

Title: Background Cosmology and Cosmological Perturbations in Horava-Lifshitz Gravity

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URL: <http://pirsa.org/09110059>

Abstract:

Background Cosmology & Cosmol. Perturbations in HL Gravity

1. Introduction
2. Bouncing universe from HL Gravity RB 0904.2571
3. Cosmol. Perturb. in HL Gravity X. Gao et al
0905.3821
& in preparation

$$ds^2 = -N^2 c^2 dt^2 + \gamma_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$$

$$S_V = \frac{2}{\kappa^2} \int d^4x \sqrt{\gamma} N (K_{ij} K^{ij} - \lambda K^2)$$

$$K_{ij} = \frac{1}{2N} (\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

$$S_V = \frac{\kappa^2}{2} \int d^4x \sqrt{\gamma} N \left[-\frac{1}{N^2} C_{ij} C^{ij} + \frac{\Lambda}{N^2} \epsilon^{ijkl} \epsilon_{ik} \nabla_j R_{l^k} \right. \\ \left. - \frac{\kappa^2 \Lambda^2}{2} \epsilon_{ij} \epsilon^{kl} R_{kl} + \frac{\Lambda^2}{4(1-3\beta)} \left(\frac{(1-4\beta)^2}{\beta} R + \Lambda R - 3\Lambda^2 \right) \right]$$

$$dx' + N'(t)$$

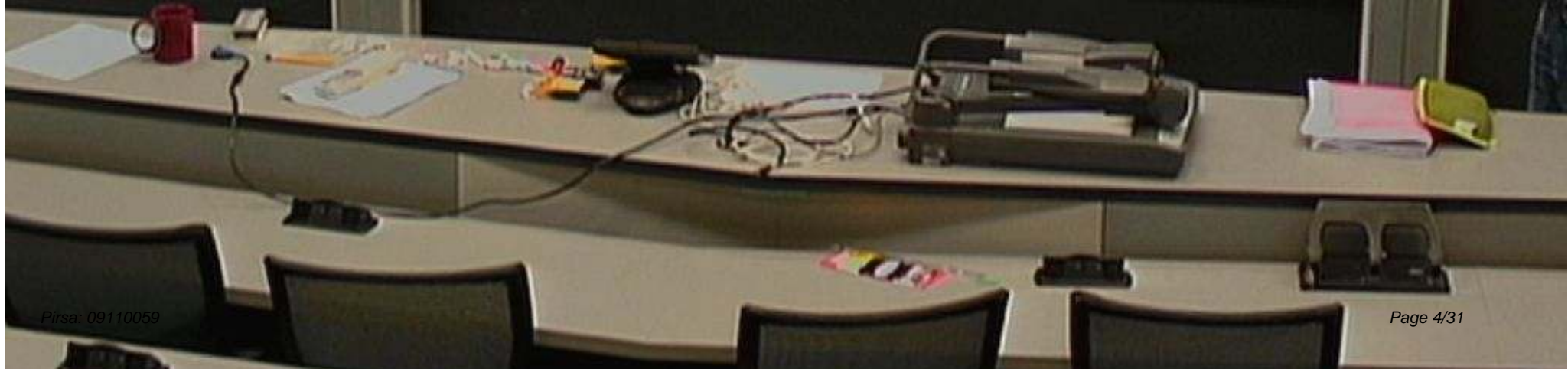
$$- (2k^2)$$

$$+ \frac{\Lambda}{W^2} \epsilon^{ijk} P_i \nabla_j P_k$$

$$\frac{\Lambda^2}{(1-\beta^2)} \left(\frac{1+\lambda^2}{f} R + \lambda R - 3\Lambda^2 \right)$$

