

Title: A "healthy" proposal for non-relativistic quantum gravity

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Abstract:

A "healthy" proposal for non-relativistic quantum gravity

Diego Blas Temiño



Switzerland

based on

D. Blas, O. Pujolàs and S. Sibiryakov ([arXiv:0906.3046](https://arxiv.org/abs/0906.3046)) JHEP10(2009)029

D. Blas, O. Pujolàs and S. Sibiryakov ([arXiv:0909.3525](https://arxiv.org/abs/0909.3525))



Outline

- ① General framework
- ② Degrees of freedom
- ③ Covariant form and strong coupling scales
- ④ First phenomenological tests
- ⑤ Conclusions and open issues



General framework

Following the proposal by P. Hořava:

0901.3775

Theory of quantum gravity with

- △ $z = 3$ scaling in the UV (power counting renormalizable).
- △ Space-time endowed with a preferred $3 + 1$ foliation (x^i, t) .
- △ Invariant under foliation preserving Diff

$$x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$$

- △ Naïvely approaching GR in the IR (in the preferred foliation).

$$\mathcal{L} = M_P^2 \sqrt{\gamma} N \left[\left(K_{ij} K^{ij} - \lambda K^2 + {}^{(3)}R \right) + O(1/M_P^2) V[\gamma_{ij}] + M_P^2 \Lambda \right]$$

- △ $\lambda \rightarrow 1$ is a problematic limit ($\delta\lambda \equiv \lambda - 1 \rightarrow 0$)

General framework

Extending the proposal by P. Hořava:

0909.3525

Theory of quantum gravity with

- △ $z = 3$ scaling in the UV (power counting renormalizable).
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- △ Invariant under foliation preserving Diff

$$x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$$

- △ Well defined dynamics in the IR and generality.

$$\begin{aligned} \mathcal{L} = M_P^2 \sqrt{\gamma} N \left(K_{ij} K^{ij} - \lambda K^2 + {}^{(3)}R + \alpha N^{-2} \gamma^{ij} \partial_i N \partial_j N \right. \\ \left. + O(1/M_P^2) V[\gamma_{ij}, \partial_i \ln N (\beta_i)] + M_P^2 \Lambda \right) \end{aligned}$$

- △ No GR in the IR
 - $\alpha, \beta_i \rightarrow 0$, non-projectable case
 - $\alpha \rightarrow \infty$, projectable case ($\partial_i N = 0$, $N = N(t)$)

Degrees of freedom: the generic case

The contributions at quadratic order come from ($a_i \equiv N^{-1} \partial_i N$)

- (dim 2) $R, \alpha a_i a^i$
- (dim 4) $R^2, R_{ij} R^{ij}, \beta_1 R \nabla_i a^i, \beta_2 a_i \Delta a^i$
- (dim 6) $(\nabla_i R_{jk})^2, (\nabla_i R)^2, \beta_3 \Delta R \nabla_i a^i, \beta_4 a_i \Delta^2 a^i$

Around Minkowski (scalar sector, tensors as in GR up to $O(M_P^{-2})$)

$$N = 1 + \phi, \quad N_i = \frac{\partial_i B}{\sqrt{\Delta}}, \quad \gamma_{ij} = \delta_{ij} - 2 \left(\delta_{ij} - \frac{\partial_i \partial_j}{\Delta} \right) \psi - 2 \frac{\partial_i \partial_j}{\Delta} E$$

After gauge fixing, the remaining scalar DOF is described by

$${}^{(2)} \mathcal{L} = M_P^2 \left\{ \frac{2(3\lambda - 1)}{\lambda - 1} \psi^2 + \psi \frac{P[M_P^{-2} \Delta, \alpha]}{Q[M_P^{-2} \Delta, \alpha]} \Delta \psi \right\}$$

△ “Regular” DOF for: $\frac{3\lambda - 1}{\lambda - 1} > 0, \frac{P[M_P^{-2} \Delta, \alpha]}{Q[M_P^{-2} \Delta, \alpha]} > 0 \quad (0 < \alpha < 2)$
 (gapless mode with $c_*^2(\Delta)$ and $c_*^2(0) \neq 1$)



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Degrees of freedom: particular limits

$${}^{(2)}\mathcal{L} = M_P^2 \left\{ \frac{2(3\lambda - 1)}{\lambda - 1} \dot{\psi}^2 + \dot{\psi} \frac{P[M_P^{-2}\Delta, \alpha]}{Q[M_P^{-2}\Delta, \alpha]} \Delta \psi \right\}$$

► Projectable case: $\alpha \rightarrow \infty$.

