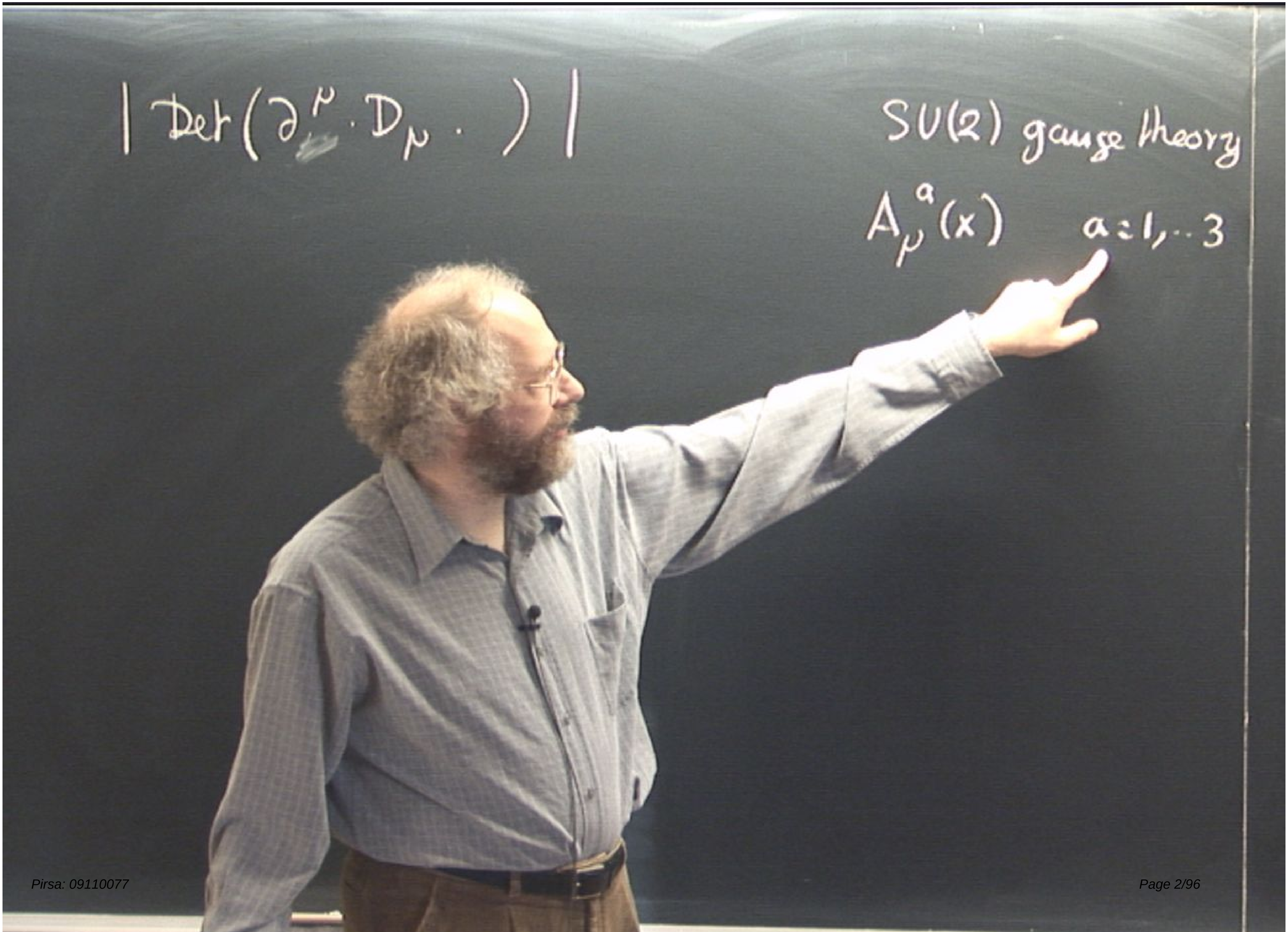
Title: Quantum Field Theory II (PHYS 603) - Lecture 13

Date: Nov 11, 2009 09:00 AM

URL: http://pirsa.org/09110077

Abstract:


| Det(∂^μ · D_μ ·) |
SU(2) gauge theory
A_μ^a(x)   a=1,..3

$$\left| \mathrm{Det}\left(\partial^{\mu} \cdot D_{\mu} \cdot \right) \right|$$

$$D_{\mu}\alpha^{a} = \partial_{\mu}\alpha^{a} + f^{abc}A_{\mu}^{b}\alpha^{c}$$

SU(2) gauge theory

$A_{\mu}^{a}(x)$ $a=1,\ldots 3$

α^{a} generator of gauge transf.

$$\left| \mathrm{Det}\left(\partial^{\mu} \cdot D_{\mu} \cdot \quad\right) \right|$$

$$D_\mu \alpha^a = \partial_\mu \alpha^a + f^{abc} A^b_\mu(x)\, \alpha^c$$

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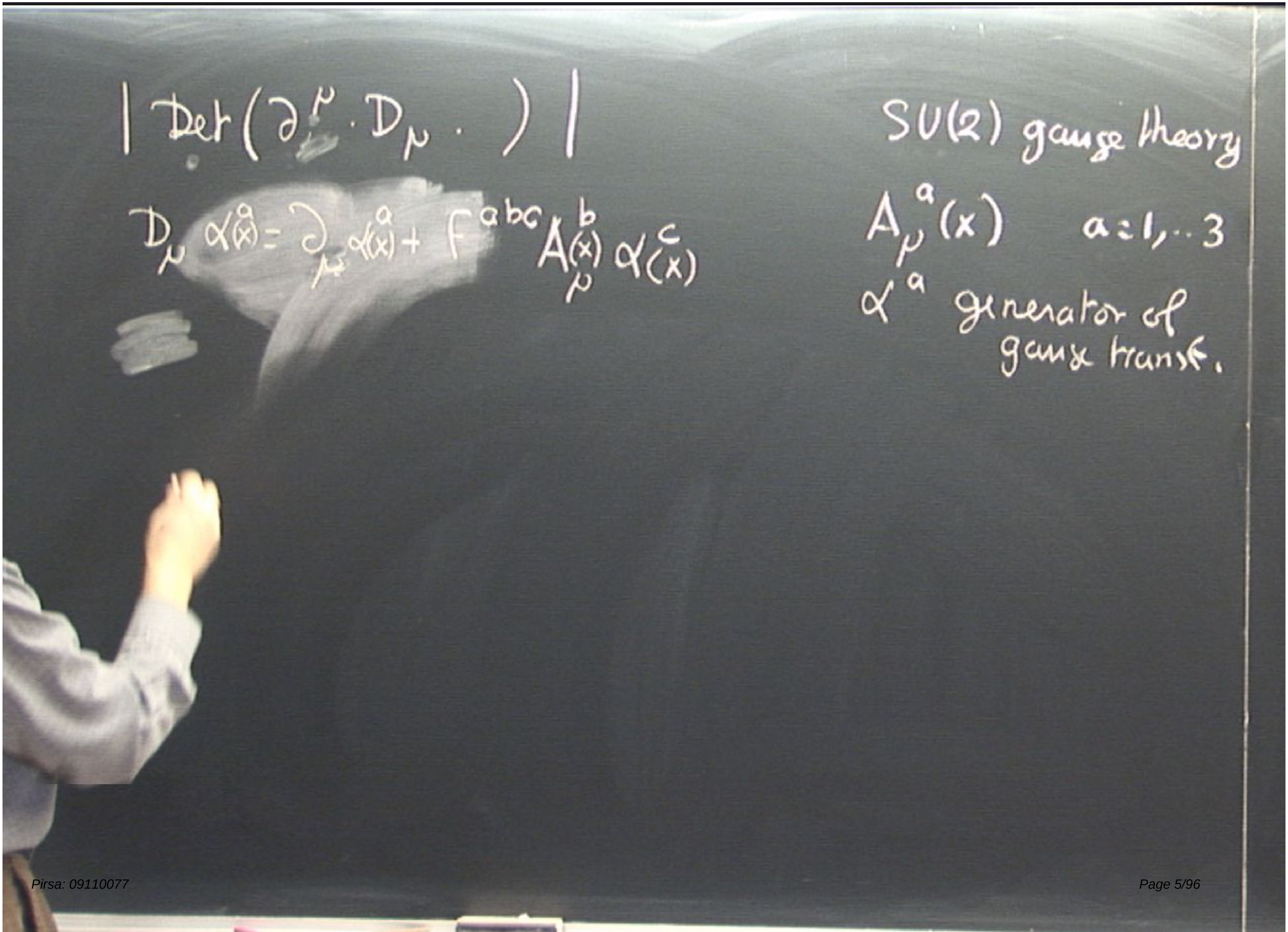
$|\mathrm{Det}(\partial^\mu \cdot D_\mu \cdot \ )|$
SU(2) gauge theory
$D_\mu \alpha^a(x) = \partial_\mu \alpha^a(x) + f^{abc} A^b_\mu(x) \alpha^c(x)$
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$$D_\mu \alpha^a(x) = \partial_\mu \alpha^a(x) + f^{abc} A^b_\mu(x)\, \alpha^c(x)$$

$$[\partial^\mu D_\mu]^{ac} = \partial^\mu \partial_\mu \delta^{ac} + f^{abc}\left[A^b_\mu(x)\,\partial^\mu + \partial^\mu A^b(x)\right]$$

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Faddeev - Popov : Use anticommuting (Grassmann) <u>scalar</u> fields

$$\left| \mathrm{Det}\left(\partial^\mu \cdot D_\mu \cdot \right) \right|$$

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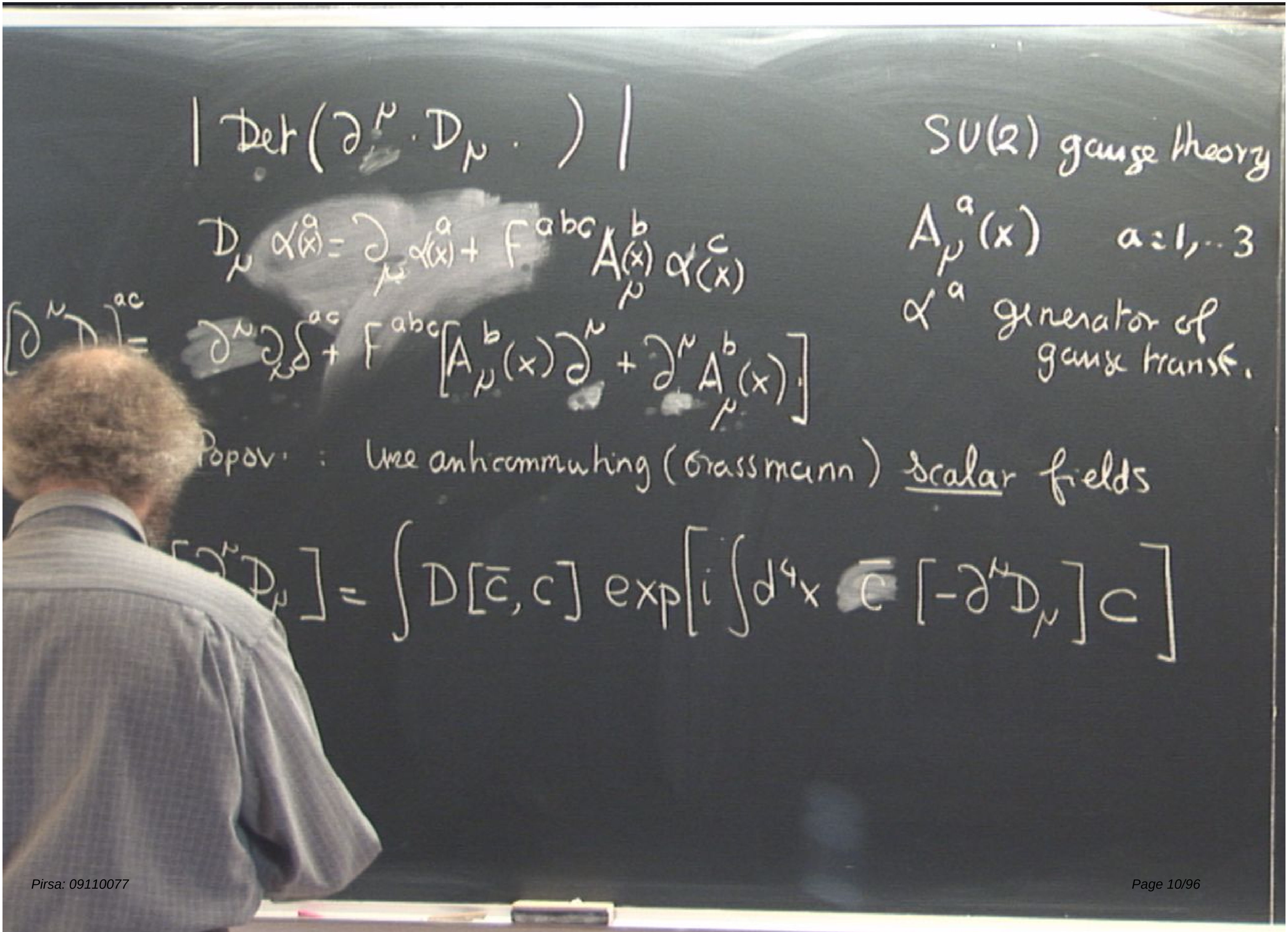
Faddeev - Popov : Use anticommuting (Grassmann) <u>scalar</u> fields

$$\mathrm{Det}\left[\partial^\mu D_\mu\right] = \int$$


$| \mathrm{Det}(\partial^\mu \cdot D_\mu \cdot \ ) |$
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$D_\mu \alpha^a(x) = \partial_\mu \alpha^a(x) + f^{abc} A^b_\mu(x) \alpha^c(x)$
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Popov: Use anticommuting (Grassmann) scalar fields
$[\partial^\mu D_\mu] = \int D[\bar{c}, c] \exp\left[i \int d^4x\, \bar{c}\, [-\partial^\mu D_\mu]\, c\right]$

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SU(2) gauge theory

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$$c = c^a(x)$$

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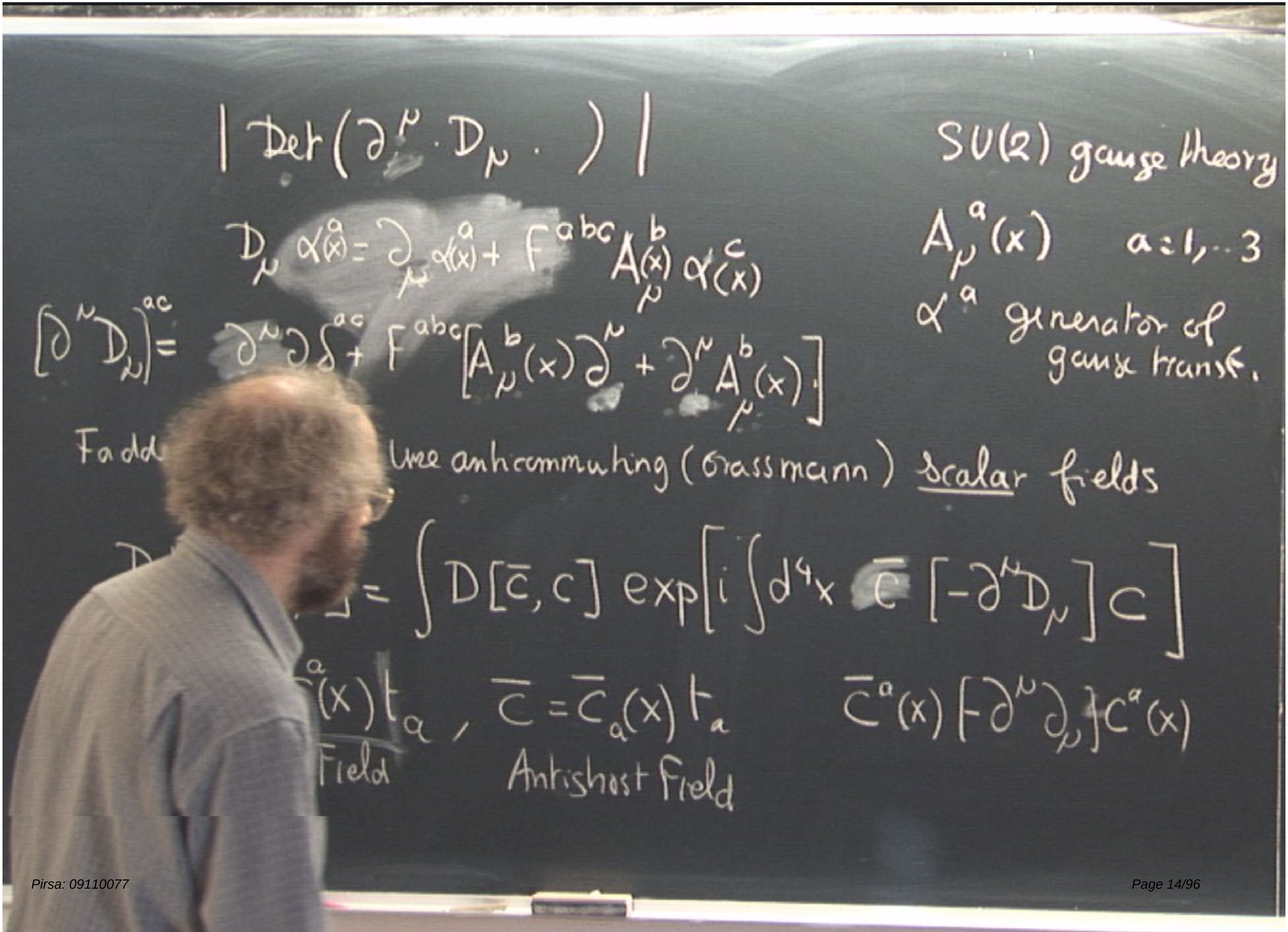
Ghost Field — Antighost Field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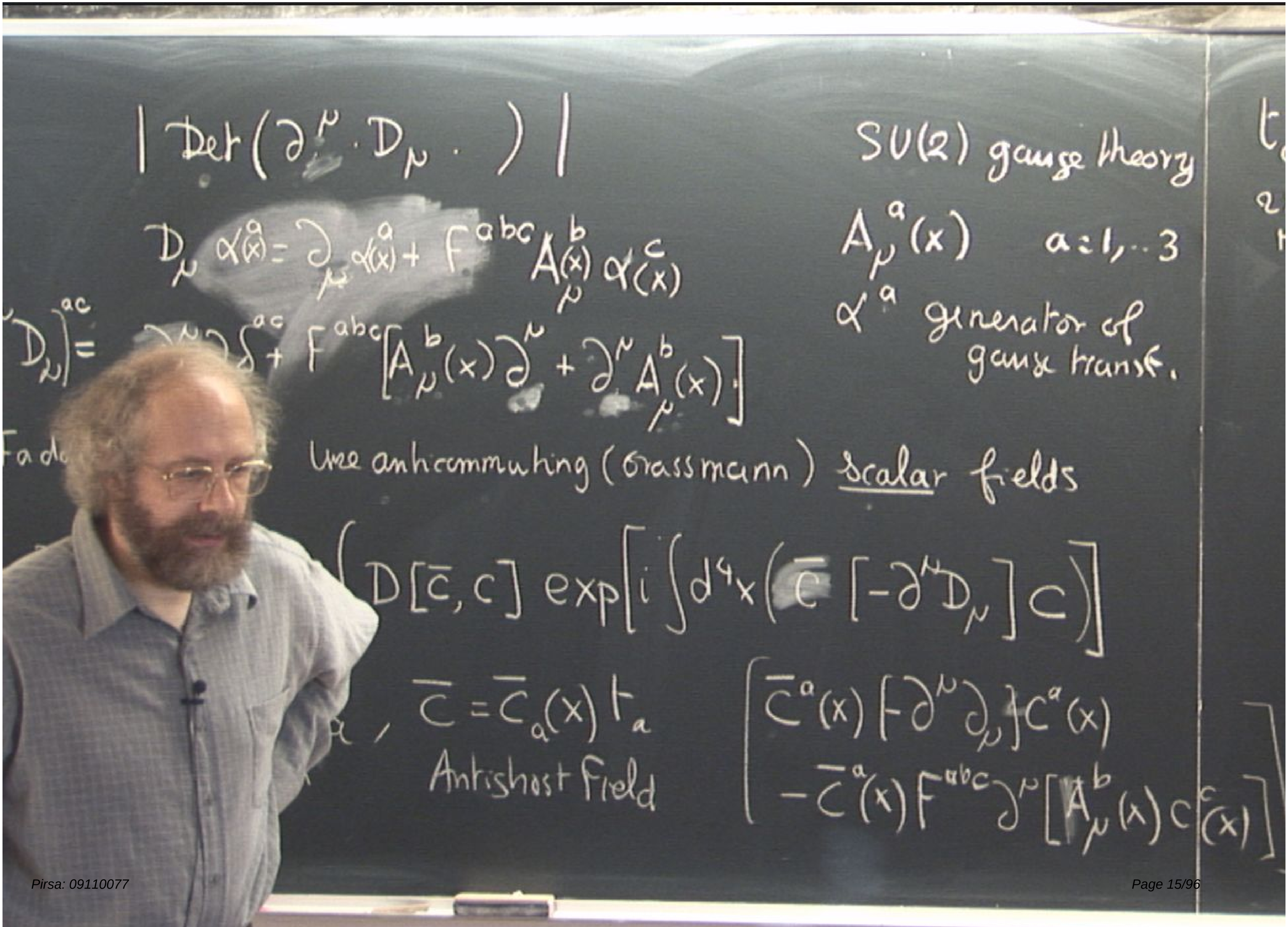
[∂^μ D_μ]^ac = ∂^μ ∂ δ^ac + F^abc [A^b_μ(x) ∂^μ + ∂^μ A^b_μ(x)]
Fadd
use anticommuting (Grassmann) scalar fields
= ∫ D[c̄, c] exp[i ∫ d^4x c̄ [−∂^μ D_μ] c]
(x) t_a , c̄ = c̄_a(x) t_a
c̄^a(x) [−∂^μ ∂_μ] c^a(x)
Field
Antighost Field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Use anticommuting (Grassmann) scalar fields
D[c̄, c] exp[i ∫ d^4x (c̄ [−∂^μ D_μ] c)]
c̄ = c̄_a(x) t_a
Antighost field
c̄^a(x) [−∂^μ ∂_μ] c^a(x)
− c̄^a(x) F^abc ∂^μ [A^b_μ(x) c^c(x)]

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$c = c^a(x)\, t_a$, $\quad \bar{c} = \bar{c}_a(x)\, t_a$

Ghost Field $\quad$ Antighost Field

$$\Big[\bar{c}^a(x)\left[-\partial^\mu \partial_\mu\right] c^a(x)$$
$$- \bar{c}^a(x) F^{abc} \partial^\mu \left[A^b_\mu(x)\, c^c(x)\right]\Big]$$

$t_a = \frac{1}{2}\sigma_a$

2x2 Pauli Matrices

C & $\bar{C}$ carry charges of SU(2)

but no spin spin = 0

(x)

$t_a = \frac{1}{2}\sigma_a$

2x2 Pauli Matrices

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but no spin spin = 0

propagator for the ghost

$\langle 0 | T\, C^a(x)\, \bar{C}^b(y) | 0 \rangle =$

$^c(x)$

$t_a = \frac{1}{2}\sigma_a$

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propagator for the ghost

$$\langle 0 | T\, C^a(x)\, \bar{C}^b(y) | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{\delta^{ab}}{p^2}$$

$c(x)]]$

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propagator for the ghost

$$\langle 0|T\, C^a(x)\bar{C}^b(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{\delta^{ab}}{p^2}$$

But !

$$\langle 0|T\, \bar{C}^b(y) C^a(x)|0\rangle$$

(x)

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<u>But!</u>

$$\langle 0|T\, \bar{c}^b(y) c^a(x)|0\rangle = -\langle 0|T\, c^a(x)\bar{c}^b(y)|0\rangle$$

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Bul !

$$\langle 0|T\,\bar{c}^b(y)c^a(x)|0\rangle = -\langle 0|T\,c^a(x)\bar{c}^b(y)|0\rangle$$

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Spin-Statistic Theorem.