$$ds^2 = - dt^2 + \bar{g}_{ij} dx^i dx^j$$

$$\bar{g}_{ij} = \frac{a^2(t)}{1 + \frac{R_{ij}^2}{f}} \delta_{ij}$$



$$dx' + N' dt$$

$$- \lambda k^2$$

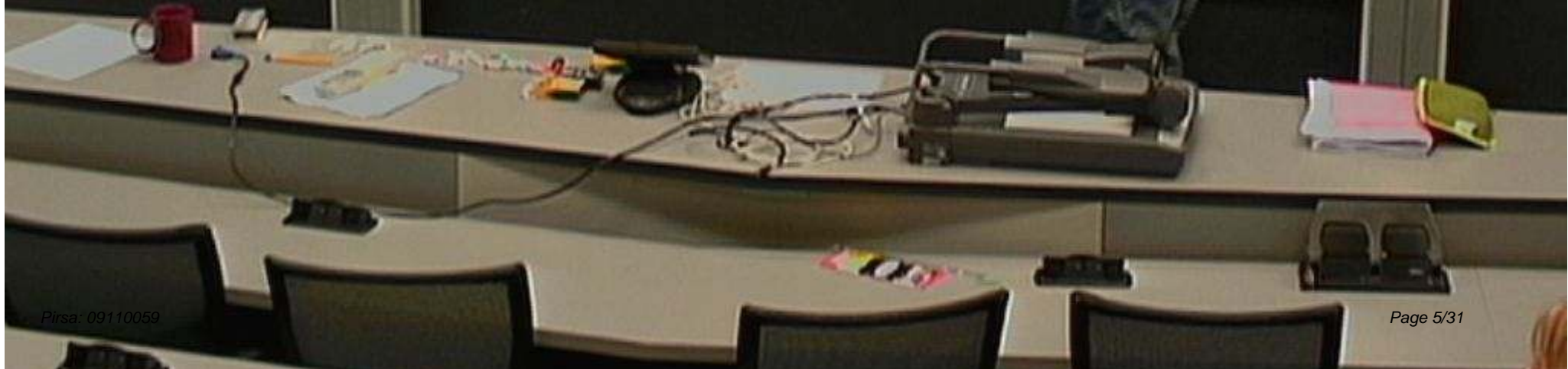
$$+ \frac{\lambda}{w} \epsilon^{ik} \partial_i \partial_k R^j$$

$$\frac{\lambda^2}{(1-\lambda^2)} \left(\frac{1-\lambda^2}{\lambda} R + \lambda R - 3\lambda^2 \right)$$

$$ds^2 = - dt^2 + \bar{\gamma}_{ij} dx^i dx^j$$

$$\bar{\gamma}_{ij} = \frac{a^2(t)}{1 + \frac{R_{ij}}{\lambda}} \delta_{ij}$$

$$\frac{6(3\lambda - 1)}{\lambda^2} H^2 = \mathcal{S}^-$$



$$dx' + W dt$$

$$-2k^2$$

$$+ \frac{\mu}{\nu} \epsilon^{ijk} \partial_j R_k^i$$

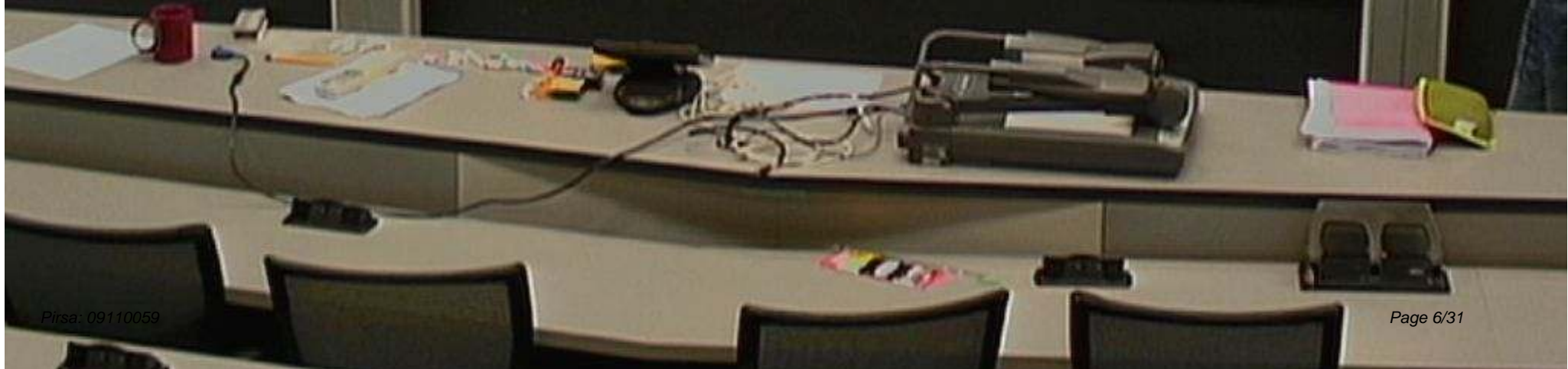
$$\frac{\mu^2}{(1-\beta^2)} \left(\frac{1+\beta^2}{\beta} R + \beta R - 3\beta^2 \right)$$

$$ds^2 = -dt^2 + \bar{g}_{ij} dx^i dx^j \quad W = \frac{p}{\rho}$$

$$\bar{g}_{ij} = \frac{a^2(t)}{1 + \frac{R^2}{f}} \delta_{ij}$$

$$\frac{6(3\alpha-1)}{k^2} H^2 = \int - \frac{3k^2 \mu^2}{8(3\alpha-1)} \left(\frac{\bar{k}}{a^2} - \Lambda \right)^2$$

$$\frac{2(3\alpha-1)}{k^2} \dot{H} = - \frac{(1+W)}{2} \int$$



$$dx' + N/H$$

$$-2k^2$$

$$+ \frac{\Lambda}{W} \epsilon^{\mu\nu} \rho_{(\mu} \nabla_{\nu)} \rho^2$$

$$\frac{\Lambda^2}{(1-3\lambda)} \left(\frac{1-4\lambda^2}{4} R + \Lambda R - 3\Lambda^2 \right)$$

$$ds^2 = -dt^2 + \bar{g}_{ij} dx^i dx^j \quad W = \int$$

$$\bar{g}_{ij} = \frac{a^2(t)}{1 + \frac{R_{ij}}{4}} \delta_{ij}$$

$$\frac{6(3\lambda-1)}{k^2} \dot{H}^2 = \int - \frac{3k^2 \Lambda^2}{8(3\lambda-1)} \left(\frac{\bar{k}}{a^2} - \Lambda \right)^2$$

