Title: Quantum Field Theory II (PHYS 603) - Lecture 11

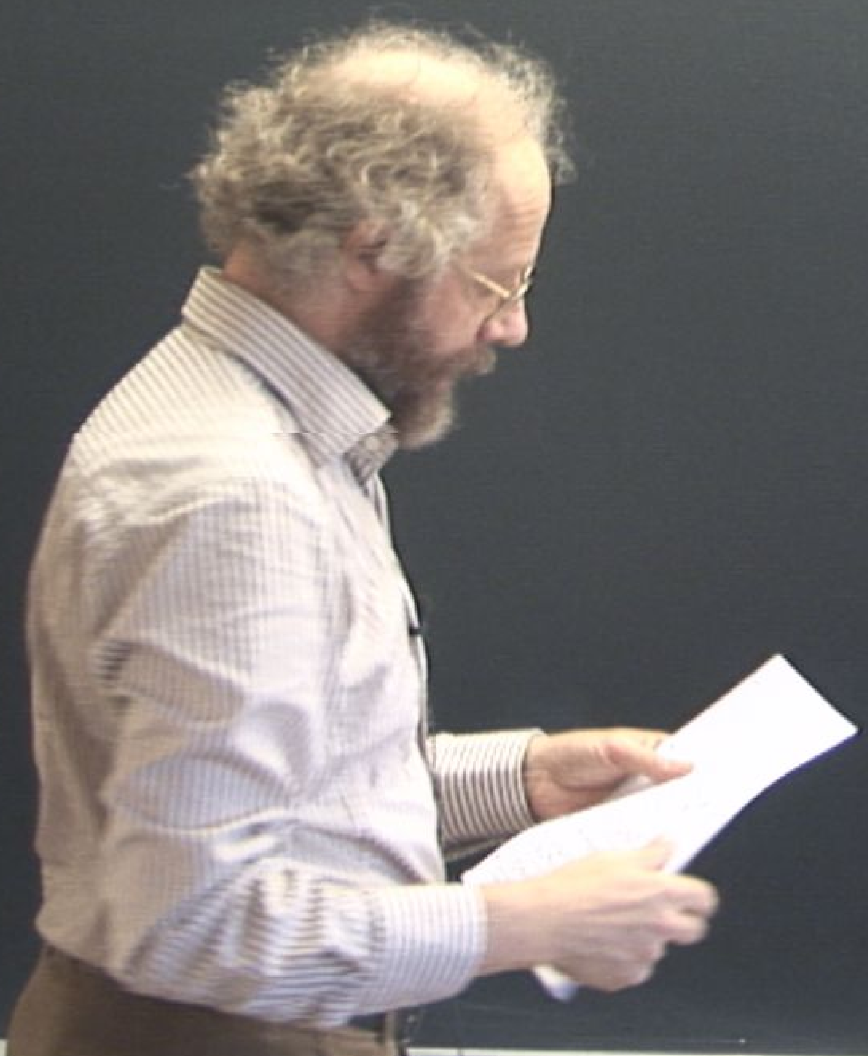
Date: Nov 09, 2009 09:00 AM

URL: http://pirsa.org/09110075

Abstract:

$A_\mu(x)$

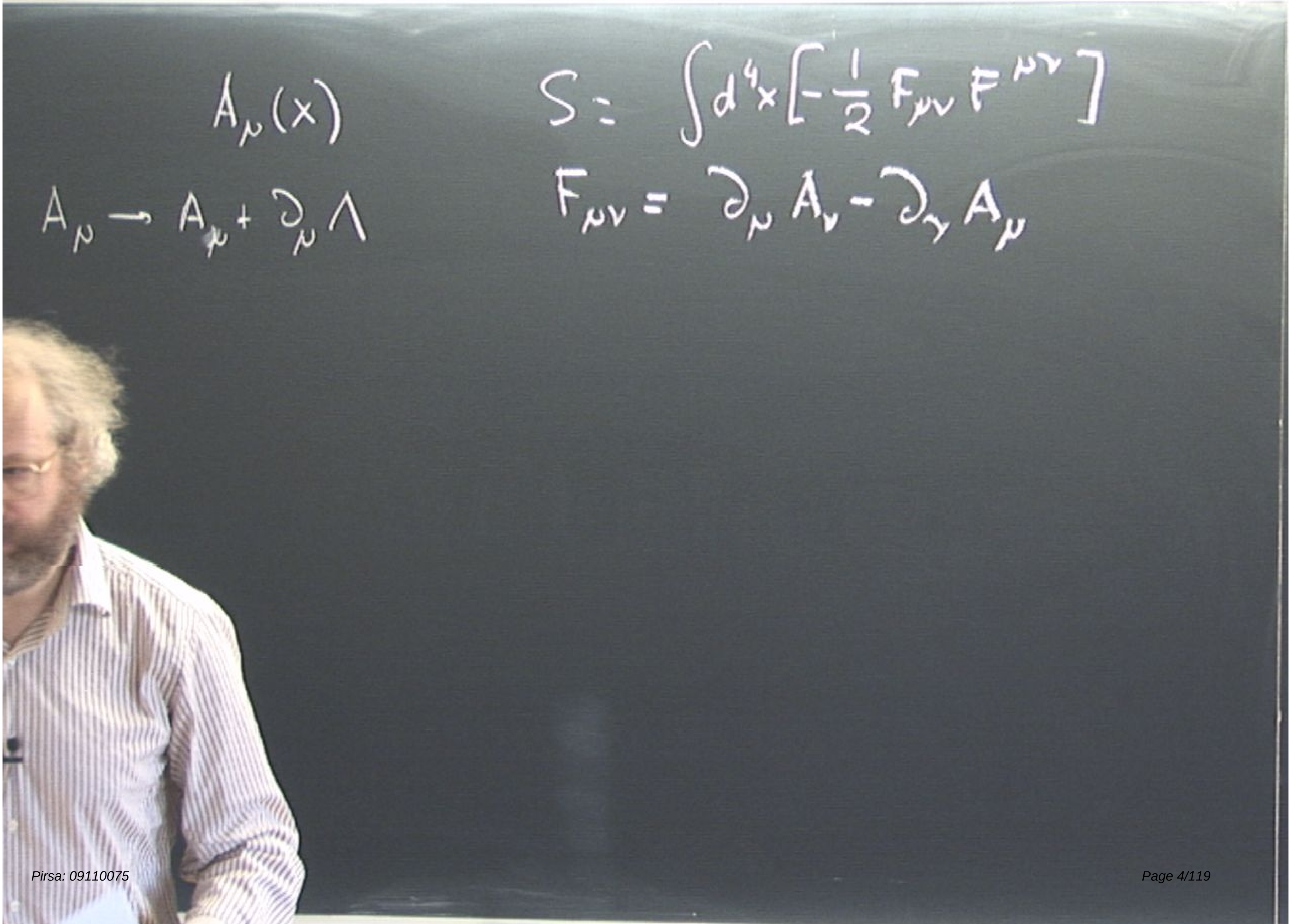
$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$



$A_\mu(x)$

$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



$$A_\mu(x)$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$

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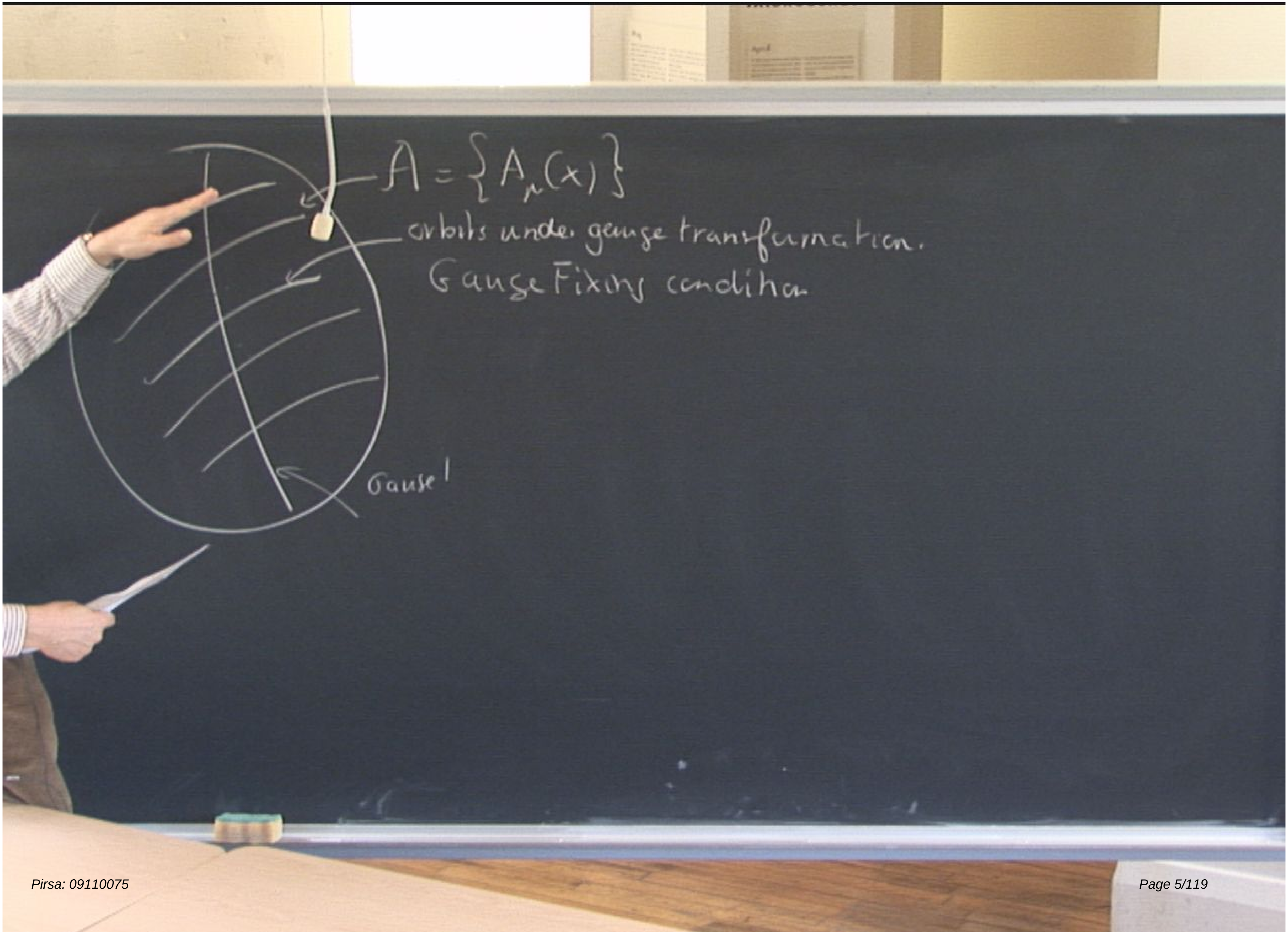


$A = \{A_\mu(x)\}$

orbits under gauge transformation.

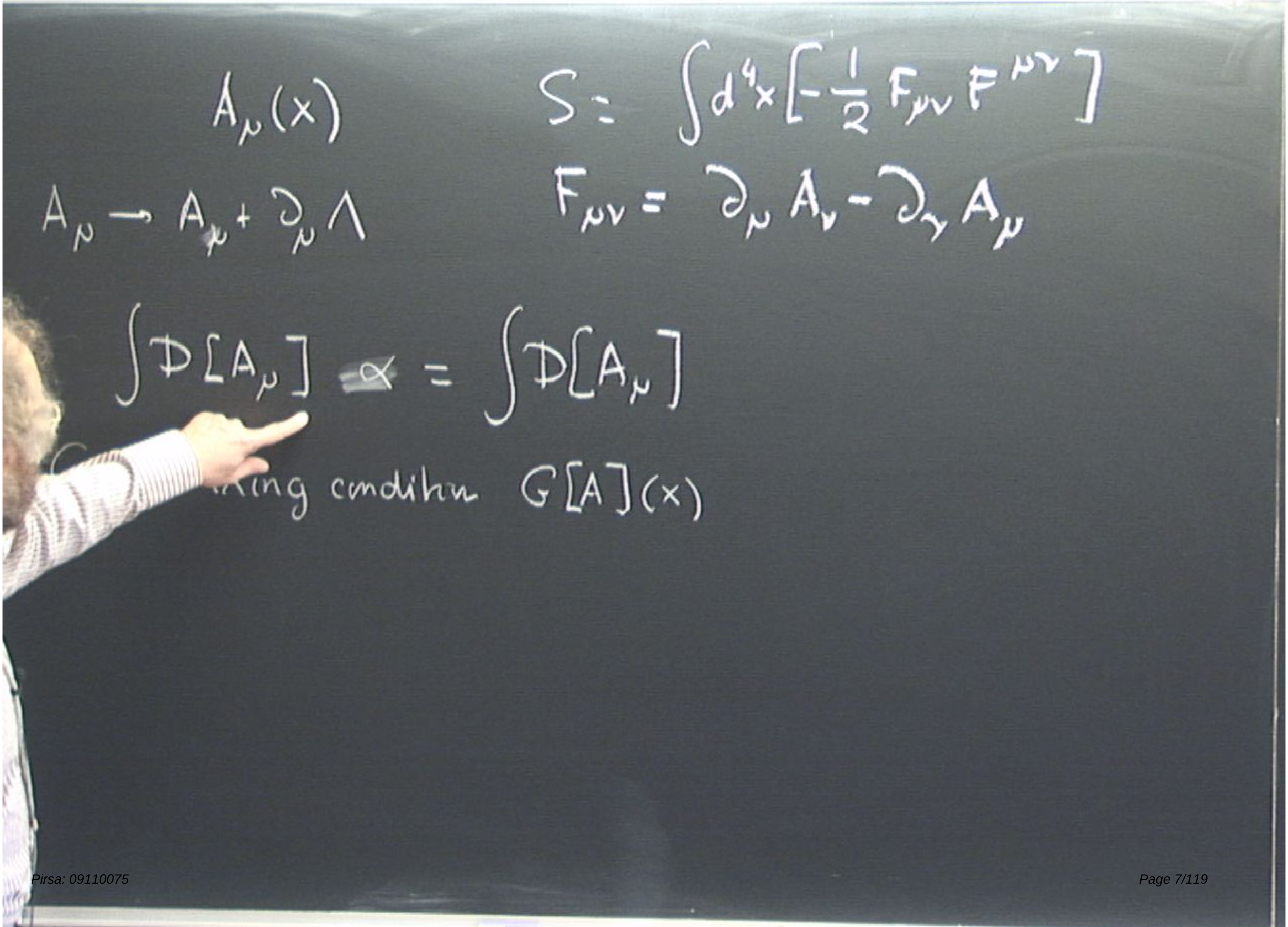
Gauge Fixing condition

Gauge!

$\mathcal{A} = \{A_\mu(x)\}$

orbits under gauge transformation.

Gauge Fixing condition
defines a "slice" in $\mathcal{A}$



$$A_\mu(x)$$

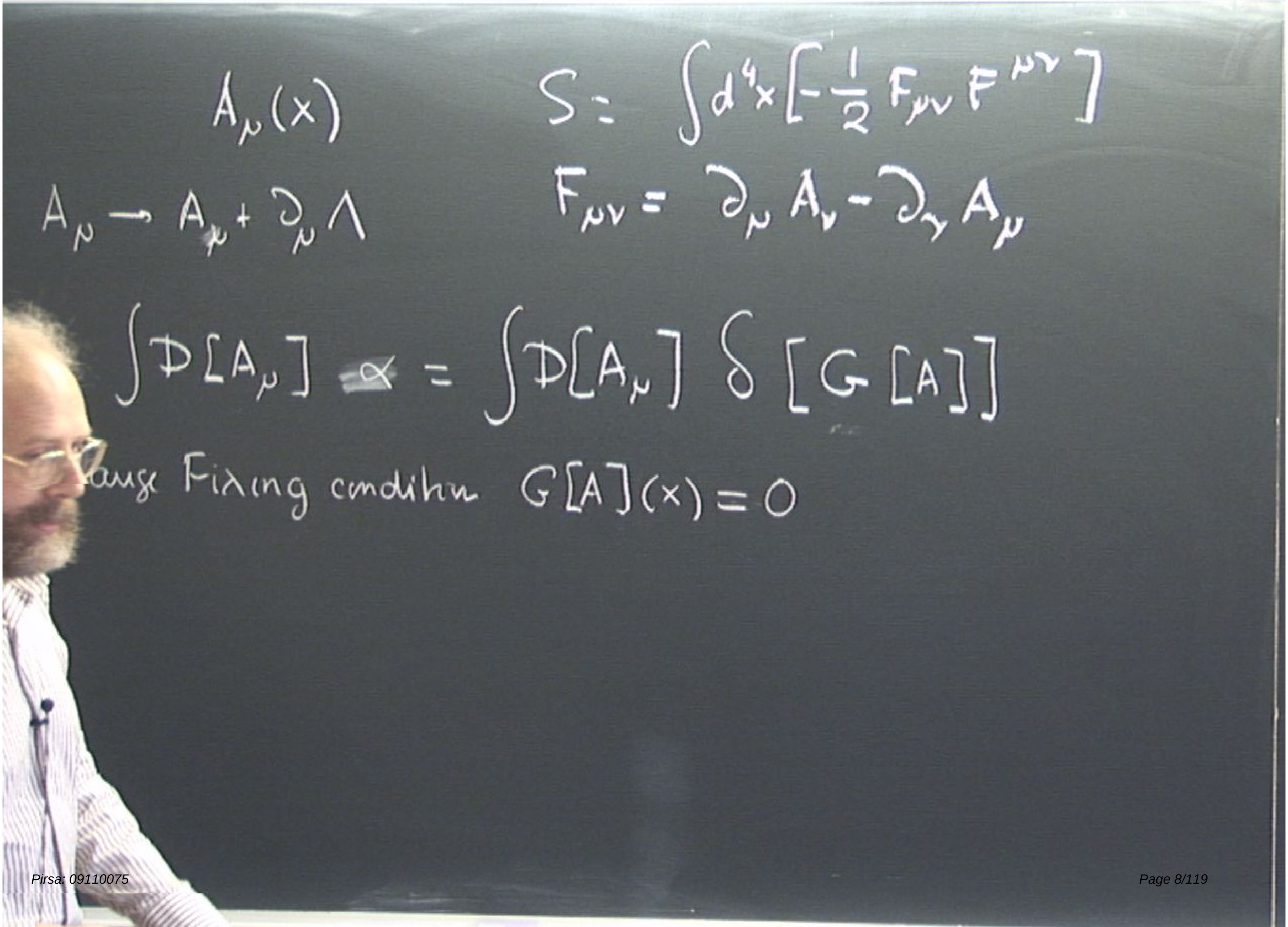
$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\int \mathcal{D}[A_\mu] \propto = \int \mathcal{D}[A_\mu]$$

xing condition $G[A](x)$



$A_\mu(x)$

$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

$A_\mu \to A_\mu + \partial_\mu \Lambda$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\int \mathcal{D}[A_\mu] \cancel{\propto} = \int \mathcal{D}[A_\mu] \, \delta\left[G[A] \right]$$

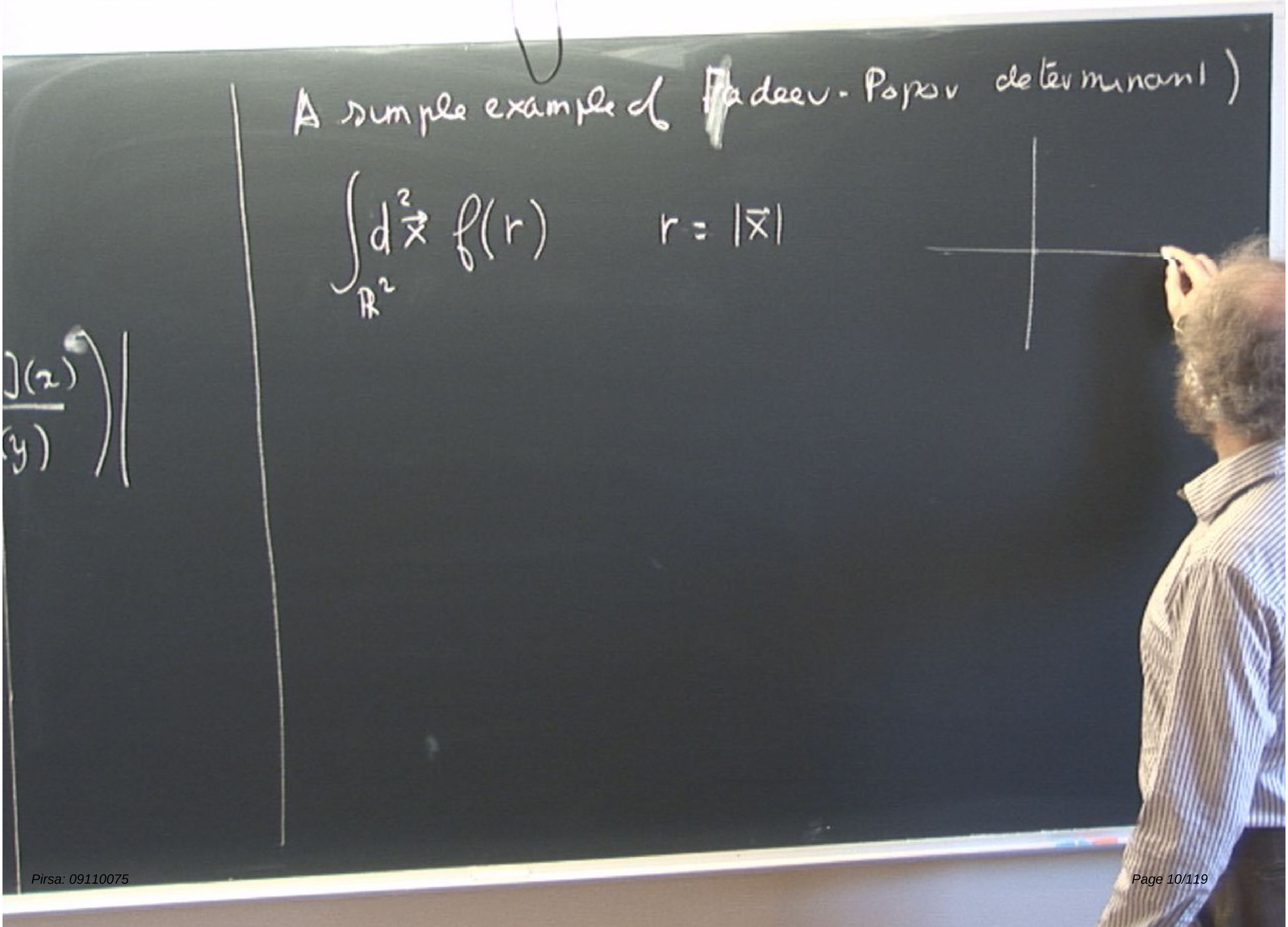
auge Fixing condition $G[A](x) = 0$

$A_\mu(x)$ $\qquad$ $S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$ $\qquad$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\int \mathcal{D}[A_\mu] \propto \int \mathcal{D}[A_\mu] \, \delta\left[G[A] \right] \left| \det\left(\frac{\delta G[A^\Lambda](x)}{\delta \Lambda(y)} \right) \right|$$

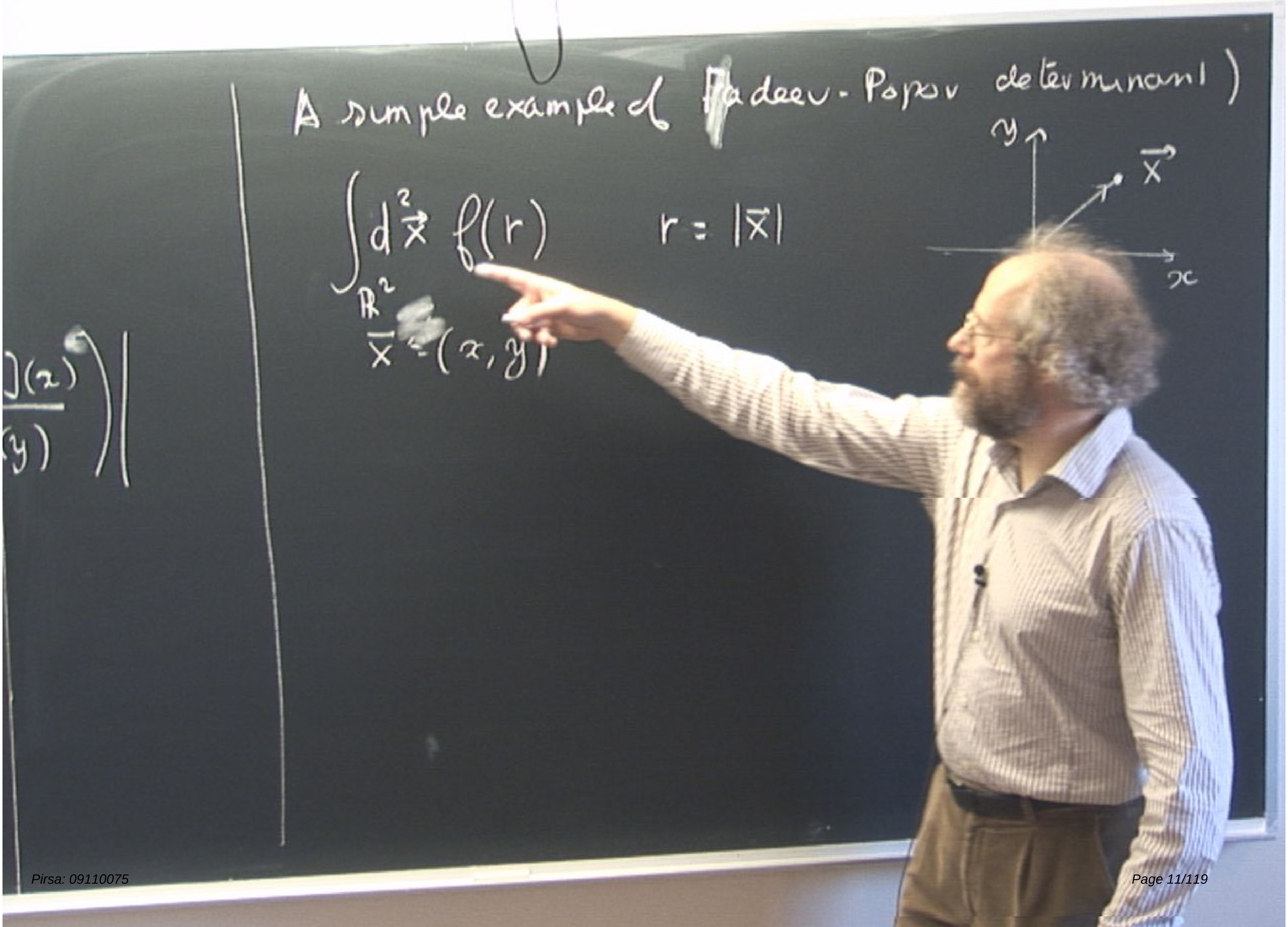
Gauge Fixing condition $G[A](x) = 0$



A simple example of (Fadeev - Popov determinant)

$$\int_{\mathbb{R}^2} d^2\vec{x}\; f(r) \qquad r = |\vec{x}|$$

$$\frac{(x)}{(y)}\Big)\Big|$$



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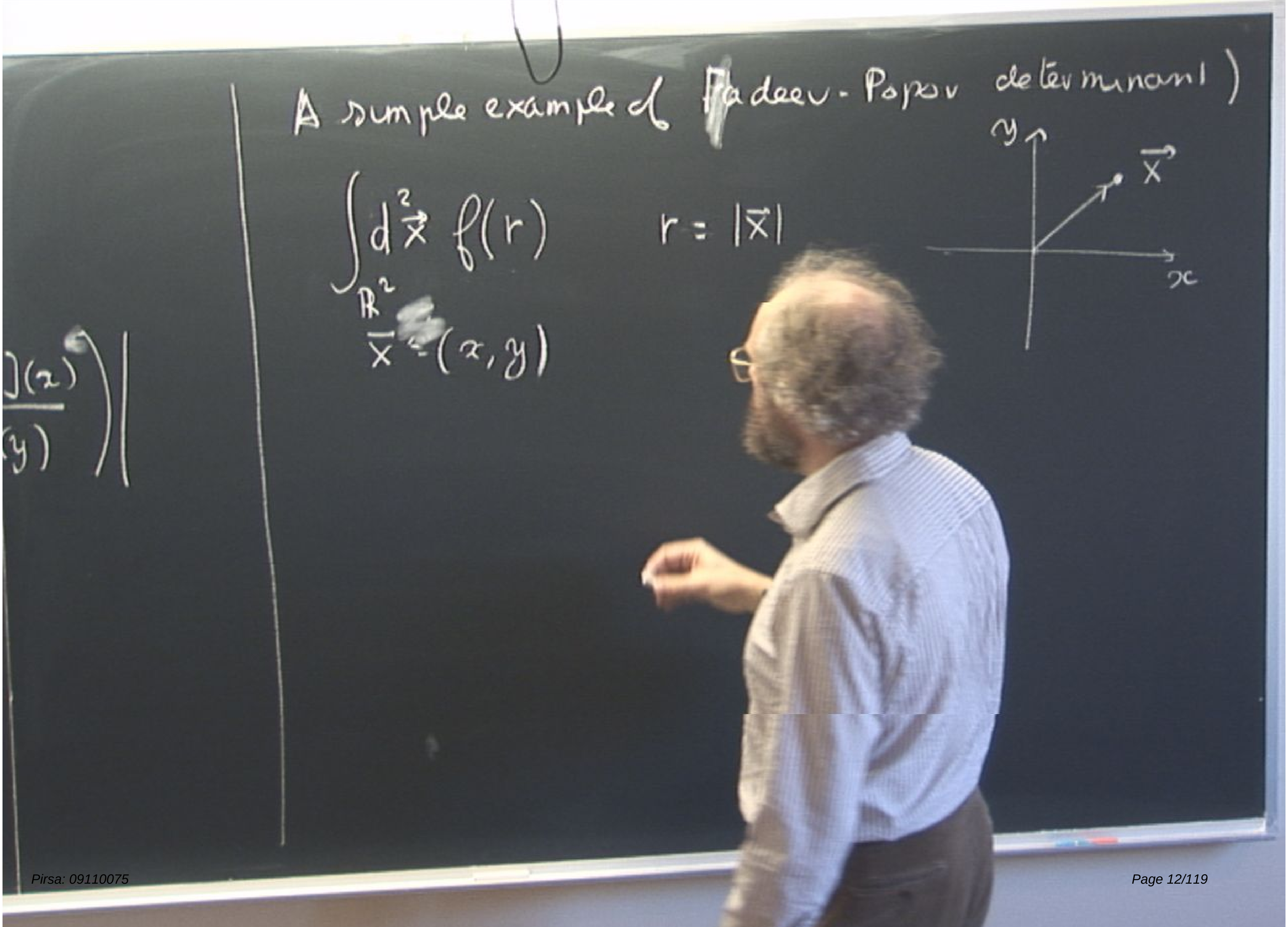

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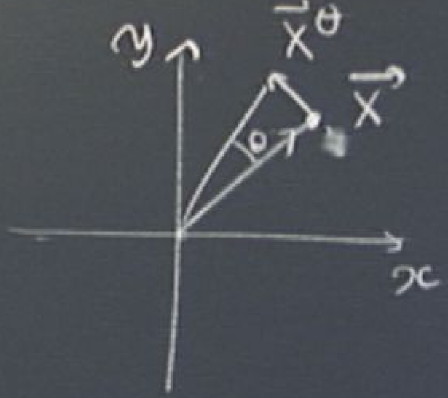

$$\int_{\mathbb{R}^2} d^2\vec{x}\, f(r) \qquad r = |\vec{x}|$$

$\vec{x} = (x, y)$

Invariance under rotations

$\vec{x} \to \vec{x}^{\theta}$

U(1)

$\frac{(x)}{(y)}\Big)\Big|$

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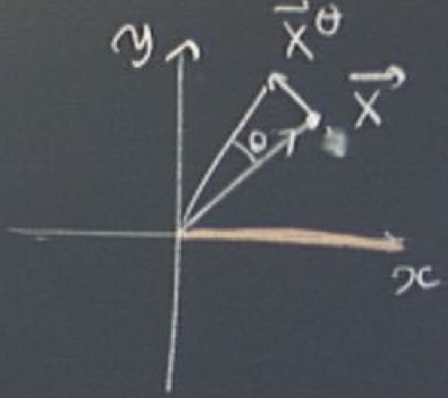

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$\vec{x} = (x, y)$ Invariance under rotations

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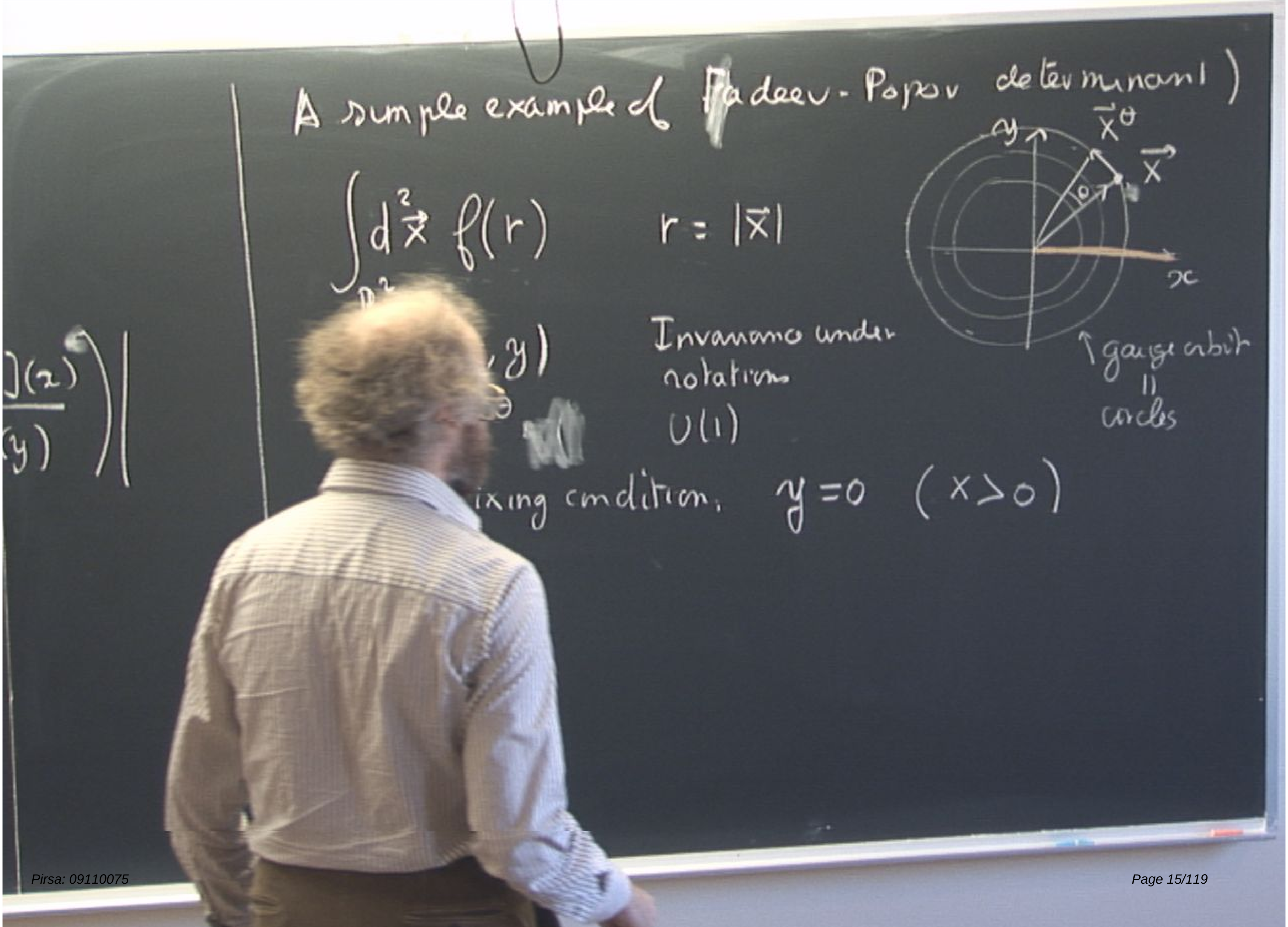
Gauge Fixing condition: $y = 0 \quad (x > 0)$



$\frac{J(x)}{(y)}))|$

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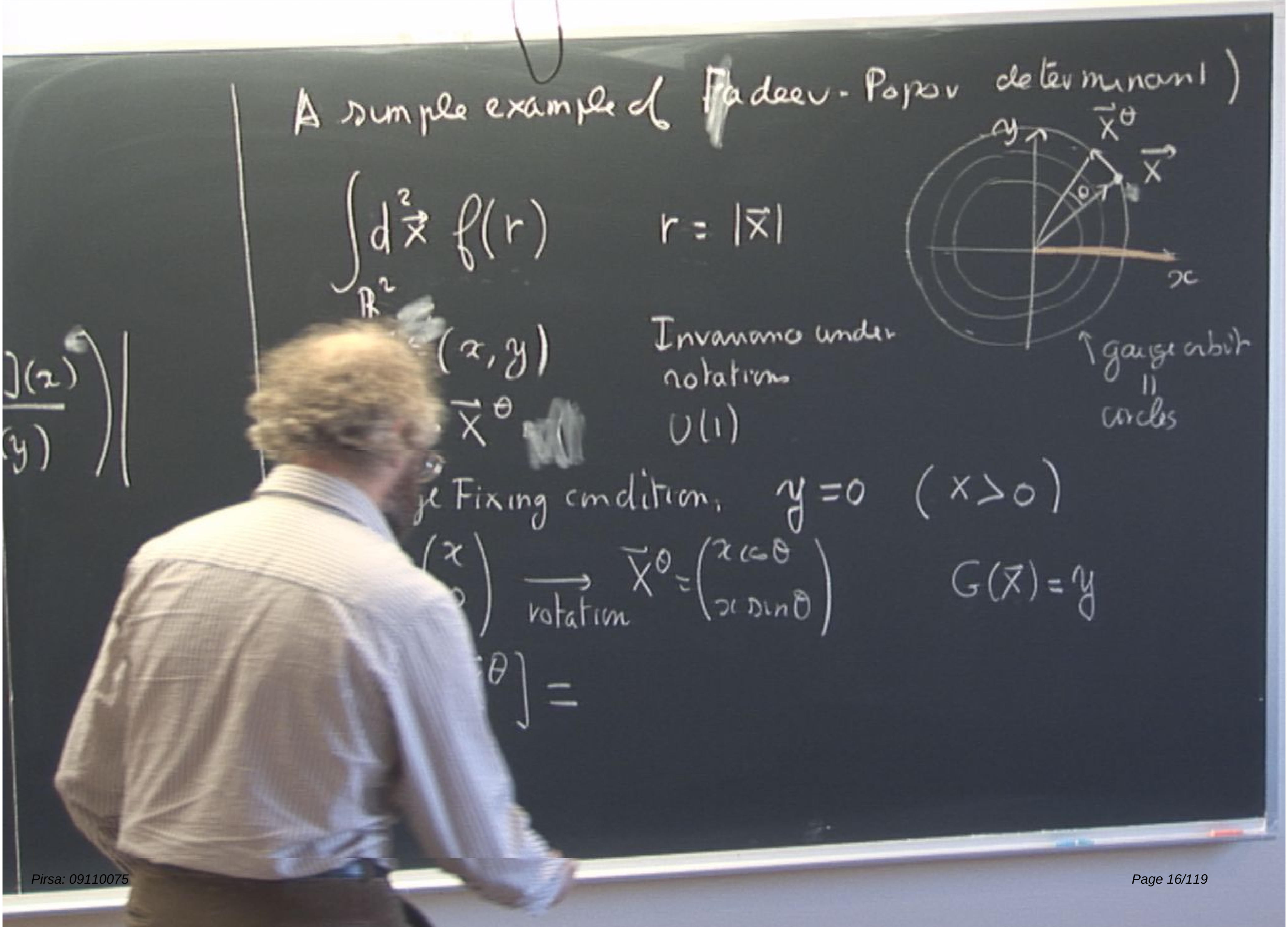