Sotiriu, Visser, Weinfurtner 0905.2798

- Imaginary sound speed at low energies: $c_*^2(0) = -\frac{\lambda-1}{3\lambda-1} < 0$
- The instability can be cut-off at $p \sim M_P$,

$$\omega^2 = c_*^2(0)p^2 + O(p^2/M_P^2)$$

still the decay rate is $\Gamma \sim |c_*(0)|M_P$, i.e. one needs $\lambda - 1 \ll 1$
 ($\Gamma \lesssim H_0$ implies $\delta\lambda_p \lesssim 10^{-120}$).

► Non-projectable case: $\alpha, \beta_i \rightarrow 0$.

- $Q[M_P^{-2}\Delta, \alpha] = 0$: singular limit (extra constraint).
- NO extra DOF around Minkowski at quadratic order :
 2 polarizations (4 initial conditions).

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Degrees of freedom: non-linear analysis

The equations of motion (gauge $N_i = 0$)

0906.3046

- Constraints:

$$\delta N : -K_{ij}K^{ij} + \lambda K^2 + V[g_{ij}, \beta_i] + \alpha f(N) = 0, \quad \delta N^i : \nabla_i K^{ij} - \lambda \nabla^j K = 0$$

- Dynamical equations:

$$\delta \gamma^{ij} : -\frac{\partial}{\partial t}(K^{ij} - \lambda \gamma^{ij} K) + \dots = 0, \quad (\dot{\gamma}^{ij} = 2NK_{ij})$$

Cauchy data $(N, \gamma_{ij}, K_{ij}) = 13$ variables

- ▶ $\alpha, \beta_i \neq 0$: δN fixes N ($\alpha \rightarrow \infty$: $f(N) = 0$, i.e. $N \equiv N(t)$)
 $13 - 4(\text{const.}) - 3(\text{resid.}) = 6$ initial conditions.

- ▶ $\alpha = \beta_i = 0$: $\partial_t \delta N \neq 0$, i.e. secondary that fixes N
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Stückelberg formalism

Diff invariance is naïvely restored by adding the DOF of the foliation $\rho(x)$:

- ▶ surfaces of the foliation defined by $\rho(x) = ct$.
- ▶ $t \mapsto f(t)$ promotes to $\rho \mapsto f(\rho)$.
- ▶ dt promotes to $u_\mu dx^\mu \equiv \frac{\partial_\mu \rho}{\sqrt{g^{\mu\nu} \partial_\mu \rho \partial_\nu \rho}} dx^\mu$, (δ_i^j to $P_\nu^\mu \equiv \delta_\nu^\mu - u^\mu u_\nu$).

The other objects

$$K_{ij} \rightarrow \mathcal{K}_{\mu\nu} = P_{\sigma\beta} \nabla^\beta u^\sigma, \quad ({}^3)R^i{}_{jkl} \rightarrow P_\alpha^\mu P_\nu^\beta P_\rho^\gamma P_\sigma^\delta ({}^4)R^\alpha{}_{\beta\gamma\delta} + \mathcal{K}_\rho^\mu \mathcal{K}_{\nu\sigma} - \mathcal{K}_\sigma^\mu \mathcal{K}_{\nu\rho}$$

△ Extended covariant action

$$\mathcal{L} = M_{\text{Pl}}^2 \left[\sqrt{-g} \left(-({}^4)R + (1 - \lambda)\mathcal{K} + \alpha P^{\mu\nu} \nabla_\mu u^\sigma \nabla_\nu u_\sigma \right) + O(1/M_{\text{Pl}}^2) V \right]$$

- Clear advantage of this formalism: GR+scalar field $\rho(x)$
- Most general action with dimension 2 operators ($[\rho] = L^{-1}$), invariant under Diff and reparametrizations of $\rho!$

