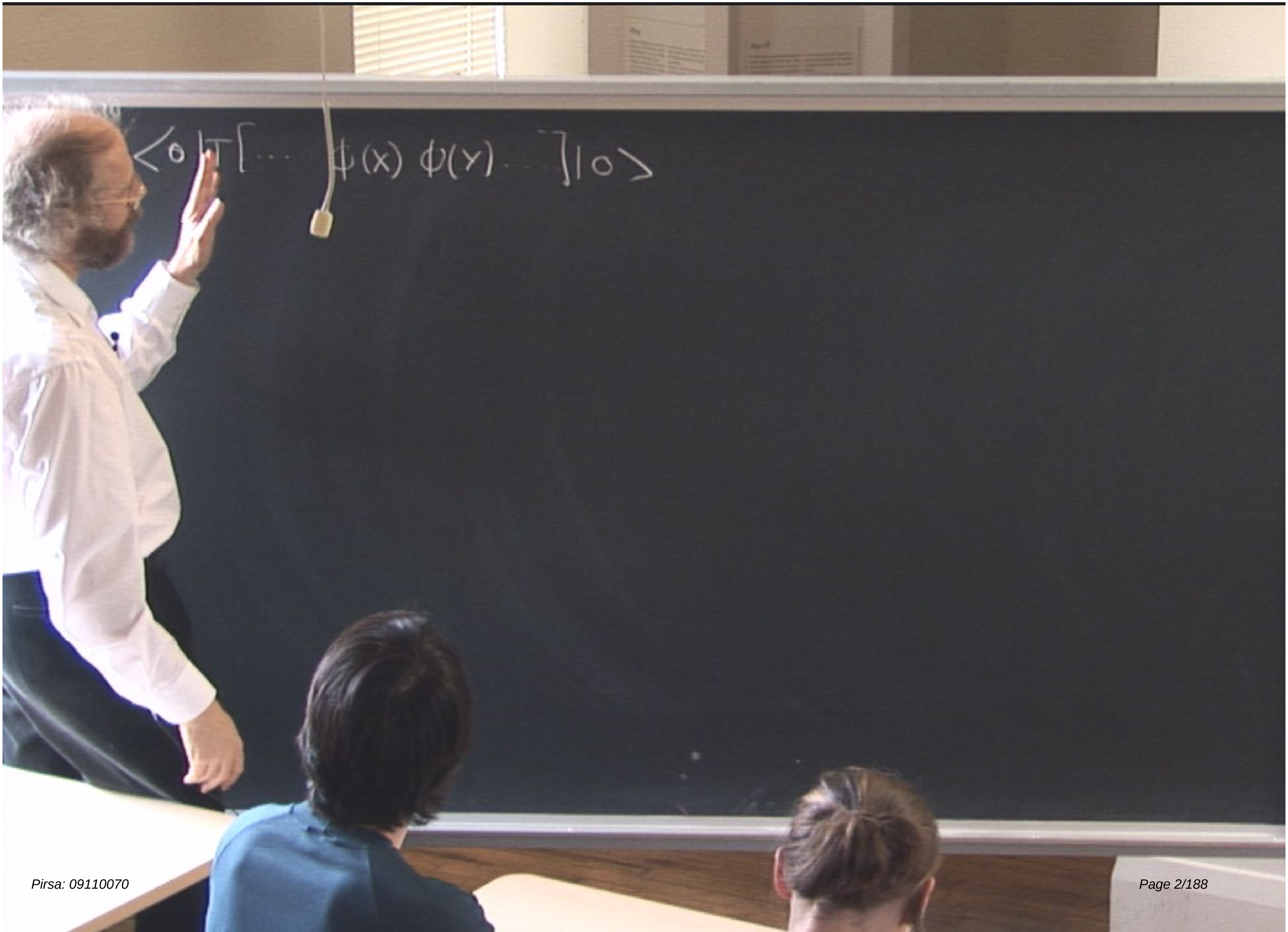
Title: Quantum Field Theory II (PHYS 603) - Lecture 8

Date: Nov 04, 2009 09:00 AM

URL: http://pirsa.org/09110070

Abstract: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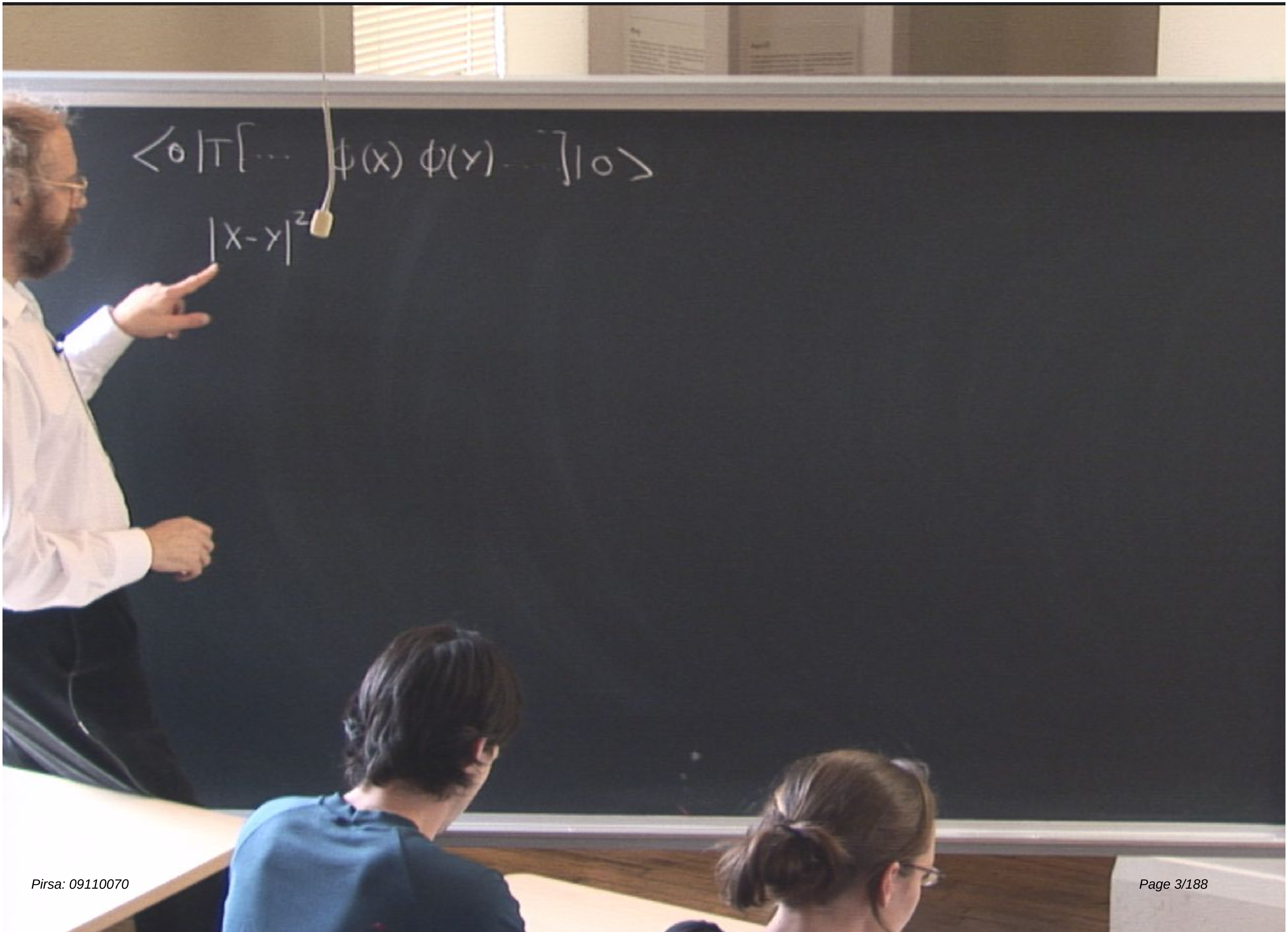
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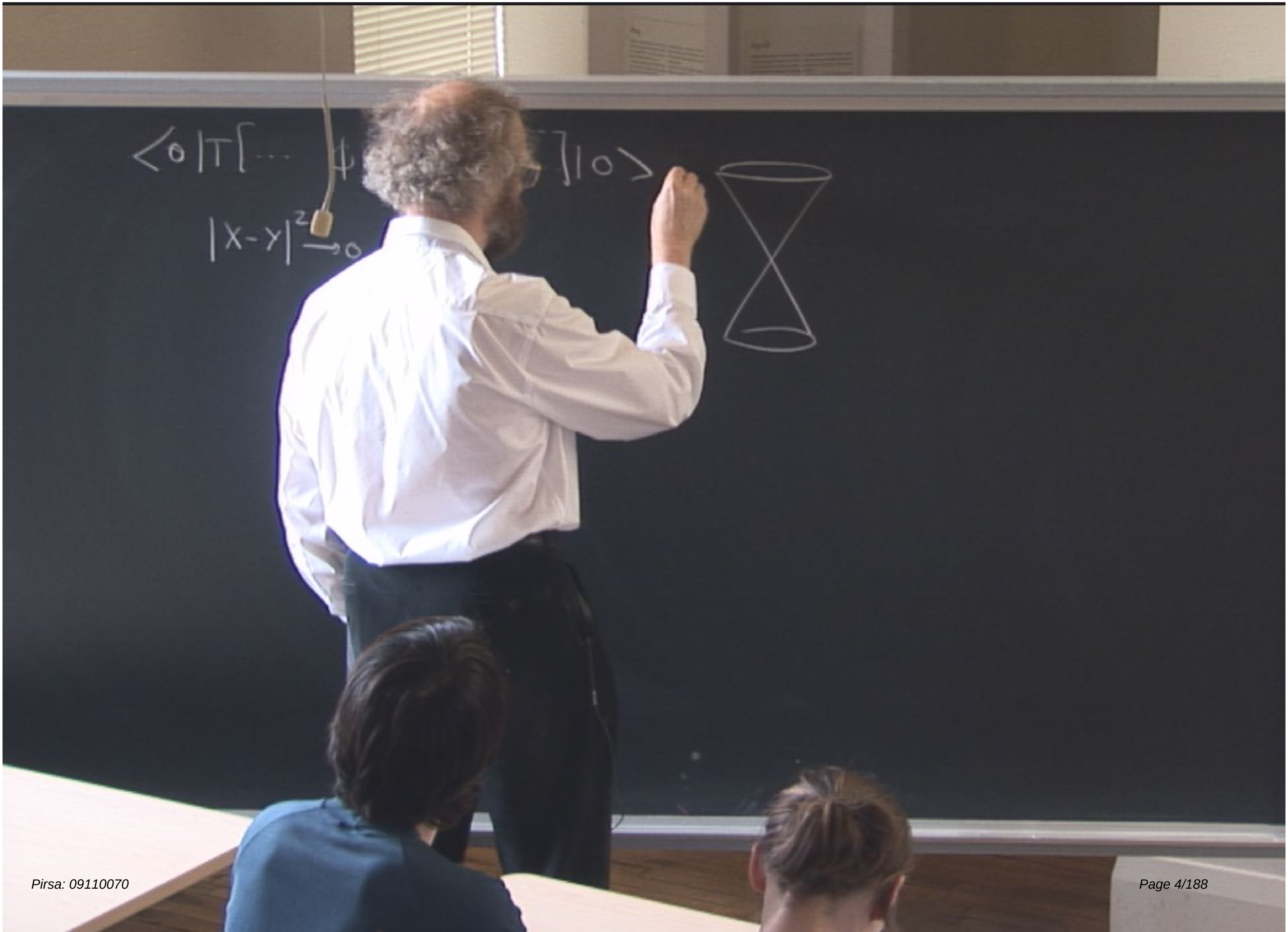
|x-y|²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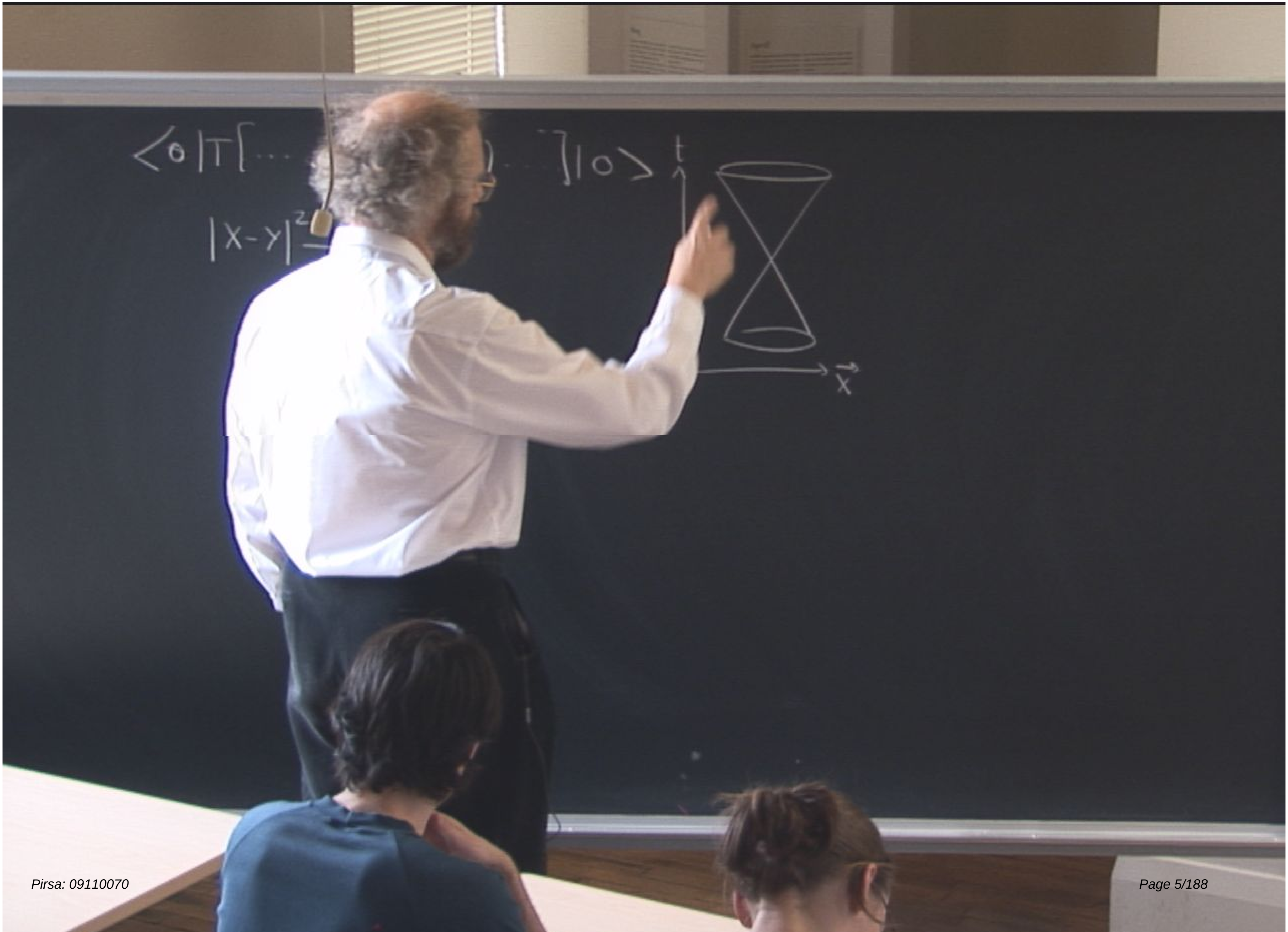
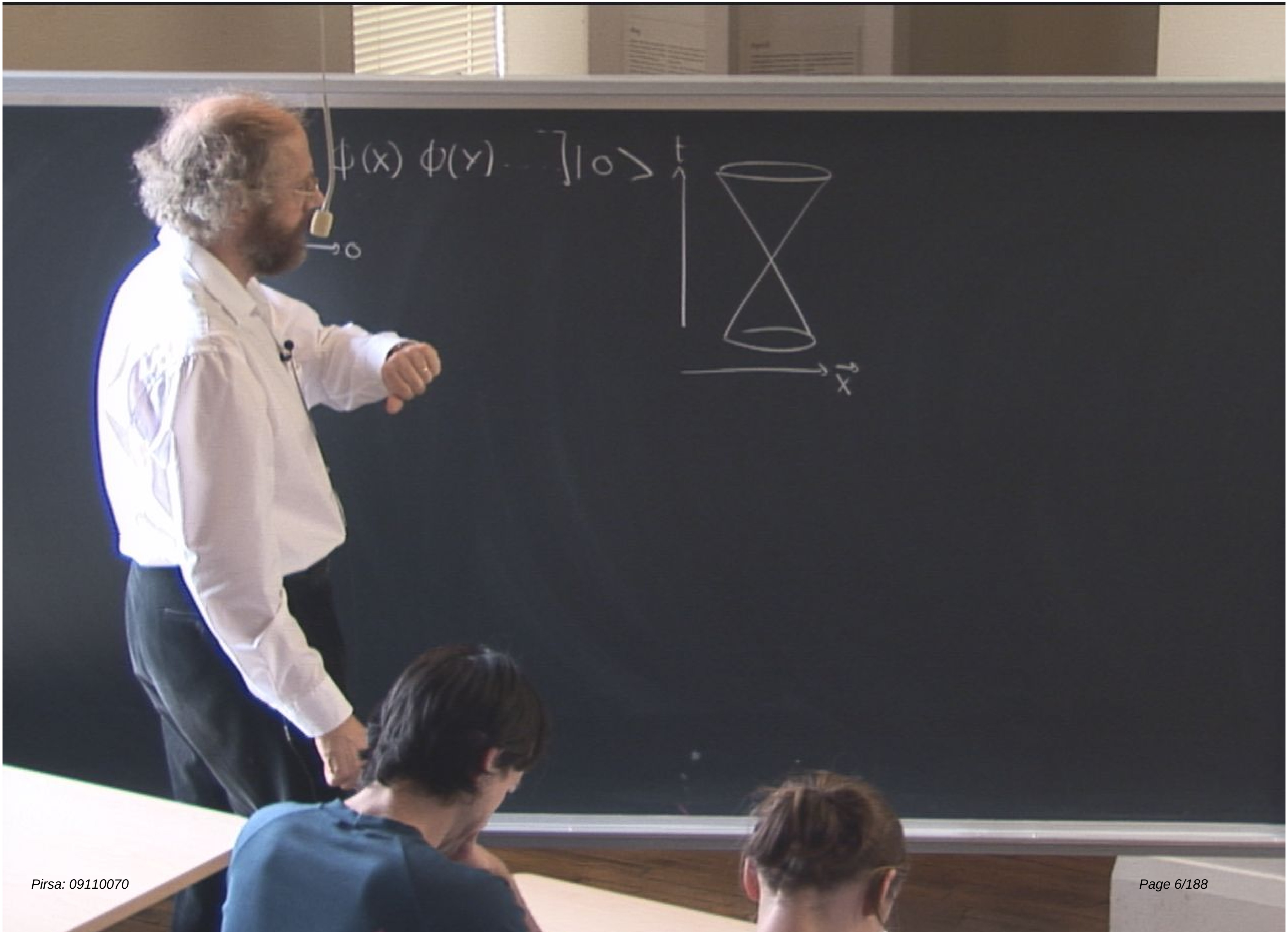
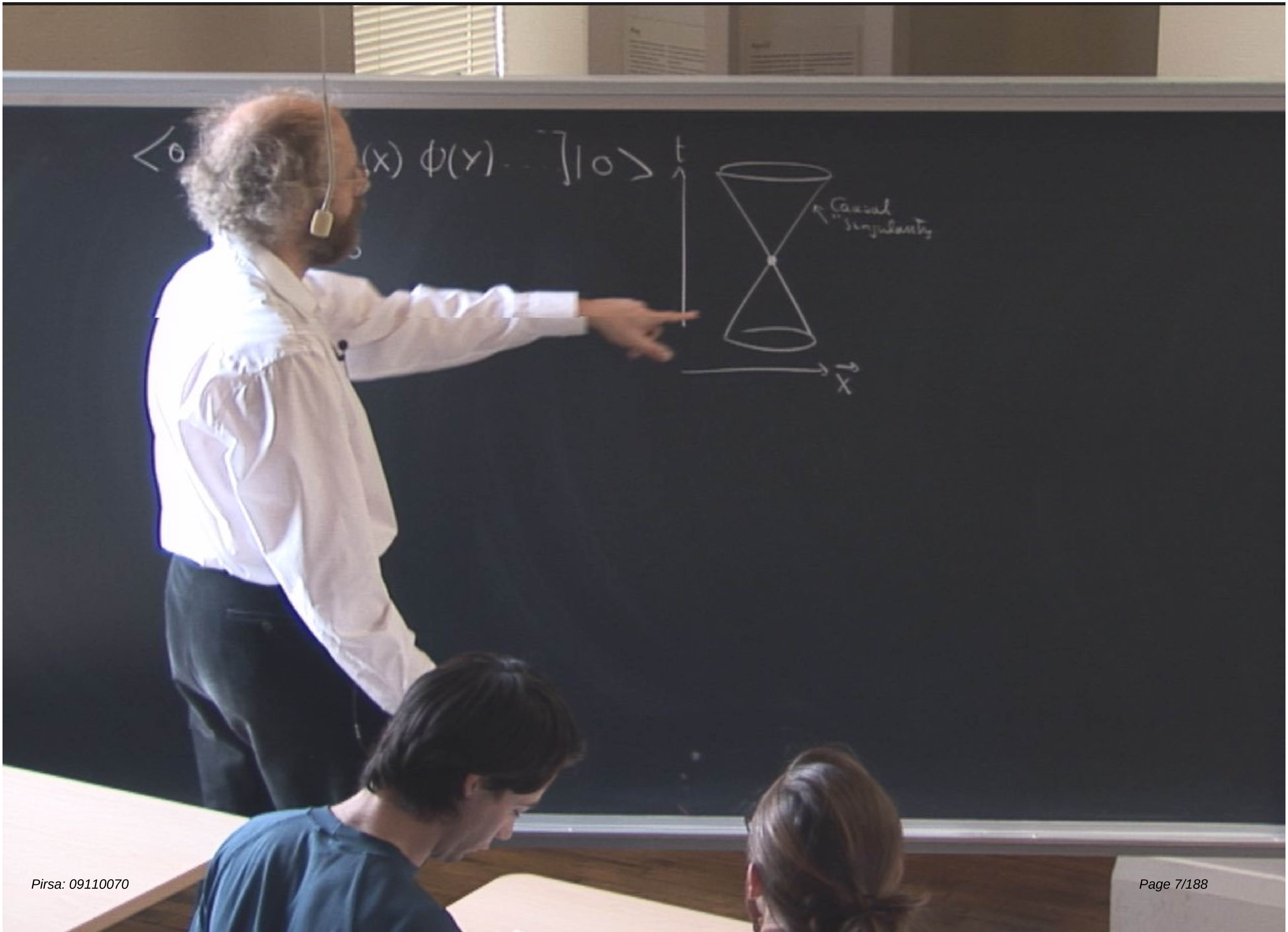
]|0>
|X-Y|² → 0

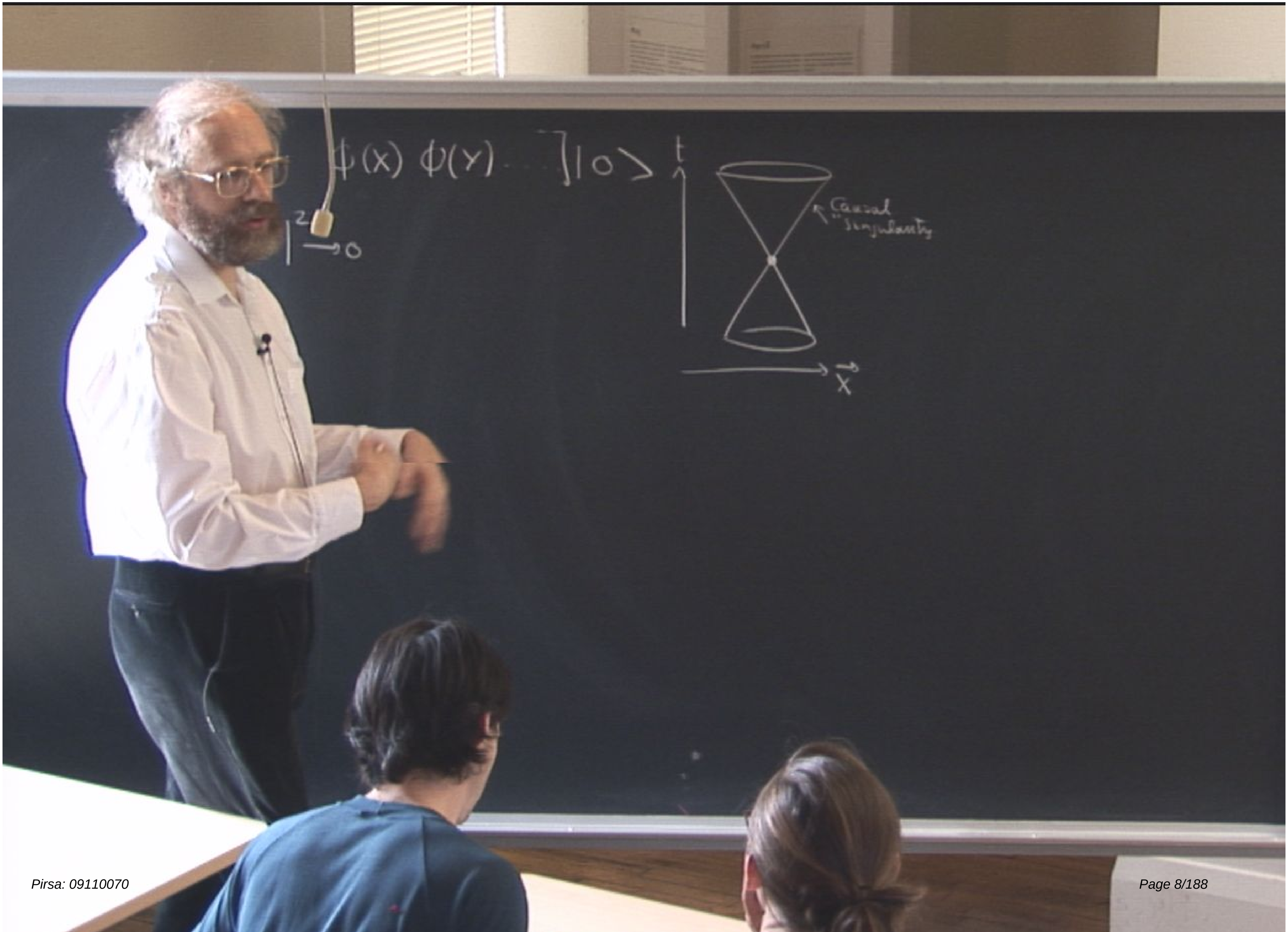

<0|T[...
...]|0>
|x-y|²
t
x⃗



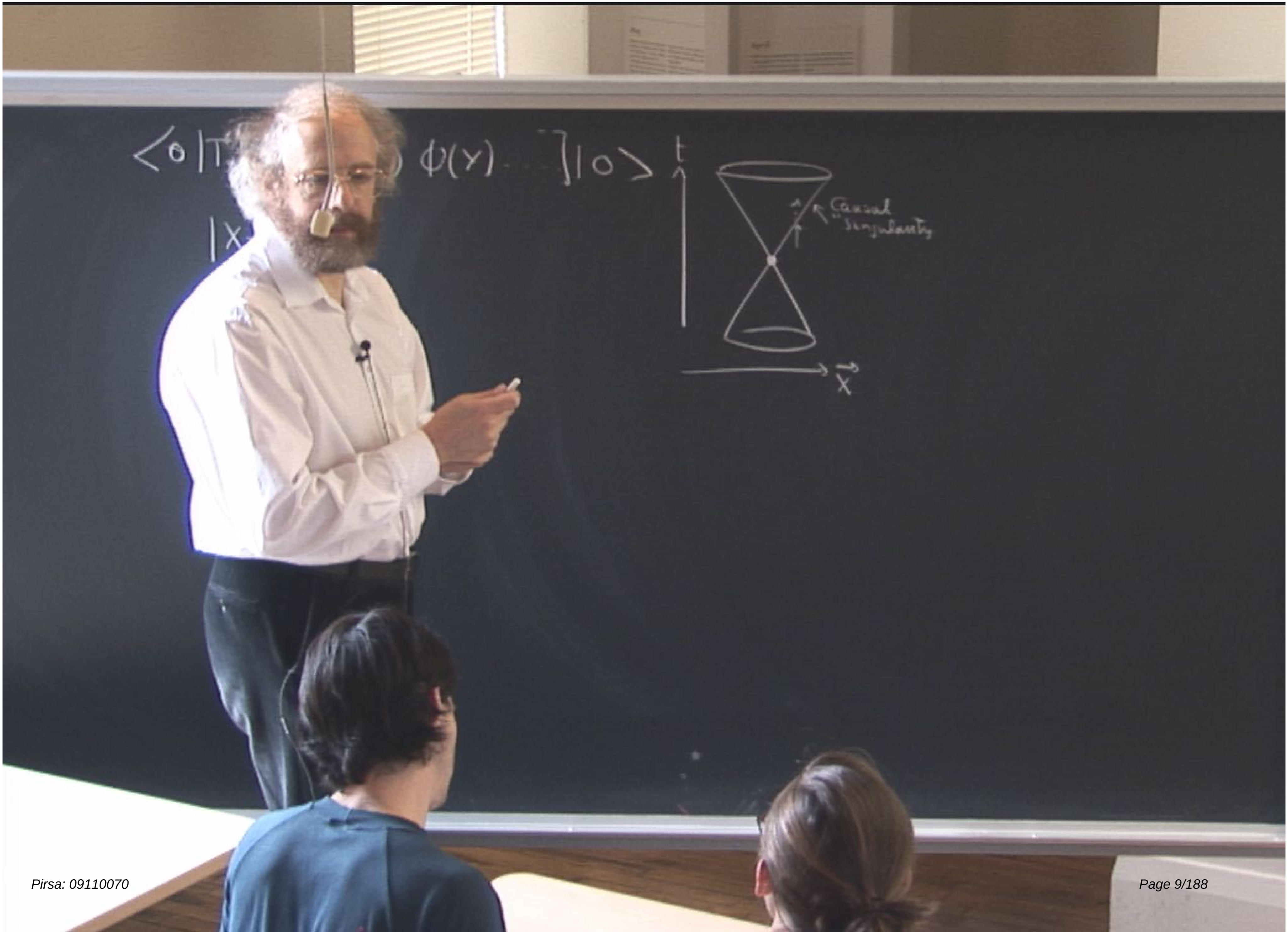
$\phi(x)\ \phi(y) \ldots]\,|0\rangle$
t
$\vec{x}$

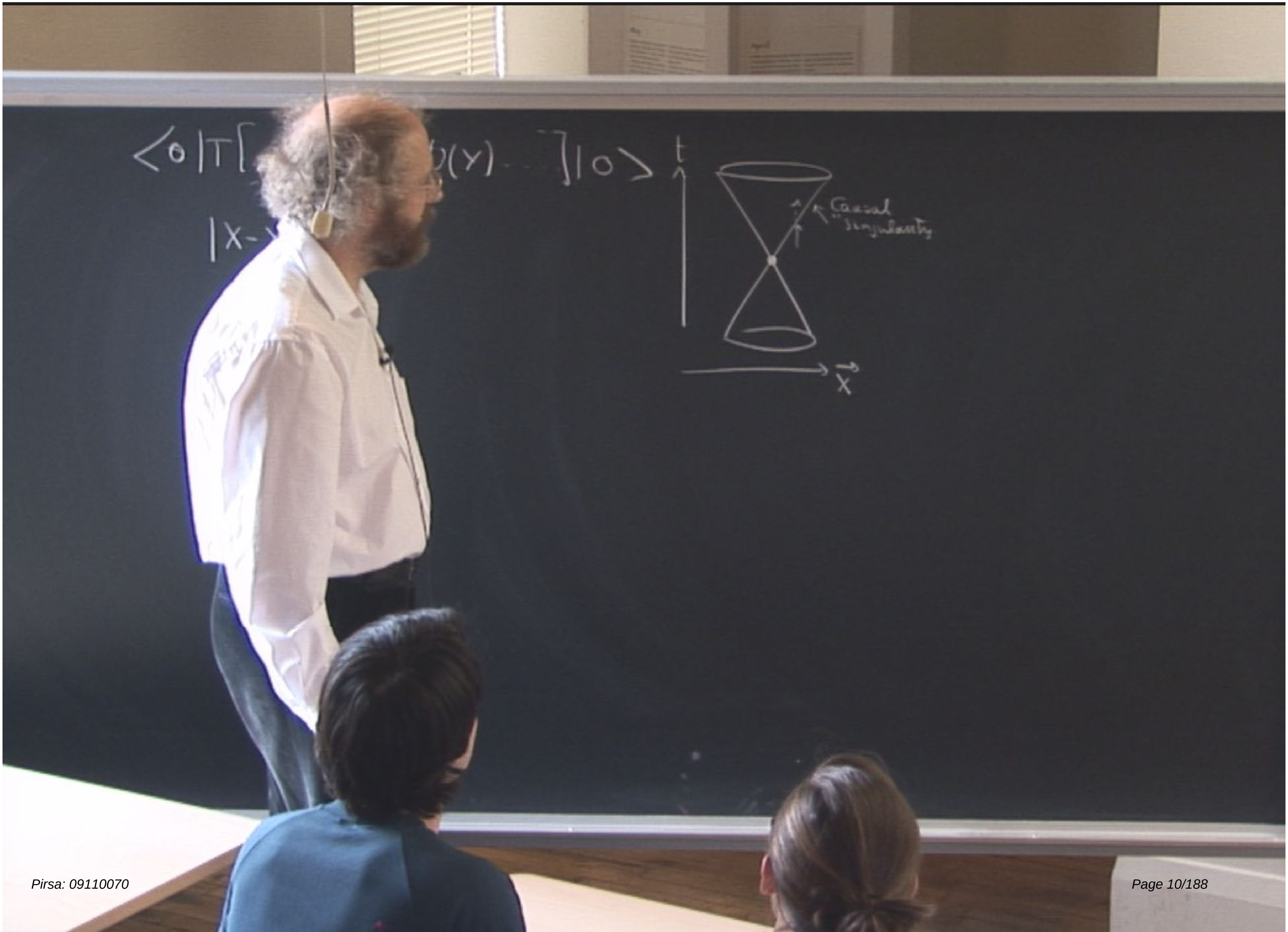


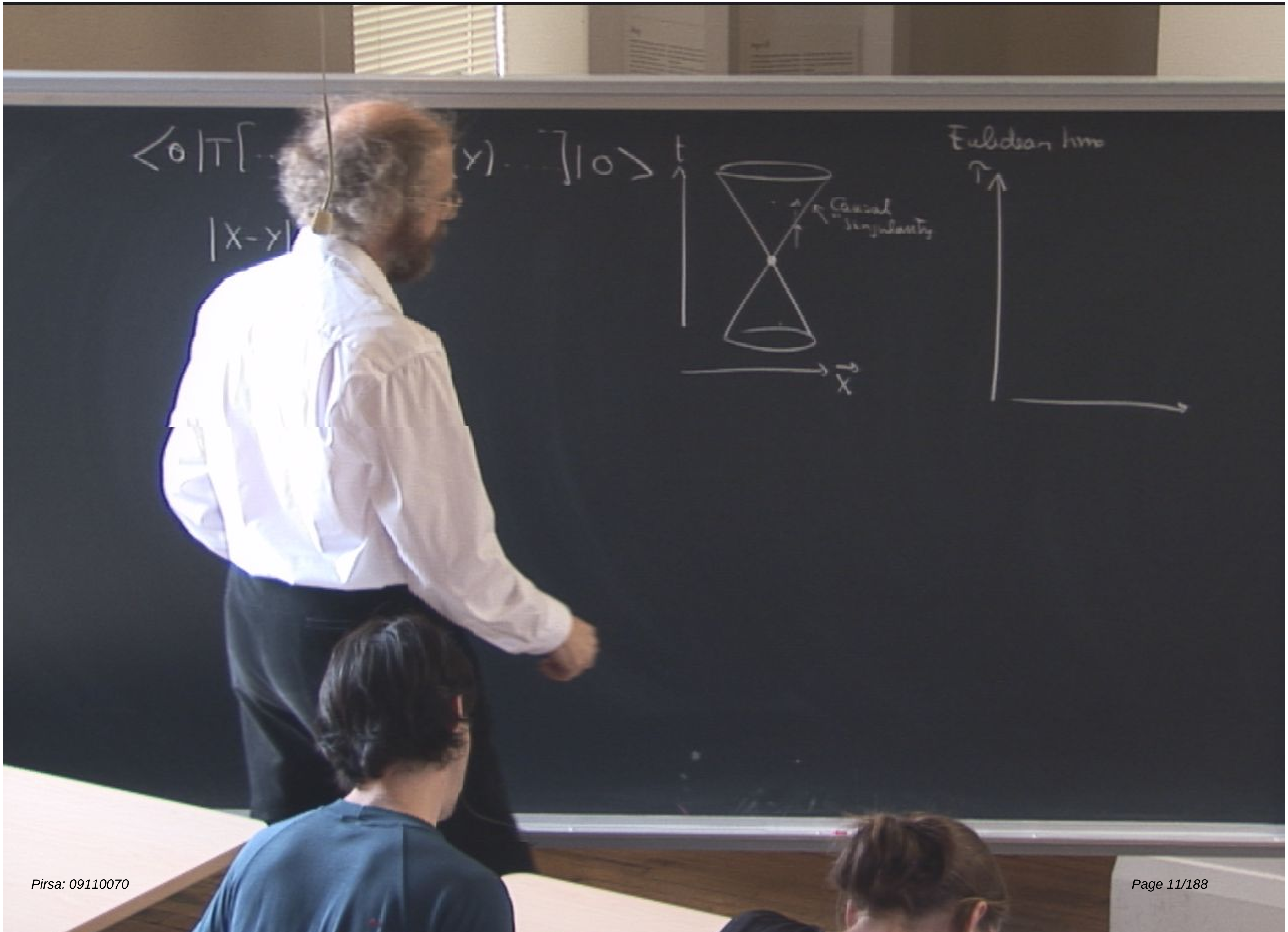
Causal
"Singularity



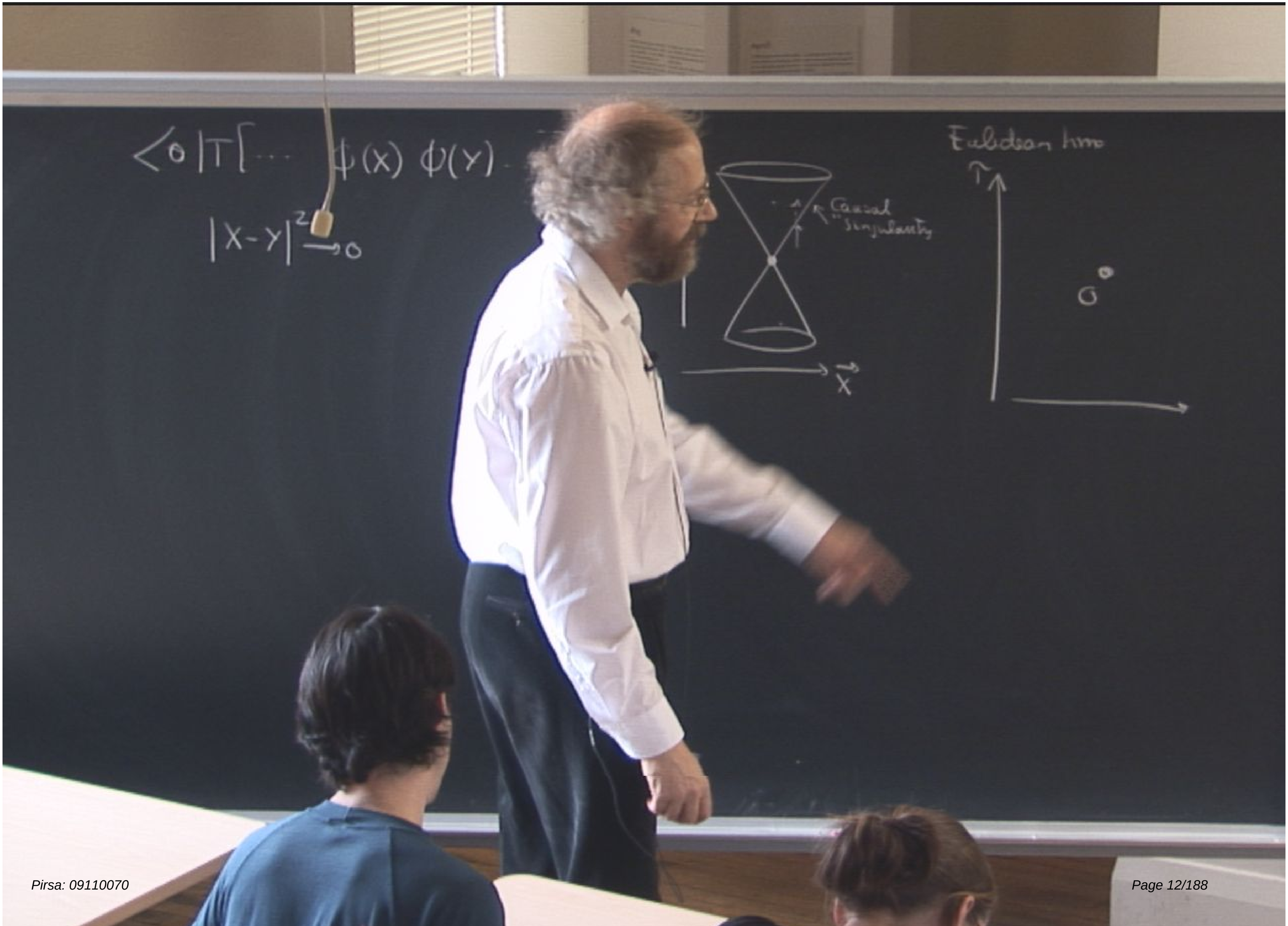
$\phi(x)\ \phi(y) \cdots]|0\rangle$
t
Causal
"Singularity
$\vec{x}$


t
x
Causal
"Singularity"

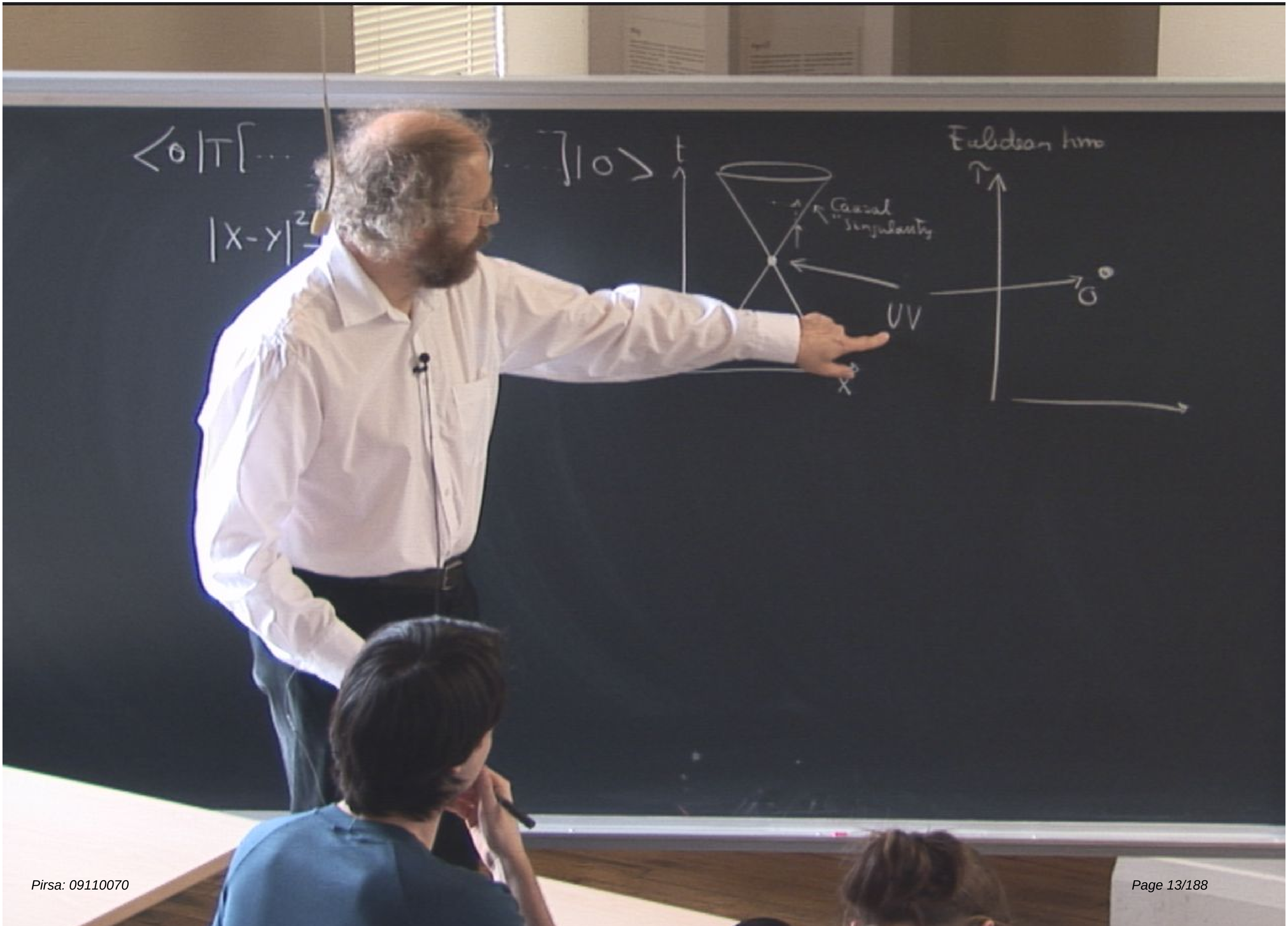

t
Causal
"Singularity"
x

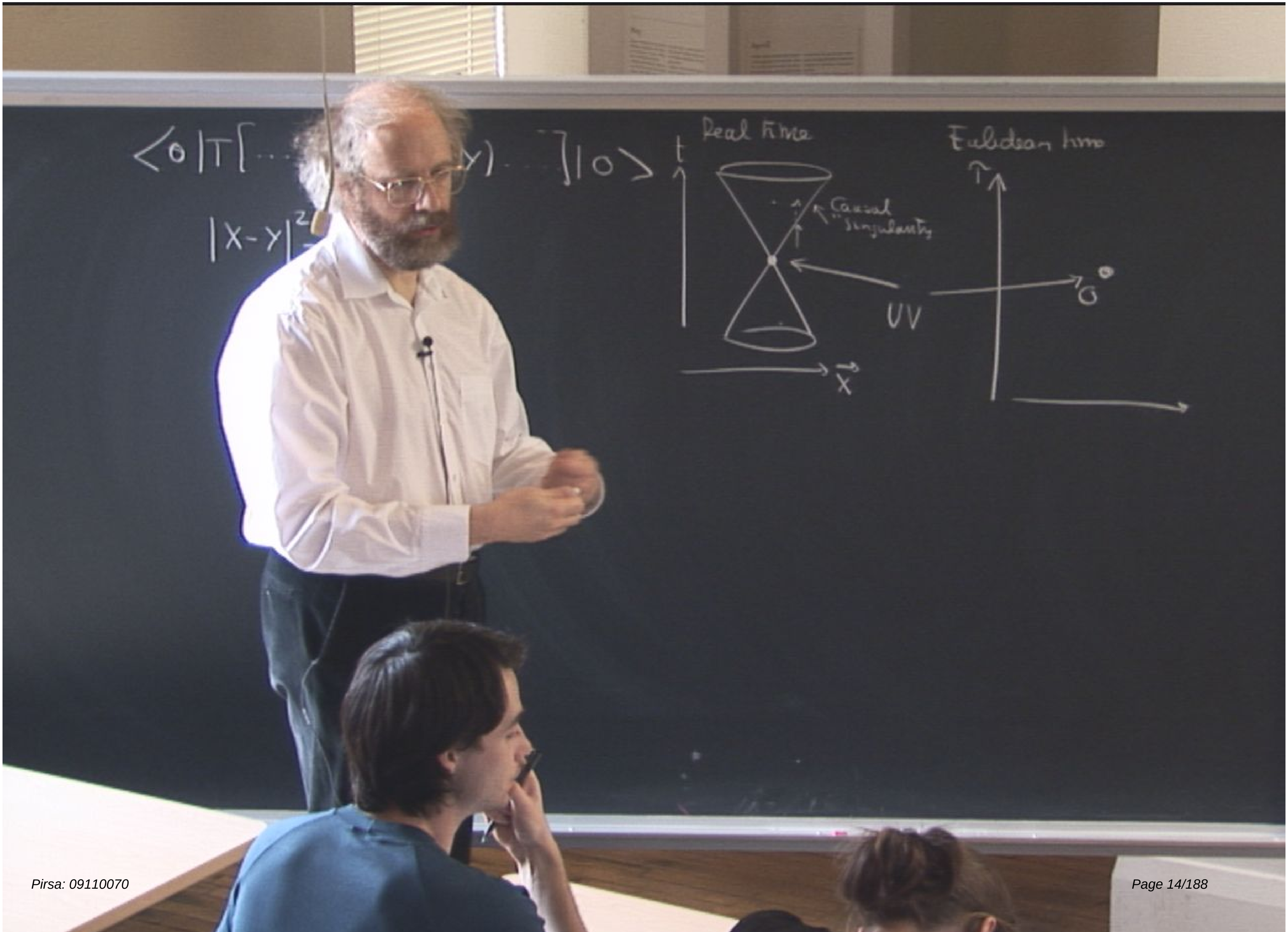


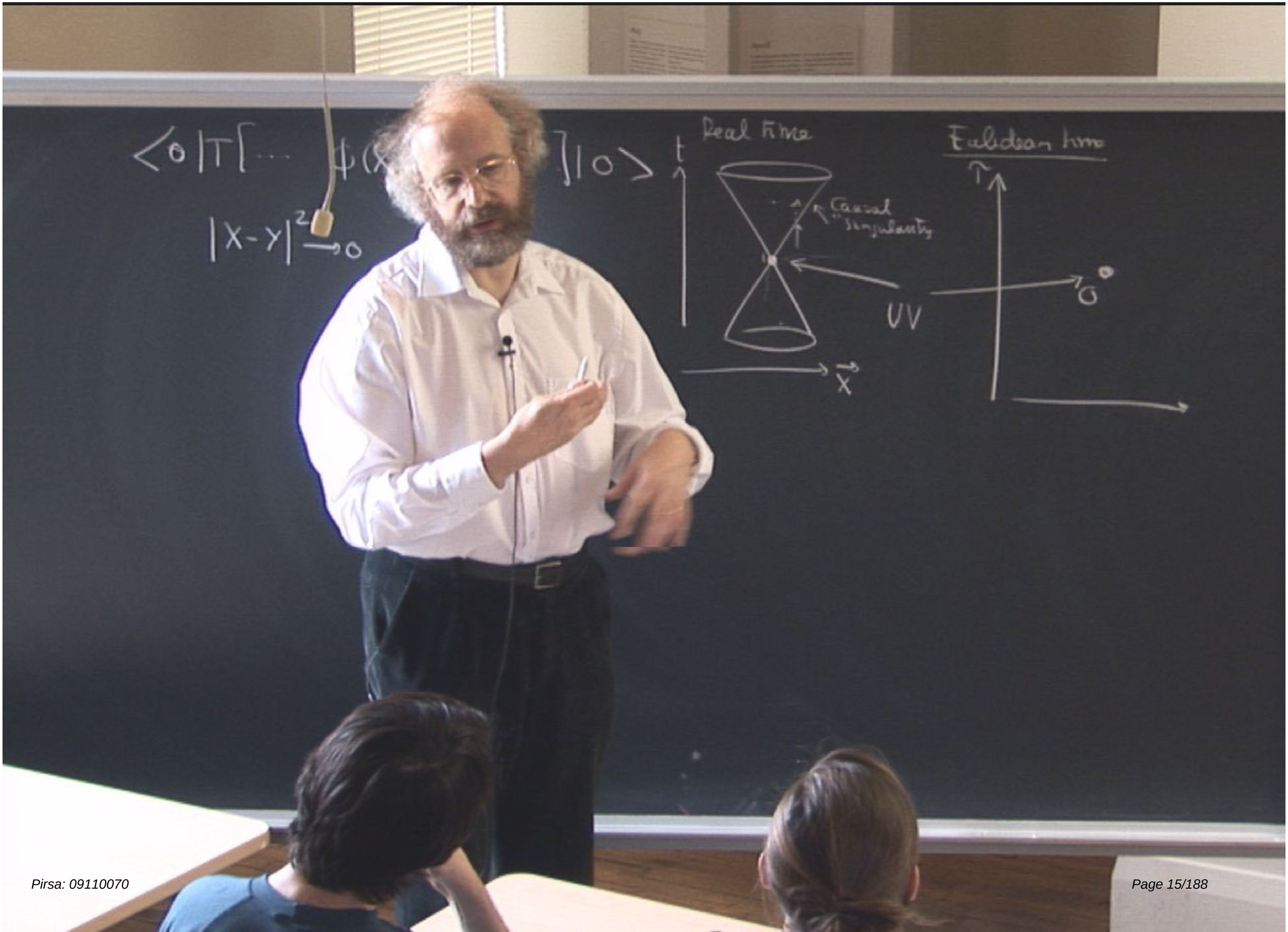
|X-y|
t
Causal
Singularity
Euclidean time