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(x, y) Invariance under rotations

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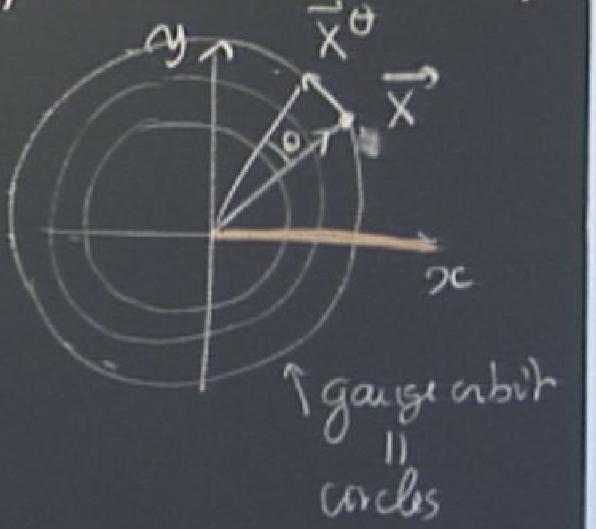
$$\begin{pmatrix} x \\ 0 \end{pmatrix} \xrightarrow[\text{rotation}]{} \vec{X}^{\theta} = \begin{pmatrix} x\cos\theta \\ x\sin\theta \end{pmatrix} \qquad G(\vec{X}) = y$$

$$\theta\Big] =$$

$$\frac{](x)}{(y)}\Big)\Big|$$

$$\frac{J(x)}{(y)}\Big)\Big|$$

A simple example of (Fadeev - Popov determinant)



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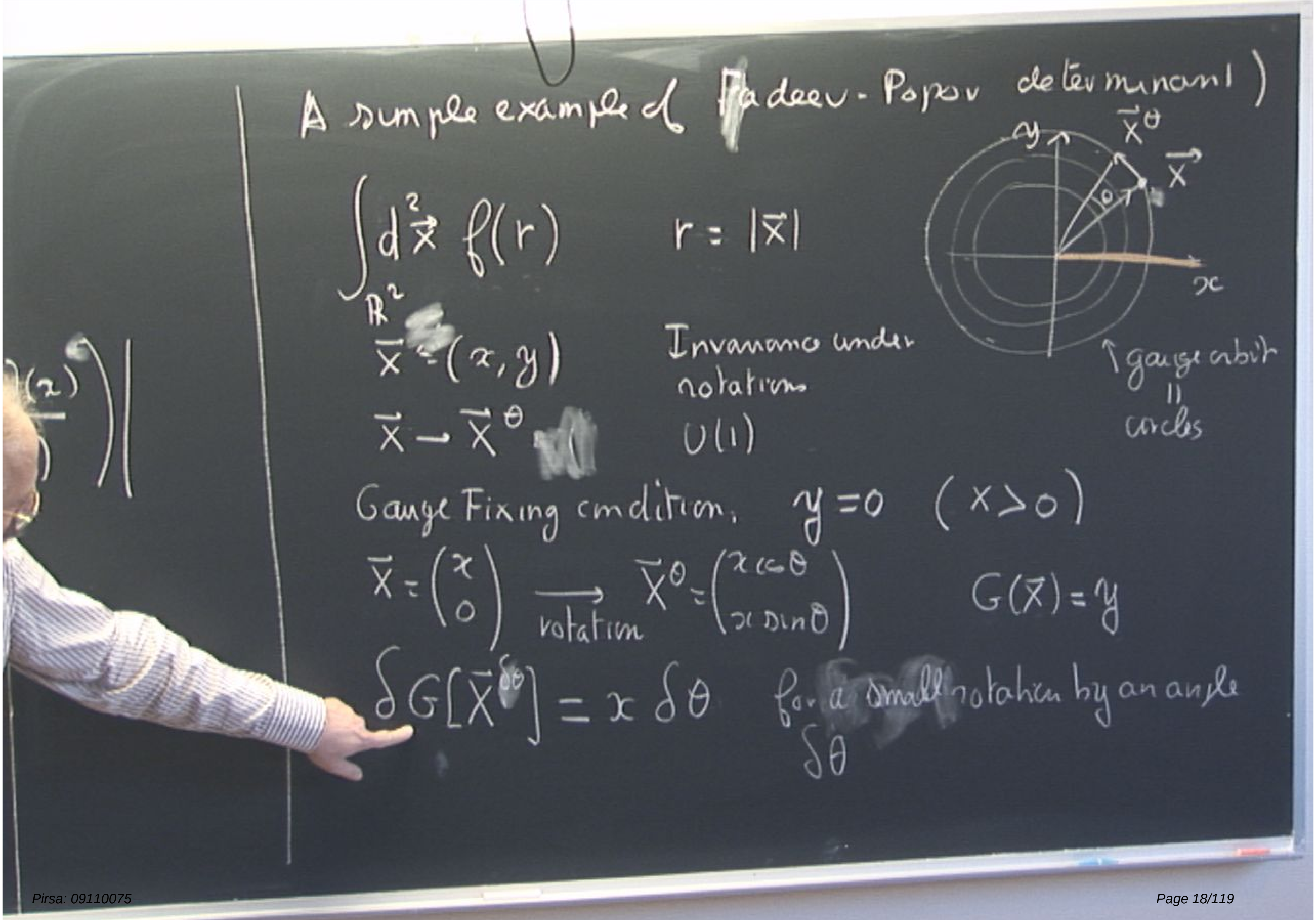
$$\delta G[\vec{x}^{\theta}] = x\,\delta\theta$$



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$\vec{x} = (x, y)$ Invariance under rotations

$\vec{x} \to \vec{x}^{\theta}$ U(1)

y, x, $\vec{x}^{\theta}$, $\vec{x}$, θ

↑ gauge orbit = circles

Gauge Fixing condition: $y = 0$ $(x > 0)$

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$\delta G[\vec{x}^{\delta\theta}] = x\,\delta\theta$ for a small rotation by an angle $\delta\theta$

$= x$

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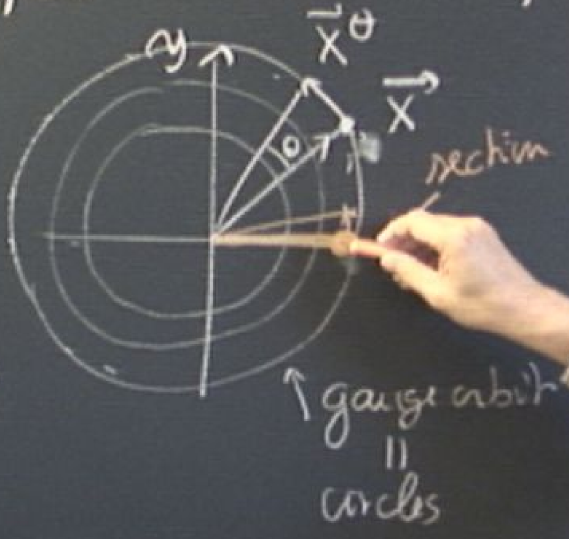

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$\vec{x} = (x, y)$ Invariance under rotations

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$\delta G[\vec{x}^{\delta\theta}] = x\,\delta\theta$ for a small rotation by an angle $\delta\theta$

$${}_{\mu\nu} F^{\mu\nu}]$$

$$\partial_{\nu} A_{\mu}$$

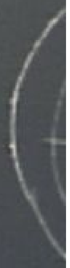
$$A]] \left| \det\left(\frac{\delta G[A^{\Lambda}](x)}{\delta \Lambda(y)} \right) \right|$$

$$\frac{\delta G[\vec{X}^{\theta}]}{\delta \theta} = x$$

$$I = \int d^2\vec{X}\, \delta(y)$$

A simple example of Fadeev - Popo

$$I = \int_{\mathbb{R}^2} d^2\vec{x}\, f(r) \qquad r = |\vec{x}|$$



$\vec{x} = (x, y)$

Invariance under rotations

$\vec{x} \to \vec{x}^{\theta}$

U(1)

Gauge Fixing condition: $y = 0$ (

$$\vec{X} = \begin{pmatrix} x \\ 0 \end{pmatrix} \xrightarrow[\text{rotation}]{} \vec{X}^{\theta} = \begin{pmatrix} x\cos\theta \\ x\sin\theta \end{pmatrix}$$

$$\delta G[\vec{X}^{\delta\theta}] = x\, \delta\theta$$

for a small $\delta\theta$

$A_\mu(x)$ $S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$

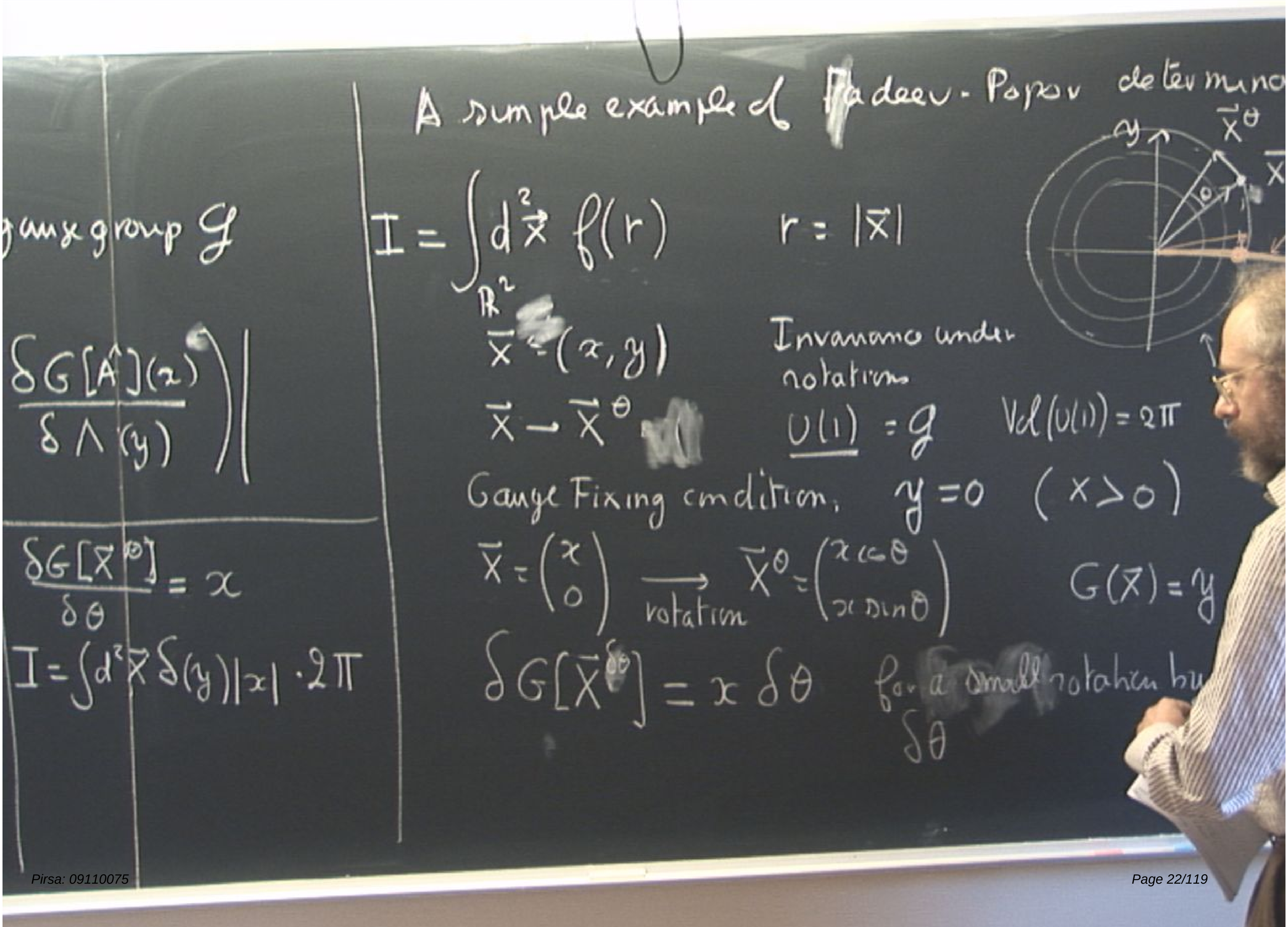
$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ gauge g

$$\int \mathcal{D}[A_\mu] = \mathrm{Vol}(g) = \int \mathcal{D}[A_\mu] \; \delta \left[G[A] \right] \left| \det \left(\frac{\delta G[A}{\delta \Lambda} \right. \right.$$

Gauge Fixing condition $G[A](x) = 0$

$$\frac{\delta G[\vec{x}}{\delta \theta}$$

$I = \int d^3$



gauge group $\mathcal{G}$

$$\frac{\delta G[A^{\Lambda}](x)}{\delta \Lambda(y)} \Big) \Big|$$

$$\frac{\delta G[\vec{x}^{\theta}]}{\delta \theta} = x$$

$$I = \int d^2\vec{x}\, \delta(y)\, |x| \cdot 2\pi$$

A simple example of Fadeev-Popov determina

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$\vec{x} = (x, y)$

Invariance under rotations

$\vec{x} \to \vec{x}^{\theta}$

$\underline{U(1)} = \mathcal{G} \qquad Vol(U(1)) = 2\pi$

Gauge Fixing condition, $y = 0 \quad (x > 0)$

$$\vec{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \xrightarrow[\text{rotation}]{} \vec{x}^{\theta} = \begin{pmatrix} x\cos\theta \\ x\sin\theta \end{pmatrix} \qquad G(\vec{x}) = y$$

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oup $\mathcal{G}$

$\frac{](x)}{(y)}\Big)\Big|$

$\theta] = x$

$\vec{x}\,\delta(y)|x| \cdot 2\pi$

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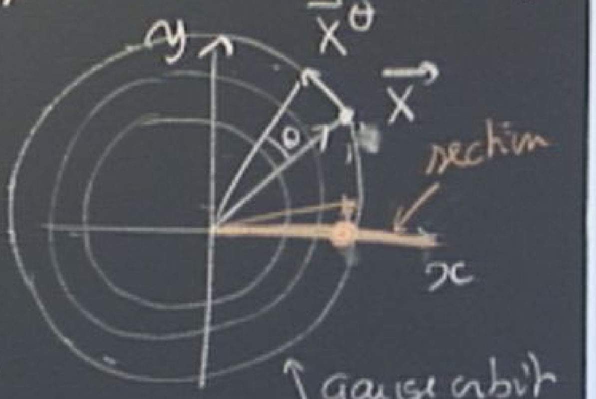
$$I = \int_{\mathbb{R}^2} d^2\vec{x}\, f(r) \qquad r = |\vec{x}|$$



$\vec{x} = (x, y)$

Invariance under rotations

$\vec{x} \to \vec{x}^\theta$

$\underline{U(1)} = \mathcal{G}$ $\quad \mathrm{Vol}(U(1)) = 2\pi$

↑ gauge orbit

$\|$

circles

Gauge Fixing condition: $y = 0 \quad (x > 0)$

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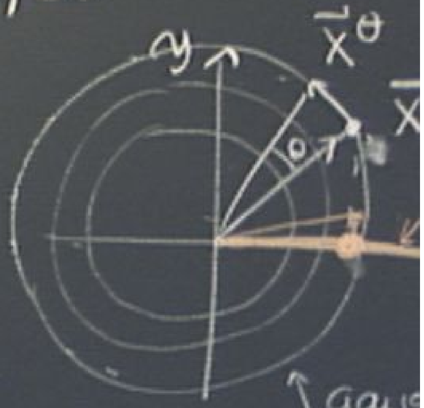
gauge group $\mathcal{G}$

$$\frac{\delta G[\hat{A}](x)}{\delta \Lambda(y)}\Big)\Big|$$

$$\frac{\delta G[\vec{x}^{\theta}]}{\delta\theta} = x$$

$$I = \int d^2\vec{x}\, \delta(y)\, |x| \cdot 2\pi$$
$$\theta(x)$$

A simple example of Fadeev-Popov determina



$$I = \int_{\mathbb{R}^2} d^2\vec{x}\, f(r) \qquad r = |\vec{x}|$$

$\vec{x} = (x, y)$ Invariance under rotations

↑ gaug
$\|$
circl

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$$\frac{\delta G[A^{\Lambda}](x)}{\delta \Lambda(y)}\Bigg)\Bigg|$$

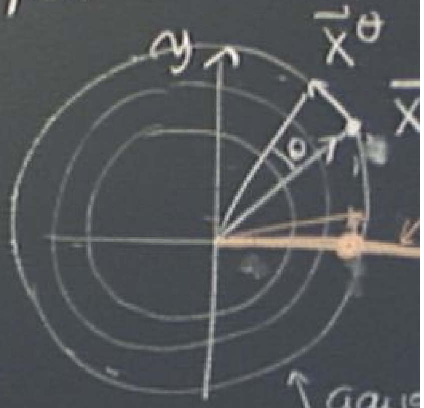
$$\frac{\delta G[\vec{X}^{\theta}]}{\delta \theta} = x$$

$$I = \int d^2\vec{x}\, \delta(y)\,|x| \cdot 2\pi\, f(|x|)$$

$\theta(x)$

$$= 2\pi \int_0^{\infty} dr\, r\, f(r)$$

A simple example of Fadeev - Popov determin



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$$\vec{x} = (x, y)$$

$$\vec{x} \to \vec{x}^{\theta}$$

Invariance under rotations

$\underline{U(1)} = \mathcal{G}$ $\quad$ $\mathrm{Vol}(U(1)) = 2\pi$

↑ gau

||

circl

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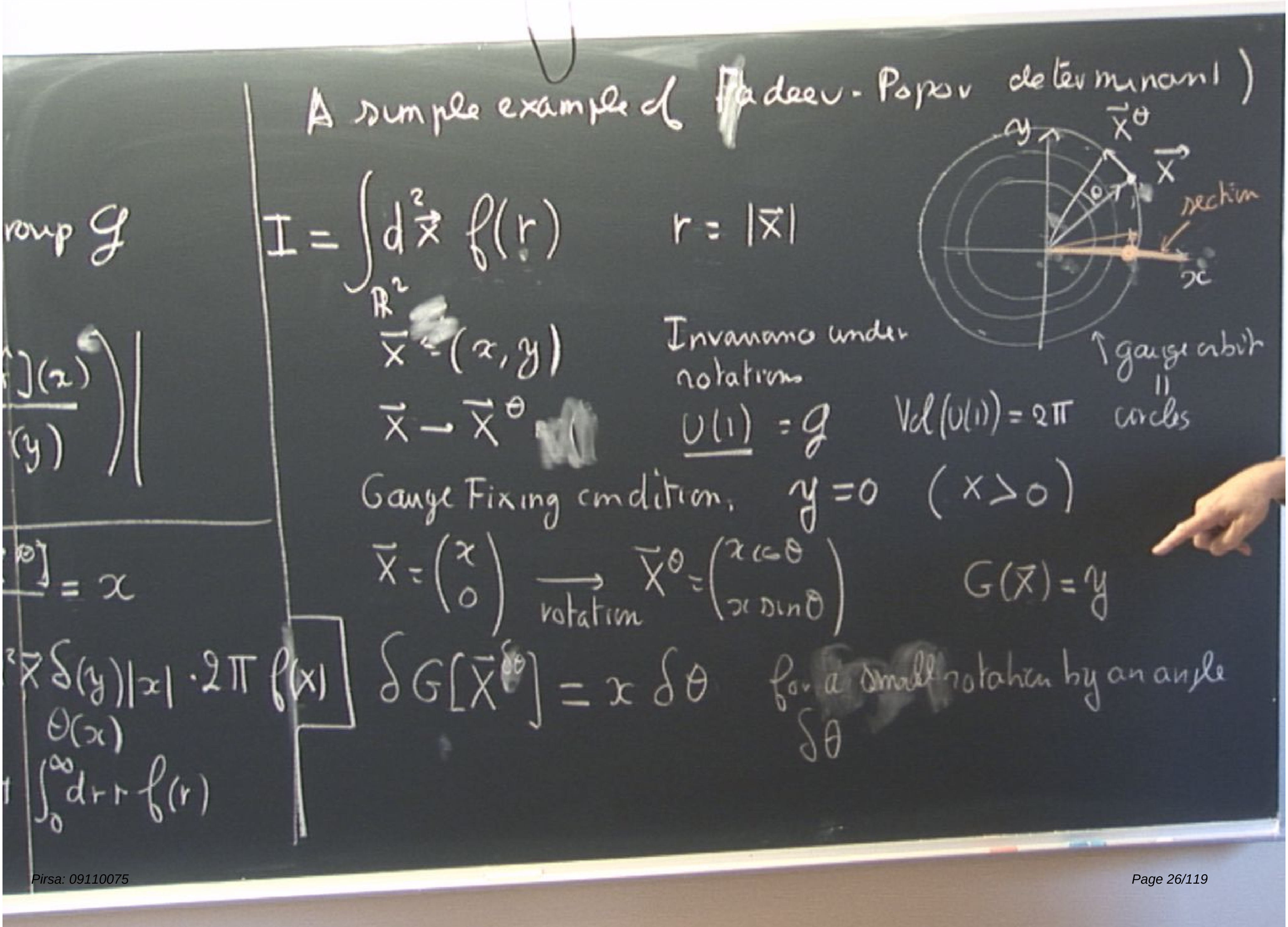

roup $\mathcal{G}$

$$](x) \over (y)$$)|

$\theta] = x$

$$^2\vec{x}\,\delta(y)\,|x|\cdot 2\pi \boxed{f(x)}$$
$\theta(x)$

$$\int_0^\infty dr\, r\, f(r)$$

A simple example of (Fadeev-Popov determinant)

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roup $\mathcal{G}$

$](x)$ / (y) $\Big)\Big|$

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$\theta(x)$

$\int_0^\infty dr\, r\, f(r)$

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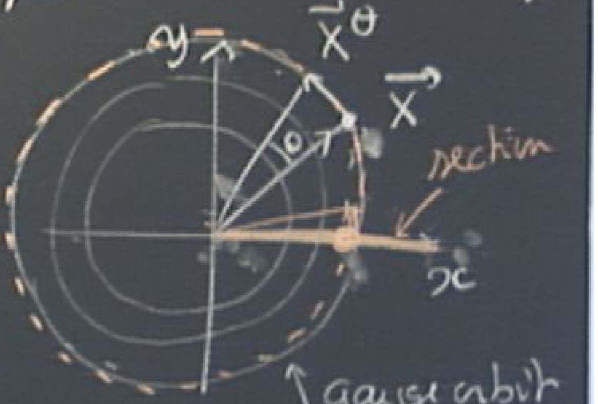

↑ gauge orbit = circles

$\vec{x} = (x, y)$

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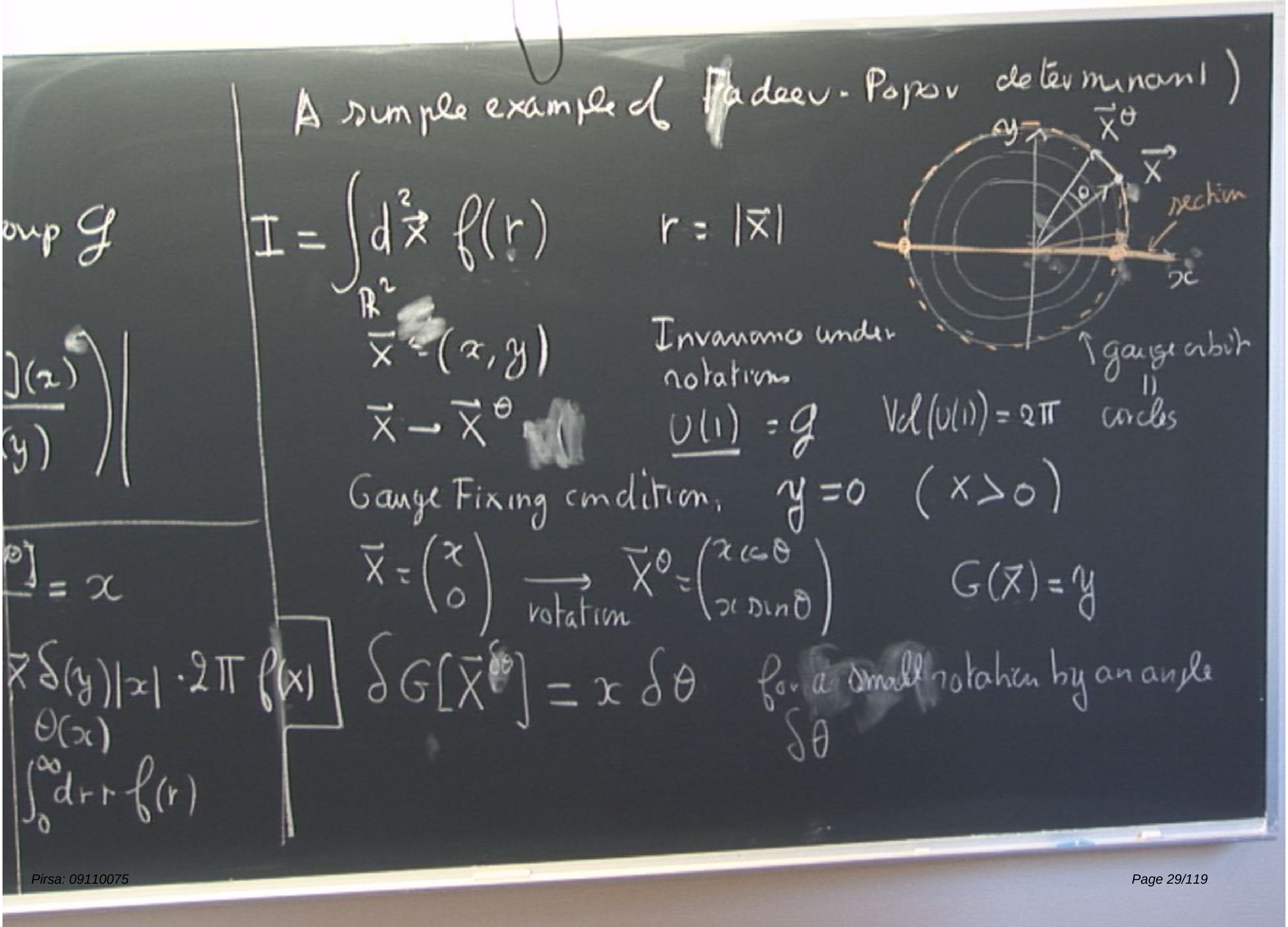
$\frac{\delta G[\vec{x}}{\delta \theta}$

$I = \int d^2\vec{x}$

$= \int$



oup $\mathcal{G}$

$$](x)$$

$$(y)\Big)\Big|$$

$$\overline{o}] = x$$

$$\vec{x}\,\delta(y)|x| \cdot 2\pi \boxed{f(x)}$$

$$\theta(x)$$

$$\int_0^\infty dr\, r\, f(r)$$

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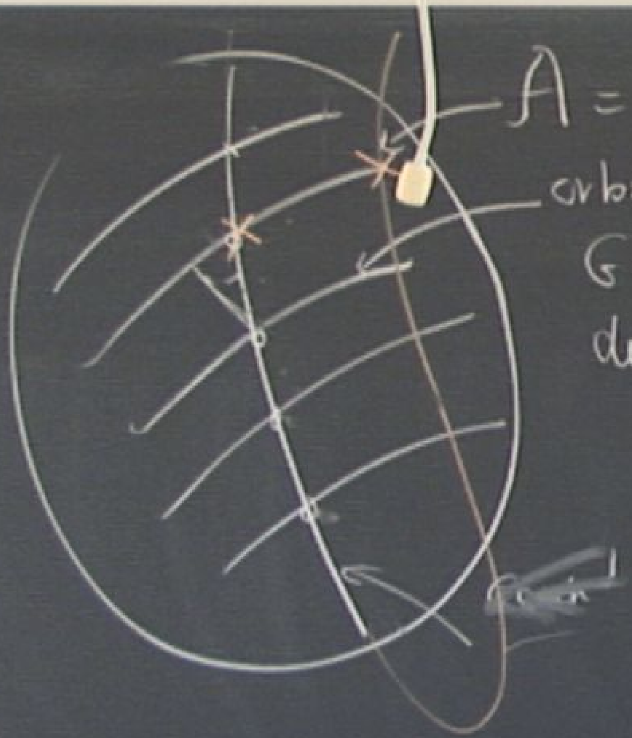
for a small rotation by an angle $\delta\theta$



$\mathcal{A} = \{ A_\mu(x) \}$

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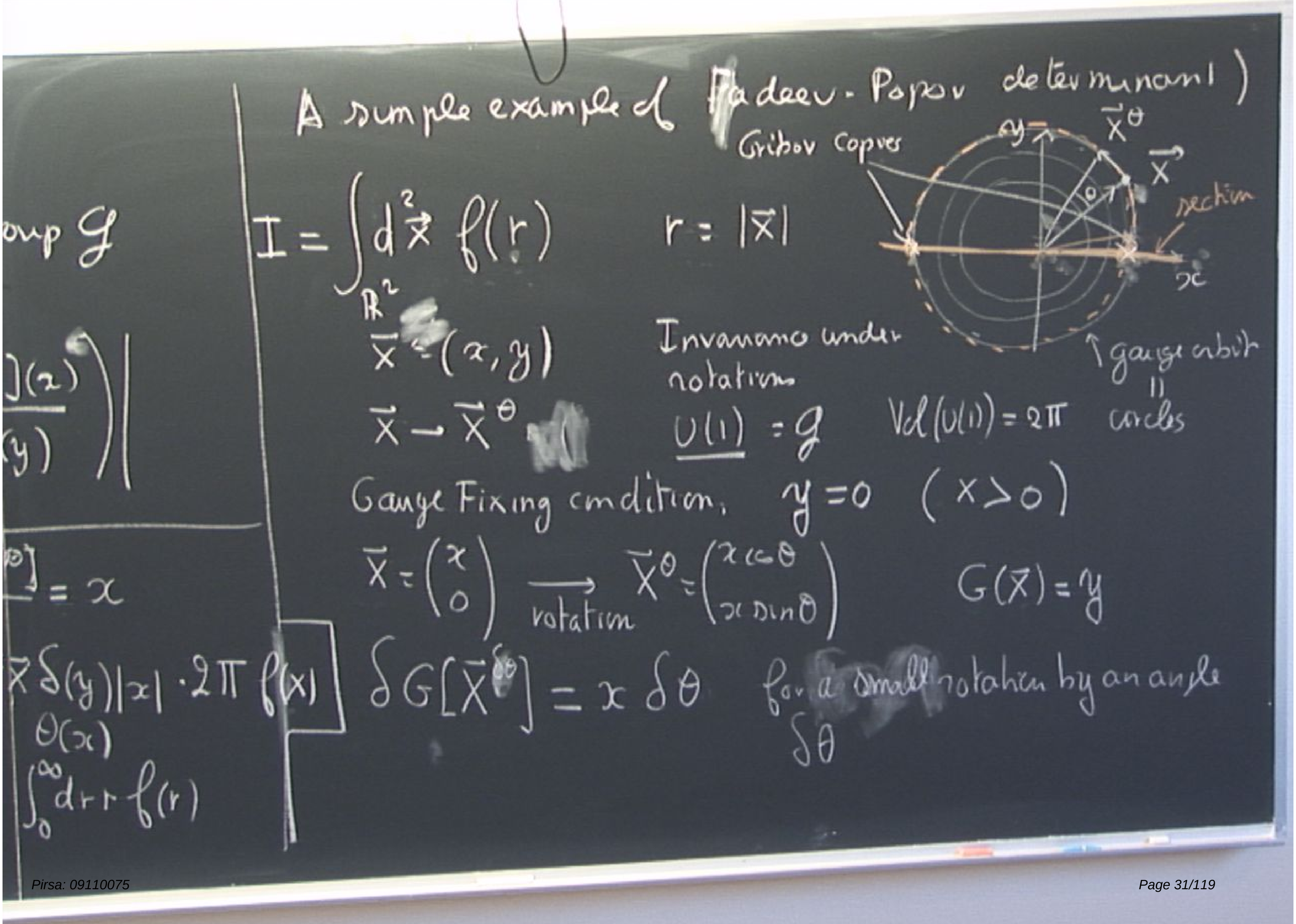
Gauge Fixing condition defines a "slice" in $\mathcal{A}$



oup $\mathcal{G}$

$](x) / (y)))|$

$] = x$

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$\Theta(x)$

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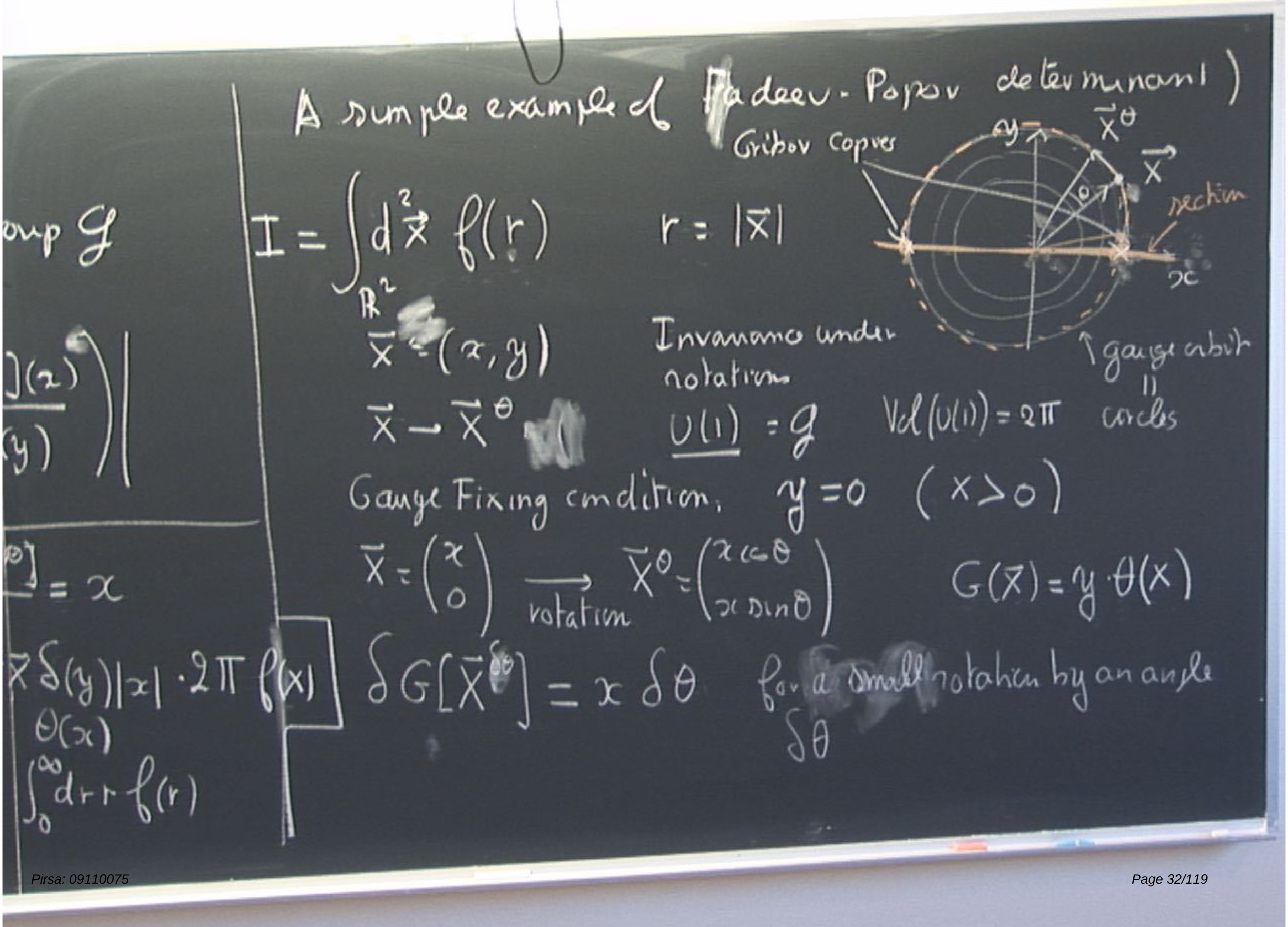
$$\vec{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \xrightarrow[\text{rotation}]{} \vec{x}^{\theta} = \begin{pmatrix} x\cos\theta \\ x\sin\theta \end{pmatrix} \qquad G(\vec{x}) = y$$

