Title: Renormalization group in Lifshitz-type theories

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Abstract:

Plan

Introduction and brief review

UV behaviour

IR behaviour

Fine-tuning problem

Conclusions

Introduction

From a particle physics point of view, it is known that theories with higher derivative quadratic operators have a better UV-behaviour with respect to standard theories, thanks to the modified free particle propagator

However, Lorentz invariance + higher derivative operators imply a lack of unitarity due to the appearance of ghost (PV-like fields)

Lifshitz-like theories, explicitly breaking Lorentz invariance, allow for the construction of unitary otherwise non-renormalizable theories

An extensive study of Lifshitz-like theories for scalars, fermions and gauge fields, as well as applications to the SM, has been made by Anselmi [Anselmi&Helat, 0707.2480; Anselmi, 0808.3470; 0808.3474; 0808.3475; 0904.1849]

The main recent interest, however, comes from the Horava proposal of applying this idea to build a renormalizable theory of gravity [Horava, 0901.3775]

I will not enter into the issue of if and how Horava theory is phenomenologically acceptable, but focus instead on the obvious general drawback of any Lifshitz-like theory:

How can we recover Lorentz invariance in the IR ?

Is there a possibility of an RGE of the Lorentz-violating parameters such that they become all small in the IR ?
[Chadia, Nielsen, NPB 217 (1983)]

How much (if any) are these theories fine-tuned ?

Our aim is to answer to these questions by looking at simple Lifshitz-like theories with scalars only

The Lorentz violating order parameters we will consider are the "speeds of light" of the scalar fields

The answer does not seem to be promising for weakly coupled theories

There is no way, in general, to have a sufficient RGE suppression, and the fine-tuned left is very severe $\sim 10^{-20}$

[Collins et al, gr-qc/0403053]

Our analysis will also clarify the IR-UV structure of Lifshitz-like theories

General properties of Lifshitz-like theories

Anisotropic scale invariance

$$t = \lambda^z t' \,, \quad x^i = \lambda x^{i\prime} \,, \quad \phi(x^i, t) = \lambda^{\frac{z-D}{2}} \phi'(x^{i\prime}, t') \,.$$

It is useful to introduce weighted scaling dimensions so that

$$[t]_w = -z \,, \quad [x^i]_w = -1 \,, \quad [\phi]_w = \frac{D-z}{2}$$

This is equivalent, but makes physics more transparent, than defining unusual natural units.

Renormalizability by power-counting still holds, substituting standard scaling dimensions with weighted scaling dimensions

$(\partial_i^z \phi)^2$ is standard irrelevant, but weighted marginal

For instance, in $D=3$ with $z=2$, the most general renormalizable Lagrangian, invariant under anisotropic scaling, is (in a “preferred frame” where spatial $SO(3)$ rotations, translations and time-reversal are unbroken symmetries)

$$\mathcal{L}_r = \frac{1}{2}\dot{\phi}^2 - \frac{a^2}{2\Lambda^2}(\Delta\phi)^2 - \frac{h_2}{48\Lambda^4}(\partial_i\phi)^2\phi^4 - \frac{g_4}{10!\Lambda^6}\phi^{10} ,$$

Weighted relevant terms are

$$\mathcal{L}_{\rm sr} = -\frac{m^2}{2}\phi^2 - \frac{c^2}{2}(\partial_i\phi)^2 + \sum_{n=1}^{3}\frac{g_n}{(2n+2)!\Lambda^{2(n-1)}}\phi^{2n+2} + \frac{h_1}{4\Lambda^2}(\partial_i\phi)^2\phi^2$$

Free particle propagator is $\quad i\left(k_0^2 - c^2k^2 - \frac{a^2}{\Lambda^2}k^4 - m^2\right)^{-1}$

Neglecting m, there are two regimes of interest.

UV behaviour: $k \gg \Lambda$

IR behaviour: $k \ll \Lambda$

UV behaviour: D=4 Theory with 1 particle

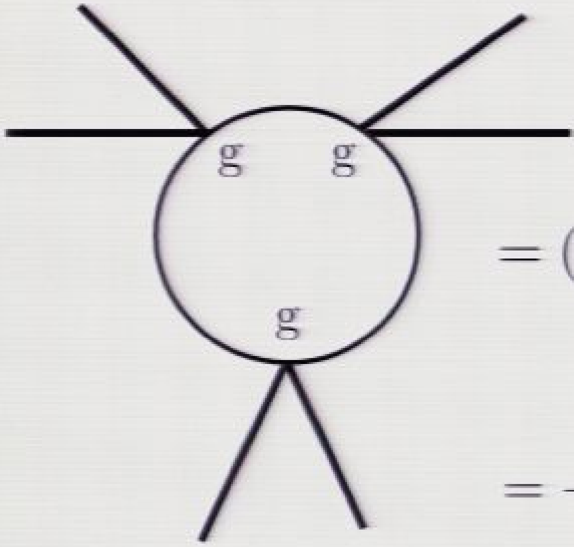
$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{a^2}{2\Lambda^2}(\Delta\phi)^2 - \frac{c^2}{2}(\partial_i\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!\Lambda}\phi^4 - \frac{g}{4\Lambda^3}(\phi\partial_i\phi)^2 - \frac{k}{6!\Lambda^4}\phi^6 .$$

The marginal couplings are a^2, g and k

Once their RG flows are known, we can study the c^2 operator

We work at 1-loop level

Use DR to the spatial directions only ($D = 4 - \epsilon$) and renormalize using MS scheme where the poles in $1/\epsilon$ are subtracted



$$\Lambda = 1$$

$$= (-ig\mu^{\epsilon})^3 \frac{15}{2} \int \frac{dq^0 d^D q}{(2\pi)^{D+1}} \frac{i^3 q^6}{(q_0^2 - a^2 q^4 - c^2 q^2 - m^2)^3}$$

$$= -\frac{15 i g^3 \mu^{3\epsilon}}{4} \frac{d^2}{(dm^2)^2} \int_0^{\infty} d\alpha \int \frac{dq_5 d^D q}{(2\pi)^{D+1}} q^6 e^{-\alpha(q_5^2 + a^2 q^4 + c^2 q^2 + m^2)}$$

$$= \frac{3 l_4}{8} \frac{g^3}{a^5} \frac{1}{\epsilon} + \text{finite} \qquad l_D \equiv \frac{\Omega_D}{(2\pi)^D}$$

