Title: Spin Systems and Emergent Gauge Fields at Lifshitz points

Date: Nov 10, 2009 09:30 AM

URL: http://pirsa.org/09110062

Abstract:

Spins and Emergent Gauge Fields at Lifshitz points

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- It is about an example of emergent gauge fields – and its dynamics in situations where the UV theory is not Lorentz invariant.
- The motivation is to find possible applications in condensed matter systems.
- However, there *could* be some lessons which may have implications to low-energy behavior of other Lifshitz theories, including gravity.

Background: CP^{N-1} models

- There is an interesting way to rewrite the $O(3)$ sigma model fields $\vec{n}$ as

$$\vec{n} = \phi^\dagger \vec{\sigma} \phi$$

- ϕ is a 2-component complex vector - SPINON
- The constraint $\vec{n} \cdot \vec{n} = 1$ then becomes the constraint

$$\phi^\dagger \phi = 1$$

- However, this is a redundant description - so there is a compact $U(1)$ gauge symmetry

$$\phi(x) \sim e^{i\theta(x)} \phi(x)$$

- Then the number of degrees of freedom work out right.
- This is the CP^1 model.

- If we have a *N* component complex vector ϕ with the conditions

$$\phi^\dagger \phi = 1 \qquad \phi(x) \sim e^{i\theta(x)}\ \phi(x)$$

we have a CP^{N-1} model.

For the original $O(3)$ sigma model, the lagrangian becomes

$$\frac{1}{2}(\partial \vec{n})^2 = \frac{1}{2}[\partial_\mu \phi^\dagger \partial^\mu \phi - j_\mu j^\mu]$$

$$j_\mu = \frac{1}{2i}[\phi^\dagger \partial_\mu \phi - (\partial_\mu \phi^\dagger)\phi]$$

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This model of course has gauge symmetry – but as yet no gauge field. We can introduce a gauge field and rewrite this

$$L = \frac{1}{2}(D_\mu \phi^\dagger)(D^\mu \phi) \quad , \qquad D_\mu = \partial_\mu + iA_\mu$$

Integrating out A_μ leads to the above lagrangian.

- The same lagrangian defines the usual CP^{N-1} model when the vector ϕ is N component.
- It is useful to absorb the overall coupling constant f into the field and impose the constraint

$$\phi^\dagger \phi = \frac{1}{f^2}$$

by a Lagrange multiplier field χ

$$L = \frac{1}{2}(D_\mu \phi^\dagger)(D^\mu \phi) + \chi(\phi^\dagger \phi - \frac{1}{f^2})$$

- We have introduced redundant degrees of freedom to describe the system – and therefore a gauge field – which has no dynamics.
- **What is the use of all this ?**

Emergent Gauge Dynamics

- In the 1980's this model in euclidean $d=2$ was popular among particle theorists as a model of dynamical mass generation.
- Like other nonlinear sigma models in $d=2$ the coupling f is asymptotically free.
- As a result it flows to strong coupling in the IR and the Lagrange multiplier field χ acquires a nonzero expectation value. This means that the spinon acquires a dynamically generated mass,

$$m_\phi = < \chi > \sim \Lambda \, e^{-1/f^2 N}$$

- Here Λ is a UV cutoff. The beta function is therefore

$$\beta(f) \sim -f^3 \, N$$

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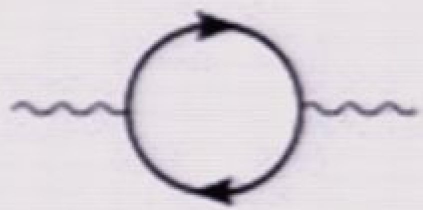
- This makes the gauge field dynamical. In fact the one loop diagram



- leads to the effective action for the gauge field

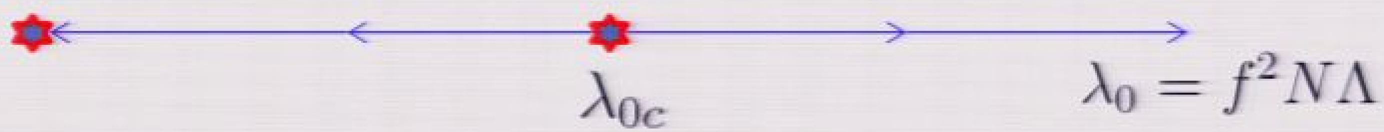
$$\frac{1}{m_\phi^2} F_{\mu\nu} F^{\mu\nu}$$

- In terms of the spin model, there was no gauge theory at all
- Introduction of redundant variables leads to a gauge invariance, but the gauge field has no dynamics at the classical level
- Quantum effects induce this dynamics.
- All these results can be seen explicitly in the 't Hooft large N expansion

$$f \to 0 \qquad N \to \infty \qquad f^2 N = \lambda = \text{finite}$$

- However in $d = 2$ gauge fields are rather boring.
- There are no photons.
- The potential between charges is always confining.
- In a sense one does not gain very much by introducing this redundancy.

- In $d = 3$ this model is of interest in condensed matter systems and the situation becomes interesting.
- Now there is a phase transition between ordered and disordered phases – there is a IR unstable non-trivial fixed point.



- The ordered phase $\lambda_0 < \lambda_{0c}$ is gapless, while the disordered phase $\lambda_0 > \lambda_{0c}$ has a gap.

$$< \sqrt{\chi} > \sim \Lambda \left(\frac{1}{\lambda_{0c}} - \frac{1}{\lambda_0} \right)$$

- Thus $< \chi > \neq 0$ only for $\lambda_0 > \lambda_{0c}$
- The beta function reflects this

$$\beta(\lambda_0) \sim -\lambda_0(\lambda_0 - \lambda_{0c})$$

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- This means that *normally* there is no excitation corresponding to a massless photon in the original spin model.
- There are, however, several situations where such monopoles are suppressed. The gauge theory – which is effectively non-compact – does not confine.
- In these situations, a gapless photon mode remains – something which would have been rather difficult to discover in the original spin language. Most dramatic – deconfined criticality *(Balents, Fisher, Sachdev, Senthil & Viswanath)* with **emergent photon**.