$t_a = \frac{1}{2}\sigma_a$

2x2 Pauli Matrices

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propagator for the ghost

$$\langle 0|T\, c^a(x)\,\bar{c}^b(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{\delta^{ab}}{p^2}$$

But!

$$\langle 0|T\, \bar{c}^b(y)\, c^a(x)|0\rangle = -\langle 0|T\, c^a(x)\,\bar{c}^b(y)|0\rangle$$

Spin-Statistic Theorem. ; Locality + Causality

Lorentz Invariance

$t_a = \frac{1}{2}\sigma_a$

2x2 Pauli Matrices

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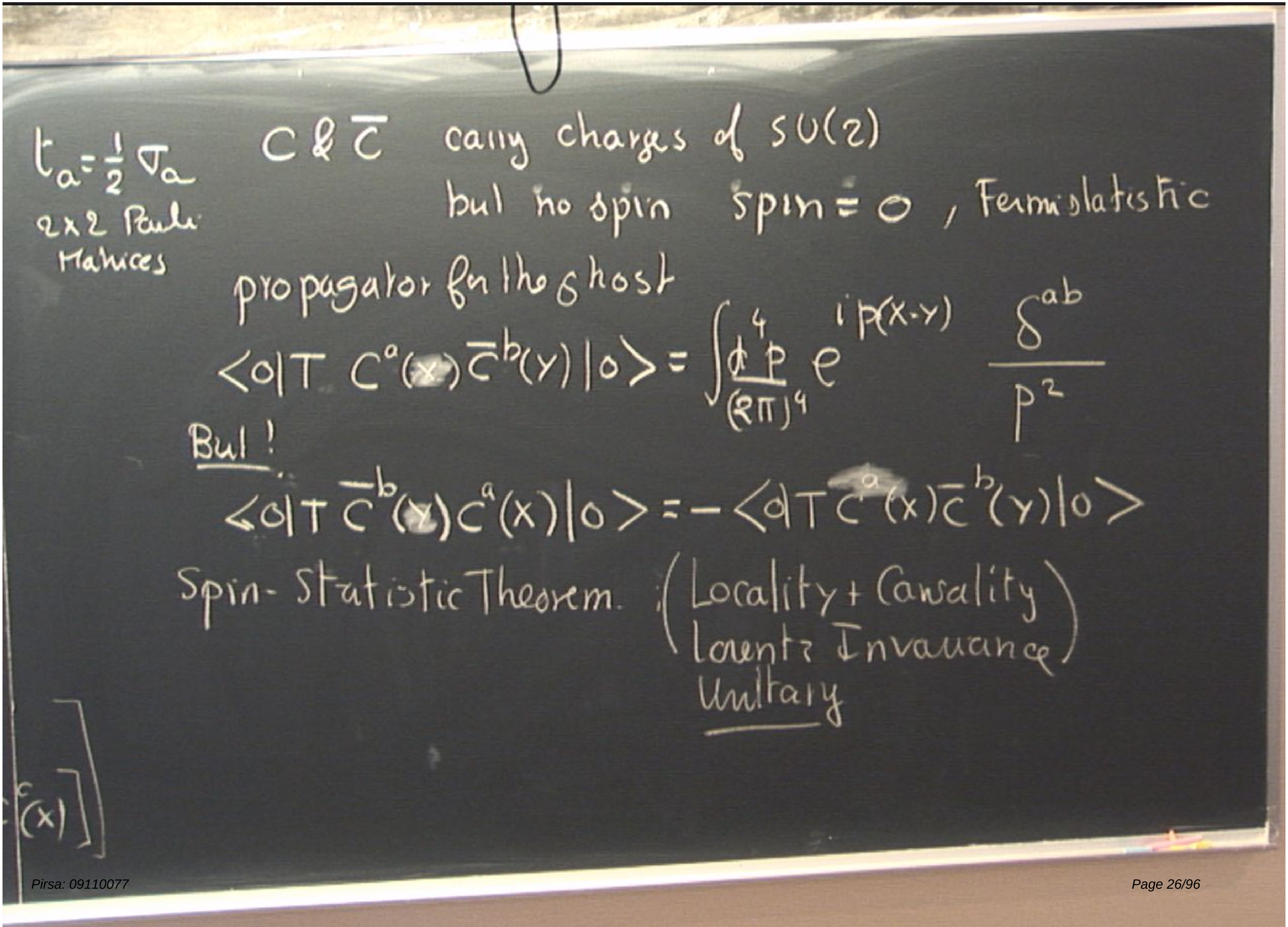
propagator for the ghost

$$\langle 0|T\, C^a(x)\,\bar{C}^b(y)\,|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{\delta^{ab}}{p^2}$$

But !

$$\langle 0|T\, \bar{C}^b(y)\, C^a(x)\,|0\rangle = -\langle 0|T\, C^a(x)\,\bar{C}^b(y)\,|0\rangle$$

Spin-Statistic Theorem. ; (Locality + Causality, Lorentz Invariance) Unitary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propagator for the ghost
$\langle 0|T\, C^a(x)\bar{C}^b(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \frac{\delta^{ab}}{p^2}$
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Spin-Statistic Theorem. (Locality + Causality
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propagator for the ghost

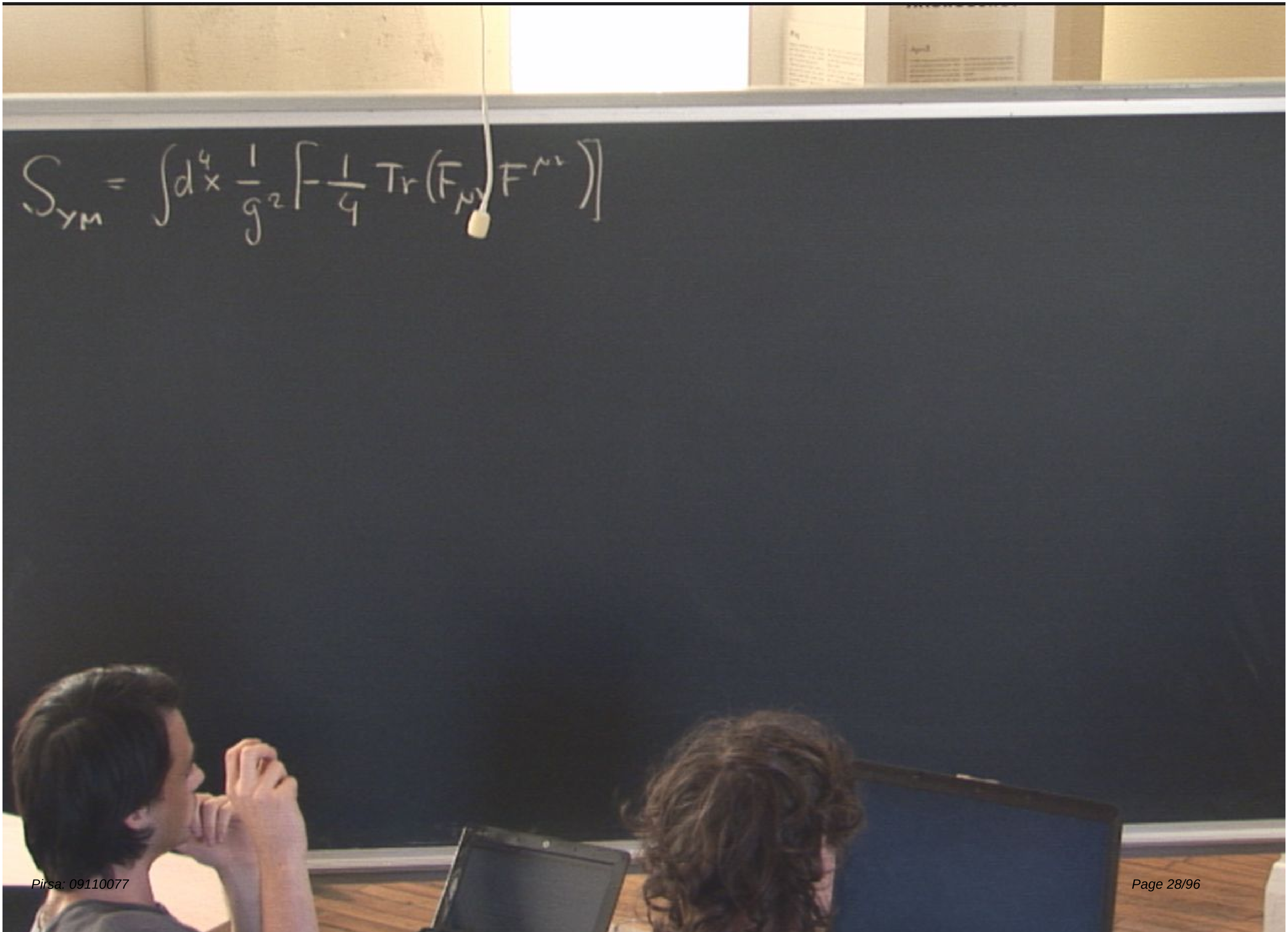
$$\langle 0|T\, c^a(x)\,\bar{c}^b(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{\delta^{ab}}{p^2}$$

<u>But !</u>

$$\langle 0|T\, \bar{c}^b(y)\,c^a(x)|0\rangle = -\langle 0|T\, c^a(x)\,\bar{c}^b(y)|0\rangle$$

Spin-Statistic Theorem.

- (Locality + Causality
- Lorentz Invariance)
- <u>Unitary</u>


$S_{YM} = \int d^4x \frac{1}{g^2} \left[-\frac{1}{4} Tr(F_{\mu\nu} F^{\mu\nu}) \right]$

$t_a = \frac{1}{2}\sigma_a$

2x2 Pauli Matrices

c & $\bar{c}$ carry charges of $SU(2)$

but no spin spin = 0, Fermi statistic

propagator for the ghost (free : $g=0$)

$$\langle 0|T\, c^a(x)\,\bar{c}^b(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\, e^{ip(x-y)}\, \frac{\delta^{ab}}{p^2}$$

<u>But!</u>

$$\langle 0|T\,\bar{c}^b(y)\,c^a(x)|0\rangle = -\langle 0|T\,c^a(x)\,\bar{c}^b(y)|0\rangle$$

Spin-Statistic Theorem
- (Locality + Causality)
- Lorentz Invariance
- Unitary

not unitary

$$\langle \Psi_{ghost}|\Psi_{ghost}\rangle \leq 0$$

$t_a = \frac{1}{2}\sigma_a$

2×2 Pauli Matrices

c & $\bar{c}$ carry charges of $SU(2)$

but no spin spin $= 0$, Fermi statistic

propagator for the ghost (free: $g = 0$)

$$\langle 0 | T\, c^a(x) \bar{c}^b(y) | 0 \rangle = \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-y)} \frac{\delta^{ab}}{p^2}$$

But!

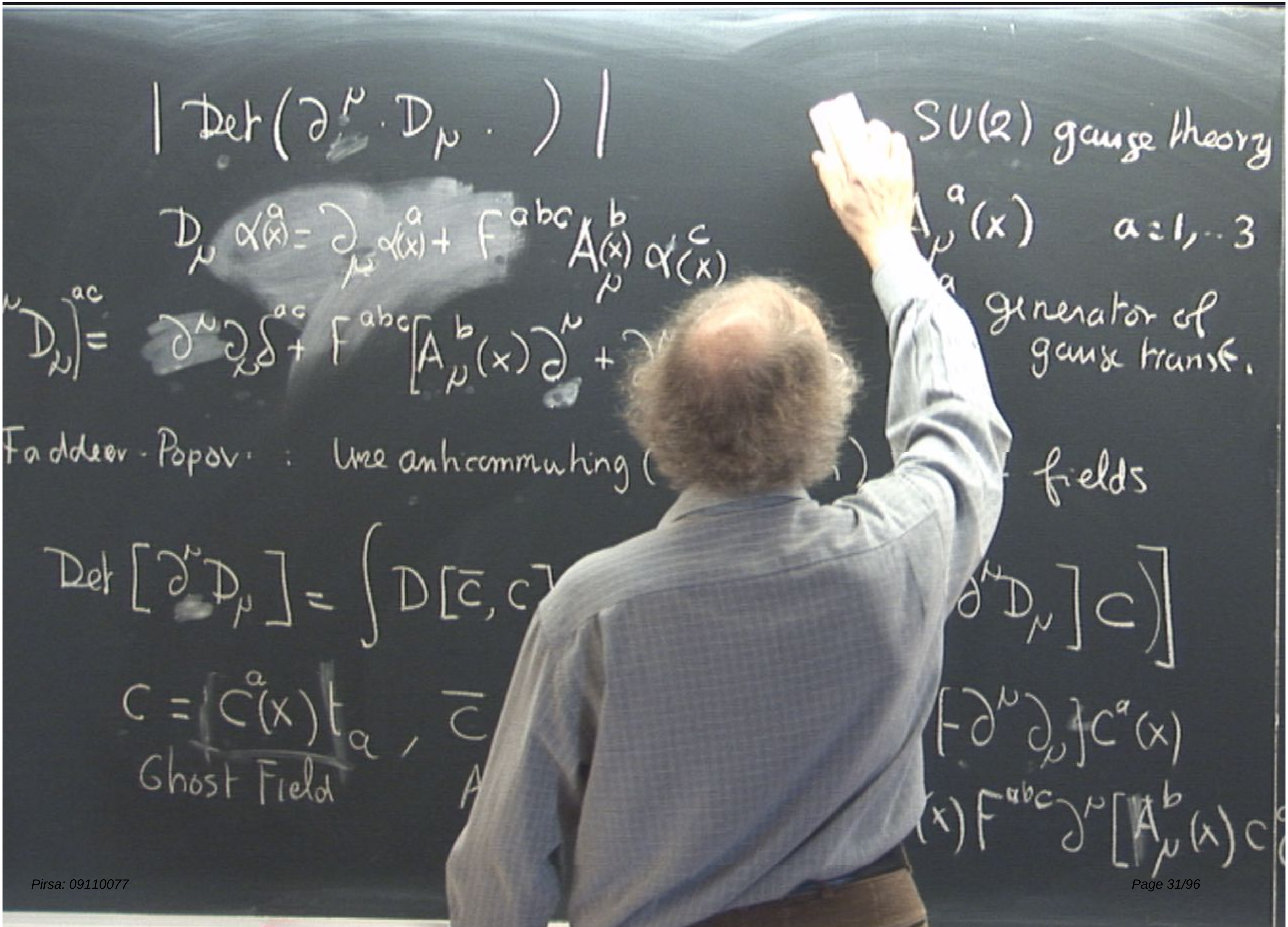
$$\langle 0 | T\, \bar{c}^b(y) c^a(x) | 0 \rangle = - \langle 0 | T\, c^a(x) \bar{c}^b(y) | 0 \rangle$$

Spin-Statistic Theorem
- (Locality + Causality
- Lorentz Invariance)
- Unitary

not unitary

$$\langle \Psi_{ghost} | \Psi_{ghost} \rangle \leq 0$$ possible

$c(x)]$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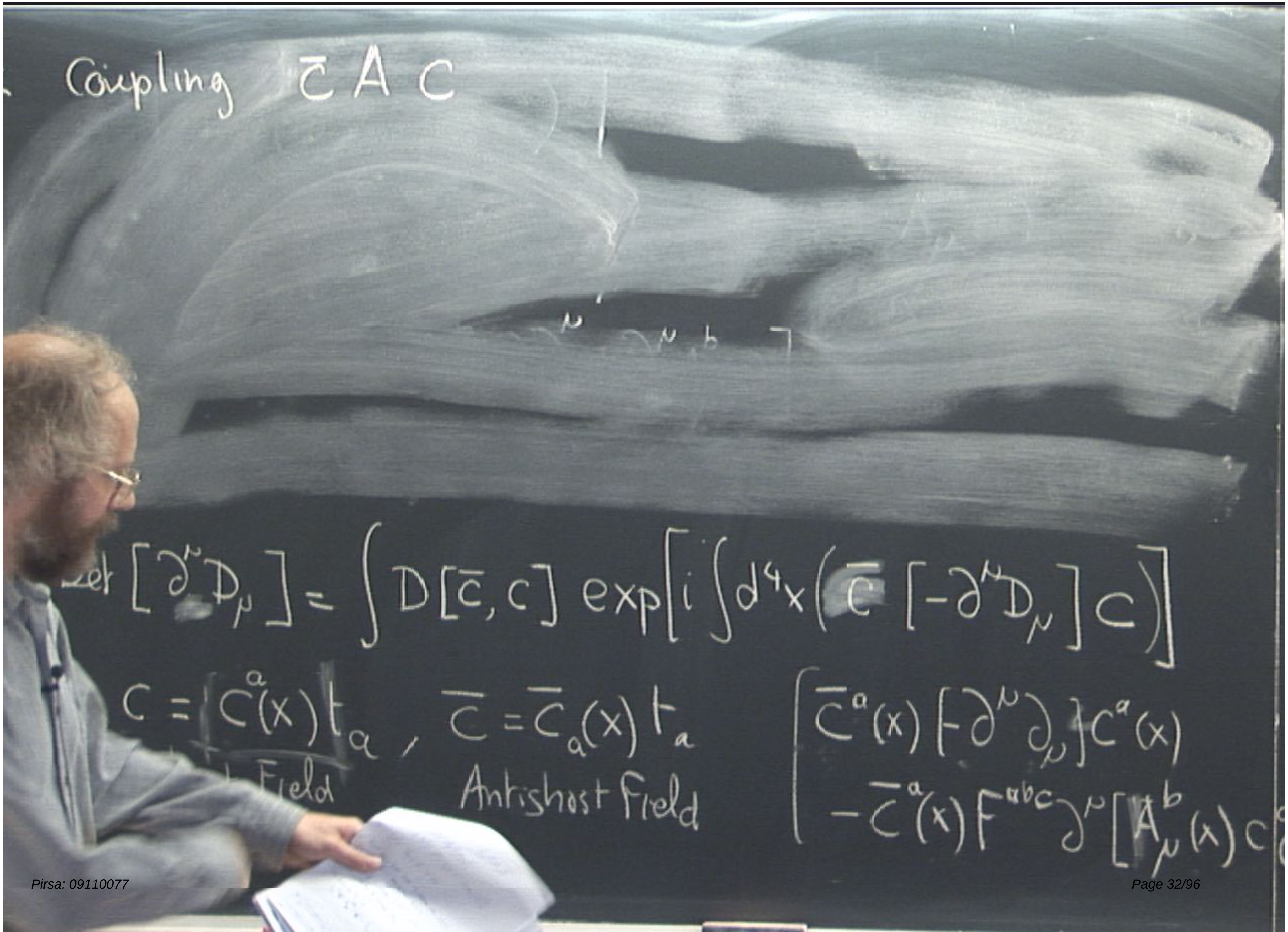
A^a_μ(x)   a = 1, .. 3
generator of gauge transf.
D_μ]^ac = ∂^ν ∂_ν δ^ac + F^abc [A^b_μ(x) ∂^ν + ...
Faddeev - Popov : Use anticommuting (...) ... fields
Det [∂^μ D_μ] = ∫ D[c̄, c] ... ∂^μ D_μ] c)]
c = c^a(x) t_a , c̄ ...
Ghost Field
... ∂^μ ∂_μ] c^a(x)
(x) F^abc ∂^μ [A^b_μ(x) c


Coupling $\bar{c} A c$
$\det[\partial^\mu D_\mu] = \int D[\bar{c}, c] \exp\left[i \int d^4x \left(\bar{c} [-\partial^\mu D_\mu] c\right)\right]$
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Field
Antighost Field
$\int \bar{c}^a(x) [-\partial^\mu \partial_\mu] c^a(x)$
$- \bar{c}^a(x) f^{abc} \partial^\mu [A^b_\mu(x) c$

Coupling $\bar{c} A c$

$\langle c\,\bar{c}\rangle$: - - - -> - - - - -

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Ghost Field Antighost Field

$$\int \bar{c}^a(x)\,[-\partial^\mu \partial_\mu]\, c^a(x) - \bar{c}^a(x)\, F^{abc}\, \partial^\mu [A^b_\mu(x)\, c$$

Coupling $\bar{c} A c$

$\langle c \bar{c} \rangle$: o - - - > - - - - o

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Ghost Field   Antighost Field

$$\int \bar{c}^a(x) [-\partial^\mu \partial_\mu] c^a(x)$$
$$- \bar{c}^a(x) F^{abc} \partial^\mu [A^b_\mu(x) c$$

Coupling $\bar{c} A c$

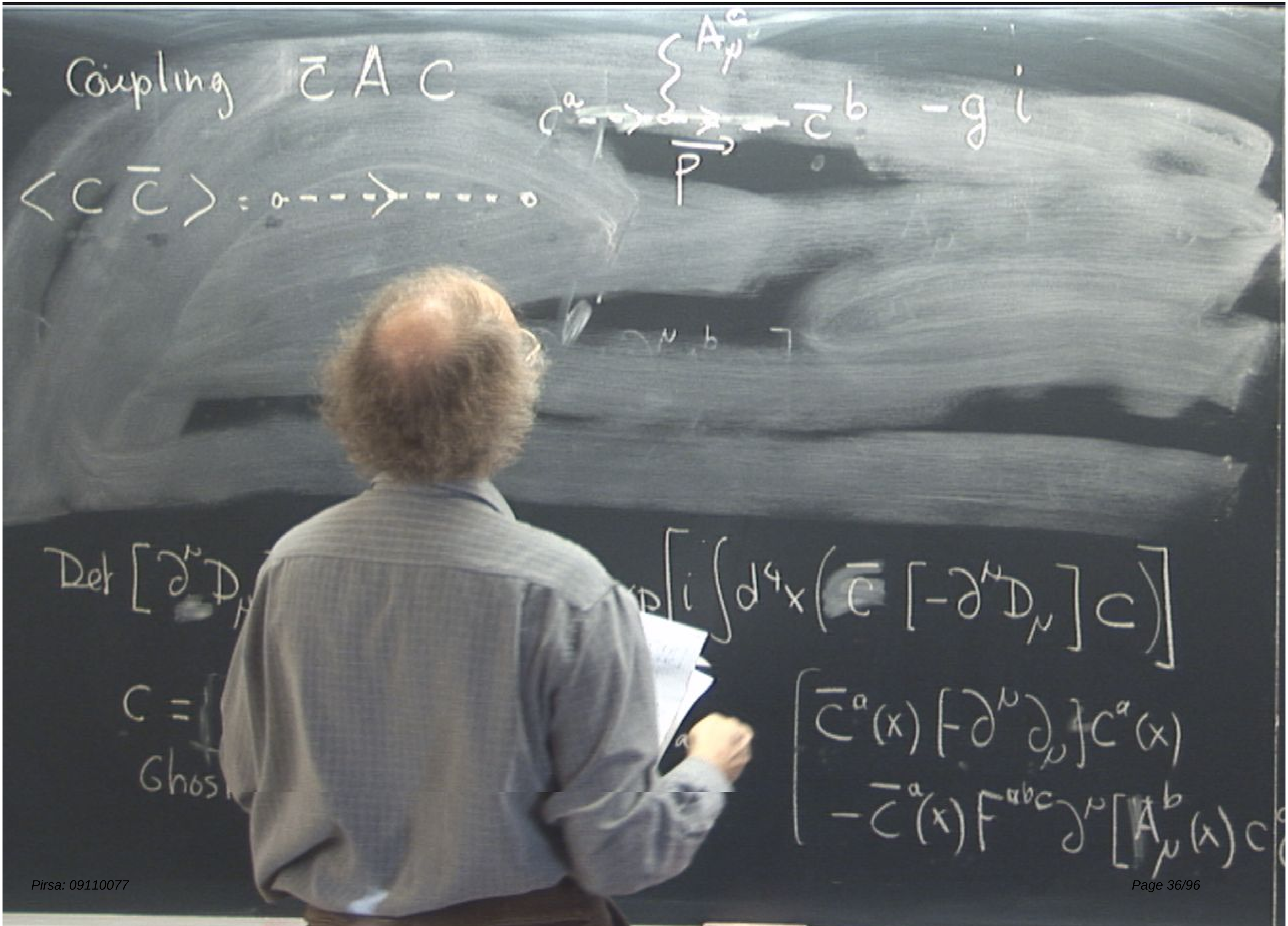
$\langle c \bar{c} \rangle$ = 

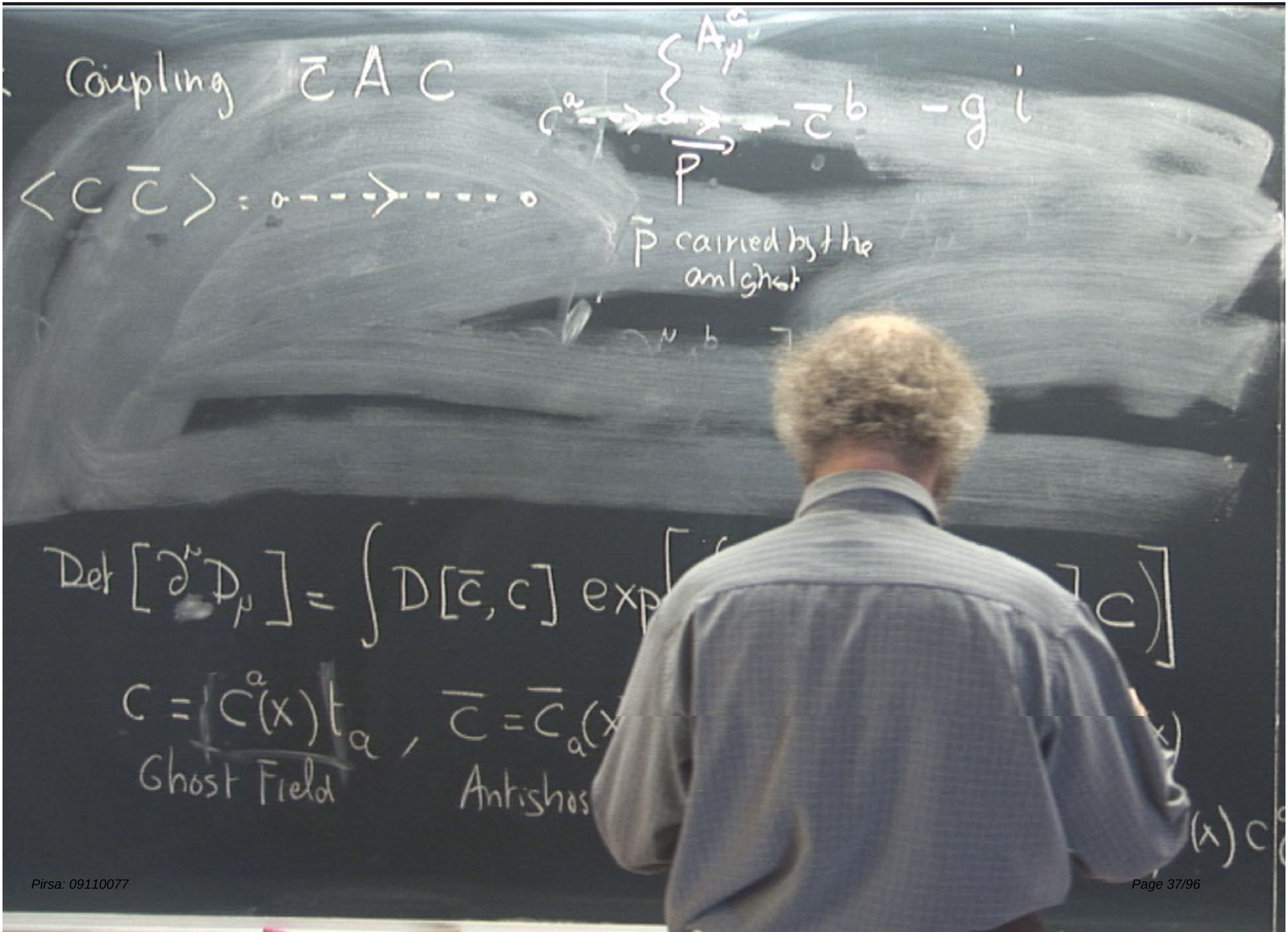
$$\mathrm{Det}\left[\partial^\mu D_\mu\right] = \int D[\bar{c}, c]\, \exp\left[i \int d^4x \left(\bar{c} \left[-\partial^\mu D_\mu \right] c \right) \right]$$

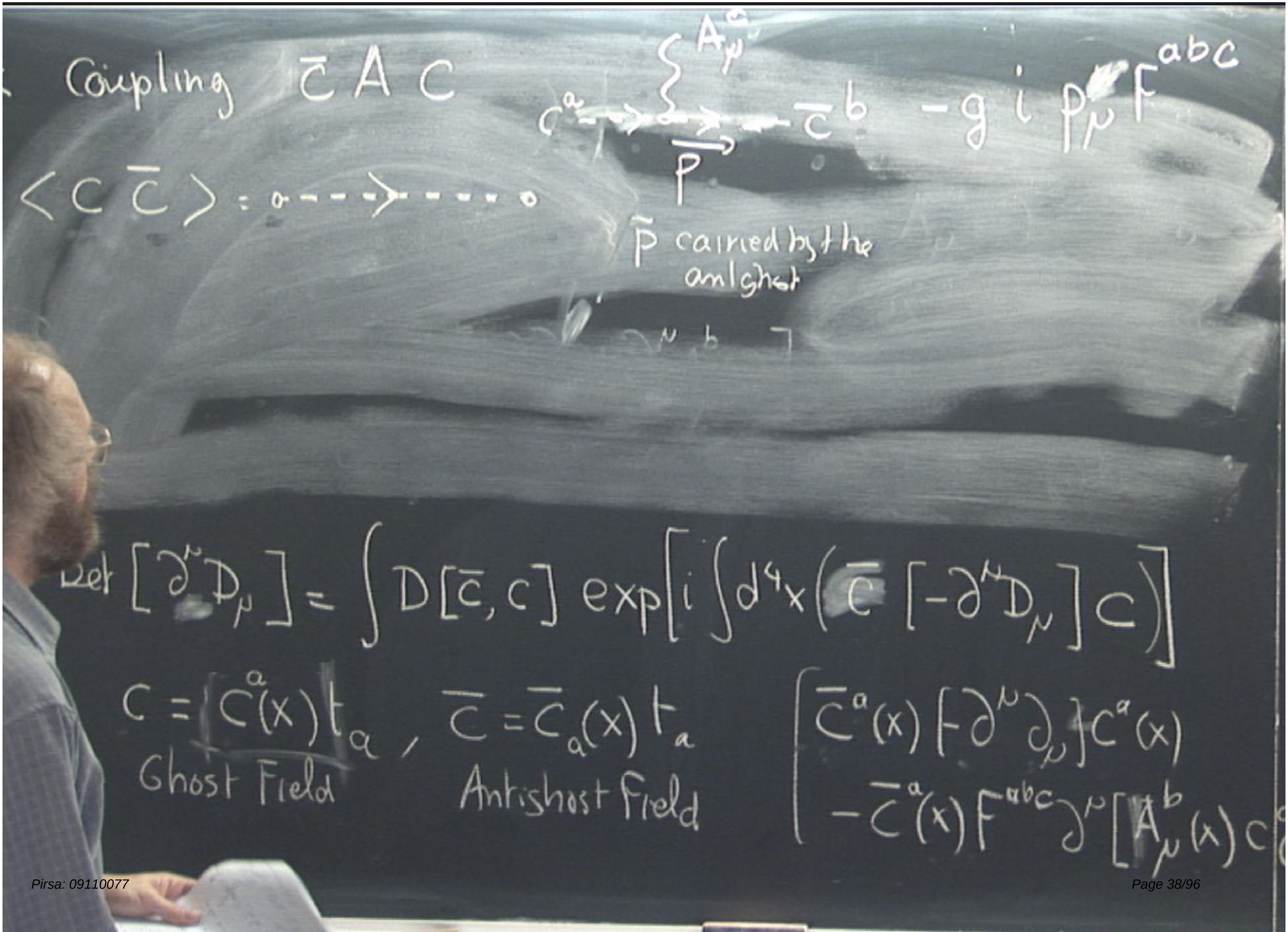
$$c = c^a(x)\, t_a \, , \quad \bar{c} = \bar{c}_a(x)\, t_a$$