By similar manipulations we get

$$\dot{k} = \beta_k = \frac{15 l_4}{8} \frac{gk}{a^3} - \frac{45 l_4}{32} \frac{g^3}{a^5}$$

$$\dot{g} = \beta_g = \frac{3 l_4}{8} \frac{g^2}{a^3}$$

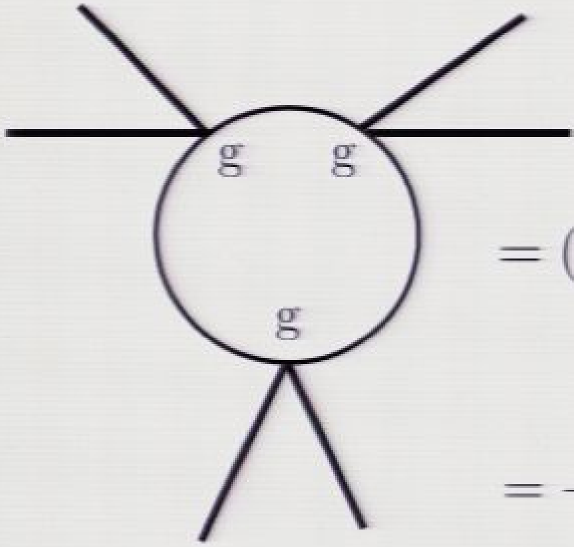
$$\dot{a} = \beta_a = 0$$

Define $$\hat{g} \equiv \frac{g}{a^3}\,, \qquad \hat{k} \equiv \frac{k}{a^4}\,.$$

$$\Longrightarrow \quad \begin{aligned} \hat{g}(t) &= \frac{\hat{g}_0}{1 - \frac{3l_4\hat{g}_0}{8}t} \qquad t = \ln\mu/\mu_0,\ \mu_0 \gg 1 \\ \hat{k}(t) &= \hat{k}_0\left(\frac{\hat{g}(t)}{\hat{g}_0}\right)^5 + \frac{5\hat{g}_0^2}{4}\left(\frac{\hat{g}(t)}{\hat{g}_0}\right)^2\left[1 - \left(\frac{\hat{g}(t)}{\hat{g}_0}\right)^3\right] \end{aligned}$$

A Landau pole appears at $E_{pole} = \mu_0 e^{\frac{8}{3l_4\hat{g}_0}}$ $\qquad 1 \ll E \ll E_{pole}$

$$\frac{dc^2}{dt} = \beta_{c^2} = \frac{l_4\hat{g}}{8}c^2 \quad \Longrightarrow \quad c^2(t) = c_0^2\left(\frac{\hat{g}(t)}{\hat{g}_0}\right)^{\frac{1}{3}}$$



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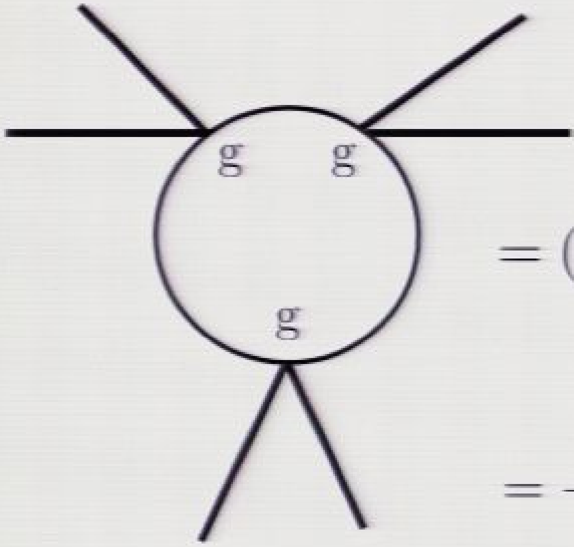

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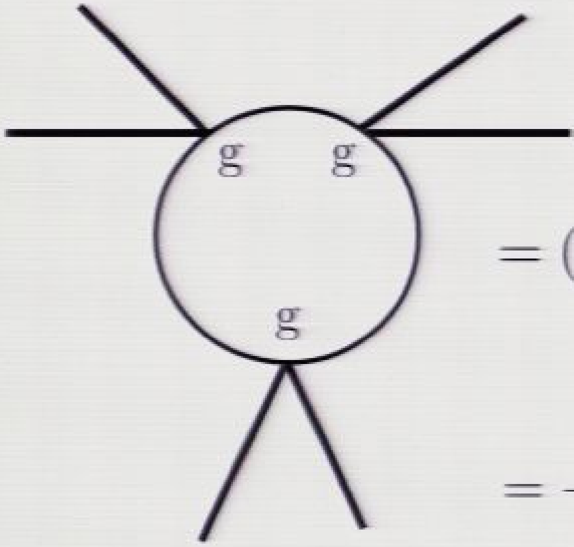

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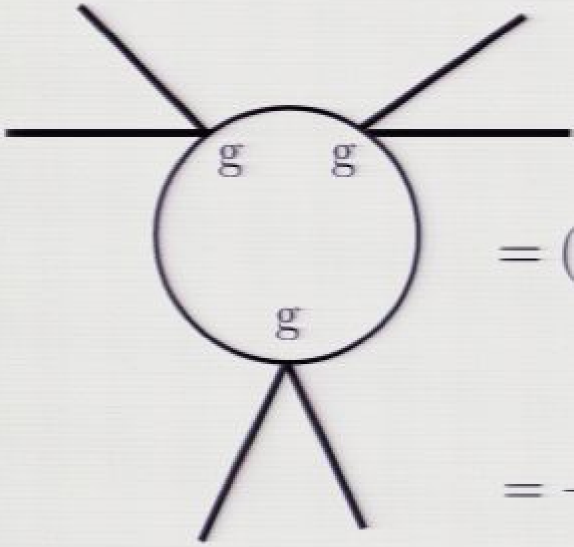

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UV behaviour: D=4 Theory with 2 particles

$$\mathcal{L}_{2\phi} = \mathcal{L}_1 + \mathcal{L}_2 - g_{12}(\phi_1\partial\phi_1)(\phi_2\partial\phi_2) - \frac{h_1}{4}(\partial\phi_1)^2\phi_2^2 - \frac{h_2}{4}(\partial\phi_2)^2\phi_1^2 - V_{12}$$

$$\begin{aligned}
\beta_{g_1} &= \frac{l_4}{8}\Big(3g_1^2 + 4g_{12}h_2 + h_1h_2 - 2h_2^2\Big) \\
\beta_{g_2} &= \frac{l_4}{8}\Big(3g_2^2 + 4g_{12}h_1 + h_1h_2 - 2h_1^2\Big) \\
\beta_{h_i} &= \frac{l_4}{8}\Big(g_{12}^2 + h_i(g_1+g_2) + h_i^2 + 2g_{12}h_i\Big), \qquad i = 1,2 \\
\beta_{g_{12}} &= \frac{l_4}{16}\Big[3g_{12}^2 + 2g_{12}(g_1+g_2) + 3g_{12}(h_1+h_2) - h_1h_2\Big]
\end{aligned}$$

$$\begin{aligned}
\beta_{c_1^2} &= \frac{l_4}{8}\Big(c_1^2 g_1 + c_2^2 h_1\Big) \\
\beta_{c_2^2} &= \frac{l_4}{8}\Big(c_2^2 g_2 + c_1^2 h_2\Big)
\end{aligned}$$