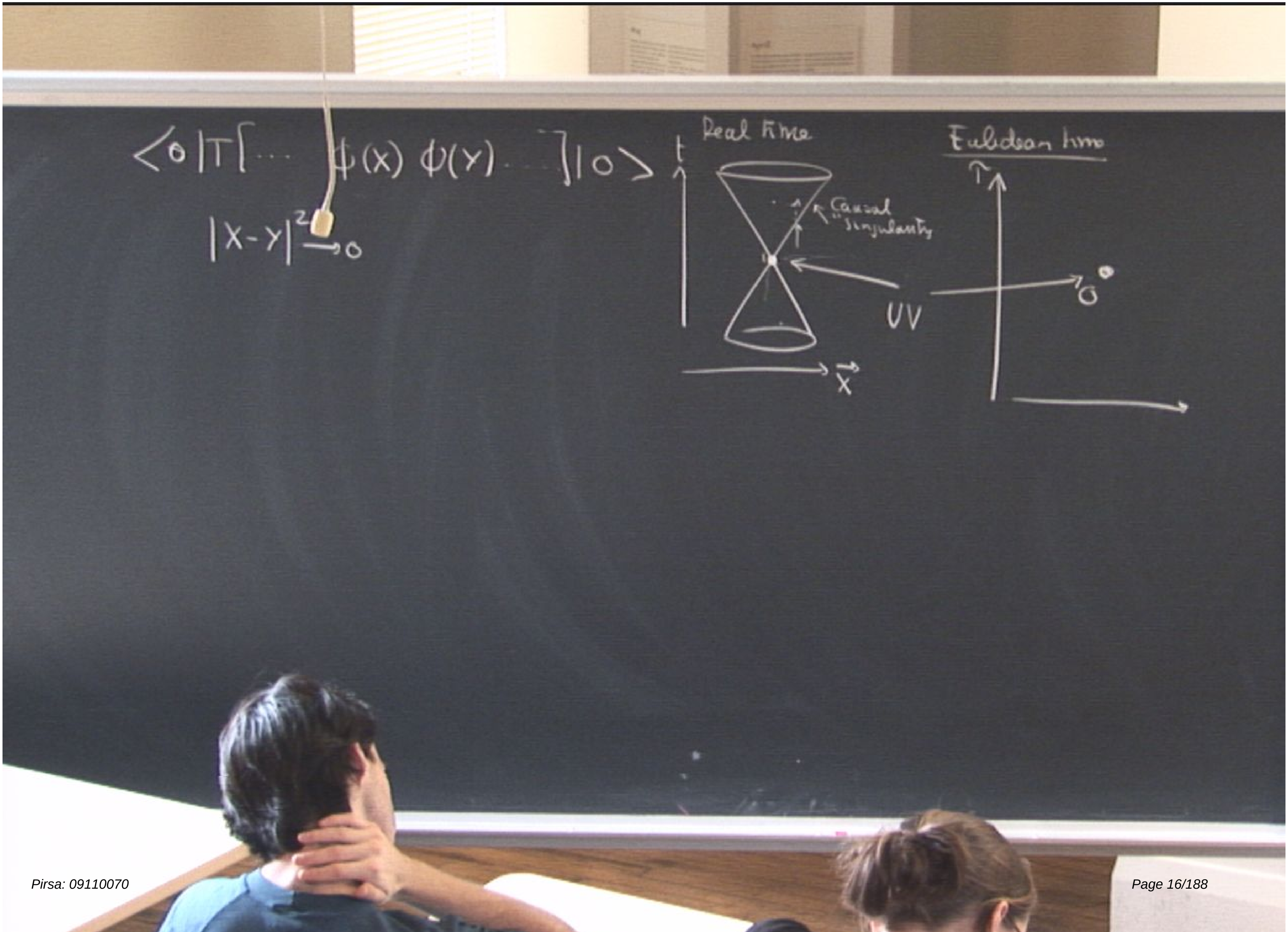


<0|T[... φ(x) φ(y)
|x-y|² → 0
Causal
Singularity
Euclidean time

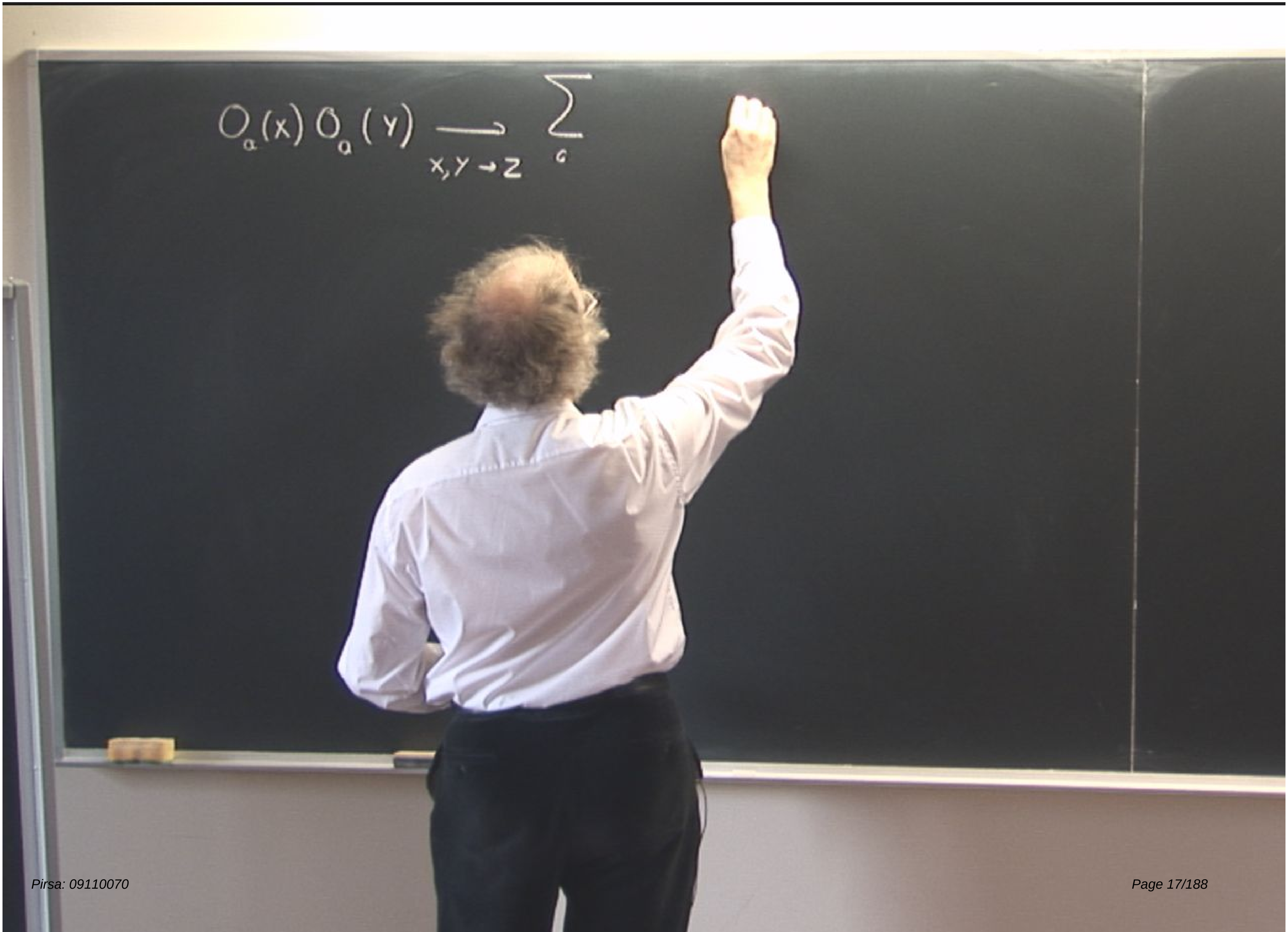

Causal
UV

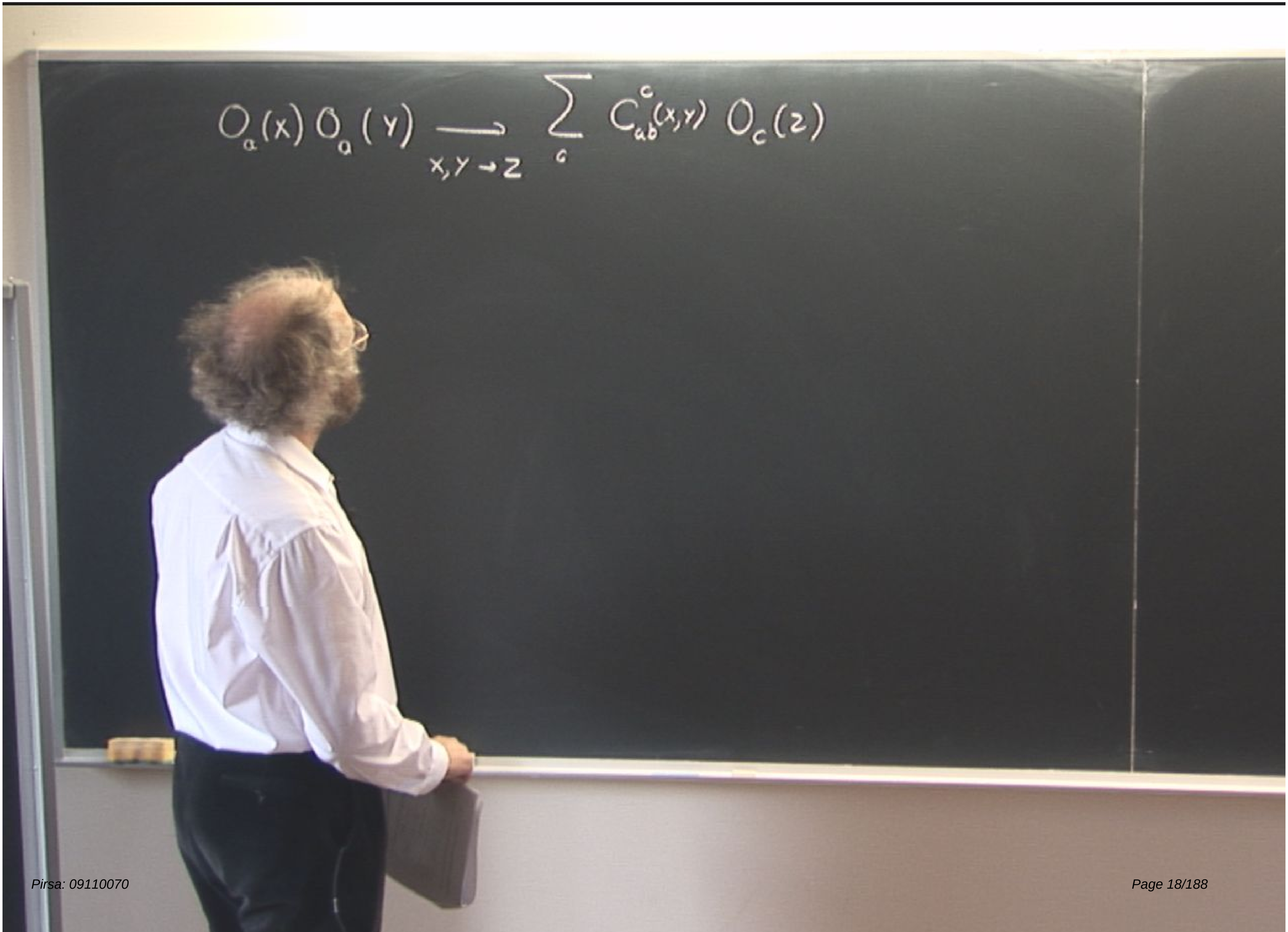

Real time
Causal
singularity
UV
Euclidean time

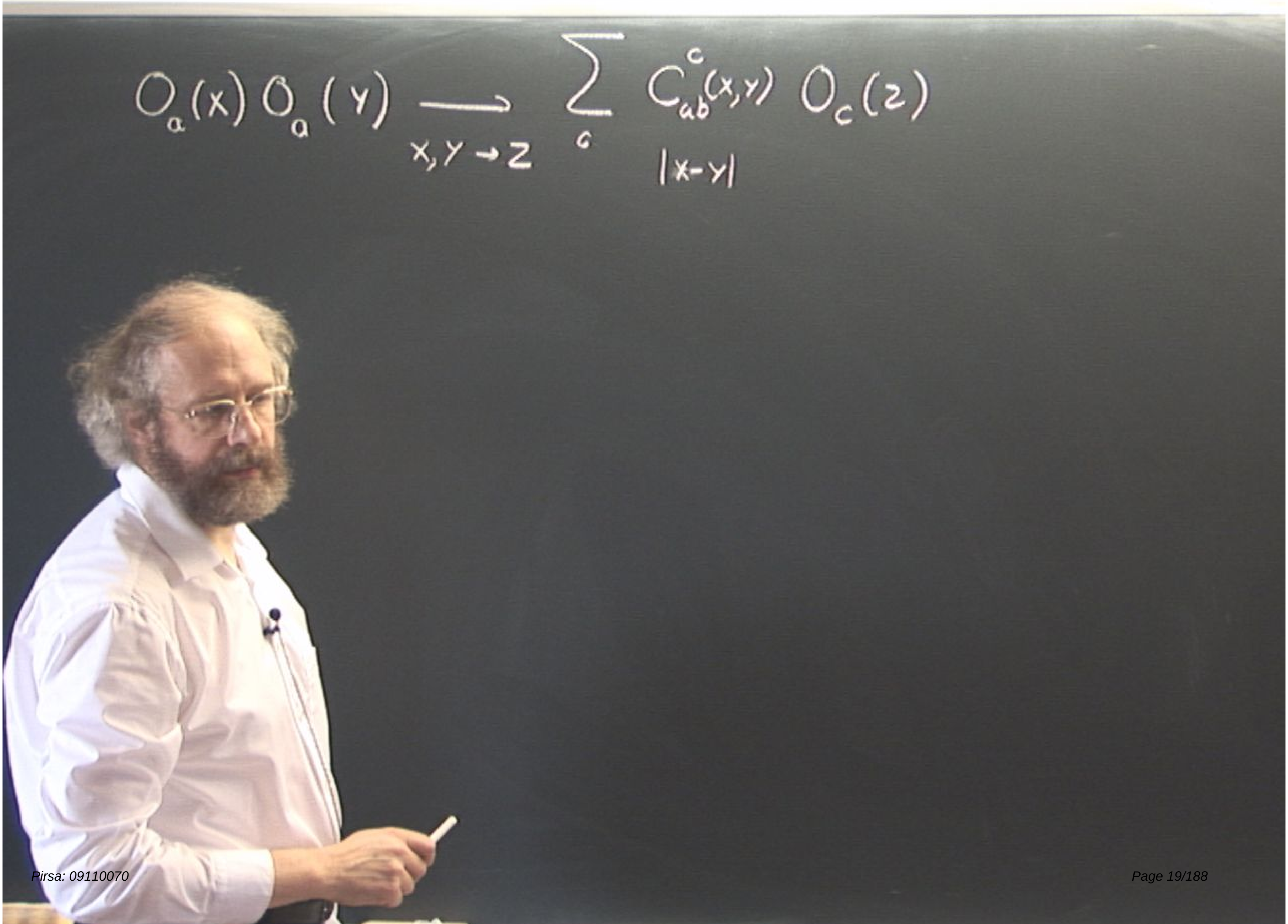

Real time
Causal
singularity
UV
Euclidean time



<0|T[... φ(x) φ(y) ...]|0>
|x-y|² → 0
Real time
t
Causal
"singularity"
UV
x
Euclidean time


Oa(x) Oa(y) ⟶ Σ
x,y → z
c


$O_a(x)\, O_a(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$


$O_a(x)\, O_a(y) \underset{x,y \to z}{\longrightarrow} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$
$|x-y|$

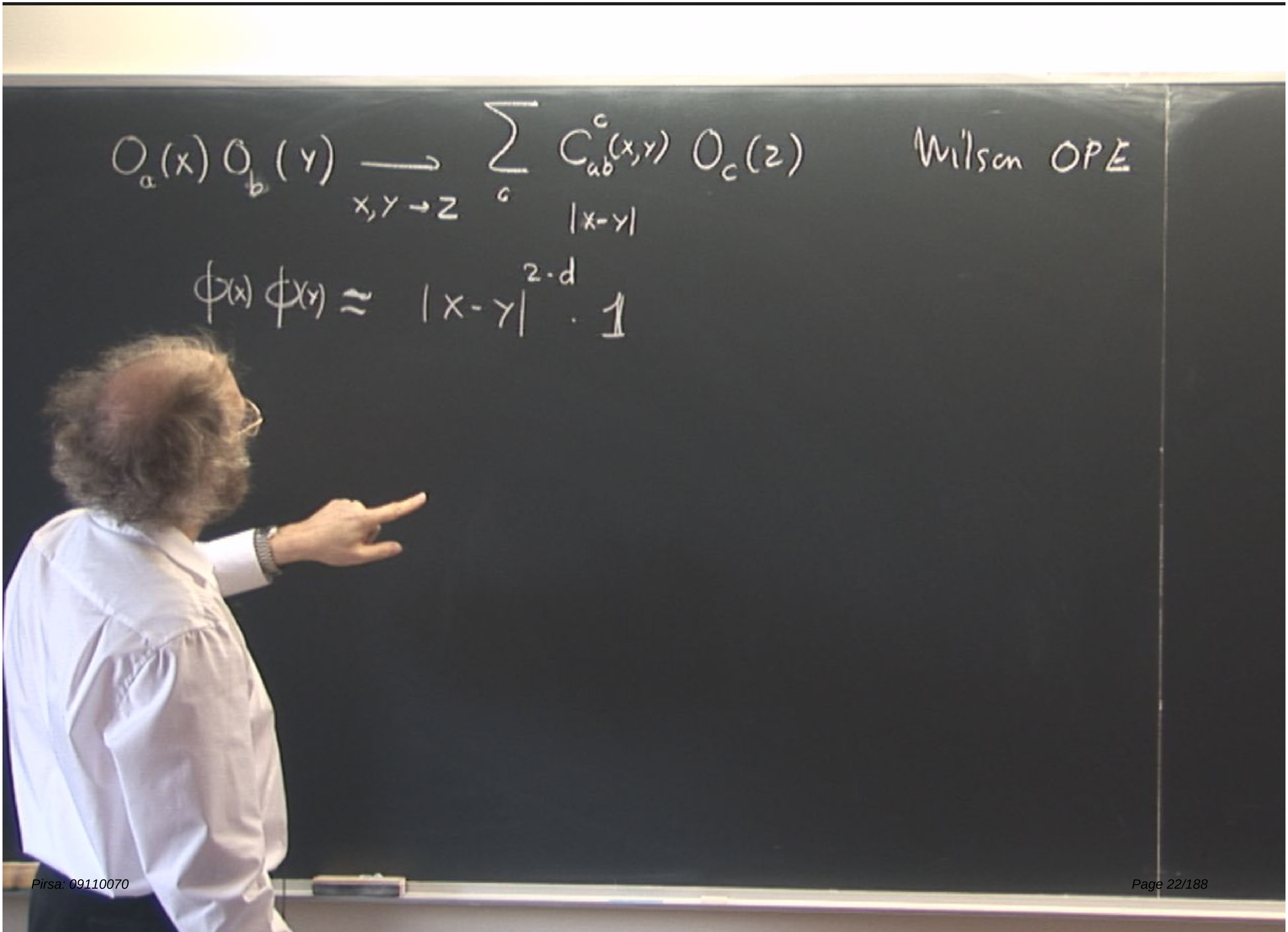
$$O_a(x)\, O_a(y) \xrightarrow[x,y\to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z) \qquad \text{Wilson OP}$$

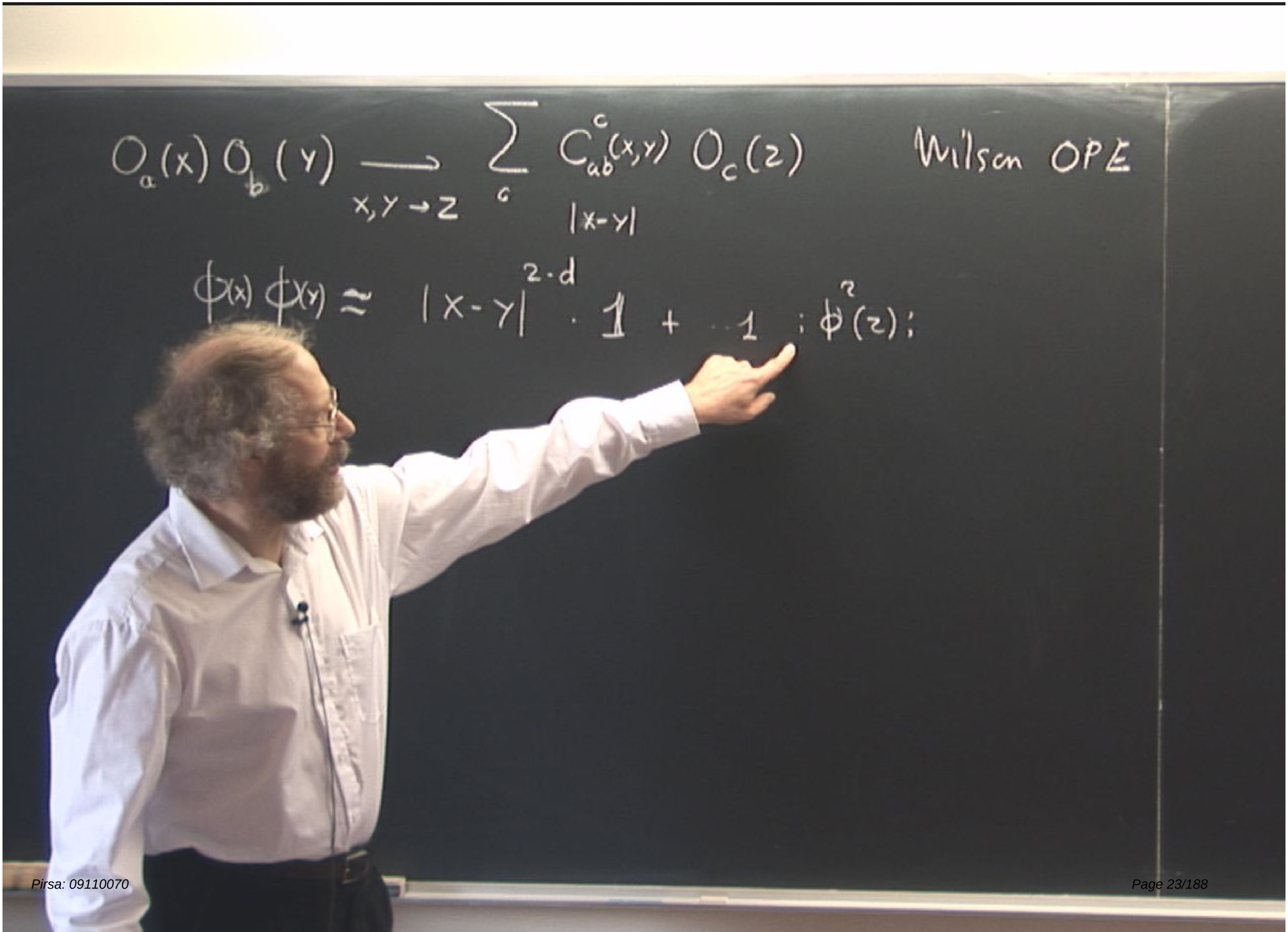
$$|x-y|$$

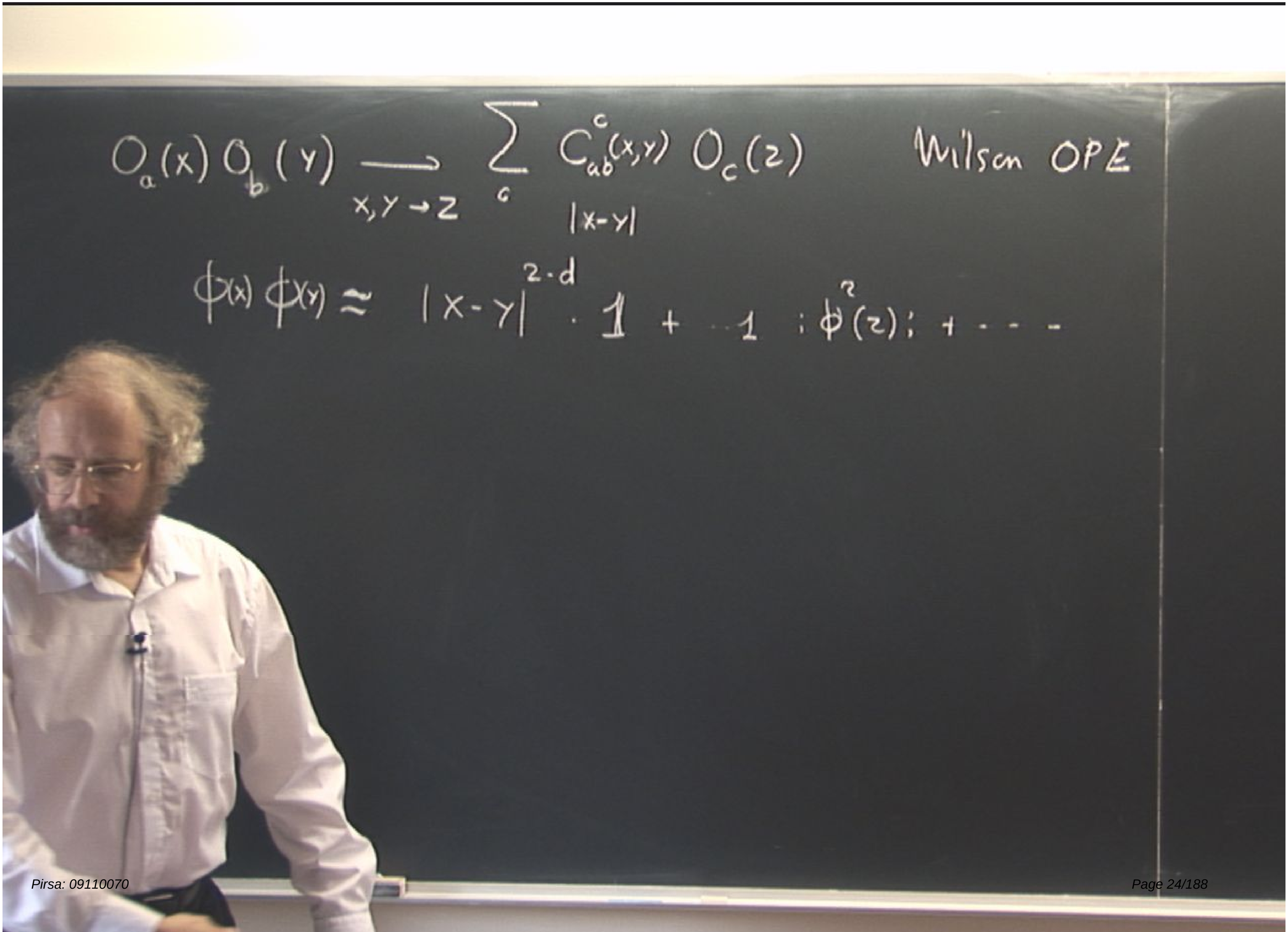
$$O_a(x)\, O_a(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z) \qquad |x-y|$$

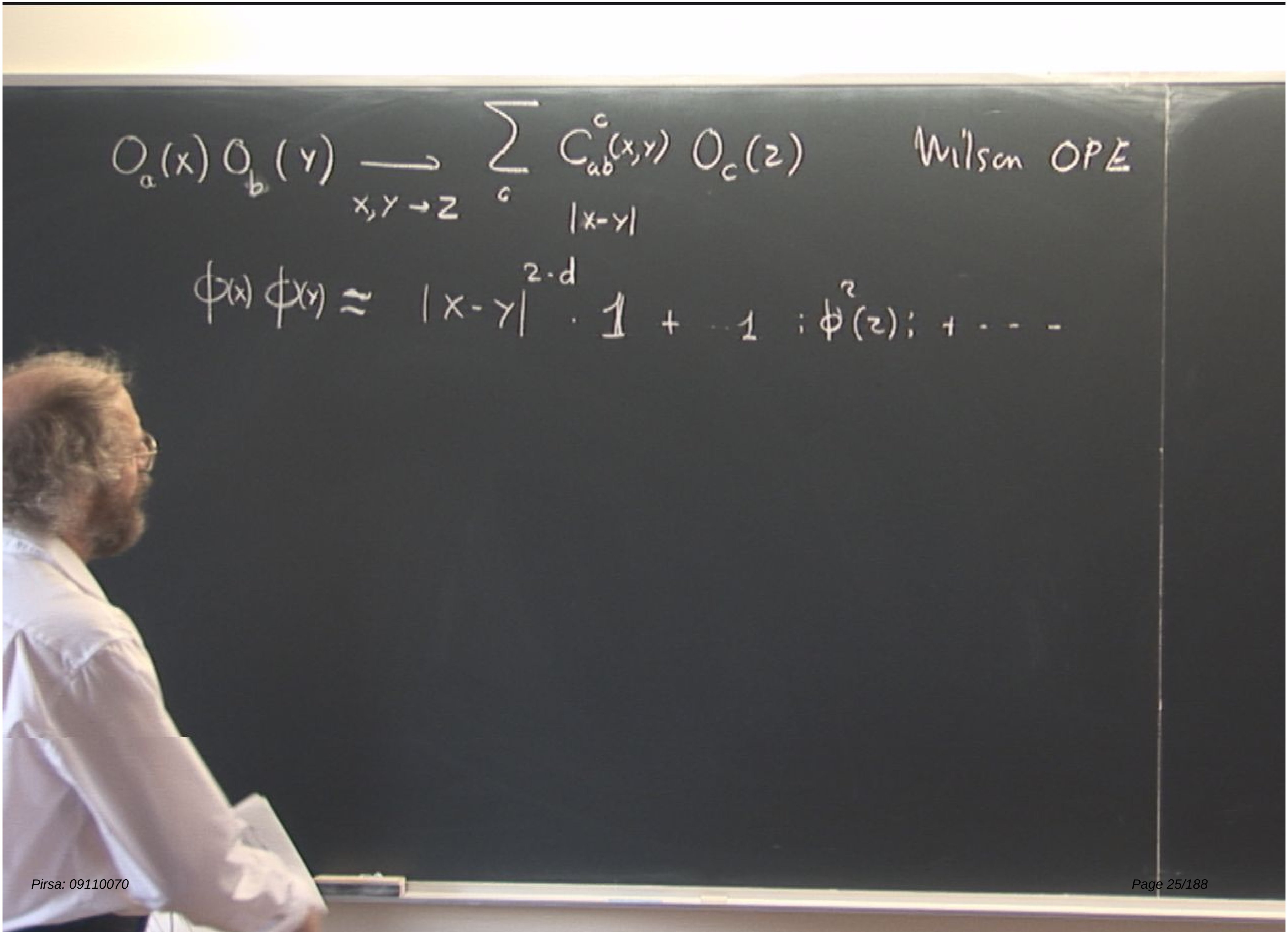
Wilson OPE

$$\phi\, \phi \approx |x-y|^{2-d} \cdot \mathbb{1}$$


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z)$
$|x-y|$
Wilson OPE
$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot 1$


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$
$|x-y|$
Wilson OPE
$\phi(x)\, \phi(y) \simeq |x-y|^{2-d} \cdot \mathbb{1} + 1 :\phi^2(z):$


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$
Wilson OPE
$|x-y|$
$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot 1 + \cdot 1 \; :\phi^2(z): + \cdots$


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$ Wilson OPE
$|x-y|$
$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot 1 + \cdot 1 \;:\phi^2(z): + \cdots$

$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z) \qquad \text{Wilson OPE}$$

$$|x-y|$$

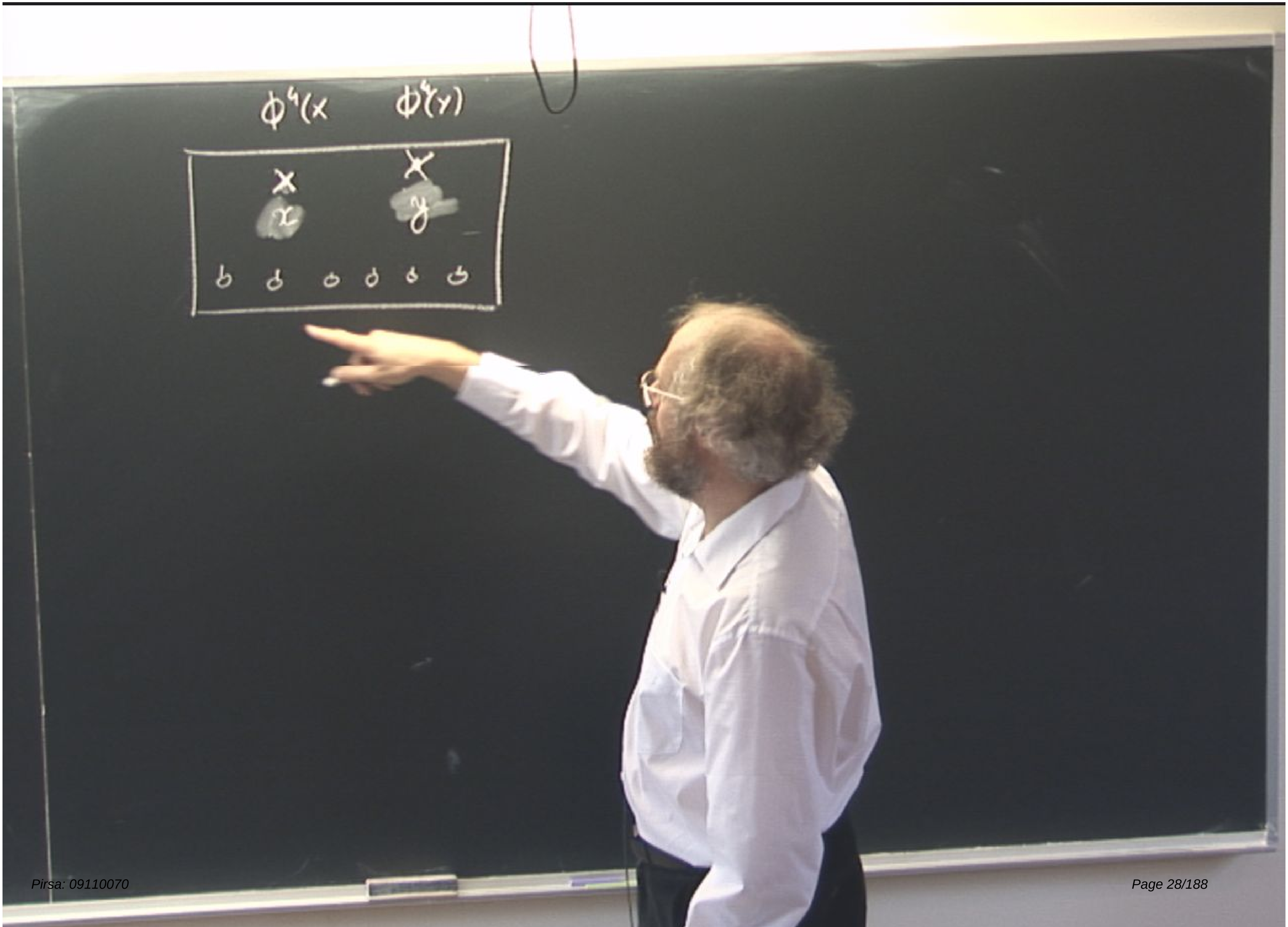
$$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + \cdot 1 \; :\phi^2(z): + \cdots$$

$$:\phi^4(x)::\phi^4(y): =$$

$$O_a(x)\, O_b(y) \underset{x,y\to z}{\longrightarrow} \sum_c \underset{|x-y|}{C_{ab}^{c}(x,y)}\, O_c(z) \qquad \text{Wilson OPE}$$

$$\phi(x)\, \phi(y) \approx |x-y|^{2-d} \cdot 1 + 1 \; :\phi^2(z): + \cdots$$

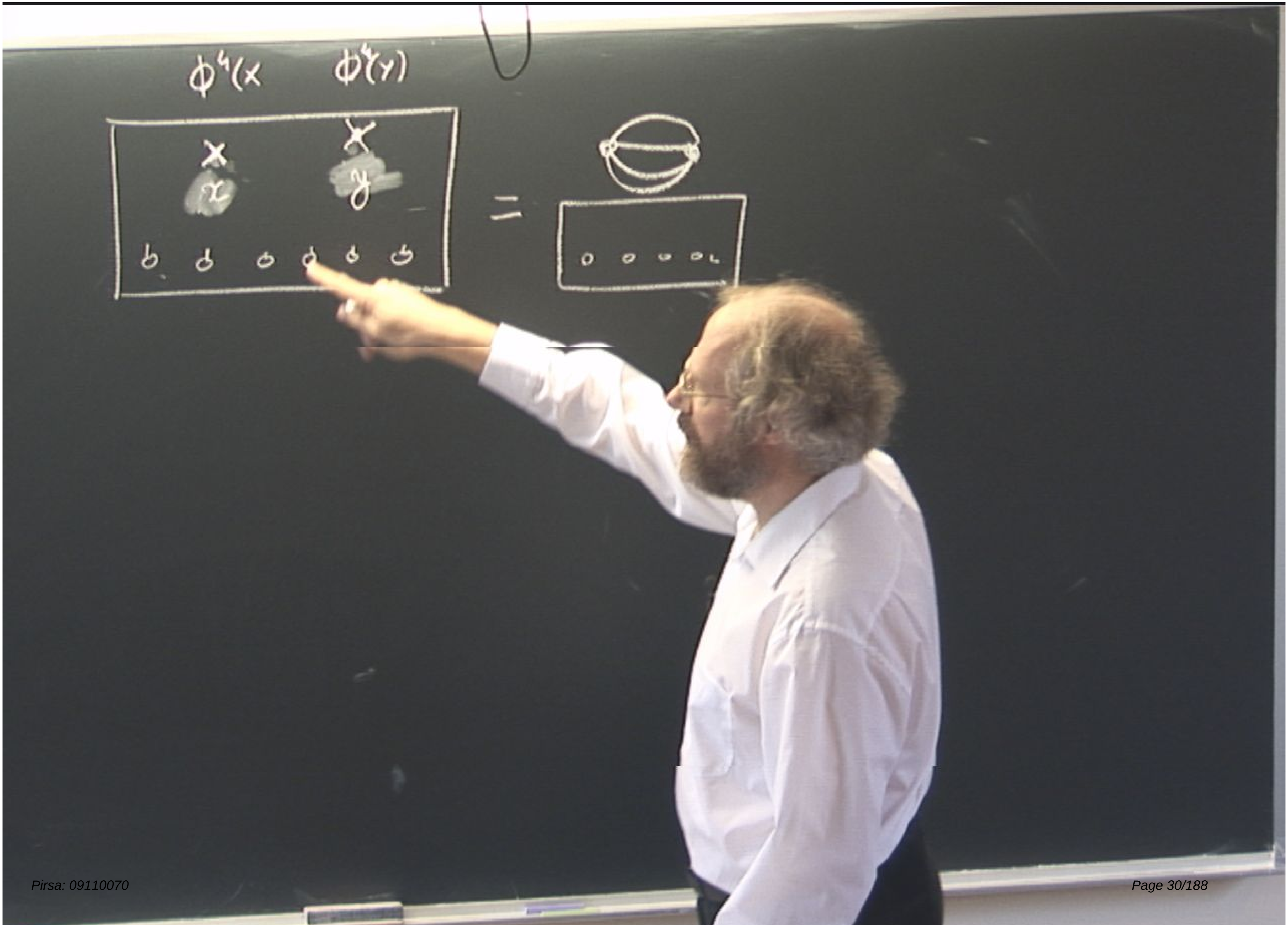
$$:\phi^4(x): :\phi^4(y): =$$

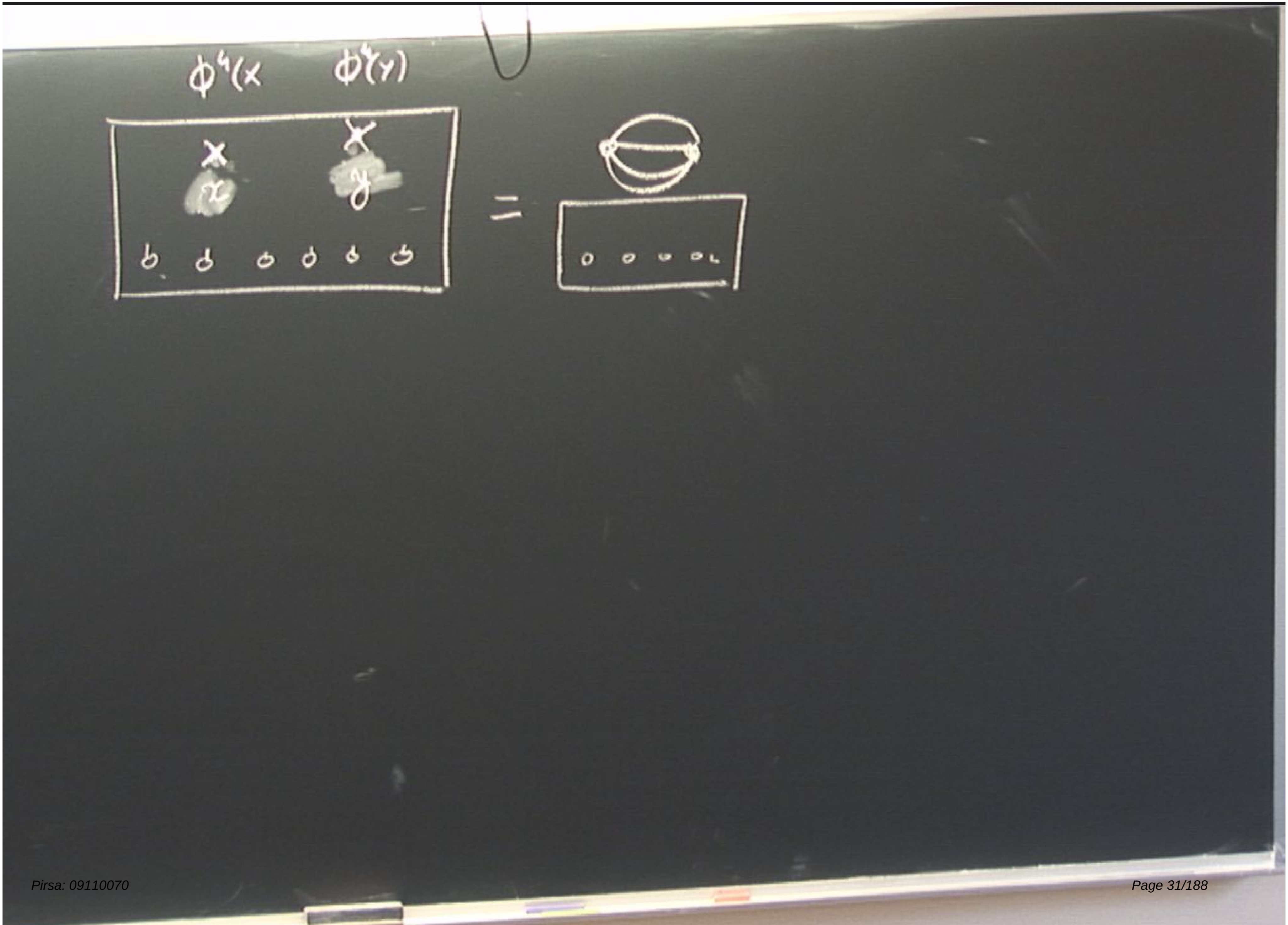

$\phi^4(x$ $\phi^4(y)$
x
y

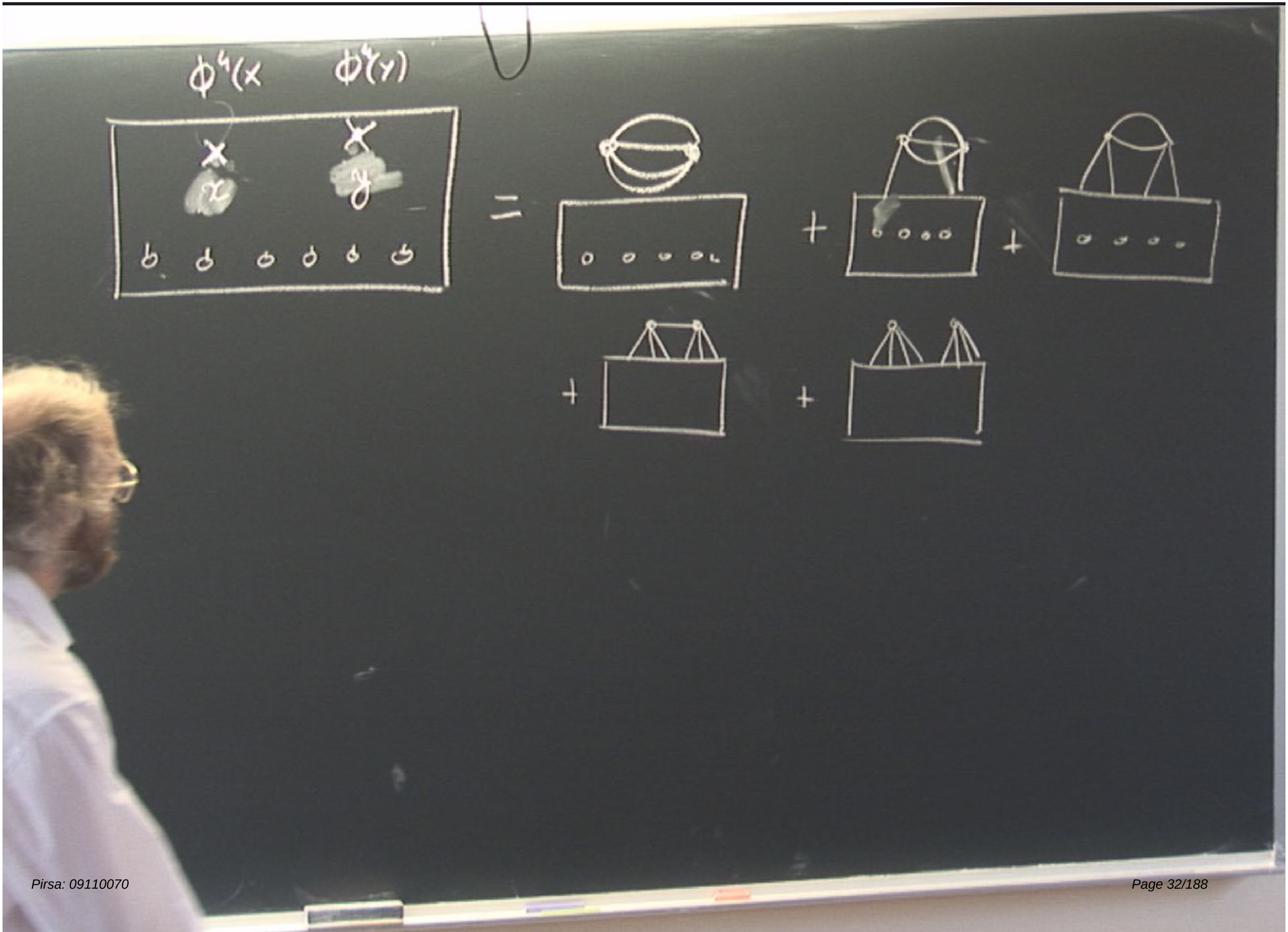
$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)_{|x-y|}\, O_c(z) \qquad \text{Wilson OPE}$$

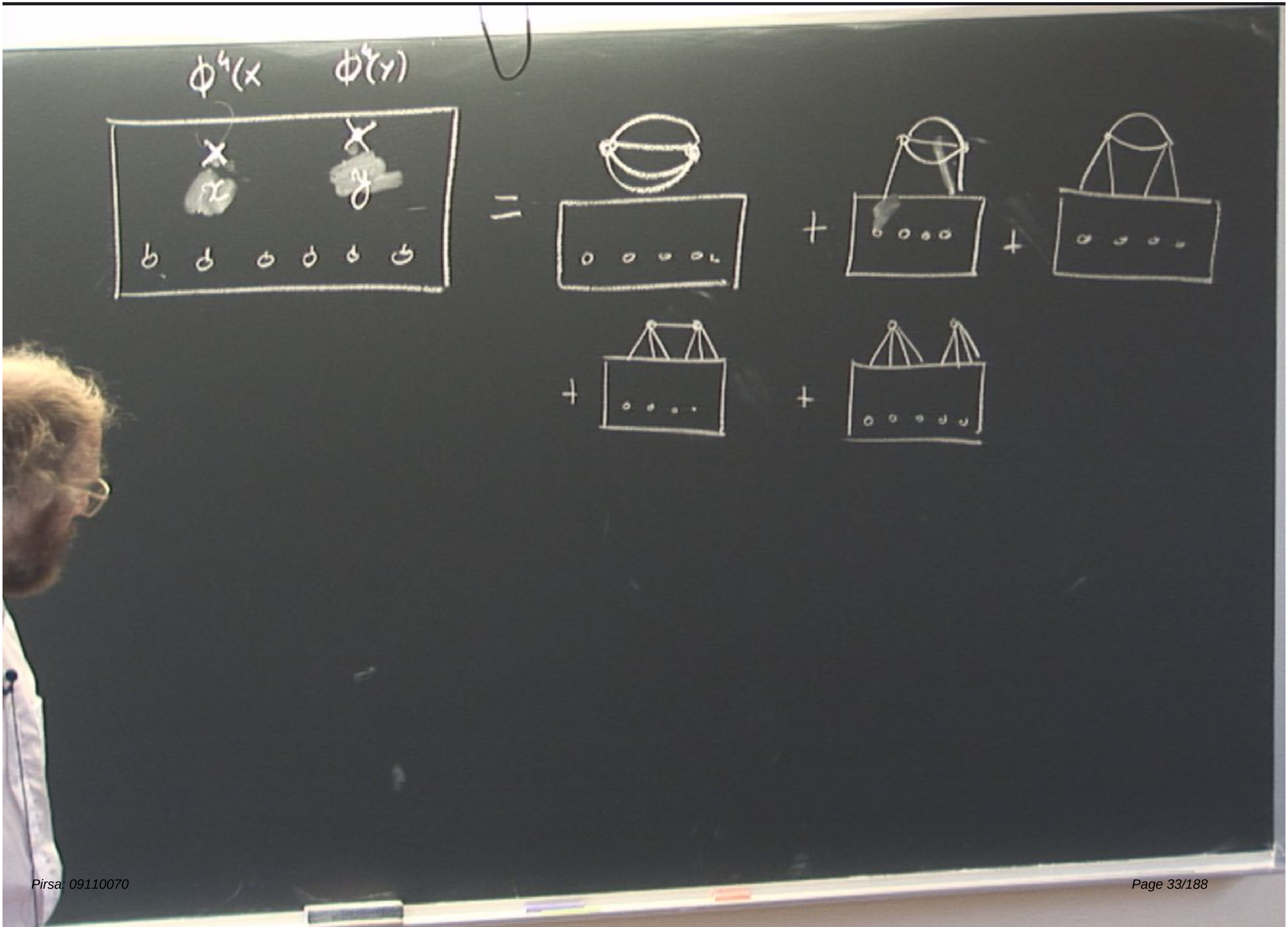
Free theory $\quad \phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot 1 + \cdots 1 \; :\phi^2(z): + \cdots$

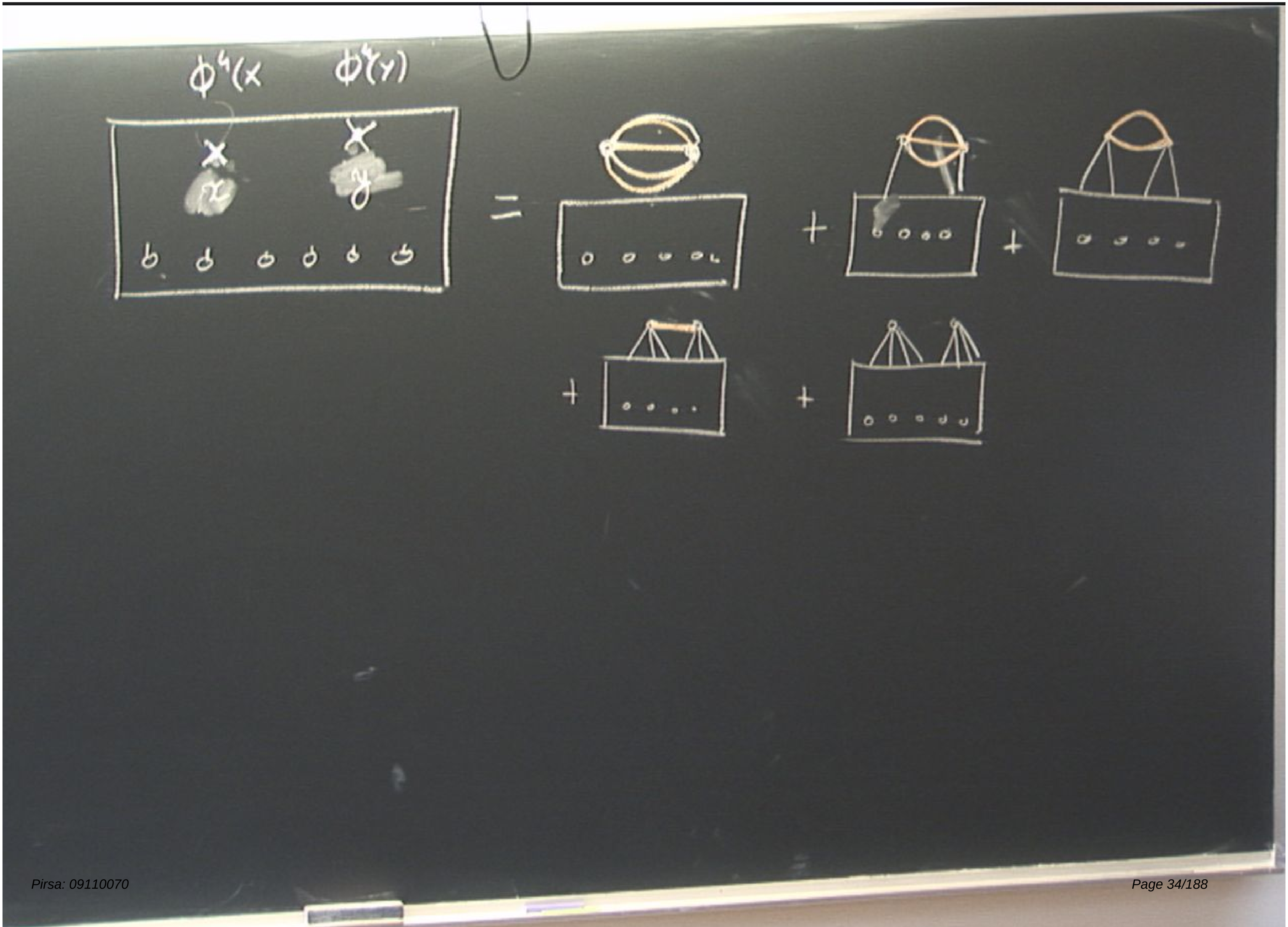
$$:\phi^4(x)::\phi^4(y): =$$

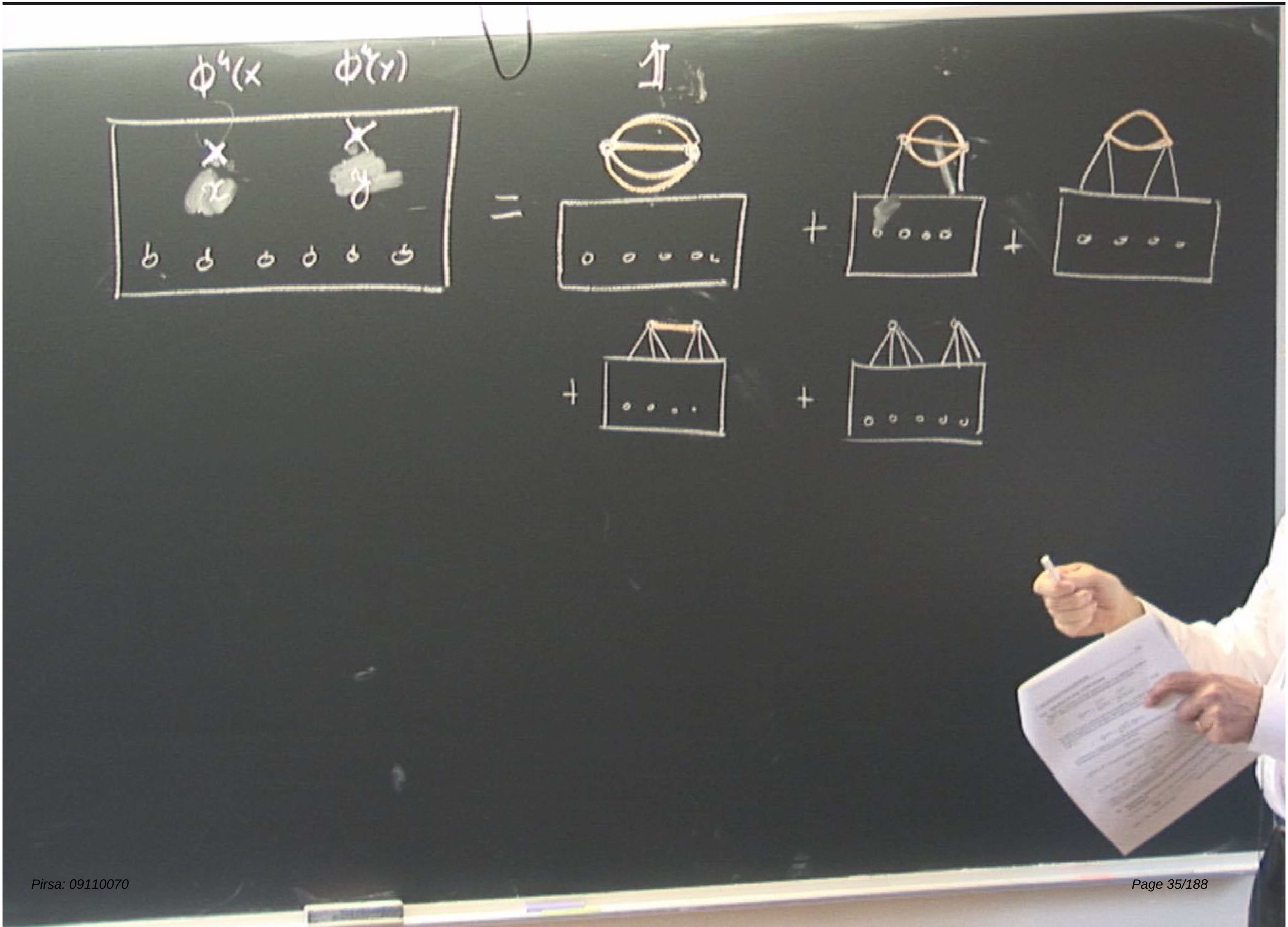

$\phi^4(x$ $\phi^4(y)$
x
y
=


$\phi^4(x$
$\phi^4(y)$
x
y
=

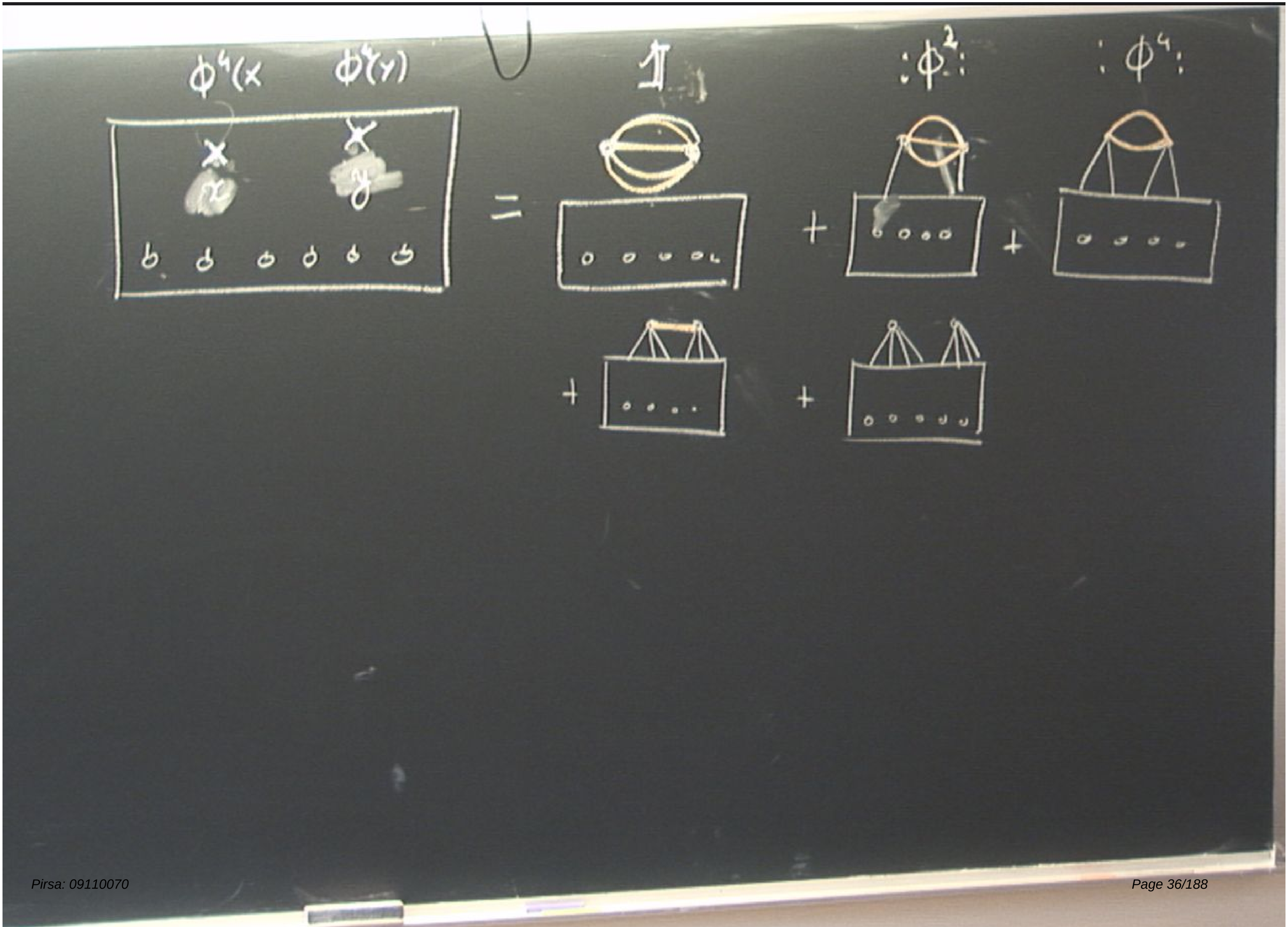

ϕ⁴(x
ϕ⁴(y)
x
y

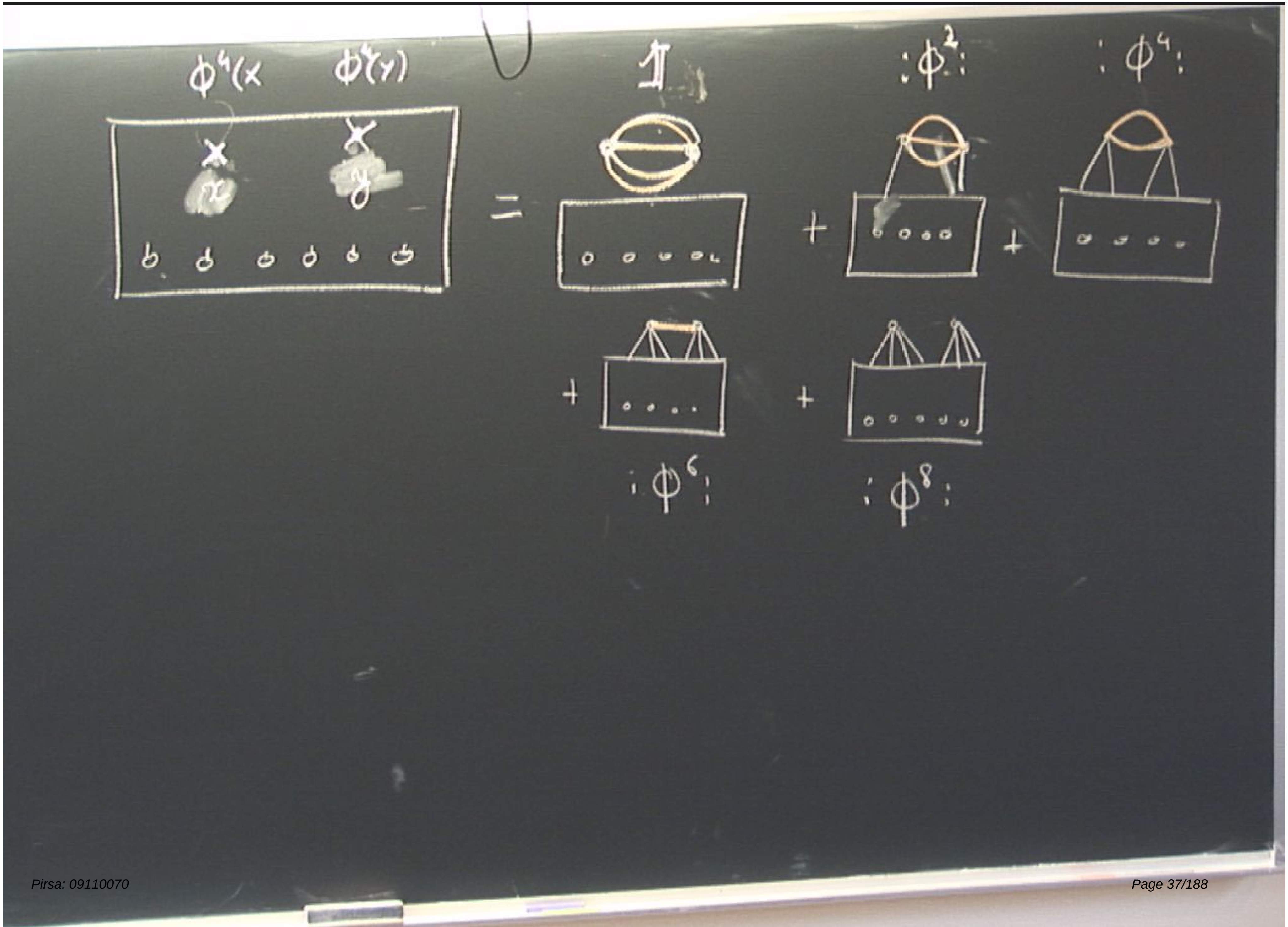

$\phi^4(x$
$\phi^4(y)$
x
y
=
+
+
+
+

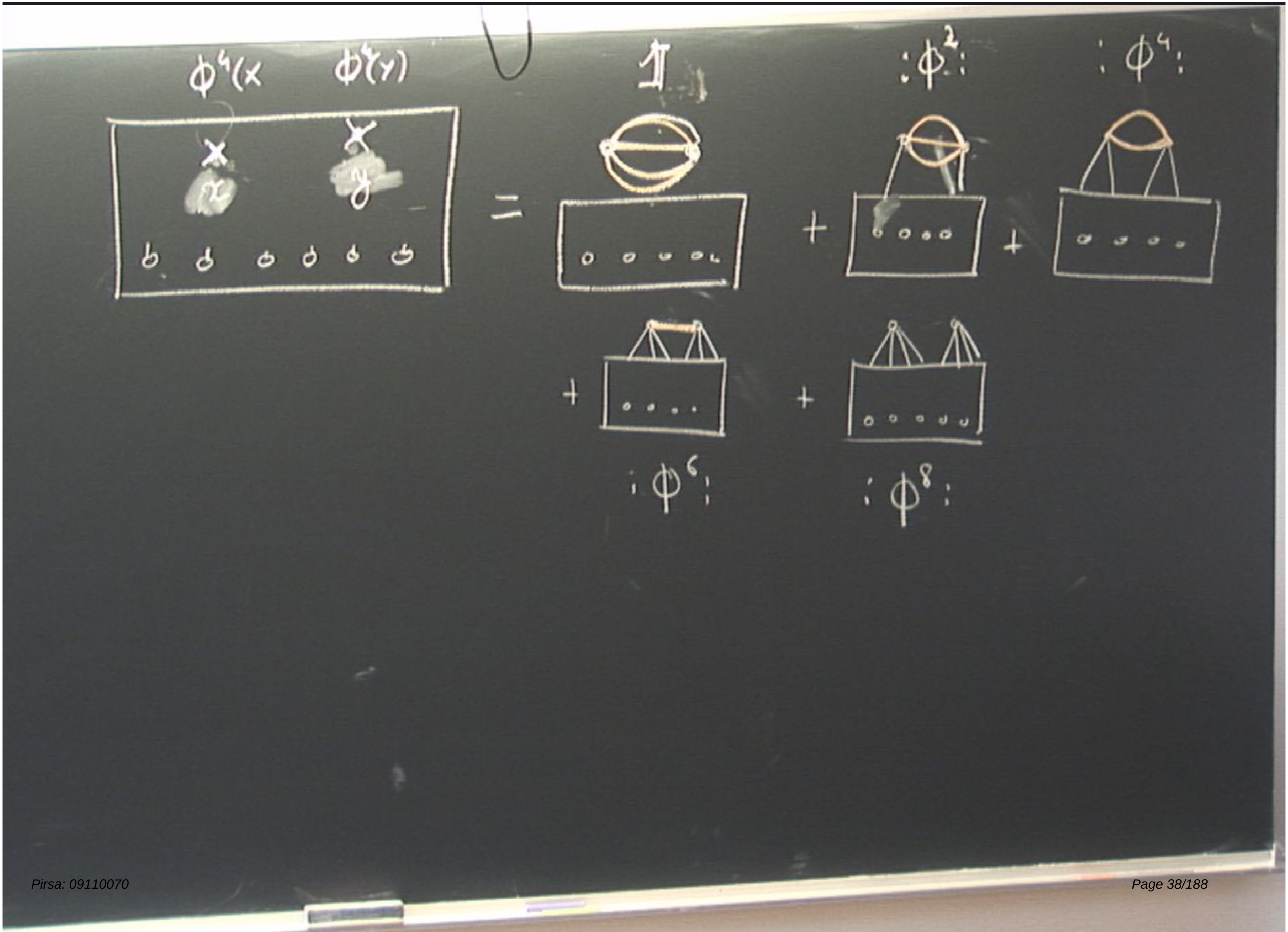

φ⁴(x
φ⁴(y)
x
y



ϕ⁴(x
ϕ⁴(y)
x
y
=
+
+
+
+


$\phi^4(x$ $\phi^4(y)$
x
y
$=$
$\mathbb{1}$
$:\phi^2:$
$:\phi^4:$
+
+
+
+


φ⁴(x
φ⁴(y)
x
y
=
𝟙
:φ²:
:φ⁴:
+
:φ⁶:
:φ⁸:



