Title: Anisotropic Conformal Infinity

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Abstract:

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Charles Melby-Thompson, UC Berkeley

(arXiv:0909.3841, with Petr Hořava)

Perimeter Institute, Nov. 10^{th} 2009

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Infinity!

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- Identifying proper boundary conditions for fields
- Analyzing causal properties of spacetimes (e.g. black holes)
- Defining asymptotic charges
- AdS/CFT correspondence
- ... etc.

Infinity in General Relativity

- Penrose's definition: boundary of the manifold under a conformal embedding in an auxiliary spacetime
$$\widetilde{g_{\mu\nu}} = \Omega^2 g_{\mu\nu} \tag{1}$$
- Uses Weyl rescalings of the spacetime metric
- Consistent because diffeomorphisms preserve the group Weyl transformations:
$$[\delta_X, \delta_\omega] = \delta_{X(\omega)} \tag{2}$$
- Useful because it preserves causal structure

Infinity in Hořava-Lifshitz Gravity

- Foliation + higher spatial derivatives $\Longrightarrow$ causal structure is degenerate

  So: Weyl transformations more restrictive than necessary

- Smaller symmetry group $\Longrightarrow$ weaker consistency conditions

- When there is anisotropy between spatial and temporal metric components, it is unnatural to scale them the same way

Much more natural to have an anisotropic concept of conformal infinity.

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Asymptotically Anisotropic Spacetimes

- Recent interest in possible duals to condensed matter systems
- Superconductors, cold atoms, Lifshitz-like fixed points

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AdS/CFT Correspondence: Very Quick Review

- Indications that quantum gravity should be holographic
- In AdS-like spaces partition function is a functional of boundary conditions on fields at spatial infinity
- Partition function can be viewed as the generating functional for a field theory living on the boundary:

$$Z_{gravity}[\Phi_1, \cdots] = \left\langle \exp\left(\frac{i}{\hbar}\int \Phi_i(x)\mathcal{O}_i(x)dV\right)\right\rangle_{CFT} \tag{3}$$

- Scale invariance $\sim$ bulk isometry of $ds^2_{AdS} = \ell^2 u^{-2}(dx^\mu dx_\mu + du^2)$.

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$$t \mapsto \lambda^z t \qquad \vec{x} \mapsto \lambda \vec{x} \tag{4}$$

- Example: Quantum gravity on "Lifshitz" backgrounds ($z = 2$):

$$ds^2 = \frac{-d\tau^2}{\rho^4} + \frac{d\vec{x}^2 + d\rho^2}{\rho^2} \tag{5}$$

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Degenerate Asymptopia of Lifshitz Space

- Metric has degenerate boundary; finite metric obtained in coordinate system

$$ds^2 = \frac{-d\tau^2 + dr^2/4}{r^2} + \frac{d\vec{x}^2}{r} \tag{6}$$

- Penrose boundary has dimension 1:

$$\widetilde{ds^2} = -d\tau^2 \tag{7}$$

- Consistent boundary conditions for fields *can* depend on $\vec{x}$ coordinates
- Penrose definition with codimension > 1 boundary does not properly describe boundary geometry — dysymptopia of Lifshitz space cannot support field theory

Anisotropic Infinity?

- Need concept of infinity compatible with geometric anisotropy
- Conservatively: use local rescalings as in usual conformal boundary
- To do this we need Weyl transformations consistent with anisotropy.

Problem: Weyl transformations need to close with diffeomorphisms (complete group must be a semi-direct product)

Natural to look to anisotropic gravity models for inspiration even when doing General Relativity!