$\delta G[\vec{x}^{\delta\theta}] = x\,\delta\theta$ for a small rotation by an angle $\delta\theta$



oup $\mathcal{G}$

$\frac{](x)}{(y)}\Big)\Big|$

$\underline{\theta]} = x$

$\vec{x}\,\delta(y)\,|x|\cdot 2\pi\, \boxed{f(|x|)}$

$\theta(x)$

$\int_0^\infty dr\, r\, f(r)$

A simple example of (Faddeev-Popov determinant)

$$I = \int_{\mathbb{R}^2} d^2\vec{x}\; f(r) \qquad r = |\vec{x}|$$

$$\vec{x} = (x, y)$$

Invariance under rotations

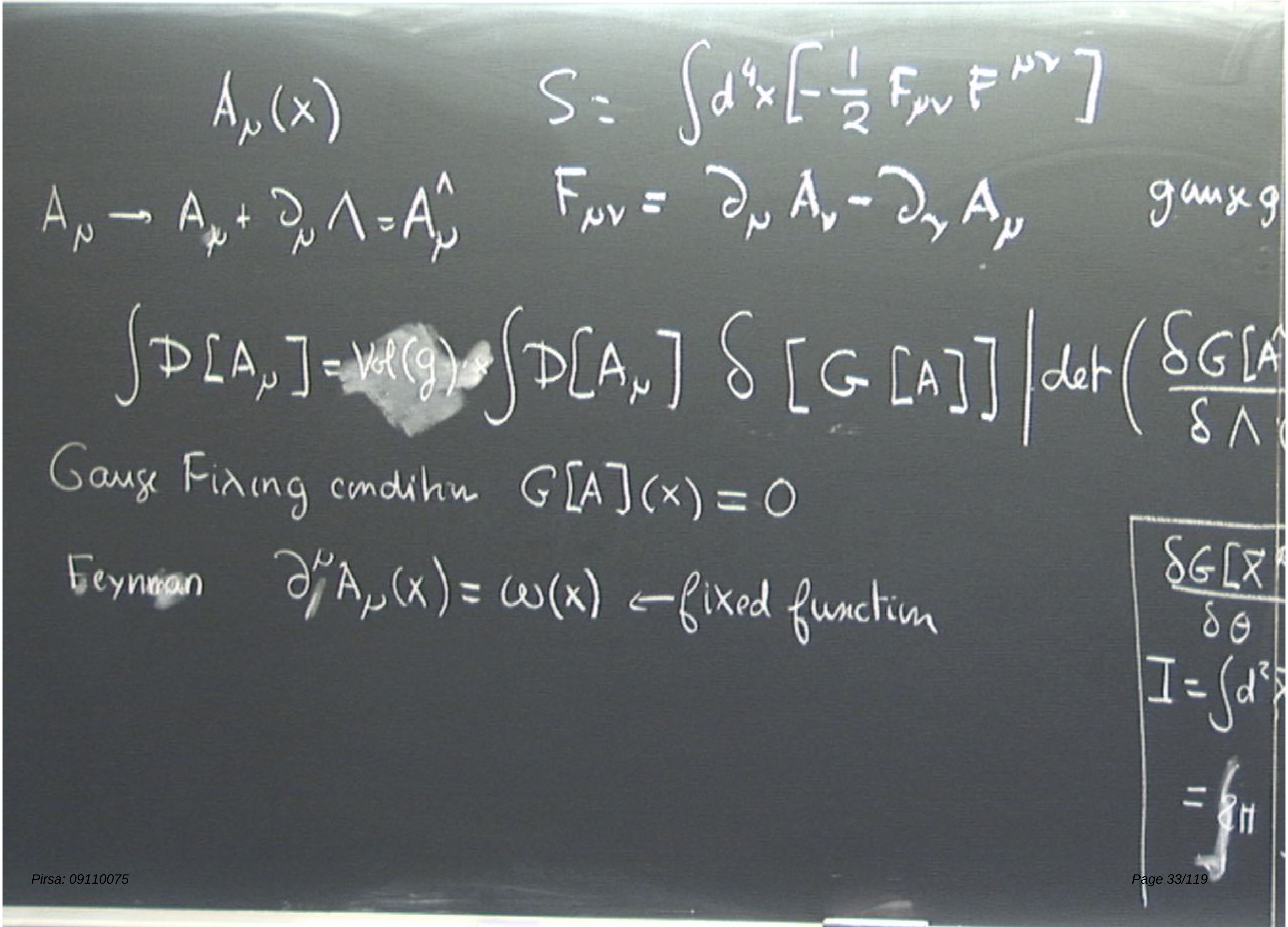
$$\vec{x} \to \vec{x}^{\theta}$$

$\underline{U(1)} = \mathcal{G}$ $\quad Vol(U(1)) = 2\pi$

Gauge Fixing condition: $y = 0 \quad (x > 0)$

$$\vec{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \xrightarrow[\text{rotation}]{} \vec{x}^{\theta} = \begin{pmatrix} x\cos\theta \\ x\sin\theta \end{pmatrix} \qquad G(\vec{x}) = y\cdot\theta(x)$$

$\delta G[\vec{x}^{\delta\theta}] = x\,\delta\theta$ for a small rotation by an angle $\delta\theta$



$A_\mu(x)$

$$S = \int d^4x \left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right]$$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

gauge g

$$\int \mathcal{D}[A_\mu] = \mathrm{Vol}(g) \times \int \mathcal{D}[A_\mu]\ \delta\left[G[A]\right] \left|\det\left(\frac{\delta G[A}{\delta \Lambda}\right.\right.$$

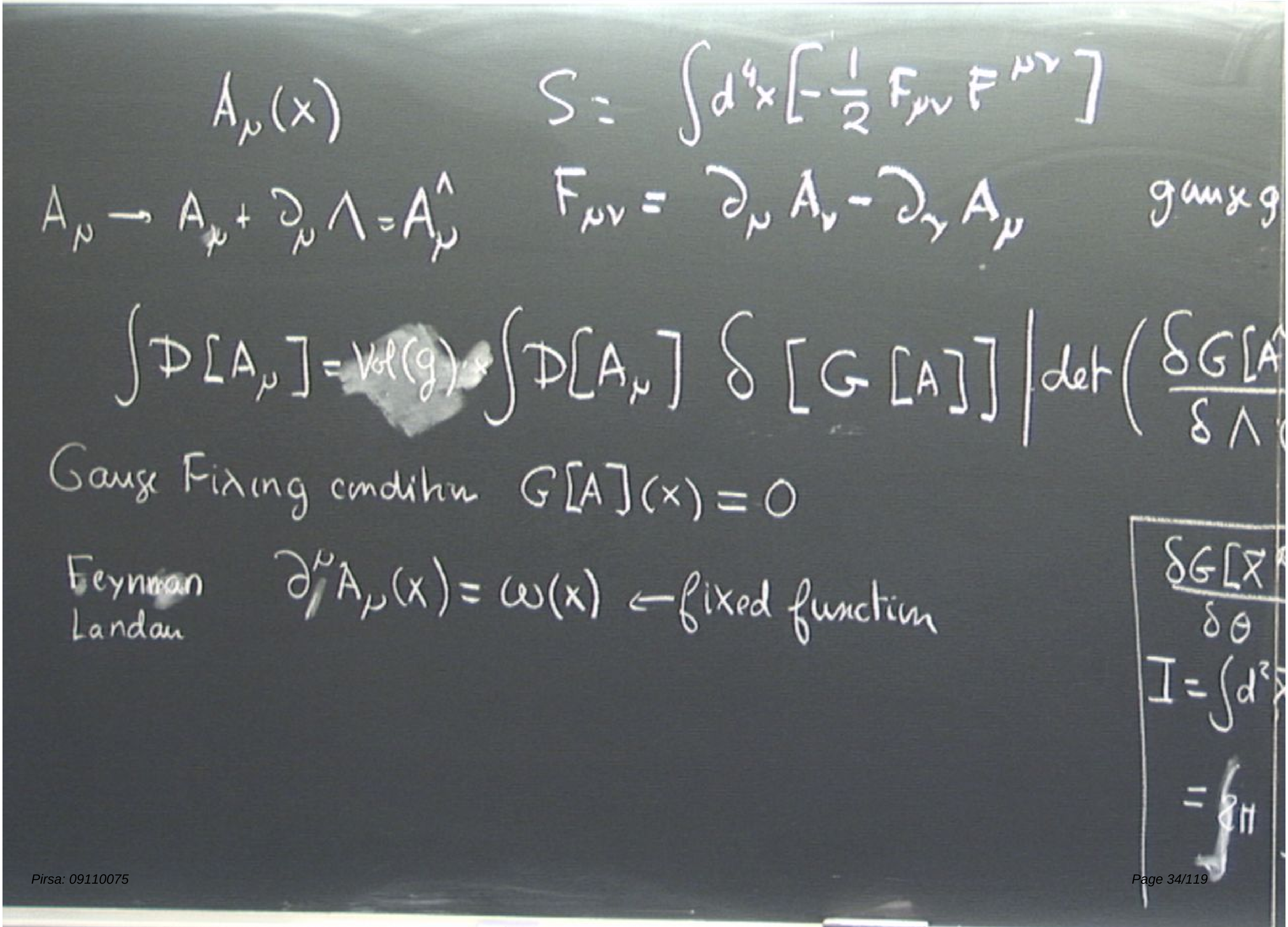
Gauge Fixing condition $G[A](x) = 0$

Feynman $\partial^\mu A_\mu(x) = \omega(x)$ ← fixed function

$$\frac{\delta G[\bar{x}}{\delta \theta}$$

$$I = \int d^2\bar{x}$$

$$= \int$$



$A_\mu(x)$ $\qquad$ $S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$ $\qquad$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad$ gauge g

$$\int D[A_\mu] = \mathrm{Vol}(g) \times \int D[A_\mu]\, \delta\left[G[A]\right] \Big| \det \Big(\frac{\delta G[A}{\delta \Lambda}$$

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Feynman
Landau $\qquad$ $\partial^\mu A_\mu(x) = \omega(x)$ ← fixed function

$\frac{\delta G[\bar{x}}{\delta \theta}$

$I = \int d^2$

$= \int$

$$A_\mu(x) \qquad S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

$$A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge g

$$\int \mathcal{D}[A_\mu] = \mathrm{Vol}(g) \times \int \mathcal{D}[A_\mu]\, \delta\left[G[A] \right] \left| \det \left(\frac{\delta G[A}{\delta \Lambda} \right. \right.$$

Gauge Fixing condition $G[A](x) = 0$

Feynman
Landau

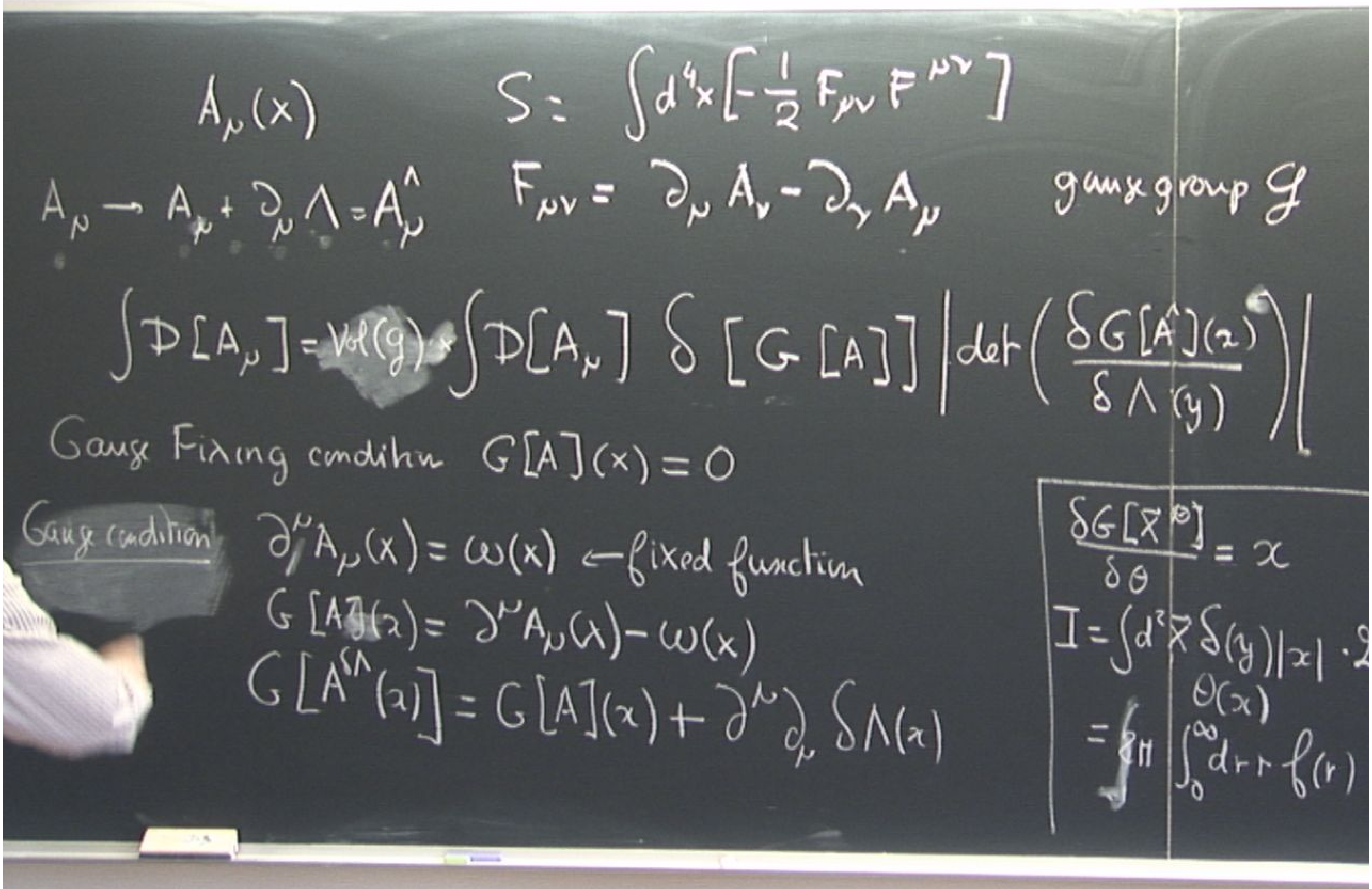
$$\partial^\mu A_\mu(x) = \omega(x) \leftarrow \text{fixed function}$$

$$G[A](x) = \partial^\mu A_\mu(x) - \omega(x)$$

$$\frac{\delta G[\bar{x}}{\delta \theta}$$

$$I = \int d^2 \bar{x}$$

$$= \int_H$$



$A_\mu(x)$

$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

gauge group $\mathcal{G}$

$$\int \mathcal{D}[A_\mu] = \mathrm{Vol}(\mathcal{G}) \times \int \mathcal{D}[A_\mu]\, \delta\left[G[A]\right] \left| \det\left(\frac{\delta G[A^\Lambda](x)}{\delta \Lambda(y)} \right) \right|$$

Gauge Fixing condition $G[A](x) = 0$

Gauge condition

$\partial^\mu A_\mu(x) = \omega(x) \leftarrow$ fixed function

$G[A](x) = \partial^\mu A_\mu(x) - \omega(x)$

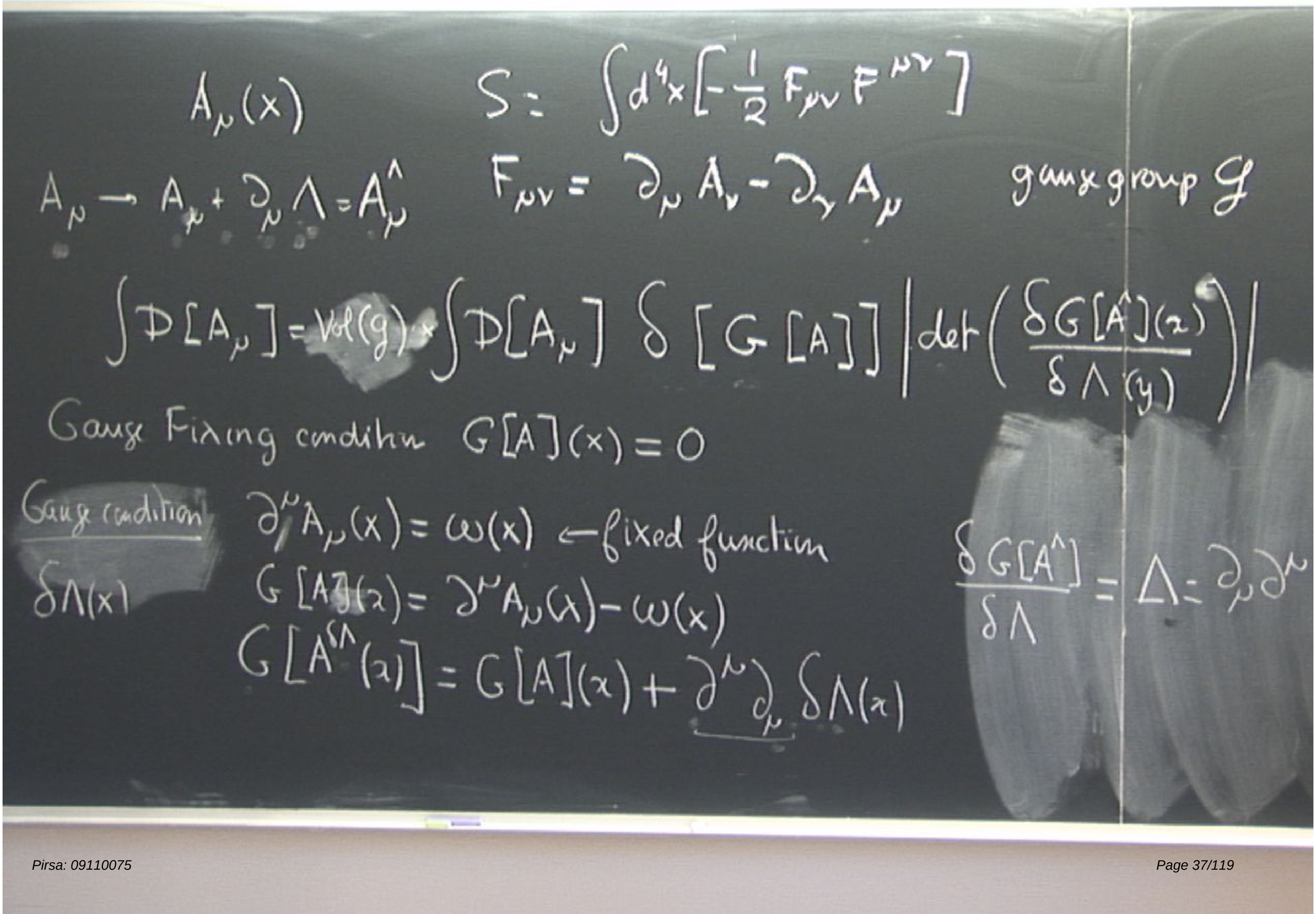
$G[A^{\delta\Lambda}(x)] = G[A](x) + \partial^\mu \partial_\mu \delta\Lambda(x)$

$$\frac{\delta G[\vec{x}^\theta]}{\delta \theta} = x$$

$$I = \int d^2\vec{x}\, \delta(y)\, |x| \cdot$$

$\theta(x)$

$$= \int 2\pi \int_0^\infty dr\, r\, f(r)$$



$A_\mu(x)$

$$S = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu}\right]$$

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Gauge Fixing condition $G[A](x) = 0$

Gauge condition

$\delta\Lambda(x)$

$$\partial^\mu A_\mu(x) = \omega(x) \leftarrow \text{fixed function}$$

$$G[A](x) = \partial^\mu A_\mu(x) - \omega(x)$$

$$G[A^{\delta\Lambda}(x)] = G[A](x) + \underbrace{\partial^\mu \partial_\mu}\, \delta\Lambda(x)$$

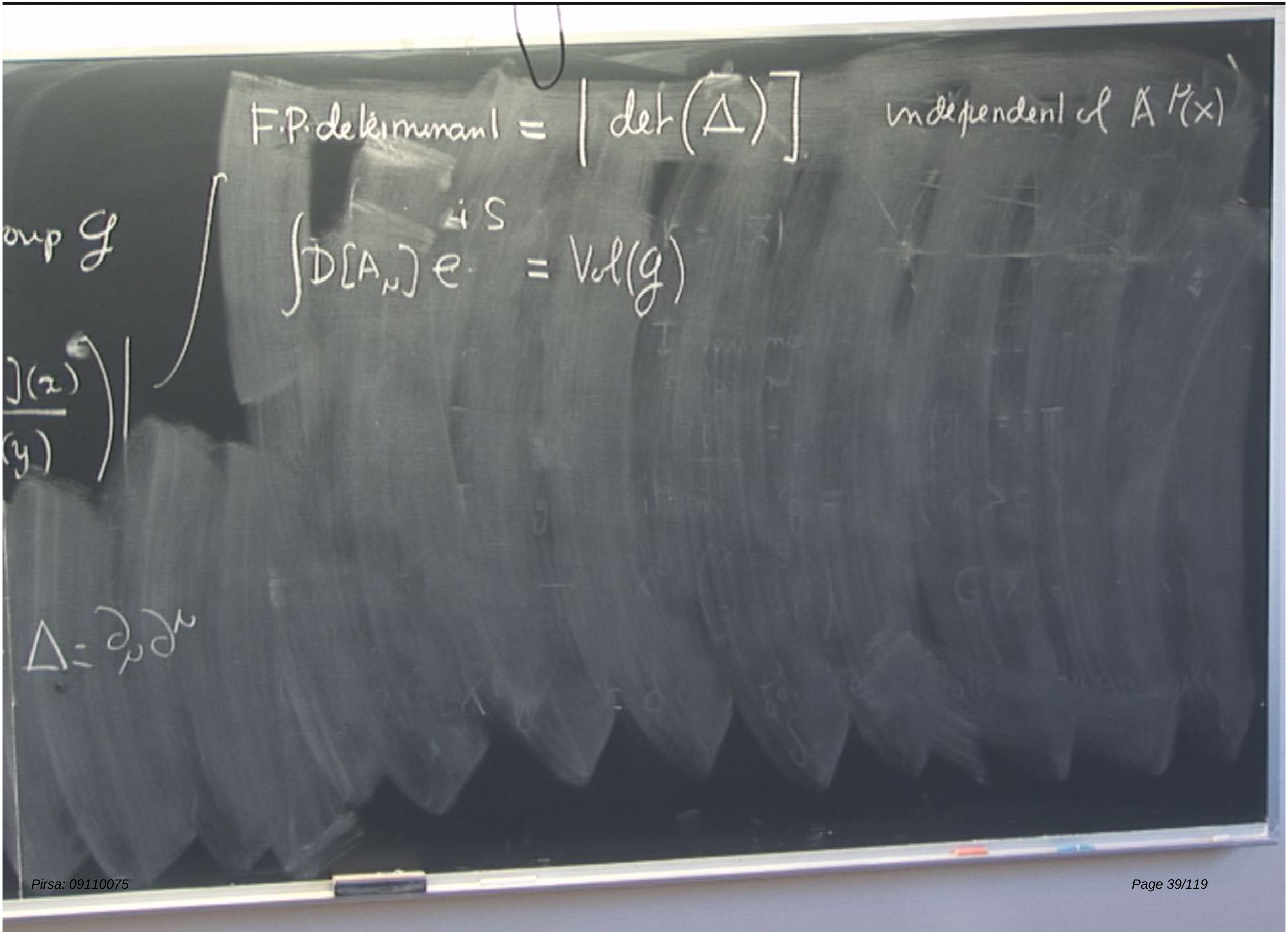
$$\frac{\delta G[A^\Lambda]}{\delta \Lambda} = \Delta = \partial_\mu \partial^\mu$$

F.P. determinant $= \left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

$\frac{J(x)}{(y)} \Big) \Big|$

$\Delta = \partial_\mu \partial^\mu$



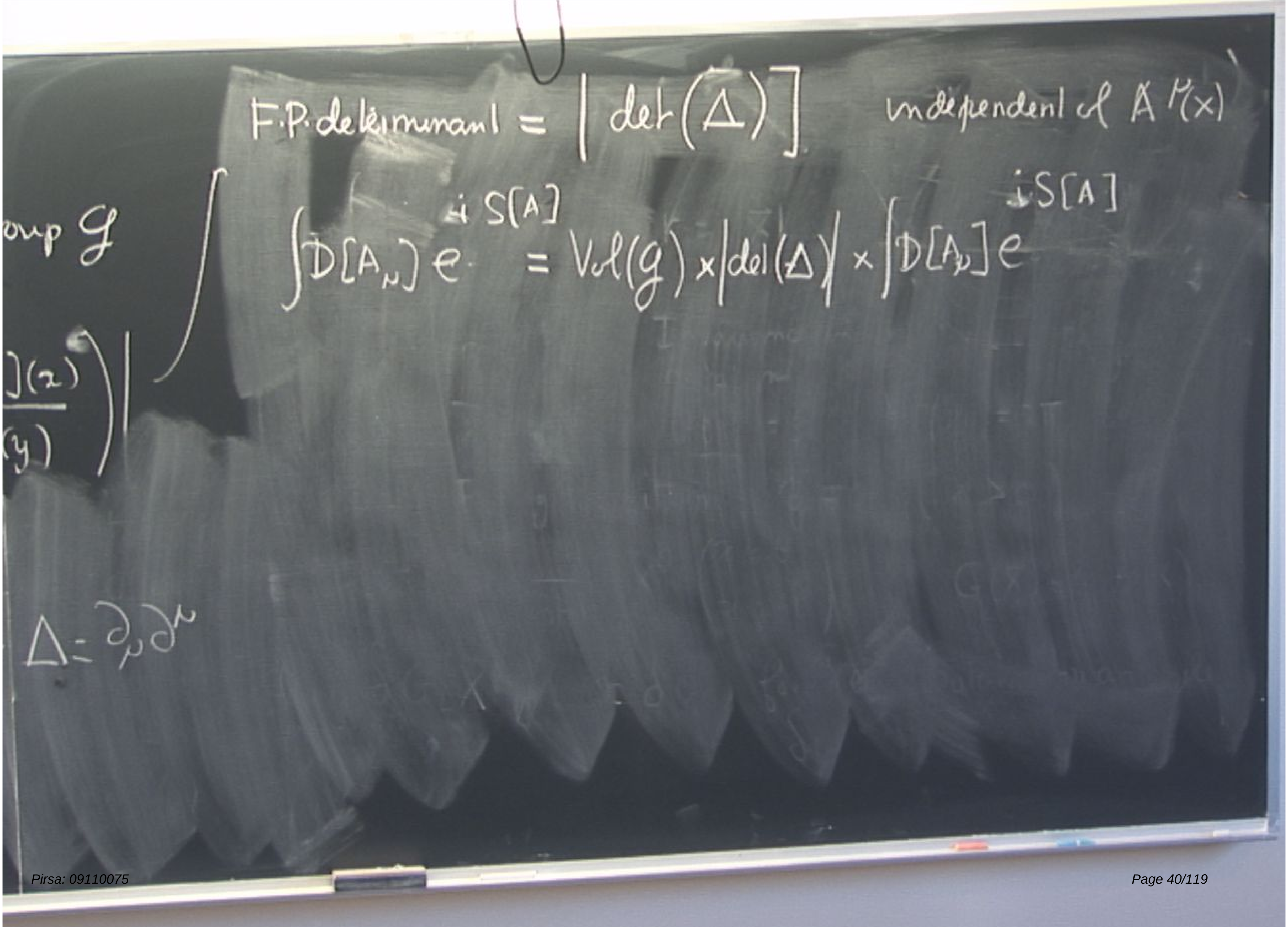
F.P. determinant $= \left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS} = \mathrm{Vol}(\mathcal{G})$$

$$\frac{](x)}{(y)}\Big)\Big|$$

$$\Delta = \partial_\mu \partial^\mu$$



F.P. determinant $= \left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS[A]} = Vol(\mathcal{G}) \times \left|\det(\Delta)\right| \times \int D[A_\nu]\, e^{iS[A]}$$

$$](x) \over (y)) \Big|$$

$\Delta = \partial_\mu \partial^\mu$

$A_\mu(x)$ $\qquad S[A] = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A^\Lambda_\mu$ $\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\qquad$ gauge g

$$\int D[A_\mu] = Vol(g) \times \int D[A_\mu]\, \delta\left[G[A]\right] \left|\det\left(\frac{\delta G[A}{\delta \Lambda}\right.\right.$$

Gauge Fixing condition $G[A](x) = 0$

Gauge condition

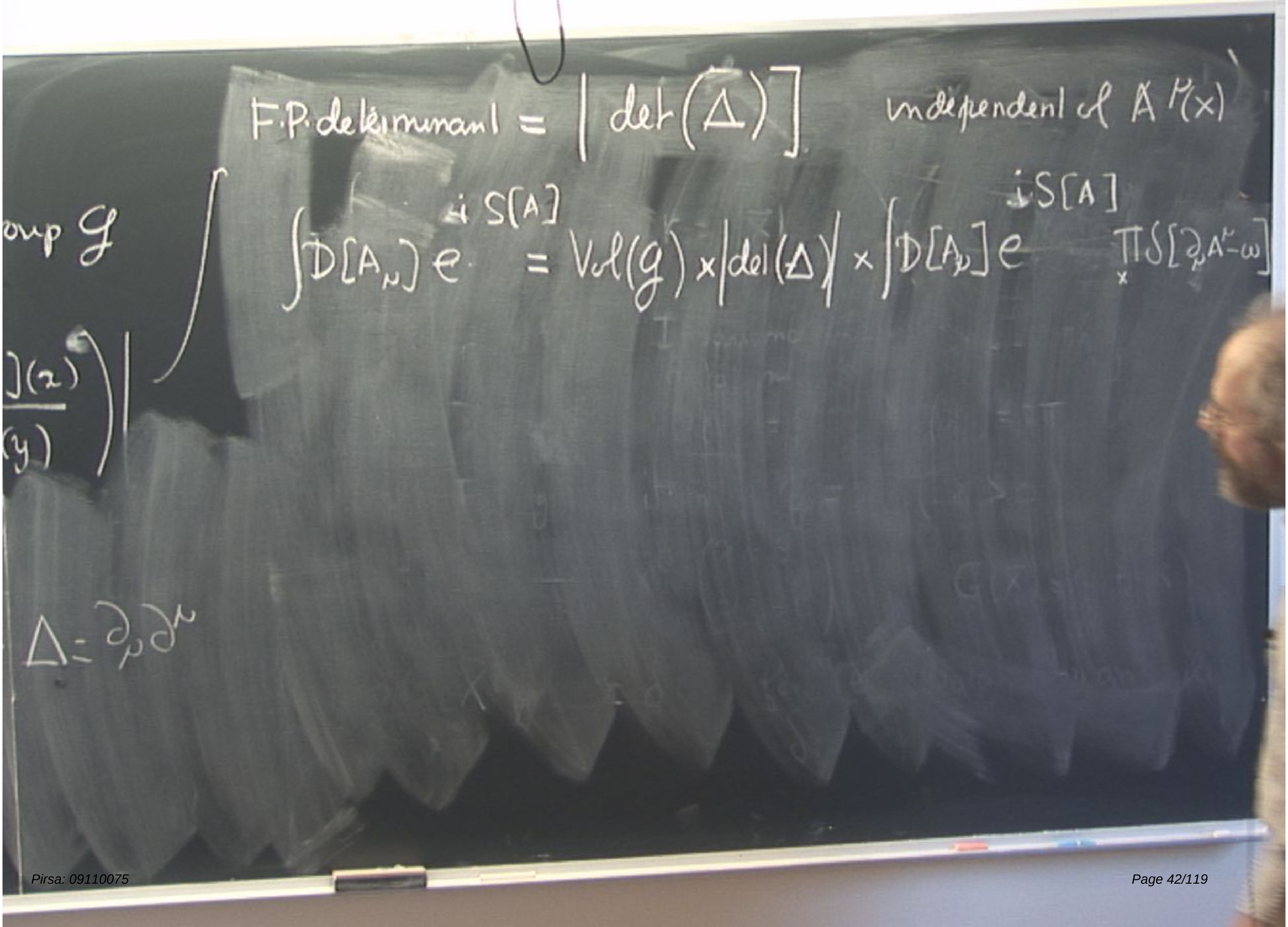
$\delta\Lambda(x)$

$\partial^\mu A_\mu(x) = \omega(x)$ ← fixed function

$G[A](x) = \partial^\mu A_\mu(x) - \omega(x)$

$$G\left[A^{\delta\Lambda}(x)\right] = G[A](x) + \underbrace{\partial^\mu \partial_\mu} \delta\Lambda(x)$$

$$\frac{\delta G[A^\Lambda]}{\delta \Lambda} =$$



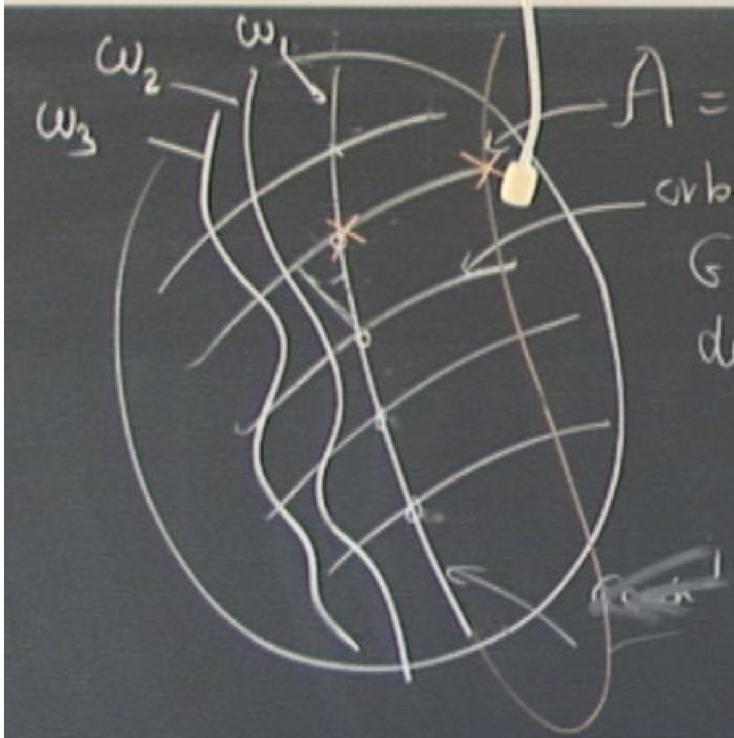
F.P. determinant $= \left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS[A]} = Vol(\mathcal{G}) \times \left|\det(\Delta)\right| \times \int D[A_\mu]\, e^{iS[A]} \prod_x \delta[\partial_\mu A^\mu - \omega]$$

$\frac{](x)}{(y)}\Big)\Big|$

$\Delta = \partial_\mu \partial^\mu$



$\mathcal{A} = \{A_\mu(x)\}$

orbits under gauge transformation.

Gauge Fixing condition defines a "slice" in $\mathcal{A}$

$$\text{F.P. determinant} = \left| \det(\Delta) \right] \qquad \text{independent of } A^{\mu}(x)$$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS[A]} = \mathrm{Vol}(\mathcal{G}) \times \left|\det(\Delta)\right| \times \int D[A_\mu]\, e^{iS[A]} \prod_x \delta[\partial_\mu A^\mu - \omega]$$

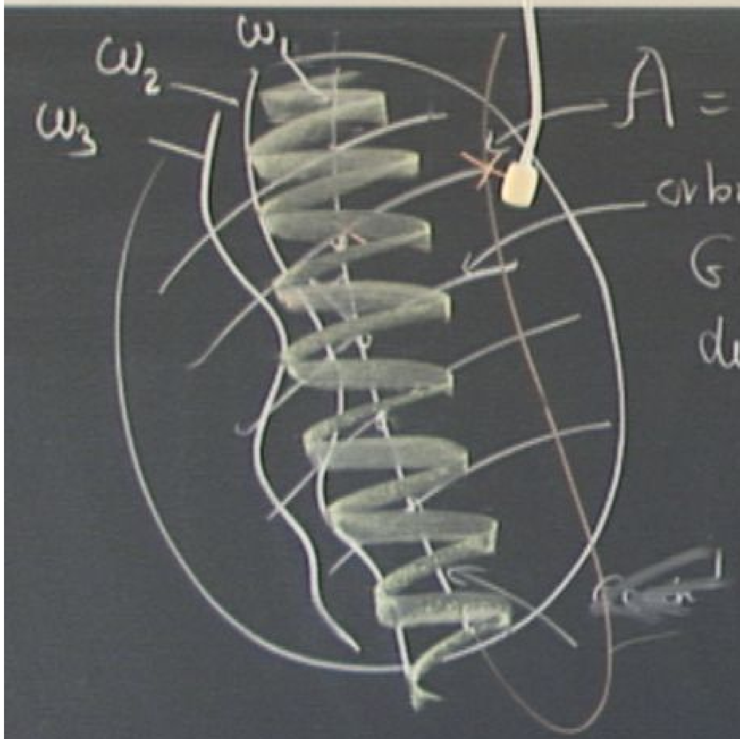
$](x)$

(y)

$$(-+++) \quad \partial^\mu A_\mu = -\partial_0 A_0 + \partial_i A_i \qquad \text{Lorentz invariant}$$

average over differents $\omega(x)$ around $\omega = 0$

$$\Delta = \partial_\mu \partial^\mu$$



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F.P. determinant = $\left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS[A]} = \mathrm{Vol}(\mathcal{G}) \times \left|\det(\Delta)\right| \times \int D[A_\mu]\, e^{iS[A]} \prod_x \delta[\partial_\mu A^\mu - \omega]$$

$](x)$

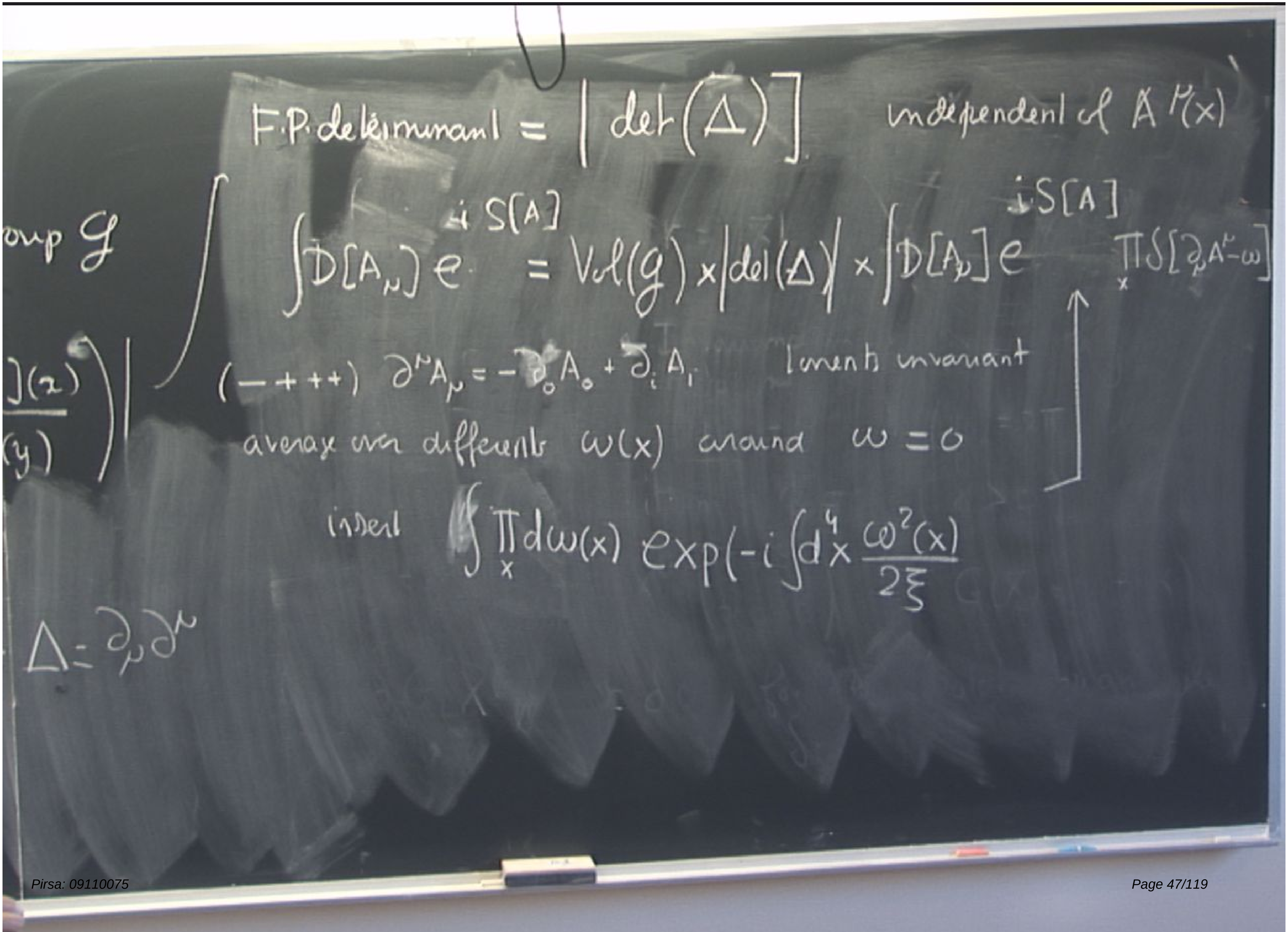
(y)

$$(-+++) \quad \partial^\mu A_\mu = -\partial_0 A_0 + \partial_i A_i$$ Lorentz invariant

average over different $\omega(x)$ around $\omega = 0$

insert $\int \prod_x d\omega(x)$

$\Delta = \partial_\mu \partial^\mu$



F.P. determinant = $\left| \det(\Delta) \right]$ independent of $A^{\mu}(x)$

oup $\mathcal{G}$

$$\int D[A_\mu]\, e^{iS[A]} = Vol(\mathcal{G}) \times \left|\det(\Delta)\right| \times \int D[A_\nu]\, e^{iS[A]} \prod_x \delta[\partial_\mu A^\mu - \omega]$$

