

Title: Cosmological implications of gravity at a Lifshitz point

Date: Nov 09, 2009 04:30 PM

URL: <http://pirsa.org/09110061>

Abstract:



# Cosmological implications of gravity at a Lifshitz point

- arXiv:0904.2190 [hep-th]
- \* arXiv:0905.0055 [hep-th]
- arXiv:0905.3563 [hep-th]
- arXiv:0906.5069 [hep-th]
- + arXiv:0909.2149 [astro-ph.CO]
- ++ arXiv:0911.xxxx [hep-th]

**Shinji Mukohyama (IPMU, U of Tokyo)**

- \* w/ K.Nakayama, F.Takahashi and S.Yokoyama
- + w/ S.Maeda and T.Shiromizu
- ++ w/ K.Izumi



# Contents of this talk

- Generation of scale-invariant cosmological perturbations
- “Dark matter” without dark matter
- Comments on scalar graviton
- Black holes and stars



# Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-ph.CO] with S. Maeda and T. Shiromizu.



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 $1+3-2+2s = 0$   
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- Renormalizability  
 $n \leq 4$



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- Renormalizability

$$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable



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- Gravity becomes renormalizable!?



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- $\omega^2 \gg H^2$  : oscillate

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oscillation  $\rightarrow$  freeze-out iff  $d(H^2/\omega^2)/t > 0$

$\omega^2 = k^2/a^2$  leads to  $d^2a/dt^2 > 0$

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Scale-invariant fluctuations!



$\ln L$

# Horizon exit and re-entry

$$a \propto t^p$$

$$1/3 < p < 1$$

wavelength  $\sim a/k$

super-horizon & scale-invariant

sound horizon  $\sim H^{-1}$

sound horizon  $\sim (M^2 H)^{-1/3}$

$H \gg M$

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# Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]

Caustic avoidance in Horava-Lifshitz gravity



# Structure of GR

- 4D diffeomorphism  $\rightarrow$   
4 constraints = 1 Hamiltonian + 3 momentum  
@ each time @ each point
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# FRW spacetime in GR

- $ds^2 = - dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$
- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- Hamiltonian constraint  
→ Friedmann eq  
E.o.m. of matter  
→ conservation eq.  
$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^n \rho_i$$
$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$
- Dynamical eq  
is not independent  
but follows from the above  $n+1$  eqs.  
$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^n P_i$$



# Structure of HL gravity

- Foliation-preserving diffeomorphism  
= 3D spatial diffeomorphism  
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Friedmann eq with  
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$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left( \sum_{i=1}^n \rho_i + \frac{C}{a^3} \right)$$



# IR limit of HL gravity

$$\frac{1}{16\pi G_N} \int N dt \sqrt{g} d^3x \left( K_{ij} K^{ij} - \lambda K^2 + R - 2\Lambda \right)$$

- Looks like GR iff  $\lambda = 1$ . So, we assume that  $\lambda = 1$  is an IR fixed point of RG flow.
- **Global Hamiltonian constraint**

$$\int d^3x \sqrt{g} (G_{\mu\nu}^{(4)} + \Lambda g_{\mu\nu}^{(4)} - 8\pi G_N T_{\mu\nu}) n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N} (\partial_t - N^i \partial_i)$$

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# Dark matter as integration constant

- Def.  $T_{\mu\nu}^{HL}$   $G_{\mu\nu}^{(4)} + \Lambda g_{\mu\nu}^{(4)} = 8\pi G_N (T_{\mu\nu} + T_{\mu\nu}^{HL})$
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$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

$\rho^{HL}$  can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity  $\rightarrow$  (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$



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- General solution to the momentum constraint and dynamical eq.

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$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

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$$(G^{(4)}_{i\mu} + \Lambda g^{(4)}_{i\mu} - 8\pi G_N T_{i\mu}) n^\mu = 0$$

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$$\frac{1}{16\pi G_N} \int N dt \sqrt{g} d^3x \left( K_{ij} K^{ij} - \lambda K^2 + R - 2\Lambda \right)$$

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# FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.

- No “local” Hamiltonian constraint

E.o.m. of matter

→ conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq  
can be integrated to give

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^n P_i$$

Friedmann eq with  
“dark matter as  
integration constant”

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left( \sum_{i=1}^n \rho_i + \frac{C}{a^3} \right)$$



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# Summary so far

- The  $z=3$  scaling solves horizon problem and leads to scale-invariant cosmological perturbations for  $a \sim t^p$  with  $p > 1/3$ .
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- The lack of local Hamiltonian constraint may explain “dark matter” without dark matter.