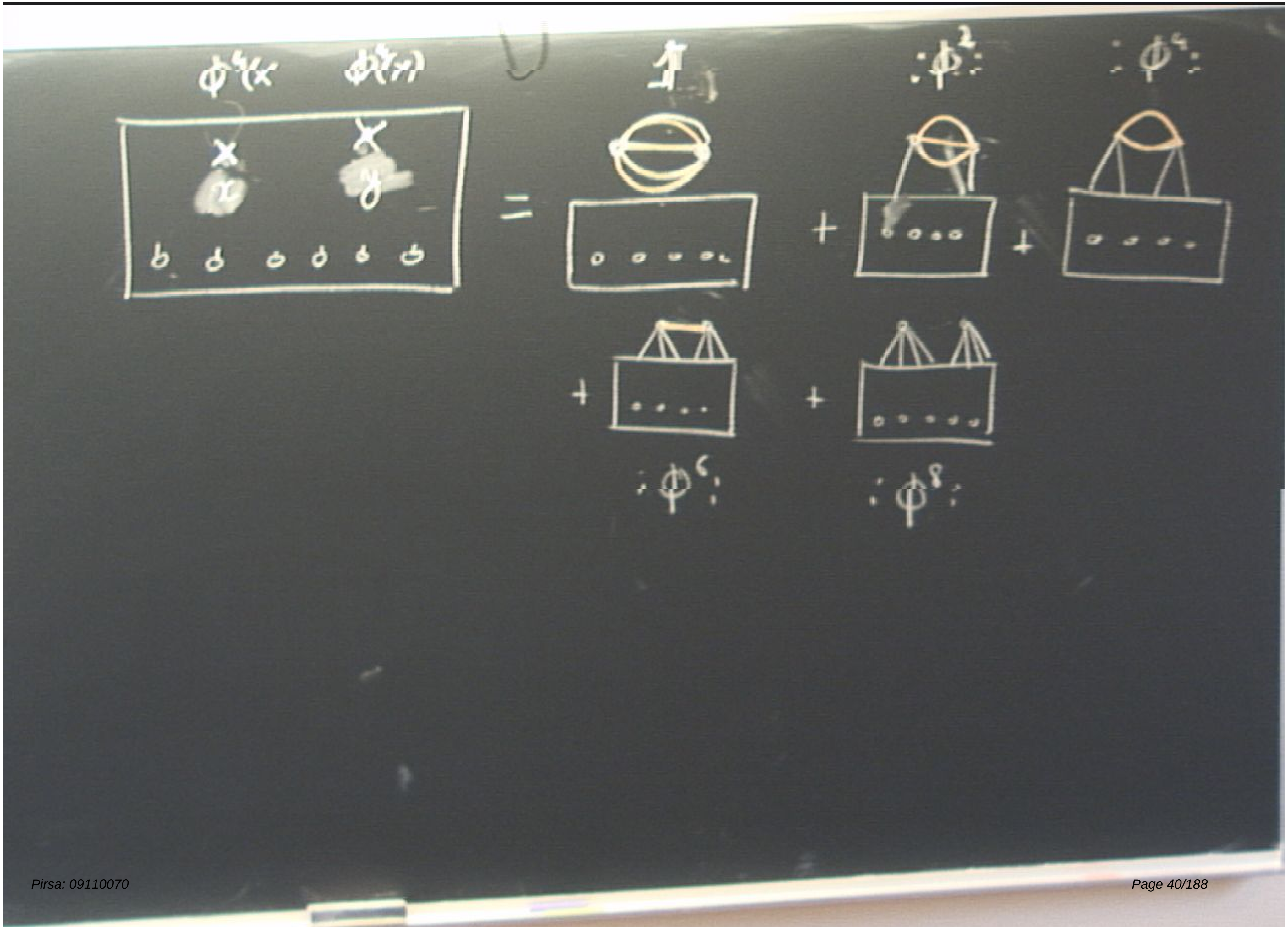
φ⁴(x φ⁴(y)
x
y
=
𝟙
:φ²:
:φ⁴:
:φ⁶:
:φ⁸:

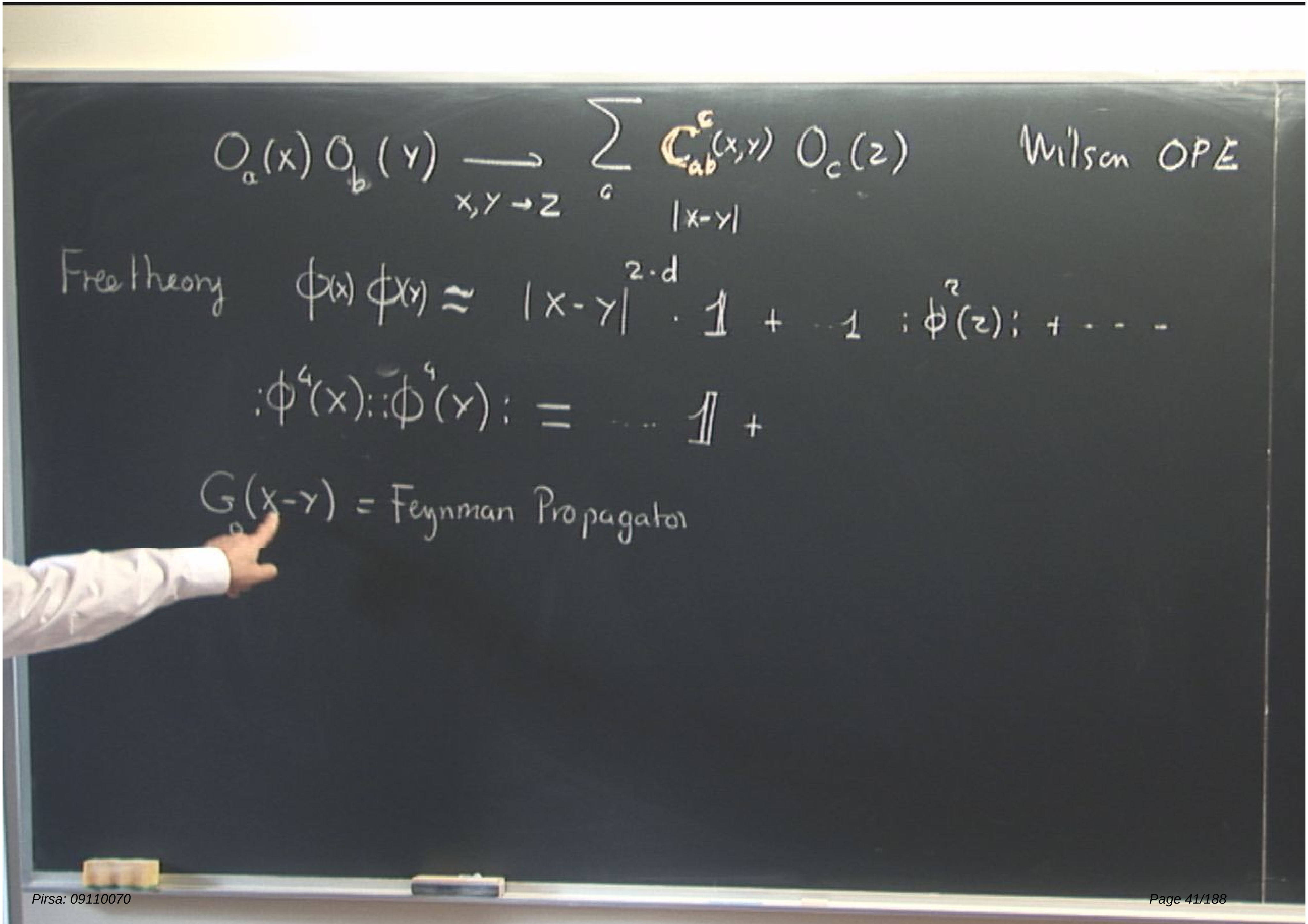
$$O_a(x)\, O_b(y) \xrightarrow[x,y\to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$$ Wilson OPE

$$|x-y|$$

e theory $\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + \cdots 1 \; :\phi^2(z): + \cdots$

$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} +$$


φ⁴(x
φ⁴(y)
=
:φ²:
:φ⁴:
:φ⁶:
:φ⁸:


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z)$ Wilson OPE
$|x-y|$
Free theory $\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + 1\ :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} +$
$G_0(x-y)$ = Feynman Propagator

$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C^c_{ab}(x,y)\, O_c(z)$$ Wilson OPE

$|x-y|$

Free theory $\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + \cdots 1 \; :\phi^2(z): + \cdots$

$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^2$$

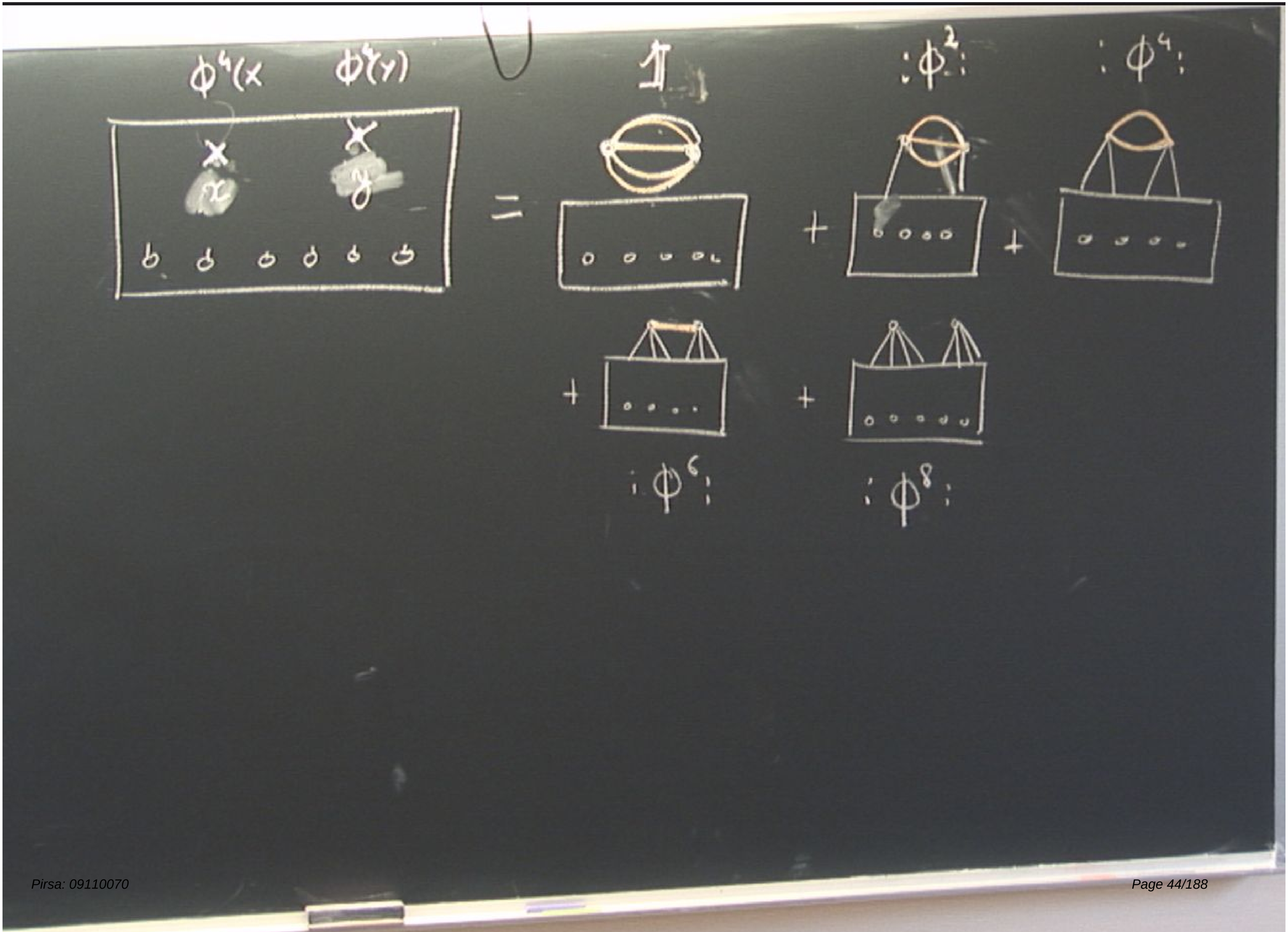
$G_0(x-y)$ = Feynman Propagator

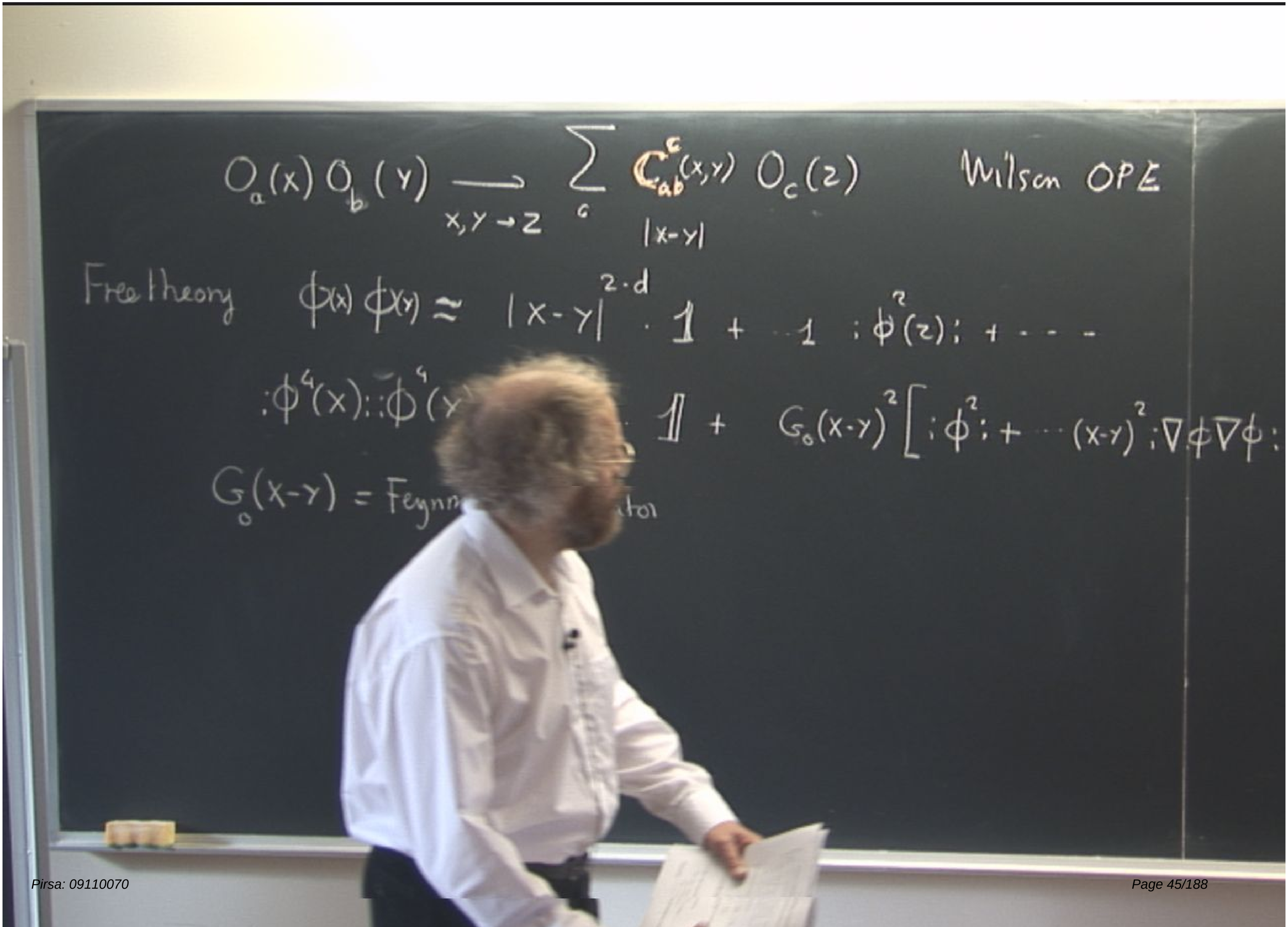
$$O_a(x)\, O_b(y) \underset{x,y \to z}{\longrightarrow} \sum_c \underset{|x-y|}{C_{ab}^{c}(x,y)}\, O_c(z)$$ Wilson OPE

Free theory $\phi(x)\,\phi(y) \simeq |x-y|^{2-d} \cdot \mathbb{1} + 1 \; :\phi^2(z): + \cdots$

$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^2 \left[:\phi^2: + \right.$$

$G_0(x-y)$ = Feynman Propagator


$\phi^4(x$ $\phi^4(y)$
x
y
$=$
$\mathbb{1}$
$+$
$:\phi^2:$
$+$
$:\phi^4:$
$+$
$+$
$:\phi^6:$
$:\phi^8:$



$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c \mathbb{C}_{ab}^c(x,y)\, O_c(z)$
$|x-y|$
Wilson OPE
Free theory
$\phi(x)\,\phi(y) \simeq |x-y|^{2-d} \cdot \mathbb{1} + \cdot 1 \; :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y$
$\mathbb{1} + G_0(x-y)^2 \left[:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: \right.$
$G_0(x-y) =$ Feynm
tor

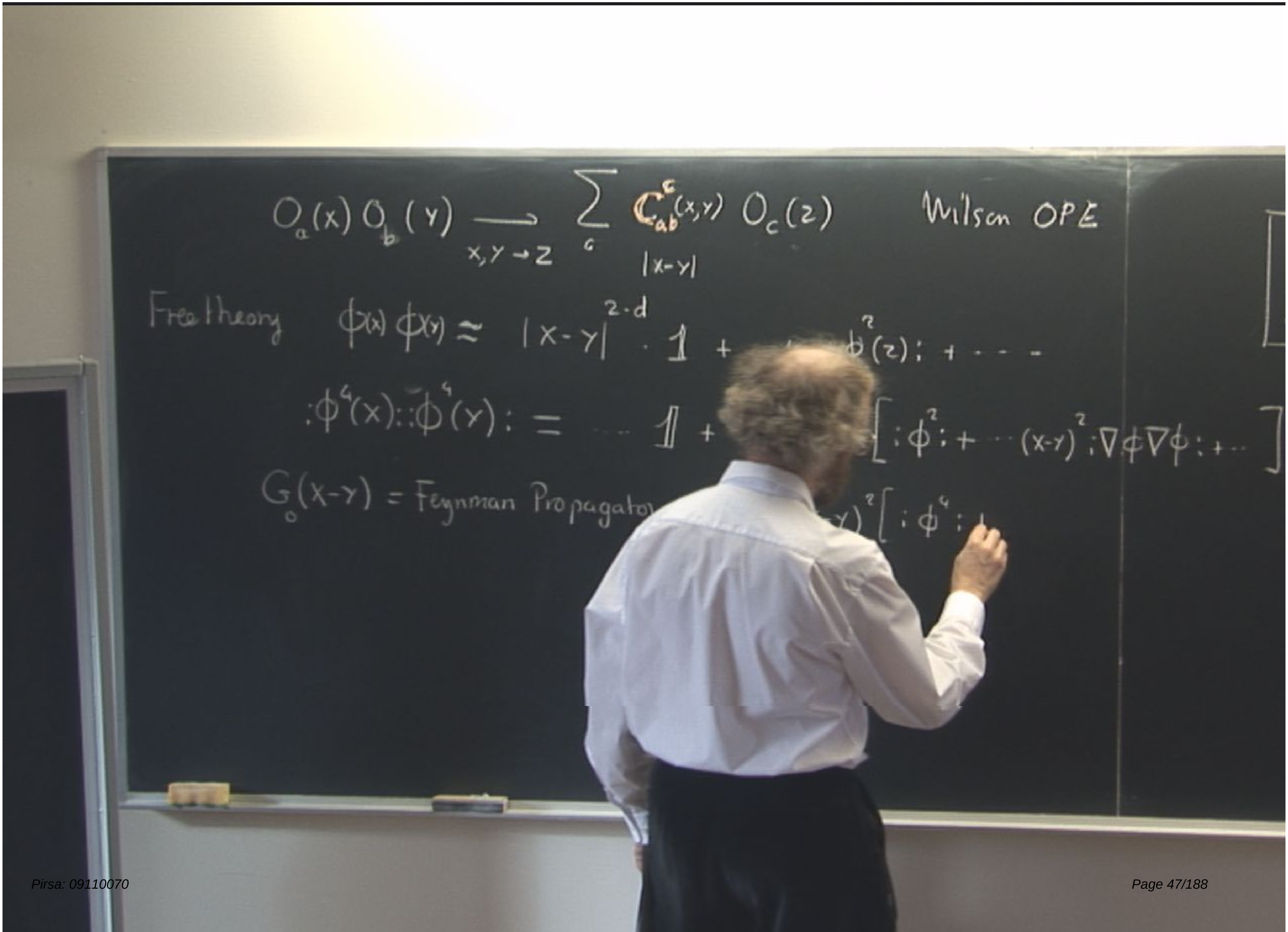
$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z) \qquad \text{Wilson OPE}$$

$$|x-y|$$

Free theory $\phi(x)\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + 1 \; :\phi^2(z): + \cdots$

$$:\phi^4(x)::\phi^4(y): = \mathbb{1} + G_0(x-y)^2 \left[:\phi^2: + (x-y)^2 :\nabla\phi\nabla\phi: + \cdots \right]$$

$G_0(x-y)$ = Feynman Propagator


O_a(x) O_b(y) → Σ_c C^c_ab(x,y) O_c(z)
x,y → z
|x−y|
Wilson OPE
Free theory
φ(x) φ(y) ≃ |x−y|^(2−d) · 1 + … φ²(z): + - - -
:φ⁴(x)::φ⁴(y): = … 1 + … [:φ²: + … (x−y)² :∇φ∇φ: + …]
G_0(x−y) = Feynman Propagator
(x−y)² [:φ⁴: +

$$O_a(x)\,O_b(y) \xrightarrow[x,y\to z]{} \sum_c \underset{|x-y|}{C_{ab}^{c}(x,y)}\, O_c(z)$$ Wilson OPE

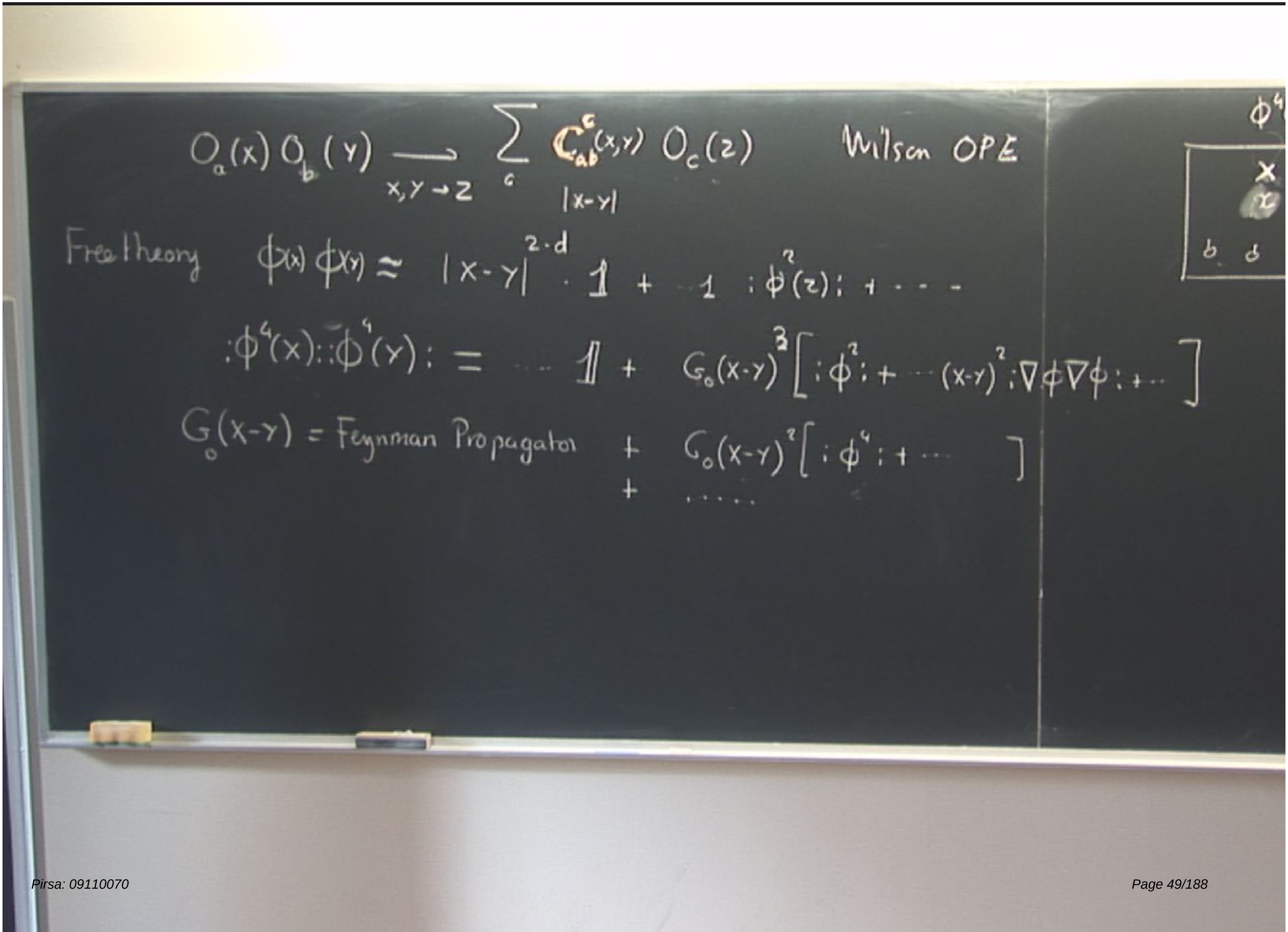
Free theory $$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + 1\ :\phi^2(z): + \cdots$$

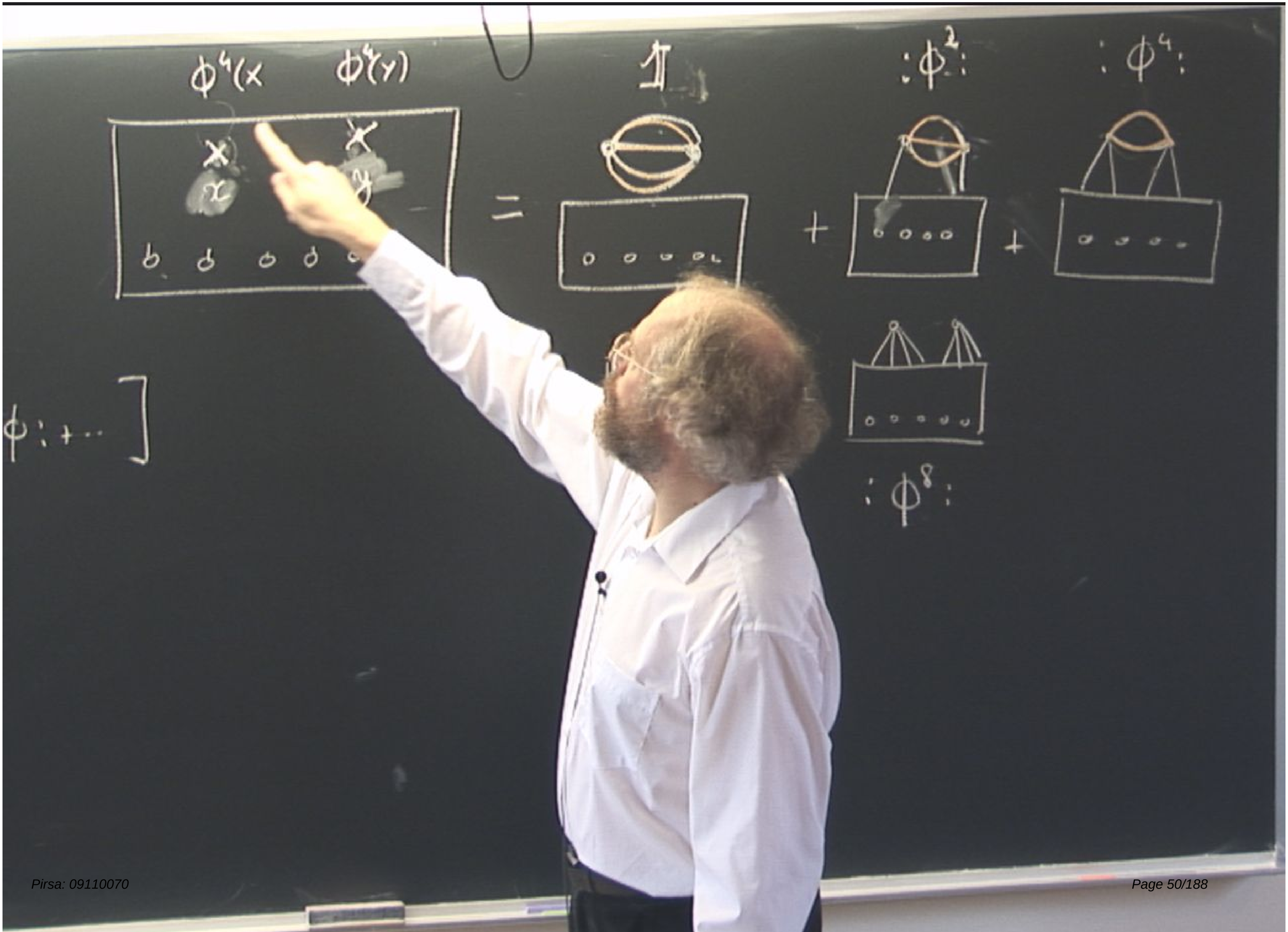
$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^3\left[:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots\right]$$

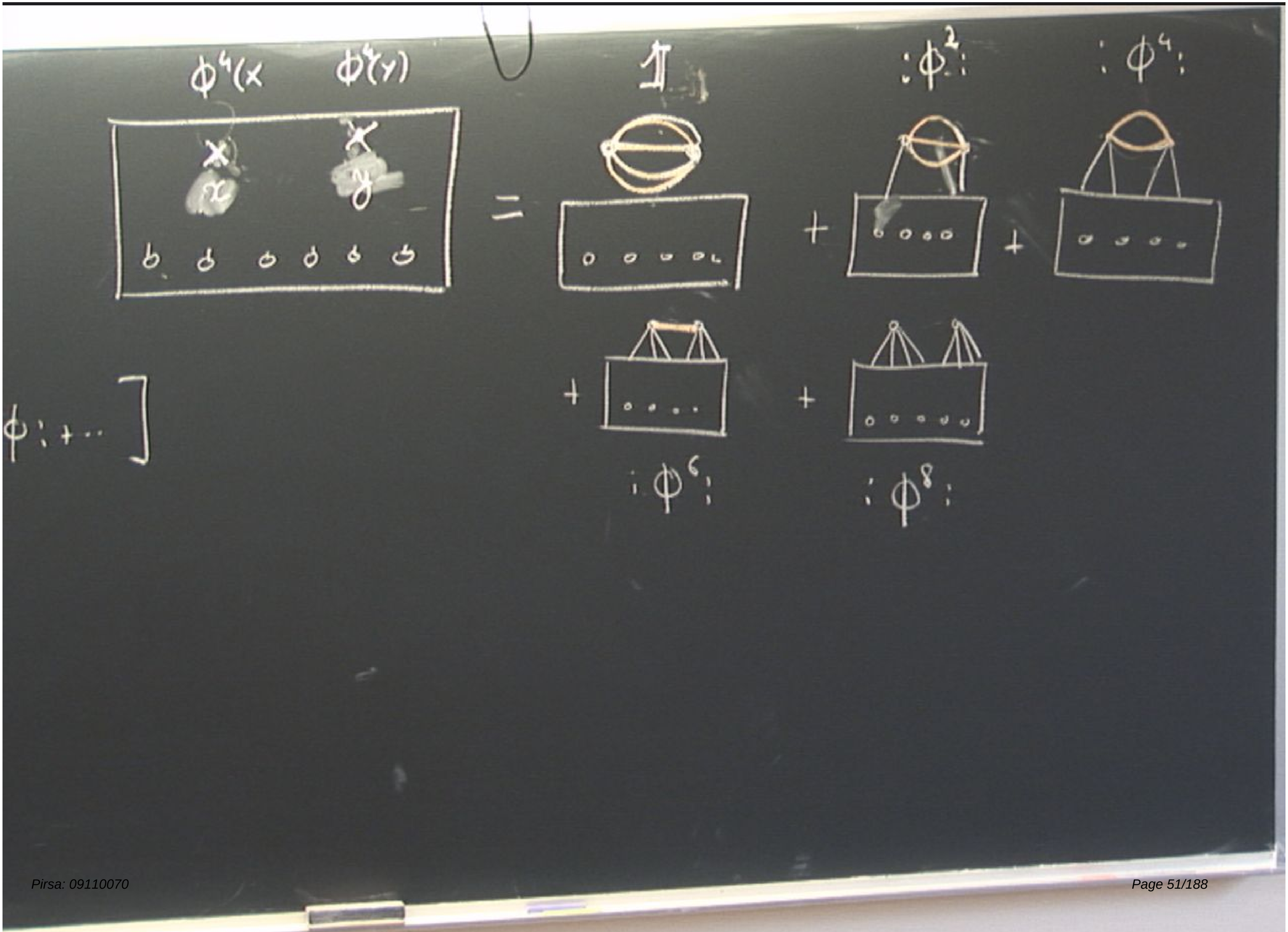
$$+\ G_0(x-y)^2\left[:\phi^4: + \cdots\right]$$

$$+\ \cdots\cdots$$

$G_0(x-y)$ = Feynman Propagator


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z)$
$|x-y|$
Wilson OPE
Free theory $\phi(x)\phi(y) \simeq |x-y|^{2-d} \cdot \mathbb{1} + 1\; :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y): = \mathbb{1} + G_0(x-y)^3 [\, :\phi^2: + (x-y)^2 :\nabla\phi\nabla\phi: + \cdots]$
$G_0(x-y)$ = Feynman Propagator
$+ G_0(x-y)^2 [\, :\phi^4: + \cdots]$
$+ \cdots\cdots$
ϕ^4
x
b d


ϕ⁴(x
ϕ⁴(y)
x
y
=
𝟙
+
:ϕ²:
+
:ϕ⁴:
:ϕ⁸:
ϕ: + ···]


φ4(x
φ4(y)
x
y
:φ2:
:φ4:
:φ6:
:φ8:

$$O_a(x)\,O_b(y) \xrightarrow[x,y\to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z) \qquad \text{Wilson OPE}$$

$$|x-y|$$

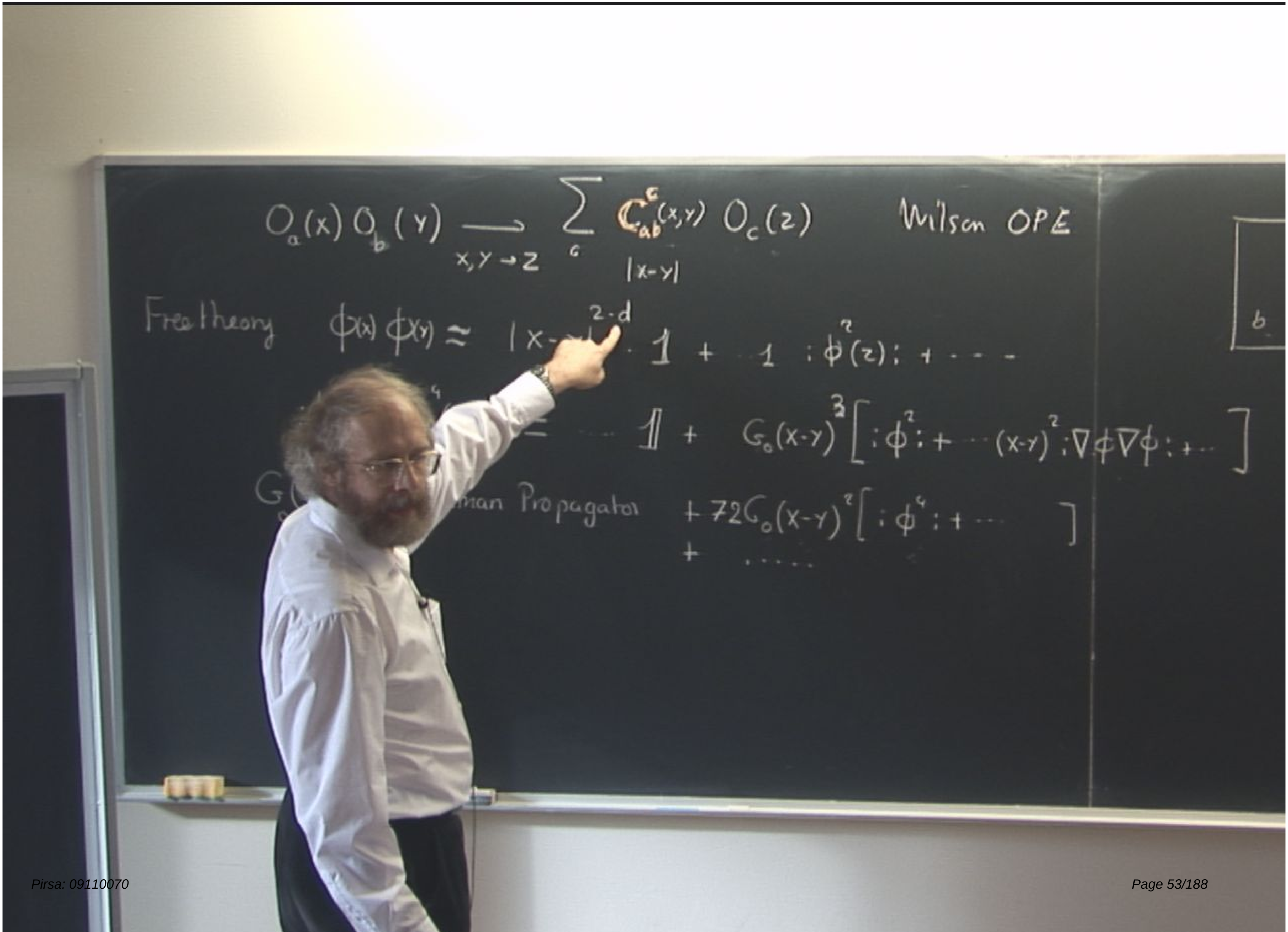
e theory $\quad \phi(x)\,\phi(y) \simeq |x-y|^{2\cdot d} \cdot \mathbb{1} + \cdots 1 \; :\phi^2(z): + \cdots$

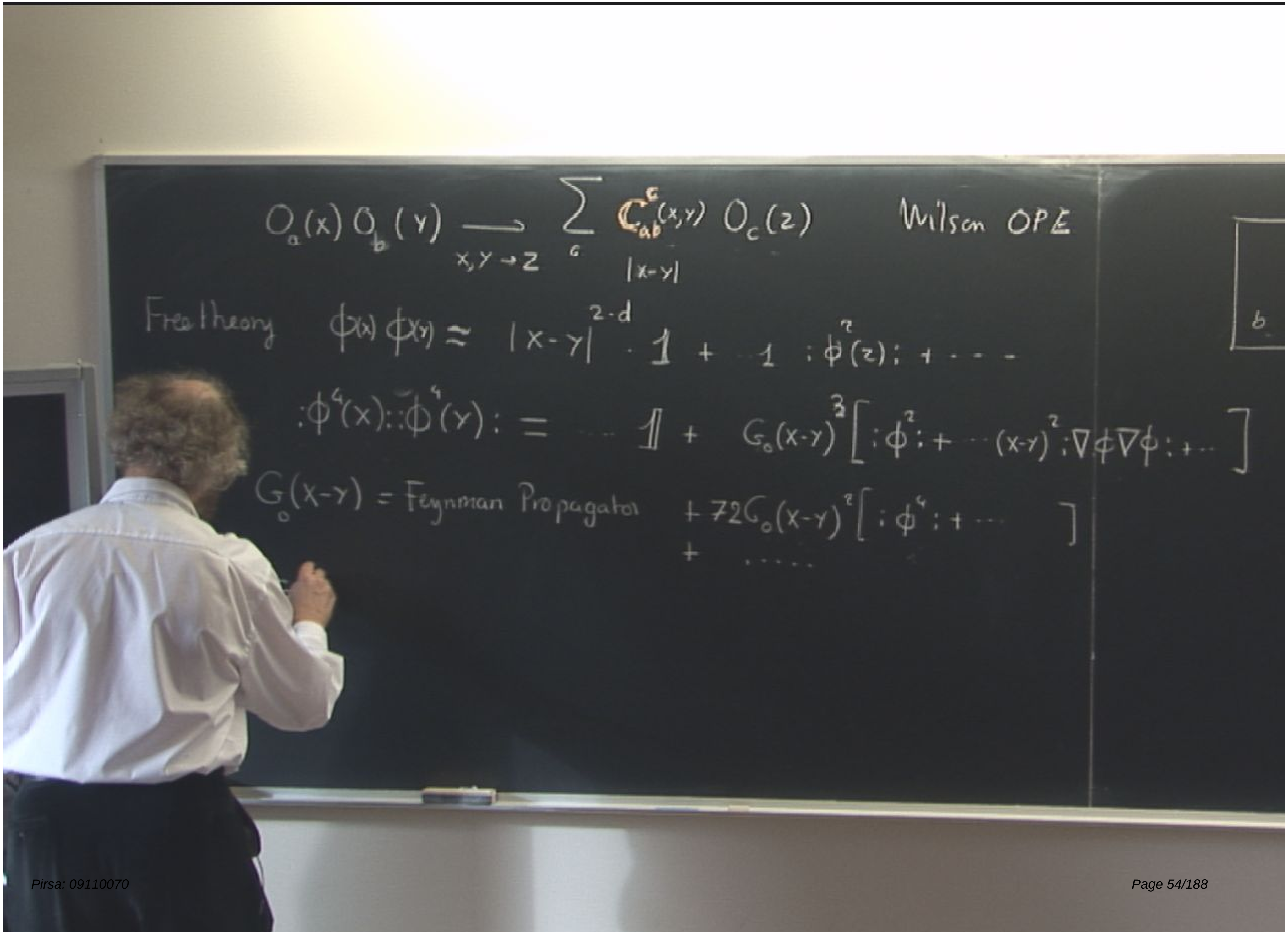
$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^3 \left[:\phi^2: + \cdots (x-y)^2 \right.$$

$G_0(x-y)$ = Feynman Propagator

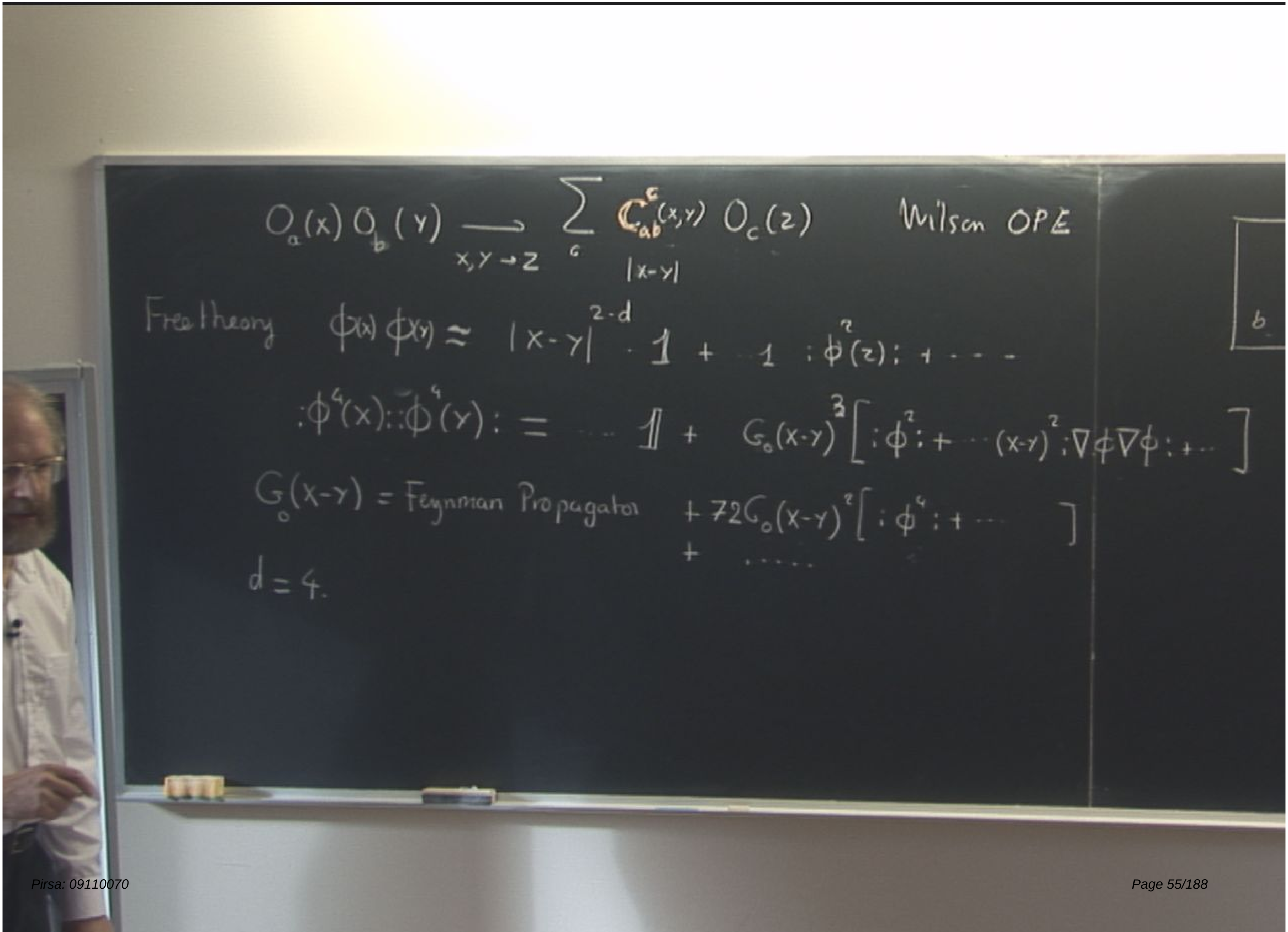
$$+ 72\, G_0(x-y)^2 \left[:\phi^4: + \cdots \right.$$