Class of exact solutions:

$$g_1(t) = g_2(t) = g(t) = \frac{g_0}{1 - x l_4 t}$$

$$h_1(t) = h_2(t) = h(t) = \frac{h_0}{1 - x l_4 t}$$

$$g_{12}(t) = \frac{g_{12,0}}{1 - x l_4 t}$$

$$c^2(t) \;=\; c_0^2 \left(\frac{g(t)}{g_0} \right)^{\frac{g_0 + h_0}{8x}}$$

$$\delta c^2(t) \;\equiv\; c_1^2(t) - c_2^2(t) = \delta c_0^2 \left(\frac{g(t)}{g_0} \right)^{\frac{g_0 - h_0}{8x}}$$

Study the RG evolution of the perturbed solutions at linear level

for $x = 3/8$, $g_{12,0} = 0$, $h_0 = 0$, $g_0 = 1$

$$g_1 = g + \delta g + \delta u\ , \qquad g_2 = g - \delta g + \delta u$$

$$h_1 = h + \delta h + \delta v\ , \qquad h_2 = h - \delta h + \delta v\ , \qquad g_{12} \rightarrow g_{12} + \delta g_{12}$$

$$\frac{\delta g(t)}{\delta g_0} = \frac{\delta u(t)}{\delta u_0} = \left(\frac{g(t)}{g_0}\right)^2$$

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$$\delta c^2(t) = \delta c_0^2 \left(\frac{g(t)}{g_0}\right)^{\frac{1}{3}} + \frac{c_0^2 l_4 \delta g_0 t}{8}\left(\frac{g(t)}{g_0}\right)^{\frac{4}{3}} + c_0^2\left[\left(\frac{g(t)}{g_0}\right)^{\frac{1}{3}} - 1\right]\delta h_0$$

1. **Not** enough to start with $\delta c_0^2 = 0$ at $\mu_0 \gg 1$ to get $\delta c^2 = 0$ near $E \sim \Lambda$
2. **Other** perturbations around fixed point must be fine-tuned

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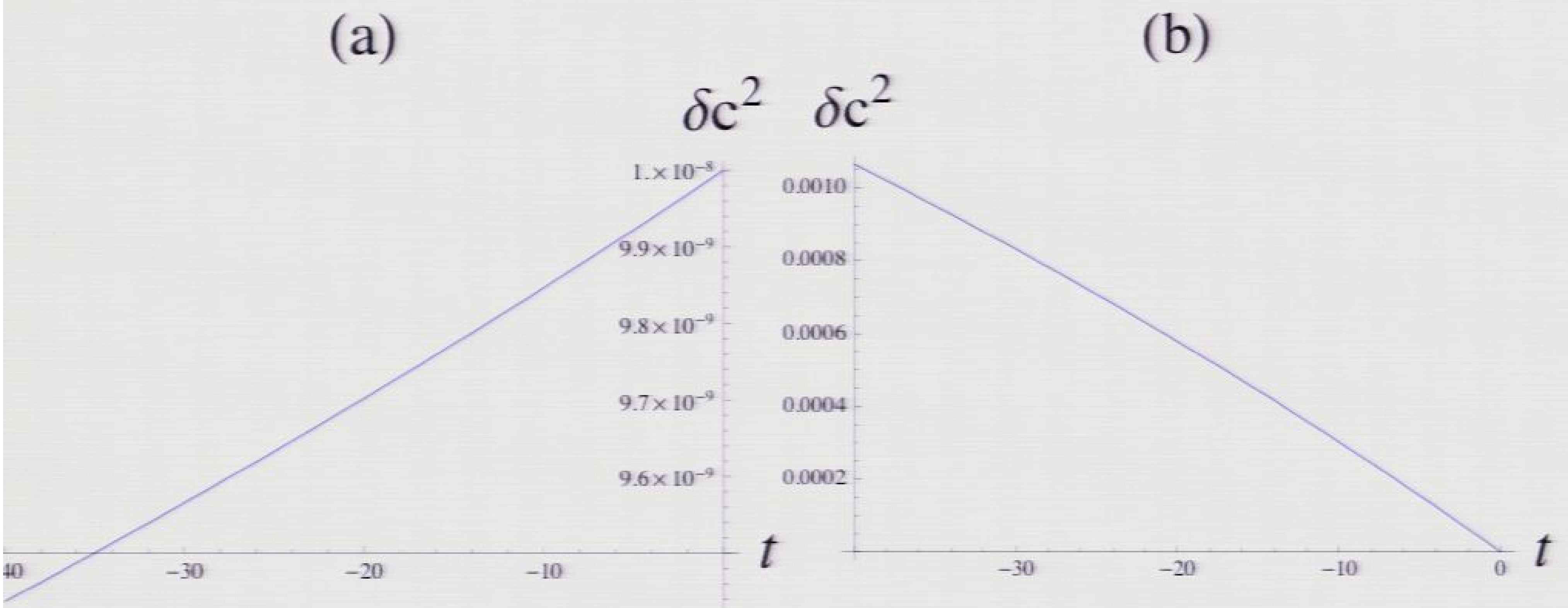

(a) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = 0$, $\delta c_0^2 = 10^{-8}$ (b) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = -10^{-2}$, $\delta c_0^2 = 0$

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$$\gamma = \frac{l_{10}\lambda^2}{64a^5} \quad \beta_{a^2} = 2\omega_a \frac{\lambda^2}{a^5}a^2, \qquad \beta_\lambda = -\omega_\lambda \frac{\lambda^2}{a^5}\lambda$$

$$\lambda(t) = \lambda_0 \left(\frac{x(t)}{x_0}\right)^{\frac{\omega_\lambda}{\omega_x}}, \quad a^2(t) = a_0^2 \left(\frac{x(t)}{x_0}\right)^{-\frac{2\omega_a}{\omega_x}}$$

$$x = \frac{\lambda^2}{a^5}$$

λ and effective coupling x are UV free.