$](x)$

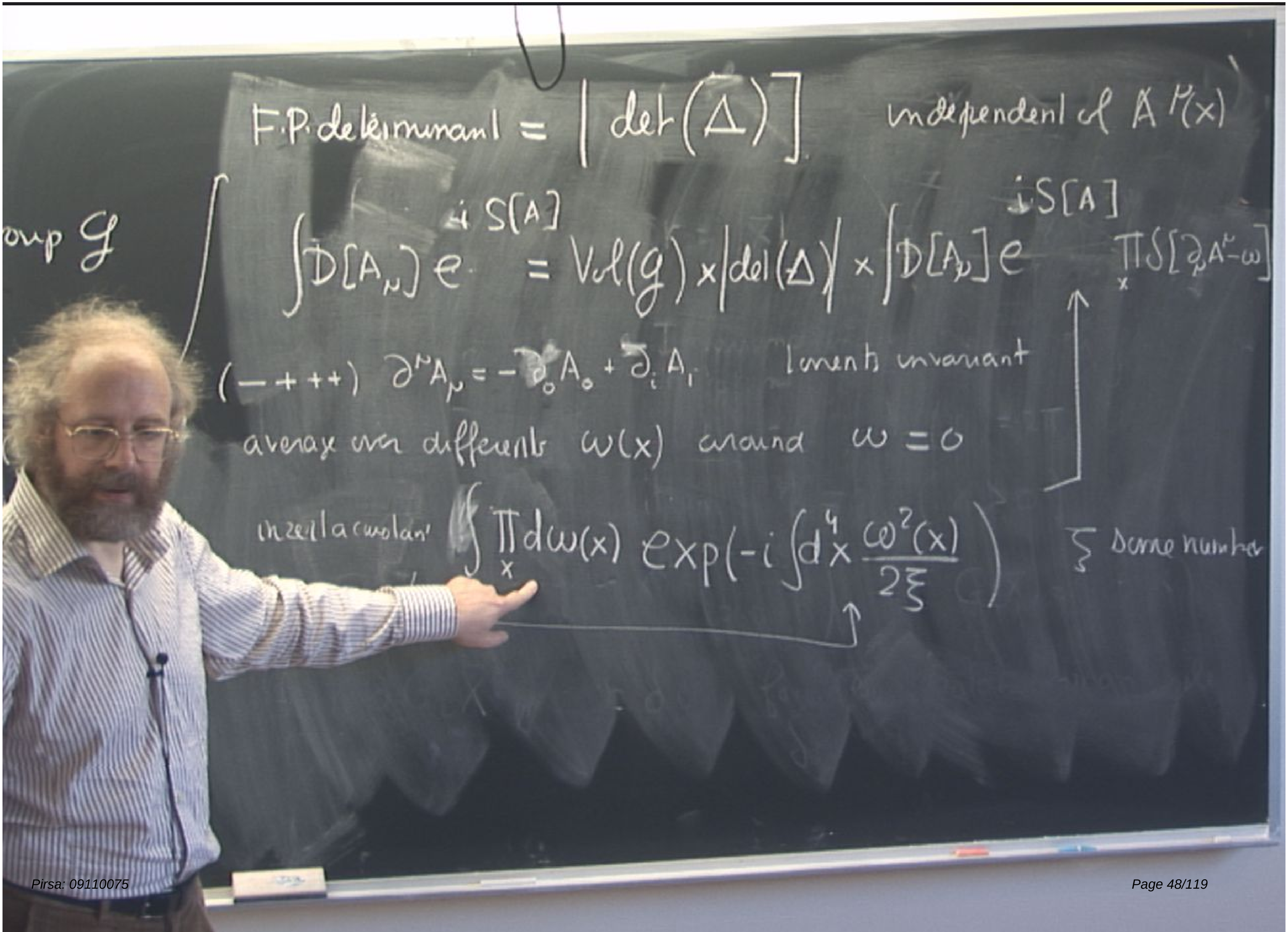
(y)

$(-+++) \quad \partial^\mu A_\mu = -\partial_0 A_0 + \partial_i A_i$ Lorentz invariant

average over differents $\omega(x)$ around $\omega = 0$

insert $\int \prod_x d\omega(x)\, \exp(-i \int d^4x \frac{\omega^2(x)}{2\xi}$

$\Delta = \partial_\mu \partial^\mu$



F.P. determinant = $\left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

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$(-+++)$ $\partial^\mu A_\mu = -\partial_0 A_0 + \partial_i A_i$ Lorentz invariant

average over differents $\omega(x)$ around $\omega = 0$

inzeilla cuolan' $\int \prod_x d\omega(x)\, \exp\left(-i \int d^4x \frac{\omega^2(x)}{2\xi}\right)$ ξ some number

F.P. determinant = $\left| \det(\Delta) \right]$ independent of $A^\mu(x)$

oup $\mathcal{G}$

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$(-+++)$ $\partial^\mu A_\mu = -\partial_0 A_0 + \partial_i A_i$ lorentz invariant

average over differents $\omega(x)$ around $\omega = 0$

[illegible] $\int \prod_x d\omega(x)\, \exp\left(-i \int d^4x \frac{\omega^2(x)}{2\xi}\right)$ ξ some number

(y)

$\Delta = \partial_\mu \partial^\mu$

$A_\mu(x)$

$$S[A] = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu}\right]$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge g

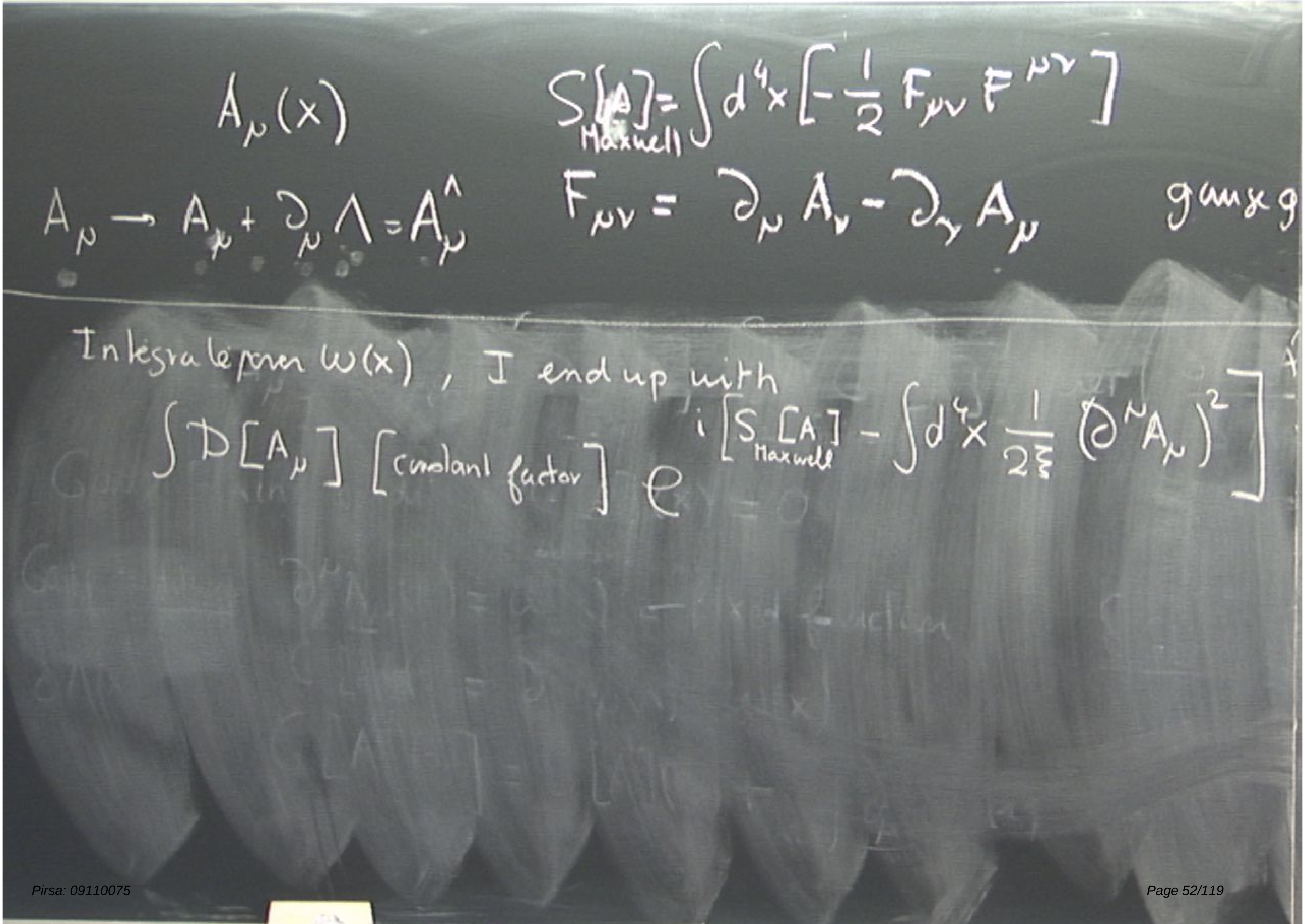
Integrate over $\omega(x)$

$A_\mu(x)$ $\quad S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $\quad$ gauge g

Integrate over $\omega(x)$, I end up with

$$\int \mathcal{D}[A_\mu] \left[\text{constant factor} \right] e^{i S_{\text{Maxwell}}[A]}$$



$A_\mu(x)$

$$S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} \right]$$

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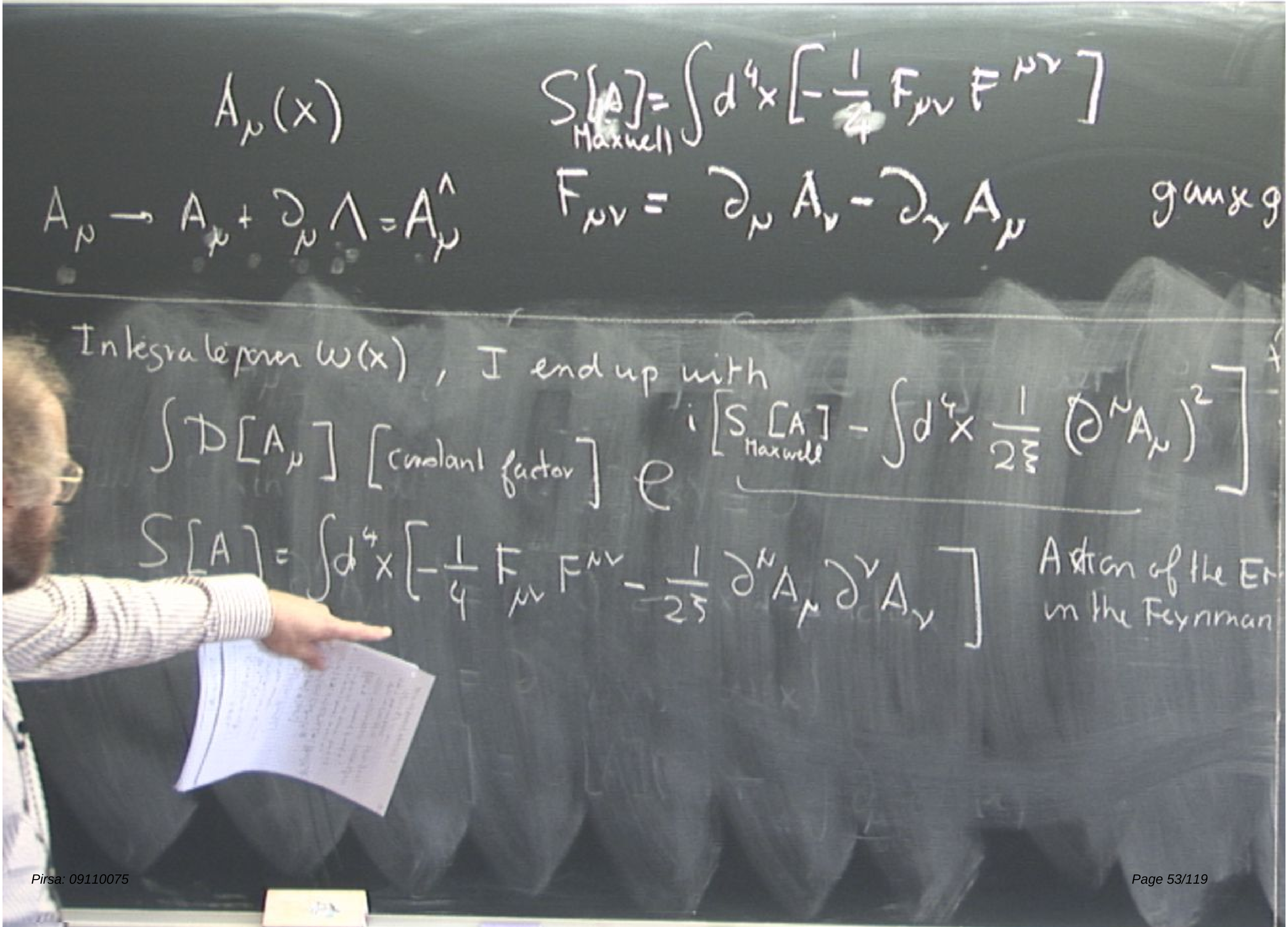
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge g

Integrate over $\omega(x)$, I end up with

$$\int \mathcal{D}[A_\mu] \left[\text{constant factor}\right] e^{i\left[S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi} (\partial^\mu A_\mu)^2\right]}$$



$A_\mu(x)$

$$S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda = A^\Lambda_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge g

Integrate over $\omega(x)$, I end up with

$$\int D[A_\mu] \, [\text{constant factor}] \, e^{i\left[\underline{S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi} (\partial^\mu A_\mu)^2}\right]}$$

$$S[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu \right]$$

Action of the E
in the Feynman

$A_\mu(x)$

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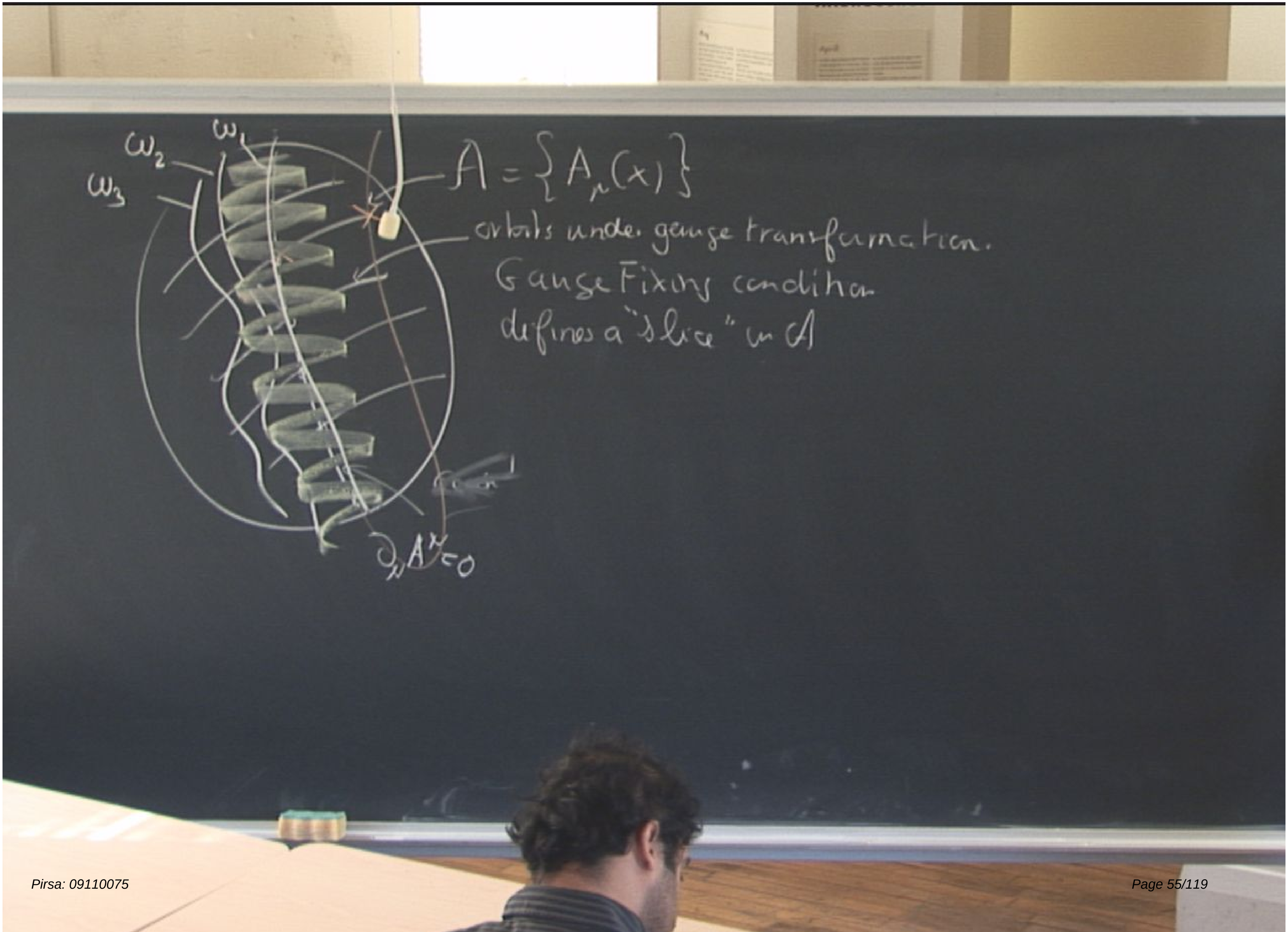
$$S[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu \right]$$

Action of the Em
in the Feynman



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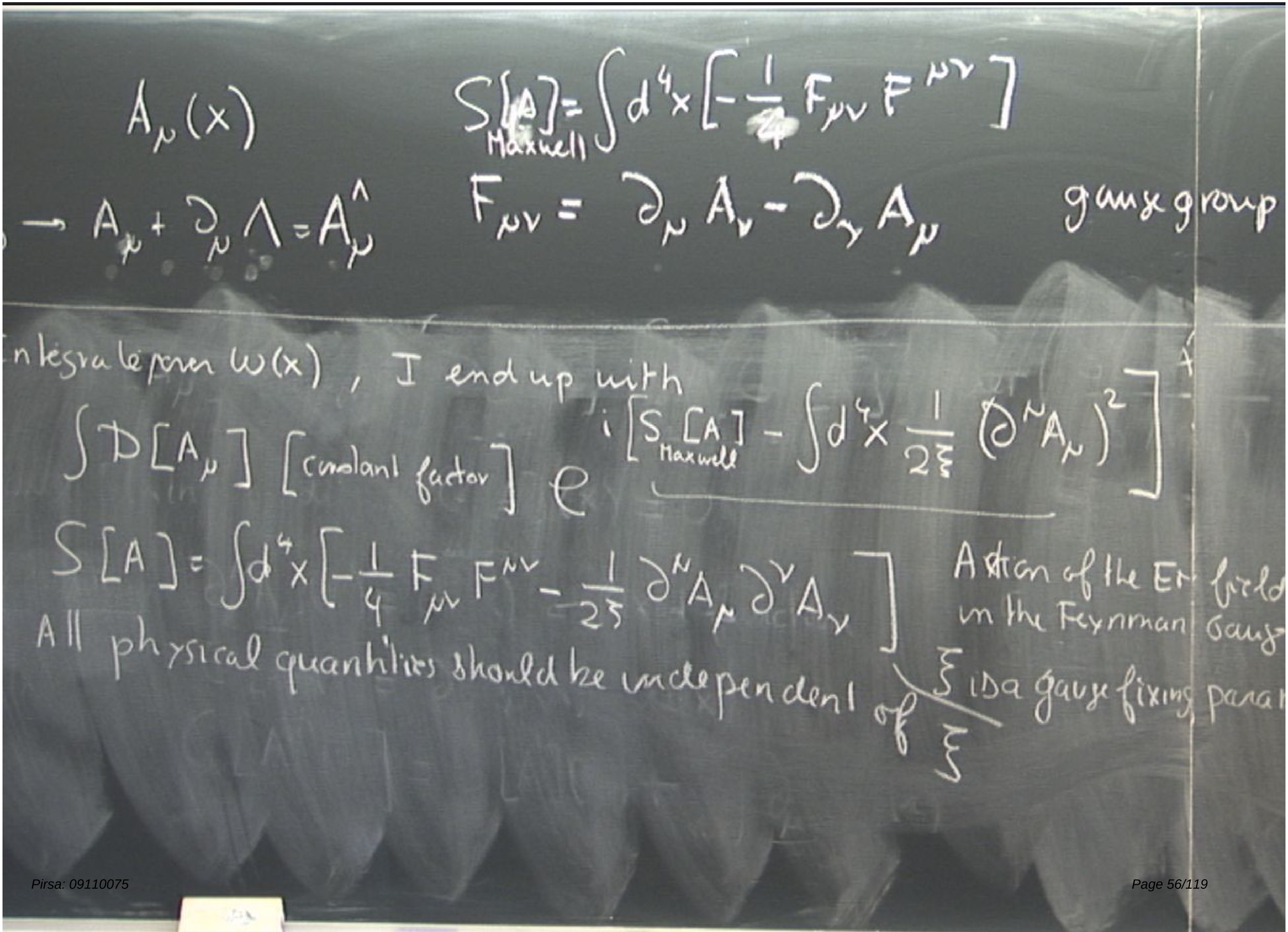
Gauge Fixing condition defines a "slice" in $\mathcal{A}$



$A_\mu(x)$

$$S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right]$$

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gauge group

ntegrate over $\omega(x)$, I end up with

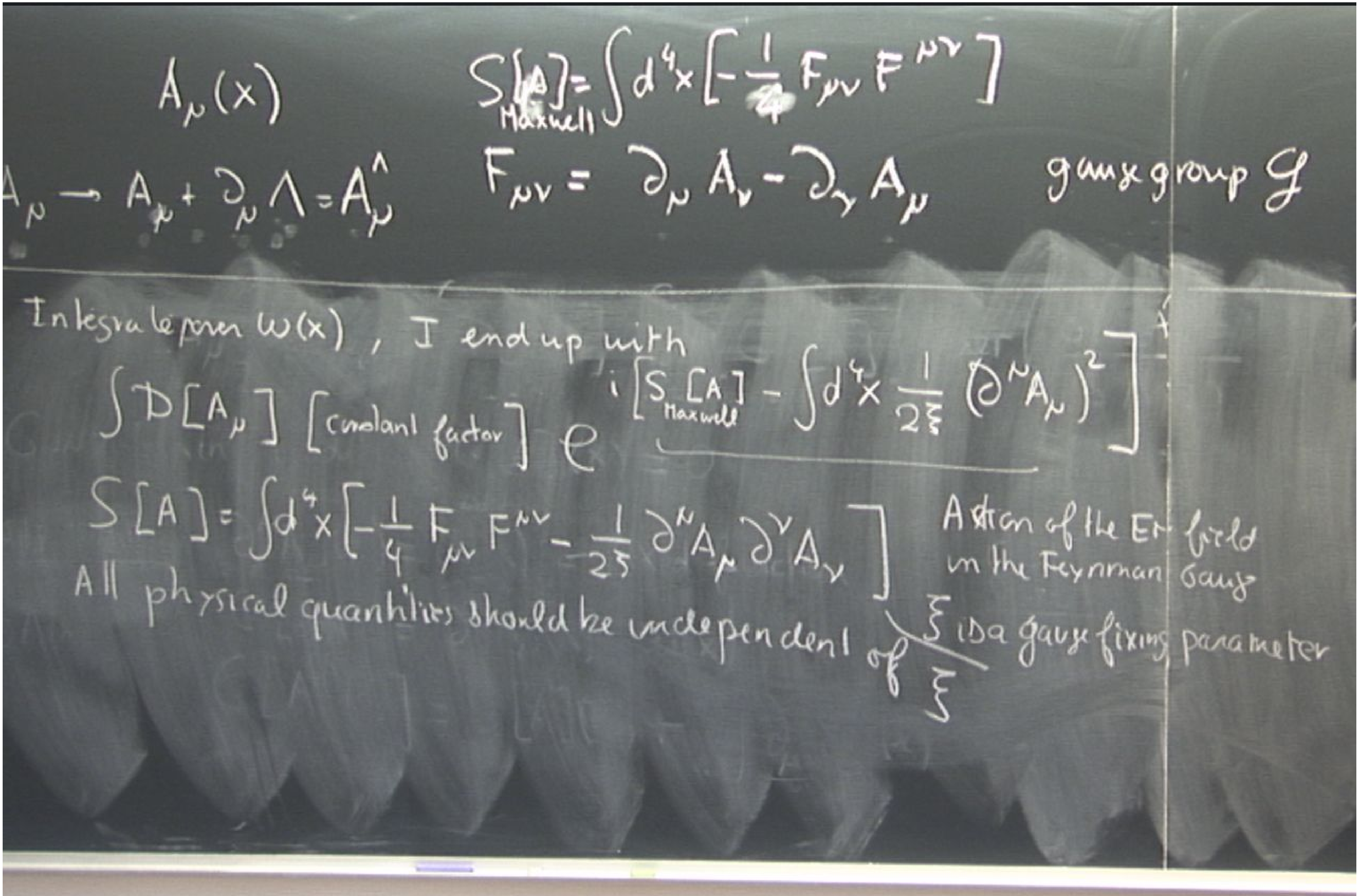
$$\int D[A_\mu] \left[\text{constant factor}\right] e^{\underline{i\left[S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi} (\partial^\mu A_\mu)^2\right]}}$$

$$S[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu\right]$$

Action of the EM field in the Feynman Gaug

All physical quantities should be independent of ξ

ξ is a gauge fixing para



$A_\mu(x)$

$$S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right]$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

gauge group $\mathcal{G}$

Integrate over $\omega(x)$, I end up with

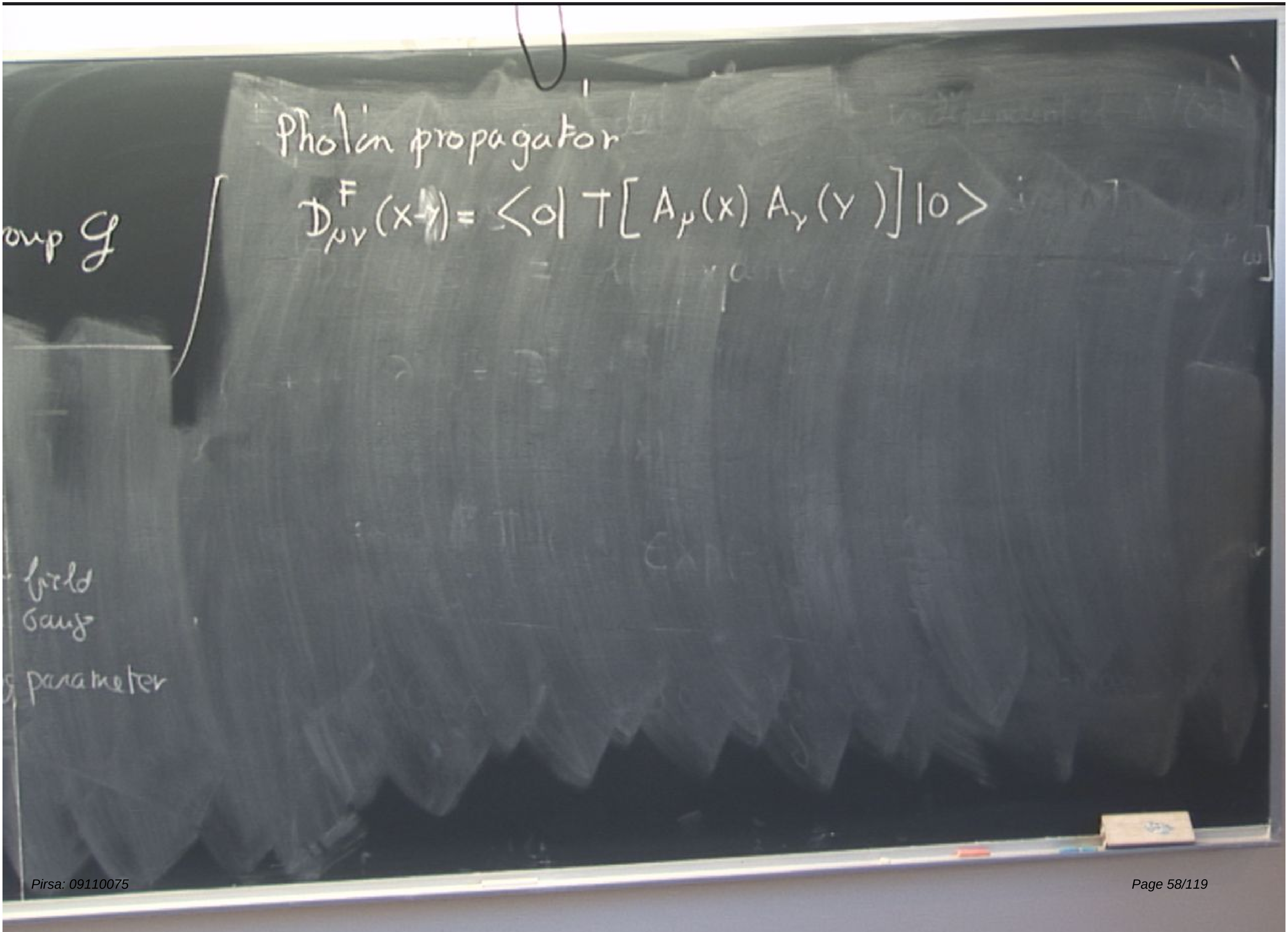
$$\int D[A_\mu]\,[\text{constant factor}]\, e^{i\underline{\left[S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi} (\partial^\mu A_\mu)^2\right]}}$$

$$S[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu\right]$$

Action of the EM field in the Feynman gauge

All physical quantities should be independent of ξ

ξ is a gauge fixing parameter



oup $\mathcal{G}$

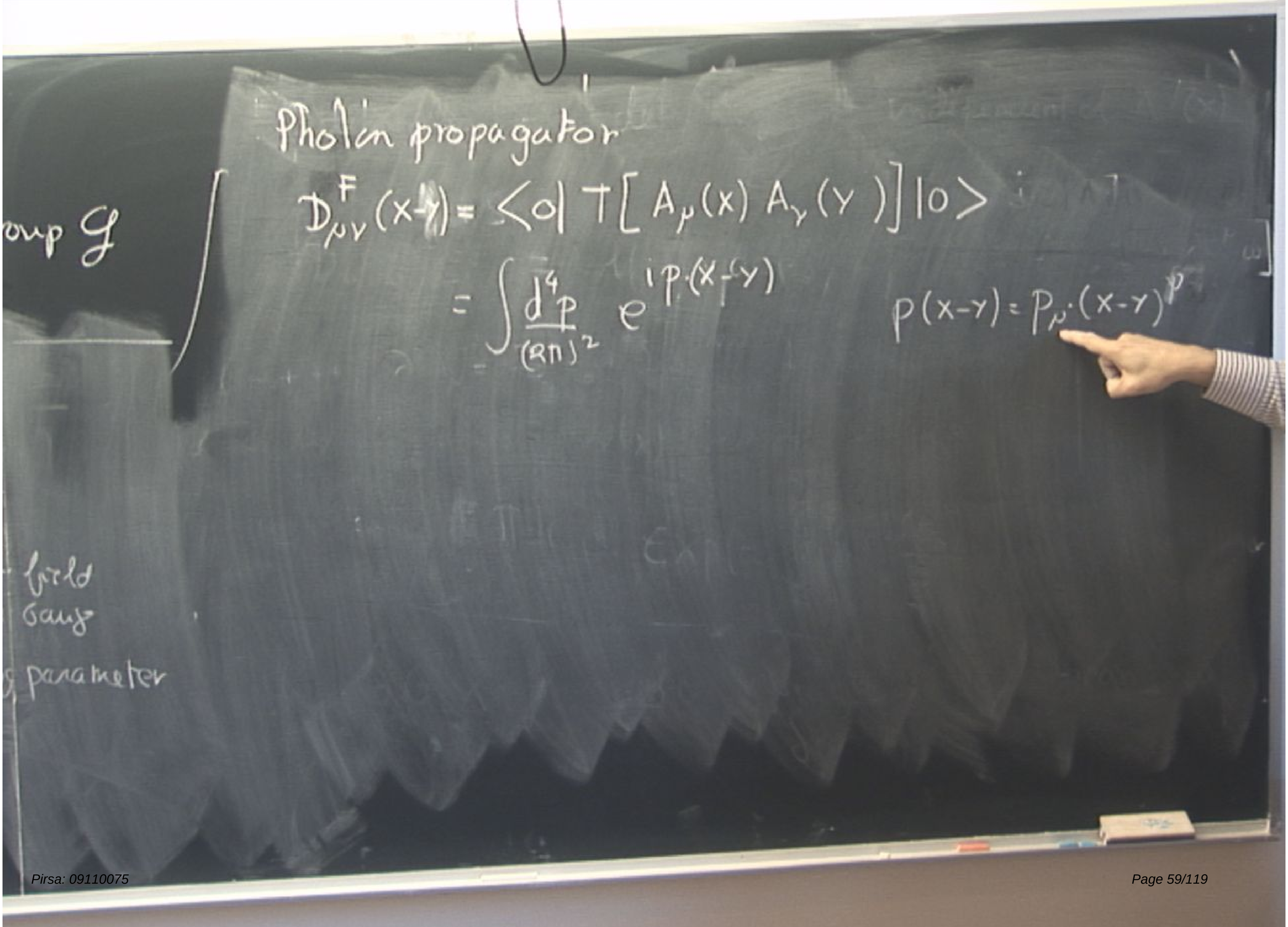
field

Gauge

parameter

Photon propagator

$$D^F_{\mu\nu}(x-y) = \langle 0|\, T\left[A_\mu(x)\, A_\nu(y) \right] |0\rangle$$