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Foliations

- Manifold foliated by spatial slices
- ADM Decomposition

$$ds^2 = -d\tau^2 + g_{ij}(dx^i + N^i d\tau)(dx^j + N^j d\tau) \tag{8}$$

- Under foliation-preserving diffeomorphism $X = (f, \xi^i)$:

$$\begin{aligned}
\delta N &= X^\mu \partial_\mu N + N\dot{f} && (9)\\
\delta N_i &= X^\mu \partial_\mu N_i + N_j \partial_i \xi^j + g_{ij}\dot{\xi}^j && (10)\\
\delta g_{ij} &= X^\mu \partial_\mu g_{ij} + g_{ik}\partial_j \xi^k + g_{kj}\partial_i \xi^k && (11)
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Anisotropic Weyl Rescalings

- Foliated manifold and a metric with an ADM-type decomposition
- Anisotropic Weyl transformations with critical exponent z:

$$N \to \Omega^z N \qquad N_i \to \Omega^2 N_i \qquad g_{ij} \to \Omega^2 g_{ij} \tag{12}$$

- Close with foliation-preserving diffeomorphisms: for $\Omega = 1 + \omega$ infinitesimal,

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Defining an Anisotropic Conformal Infinity

- We now have a natural definition if the theory is already foliated...
- ... but we want to apply this to geometries in general relativity (*e.g.*, from AdS/CFT), where diffeomorphisms do not close on anisotropic Weyl transformations.

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- Lifshitz space metric in ADM decomposition:

$$N = \frac{1}{\rho^2} \qquad N_i = 0 \qquad g_{ij} = \frac{\delta_{ij}}{\rho^2} \tag{14}$$

- Critical exponent: $z = 2$

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- Critical exponent: $z = 2$
- With $\Omega = \rho$ the rescaled metric is

$$\widetilde{ds}^2 = -d\tau^2 + d\vec{x}^2 + d\rho^2, \tag{15}$$

- and the induced boundary metric is now nice:

$$ds_B^2 = -d\tau^2 + d\vec{x}^2. \tag{16}$$

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Other Consistent Weyl Scalings

- Unsatisfactory results in other situations (*e.g.*, Warped AdS and Schrödinger spacetimes)
- We have learned the following lesson: modifying Weyl transformations requires altering the diffeomorphism group.
- Take this a step further: modify the action of diffeomorphisms on metric components. Consider a 2-dimensional foliated spacetime with coordinates (τ, x) and let the transformation under diffeomorphisms under an FPD $X = f(t)\partial_t + h(\tau, x)\partial_x$ be

$$\delta g_{tt} = X(g_{tt}) + 2h_{tt}\dot{f}(t) \qquad \delta g_{tx} = X(g_{tx}) + h_{tx}(\dot{f}(t) + h'(t,x)) \tag{17}$$

$$\delta g_{xx} = X(g_{xx}) + 2g_{xx}h'(t,x) \tag{18}$$

Other Consistent Weyl Scalings

- [These rules can be obtained from a non-relativistic limit analogous to (but distinct from) the original presentation of foliation-preserving diffeomorphisms. They also come naturally from the geometry.]

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- Postulate the anisotropic scalings

$$g_{tt} \to \Omega^2 g_{tt} \qquad g_{tx} \to \Omega g_{tx} \qquad g_{xx} \to g_{xx} \tag{19}$$

- Rescalings close with the transformation rules given in the previous slide.

Example: Warped AdS Space

$$ds^2 = -(r^2+1)du^2 + \frac{dr^2}{r^2+1} + \frac{4\nu^2}{\nu^2+3}(rdu+dv)^2 \tag{20}$$

- Asymptotic isometries:

$$\left(F(u) + \mathcal{O}\left(\frac{1}{r^2}\right)\right)\partial_u + \left(G(u) + \mathcal{O}\left(\frac{1}{r}\right)\right)\partial_v - \left(rF'(u) + \mathcal{O}(1)\right)\partial_r$$

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- Allowed diffeomorphisms are asymptotically foliation-preserving

Example: Warped AdS Space

- Diffeomorphism symmetries consistent with anisotropic conformal weights:

$$[g_{uu}] = 2 \qquad [g_{uv}] = 1 \qquad [g_{vv}] = 0 \tag{22}$$

- Using these weights one finds the metric at anisotropic infinity to be

$$\widetilde{ds^2_B} = -du^2 + \frac{4\nu^2}{\nu^2+3}(du+dv)^2 \tag{23}$$

Schrödinger Space

- We can also consider the slightly more complicated space

$$ds^2 = -\frac{dt^2}{u^4} + \frac{2dtd\theta + d\vec{x}^2 + du^2}{u^2} \tag{24}$$

- (Proposed dual geometry to *e.g.* fermions at unitarity.)

