Title: Cosmological implications of gravity at a Lifshitz point

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Abstract:

INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Cosmological implications of gravity at a Lifshitz point

arXiv:0904.2190 [hep-th]
* arXiv:0905.0055 [hep-th]
arXiv:0905.3563 [hep-th]
arXiv:0906.5069 [hep-th]
+ arXiv:0909.2149 [astro-ph.CO]
++ arXiv:0911.xxxx [hep-th]

Shinji Mukohyama (IPMU, U of Tokyo)

* w/ K.Nakayama, F.Takahashi and S.Yokoyama
+ w/ S.Maeda and T.Shiromizu
++ w/ K.Izumi

Contents of this talk

- Generation of scale-invariant cosmological perturbations
- “Dark matter” without dark matter
- Comments on scalar graviton
- Black holes and stars

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

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- Scaling dim of ϕ

  $t \to b\,t$ ($E \to b^{-1}E$)

  $x \to b\,x$

  $\phi \to b^s\,\phi$

  $1+3-2+2s = 0$

  $s = -1$

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- Renormalizability

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- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

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- $s = 0$ if $z = 3$

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- Gravity becomes renormalizable!?

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  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

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  ⇒ $\delta\phi \propto E \sim H$

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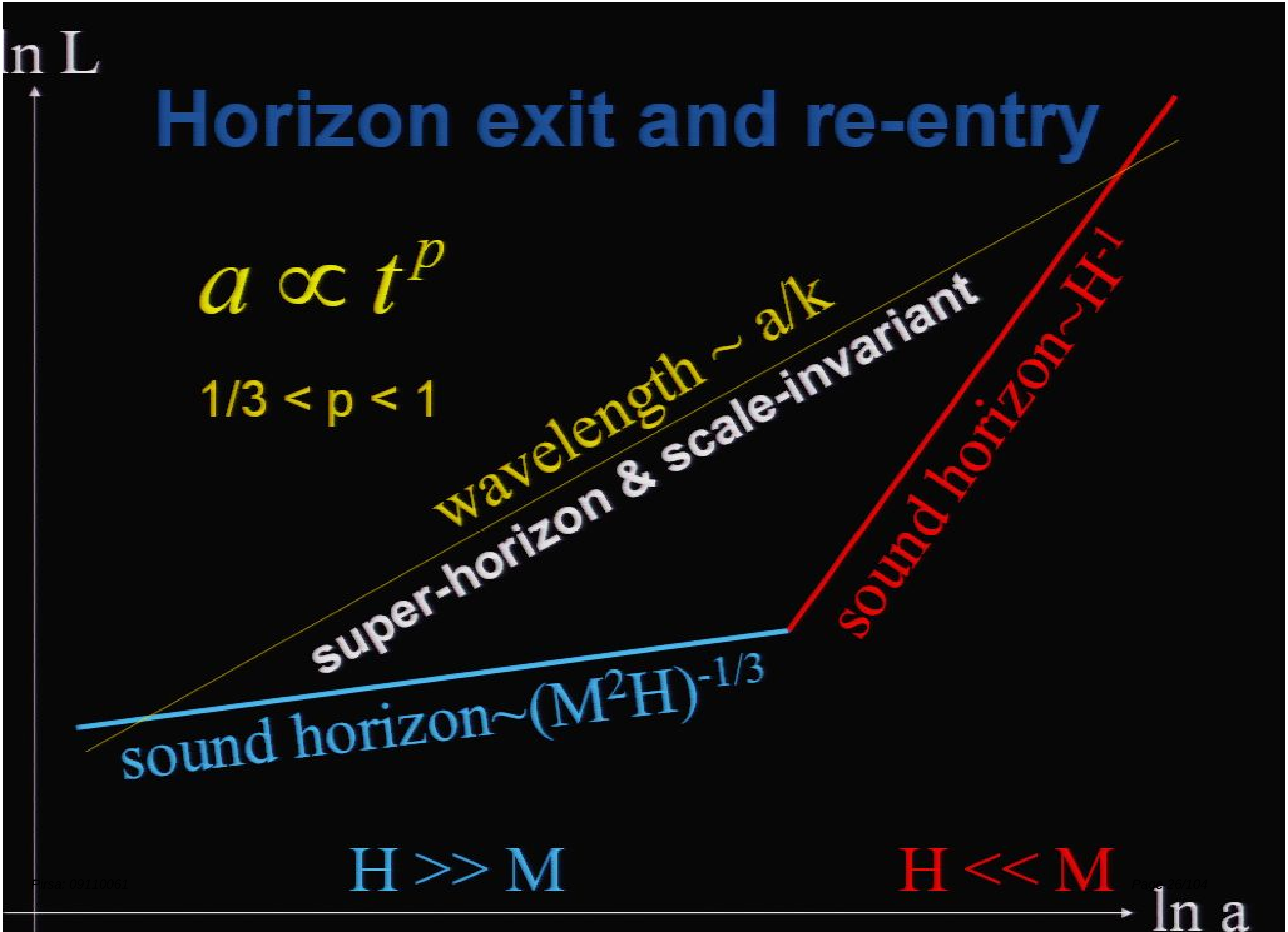
  Scale-invariant fluctuations!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ln L
$a \propto t^p$
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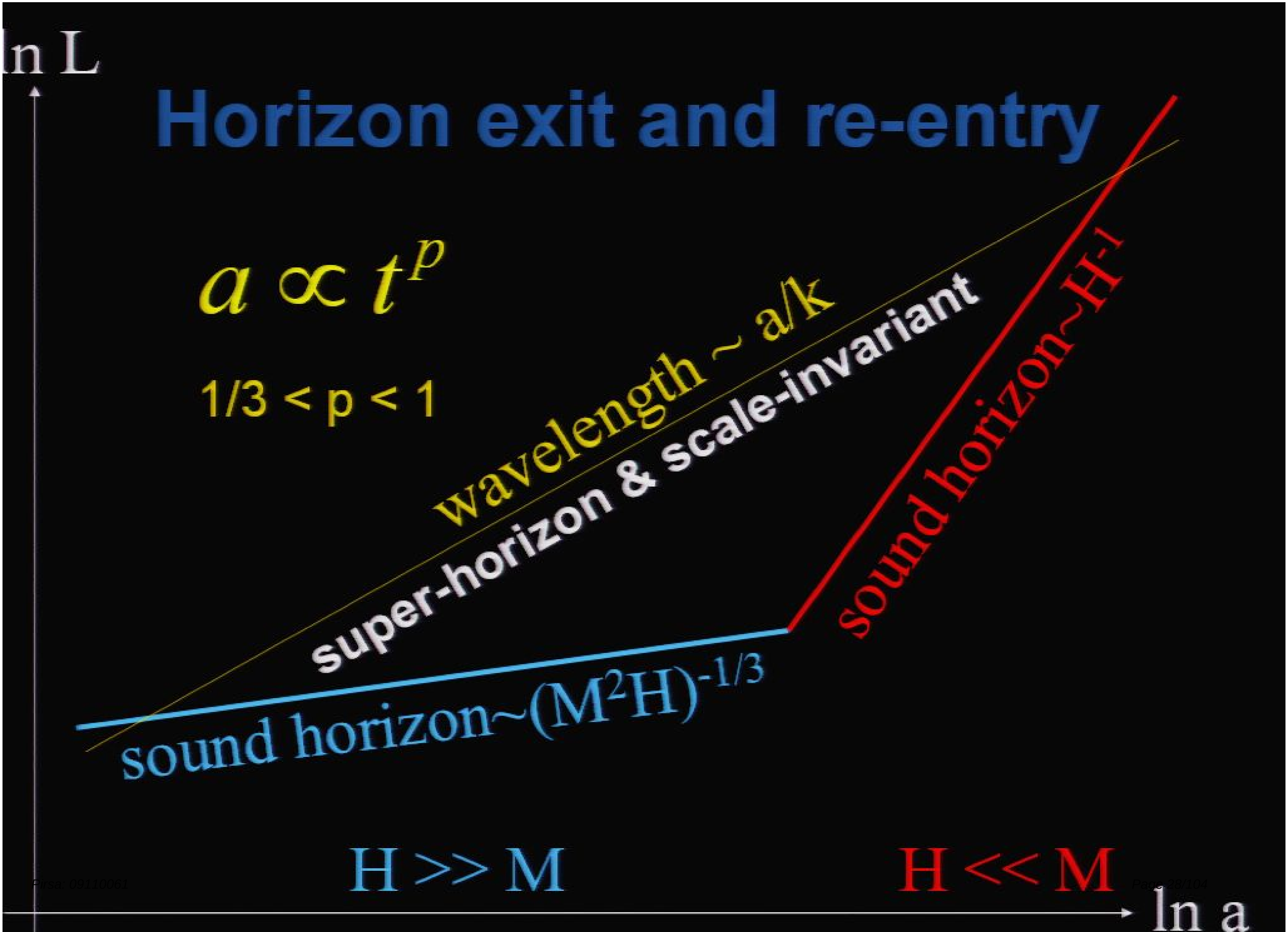
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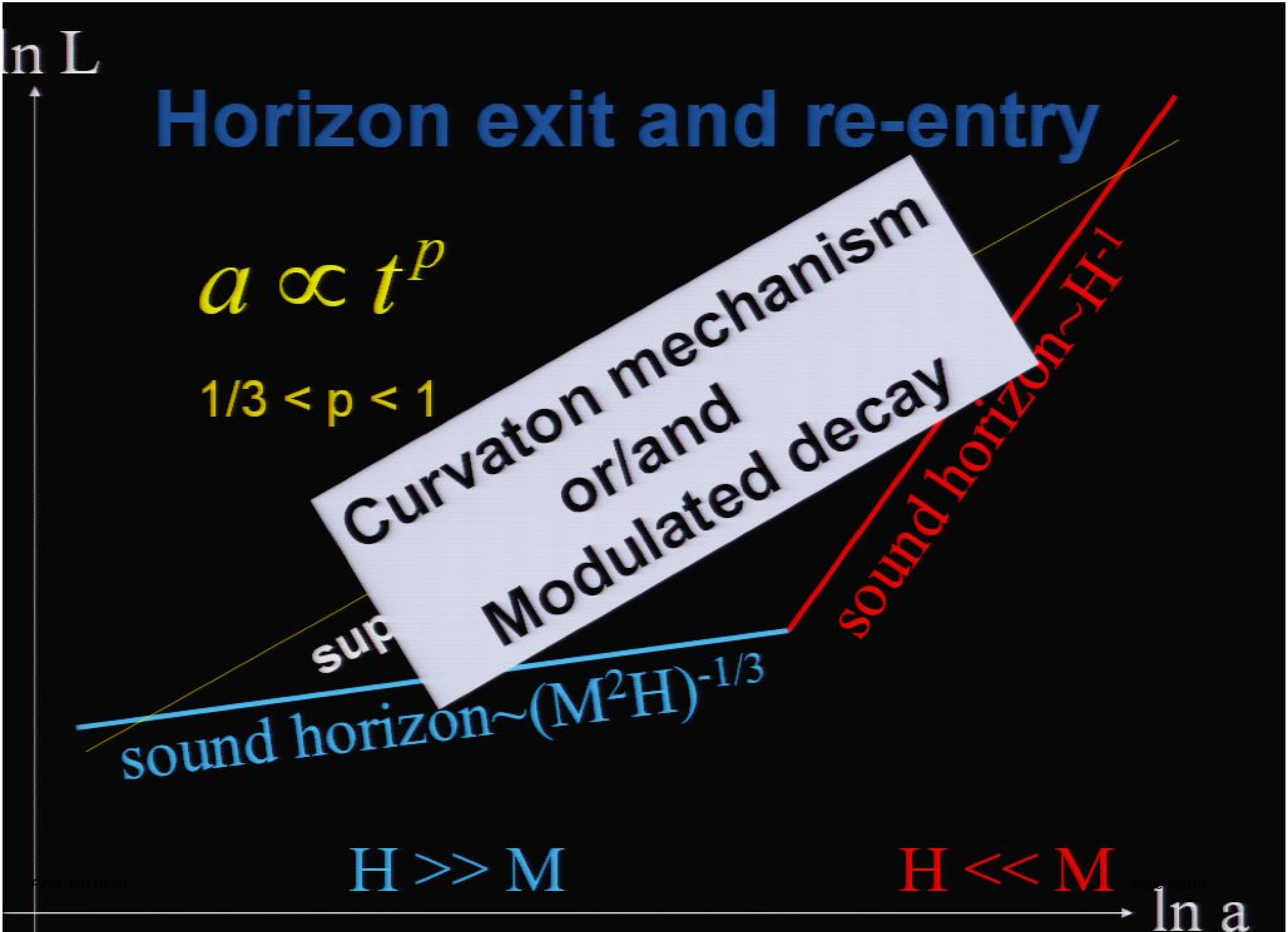
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See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of GR

