Title: Quantum Field Theory II (PHYS 603) - Lecture 9

Date: Nov 05, 2009 01:30 PM

URL: http://pirsa.org/09110071

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

$\langle \phi(x_1) \cdots \phi(x_N) \rangle$

finite in the limit

where $\Lambda \to \infty$

Λ is an UV regulator

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle \longleftarrow \int D[\phi]\, e^{\frac{i}{\hbar} S[\phi]}$$

massl

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Is there an action $S[\phi]$ such that The Green Functions (hence the S matrix elements, etc..) exist ?

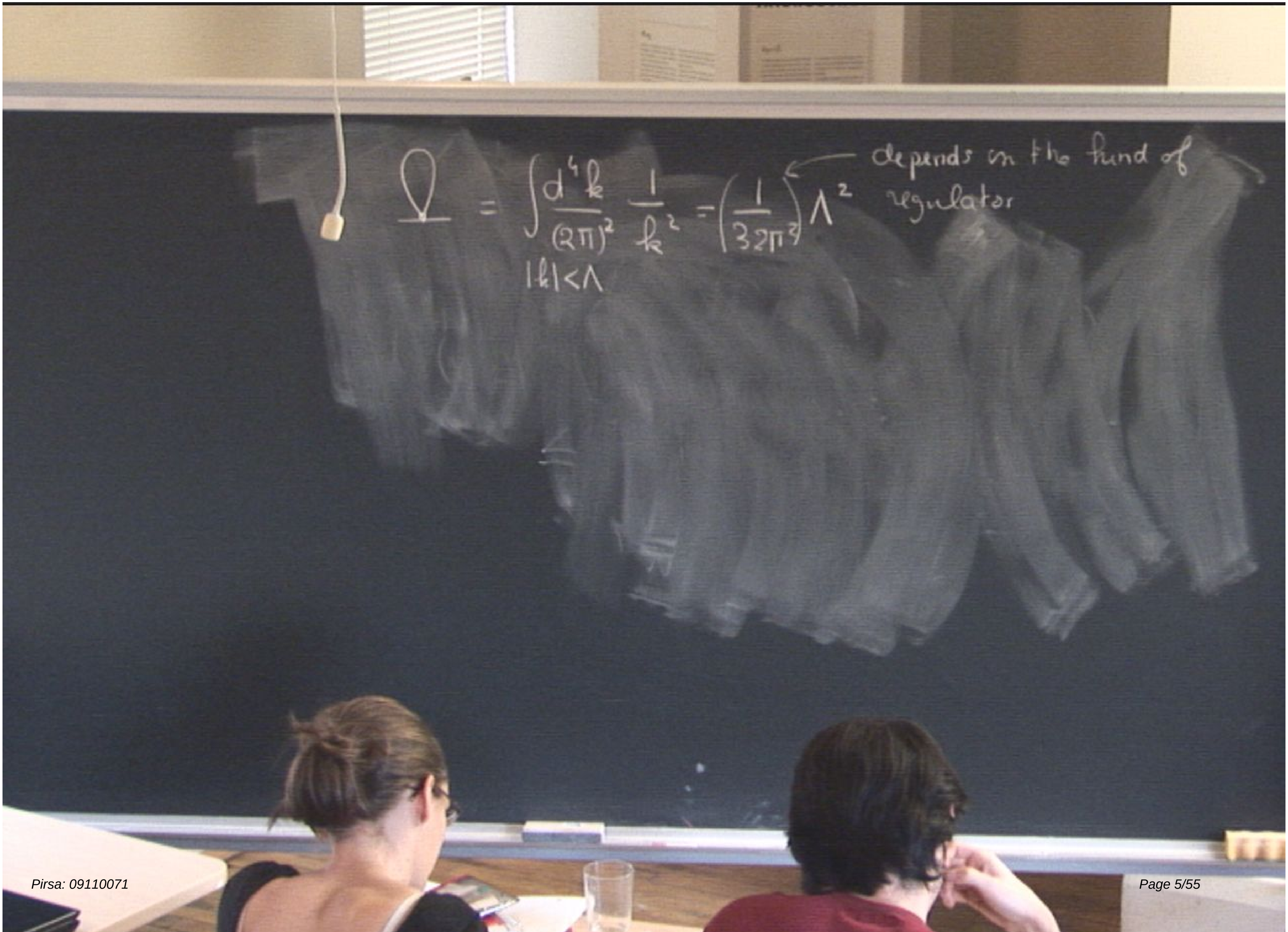
$$S_{classical}[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{g}{4} \phi^4 \right]$$

$$S[\phi] = S_{classical}[\phi] + \Delta_1 S[\phi]$$

counterterm

$$\Delta_1 S[\phi] = \int d^4x \, B_1 \phi^2$$

$$B_1 = -g\frac{\hbar}{2}$$


= ∫ d^4k/(2π)^2 1/k^2 = (1/32π^2) Λ^2
|k|<Λ
depends on the kind of regulator

$$\langle \phi(x_1) \cdots \phi(x_N) \rangle \longleftarrow \int D[\phi]\, e^{-\frac{i}{\hbar} S[\phi]}$$

mass...