- We assign the scalings from warped AdS to t and θ and the scaling $g_{ij} \to \Omega^2 g_{ij}$, obtaining the metric at anisotropic conformal infinity:

$$\widetilde{ds^2_B} = -dt^2 + 2dt\, d\theta + d\vec{x}^2. \tag{25}$$

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Near-Horizon Extreme Kerr

- Metric in the near-horizon limit:

$$ds^2 = 2GJ\Omega^2(\theta)\left(\frac{-dt^2 + du^2}{u^2} + d\theta^2 + \Lambda^2(\theta)(d\phi^2 + \frac{dt}{y})^2\right) \qquad (26)$$

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Anisotropic Conformal Equivalence

- New concept of conformal transformation $\Longrightarrow$ new concept of anisotropic conformal equivalence classes of metrics
- Anisotropic conformal equivalence: two metrics are equivalent if related by an anisotropic Weyl transformation
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Generalized Warped AdS Space

- Now we generalize warped AdS to spacetimes of the following form:

$$g_{uu} = \gamma_{uu} r^2 + O(r) \qquad g_{uv} = \gamma_{uv} r + O(1) \qquad g_{vv} = \gamma_{vv} \tag{27}$$

$$g_{ur} = O(r^{-1}) \qquad g_{vr} = O(r^{-2}) \qquad g_{rr} = \frac{1}{r^2} + O(r^{-3}) \tag{28}$$

where $\gamma_{\mu\nu} = \mathcal{O}(1)$ as $r \to \infty$

- Phase space is preserved by all diffeomorphisms of the form

$$\left(F(u) + \mathcal{O}\left(\frac{1}{r^2}\right)\right)\partial_u + \left(G(u,v) + \mathcal{O}\left(\frac{1}{r}\right)\right)\partial_v - \left(rH(u) + \mathcal{O}(1)\right)\partial_r \tag{29}$$

Warped AdS Space: Boundary Geometry

- Boundary "metric" is then $\gamma_{\mu\nu}$

Warped AdS Space: Boundary Geometry

- Boundary "metric" is then $\gamma_{\mu\nu}$
- Using the leading-order terms and the vector fields above, we find that the induced behavior on the boundary metric is:

$$\begin{aligned} \delta\gamma_{uu} &= X(\gamma_{uu}) + 2\gamma_{uu}\partial_u F(u) & (30) \\ \delta\gamma_{uv} &= X(\gamma_{uv}) + \gamma_{uv}(\partial_u F(u) + \partial_v G(u,v)) & (31) \\ \delta\gamma_{vv} &= X(\gamma_{vv}) + 2\gamma_{vv}\partial_v G(u,v). & (32) \end{aligned}$$

  as we saw previously.

- Look for boundary metrics with diffeomorphisms that are anisotropic conformal mappings...

Warped AdS Space and Anisotropic Conformal Maps

- Simplest case: γ_{uu}, γ_{uv}, γ_{vv} constant (= warped AdS)
- We need a vector field X and an anisotropic rescaling $1+\omega$ satisfying

$$\delta_X \gamma_{\mu\nu} = \delta_\omega \gamma_{\mu\nu} = z^{(\mu\nu)} \omega \gamma_{\mu\nu}$$

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- Solution to this problem:

$$X = F(u)\partial_u + G(u)\partial_v \qquad \omega = F'(u). \tag{34}$$

- Just as in AdS space, the solution to this geometric problem (finding the maps that preserve the boundary metric up to conformal rescaling) are precisely the maps induced on the boundary by the asymptotic isometries.