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Lifshitz CP^{N-1} models

- The general model is defined by

$$S_L = \frac{1}{2}\int dt \int d^d x \ \left[(D_0\vec{\phi})^\star (D^0\vec{\phi}) + \alpha (D_i\vec{\phi})^\star (D^i\vec{\phi}) + |\mathcal{D}^z\vec{\phi}|^2\right]$$

- With the usual constraint $\phi^\dagger \phi = \frac{1}{f^2}$

  $\mathcal{D}^z$ is $O(d)$ invariant and contains z derivatives. For $z = 2$

$$|\mathcal{D}^z\phi|^2 \equiv a\ (D_i D_j\vec{\phi})^\star \cdot (D_i D_j\vec{\phi}) + b\ (D^2\vec{\phi})^\star \cdot (D^2\vec{\phi})$$

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The point $\alpha = 0$ has Lifshitz scaling with dynamical critical exponent *z*, under which $t \rightarrow \gamma^z t \quad x \rightarrow \gamma t$

$$[\vec{\phi}] \sim [L]^{\frac{z-d}{2}} \qquad [f] \sim [L]^{\frac{d-z}{2}}$$

- The coupling is now asymptotically free for all $d = z$
- This may be seen in the large-N expansion as follows. First introduce a lagrange multiplier field as usual

$$\frac{\mathcal{L}}{2} = (D_0\vec{\phi})^2 + \alpha(D_i\vec{\phi})^2 + (\mathcal{D}^z\vec{\phi})^2 + \chi(\vec{\phi}^\dagger \cdot \vec{\phi} - \frac{1}{f^2})$$

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$$S_{eff} = \mathrm{Tr}\log\ [-D_0^2 - \alpha D_i^2 + (-1)^z(\mathcal{D}^z)^2 + \chi] + \frac{1}{2f^2}\int dt d^d x\ \chi$$

- To leading order in large-*N*, the functional integral over χ and A_μ is dominated by a saddle point with vanishing gauge field and a constant $\chi(t,x) = \chi_0$. The saddle point equation is

$$2N\int \frac{d\omega d^2k}{(2\pi)^3}\frac{1}{\omega^2 + \alpha\vec{k}^2 + \vec{k}^{2z} + \chi_0} = \frac{1}{f^2}$$

- At a Lifshitz line, $\alpha = 0$ the solution is

$$m^2 = \chi_0 = \Lambda^{2z} \exp[-\frac{A}{f^2 N}]$$

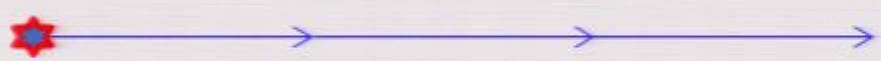
- This immediately shows asymptotic freedom

$$\Lambda \frac{df}{d\Lambda} = -\frac{f^3 N}{A}$$

- The leading *1/N* correction is obtained by expanding around this saddle point solution

$$\chi(t,x) = \chi_0 + \frac{1}{\sqrt{N}} \delta\chi \qquad A_\mu(t,x) \to \frac{1}{\sqrt{N}} A_\mu(t,x)$$

- We now evaluate the effective action for the gauge field explicitly for $z = d = 2.$



The Gauge Field Action

- We need to evaluate a determinant,

$$\mathrm{tr}\ \log[-D_0^2 + (\mathcal{D}^2)^2 + m^2]$$

- Using de-Witt-Schwinger representation

$$S_{eff} = -N \int_0^\infty \frac{ds}{s} e^{-m^2 s} \mathrm{tr} e^{-sH}$$

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- The quantity κ is a renormalization scale.

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This is z=2 electrodynamics (Horava, 2008)

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- The quantity κ is a renormalization scale.
- For generic a and b, the gauge dynamics becomes lorentz invariant at low energies – with a scale dependent speed of light.
- However for ***a = 0*** something special happens.
- The lowest term in B is now $(\partial_i B)^2$
- In fact, in this case for constant B and for $F_{i0} = 0$, there are no terms at all which are purely power laws in B.
- The leading term for small B turns out to be

$$\frac{S_{eff}(B) - S_{eff}(0)}{VT} \simeq \frac{B^{\frac{3}{2}}\ m^{\frac{1}{2}}\ b^{\frac{1}{4}}}{4\pi^2\sqrt{2}}\ e^{-\frac{\pi m}{B\sqrt{b}}}$$

- This is non-analytic in *B* – vanishes faster than any power.

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$$H = -D_0^2 + (a+b)(D^2)^2 + aB^2 - ia\epsilon^{ij}(\partial_i B) D_j$$

- Where $B = \epsilon^{ij} F_{ij}$
- This is evaluated by known techniques. The lowest terms in a power series expansion of the field strength yields the effective lagrangian

$$\mathcal{L} = \frac{a+b}{12m}[F_{0i}^2 + \frac{1}{10}(\partial_i B)^2] + \frac{a}{4}\log(\kappa/m) B^2 + \cdots$$

Heat Kernel Calculation

- Basic technique : represent the trace as

$$\mathrm{Tr} e^{-s\mathcal{O}} = \int dt d^2x \int \frac{d\omega d^2k}{(2\pi)^3}\, e^{-i(\omega t + k\cdot x)}\; e^{-s\mathcal{O}}\;\; e^{i(\omega t + k\cdot x)}$$

- Then use

$$e^{-i(\omega t + k\cdot x)}\; D_\mu\; e^{i(\omega t + k\cdot x)} = ik_\mu + D_\mu$$

repeatedly and expand in powers of the field strength.

$$\mathcal{L} = \frac{a+b}{12m}[F_{0i}^2 + \frac{1}{10}(\partial_i B)^2] + \frac{a}{4}\log(\kappa/m)B^2 + \cdots$$

- The quantity κ is a renormalization scale.
- For generic a and b, the gauge dynamics becomes lorentz invariant at low energies – with a scale dependent speed of light.
- However for ***a* = 0** something special happens.
- The lowest term in B is now $(\partial_i B)^2$
- In fact, in this case for constant B and for $F_{i0} = 0$, there are no terms at all which are purely power laws in B.
- The leading term for small B turns out to be

$$\frac{S_{eff}(B) - S_{eff}(0)}{VT} \simeq \frac{B^{\frac{3}{2}}\ m^{\frac{1}{2}}\ b^{\frac{1}{4}}}{4\pi^2\sqrt{2}}\ e^{-\frac{\pi m}{B\sqrt{b}}}$$

- This is non-analytic in *B* – vanishes faster than any power.