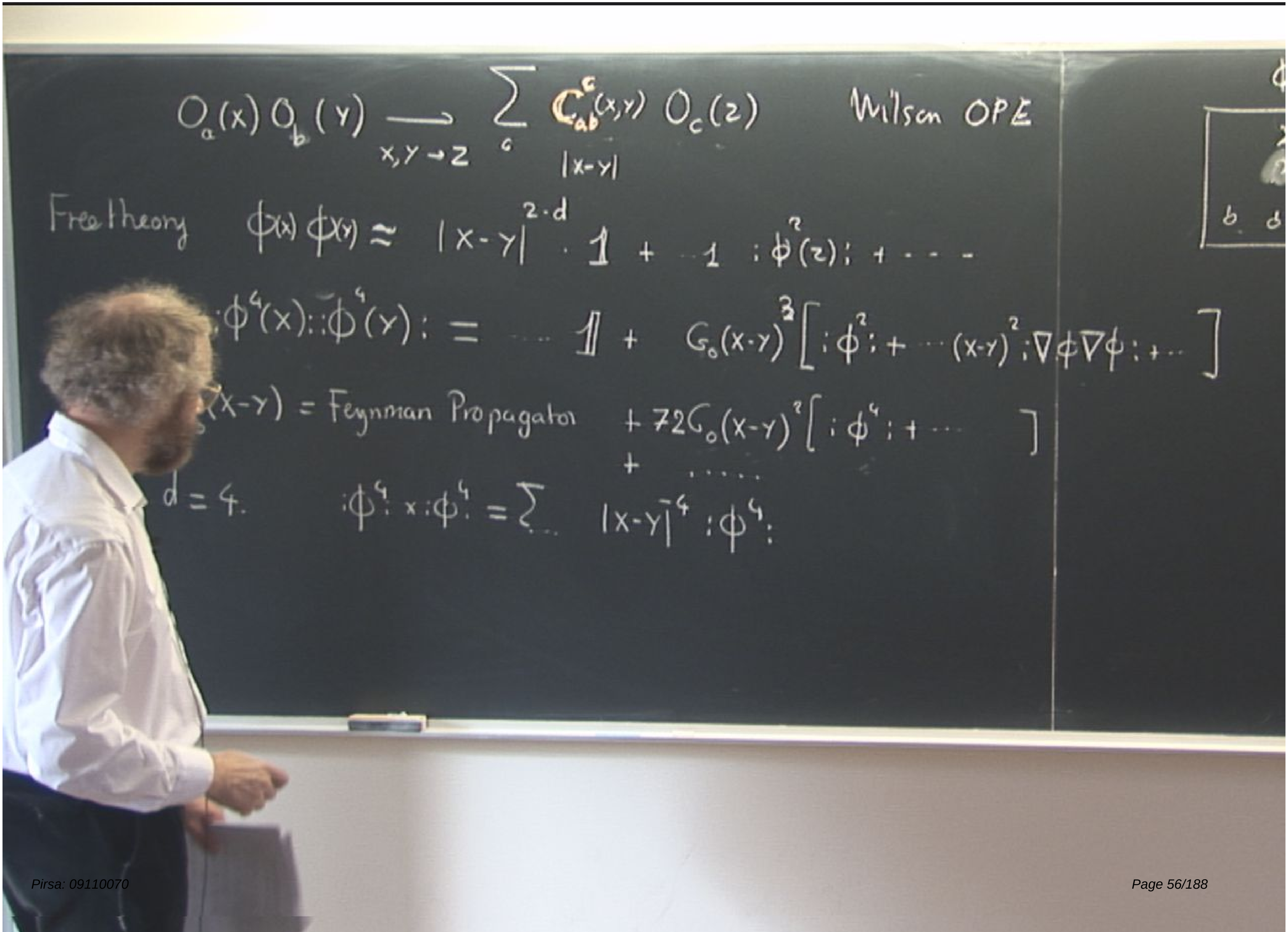
$$+ \quad \cdots\cdots$$

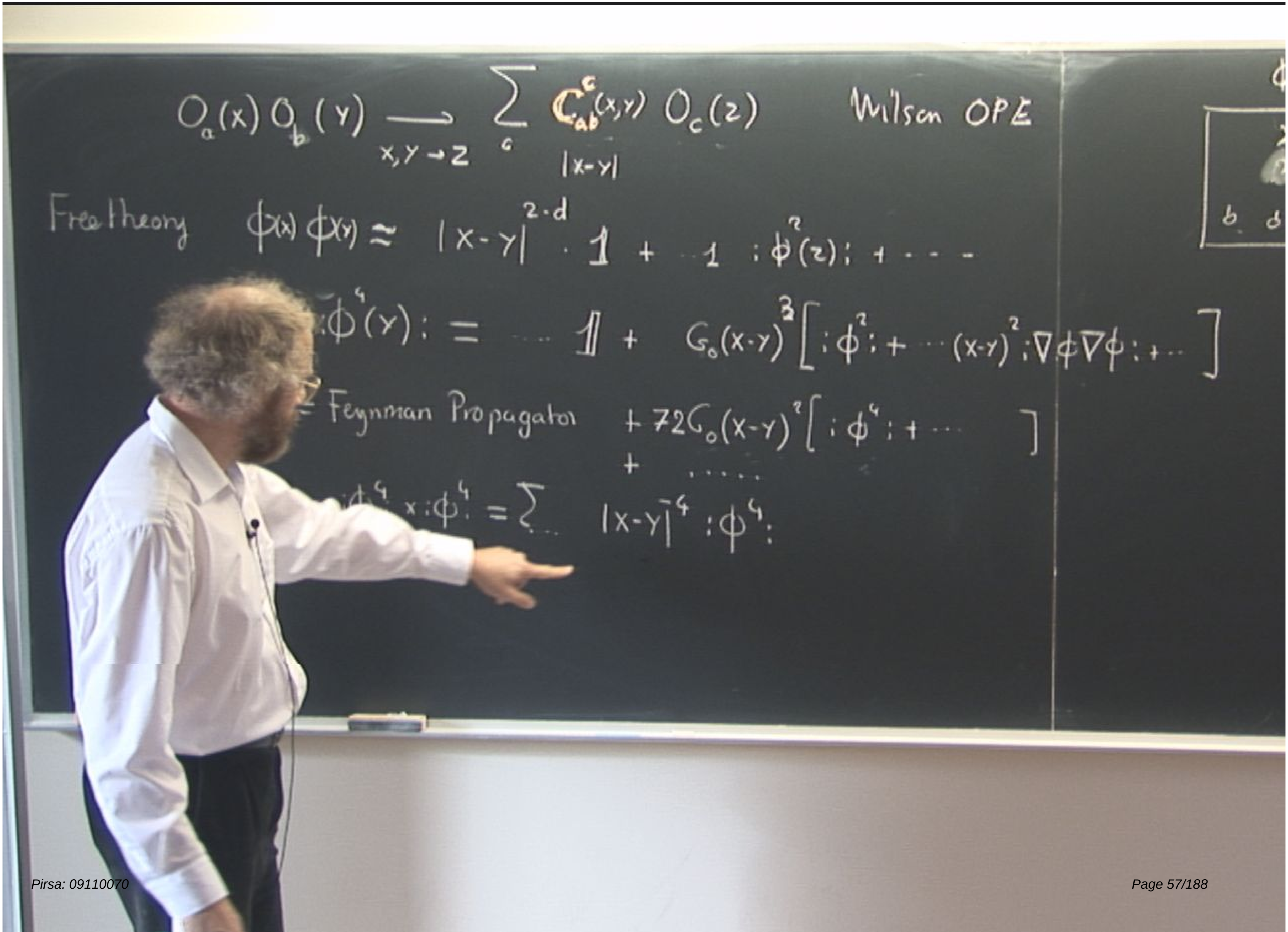

Wilson OPE
x,y → z
|x-y|
Free theory
Propagator



$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^{c}(x,y)\, O_c(z)$ Wilson OPE
$|x-y|$
Free theory $\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + \cdots 1 \; :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^3 \left[:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots \right]$
$G_0(x-y)$ = Feynman Propagator
$+ 72\, G_0(x-y)^2 \left[:\phi^4: + \cdots \right]$
$+ \cdots\cdots$
b


$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z)$
$|x-y|$
Wilson OPE
Free theory $\phi(x)\phi(y) \simeq |x-y|^{2-d} \cdot 1 + 1 \; :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y): = \cdots 1 + G_0(x-y)^3 [:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots]$
$+ 72 G_0(x-y)^2 [:\phi^4: + \cdots]$
$+ \cdots\cdots$
$G_0(x-y)$ = Feynman Propagator
$d = 4$.
b


Oa(x) Ob(y) → Σ Cab^c(x,y) Oc(z) Wilson OPE
x,y → z c |x-y|
Free theory φ(x) φ(y) ≈ |x-y|^(2-d) · 1 + 1 :φ²(z): + - - -
:φ⁴(x)::φ⁴(y): = 1 + G₀(x-y)³ [:φ²: + (x-y)² :∇φ∇φ: + ..]
(x-y) = Feynman Propagator + 72G₀(x-y)² [:φ⁴: + ...]
+ .....
d = 4. :φ⁴: x :φ⁴: = Σ |x-y|⁻⁴ :φ⁴:


Wilson OPE
Free theory
Feynman Propagator

$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z) \qquad \text{Wilson OPE}$$

$$|x-y|$$

Free theory $$\phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + 1 \; :\phi^2(z): + \cdots$$

$$:\phi^4(x)::\phi^4(y): = \cdots \mathbb{1} + G_0(x-y)^3 \left[:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots \right]$$

$$+ 72\, G_0(x-y)^2 \left[:\phi^4: + \cdots \right]$$

$$+ \cdots\cdots$$

$G_0(x-y)$ = Feynman Propagator

$d = 4.$ $$:\phi^4: \times :\phi^4: = \sum |x-y|^{-4} :\phi^4:$$

$$O_a(x)\, O_b(y) \xrightarrow[x,y \to z]{} \sum_c C_{ab}^c(x,y)\, O_c(z) \qquad \text{Wilson OPE}$$

$$|x-y|$$

Free theory $\quad \phi(x)\,\phi(y) \approx |x-y|^{2-d} \cdot \mathbb{1} + 1 \; :\phi^2(z): + \cdots$

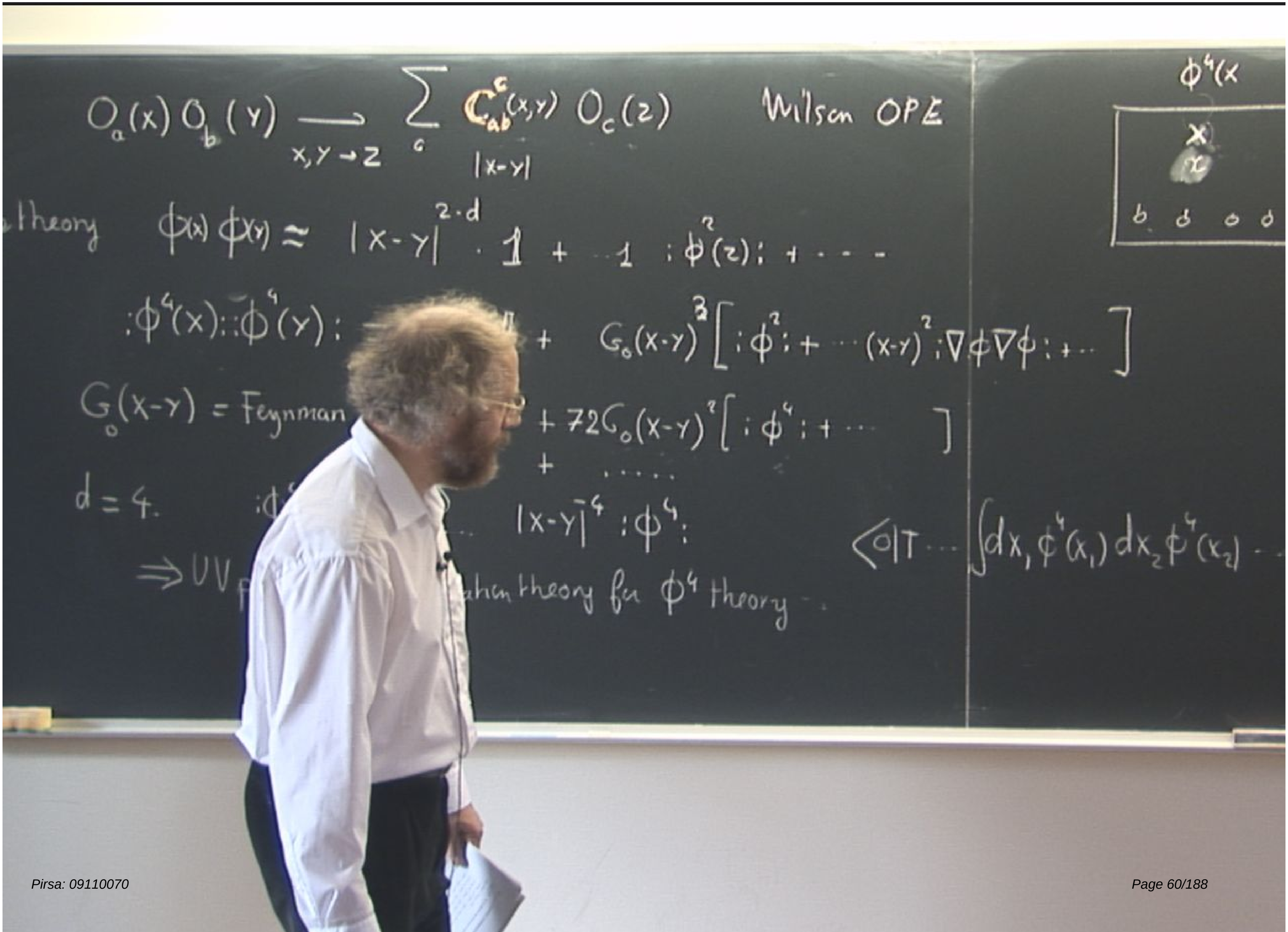
$$:\phi^4(x)::\phi^4(y): \; = \; \cdots \mathbb{1} + G_0(x-y)^3 \left[:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots \right]$$

$G_0(x-y) =$ Feynman Propagator $\quad + \, 72\, G_0(x-y)^2 \left[:\phi^4: + \cdots \right]$

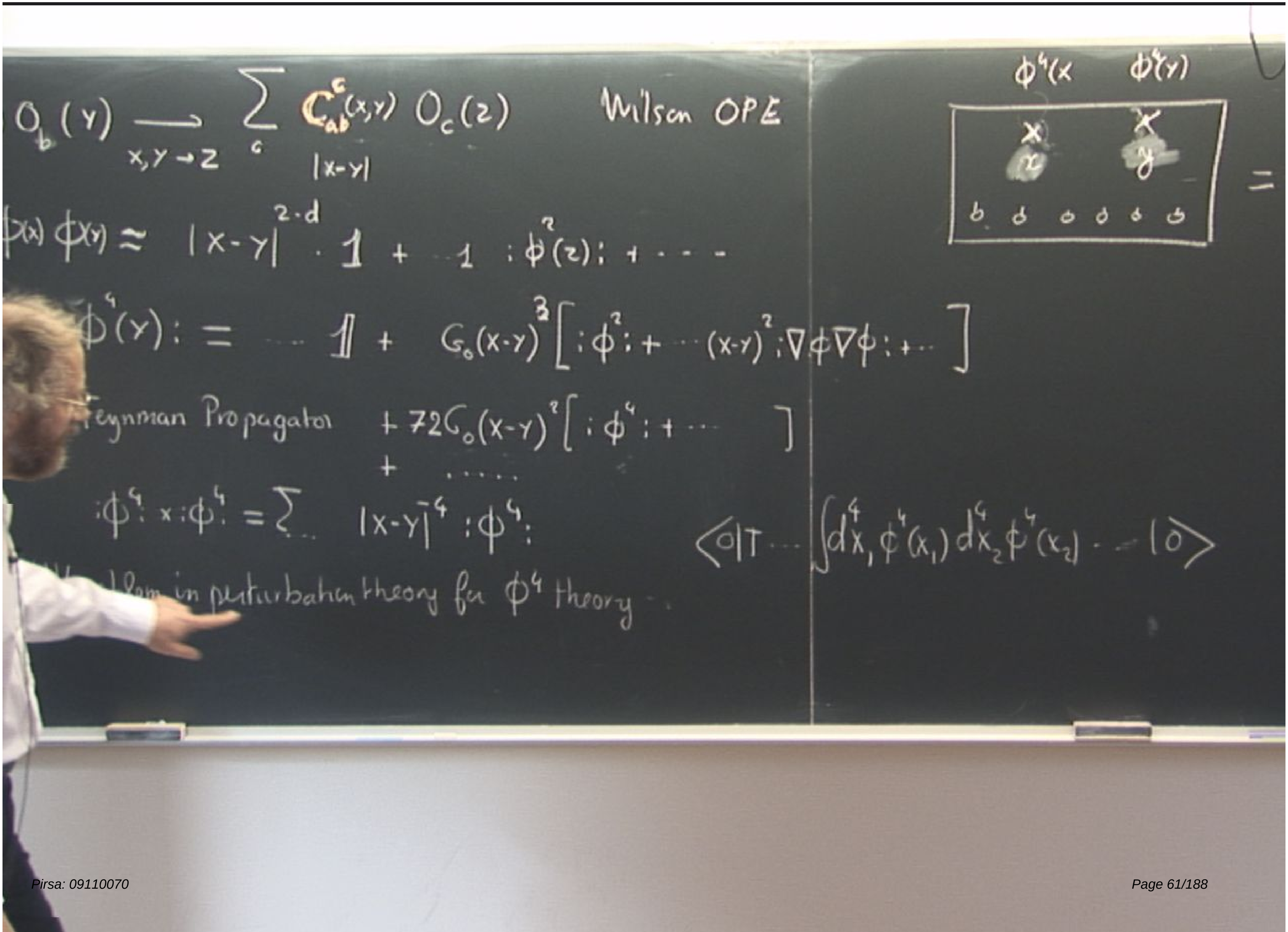
$$+ \; \cdots\cdots$$

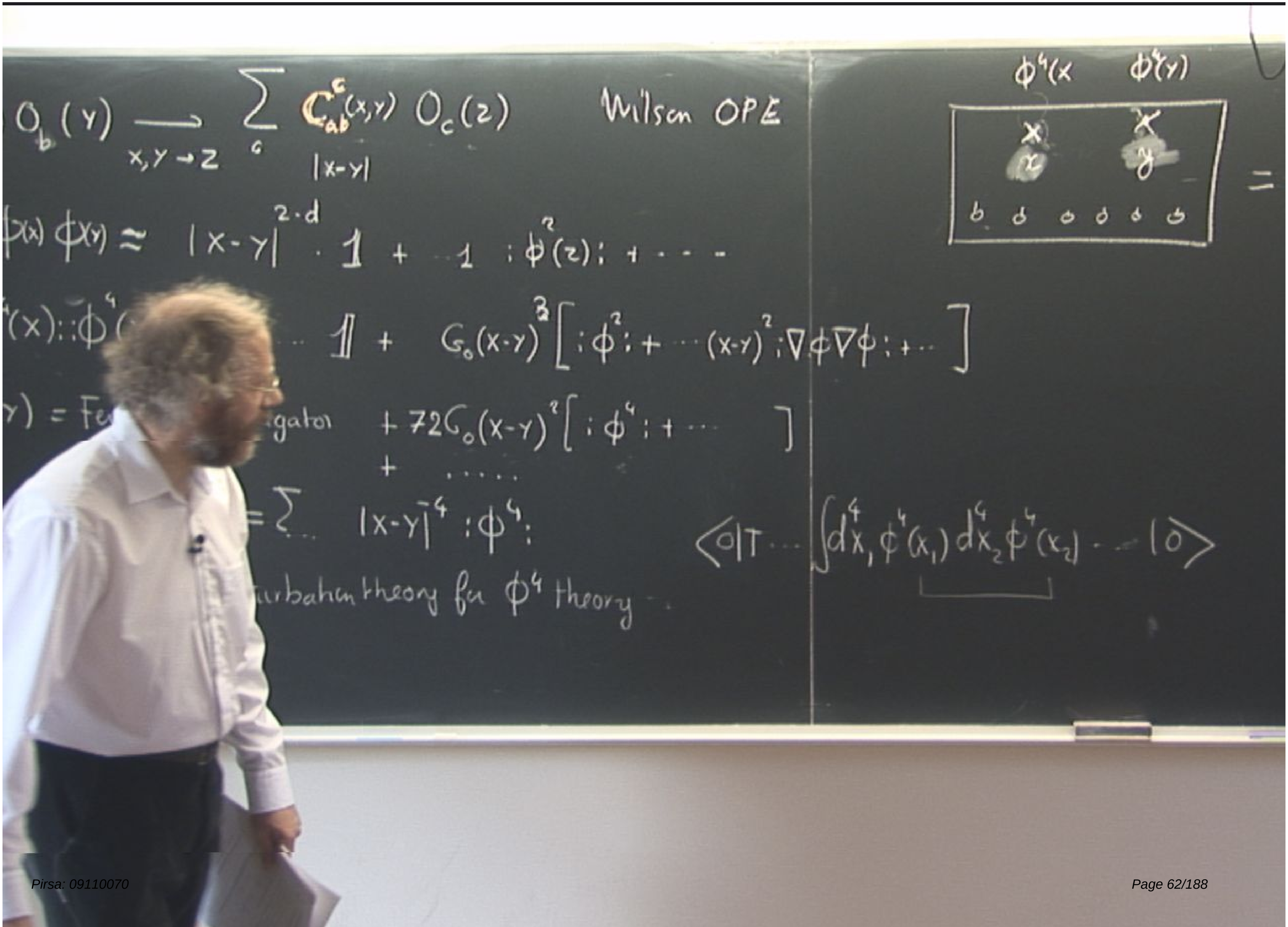
$d = 4. \qquad :\phi^4: \times :\phi^4: = \sum \; |x-y|^{-4} :\phi^4:$

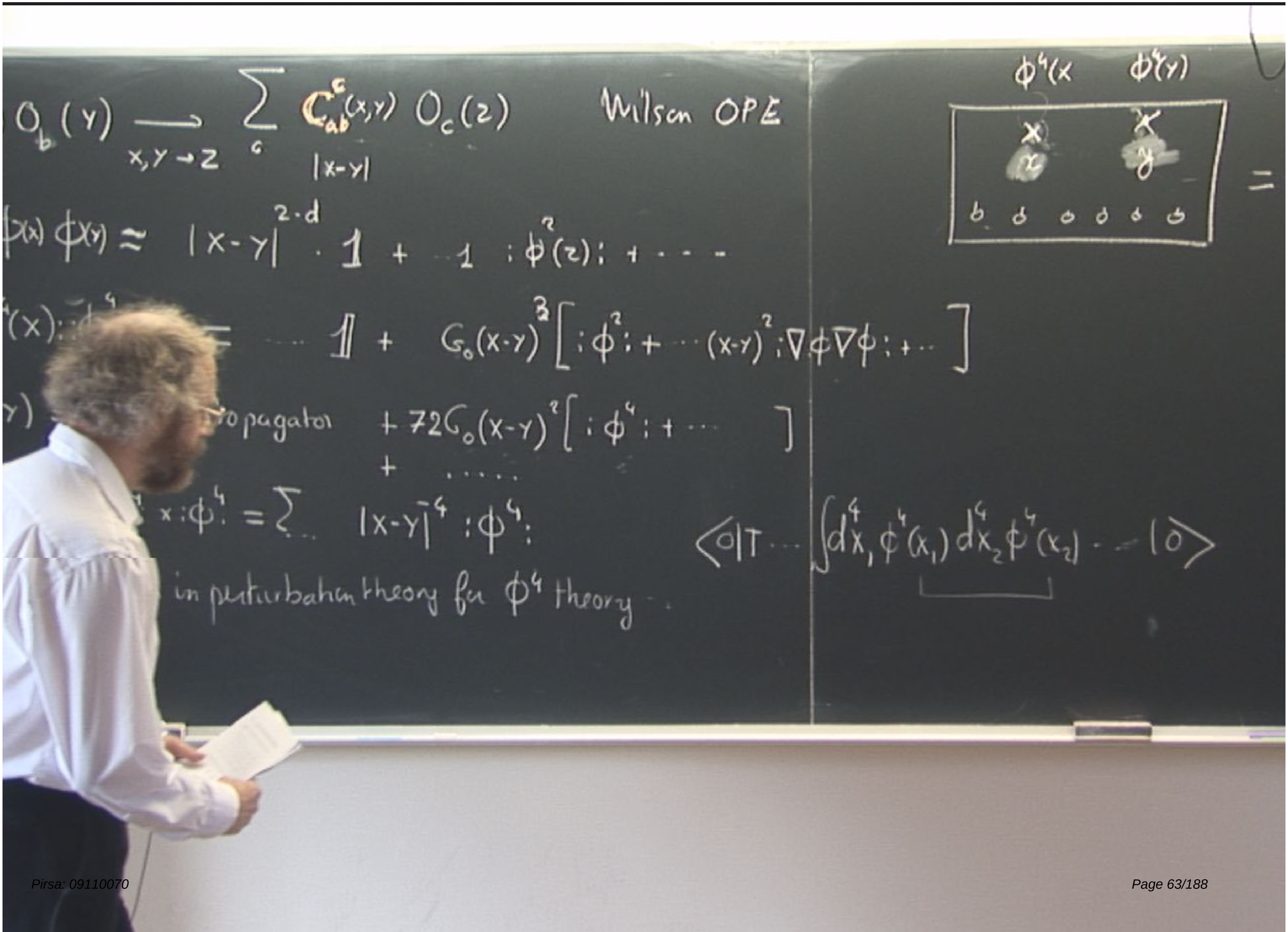
$\Rightarrow$ UV problem in perturbation theory for ϕ^4 theory

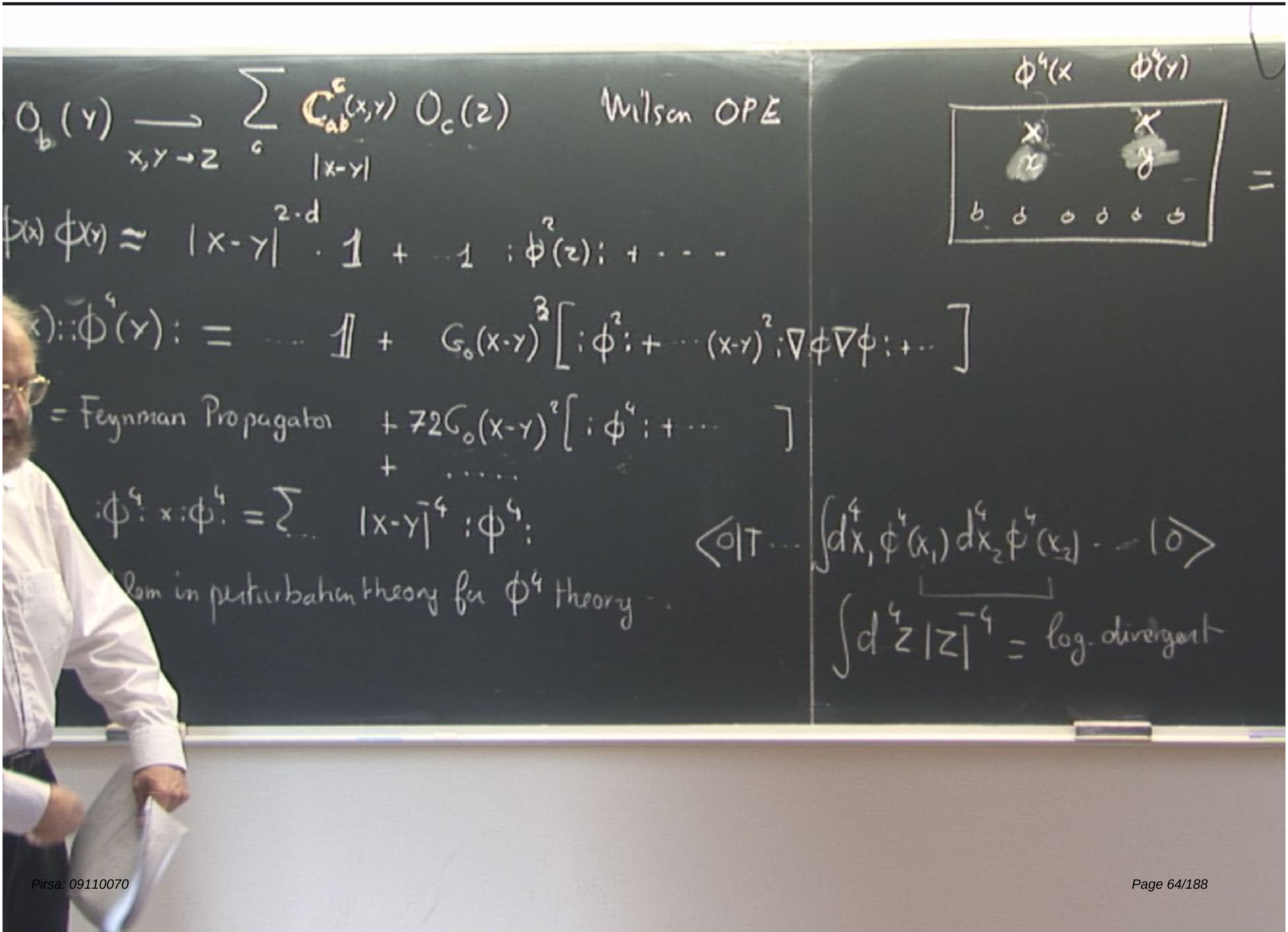


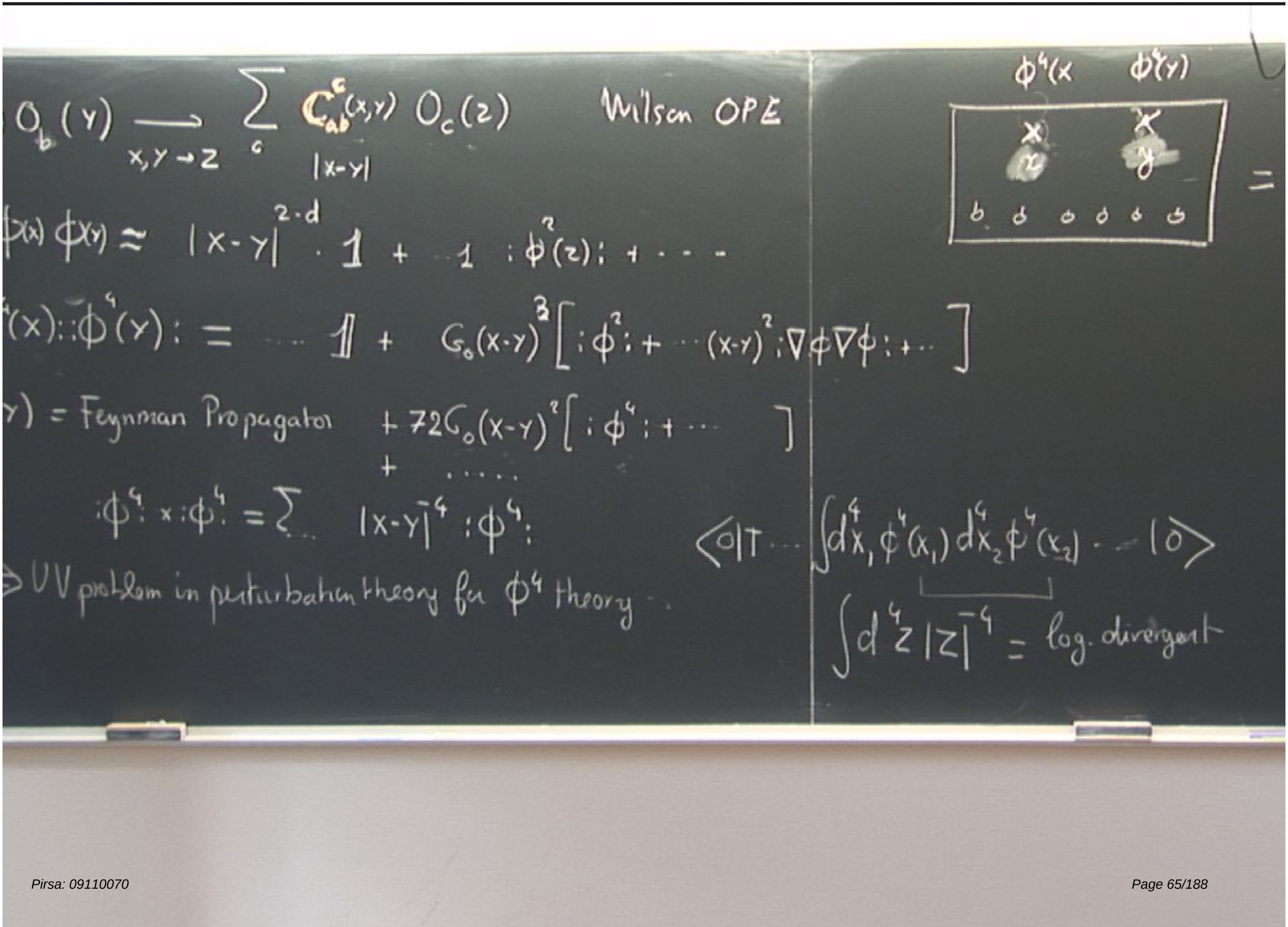
$O_a(x)\,O_b(y) \xrightarrow[x,y\to z]{} \sum_c C_{ab}^c(x,y)\,O_c(z)$
Wilson OPE
$|x-y|$
theory $\phi(x)\phi(y) \approx |x-y|^{2-d}\cdot 1 + 1\ :\phi^2(z): + \cdots$
$:\phi^4(x)::\phi^4(y):$
$+\ G_0(x-y)^3\,[\,:\phi^2: + \cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots\,]$
$G_0(x-y)$ = Feynman
$+\ 72\,G_0(x-y)^2\,[\,:\phi^4: + \cdots\,]$
$+\ \cdots$
$d = 4.$
$|x-y|^{-4}\ :\phi^4:$
$\Rightarrow$ UV
theory for ϕ^4 theory
$\langle 0|T \cdots \int dx_1\,\phi^4(x_1)\,dx_2\,\phi^4(x_2) \cdots$
$\phi^4(x$

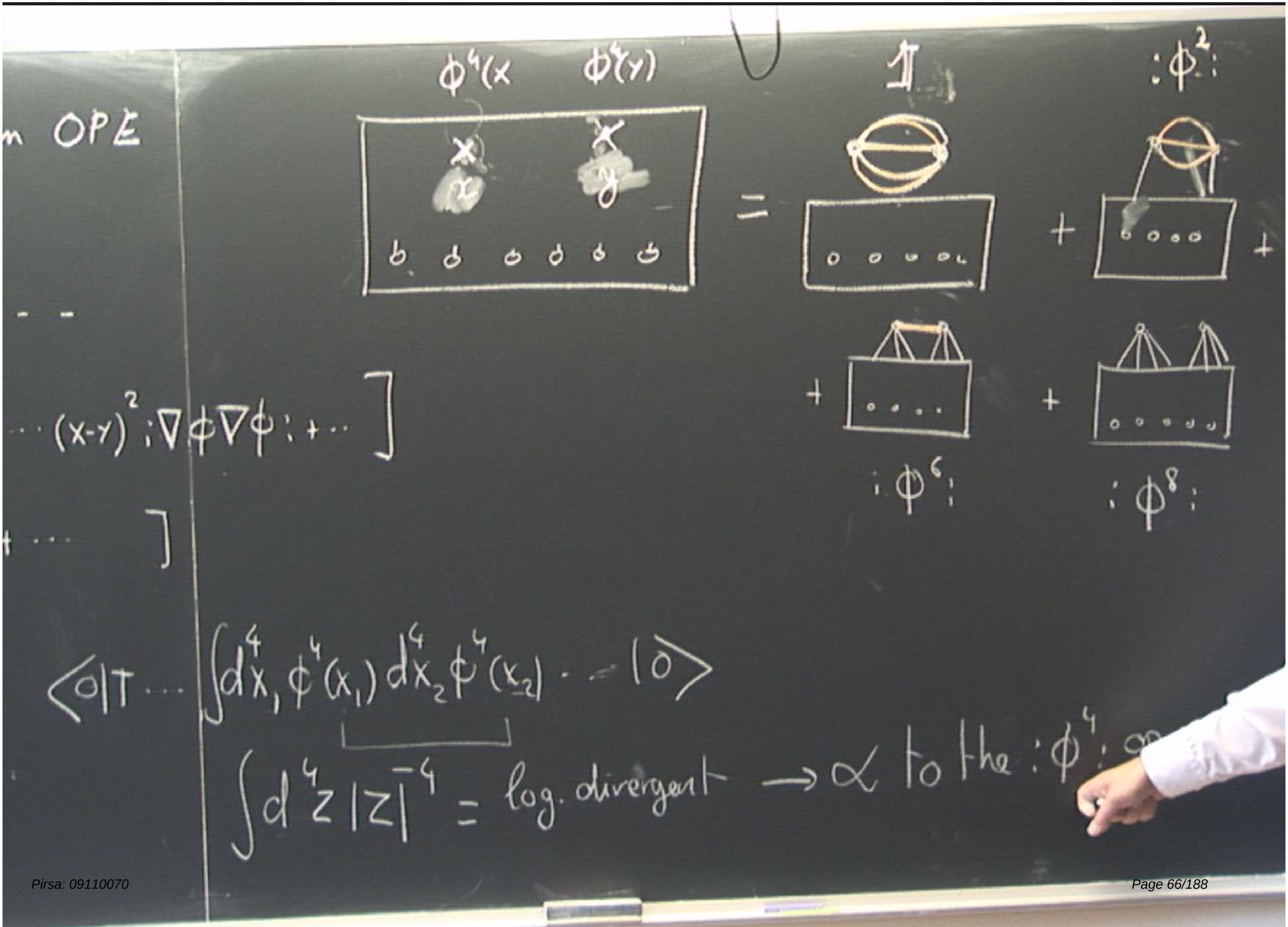

Wilson OPE
x,y → z
|x-y|
Feynman Propagator
in perturbation theory for φ⁴ theory

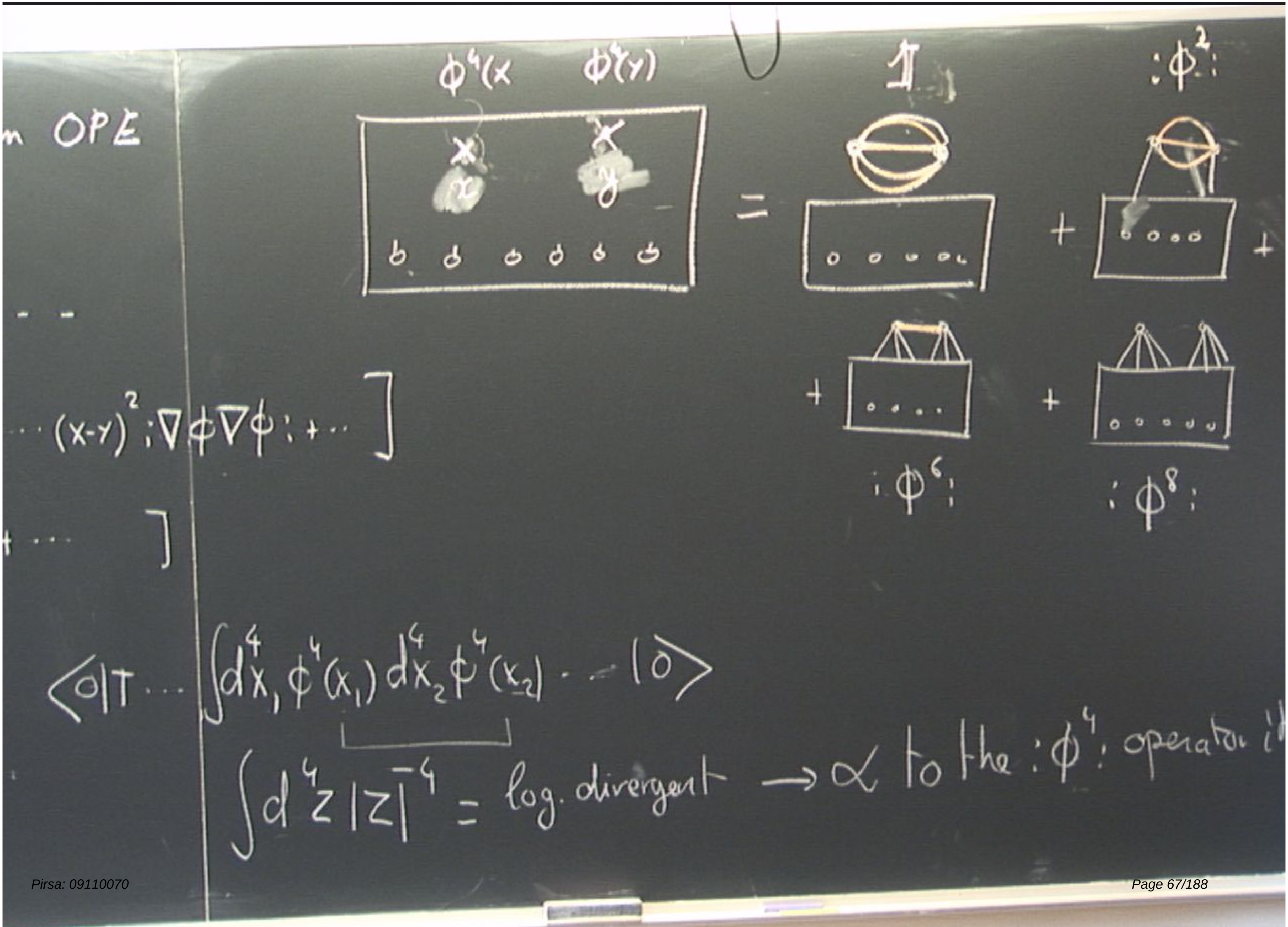

Wilson OPE
x, y → z
|x-y|
+ 72 $G_0(x-y)^2 [:\phi^4: + \cdots]$
Perturbation theory for ϕ^4 theory
$\langle 0 | T \cdots \int d^4x_1 \phi^4(x_1) d^4x_2 \phi^4(x_2) \cdots | 0 \rangle$
$\phi^4(x$
$\phi^4(y)$

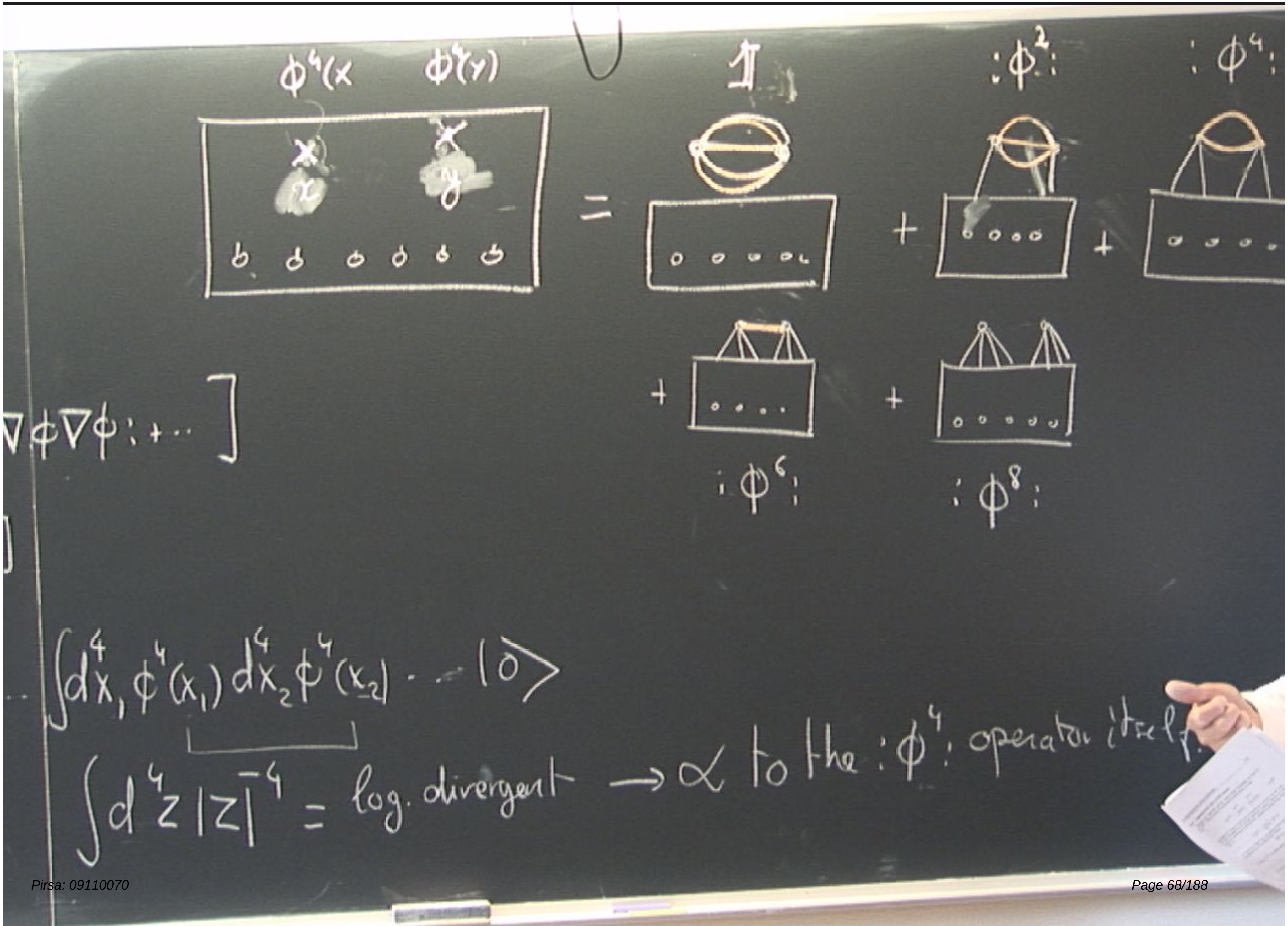

Wilson OPE
x, y → z
|x-y|
propagator
in perturbation theory for ϕ⁴ theory
⟨0|T ∫d⁴x₁ ϕ⁴(x₁) d⁴x₂ ϕ⁴(x₂) |0⟩

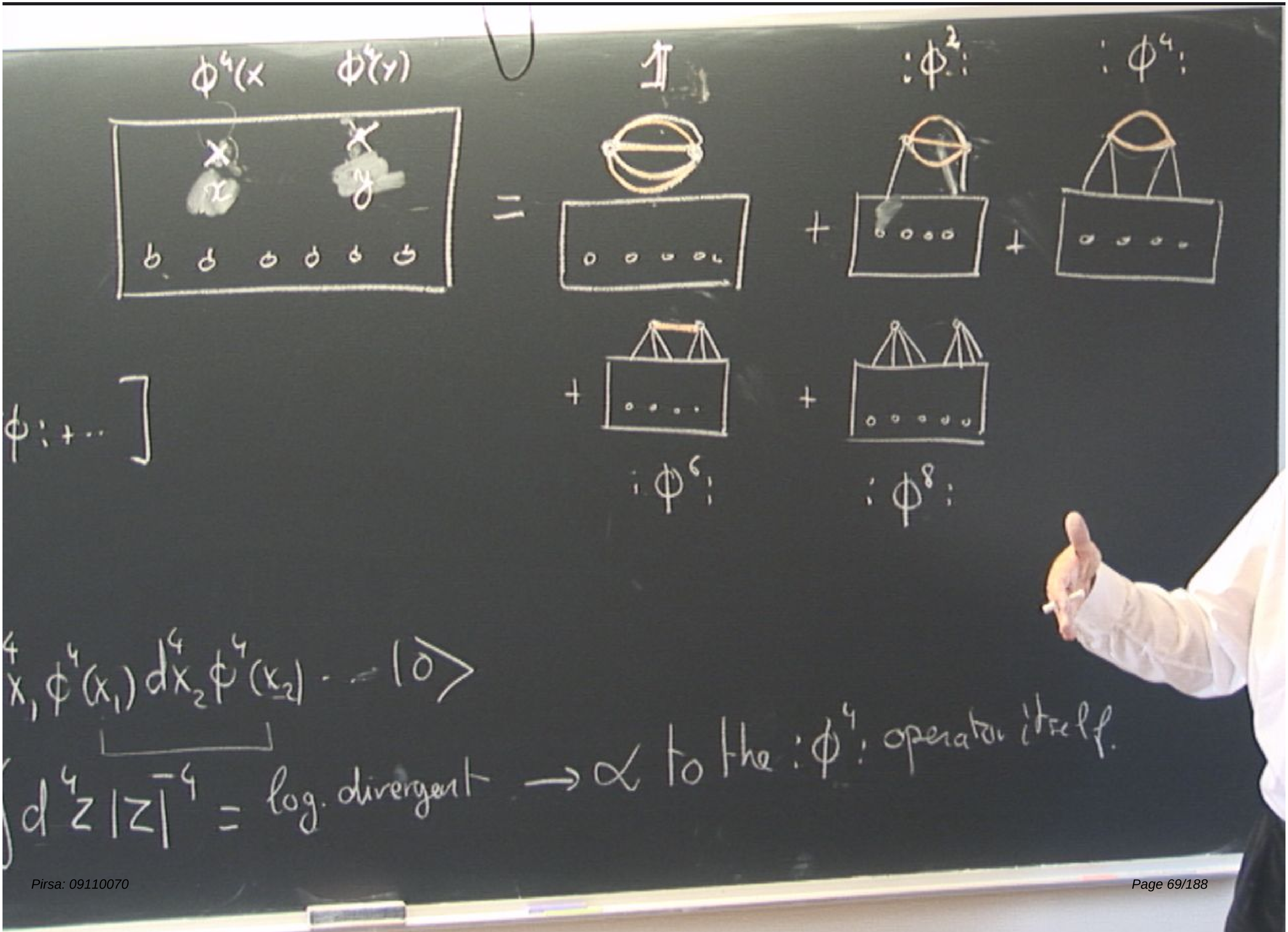

O_b (y) ⟶ Σ_c C^c_ab(x,y) O_c(z) Wilson OPE
x,y → z |x-y|
φ(x) φ(y) ≈ |x-y|^(2-d) · 1 + 1 :φ²(z): + . . .
:φ⁴(y): = 1 + G₀(x-y)³ [:φ²: + (x-y)² :∇φ∇φ: + . .]
= Feynman Propagator + 72G₀(x-y)² [:φ⁴: + . . .]
+ . . . . .
:φ⁴: x :φ⁴: = Σ |x-y|⁻⁴ :φ⁴:
in perturbation theory for φ⁴ theory
φ⁴(x φ⁴(y)
x x
b
⟨0|T . . . ∫d⁴x₁ φ⁴(x₁) d⁴x₂ φ⁴(x₂) . . . |0⟩
∫d⁴z |z|⁻⁴ = log. divergent

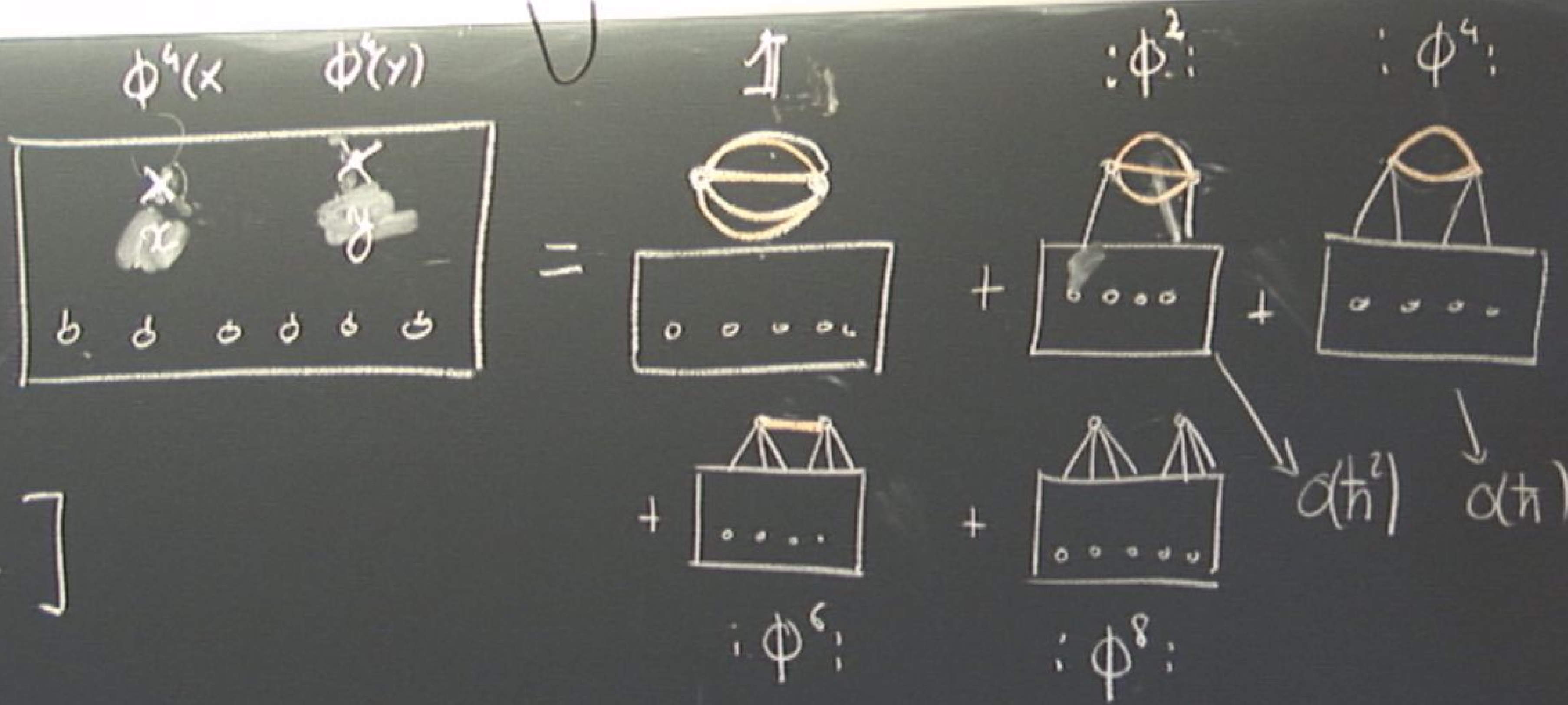

O_b(y) → (x,y → z) Σ_c C_ab^c(x,y) O_c(z)   Wilson OPE
|x-y|
φ(x) φ(y) ≃ |x-y|^(2-d) · 1 + 1 :φ²(z): + · · · -
(x)::φ⁴(y): = ··· 1 + G₀(x-y)³ [:φ²: + ··· (x-y)² :∇φ∇φ: + ··]
y) = Feynman Propagator   + 72 G₀(x-y)² [:φ⁴: + ···]
+ . . . . . .
:φ⁴: x :φ⁴: = Σ |x-y|⁻⁴ :φ⁴:
UV problem in perturbation theory for φ⁴ theory · ·
φ⁴(x   φ⁴(y)
x   y
⟨0|T ··· ∫d⁴x₁ φ⁴(x₁) d⁴x₂ φ⁴(x₂) ··· |0⟩
∫d⁴z |z|⁻⁴ = log. divergent


n OPE
$\phi^4(x$ $\phi^4(y)$
x
y
$\mathbb{1}$
$:\phi^2:$
$:\phi^6:$
$:\phi^8:$
$\cdots (x-y)^2 :\nabla\phi\nabla\phi: + \cdots]$
$+ \cdots]$
$\langle 0|T \cdots \int d^4x_1 \phi^4(x_1) d^4x_2 \phi^4(x_2) \cdots |0\rangle$
$\int d^4z |z|^{-4}$ = log. divergent $\rightarrow \propto$ to the $:\phi^4:$


n OPE
φ⁴(x φ⁴(y)
x y
𝟙 :φ²:
:φ⁶: :φ⁸:
··· (x-y)² :∇φ∇φ: + ···]
+ ···]
⟨0|T ··· ∫d⁴x₁ φ⁴(x₁) d⁴x₂ φ⁴(x₂) ··· |0⟩
∫d⁴z |z|⁻⁴ = log. divergent → ∝ to the :φ⁴: operator i


$\phi^4(x$ $\phi^4(y)$
$\mathbb{1}$ $:\phi^2:$ $:\phi^4:$
$:\phi^6:$ $:\phi^8:$
$\nabla\phi\nabla\phi: + \cdots]$
$\int d^4x_1 \phi^4(x_1) d^4x_2 \phi^4(x_2) \cdots |0\rangle$
$\int d^4z |z|^{-4}$ = log. divergent $\rightarrow \propto$ to the $:\phi^4:$ operator itself.


ϕ⁴(x ϕ⁴(y)
x y
=
𝟙 :ϕ²: :ϕ⁴:
+ +
+ +
:ϕ⁶: :ϕ⁸:
ϕ: + ...]
x₁ ϕ⁴(x₁) d⁴x₂ ϕ⁴(x₂) ... |0>
∫d⁴z |z|⁻⁴ = log. divergent → ∝ to the :ϕ⁴: operator itself.



$\phi: + \cdots \,]$

$^4x_1\, \phi^4(x_1)\, d^4x_2\, \phi^4(x_2) \cdots |0\rangle$

$\int d^4z\, |z|^{-4}$ = log. divergent $\longrightarrow \propto$ to the $:\phi^4:$ operator itself.

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

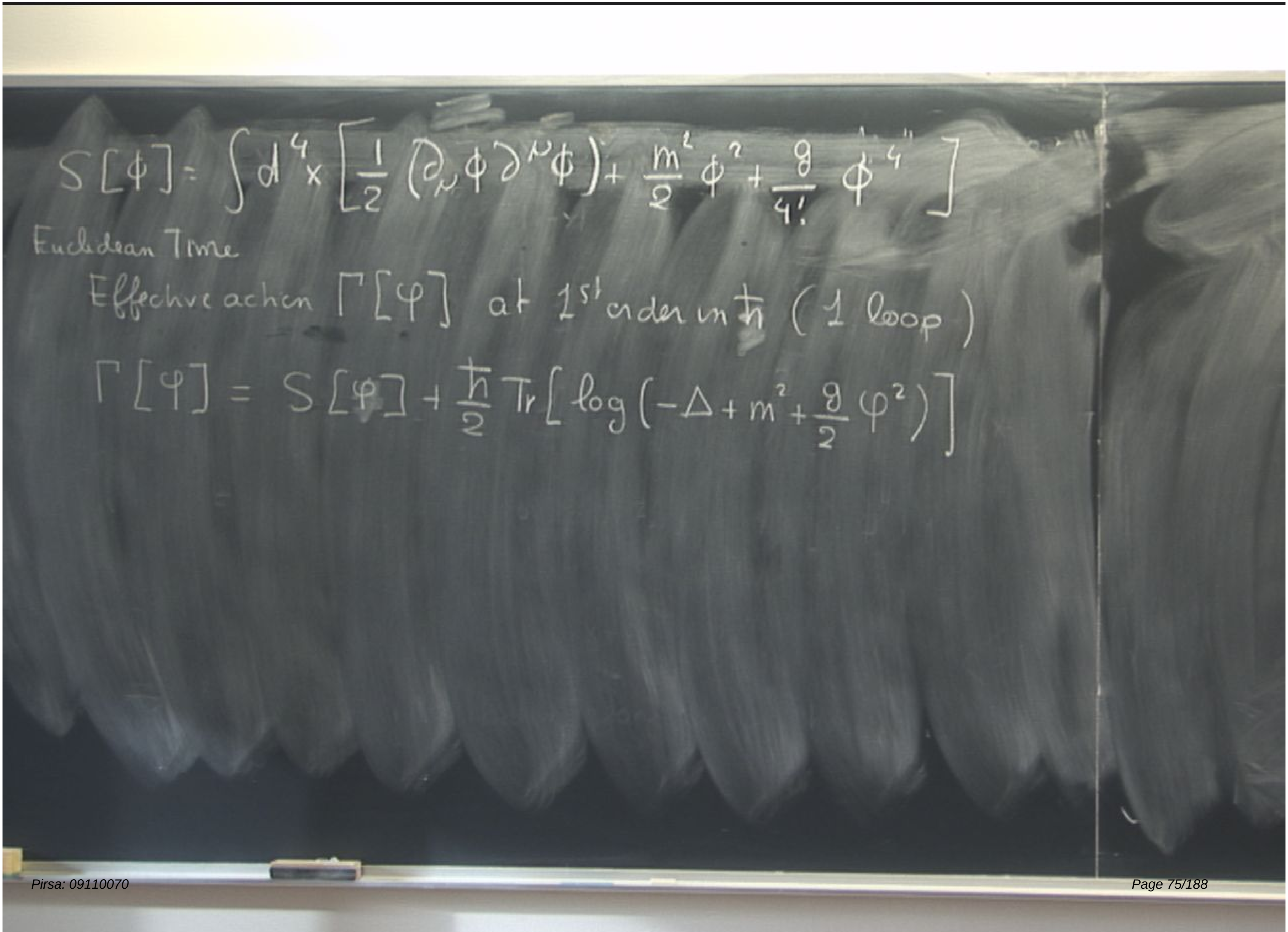
Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)


$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$
Euclidean Time
Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)
$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right]$

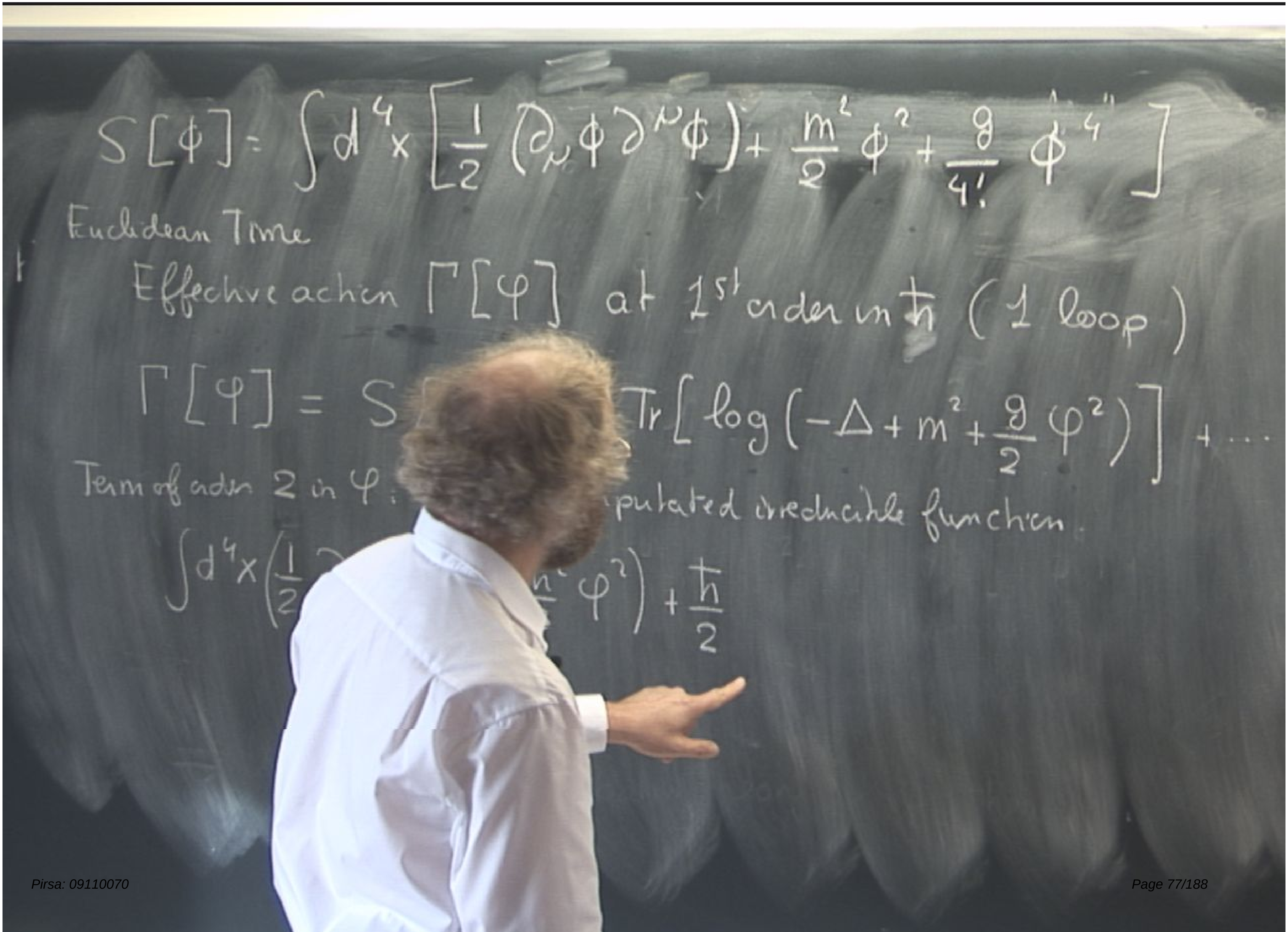
$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\nu \phi \partial^\nu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right) \right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.