Conformal Boundary of Warped AdS Black Holes

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- Warped AdS supports black holes:

$$\frac{ds^2}{\ell^2} = \frac{r}{4}\left[3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+ r_-(\nu^2+3)}\right] d\theta^2$$
$$+ \left[2\nu r - \sqrt{r_+ r_-(\nu^2+3)}\right] d\tau d\theta + d\tau^2 + \frac{dr^2}{(\nu^2+3)(r-r_+)(r-r_-)} \quad (35)$$

- Like BTZ black holes, warped AdS black holes are quotients of warped AdS, so they are locally isometric to it

Conformal Boundary of Warped AdS Black Holes

- Local isometry from black hole space to global warped AdS:

$$u = \arctan\left(\frac{2\sqrt{(r-r_+)(r-r_-)}}{2r-r_+-r_-}\sinh\left[\frac{(\nu^2+3)(r_+-r_-)}{4}\theta\right]\right) \quad (36)$$

$$v = \frac{\nu^2+3}{4\nu}\left(2\tau+\left[\nu(r_++r_-)-\sqrt{r_+r_-(\nu^2+3)}\right]\theta\right) \quad (37)$$

$$-\operatorname{arccoth}\left(\frac{2r-r_+-r_-}{r_+-r_-}\sinh\left[\frac{(\nu^2+3)(r_+-r_-)}{4}\theta\right]\right) \quad (38)$$

$$\rho = \frac{2\sqrt{(r-r_+)(r-r_-)}}{r_+-r_-}\cosh\left[\frac{(\nu^2+3)(r_+-r_-)}{4}\theta\right] \quad (39)$$

Conformal Boundary of Warped AdS Black Holes

- These are not foliation-preserving
- However, they are asymptotically foliation-preserving:

$$u \sim \operatorname{arctan} \sinh \left[\frac{(\nu^2+3)(r_+ - r_-)}{4} \theta \right] \tag{40}$$

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Conformal Boundary of Warped AdS Black Holes

- The induced conformal class of metrics is

$$ds_B^2 = \ell^2 \left(dt^2 + 2\nu dt d\theta + \frac{3(\nu^2 - 1)}{4} d\theta^2 \right) \tag{43}$$

- Using the rescaled coordinates

$$\tau = \frac{2\nu t}{\nu^2 + 3} \qquad \theta = \frac{\ell^2}{\nu^2 + 3}\phi \tag{44}$$

one obtains

$$ds_B^2 = \frac{\ell^2}{4(\nu^2 + 3)} \left(-\ell^4 d\theta^2 + \frac{4\nu^2}{\nu^2 + 3}(2d\tau + \ell^2 d\theta)^2 \right) \tag{45}$$

Conformal Boundary of Warped AdS Black Holes

- Finally one can perform an anisotropic rescaling with $\Omega^2 = \frac{2}{\ell^2}$ to obtain the vacuum boundary metric

$$\widetilde{ds^2_B} = \frac{\ell^2}{\nu^2+3}\left(\frac{4\nu^2}{\nu^2+3}(d\tau + d\theta)^2 - d\theta^2\right). \tag{46}$$

Summary

- Both Hořava-Lifshitz gravity and standard general relativity require more general concept of infinity; structures arising in HL gravity provide a natural solution to this problem
- General relativity: need to pay attention to the group of symmetries (vs. diffeomorphisms)
- Provides consistent geometric framework in which interesting gravity solutions (Warped AdS_3, Schrödinger space, etc.) can be given sensible geometric boundaries — holographic renormalization, stress tensor, etc.
- Framework for evaluating partition function of anisotropic gravity models/gravity on anisotropic backgrounds and may give additional information about holography

Partition Function of an Anisotropic Gravity Model?

- Additional gauge symmetry kills scalar degree of freedom, leaving no propagating modes
- Classical solutions are all locally isomorphic (c.f. black holes in AdS_3)
- Usual prescription: sum over contributions of all classical geometries with a given conformal infinity
- For HL theories: sum of contributions from all classical geometries with a given anisotropic conformal infinity
- Holography? Geometric structure of boundary theory?
- Similar questions arise in TMG on warped AdS_3 backgrounds.