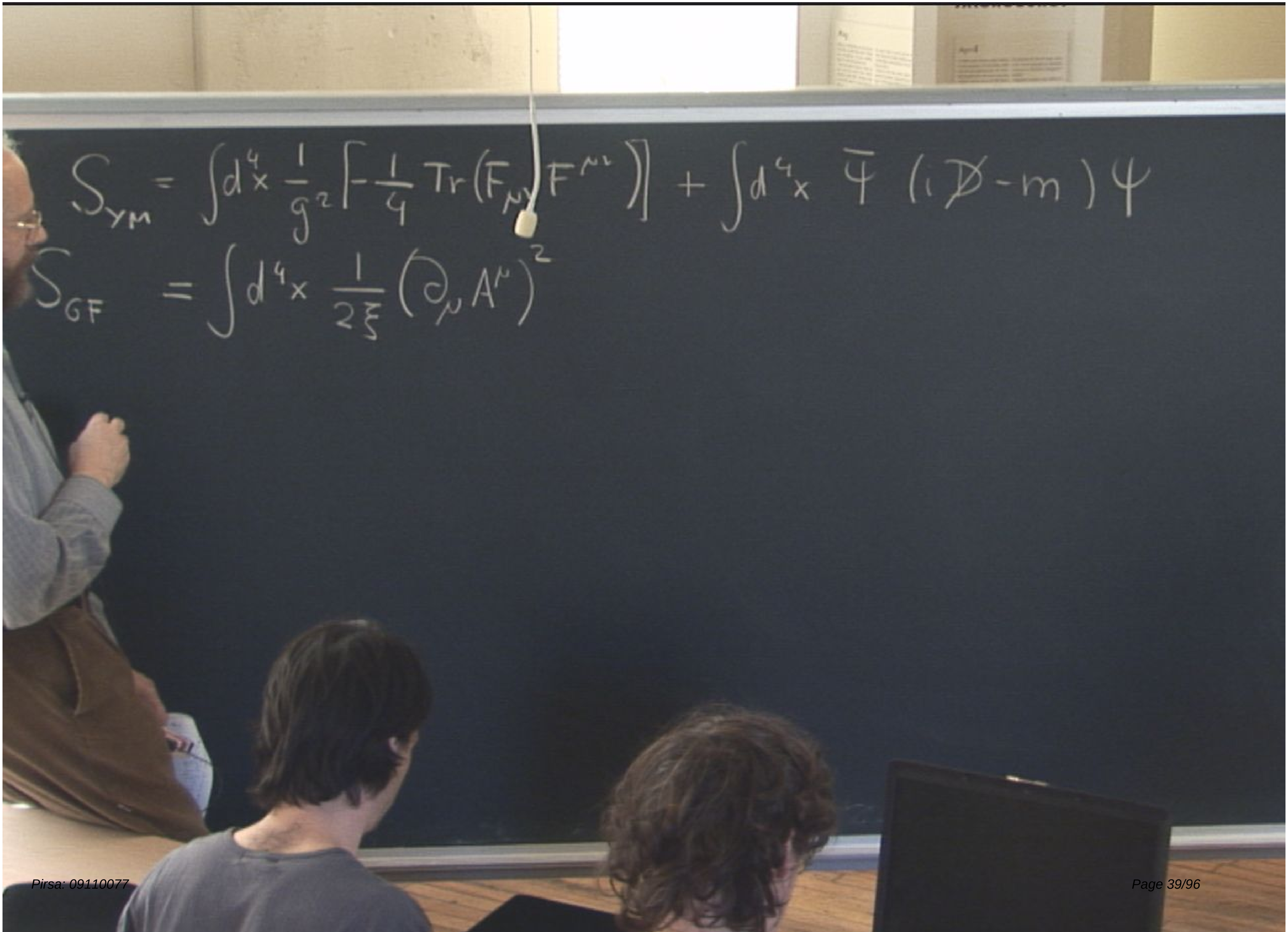
Ghost Field — Antighost Field

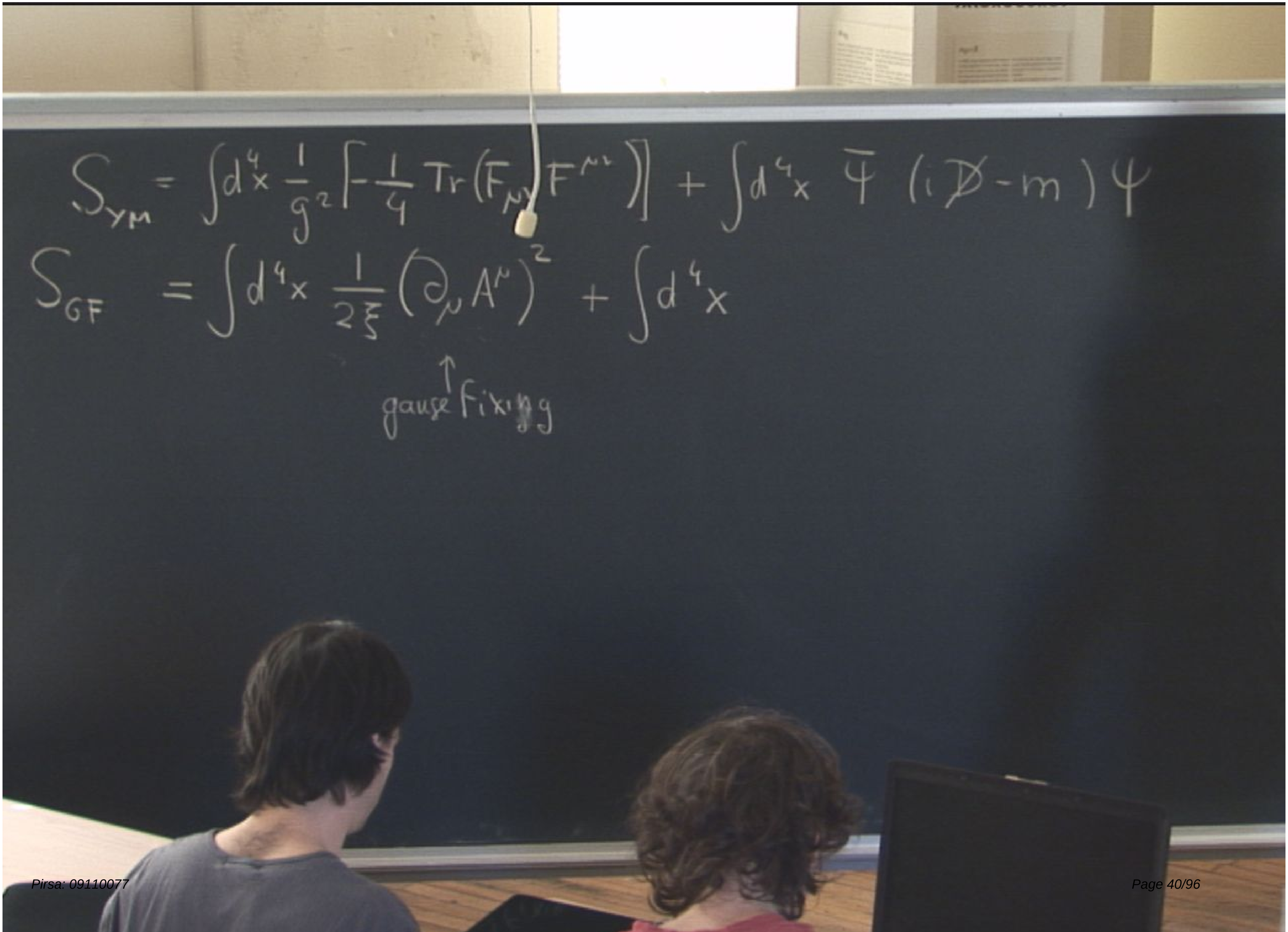
$$\int \bar{c}^a(x) \left[-\partial^\mu \partial_\mu \right] c^a(x) - \bar{c}^a(x) f^{abc} \partial^\mu \left[A^b_\mu(x)\, c \right.$$


Coupling C̄ A C
-g i
< C C̄ > :
Det [∂^μ D_μ
exp [i ∫ d^4 x (C̄ [-∂^μ D_μ] C)]
C =
Ghos
C̄^a (x) [-∂^μ ∂_μ] C^a (x)
- C̄^a (x) F^abc ∂^μ [A^b_μ (x) C

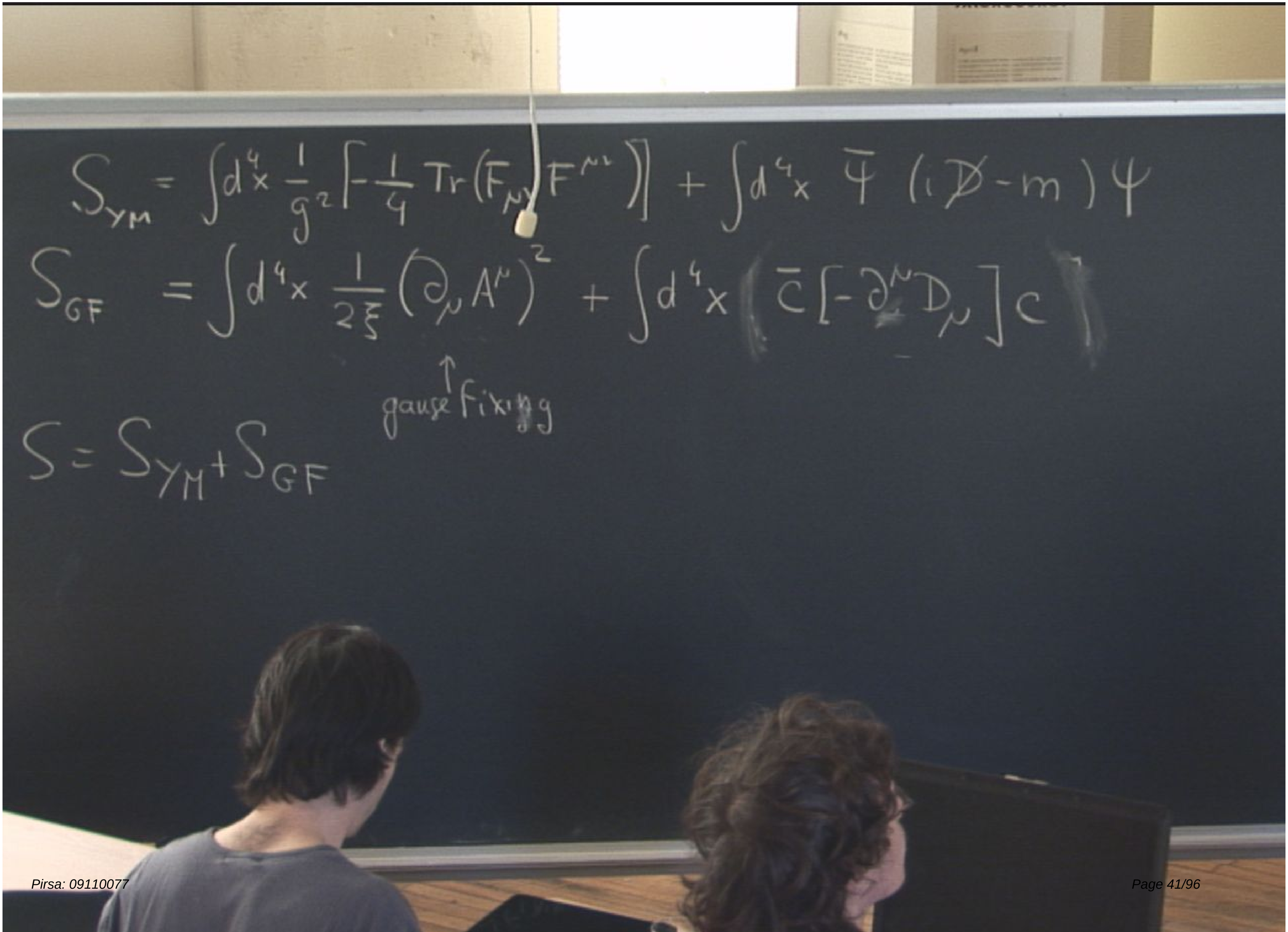

Coupling C̄ A C
⟨C C̄⟩ =
−g i
p̃ carried by the antighost
Det [∂^μ D_μ] = ∫ D[c̄, c] exp
C = C^a(x) t_a , C̄ = C̄_a(x
Ghost Field

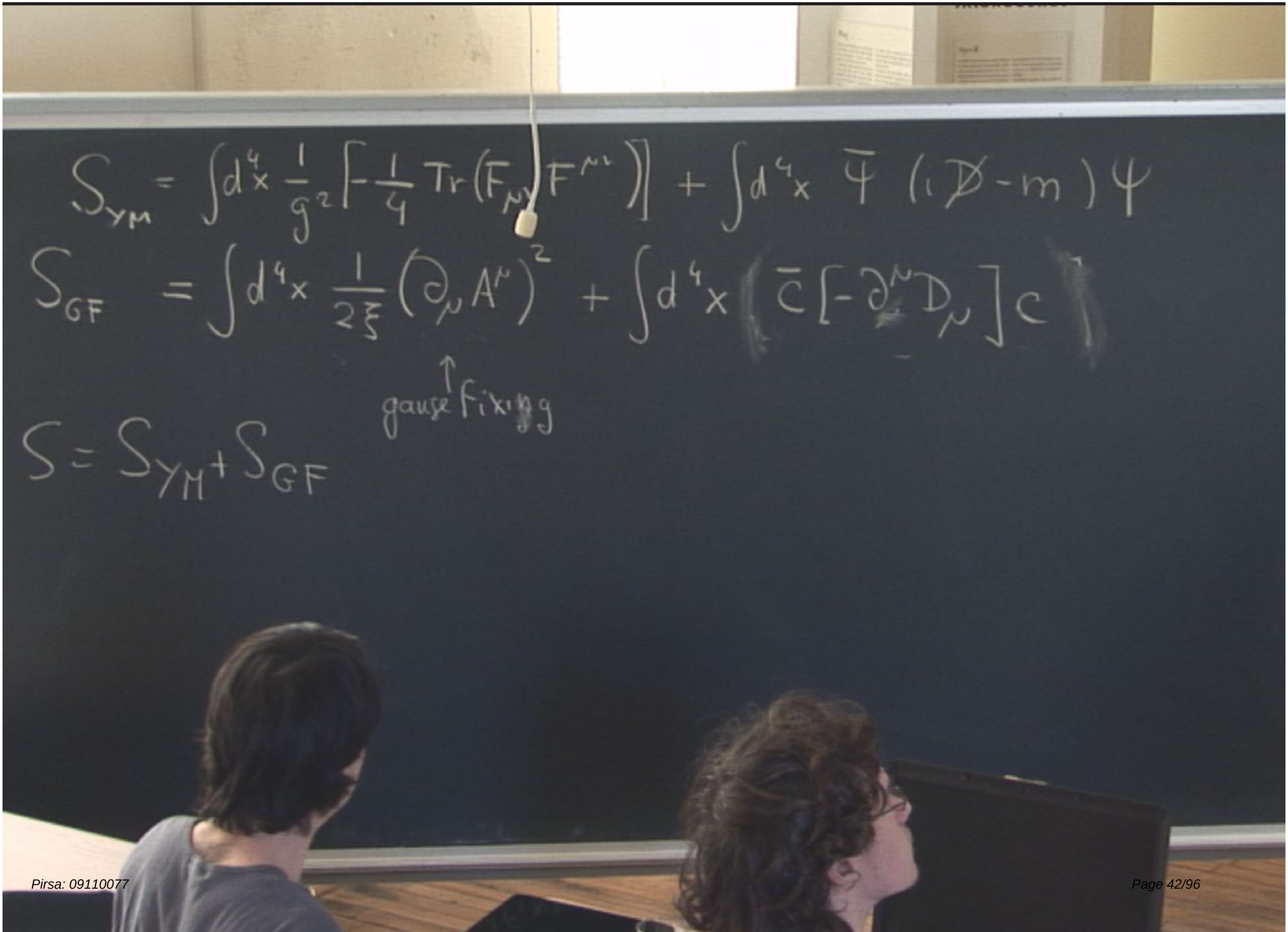

Coupling $\bar{c} A c$
$\langle c \bar{c} \rangle$
A^a_μ
c^a $\bar{c}^b$ $-g\, i\, p_\mu F^{abc}$
p
$\tilde{p}$ carried by the antighost
$\mathrm{Det}[\partial^\mu D_\mu] = \int D[\bar{c}, c] \exp\left[i \int d^4x \left(\bar{c}\, [-\partial^\mu D_\mu]\, c\right)\right]$
$c = c^a(x)\, t_a$, $\bar{c} = \bar{c}_a(x)\, t_a$
Ghost Field
Antighost Field
$\bar{c}^a(x)\, [-\partial^\mu \partial_\mu]\, c^a(x)$
$- \bar{c}^a(x)\, F^{abc} \partial^\mu [A^b_\mu(x)\, c$


S_YM = ∫d^4x 1/g^2 [-1/4 Tr(F_μν F^μν)] + ∫d^4x Ψ̄ (i D̸ - m) Ψ
S_GF = ∫d^4x 1/(2ξ) (∂_μ A^μ)^2

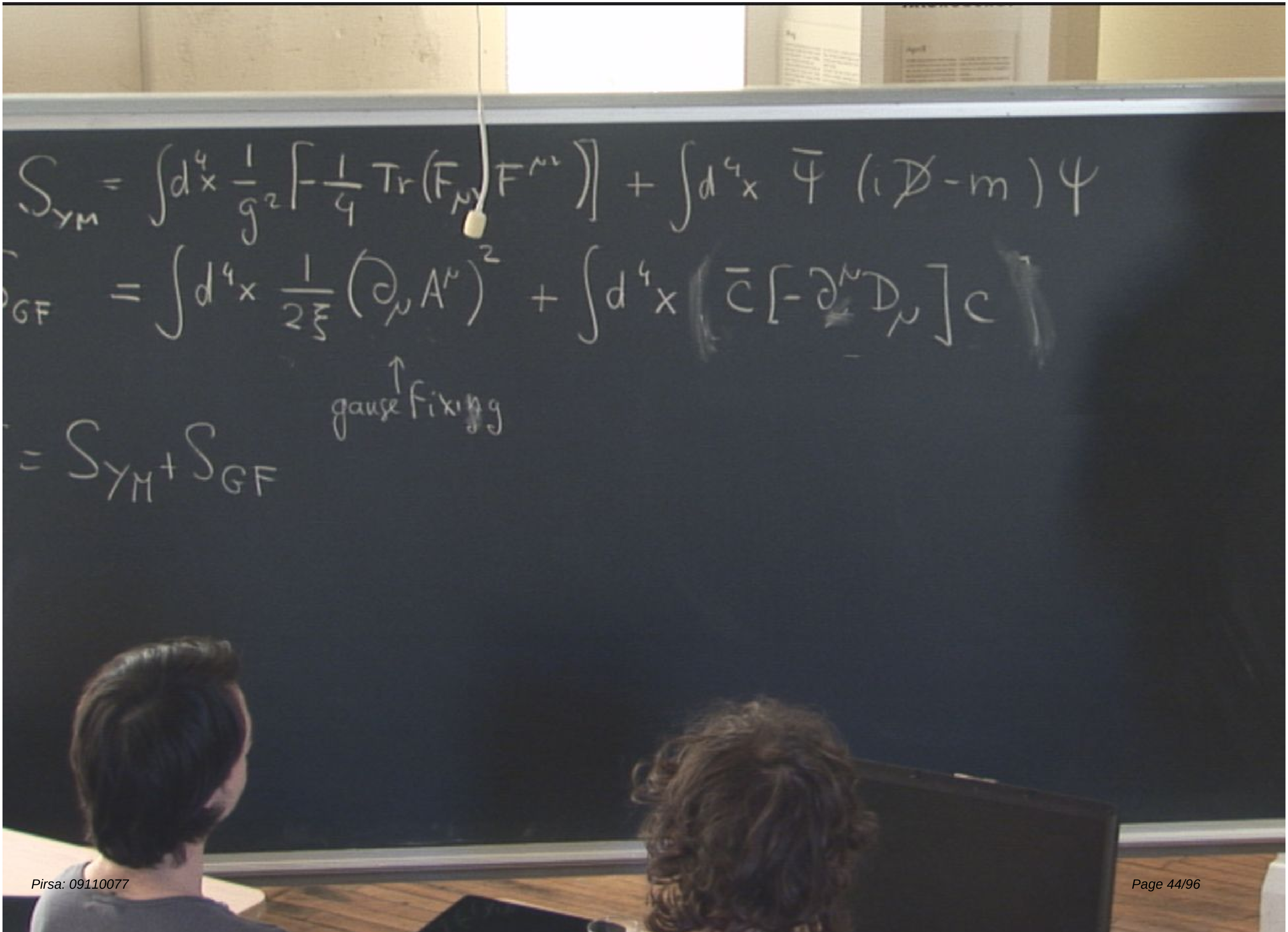


$S_{YM} = \int d^4x \frac{1}{g^2}\left[-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\right] + \int d^4x\, \bar{\psi}\,(i\not{D} - m)\,\psi$
$S_{GF} = \int d^4x\, \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \int d^4x$
gauge fixing


$S_{YM} = \int d^4x \frac{1}{g^2}\left[-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\right] + \int d^4x\, \bar{\Psi}(i\not{D} - m)\Psi$
$S_{GF} = \int d^4x \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \int d^4x\, (\bar{c}[-\partial^\mu D_\mu]c)$
gauge fixing
$S = S_{YM} + S_{GF}$


$S_{YM} = \int d^4x \, \frac{1}{g^2} \left[-\frac{1}{4} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \right] + \int d^4x \, \bar{\Psi} \, (i \not{D} - m) \Psi$
$S_{GF} = \int d^4x \, \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \int d^4x \, \bar{c} [-\partial^\mu D_\mu] c$
gauge fixing
$S = S_{YM} + S_{GF}$

What are the Feynman rules of such a theory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gauge fixing
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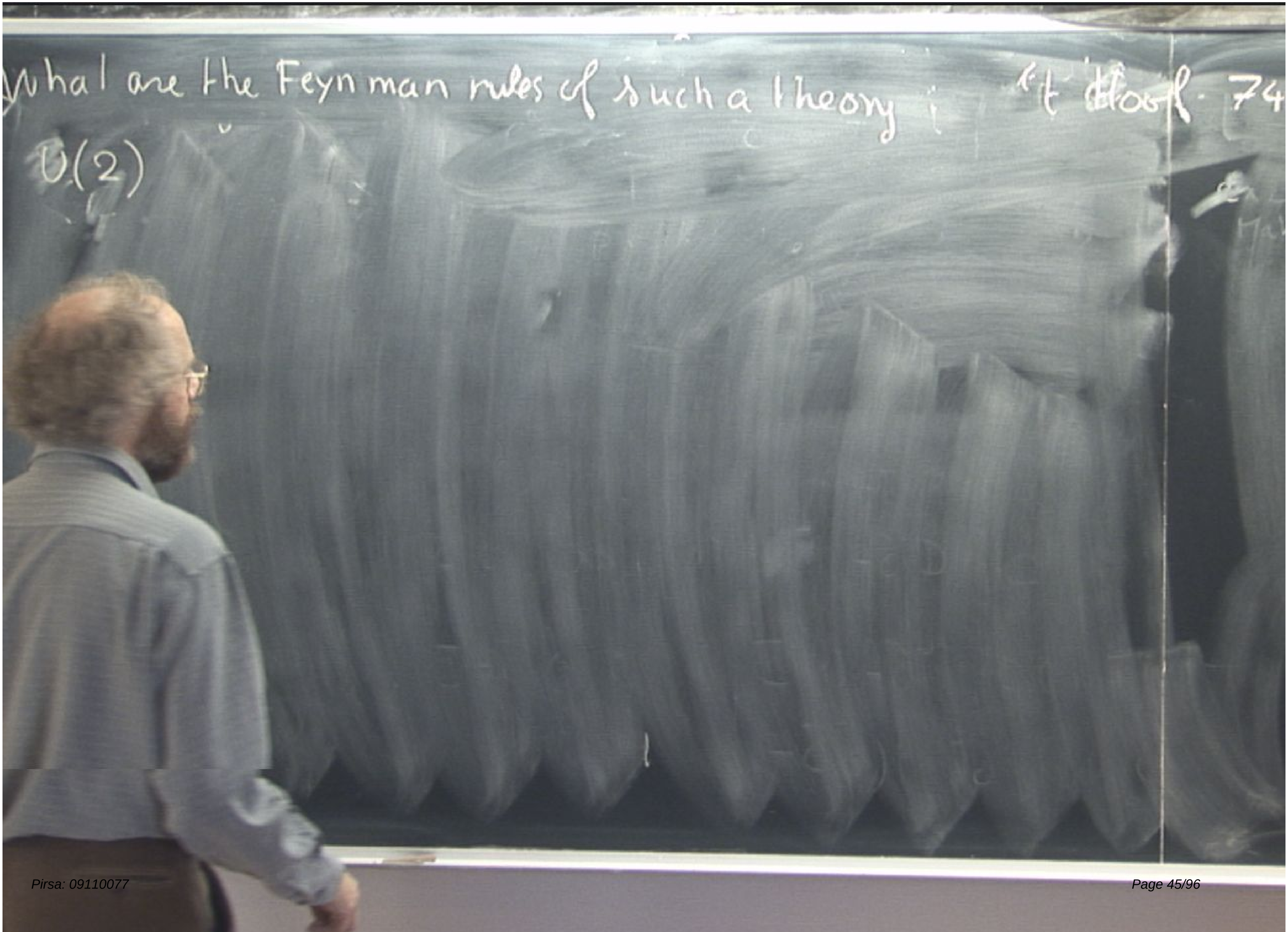
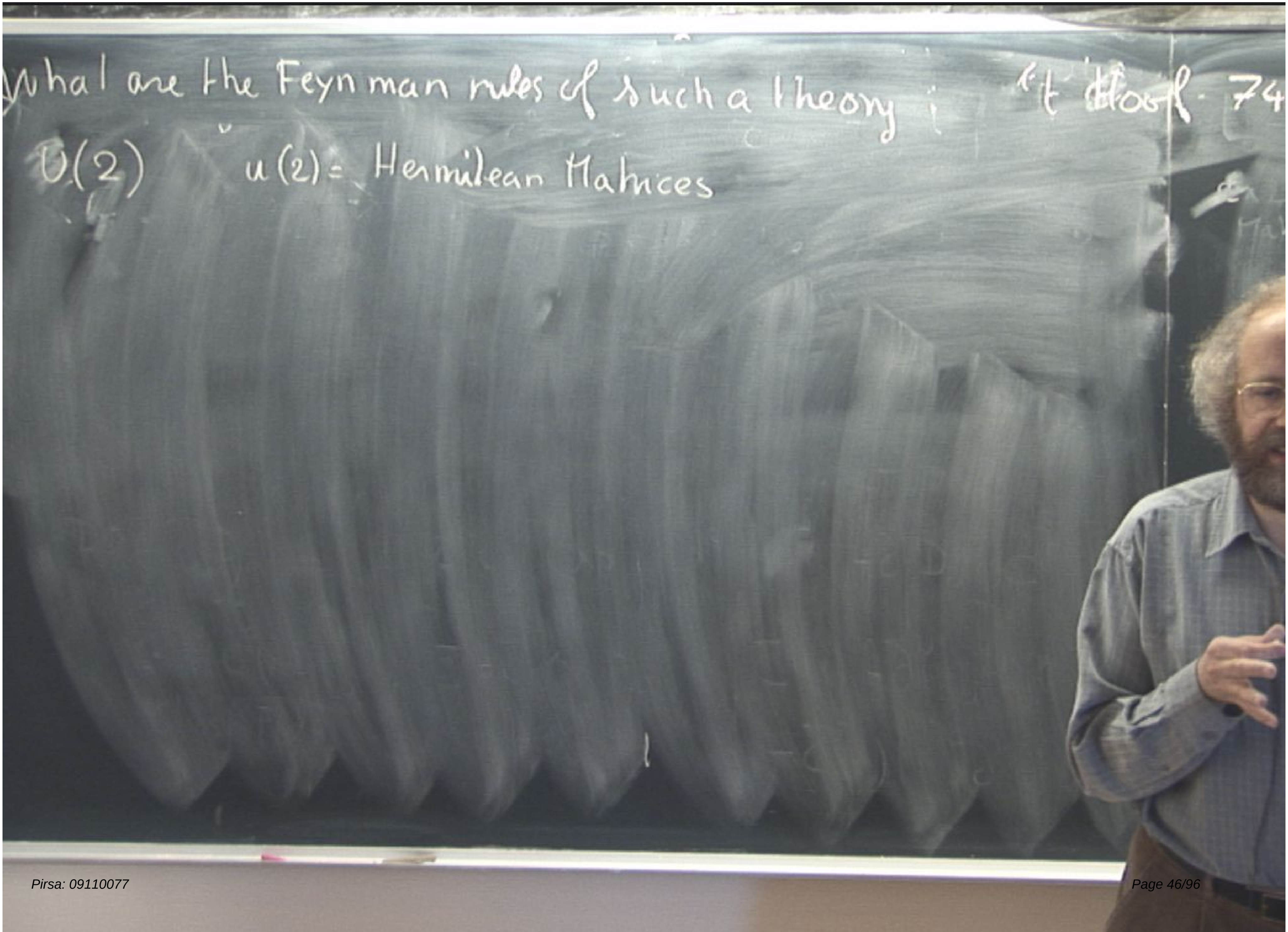
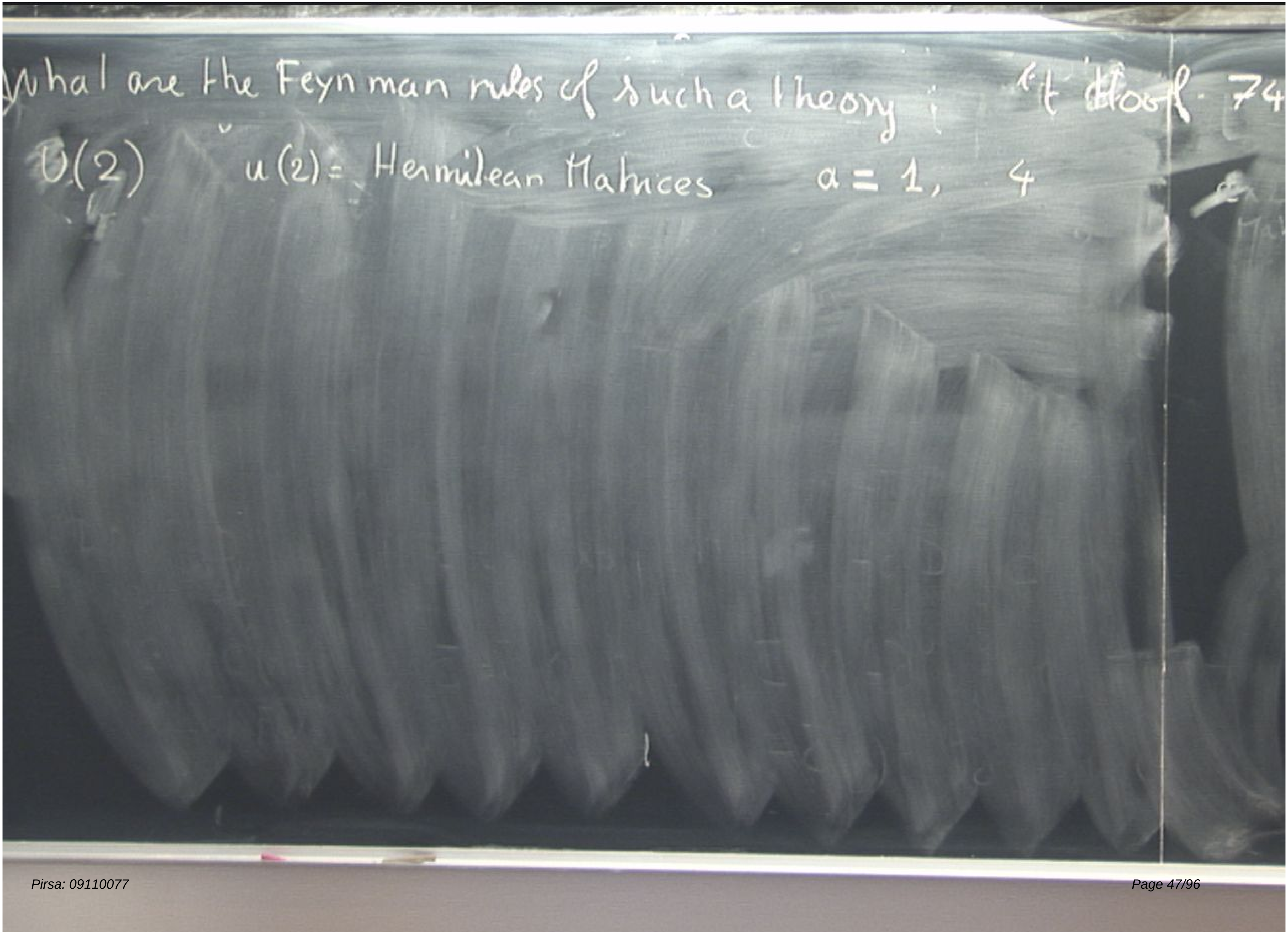