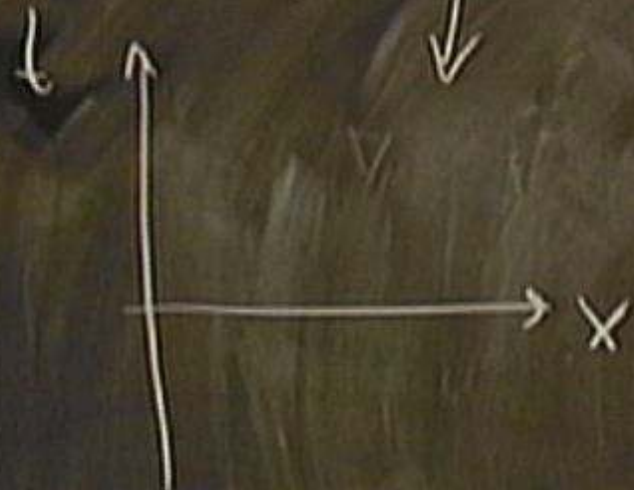
$$\frac{2(3\lambda-1)}{k^2} \ddot{H} = - \frac{(1+W)}{2} \int + \frac{k^2 \Lambda^2}{4(3\lambda-1)} \left(\frac{\bar{k}}{a^2} - \Lambda \right) \frac{\bar{k}}{a^2}$$