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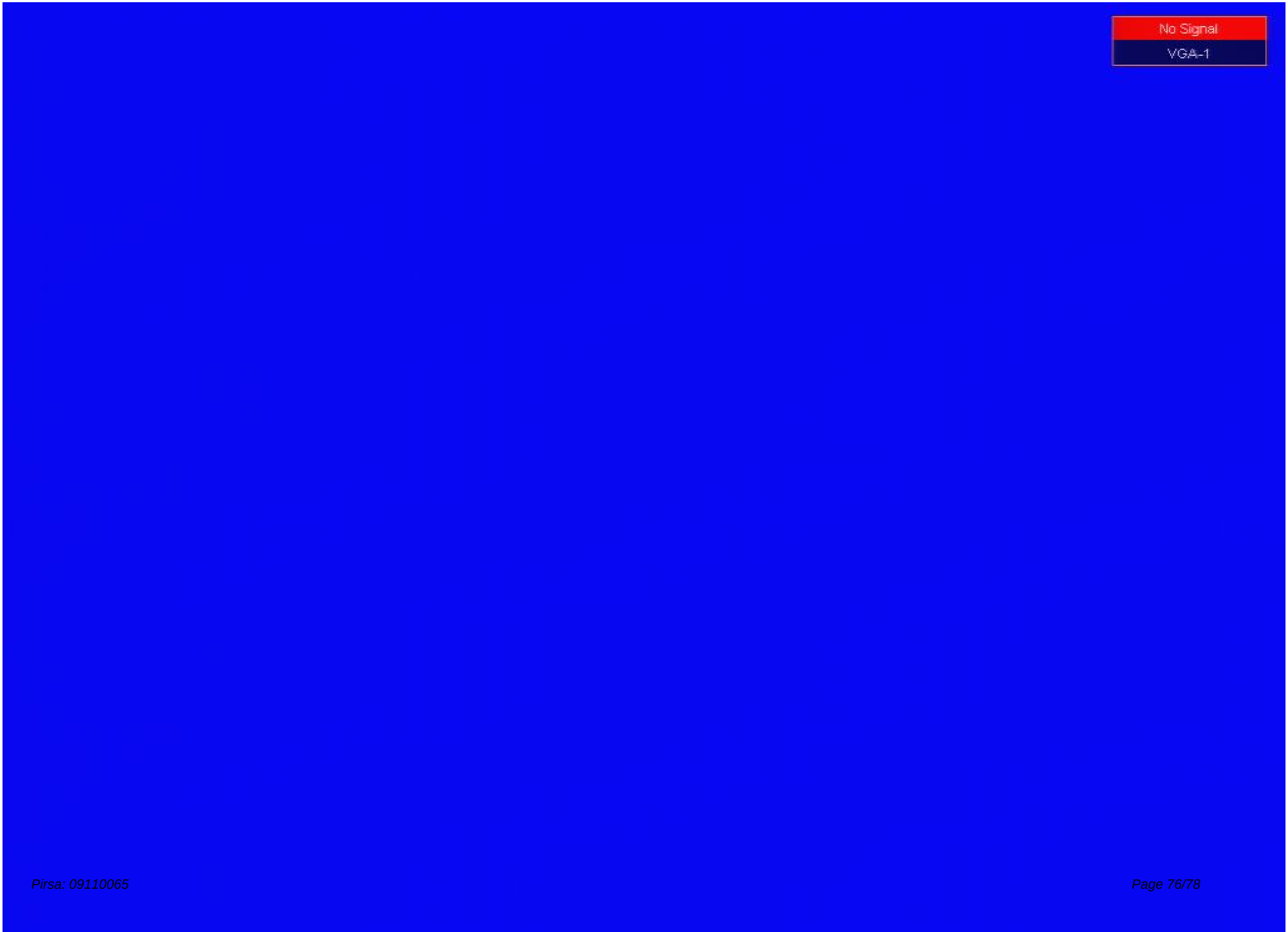
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