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda^2} + \underbrace{p^2 \log\Lambda}_{} + m^2 \cancel{\log}\Lambda$$

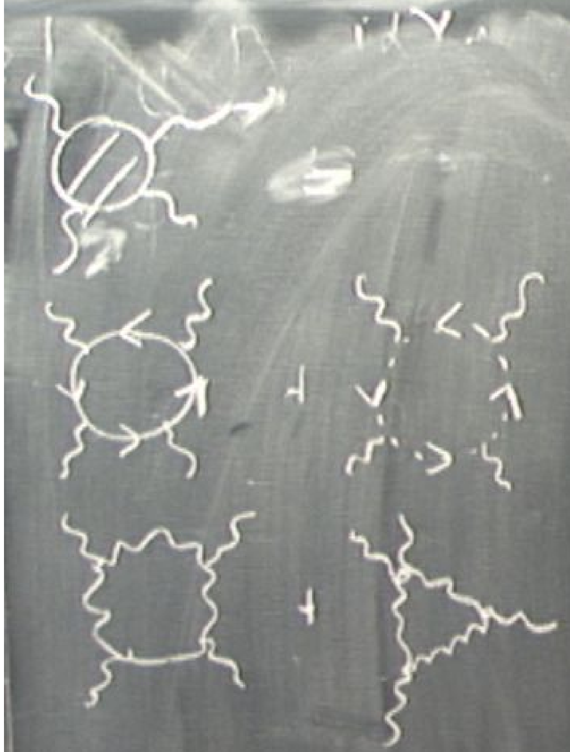
gauge symmetry $U(1)$ $p^\mu p^\nu - g^{\mu\nu} p^2$

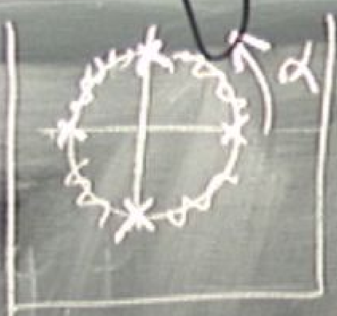


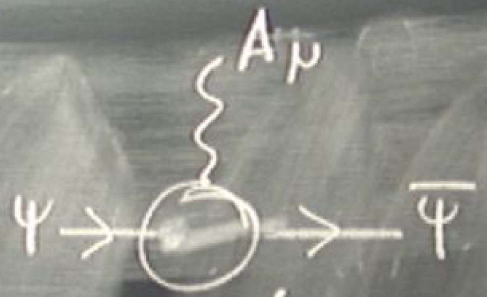
$$\simeq \log\Lambda \, \partial A \cdot A \cdot A$$

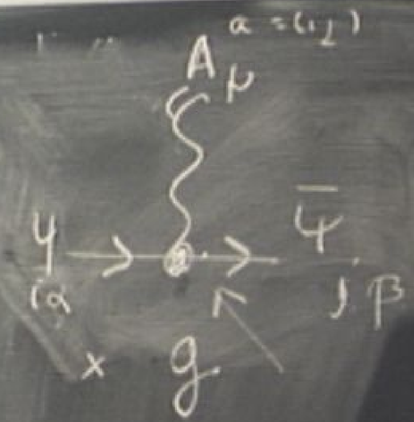


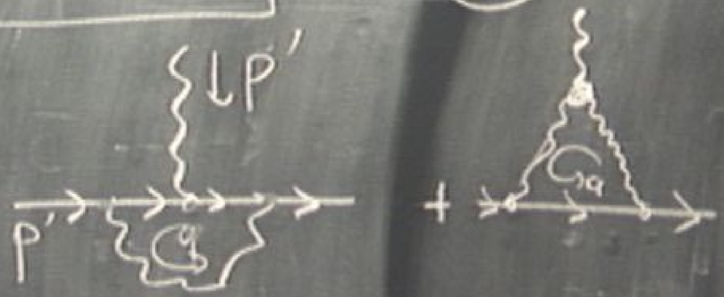
α
A_μ
ψ
Ψ̄
α = (1,2)
α
β
x
g
p'
p'
q
∫d^4q 1/q^2 1/q 1/q + … ≃ log Λ