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- ▶ For $\nabla_i \bar{K}^{jk} \sim L^{-2} \neq 0$, and perturbations with ω , $p \gg L^{-1}$,
 - For $\omega \sim p$: **2** polarizations (TT part).
 - For $\omega \gg p$: **1** polarization (κ) with **first order DE**,

$$\omega = \frac{\bar{N} p^4}{(1 - 3\lambda) p^i \partial_j \bar{K}} \sim (pL)^2 p \gg p.$$



Stückelberg formalism

Diff invariance is naïvely restored by adding the DOF of the foliation $\rho(x)$:

- ▶ surfaces of the foliation defined by $\rho(x) = ct$.
- ▶ $t \mapsto f(t)$ promotes to $\rho \mapsto f(\rho)$.
- ▶ dt promotes to $u_\mu dx^\mu \equiv \frac{\partial_\mu \rho}{\sqrt{g^{\mu\nu} \partial_\mu \rho \partial_\nu \rho}} dx^\mu$, (δ_i^j to $P_\nu^\mu \equiv \delta_\nu^\mu - u^\mu u_\nu$).

The other objects

$$K_{ij} \rightarrow \mathcal{K}_{\mu\nu} = P_{\sigma\beta} \nabla^\beta u^\sigma, \quad (3) \quad R^i_{jkl} \rightarrow P_\alpha^\mu P_\nu^\beta P_\rho^\gamma P_\sigma^\delta (4) \quad R^\alpha_{\beta\gamma\delta} + \mathcal{K}_\rho^\mu \mathcal{K}_{\nu\sigma} - \mathcal{K}_\sigma^\mu \mathcal{K}_{\nu\rho}$$

△ Extended covariant action

$$\mathcal{L} = M_{\tilde{P}}^2 \left[\sqrt{-g} \left(-{}^{(4)}R + (1 - \lambda)\mathcal{K} + \alpha P^{\mu\nu} \nabla_\mu u^\sigma \nabla_\nu u_\sigma \right) + O(1/M_{\tilde{P}}^2) V \right]$$

- Clear advantage of this formalism: GR+scalar field $\rho(x)$
- Most general action with dimension 2 operators ($[\rho] = L^{-1}$), invariant under Diff and reparametrizations of ρ !



Stückelberg formalism: EOM

Equations of motion:

- ▶ 10 coming from $\delta g_{\mu\nu}$ (4 constraints)
- ▶ 1 coming from $\delta\rho$: $\nabla_\mu J^\mu = 0$ with $u_\mu J^\mu = 0$.

Different gauge choices

- **Unitary gauge**: $\rho(x) = t$: non covariant results reproduced.
 - ▶ Only **3** residual tr: $\delta\rho = \xi_0 = 0$. Extra DOF in the metric!
 - ▶ The EOM for N turns into the **secondary constraint**

$$\nabla_\mu J^\mu = \nabla_i((1 - \lambda)N\nabla^i K + \dots) = 0.$$

- Any other gauge has **4** residual inv. BUT
 - △ The EOM for ρ is in general of forth order!

$$\mathcal{K}^2 \sim (\square\rho + \dots)^2$$

Puzzle solved (at linear order) choosing the frame (background)

$\bar{\rho} = t$: the EOM for $\zeta = \rho - \bar{\rho}$ are of first order in time.

△ NOT COVARIANT: the Cauchy problem has a preferred foliation!

Stückelberg formalism: instability and strong coupling

Phenomenologically we expect $|\delta\lambda| \ll 1$ (scalar+tensor $\delta\lambda \sim 10^{-7}$)
 Thus the **decoupling limit** is physical: $\mathcal{L} = M_P^2 [\sqrt{-g}R + (1 - \lambda)P(\phi)]$

$$M_P \rightarrow \infty, \quad M_P(\lambda - 1)^{1/2} \sim 10^{15} \text{ GeV fixed.}$$

The EOM for ζ (only piece with **interactions**) is ($\omega, p \gg \bar{K} \sim L^{-1}$)

$$M_P^2(\lambda - 1) [2\partial^i \bar{K} \partial_i \dot{\zeta} + \bar{N} \Delta^2 \zeta + 6\partial^i \bar{N} \partial_i \Delta \zeta + \partial_t (\Delta \zeta)^2] = 0.$$

- ▶ Dispersion relation: $\omega = -\frac{Np^4}{2p^i \partial_i K} + O(pL) \sim (pL)^2 p + i(pL)p$
 Very fast instability!
- ▶ After canonical normalization we find a strong coupling scale

$$p \lesssim \Lambda_{np} \equiv L^{-3/4} |1 - \lambda|^{1/8} M_P^{1/4}$$

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Projectable case: strong coupling