non-singular bounce

$$H = 0$$

$$\dot{H} > 0$$

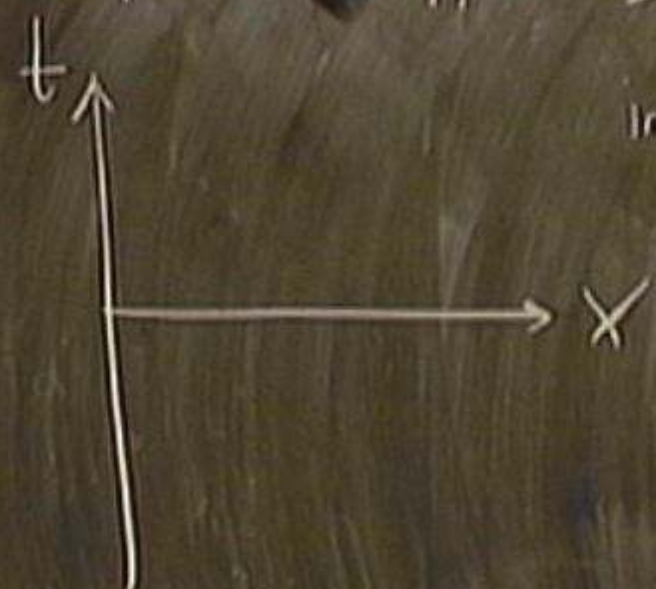
non-singular bounce



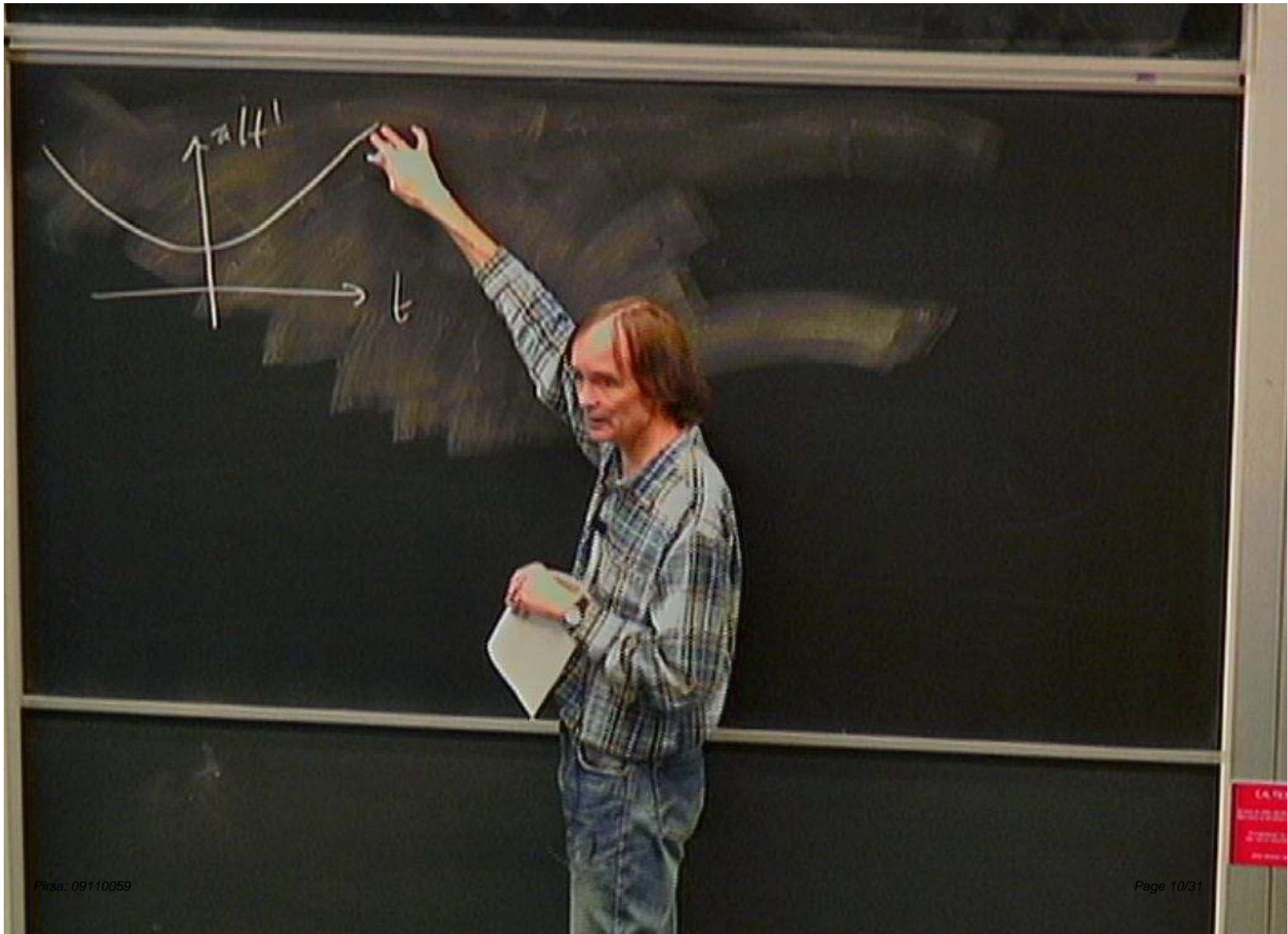
$$H = 0$$