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Constant B calculation

- For constant B and $F_{i0} = 0$ the operator is

$$H(B) = -D_0^2 + (a+b)(-D_1^2 - D_2^2)^2 + aB^2$$

- Choose a gauge

$$A_0 = A_1 = 0 \qquad A_2 = B\,x^1$$

- The non-trivial part is the square of the Landau hamiltonian of a charged particle in a constant magnetic field. The eigenvalues :

$$\kappa(p_0, n) = p_0^2 + (a+b)B^2(2n+1)^2 + aB^2$$

- The degeneracies are

$$d(n) = \frac{BL_1L_2}{2\pi}$$

Theta Function

- This gives

$$\mathrm{Tr}\; e^{-sH(B)} = \frac{VT}{16\pi s\,\sqrt{a+b}} e^{-saB^2}\;\vartheta_4\left[0\mid i\pi/(4B^2 s(a+b))\right]$$

$$= \frac{VT}{16\pi s\,\sqrt{a+b}} e^{-saB^2}\sum_{k=-\infty}^{\infty}(-1)^k\, e^{-\frac{\pi^2 k^2}{4sB^2(a+b)}}$$

- Integration over s yields an effective action

$$\frac{S_{eff}(B)}{VT} = -\sum_{k\neq 0}(-1)^k \frac{Bm}{4\pi^2 k}\sqrt{1+\frac{aB^2}{m^2}}\,K_1\left(\frac{\pi k m}{B}\sqrt{\frac{1+\frac{aB^2}{m^2}}{a+b}}\right)$$

$$-\frac{1}{16\pi\sqrt{a+b}}\int_0^\infty \frac{ds}{s^2} e^{-m^2 s}\left(e^{-saB^2}-1\right)$$

- For $a = 0$ there are no terms which vanish as any power of B

Perturbative *z=2* Electrodynamics

- We now investigate the IR non-perturbative behavior of the model $$S = \frac{1}{2g^2}\int dt\, d^2x\, \left[F_{0i}F^{0i} + \frac{1}{2}(\partial_k F_{ij})(\partial^k F^{ij})\right]$$
- Define gauge invariant field strengths

$$H_\mu(t,\vec{x}) = \frac{1}{2}\epsilon_{\mu\nu\lambda}F^{\nu\lambda}(t,\vec{x}) = \int \frac{d\omega d^2\vec{k}}{(2\pi)^3} H_\mu(\omega,\vec{k})e^{-i(\omega t+\vec{k}\cdot\vec{x})}$$

The perturbative propagators have poles at $\omega = \pm i\vec{k}^2$

$$\begin{aligned}
< H_0(\omega,\vec{k})H_0(-\omega,-\vec{k}) > &= \frac{\vec{k}^2}{\omega^2+\vec{k}^4} \\
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< H_i(\omega,\vec{k})H_j(-\omega,-\vec{k}) > &= \delta_{ij} - \frac{k_i k_j \vec{k}^2}{\omega^2+\vec{k}^4}
\end{aligned}$$

Monopole Instantons

- Monopole instantons are solutions of the (Euclidean) equations of motion which violate Bianchi identity.
- The equations of motion are

$$\partial_i F^{0i} = 0 \qquad \Rightarrow \qquad F_{0i} = \epsilon_{ij}\partial^j \chi$$

$$\partial^0 F_{0i} + \nabla^2 \partial^j F_{ji} = 0 \qquad \Rightarrow \qquad H_0 = -\frac{\partial_0}{\nabla^2}\chi$$

- Where $\nabla^2 \equiv \partial_i \partial^i$ and we have used a freedom to shift χ by a function of time.
- The Bianchi identity is replaced by

$$\partial_\mu H^\mu = \rho(t, \vec{x}) \qquad \Rightarrow \qquad \partial_0 H_0 + \nabla^2 \chi = \rho(t, \vec{x})$$

- Where $\rho(t, \vec{x})$ is the monopole density.
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- These are of course instantons.

- The potential due to a monopole distribution can be now easily solved

$$\chi(t,\vec{x}) = \int dt'\ d^2x'\ G_0(t-t',\vec{x}-\vec{x}')\ \rho(t',\vec{x}')$$

- $G_0(t-t',\vec{x}-\vec{x}')$ is the Green's function which replaces Coulomb law

$$\left[-\frac{\partial_0^2}{\nabla^2}+\nabla^2\right] G_0(t-t',\vec{x}-\vec{x}') = \delta(t-t')\ \delta^{(2)}(\vec{x}-\vec{x}')$$

$$G_0(t,\vec{x}) = 2\pi \int \frac{d\omega d^2\vec{k}}{(2\pi)^3} e^{-i(\omega t+\vec{k}\cdot\vec{x})}\ \frac{\vec{k}^2}{\omega^2+\vec{k}^4}$$

- The action for a give monopole distribution is

$$S_\rho = \frac{1}{2g^2}\int \frac{d\omega d^2\vec{k}}{(2\pi)^3}\frac{\vec{k}^2}{\omega^2+\vec{k}^4}\rho(\omega,\vec{k})\rho(-\omega,-\vec{k})$$

- Dirac quantization $\quad \rho(t,\vec{x}) = q\delta(t)\delta^2(\vec{x}) \quad \Rightarrow \quad q = \frac{2\pi n}{g}$

- For a single monopole at the origin

$$S_1 = \frac{1}{2g^2} \int \frac{d\omega d^2\vec{k}}{(2\pi)^3} \frac{\vec{k}^2}{\omega^2 + \vec{k}^4}$$

- This is UV divergent because of self energy, but IR finite. This means that the entropy factor always dominates for large volumes – these **instantons proliferate the vacuum for all values of the gauge coupling**.

Sine-Gordon Representation

- A normal Coulomb gas has a dual representation in terms of a sine-Gordon theory – this is what happens for a monopole gas in standard 2+1 dimensional elctrodynamics.
- In our case, the interaction is not Coulomb – and we get a novel non-relativistic version of sine-Gordon theory.
- The partition function of a monopole gas may be written as

$$e^{-S_\rho} = \int D\phi_1 \; D\phi_2 \; e^{-S[\phi_1,\phi_2]}$$

$$S[\phi_1,\phi_2] = \int d^3x \; \left[i\phi_1 \partial_0 \phi_2 + \frac{1}{2}\{(\nabla\phi_1)^2 + (\nabla\phi_2)^2\} - \frac{i}{g}\rho\phi_1 \right]$$

- Note that ϕ_2 is canonically conjugate to ϕ_1.