- 4D diffeomorphism →
  4 constraints = 1 Hamiltonian + 3 momentum
  @ each time @ each point
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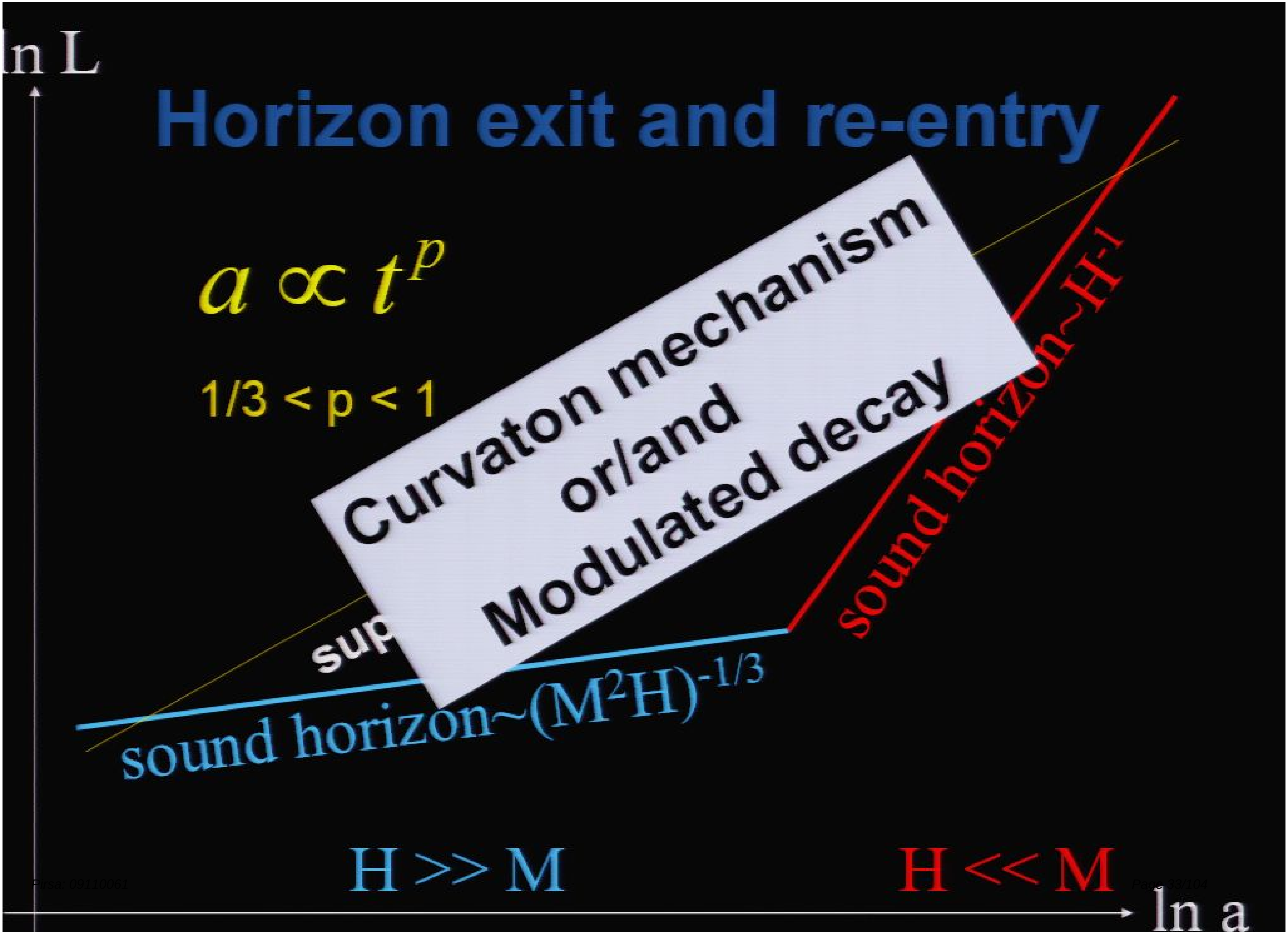
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