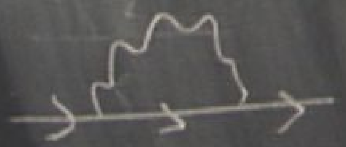


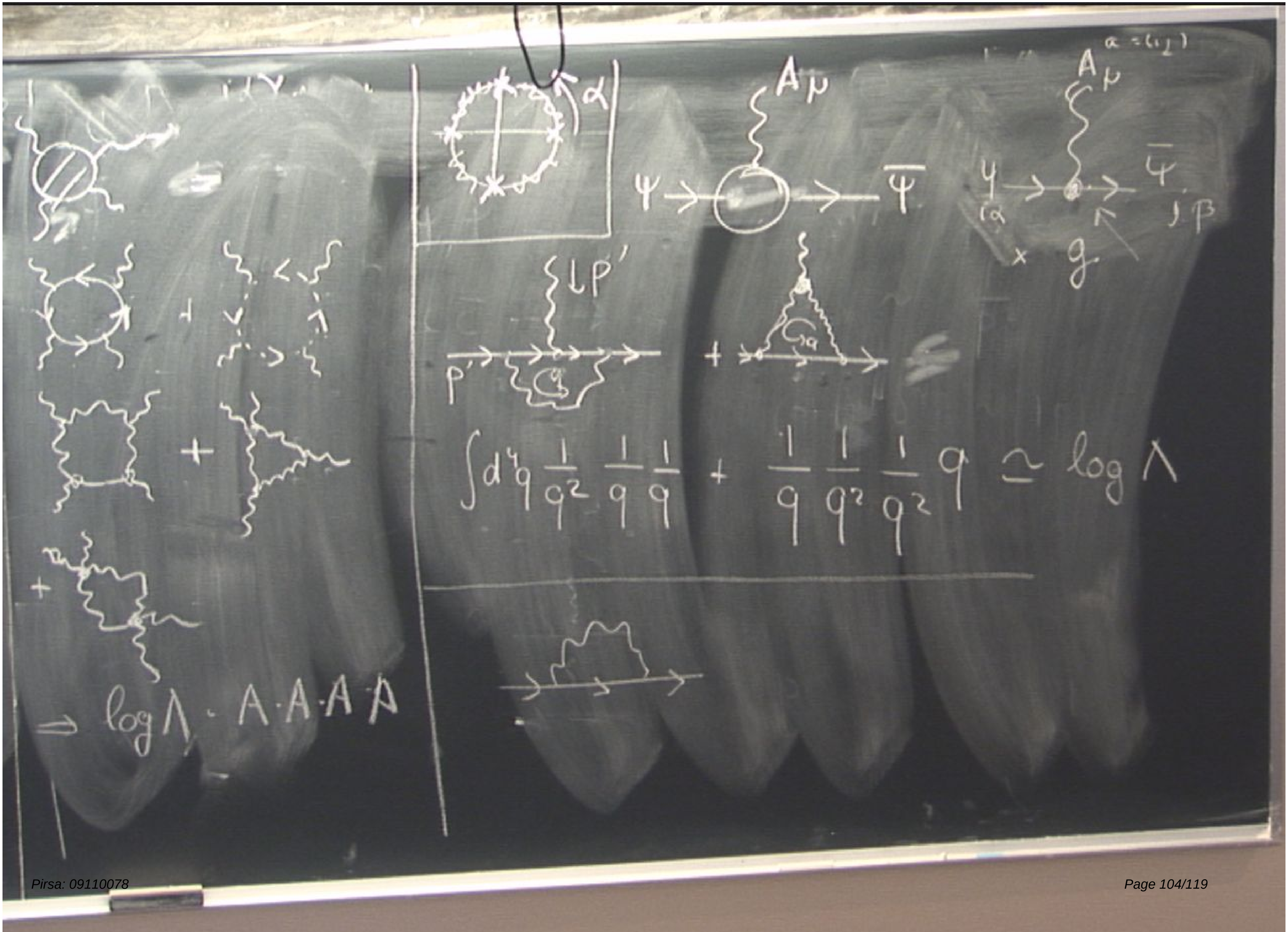






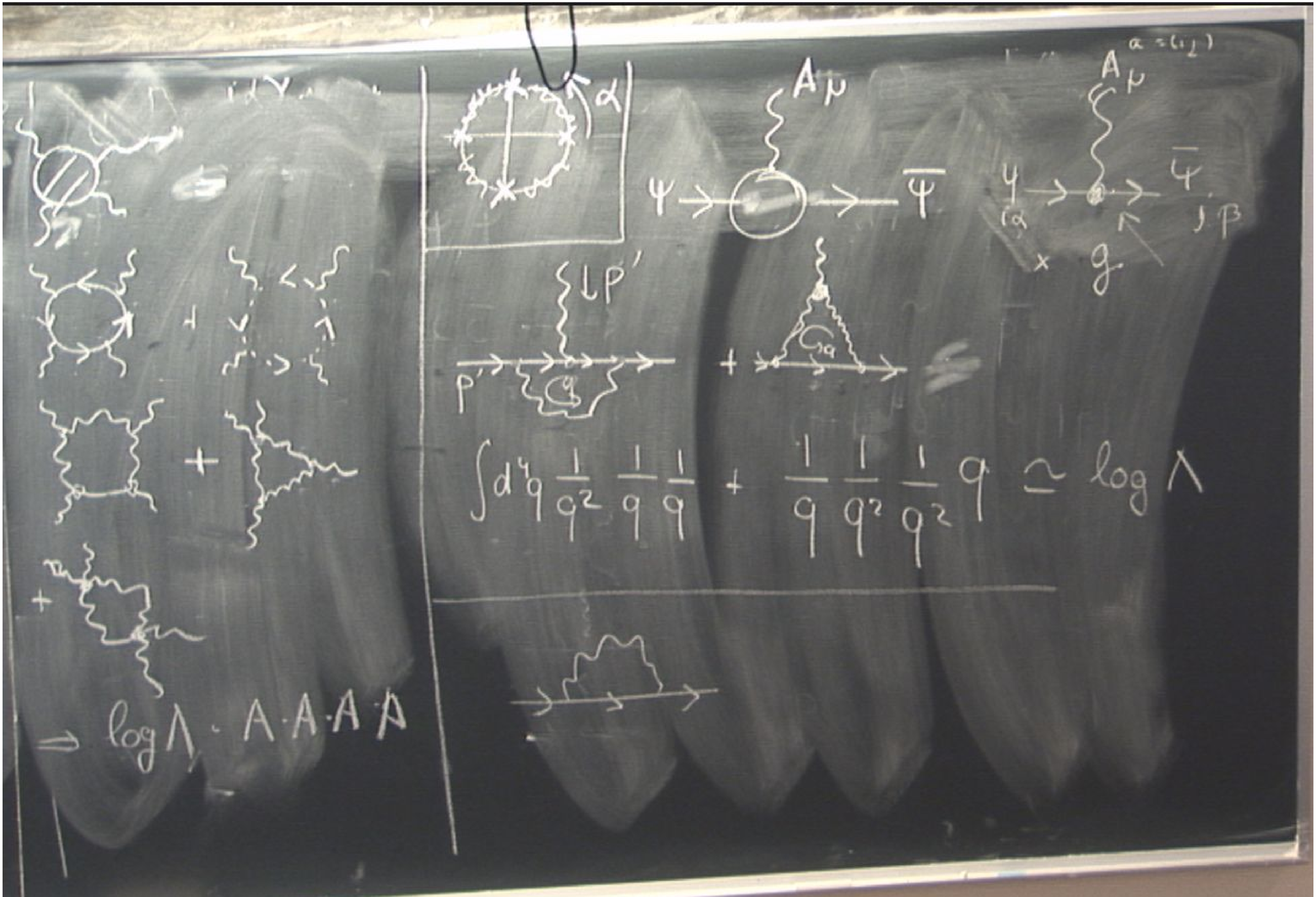
$$\int d^4q \frac{1}{q^2}\frac{1}{q}\frac{1}{q} + \frac{1}{q}\frac{1}{q^2}\frac{1}{q^2}\, q \simeq \log \Lambda$$





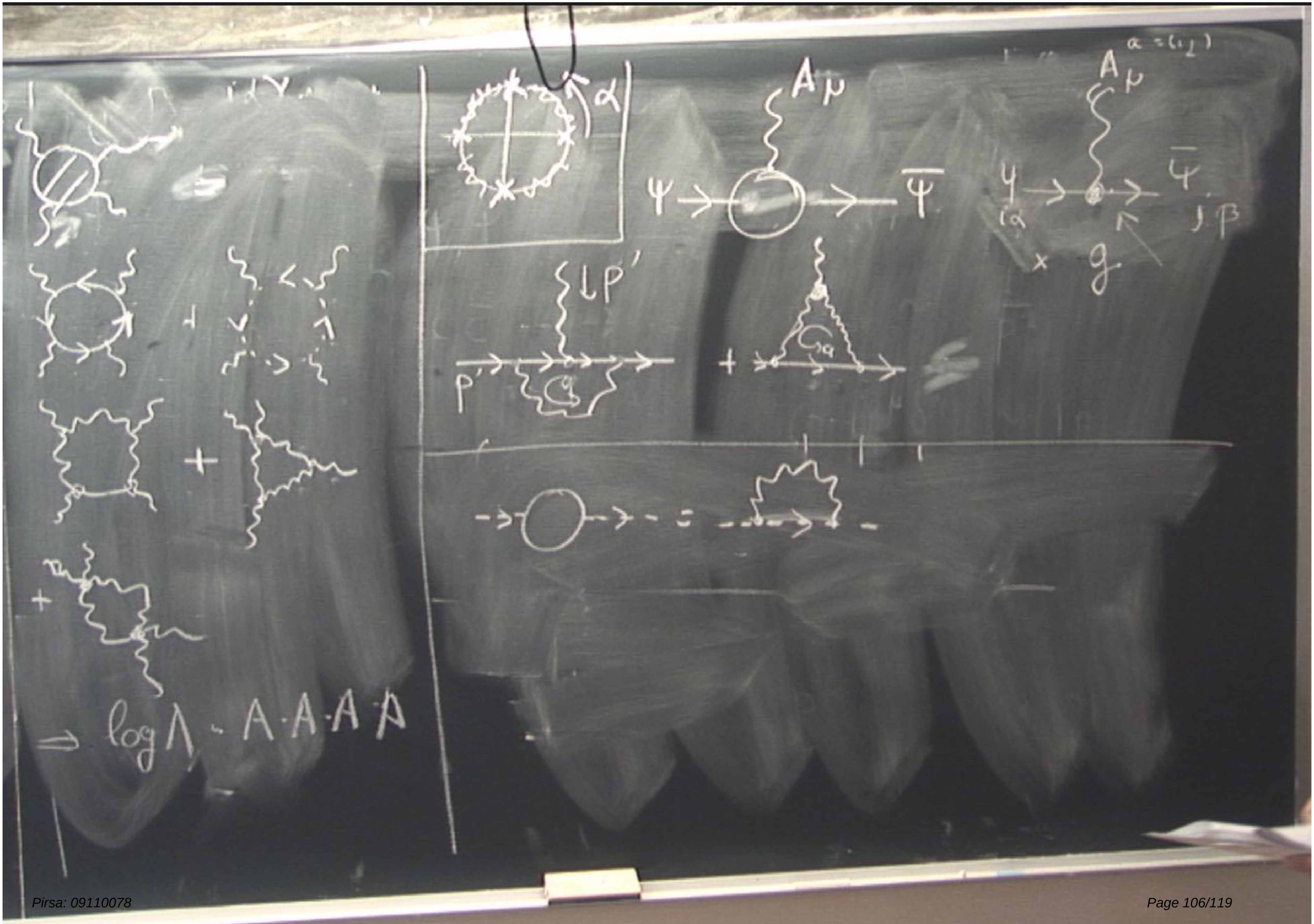
$$\Rightarrow \log\Lambda \cdot A \cdot A \cdot A \cdot A$$

$$\int d^4q \, \frac{1}{q^2}\,\frac{1}{q}\,\frac{1}{q} + \frac{1}{q}\,\frac{1}{q^2}\,\frac{1}{q^2}\,q \simeq \log\Lambda$$



$$\int d^4q \frac{1}{q^2}\frac{1}{q}\frac{1}{q} + \frac{1}{q}\frac{1}{q^2}\frac{1}{q^2}q \simeq \log\Lambda$$