- The dominant contribution to the partition function is due to minimally charged monopoles and anti-monopoles

$$Z_g = \sum_{N_\pm} \frac{\zeta^{N_+ + N_-}}{N_+! N_-!} \int \prod_{a=1}^{N_+} d^3x_a \int \prod_{b=1}^{N_-} d^3x_b \, e^{-\frac{4\pi^2}{g^2} \sum_{ij} n_i n_j G_{ij}}$$

where $n_i = \pm 1$ and ζ is the fugacity of monopoles,

$$\zeta \sim g^4 \, e^{-\frac{1}{g^2 a}}$$

a being a UV cutoff. This may be now written as

$$Z_g = \int D\phi_1 D\phi_2 \, e^{-S_{SG}(\phi_1, \phi_2)}$$

$$\mathcal{L}_{SG}(\phi_1, \phi_2) = \frac{g^2}{4\pi^2} \left[i\phi_1 \partial_t \phi_2 + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}(\nabla\phi_2)^2 - M^2 \cos\phi_1 \right]$$

where

$$M^2 = \frac{8\pi^2 \zeta}{g^2}$$

- Upon continuation to Lorentzian signature, the hamiltonian density is

$$\mathcal{H} = \frac{4\pi^2}{g^2}(\nabla\Pi_1)^2 + \frac{g^2}{4\pi^2}(\nabla\phi_1)^2 - \frac{g^2M^2}{2\pi^2}\cos\phi_1$$

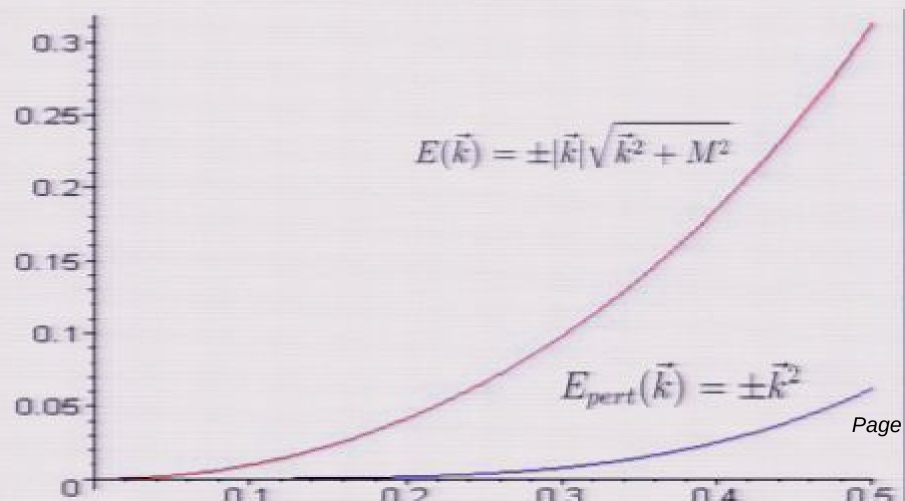
- The spectrum of small fluctuations

$$E(\vec{k}) = \pm|\vec{k}|\sqrt{\vec{k}^2 + M^2}$$

- Recall that the perturbative spectrum is

$$E_{pert}(\vec{k}) = \pm\vec{k}^2$$

- The monopole gas has introduced a mass scale M, and has removed the original gapless mode. However, a new gapless mode has taken its place.



The Full Propagator

- This new gapless mode is present in the full propagator of the gauge invariant field strength.
- The total field strength is a sum of the monopole contribution and fluctuations

$$H_\mu = H_\mu^M + h_\mu$$

- Since the theory is quadratic,

$$< H_\mu H_\nu >_{tot} = < H_\mu^M H_\nu^M > + < h_\mu h_\nu >$$

- And the correlator of fluctuations is the same as the perturabtive result.
- So we need to calculate the monopole contribution.

- The monopole contributions to the field strength are

$$H_i^M = -ik_i\chi = -\frac{k_i\vec{k}^2}{\omega^2+\vec{k}^4}\rho(\omega,\vec{k})$$

$$H_0^M = i\frac{\omega}{\vec{k}^2}\chi(\omega,\vec{k}) = \frac{\omega}{\omega^2+\vec{k}^4}\rho(\omega,\vec{k})$$

- We need to calculate the correlators of $\rho(\omega,\vec{k})$ in the monopole gas.

- Recall the representation for the action for monopole gas

$$e^{-S_\rho} = \int D\phi_1\ D\phi_2\ e^{-S[\phi_1,\phi_2]}$$

$$S[\phi_1,\phi_2] = \int d^3x\ \left[i\phi_1\partial_0\phi_2 + \frac{1}{2}\{(\nabla\phi_1)^2 + (\nabla\phi_2)^2\} - \frac{i}{g}\rho\phi_1 \right]$$

- Thus the generating functional for correlators of $\rho(\omega,\vec{k})$

$$Z[J] = < \exp[i\int d^3x\ J(x)\rho(x)] >_{mgas}$$

may be obtained by following the same steps which led to the sine-Gordon representation - by shifting ϕ_1

$$Z[J] = \int D\phi_1 D\phi_2\ e^{-S_{SG}(\phi_1 - \frac{2\pi J}{g}, \phi_2)}$$

- $$< \rho(\omega,\vec{k})\rho(-\omega,-\vec{k}) > = \frac{M^2(\omega^2 + \vec{k}^4)}{\omega^2 + \vec{k}^2(\vec{k}^2 + M^2)}$$

- This leads to the following monopole contributions to the field strength correlators

$$
\begin{aligned}
< H_0(\omega,\vec{k})H_0(-\omega,-\vec{k}) >_{monopole} &= \frac{\omega^2 M^2}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)} \\
< H_0(\omega,\vec{k})H_i(-\omega,-\vec{k}) >_{monopole} &= \frac{M^2\vec{k}^2\omega k_i}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)} \\
< H_i(\omega,\vec{k})H_j(-\omega,-\vec{k}) >_{monopole} &= \frac{M^2(k_i k_j \vec{k}^4)}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)}
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- The total correlator becomes

$$< H_0(\omega, \vec{k}) H_0(-\omega, -\vec{k}) >_{total} \;=\; \frac{\vec{k}^2 + M^2}{\omega^2 + M^2\vec{k}^2 + \vec{k}^4}$$

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- The original gapless pole has been removed – but a new gapless pole has emerged – corresponding to the spectrum of the sine-Gordon theory.
- This is in contrast to the usual *(z=1)* theory where

$$< H_\mu H_\nu >= \delta_{\mu\nu} - \frac{k_\mu k_\nu}{\vec{k}^2 + M^2}$$

- Even though the dispersion relation becomes relativistic for low momenta,

$$\omega \sim \pm M|\vec{k}| \qquad\qquad |\vec{k}| \ll M$$

- Lorentz invariance is not really recovered since the various correlators do not rotate into each other properly. However there is a non-local (in position space) redefinition of the fields $\tilde{H}_\mu$ and the momenta which lead to correlators which look Lorentz invariant

$$\tilde{H}_0(\omega, \vec{k}) = \frac{H_0(\omega, \vec{k})}{|\vec{k}|}, \tilde{H}_i = H_i \qquad\qquad k_0 \equiv \frac{\omega}{|\vec{k}|}$$

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Wilson Loops

- The standard z=1 electrodynamics in 2+1 dimensions has a mass gap – the theory also confines.
- The expectation value of a time-like Wilson loop

$$W_C = \exp\left(ie\int_C A_\mu dx^\mu\right)$$

obeys an area law – signifying that non-dynamical charges have a linear potential between them.