$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$
Euclidean Time
Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)
$\Gamma[\varphi] = S$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right) \right] + \dots$
Term of order 2 in φ :
putated irreducible function.
$\int d^4x \left(\frac{1}{2}\right.$
$\left.\varphi^2\right) + \frac{\hbar}{2}$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1st order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xx} \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \right]_{xy} \varphi(y)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2 \right.\right.$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(y)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} \left(\partial_\mu \phi \, \partial^\mu \phi \right) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr} \left[\log \left(-\Delta + m^2 + \frac{g}{2} \varphi^2 \right) \right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x) \left[\left(-\Delta + m^2 \right) + \hbar \frac{g}{2} \right] \varphi(y)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2} \varphi^2 \right) \right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(y)$$

$$\Gamma^{(2)}(x,y)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1st order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right) \right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(y)$$

$\Gamma^{(2)}(x,y)$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$$

Euclidean Time

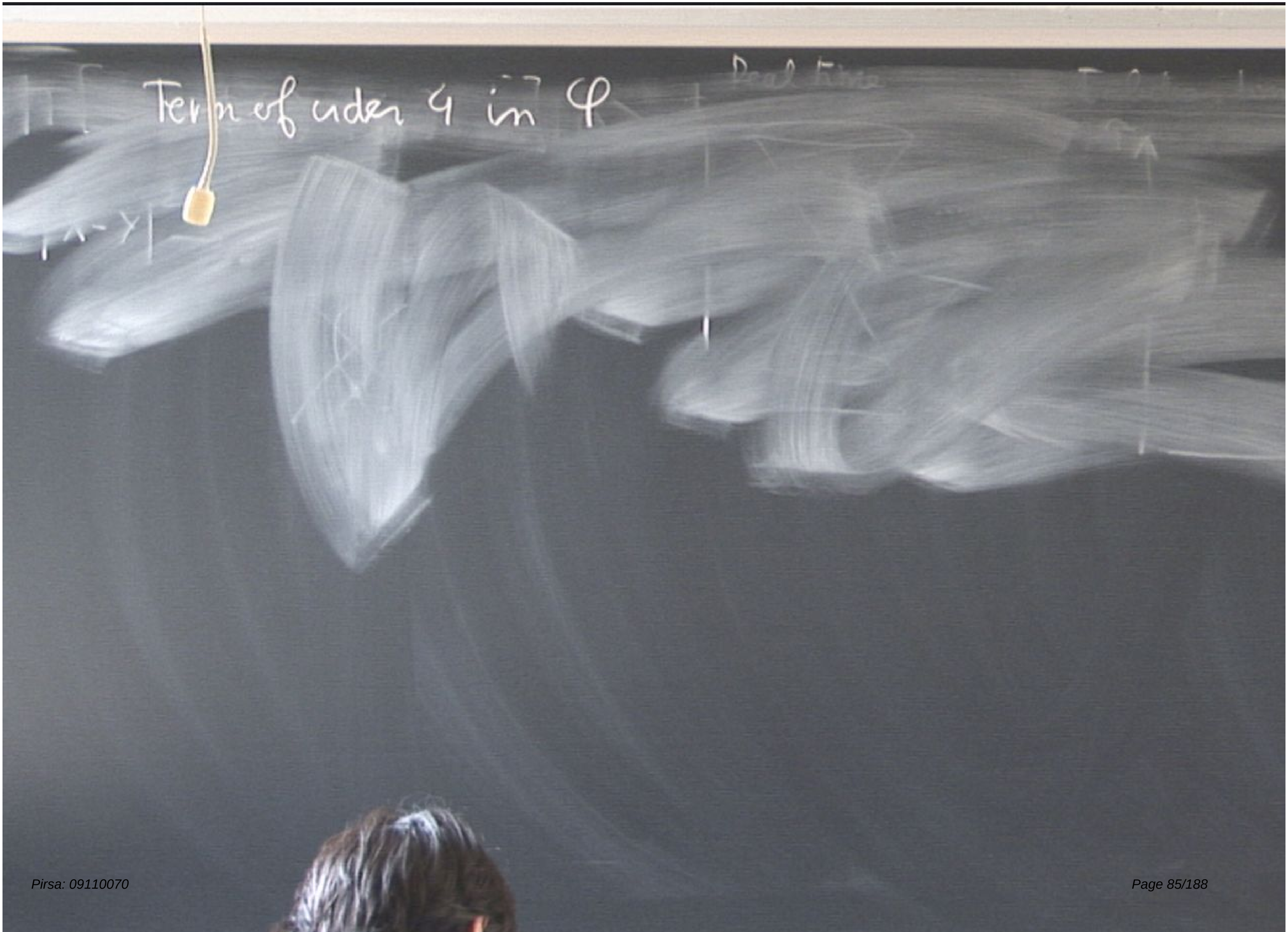
Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2}\varphi^2 \right) + \frac{\hbar}{2}\frac{g}{2} \underbrace{\int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xx}} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x) \underbrace{\left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right]}_{\Gamma^{(2)}(x,y)} \varphi(y)$$


Term of order 4 in φ

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2} \varphi^2\right) \right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(y)$$

$\Gamma^{(2)}(x,y)$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

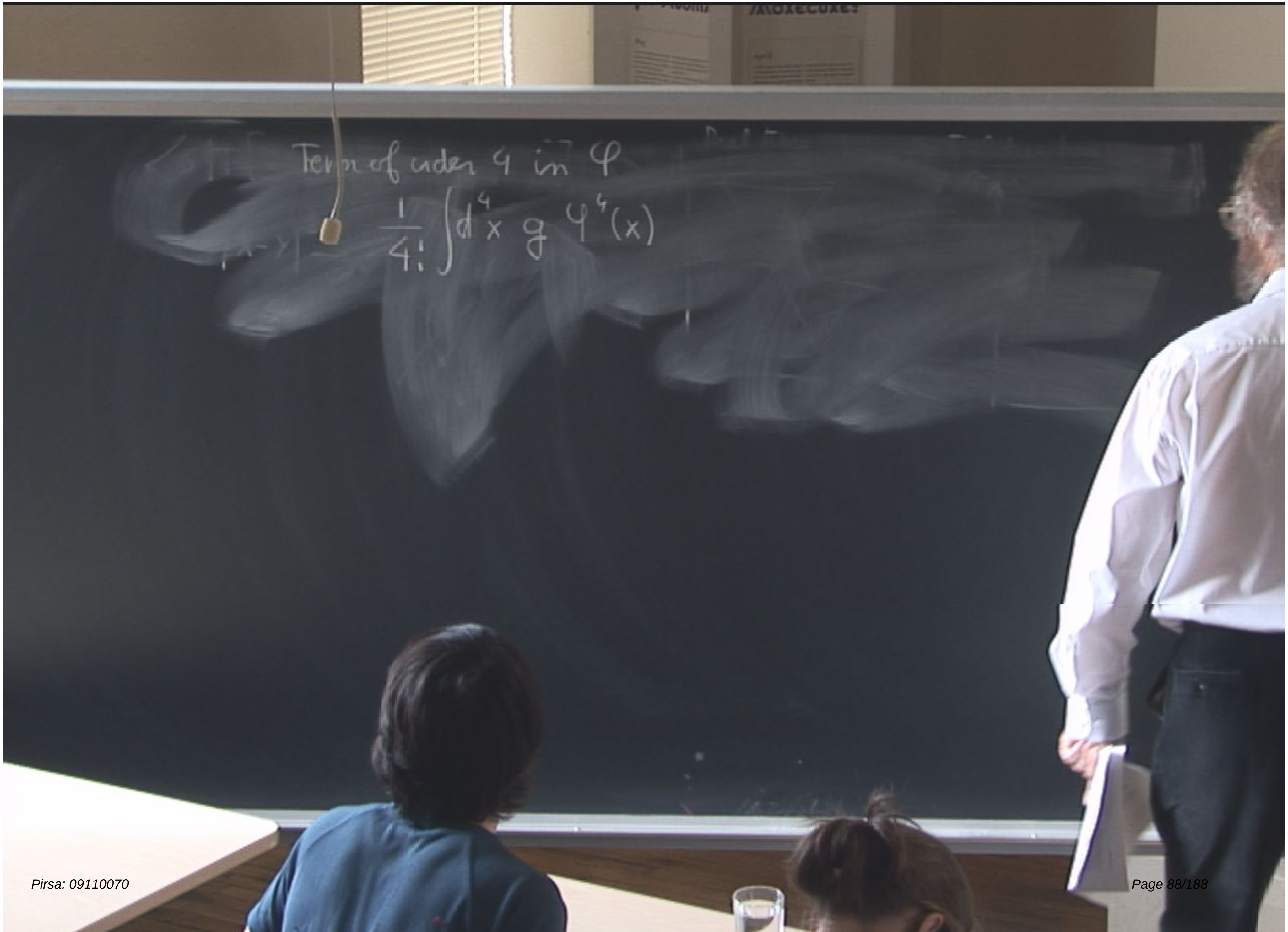
Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

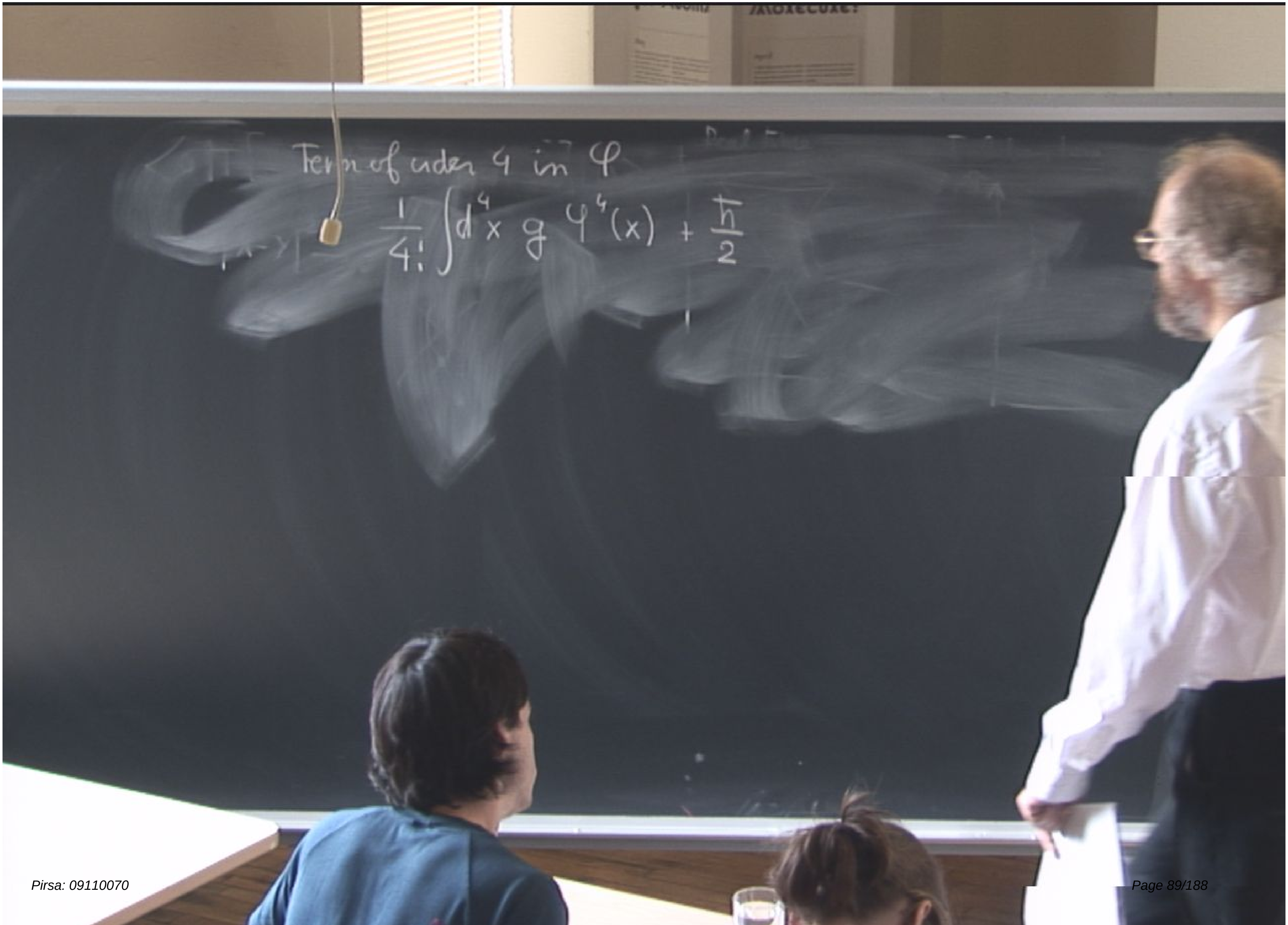
$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

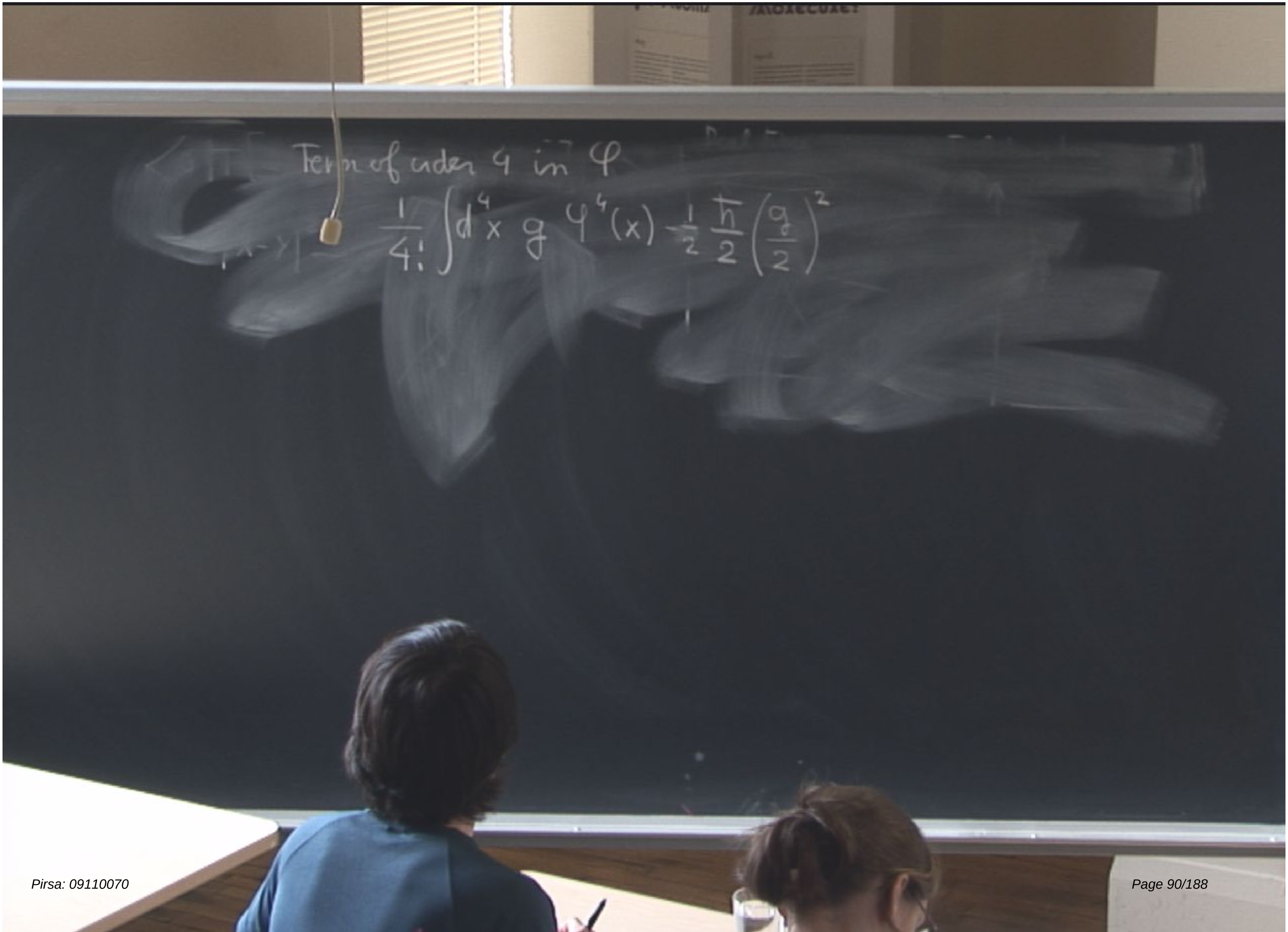
Term of order 2 in φ: 2 point amputated irreducible function.

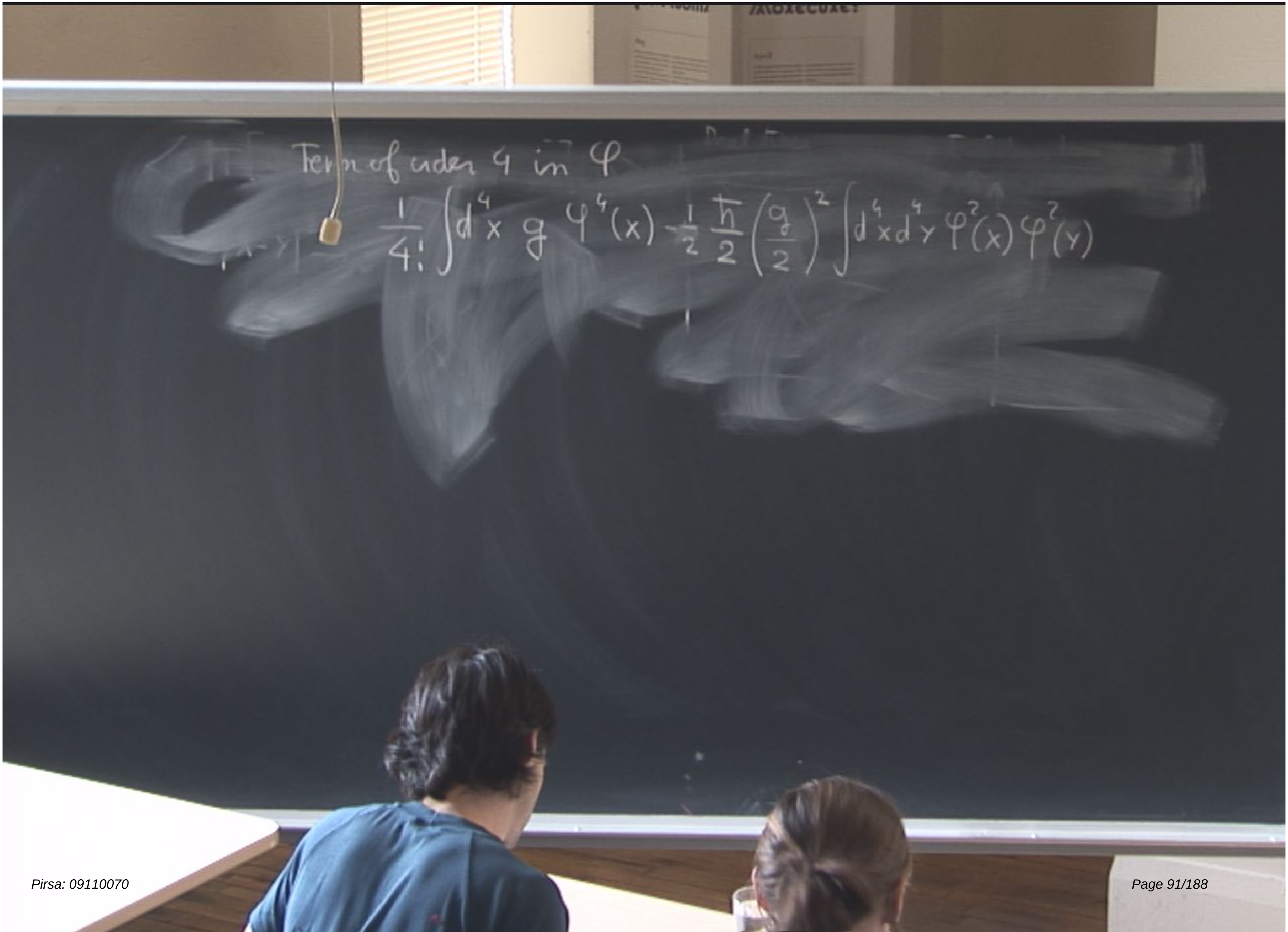
$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

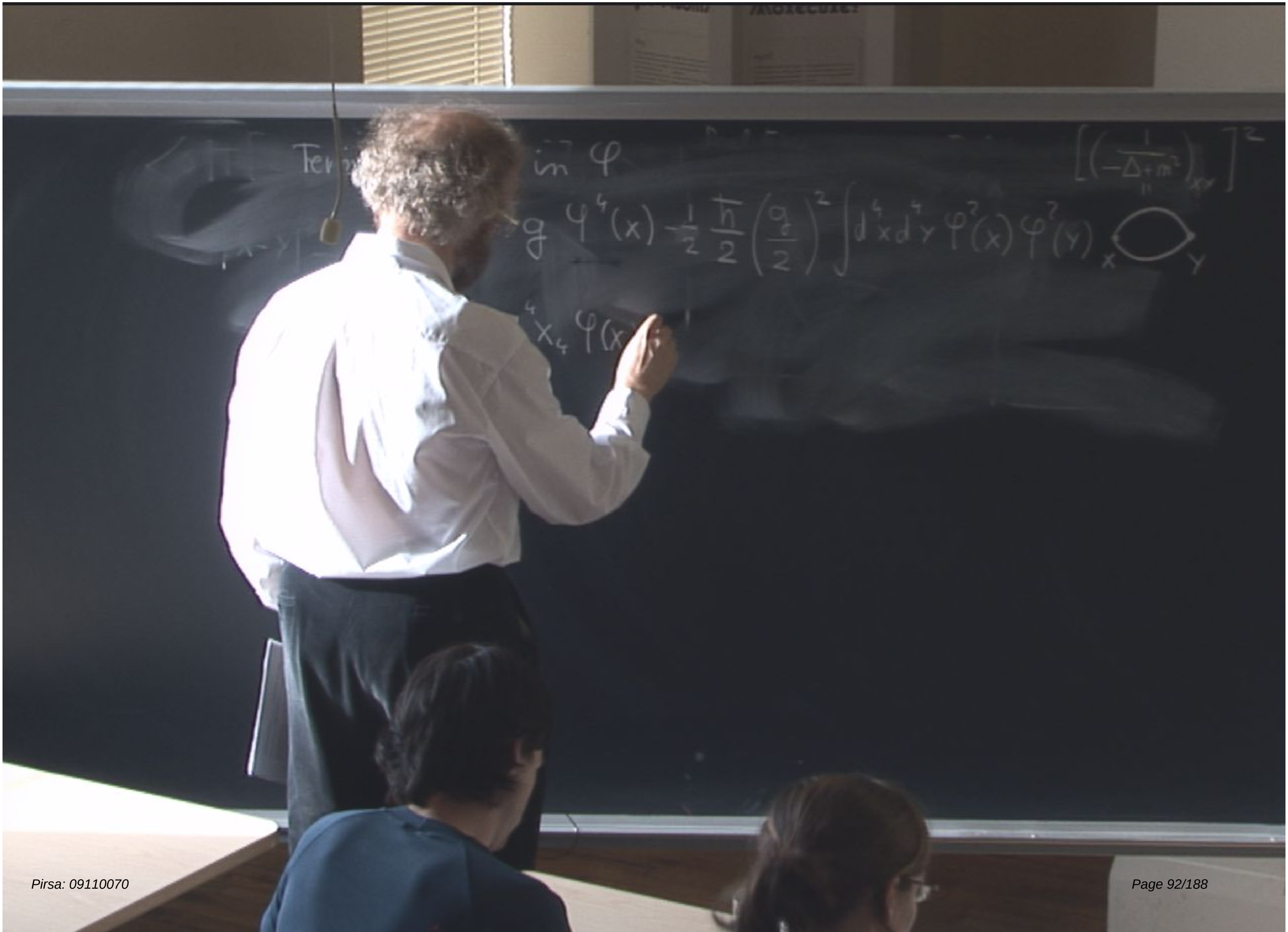
$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x) \underbrace{\left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right]}_{\Gamma^{(2)}(x,y)} \varphi(y)$$

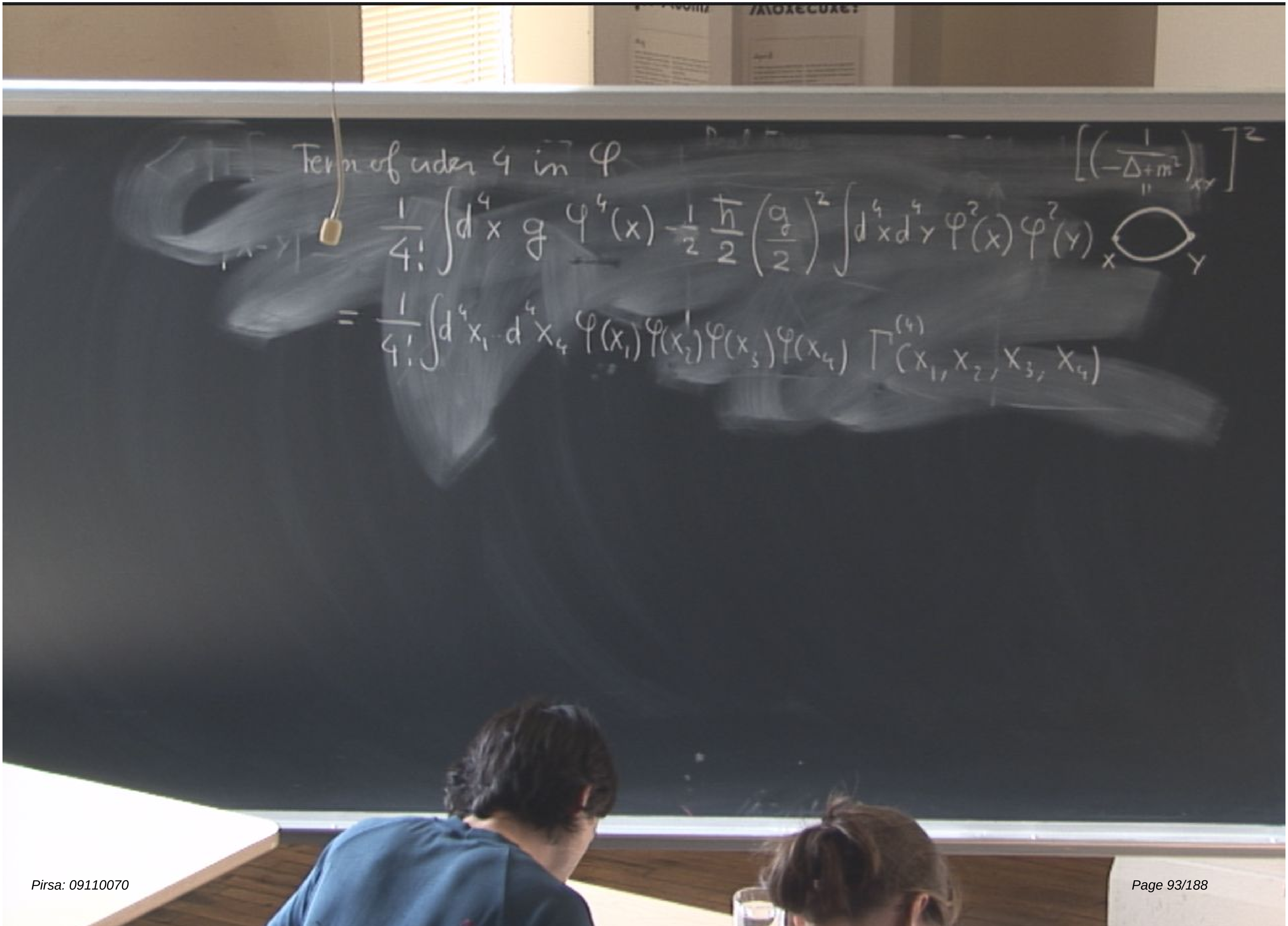

Term of order 4 in φ
1/4! ∫d⁴x g φ⁴(x)


Term of order 4 in φ
1/4! ∫d⁴x g φ⁴(x) + ħ/2


Term of order 4 in φ
1/4! ∫d⁴x g φ⁴(x) − 1/2 ħ/2 (g/2)²


Term of order 4 in φ
1/4! ∫d⁴x g φ⁴(x) − 1/2 ħ/2 (g/2)² ∫d⁴x d⁴y φ²(x) φ²(y)


in φ
g φ⁴(x) − 1/2 ħ/2 (g/2)² ∫ d⁴x d⁴y φ²(x) φ²(y)
x
y


Term of order 4 in φ
$\frac{1}{4!}\int d^4x\, g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int d^4x\, d^4y\, \varphi^2(x)\varphi^2(y)$
$\left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$
x
y
$= \frac{1}{4!}\int d^4x_1 \cdots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\, \Gamma^{(4)}(x_1, x_2, x_3, x_4)$

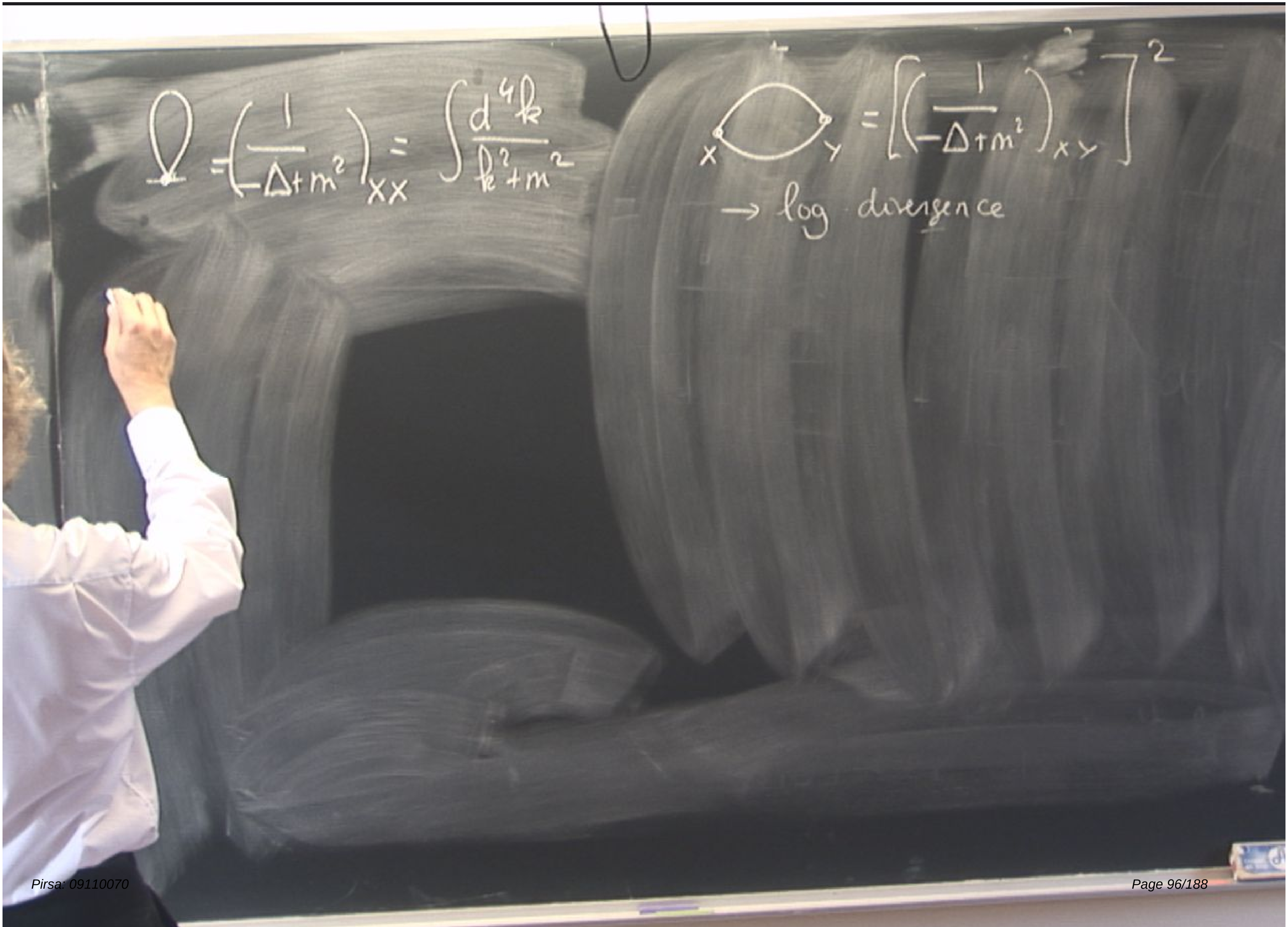
$$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$$

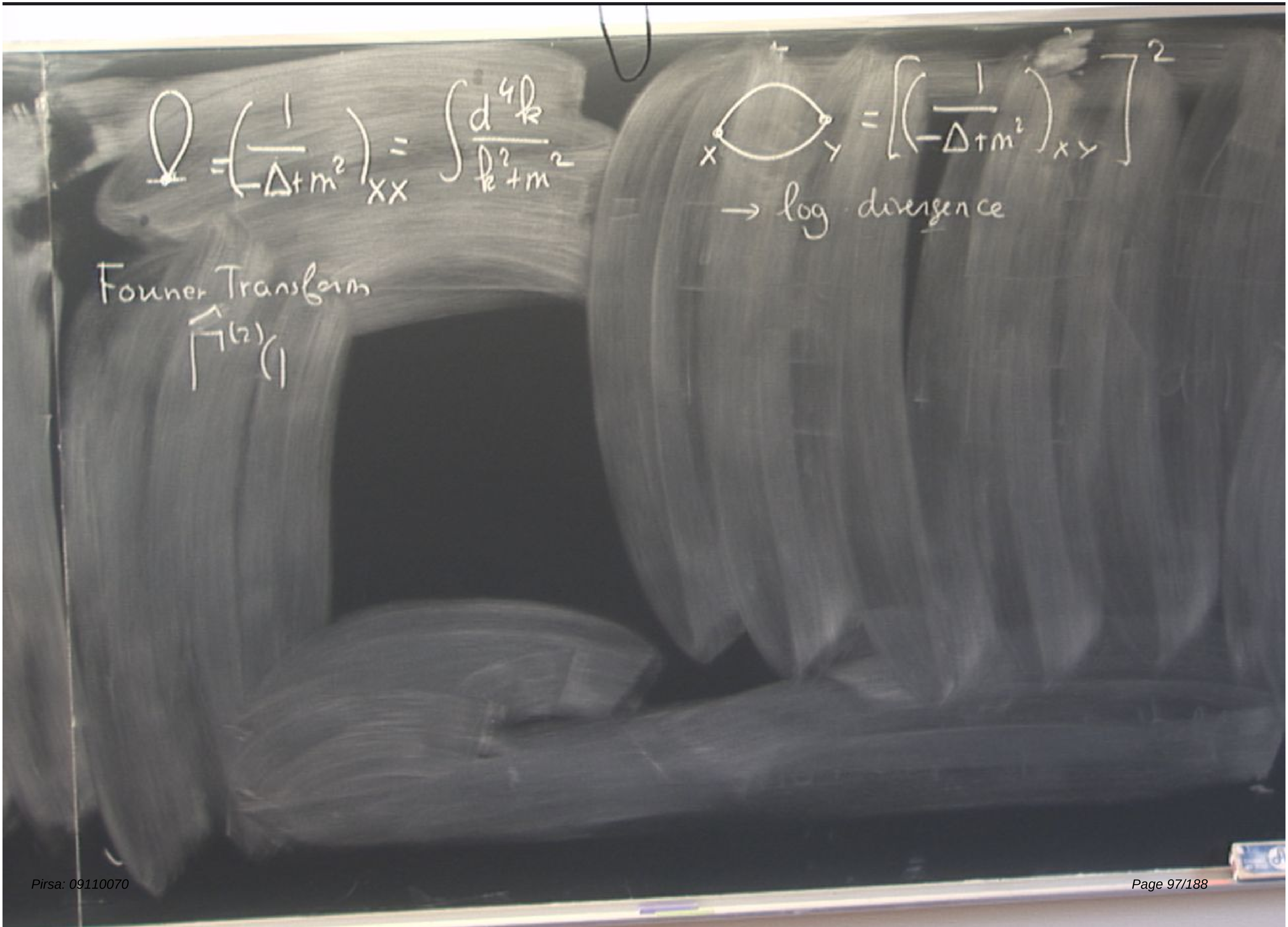
$$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$$

$$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2 + m^2}$$

$$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$$

$\rightarrow$ log divergence


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
x
y
$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$
→ log divergence


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
x y $= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$
→ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}($

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr} \left[\log \left(-\Delta + m^2 + \frac{g}{2} \varphi^2 \right) \right] + \ldots$$

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$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(x_2)$$

$$\Gamma^{(2)}(x_1, x_2)$$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2} \varphi^2 \right) \right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \underbrace{\int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xx}} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x_1) \underbrace{\left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right]}_{\Gamma^{(2)}(x_1, x_2)} \varphi(x_2)$$

$$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2 + m^2}$$

$$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$$

$\rightarrow$ log divergence

Fourier Transform

$$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1 + p_2)$$

 $= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$

→ log divergence

Fourier Transform

$$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2 + m^2 + \frac{\hbar}{2} g \right.$$  $\left.\right]$

$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$

→ log divergence

Fourier Transform

$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2} g\right.$  $\left.\right]$

$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)$

 $= \left(\frac{1}{-\Delta+m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

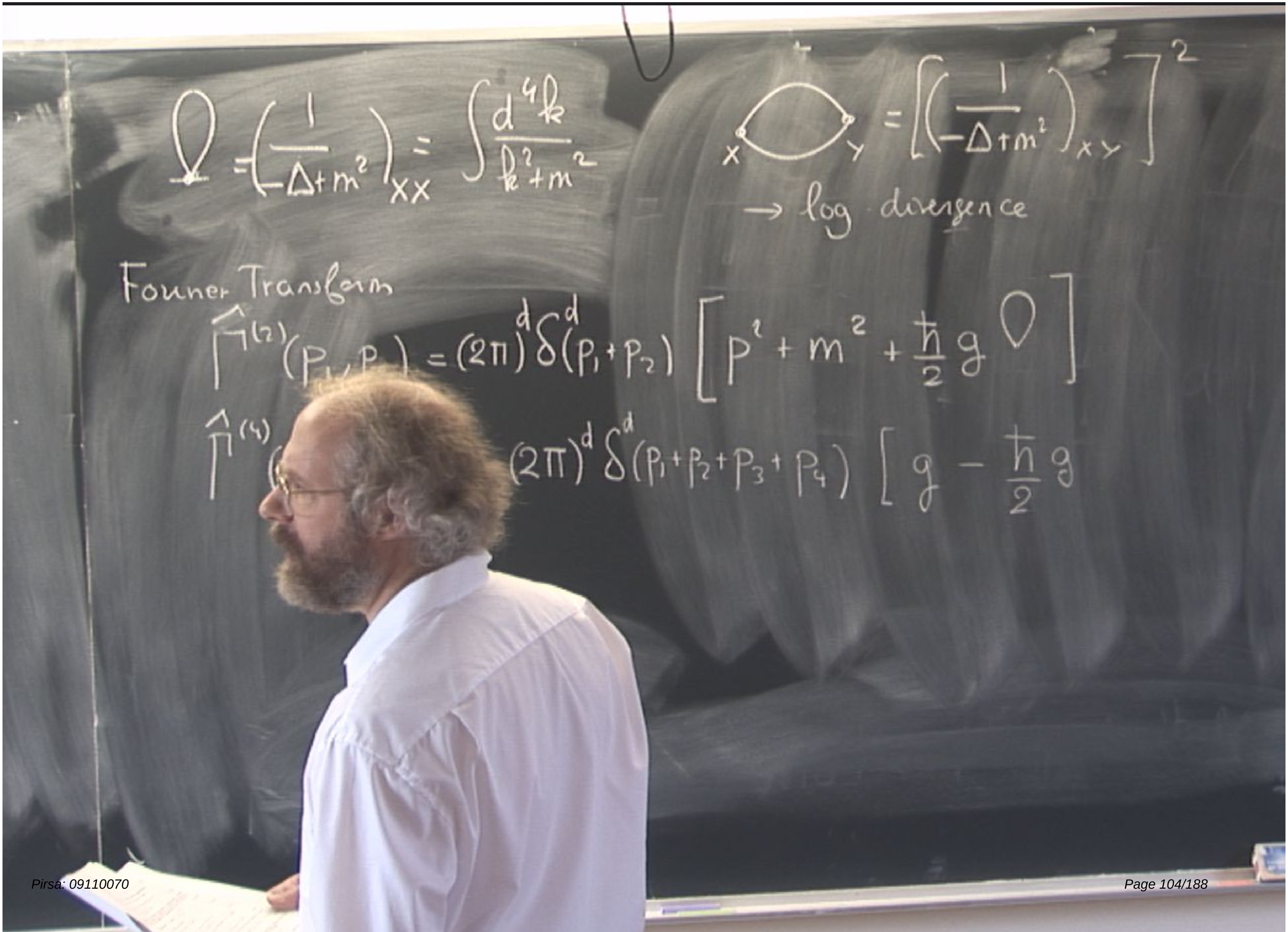
$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$

→ log divergence

Fourier Transform

$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1+p_2) \left[p^2 + m^2 + \frac{\hbar}{2} g\right.$  $\left.\right]$

$\hat{\Gamma}^{(4)}(p_1, p_2, p_3, p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4) \left[g \right.$


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
x y
$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$
→ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1+p_2) \left[p^2 + m^2 + \frac{\hbar}{2} g \right]$
$\hat{\Gamma}^{(4)}$
$(2\pi)^d \delta^d(p_1+p_2+p_3+p_4) \left[g - \frac{\hbar}{2} g \right.$

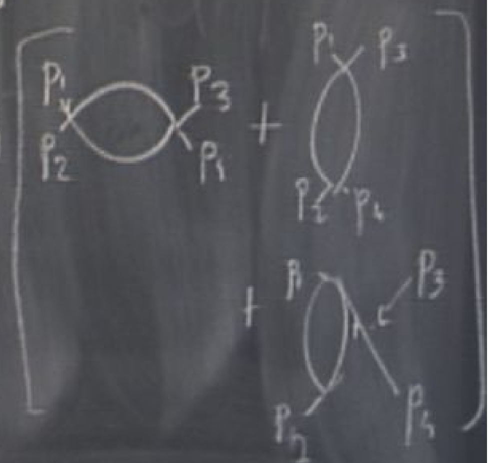
 $= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

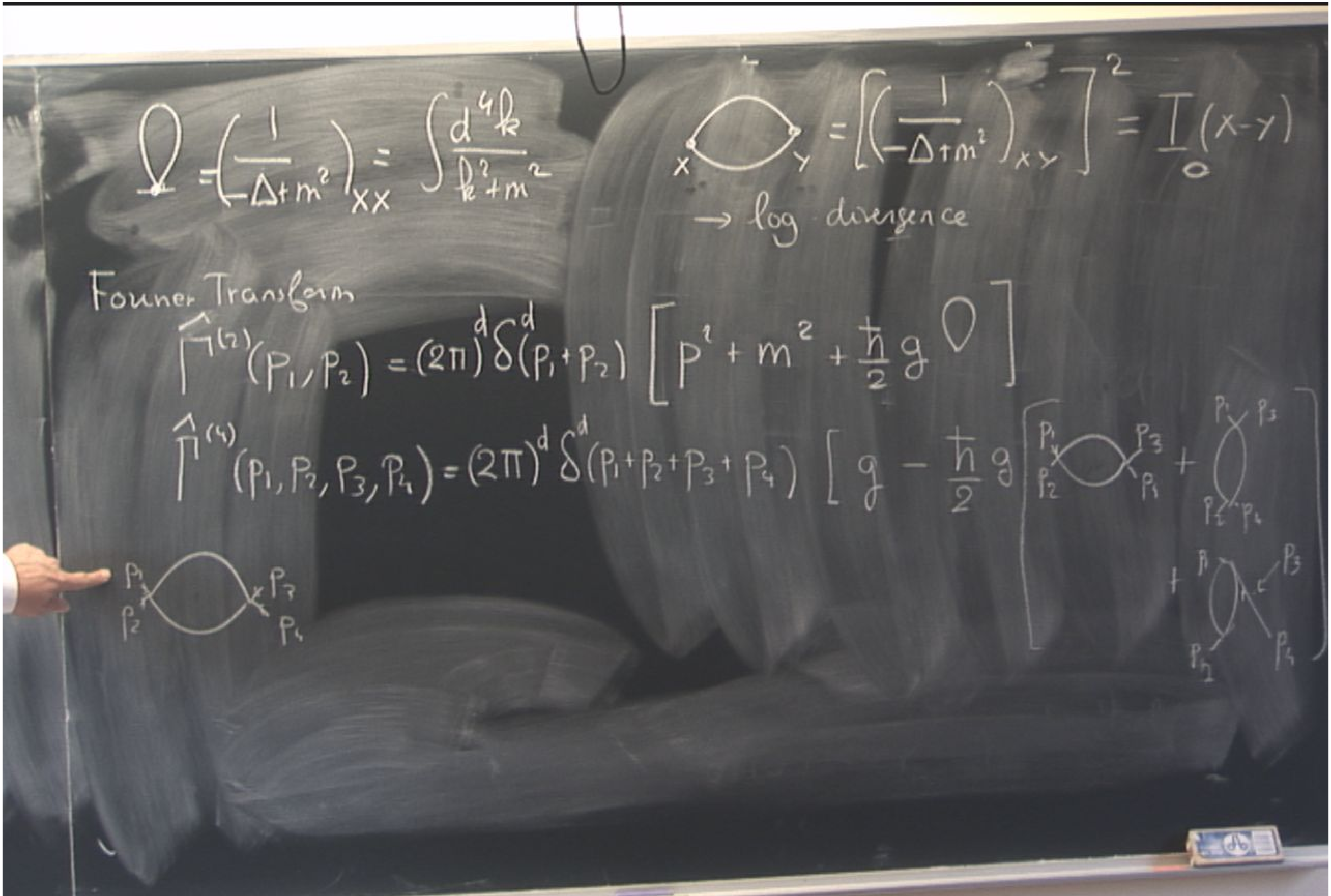
 $= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2$

$\rightarrow$ log divergence

Fourier Transform

$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1 + p_2)\left[p^2 + m^2 + \frac{\hbar}{2} g\right.$  $\left.\right]$

$\hat{\Gamma}^{(4)}(p_1, p_2, p_3, p_4) = (2\pi)^d \delta^d(p_1 + p_2 + p_3 + p_4)\left[g - \frac{\hbar}{2} g\right.$  $\left.\right]$


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
$x \quad y = \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$
$\rightarrow$ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g\right]$
$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2}g\right]$
p_1 p_2 p_3 p_4

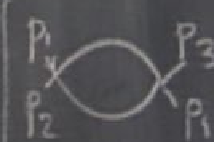
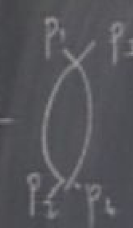
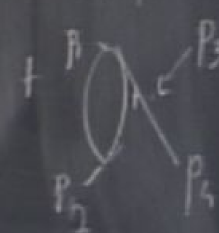
 $= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

 $= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2 = I_0(x-y)$

$\rightarrow$ log divergence

Fourier Transform

$$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g \right.$$  $\left.\right]$

$$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2}g \right.$$ $\Big[$  $+$  $+$  $\Big]$

 $= \hat{I}_0$

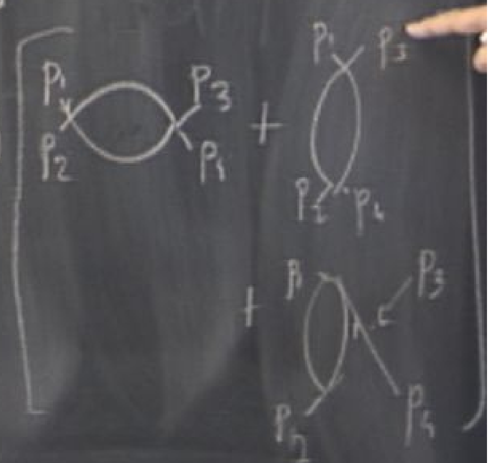
 $= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4 k}{k^2 + m^2}$

 $= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2 = I_0(x-y)$

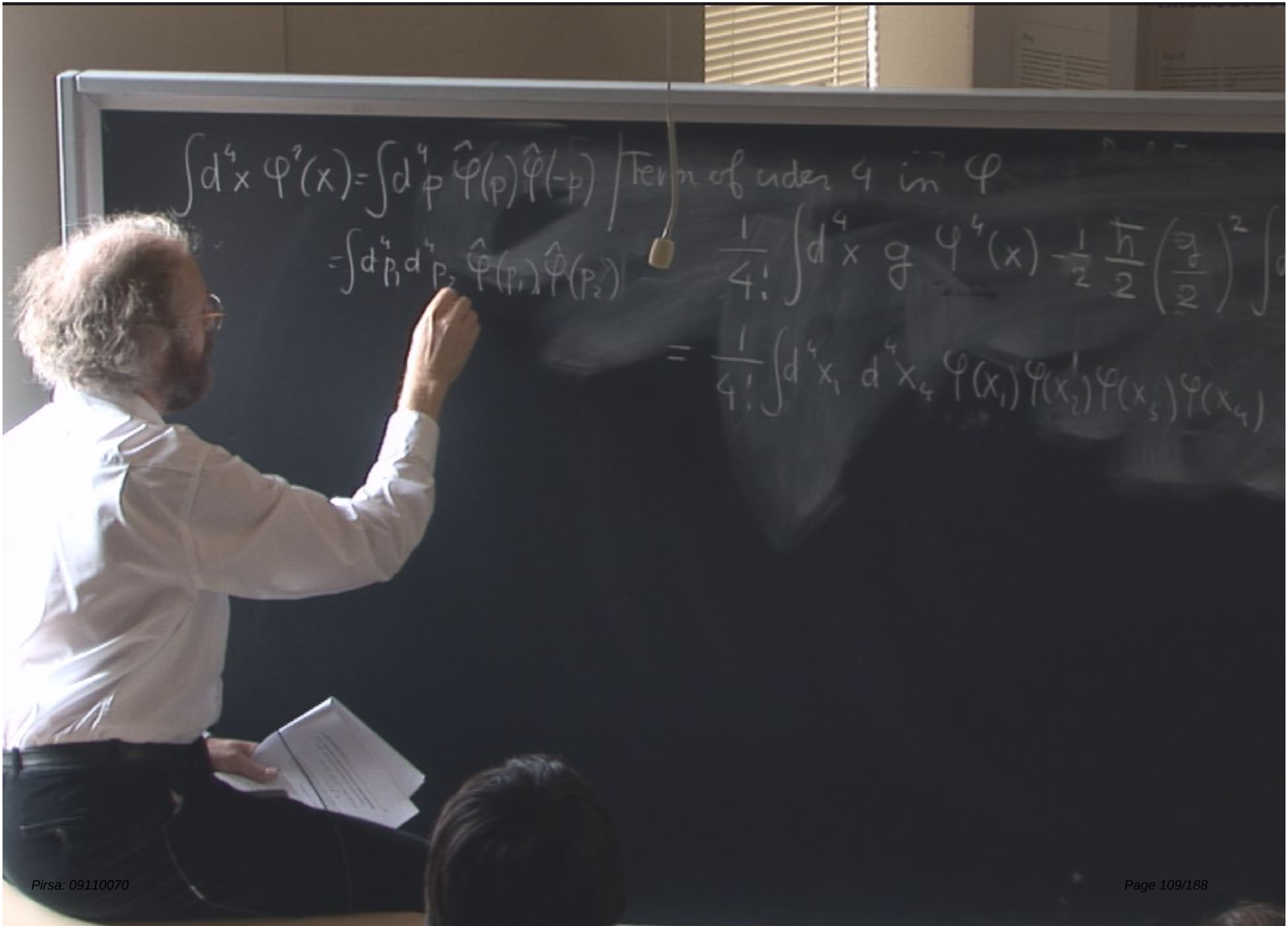
$\rightarrow$ log divergence

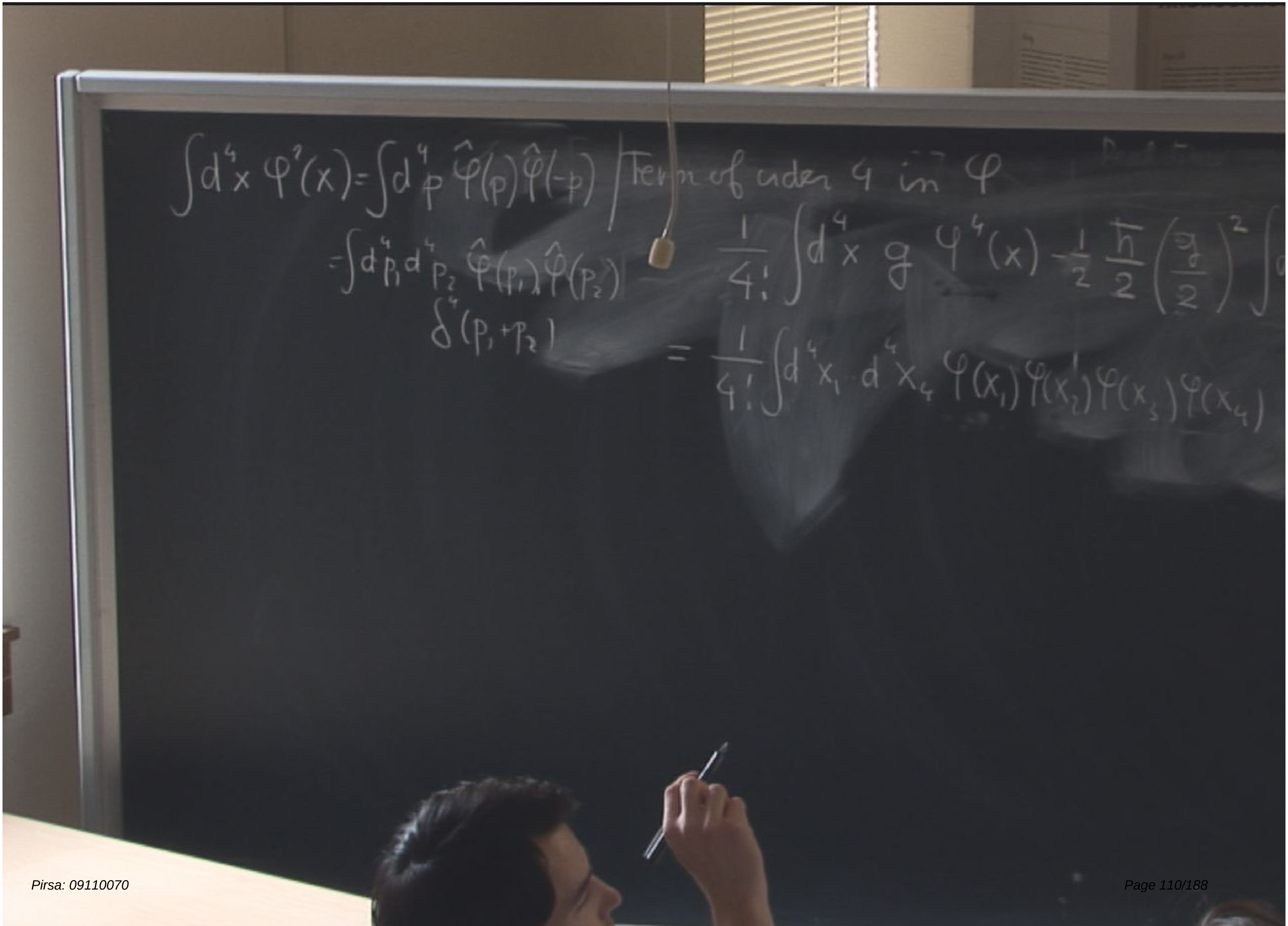
Fourier Transform

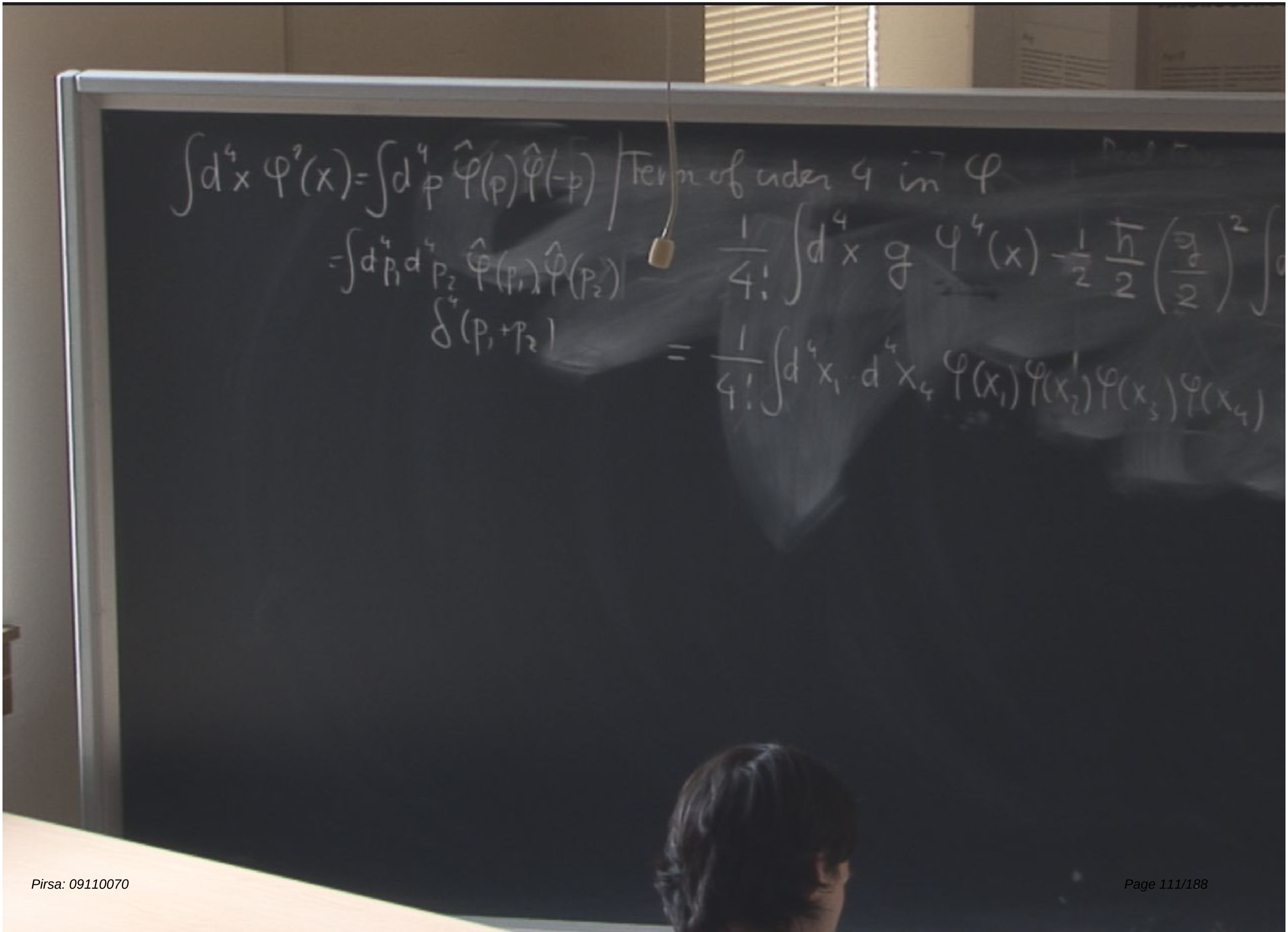
$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1 + p_2) \left[p^2 + m^2 + \frac{\hbar}{2} g \right.$  $\left. \right]$