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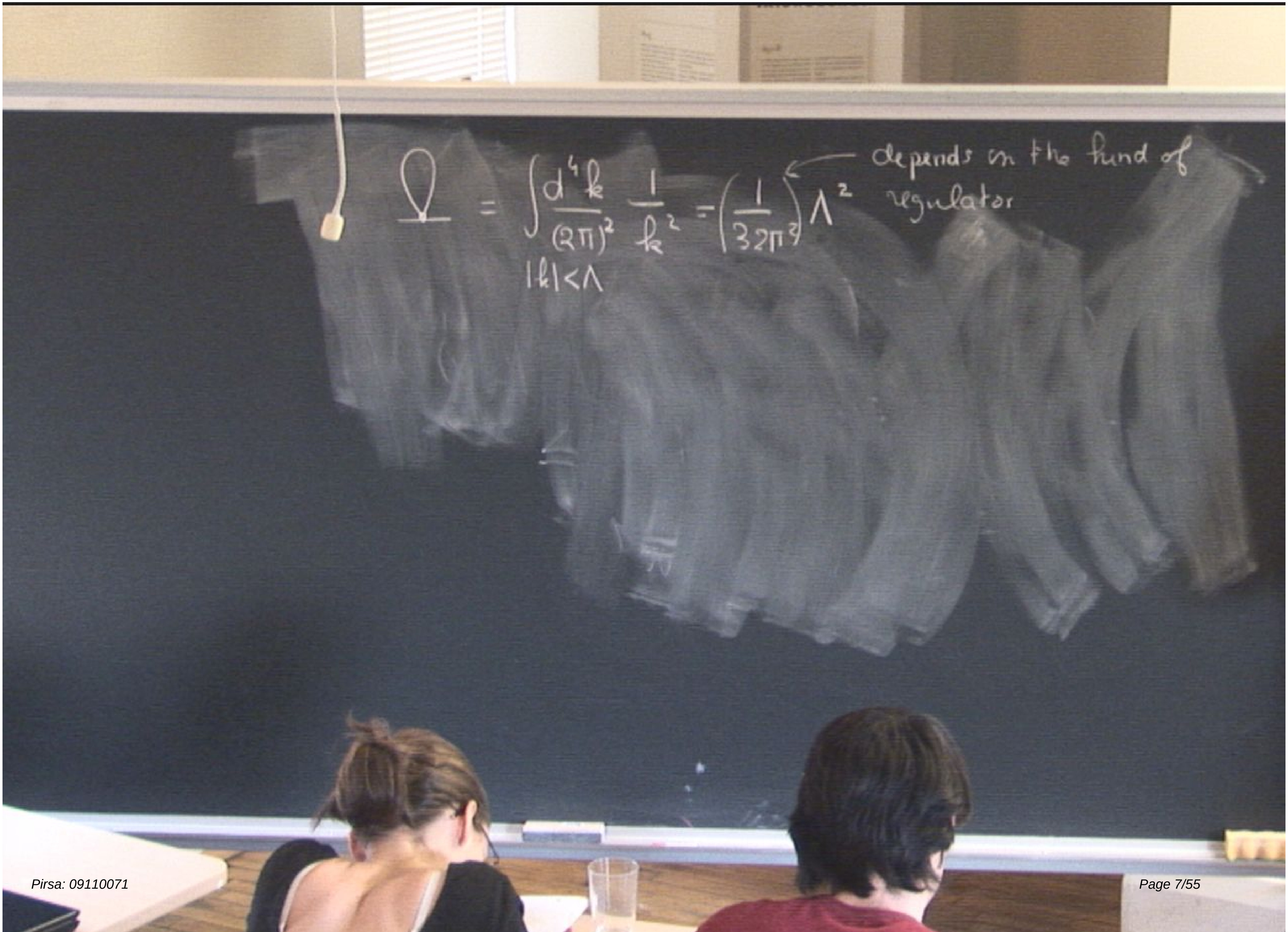
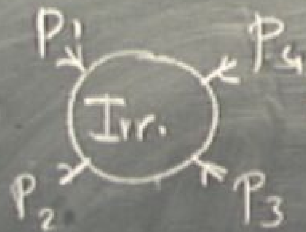
$$\Delta_1 S[\phi] = \int d^4x\, B_1 \phi^2 \qquad B_1 = -\frac{g\hbar}{2}$$

$$\hat{\Gamma}^{(2)}(p) = \text{Irr} = p^2 + \frac{g\hbar}{2} + B_1 = p^2$$


$= \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = \left(\frac{1}{32\pi^2}\right)\Lambda^2$
depends on the kind of regulator

4-point irreducible function. (term of order 4 in φ in effective action)

$\hat{\Gamma}^{(4)}(p_i) =$ 

$= (2\pi)^4 \delta^4(\Sigma p_i) \, [\times g$

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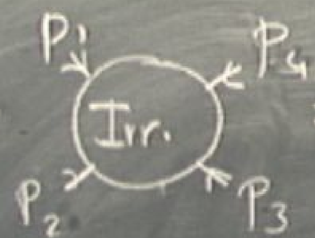
$\hat{\Gamma}^{(4)}(p_i) =$  $= (2\pi)^4 \delta^4(\Sigma p_i) \left[\times g + \left(-\frac{g^2 \hbar}{2}\right) \right.$ ( $+ \;'' + \;'' \left. \right) \Big]$;

$\frac{1}{2}$  $= J(p) = \int \frac{d^4 q}{(2\pi)^2} \frac{1}{q^2 (p+q)^2}$

$p = p_1 + p_2,\; p_1 + p_3$ or $p_1 + p_4$

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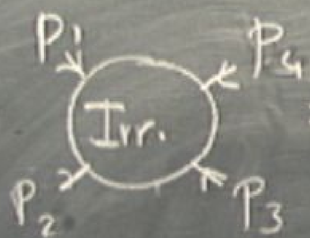
$\hat{\Gamma}^{(4)}(p_i) =$  $= (2\pi)^4 \delta^4(\Sigma p_i) \Big[g + \left(-\frac{g^2 \hbar}{2}\right)\Big($  $+ \;'' + \;''\Big)\Big]$

3 diagrams

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$p = p_1 + p_2, \; p_1 + p_3$ or $p_1 + p_4 \quad |q| < \Lambda$

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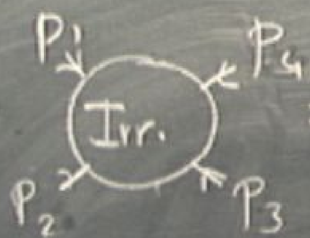
$$\hat{\Gamma}^{(4)}(p_i) =$$ 

$$= (2\pi)^4 \delta^4\left(\sum p_i\right) \left[\times g + \left(-\frac{g^2\hbar}{2}\right)\left(\right.\right.$$  $$\left.\left. + \;'' \; + \; '' \right)\right] ;$$

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↑ universal (independent of the regulator used)

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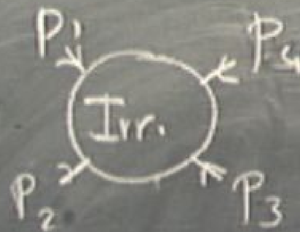
$\hat{\Gamma}^{(4)}(p_i) =$  $= (2\pi)^4 \delta^4(\Sigma p_i) \Big[\times g + \left(-g^2 \frac{\hbar}{2}\right)\Big($  $+ \;'' + \;''\Big)$

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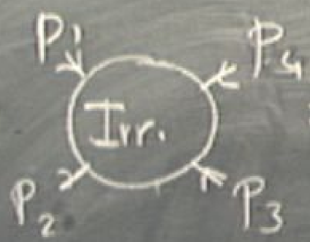

$$= (2\pi)^4 \delta^4\left(\Sigma p_i\right) \left[\times g + \left(-\frac{g^2 \hbar}{2}\right)\left(\quad + \;\; " \;\; + \;\; " \right) \right];$$

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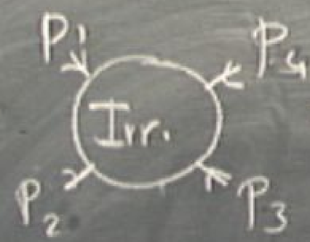

$$\hat{\Gamma}^{(4)}(p_i) = \text{[Irr.]} = (2\pi)^4 \delta^4(\Sigma p_i) \left[\times g + \left(-g^2 \frac{\hbar}{2}\right)\left(\text{[loop]} + '' + '' \right) \right];$$