Photon propagator

oup $\mathcal{G}$

$$D^F_{\mu\nu}(x-y) = \langle 0| T[\, A_\mu(x)\, A_\nu(y)\,] |0\rangle$$

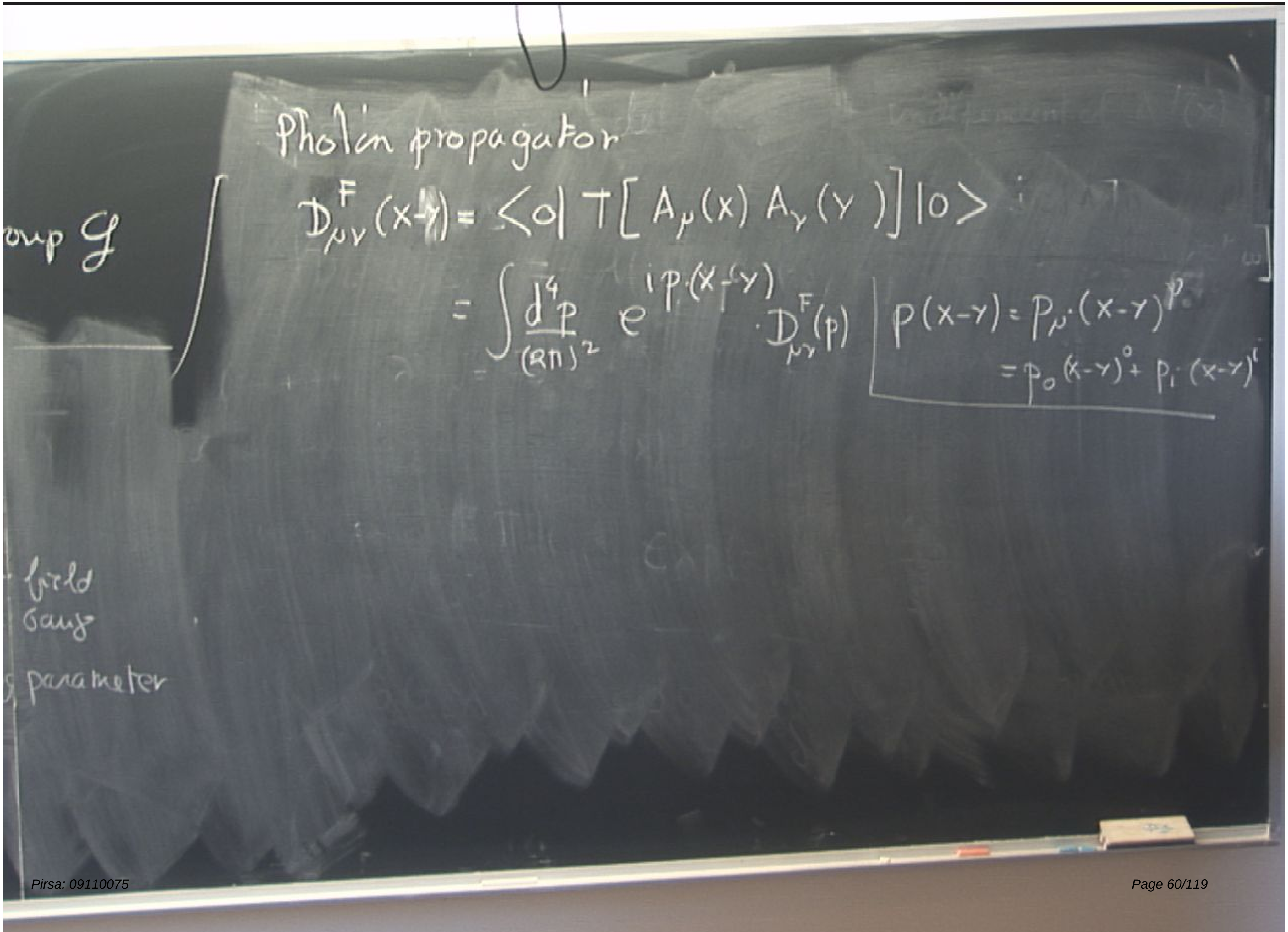
$$= \int \frac{d^4p}{(2\pi)^2}\, e^{ip\cdot(x-y)}$$

$$p(x-y) = p_\mu\cdot(x-y)^\mu$$

field

Gauge

parameter



Photon propagator

$$D^F_{\mu\nu}(x-y) = \langle 0 | T[A_\mu(x) A_\nu(y)] | 0 \rangle$$

$$= \int \frac{d^4p}{(2\pi)^2} e^{ip\cdot(x-y)} \cdot D^F_{\mu\nu}(p)$$

$$p(x-y) = p_\mu \cdot (x-y)^\mu$$

$$= p_0 (x-y)^0 + p_1 \cdot (x-y)^1$$

oup $\mathcal{G}$

field

Gauge

parameter

oup $\mathcal{G}$

field

Gauge

parameter

Photon propagator

$$D^F_{\mu\nu}(x-y) = \langle 0| T[A_\mu(x)\, A_\nu(y)]|0\rangle$$

$$= \int \frac{d^4p}{(2\pi)^2}\, e^{ip\cdot(x-y)} \cdot D^F_{\mu\nu}(p)$$

$$p(x-y) = p_\nu\cdot(x-y)^\nu$$

$$= p_0(x-y)^0 + p_i\cdot(x-y)^i$$

$$p^2 = p_\nu p^\nu = -p_0^2 + \vec{p}^{\,2}$$

$$D^F_{\mu\nu}(p) = \frac{-i}{p^2 - i\epsilon}\Big($$

oup $\mathcal{G}$

field

Gauge

parameter

Photon propagator

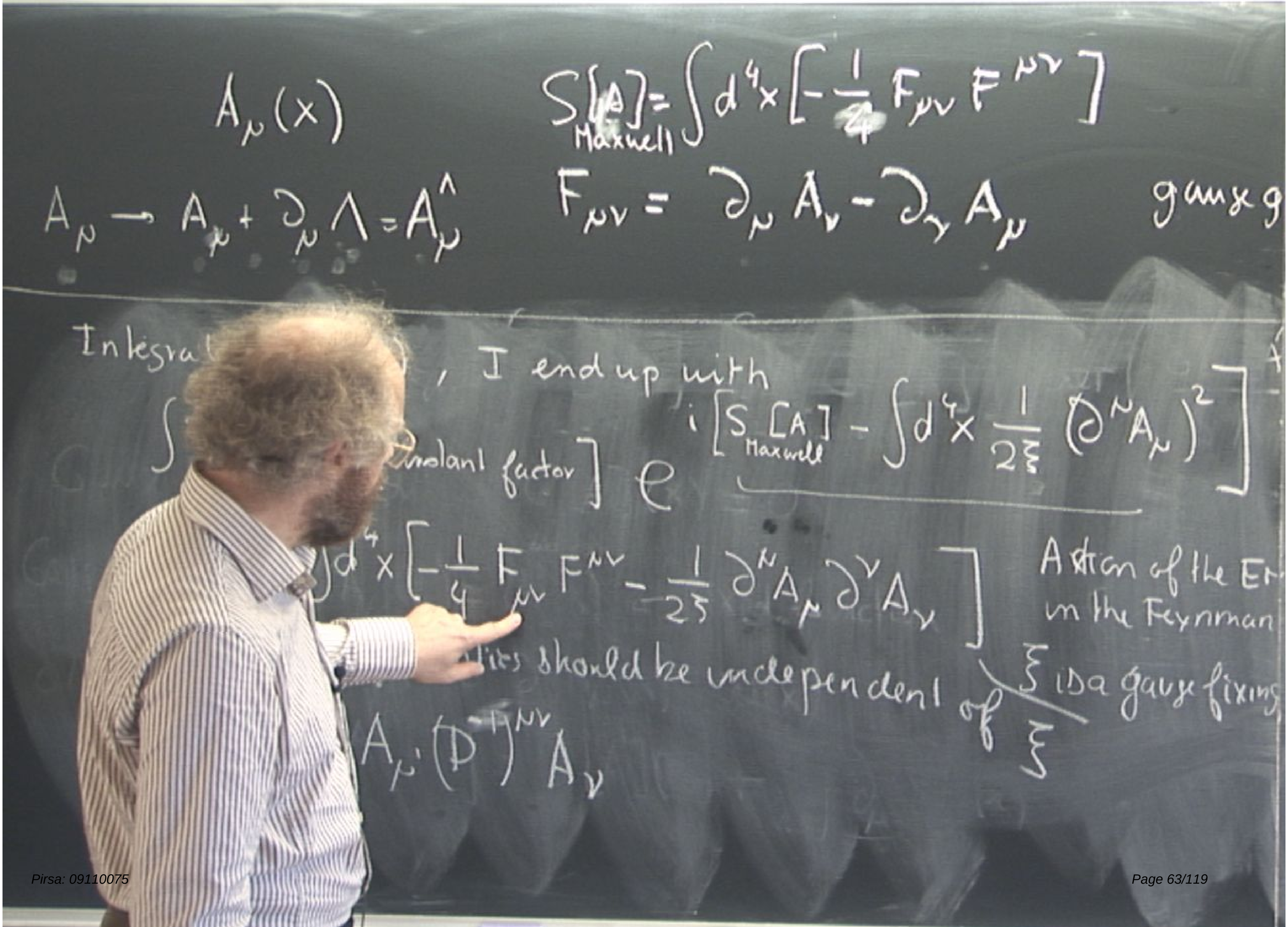
$$D^F_{\mu\nu}(x-y) = \langle 0|\, T[\, A_\mu(x)\, A_\nu(y)\,]\, |0\rangle$$

$$= \int \frac{d^4p}{(2\pi)^2}\, e^{ip\cdot(x-y)} \cdot D^F_{\mu\nu}(p)$$

$$p(x-y) = p_\mu\cdot(x-y)^\mu$$
$$= p_0 (x-y)^0 + p_i\cdot(x-y)^i$$

$$\boxed{D^F_{\mu\nu}(p) = \frac{-i}{p^2-i\epsilon}\left(\eta_{\mu\nu} + (\xi-1)\frac{p_\mu p_\nu}{p^2}\right)}$$

$$p^2 = p_\mu p^\nu = -p_0^2 + \vec{p}^{\,2}$$



$A_\mu(x)$

$S_{\text{Maxwell}}[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$

$A_\mu \to A_\mu + \partial_\mu \Lambda = A_\mu^\Lambda$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

gauge g

Integra , I end up with

$\int$ nstant factor] $e^{i \left[\underline{S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi} (\partial^\mu A_\mu)^2} \right]}$

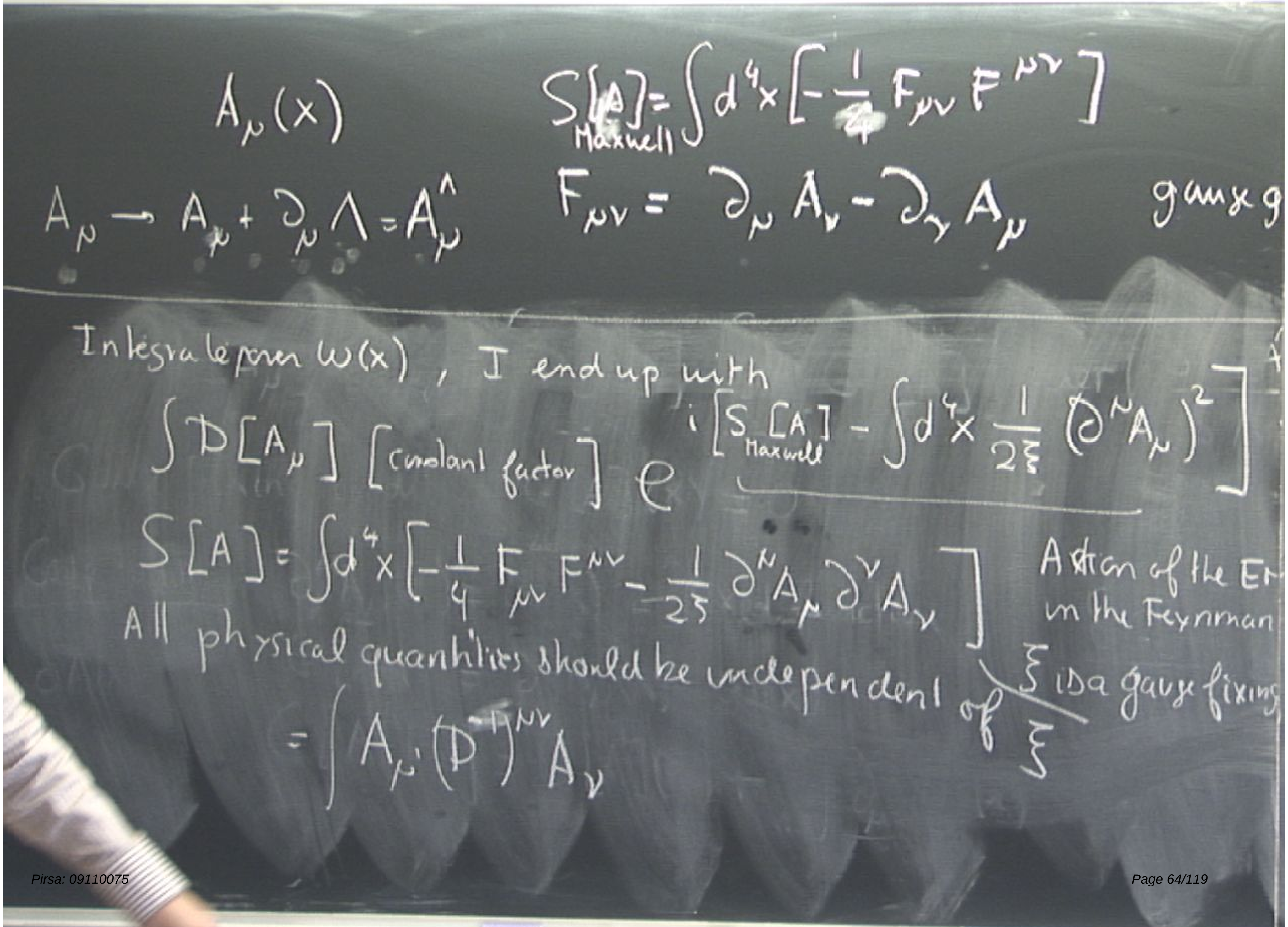
$\int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu \right]$

Action of the E
in the Feynman

ities should be independent of ξ

ξ is a gauge fixing

$A_\mu (D^{-1})^{\mu\nu} A_\nu$



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gauge g

Integrate over $\omega(x)$, I end up with

$$\int D[A_\mu]\,[\text{constant factor}]\, e^{i\left[\underline{S_{\text{Maxwell}}[A] - \int d^4x \frac{1}{2\xi}(\partial^\mu A_\mu)^2}\right]}$$

$$S[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} \partial^\mu A_\mu \partial^\nu A_\nu\right]$$

Action of the E

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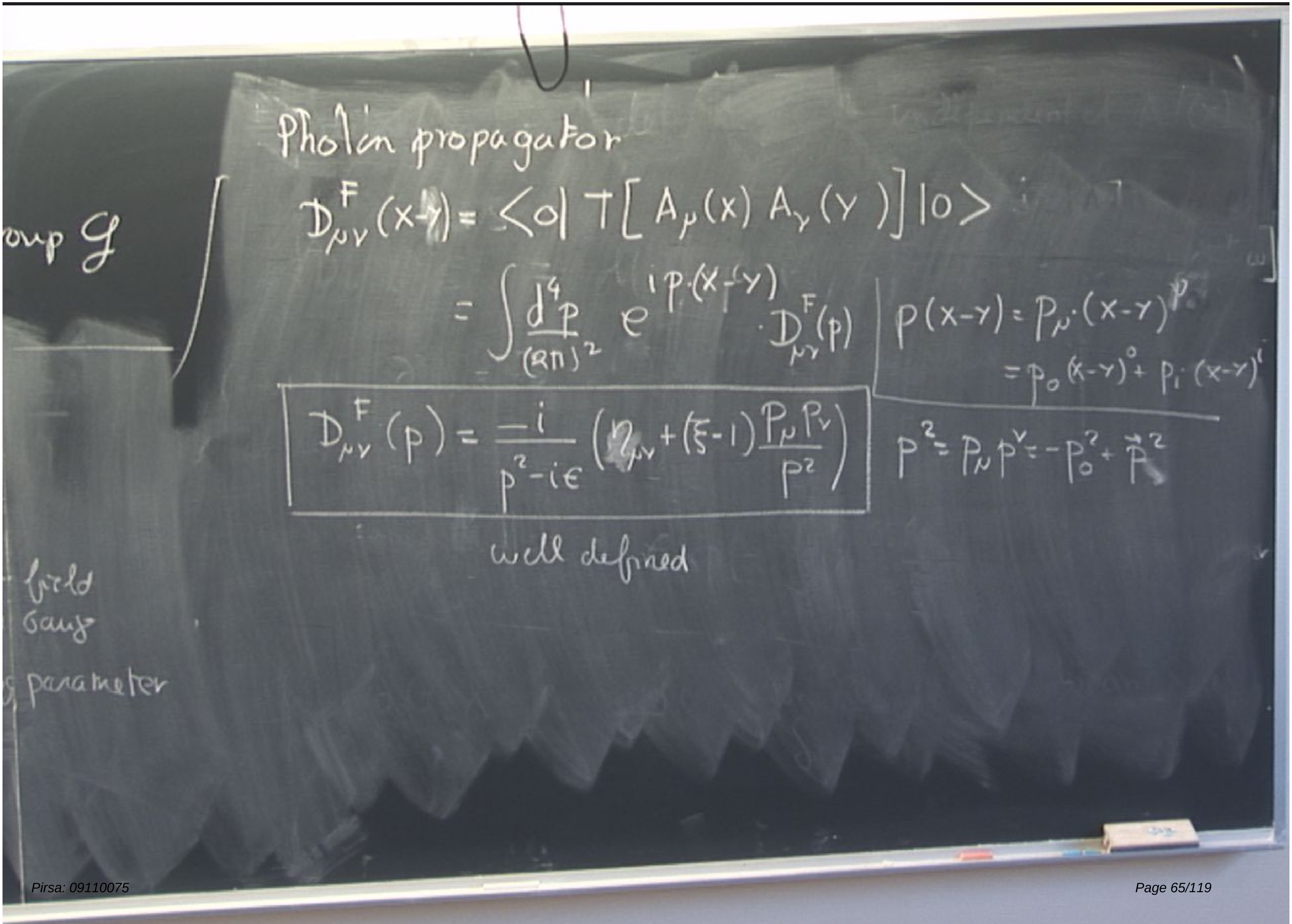
All physical quantities should be independent of ξ

ξ is a gauge fixing

$$= \int A_\mu (D^{-1})^{\mu\nu} A_\nu$$



oup $\mathcal{G}$

field

Gauge

parameter

Photon propagator

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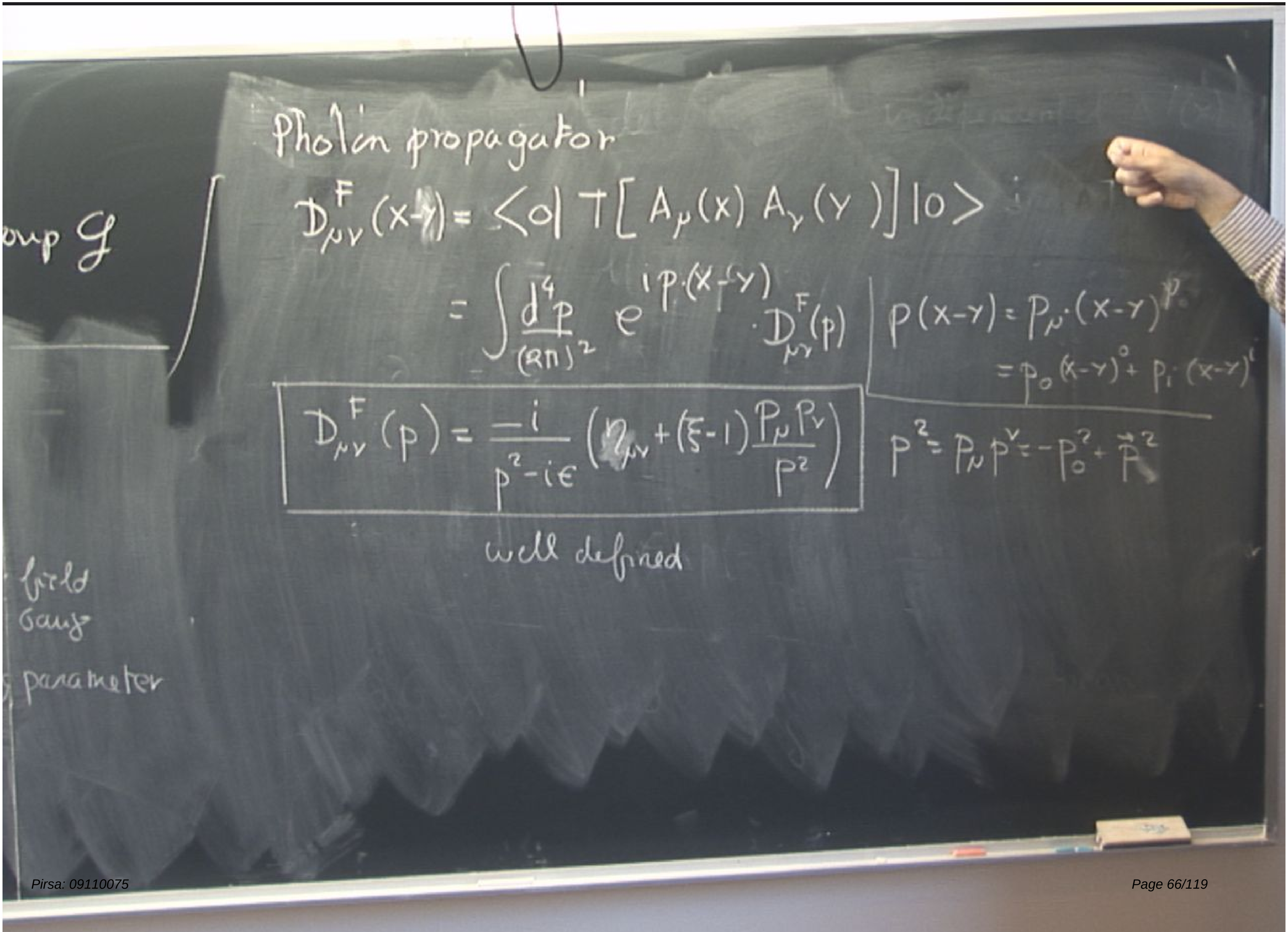
well defined



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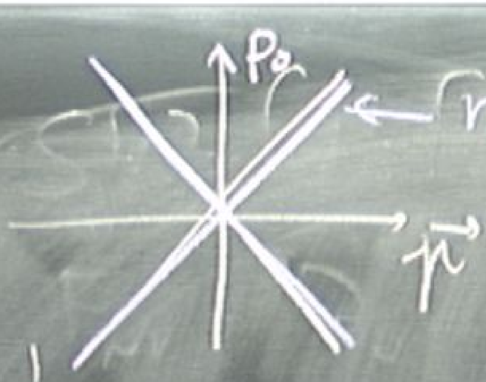
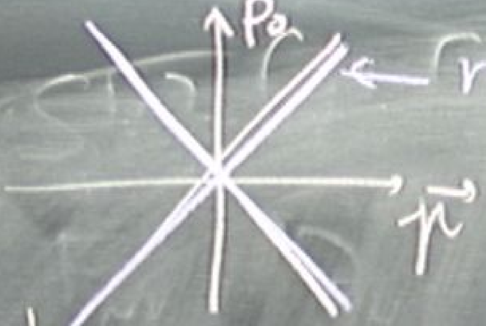


← mass shell for photons

photons are massless !

pole at $p^2 = 0$



photons are massless !

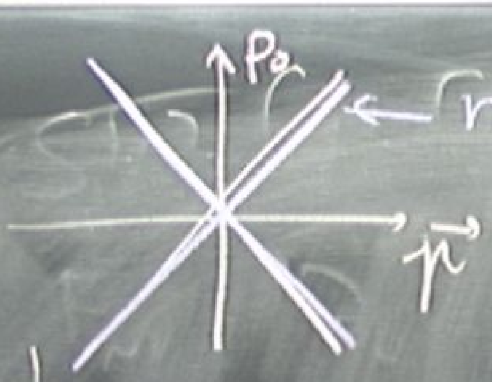
A mode for the e.m field

$$A_\mu(x) = e^{i p \cdot x} \epsilon_\mu$$

$p = (p_0, \vec{p})$ with $-p_0^2 + \vec{p}^2 = 0$

ϵ_μ polarization vector of the photon

pole at $p^2 = 0$



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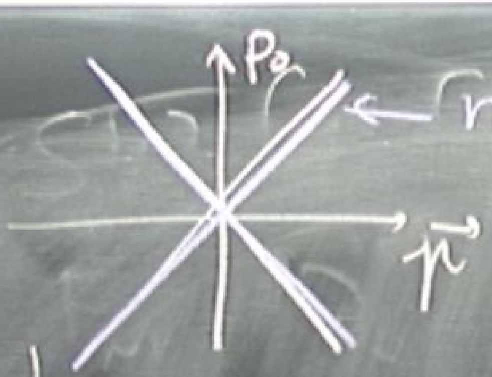

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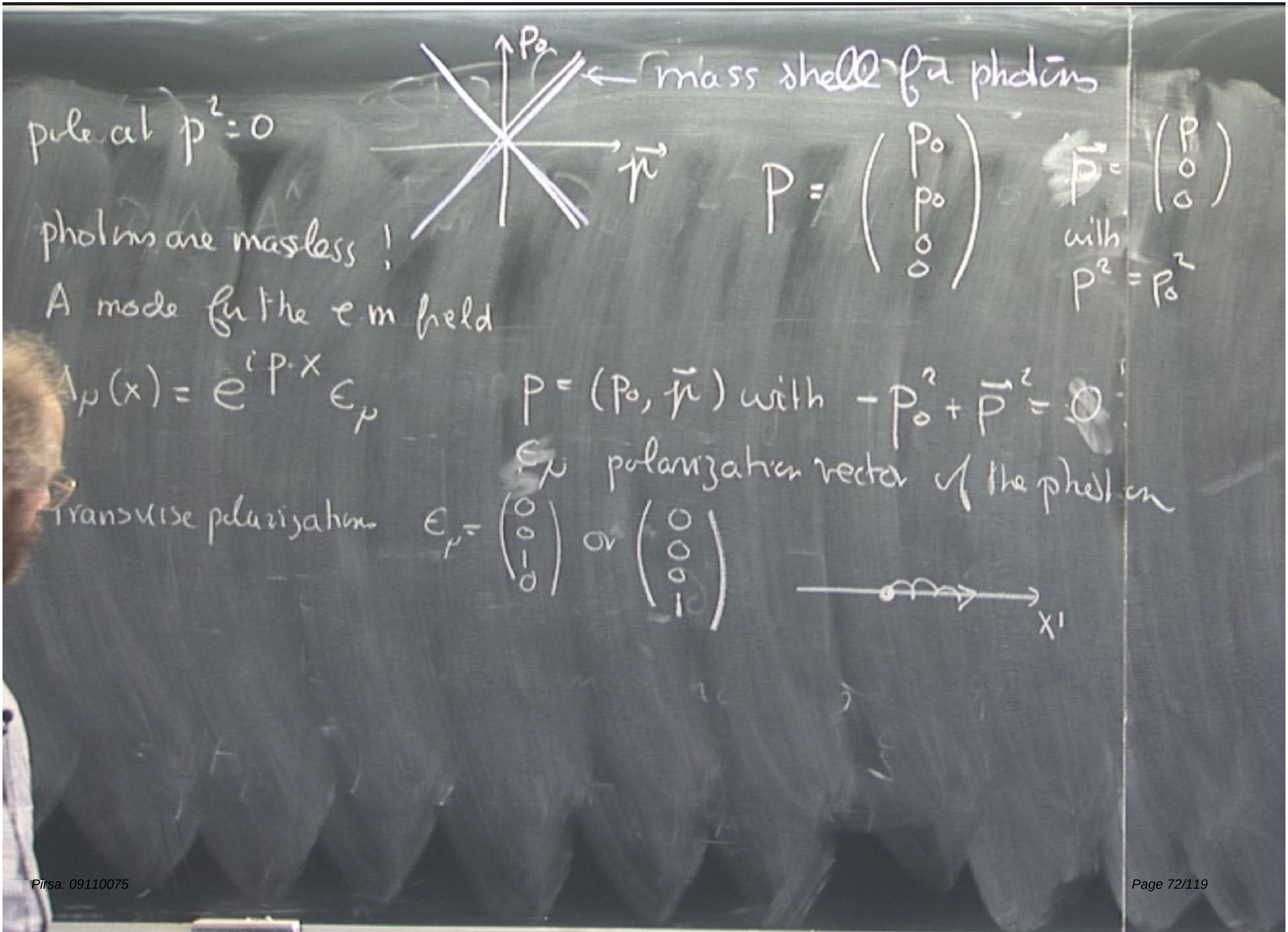

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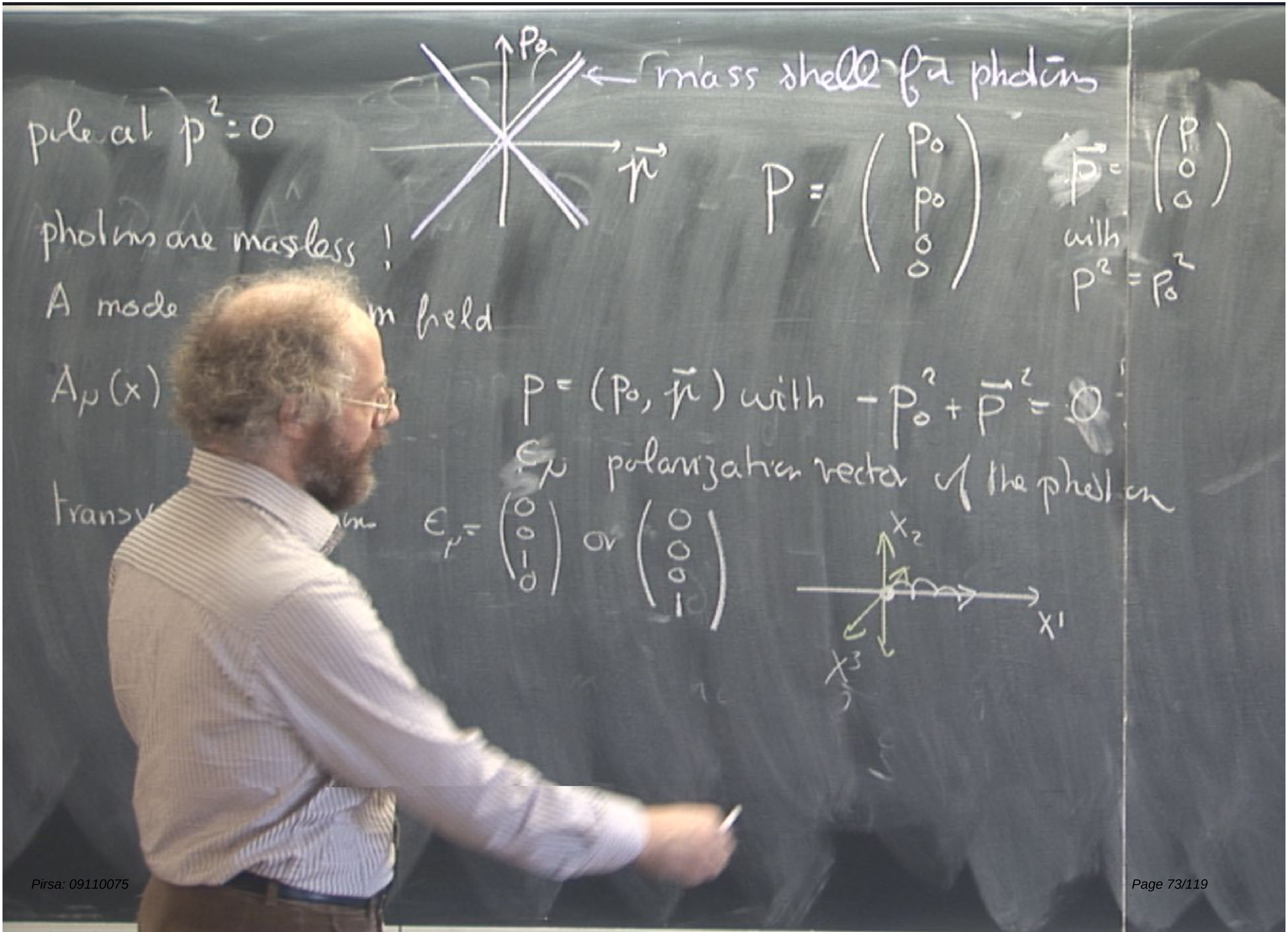

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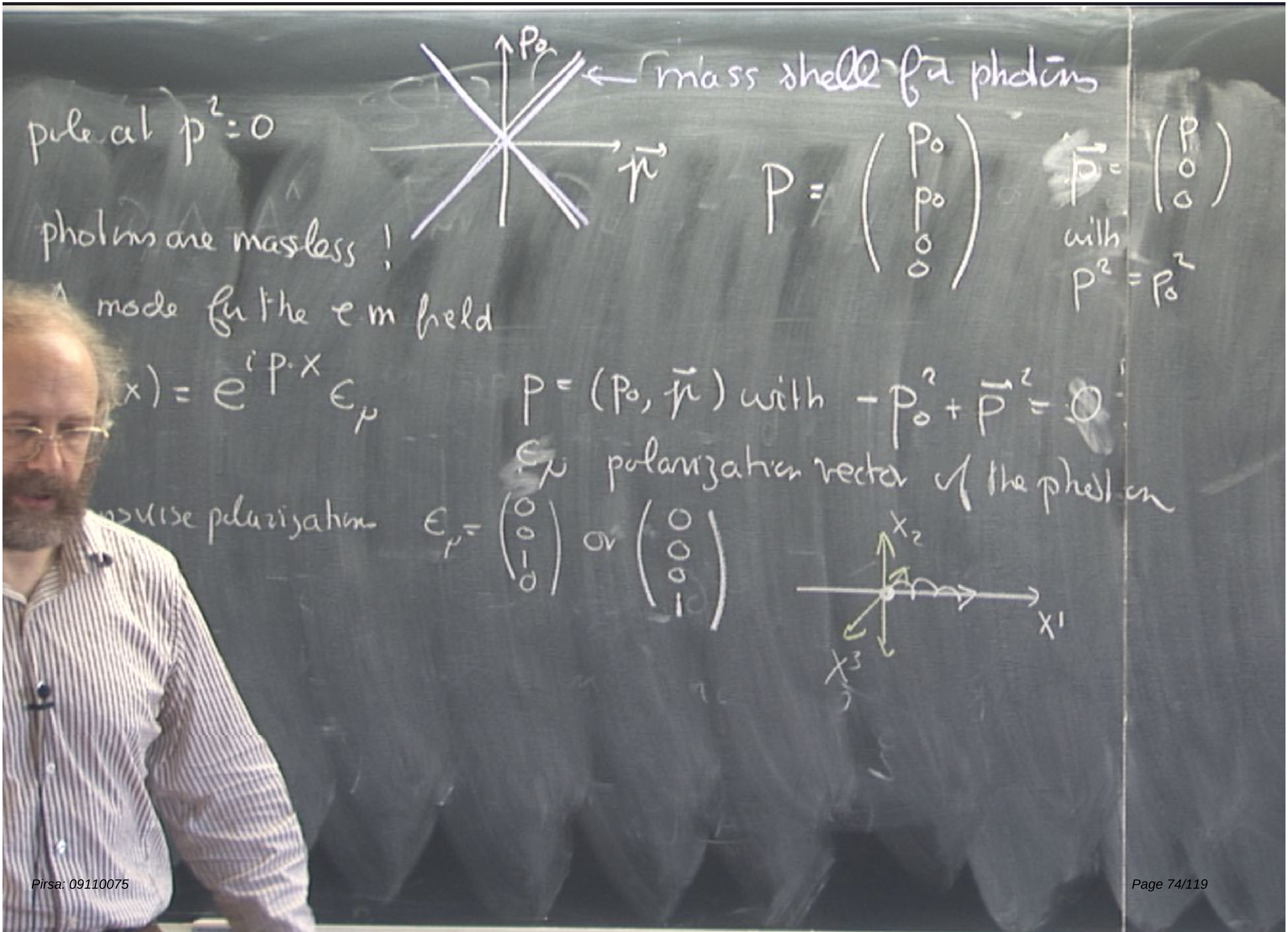

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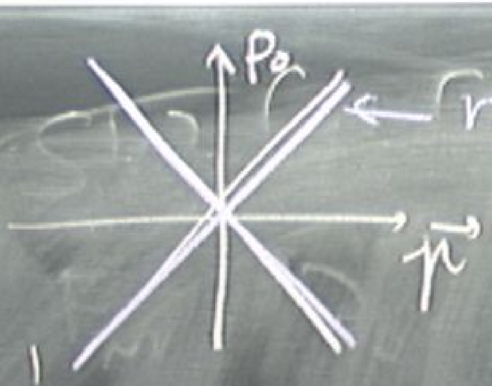

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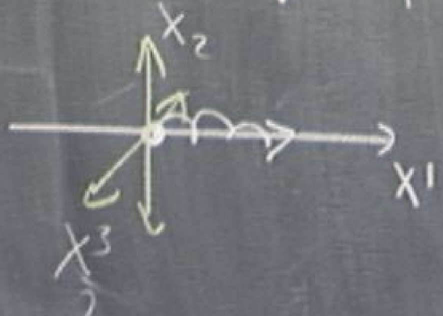

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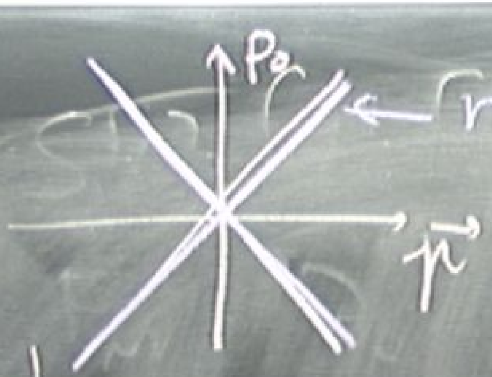

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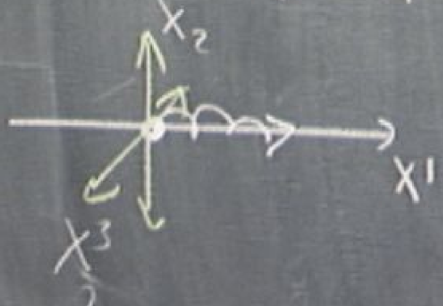

$\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}$

$= p_0^2$

lcn

Photon propagator

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$$\epsilon^\mu D_{\mu\nu}(p)\epsilon^\nu = -i\,\frac{\epsilon^\mu \epsilon_\mu}{p^2 - i\epsilon}$$ independent of ξ

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on

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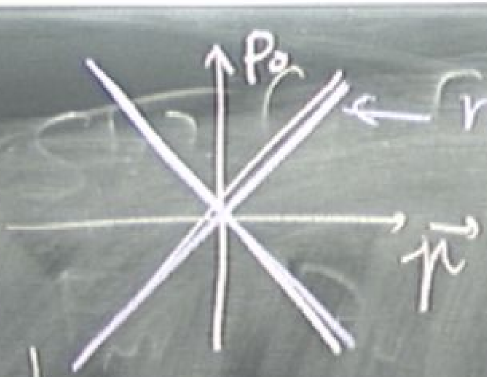

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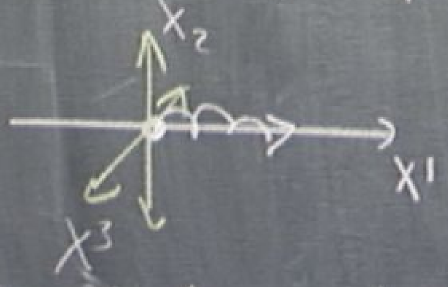
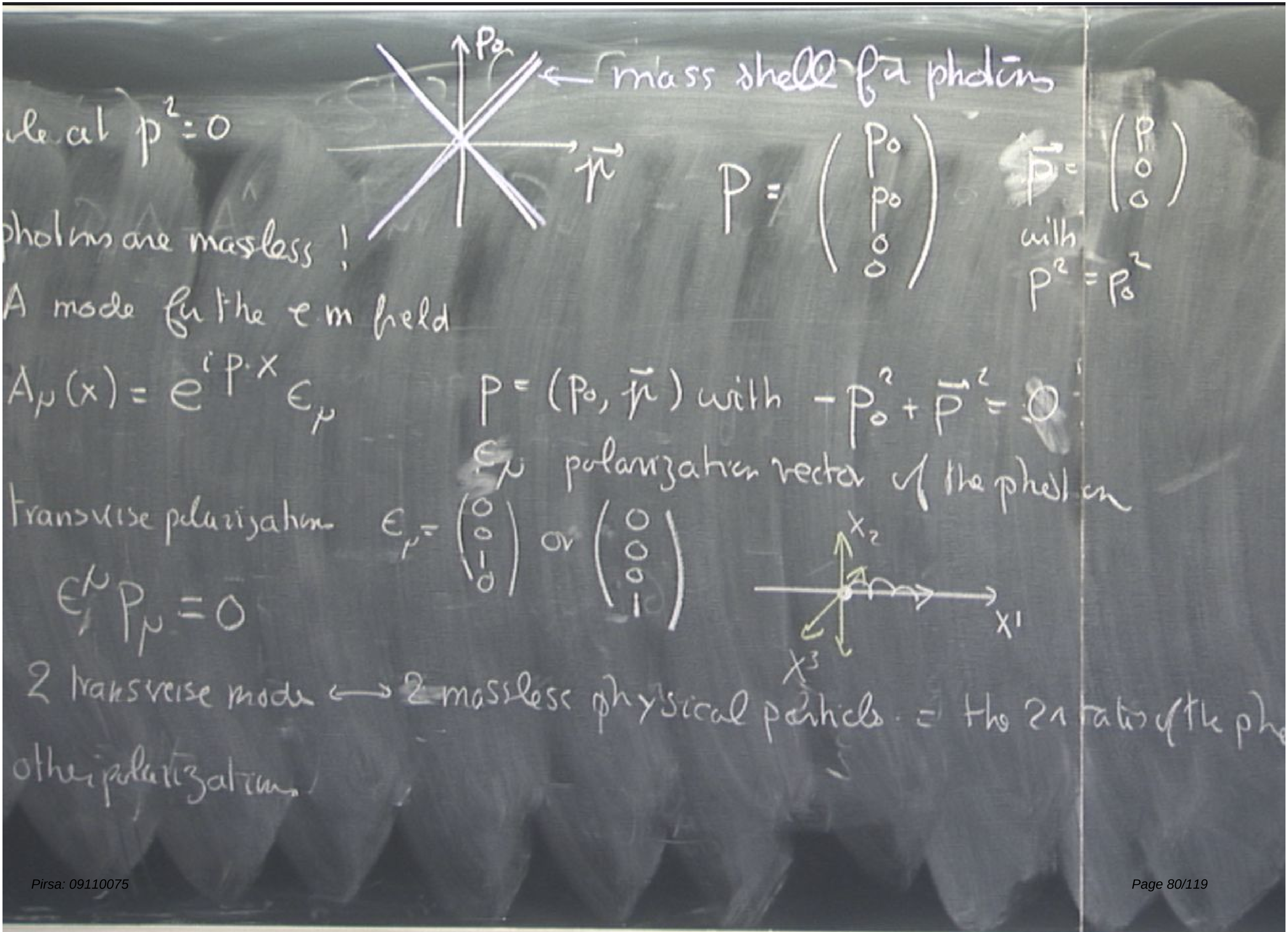