$$\Rightarrow \log\Lambda \cdot A \cdot A \cdot A \cdot A$$


α
A_μ
Ψ
Ψ̄
a = (1,2)
A_μ
ψ
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g
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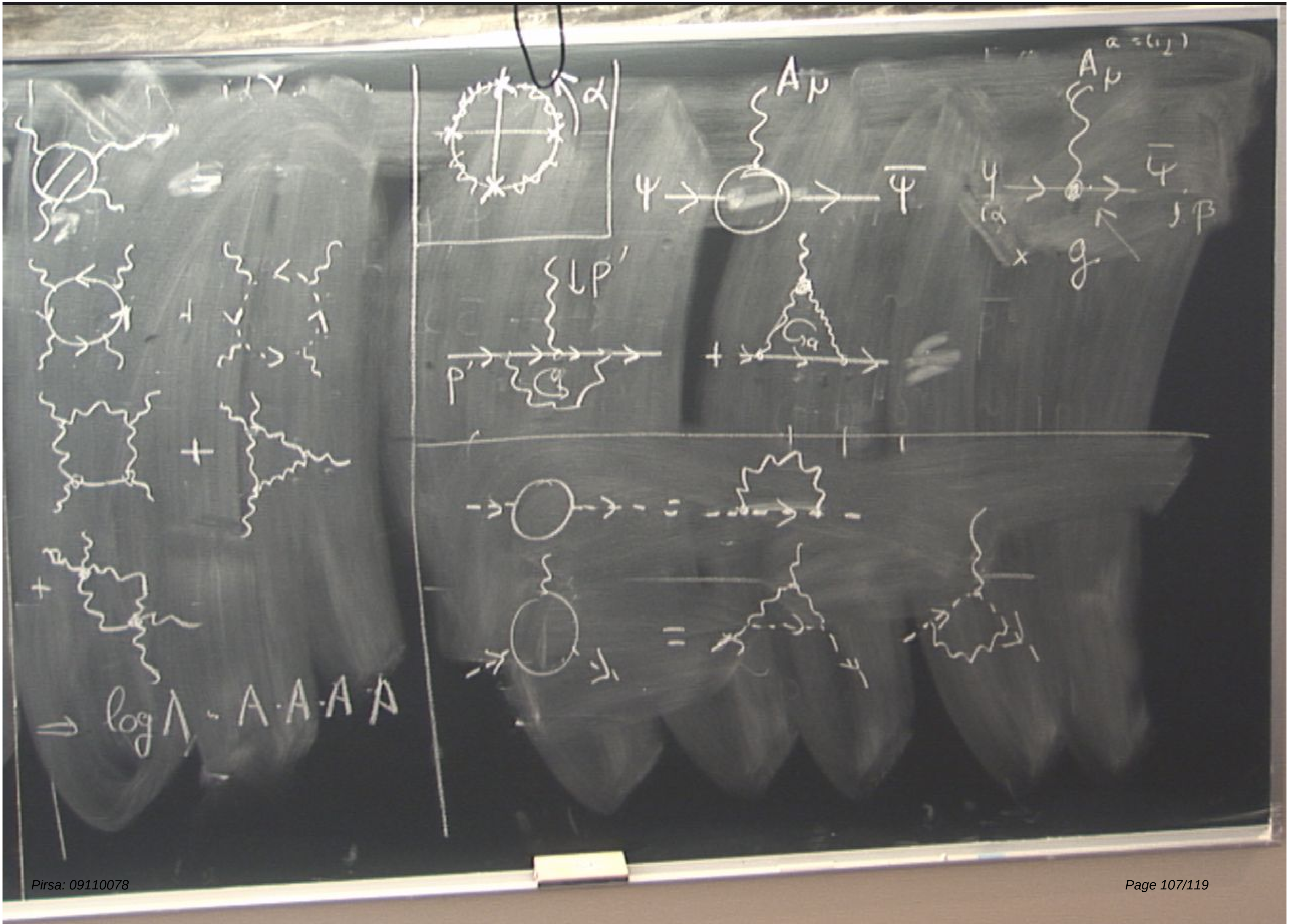
q
G_a
⇒ log Λ ~ A·A·A·A


α
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a = (1,2)
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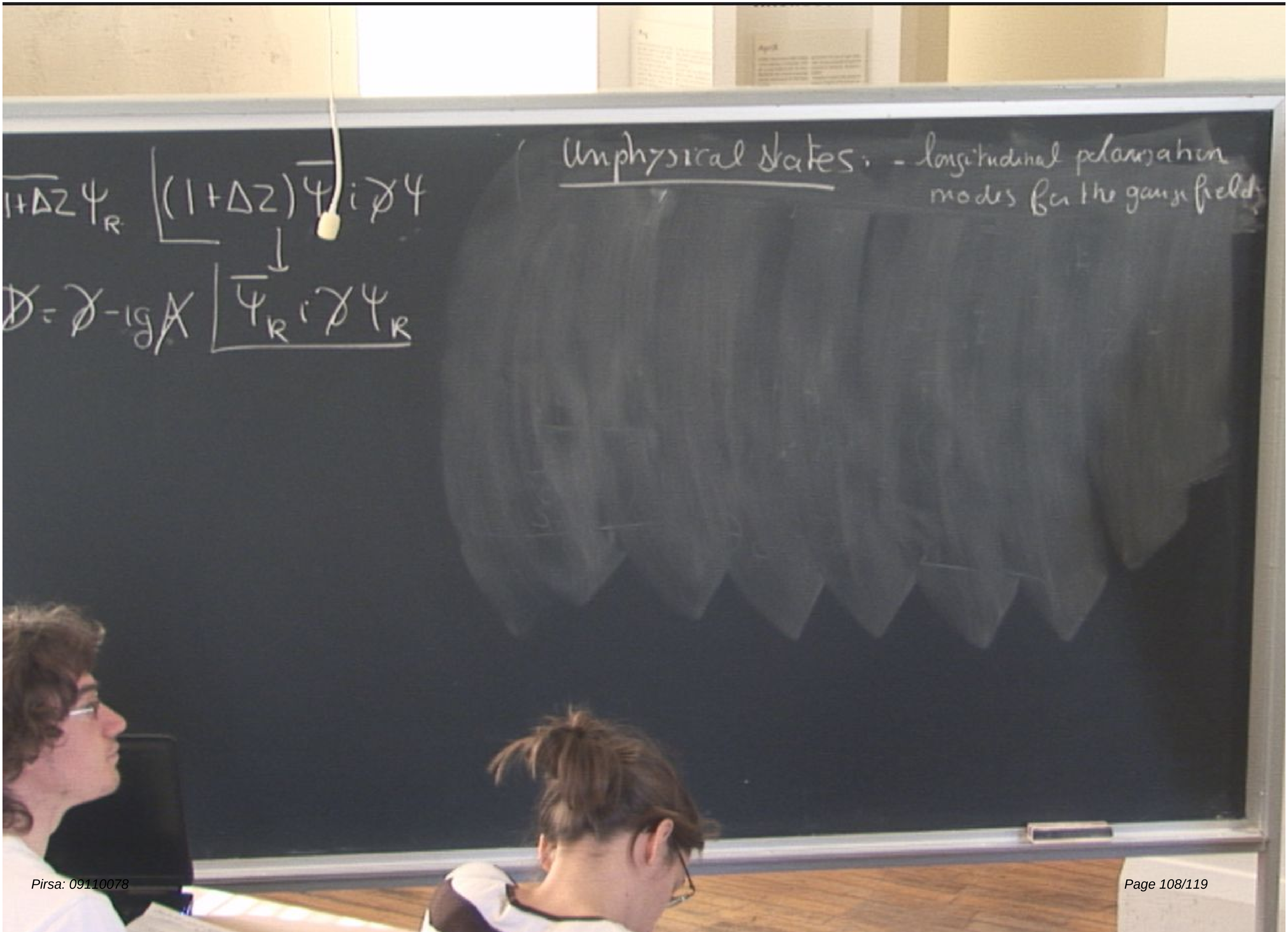
⇒ log Λ · A·A·A·A