$$\dot{H} > 0$$

inflation

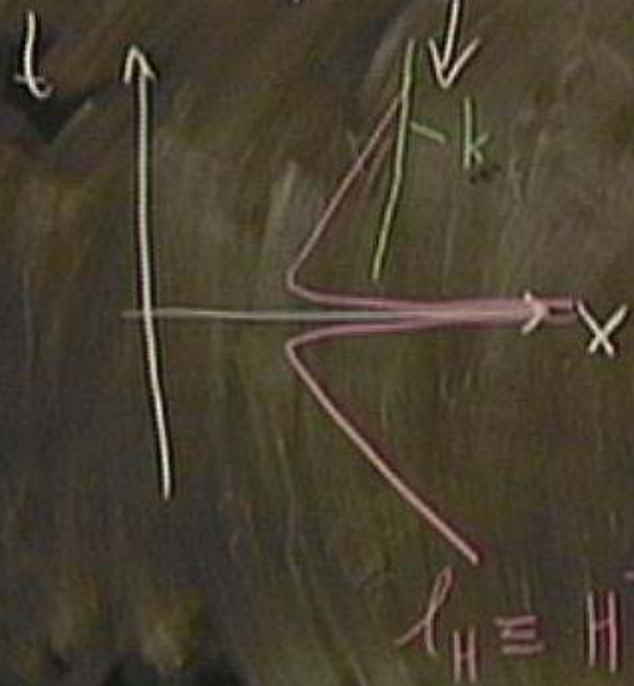


$$l_H \equiv H^{-1}$$



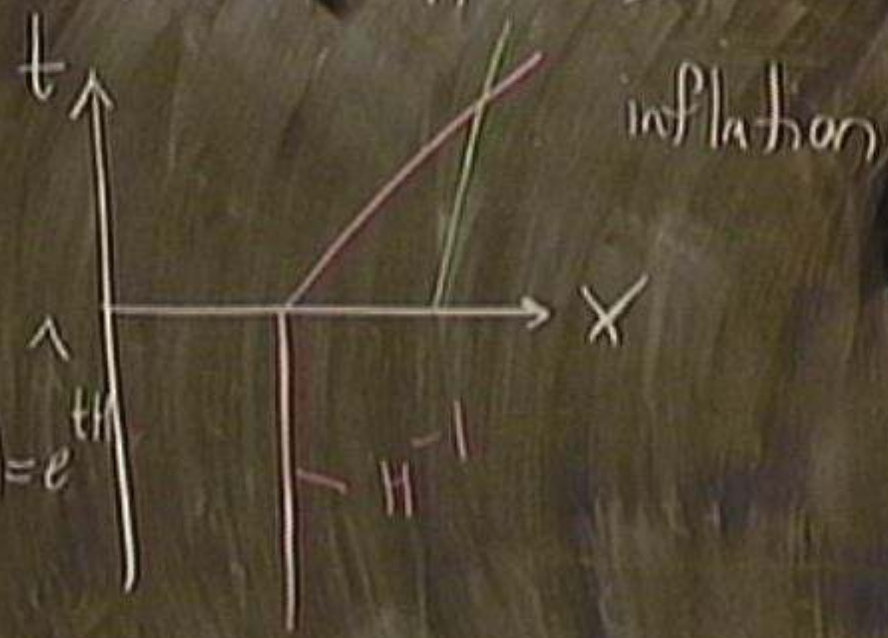
CAUTION
DO NOT TOUCH
EQUIPMENT
OR SURFACES

