Title: Quantum gravity without Lorentz invariance

Date: Nov 08, 2009 04:30 PM

URL: http://pirsa.org/09110057

Abstract:

From sound waves to gravity?

Hořava-Lifshitz Gravity

by

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The concept of anisotropic scaling - I

What is anisotropic scaling..?

- Lorentz symmetry breaking at the non-perturbative level
- splitting of spacetime into space and time in the *ultraviolet*
- only higher (then two) spatial space derivatives added

Kinematic framework for Lorentz violation

$$\mathbf{E^2 = m^2\, c^4 + p^2\, c^2 + c^4 \left\{ \eta_1\, M_{Pl}\, p/c + \eta_2\, p^2/c^2 + \sum_{n\geq 3} \eta_n \frac{(p/c)^n}{M_{Pl}^{n-2}} \right\}}$$

naturalness problem..?

Lorentz symmetry: *NOT exact ➡ emergent* symmetry

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naturalness problem..?

Lorentz symmetry: *NOT exact ➡ emergent* symmetry

$$E^2 = F(p,m)$$

$$E^2 = m^2c^2 + p^2c^2 + F^{(1)}_i p^i + F^{(2)}_{ij} p^i p^j,$$

The concept of anisotropic scaling - I

What is anisotropic scaling..?

- Lorentz symmetry breaking at the non-perturbative level
- splitting of spacetime into space and time in the *ultraviolet*
- only higher (then two) spatial space derivatives added

Kinematic framework for Lorentz violation

$$\mathbf{E^2 = m^2\, c^4 + p^2\, c^2 + c^4 \left\{ \eta_1\, M_{Pl}\, p/c + \eta_2\, p^2/c^2 + \sum_{n\geq 3} \eta_n \frac{(p/c)^n}{M_{Pl}^{n-2}} \right\}}$$

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The concept of anisotropic scaling - II

What is the degree 'z' of anisotropy..?

• z tells us the anisotropy between the highest order spatial versus the highest order temporal derivatives in our *modified* field theory;

• dimensional analysis: introduce "dynamical critical exponent" z such that: $[dx] = [\kappa]^{-1}$ $\qquad$ $[dt] = [\kappa]^{-z}$

Why are we interested? → d=z *special*

Anisotropic scaling as qft regulator - (a)

Matt Visser, PRD D80:025011, Lorentz symmetry breaking as a quantum field theory regulator, 2009

Idea:

- take LSB as QFT regulator: "how much" LSB is necessary to FULLY regulate the theory and make it finite?

Ad-hoc modified action (free-field term):

$$S_{\text{free}} = \int \left\{ \dot{\phi}^2 - \phi(-\Delta)^z \phi \right\} dt\, d^d x \quad \Rightarrow \quad [\phi] = [dx]^{(z-d)/2}$$

- define formal symbols κ and m having the dimension of momentum and energy

$$[\phi] = [\kappa]^{(d-z)/2} = [m]^{(d-z)/(2z)}$$

Anisotropic scaling as qft regulator - (b)

Matt Visser, PRD D80:025011, Lorentz symmetry breaking as a quantum field theory regulator, 2009

Modified action (kinetic term):

$$S_{\text{free}} = \int \left\{ \dot{\phi}^2 - \phi \left[m^2 - c^2 \Delta + \cdots + (-\Delta)^z \right] \phi \right\} dt \, d^d x$$

Modified action (polynomial potential term):

$$S_{\text{interaction}} = \int P(\phi) \, dt \, d^d x = \int \left\{ \sum_{n=1}^{N} g_n \, \phi^n \right\} dt \, d^d x$$

=> $$[\phi] = [\kappa]^{(d-z)/2} = [m]^{(d-z)/(2z)}$$

Anisotropic scaling as qft regulator - (c)

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Coupling constants:

$$[g_n] = [\kappa]^{d+z-n(d-z)/2} = [m]^{[d+z-n(d-z)/2]/z}$$

UV-behaviour:

=> theory is renormalizable as long as couplings have non-negative momentum dimensions:

$$d + z - \frac{n(d-z)}{2} \geq 0$$

Notice!!!:

$$N = \infty; \qquad \text{(provided } z \geq d\text{)}$$

Anisotropic scaling as qft regulator - (d)

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Superficial degree of divergence:

- If the superficial degree of divergence is negative then the Feynman diagram is convergent...

Each loop: $$\int d\omega_l \, d^d k_l \cdots \to [d\omega][dk]^d = [\kappa]^{d+z}$$

Each propagator: $$G(\omega, \vec{k}) \to [\kappa]^{-2z}$$

Total: $$[\kappa]^{(d+z)\,L-2\,I\,z}$$

✓ (z=d)

Superficial degree of freedom:

$$\delta = (d-z)\,L - 2\,(I-L)\,z \to \delta \le (d-z)\,L$$

Anisotropic scaling in emergent spacetimes - (ESI)

Emergent spacetimes, PhD thesis, arXiv:0711.4416v1, 2007; and proceedings: arXiv:0905.4530v1, 2009.

Toy models for an effective/emergent/induced spacetime:

Geometry (IR) & anisotropic scaling (UV)

Bose-Einstein condensate: $\omega_k^2 = c_0^2\, k^2 + \epsilon_{\mathrm{qp}}^2\, k^4$

Electromagnetic waveguide: $\omega_k^2 = \frac{4}{L\,C}\sin^2\left(\frac{k\,\Delta x}{2}\right) \approx c^2\,k^2 - \frac{\Delta x^4}{12\,L\,C}\,k^4.$

Anisotropic scaling in emergent spacetimes - (ES11)

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Ultra-cold gas of weakly interaction Bosons:

(I) microscopic system consisting

(II) mean-field regime where the microscopic degrees of freedom give way to collective variables

(III) small perturbations are dominated by symmetric second rank tensor

(IV) Lorentz symmetry is an emergent symmetry in the IR and with anisotropic scaling in the UV

(V) degree of anisotropy depends on various things...

Anisotropic scaling in emergent spacetimes - (ES11)

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(I) microscopic system:

$$\hat{\mathbf{H}} = \int \mathrm{dx} \left(-\hat{\psi}^\dagger \frac{\hbar^2}{2\mathbf{m}} \nabla^2 \hat{\psi} + \hat{\psi}^\dagger \mathbf{V}_{\mathrm{ext}} \hat{\psi} + \frac{\mathbf{U}}{2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \right)$$

pseudo-contact potential

$$\mathbf{U} = \frac{4\pi\hbar^2 \mathbf{a}}{\mathbf{m}}$$

❖ boson commutators:

$$\left[\hat{\psi}(\mathbf{t},\mathbf{x}), \hat{\psi}(\mathbf{t},\mathbf{x}')\right] = \left[\hat{\psi}^\dagger(\mathbf{t},\mathbf{x}), \hat{\psi}^\dagger(\mathbf{t},\mathbf{x}')\right] = \mathbf{0}\,; \quad \left[\hat{\psi}(\mathbf{t},\mathbf{x}), \hat{\psi}^\dagger(\mathbf{t},\mathbf{x}')\right] = \delta(\mathbf{x}-\mathbf{x}')$$

❖ SO(2)-symmetry: $\hat{\psi} \rightarrow \hat{\psi}\, \exp(\mathbf{i}\alpha)$

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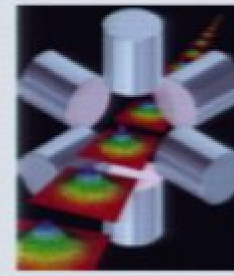

❖ SO(2)-symmetry: $\hat{\psi} \to \hat{\psi}\,\exp(\mathbf{i}\alpha)$

first order phase-transition

$$\langle\hat{\psi}(\mathbf{t},\mathbf{x})\rangle = \psi(\mathbf{t},\mathbf{x}) = \sqrt{\mathbf{n_0}(\mathbf{t},\mathbf{x})}\;\exp\left(\mathbf{i}\,\phi_{\mathbf{0}}(\mathbf{t},\mathbf{x})\right) \neq \mathbf{0}$$

(II) mean-field regime where the microscopic degrees of freedom give way to collective variables

condensate density $\mathbf{n_0} \equiv \mathbf{n_0}(\mathbf{t},\mathbf{x})$ and the phase $\phi_{\mathbf{0}} \equiv \phi_{\mathbf{0}}(\mathbf{t},\mathbf{x})$

linearise around ground state of the system...