$\overline{1+\Delta Z}\,\psi_R$

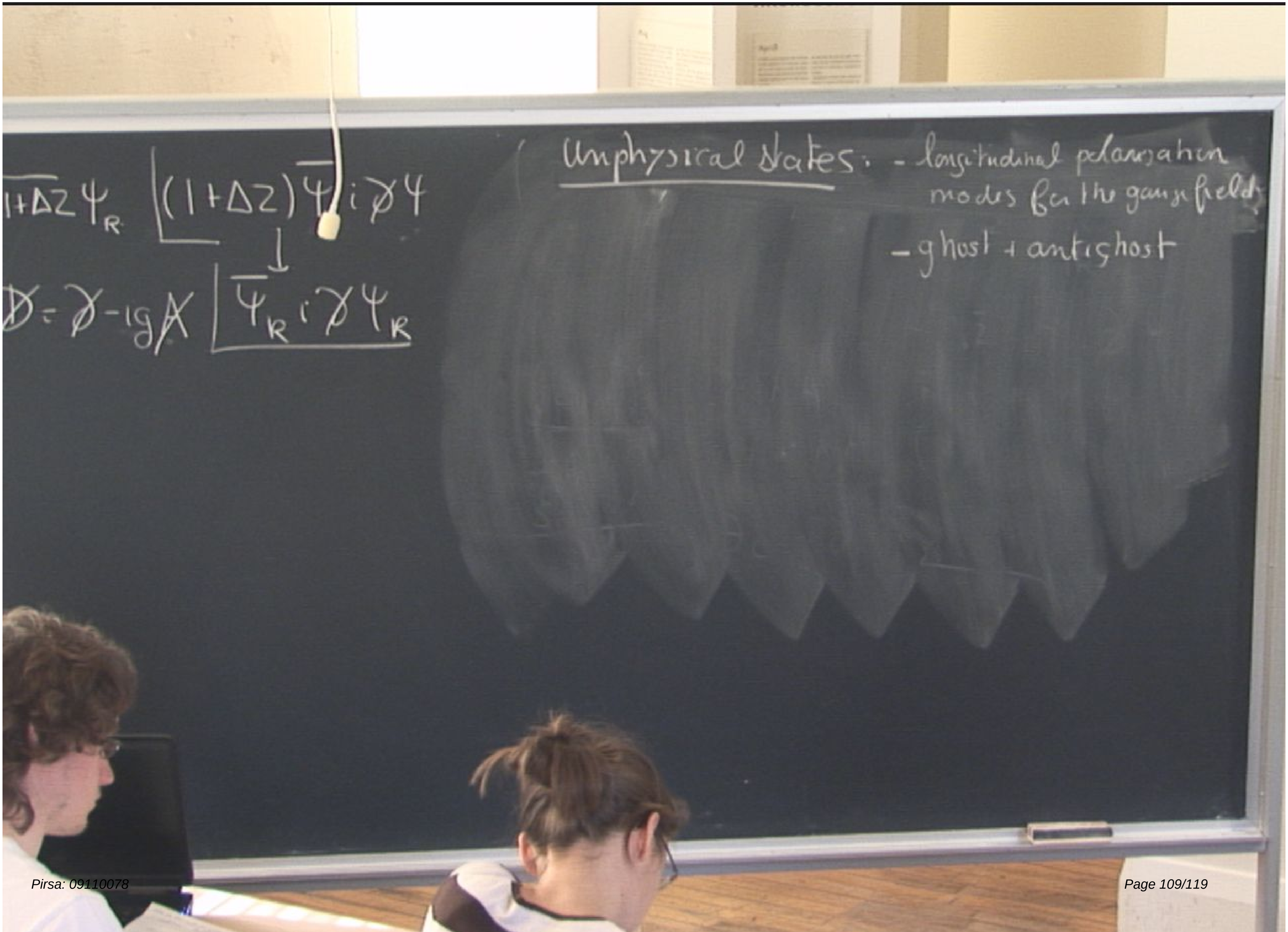
$(1+\Delta Z)\,\bar{\psi}\, i \not{\partial} \psi$

$\downarrow$

$\bar{\psi}_R\, i \not{\partial} \psi_R$

$\not{D} = \not{\partial} - ig \not{A}$

Unphysical States: - longitudinal polarization modes for the gauge field



$\overline{1+\Delta Z}\,\Psi_R$

$(1+\Delta Z)\overline{\Psi}\, i\not\partial\Psi$

$\downarrow$

$\not D = \not\partial - ig\not A$

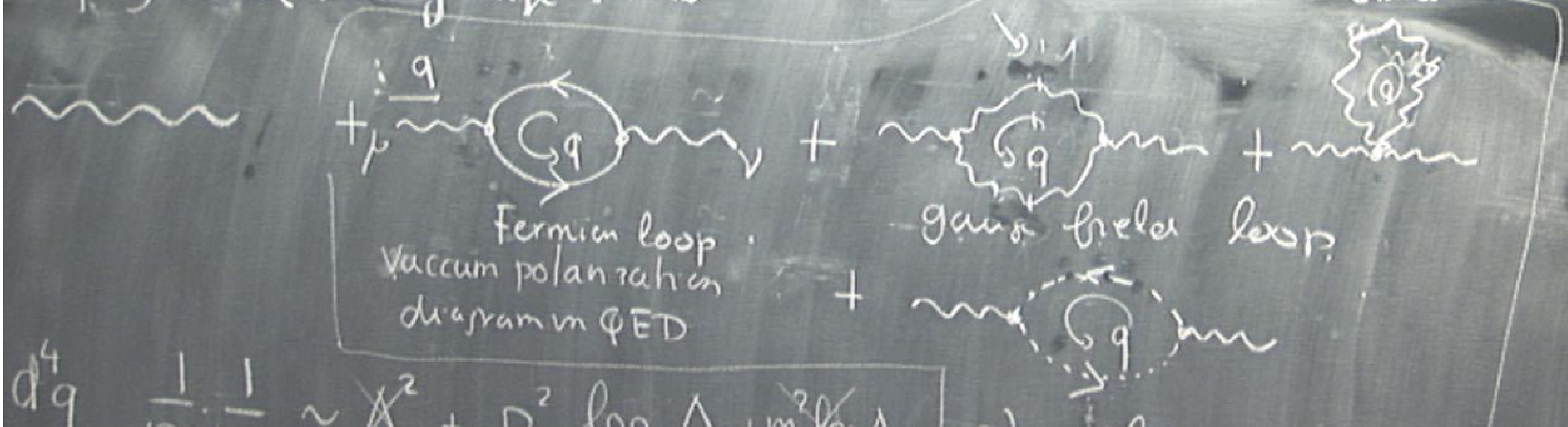
$\overline{\Psi}_R\, i\not\partial\,\Psi_R$

Unphysical States: - longitudinal polarisation modes for the gauge fields

- ghost + antighost

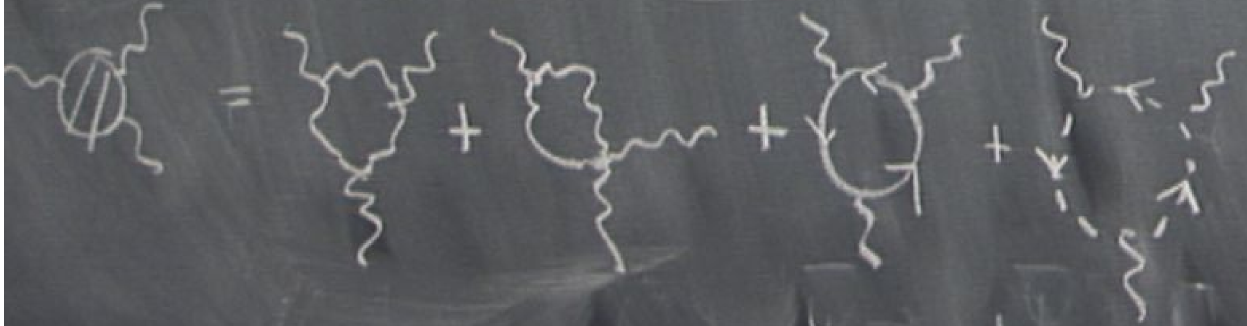
Propagator of the gauge Field

intermediate states ← negative norm states



$$d^4q \quad \frac{1}{|q|}\,\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log\Lambda} + m^2 \cancel{\log}\Lambda$$

gauge symmetry $U(1)$ $\quad p^\mu p^\nu - g^{\mu\nu} p^2$

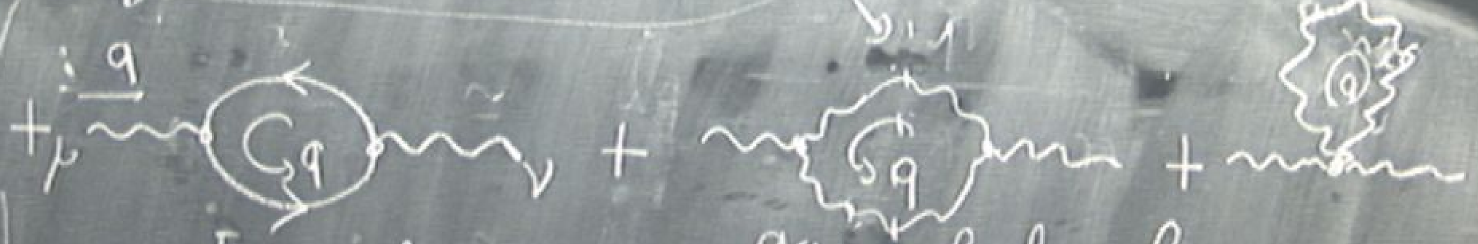


$$\simeq \log\Lambda \; \partial A \cdot A \cdot A$$

Propagator of the gauge Field

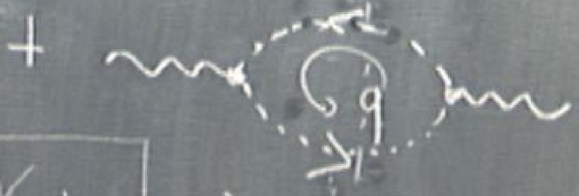
Unitarity:

intermediate states ← negative norm states



Fermion loop:
Vaccum polarization diagram in QED

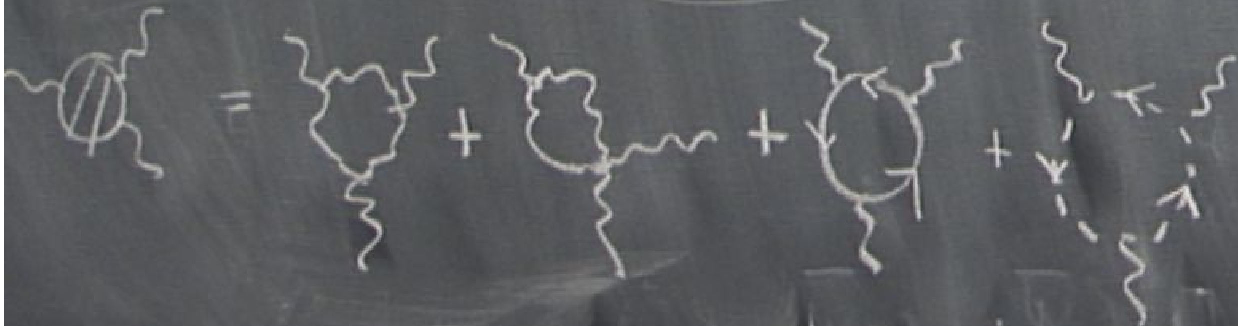
gauge field loop



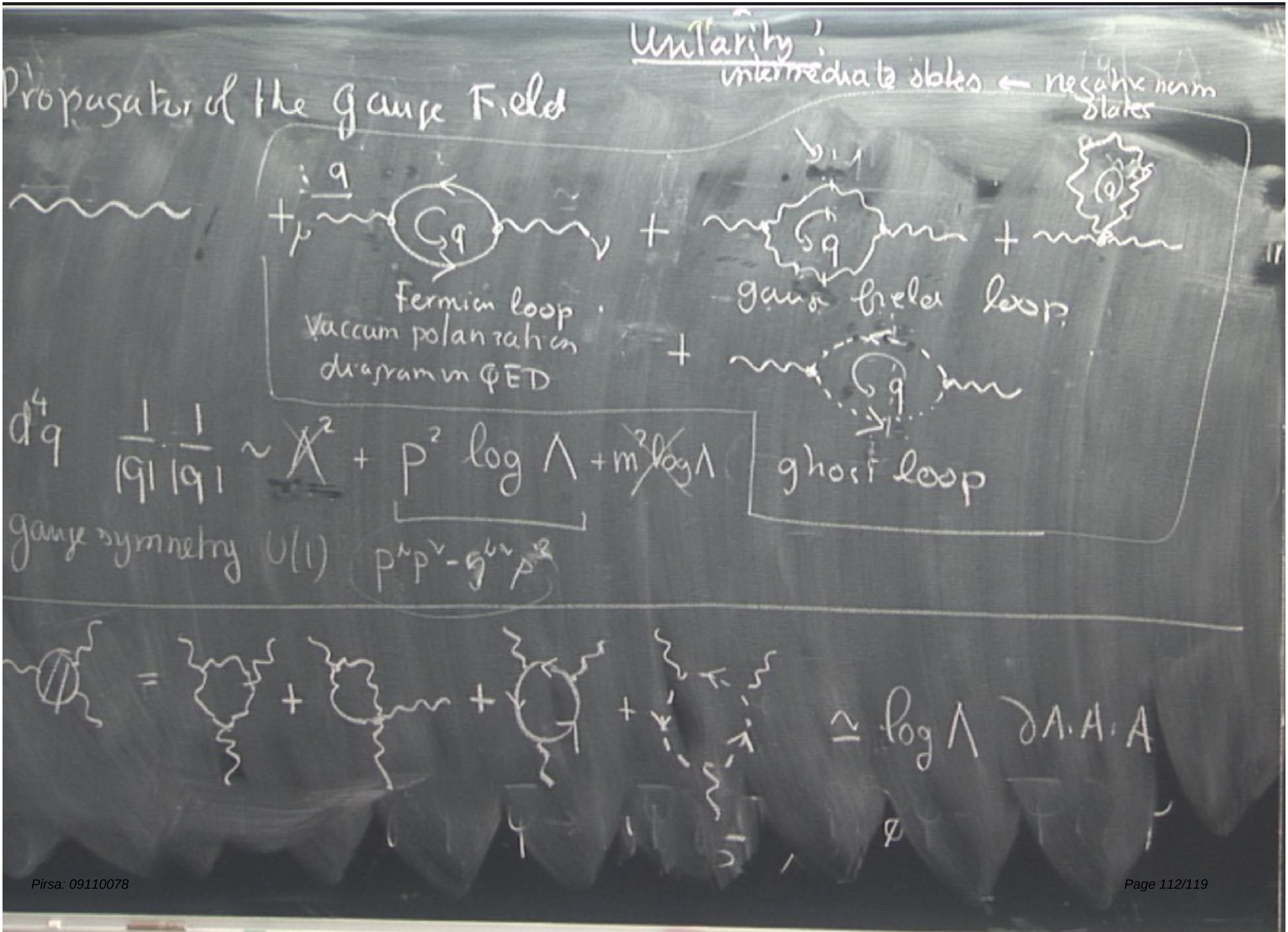
ghost loop

$$d^4q \, \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log\Lambda} + m^2\cancel{\log}\Lambda$$

gauge symmetry U(1) $\boxed{p^\mu p^\nu - g^{\mu\nu} p^2}$



$\simeq \log\Lambda \; \partial A \cdot A \cdot A$



Propagator of the gauge Field

Unitarity: intermediate states ← negative norm states

Fermion loop . Vaccum polarization diagram in QED

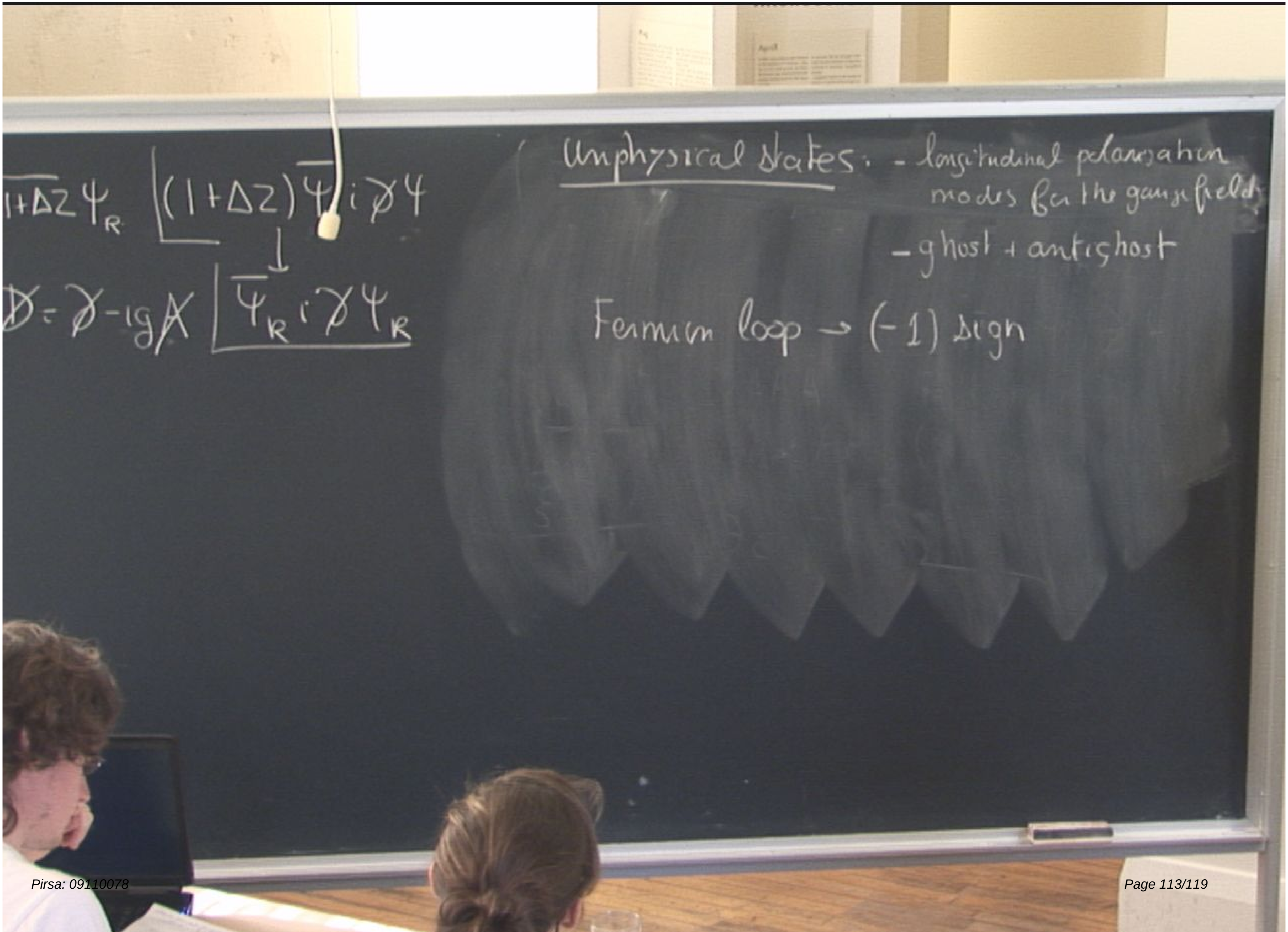
gauge field loop

ghost loop

$$d^4q \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda^2} + \underbrace{p^2 \log \Lambda} + m^2 \cancel{\log \Lambda}$$

gauge symmetry U(1) $p^\mu p^\nu - g^{\mu\nu} p^2$

$$\simeq \log \Lambda \ \partial A \cdot A \cdot A$$



$\overline{1+\Delta Z}\,\psi_R$ $\quad \lfloor (1+\Delta Z)\bar{\psi}\, i \not{\partial} \psi$