non-singular bounce

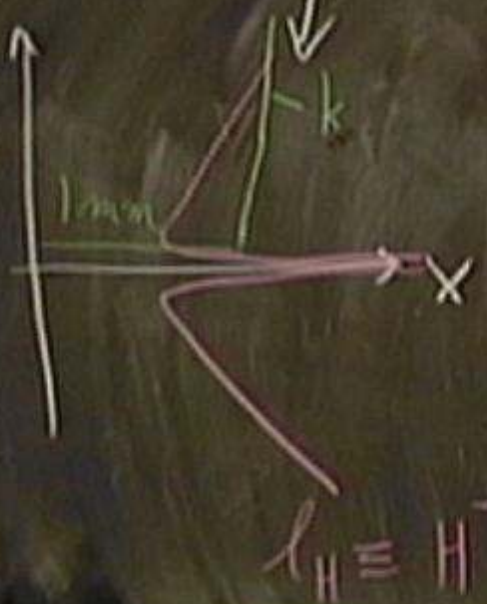


$$H = 0$$

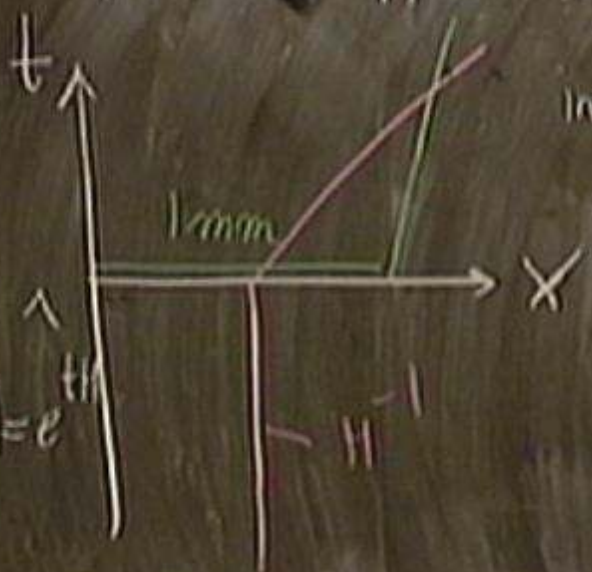
$$\dot{H} > 0$$



non-singular bounce



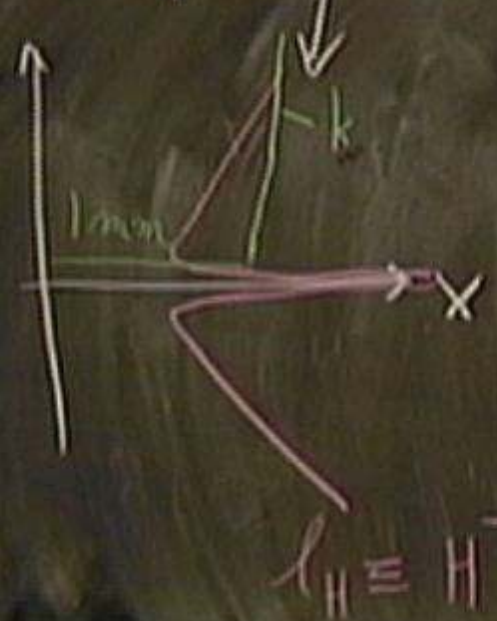
$$H = 0 \quad \dot{H} > 0$$



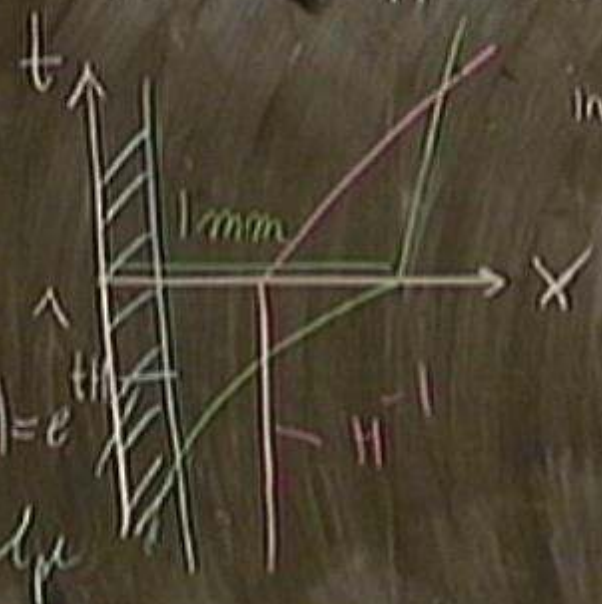
inflation



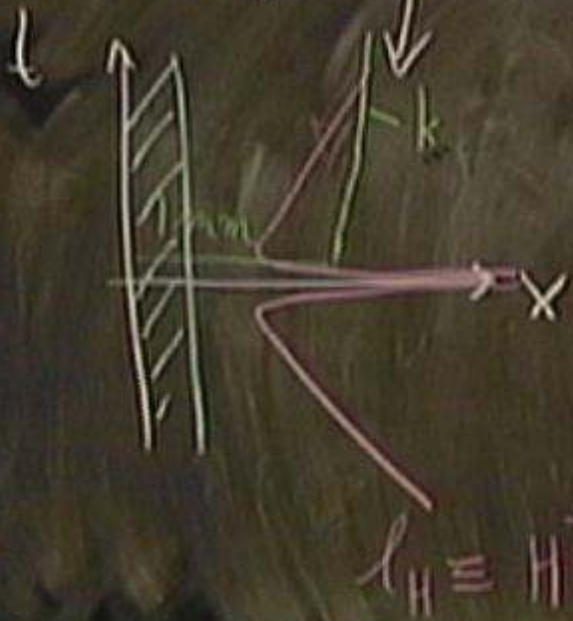
non-singular bounce



$H=0$ $\dot{H} > 0$

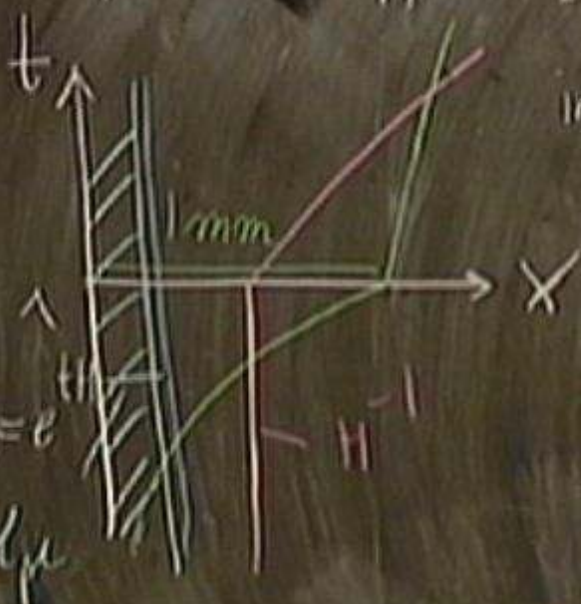


non-singular bounce



$$H = 0$$

$$\dot{H} > 0$$



inflation

$$a(t) = e^{Ht}$$

Claim 1 vacuum init. fluctuations in contracted phase
→ scale-inv. spectrum after bounce



Claim 1 vacuum init. Flucts in contrast. phase

→ scale-inv. spectrum after bounce

Claim 2. specific predictions. $\mathcal{O}(H)$ amplitude
& specific shape of 3-pt. fct.

Framework -

$$ds^2 = -a^2(\eta) \left[(1+2\phi) d\eta^2 - (1-2\phi) dx^2 \right]$$

$$\phi(x, \eta)$$

$$\zeta = \frac{a}{z} \delta\psi + \phi$$

Framework -

$$ds^2 = -a^2(\eta) \left[(1+2\phi) d\eta^2 - (1-2\phi) dx^2 \right]$$

$\phi(x, \eta)$

$$\psi = \frac{a}{z} \delta\psi + \phi \quad z = a \frac{\psi'}{\mathcal{H}}$$

$$' = \frac{\partial}{\partial \eta} \quad \mathcal{H} = \frac{a'}{a}$$

framework.

$$ds^2 = -a^2(\eta) \left[(1+2\phi) d\eta^2 - (1-2\phi) dx^2 \right]$$

$\phi(x, \eta)$

Chr. Pl.