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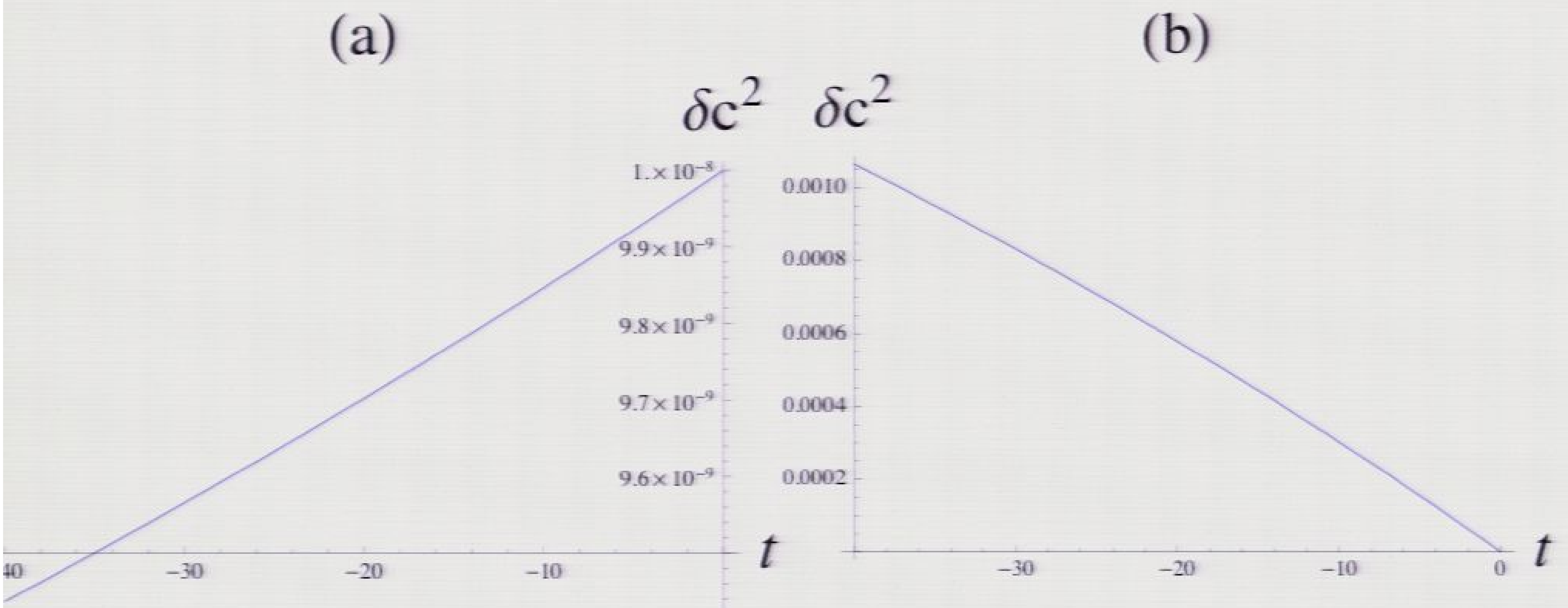

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$$g_1(t) = g_2(t) = g(t) = \frac{g_0}{1 - xl_4t}$$

$$h_1(t) = h_2(t) = h(t) = \frac{h_0}{1 - xl_4t}$$

$$g_{12}(t) = \frac{g_{12,0}}{1 - xl_4t}$$

$$c^2(t) = c_0^2 \left(\frac{g(t)}{g_0} \right)^{\frac{g_0+h_0}{8x}}$$

$$\delta c^2(t) \equiv c_1^2(t) - c_2^2(t) = \delta c_0^2 \left(\frac{g(t)}{g_0} \right)^{\frac{g_0-h_0}{8x}}$$

Study the RG evolution of the perturbed solutions at linear level

for $x = 3/8$, $g_{12,0} = 0$, $h_0 = 0$, $g_0 = 1$

$$g_1 = g + \delta g + \delta u \,, \qquad g_2 = g - \delta g + \delta u$$

$$h_1 = h + \delta h + \delta v \,, \qquad h_2 = h - \delta h + \delta v \,, \qquad g_{12} \to g_{12} + \delta g_{12}$$

$$\frac{\delta g(t)}{\delta g_0} = \frac{\delta u(t)}{\delta u_0} = \left(\frac{g(t)}{g_0}\right)^2$$

$$\frac{\delta h(t)}{\delta h_0} = \frac{\delta v(t)}{\delta v_0} = \frac{\delta g_{12}(t)}{\delta g_{12,0}} = \left(\frac{g(t)}{g_0}\right)^{\frac{2}{3}}$$

$$\delta c^2(t) = \delta c_0^2 \left(\frac{g(t)}{g_0}\right)^{\frac{1}{3}} + \frac{c_0^2 l_4 \delta g_0 t}{8}\left(\frac{g(t)}{g_0}\right)^{\frac{4}{3}} + c_0^2\left[\left(\frac{g(t)}{g_0}\right)^{\frac{1}{3}} - 1\right]\delta h_0$$

1. **Not** enough to start with $\delta c_0^2 = 0$ at $\mu_0 \gg 1$ to get $\delta c^2 = 0$ near $E \sim \Lambda$
2. **Other** perturbations around fixed point must be fine-tuned

Define $$\hat{g} \equiv \frac{g}{\sigma^3}, \qquad \hat{k} \equiv \frac{k}{\sigma^4}.$$

$$\mathcal{L}_{2\phi} = \mathcal{L}_1 + \mathcal{L}_2 - g_{12}(\phi_1 \partial \phi_1)(\phi_2 \partial \phi_2) - \frac{h_1}{4}(\partial \phi_1)^2 \phi_2^2 - \frac{h_2}{4}(\partial \phi_2)^2 \phi_1^2 - V_{12}$$

$$\begin{aligned}
\beta_{g_1} &= \frac{l_4}{8}\Big(3g_1^2 + 4g_{12}h_2 + h_1 h_2 - 2h_2^2\Big) \\
\beta_{g_2} &= \frac{l_4}{8}\Big(3g_2^2 + 4g_{12}h_1 + h_1 h_2 - 2h_1^2\Big) \\
\beta_{h_i} &= \frac{l_4}{8}\Big(g_{12}^2 + h_i(g_1 + g_2) + h_i^2 + 2g_{12}h_i\Big), \qquad i = 1, 2 \\
\beta_{g_{12}} &= \frac{l_4}{16}\Big[3g_{12}^2 + 2g_{12}(g_1 + g_2) + 3g_{12}(h_1 + h_2) - h_1 h_2\Big]
\end{aligned}$$

$$\begin{aligned}
\beta_{c_1^2} &= \frac{l_4}{8}\Big(c_1^2 g_1 + c_2^2 h_1\Big) \\
\beta_{c_2^2} &= \frac{l_4}{8}\Big(c_2^2 g_2 + c_1^2 h_2\Big)
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UV behaviour: D=4 Theory with 2 particles

$$\mathcal{L}_{2\phi} = \mathcal{L}_1 + \mathcal{L}_2 - g_{12}(\phi_1\partial\phi_1)(\phi_2\partial\phi_2) - \frac{h_1}{4}(\partial\phi_1)^2\phi_2^2 - \frac{h_2}{4}(\partial\phi_2)^2\phi_1^2 - V_{12}$$

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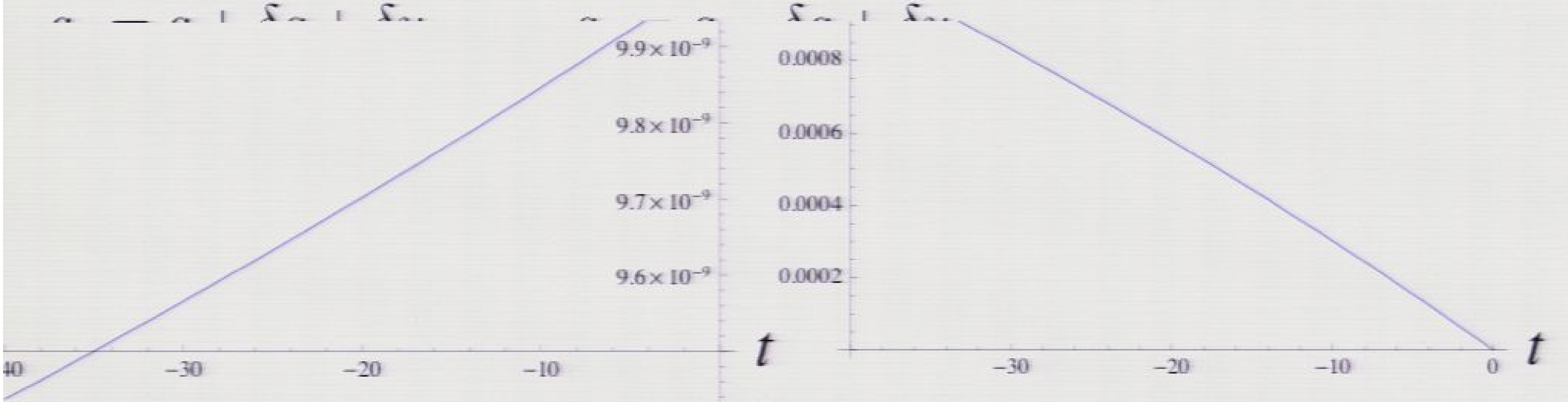