- We now want to investigate what happens in our theory.

- The Wilson loop may be written as

$$W_C = \exp\left(ie\int_C A_\mu dx^\mu\right) = \exp\left(ie\int_S H_\mu d\sigma^\mu\right)$$

- Therefore this factorizes into a monopole contribution and a fluctuation contribution.

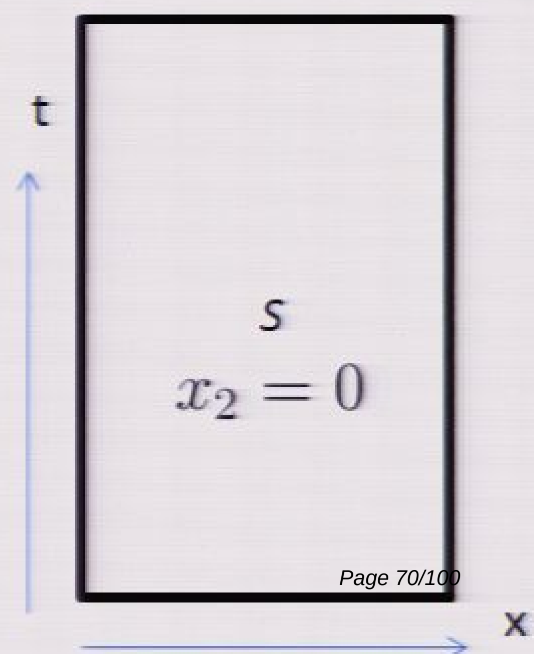
$$< W(C) >=< W(C) >_{mon}< W(C) >_{quant}$$

- The monopole contribution is

$$\int_S H_\mu d\sigma^\mu = \int d^3x \rho(x)\eta_C(x)$$

- Where η_C is determined by the loop.
- The calculation is identical to that of $Z[J]$

$$[W_C]_{classical} = Z[J = e\eta_C]$$



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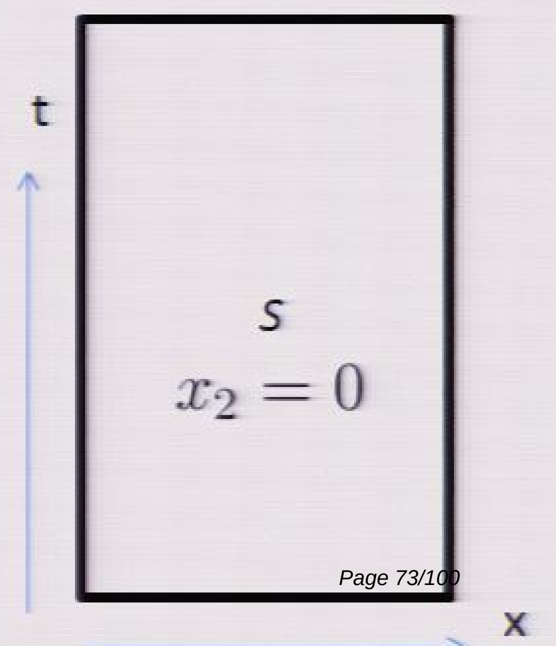

- Use the sine-Gordon representation for this

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- For a time-like Wilson loop at $x_2 = 0$ the quantity η_C is

$$\eta_C = \frac{\partial}{\partial x_2}\int dt' d^2x' G_0(t - t', \vec{x} - \vec{x}')\delta(x_2')\Theta_S(t'x_1')$$

- The quantity Θ_S is non-zero only inside the surface *S.*
- The dimensionless coupling of the sine-Gordon theory is given by

$$\frac{M}{g} \sim \frac{\zeta}{g^4} \sim e^{-1/(g^2a^2)}$$

- Thus when $g_0 = ga \ll 1$ the integral over ϕ_1 and ϕ_2 can be performed by saddle point.

- The quantity η_C is in fact the potential due to dipole layer

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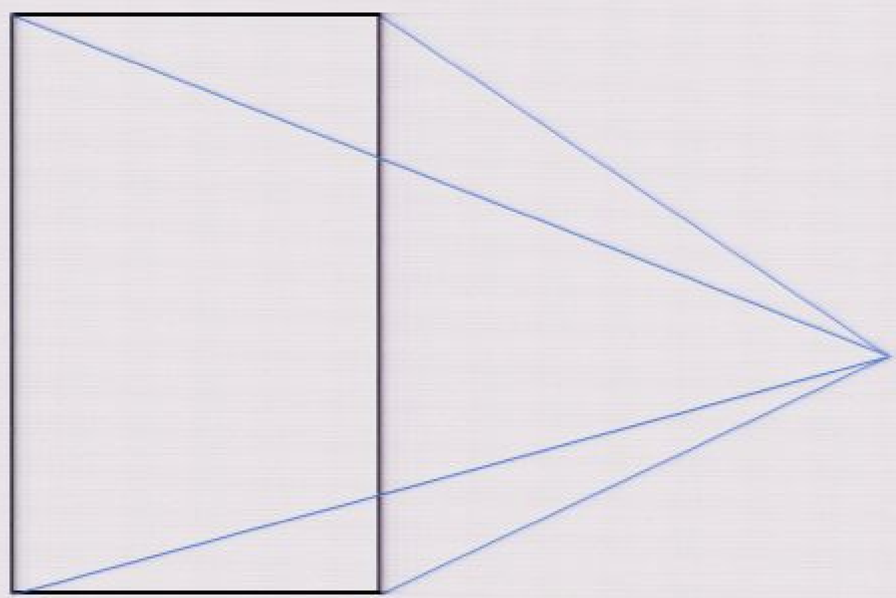

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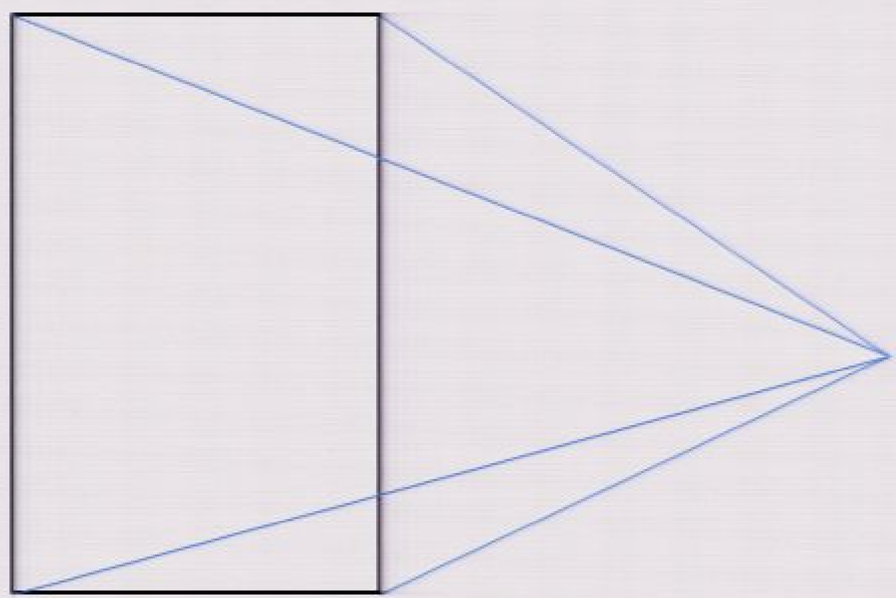