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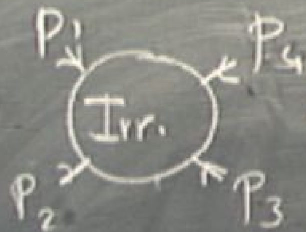
$\hat{\Gamma}^{(4)}(p_i) =$ 

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coupling constant renormalization

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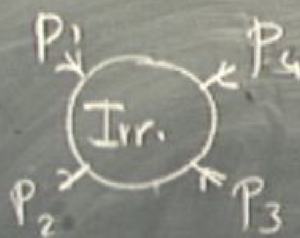
$\hat{\Gamma}^{(4)}(p_i) =$ 

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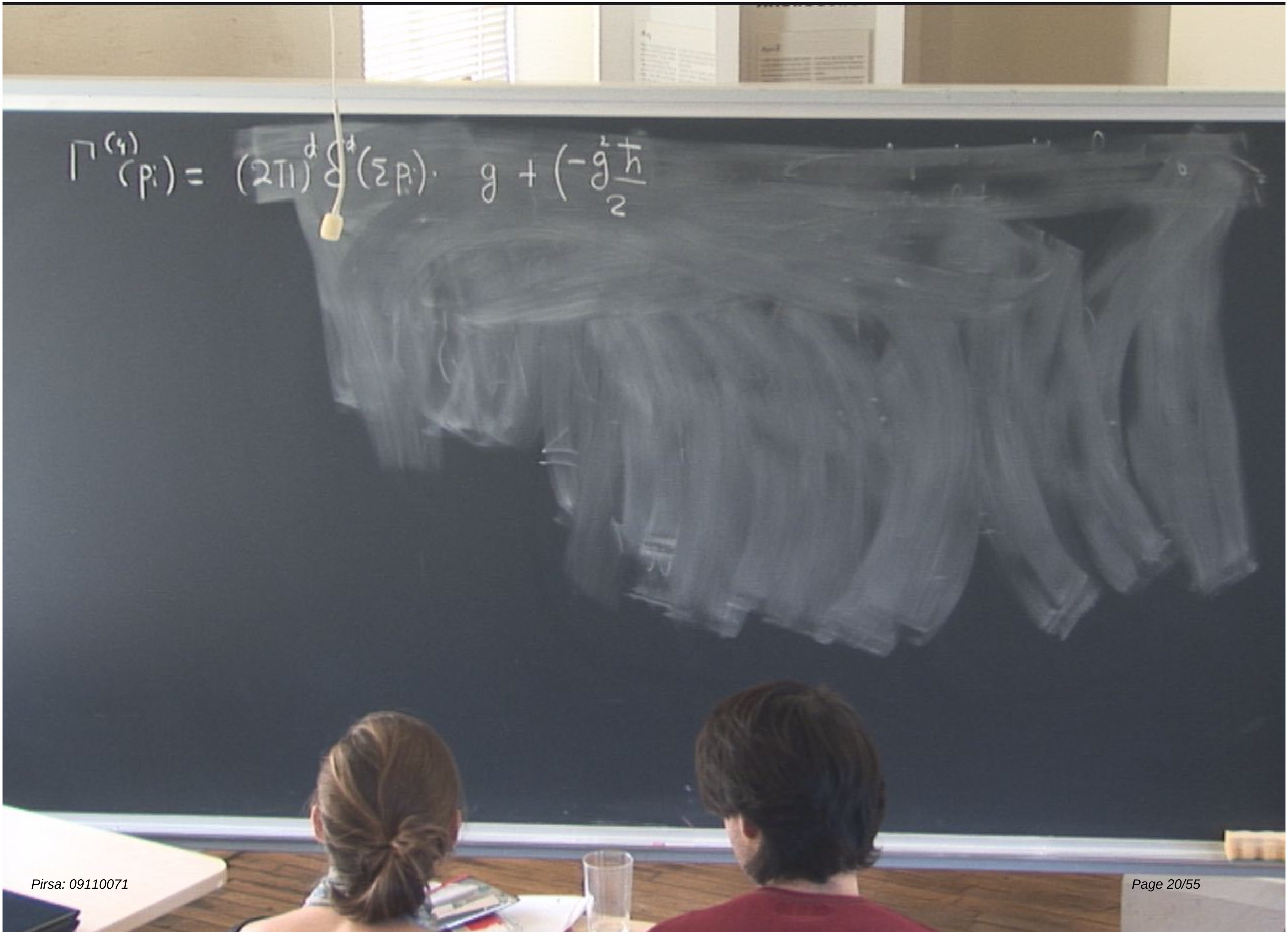
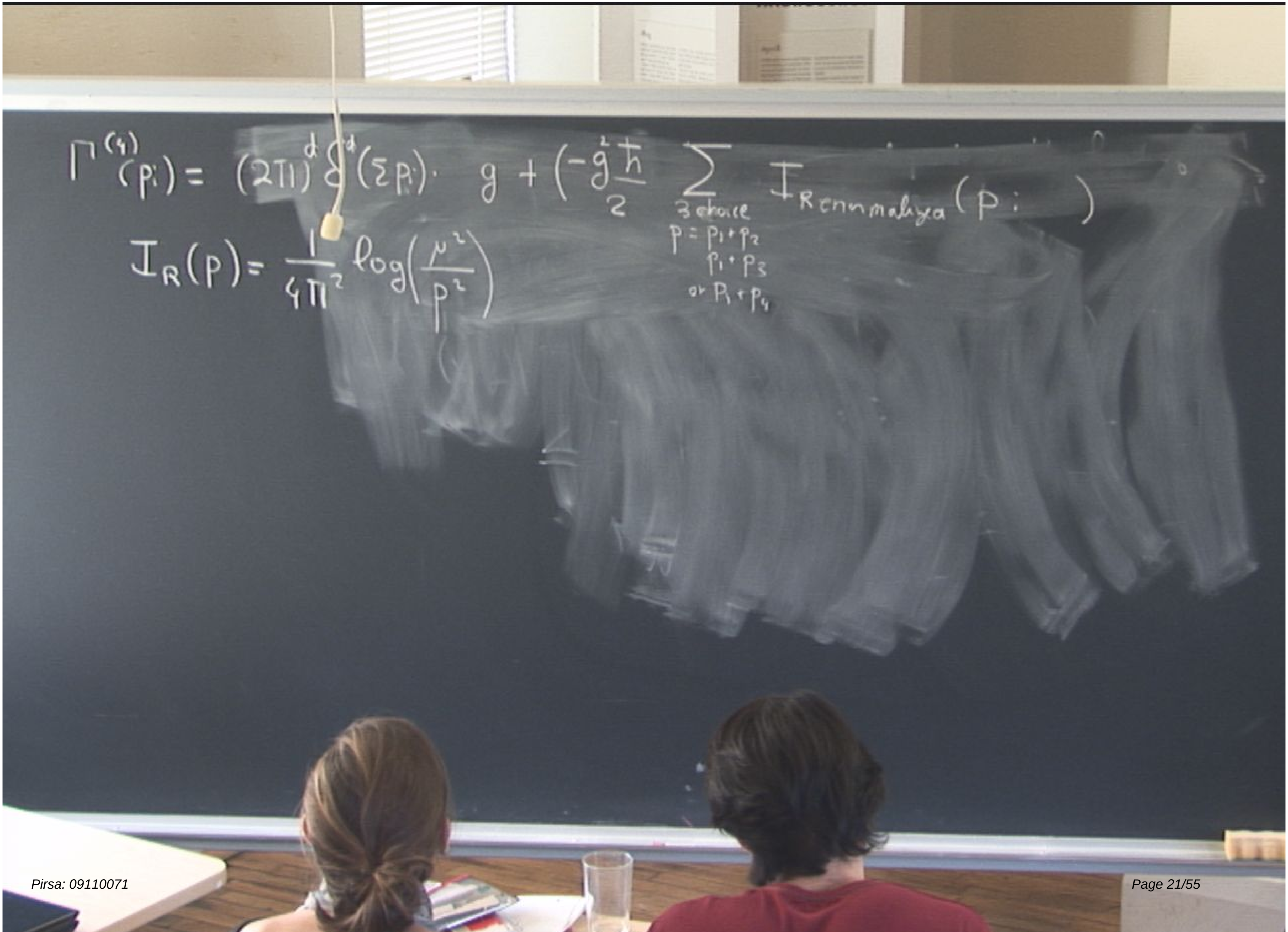
$$C_1 = \hbar g^2 \frac{1}{2} 3 \log\left(\Lambda^2 / \mu^2\right)$$