What are the Feynman rules of such a theory ;
't Hooft 74
U(2)

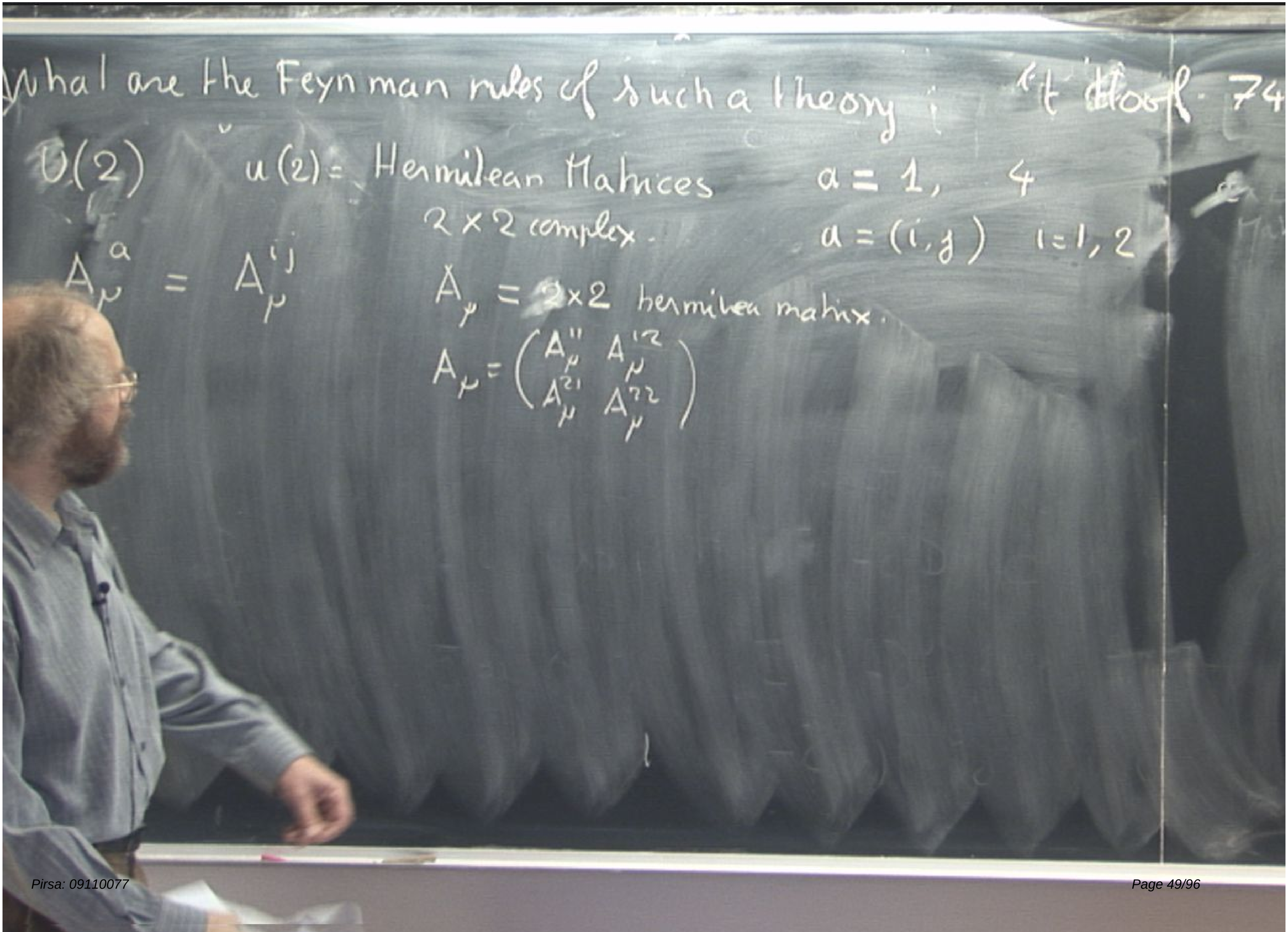

What are the Feynman rules of such a theory ; 't Hooft 74
U(2)
u(2) = Hermitean Matrices


What are the Feynman rules of such a theory ; 't Hooft 74
U(2) u(2) = Hermitean Matrices a = 1, 4

What are the Feynman rules of such a theory ; 't Hooft 74

$U(2)$ $u(2) =$ Hermitean Matrices $a = 1, \ldots 4$

$a = (i, j)$ $i = 1, 2$


What are the Feynman rules of such a theory ; 't Hooft 74
U(2) u(2) = Hermitean Matrices a = 1, 4
2 × 2 complex. a = (i, j) i = 1, 2
$A_\mu^a = A_\mu^{ij}$
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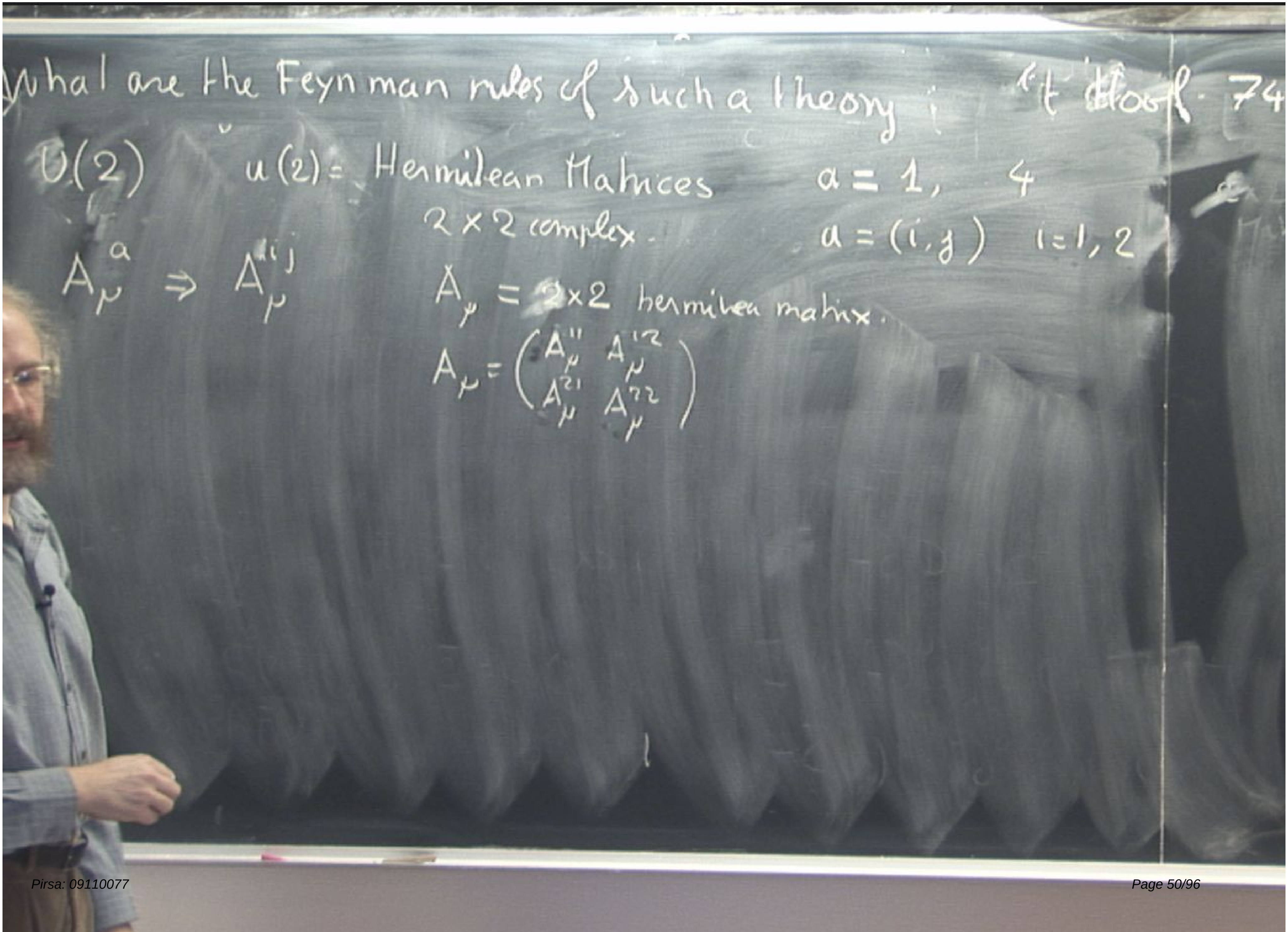
$A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix}$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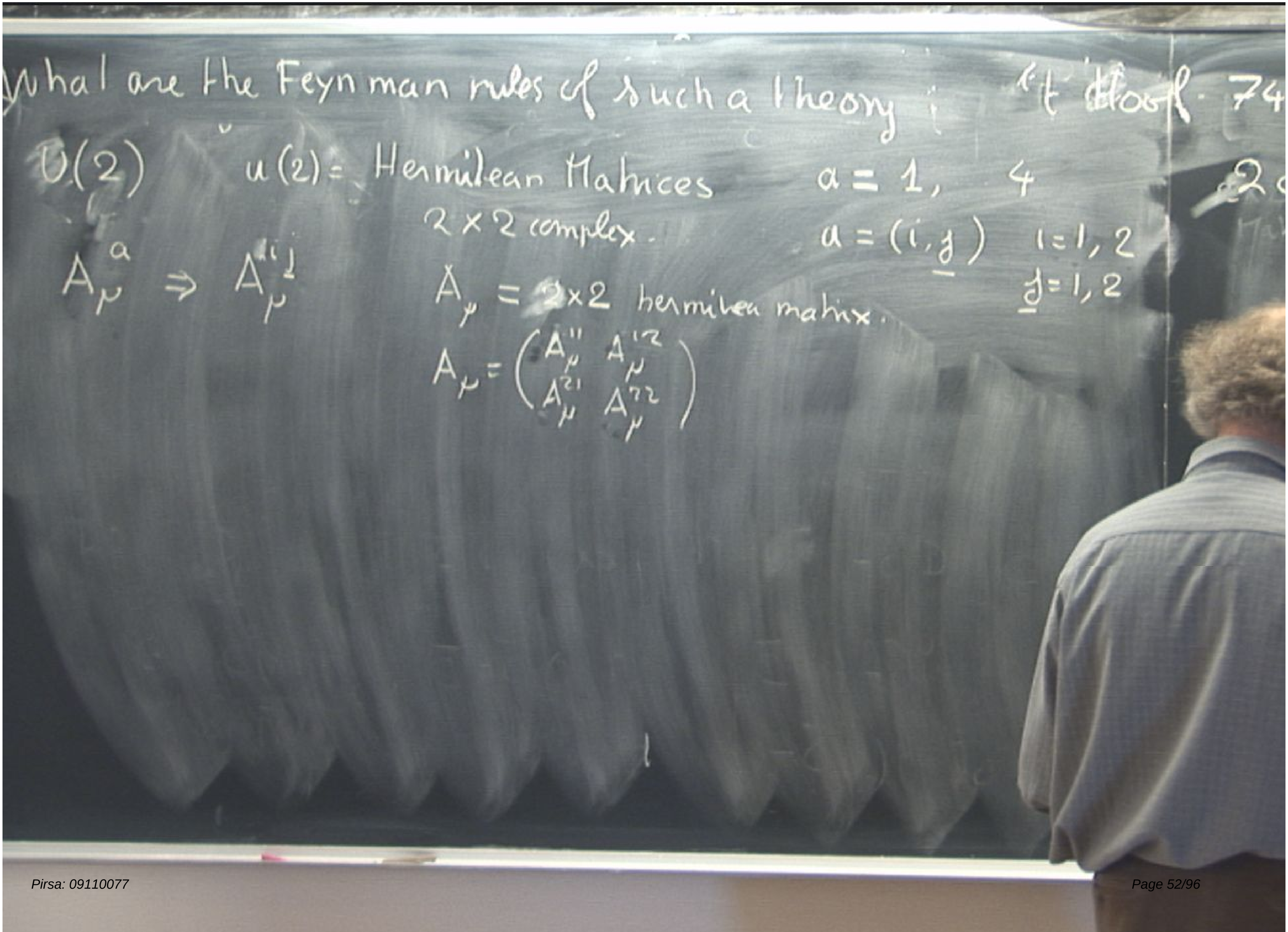
What are the Feynman rules of such a theory ? 't Hooft 74
U(2) u(2) = Hermitean Matrices a = 1, 4
2 x 2 complex.
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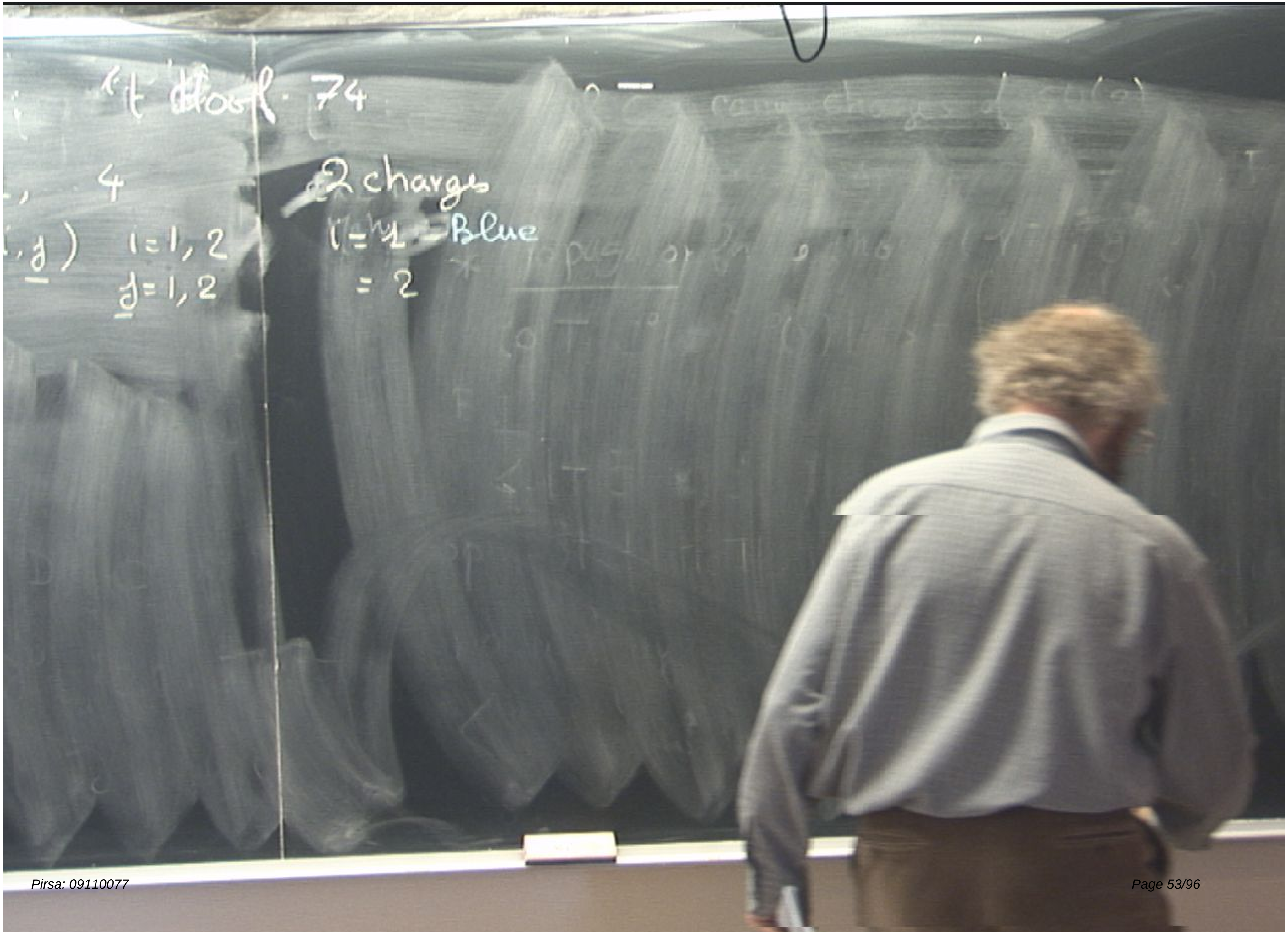
j = 1, 2
A_μ^a ⇒ A_μ^{ij}
A_μ = 2x2 hermitean matrix.
A_μ = (A_μ^11 A_μ^12 / A_μ^21 A_μ^22)


74
4
2 charges
i = 1, 2
j = 1, 2
Blue
= 2

74

ψ^i

2 charges

$\underline{i} = 1$ Blue $\underline{i} = 1$ antiblue

$\underline{i} = 2$ Red $\underline{i} = 2$ anti-red

What are the Feynman rules of such a theory ? 't Hoo[ft]

$U(2)$ $\quad u(2) =$ Hermitean Matrices $\quad a = 1, \ldots, 4$

2 × 2 complex. $\quad a = (i, j) \quad i = 1, 2$

$j = 1, 2$

$$A_\mu^{a} \Rightarrow A_\mu^{ij}$$

$A_\mu = 2\times 2$ hermitean matrix.

$$A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix} = A_\mu^{ij}\, t_{ij}$$

What are the Feynman rules of such a theory? 't Hooft

$U(2)$ $u(2)$ = Hermitean Matrices $a = 1, \ldots 4$

2 × 2 complex. $a = (i, j)$ $i = 1, 2$

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$A_\mu^{ij} = \overline{A_\mu^{ji}}$ $A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix} = A_\mu^{ij} \cdot t_{ij}$

What are the Feynman rules of such a theory? ... t Hooft

$U(2)$ $\quad u(2) =$ Hermitean Matrices $\quad a = 1, \ldots 4$

2×2 complex. $\quad a = (i, \underline{j})$ $\quad i = 1, 2$

$\underline{j} = 1, 2$

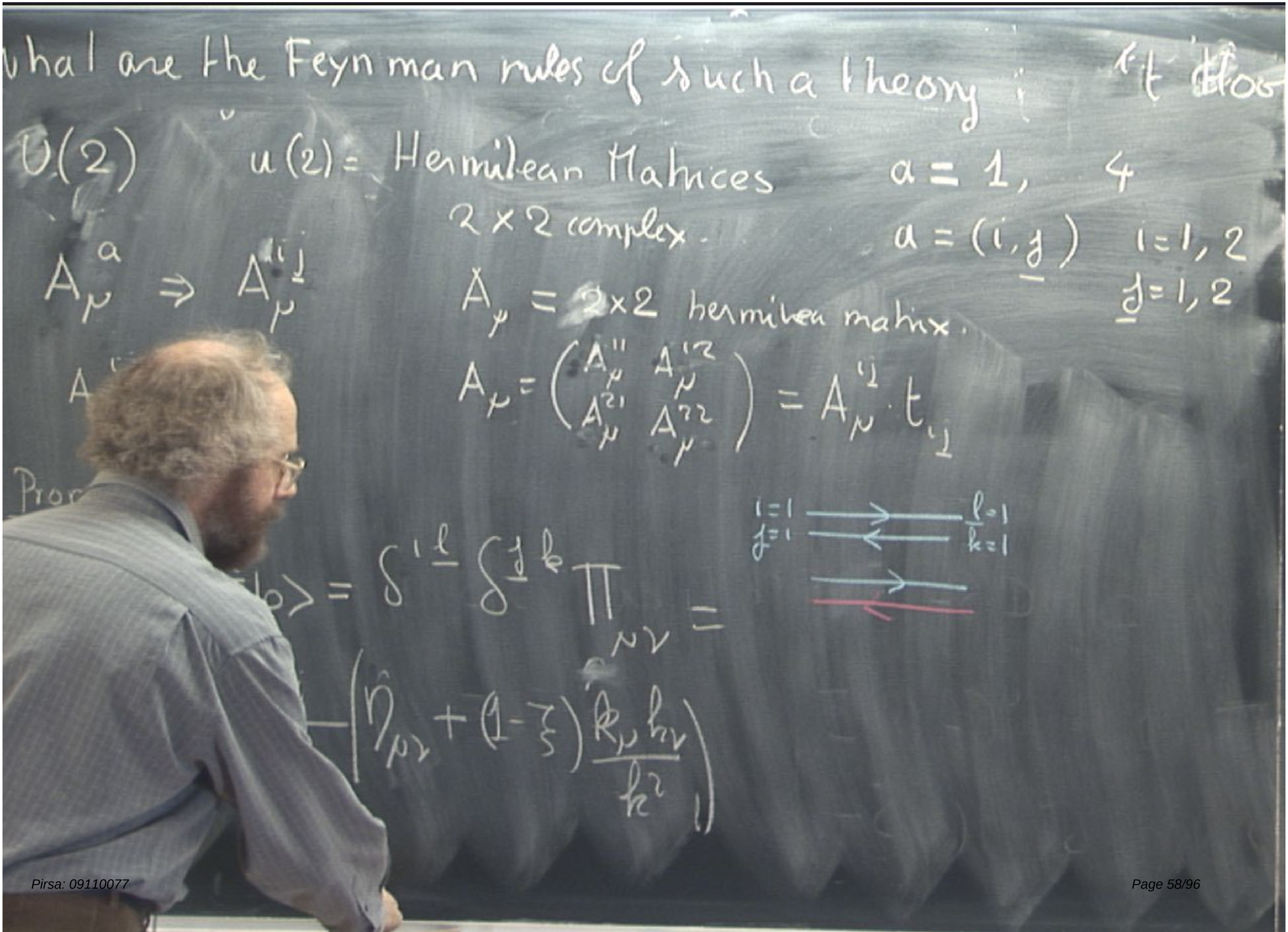
$A_\mu^a \Rightarrow A_\mu^{i\underline{j}}$ $\quad A_\mu = 2 \times 2$ hermitean matrix.