$$\zeta = \frac{a}{z} \delta\psi + \phi \quad z = a \frac{\psi'}{\mathcal{H}} \sim a$$

$$' = \frac{d}{d\eta} \quad \mathcal{H} = \frac{a'}{a}$$

$$v = z \}$$

Sasaki-Mukhanov canonical var.

$$P_f(k) = k^3 |f(k)|^2$$



$$P_{\zeta}(k) = k^3 |f(k)|^2 \sim k^0 \text{ scale-inv.}$$



vacuum spectrum

$$P_{\zeta}(k) \sim k^3 k^{-1} \sim k^2$$

$$V_k'' + \left(k^2 - \frac{\ddot{z}}{z}\right) V_k = 0$$

expanding: $V_k \sim H^2 z \Rightarrow \int k \sim \text{const}$

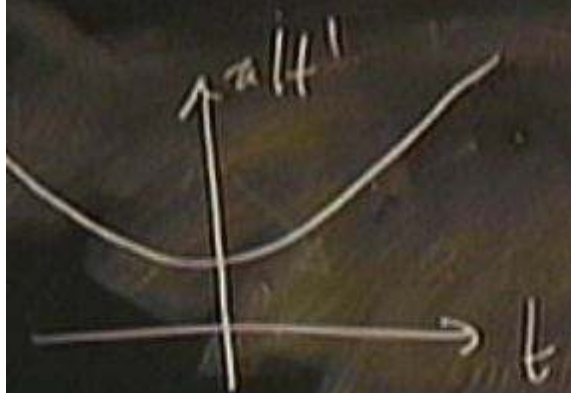
$$S_{\text{tot}} = S_k + S_V + S_{\text{matter}}$$

$$V_k'' + \left(k^2 - \left(\frac{\ddot{z}}{z} \right) \right) V_k = 0$$

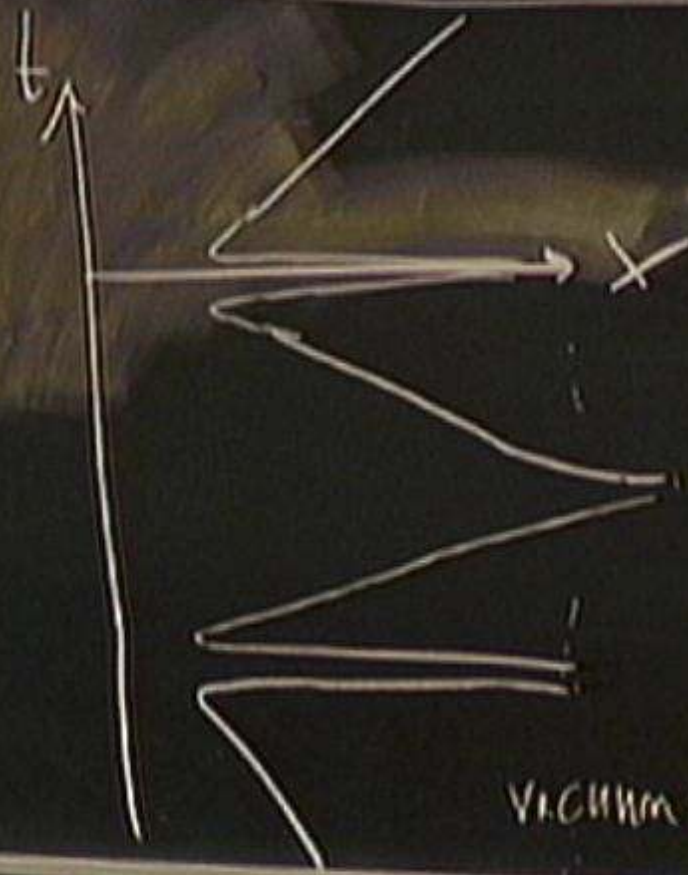
expanding: $V_k \sim H^2 z \rightarrow \int k \sim \text{const}$

contrast matter: $V_k \sim z^{-1/2} \rightarrow \int k \sim z^{-3/2}$

$$S_{\text{tot}} = S_k + S_V + S_{\text{matter}}$$



d) cyclic matter bounce?



VICINIA SP.