"new" mass scale μ

renormalization scale


$\Gamma^{(4)}(p_i) = (2\pi)^d \delta^d(\Sigma p_i) \cdot \ g + \left(-g^2 \frac{\hbar}{2}\right.$


$\Gamma^{(4)}(p_i) = (2\pi)^d \delta^d(\Sigma p_i) \cdot g + \left(-g^2 \frac{\hbar}{2} \sum I_{Renormalized}(p_i \quad)\right)$
3 choice
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or $p_1 + p_4$
$I_R(p) = \frac{1}{4\pi^2} \log\left(\frac{\mu^2}{p^2}\right)$

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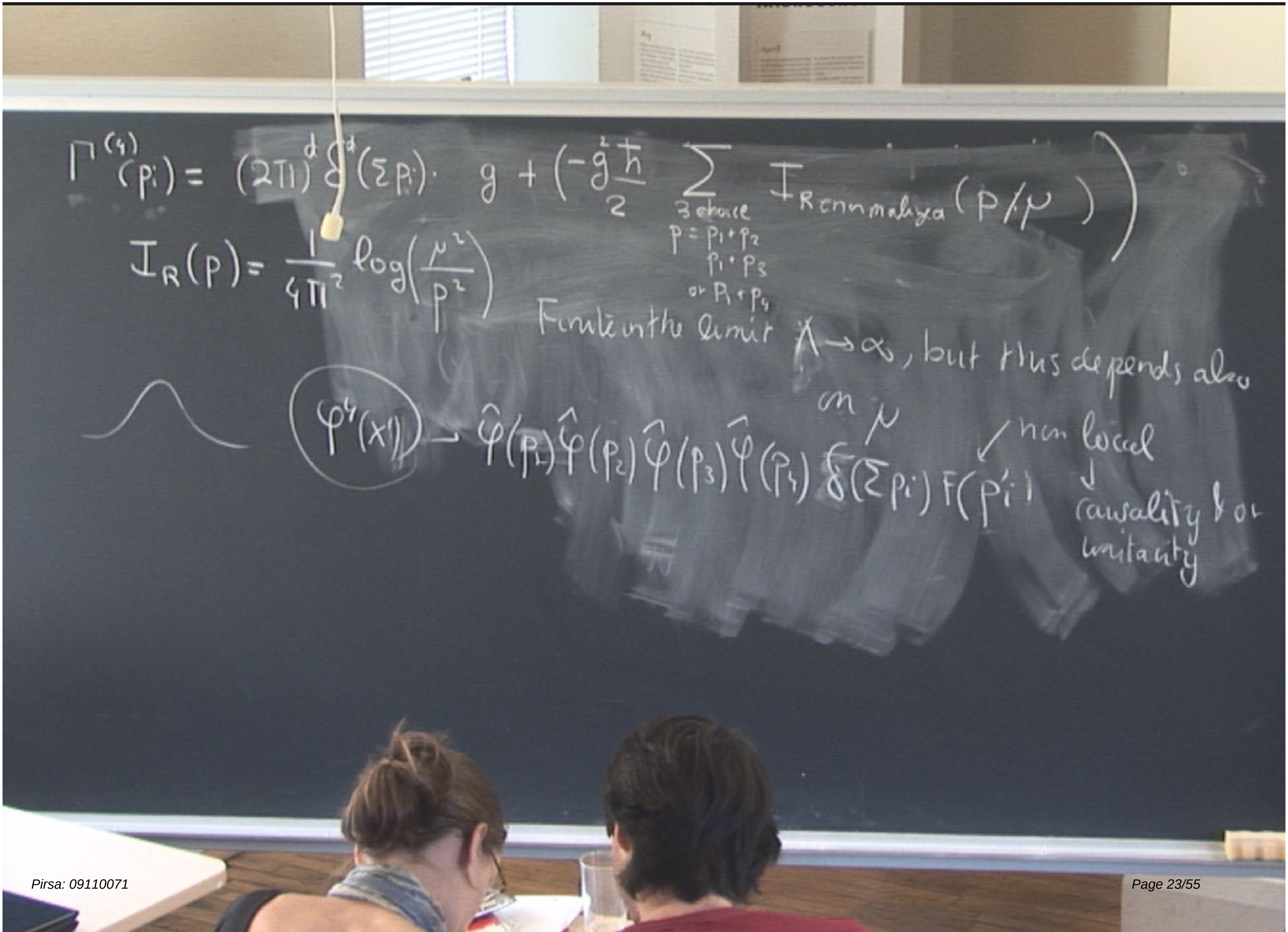
Finite in the limit $\Lambda \to \infty$, but this depends also on μ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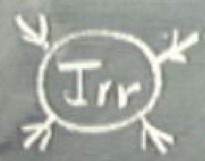
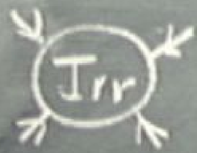
non local
causality &/or unitarity