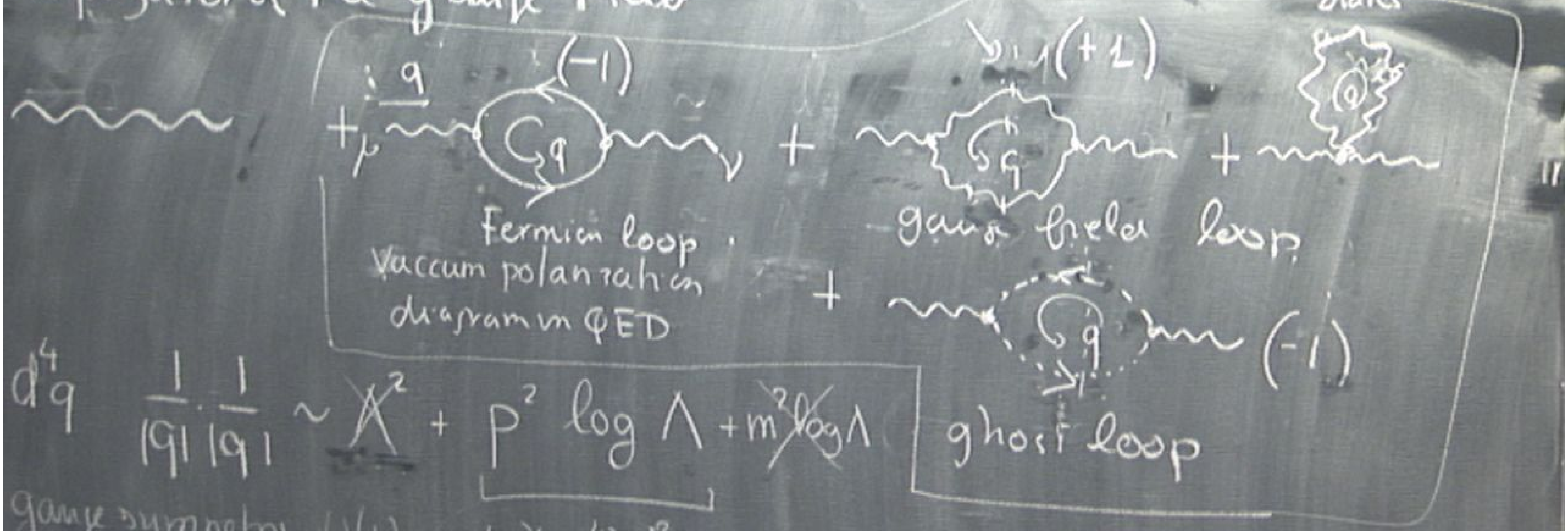
$\not{D} = \not{\partial} - ig\not{A}$ $\quad \downarrow \lfloor \bar{\psi}_R\, i \not{\partial} \psi_R$

Unphysical States: - longitudinal polarization modes for the gauge field
- ghost + antighost

Fermion loop → (-1) sign

Propagator of the gauge Field

Unitarity: intermediate states ← negative norm states

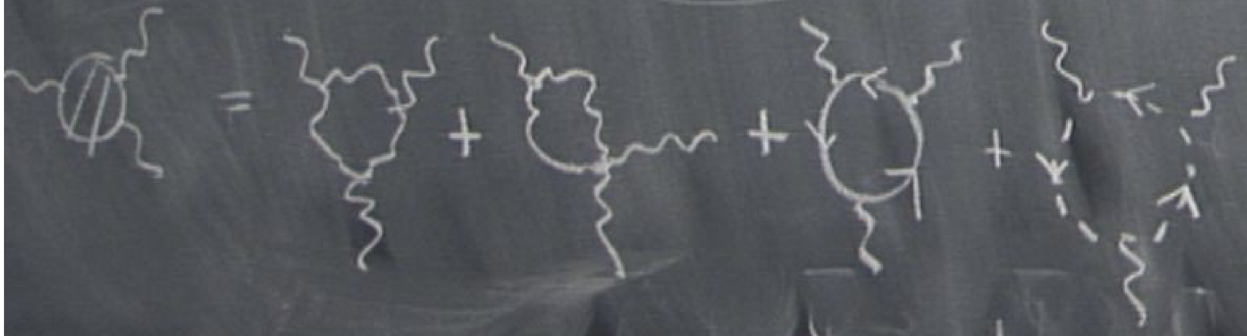


\+ Fermion loop + gauge field loop + ghost loop

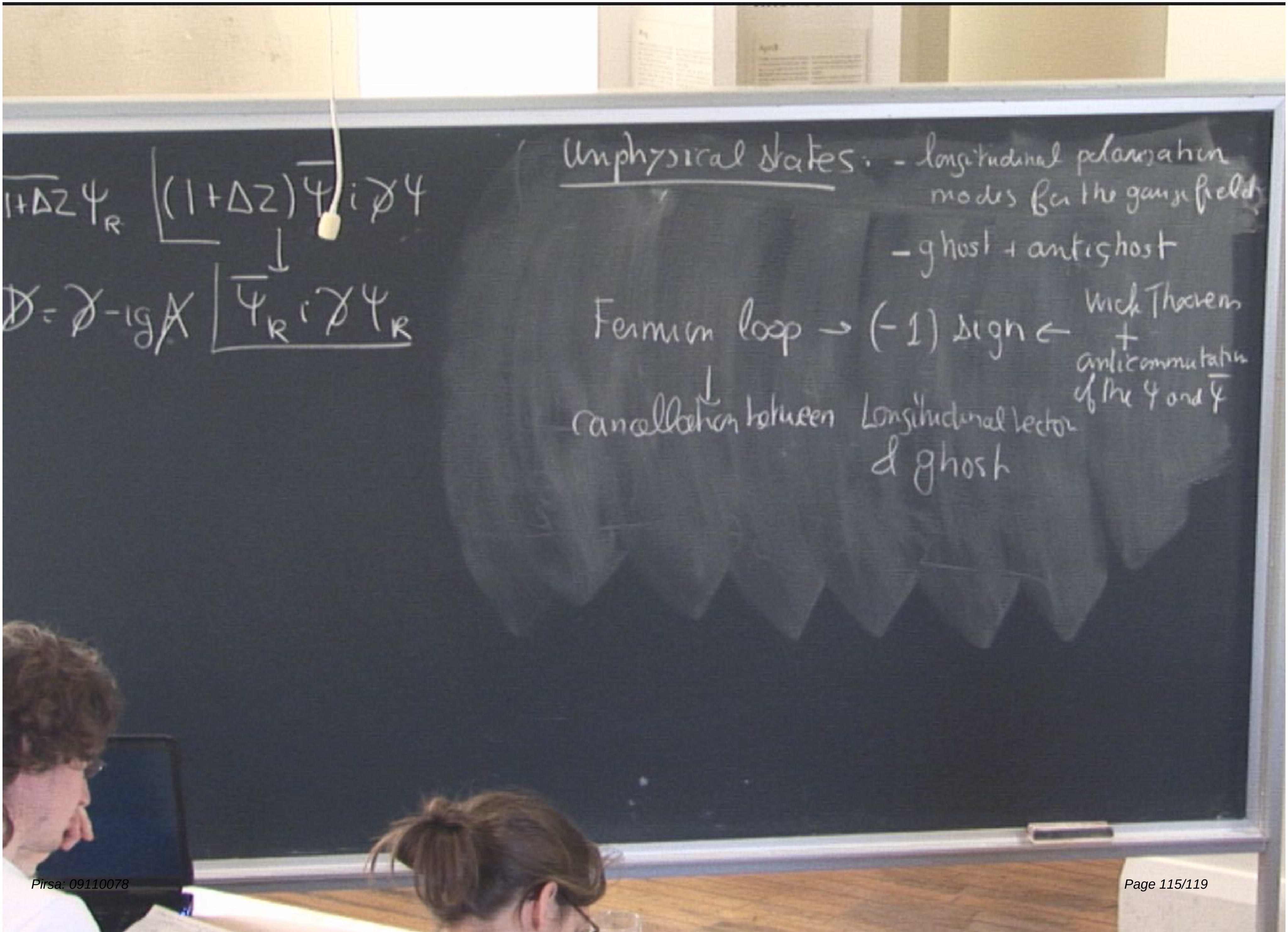
Vaccum polarization diagram in QED

$$d^4q \; \frac{1}{|q|}\frac{1}{|q|} \sim \cancel{\Lambda}^2 + \underbrace{p^2 \log\Lambda} + m^2\cancel{\log\Lambda}$$