$\hat{\Gamma}^{(4)}(p_1, p_2, p_3, p_4) = (2\pi)^d \delta^d(p_1 + p_2 + p_3 + p_4) \left[g - \frac{\hbar}{2} g \right.$  $\left. \right]$

 $= \hat{I}_0(p_1 + p_2)$


$\int d^4x\, \varphi^2(x) = \int d^4p\, \hat{\varphi}(p)\hat{\varphi}(-p)$
$= \int d^4p_1 d^4p_2\, \hat{\varphi}(p_1)\hat{\varphi}(p_2)$
Term of order 4 in φ
$\frac{1}{4!}\int d^4x\, g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int$
$= \frac{1}{4!}\int d^4x_1 \cdots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$


$\int d^4x\, \varphi^2(x) = \int d^4p\, \hat{\varphi}(p)\hat{\varphi}(-p)$
$= \int d^4p_1 d^4p_2\, \hat{\varphi}(p_1)\hat{\varphi}(p_2)$
$\delta^4(p_1+p_2)$
Term of order 4 in φ
$\frac{1}{4!}\int d^4x\; g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int$
$= \frac{1}{4!}\int d^4x_1 \cdots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$


$\int d^4x\, \varphi^2(x) = \int d^4p\, \hat{\varphi}(p)\hat{\varphi}(-p)$
$= \int d^4p_1 d^4p_2\, \hat{\varphi}(p_1)\hat{\varphi}(p_2)\, \delta^4(p_1+p_2)$
Term of order 4 in φ
$\frac{1}{4!}\int d^4x\, g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int$
$= \frac{1}{4!}\int d^4x_1 \ldots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu\phi\,\partial^\mu\phi) + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4\right]$$

Euclidean Time

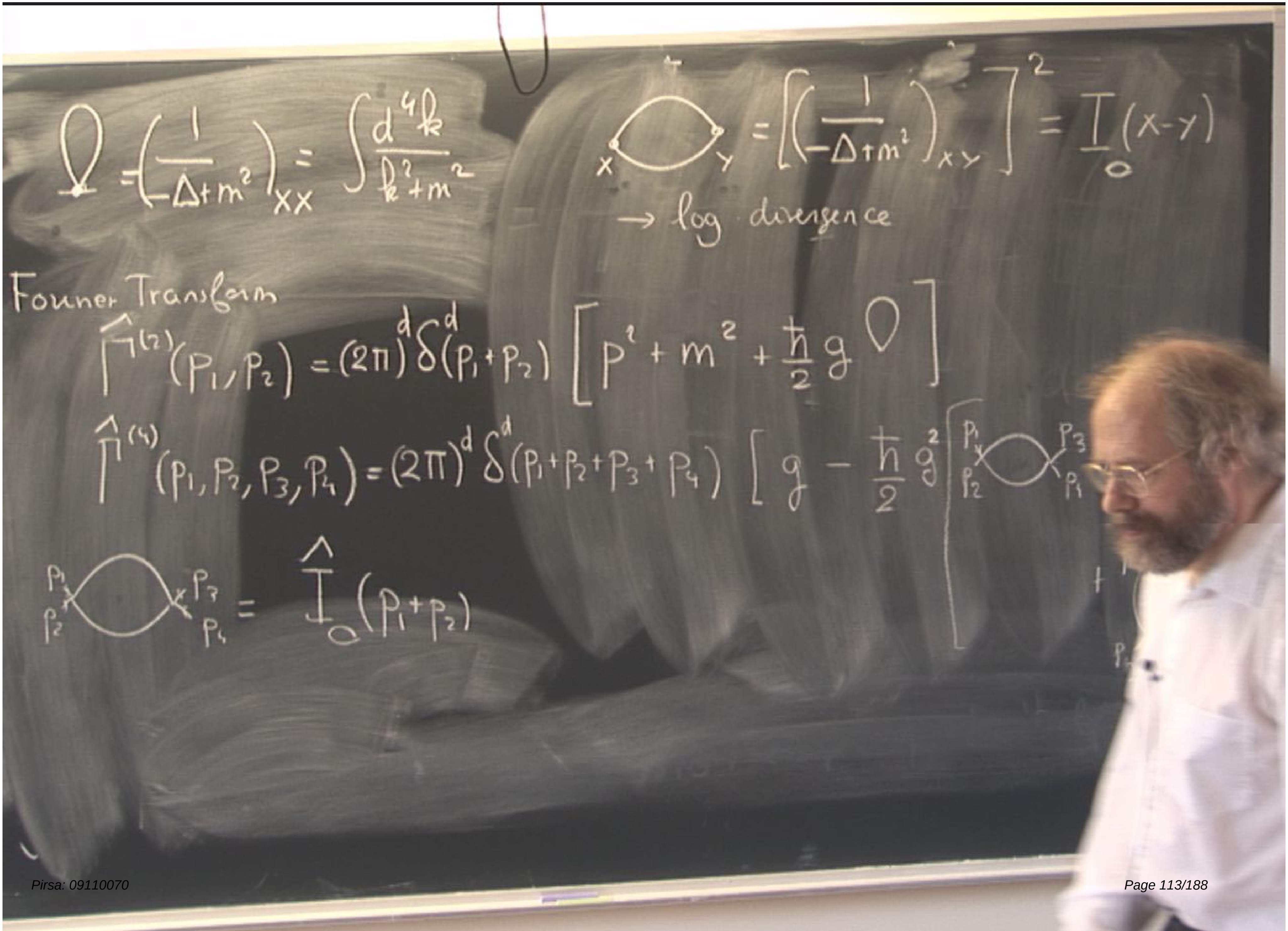
Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

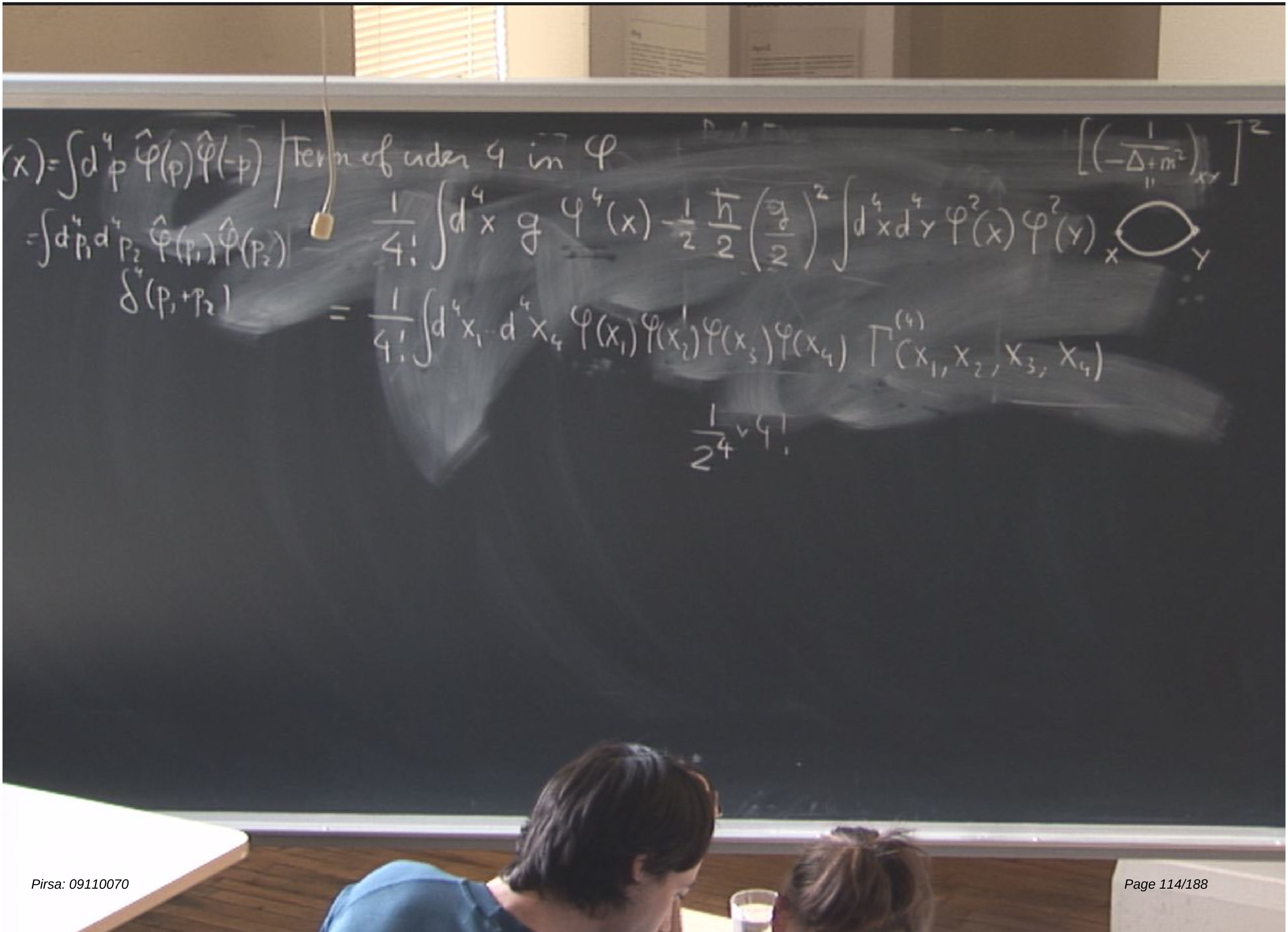
$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

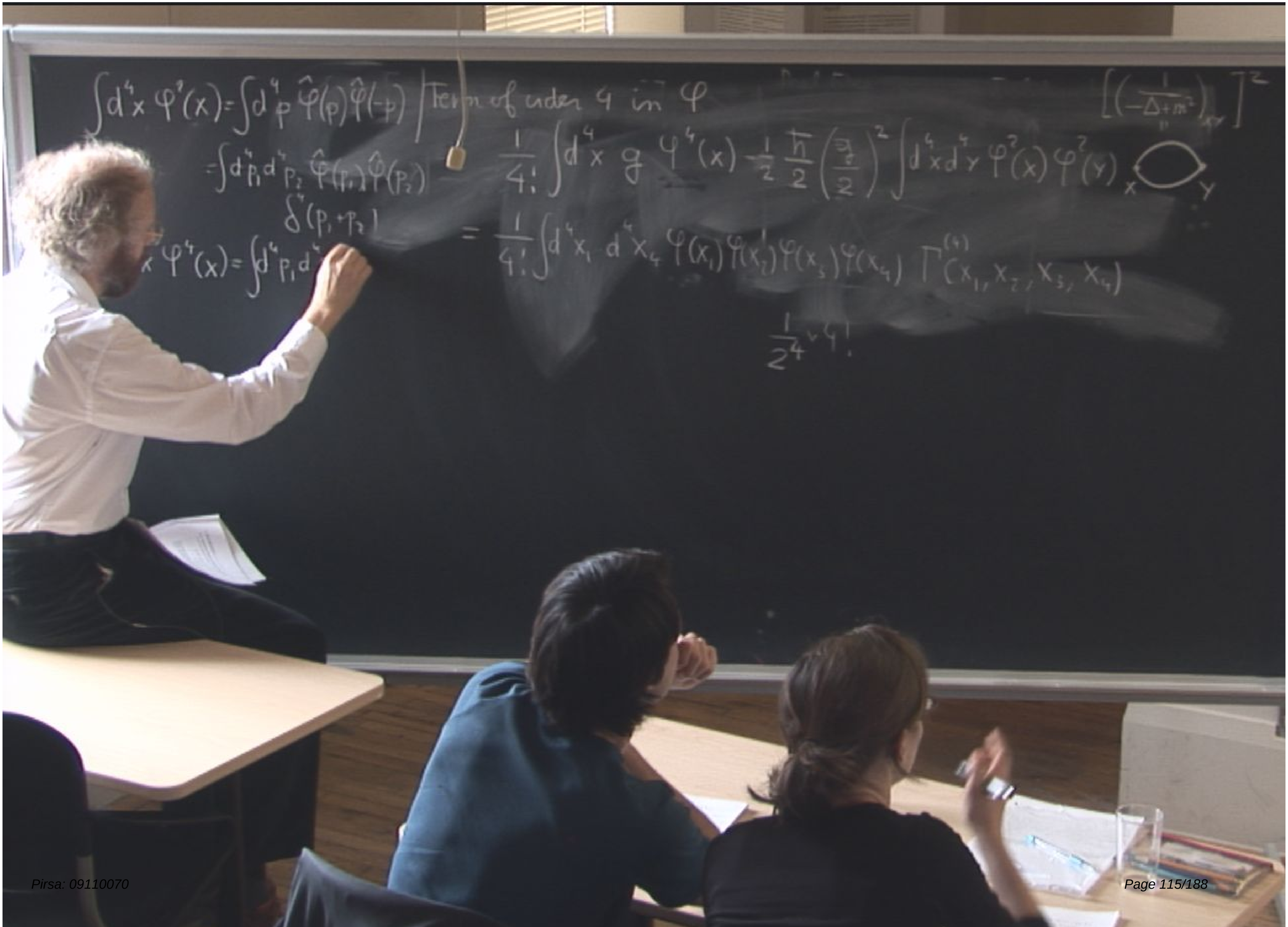
Term of order 2 in φ: 2 point amputated irreducible function.

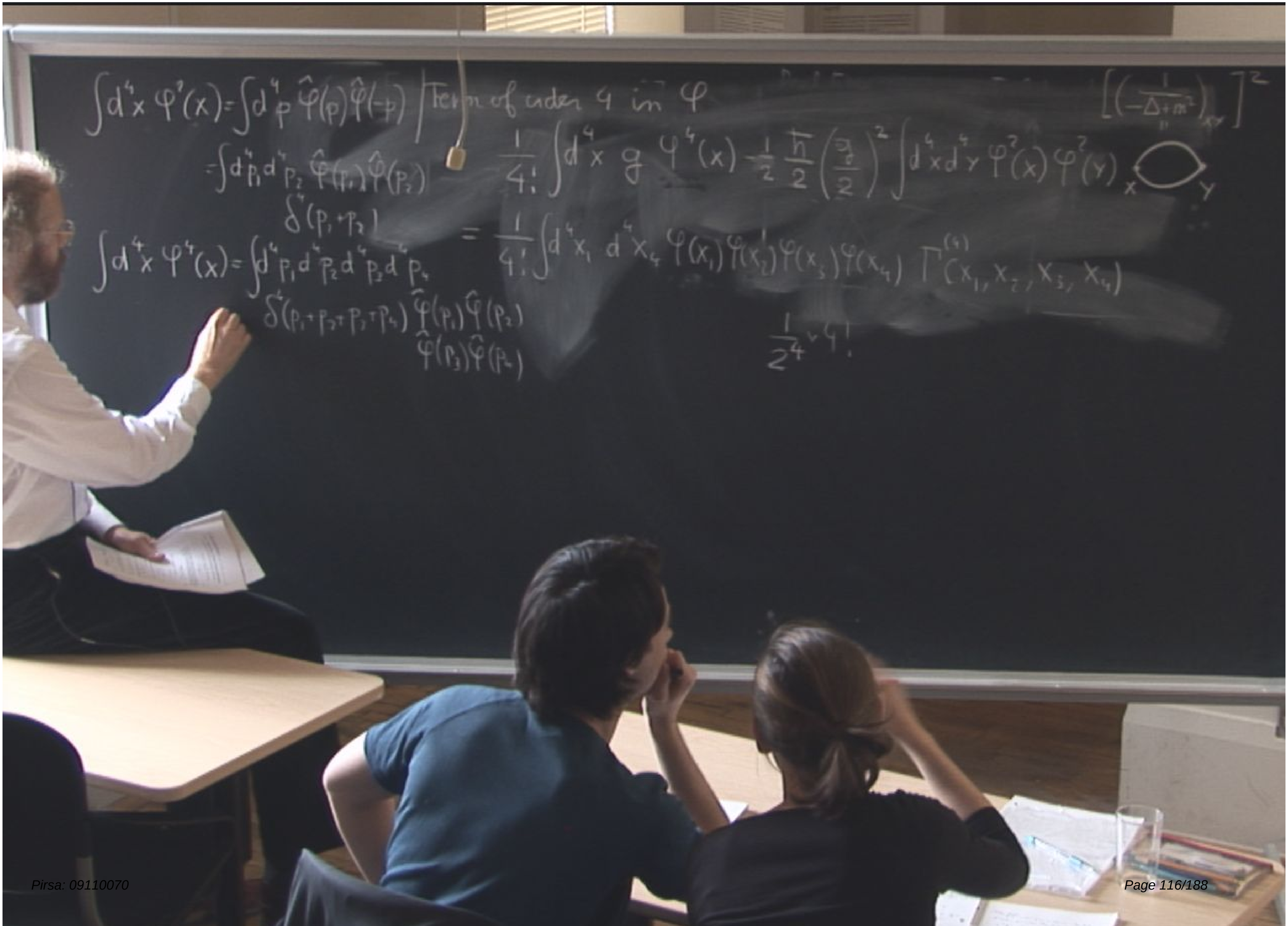
$$\int d^4x\left(\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi + \frac{m^2}{2}\varphi^2\right) + \frac{\hbar}{2}\frac{g}{2}\underbrace{\int d^4x\left(\frac{1}{-\Delta+m^2}\right)_{xx}}\cdot\varphi^2(x)$$

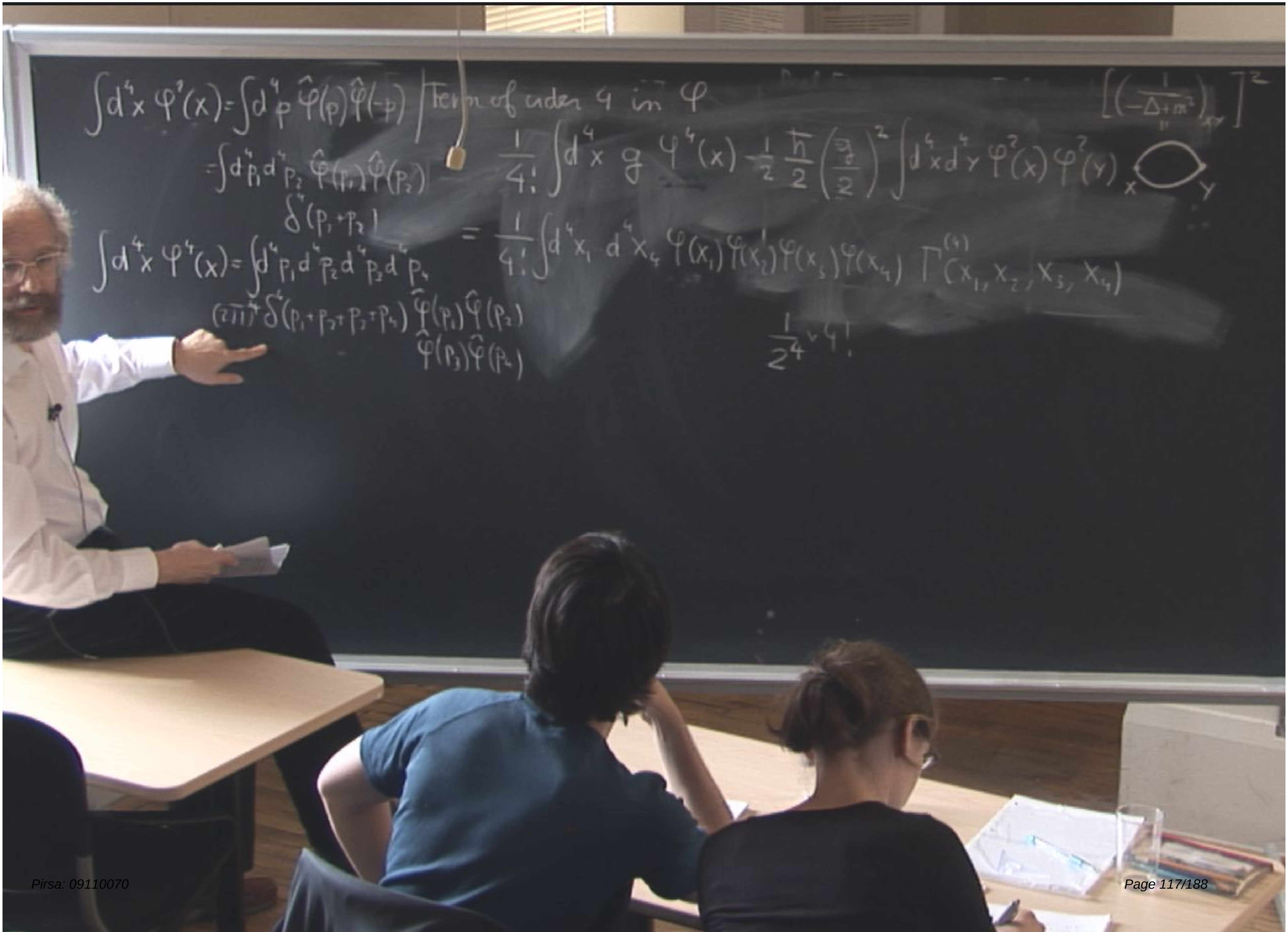
$$= \frac{1}{2}\int d^4x\,d^4y\,\varphi(x_1)\underbrace{\left[(-\Delta+m^2) + \hbar\frac{g}{2}\bigcirc\right]}_{\Gamma^{(2)}(x_1,x_2)}\varphi(x_2)$$

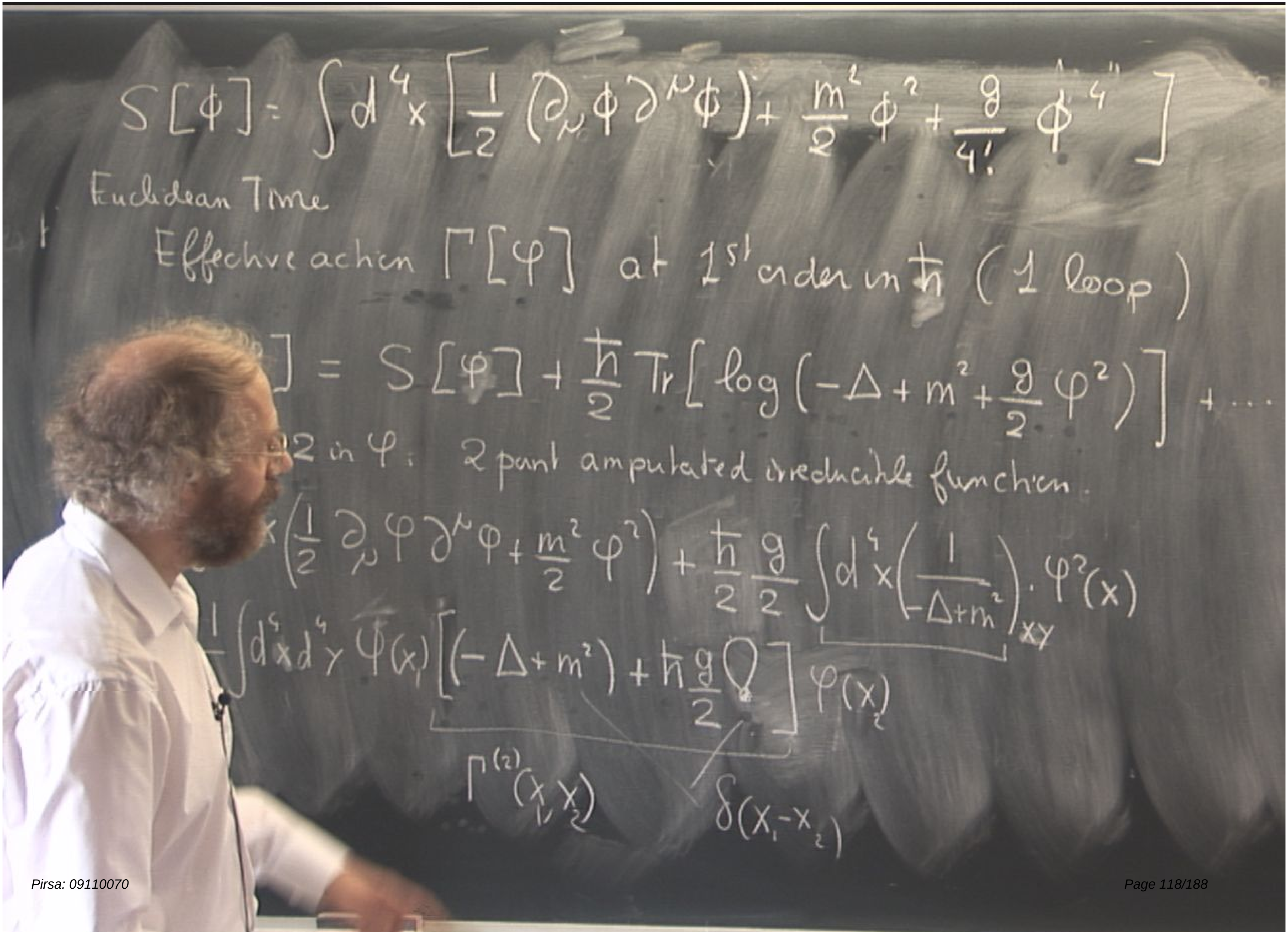

= (1 / -Δ+m²)xx = ∫ d⁴k / k²+m²
x y = [(1 / -Δ+m²)xy]² = I₀(x-y)
→ log divergence
Fourier Transform
Γ̂⁽²⁾(p₁, p₂) = (2π)ᵈ δᵈ(p₁+p₂) [p² + m² + ħ/2 g]
Γ̂⁽⁴⁾(p₁, p₂, p₃, p₄) = (2π)ᵈ δᵈ(p₁+p₂+p₃+p₄) [g − ħ/2 g²
p₁ p₂ p₃ p₄ = Î₀(p₁+p₂)


$(x) = \int d^4p\, \hat{\varphi}(p)\hat{\varphi}(-p)$
$= \int d^4p_1 d^4p_2\, \hat{\varphi}(p_1)\hat{\varphi}(p_2)\, \delta^4(p_1+p_2)$
Term of order 4 in φ
$\frac{1}{4!}\int d^4x\, g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int d^4x\, d^4y\, \varphi^2(x)\varphi^2(y)$
$\left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$
x
y
$= \frac{1}{4!}\int d^4x_1 \cdots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\, \Gamma^{(4)}(x_1, x_2, x_3, x_4)$
$\frac{1}{2^4} \cdot 4!$


Term of order 4 in φ


Term of order 4 in φ


∫d⁴x φ²(x) = ∫d⁴p φ̂(p) φ̂(−p)
Term of order 4 in φ
=∫d⁴p₁ d⁴p₂ φ̂(p₁) φ̂(p₂) δ⁴(p₁+p₂)
∫d⁴x φ⁴(x) = ∫d⁴p₁ d⁴p₂ d⁴p₃ d⁴p₄ (2π)⁴ δ⁴(p₁+p₂+p₃+p₄) φ̂(p₁) φ̂(p₂) φ̂(p₃) φ̂(p₄)
1/4! ∫d⁴x g φ⁴(x) − 1/2 ħ/2 (g/2)² ∫d⁴x d⁴y φ²(x) φ²(y)
= 1/4! ∫d⁴x₁ … d⁴x₄ φ(x₁) φ(x₂) φ(x₃) φ(x₄) Γ⁽⁴⁾(x₁, x₂, x₃, x₄)
1/2⁴ · 4!
x
y


S[φ] = ∫d⁴x [1/2 (∂_μ φ ∂^μ φ) + m²/2 φ² + g/4! φ⁴]
Euclidean Time
Effective action Γ[φ] at 1st order in ħ (1 loop)
] = S[φ] + ħ/2 Tr[log(−Δ + m² + g/2 φ²)] + ...
2 in φ : 2 point amputated irreducible function.
(1/2 ∂_μ φ ∂^μ φ + m²/2 φ²) + ħ/2 g/2 ∫d⁴x (1/(−Δ+m²))_xy · φ²(x)
1/2 ∫d⁴x d⁴y φ(x₁) [(−Δ + m²) + ħ g/2] φ(x₂)
Γ^(2)(x₁, x₂)
δ(x₁ − x₂)

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2} \varphi^2 \right) \right] + \dots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xx} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x \, d^4y \, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(x_2)$$

$\Gamma^{(2)}(x_1, x_2)$ $\qquad$ $\delta(x_1 - x_2)$

$$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi\, \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

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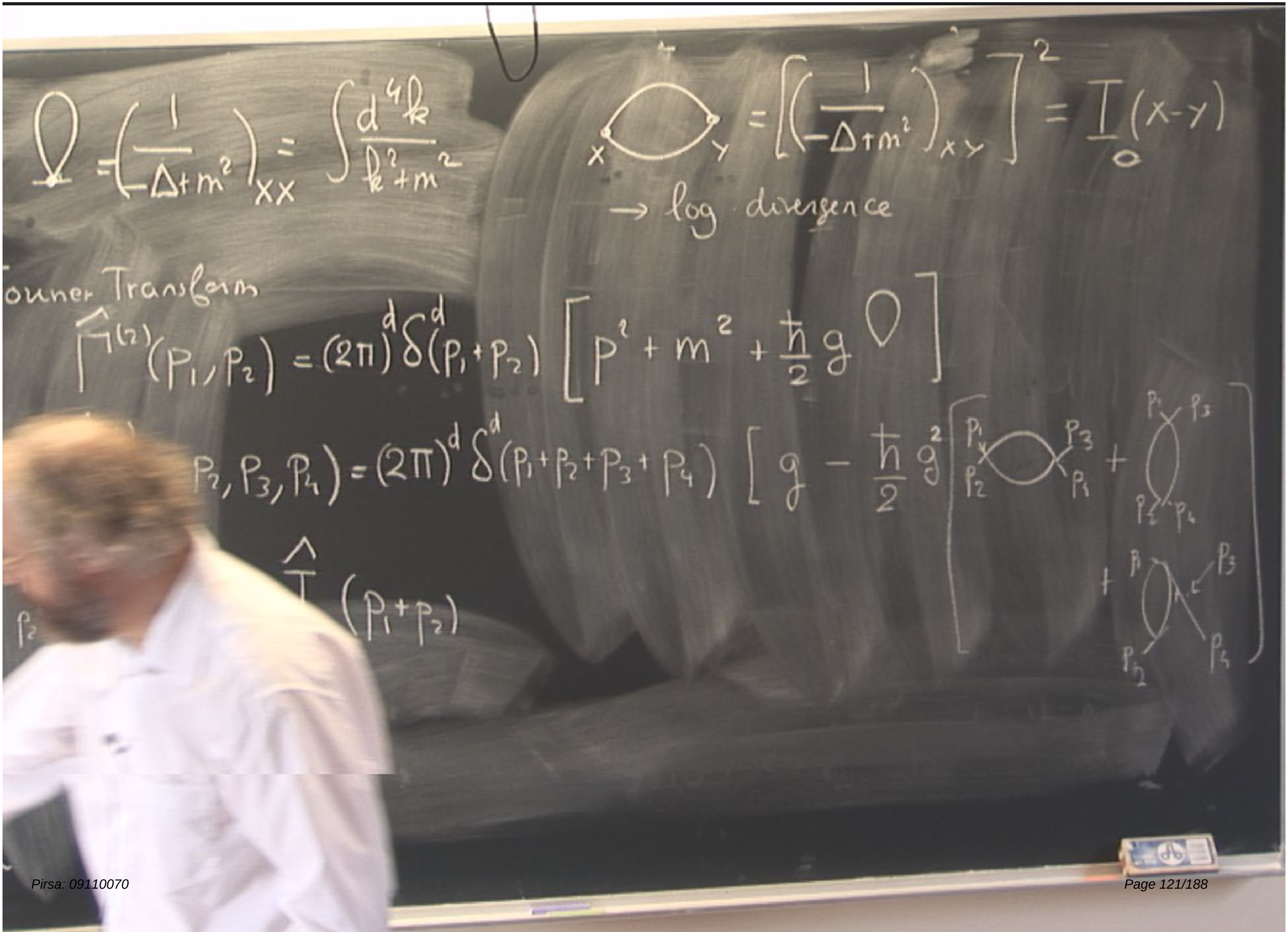
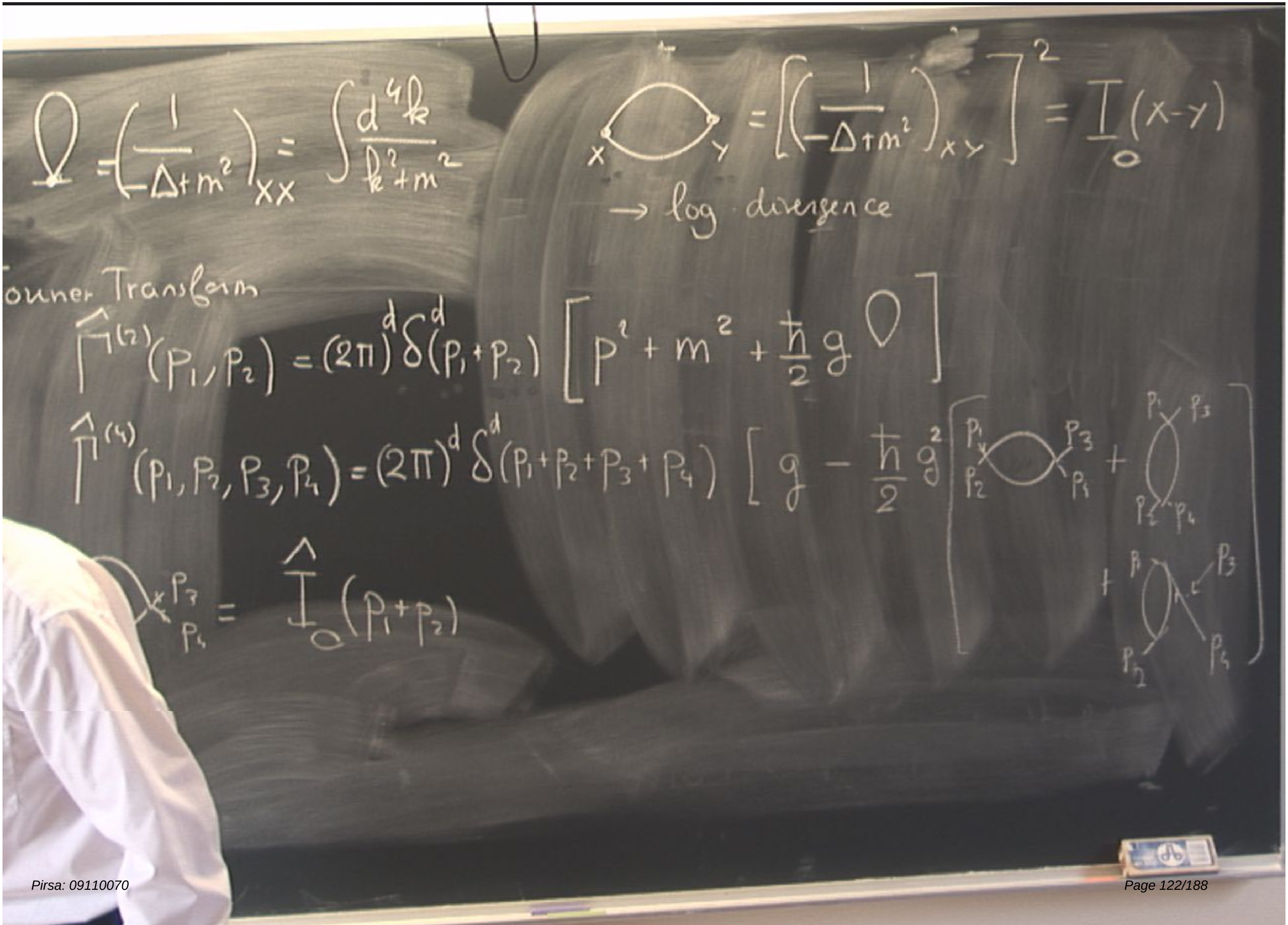
Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi\, \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xx} \cdot \varphi^2(x)$$

$$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(x_2)$$

$\Gamma^{(2)}(x_1, x_2)$ $\qquad$ $\delta(x_1 - x_2)$


= (1 / (-Δ+m²))_XX = ∫ d⁴k / (k²+m²)
= [(1 / (-Δ+m²))_XY]² = I_0(X-Y)
X Y
→ log divergence
ourier Transform
Γ^(2)(P1, P2) = (2π)^d δ^d(P1+P2) [P² + m² + ħ/2 g]
P2, P3, P4) = (2π)^d δ^d(P1+P2+P3+P4) [g - ħ/2 g²
P1 P3 P2 P4
+
Î (P1+P2)


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2 + m^2}$
$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2 = I_0(x-y)$
$\rightarrow$ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1 + p_2) \left[p^2 + m^2 + \frac{\hbar}{2} g \right]$
$\hat{\Gamma}^{(4)}(p_1, p_2, p_3, p_4) = (2\pi)^d \delta^d(p_1 + p_2 + p_3 + p_4) \left[g - \frac{\hbar}{2} g^2 \right]$
$= \hat{I}_0(p_1 + p_2)$

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

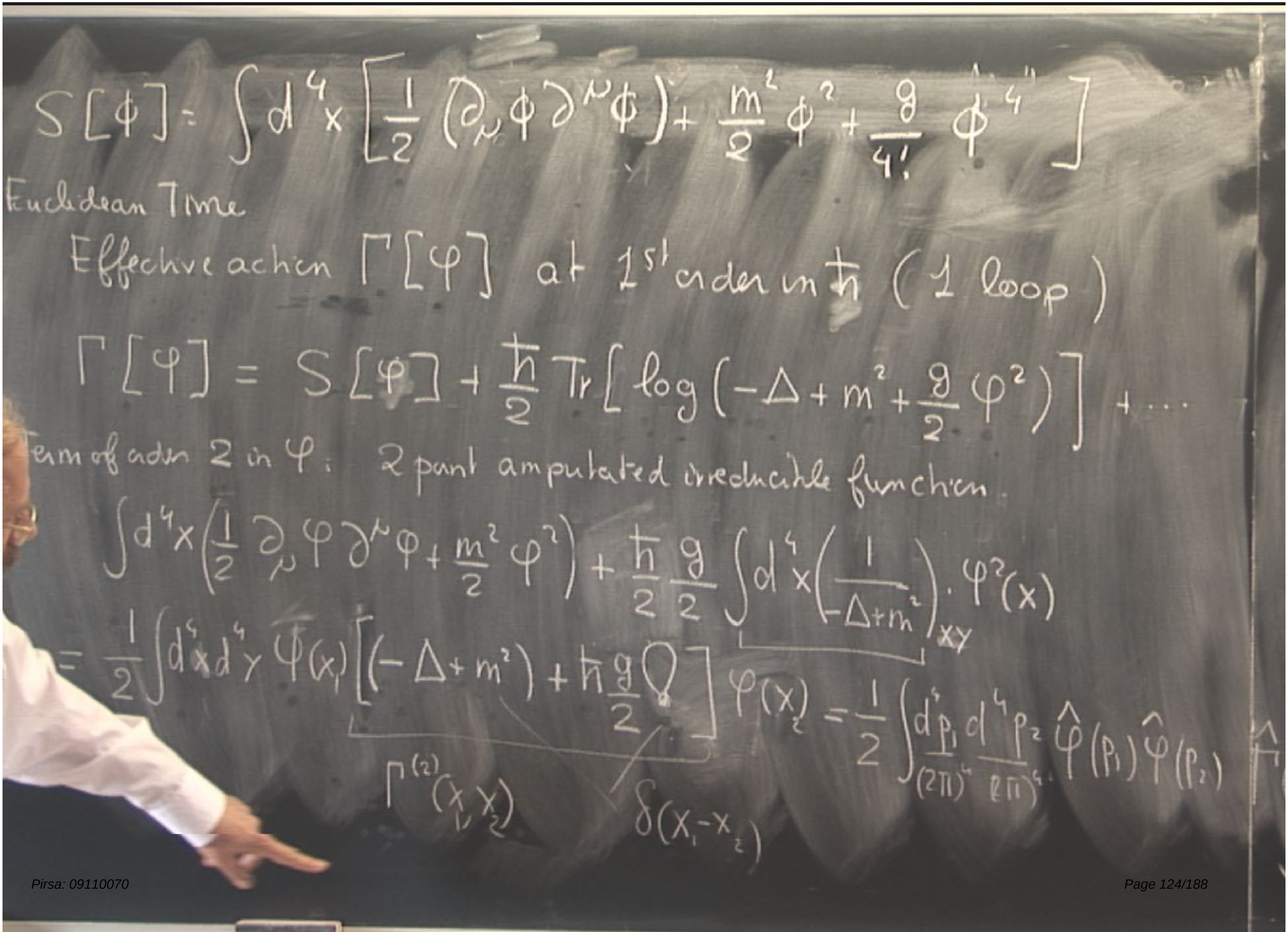
$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right) \right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2}\varphi^2 \right) + \frac{\hbar}{2}\frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$$

$$\frac{1}{2}\int d^4x_1 d^4x_2\, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(x_2) = \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \hat{\varphi}(p_1) \hat{\varphi}(p_2)$$

$\Gamma^{(2)}(x_1, x_2)$ $\qquad \delta(x_1 - x_2)$


$S[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$
Euclidean Time
Effective action $\Gamma[\varphi]$ at 1st order in $\hbar$ (1 loop)
$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \dots$
Term of order 2 in φ: 2 point amputated irreducible function.
$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 \right) + \frac{\hbar}{2} \frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2} \right)_{xy} \cdot \varphi^2(x)$
$= \frac{1}{2} \int d^4x\, d^4y\, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right] \varphi(x_2) = \frac{1}{2} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \hat{\varphi}(p_1) \hat{\varphi}(p_2)$
$\Gamma^{(2)}(x_1, x_2)$
$\delta(x_1 - x_2)$

$$S[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4\right]$$

Euclidean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

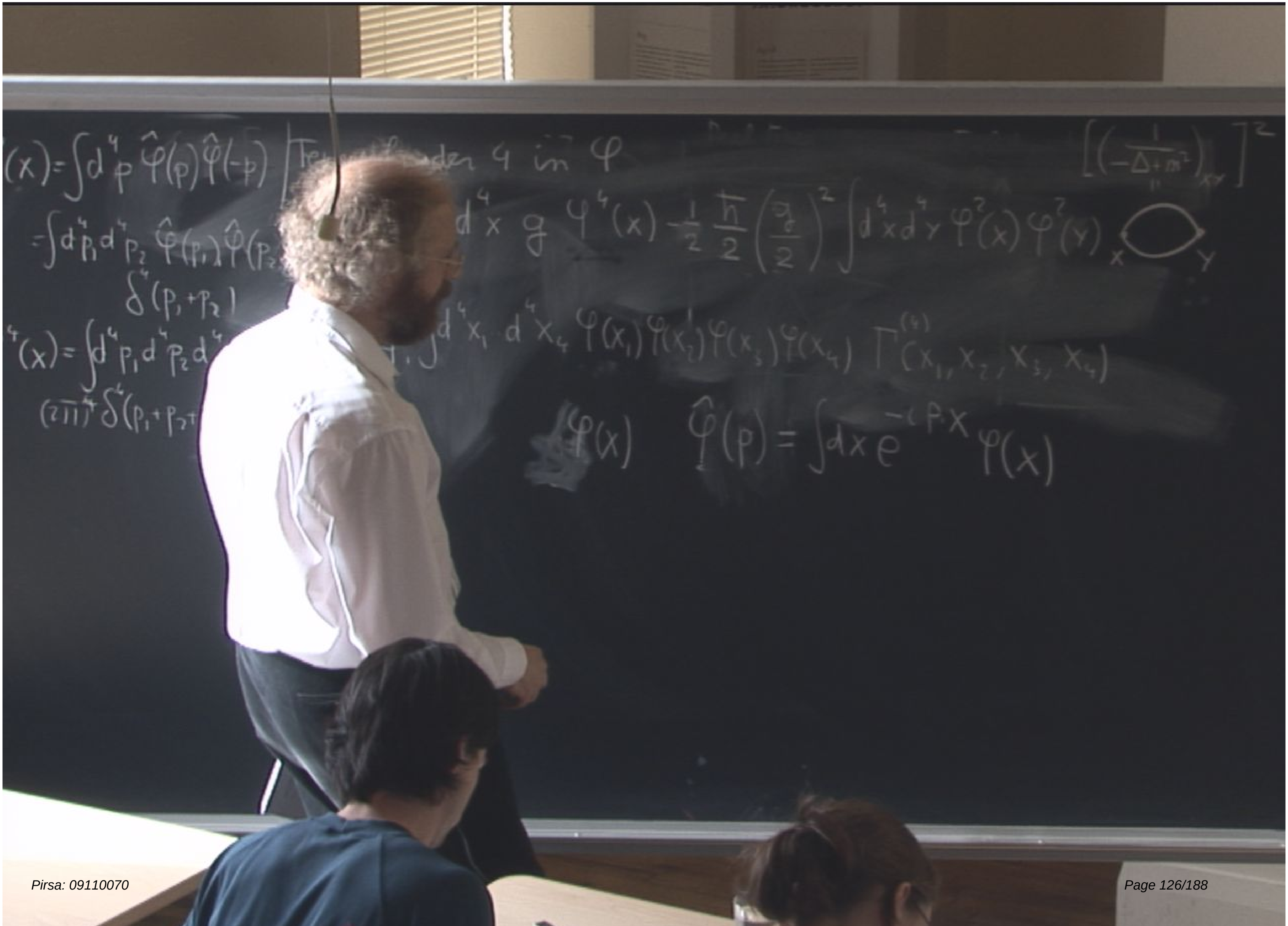
$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

Term of order 2 in φ: 2 point amputated irreducible function.