$\Gamma^{(4)}$ = 

= 2+2 particle element of the S matrix

$\Gamma^{(4)}$ = 

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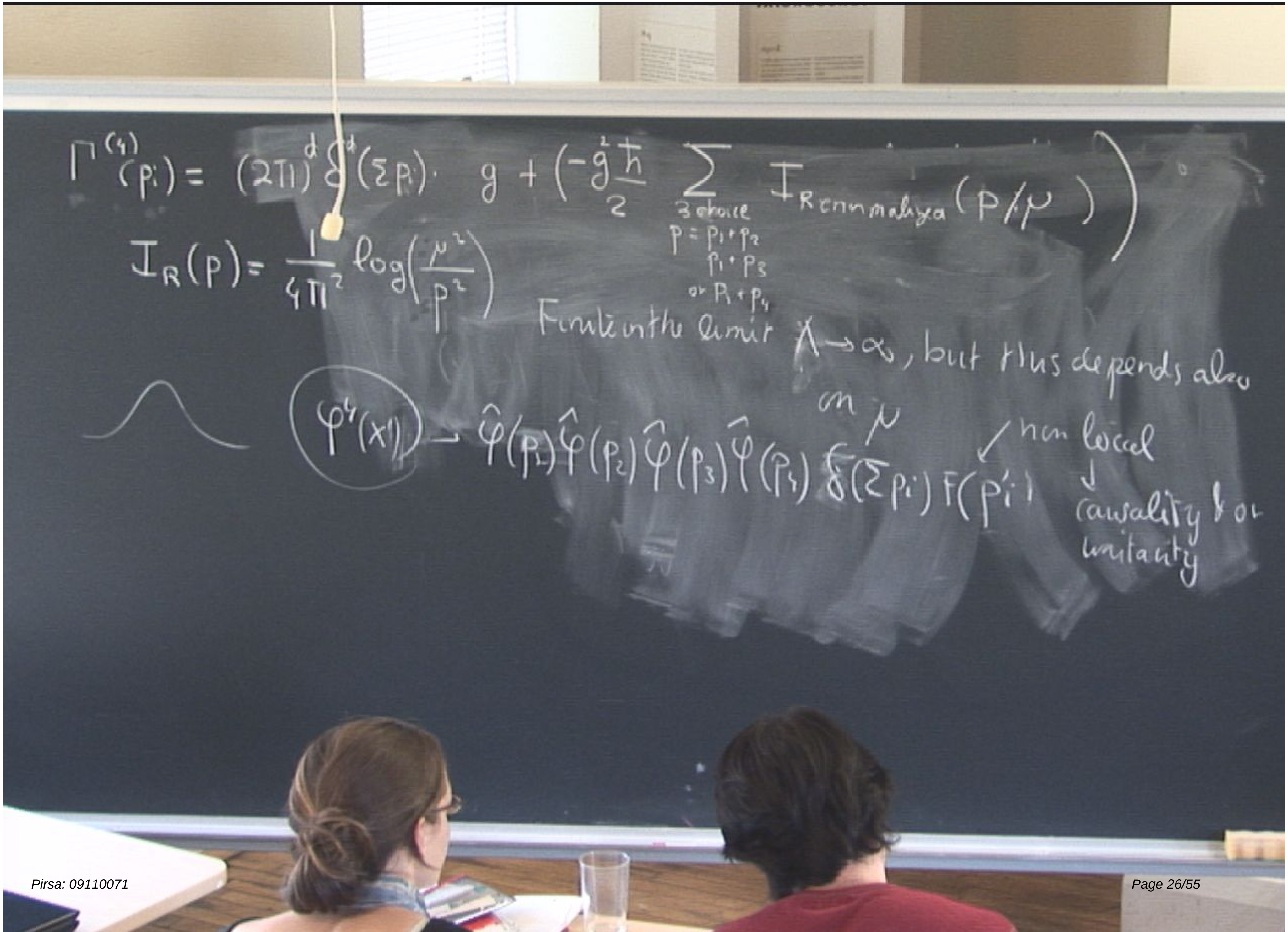
$$|p_1 + p_2| = |p_2 + p_3| = |p_1 + p_4| = \mu$$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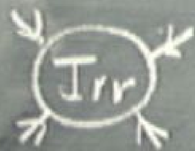
$\Gamma^{(4)} =$ 

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for this choice of momenta for the external particles

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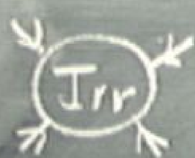
$\Gamma^{(4)} =$  $=$ 2+2 particle element of the S matrix

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on shell

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mass $= 0 = m_R$ renormalized mass

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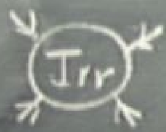
$\Gamma^{(4)}\Big/ =$  $=$ 2+2 particle element of the S matrix

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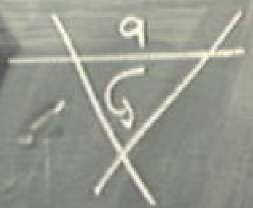
No additional UV singularities at order $\hbar$ (1 loop) ( = Finite)



$$\int_{|q|<\Lambda} d^4q \left(\frac{1}{q^2}\right)^3$$

The same theory, with the same Observables

How does the "renormalized coupling constant" depends on the energy scale at which it is measured

The same theory, with the same Observables

How does the "renormalized coupling constant" depends on the energy scale at which it is measured μ

: renormalized with a renormalization scale μ_0, cc is g_0

if $\mu \neq 0$, what is measure is $g(\mu)$

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$$g(\mu) - g(\mu)^2 \cdot \frac{\hbar}{2} \frac{3}{4\pi^2} \log(\mu^2) = g_0 - g_0^2 \cdot \frac{\hbar}{2} \frac{3}{4\pi^2} \log(\mu_0^2)$$

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write a differential equation for $g(\mu) \Rightarrow \beta$- function

Gell-Mann & Low QED

$$\mu \frac{d}{d\mu} g(\mu) := \beta_g(g(\mu))$$

β-function for the coupling g

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β function for the ϕ^4 theory

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β-function for the coupling g

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β function for the ϕ^4 theory

universal (independant of the renormalization scheme choozen.

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$$= \hbar g^2 \frac{3}{16\pi^2}$$

β function for the ϕ^4 theory

universal (independant of the renormalization scheme choozen.