$A_\mu^{i\underline{j}} = \overline{A_\mu^{j\underline{i}}}$ $\quad A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix} = A_\mu^{i\underline{j}} \cdot t_{i\underline{j}}$

Propagators:

$$\langle 0 | T A_\mu^{i\underline{j}} A_\nu^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{\underline{j}k} \Pi_{\mu\nu} =$$

$$\Pi_{\mu\nu} = \frac{-i}{p^2} \left(\eta_{\mu\nu} + (1-\xi) \frac{k_\mu k_\nu}{k^2} \right)$$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Pro
> = δ^il δ^jk Π_μν =
(η_μν + (1 − ξ) k_μ k_ν / k^2)
i=1 l=1
j=1 k=1

What are the Feynman rules of such a theory ; 't Hooft

$U(2)$ $\quad u(2) =$ Hermitean Matrices $\quad a = 1, \ldots, 4$

2×2 complex. $\quad a = (i, j) \quad i = 1, 2 \quad j = 1, 2$

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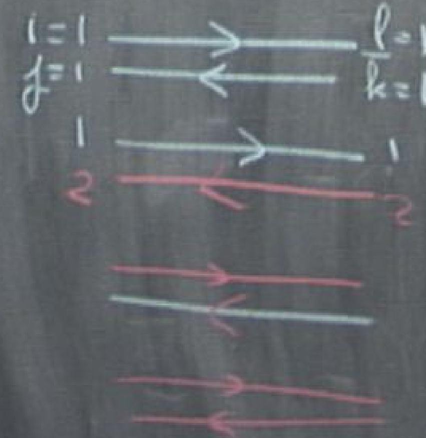

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$U(2)$ $\quad u(2) =$ Hermitean Matrices $\quad a = 1, \ldots 4$

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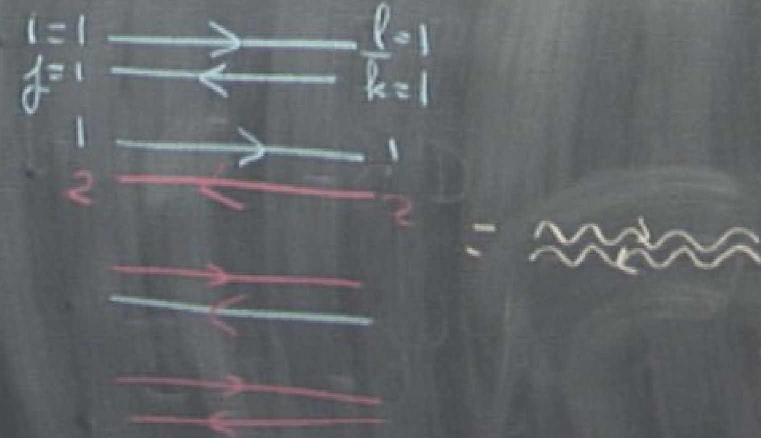
...ctors :

$\langle A^{ij} A^{kl} \rangle = \delta^{il} \delta^{jk} \Pi_{\mu\nu} =$



$\Pi_{\mu\nu} = \frac{-i}{p^2}\left(\eta_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)$

What are the Feynman rules of such a theory ; 't Hooft

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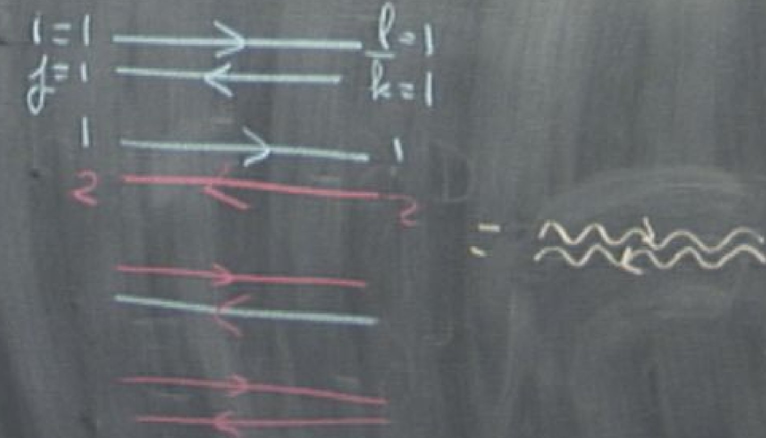
Propagators:

$$\langle 0 | T A_\mu^{ij} A_\nu^{kl} | 0 \rangle = \delta^{il} \delta^{jk} \Pi_{\mu\nu} =$$



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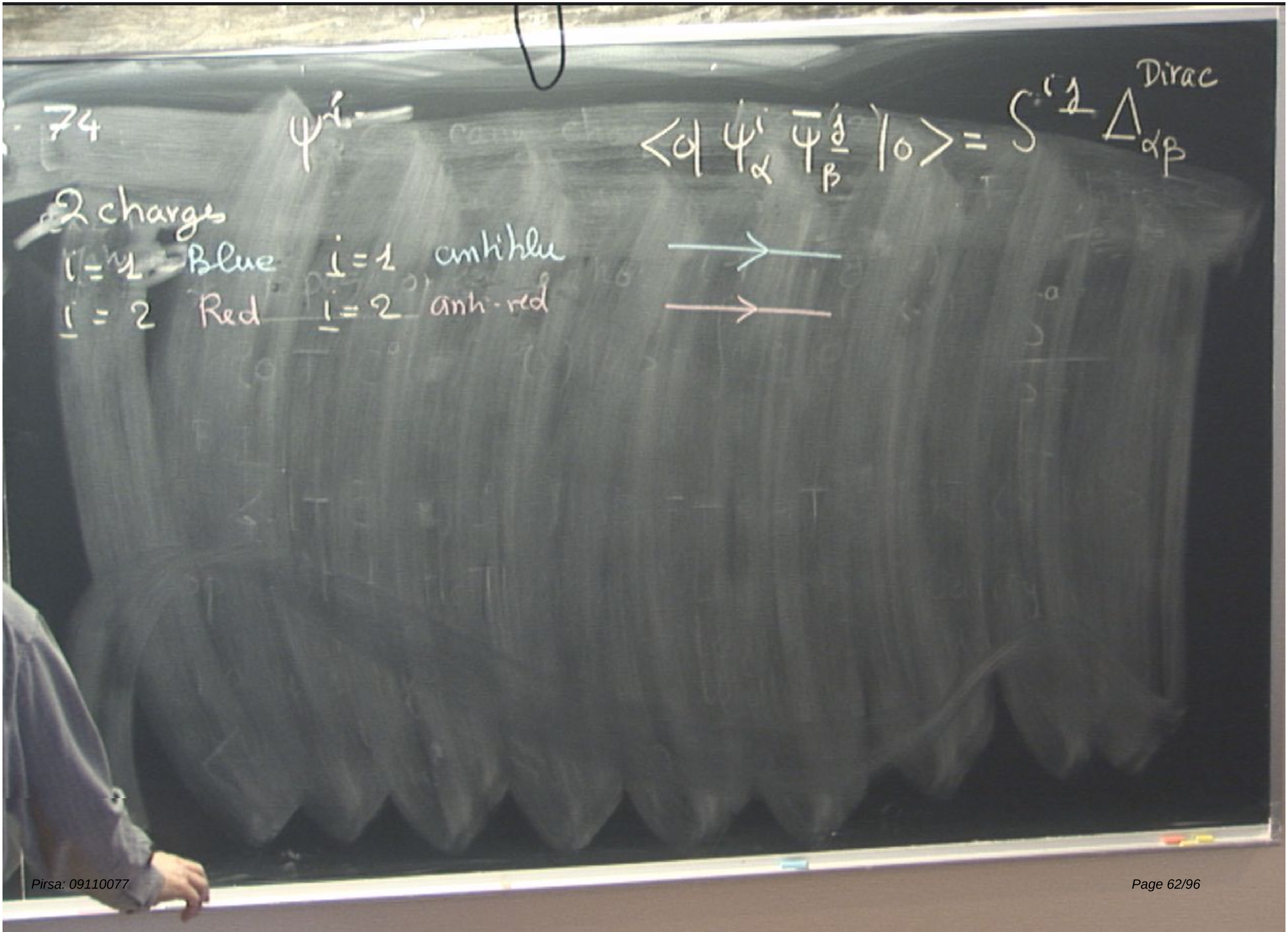
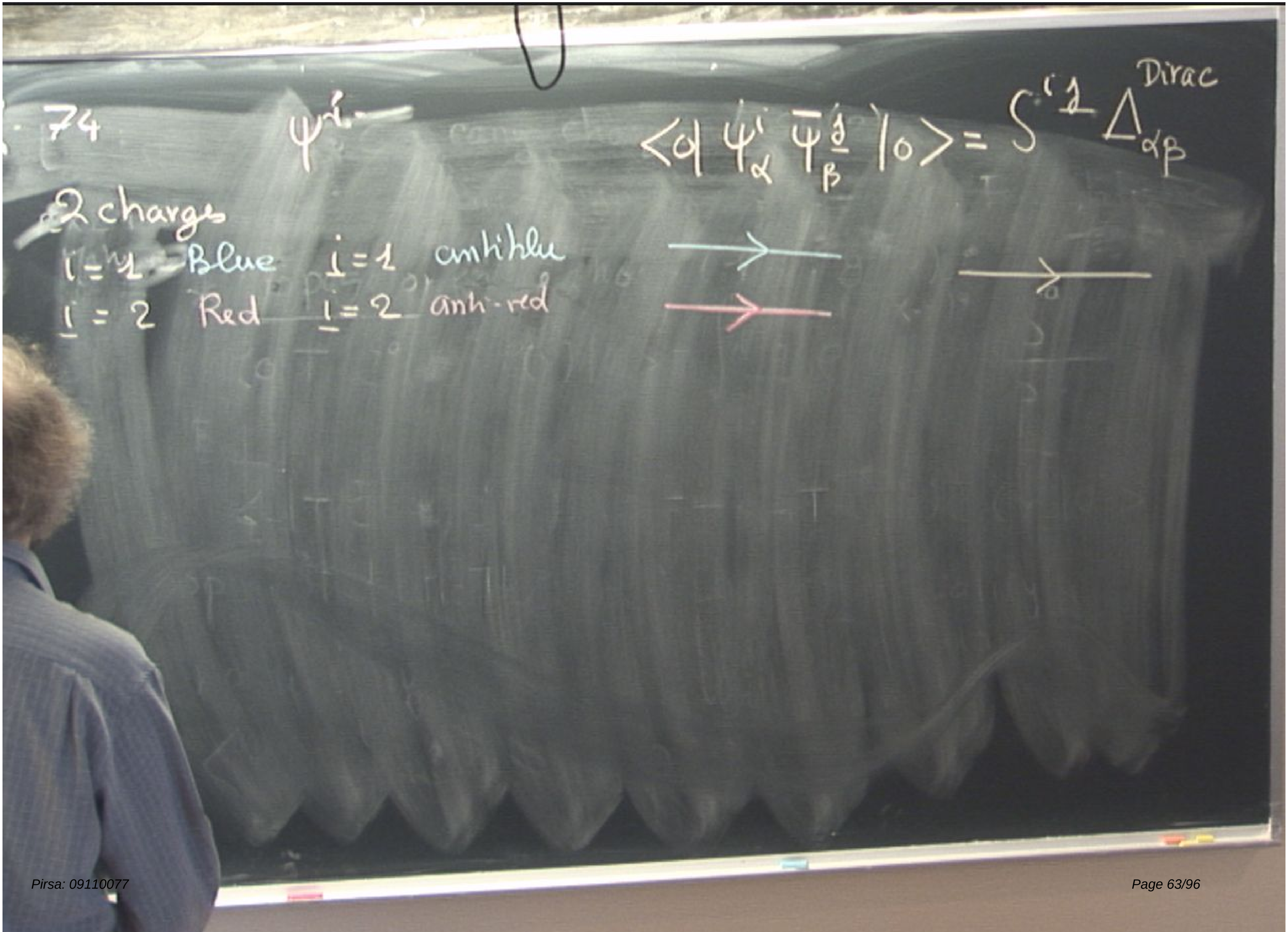
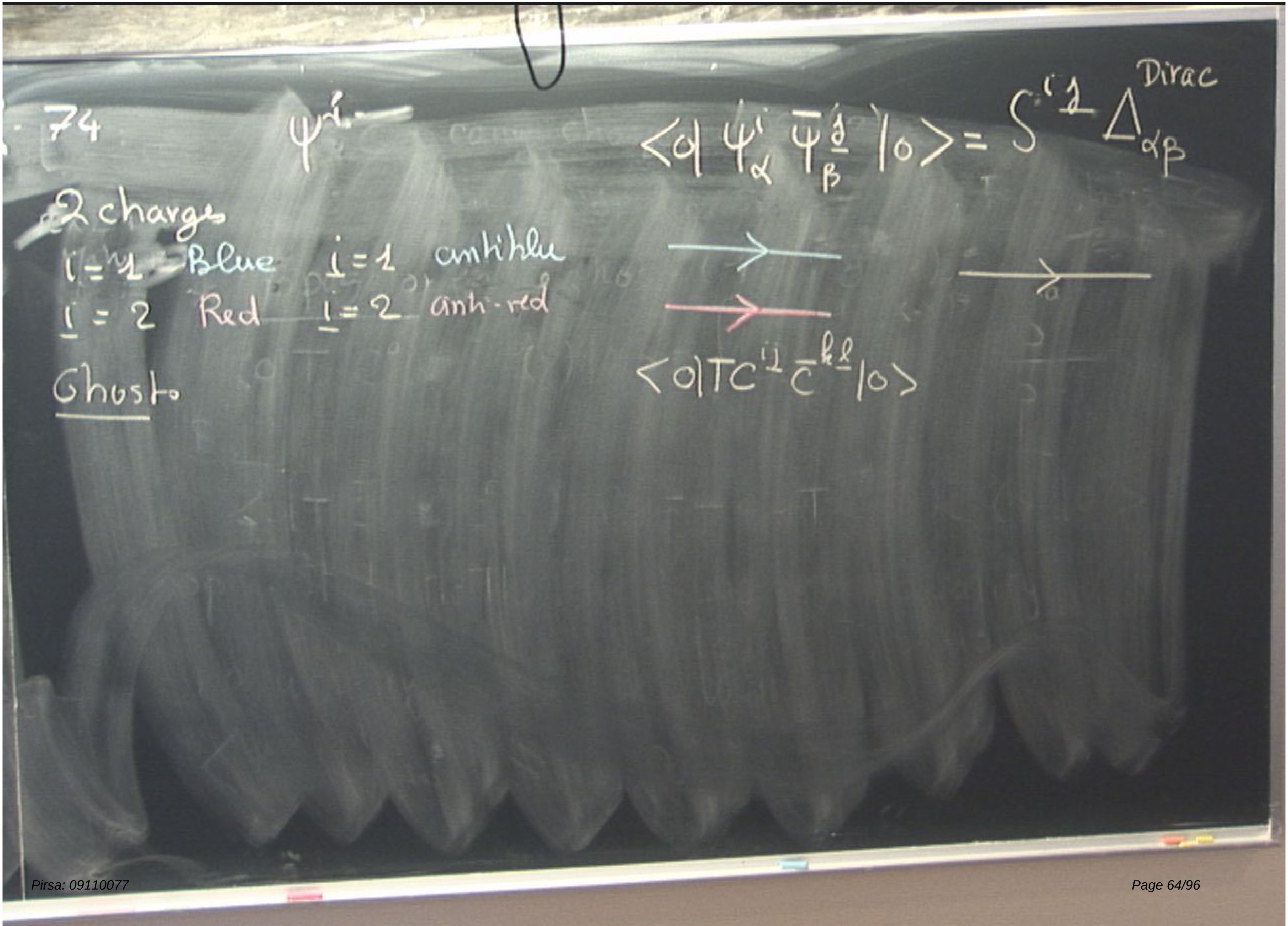

74
ψ^i
2 charges
i = 1 Blue   i = 1 antiblue
i = 2 Red   i = 2 anti-red
<0| ψ^i_α ψ̄^j_β |0> = δ^ij Δ^Dirac_αβ


74
ψ^i
⟨0| ψ^i_α ψ̄^j_β |0⟩ = δ^{ij} Δ^{Dirac}_{αβ}
2 charges
i = 1 Blue
i = 1 anhihlu
i = 2 Red
i = 2 anh-red


74
Dirac
ψ^i
$\langle 0|\, \psi^i_\alpha\, \bar{\psi}^j_\beta\, |0\rangle = \delta^{ij}\, \Delta_{\alpha\beta}$
2 charges
i = 1 Blue
i = 1 antiblue
i = 2 Red
i = 2 anti-red
$\langle 0|T c^{ij}\, \bar{c}^{kl}\, |0\rangle$
Ghosts

74

ψ^{i}

2 charges

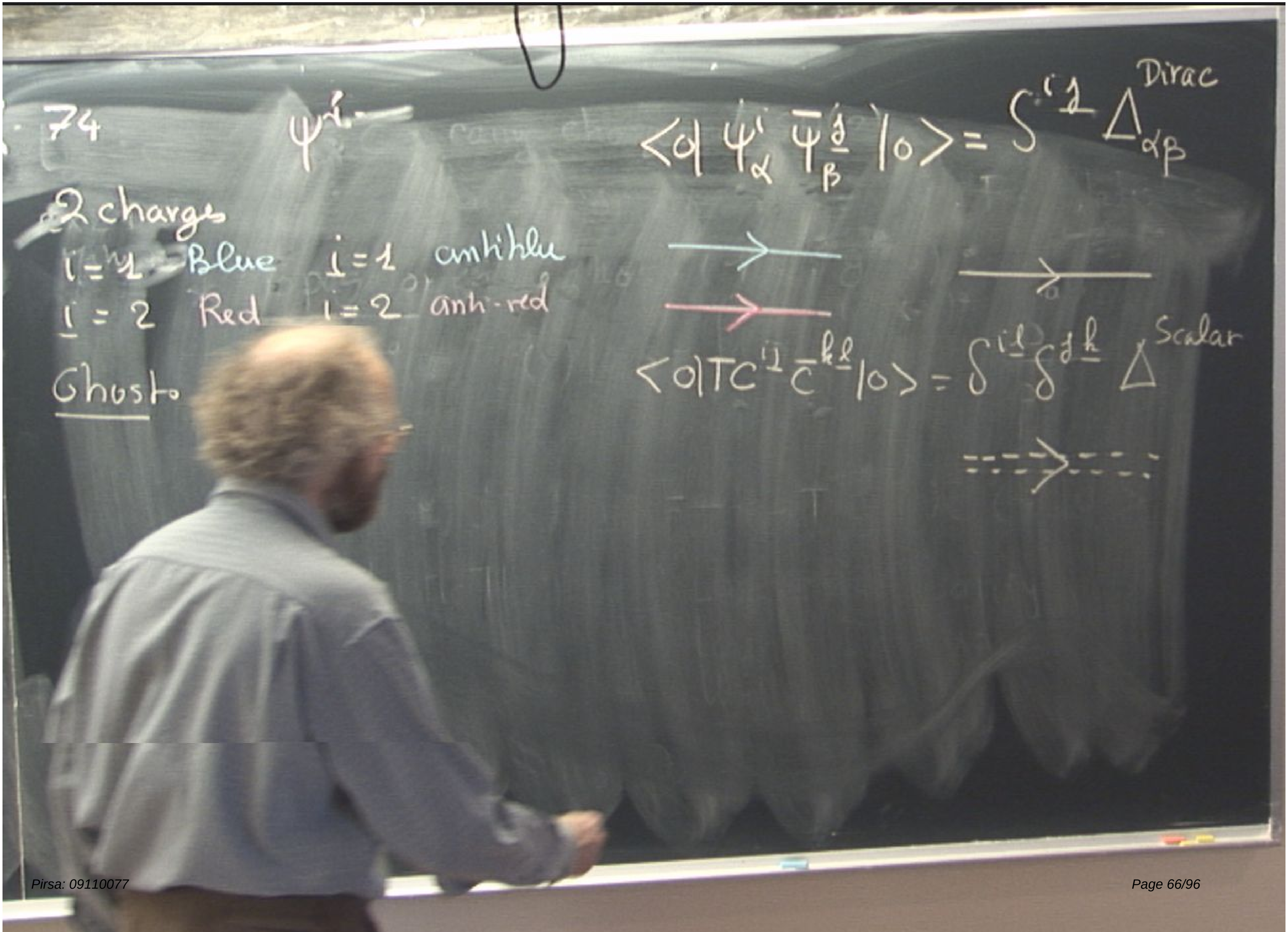
$i = 1$ Blue $\quad \underline{i} = 1$ antiblue

$i = 2$ Red $\quad \underline{i} = 2$ anti-red

$$\langle 0 | \psi^{i}_{\alpha} \bar{\psi}^{\underline{j}}_{\beta} | 0 \rangle = \delta^{i\underline{j}} \Delta^{\text{Dirac}}_{\alpha\beta}$$

Ghosts

$$\langle 0 | T c^{i\underline{j}} \bar{c}^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{\underline{j}k} \Delta^{\text{Scalar}}$$


74
2 charges
i = 1 Blue
i = 2 Red
i = 1 antiblue
i = 2 anti-red
Ghosts
Dirac
Scalar

74

ψ^{i}

$$\langle 0|\, \psi^{i}_{\alpha}\, \bar{\psi}^{\underline{j}}_{\beta}\, |0\rangle = \delta^{i\underline{j}}\, \Delta^{\text{Dirac}}_{\alpha\beta}$$

2 charges

$i=1$ Blue $\quad \underline{i}=1$ antiblue

$i=2$ Red $\quad \underline{i}=2$ anti-red

Ghosts

$$\langle 0|T c^{i\underline{j}}\, \bar{c}^{k\underline{l}}|0\rangle = \delta^{i\underline{l}}\, \delta^{\underline{j}\underline{k}}\, \Delta^{\text{Scalar}}$$

74

ψ^{i} — **Fermion Propagator** $\langle 0|\, \psi^{i}_{\alpha}\, \bar{\psi}^{\underline{j}}_{\beta}\, |0\rangle = \delta^{i\underline{j}}\, \Delta^{\text{Dirac}}_{\alpha\beta}$

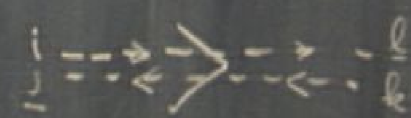
2 charges

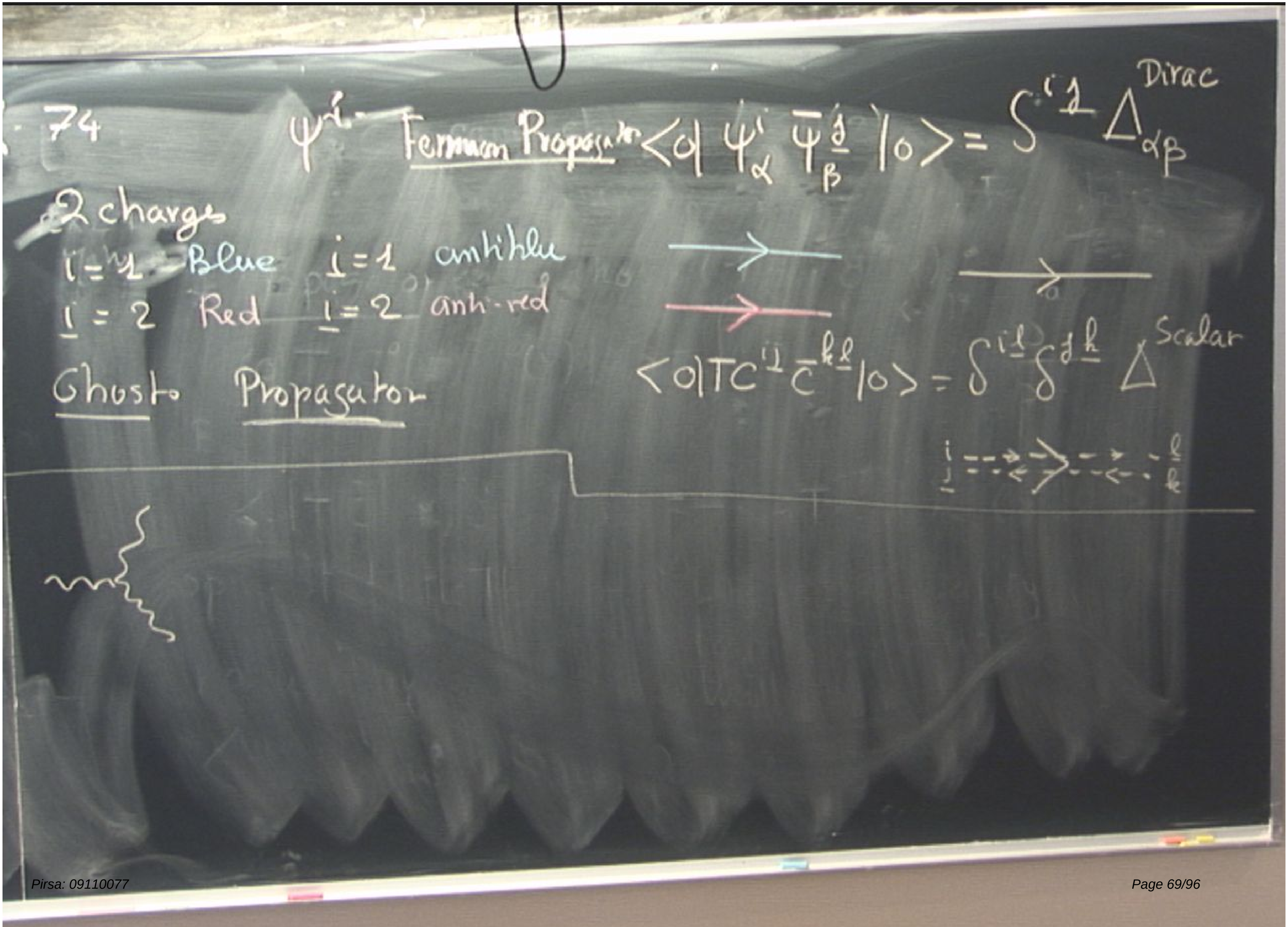
$i = 1$ Blue $\quad \underline{i} = 1$ antiblue

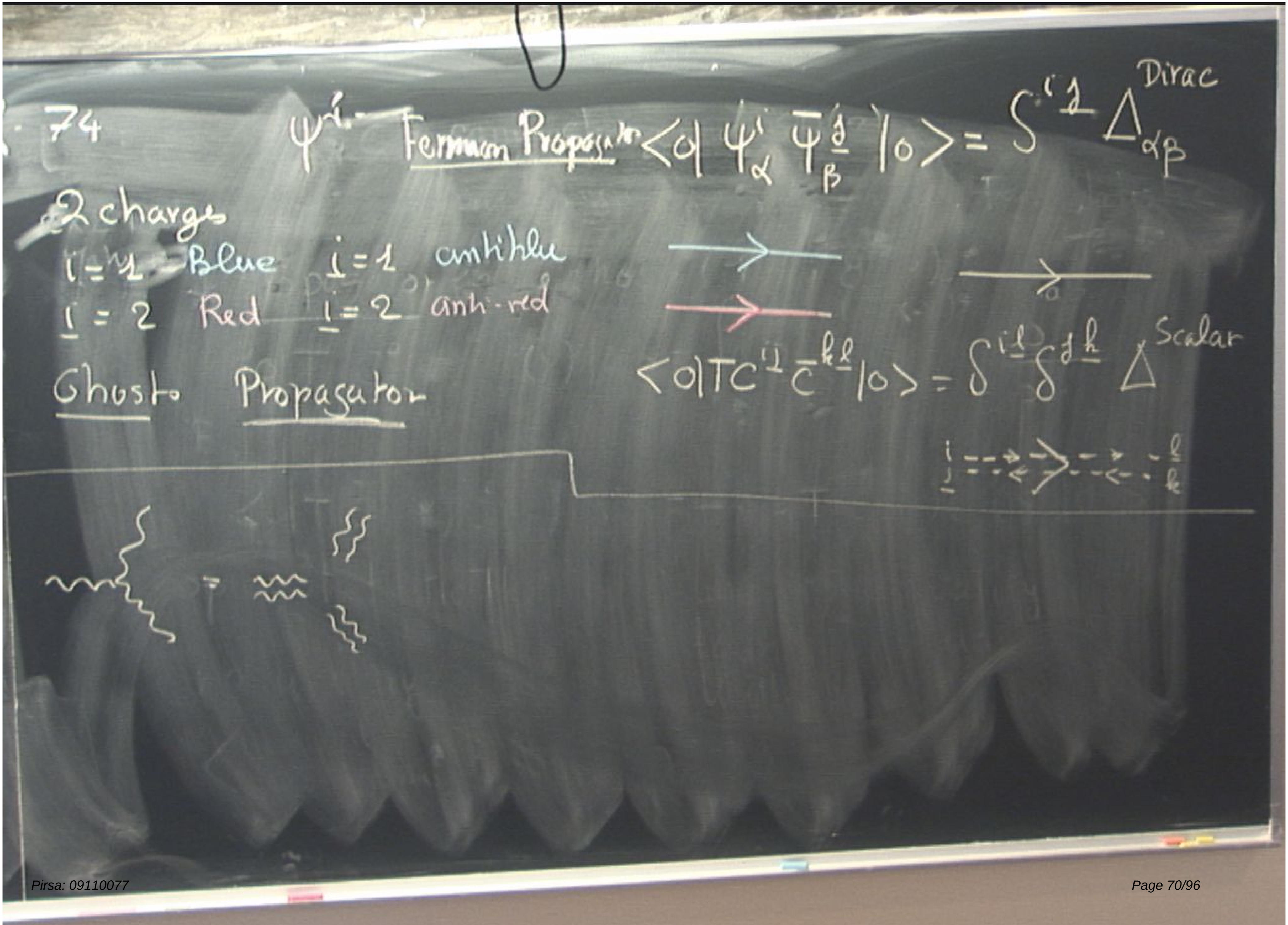
$\underline{i} = 2$ Red $\quad \underline{i} = 2$ anti-red