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$$\psi(t,\mathbf{x}) + \delta\hat{\psi}(t,\mathbf{x}) \quad \rightarrow \quad \psi(t,\mathbf{x}) \left(1 + \frac{1}{2}\frac{\hat{n}}{n_0} + i\hat{\phi}\right)$$

$$\psi^*(t,\mathbf{x}) + \delta\hat{\psi}^\dagger(t,\mathbf{x}) \quad \rightarrow \quad \psi^*(t,\mathbf{x}) \left(1 + \frac{1}{2}\frac{\hat{n}}{n_0} - i\hat{\phi}\right)$$

excitation spectrum

$$\frac{1}{\sqrt{|\det(\mathbf{g_{ab}})|}}\, \partial_\mathbf{a} \left(\sqrt{|\det(\mathbf{g_{ab}})|}\, \mathbf{g^{ab}}\, \partial_\mathbf{b}\hat{\phi}\right) = \mathbf{0}$$

(III) small perturbations are dominated by symmetric second rank tensor

$$\mathbf{g_{ab}} = \left(\frac{\mathbf{c_0}}{\mathbf{U}/\hbar}\right)^{\frac{2}{d-1}} \begin{bmatrix} -\left(c_0^2 - \mathbf{v}^2\right) & -v_x & -v_y & -v_z \\ -v_x & 1 & 0 & 0 \\ -v_y & 0 & 1 & 0 \\ -v_z & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v} = \frac{\hbar}{\mathbf{m}}\nabla\phi_\mathbf{0}$$

$$\mathbf{c_0^2} = \frac{\mathbf{n_0 U}}{\mathbf{m}}$$

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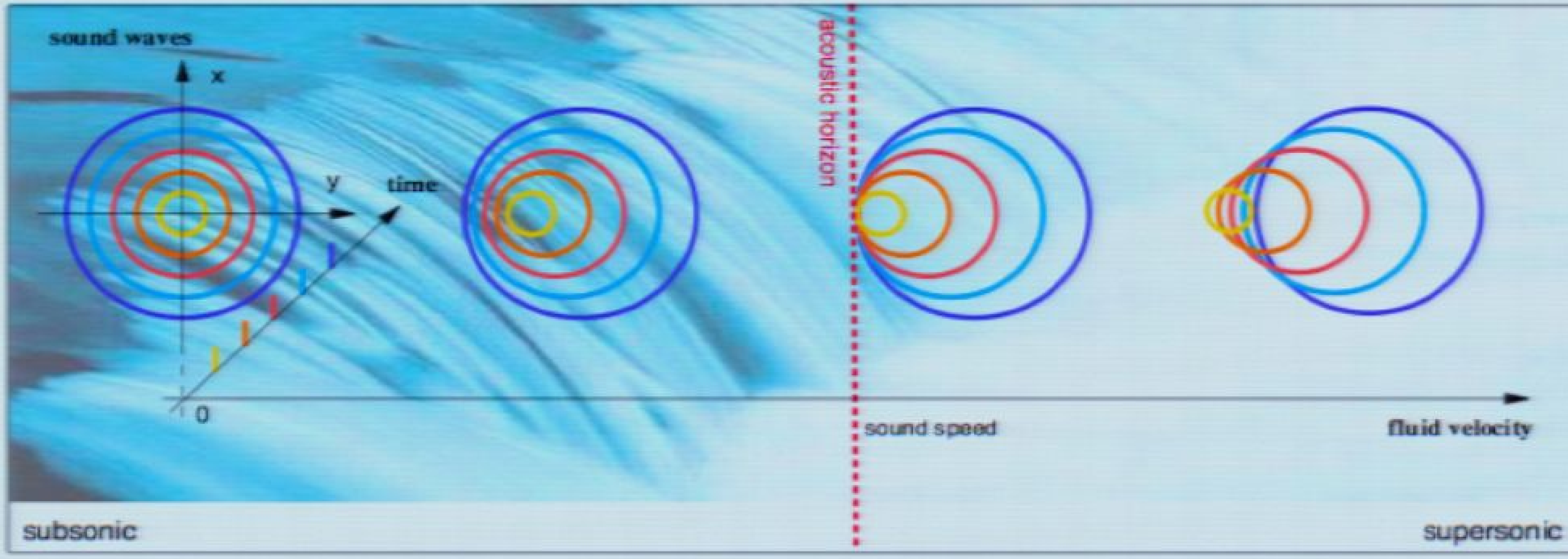

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- integro-differential equation: $\partial_{\mathbf{a}}\left(\mathbf{f^{ab}}\,\partial_{\mathbf{b}}\hat{\phi}\right) = \mathbf{0}$

$$\mathbf{f^{ab}} = \hbar\left[\begin{array}{c|c} -\widetilde{U}^{-1} & -\widetilde{U}^{-1}v^j \\ \hline -v^i\widetilde{U}^{-1} & \frac{n_0}{m}\delta^{ij} - v^i\widetilde{U}^{-1}v^j \end{array}\right]$$

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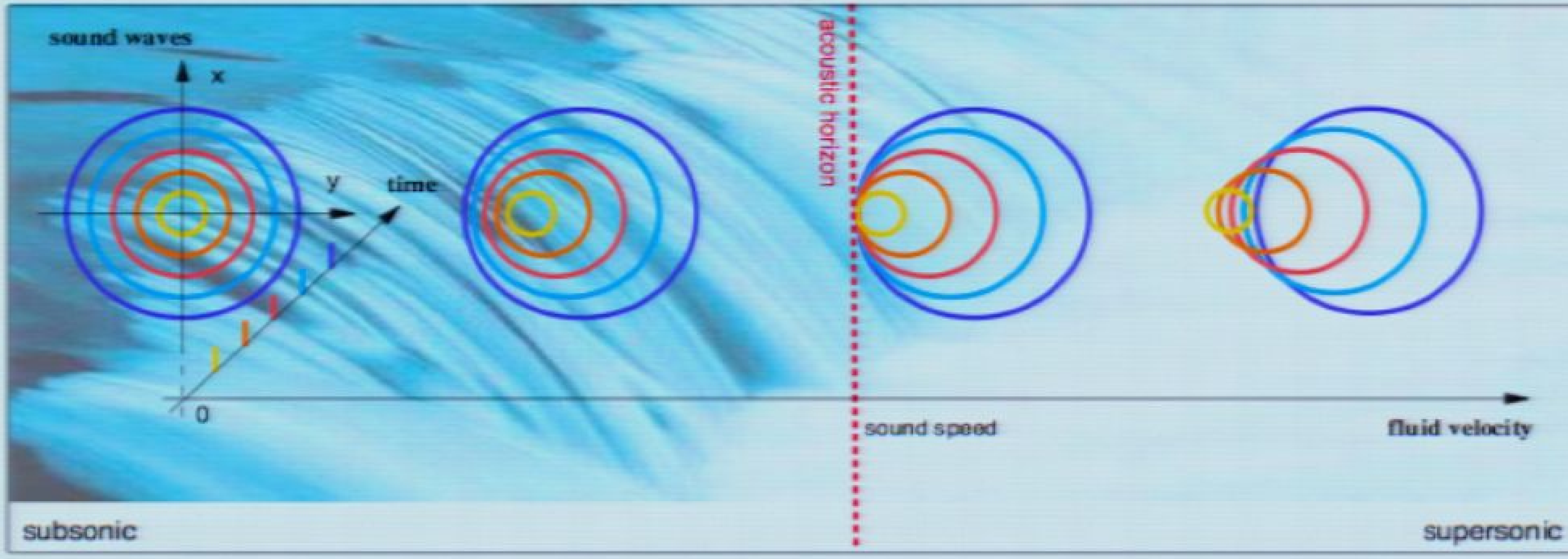

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Anisotropic scaling in emergent spacetimes - (ES11)

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In some cases...

(IV) Lorentz symmetry is an emergent symmetry in the IR and with anisotropic scaling in the UV

$$\partial_t\partial_t\hat{\phi} + (f^{tt})^{-1} f^{ij}\partial_i\partial_j\hat{\phi} = 0$$

$$\updownarrow$$

$$S^{(1)}_{BEC} \approx \int \left\{\dot{\phi}^2 - \phi\left[-c_0^2\triangle + \gamma_{qp}^2\triangle^2\right]\phi\right\} dt\, d^dx$$

here: z=2

$$E^2 = m^2 c^4 + p^2 c^2 + c^4 \left\{\eta_1 M_{Pl}\, p/c + \eta_2\, p^2/c^2 + \sum_{n\geq 3} \eta_n \frac{(p/c)^n}{M_{Pl}^{n-2}}\right\}$$

naturalness problem..?