There are higher loop corrections to the β-function

$$\boxed{\mu \frac{d}{d\mu} g(\mu) := \beta_g(g(\mu)) = \hbar g^2 \frac{3}{16\pi^2}}$$

β- function for the coupling g

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There are higher loop corrections to the β- function

sign of the 1 loop coefficient of the β function is > 0 for ϕ^4, & QE[D]

high energies $g(\mu) \nearrow$

low energies $g(\mu) \searrow$

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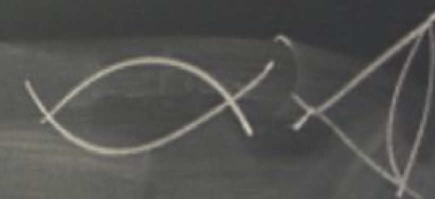
high energies $g(\mu) \nearrow$ UV asymptotic "slavery"

low energies $g(\mu) \searrow$ IR asymptotic freedom

The same theory, with the same Observables



How does the "renormalized coupling constant" depends on the energy scale at which it is measured μ

: renormalized with a renormalization scale μ_0, cc is g_0

if $\mu \neq 0$, what is measure is $g(\mu)$

$$g(\mu) - g(\mu)^2 \cdot \frac{\hbar}{2} \frac{3}{4\pi^2} \log(\mu^2) = g_0 - g_0^2 \cdot \frac{\hbar}{2} \frac{3}{4\pi^2} \log(\mu_0^2)$$

$g(\mu)$ = running coupling constant

write a differential equation for $g(\mu) \Rightarrow \beta$- function

Gell-Mann & Low QED

- Integrate: $g(\mu) = " \dfrac{g_0}{1 - g_0 \cdot \frac{3}{16 \cdot \pi^2} \cdot \log\left(\mu / \mu_0\right)} "$

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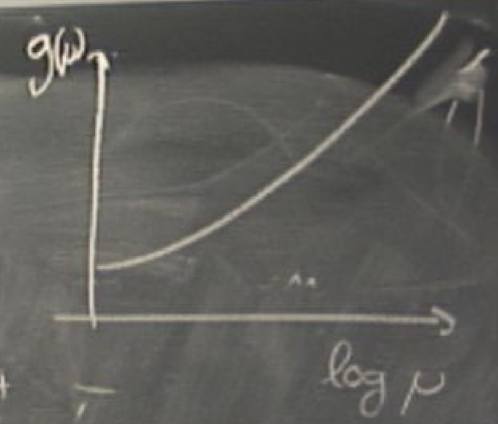
$$= g_0 + g_0^2 (\;) \log + g_0^3 (\;) \log^2 + \dots$$



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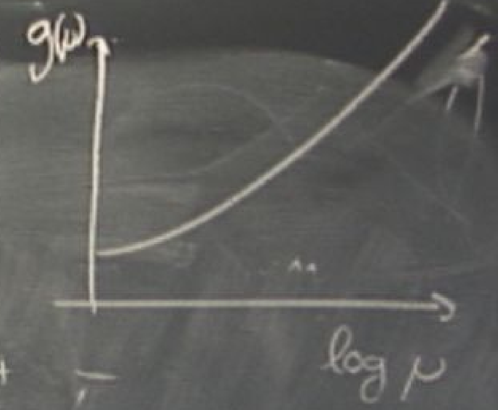

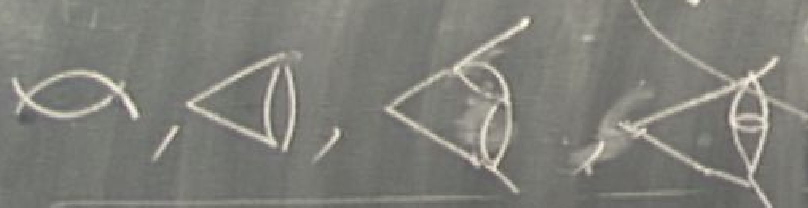

$$\boxed{\mu = \mu_0 \exp\left(\frac{16\pi^2}{3 \cdot g_0 \hbar}\right)},$$

there is a pole

$g(\mu) \nearrow \infty$

perturbation theory breaks down!

- Integrate : $$\boxed{g(\mu) = \frac{g_0}{1 - g_0 \hbar \frac{3}{16\pi^2} \log(\mu/\mu_0)}}$$

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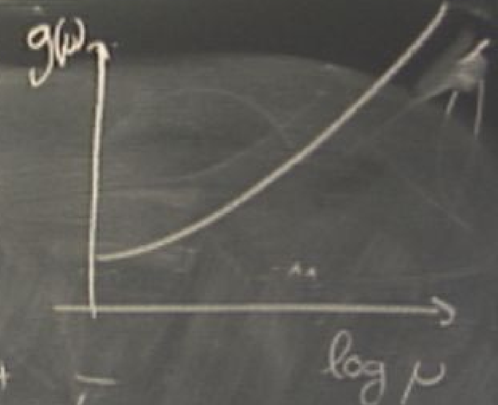
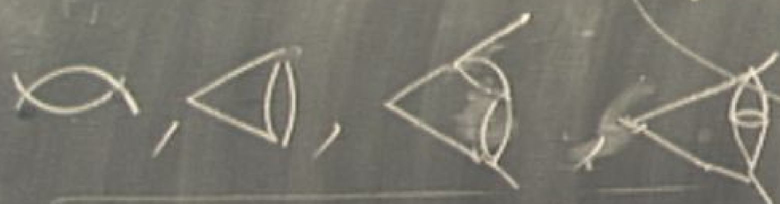




there is a pole (landau)

$$\boxed{\mu_{LP} = \mu_0 \exp\left(\frac{16\pi^2}{3 \cdot g_0 \hbar}\right)}, \quad g(\mu) \nearrow \infty$$

perturbation theory breaks down !

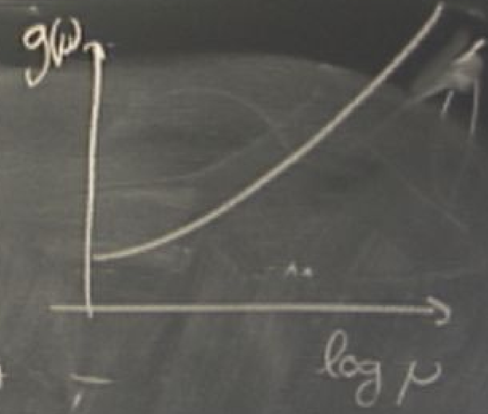
QED. Landau Pole

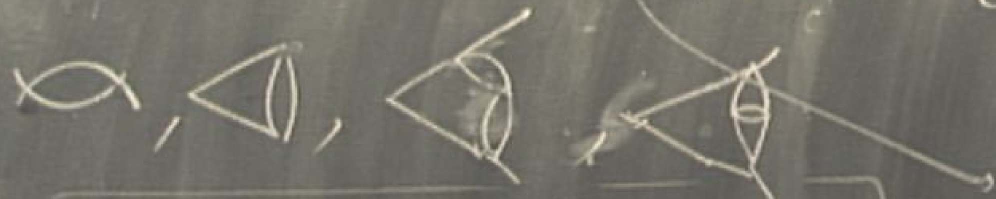
Problem for the Higgs Sector

Unitarity ?