(a) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = 0$, $\delta c_0^2 = 10^{-8}$ (b) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = -10^{-2}$, $\delta c_0^2 = 0$

UV behaviour: D=10 Theory with 1 particle

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{a^2}{2}(\Delta\phi)^2 - \frac{c^2}{2}(\partial_i\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\phi^3$$

$$\gamma = \frac{l_{10}\lambda^2}{64a^5} \quad \beta_{a^2} = 2\omega_a\frac{\lambda^2}{a^5}a^2, \quad \beta_\lambda = -\omega_\lambda\frac{\lambda^2}{a^5}\lambda$$

$$\lambda(t) = \lambda_0\left(\frac{x(t)}{x_0}\right)^{\frac{\omega_\lambda}{\omega_x}}, \quad a^2(t) = a_0^2\left(\frac{x(t)}{x_0}\right)^{-\frac{2\omega_a}{\omega_x}}$$

$$x = \frac{\lambda^2}{a^5}$$

λ and effective coupling x are UV free.

$$c^2(t) = c_0^2\left(\frac{x(t)}{x_0}\right)^{-\frac{\omega_c}{\omega_x}}$$

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functions involved. Particular fixed-point solution and linear perturbations

$$\begin{aligned}\lambda &= \bar{\lambda} + \delta\lambda\,, & \tilde{\lambda} &= \bar{\lambda} - \delta\lambda \\ a^2 &= \bar{a}^2 + \delta a^2\,, & \tilde{a}^2 &= \bar{a}^2 - \delta a^2 \\ c^2 &= \bar{c}^2 + \delta c^2\,, & \tilde{c}^2 &= \bar{c}^2 - \delta c^2\end{aligned}$$

$$\bar{\lambda}(t) = \bar{\lambda}_0 \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{\frac{\omega_\lambda}{\omega_x}}, \quad \bar{a}^2(t) = \bar{a}_0^2 \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{2\omega_a}{\omega_x}}, \quad \bar{c}^2(t) = \bar{c}_0^2 \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{\omega_c}{\omega_x}}$$

$$\begin{aligned}
\delta\lambda(t) &= \delta\lambda_0 \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{\frac{\omega_{\delta\lambda}}{\omega_x}} \\
\delta a^2(t) &= \left(\delta a_0^2 - \frac{a_0^2 \delta\lambda_0}{\bar{\lambda}_0}\right) \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{4}{57}} + \frac{a_0^2 \delta\lambda_0}{\bar{\lambda}_0} \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{62}{285}} \\
\delta c^2(t) &= \left(\delta c_0^2 - \frac{c_0^2 \delta\lambda_0}{\bar{\lambda}_0}\right) \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{4}{57}} + \frac{c_0^2 \delta\lambda_0}{\bar{\lambda}_0} \left(\frac{\bar{x}(t)}{\bar{x}_0}\right)^{-\frac{28}{57}}
\end{aligned}$$

The RG evolution of δc^2 in the UV does **not** help in alleviating the fine-tuning needed to get δc small enough for energies near Λ.

IR behaviour

n conventional theories the MS scheme has to be used with care in presence of mass terms when $E \ll m$, since the decoupling of massive particles is not manifest. Indeed, the MS β-functions, being mass–independent, are formally the ame for any E, while for $E \ll m$ the physical coupling does not run anymore

For Lifshitz-like theories, situation more complicated

$$i\left(k_0^2 - c^2k^2 - \frac{a^2}{\Lambda^2}k^4 - m^2\right)^{-1}$$

suggests that IR effects arise for $k \sim c\Lambda/a$, the scale $c\Lambda/a$ playing the role of m

$\Longrightarrow$ use physical scheme (momentum subtraction)

Recall β in usual ϕ^4 theory

Let $\Gamma^{(4)}$ be the tree+one-loop+counterterms 1PI four-point function

Define the physical coupling λ as

$$\Gamma^{(4)}(s = t = u = -\mu^2) \equiv \Gamma^{(4)}(\mu) = -\lambda$$

Thus $$\Gamma^{(4)}(s, t, u) = -\lambda + \Gamma_l^{(4)}(s, t, u) - \Gamma_l^{(4)}(\mu)$$

At one-loop level $\gamma = 0$ and CS equation gives $\beta_\lambda = -\mu \dfrac{\partial \Gamma_l^{(4)}}{\partial \mu}$

$$\beta_\lambda = \frac{3\lambda^2}{16\pi^2} \int_0^1 dx \frac{\mu^2 x(1-x)}{m^2 + \mu^2 x(1-x)}$$

m acts as an IR regulator to the one-loop graph, which would otherwise have an IR divergence when $\mu \to 0$.

IR behaviour: D=10 Theory with 1 particle

Define the renormalized field ϕ and the couplings λ, a^2 and c^2 as

$$\left.\frac{\partial\Gamma^{(2)}}{\partial(p^0)^2}\right|_0 = 1; \quad \left.\frac{1}{4!}\frac{\partial^4\Gamma^{(2)}}{\partial p^4}\right|_0 = -a^2; \quad \left.\frac{1}{2}\frac{\partial^2\Gamma^{(2)}}{\partial p^2}\right|_0 = -c^2$$

$$\Gamma^{(3)}[(p_1^0)^2 = -\omega(\mu^2), p_{2,3}^0 = \vec{p}_{1,2,3} = 0] \equiv \Gamma^{(3)}(\mu) = -\lambda$$

$$\omega(\mu^2) = a^2\mu^4 + c^2\mu^2 \simeq \begin{cases} a^2\mu^4 & (UV) \\ c^2\mu^2 & (IR) \end{cases}$$

$$\gamma = -\frac{1}{2}\mu\frac{\partial\dot{\Gamma}^{(2)}_{l,0}}{\partial\mu}, \qquad \beta_\lambda = -\mu\frac{\partial\Gamma^{(3)}_{l,0}}{\partial\mu} + 3\lambda\gamma$$