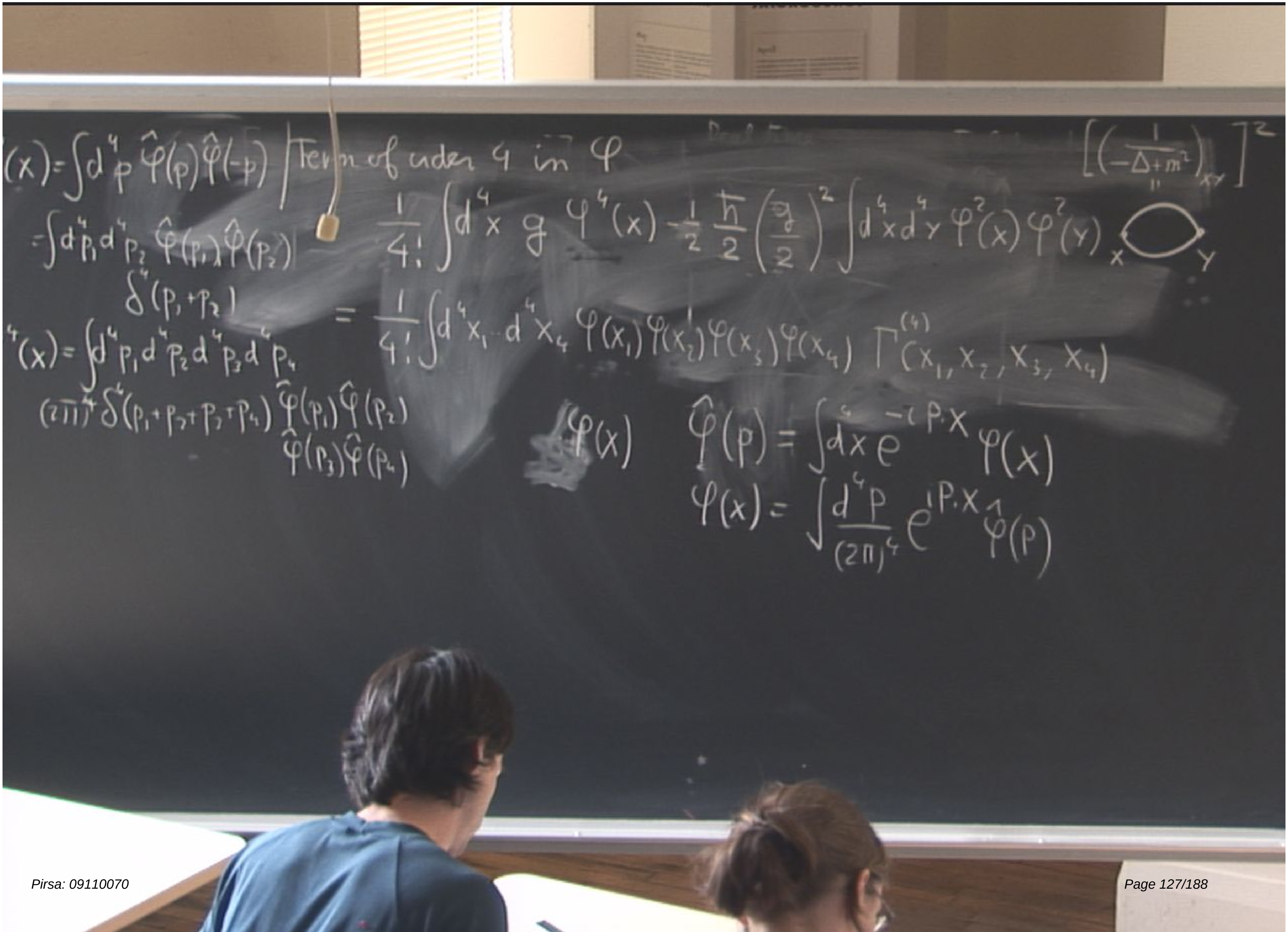
$$\int d^4x \left(\frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2}\varphi^2\right) + \frac{\hbar}{2}\frac{g}{2}\int d^4x \left(\frac{1}{-\Delta + m^2}\right)_{xy} \cdot \varphi^2(x)$$

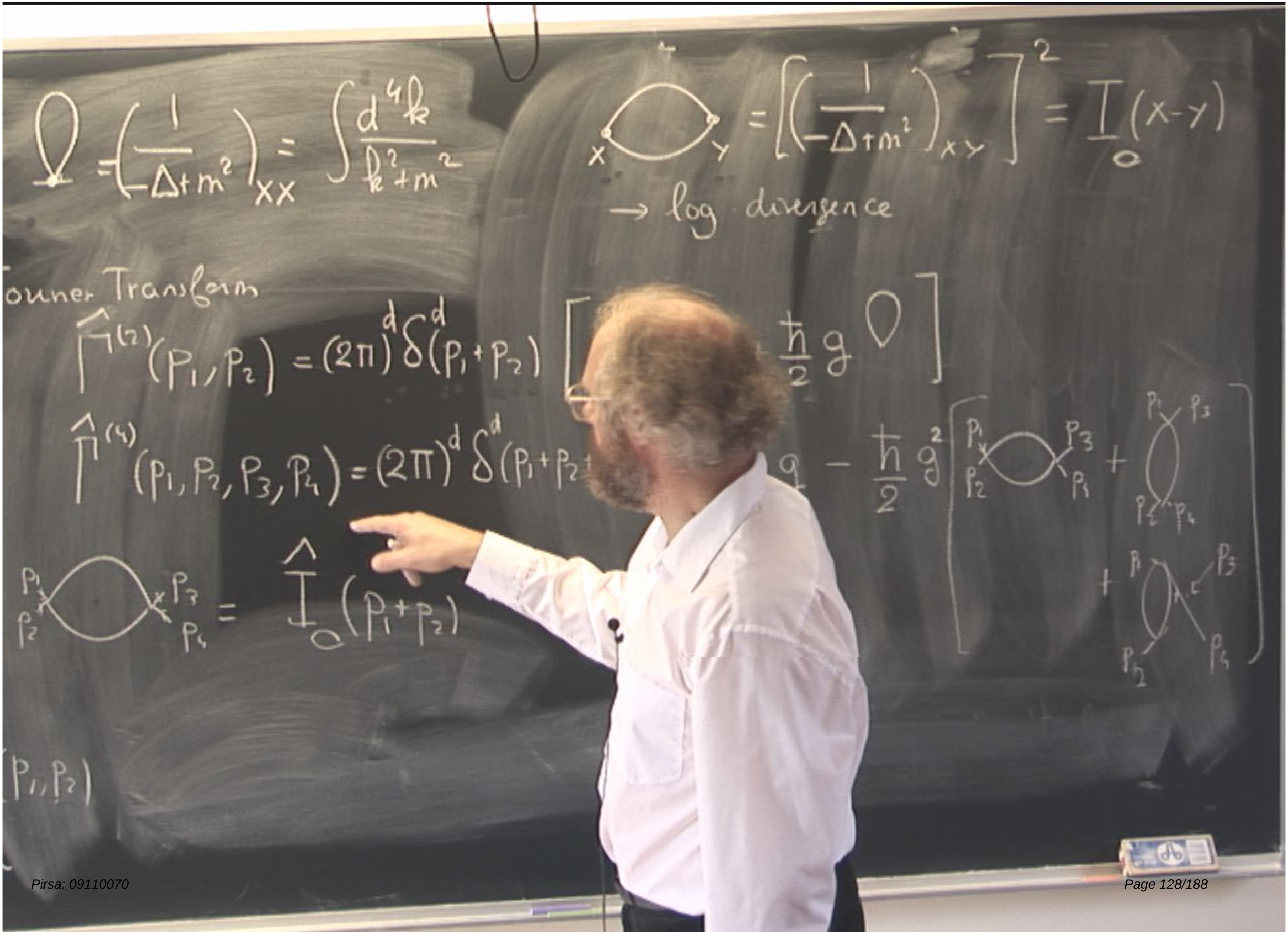
$$= \frac{1}{2}\int d^4x\, d^4y\, \varphi(x_1)\left[(-\Delta + m^2) + \hbar \frac{g}{2} \bigcirc \right]\varphi(x_2) = \frac{1}{2}\int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\hat{\varphi}(p_1)\hat{\varphi}(p_2)$$

$\Gamma^{(2)}(x_1, x_2)$ $\delta(x_1 - x_2)$

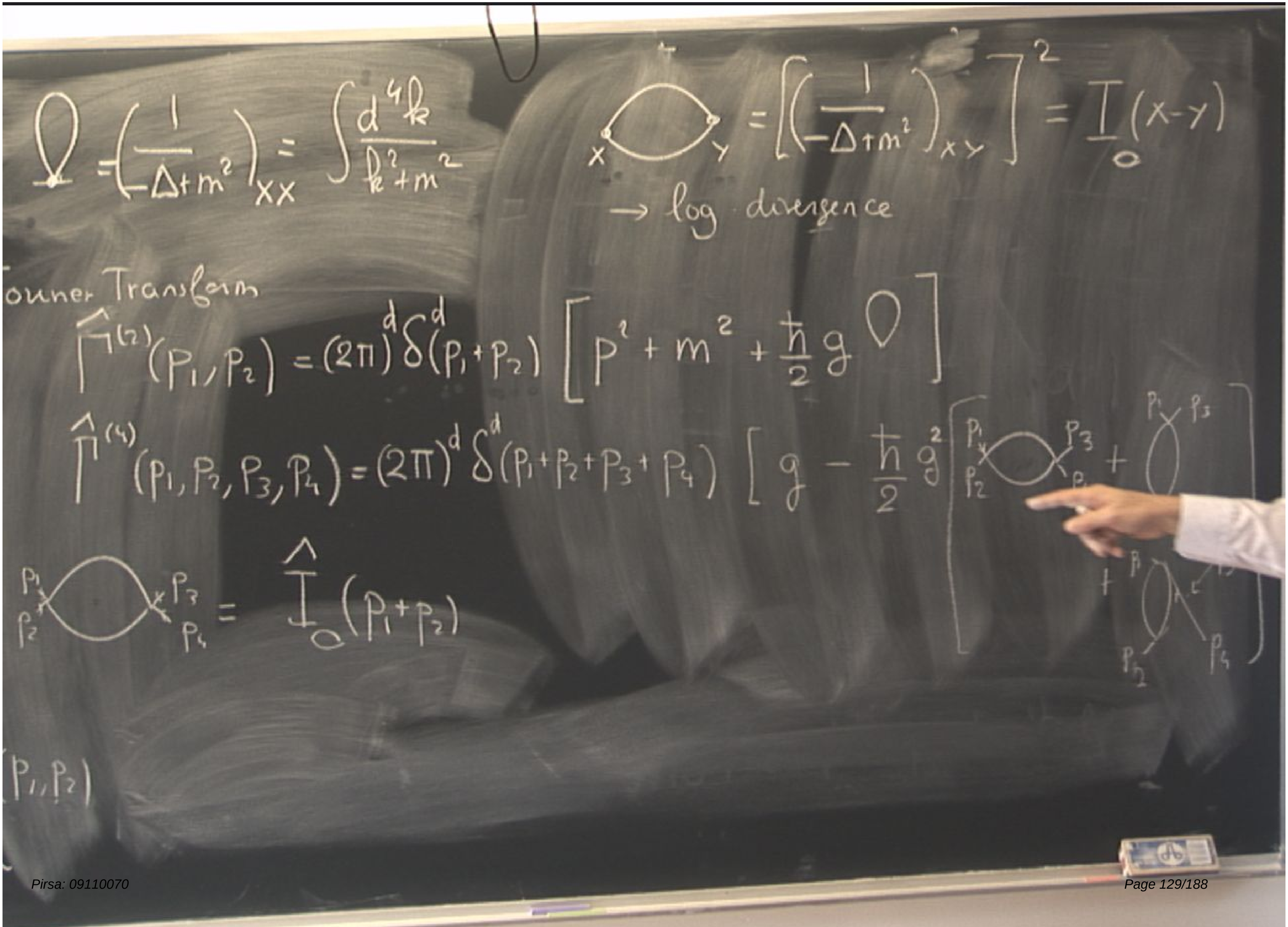


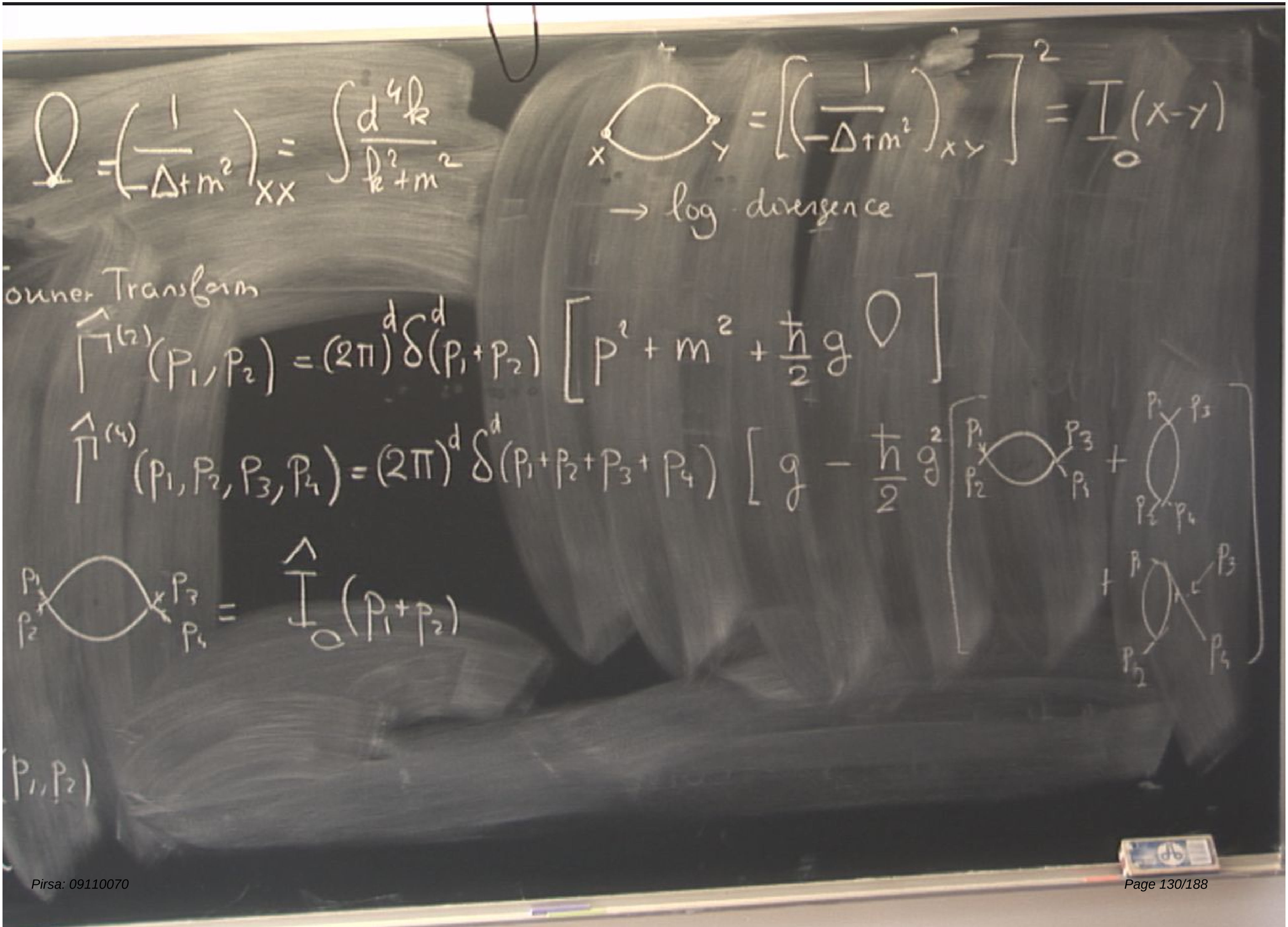
in φ
φ(x)
φ̂(p) = ∫dx e^{-ip·x} φ(x)


$(x) = \int d^4p\, \hat\varphi(p)\hat\varphi(-p)$
Term of order 4 in φ
$= \int d^4p_1 d^4p_2\, \hat\varphi(p_1)\hat\varphi(p_2)\, \delta^4(p_1+p_2)$
$\frac{1}{4!}\int d^4x\, g\, \varphi^4(x) - \frac{1}{2}\frac{\hbar}{2}\left(\frac{g}{2}\right)^2 \int d^4x\, d^4y\, \varphi^2(x)\varphi^2(y)$
$\left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2$
x
y
$= \frac{1}{4!}\int d^4x_1 \cdots d^4x_4\, \varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\, \Gamma^{(4)}(x_1,x_2,x_3,x_4)$
$^4(x) = \int d^4p_1 d^4p_2 d^4p_3 d^4p_4\, (2\pi)^4 \delta^4(p_1+p_2+p_3+p_4)\, \hat\varphi(p_1)\hat\varphi(p_2)\hat\varphi(p_3)\hat\varphi(p_4)$
$\varphi(x)$
$\hat\varphi(p) = \int d^4x\, e^{-ip\cdot x}\varphi(x)$
$\varphi(x) = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot x}\hat\varphi(p)$



→ log divergence
ouner Transform


= (1/(−Δ+m²))_XX = ∫ d⁴k/(k²+m²)
x y
= [(1/(−Δ+m²))_XY]² = I_0(x−y)
→ log divergence
ourier Transform
Γ̂^(2)(p₁, p₂) = (2π)^d δ^d(p₁+p₂) [p² + m² + ħ/2 g]
Γ̂^(4)(p₁, p₂, p₃, p₄) = (2π)^d δ^d(p₁+p₂+p₃+p₄) [g − ħ/2 g² [+ +]]
p₁ p₃ p₂ p₄ = Î_0(p₁+p₂)
(p₁, p₂)


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
x y
$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$
→ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g\right]$
$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2}g^2\right]$
p_1 p_3 p_2 p_4 +
p_1 p_3 p_2 p_4
+ p_1 p_3 p_2 p_4
p_1 p_3 p_2 p_4 $= \hat{I}_0(p_1+p_2)$
(p_1,p_2)

$[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$

...idean Time

Effective action $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ (1 loop)

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)\right] + \cdots$$

...n of order 2 in φ: 2 point amputated irreducible function.

$$\int d^4x \left(\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2}\varphi^2\right) + \frac{\hbar}{2}\frac{g}{2} \int d^4x \left(\frac{1}{-\Delta + m^2}\right)_{xy} \cdot \varphi^2(x)$$

$$= \frac{1}{2}\int d^4x\, d^4y\, \varphi(x_1) \left[(-\Delta + m^2) + \hbar \frac{g}{2} \, \mathrm{Q} \right] \varphi(x_2) = \frac{1}{2} \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \hat{\varphi}(p_1) \hat{\varphi}(p_2) \; \hat{\Gamma}(p_1, \ldots$$

$\Gamma^{(2)}(x_1, x_2)$ $\quad \delta(x_1 - x_2)$

For...

p_1

p_2

$$\bigcirc = \left(\frac{1}{-\Delta+m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$$

$$\underset{x\quad\quad y}{\bigcirc} = \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$$

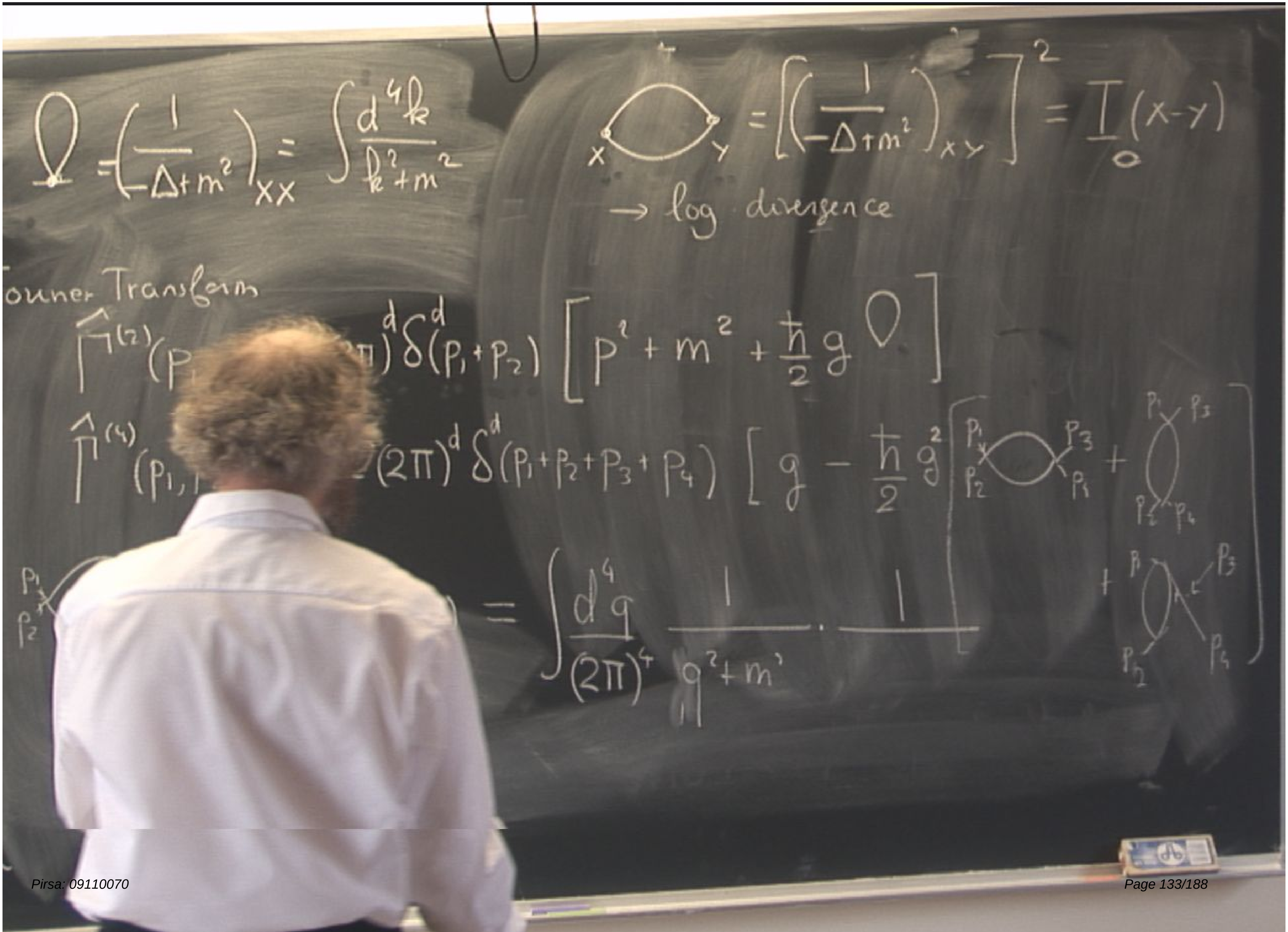
→ log divergence

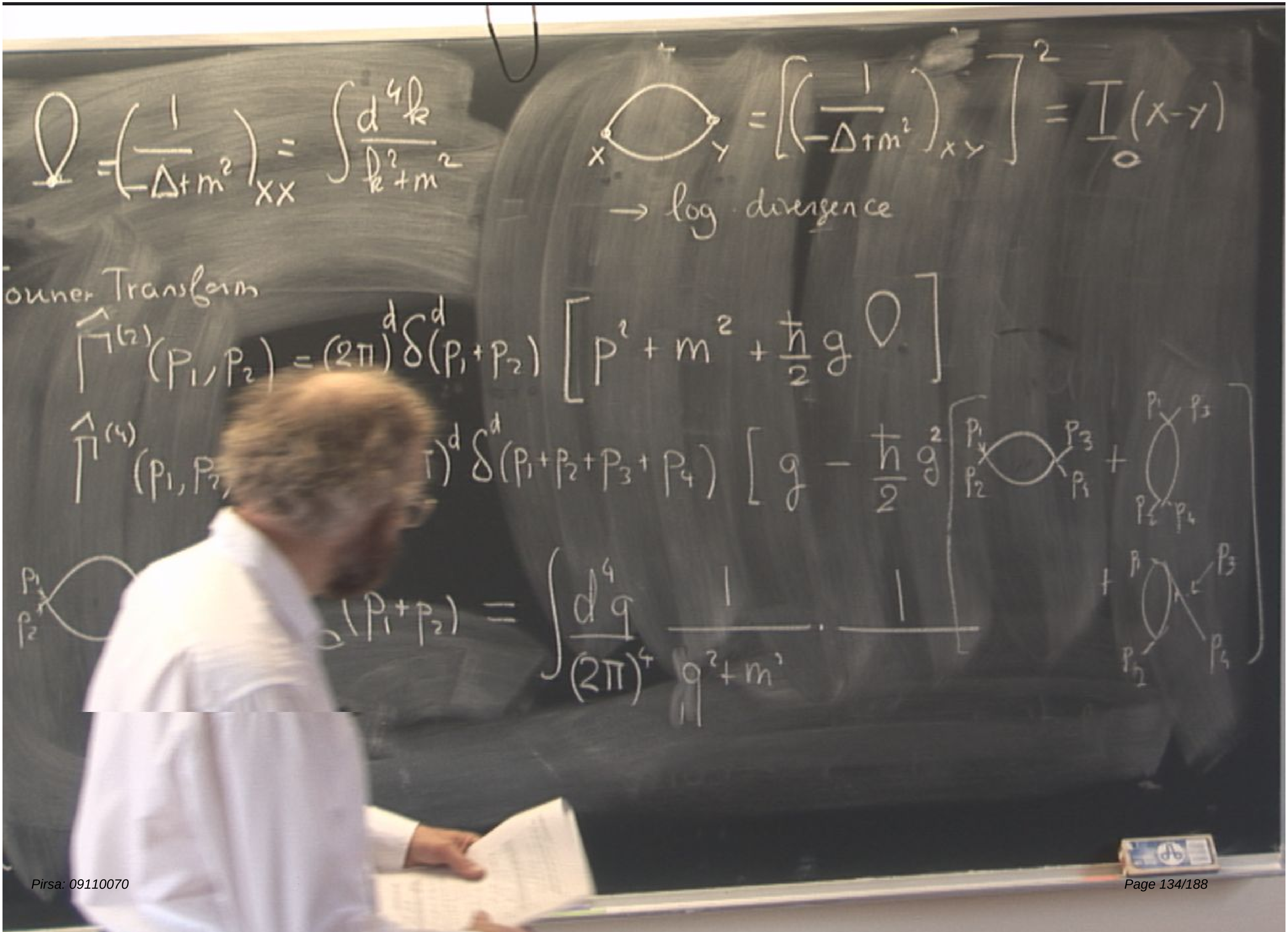
Fourier Transform

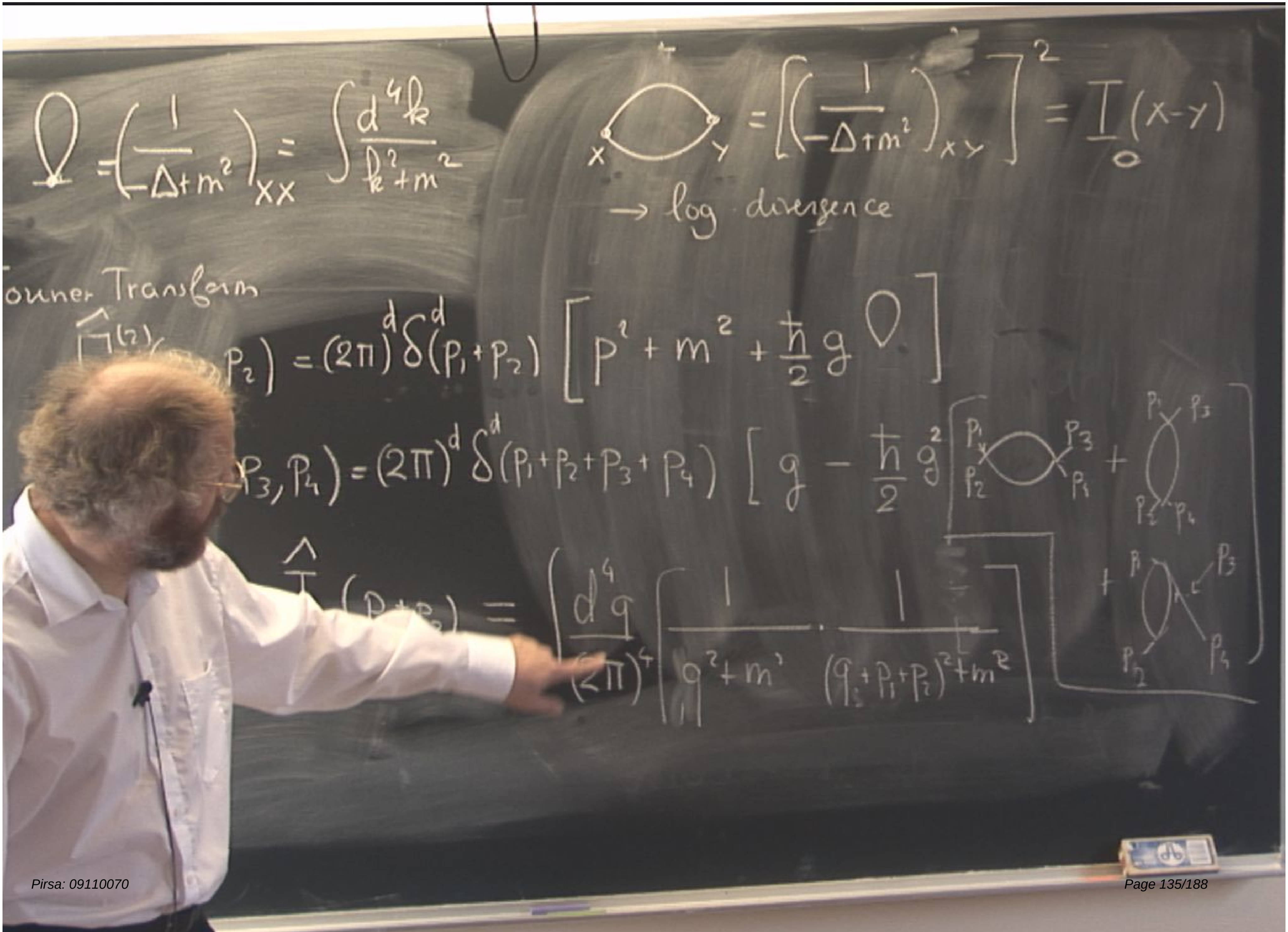
$$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d\,\delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}\,g\,\bigcirc\right]$$

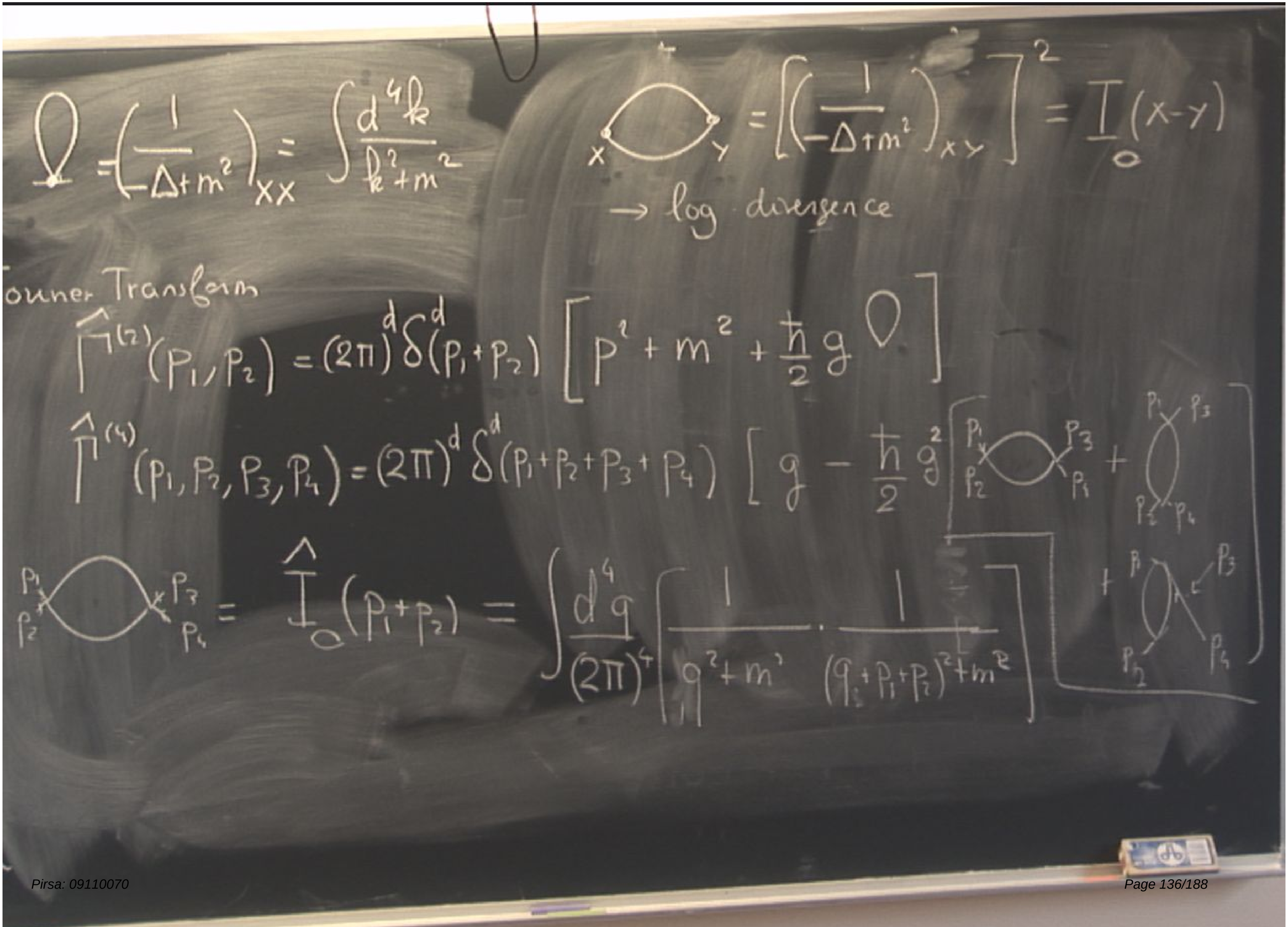
$$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d\,\delta^d(p_1+p_2+p_3+p_4)\left[g-\frac{\hbar}{2}\,g^2\left[\,{}^{p_1}_{p_2}\!\bigcirc\!{}^{p_3}_{p_4} + {}^{p_1\ p_3}_{p_2\ p_4} + {}^{p_1\ p_3}_{p_2\ p_4}\right]\right]$$

$${}^{p_1}_{p_2}\!\bigcirc\!{}^{p_3}_{p_4} = \hat{I}_0(p_1+p_2)$$


= (1 / (−Δ+m²))_XX = ∫ d⁴k / (k²+m²)
x y
= [(1 / (−Δ+m²))_XY]² = I_0(x−y)
→ log divergence
ourier Transform
d d
δ(p₁+p₂) [p² + m² + ħ/2 g]
(2π)^d δ^d(p₁+p₂+p₃+p₄) [g − ħ/2 g²
p₁ p₃ p₂ p₄
= ∫ d⁴q/(2π)⁴ 1/(q²+m²) · 1


= (1 / -Δ + m²)_XX = ∫ d⁴k / k² + m²
= [(1 / -Δ + m²)_XY]² = I_0 (X − Y)
→ log divergence
ourier Transform
Γ^(2)(p₁, p₂) = (2π)^d δ^d(p₁ + p₂) [p² + m² + ħ/2 g]
Γ^(4)(p₁, p₂ … (2π)^d δ^d(p₁ + p₂ + p₃ + p₄) [g − ħ/2 g²
p₁ p₂ p₃ p₄
(p₁ + p₂) = ∫ d⁴q / (2π)⁴ · 1 / q² + m² · 1


= $\left(\frac{1}{-\Delta+m^2}\right)_{xx}$ = $\int \frac{d^4k}{k^2+m^2}$
= $\left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$
x y
→ log divergence
ourier Transform
$\hat{\Gamma}^{(2)}$
$p_2) = (2\pi)^d \delta^d(p_1+p_2) \left[p^2 + m^2 + \frac{\hbar}{2} g \right]$
$p_3, p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4) \left[g - \frac{\hbar}{2} g^2 \right.$
p_1 p_3 p_2 p_4
$\hat{I}$
$= \int \frac{d^4q}{(2\pi)^4} \left[\frac{1}{q^2+m^2} \cdot \frac{1}{(q+p_1+p_2)^2+m^2}\right]$


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
$= \left[\left(\frac{1}{-\Delta + m^2}\right)_{xy}\right]^2 = I_0(x-y)$
x y
→ log divergence
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1, p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2 + m^2 + \frac{\hbar}{2} g \right]$
$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2} g^2 \left[\quad + \quad + \quad \right]\right]$
p_1 p_2 p_3 p_4
$= \hat{I}_0(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2}\cdot\frac{1}{(q+p_1+p_2)^2+m^2}\right]$

 $= \left(\frac{1}{-\Delta+m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

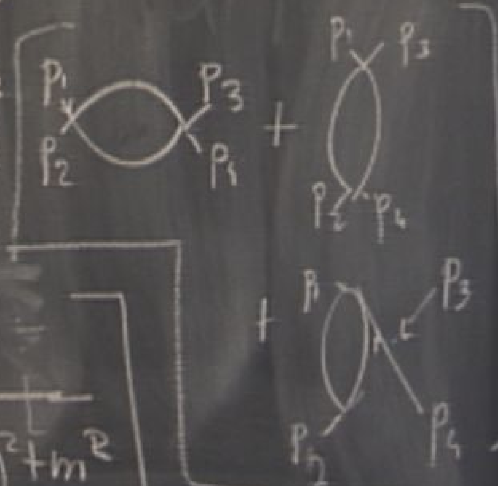
$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_o(x-y)$

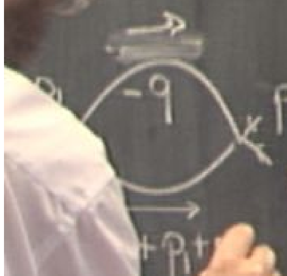
→ log divergence

ourier Transform

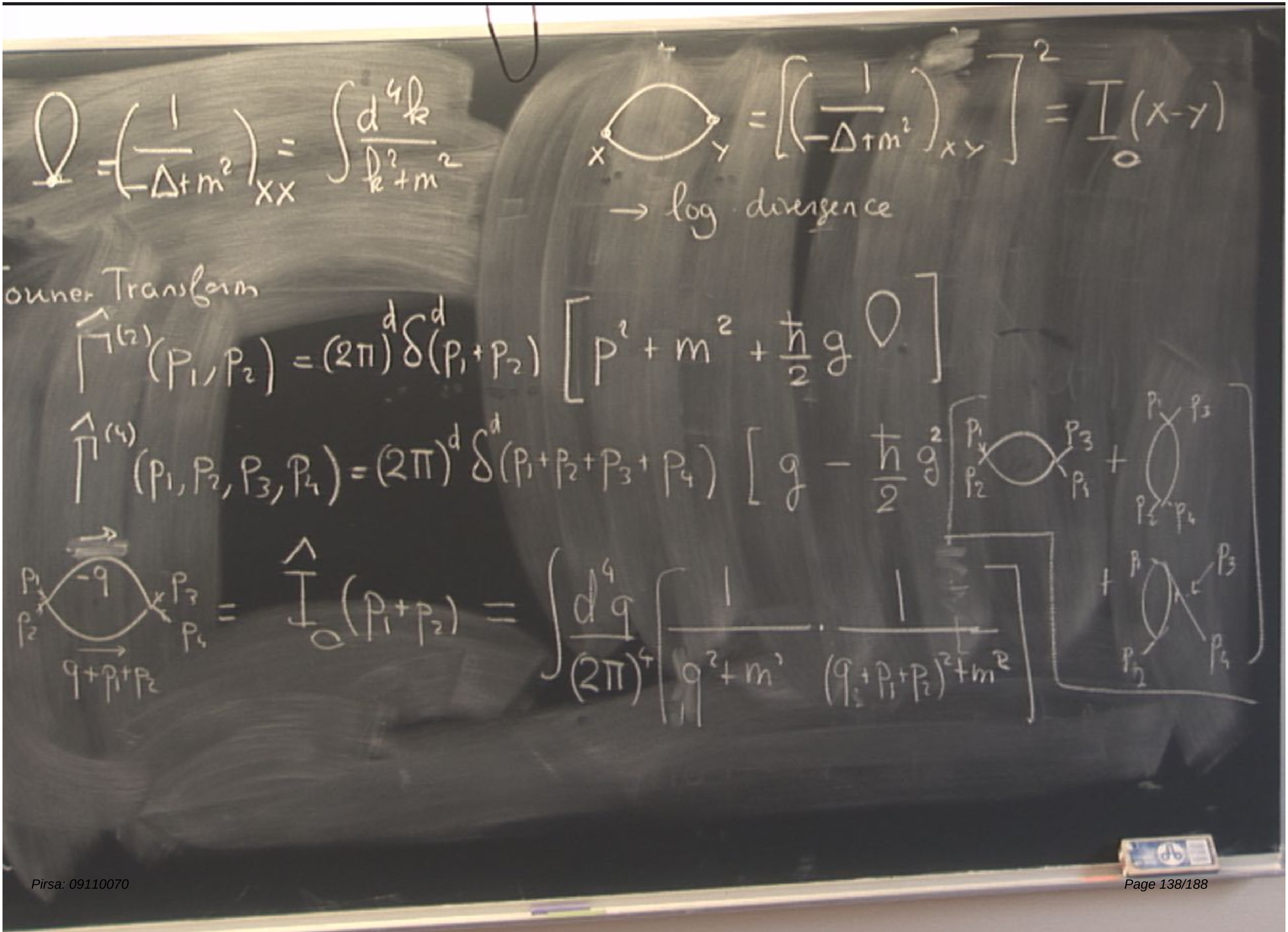
$$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d\,\delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g\,\bigcirc\right]$$

$$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d\,\delta^d(p_1+p_2+p_3+p_4)\left[g-\frac{\hbar}{2}g^2\left[\ \ldots\ \right]\right]$$





$$= \hat{I}_o(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2}\cdot\frac{1}{(q+p_1+p_2)^2+m^2}\right]$$


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$
x y
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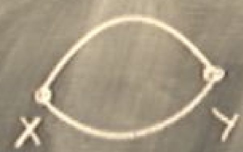
Fourier Transform
$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2} g \right]$
$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2} g^2 \right]$
p1 p2 p3 p4
-q
q+p1+p2
$= \hat{I}_0(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2} \cdot \frac{1}{(q+p_1+p_2)^2+m^2}\right]$


$$= \left(\frac{1}{-\Delta+m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$$



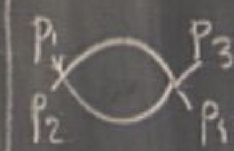
$$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$$

→ log divergence

Fourier Transform

$$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d\,\delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g\right.$$

$$\left.\vphantom{\frac{\hbar}{2}}\right]$$

$$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d\,\delta^d(p_1+p_2+p_3+p_4)\left[g-\frac{\hbar}{2}g^2\left[\vphantom{\frac{1}{2}}\right.\right.$$


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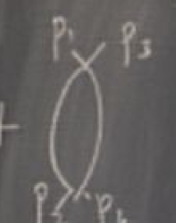

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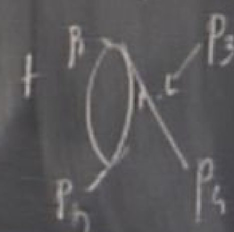


$$\left.\left.\vphantom{\frac{1}{2}}\right]\right]$$

$$= \hat{I}_0(p_1+p_2) = \int\frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2}\cdot\frac{1}{(q+p_1+p_2)^2+m^2}\right]$$

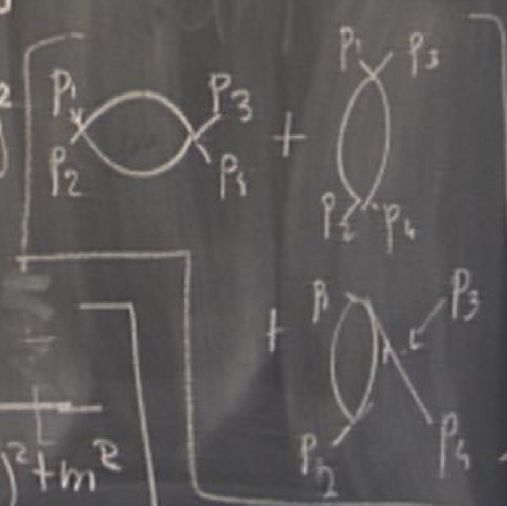
 $= \left(\frac{1}{-\Delta+m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$

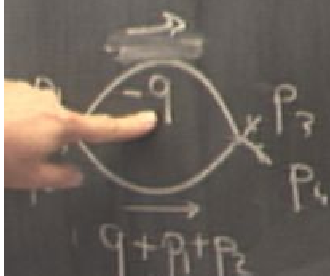
$= \left[\left(\frac{1}{-\Delta+m^2}\right)_{xy}\right]^2 = I_0(x-y)$

→ log divergence

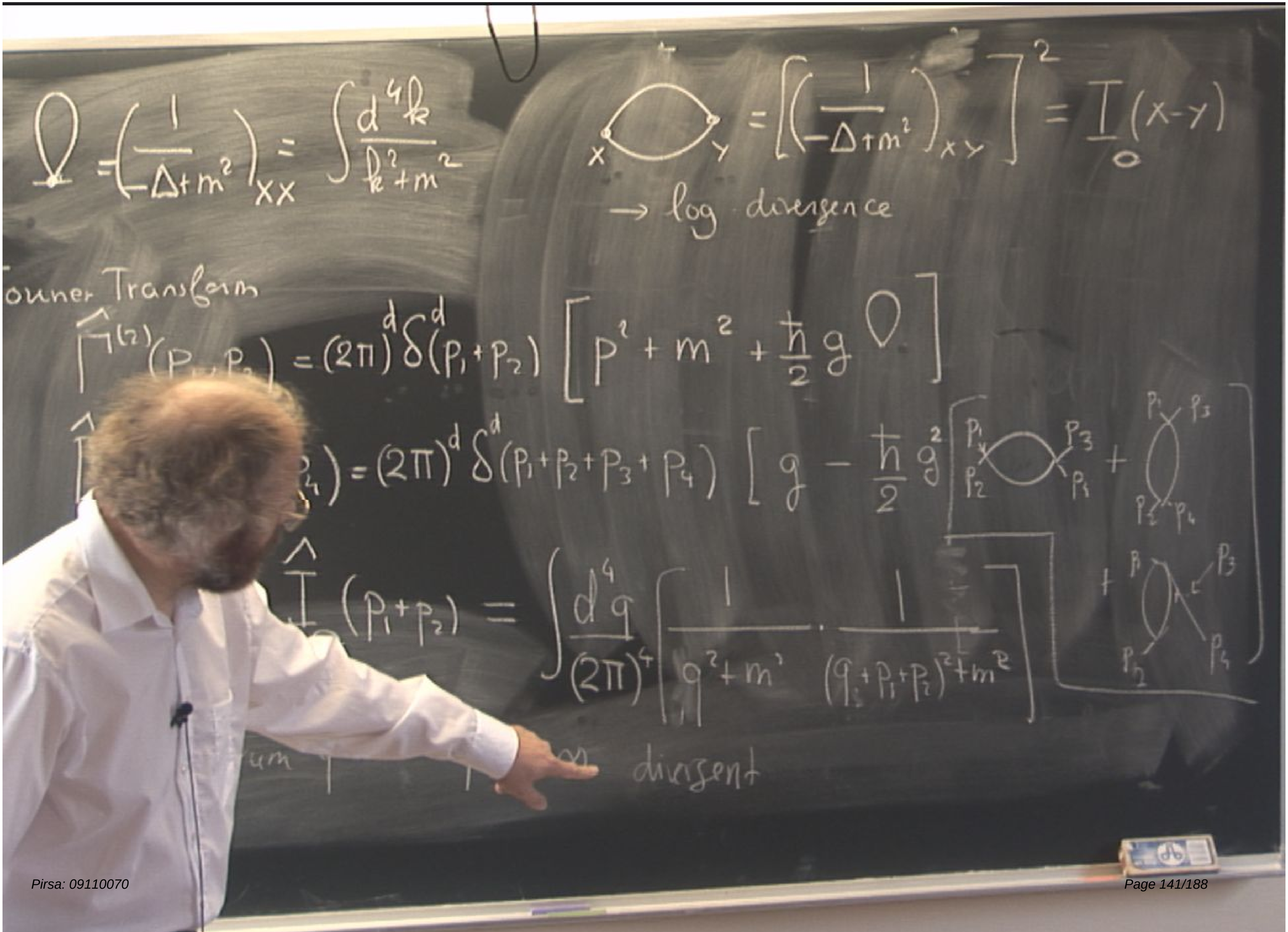
Fourier Transform

$\hat{\Gamma}^{(2)}(p_1,p_2) = (2\pi)^d \delta^d(p_1+p_2)\left[p^2+m^2+\frac{\hbar}{2}g\right.$  $\left.\right]$

$\hat{\Gamma}^{(4)}(p_1,p_2,p_3,p_4) = (2\pi)^d \delta^d(p_1+p_2+p_3+p_4)\left[g - \frac{\hbar}{2}g^2\right.$  $\left.\right]$



$= \hat{I}_0(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2}\cdot\frac{1}{(q+p_1+p_2)^2+m^2}\right]$


$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$
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x
y
→ log divergence
ourier Transform
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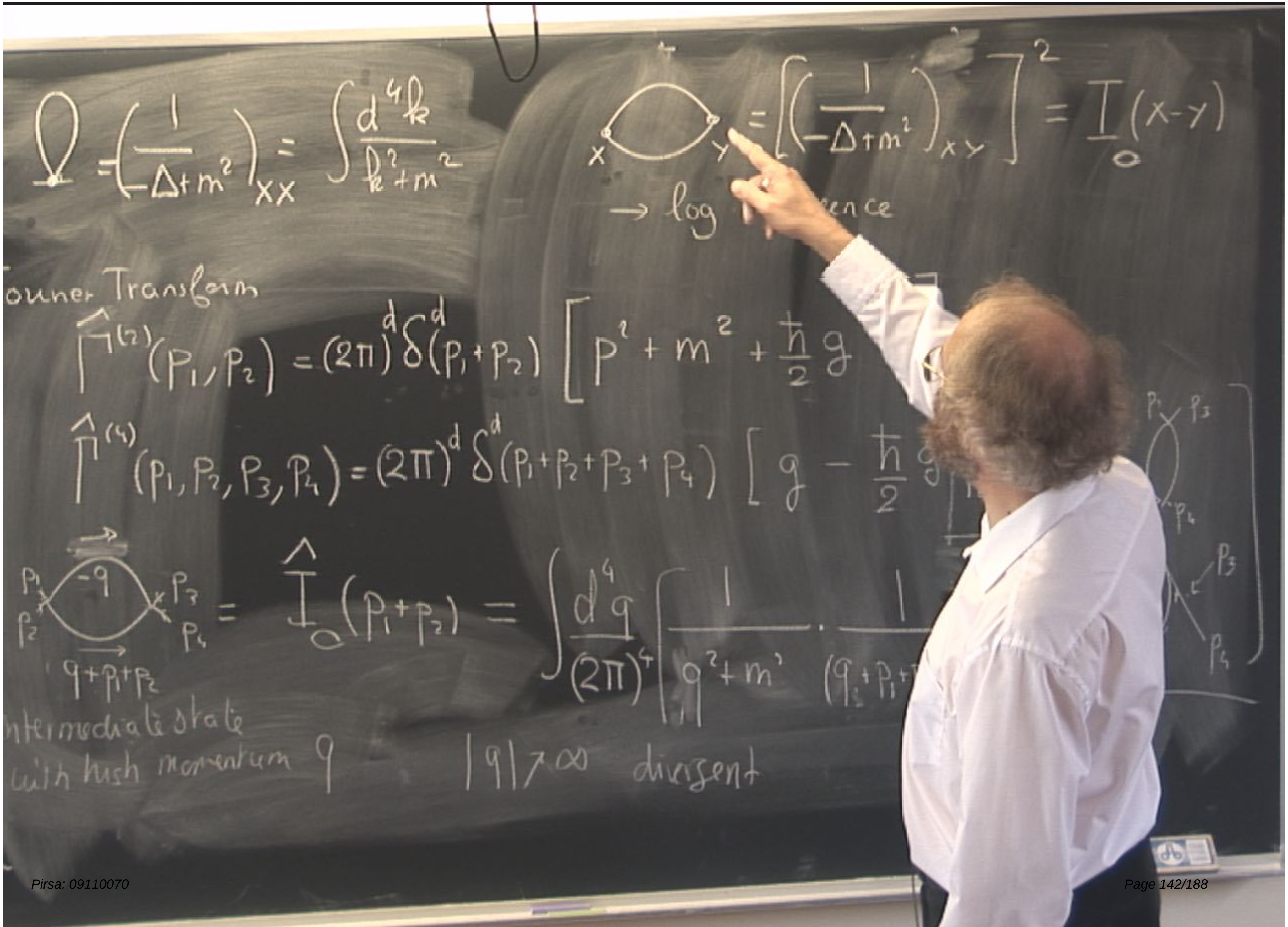
p_1
p_2
p_3
p_4
$\hat{I}_0(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2} \cdot \frac{1}{(q+p_1+p_2)^2+m^2}\right]$
divergent


ourier Transform
→ log
ntermediate state
with high momentum q
|q| → ∞ divergent

$$= \left(\frac{1}{-\Delta + m^2}\right)_{xx} = \int \frac{d^4k}{k^2+m^2}$$

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ourier Transform

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$-q$, $q+p_1+p_2$

ntermediate state with high momentum q

$|q| \to \infty$ divergent at high momenta

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→ log divergence

Fourier Transform

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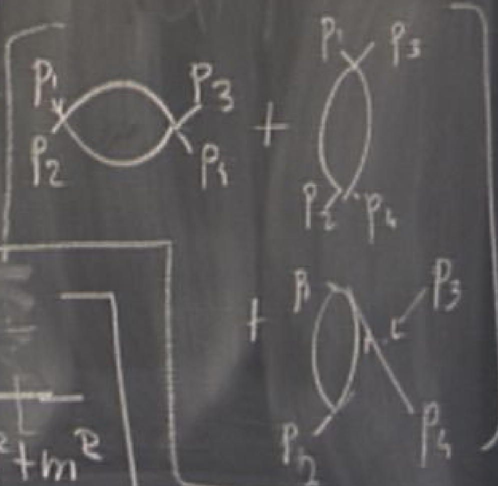
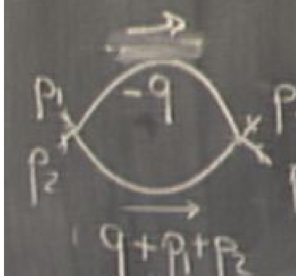


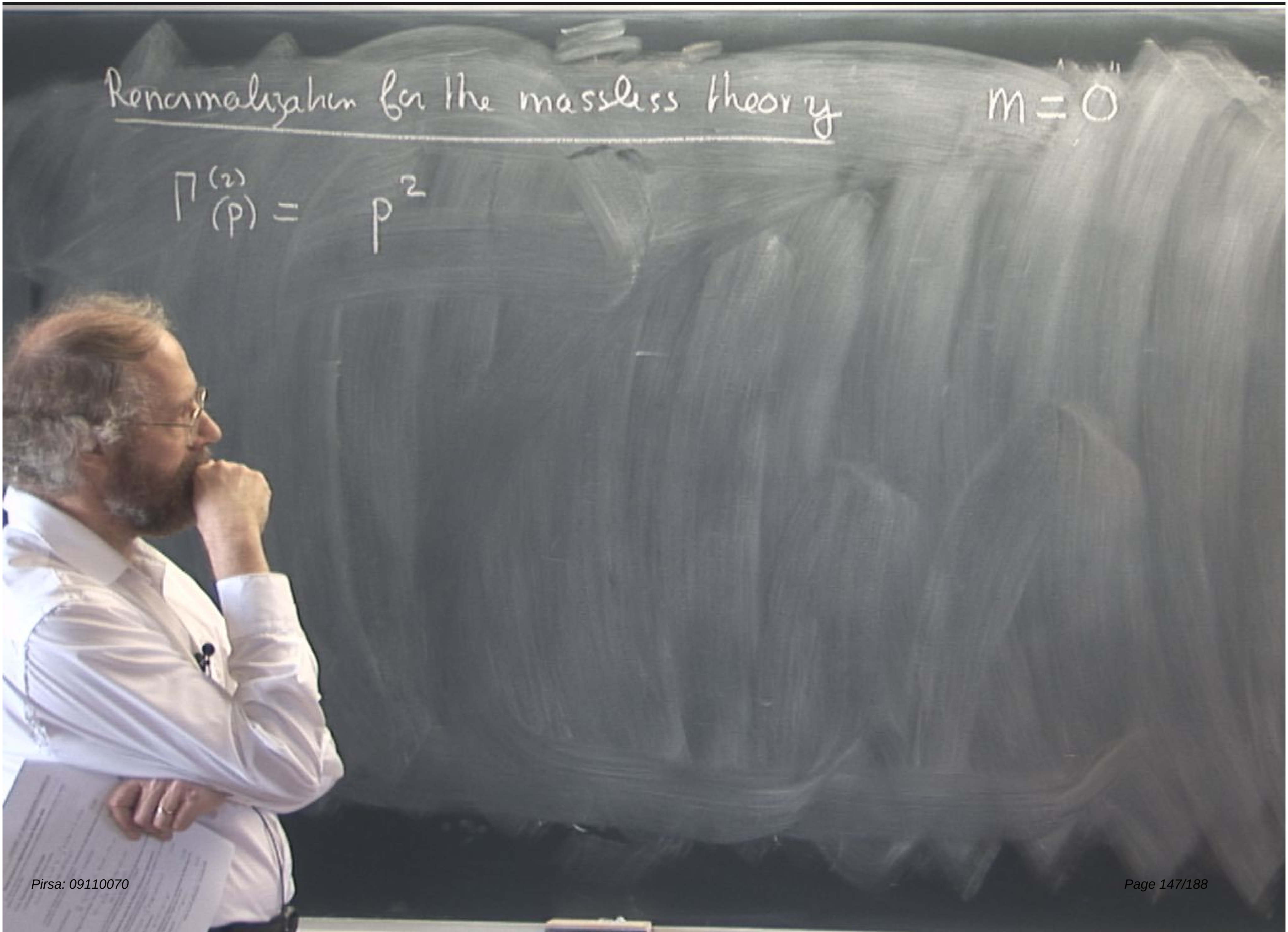
 $= \hat{I}_0(p_1+p_2) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{q^2+m^2}\cdot\frac{1}{(q+p_1+p_2)^2+m^2}\right]$

Intermediate state with high momentum q

$|q|\to\infty$ divergent at high momenta

Renormalization for the massless theory

Renormalization for the massless theory $m = 0$


Renormalization for the massless theory
m = "0"
Γ(2)(p) = p2

Renormalization for the massless theory $m=0$

$$\Gamma^{(2)}(p) = p^2 +$$

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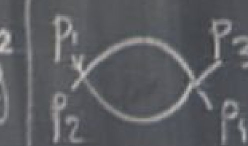
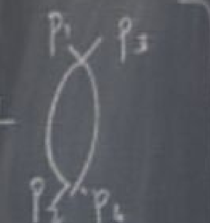


+



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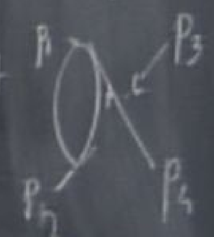

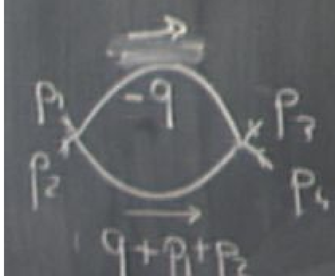

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Intermediate state with high momentum q

$|q| \to \infty$ divergent at high momenta

Renormalization for the massless theory $m = 0$

$$\Gamma^{(2)}(p) = p^2 +$$

Term of $(-+++)$

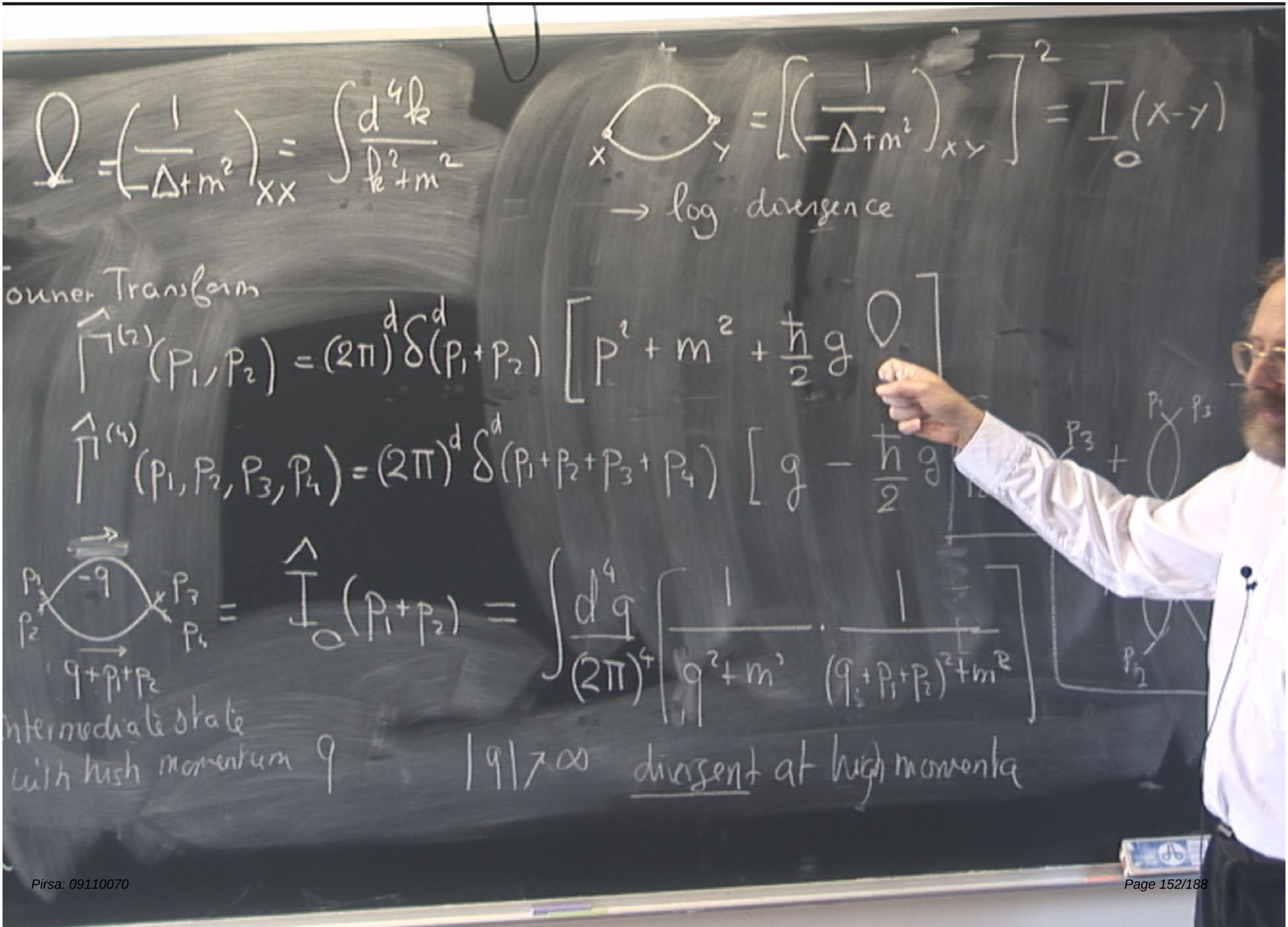
Physical mass of the 1 particle state

$$\Gamma^{(2)}(p) = 0 \quad \text{when} \quad p^2 = -m_{phys}^2$$

$$p^2 = -E^2 + \vec{p}^{\,2} \qquad E^2 = \vec{p}^{\,2} + m^2$$

$\varphi(p_1)$

$\varphi(p_3)$



= (1/(−Δ+m²))_XX = ∫ d⁴k/(k²+m²)
= [(1/(−Δ+m²))_XY]² = I_0(X−Y)
→ log divergence
Fourier Transform
Γ^(2)(p1, p2) = (2π)^d δ^d(p1+p2) [p² + m² + ħ/2 g]
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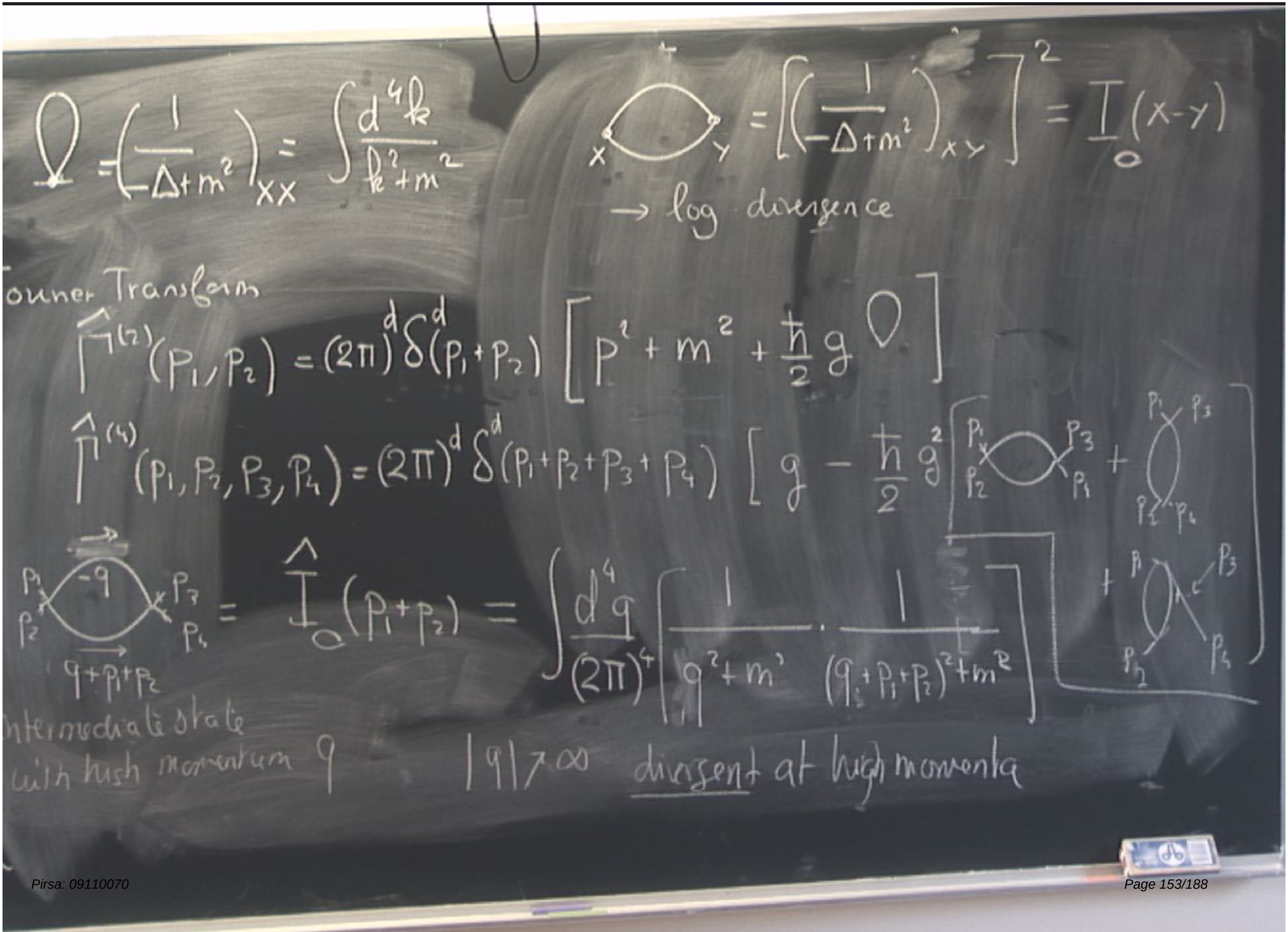
-q
q+p1+p2
= I_0(p1+p2) = ∫ d⁴q/(2π)⁴ [1/(q²+m²) · 1/((q+p1+p2)²+m²)]
Intermediate state
with high momentum q
|q| → ∞ divergent at high momenta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−q
q+p₁+p₂
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intermediate state
with high momentum q
|q| → ∞ divergent at high momenta