Integrate: $\boxed{g(\mu) \text{ "=" } \frac{g_0}{1 - g_0 \hbar \frac{3}{16 \cdot \pi^2} \log\left(\mu/\mu_0\right)}}$

$= g_0 + g_0^2(\;)\log + g_0^3(\;)\log^2 + \dots$





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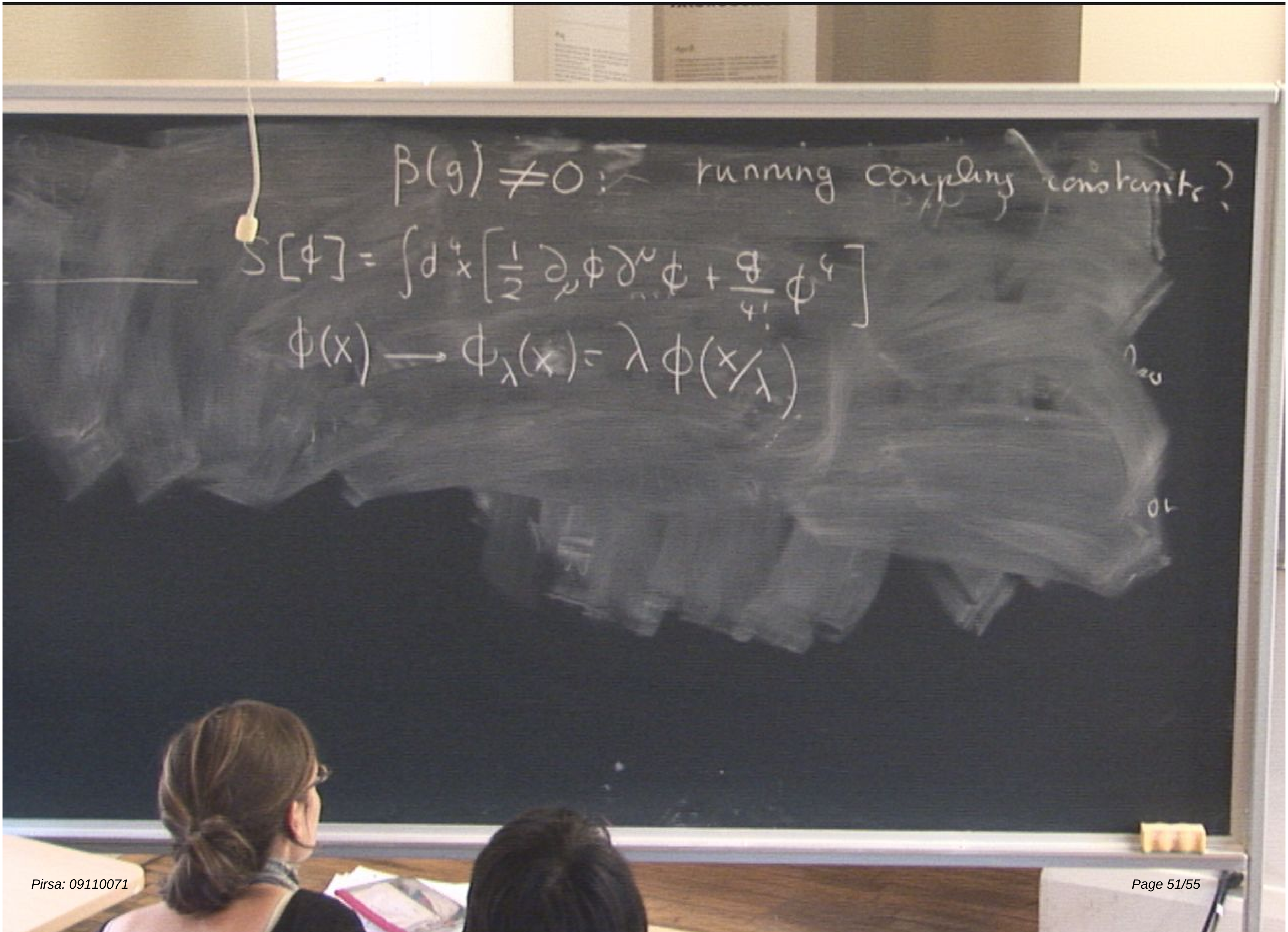
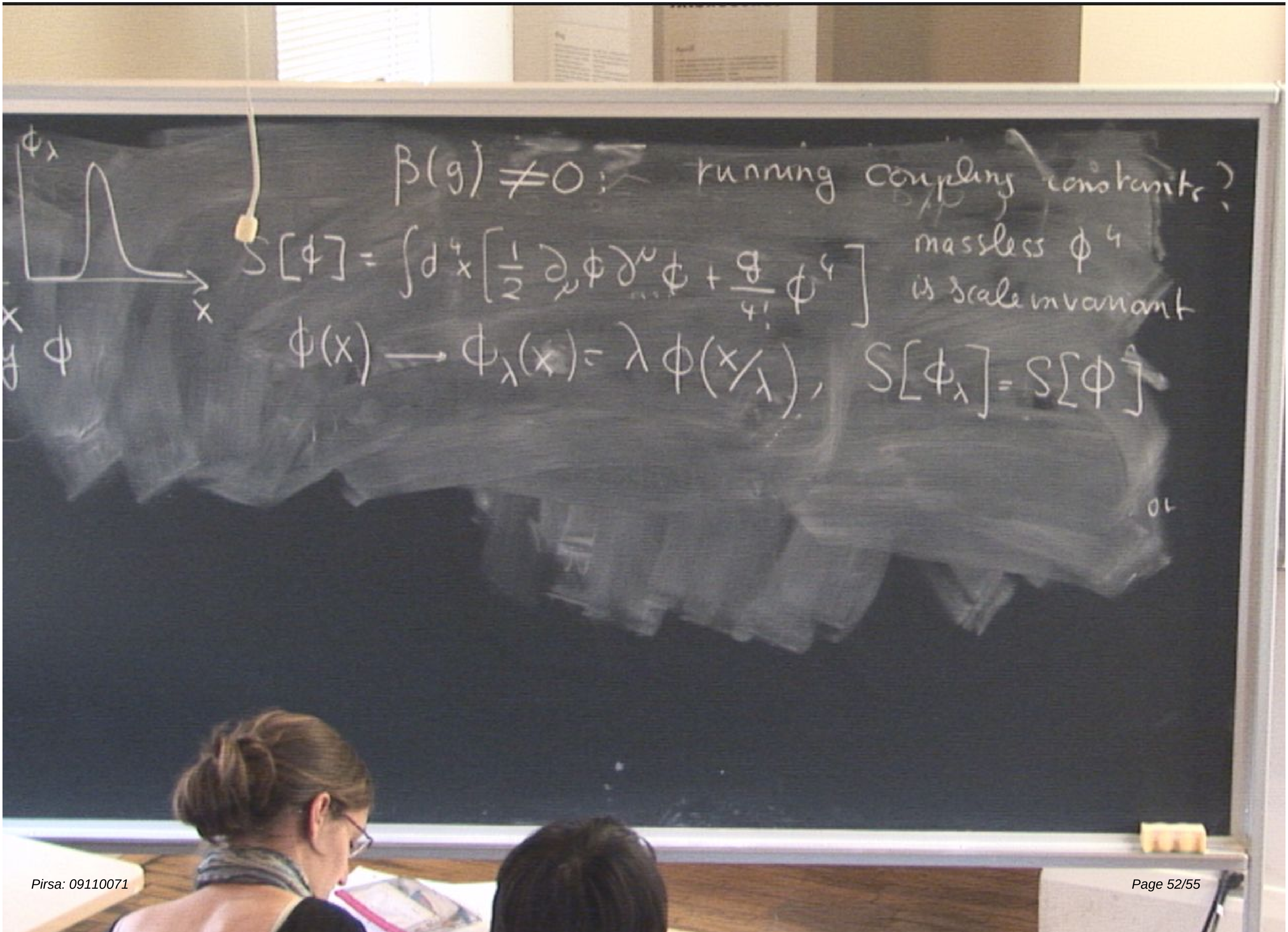
QED. Landau Pole