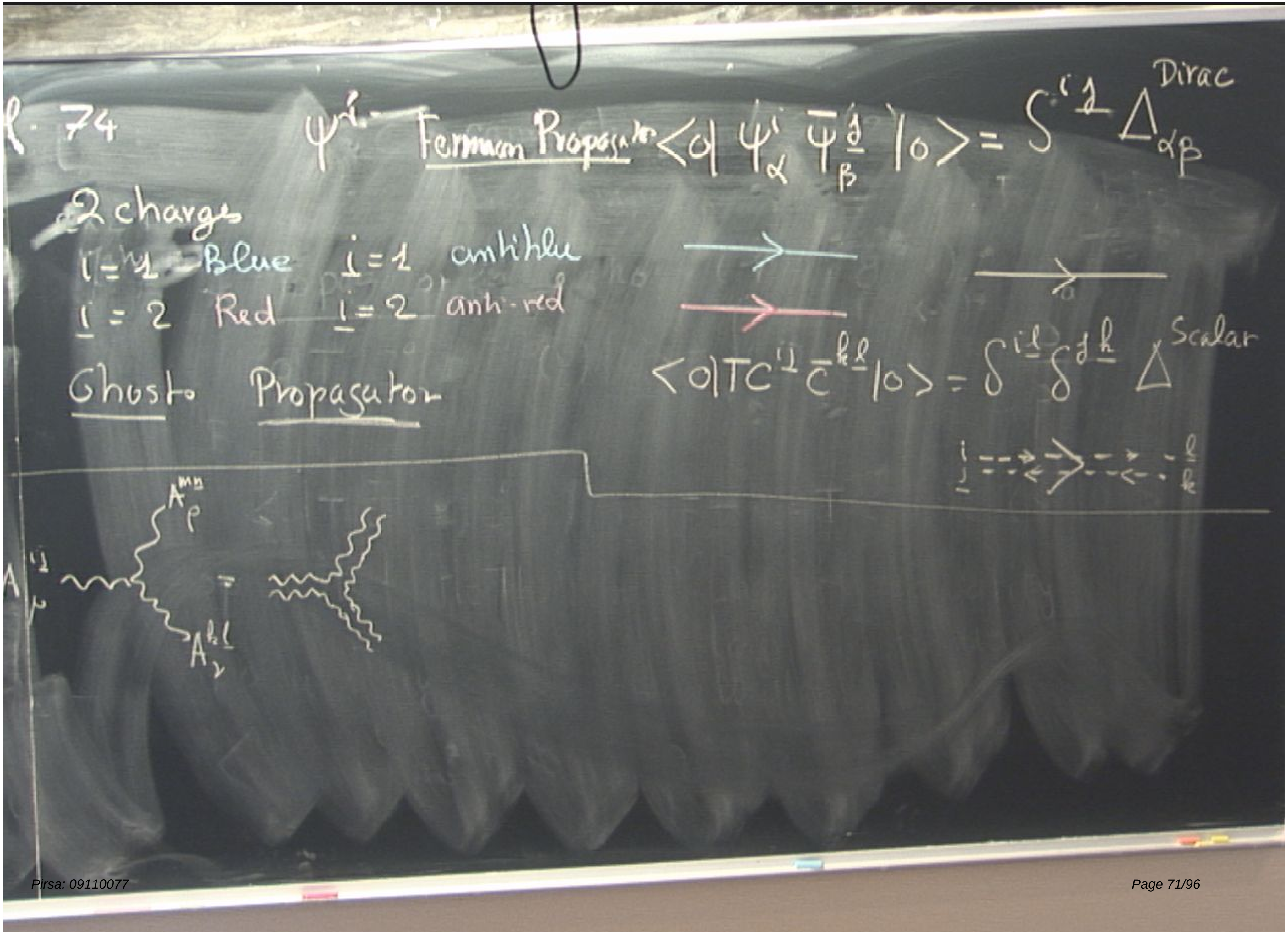
Ghosts **Propagator**

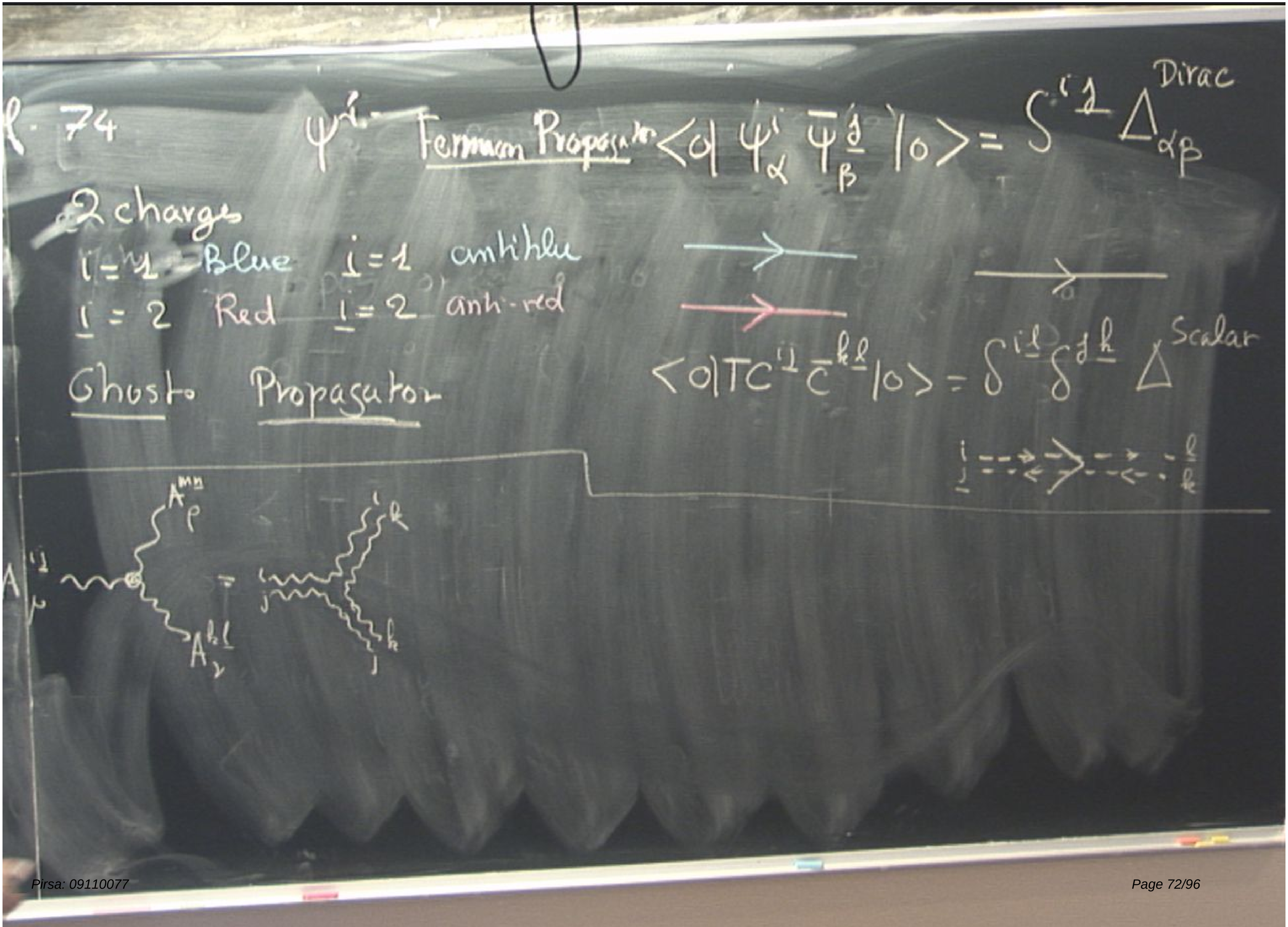
$\langle 0|T c^{i\underline{j}}\, \bar{c}^{k\underline{l}}\, |0\rangle = \delta^{i\underline{l}}\, \delta^{\underline{j}\underline{k}}\, \Delta^{\text{Scalar}}$




74
Fermion Propagator $\langle 0| \psi^i_\alpha \bar{\psi}^j_\beta |0\rangle = \delta^{ij} \Delta^{Dirac}_{\alpha\beta}$
2 charges
i = 1 Blue i = 1 anti-blue
i = 2 Red i = 2 anti-red
Ghosts Propagator
$\langle 0|T C^{ij} \bar{C}^{kl} |0\rangle = \delta^{il} \delta^{jk} \Delta^{Scalar}$
i j l k


74
Fermion Propagator
Dirac
2 charges
i = 1 Blue
i = 1 antiblue
i = 2 Red
i = 2 anti-red
Scalar
Ghosts Propagator


Dirac
Fermion Propagator
2 charges
i = 1 Blue
i = 2 Red
anti-blue
anti-red
Ghosts Propagator
Scalar


74
Fermion Propagator
Dirac
Scalar
2 charges
i = 1 Blue i = 1 antiblue
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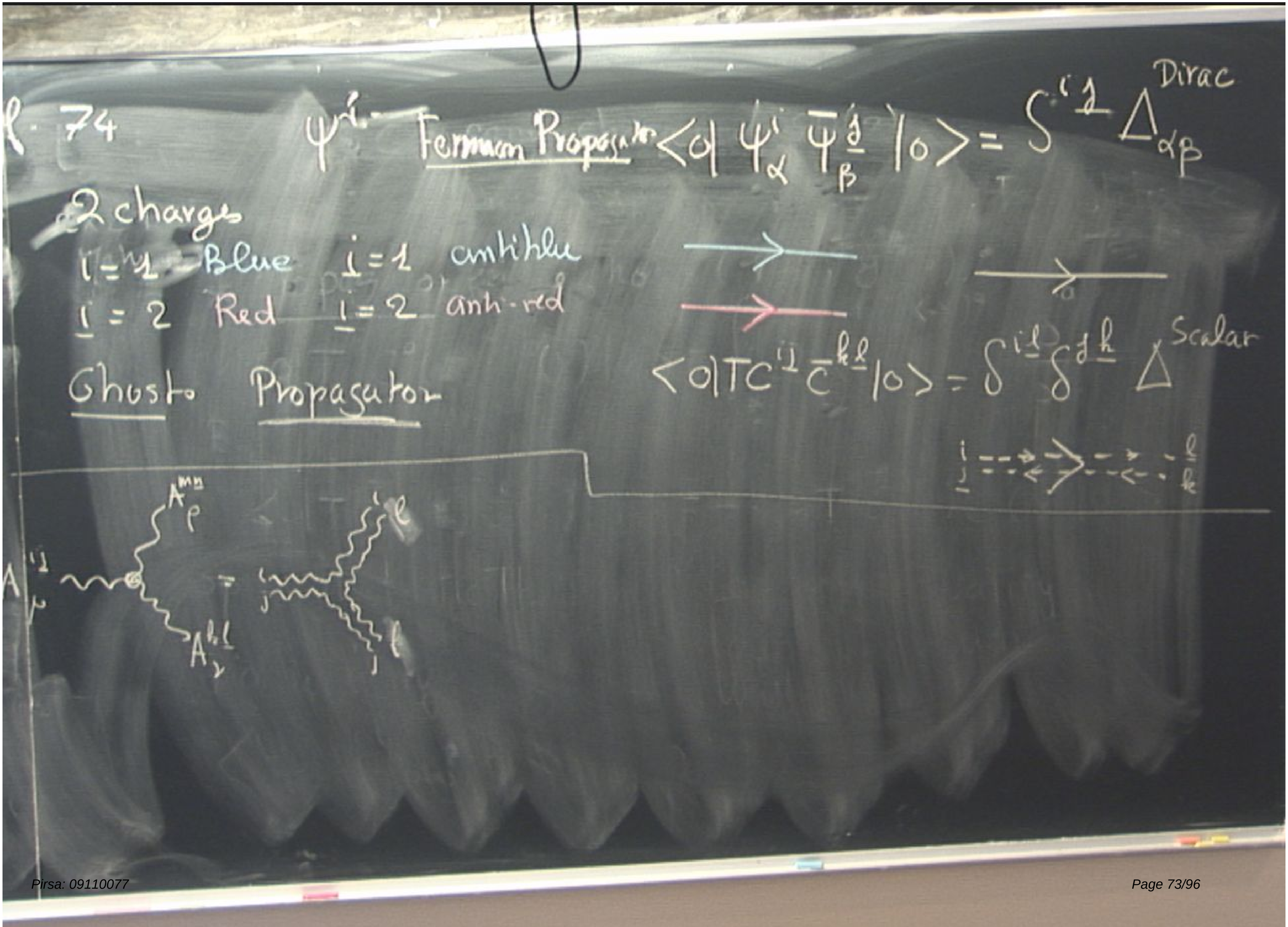
Ghosts Propagator


Dirac
Fermion Propagator
2 charges
i = 1 Blue
j = 1 antiblue
i = 2 Red
j = 2 anti-red
Scalar
Ghosts
Propagator

l. 74

ψ^{i} Fermion Propagator $\langle 0| \psi^{i}_{\alpha} \bar{\psi}^{j}_{\beta} |0\rangle = \delta^{ij} \Delta^{Dirac}_{\alpha\beta}$

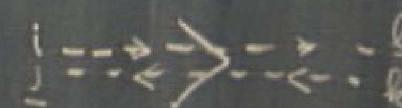
2 charges

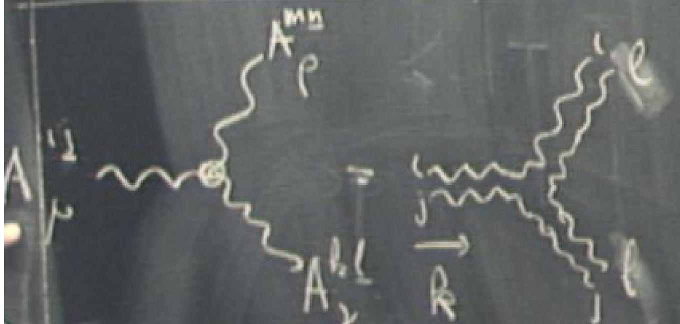
$i = 1$ Blue $\quad \underline{j} = 1$ antiblue

$\underline{i} = 2$ Red $\quad \underline{j} = 2$ anti-red

Ghosts Propagator

$\langle 0|T c^{i\underline{j}} \bar{c}^{k\underline{l}}|0\rangle = \delta^{i\underline{l}}\delta^{j\underline{k}} \Delta^{Scalar}$





$-g\,\delta^{\mu\nu}$

74

ψ^{i}. Fermion Propagator $\langle 0|\, \psi^{i}_{\alpha}\, \bar{\psi}^{\underline{j}}_{\beta}\, |0\rangle = \delta^{i\underline{j}}\, \Delta^{\text{Dirac}}_{\alpha\beta}$

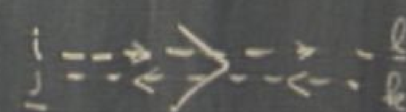
2 charges

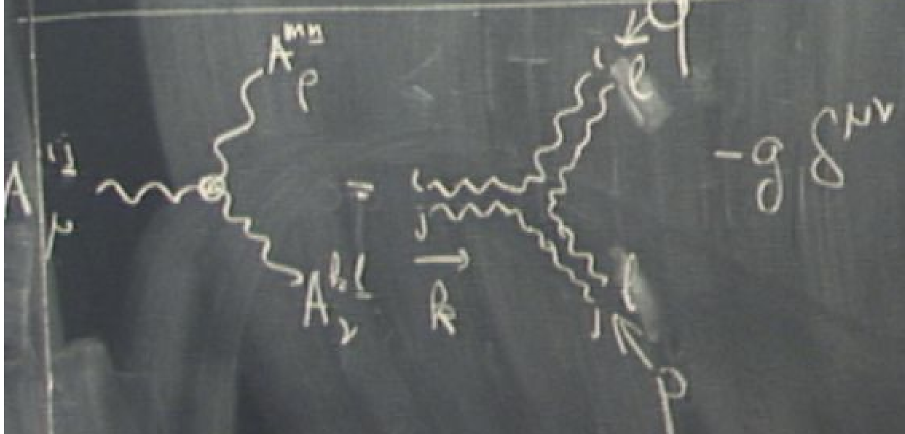
$i = 1$ Blue $\quad \underline{j} = 1$ antiblue

$i = 2$ Red $\quad \underline{j} = 2$ anti-red

Ghosts Propagator

$\langle 0|\, T\, c^{i\underline{j}}\, \bar{c}^{k\underline{l}}\, |0\rangle = \delta^{i\underline{l}}\, \delta^{\underline{j}k}\, \Delta^{\text{Scalar}}$





ℓ. 74

ψ^{i} Fermion Propagator $\langle 0 | \psi^{i}_{\alpha} \bar{\psi}^{j}_{\beta} | 0 \rangle = \delta^{ij} \Delta^{\text{Dirac}}_{\alpha\beta}$

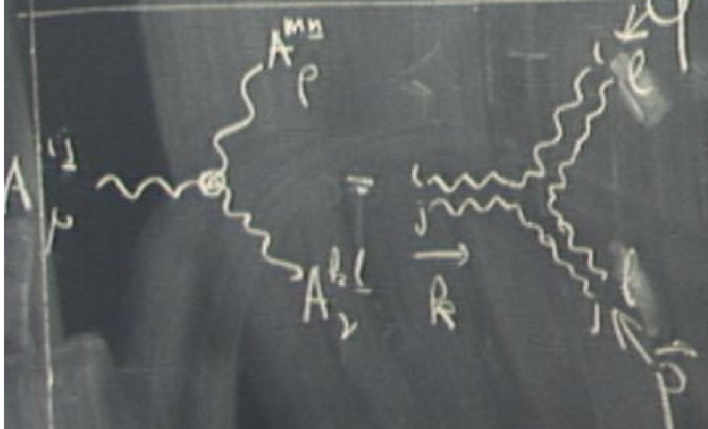
2 charges

$i = 1$ Blue $\underline{i} = 1$ antiblue

$i = 2$ Red $\underline{i} = 2$ anti-red

Ghosts Propagator

$\langle 0 | T c^{i\underline{j}} \bar{c}^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{\underline{j}k} \Delta^{\text{Scalar}}$



$-g\left(\delta^{\mu\nu}(k-p)^{\nu} + \delta^{\nu\rho}(q-p)^{\nu} + \delta^{\rho\nu}(p-k)^{\nu}\right]$

l. 74

ψ^{i}. **Fermion Propagator** $\langle 0|\, \psi^{i}_{\alpha}\, \bar{\psi}^{j}_{\beta}\, |0\rangle = \delta^{ij}\, \Delta^{\text{Dirac}}_{\alpha\beta}$

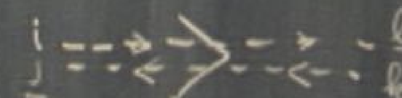
2 charges

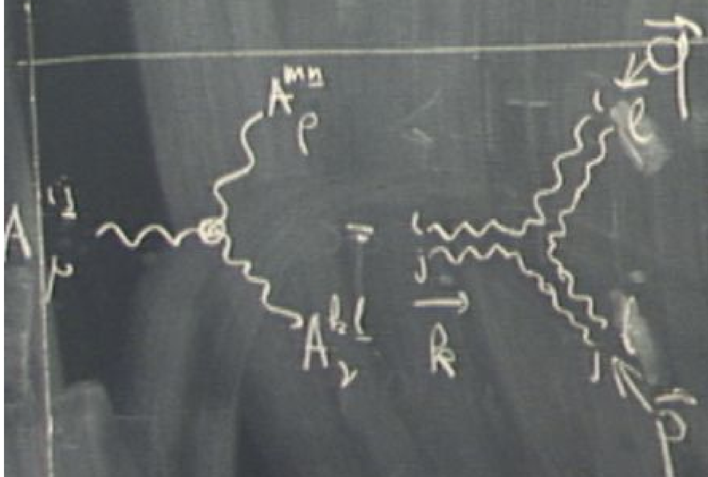
$i = 1$ Blue $\quad \underline{j} = 1$ antiblue

$i = 2$ Red $\quad \underline{j} = 2$ anti-red

Ghosts Propagator

$$\langle 0|\, T c^{i\underline{j}}\, \bar{c}^{k\underline{l}}\, |0\rangle = \delta^{i\underline{l}}\, \delta^{j\underline{k}}\, \Delta^{\text{Scalar}}$$





$$= -g\left(\delta^{\mu\nu}(k-p)^{\nu} + \delta^{\nu\rho}(q-p)^{\nu} + \delta^{\rho\nu}(p-k)^{\nu}\right]$$

l · 74

ψ^i <u>Fermion Propagator</u> $\langle 0|\, \psi^i_\alpha\, \bar{\psi}^j_\beta\, |0\rangle = \delta^{ij}\, \Delta^{\text{Dirac}}_{\alpha\beta}$

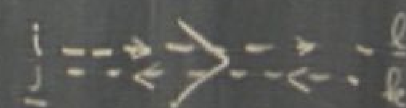
2 charges

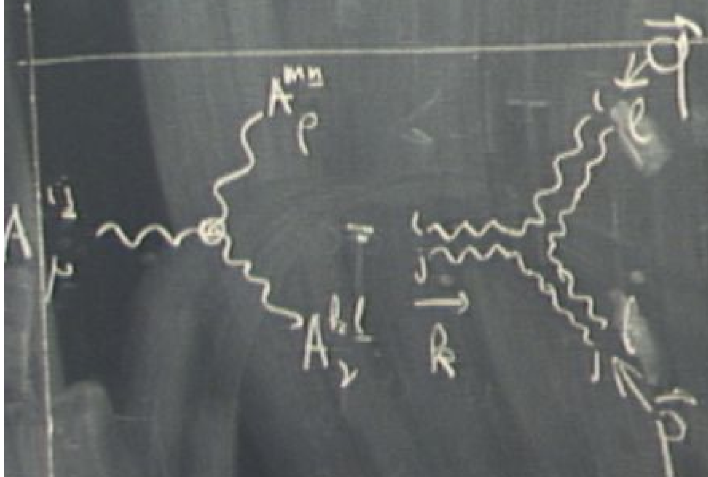
$i = 1$ Blue $\quad \underline{j} = 1$ antiblue

$\underline{i} = 2$ Red $\quad \underline{j} = 2$ anti-red

<u>Ghosts</u> <u>Propagator</u>

$\langle 0|T\, c^{i\underline{j}}\, \bar{c}^{k\underline{l}}\, |0\rangle = \delta^{i\underline{l}}\, \delta^{j\underline{k}}\, \Delta^{\text{Scalar}}$





$-g\left(\delta^{\mu\nu}(k-p)^{\rho} + \delta^{\nu\rho}(q-p)^{\mu} + \delta^{\rho\mu}(p-k)^{\nu}\right]$

ℓ. 74

ψ^i — Fermion Propagator $\langle 0 | \psi^i_\alpha \bar{\psi}^{\underline{j}}_\beta | 0 \rangle = \delta^{i\underline{j}} \Delta^{\text{Dirac}}_{\alpha\beta}$

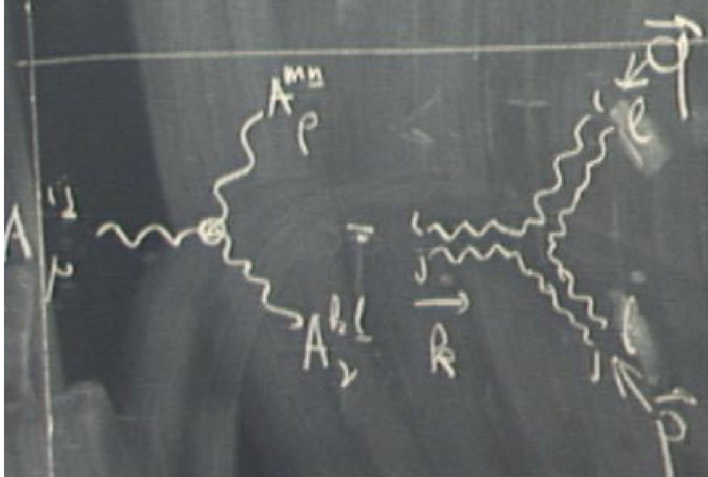
2 charges

$i = 1$ Blue $\quad \underline{i} = 1$ anti-blue

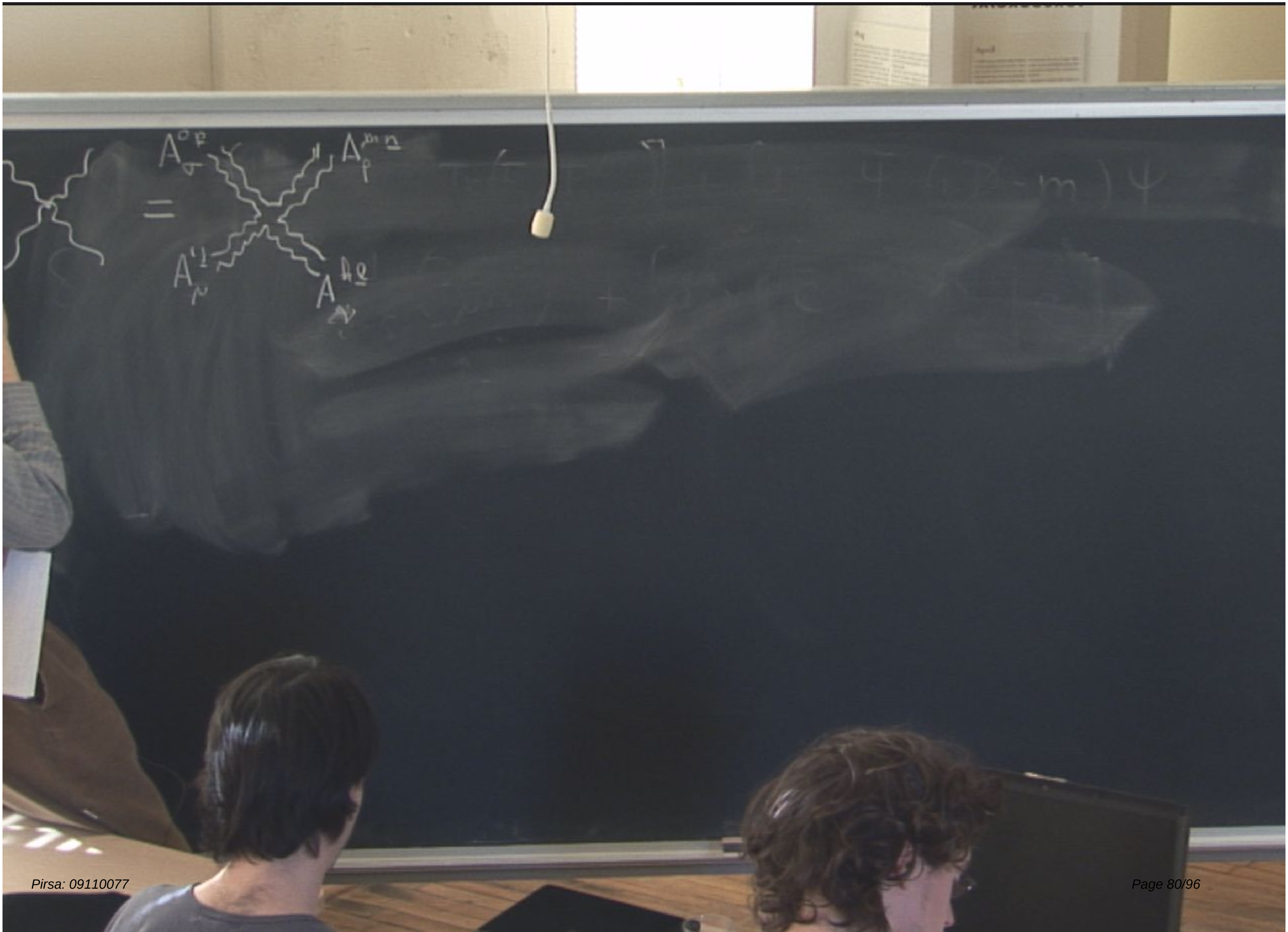
$i = 2$ Red $\quad \underline{i} = 2$ anti-red