Claim 1

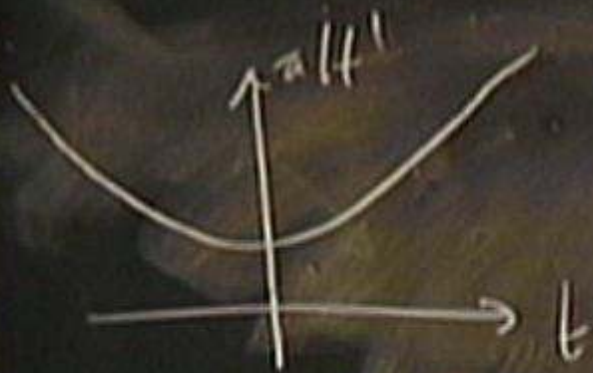
vacuum init. Flucts in contact, phase

→ scale-inv. spectrum after bounce

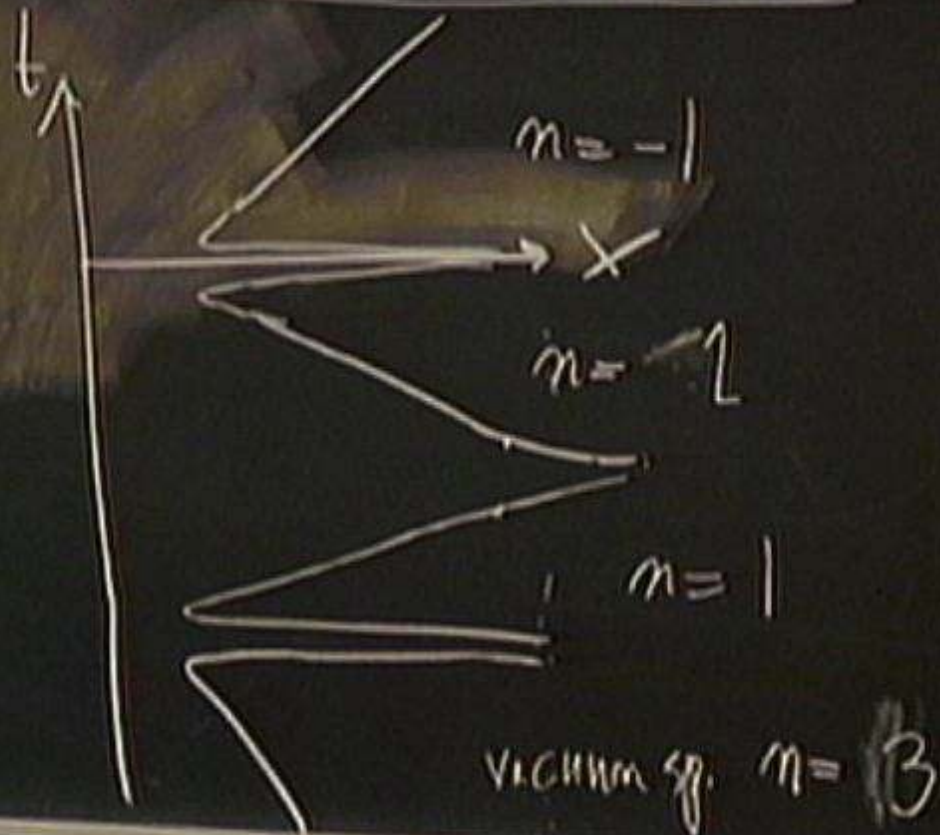
Claim 2

specific predictions. $\mathcal{O}(H)$ amplitude
& specific shape of 3-pt. fct.

$$P_{\xi}(k) \sim k^{n-1}$$



d) cyclic matter bounce?



Formalism

$$\begin{aligned} \delta \gamma_{00} &= -2\phi && \phi(x,t) && \text{scalar} \\ \delta \gamma_{0i} &= a^2 B && B(x,t) && \\ \delta \gamma_{ij} &= a^2 (\psi \delta_{ij} + E_{ij}) && \psi(x,t) E(x,t) && \\ \delta \varphi(x,t) &&& && \\ 5 \text{ DOF} &&& && \end{aligned}$$

GR

5 DOF

-2 GDF

-1 no anisotropic stress

-1 energy constraint

1 DOF

HL Gravity

5 DOF

$$x^{\dot{i}} \rightarrow x^{\dot{i}} + p^i$$

$$t^3 \rightarrow \cancel{t^3} + \cancel{f^0(x,t)}$$

GR

5 DOF

-2 GDF

-1 no anisotropic stress

-1 energy constraint

1 DOF

HL Gravity

5 DOF

-1 GDF

-1 no anisotropic stress

-1 energy const

-1 momentum const. $X^{\ddot{a}} \rightarrow X^{\ddot{a}} + f^{\ddot{a}}$

1 DOF

~~$t^{\ddot{a}} \rightarrow t^{\ddot{a}} + f^{\ddot{a}}(x, t)$~~

step 1 gauge choice $E=0$

step 2

$$\phi =$$

$$B =$$

$$\frac{f(\psi, \delta\psi)}{32H^2(\lambda-1) - 4K^2(\lambda-1)\dot{\psi}_0^2}$$

→ scale [REDACTED] after bounce

Claim 2: specific predictions. $\mathcal{O}(1)$ amplitude
& specific shape of 3-pt. fct.

$$P_{\xi}(k) \sim k^{n-1}$$