2 transverse modes ⟷ 2 massless physical particles = the 2 rotations of the ph



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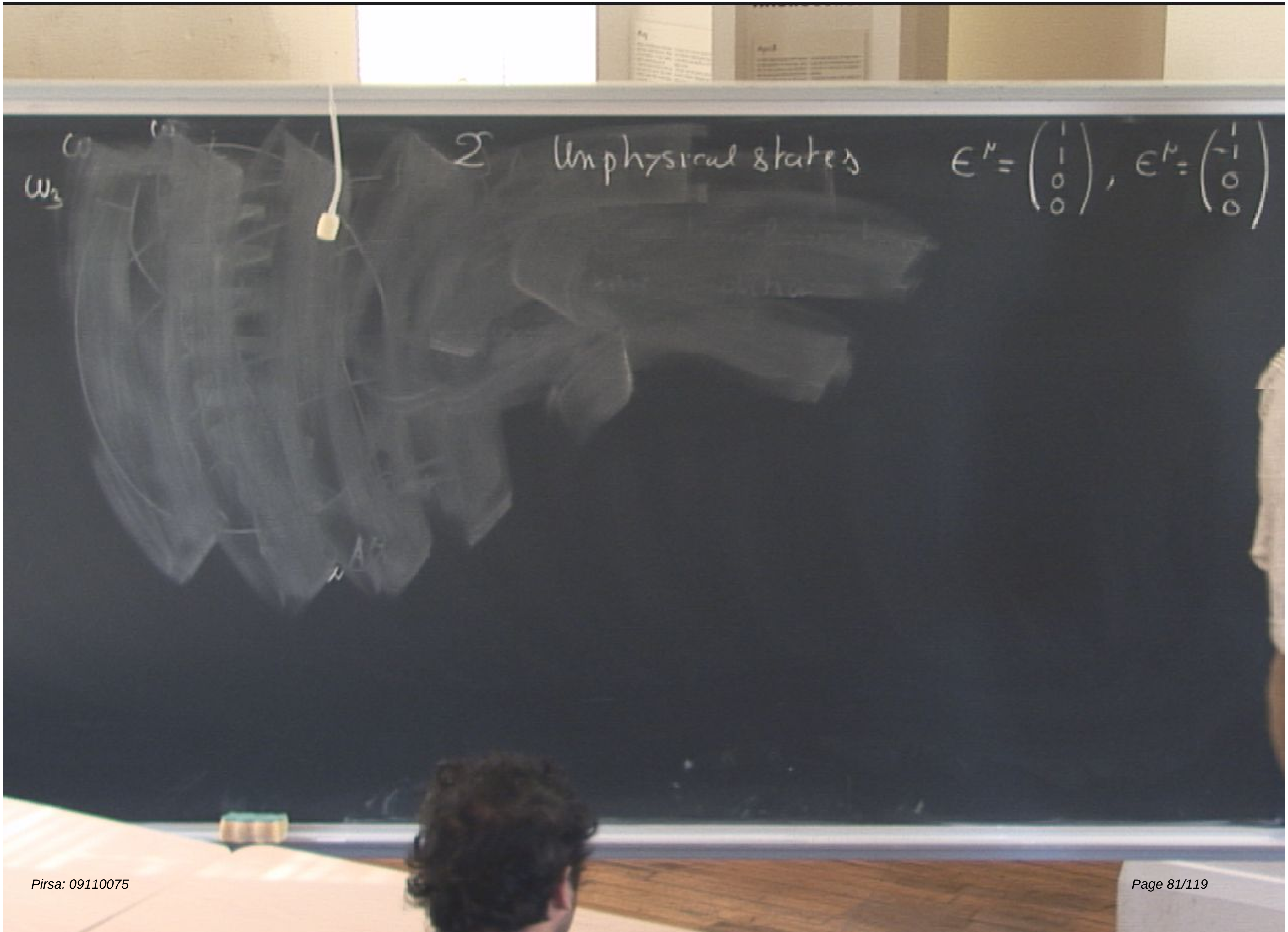
other polarizations

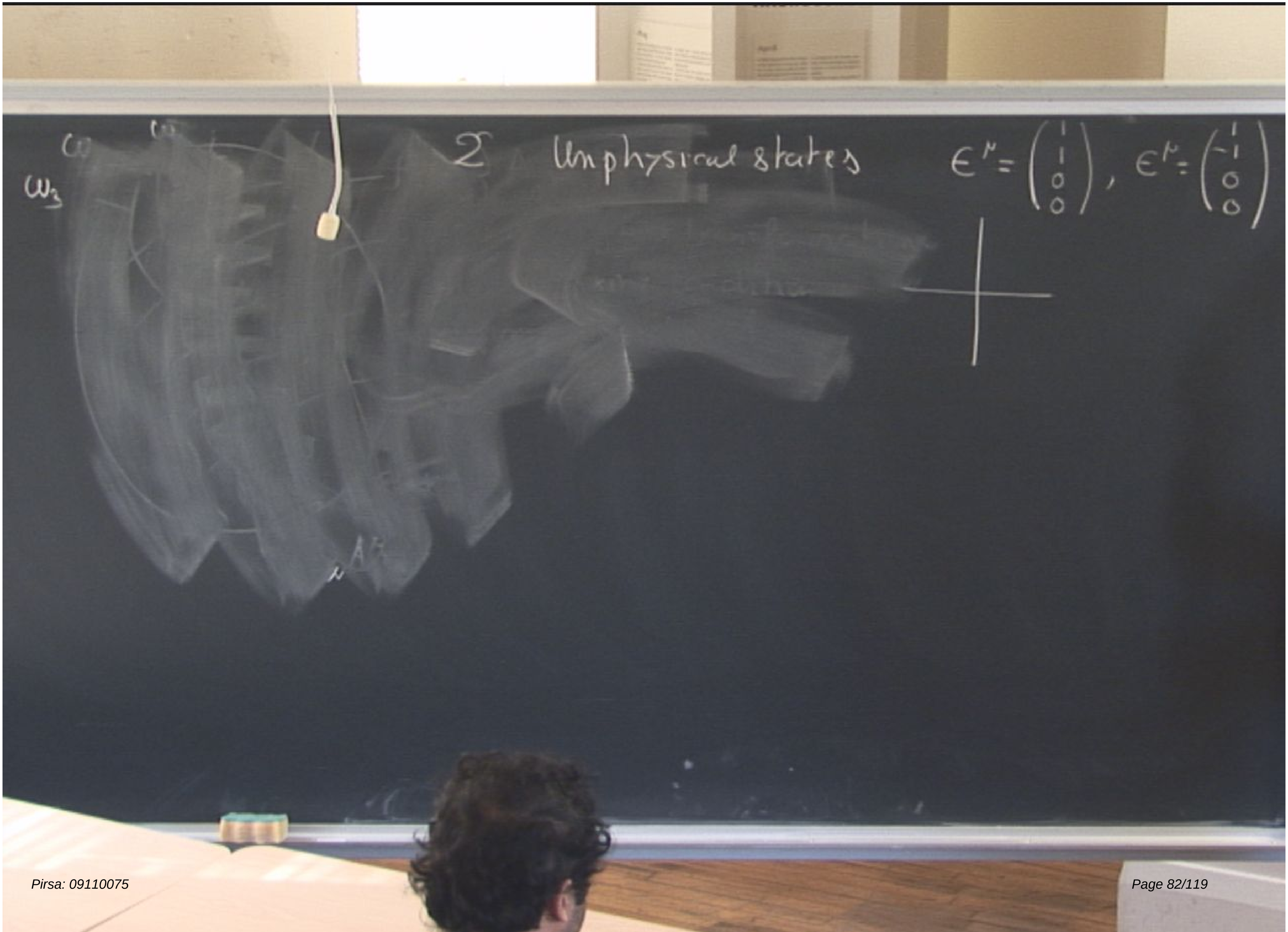


ω ω Σ

ω_3

Unphysical states

$$\epsilon^\mu = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \epsilon^\mu = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$



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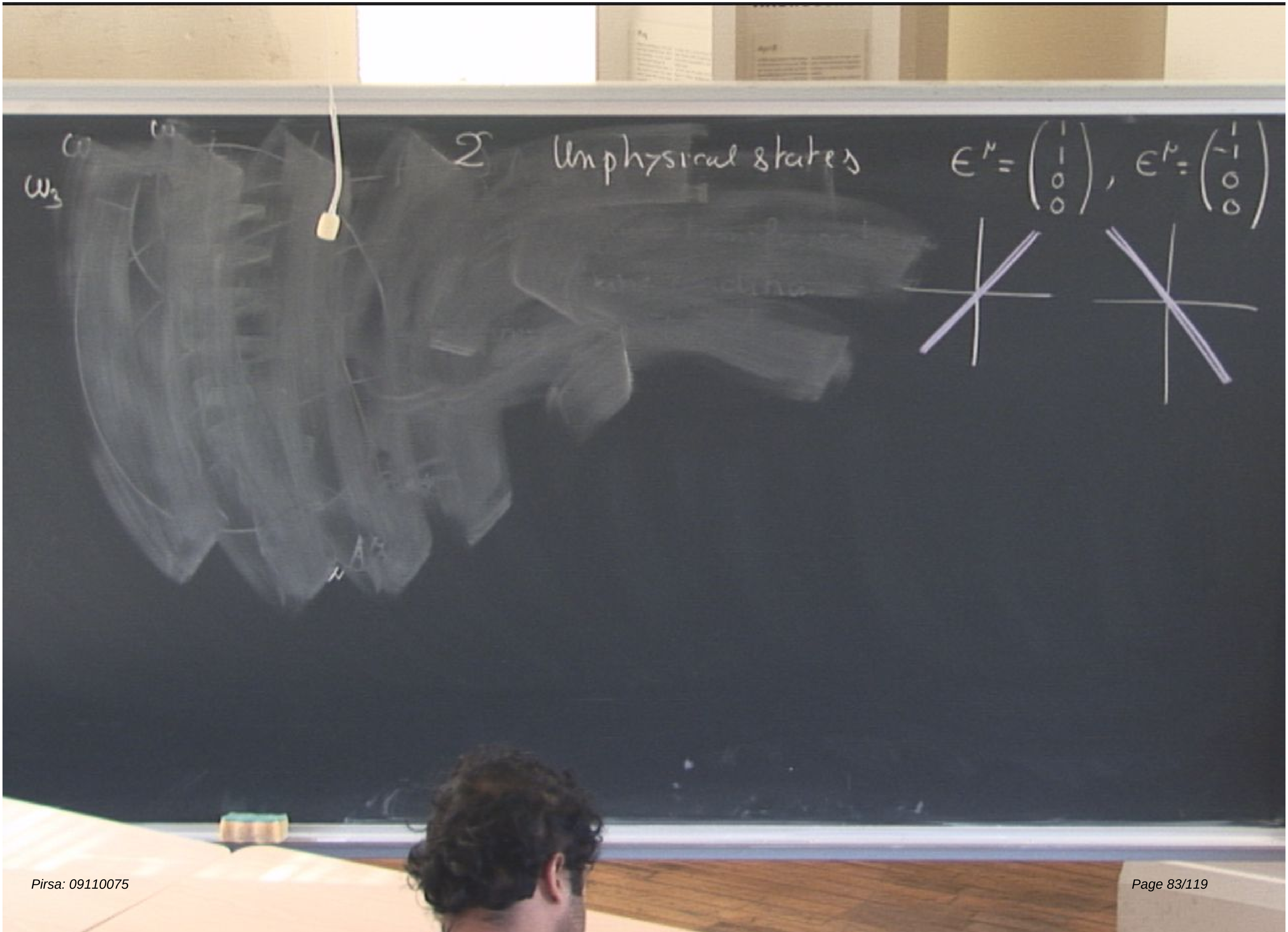

ω ω

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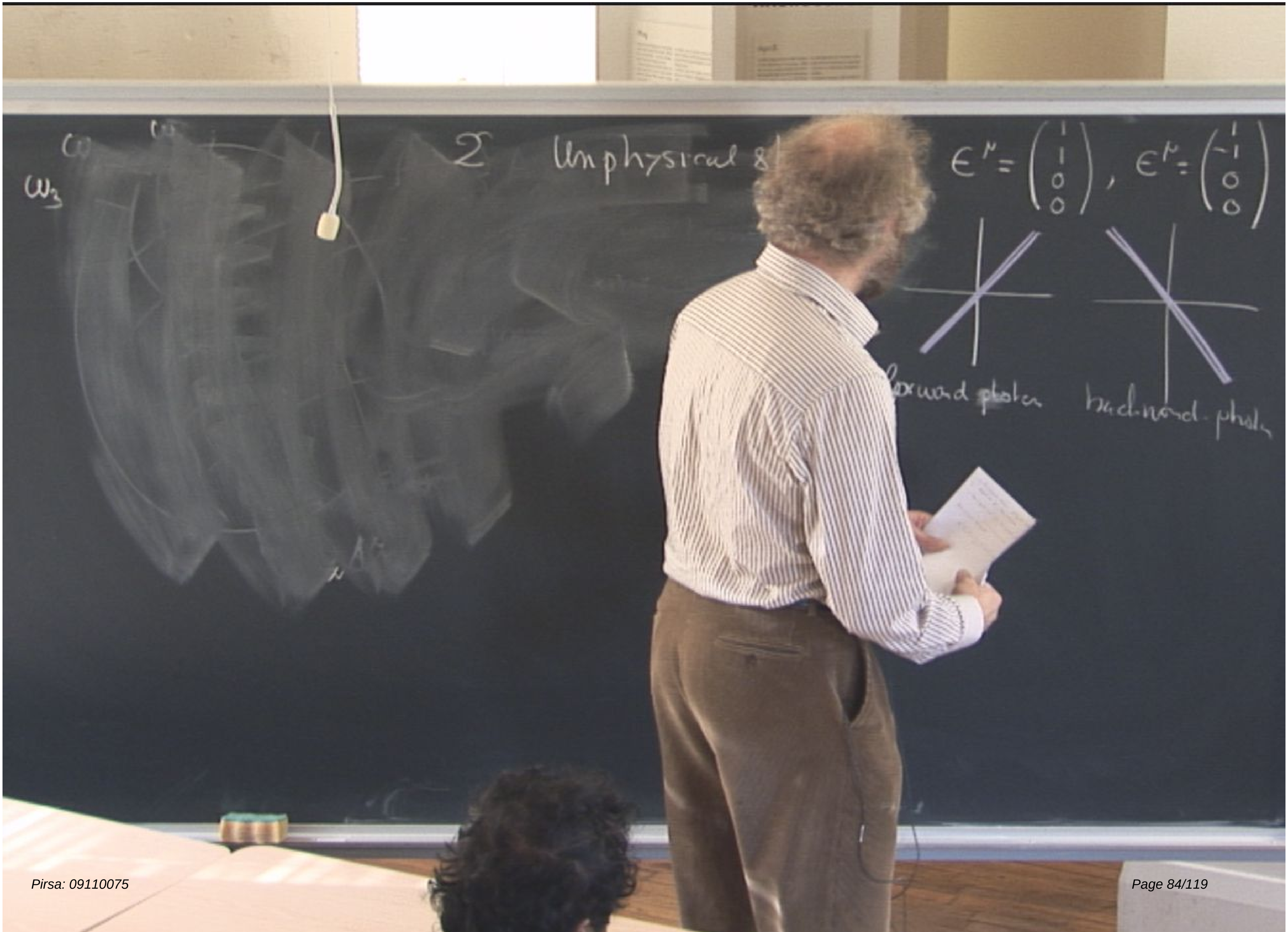

ω_3
Σ
Unphysical &
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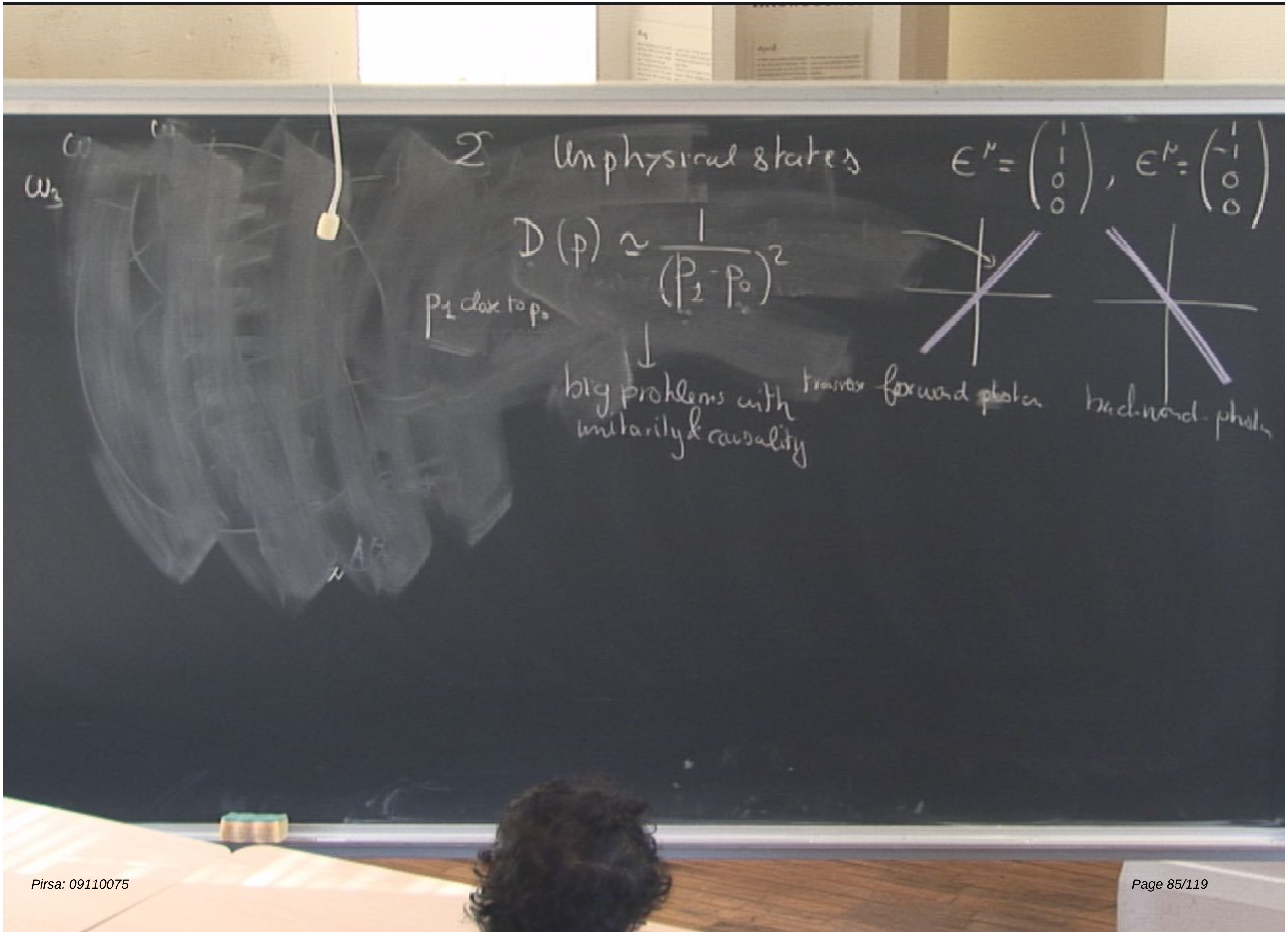
forward photon
backward photon



ω ω

ω_3

2 Unphysical states

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$$D(p) \simeq \frac{1}{(p_1 - p_0)^2}$$

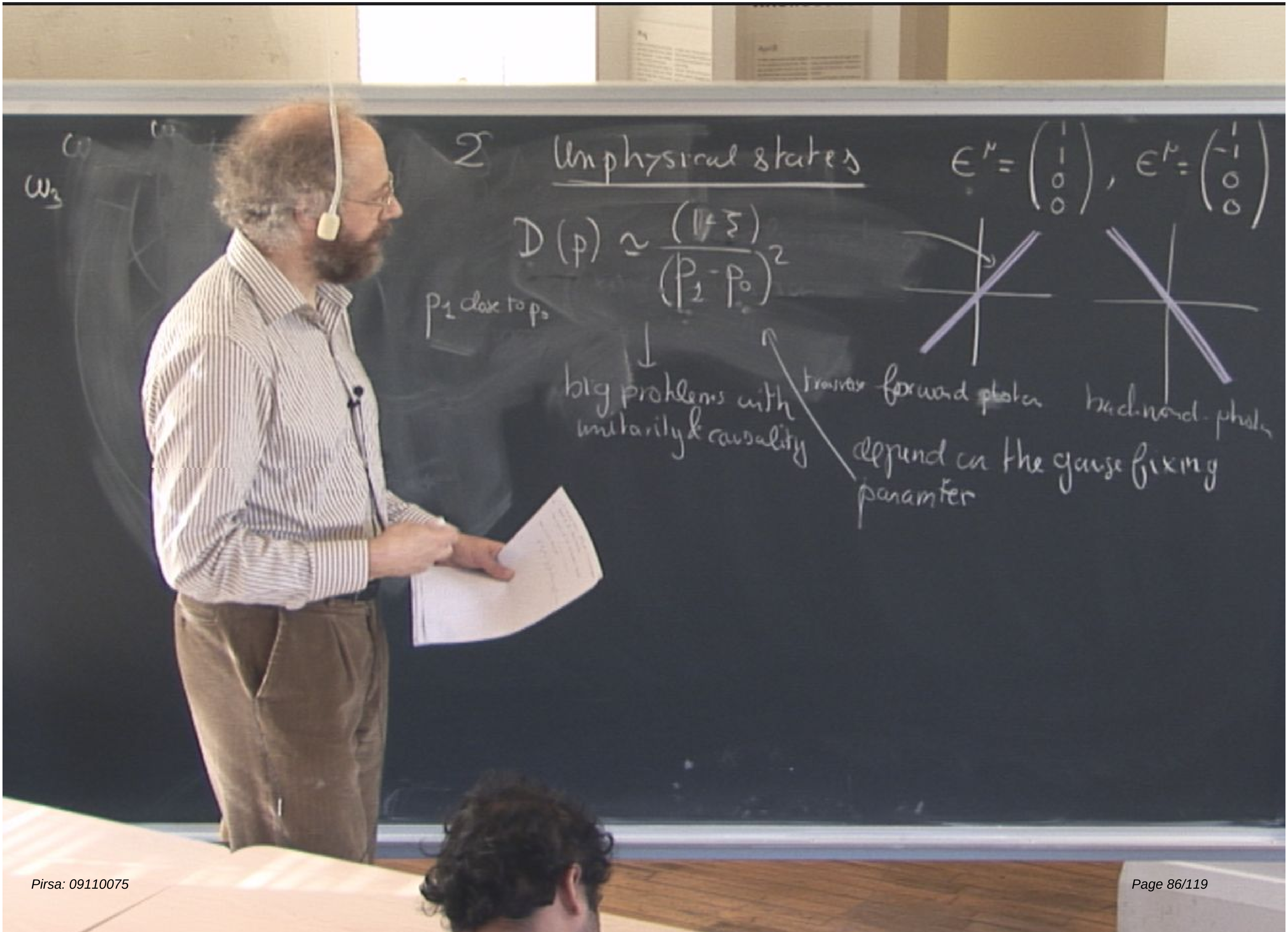
p_1 close to p_0

$\downarrow$

big problems with unitarity & causality

transverse forward photon

backward photon



Unphysical states

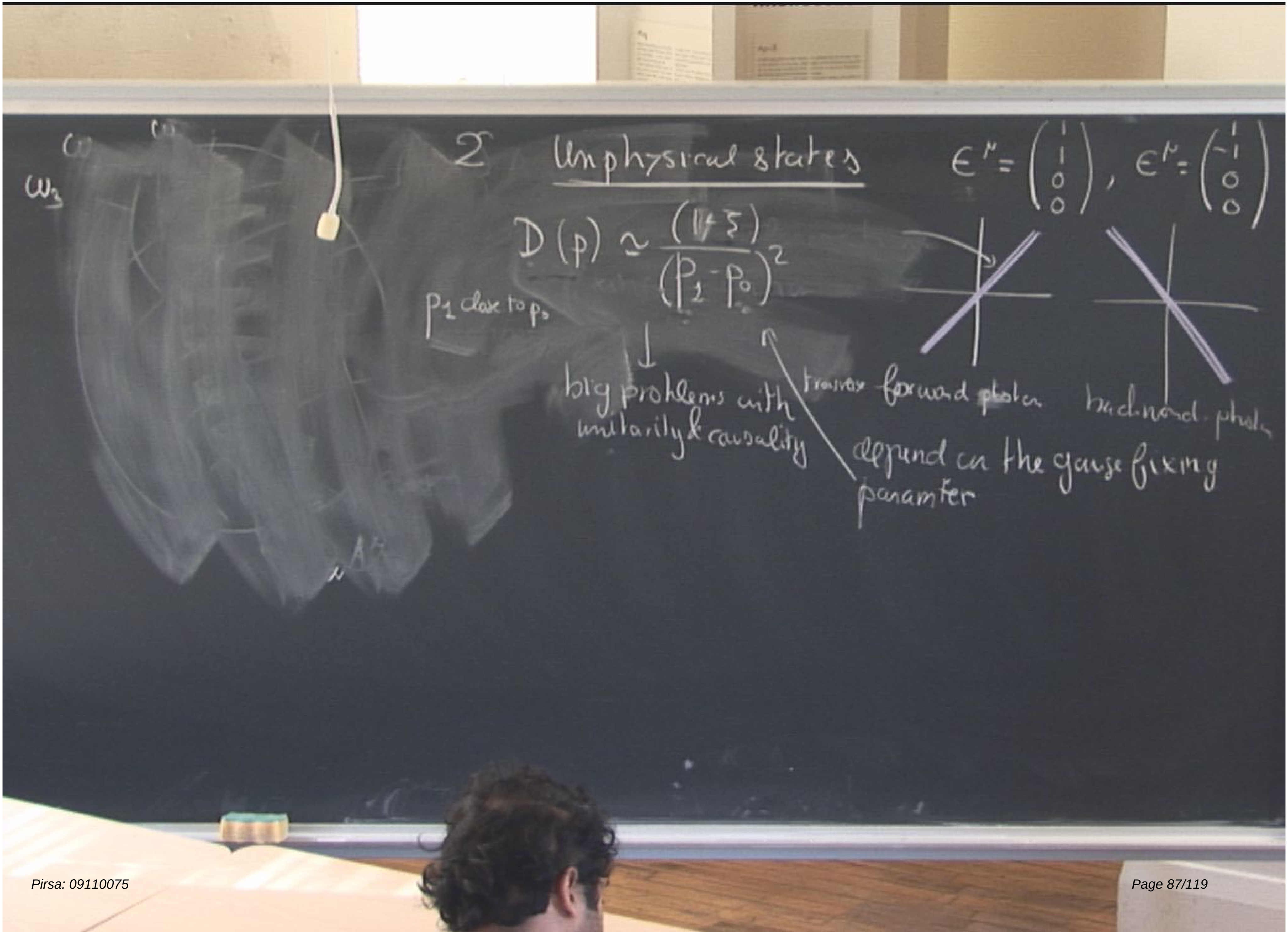
$$D(p) \simeq \frac{(1+\xi)}{(p_1 - p_0)^2}$$

p_1 close to p_0

big problems with unitarity & causality

depend on the gauge fixing paramter

$$\epsilon^\mu = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \epsilon^\mu = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$



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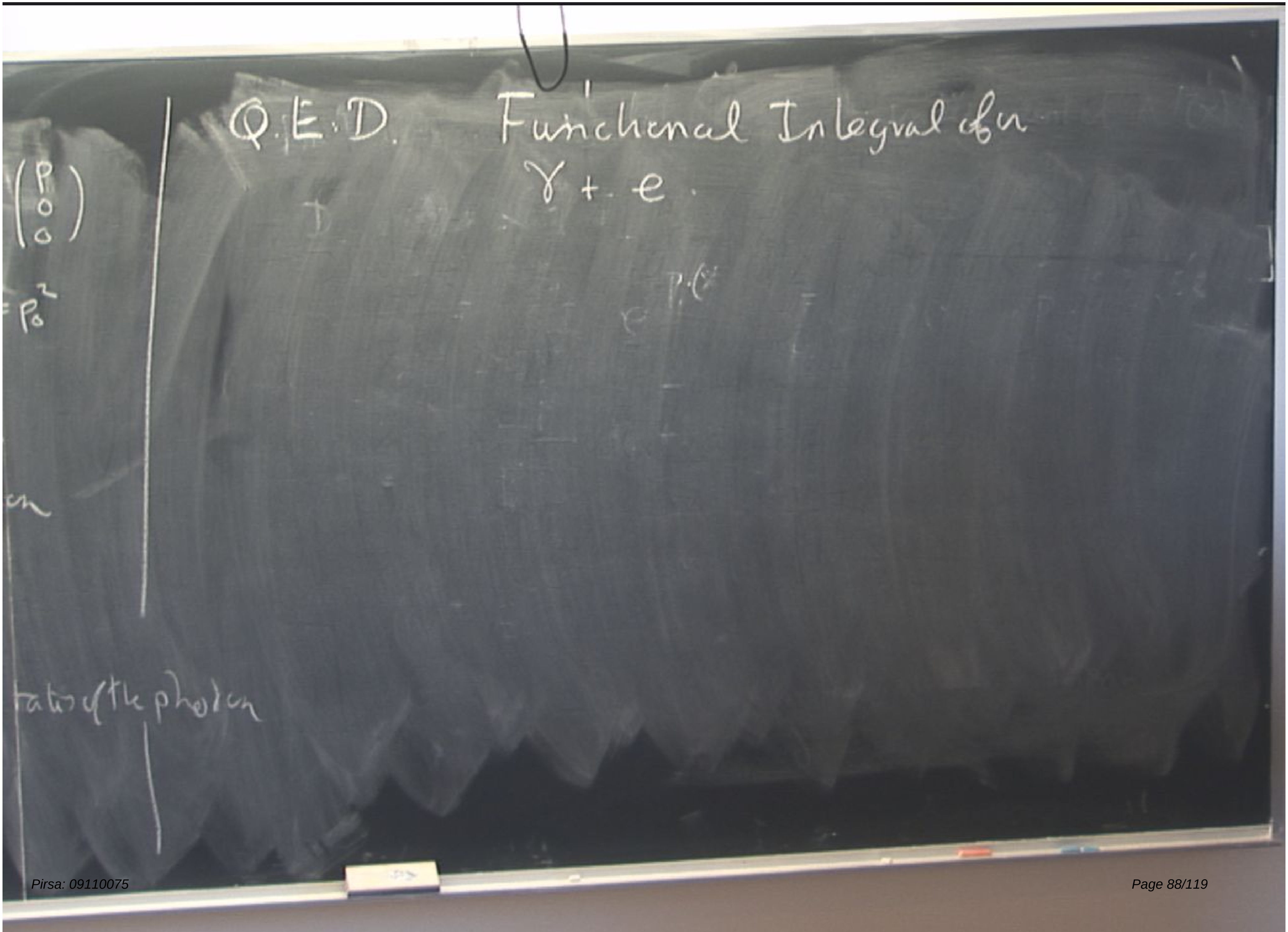


Q.E.D. Functional Integral for

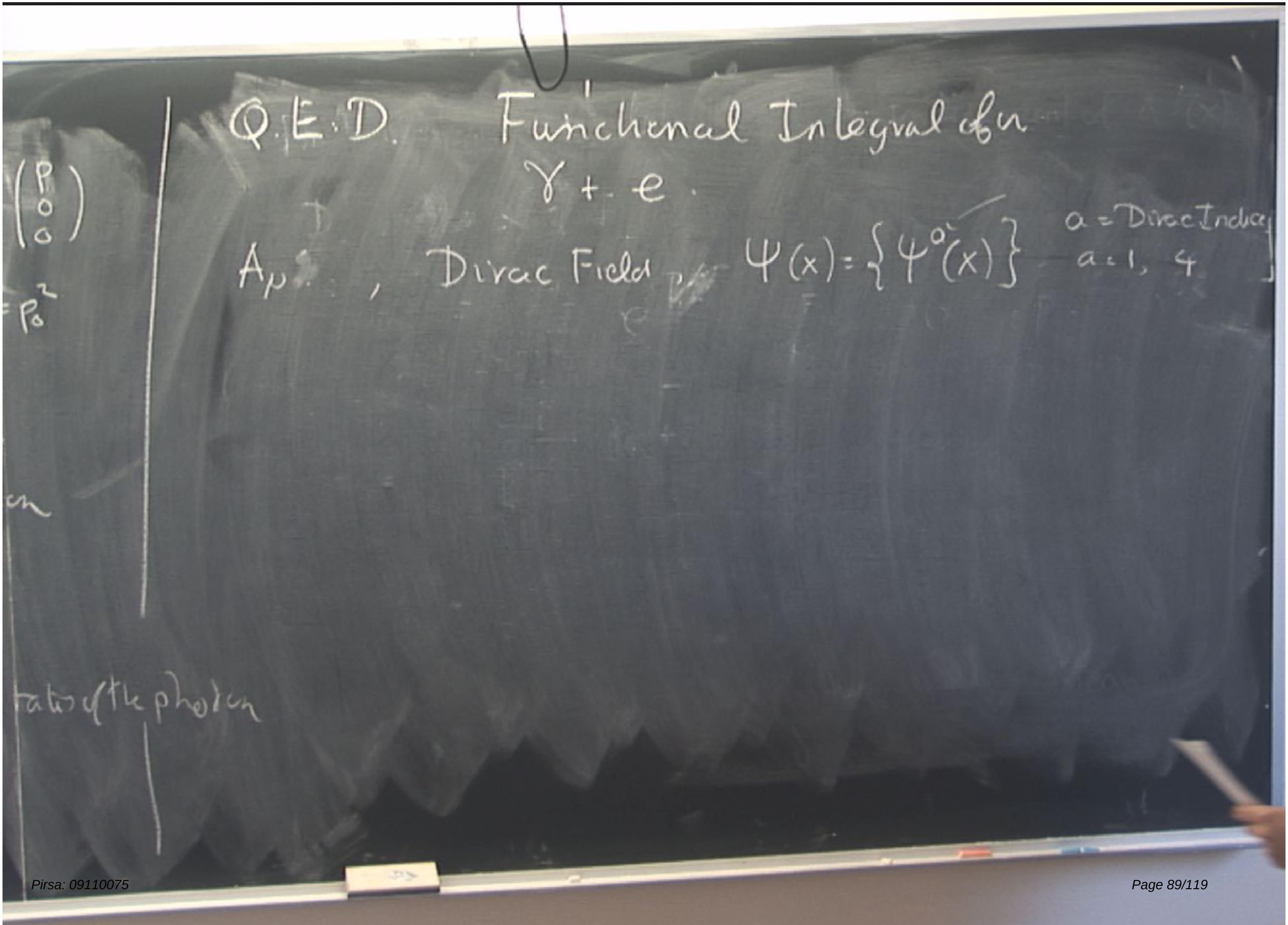
$\gamma + e$

$\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}$

$= p_0^2$

cn

tates of the photon



$\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}$

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cn

tates of the photon

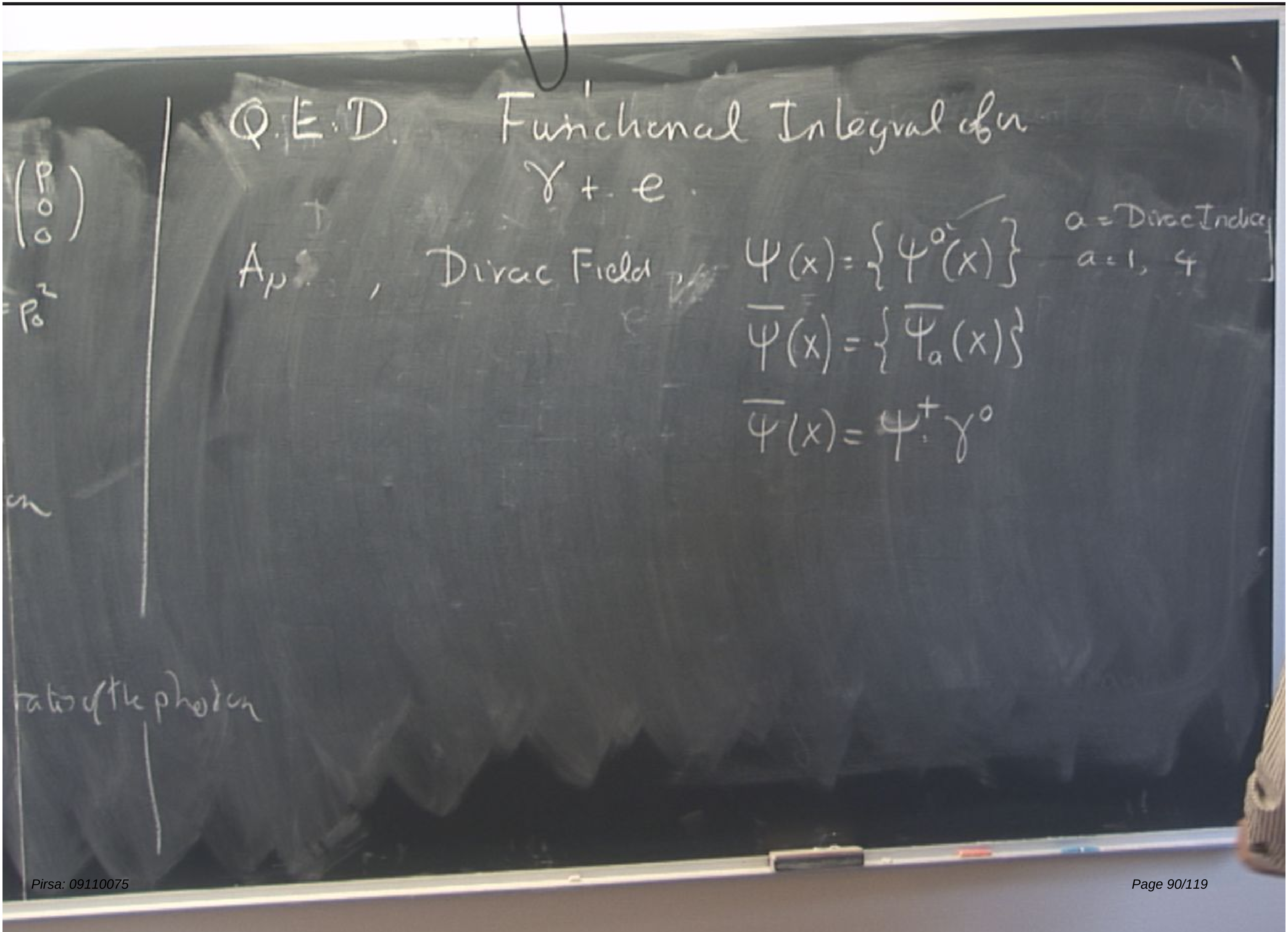
Q.E.D. Functional Integral for

$\gamma + e$.