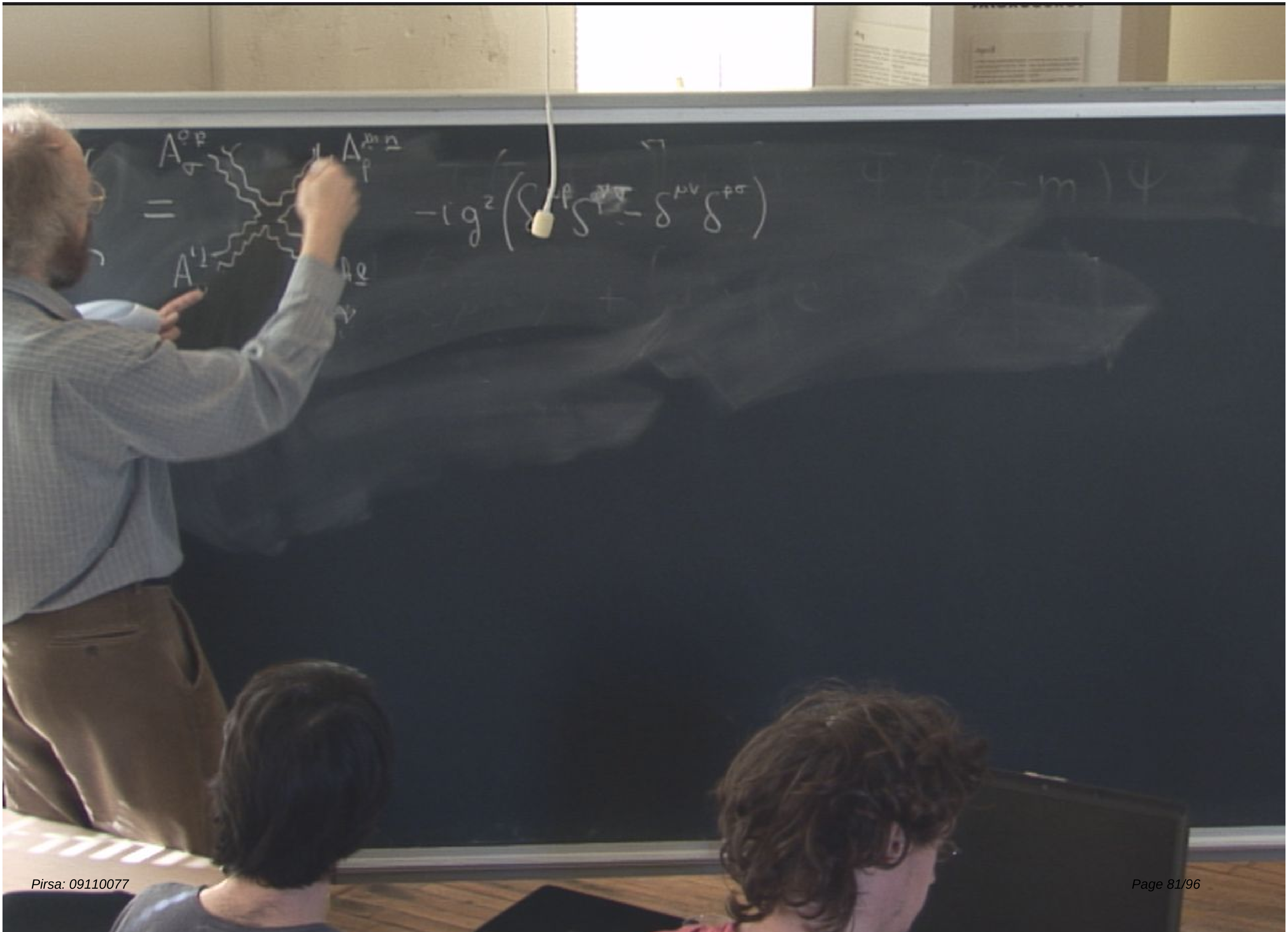
Ghosts Propagator

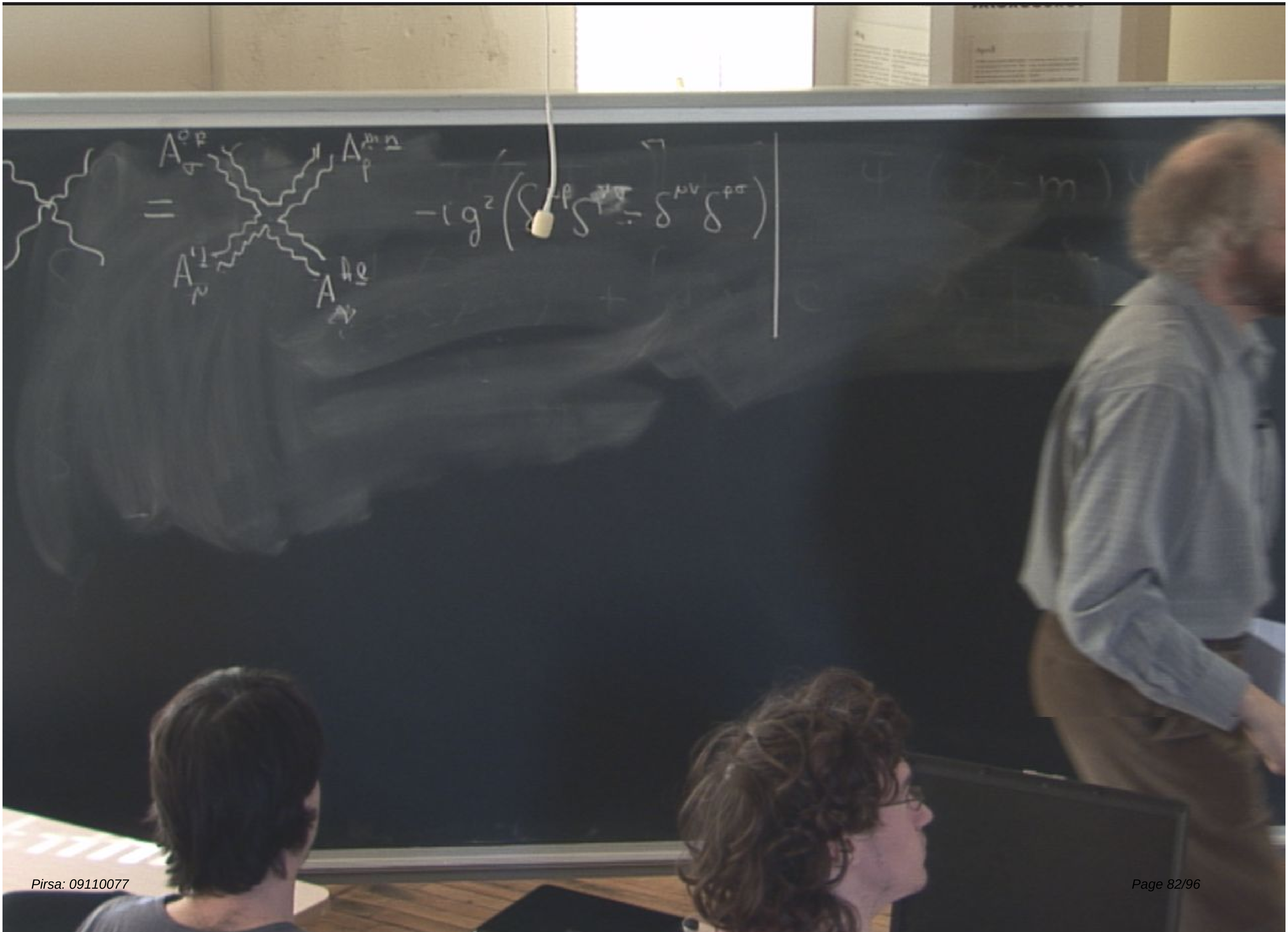
$\langle 0 | T c^{i\underline{j}} \bar{c}^{k\underline{\ell}} | 0 \rangle = \delta^{i\underline{\ell}} \delta^{\underline{j} k} \Delta^{\text{Scalar}}$

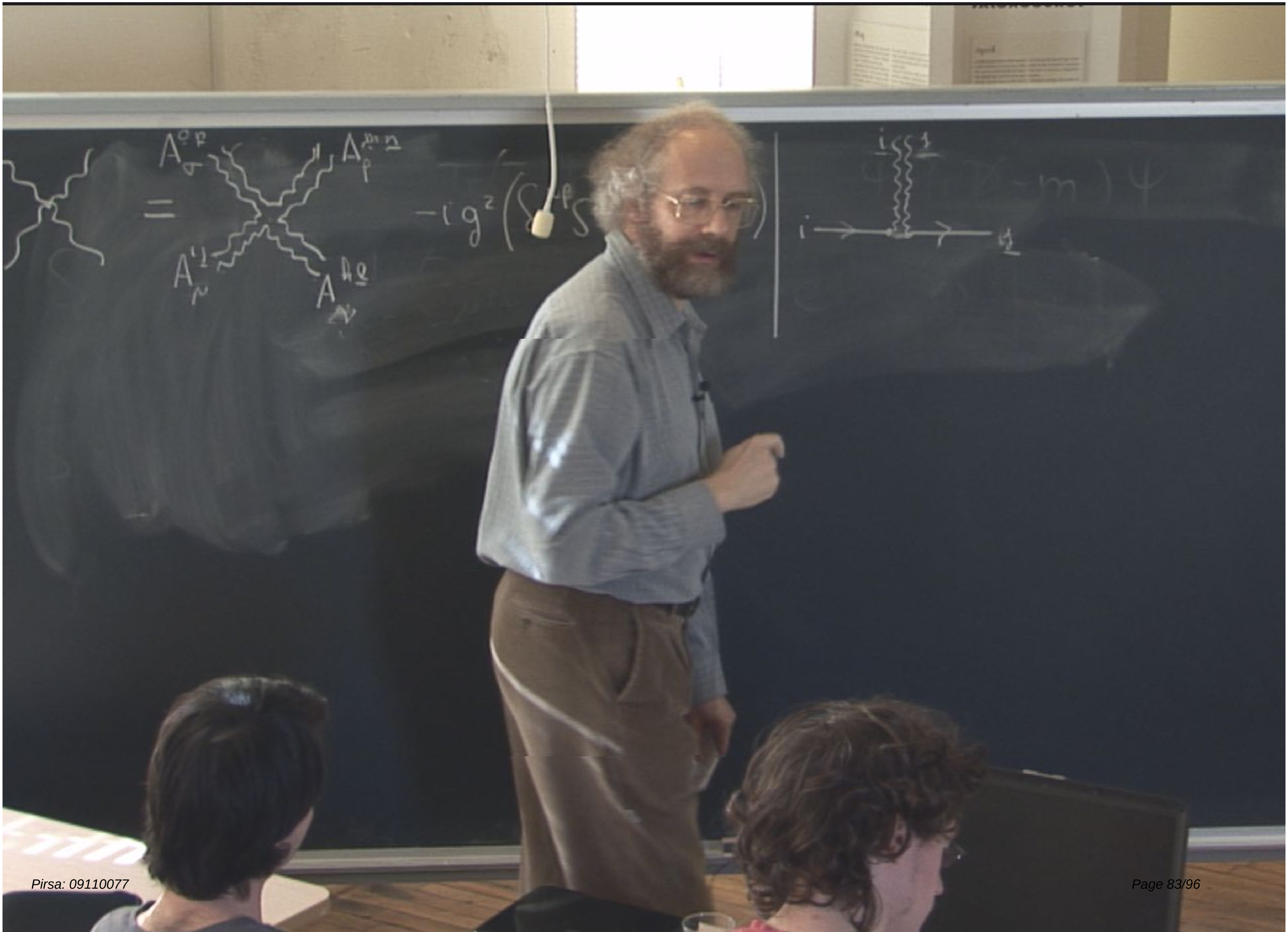


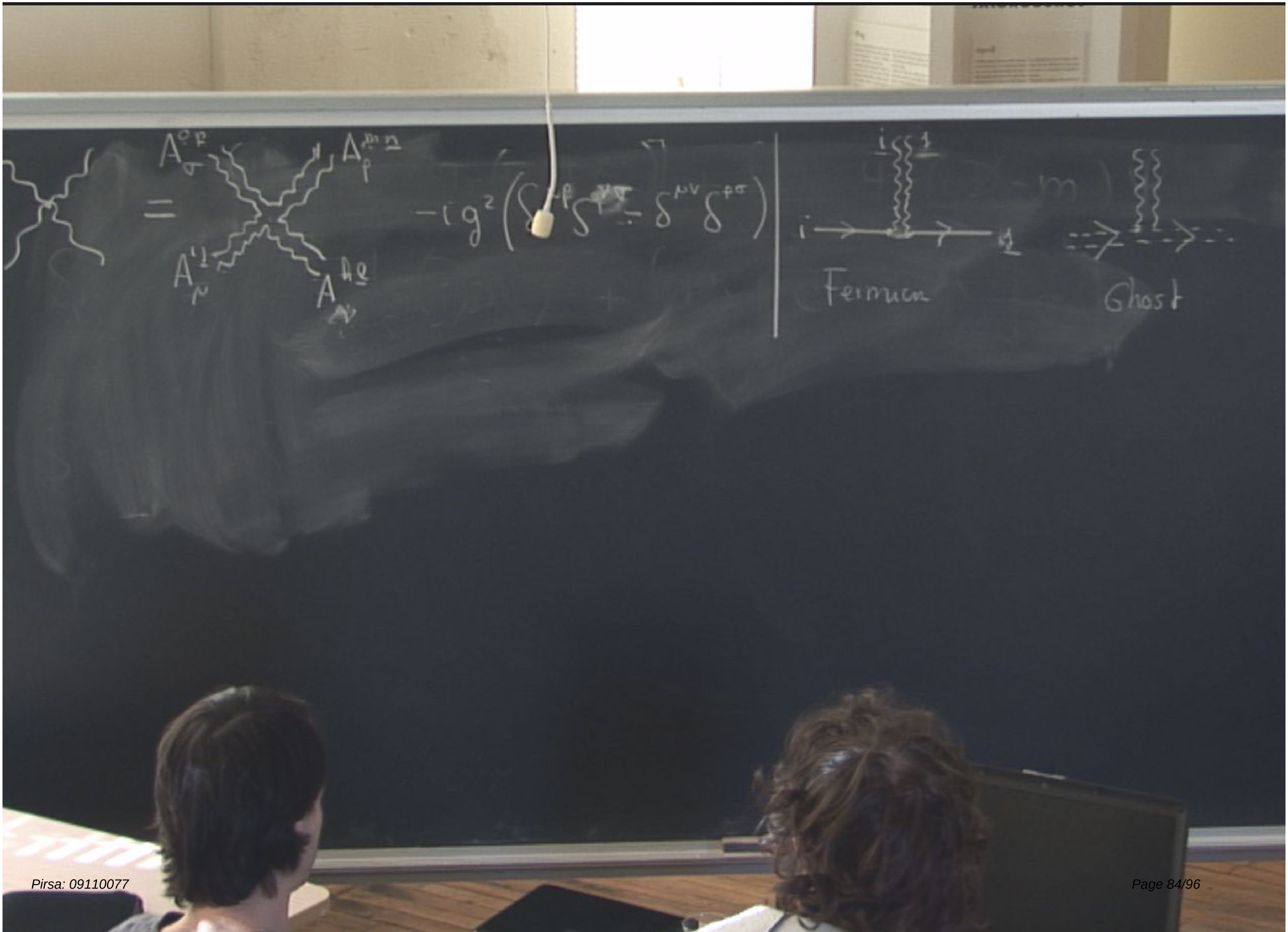
$-g\left(\delta^{\mu\nu}(k-p)^\rho + \delta^{\nu\rho}(q-p)^\mu + \delta^{\rho\mu}(q-k)^\nu\right]$