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$$\mathbf{E}^2 = \mathbf{m}^2\, \mathbf{c}^4 + \mathbf{p}^2\, \mathbf{c}^2 + \mathbf{c}^4 \left\{ \eta_1\, \mathrm{M_{Pl}}\, \mathbf{p}/\mathbf{c} + \eta_2\, \mathbf{p}^2/\mathbf{c}^2 + \sum_{\mathbf{n} \geq 3} \eta_{\mathbf{n}} \frac{(\mathbf{p}/\mathbf{c})^{\mathbf{n}}}{\mathrm{M}_{\mathrm{Pl}}^{\mathbf{n}-2}} \right\}$$

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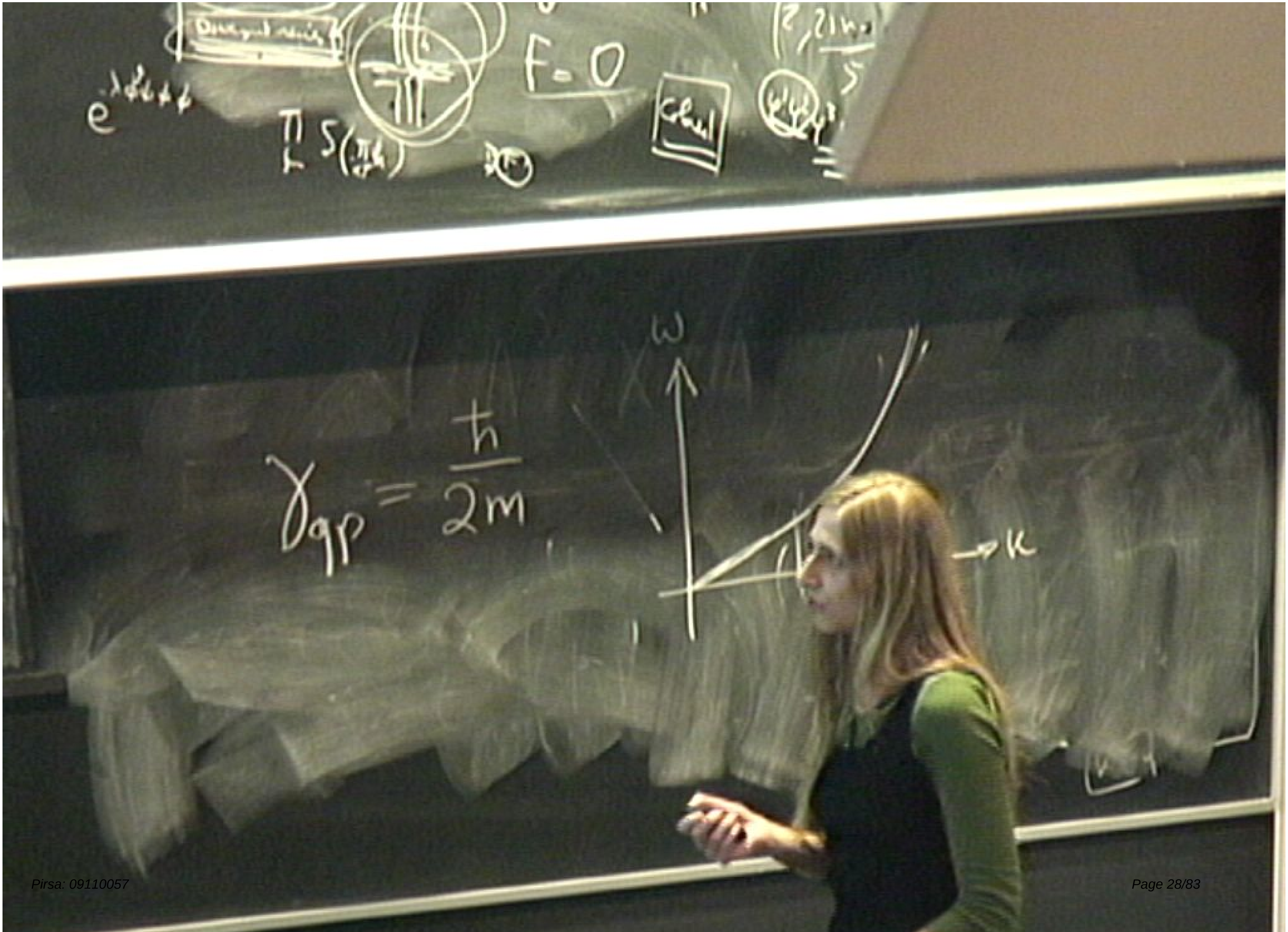
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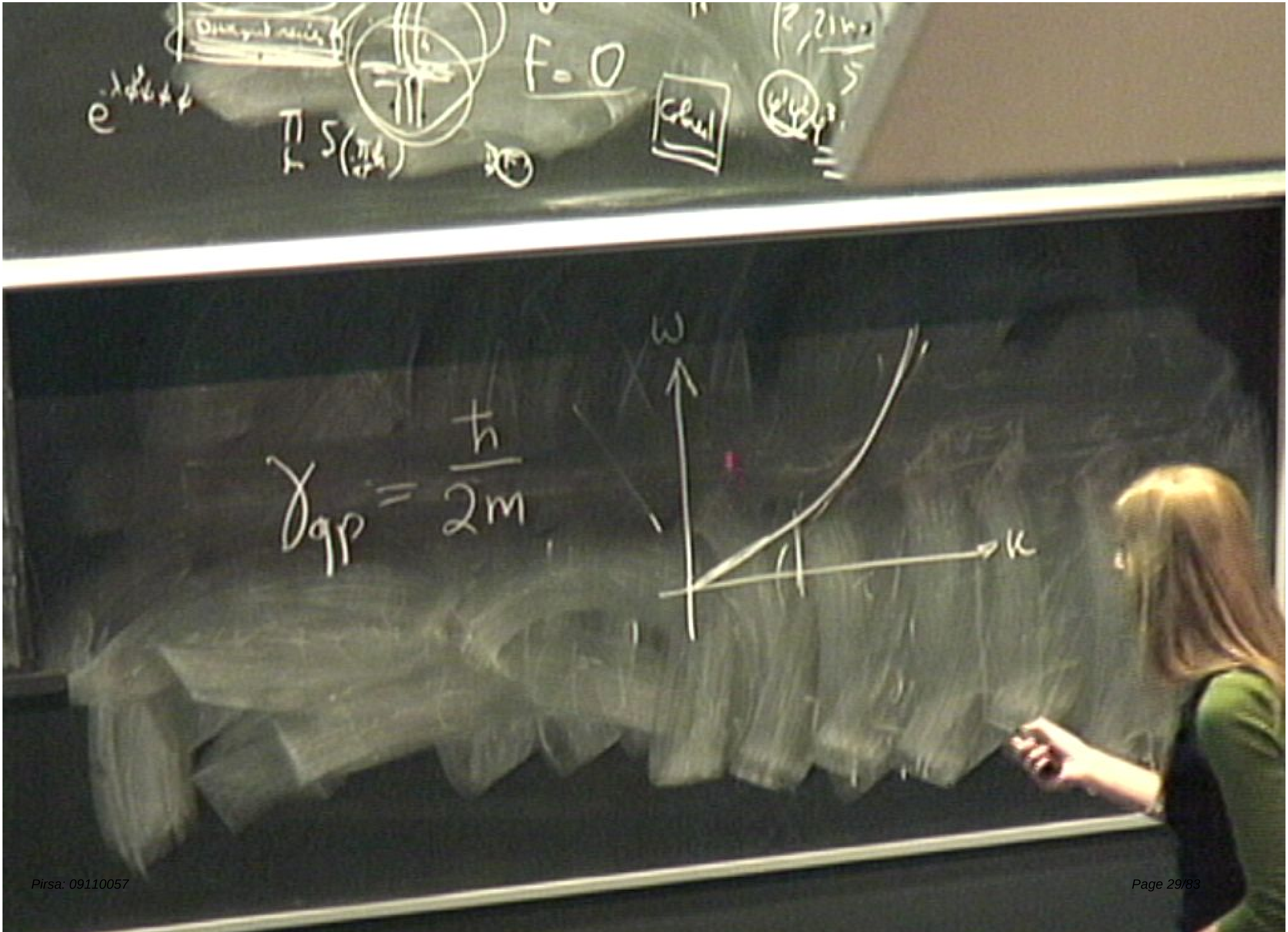
ω
k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- integro-differential equation: $\partial_{\mathbf{a}}\left(\mathbf{f}^{\mathbf{ab}}\,\partial_{\mathbf{b}}\hat{\phi}\right) = \mathbf{0}$

$$\mathbf{f}^{\mathbf{ab}} = \hbar\left[\begin{array}{c|c} -\widetilde{U}^{-1} & -\widetilde{U}^{-1}v^j \\ \hline -v^i\widetilde{U}^{-1} & \frac{n_0}{m}\delta^{ij} - v^i\widetilde{U}^{-1}v^j \end{array}\right]$$

Anisotropic scaling in emergent spacetimes - (ES11)

Emergent spacetimes, PhD thesis, arXiv:0711.4416v1, 2007; and proceedings: arXiv:0905.4530v1, 2009.

In some cases...

(IV) Lorentz symmetry is an emergent symmetry in the IR and with anisotropic scaling in the UV

$$\partial_t \partial_t \hat{\phi} + (f^{tt})^{-1} f^{ij} \partial_i \partial_j \hat{\phi} = 0$$

$$\updownarrow$$

$$S^{(1)}_{\mathrm{BEC}} \approx \int \left\{ \dot{\phi}^2 - \phi \left[-c_0^2 \triangle + \gamma_{qp}^2 \triangle^2 \right] \phi \right\} dt\, d^d x$$

here: z=2

$$E^2 = m^2 c^4 + p^2 c^2 + c^4 \left\{ \eta_1 M_{\mathrm{Pl}}\, p/c + \eta_2\, p^2/c^2 + \sum_{n \geq 3} \eta_n \frac{(p/c)^n}{M_{\mathrm{Pl}}^{n-2}} \right\}$$

naturalness problem..?