Renormalization for the massless theory $\quad m = 0 \quad T($

$$\Gamma^{(2)}(p) = p^2 + \frac{\hbar g}{2} T(0)$$

$T(0)$ is ∞

Renormalization for the massless theory $\quad m = 0 \quad T(\ldots$

$$\Gamma^{(2)}(p) = p^2 + \frac{\hbar g}{2} T(0) \qquad T(0) \text{ is } \infty$$

Introduce an UV regulator so that $|k| < \Lambda$

Renormalization for the massless theory $\quad m = 0 \quad T(m) =$

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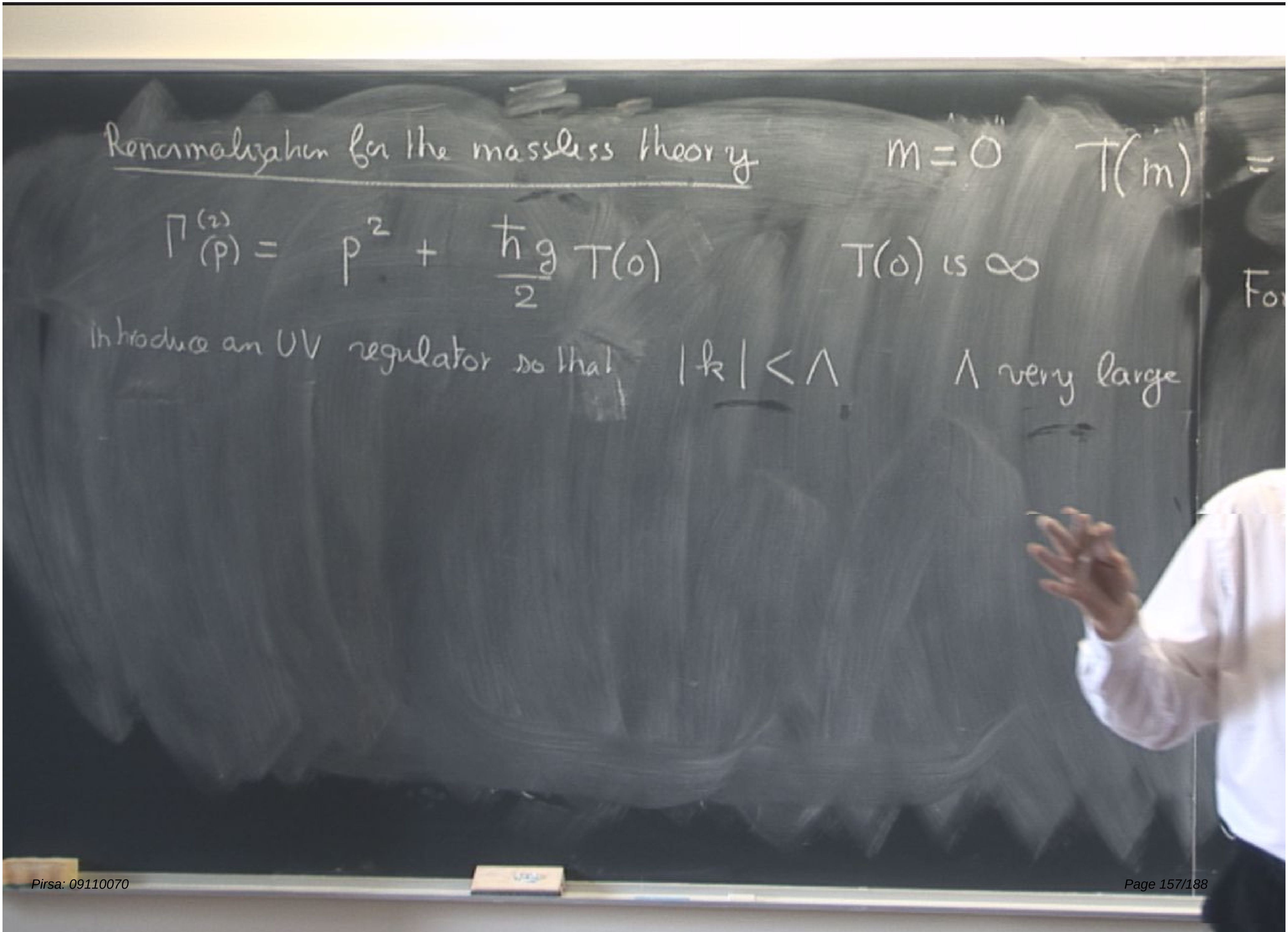
Introduce an UV regulator so that $|k| < \Lambda$ $\quad$ Λ very large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Fo

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p_1
p_2

inter
wi

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$$T(0) = \int_{|k|<\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = \Lambda^2$$

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← regulator dependent

Fo…

p_1

p_2

inter…

wi…

Renormalization for the massless theory $m=0$ $T(m) =$

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Instead of starting from a theory with mass $m^2=0$, I start with a theory

For

Renormalization for the massless theory $m = 0$ $T(m)$ =

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Instead of starting from a theory with mass $m^2 = 0$, I start with a theory with a $\text{mass}^2 = -\frac{\hbar g}{2} T(0) \simeq -\frac{\hbar g}{2} \Lambda^2$

Fo…

p_1 … p_2 …

inter…

wi…

Renormalization for the massless theory

$m = 0$ $\qquad$ $T(m) =$

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Introduce an UV regulator so that $|k| < \Lambda$ $\qquad$ Λ very large

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$\leftarrow$ regulator dependent

Instead of starting from a theory with mass $m^2 = 0$, I start with a theory with a $\text{mass}^2 = -\frac{\hbar g}{2} T(0) \simeq -\frac{\hbar g}{2} \Lambda^2 = m_B^2$ "Bare" mass

Renormalization for the massless theory $m=0$ $T(m) =$

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Introduce an UV regulator so that $|k| < \Lambda$ Λ very large

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For

p_1
p_2

inter
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Fo

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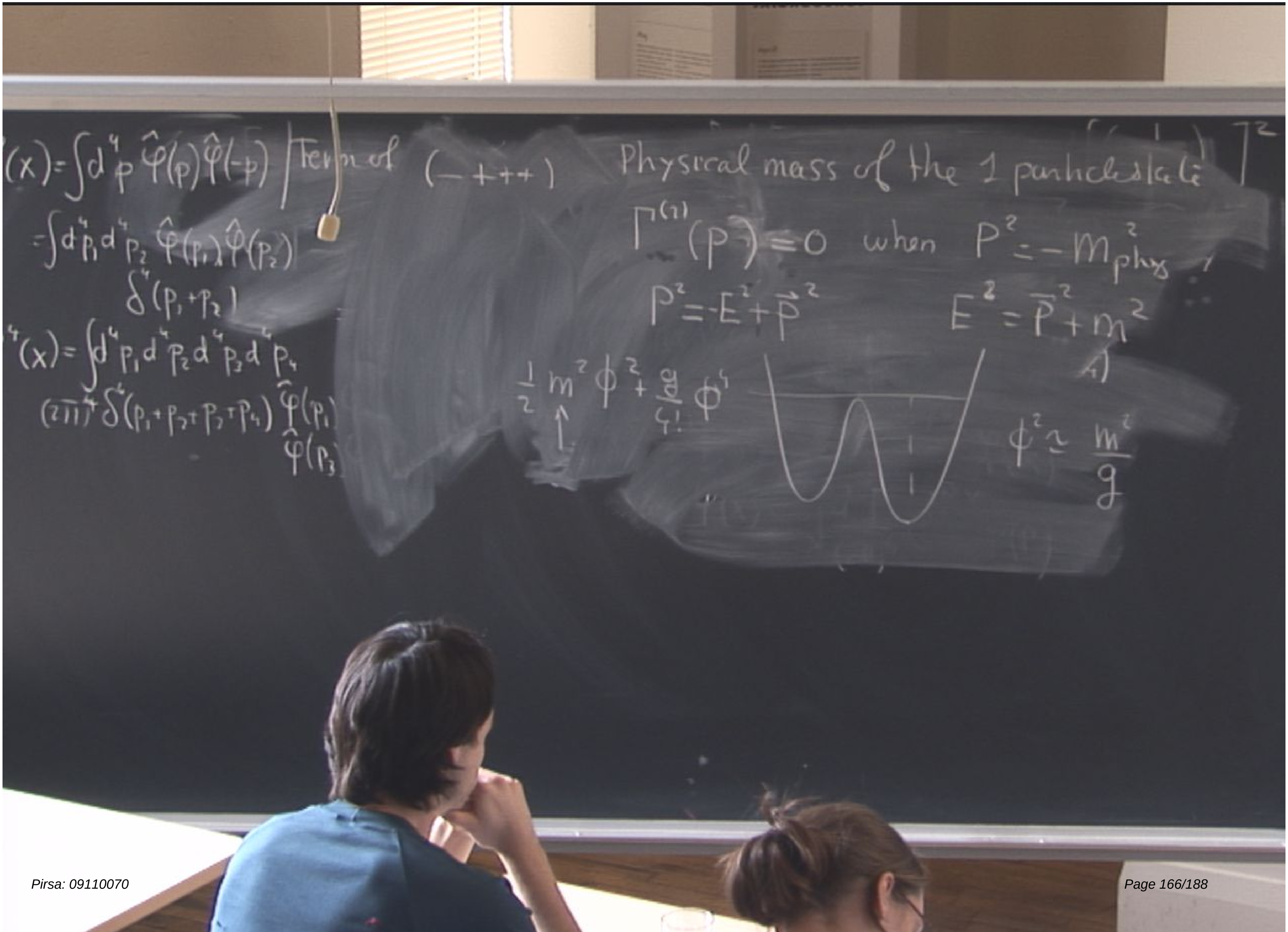
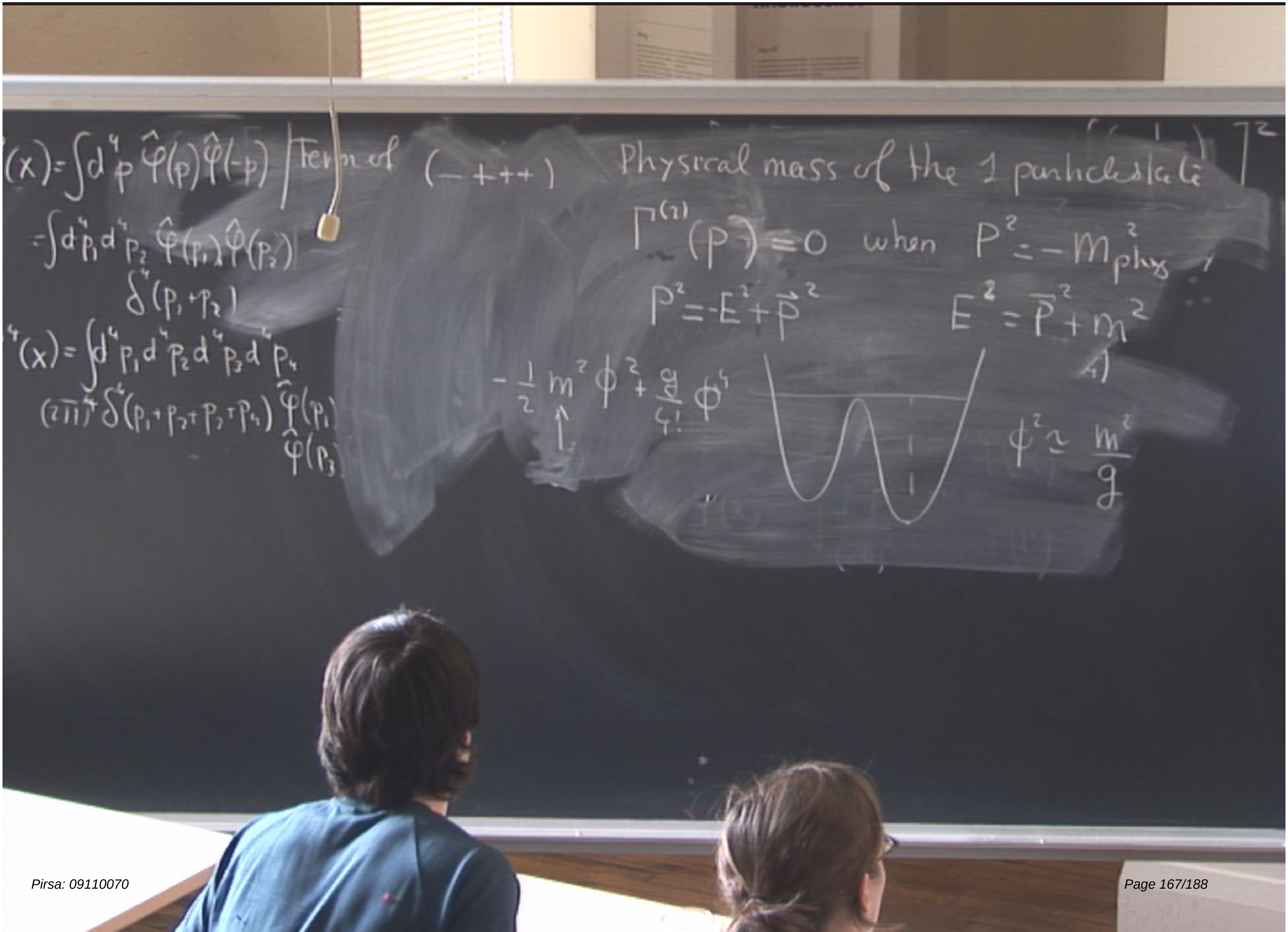

Term of
(− + + +)
Physical mass of the 1 particle state
Γ(2)(P) = 0 when P² = −m²phys
P² = −E² + p⃗²
E² = p⃗² + m²
½ m² φ² + g/4! φ⁴
φ² ≃ m²/g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Renormalization for the massless theory $m = 0$ T(

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$$\langle \phi^2 \rangle_{\text{at scale } \Lambda} \simeq \frac{\hbar g \Lambda^2}{2g} \simeq \Lambda^2$$

Renormalization for the massless theory $\quad$ $m = 0$ $\quad$ T(

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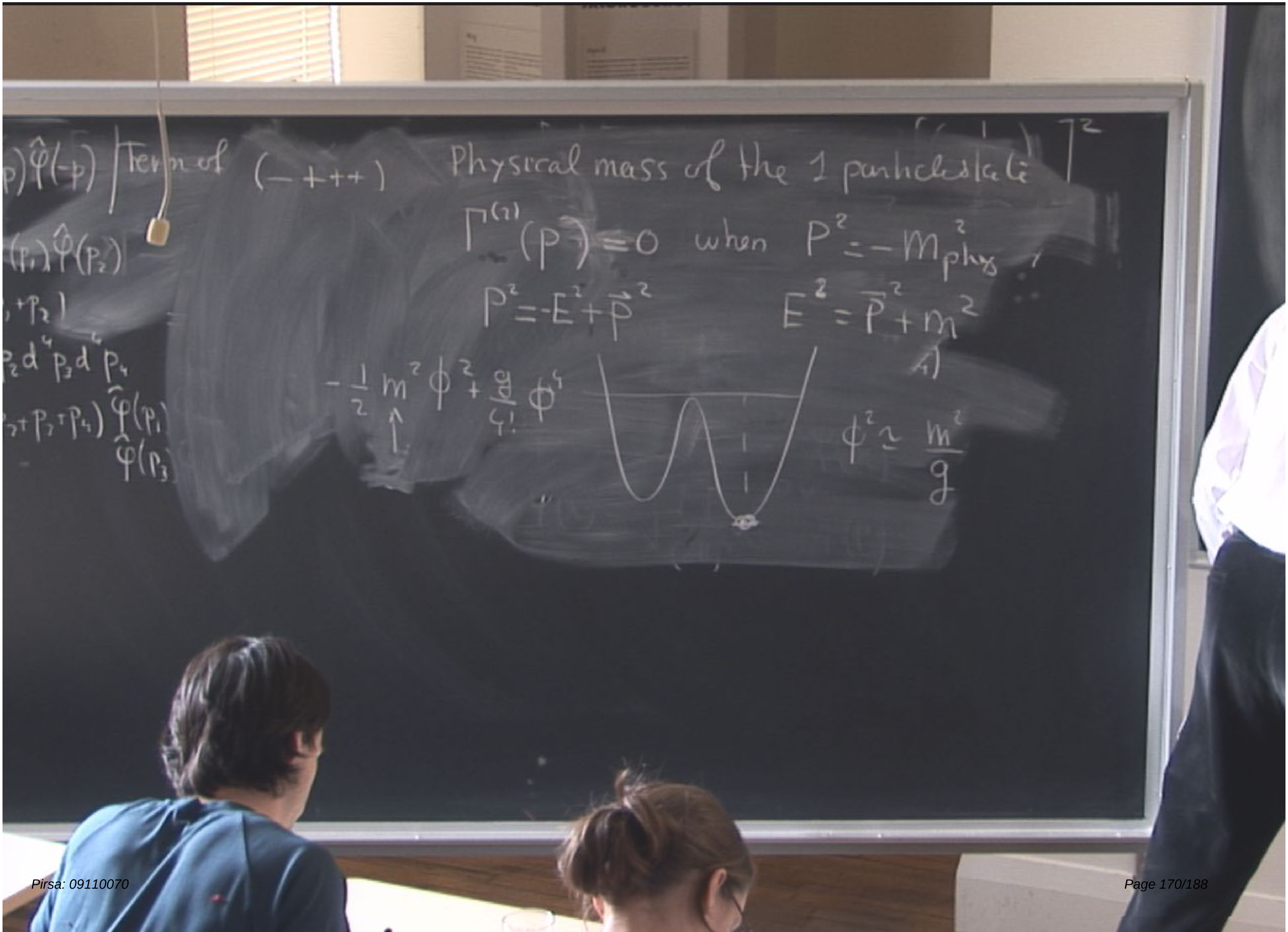
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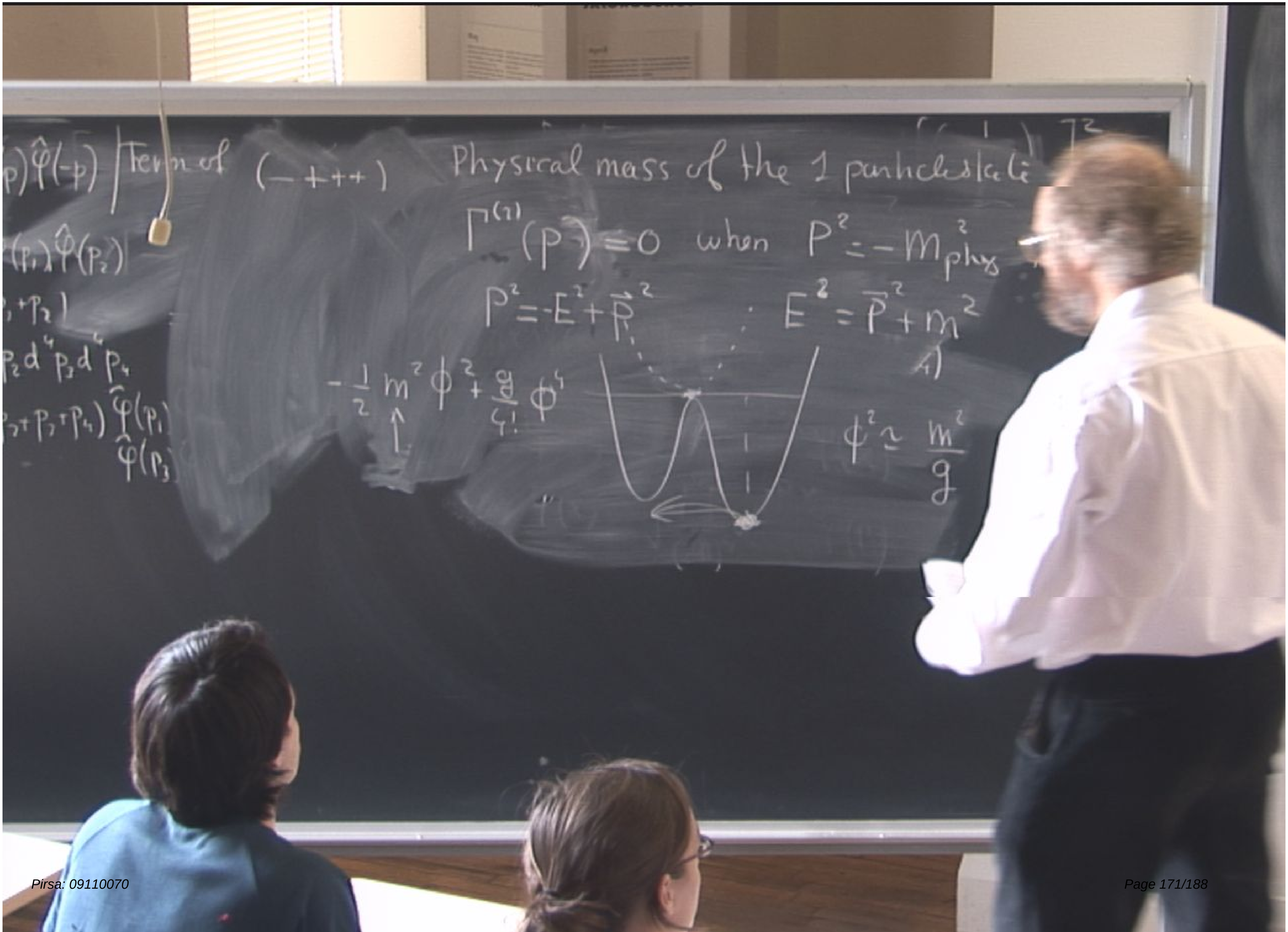

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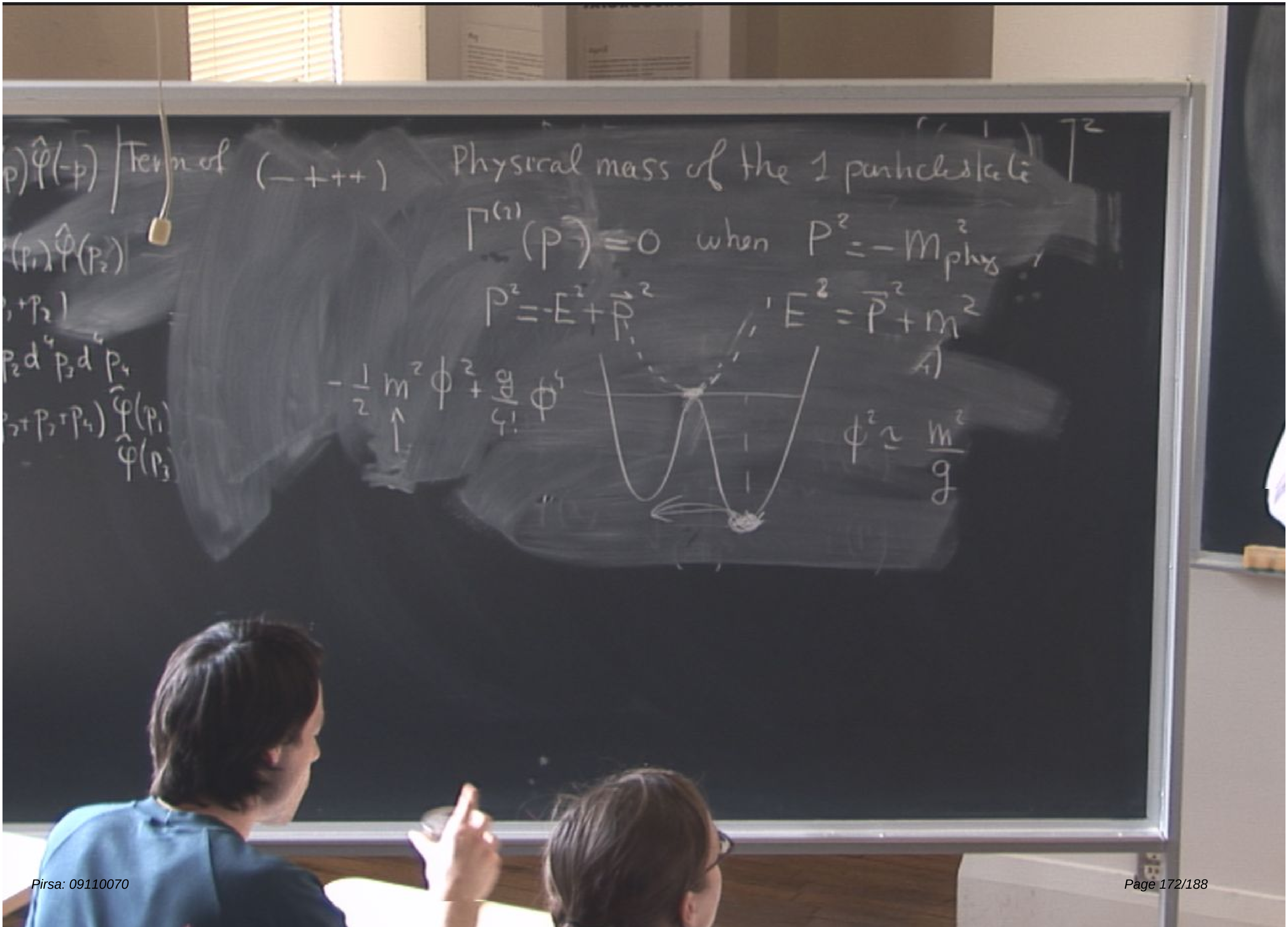
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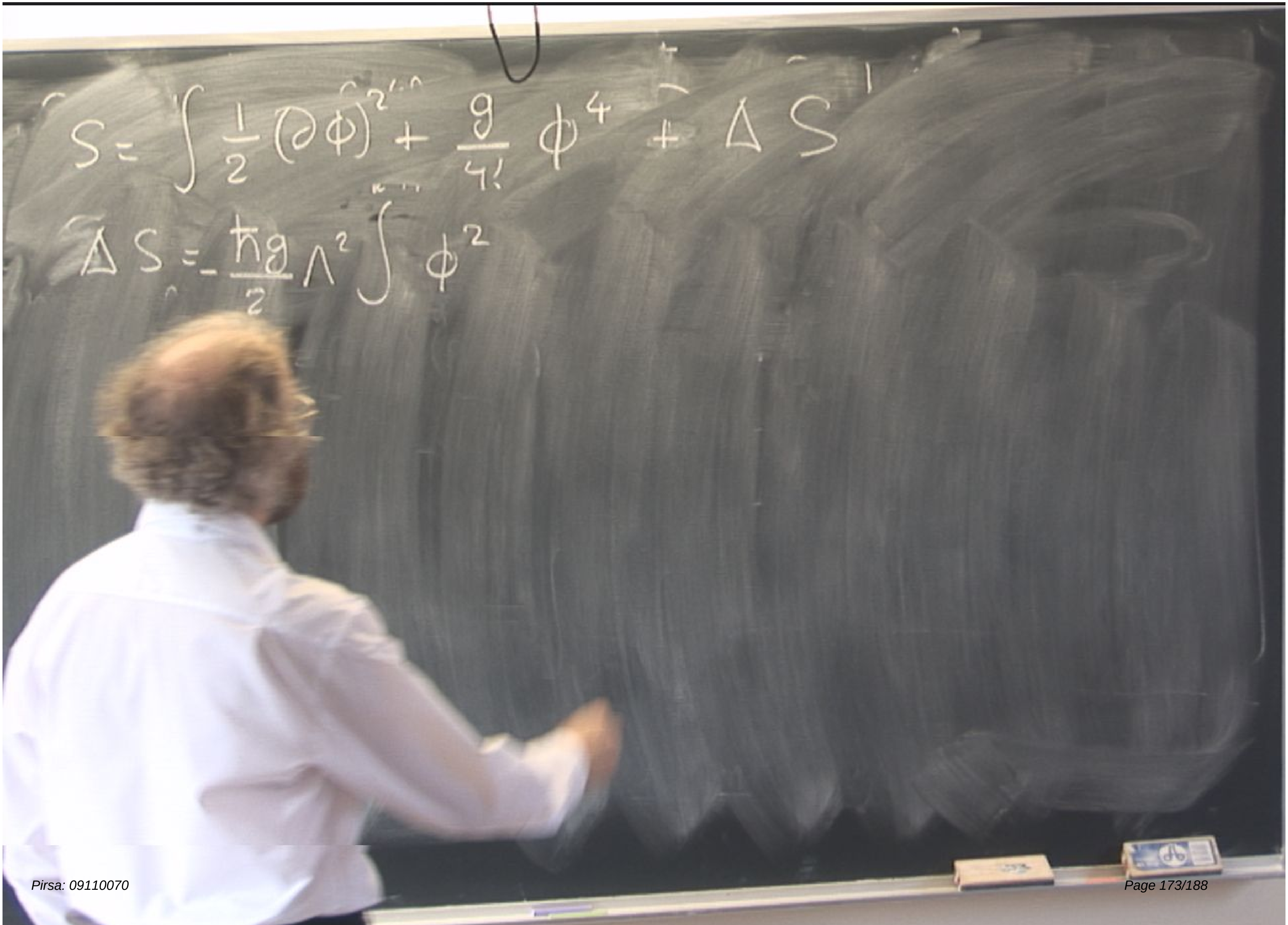
$-\frac{1}{2} m^2 \phi^2 + \frac{g}{4!} \phi^4$
$\phi^2 \simeq \frac{m^2}{g}$


$S = \int \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4 + \Delta S$
$\Delta S = -\frac{\hbar g}{2}\Lambda^2 \int \phi^2$

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$$\Delta S = -\frac{\hbar g}{2}\Lambda^2 \int \phi^2 \quad \text{counterterm}$$

$\Rightarrow$ Then the theory is such that the physical mass is of order $O(1)$, and moreover $m^2_{phys} = 0$

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Renormalization for the massless theory $m = 0$ $T($

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$$T(0) = \int_{|k|<\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = \frac{1}{32\pi^2} \Lambda^2$$

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$$S = \int \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4 + \Delta S$$

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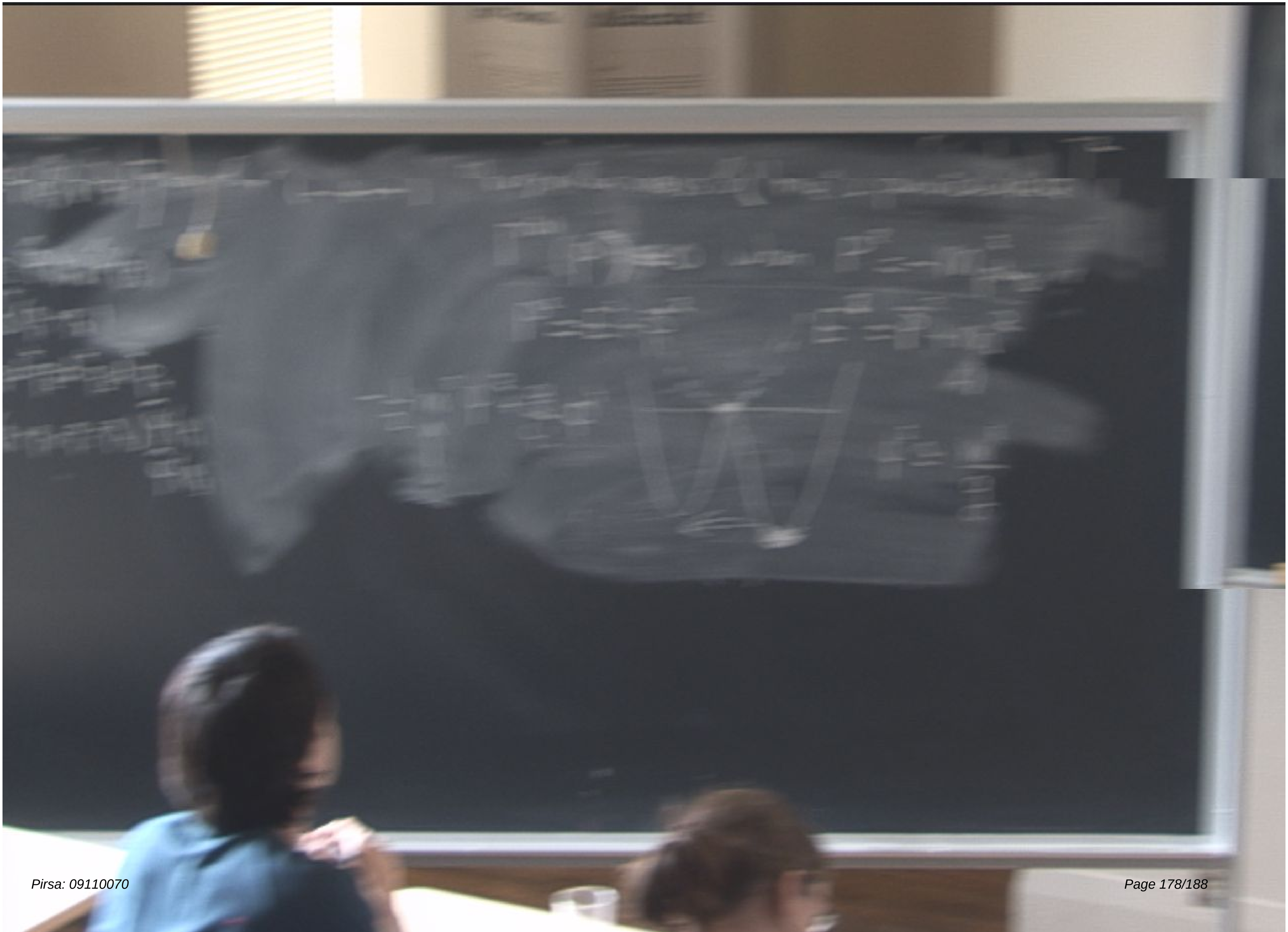
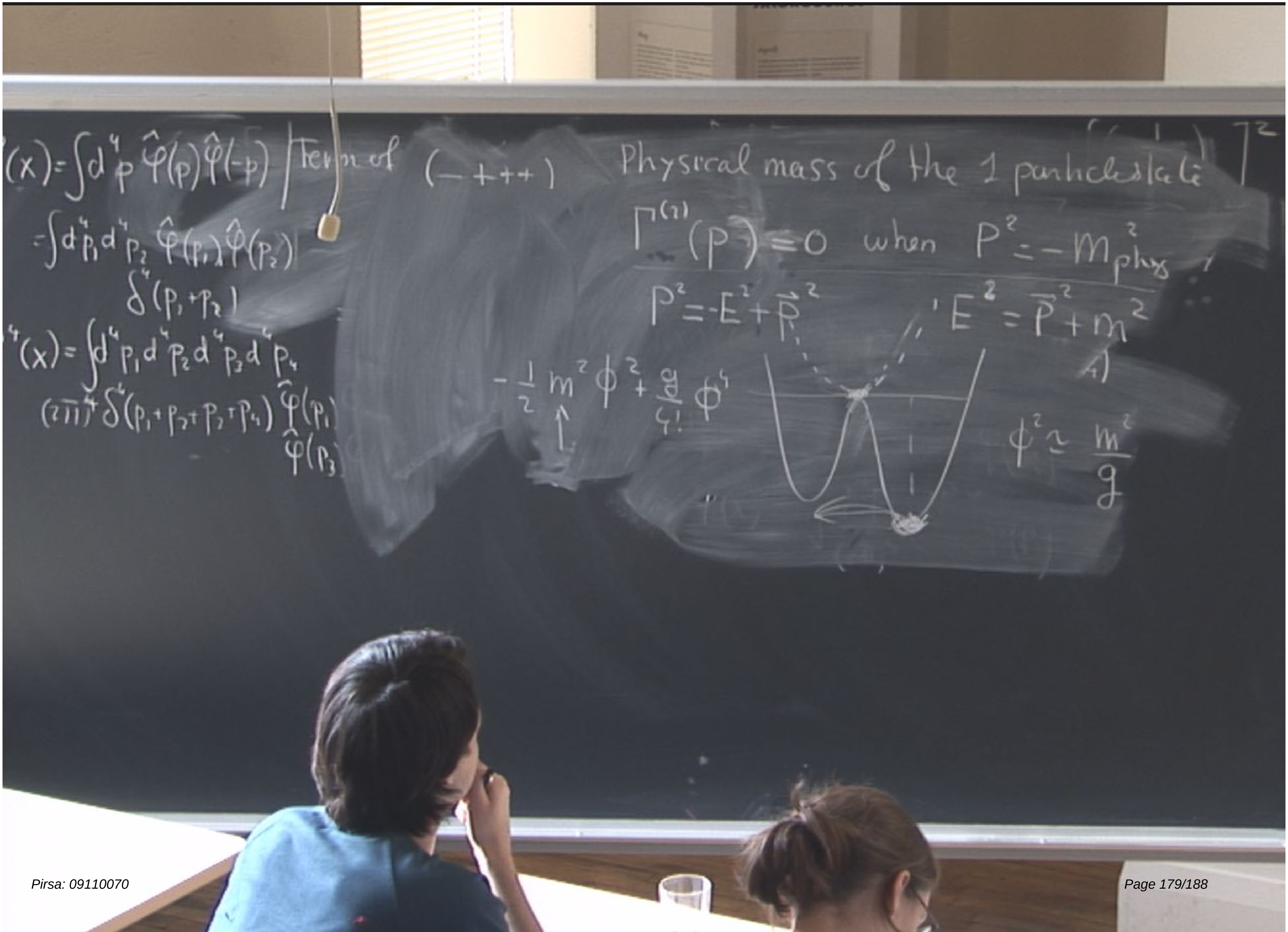
$\Rightarrow$ Then the theory is such that the physical mass is ~~of~~ order $O(1)$, and moreover

$$\boxed{m^2_{phys} = 0}$$

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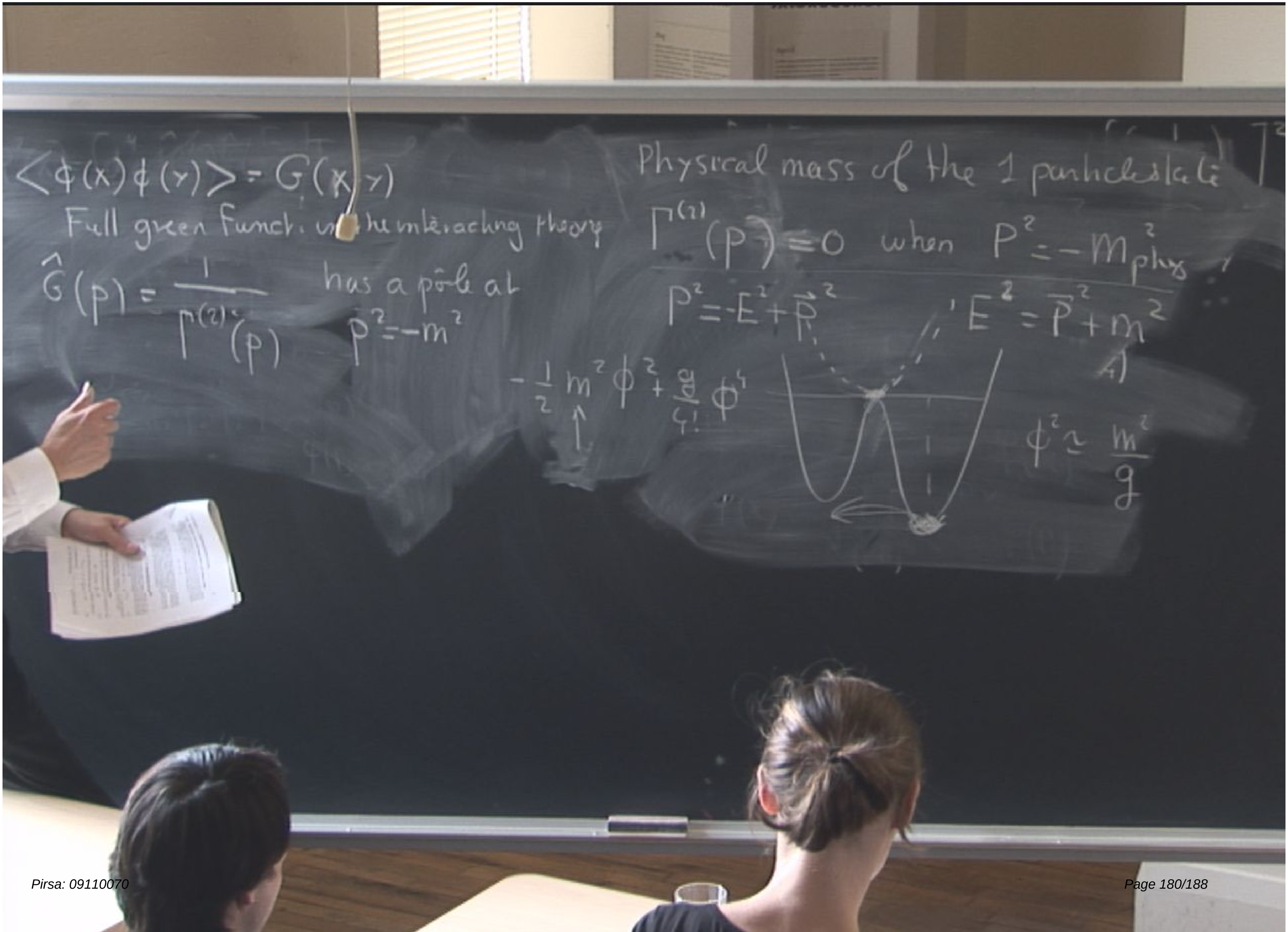
P2 = −E2 + P2
E2 = P2 + m2
−1/2 m2 φ2 + g/4! φ4
φ2 ≃ m2/g


$\langle \phi(x)\phi(y) \rangle = G(x,y)$
Full green function in the interacting theory
$\hat{G}(p) = \frac{1}{\Gamma^{(2)}(p)}$ has a pole at $p^2 = -m^2$
$-\frac{1}{2} m^2 \phi^2 + \frac{g}{4!} \phi^4$
Physical mass of the 1 particle state
$\Gamma^{(2)}(p) = 0$ when $p^2 = -m_{phys}^2$
$p^2 = -E^2 + \vec{p}^2$
$E^2 = \vec{p}^2 + m^2$
$\phi^2 \simeq \frac{m^2}{g}$

Renormalization for the massless theory $\quad m = 0 \quad T($

$$\Gamma^{(2)}(p) = p^2 + \frac{\hbar g}{2} T(0) \qquad T(0) \text{ is } \infty$$

introduce an UV regulator so that $|k| < \Lambda$ $\quad \Lambda$ very l

$$T(0) = \int_{|k|<\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} = \frac{1}{32\pi^2} \Lambda^2$$

$\leftarrow$ regulator dependent

Instead of starting from a theory with mass $m^2 = 0$, I start with a the[ory] with a $\text{mass}^2 = -\frac{\hbar g}{2} T(0) \simeq -\frac{\hbar g}{2} \Lambda^2 = m_B^2$ "Bare" ma[ss]

$$\langle \phi^2 \rangle_{\substack{\text{at scale} \\ \Lambda}} \simeq \frac{\hbar g \Lambda^2}{2g} \simeq \Lambda^2$$

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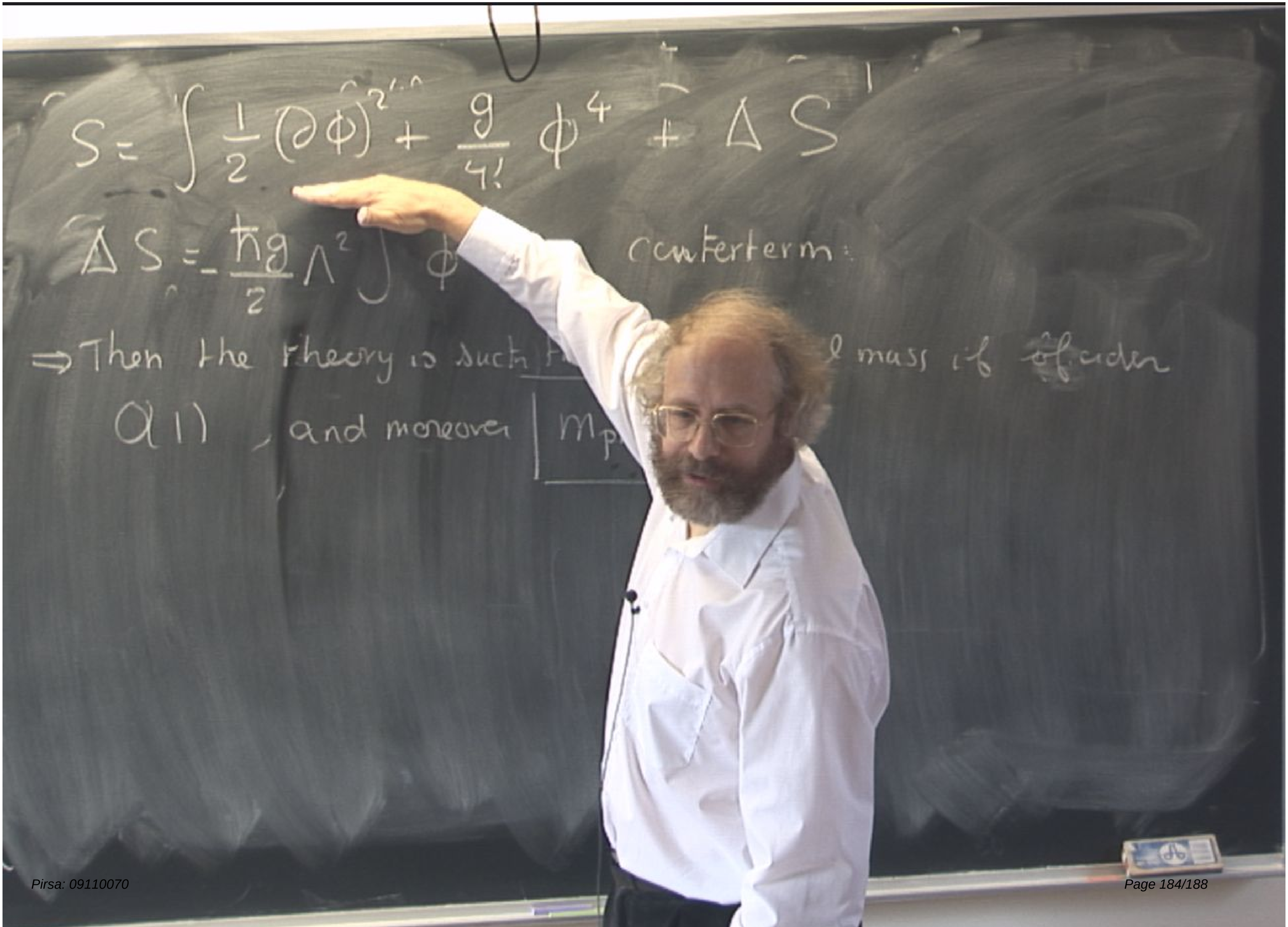
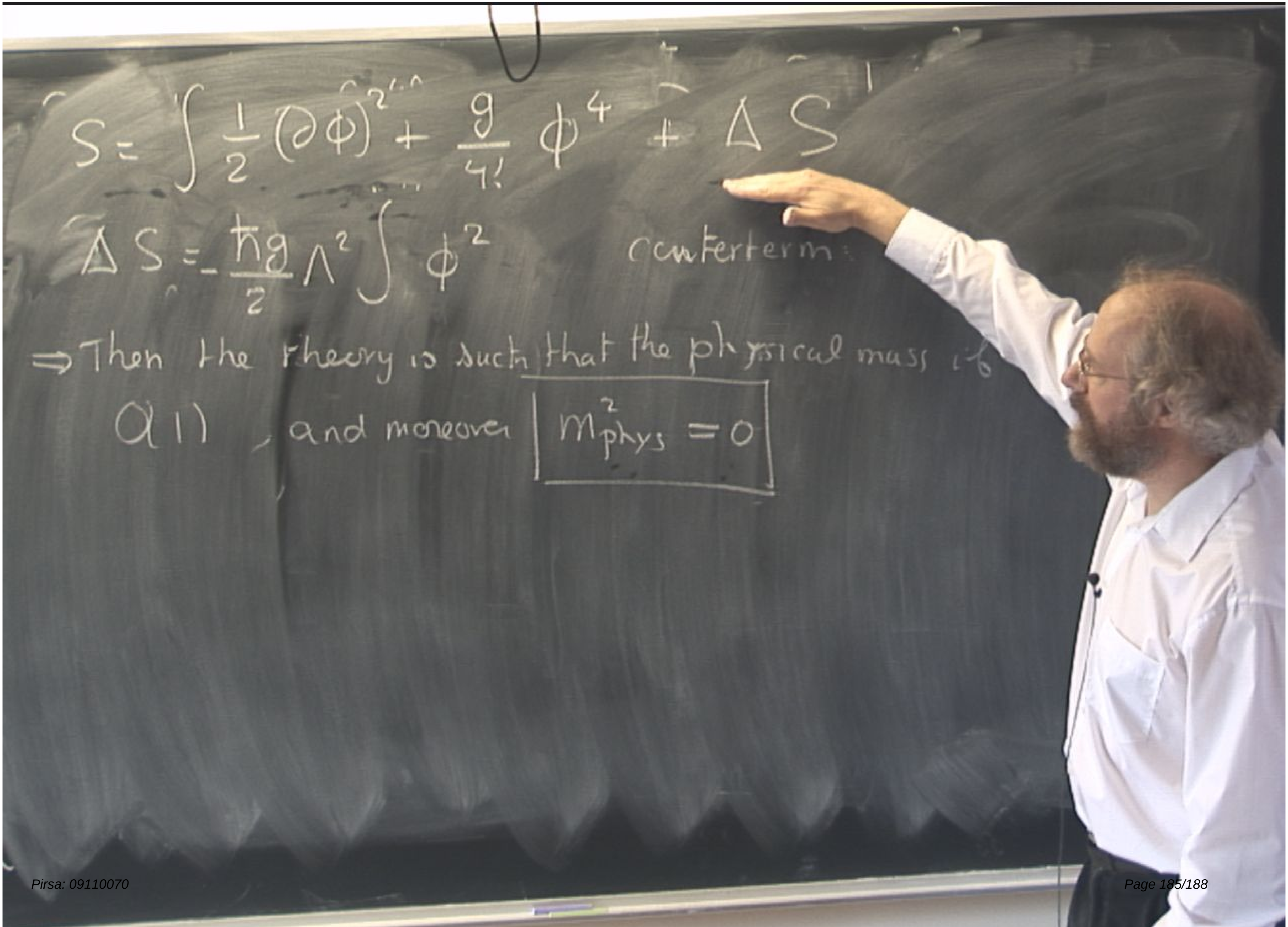

S = ∫ 1/2 (∂φ)² + g/4! φ⁴ + ΔS
ΔS = - ħg/2 Λ² ∫ φ
counterterm:
⇒ Then the theory is such
mass if of order
O(1)
, and moreover

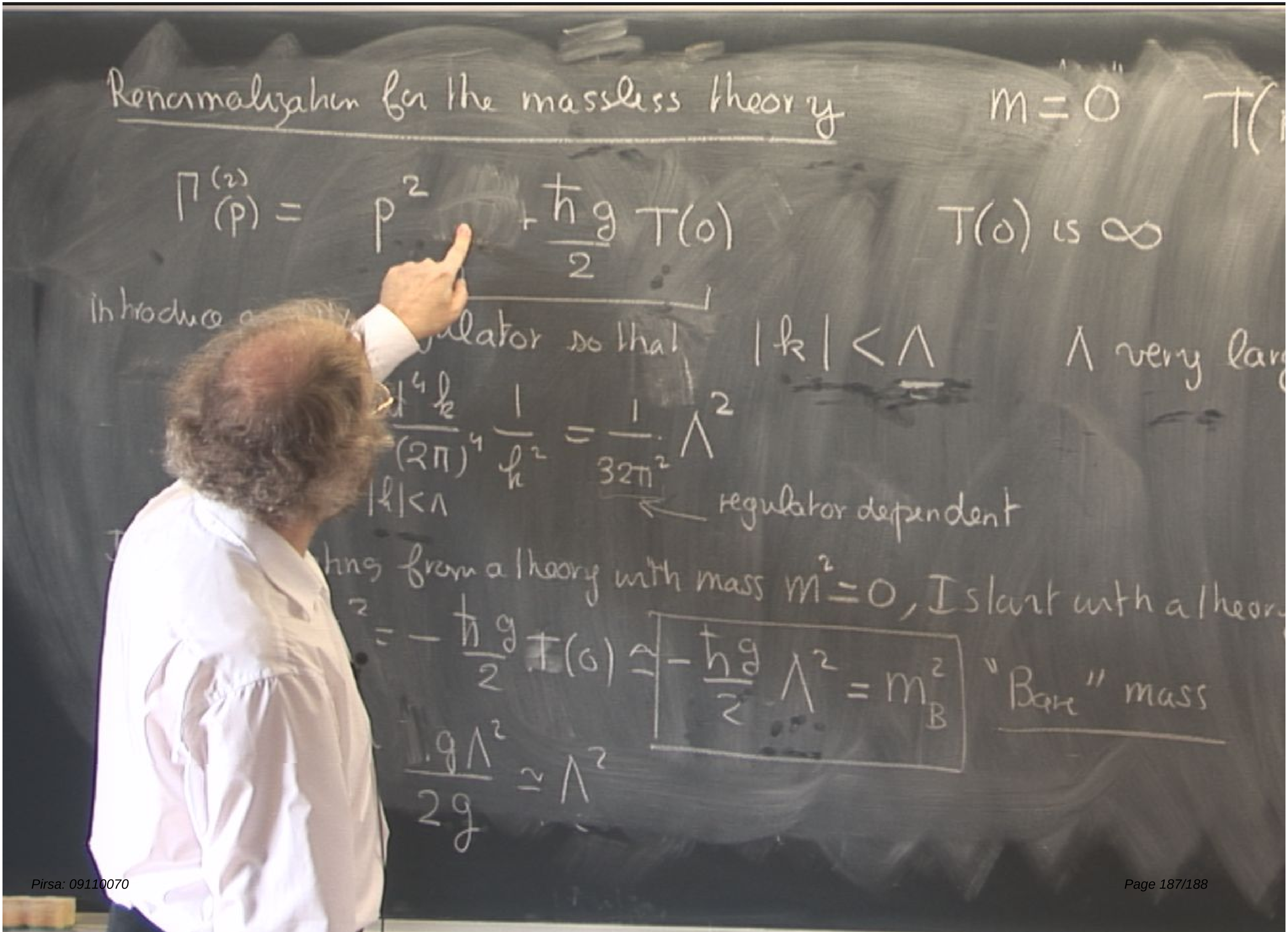

$S = \int \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4 + \Delta S$
$\Delta S = -\frac{\hbar g}{2}\Lambda^2 \int \phi^2$
counterterm
⇒ Then the theory is such that the physical mass is
$O(1)$, and moreover
$m^2_{phys} = 0$

$$S = \int \frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4 + \Delta S$$

$$\Delta S = -\frac{\hbar g}{2}\Lambda^2 \int \phi^2$$ counterterm:

⇒ Then the theory is such that the physical mass is ~~of~~ order $O(1)$, and moreover $\boxed{m^2_{phys} = 0}$

$$S + \Delta S \Rightarrow \Gamma^{(2)}(p) = p^2 + O(\hbar^2)$$


Renormalization for the massless theory
m = 0
Γ(2)(p) = p2 + ħg/2 T(0)
T(0) is ∞
so that |k| < Λ
Λ very
= 1/32π2 Λ2
|k|<Λ
regulator dependent
from a theory with mass m2 = 0, I start with a theory
= − ħg/2 T(0) ≃ − ħg/2 Λ2 = mB2
"Bare" mass
≃ Λ2

Renormalization for the massless theory $\quad m = 0 \quad T($

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