Title: Horava-Lifshitz gravity: What's the matter?

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Abstract:

Outline

1. The idea
2. Hořava model ($z = 3$)
3. Minimally coupled theory

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4. Nonminimal coupling
5. Conclusions and open issues
6. Shadows on a wall

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Example: Lifshitz scalar (1941)

$$S_{\mathrm{Lifshitz}} = \frac{1}{2}\int \mathrm{d}t\mathrm{d}^D x \left[\dot{\phi}^2 - \frac{1}{4}(\Delta\phi)^2\right]$$

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It defines an *anisotropic scaling* between time and space:

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$$t \to b^z t\,, \qquad \mathbf{x} \to b\mathbf{x}\,, \qquad [t] = -z, \quad [x^i] = -1, \quad [\phi] = \frac{D-z}{2}$$

- The critical exponent z determines the dim. D at which the propagator becomes logarithmic, critical behaviour of correlation functions near a phase transition.

UV completion

Hořava, arXiv:0812.4287, arXiv:0901.3775

- The meeting point of phase boundaries in multicritical phenomena is called Lifshitz point.

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- 3 d.o.f.: extra scalar h, trace of the graviton.

Projectability and scalar mode

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- For simplicity we consider the projectable version but all arguments below hold also in the non-projectable case.

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Alternative: Minimal coupling prescription, scalar and gravity sectors factorize:

$$\pi_{ij} = \frac{\delta W_g}{\delta g^{ij}}, \qquad \pi_\phi = \frac{\delta W_\phi}{\delta \phi}$$

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$\Rightarrow$ It may be desirable to abandon it.

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Relevant deformations push the system towards the IR f.p.:

$$S_g \sim \frac{2}{\kappa^2}\int \mathrm{d}t\mathrm{d}^3x\,\sqrt{g}N\left[K_{ij}K^{ij} - \lambda K^2 + c^2(R - 3\Lambda_W)\right]$$

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$$c \equiv \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G \equiv \frac{\kappa^2}{32\pi c}$$

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$3D$ action:

$$W_\phi = \frac{1}{2}\int \mathrm{d}^3x\sqrt{g}\left[-\sigma_3\phi\Delta^{3/2}\phi - \sigma_2\phi\Delta\phi + m\phi^2\right].$$

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Pseudo-differential operators $\Delta^\alpha = [g_{ij}(t,\mathbf{x})\nabla^i\nabla^j]^\alpha$, $\alpha \in \mathbb{C}$, studied since the late 60's [Seeley 1967; Hörmander 1968].

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- Difficult to represent fractional operators and define functional calculus. Initialized calculus [Lorenzo and Hartley 2000–2008].

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- $\partial^\alpha \mathrm{const} \neq 0$; $\Delta^{\alpha+\beta} \neq \Delta^\alpha \Delta^\beta$ unless α or β natural; solutions of the fractional wave equation do not solve ordinary wave equation, continuum spectrum of massive modes [Barci et al. 1998].

Fractional functional calculus

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- Integration by parts $\Leftrightarrow$ self-adjoint definition of Δ^α:

$$\int \mathrm{d}^3x\,\sqrt{g}\,A\Delta^\alpha B = \int \mathrm{d}^3x\,\sqrt{g}\,(\Delta^\alpha A)B + \dots$$

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Minimally coupled theory – Cosmology

Bounce

Minimally coupled theory – Scalar sector

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Possibility of a bounce at

$$\rho_* = \frac{c|\Lambda|}{8\pi G} + \frac{3c\mathsf{K}}{8\pi G a_*^2}\left(1 + \frac{B^2}{a_*^2 c^2\tilde{\mathsf{K}}}\right)$$

Minimally coupled theory – Cosmology

Tensor linear perturbations

Minimally coupled theory – Cosmology

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$$\delta^{(2)} S_g = -\frac{1}{2\kappa^2}\int \mathrm{d}\tau\,\mathrm{d}^3x\, a^2 \left[h^{ij}h''_{ij} - \left(\frac{\kappa^2}{2\nu^2}\right)^2 a^2 h_{ij}\Delta^3 h^{ij}\right]$$

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Corley–Jacobson dispersion relation as in trans-Planckian cosmology [Brandenberger and Martin 2000–2003].

Scale-invariant tensor spectrum ($p \lesssim -1$):

$$k^3 P_T = k^3 \frac{|v_k|^2}{a^2} = k^{2(1+p)}, \quad n_T \equiv \frac{\mathrm{d}\ln(k^3 P_T)}{\mathrm{d}\ln k} = 2(1+p).$$

Minimally coupled theory – Cosmology

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Perturbed KG equation for a test scalar field $u_k = a\delta\phi_k$:

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$$k^3 P_S = k^{2(1+p)} \mathrm{e}^{Ak^z} \cos^2\left[2\pi|p| - \frac{\pi}{4} - C^{\frac{1}{p(z-1)}} k^{1+\frac{1}{p}}\right],$$

where $A \gg 1$ for wavenumbers $k \sim 2\pi$.