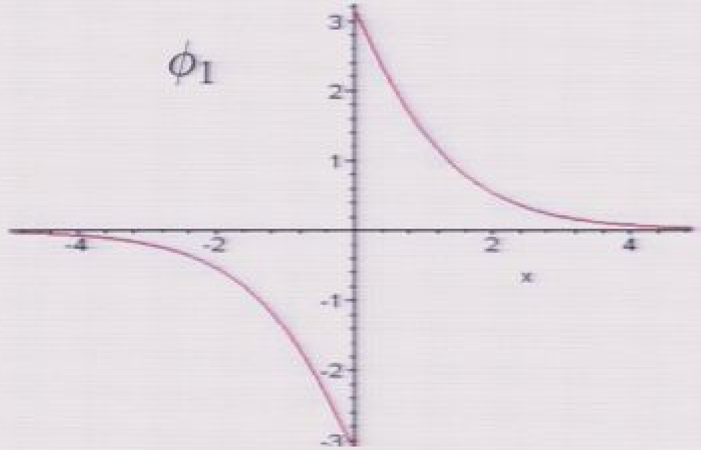
- Integrate over the gaussian variable ϕ_2 first. The saddle point equation for ϕ_1 is then

$$\Big(-\frac{\partial_0^2}{\nabla^2} + \nabla^2 \Big)\phi_1 = 2\pi \frac{e}{g} \delta'(x_2)\Theta_S + M^2 \sin\phi_1$$

- Where we have used the equation satisfied by the Green's function.
- The source term requires ϕ_1 to be discontinuous as one crosses the surface spanned by the loop at $x_2 = 0$
- If the time extent of the loop, *T* is very large and the space extent *L* is large as well, the term involving ∂_0^2/∇^2 may be ignored – one gets a ODE, whose solution is

$$\phi_1(x_2) = 4sgn(x_2)\tan^{-1}\Big(e^{-M|x_2|}\tan\big(\frac{\pi e}{4g}\big)\Big)$$

- When the charge $e = (2n+1)g$ this solution is non-trivial



- Clearly the saddle point value of the action involved in the calculation of the Wilson loop is proportional to the area *TL*

$$< W(C) >_{mon} \sim e^{-\sigma\, TL}$$

- With the string tension $\sigma \sim Mg^2$
- When the charge $e = 2n\ g$ the approximation of ignoring dependence on t and x_1 is not adequate – the answer still evaluates to an area law.

Wilson Loops : II

- We have performed a direct calculation of the Wilson loop in the linearized approximation to the saddle point equation.
- For time-like Wilson loops we verify the behavior

$$< W(C) >_{mon} \sim e^{-\sigma \, TL}$$

- For space-like Wilson loops we find

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- Finally, the fluctuations contribute a subleading term proportional to the perimeter.

Summary of Results

- By considering special multicritical points of CP^{N-1} sigma models we were led to *z=2* electrodynamics in *d=3*.
- Monopole Instantons proliferate the vacuum for any value of the gauge coupling. They generate a mass scale M which is exponentially small compared to the mass scale set by the coupling constant.
- However, unlike the standard *z=1* theory, they are not able to disorder the vacuum – the spectrum of the theory is still gapless.
- Nevertheless, the spinons are confined.

CONFINED CRITICALITY

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CONFINED CRITICALITY

Proviso

- In the context of the original spin models, there were terms which are non-analytic in B.
- These terms vanish faster than any power of B for small B – hence may be thought to be infinitely irrelevant.
- They, however break the shift symmetry of B which is responsible for the presence of a gapless spectrum.
- It is important to investigate if these terms lead to a mass gap.

Outlook

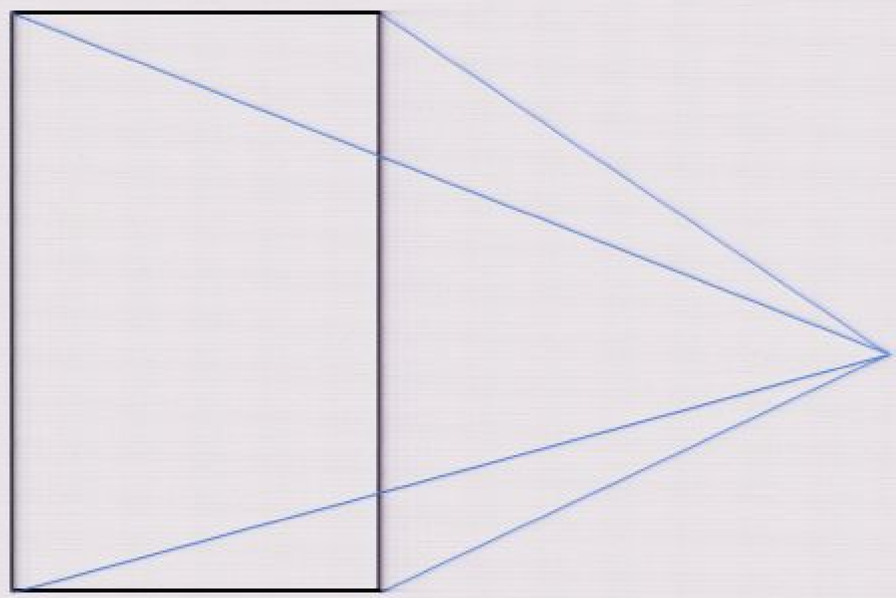
- The most urgent task is to find microscopic models whose parameters can be tuned to this kind of multicritical point.
- The dream is to find real systems which are modelled by such a microscopic model.
- At a more modest theoretical level, it may be important to generalize our work to situations where monopole instantons have different phases on different plaquettes – e.g. models of deconfined criticality.
- Similar emergence of non-abelian gauge fields from gauged sigma models ?