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naturalness problem..?

Anisotropic scaling in emergent spacetimes - (ES II)

Stefaon Liberati, Matt Visser. and SW, PRL 96 151301, Naturalness in emergent spacetime, 2006.

Two component BEC... $\hat{H} = \int d\mathbf{r} \left\{ \sum_{i=1,2} \left(-\hat{\Psi}_i^\dagger \frac{\hbar^2 \nabla^2}{2m_i} \hat{\Psi}_i + \hat{\Psi}_i^\dagger V_{\mathrm{ext},i}(\mathbf{r}) \hat{\Psi}_i \right) + \frac{1}{2} \sum_{i,j=1,2} \left(U_{ij} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_i \hat{\Psi}_j + \lambda \hat{\Psi}_i^\dagger (\sigma_x)_{ij} \hat{\Psi}_j \right) \right\}$

(IV) Lorentz symmetry is an emergent symmetry in the IR and with anisotropic scaling in the UV

here: z=∞

$$\omega^2 = \omega_0^2 + (1+\eta_2)\, c^2\, k^2 + \eta_4 \left(\frac{\hbar}{M_{\mathrm{LIV}}} \right)^2 k^4 + \dots$$

$$\eta_{2,\mathrm{I/II}} \approx \left(\frac{m_{\mathrm{I/II}}}{M_{\mathrm{LIV}}} \right)^2 = \left(\frac{\text{quasiparticle mass}}{\text{effective Planck scale}} \right)^2 ;$$

$$\eta_{4,\mathrm{I/II}} \approx 1;$$

$$\mathbf{E^2 = m^2\, c^4 + p^2\, c^2 + c^4 \left\{ \eta_1\, M_{Pl}\, p/c + \eta_2\, p^2/c^2 + \sum_{n \geq 3} \eta_n \frac{(p/c)^n}{M_{Pl}^{n-2}} \right\}}$$

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How can we map this onto gravity?

spacetime

+ anisotropic scaling!

Gravity Dynamics

+ *anisotropic scaling?*

Quantum gravity:

Might even be a class of quantum gravity models, e.g. CDT, ...

~~Detailed balance~~

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Broad class of systems with ***completely*** different dynamics:

electromagnetic waveguide, fluids, ultra-cold gas of Bosons and Fermions;

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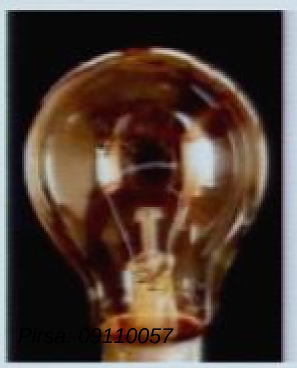
Introduction: Horava-Lifshitz Gravity in a nutshell - I

Petr Horava, Phys. Rev. D79, 084008, Quantum Gravity at a Lifshitz Point, 2009

Gravity as quantum field theory:

- gravitational coupling constant is dimensionful: $[G_N] = -2$
- graviton propagator scales as: $\dfrac{1}{k^a k_a} = \dfrac{1}{\sqrt{\omega^2 - \mathbf{k}^2}}$
- with the #loops the QFT requires counter terms with ever increasing degree in curvature

⚠ EG as QFT requires a UV completion!!!



Horava suggested an ad-hoc <u>condensed matter inspired</u> QFT where <u>gravity is emergent in the infrared</u> and the UV-behaviour perturbative renormalizable. **Price to pay**: Giving up on Lorentz symmetry as an exact symmetry.

Introduction: Horava-Lifshitz Gravity in a nutshell - II

Petr Horava, Phys. Rev. D79, 084008, Quantum Gravity at a Lifshitz Point, 2009

Anisotropic scaling mapped onto gravity:

- introduce "dynamical critical exponent" z such that:

$$[dx] = [\kappa]^{-1} \qquad [dt] = [\kappa]^{-z}$$

✓ **Dimensional analysis shows:** $[g_K] = [\kappa]^{(d-z)}$

- ultra-violet fixed point at, $z = 3$, and Lorentz symmetry emergent at infrared fixed point, $z = 1$.

Introduction: Horava-Lifshitz Gravity in a nutshell - III

Petr Horava, Phys. Rev. D79, 084008, Quantum Gravity at a Lifshitz Point, 2009

Higher curvature terms added:

- only higher spatial curvature terms, three-curvature terms...

$$\frac{1}{\omega^2 - c^2\mathbf{k}^2 - G(\mathbf{k}^2)^z}$$

✓ **Propagator shows good UV-behaviour!**

- compare with 4d modified gravity propagator; here we have the massless graviton excitation + a ghost excitation (implying unitarity violations):

$$\frac{1}{k^2 - G_N k^4} = \frac{1}{k^2} - \frac{1}{k^2 - 1/G_N}$$

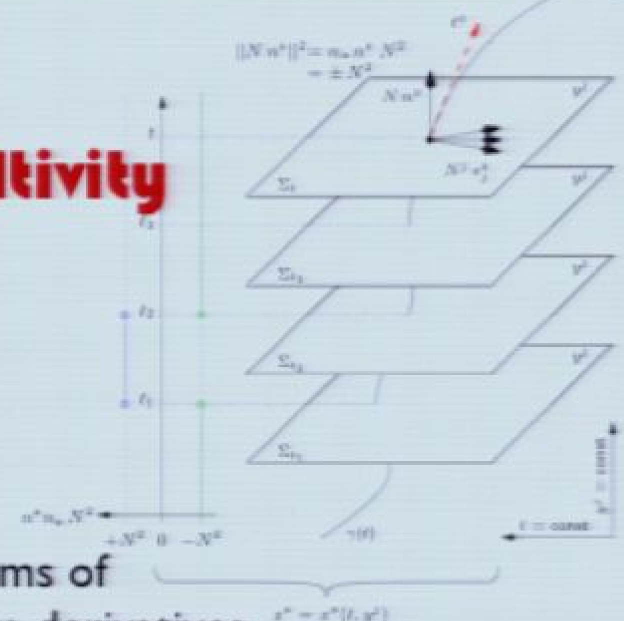
QG-IV: Overview

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

- Anisotropic scaling
- Modified Einstein-Hilbert action
- Equations of motion
- Graviton propagator

QG-LV: Anisotropic scaling - (i)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.



- **Hamiltonian formulation of General Realtivity**

$$S = \int \left[\mathscr{T}(K) - \mathscr{V}(g)\right] \sqrt{g}\, N\, \mathrm{d}^3x\, \mathrm{d}t$$

kinetic term

potential term:
scalar invariants in terms of
Riemann tensor and its derivatives

$$\mathscr{T}(K) = g_K \left\{(K^{ij}K_{ij} - K^2) + \xi K^2\right\}$$

+ anisotropic scaling!

- **ADM decomposition: lapse, shift, and induced metric**

$$\mathrm{d}s^2 = -N^2c^2\mathrm{d}t^2 + g_{ij}(\mathrm{d}x^i - N^i\mathrm{d}t)(\mathrm{d}x^j - N^j\mathrm{d}t)$$

QG-LV: Anisotropic scaling - (ii)

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Indication for improved UV-behaviour:

=> With anisotropic scaling it is possible to make the gravitational coupling constant DIMENSIONLESS:

$$[g_K] = [\kappa]^{(d-z)}$$

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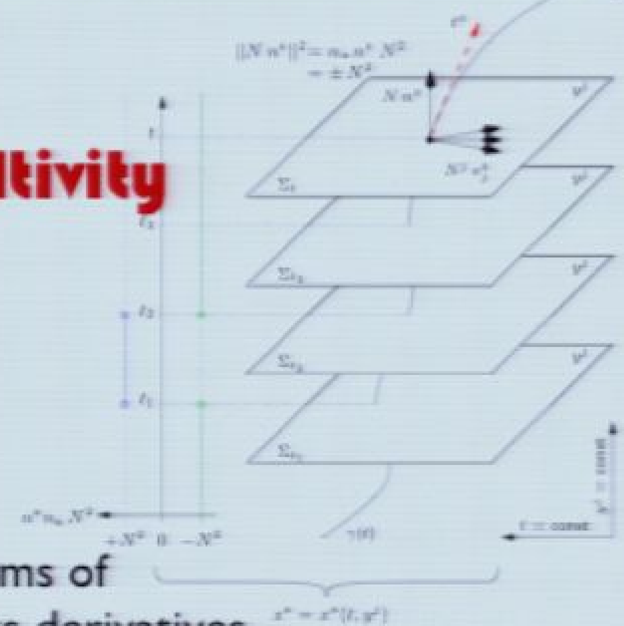

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$$d = z$$

anisotropic scaling also effects potential term...