## Contents of the rest of this talk

- Comments on scalar graviton
- Black holes and stars



# Propagating d.o.f.

- Minkowski + perturbation

$$N = 1, N^i = 0, g_{ij} = \delta_{ij} + h_{ij}$$

- Residual gauge freedom =  
time-independent spatial diffeo.

- Momentum constraint

$$\partial_t \partial_i H_{ij} = 0 \quad H_{ij} \equiv h_{ij} - \lambda h \delta_{ij}$$

- Fix the residual gauge freedom by setting

$$\partial_i H_{ij} = 0 \quad \text{at some fixed time surface.}$$

- Decompose  $H_{ij}$  into trace and traceless parts

TT part : 2 d.o.f. (usual tensor graviton)

Trace part : 1 d.o.f. (scalar graviton)



# Scalar graviton and $\lambda \rightarrow 1$

$$h_{ij} = \tilde{H}_{ij} + \frac{1-\lambda}{2(1-3\lambda)} H \delta_{ij} - \frac{\partial_i \partial_j}{2\partial^2} H$$

- In the limit  $\lambda \rightarrow 1$ , the scalar graviton  $H$  becomes pure gauge. So, it **decouples**.
- However, its kinetic term will vanish

$$I_{kin} \sim \int dt d^3x \left[ (\partial_t \tilde{H}_{ij})^2 + \frac{\lambda - 1}{2(3\lambda - 1)} (\partial_t H)^2 \right]$$

and  $H$  gets **strongly self-coupled**.

- This is not a problem in renormalizable theories if there is “Vainshtein effect”, i.e. decoupling of the strongly-coupled sector from the rest of the world. c.f. QCD+QED



# Linear instability of scalar graviton

Appendix C of arXiv:0911.1xxxx with K.Izumi

- Sign of (time) kinetic term  $(\lambda-1)/(3\lambda-1) > 0$ .
- The dispersion relation in flat background  
 $\omega^2 = k^2 \times [c_s^2 + O(k^2/M^2)]$  with  $c_s^2 = -(\lambda-1)/(3\lambda-1) < 0$   
→ **IR instability in linear level**  
(Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability of “DM as integration const” if  
 $t_J \sim (G_N \rho)^{-1/2} < t_L \sim L/|c_s|$ .
- Tamed by Hubble friction or/and  $O(k^2/M^2)$  terms if  
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- Thus, the linear instability **does not show up if**  
 **$|c_s| < \text{Max}[|\Phi|^{1/2}, HL, 1/(ML)]$** . ( $\Phi \sim -G_N \rho L^2$ )  
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- **Phenomenological constraint on properties of RG flow.**



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# Black holes and stars



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- **As a first step, let us consider stellar solutions.**



# Stellar center is dynamical in Horava-Lifshitz gravity

arXiv:0911.xxxx [hep-th]  
with K.Izumi



# Basic setup

## Painlevé-Gullstrand coordinate

$$N = 1 \quad N^i \partial_i = \beta(x) \partial_x$$

$$g_{ij} dx^i dx^j = dx^2 + r^2(x) d\Omega_2^2.$$

## Matter sector

$$T_{\mu\nu} = \rho(x) u_\mu u_\nu + P(x) \left[ g_{\mu\nu}^{(4)} + u_\mu u_\nu \right]$$

$$u^\mu = \frac{\xi^\mu}{\sqrt{1 - \beta^2}} \quad \xi^\mu = \left( \frac{\partial}{\partial t} \right)^\mu$$

- The energy density  $\rho$  is a piecewise-continuous non-negative function of the pressure  $P$ .
- The central pressure  $P_c$  is positive.



## No static star solution

- Momentum conservation equation
 
$$P'(1 - \beta^2) + (\rho + P)(1 - \beta^2)' = 0$$
- Global-staticity  $\rightarrow 1 - \beta^2 > 0$  everywhere.
- Regularity of  $K^x_x \rightarrow \beta'$  is finite  $\rightarrow P'$  is also finite  $\rightarrow \beta(x)$  and  $P(x)$  are continuous  $\rightarrow \rho(x) + P(x)$  is piecewise-continuous.
- $P_c > 0$  &  $P$  continuous &  $\rho$  non-negative  $\rightarrow \rho + P > 0$  in a neighborhood of the center.
- Define  $x_0$  as the minimal value for which at least one of  $(\rho + P)|_{x=x_0}$ ,  $\lim_{x \rightarrow x_0 - 0}(\rho + P)$  and  $\lim_{x \rightarrow x_0 + 0}(\rho + P)$  is non-positive.