THANK YOU

- The quantity η_C is in fact the potential due to dipole layer

$$\eta_C = \frac{\partial}{\partial x_2} \int dt' d^2x' G_0(t-t', \vec{x}-\vec{x}')\delta(x_2')\Theta_S(t'x_1')$$

- In standard z=1 elctrodynamics, G_0 is the Coulomb propagator and η_C is the solid angle subtended by the loop



- The total correlator becomes

$$< H_0(\omega,\vec{k})H_0(-\omega,-\vec{k}) >_{total} \;=\; \frac{\vec{k}^2+M^2}{\omega^2+M^2\vec{k}^2+\vec{k}^4}$$

$$< H_0(\omega,\vec{k})H_i(-\omega,-\vec{k}) >_{total} \;=\; -\frac{\omega k_i}{\omega^2+M^2\vec{k}^2+\vec{k}^4}$$

$$< H_i(\omega,\vec{k})H_j(-\omega,-\vec{k}) >_{total} \;=\; \delta_{ij}-\frac{k_i k_j \vec{k}^2}{\omega^2+M^2\vec{k}^2+\vec{k}^4}$$

- The original gapless pole has been removed – but a new gapless pole has emerged – corresponding to the spectrum of the sine-Gordon theory.
- This is in contrast to the usual *(z=1)* theory where

$$< H_\mu H_\nu >= \delta_{\mu\nu}-\frac{k_\mu k_\nu}{\vec{k}^2+M^2}$$

- The monopole contributions to the field strength are

$$H_i^M = -ik_i\chi = -\frac{k_i\vec{k}^2}{\omega^2+\vec{k}^4}\rho(\omega,\vec{k})$$

$$H_0^M = i\frac{\omega}{\vec{k}^2}\chi(\omega,\vec{k}) = \frac{\omega}{\omega^2+\vec{k}^4}\rho(\omega,\vec{k})$$

- We need to calculate the correlators of $\rho(\omega,\vec{k})$ in the monopole gas.

Sine-Gordon Representation

- A normal Coulomb gas has a dual representation in terms of a sine-Gordon theory – this is what happens for a monopole gas in standard 2+1 dimensional elctrodynamics.
- In our case, the interaction is not Coulomb – and we get a novel non-relativistic version of sine-Gordon theory.
- The partition function of a monopole gas may be written as

$$e^{-S_\rho} = \int D\phi_1 \; D\phi_2 \; e^{-S[\phi_1,\phi_2]}$$

$$S[\phi_1,\phi_2] = \int d^3x \; \left[i\phi_1 \partial_0 \phi_2 + \frac{1}{2}\{(\nabla\phi_1)^2 + (\nabla\phi_2)^2\} - \frac{i}{g}\rho\phi_1 \right]$$

- Note that ϕ_2 is canonically conjugate to ϕ_1.

The Full Propagator

- This new gapless mode is present in the full propagator of the gauge invariant field strength.
- The total field strength is a sum of the monopole contribution and fluctuations

$$H_\mu = H_\mu^M + h_\mu$$

- Since the theory is quadratic,

$$< H_\mu H_\nu >_{tot} = < H_\mu^M H_\nu^M > + < h_\mu h_\nu >$$

- And the correlator of fluctuations is the same as the perturabtive result.
- So we need to calculate the monopole contribution.

- This leads to the following monopole contributions to the field strength correlators

$$
\begin{aligned}
< H_0(\omega,\vec{k})H_0(-\omega,-\vec{k}) >_{monopole} &= \frac{\omega^2 M^2}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)} \\
< H_0(\omega,\vec{k})H_i(-\omega,-\vec{k}) >_{monopole} &= \frac{M^2\vec{k}^2\omega k_i}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)} \\
< H_i(\omega,\vec{k})H_j(-\omega,-\vec{k}) >_{monopole} &= \frac{M^2(k_i k_j \vec{k}^4)}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)}
\end{aligned}
$$

- Integrate over the gaussian variable ϕ_2 first. The saddle point equation for ϕ_1 is then

$$\Big(-\frac{\partial_0^2}{\nabla^2} + \nabla^2 \Big)\phi_1 = 2\pi \frac{e}{g}\delta'(x_2)\Theta_S + M^2 \sin\phi_1$$

- Where we have used the equation satisfied by the Green's function.
- The source term requires ϕ_1 to be discontinuous as one crosses the surface spanned by the loop at $x_2 = 0$
- If the time extent of the loop, *T* is very large and the space extent *L* is large as well, the term involving ∂_0^2/∇^2 may be ignored – one gets a ODE, whose solution is

$$\phi_1(x_2) = 4sgn(x_2)\tan^{-1}\Big(e^{-M|x_2|}\tan(\frac{\pi e}{4g})\Big)$$

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$$< H_0(\omega,\vec{k})H_0(-\omega,-\vec{k}) >_{monopole} = \frac{\omega^2 M^2}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)}$$

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$$< H_i(\omega,\vec{k})H_j(-\omega,-\vec{k}) >_{monopole} = \frac{M^2(k_i k_j \vec{k}^4)}{(\omega^2+\vec{k}^4)(\omega^2+M^2\vec{k}^2+\vec{k}^4)}$$

- Recall the representation for the action for monopole gas

$$e^{-S_\rho} = \int D\phi_1 \; D\phi_2 \; e^{-S[\phi_1,\phi_2]}$$

$$S[\phi_1,\phi_2] = \int d^3x \; \left[i\phi_1\partial_0\phi_2 + \frac{1}{2}\{(\nabla\phi_1)^2 + (\nabla\phi_2)^2\} - \frac{i}{g}\rho\phi_1 \right]$$

- Thus the generating functional for correlators of $\rho(\omega,\vec{k})$

$$Z[J] = < \exp[i \int d^3x \; J(x)\rho(x)] >_{mgas}$$

may be obtained by following the same steps which led to the sine-Gordon representation - by shifting ϕ_1

$$Z[J] = \int D\phi_1 D\phi_2 \; e^{-S_{SG}(\phi_1 - \frac{2\pi J}{g}, \phi_2)}$$

- $$< \rho(\omega,\vec{k})\rho(-\omega,-\vec{k}) > = \frac{M^2(\omega^2 + \vec{k}^4)}{\omega^2 + \vec{k}^2(\vec{k}^2 + M^2)}$$

- Upon continuation to Lorentzian signature, the hamiltonian density is

$$\mathcal{H} = \frac{4\pi^2}{g^2}(\nabla\Pi_1)^2 + \frac{g^2}{4\pi^2}(\nabla\phi_1)^2 - \frac{g^2 M^2}{2\pi^2}\cos\phi_1$$