Notice: The kinetic term is a generic feature of any Horava-like model!!!

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QG-IV: Modified Einstein-Hilbert action - (A)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

• Specializing to 3+1 dimension: $[\mathscr{V}(g)] \to [\kappa]^6$

• remember that the induced metric is dimensionless and thus Riemann had dim of $[\kappa]^2$:

$$\left\{(\text{Riemann})^3, [\nabla(\text{Riemann})]^2, (\text{Riemann})\nabla^2(\text{Riemann}), \nabla^4(\text{Riemann})\right\}.$$

• in d=3 the Weyl tensor automatically vanishes => decompose Riemann tensor into Ricci tensor, Ricci scalar, plus metric:

$$\left\{(\text{Ricci})^3, [\nabla(\text{Ricci})]^2, (\text{Ricci})\nabla^2(\text{Ricci}), \nabla^4(\text{Ricci})\right\}.$$

Main difference to Horava's original approach: we consider all possible terms, but hold on to parity...

QG-LV: Anisotropic scaling - (iii)

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QG-LV: Modified Einstein-Hilbert action - (B)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

- **Application of:**
  - Integration by parts and discarding surface terms.
  - Commutator identities.
  - Bianchi identities.
  - Special relations appropriate to 3 dimensions. (Weyl vanishes; properties of Cotton tensor.)

Compare with Horavas detailed balance, involving a particular combinations of these five terms: $C^i{}_j\, C^j{}_i$

$[\kappa]^5$: (Cotton) × (Einstein) = (Cotton) × (Ricci)

PROJECTABILITY!!!

- **Five independent terms terms:**

$$[\kappa]^6 : \quad R^3, \quad R\, R^i{}_j R^j{}_i, \quad R^i{}_j R^j{}_k R^k{}_i; \quad R\, \nabla^2 R, \quad \nabla_i R_{jk}\, \nabla^i R^{jk}$$

+ all possible lower-dimension terms (relevant operators, super-renormalizable by power-counting):

$$[\kappa]^0 : \ 1; \qquad [\kappa]^2 : \ R; \qquad [\kappa]^4 : \ R^2; \ R^{ij} R_{ij}.$$

QG-LIV: Modified Einstein-Hilbert action - (C)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Full classical action: $S = \int [\mathscr{T}(K) - \mathscr{V}(g)] \sqrt{g}\, N\, \mathrm{d}^3x\, \mathrm{d}t,$

$$\begin{aligned} \mathscr{V}(g) &= g_0\,\zeta^6 + g_1\,\zeta^4\, R + g_2\,\zeta^2\, R^2 + g_3\,\zeta^2\, R_{ij}R^{ij} \\ &\quad + g_4\, R^3 + g_5\, R(R_{ij}R^{ij}) + g_6\, R^i{}_j R^j{}_k R^k{}_i \\ &\quad + g_7\, R\nabla^2 R + g_8\, \nabla_i R_{jk}\, \nabla^i R^{jk} \end{aligned}$$

Splitting up into EH and LIV terms:

$$S_{\mathrm{EH}} = \int \left\{ (K^{ij}K_{ij} - K^2) + \zeta^4 R - g_0\,\zeta^6 \right\} \sqrt{g}\, N\, \mathrm{d}^3x\, \mathrm{d}t,$$

$$\begin{aligned} S_{\mathrm{LV}} &= \int \Big\{ \xi\, K^2 - g_2\,\zeta^2\, R^2 - g_3\,\zeta^2\, R_{ij}R^{ij} \\ &\quad - g_4\, R^3 - g_5\, R(R_{ij}R^{ij}) - g_6\, R^i{}_j R^j{}_k R^k{}_i \\ &\quad - g_7\, R\nabla^2 R - g_8\, \nabla_i R_{jk}\, \nabla^i R^{jk} \Big\} \sqrt{g}\, N\, \mathrm{d}^d x\, \mathrm{d}t. \end{aligned}$$

QG-IV: Modified Einstein-Hilbert action - (D)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Going to "physical" units (c -> 1). [dx]=[dt]:

$$S_{\rm EH} = \zeta^2 \int \left\{ (K^{ij} K_{ij} - K^2) + R - g_0\, \zeta^2 \right\} \sqrt{g}\; N\; {\rm d}^3x\; {\rm d}t$$

- **the Planck scale in our model**

$$(16 \pi G_{\rm Newton})^{-1} = \zeta^2$$

- **the cosmological constant**

$$\Lambda = \frac{g_0\, \zeta^2}{2}$$

Horavas original model: A non-zero Newton constant requires a non-zero cosmological constant, of the wrong sign to be compatible with cosmological observations...

QG-LV: Equations of motion - (overview)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

$$S = \int \left[\mathscr{T}(K) - \mathscr{V}(g)\right] \sqrt{g}\, N\, \mathrm{d}^3x\, \mathrm{d}t$$

- **Hamiltonian constraint:** vary w.r.t. lapse
- **Super-momentum constraint:** vary w.r.t. shift
- **Dynamical equations:** vary w.r.t. induced / spatial metric

At every stage we will discuss the differences in our model compared with standard General Relativity...

QG-LV: Equations of motion - (I)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Hamiltonian constraint: varying w.r.t. lapse

$$H = \int \sqrt{g}\, \mathscr{H}(K, g)\, \mathrm{d}^3x = \int \sqrt{g}\, \{\mathscr{T}(K) + \mathscr{V}(g)\}\, \mathrm{d}^3x = 0$$

NEW!

- LIV term in kinetic term;
- modified more complicated form of potential;
- "projectability" condition on Lapse leads to spatially integrand Hamiltonian constraint;

  => Generic to all Horava models! **poss. solution...**

QG-LV: Equations of motion - (2)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Super-momentum constraint: varying w.r.t. shift

- super-momentum

NEW!

$$\pi^{ij} = \frac{\partial \mathscr{T}(K)}{\partial \dot{g}_{ij}} = -\left\{K^{ij} - Kg^{ij} + \xi K g^{ij}\right\}$$

- super-momentum constraint

$$\nabla_i \pi^{ij} = 0$$

=> Generic to all Horava models!

QG-LIV: Equations of motion - (3)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Dynamical equations: varying w.r.t. induced metric

$$
\begin{aligned}
\frac{1}{\sqrt{g}}\,\partial_t(\sqrt{g}\,\pi^{ij}) \;=\;& -2\left\{(K^2)^{ij} - K K^{ij} + \xi K K^{ij}\right\} \\
& + \frac{1}{2}\mathscr{T}(K)\, g^{ij} + (\nabla_m N^m)\,\pi^{ij} + [\text{Ł}_{\vec{N}}\pi]^{ij} \\
& + \frac{1}{\sqrt{g}}\frac{\delta S_{\mathscr{V}}}{\delta g_{ij}}.
\end{aligned}
$$

NEW!

- LIV term in kinetic term;
- modified more complicated form of potential;
- "projectability" condition no spatial derivatives of lapse

=> Generic to all Horava models!

QG-LV: Equations of motion - (appendix)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Our specific forcing term:

$$F^{ij} = \frac{1}{\sqrt{g}} \frac{\delta S_{\mathcal{V}}}{\delta g_{ij}}$$

> is the most general symmetric conserved tensor one can build out of the metric and 0, 2, 4, or 6 derivatives

$$F^{ij} = \sum_{s=0}^{8} g_s \, \zeta^{n_s} \, (F_s)^{ij}$$

g_0: $1 \to$

$$(F_0)_{ij} = -\frac{1}{2} g_{ij}. \tag{1}$$

g_1: $R \to$

$$(F_1)_{ij} = G_{ij}. \tag{2}$$

g_2: $R^2 \to$

$$(F_2)_{ij} = 2RR_{ij} - \frac{1}{2}R^2 g_{ij} - 2[\nabla_i\nabla_j - g_{ij}\nabla^2]R. \tag{3}$$

g_3: $R_{mn}R^{mn} \to$

$$(F_3)_{ij} = \frac{1}{2}(R_{mn}R^{mn})g_{ij} + \nabla^2 R_{ij} + \frac{1}{2}g_{ij}\nabla^2 R - \nabla_i\nabla_j R + 3RR_{ij} - 4(R^2)_{ij} - R^2 g_{ij}. \tag{4}$$

g_4: $R^3 \to$

$$(F_4)_{ij} = 3R^2 R_{ij} - \frac{1}{2}R^3 g_{ij} - 3[\nabla_i\nabla_j - g_{ij}\nabla^2]R^2. \tag{5}$$

g_5: $R\,(R_{mn}R^{mn}) \to$

$$\begin{aligned}(F_5)_{ij} &= R_{ij}(R_{mn}R^{mn}) + 2R(R^2)_{ij} - \frac{1}{2}R\,(R_{mn}R^{mn})g_{ij} \\ &\quad + \left[\nabla^2(RR_{ij}) + \nabla_m\nabla_n(RR^{mn})g_{ij} - \nabla^k\nabla_i(RR_{jk}) - \nabla^k\nabla_j(RR_{ki})\right] \\ &\quad + \left[g_{ij}\nabla^2 + \nabla_i\nabla_j\right](R_{mn}R^{mn}).\end{aligned} \tag{6}$$