Problem for the Higgs Sector $\downarrow$

Unitarity ?

Gauge theories QCD $\beta < 0$


$\beta(g) \neq 0$: running coupling constants?
$S[\phi] = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{g}{4!} \phi^4 \right]$
$\phi(x) \longrightarrow \phi_\lambda(x) = \lambda \phi(x/\lambda)$


ϕ_λ
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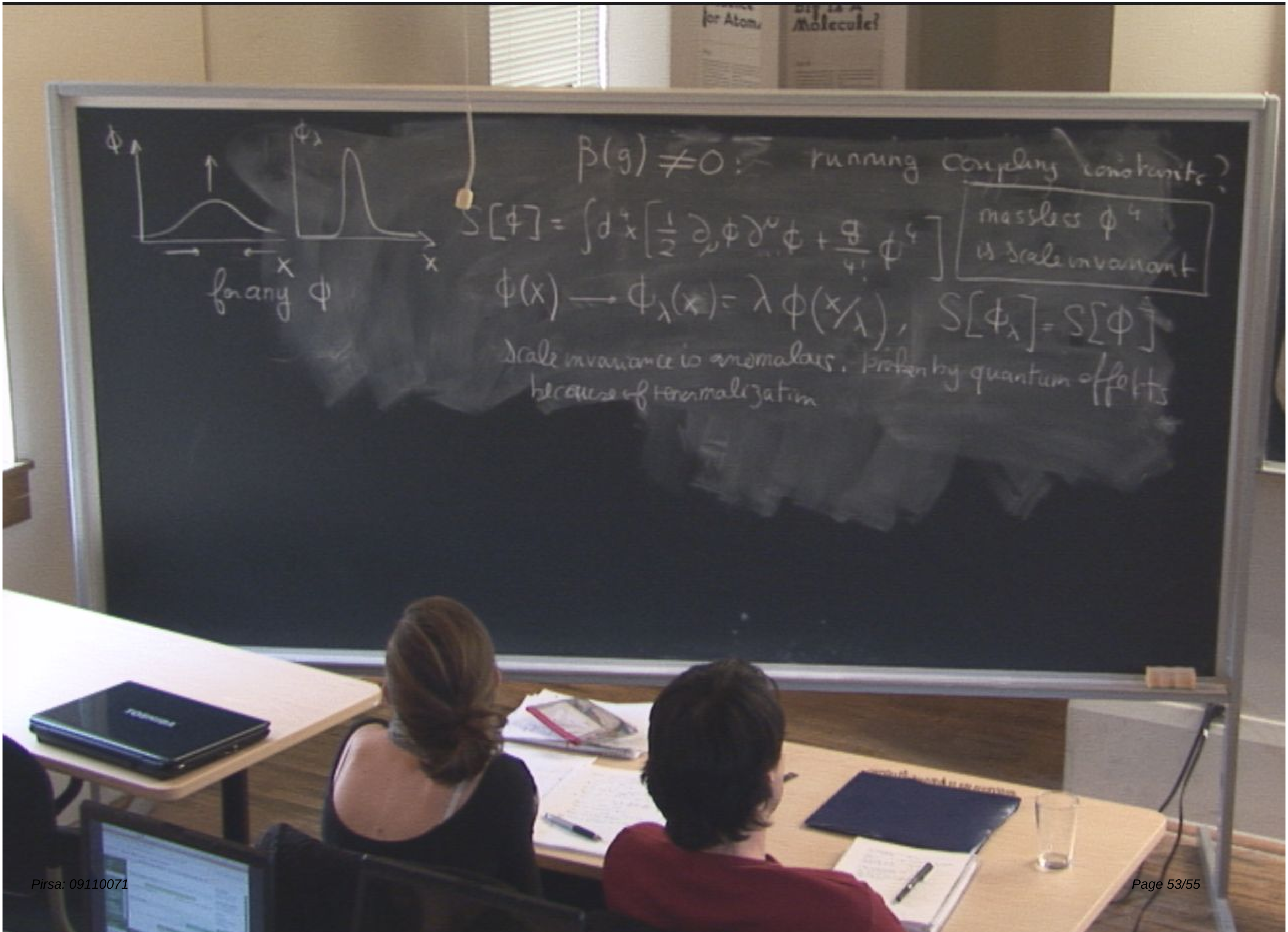
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$\phi(x) \longrightarrow \phi_\lambda(x) = \lambda\,\phi(x/\lambda)$, $S[\phi_\lambda] = S[\phi]$
for any ϕ
Scale invariance is anomalous. Broken by quantum effects
because of renormalization

$$\boxed{\mu \frac{d}{d\mu} g(\mu) := \beta_g(g(\mu)) = \hbar\, g^3(\mu) \frac{3}{16\pi^2}}$$

Symmetry $\Longrightarrow$ Conserved current

$$\partial_\mu J^\mu_{\text{Scale}}(x) = 0$$

$$\boxed{\mu \frac{d}{d\mu} g(\mu) := \beta_g(g(\mu)) = \hbar g^2(\mu) \frac{3}{16\pi^2}}$$

Symmetry $\Longrightarrow$ Conserved current

$$\partial_\mu J^\mu_{\text{Scale}}(x) = 0 + \hbar \cdot \beta_g(g)\, \phi^4(x)$$