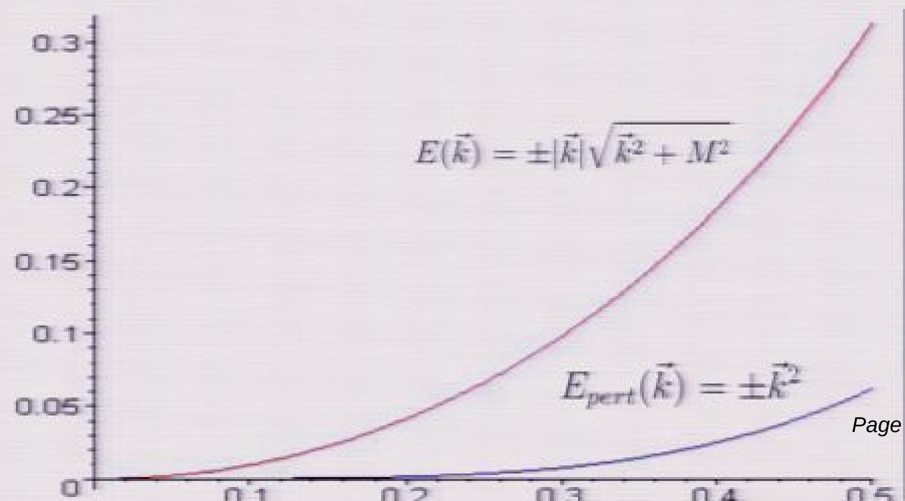
- The spectrum of small fluctuations

$$E(\vec{k}) = \pm|\vec{k}|\sqrt{\vec{k}^2 + M^2}$$

- Recall that the perturbative spectrum is

$$E_{pert}(\vec{k}) = \pm\vec{k}^2$$

- The monopole gas has introduced a mass scale M, and has removed the original gapless mode. However, a new gapless mode has taken its place.



- The dominant contribution to the partition function is due to minimally charged monopoles and anti-monopoles

$$Z_g = \sum_{N_\pm} \frac{\zeta^{N_+ + N_-}}{N_+! N_-!} \int \prod_{a=1}^{N_+} d^3x_a \int \prod_{b=1}^{N_-} d^3x_b \; e^{-\frac{4\pi^2}{g^2}\sum_{ij} n_i n_j G_{ij}}$$

where $n_i = \pm 1$ and ζ is the fugacity of monopoles,

$$\zeta \sim g^4 \; e^{-\frac{1}{g^2 a}}$$

a being a UV cutoff. This may be now written as

$$Z_g = \int D\phi_1 D\phi_2 \; e^{-S_{SG}(\phi_1,\phi_2)}$$

$$\mathcal{L}_{SG}(\phi_1,\phi_2) = \frac{g^2}{4\pi^2}\left[i\phi_1 \partial_t \phi_2 + \frac{1}{2}(\nabla\phi_1)^2 + \frac{1}{2}(\nabla\phi_2)^2 - M^2 \cos\phi_1 \right]$$

where

$$M^2 = \frac{8\pi^2\zeta}{g^2}$$

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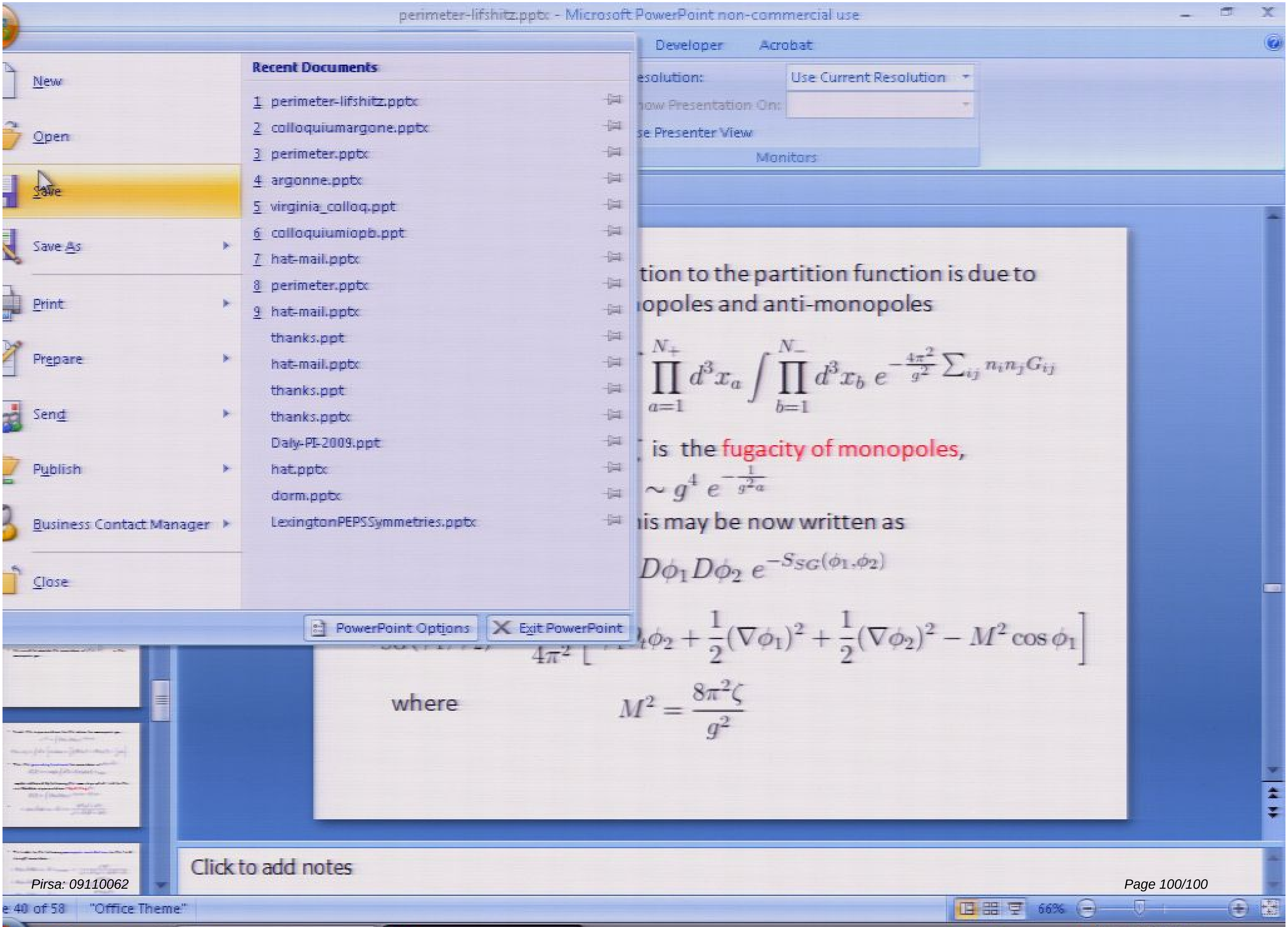
Click to add notes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