g_6: $R^m{}_n R^n{}_p R^p{}_m \to$

$$\begin{aligned}(F_6)_{ij} &= 3(R^3)_{ij} - \frac{1}{2}(R^l{}_m R^m{}_n R^n{}_l)g_{ij} \\ &\quad + \frac{3}{2}\left[\nabla^2(R^2)_{ij} + \nabla_n\nabla_m(R^2)^{nm}g_{ij} - \nabla^n\nabla_i(R^2)_{jn} - \nabla_n\nabla_j(R^2)_{in}\right].\end{aligned} \tag{7}$$

g_7: $(\nabla R)^2 = (\nabla_m R)g^{mn}(\nabla_n R) \to$

$$(F_7)_{ij} = -2[g_{ij}\nabla^2 - \nabla_i\nabla_j]\nabla^2 R - 2\nabla^2 RR_{ij} + \nabla_i R\nabla_j R - \frac{1}{2}(\nabla R)^2 g_{ij}. \tag{8}$$

g_8: $(\nabla_p R_{mn})(\nabla^p R^{mn}) \to$

$$\begin{aligned}(F_8)_{ij} &= \nabla^4 R_{ij} + (\nabla_p\nabla_q\nabla^2 R^{pq})g_{ij} - \nabla_p\nabla_i\nabla^2 R^p{}_j - \nabla_p\nabla_j\nabla^2 R^p{}_i \\ &\quad - \nabla_i R^m{}_n\nabla_j R^n{}_m - \frac{1}{2}(\nabla_p R_{mn})(\nabla^p R^{mn})g_{ij} - 2(\nabla^q R_i{}^p)(\nabla_q R_{jp}).\end{aligned} \tag{9}$$

QG-LV: Graviton propagator - (overview)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Now we study excitations in our "condensed matter system" to see if there is a connection to GR..?

Linearized equation of motion

- **Lin. Hamiltonian constraint**
- **Lin. Super-momentum constraint**
- **Lin. Dynamical equations**

+

Gauge fixing NEW!

- isometry under inf. coordinate changes

obtain the true degrees of freedom after eliminating coordinate artefacts

To do: Linearization, Gauge fixing, Scalar-tensor decomposition...

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QG-IV: Graviton propagator - (GP1)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

linearizing around flat spacetime:

$$^0g_{ij} = \delta_{ij}; \qquad ^0N_i = 0; \qquad ^0N = 1$$

linearized perturbations:

$$g_{ij} = \delta_{ij} + \epsilon\, h_{ij}; \qquad N_i = \epsilon\, n_i; \qquad N = 1 + \epsilon\, n(t)$$

$$g^{ij} = \delta^{ij} - \epsilon\, h^{ij} + O(\epsilon^2); \quad N^i = \epsilon\, n^i + O(\epsilon^2); \quad N = 1 + \epsilon\, n(t)$$

Now expand the extrinsic curvature K_{ij}, the conjugate momentum π_{ij}, the intrinsic Ricci curvature R_{ij} and forcing term F_{ij} as a series in ϵ of the form

$$X =^0 X + \epsilon\, {}^1X + O(\epsilon^2)$$

In fact K_{ij}, π_{ij}, R_{ij}, and F_{ij} all vanish at zeroth order and the first non-trivial contribution arises at order ϵ.

QG-IV: Graviton propagator - (GP2)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

- **Lin. Hamiltonian constraint:** $${}^1H \equiv g_1 \int {}^1R \, \mathrm{d}^3x = 0$$

- **Lin. Super-momentum constraint:**

Gauge fixing

$$\partial^i \left\{ \dot{h}_{ij} - (1-\xi)\delta_{ij}\dot{h} \right\} = \boxed{\partial^2 n_j - (1-2\xi)\partial_j(\vec{\partial} \cdot \vec{n})}$$

- **Lin. Dynamical equations:**

$$\frac{1}{2}\left\{ \ddot{h}_{ij} - (1-\xi)\delta_{ij}\ddot{h} \right\} = \frac{1}{2}\left\{ \partial_i \dot{n}_j + \partial_j \dot{n}_i - 2(1-\xi)\delta_{ij}(\vec{\partial} \cdot \dot{\vec{n}}) \right\} + {}^1F_{ij}$$

QG-LV: Graviton propagator - (GP3)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Gauge fixing

$$t \to t + \epsilon\chi^0(t) + \ldots; \qquad x^i \to x^i + \epsilon\chi^i(t,x) + \ldots$$

Notice: reduced diffeomorphism

$$\begin{aligned} N(t) &\to N(t) - \epsilon\,\dot{\chi}^0(t) + \ldots; \\ N_i &\to N_i + \epsilon\,\dot{\chi}_i(t,x) + \ldots; \\ g_{ij} &\to g_{ij} + \epsilon\left[\partial_i\chi_j(t,x) + \partial_j\chi_i(t,x)\right] + \ldots \end{aligned}$$

$$n \to n - \dot{\chi}^0(t)$$

$$n_i \to n_i + \dot{\chi}_i(t,x)$$

$$h_{ij} \to h_{ij} + \{\partial_i\chi_j(t,x) + \partial_j\chi_i(t,x)\}$$

$$h \to h + 2\,\partial_i\chi^i(t,x)$$

Synchronous gauge:

$$n \equiv 0; \qquad n_i \equiv 0;$$

$$h_{ij} \to h_{ij} + \partial_i\bar{\chi}_j(x) + \partial_j\bar{\chi}_i(x);$$

$$h \to h + 2\,\partial_i\bar{\chi}^i(x).$$

Residual gauge:

Scalar tensor decomposition!

QG-LV: Graviton propagator - (GP4)

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Spin-0 scalar graviton:

$$\left(1-\frac{3}{2}\xi\right)\ddot{h}=-\xi\left\{\frac{1}{2}g_1\partial^2+\left(-4g_2-\frac{3}{2}g_3\right)\partial^4+\left(4g_7-\frac{3}{2}g_8\right)\partial^6\right\}h$$

Spin-2 tensor graviton:

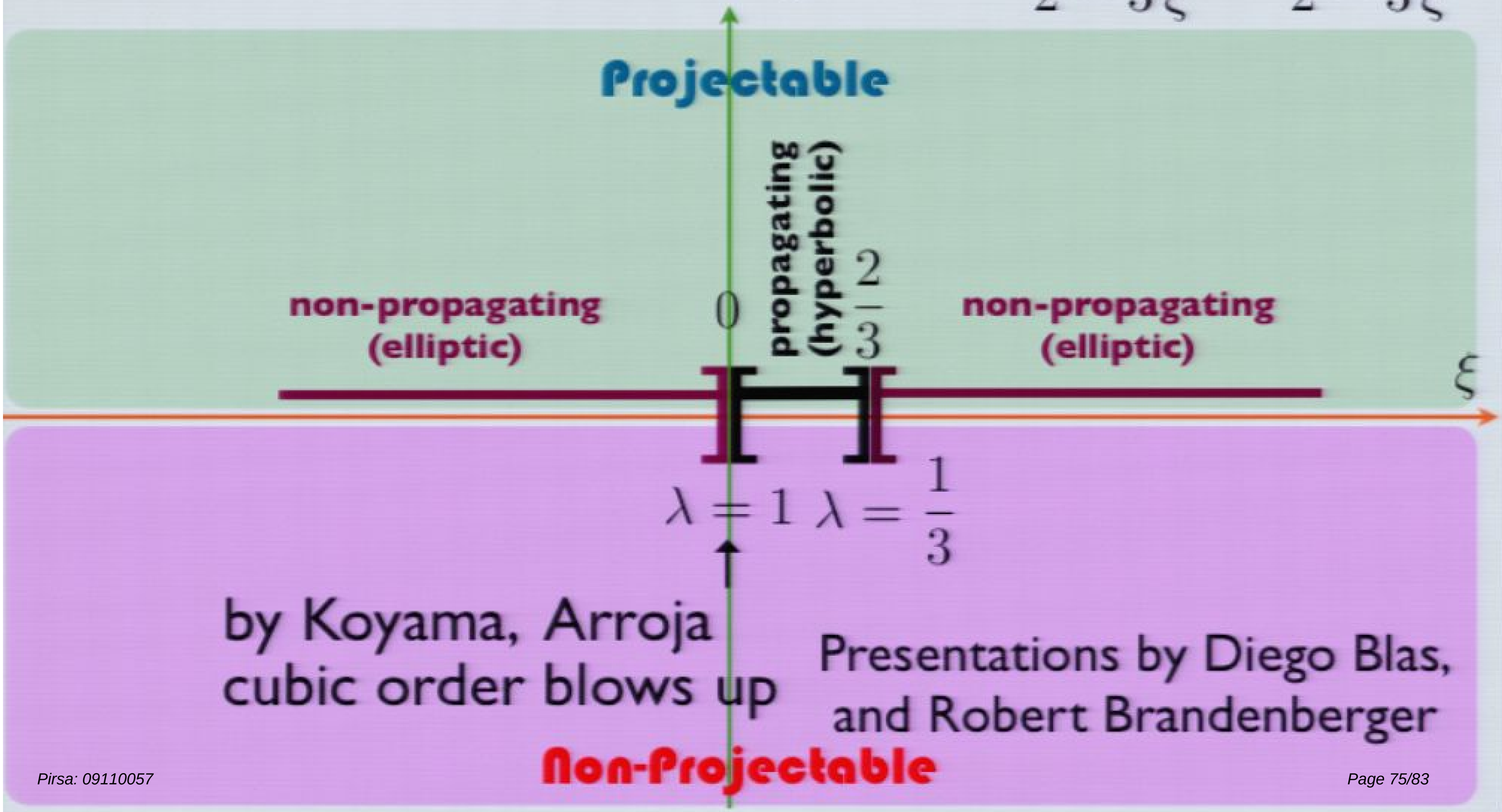
$$\ddot{\widetilde{H}}_{ij}=-\left[g_1\partial^2+g_3\partial^4+g_8\partial^6\right]\widetilde{H}_{ij}$$