gauge symmetry U(1) $p^\mu p^\nu - g^{\mu\nu}p^2$



$\simeq \log\Lambda \; \partial A \cdot A \cdot A$



$\overline{1+\Delta Z}\,\psi_R$

$(1+\Delta Z)\bar{\psi}\, i\not{\partial}\psi$

$\downarrow$

$\not{D} = \not{\partial} - ig\not{A}$

$\bar{\psi}_R\, i\not{\partial}\psi_R$

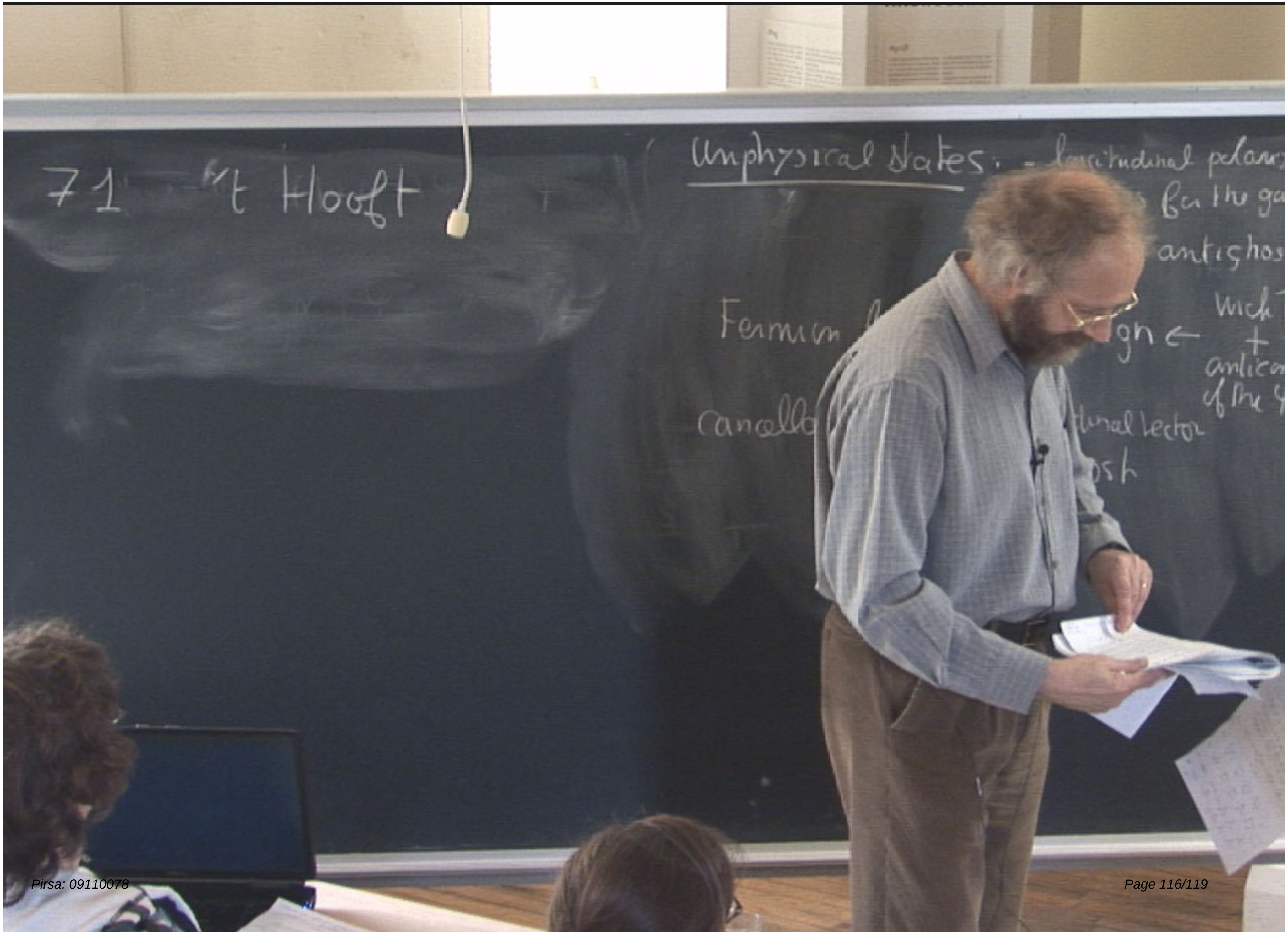
Unphysical states: - longitudinal polarisation modes for the gauge field

- ghost + antighost

Fermion loop → (-1) sign ← Wick Theorem + anticommutation of the ψ and $\bar{\psi}$

↓

cancellation between Longitudinal vector & ghost


71 't Hooft
Unphysical States: - longitudinal polari
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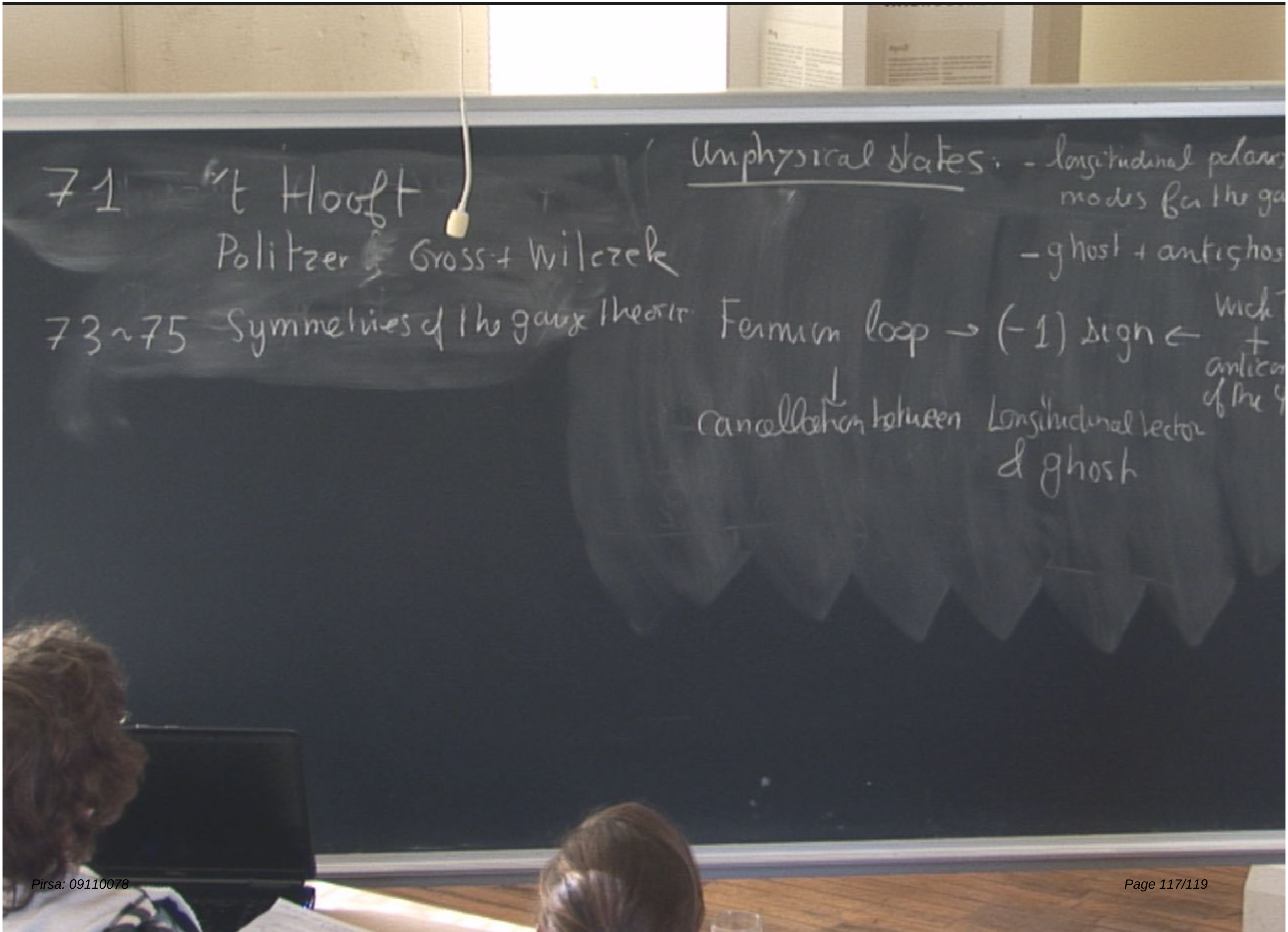