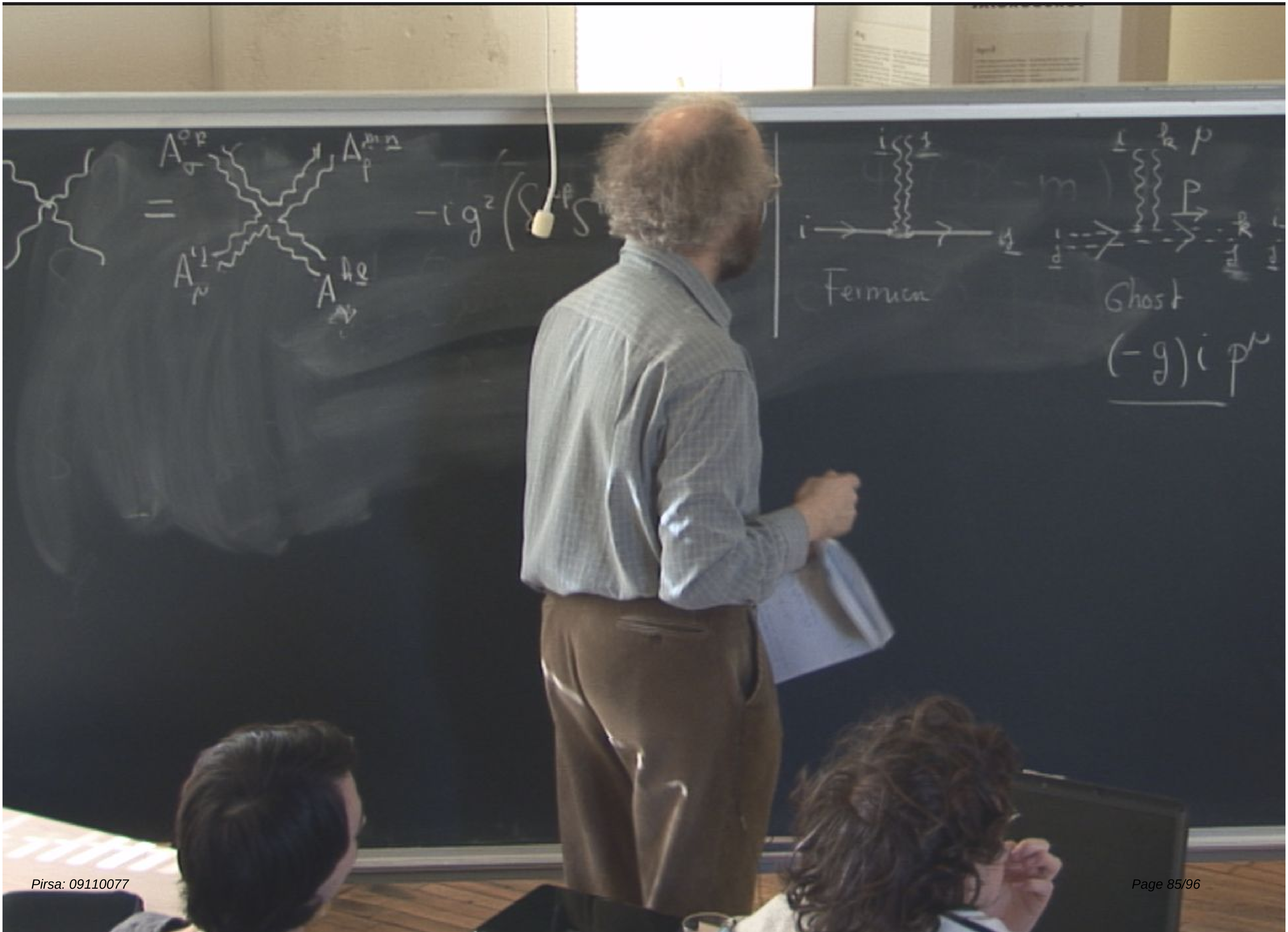


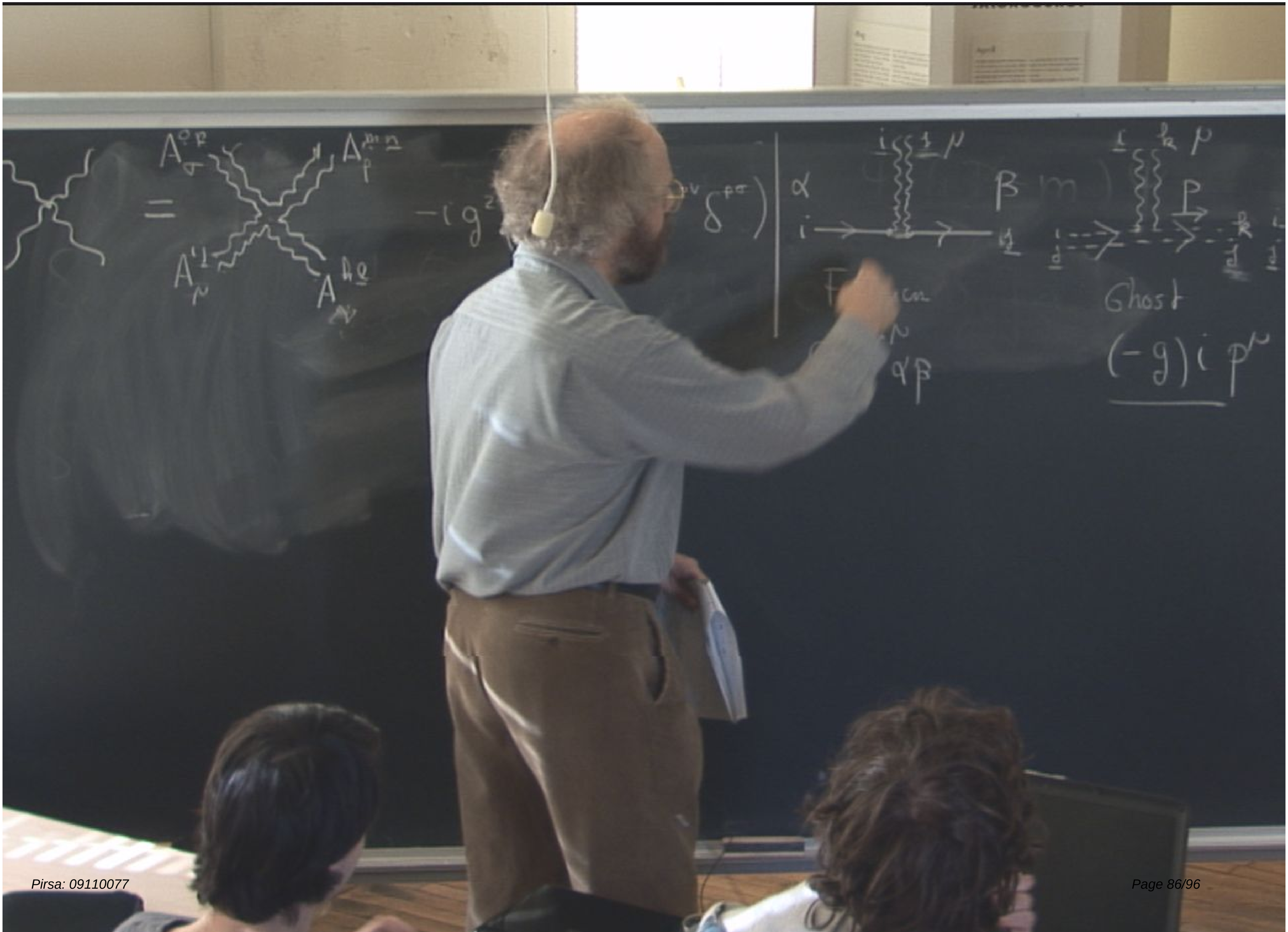


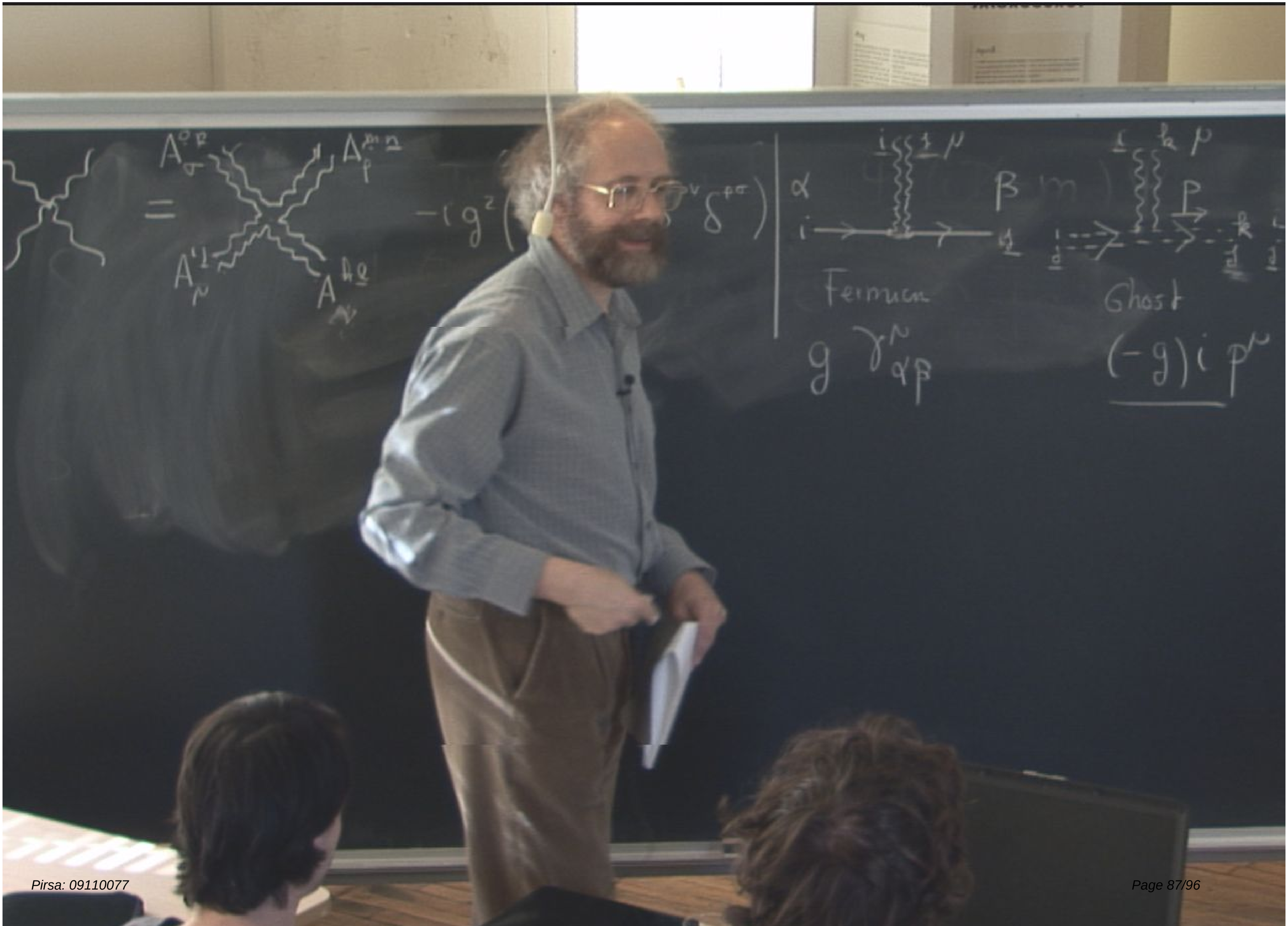


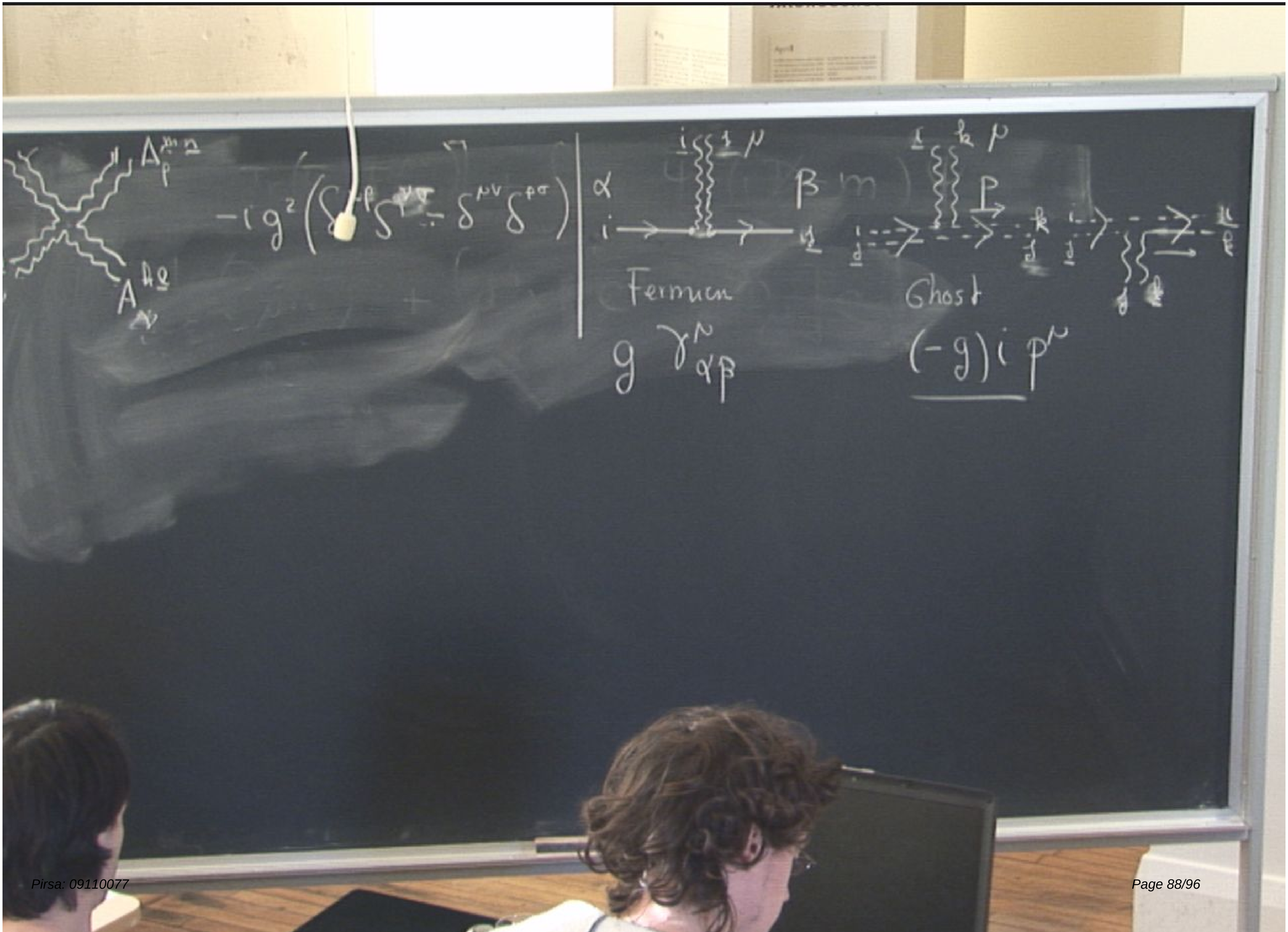



Fermion
Ghost


Fermion
Ghost
(-g)i p^μ




Fermion
Ghost


Fermion
g γ^μ_αβ
Ghost
(-g) i p^μ

74

ψ^{i} — Fermion Propagator $\langle 0 | \psi^{i}_{\alpha} \bar{\psi}^{j}_{\beta} | 0 \rangle = \delta^{ij} \Delta^{\text{Dirac}}_{\alpha\beta}$

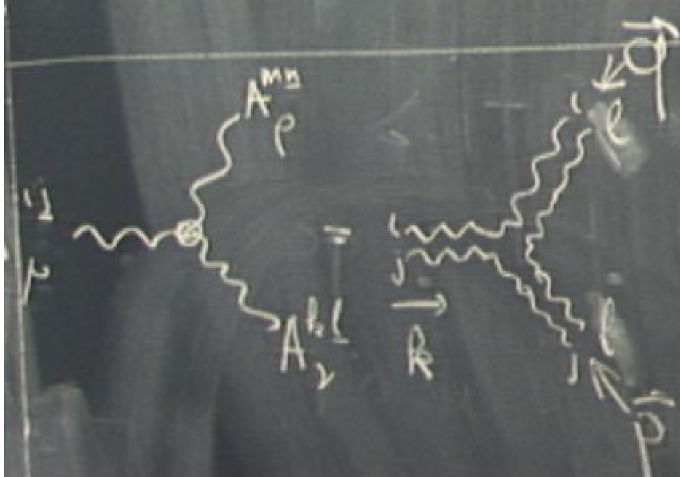
2 charges

$i = 1$ Blue $\underline{i} = 1$ antiblue

$i = 2$ Red $\underline{i} = 2$ anti-red

Ghosts Propagator

$\langle 0 | T C^{i\underline{j}} \bar{C}^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{j\underline{k}} \Delta^{\text{Scalar}}$



$-g\left(\delta^{\mu\nu}(k-p)^{\rho} + \delta^{\nu\rho}(q-p)^{\mu} + \delta^{\rho\mu}(q-k)^{\nu}\right]$

– The charges are conserved

–

74

ψ^{i} — **Fermion Propagator** $\langle 0 | \psi^{i}_{\alpha} \bar{\psi}^{j}_{\beta} | 0 \rangle = \delta^{ij} \Delta^{\text{Dirac}}_{\alpha\beta}$

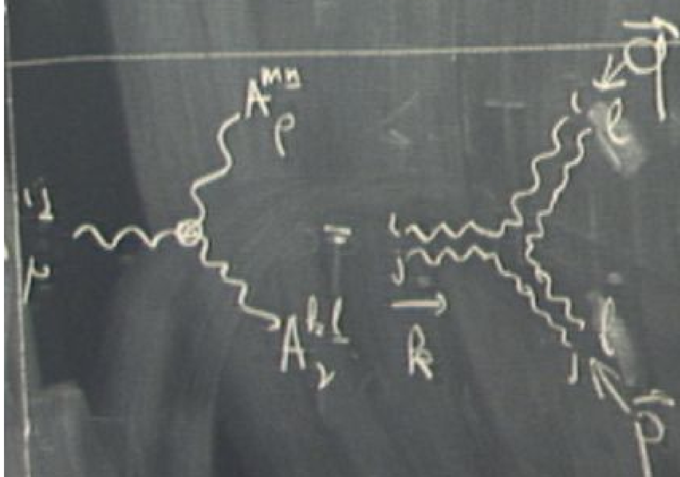
2 charges

$i = 1$ Blue $\quad \underline{i} = 1$ antiblue

$i = 2$ Red $\quad \underline{i} = 2$ anti-red

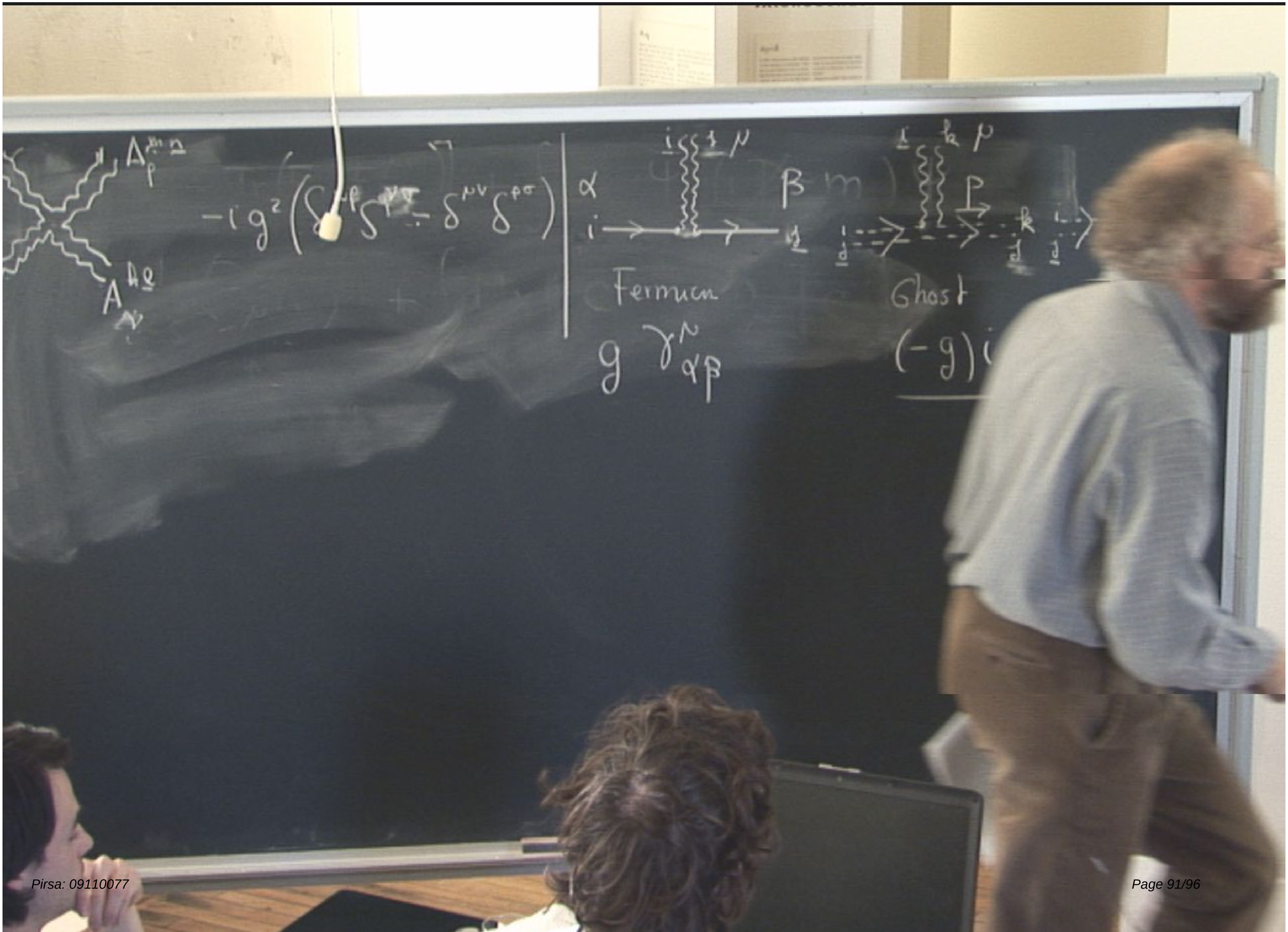
Ghosts Propagator

$\langle 0 | T c^{ij} \bar{c}^{kl} | 0 \rangle = \delta^{il} \delta^{jk} \Delta^{\text{Scalar}}$



$$= -g\left[\delta^{\mu\nu}(k-p)^{\rho} + \delta^{\nu\rho}(q-p)^{\mu} + \delta^{\rho\mu}(q-k)^{\nu}\right]$$

- The charges are conserved
- symmetric in $i = 1$ or 2


Fermion
Ghost

74

ψ^{i} — Fermion Propagator $\langle 0 | \psi^{i}_{\alpha} \bar{\psi}^{\underline{j}}_{\beta} | 0 \rangle = \delta^{i\underline{j}} \Delta^{\text{Dirac}}_{\alpha\beta}$

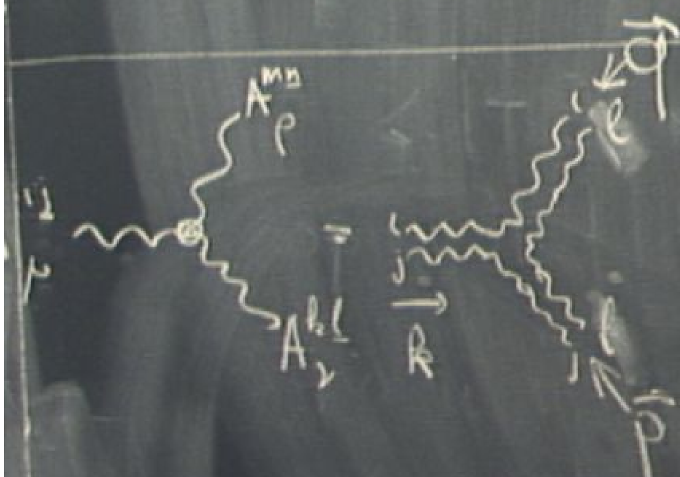
2 charges

$i = 1$ Blue $\underline{i} = 1$ antiblue

$i = 2$ Red $\underline{i} = 2$ anti-red

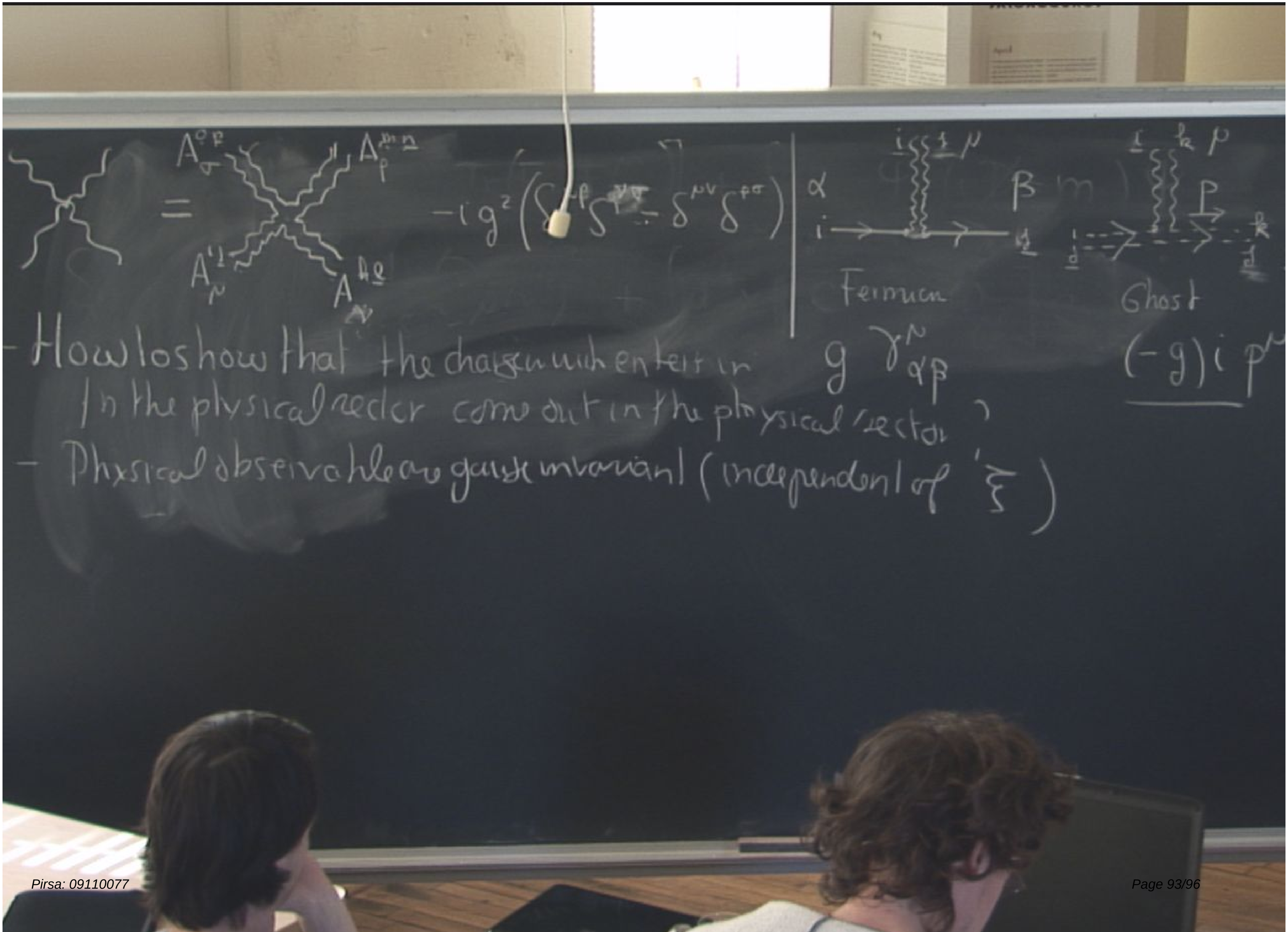
Ghosts Propagator

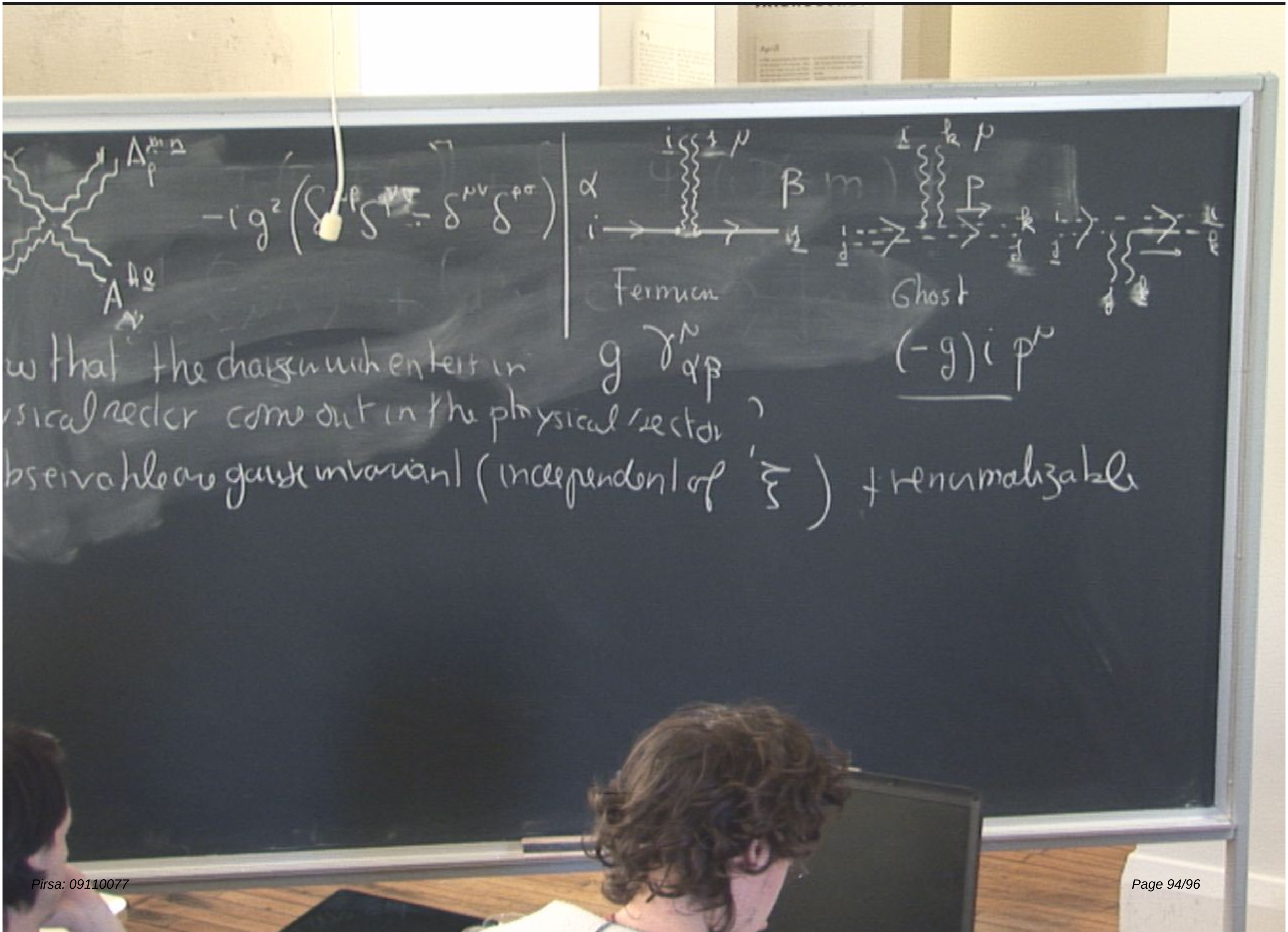
$$\langle 0 | T c^{i\underline{j}} \bar{c}^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{\underline{j}k} \Delta^{\text{Scalar}}$$



$$-g\left(\delta^{\mu\nu}(k-p)^{\rho} + \delta^{\nu\rho}(q-p)^{\mu} + \delta^{\rho\mu}(q-k)^{\nu}\right]$$

– The charges are conserved
– symmetric in $i = 1$ or 2 $\Rightarrow$ $U(2)$


Fermion
Ghost
- How to show that the charges which enter in
In the physical sector come out in the physical sector?
- Physical observables are gauge invariant (independent of ξ)


Fermion
Ghost

What are the Feynman rules of such a theory ; 't Hoo[ft]

$U(2)$ $\quad u(2) =$ Hermitean Matrices $\quad a = 1, \ldots 4$

2×2 complex. $\quad a = (i, \underline{j}) \quad i = 1, 2$

$\underline{j} = 1, 2$

$$A_\mu^a \Rightarrow A_\mu^{i\underline{j}}$$

A_μ = 2×2 hermitean matrix.

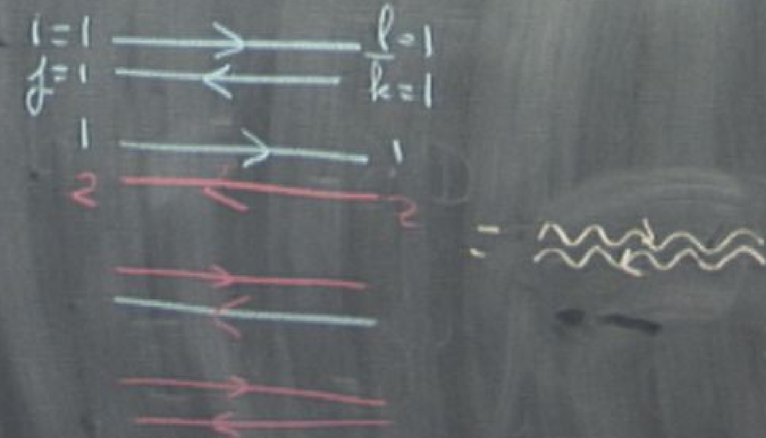
$$A_\mu^{i\underline{j}} = \overline{A_\mu^{j\underline{i}}}$$

$$A_\mu = \begin{pmatrix} A_\mu^{11} & A_\mu^{12} \\ A_\mu^{21} & A_\mu^{22} \end{pmatrix} = A_\mu^{i\underline{j}} \cdot t_{i\underline{j}}$$

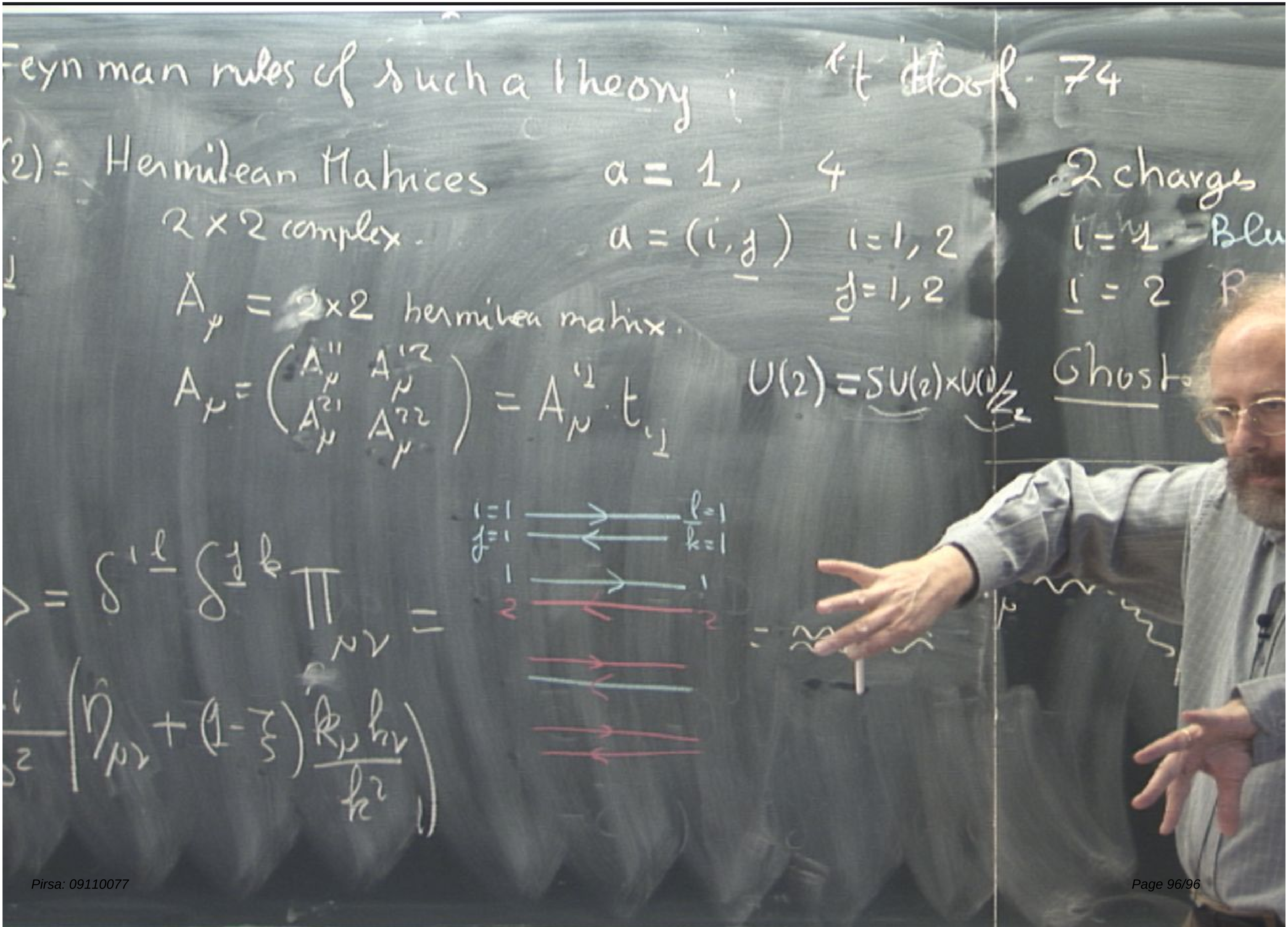
$U(2) = SU(2) \times U(1)$

Propagators :

$$\langle 0 | T A_\mu^{i\underline{j}} A_\nu^{k\underline{l}} | 0 \rangle = \delta^{i\underline{l}} \delta^{\underline{j}k} \Pi_{\mu\nu} =$$



$$\Pi_{\mu\nu} = \frac{-i}{p^2} \left(\eta_{\mu\nu} + (1 - \xi) \frac{k_\mu k_\nu}{k^2} \right)$$


Feynman rules of such a theory
't Hooft 74
Hermitean Matrices
2 × 2 complex
a = 1, 4
a = (i, j)
i = 1, 2
j = 1, 2
2 charges
i = 1 Blu
i = 2
A_μ = 2×2 hermitean matrix
U(2) = SU(2)×U(1)/Z_2
Ghost
i=1
j=1
l=1
k=1