=> both are sixth-order trans-Bogoliubov dispersion relations: $\dfrac{1}{\omega^2-c^2\mathbf{k}^2-G(\mathbf{k}^2)^z}$

QG-LV: analysis of scalar mode

Thomas Sotriou, Matt Visser, S.W., PRL102:251601 (arXiv:0904.4464) and arXiv:0905.2798, 2009.

Different parameter regimes: $c^2_{\text{spin}-0} = -\dfrac{\xi\, g_1}{2-3\,\xi} \to \dfrac{\xi}{2-3\,\xi}$



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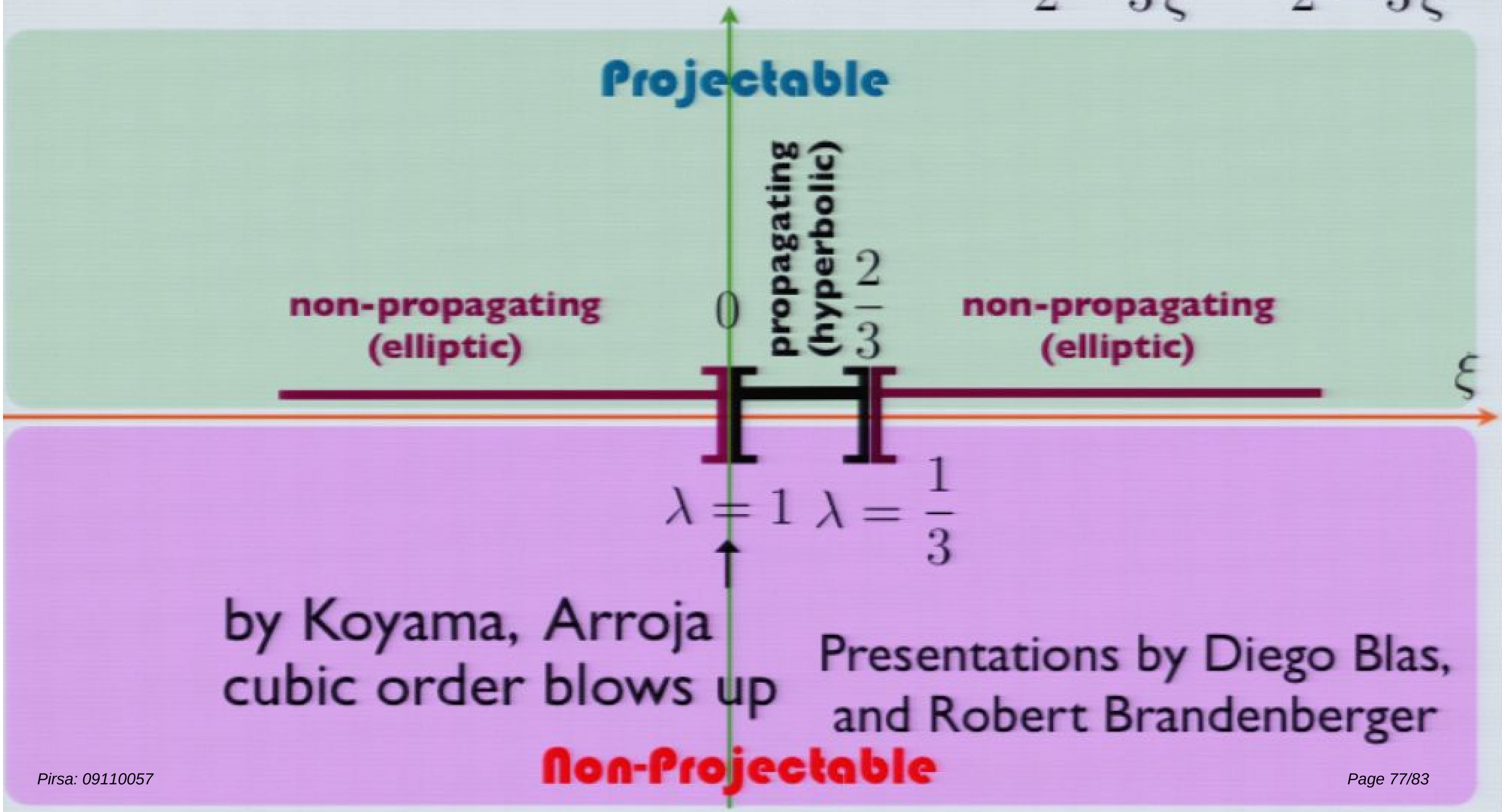

Advertisement for upcoming talk on the topic...

Future work:
spectral dimensions &
emergent field theories
and uv-behaviour & principles of
emergence..?

- Excitations
- UV-behaviour
- Detailed balance
- How to add matter to the theory
- Exact solutions
- Projectability

Gravity
Emergent

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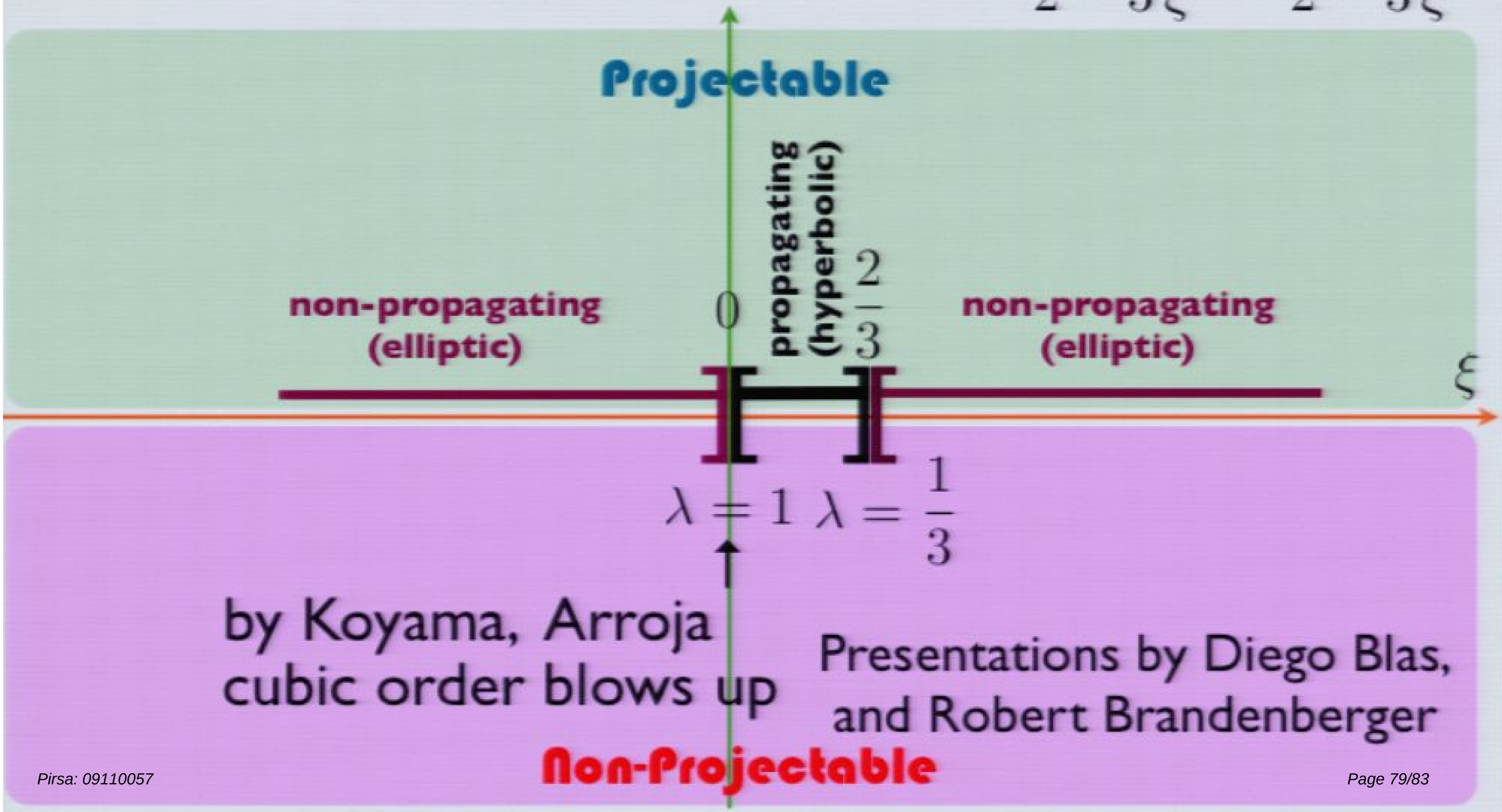