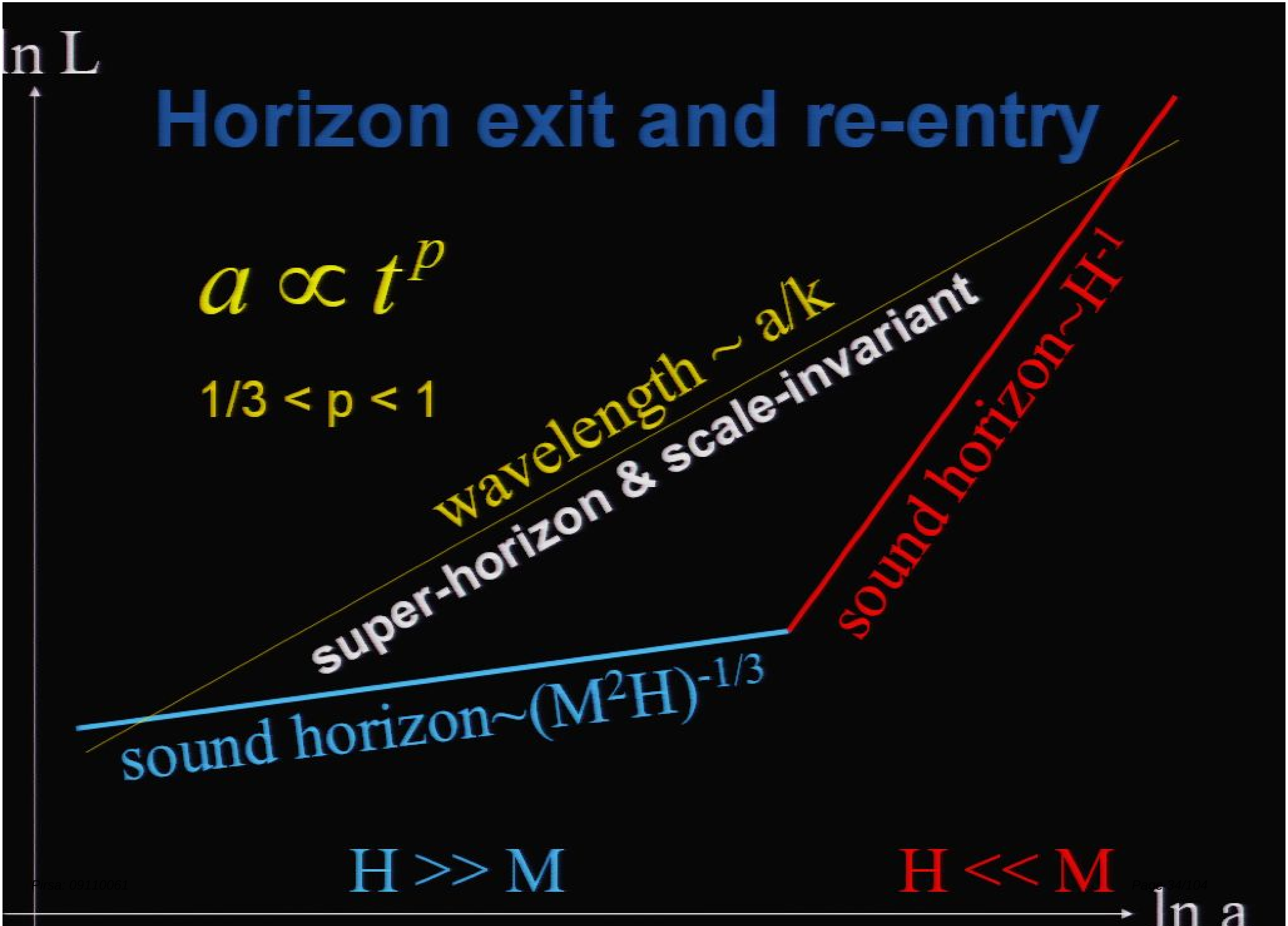
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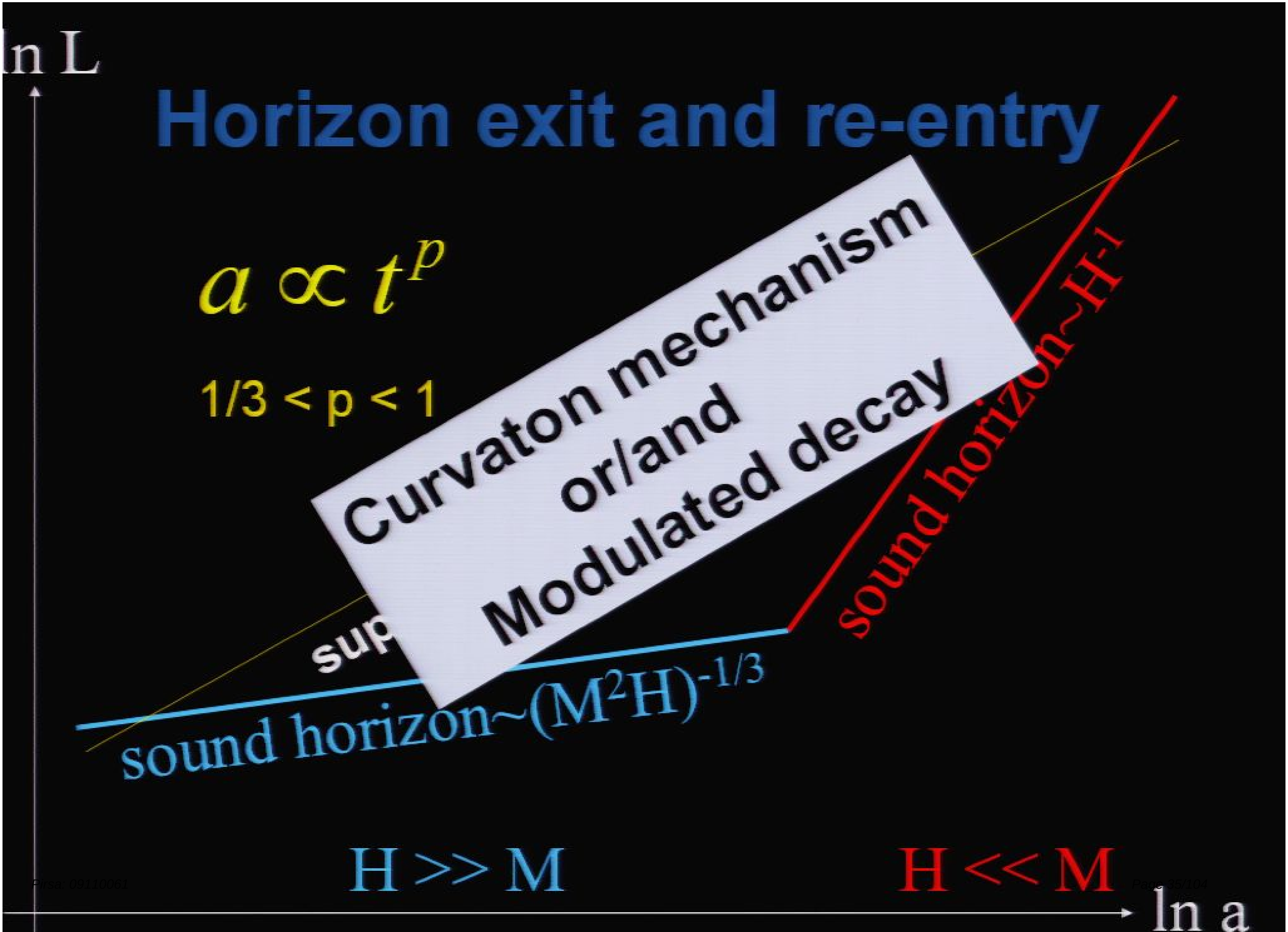