71 't Hooft

Politzer & Gross + Wilczek

73~75 Symmetries of the gauge theories

Unphysical states: - longitudinal polariz
modes for the ga

- ghost + antighos

Fermion loop → (-1) sign ← Wick T
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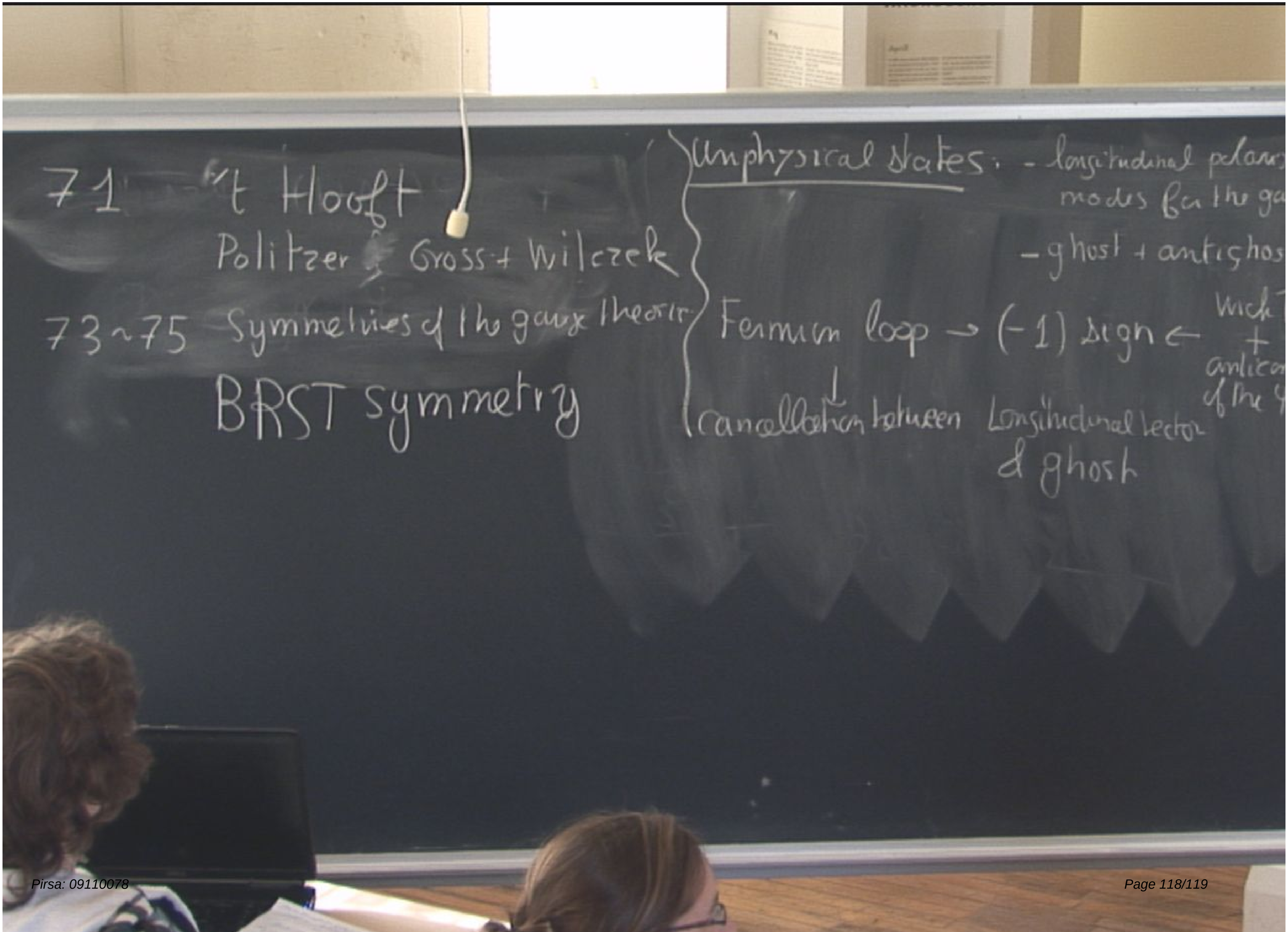
anticom
of the

↓

cancellation between Longitudinal vector
& ghost



71 't Hooft

Politzer & Gross + Wilczek

73~75 Symmetries of the gauge theories

BRST symmetry

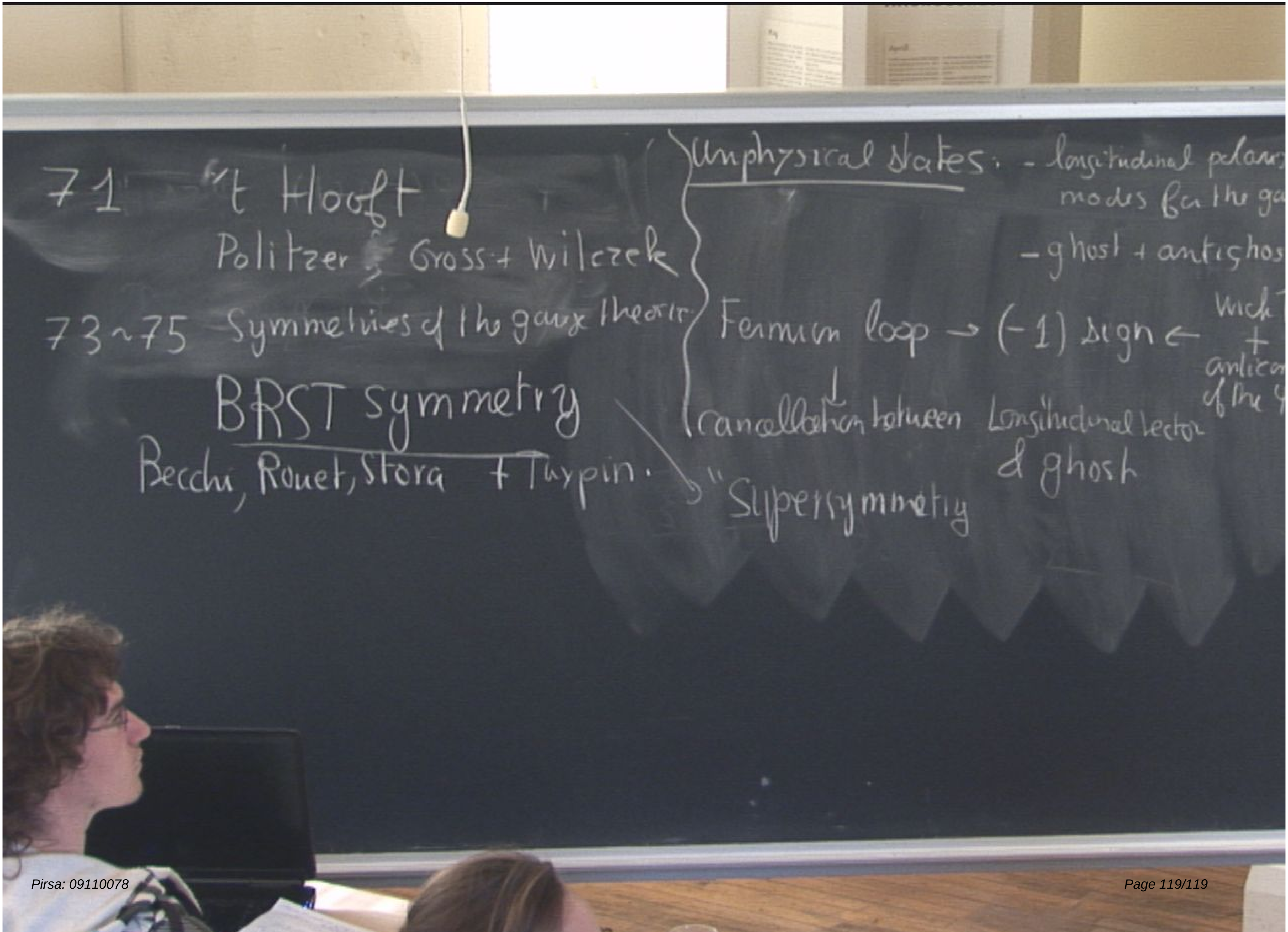
Unphysical states: - longitudinal polar
modes for the ga

- ghost + antighos

Fermion loop → (-1) sign ← Wick
\+
antico
of the

↓

cancellation between Longitudinal vector
& ghost



71 't Hooft

Politzer, Gross + Wilczek

73~75 Symmetries of the gauge theories

BRST symmetry

Becchi, Rouet, Stora + Tyupin.

Unphysical states: - longitudinal polari
modes for the ga

- ghost + antighos

Fermion loop → (-1) sign ← Wick
+
anticom
of the

↓

cancellation between longitudinal vector
& ghost

"Supersymmetry