$$\beta_{a^2} = -\frac{1}{4!}\mu\frac{\partial\Gamma^{(2)''''}_{l,0}}{\partial\mu} + 2\gamma a^2, \quad \beta_{c^2} = -\frac{1}{2}\mu\frac{\partial\Gamma^{(2)''}_{l,0}}{\partial\mu} + 2\gamma c^2$$

$$\begin{aligned}\gamma &= 3\lambda^2 \int \frac{d^{10}k dk_E}{(2\pi)^{11}} \int_0^1 dx \frac{\mu^2\omega'(\mu^2)x^2(1-x)^2}{[k_E^2 + a^2k^4 + c^2k^2 + \omega(\mu^2)x(1-x)]^4} \\ &= \frac{15 l_{10}\lambda^2}{32} \int_0^1 dx \int_0^\infty dk \frac{k^9\mu^2\omega'(\mu^2)x^2(1-x)^2}{[a^2k^4 + c^2k^2 + \omega(\mu^2)x(1-x)]^{7/2}}\,.\end{aligned}$$

$$\begin{aligned}\gamma^{(\mathrm{UV})} &\simeq \frac{l_{10}\lambda^2}{64a^5}\,, \qquad \mu \gg \frac{c}{a}\,\Lambda \\ \gamma^{(\mathrm{IR})} &\simeq \frac{l_{10}\lambda^2}{480a^3c^2}\frac{\mu^2}{\Lambda^2}\,, \quad \mu \ll \frac{c}{a}\,\Lambda\,.\end{aligned}$$

c^2k^2 term is an IR regulator forbidding IR singularities for $k \to 0$

IR finiteness is responsible for the freezing of the coupling

$$\beta_\lambda^{(\mathrm{IR})} \sim \beta_{a^2}^{(\mathrm{IR})} \sim \beta_{c^2}^{(\mathrm{IR})} \sim \frac{\mu^2}{\Lambda^2}$$

Generalization straightforward

Lesson: weighted marginal operators become inoperative for $\mu \ll \Lambda$

Are c^2 and δc^2 frozen in the IR ?

No, because in the IR their running is governed by weighted relevant operators which become standard marginal in the IR

Example: Yukawa theory in D=3

$$\mathcal{L} = \frac{1}{2}(\dot{\phi})^2 - \frac{c_\phi^2}{2}(\partial_i \phi)^2 + \bar{\psi}(\partial\!\!\!/_0 - c_\psi \vec{\partial\!\!\!/} \psi) - g\bar{\psi}\psi\phi$$

Assume only remnant of the non-Lorentz invariance of the UV theory is a tiny difference in the speed of light of fermion and scalar: $c_\phi = c_\psi + \delta c$, with $\delta c \ll 1$

By simple exercise we get

$$\beta_{c_\psi} = -\frac{g^2}{24\pi^2}\delta c\,, \qquad \beta_{\delta c} = \frac{5g^2}{24\pi^2}\delta c\,, \qquad \beta_g = \frac{5g^3}{16\pi^2}$$

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A Lorentz symmetry breaking term in the IR theory (coming from the underlying UV theory) induces, by quantum effects, an energy-dependent speed of light

Fine-tuning

For ordinary particles $\delta c < 10^{-(21\div23)}$

[Coleman-Glashow, hep-ph/9812418]

For photons, FERMI observation of GRB 080916C gives

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Conclusions

Ve have analyzed the one-loop RG evolution of Lifshitz-like theories,
by focusing on specific Lorentz violating scalar field theories
and on the the spatial kinetic term $c^2_{1,2}(\vec{\partial}\phi_{1,2})^2$ operators

Two regimes of interest

1) $E \gg \Lambda$ **UV regime**

Operators classified by their weighted scaling dimensions

RGE governed by weighted marginal operators

2) $E \ll \Lambda$ **IR regime**

Operators classified by their standard scaling dimensions

RGE governed by standard marginal operators

Weighted marginal couplings freeze

UV: c^2 and δc^2 logarithmically run with the energy

(governed by **weighted** marginal operators)

IR: c^2 and δc^2 logarithmically run with the energy

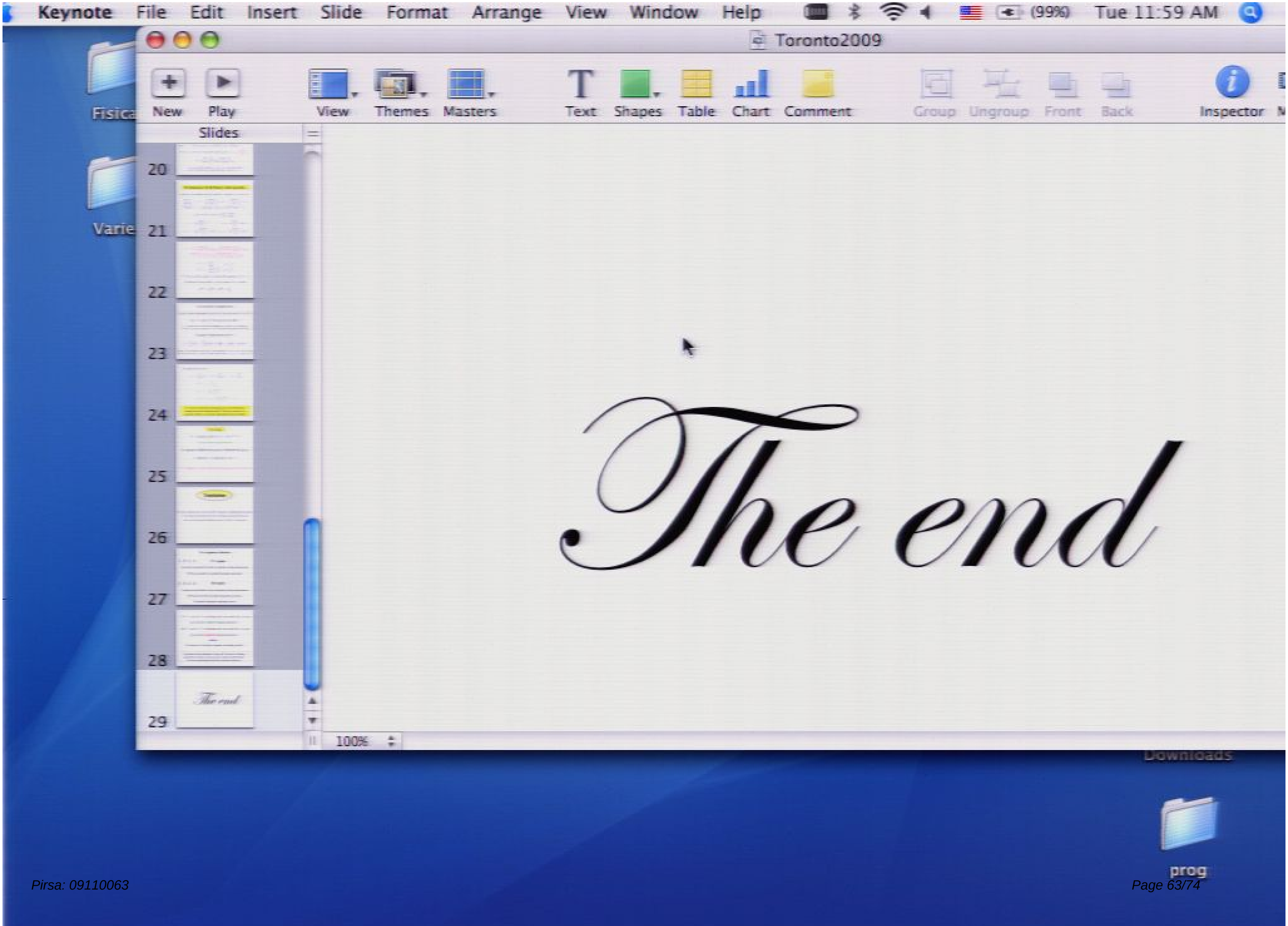
(governed by **standard** marginal operators)

unless

all sources of Lorentz symmetry breaking vanish

A dynamical mechanism to keep all Lorentz violating parameters small, is necessary to make Lifshitz-like theories promising theories of particle physics