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$\Rightarrow$ Abandoning detailed balance, the sign gets fixed.

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$$W = \frac{1}{\nu^2}\int \omega_3(\Gamma) + \int d^3x\sqrt{g}\left[\mu R + s_0 g^{ij}\phi\Delta^{1/2}R_{ij} - 2L(\phi)\right]$$

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Two future directions

Lorentz violation

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- Issue recently confirmed by Iengo et al. 2009 for Lifshitz scalar theories.

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- Integrals on net fractals (e.g., self-similar or cookie-cutter sets) can be approximated by fractional integrals [Ren et al. 2003], natural to consider fractional integrals over a space with fractional dimension.
- Stieltjes actions admit a neat geometrical and maybe physical interpretation [Bullock 1988, Podlubny 2001].

Left-sided Riemann–Liouville fractional integral

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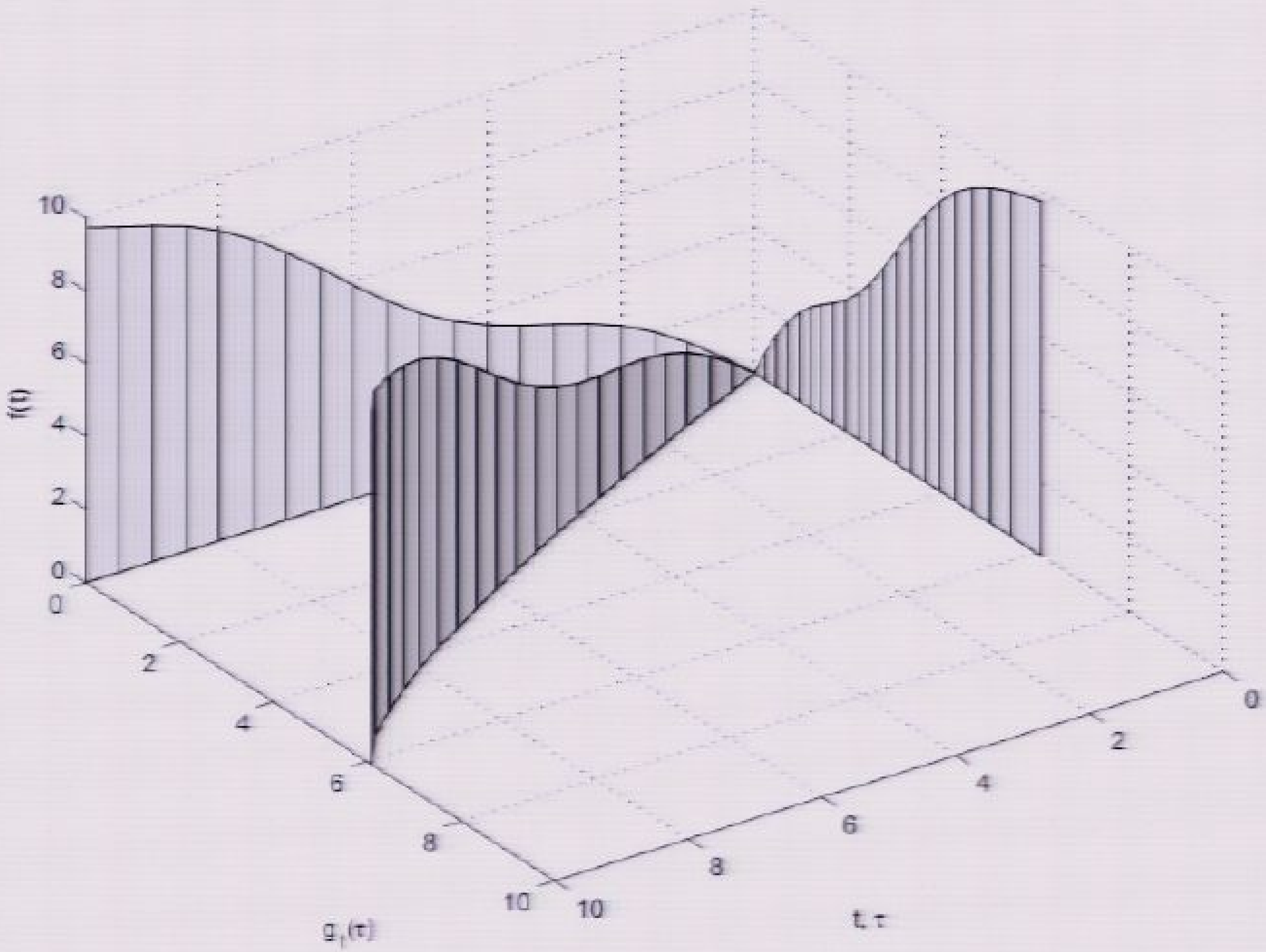
Anisotropic scaling natural in (Lebesgue–)Stieltjes integrals!

Fence shadows (from Podlubny arXiv:math/0110241)



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- Changing t, also the relation between measured and "cosmic" time changes!

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- Concrete definition of a universe with UV fractal structure.

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