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FRW spacetime in GR

- $ds^2 = - dt^2 + a^2(t)\,(dx^2 + dy^2 + dz^2)$
- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- Hamiltonian constraint
  → Friedmann eq

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} \rho_i$$

  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  is not independent
  but follows from the above n+1 eqs.

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
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$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g} d^3x \left(K_{ij}K^{ij} - \lambda K^2 + R - 2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
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$$\int d^3x \sqrt{g}(G_{\mu\nu}^{(4)} + \Lambda g_{\mu\nu}^{(4)} - 8\pi G_N T_{\mu\nu}) n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N}(\partial_t - N^i \partial_i)$$

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Dark matter as integration constant

- Def. $T^{HL}{}_{\mu\nu}$ $\quad G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad\qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

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- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

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$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

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$$+ \text{(higher curvature corrections)}$$
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FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No "local" Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq can be integrated to give Friedmann eq with "dark matter as integration constant"

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

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- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p > 1/3$.
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Contents of the rest of this talk

- Comments on scalar graviton
- Black holes and stars

Propagating d.o.f.

- Minkowski + perturbation
  $N = 1,\ N^i = 0,\ g_{ij} = \delta_{ij} + h_{ij}$
- Residual guage freedom =
  time-independent spatial diffeo.
- Momentum constraint
  $\partial_t \partial_i H_{ij} = 0$ $\qquad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$
- Fix the residual guage freedom by setting
  $\partial_i H_{ij} = 0$ at some fixed time surface.
- Decompose H_{ij} into trace and traceless parts
  TT part : 2 d.o.f. (usual tensor graviton)
  Trace part : 1 d.o.f. (scalar graviton)

Scalar graviton and λ → 1

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H \delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit λ → 1, the scalar graviton H becomes pure gauge. So, it decouples.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[(\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and H gets strongly self-coupled.

- This is not a problem in renormalizable theories if there is "Vainshtein effect", i.e. decoupling of the strongly-coupled sector from the rest of the world.   c.f. QCD+QED

Linear instability of scalar graviton

Appendix C of arXiv:0911.xxxx with K.Izumi

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = k^2 \times [c_s^2 + O(k^2/M^2)]$ with $c_s^2 = -(\lambda-1)/(3\lambda-1) < 0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability of "DM as integration const" if
  $t_J \sim (G_N\rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/(|c_s|M)$.
- Thus, the linear instability does not show up if
  $|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL, 1/(ML)]$. $(\Phi \sim -G_N\rho L^2)$
  $L > 0.01\mathrm{mm}$ (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

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- As a first step, let us consider stellar solutions.

Stellar center is dynamical in Horava-Lifshitz gravity

arXiv:0911.xxxx [hep-th]
with K.Izumi

Basic setup

Painlevé-Gullstrand coordinate

$$N = 1 \qquad N^i \partial_i = \beta(x)\partial_x$$

$$g_{ij}dx^i dx^j = dx^2 + r^2(x)d\Omega_2^2.$$

Matter sector

$$T_{\mu\nu} = \rho(x)u_\mu u_\nu + P(x)\left[g^{(4)}_{\mu\nu} + u_\mu u_\nu\right]$$

$$u^\mu = \frac{\xi^\mu}{\sqrt{1-\beta^2}} \qquad \xi^\mu = \left(\frac{\partial}{\partial t}\right)^\mu$$

- The energy density ρ is a piecewise-continuous non-negative function of the pressure P.
- The central pressure P_c is positive.

No static star solution

- Momentum conservation equation

  $$P'(1-\beta^2)+(\rho+P)(1-\beta^2)'=0$$

- Global-staticity → $1-\beta^2 > 0$ everywhere.
- Regularity of K^x_x → β' is finite → P' is also finite → $\beta(x)$ and $P(x)$ are continuous → $\rho(x)+P(x)$ is piecewise-continuous.
- $P_c>0$ & P continuous & ρ non-negative → $\rho+P>0$ in a neighborhood of the center.
- Define x_0 as the minimal value for which at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive.

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$$\ln(1-\beta_0^2) - \ln(1-\beta_c^2) = -\int_{P_c}^{P_0} \frac{dP}{\rho(P)+P}$$

- L.h.s. is non-positive ← β_c=0 & r_c'=1 ← regularity of R & $K^\theta{}_\theta$
- R.h.s. is positive ← P_0 is non-positive ← ρ is non-negative & at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive & P(x) is continuous
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- L.h.s. is non-positive ← β_c=0 & r_c'=1 ← regularity of R & $K^\theta{}_\theta$
- R.h.s. is positive ← P_0 is non-positive ← ρ is non-negative & at least one of $(\rho + P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive & P(x) is continuous
- Contradiction! → no spherically-symmetric globally-static solutions → stellar center is dynamical
- The proof is insensitive to the structure of higher-derivative terms → valid for any z

No static star solution

- Momentum conservation equation

$$P'(1-\beta^2)+(\rho+P)(1-\beta^2)'=0$$

- Global-staticity → $1-\beta^2 > 0$ everywhere.
- Regularity of K^x_x → β' is finite → P' is also finite → $\beta(x)$ and $P(x)$ are continuous → $\rho(x)+P(x)$ is piecewise-continuous.
- $P_c>0$ & P continuous & ρ non-negative → $\rho+P>0$ in a neighborhood of the center.
- Define x_0 as the minimal value for which at least one of $(\rho+P)|_{x=x_0}$, $\lim_{x\to x_0-0}(\rho+P)$ and $\lim_{x\to x_0+0}(\rho+P)$ is non-positive.