A_μ , Dirac Field , $\Psi(x) = \{\Psi^a(x)\}$

a = Dirac Indices

a = 1, 4



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on

ates of the photon

Q.E.D. Functional Integral for

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A_μ , Dirac Field ,

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$\bar{\Psi}(x) = \{\bar{\Psi}_a(x)\}$

$\bar{\Psi}(x) = \Psi^\dagger \gamma^0$

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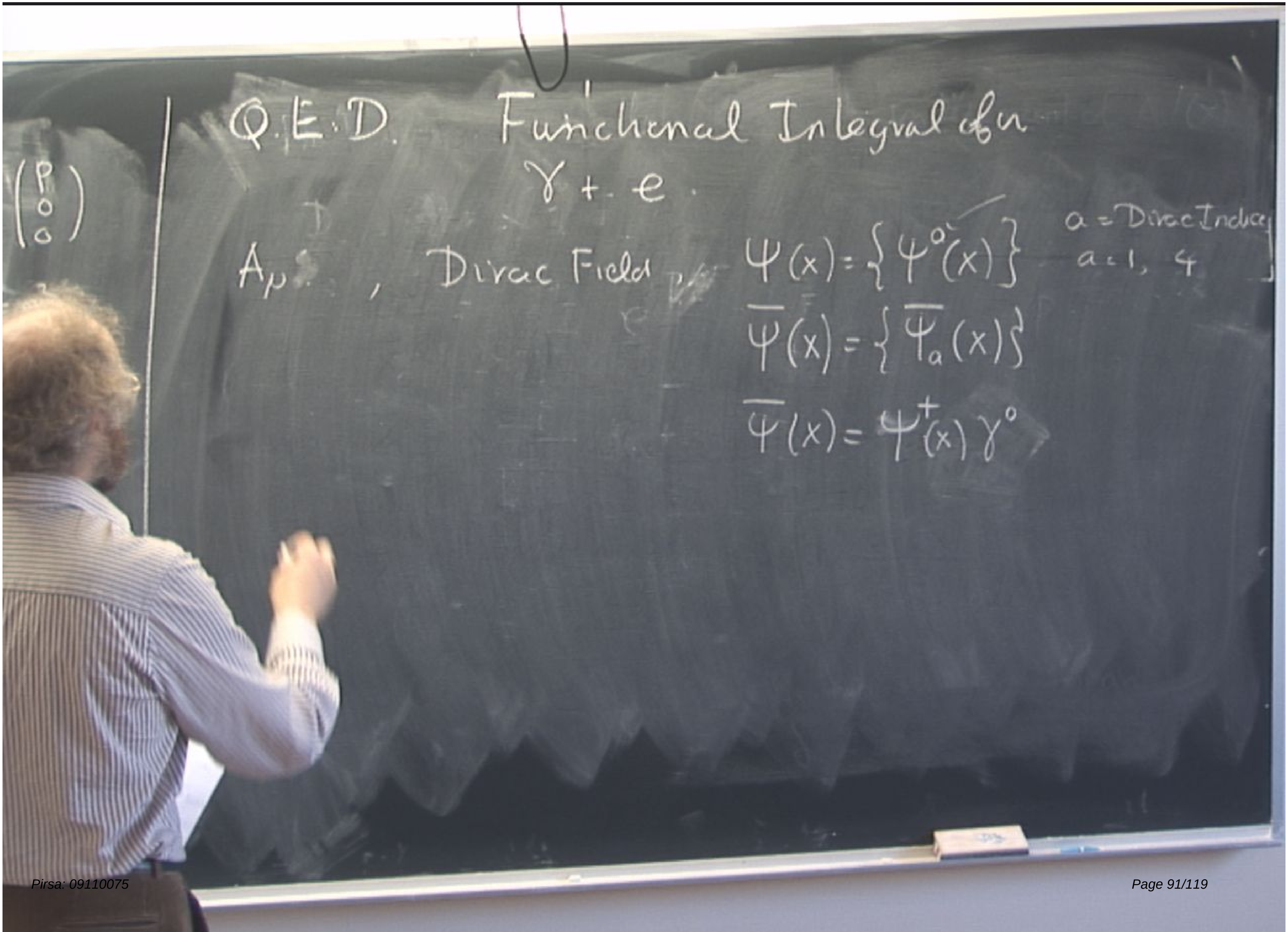

$\begin{pmatrix}p\\0\\0\end{pmatrix}$

Q.E.D. Functional Integral for $\gamma + e$.

A_μ , Dirac Field ,

$$\Psi(x) = \{\Psi^a(x)\}$$

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a = 1, 4

$$\overline{\Psi}(x) = \{\overline{\Psi}_a(x)\}$$

$$\overline{\Psi}(x) = \Psi^\dagger(x)\gamma^0$$

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tates of the photon

Q.E.D. Functional Integral for $\gamma + e$

A_μ , Dirac Field , $\Psi(x) = \{\Psi^a(x)\}$

a = Dirac Indice
a = 1, 4

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}\mathbb{1}$$

$$\overline{\Psi}(x) = \{\overline{\Psi}_a(x)\}$$

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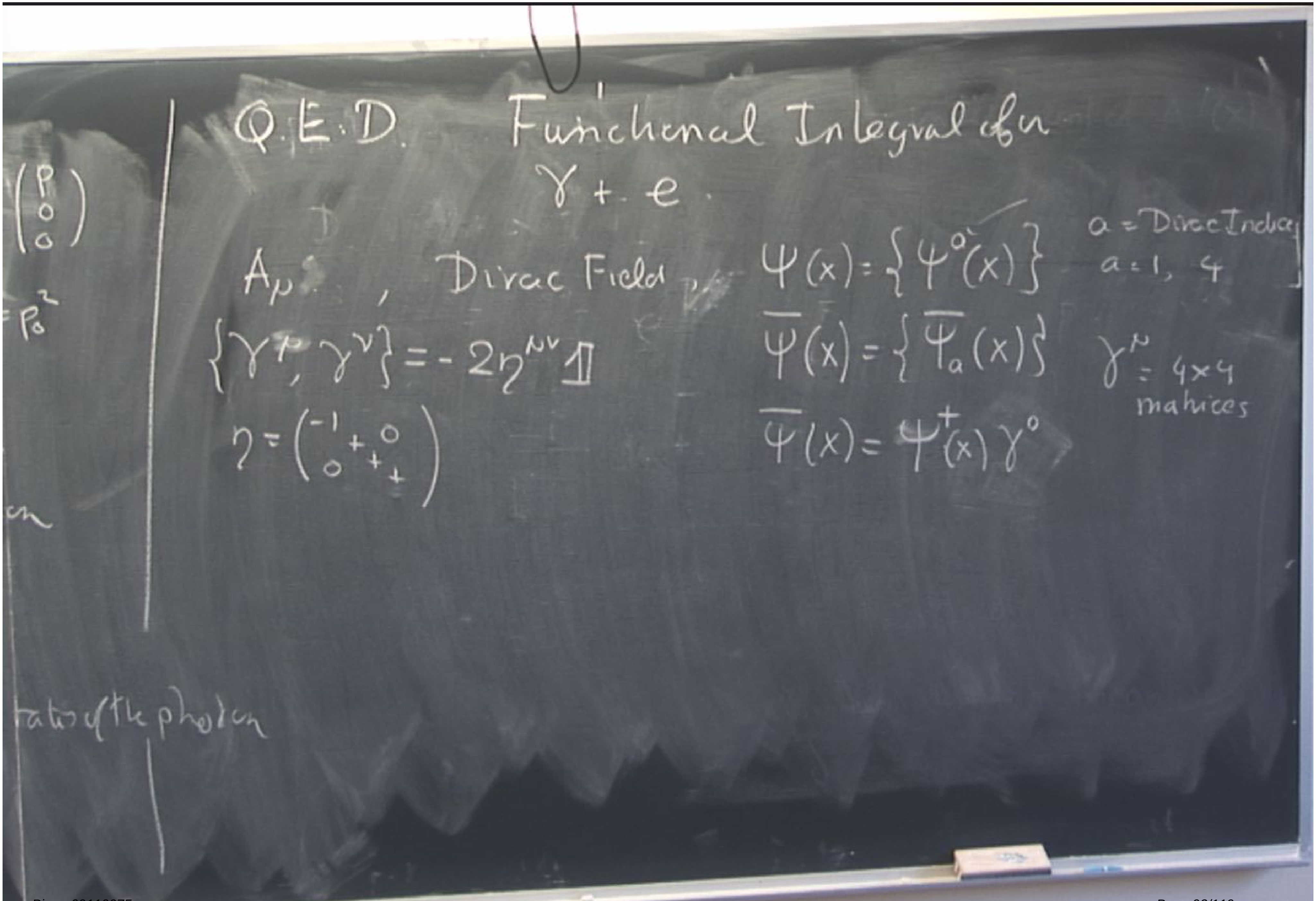

$\begin{pmatrix} p \\ 0 \\ 0 \end{pmatrix}$

$= p_0^2$

cn

tates of the photon

Q.E.D. Functional Integral for $\gamma + e$

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a = 1, 4

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$\gamma^\mu = 4\times 4$ matrices

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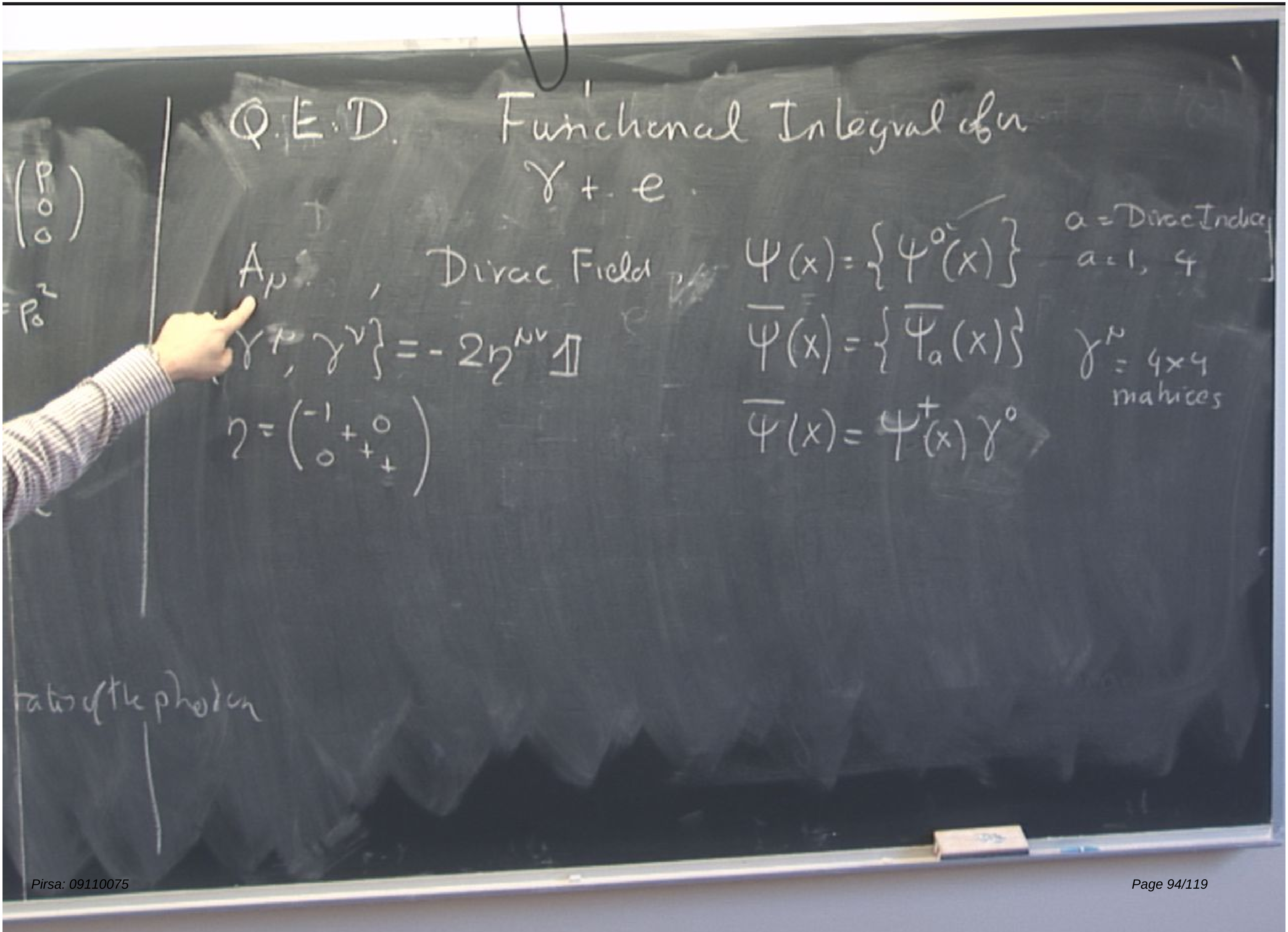

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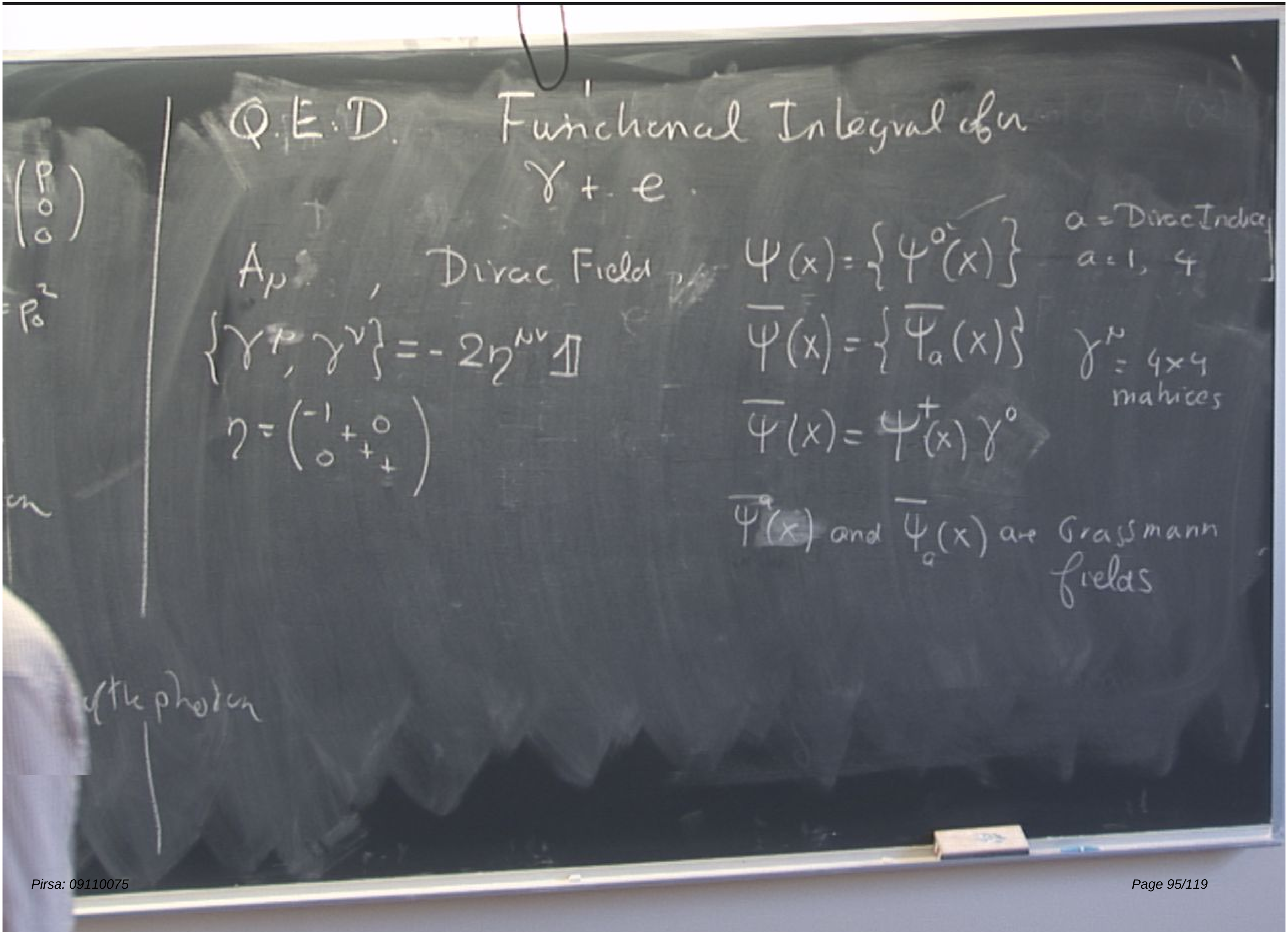

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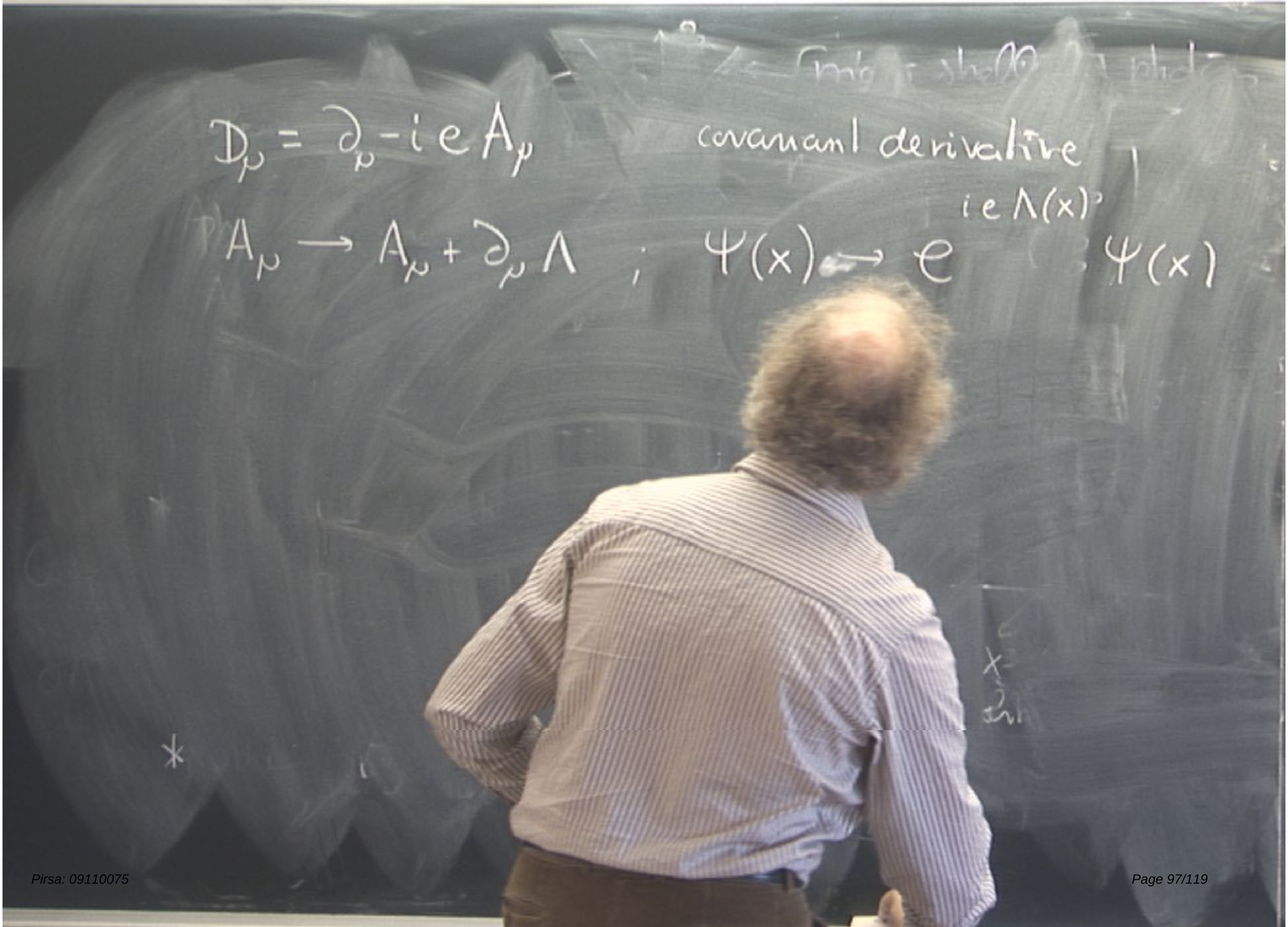
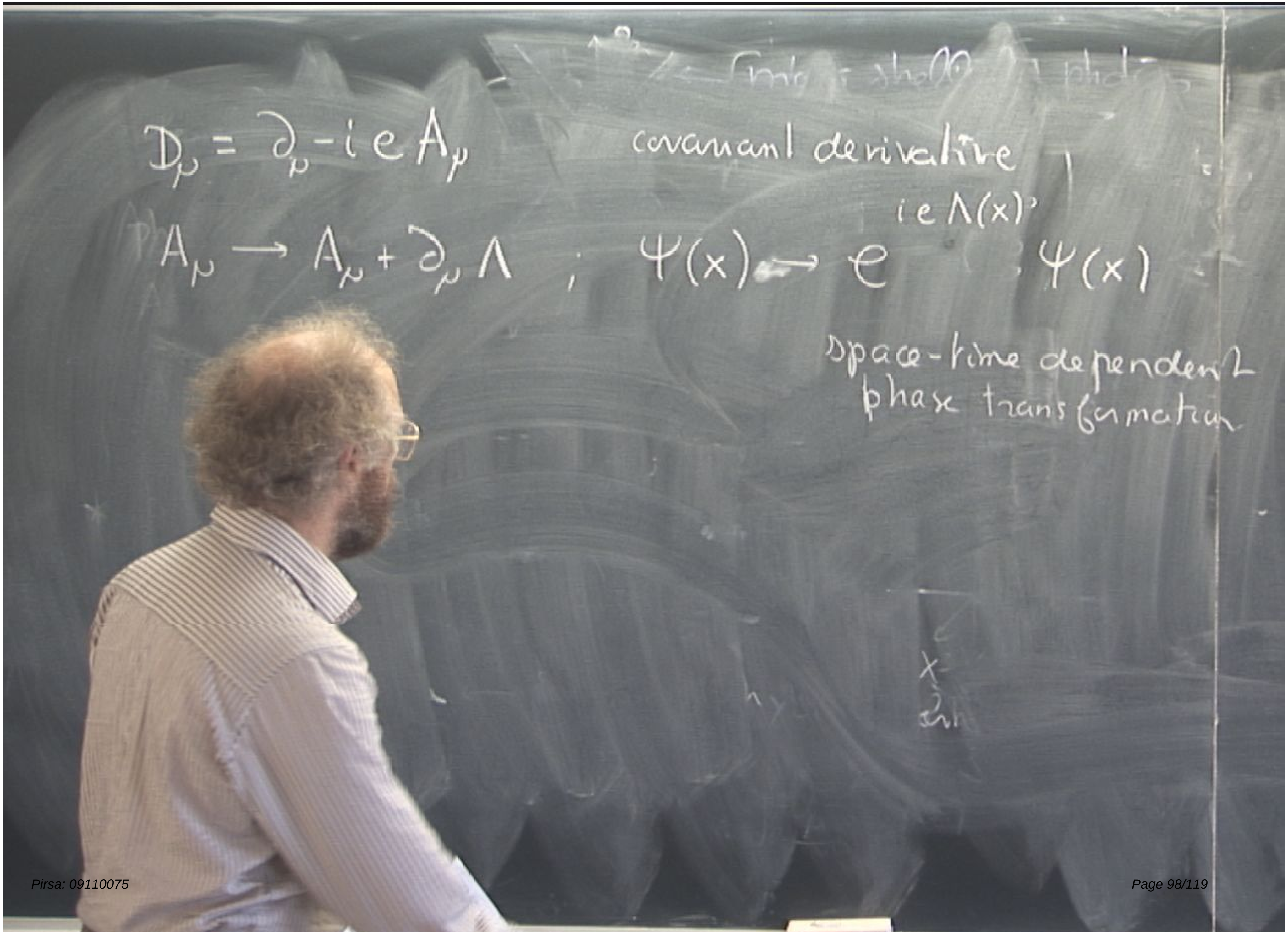


$D_\mu = \partial_\mu - ieA_\mu$ covariant derivative

$A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda \; ; \; \Psi(x) \longrightarrow e^{ie\Lambda(x)} \Psi(x)$



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space-time dependent
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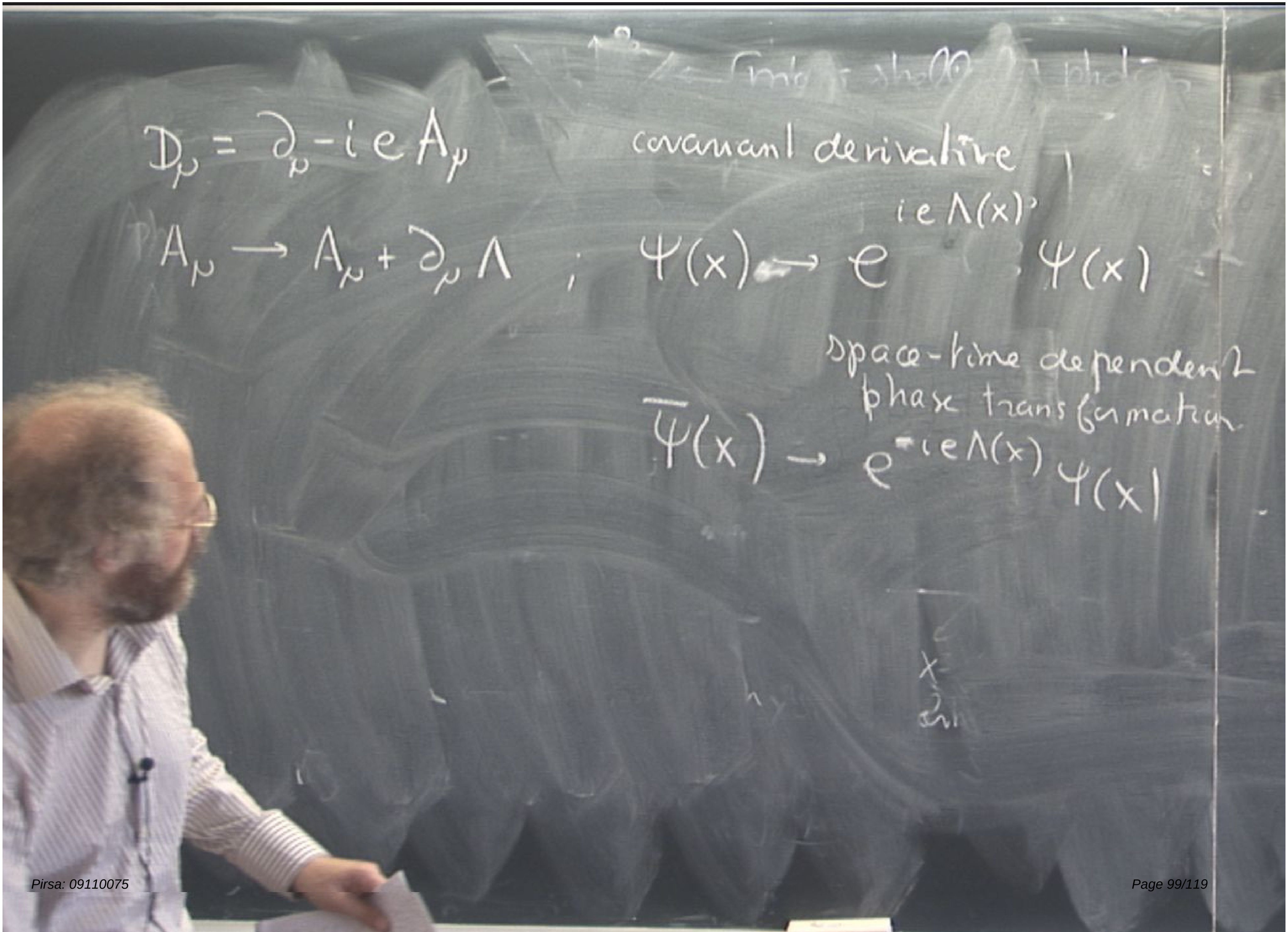

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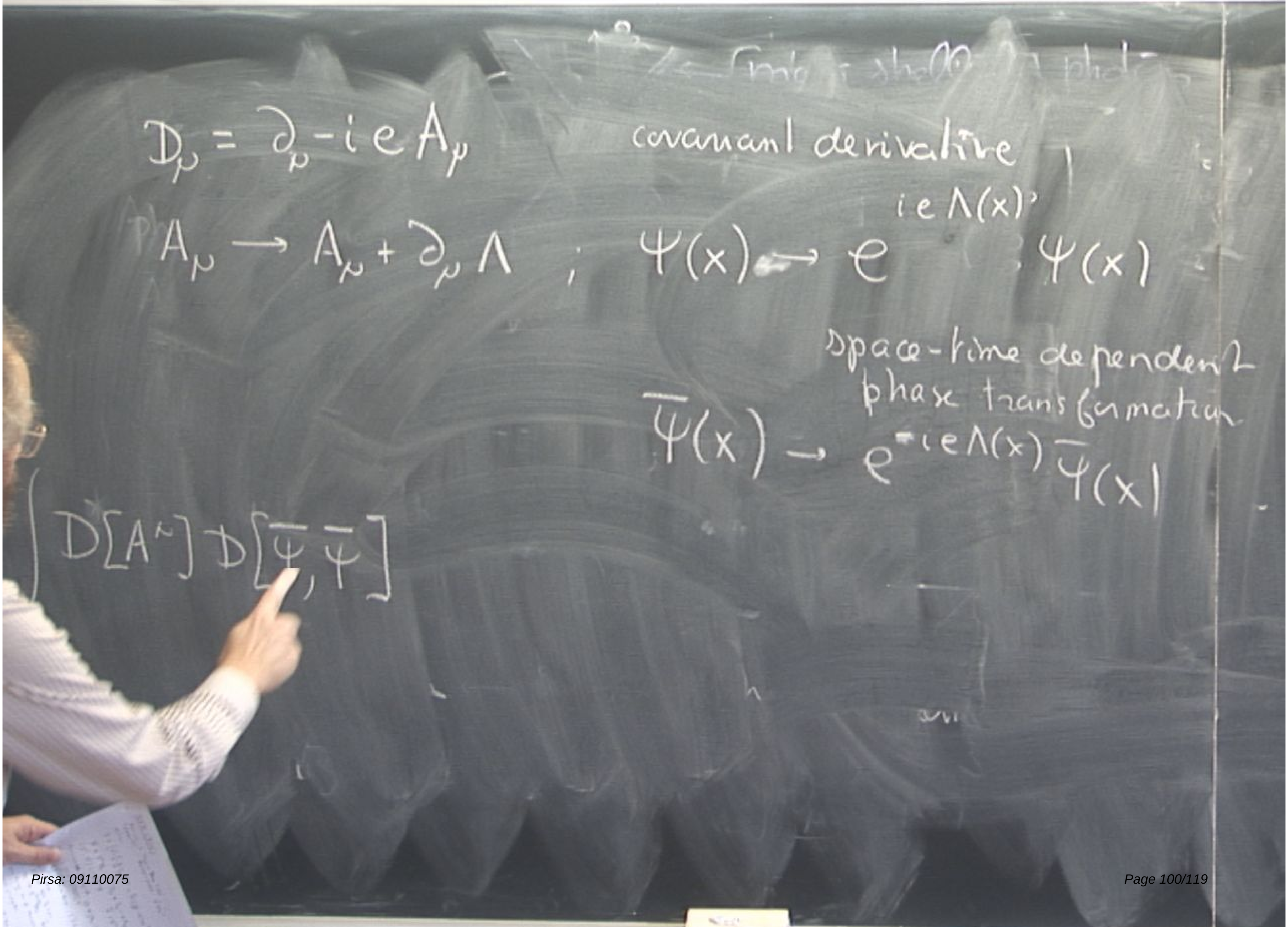

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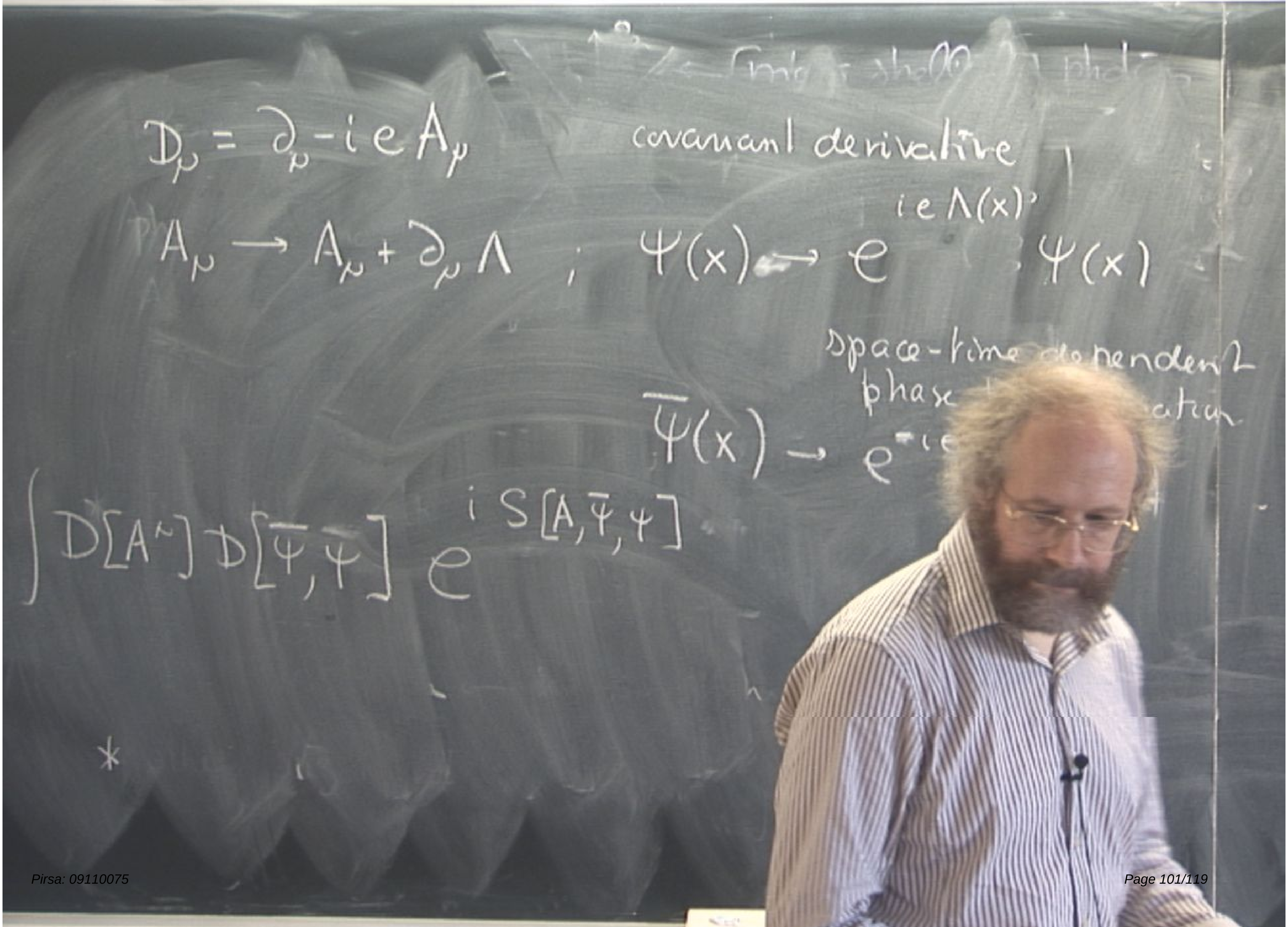

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space-time dependent
phase [illegible]ation

$$\bar{\Psi}(x) \rightarrow e^{-ie}$$

$$\int D[A^\mu]\, D[\bar{\Psi},\Psi]\, e^{iS[A,\bar{\Psi},\Psi]}$$

*

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has to be the same $\Lambda(x)$

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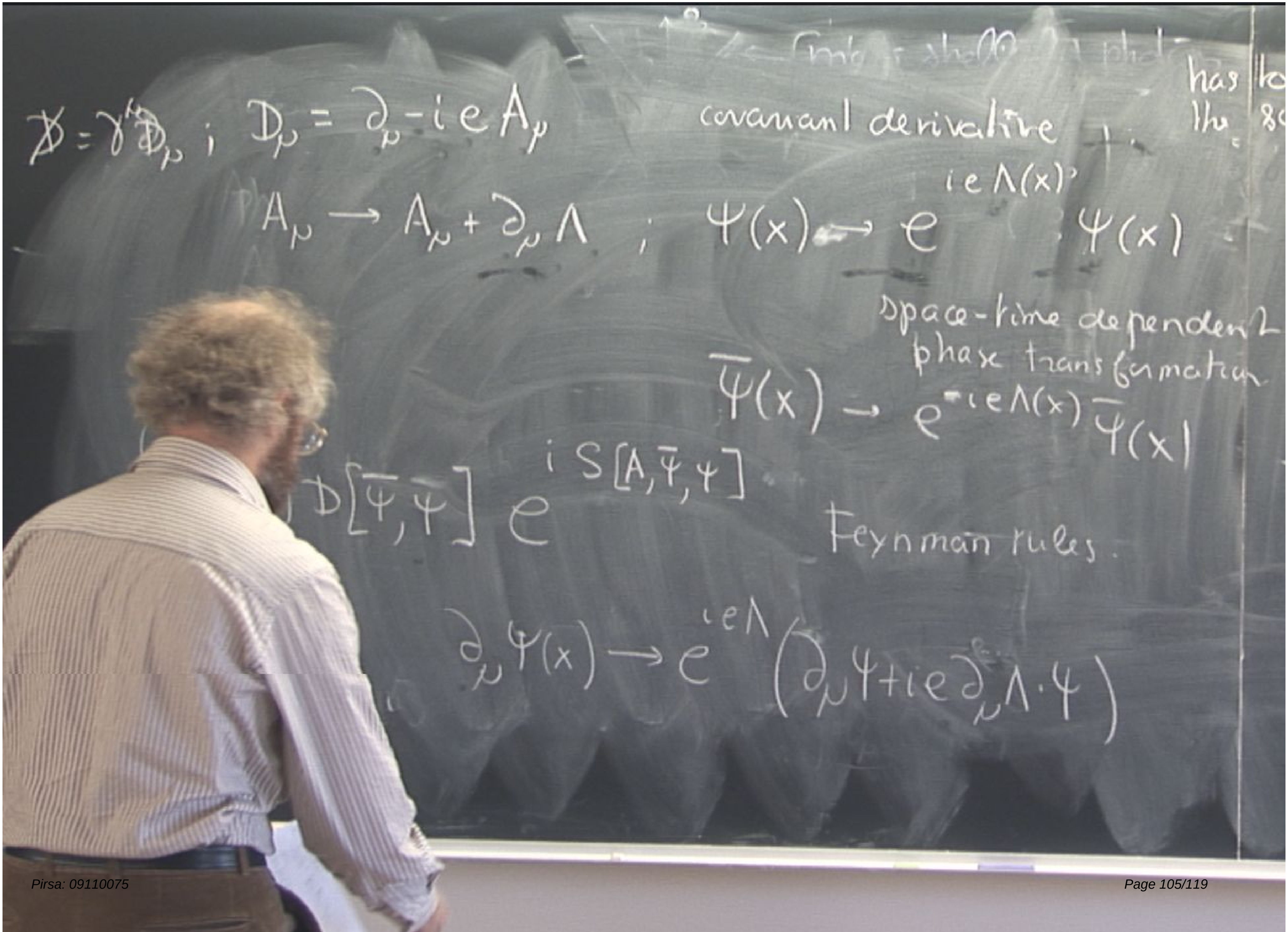