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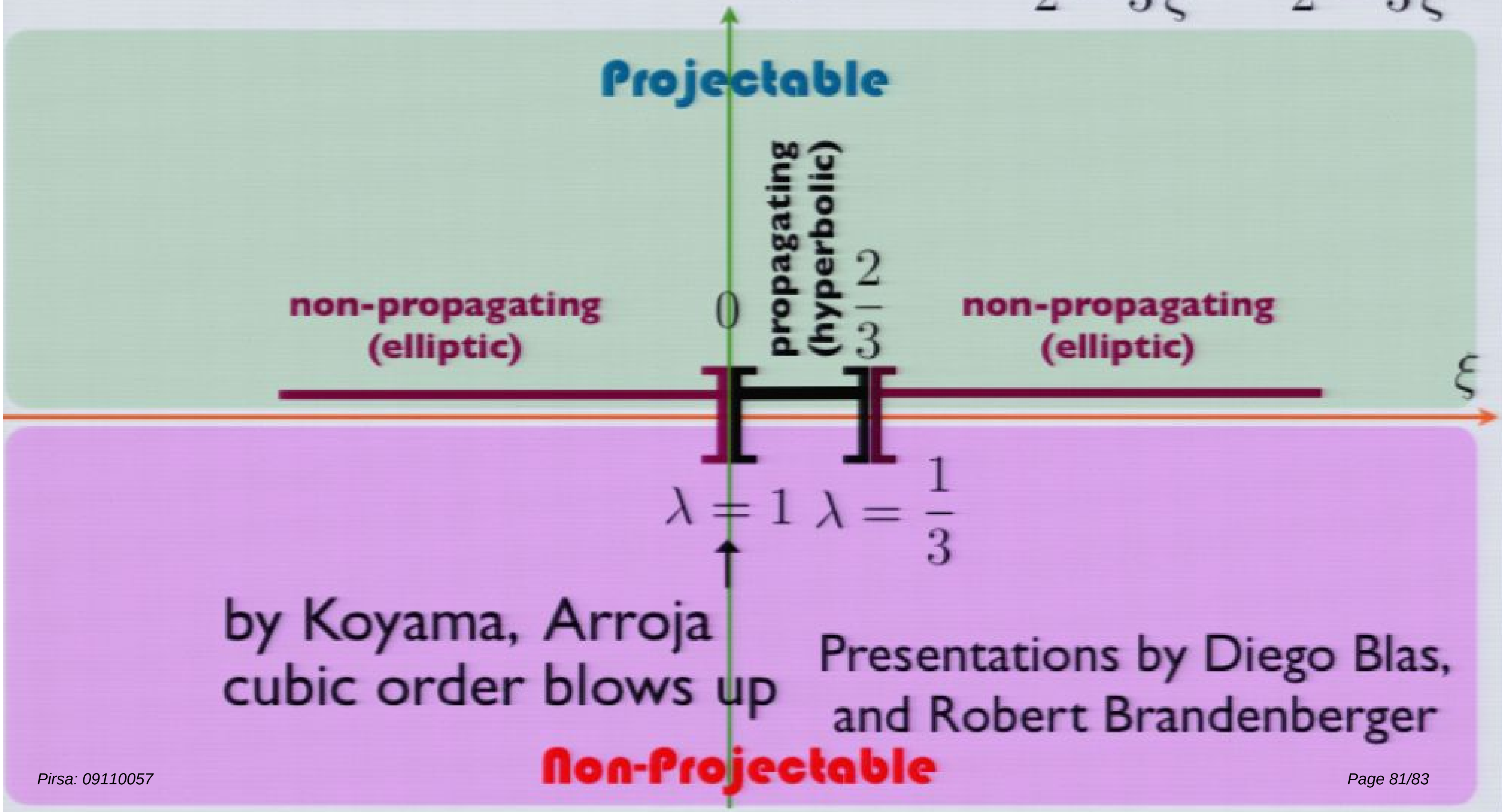

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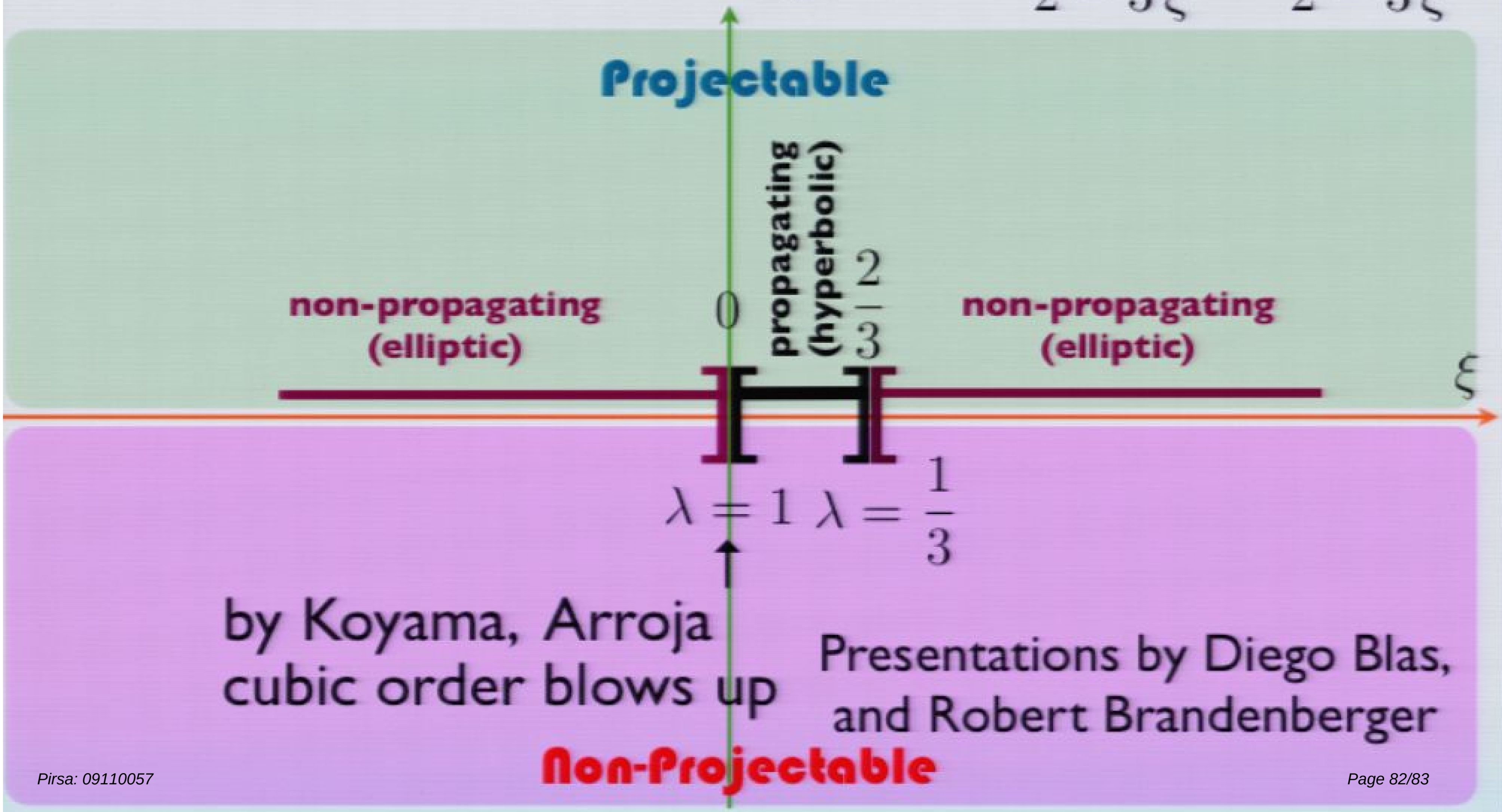

No Signal
VGA-1