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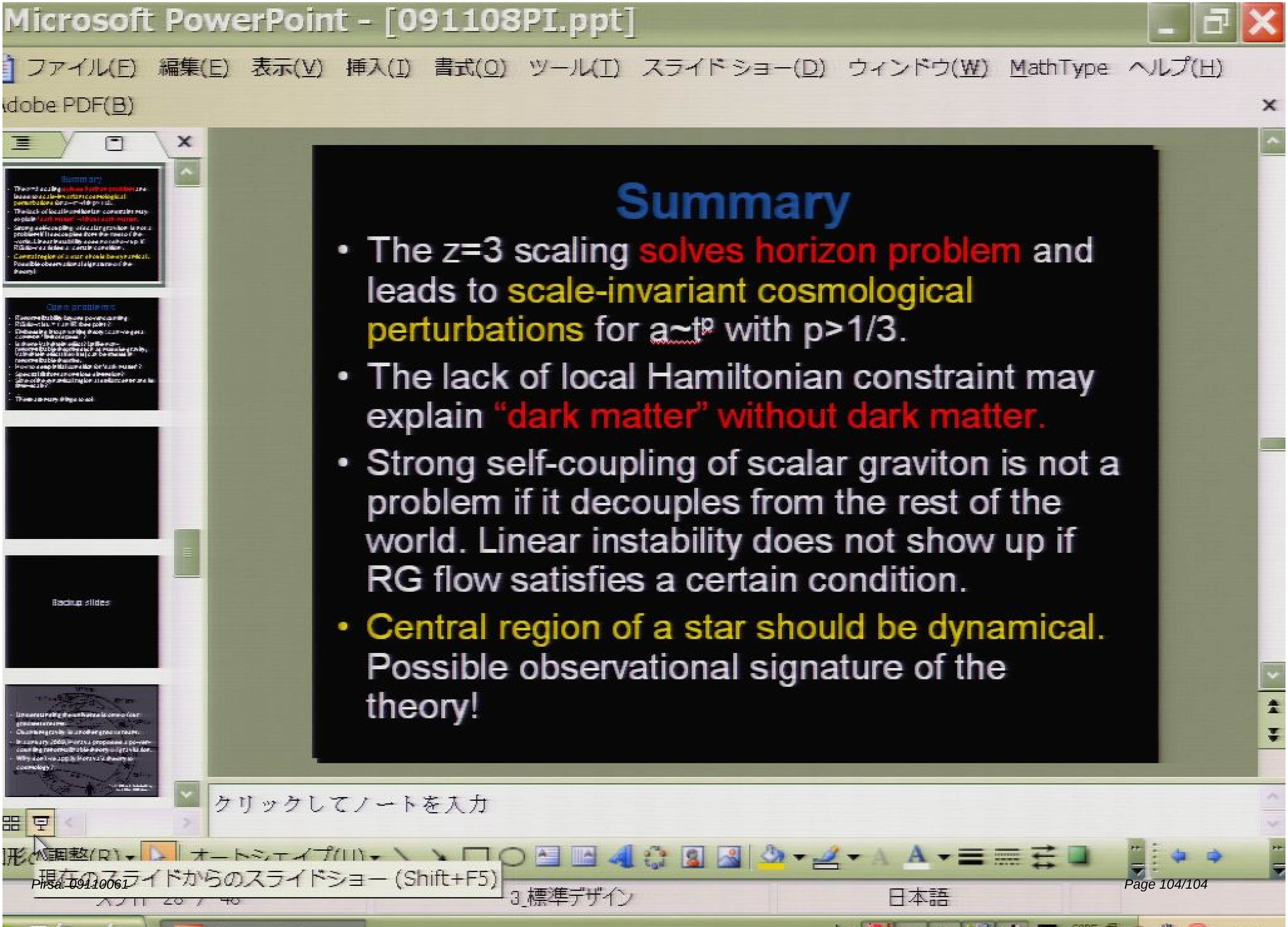
Summary

- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- The lack of local Hamiltonian constraint may explain "dark matter" without dark matter.
- Strong self-coupling of scalar graviton is not a problem if it decouples from the rest of the world. Linear instability does not show up if RG flow satisfies a certain condition.
- Central region of a star should be dynamical. Possible observational signature of the theory!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