The end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UV: c^2 and δc^2 logarithmically run with the energy

(governed by **weighted** marginal operators)

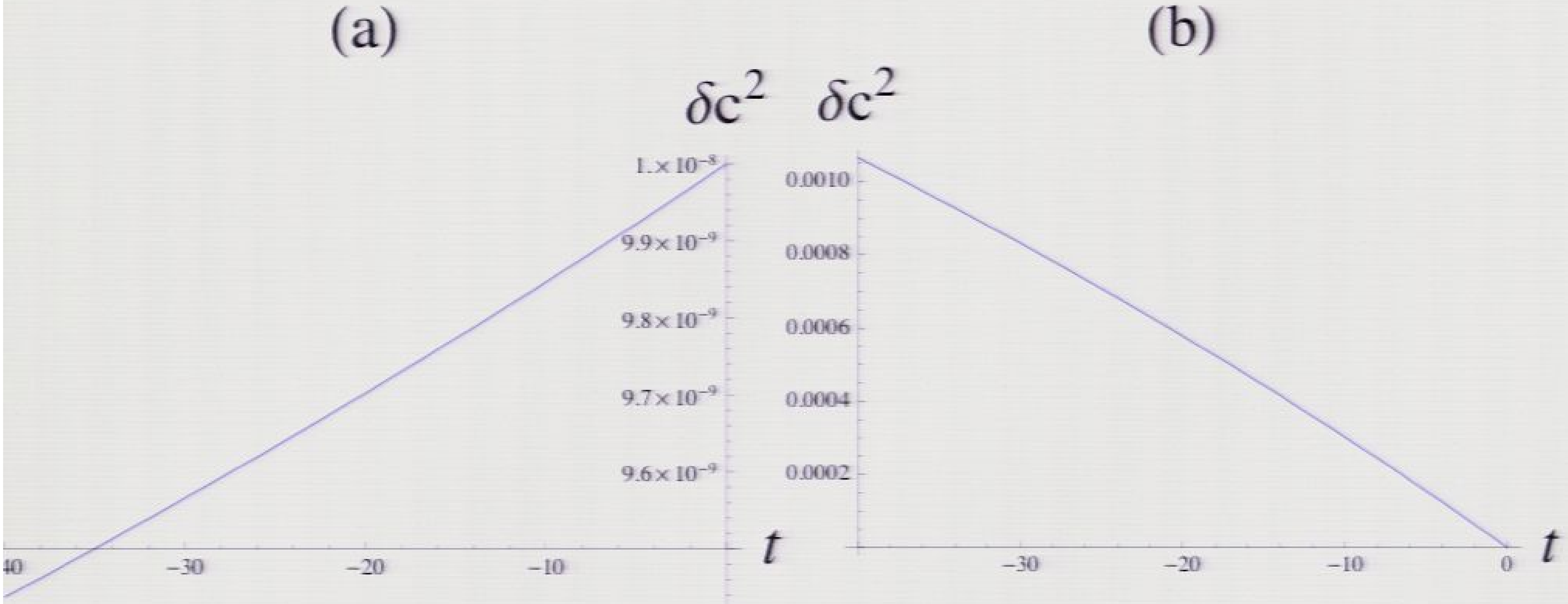
IR: c^2 and δc^2 logarithmically run with the energy

(governed by **standard** marginal operators)

unless

all sources of Lorentz symmetry breaking vanish

A dynamical mechanism to keep all Lorentz violating parameters small, is necessary to make Lifshitz-like theories promising theories of particle physics



(a) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = 0$, $\delta c_0^2 = 10^{-8}$ (b) $c_0^2 = 1$, $\delta g_0 = \delta h_0 = -10^{-2}$, $\delta c_0^2 = 0$

Introduction

From a particle physics point of view, it is known that theories with higher derivative quadratic operators have a better UV-behaviour with respect to standard theories, thanks to the modified free particle propagator

However, Lorentz invariance + higher derivative operators imply a lack of unitarity due to the appearance of ghost (PV-like fields)

Lifshitz-like theories, explicitly breaking Lorentz invariance, allow for the construction of unitary otherwise non-renormalizable theories

An extensive study of Lifshitz-like theories for scalars, fermions and gauge fields, as well as applications to the SM, has been made by Anselmi [Anselmi&Helat, 0707.2480; Anselmi, 0808.3470; 0808.3474; 0808.3475; 0904.1849]

The main recent interest, however, comes from the Horava proposal of applying this idea to build a renormalizable theory of gravity [Horava, 0901.3775]

Renormalization Group in Lifshitz-Type Theories

Marco Serone, SISSA, Trieste

In collaboration with Jorge G. Russo and Roberto Iengo, based on 0906.3477 [hep-th]