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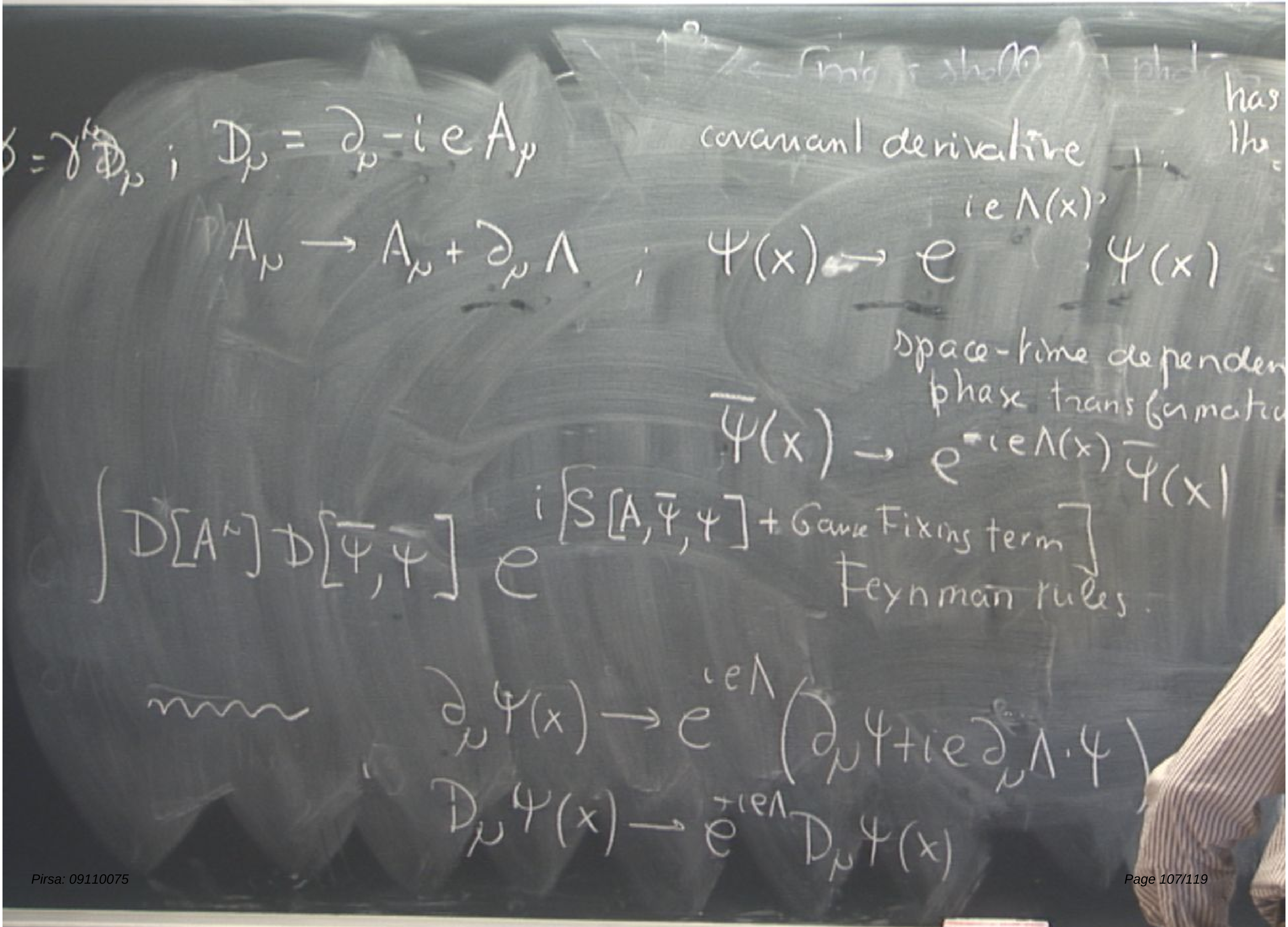

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2

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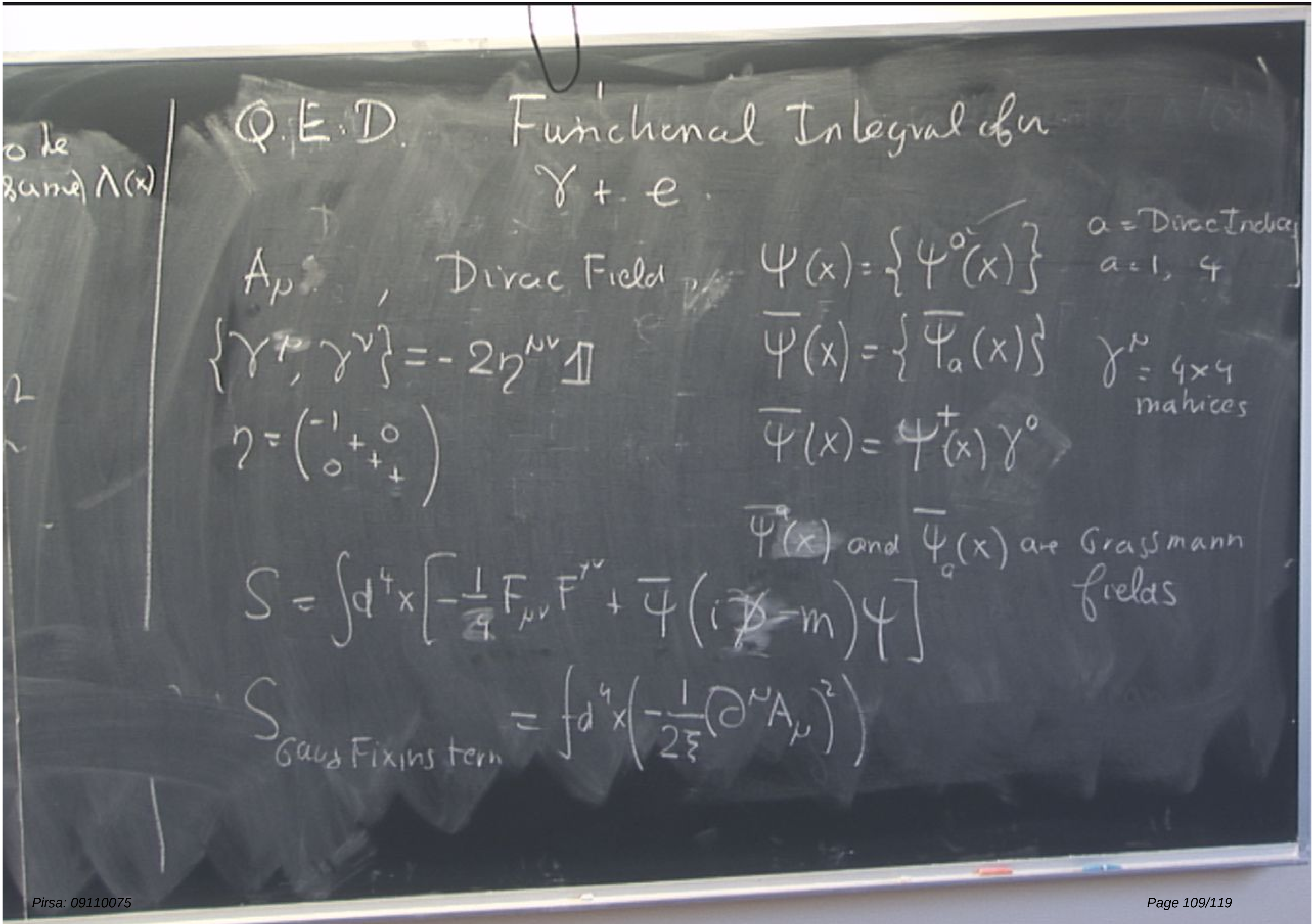

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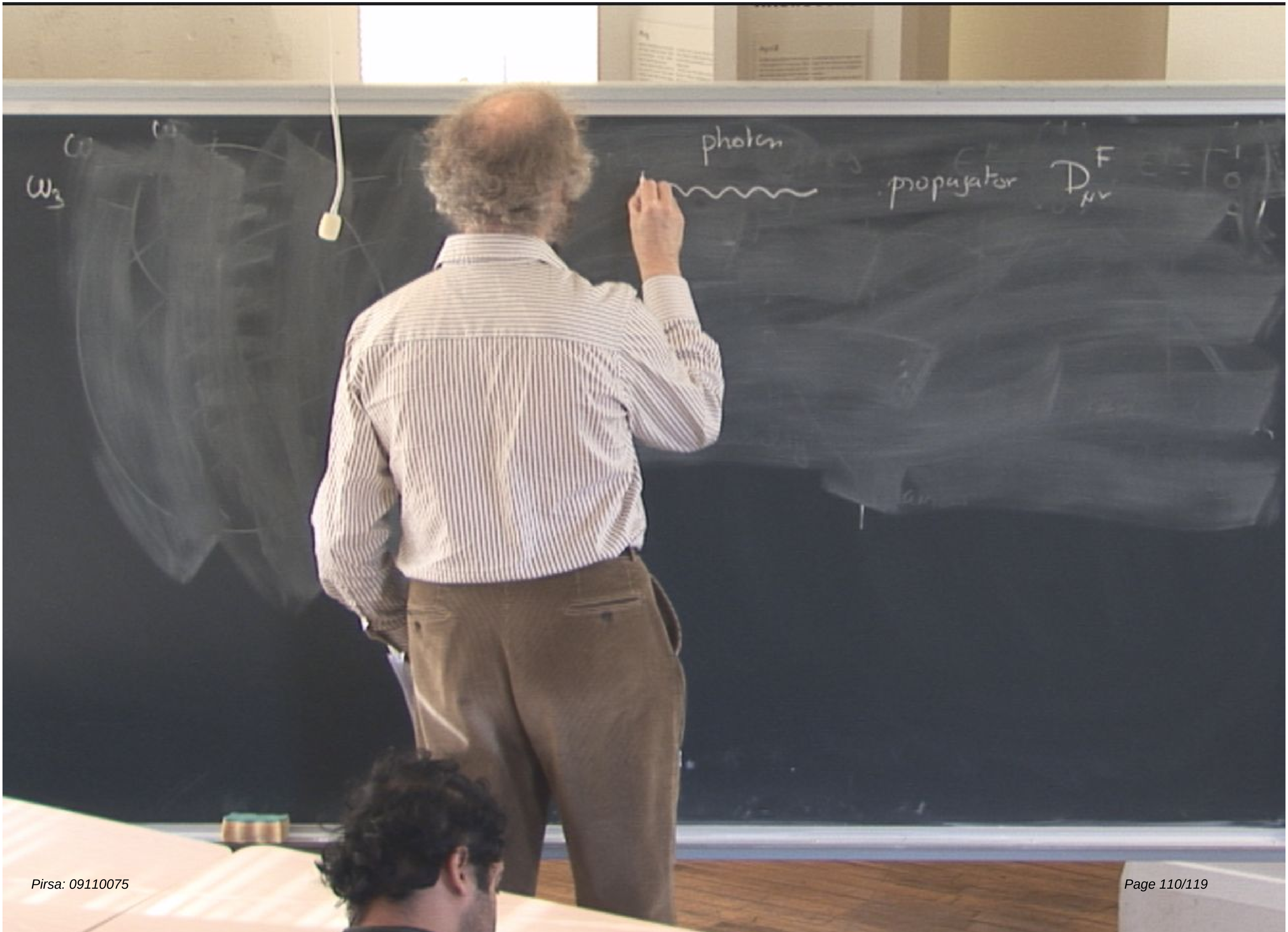
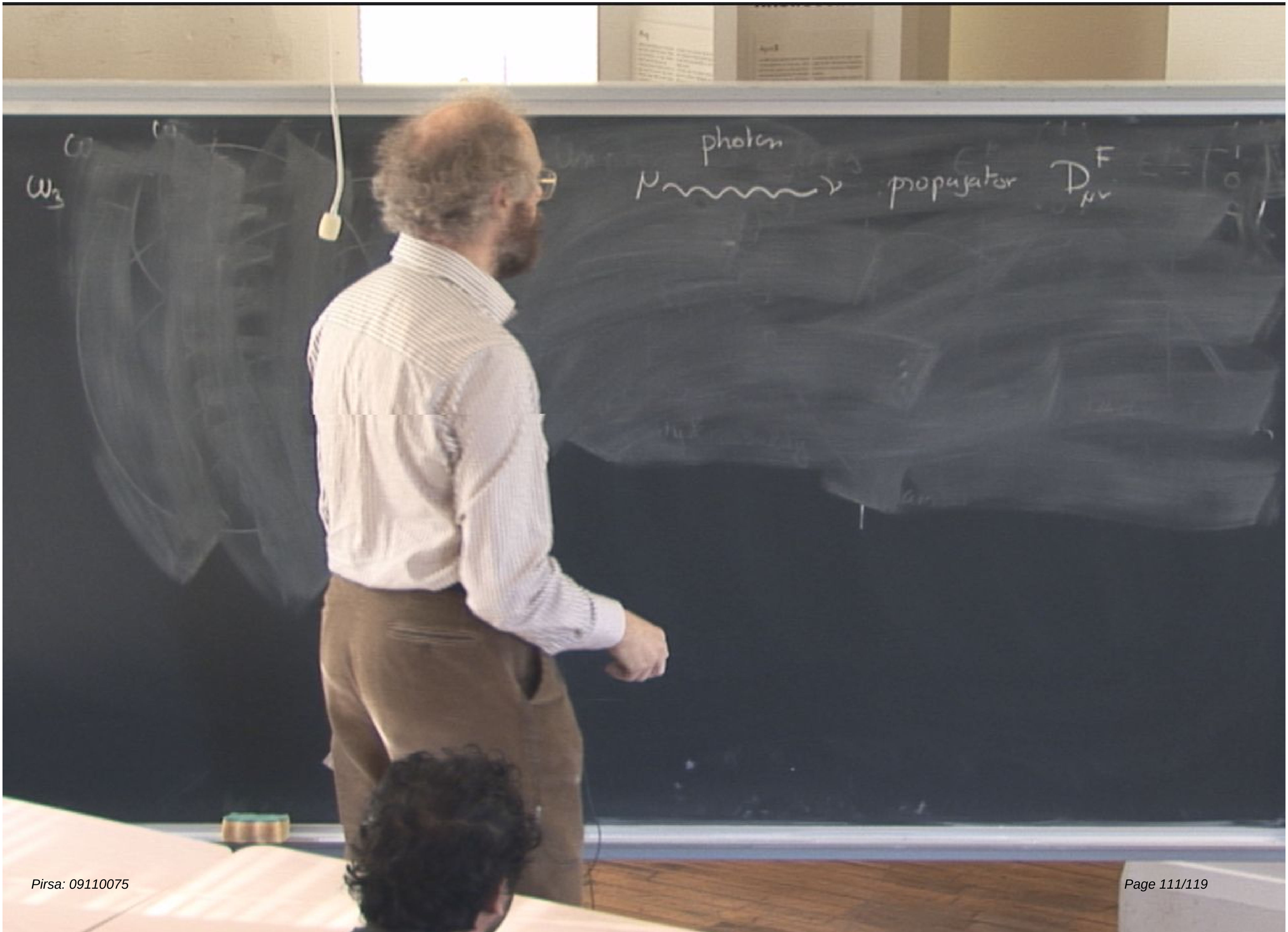

ω_3
photon
propagator $D^F_{\mu\nu}$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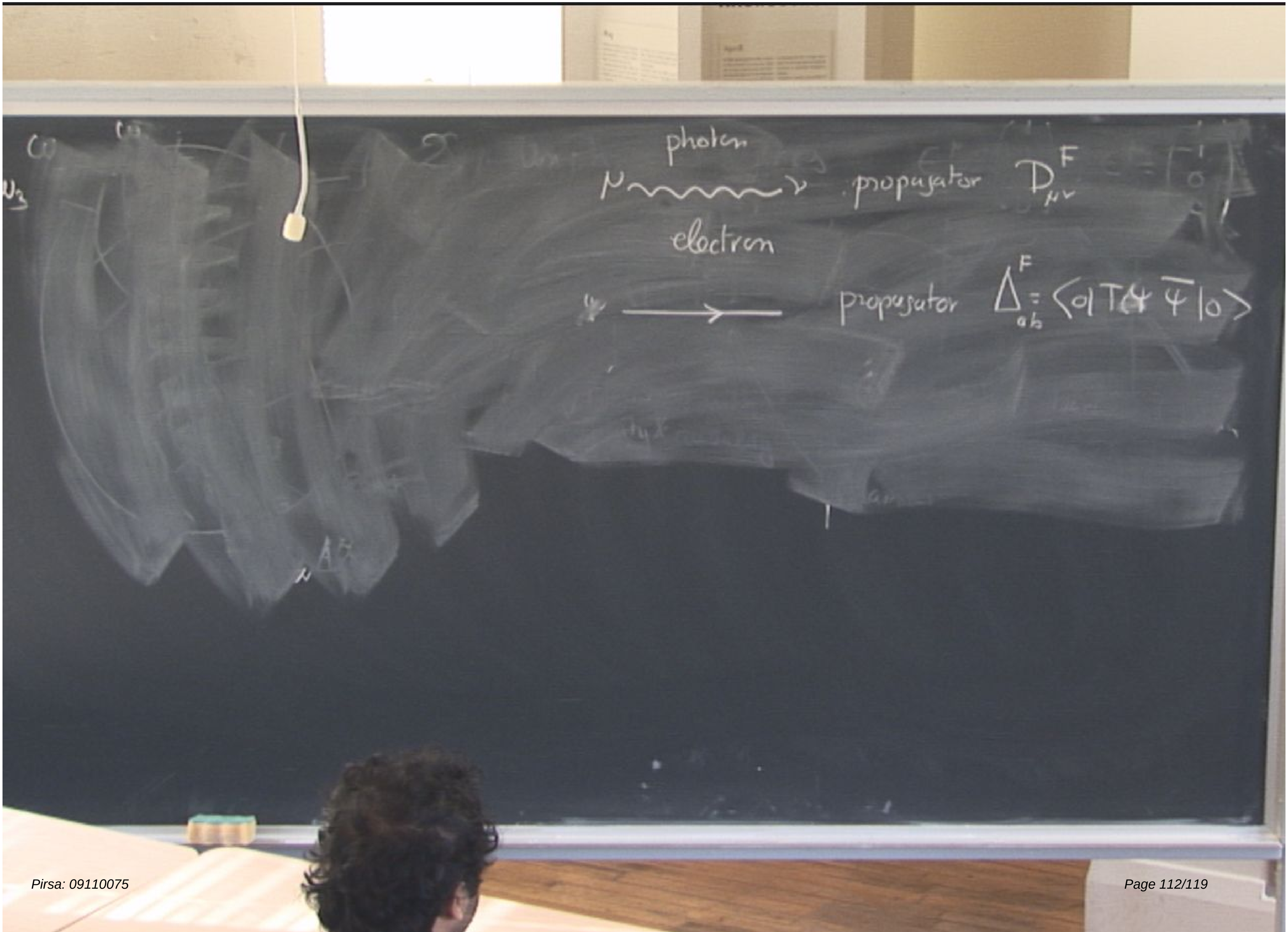
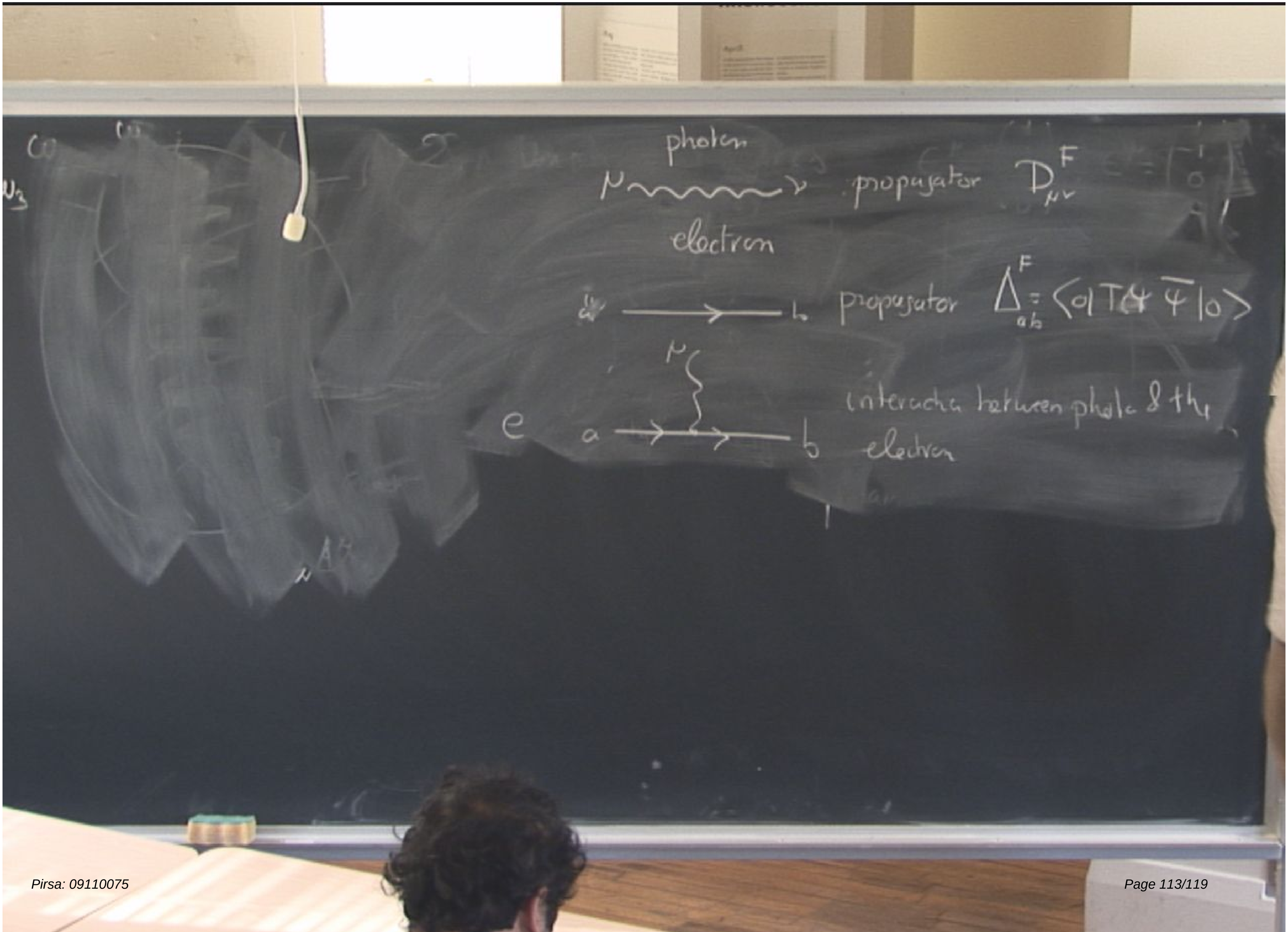


photon

$\mu \sim\sim\sim\sim\sim \nu$ propagator $D^F_{\mu\nu}$

electron

$\psi \longrightarrow$ propagator $\Delta^F_{ab} = \langle 0 | T(\psi \overline{\psi}) | 0 \rangle$



propagator $D^F_{\mu\nu}$

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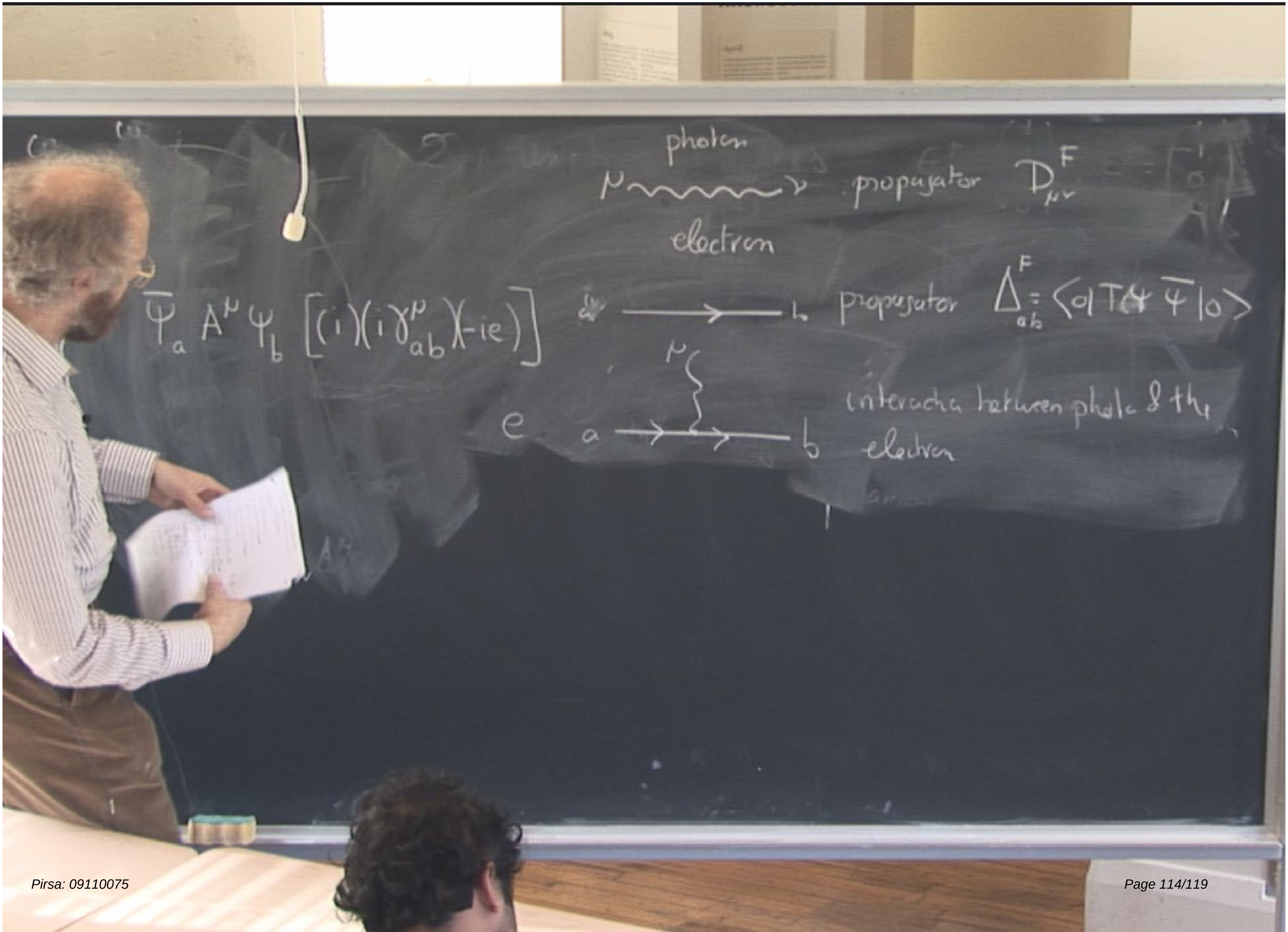
interaction between photon & the electron



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$\bar\Psi_a A^\mu \Psi_b \left[(i)(i\gamma^\mu_{ab})(-ie) \right]$

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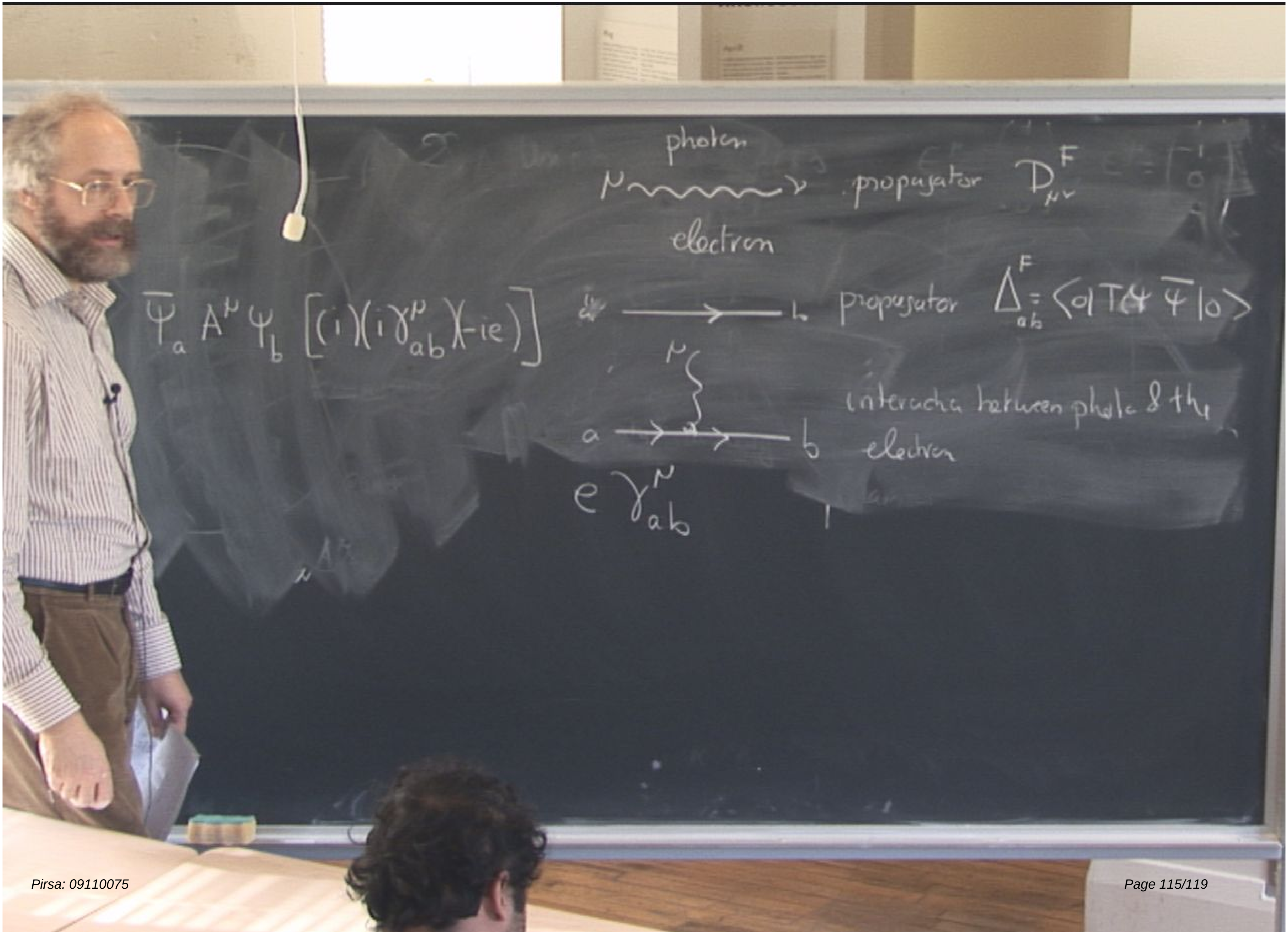
photon
μ ν propagator $D^F_{\mu\nu}$
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a b interaction between photon & the electron
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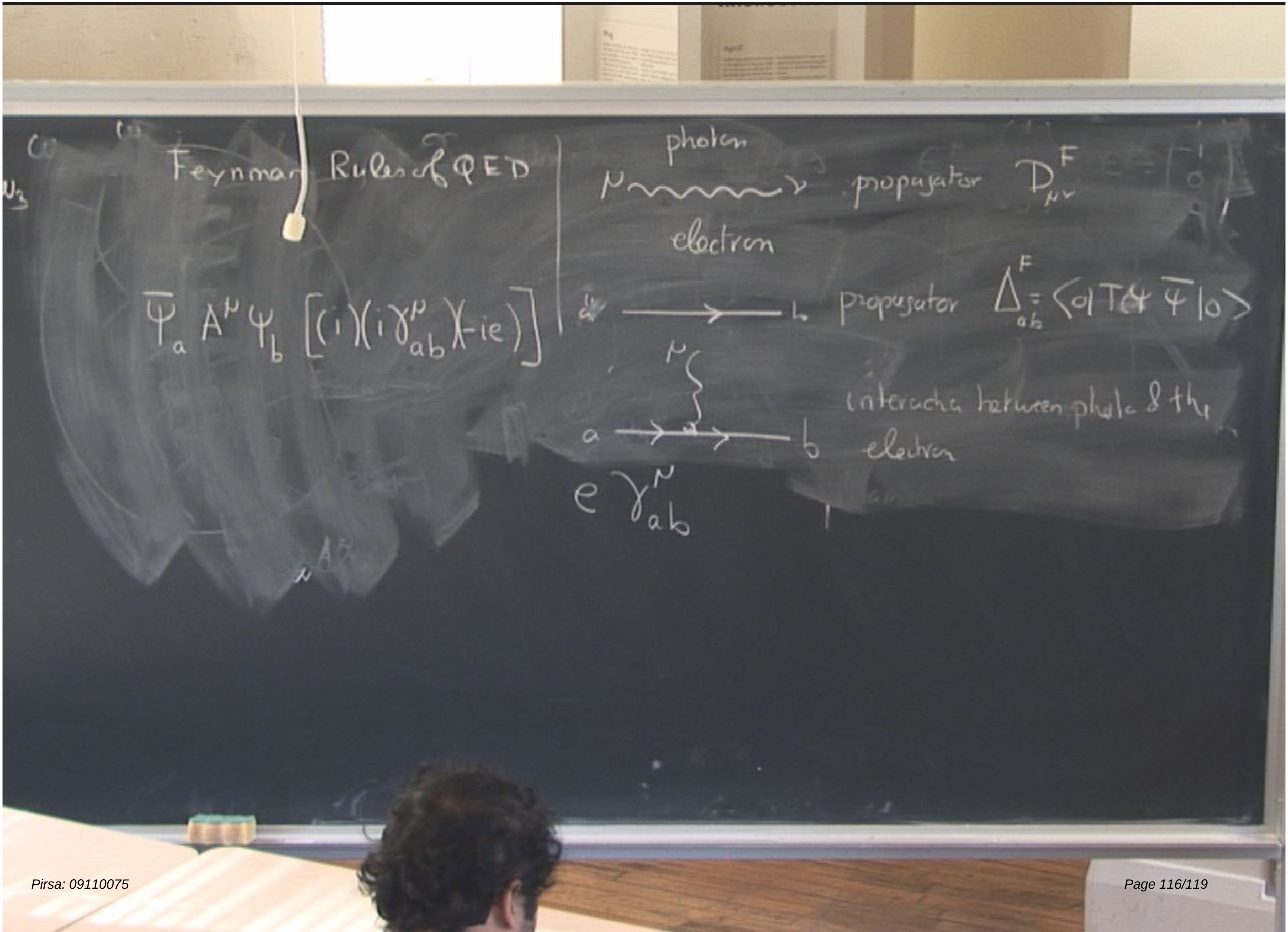

Feynman Rules of QED

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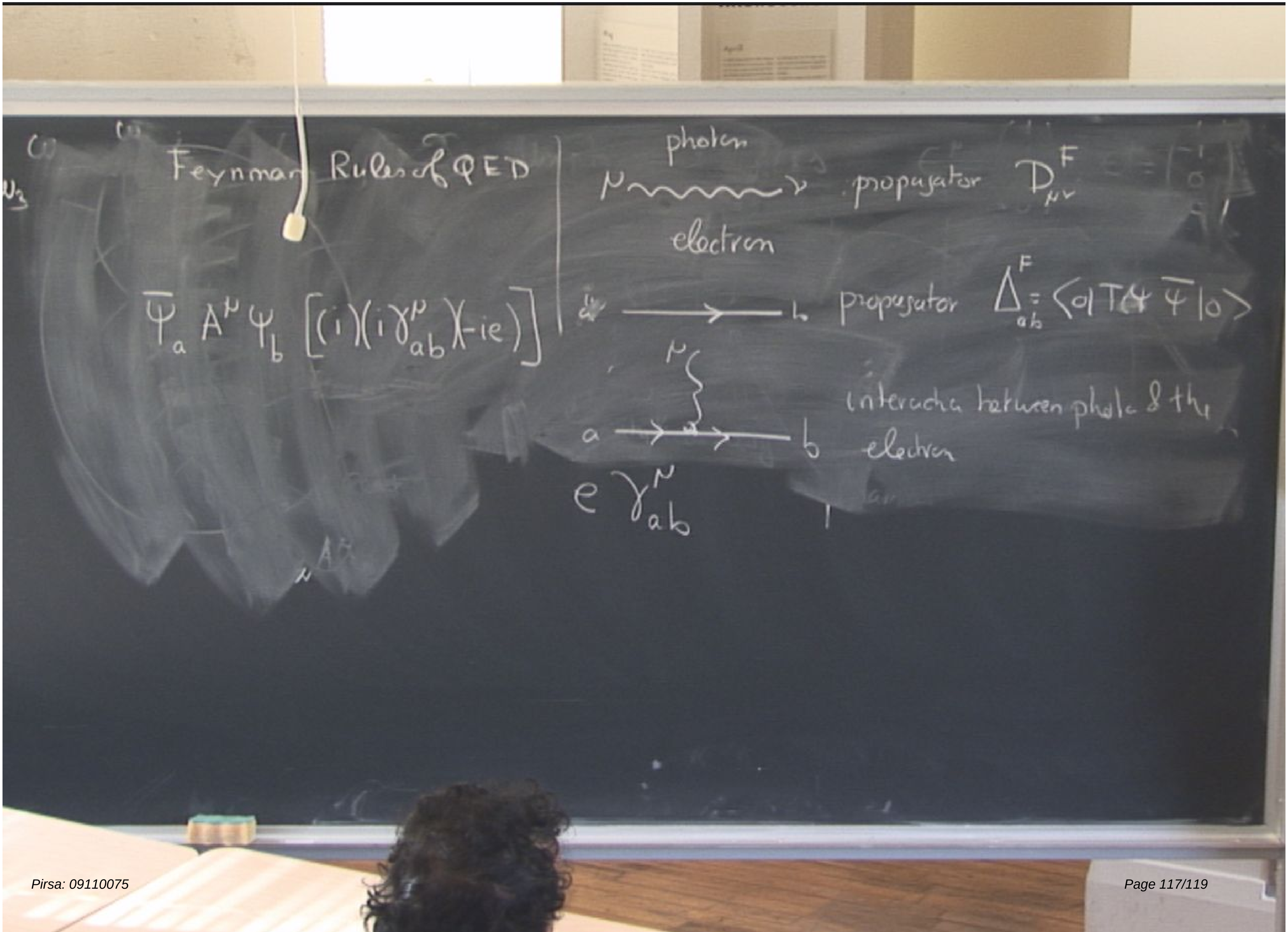

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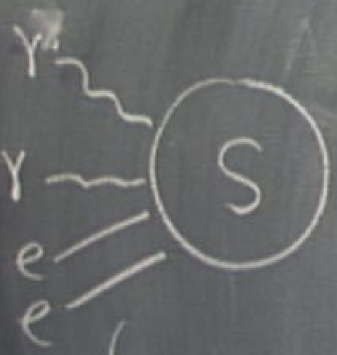

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