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$$\ln(1 - \beta_0^2) - \ln(1 - \beta_c^2) = - \int_{P_c}^{P_0} \frac{dP}{\rho(P) + P}$$

- L.h.s. is non-positive  $\leftarrow \beta_c=0$  &  $r_c'=1$   $\leftarrow$  regularity of  $R$  &  $K^\theta_\theta$
- R.h.s. is positive  $\leftarrow P_0$  is non-positive  $\leftarrow \rho$  is non-negative & at least one of  $(\rho + P)|_{x=x_0}$ ,  $\lim_{x \rightarrow x_0-0}(\rho + P)$  and  $\lim_{x \rightarrow x_0+0}(\rho + P)$  is non-positive &  $P(x)$  is continuous
- Contradiction!  $\rightarrow$  **no spherically-symmetric globally-static solutions**  $\rightarrow$  stellar center is dynamical
- The proof is insensitive to the structure of **higher-derivative terms**  $\rightarrow$  valid for any  $z$



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- Regularity of  $K^x_x \rightarrow \beta'$  is finite  $\rightarrow P'$  is also finite  $\rightarrow \beta(x)$  and  $P(x)$  are continuous  $\rightarrow \rho(x) + P(x)$  is piecewise-continuous.
- $P_c > 0$  &  $P$  continuous &  $\rho$  non-negative  $\rightarrow \rho + P > 0$  in a neighborhood of the center.
- Define  $x_0$  as the minimal value for which at least one of  $(\rho + P)|_{x=x_0}$ ,  $\lim_{x \rightarrow x_0 - 0}(\rho + P)$  and  $\lim_{x \rightarrow x_0 + 0}(\rho + P)$  is non-positive.



$$\ln(1 - \beta_0^2) - \ln(1 - \beta_c^2) = - \int_{P_c}^{P_0} \frac{dP}{\rho(P) + P}$$

- L.h.s. is non-positive  $\leftarrow \beta_c=0$  &  $r_c'=1$   $\leftarrow$  regularity of  $R$  &  $K_{\theta}^{\theta}$
- R.h.s. is positive  $\leftarrow P_0$  is non-positive  $\leftarrow \rho$  is non-negative & at least one of  $(\rho + P)|_{x=x_0}$ ,  $\lim_{x \rightarrow x_0-0}(\rho + P)$  and  $\lim_{x \rightarrow x_0+0}(\rho + P)$  is non-positive &  $P(x)$  is continuous
- Contradiction!  $\rightarrow$  **no spherically-symmetric globally-static solutions**  $\rightarrow$  stellar center is dynamical
- The proof is insensitive to the structure of **higher-derivative terms**  $\rightarrow$  valid for any  $z$



# No static star solution

- Momentum conservation equation  

$$P'(1 - \beta^2) + (\rho + P)(1 - \beta^2)' = 0$$
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# Summary

- The  $z=3$  scaling solves horizon problem and leads to scale-invariant cosmological perturbations for  $a \sim t^p$  with  $p > 1/3$ .
- The lack of local Hamiltonian constraint may explain “dark matter” without dark matter.
- Strong self-coupling of scalar graviton is not a problem if it decouples from the rest of the world. Linear instability does not show up if RG flow satisfies a certain condition.
- Central region of a star should be dynamical. Possible observational signature of the theory!



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- Central region of a star should be dynamical. Possible observational signature of the theory!

Open problems

- Can we solve the horizon problem?
- Can we solve the flatness problem?
- Can we solve the  $z=3$  scaling problem?
- Can we solve the  $a \sim t^p$  problem?
- Can we solve the  $p > 1/3$  problem?
- Can we solve the  $p < 1/3$  problem?
- Can we solve the  $p = 1/3$  problem?
- Can we solve the  $p = 0$  problem?
- Can we solve the  $p = -1$  problem?
- Can we solve the  $p = -2$  problem?
- Can we solve the  $p = -3$  problem?
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Backup slides

Can we solve the  $p = -101$  problem?

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クリックしてノートを入力