Perimeter Institute, November 10 2009

Plan

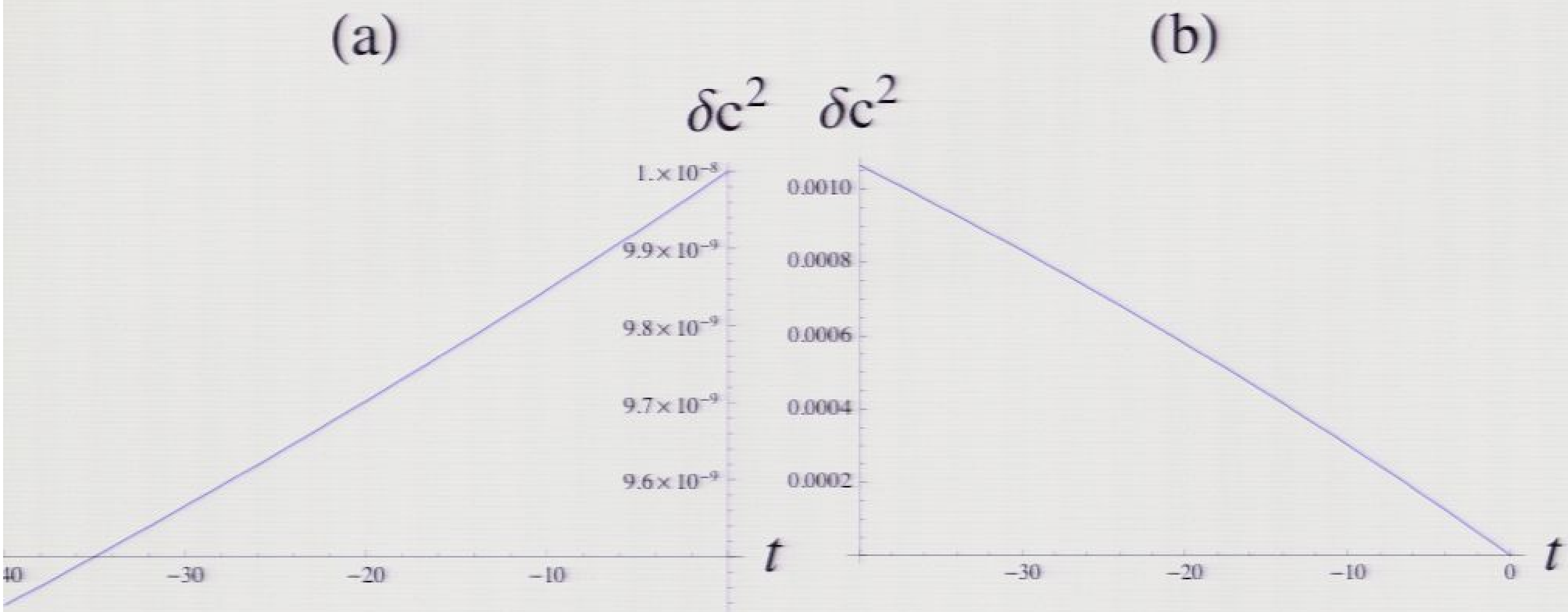
Introduction and brief review

UV behaviour

IR behaviour

Fine-tuning problem

Conclusions



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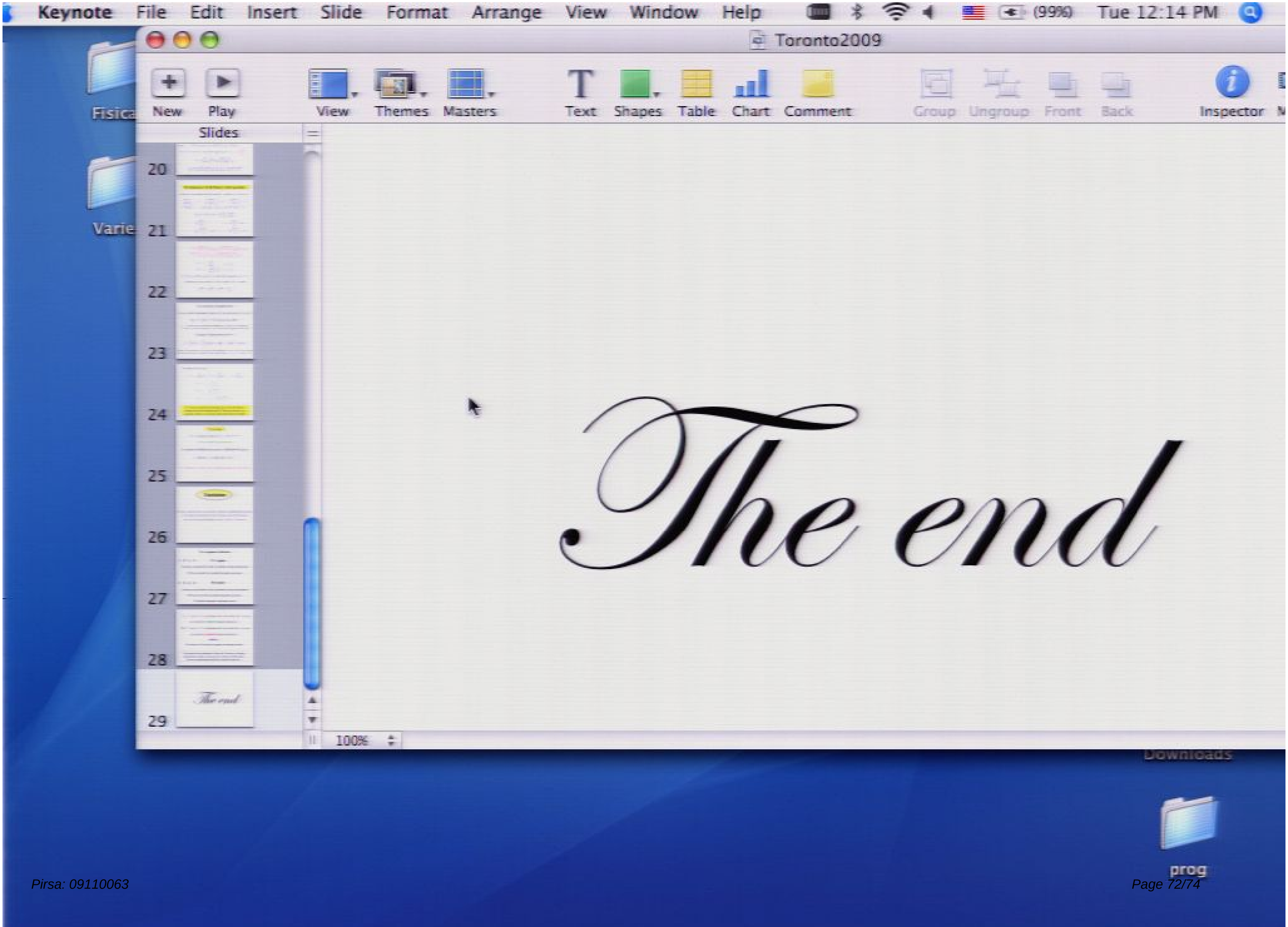
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By simple exercise we get

$$\beta_{c_\psi} = -\frac{g^2}{24\pi^2}\delta c\,, \quad \beta_{\delta c} = \frac{5g^2}{24\pi^2}\delta c\,, \quad \beta_g = \frac{5g^3}{16\pi^2}$$

$$\begin{aligned}
\alpha(t) &= \frac{\alpha_0}{1 - \frac{5}{2\pi}\alpha_0 t} \\
\delta c(t) &= \delta c_0 \left(\frac{\alpha(t)}{\alpha_0}\right)^{\frac{7}{15}} \\
c_\psi(t) &= c_{\psi,0} - \frac{\delta c_0}{7}\left[\left(\frac{\alpha(t)}{\alpha_0}\right)^{\frac{7}{15}} - 1\right]
\end{aligned}$$

A Lorentz symmetry breaking term in the IR theory (coming from the underlying UV theory) induces, by quantum effects, an energy-dependent speed of light

Fine-tuning

For ordinary particles $\delta c < 10^{-(21 \div 23)}$

[Coleman-Glashow, hep-ph/9812418]

For photons, FERMI observation of GRB 080916C gives

$$|c^2(1\text{MeV}) - c^2(10\text{GeV})| \lesssim 10^{-17}$$

Severe tuning for each Lorentz-violating parameter in the theory