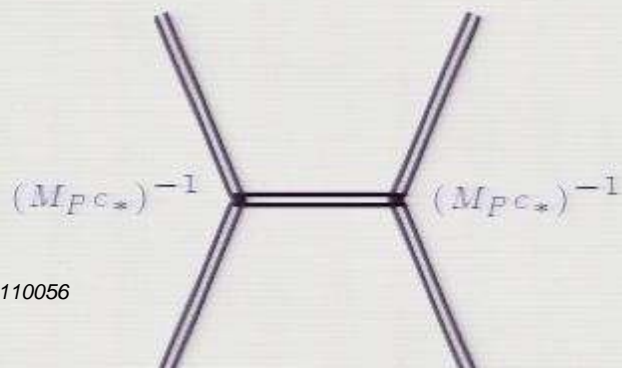
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From stability we found $c_*^2 < 0$ and $|c_*| \leq 10^{-61}$ ($|c_*| \sim \sqrt{\lambda - 1}$).
 No need for the Stückelberg formalism to find a first strong coupling scale:

Koyama, Arroja 0910.1998

$$\Lambda_p \sim M_P |c_*| \sim 10^{-34} \text{ eV}$$



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0906.3046

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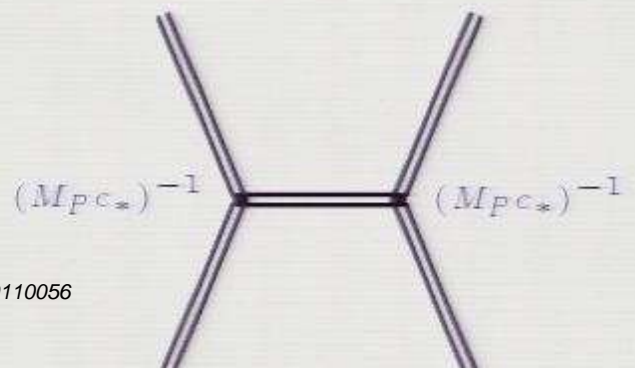
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Remaining “healthy” possibilities
(no strong coupling or instabilities):

$$0 < \alpha < 2.$$

- ▶ Scalar-tensor theory (also in the IR)
 - **Aether**-like theory with $u_\mu \equiv \frac{\partial_\mu \rho}{\sqrt{g^{\sigma\nu} \partial_\sigma \rho \partial_\nu \rho}}$.
- ▶ Invariant under reparametrizations of the scalar.
- ▶ Breaking of Lorentz invariance controlled by the scalar.

First tests

△ Newton's law and light deviation (linear)

- For sources static in the preferred frame, same as in GR with

$$G_N = \frac{1}{8\pi M_P^2 (1 - \alpha/2)}$$

No constraints!

- For sources moving wrt the preferred frame, effects of order $\alpha, \delta\lambda$. PPN constraints are $O(10^{-7}) - O(10^{-4})$.

Foster and Jacobson 06; Mattingly 05

△ Homogeneous Cosmology: as GR with $(\partial_i N = 0)$

$$G_c = \frac{2}{8\pi M_P^2 (3\lambda - 1)}$$

- From BBN (${}^4\text{He}$ abundance): $G_N/G_c = 1 + O(10^{-2})$

Conclusions

- A “healthy” non-relativistic theory of quantum gravity is possible.
- For the model to remain weakly coupled, the massless modes in the UV must appear also in the IR.
- This is the case in the most general situation $0 < \alpha < \infty$: IR limit is GR with an additional scalar mode and a preferred frame.
- First phenomenological constraints $O(10^{-7})$ ($\Lambda_w \sim 10^{15}$ GeV).

Open issues

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- ▶ Recovery of the Lorentz invariance in the IR.
(Perturbative or non perturbative mechanism to generate a mass-gap for ρ ? symmetry? getting just 2 DOF?)
- ▶ More phenomenological test.
(Cosmology, preferred frame, Čerenkov...).
- ▶ Exact solutions and black holes.
(IR solution similar to the aether theories where there are some known solutions. Also, there are black hole solutions with no hair! BH Thermodynamics?)