Title: A "healthy" proposal for non-relativistic quantum gravity

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Abstract:

A "healthy" proposal for non-relativistic quantum gravity

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based on

D. Blas, O. Pujolàs and S. Sibiryakov (*arXiv:0906.3046*) JHEP10(2009)029

D. Blas, O. Pujolàs and S. Sibiryakov (*arXiv:0909.3525*)

Outline

1. General framework
2. Degrees of freedom
3. Covariant form and strong coupling scales
4. First phenomenological tests
5. Conclusions and open issues

General framework

Following the proposal by P. Hořava: 0901.3775

Theory of quantum gravity with

- △ $z = 3$ scaling in the UV (power counting renormalizable).
- △ Space-time endowed with a preferred $3+1$ foliation (x^i, t).
- △ Invariant under foliation preserving Diff

$$x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$$

- △ Naïvely approaching GR in the IR (in the preferred foliation).

$$\mathcal{L} = M_P^2 \sqrt{\gamma} N \left[\left(K_{ij} K^{ij} - \lambda K^2 + {}^{(3)}R \right) + O(1/M_P^2) V[\gamma_{ij}] + M_P^2 \Lambda \right]$$

⚠ $\lambda \to 1$ is a problematic limit $(\delta\lambda \equiv \lambda - 1 \to 0)$

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$$x^i \mapsto \tilde{x}^i(x^j, t), \quad t \mapsto f(t).$$

△ Well defined dynamics in the IR and generality.

$$\mathcal{L} = M_P^2 \sqrt{\gamma} N \Big(K_{ij} K^{ij} - \lambda K^2 + {}^{(3)}R + \alpha N^{-2} \gamma^{ij} \partial_i N \partial_j N$$
$$+ O(1/M_P^2)\, V[\gamma_{ij}, \partial_i \ln N \; (\beta_i)] + M_P^2 \Lambda \Big)$$

⚠ No GR in the IR

- α, $\beta_i \to 0$, non-projectable case
- $\alpha \to \infty$, projectable case $(\partial_i N = 0,\ N = N(t))$

Degrees of freedom: the generic case

The contributions at quadratic order come from ($a_i \equiv N^{-1}\partial_i N$)

- (dim 2) R, $\alpha\, a_i a^i$
- (dim 4) R^2, $R_{ij}R^{ij}$, $\beta_1 R\nabla_i a^i$, $\beta_2\, a_i \Delta a^i$
- (dim 6) $(\nabla_i R_{jk})^2$, $(\nabla_i R)^2$, $\beta_3 \Delta R \nabla_i a^i$, $\beta_4\, a_i \Delta^2 a^i$

Around Minkowski (scalar sector, tensors as in GR up to $O(M_P^{-2})$)

$$N = 1 + \phi,\ N_i = \frac{\partial_i}{\sqrt{\Delta}} B,\ \gamma_{ij} = \delta_{ij} - 2\left(\delta_{ij} - \frac{\partial_i \partial_j}{\Delta}\right)\psi - 2\frac{\partial_i\partial_j}{\Delta} E$$

After gauge fixing, the remaining scalar DOF is described by

$$^{(2)}\mathcal{L} = M_P^2 \left\{ \frac{2(3\lambda - 1)}{\lambda - 1} \dot{\psi}^2 + \psi \frac{P[M_P^{-2}\Delta, \alpha]}{Q[M_P^{-2}\Delta, \alpha]} \Delta \psi \right\}$$

$\triangle$ "Regular" DOF for: $\frac{3\lambda - 1}{\lambda - 1} > 0$, $\frac{P[M_P^{-2}\Delta,\alpha]}{Q[M_P^{-2}\Delta,\alpha]} > 0 \quad (0 < \alpha < 2)$

(gapless mode with $c_*^2(\Delta)$ and $c_*^2(0) \neq 1$)

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Degrees of freedom: particular limits

$$^{(2)}\mathcal{L} = M_P^2 \left\{ \frac{2(3\lambda-1)}{\lambda-1}\dot{\psi}^2 + \psi \frac{P[M_P^{-2}\Delta, \alpha]}{Q[M_P^{-2}\Delta, \alpha]} \Delta \psi \right\}$$

- Projectable case: $\alpha \to \infty$. Sotiriu, Visser, Weinfurtner 0905.2798
  - Imaginary sound speed at low energies: $c_*^2(0) = -\frac{\lambda-1}{3\lambda-1} < 0$
  - The instability can be cut-off at $p \sim M_P$,

$$\omega^2 = c_*^2(0)p^2 + O(p^2/M_P^2)$$

still the decay rate is $\Gamma \sim |c_*(0)|M_P$, i.e. one needs $\lambda - 1 \ll 1$ ($\Gamma \lesssim H_0$ implies $\delta\lambda_p \lesssim 10^{-120}$).

- Non-projectable case: $\alpha, \ \beta_i \to 0$.
  - $Q[M_P^{-2}\Delta, \alpha] = 0$: singular limit (extra constraint).
  - NO extra DOF around Minkowski at quadratic order : 2 polarizations (4 initial conditions).

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Degrees of freedom: non-linear analysis

The equations of motion (gauge $N_i = 0$) 0906.3046

- Constraints:

$$\delta N : -K_{ij}K^{ij} + \lambda K^2 + V[g_{ij}, \beta_i] + \alpha f(N) = 0, \quad \delta N^i : \nabla_i K^{ij} - \lambda \nabla^j K = 0$$

- Dynamical equations:

$$\delta\gamma^{ij} : -\frac{\partial}{\partial t}(K^{ij} - \lambda\gamma^{ij}K) + \cdots = 0, \quad (\dot{\gamma}^{ij} = 2NK_{ij})$$

Cauchy data $(N, \gamma_{ij}, K_{ij}) = 13$ variables

- $\alpha, \beta_i \neq 0$: δN fixes N ($\alpha \to \infty$: $f(N) = 0$, i.e. $N \equiv N(t)$)
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- $\alpha = \beta_i = 0$: $\partial_t \delta N \neq 0$, i.e. secondary that fixes N
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⚠ Clash with the analysis of Minkowski at quadratic order!

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Stückelberg formalism

Diff invariance is naïvely restored by adding the DOF of the foliation $\rho(x)$:

- surfaces of the foliation defined by $\rho(x) = ct.$
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△ Extended covariant action

$$\mathcal{L} = M_P^2\left[\sqrt{-g}\left(-{}^{(4)}R + (1-\lambda)\mathcal{K} + \alpha P^{\mu\nu}\nabla_\mu u^\sigma \nabla_\nu u_\sigma\right) + O(1/M_P^2)\, V\right]$$

- Clear advantage of this formalism: GR+scalar field $\rho(x)$
- Most general action with dimension 2 operators ($[\rho] = L^{-1}$), invariant under Diff and reparametrizations of ρ!

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  The other objects

$$K_{ij} \to \mathcal{K}_{\mu\nu} = P_{\sigma\beta}\nabla^\beta u^\sigma, \quad {}^{(3)}R^i{}_{jkl} \to P_\alpha^\mu P_\nu^\beta P_\rho^\gamma P_\sigma^\delta {}^{(4)}R^\alpha{}_{\beta\gamma\delta} + \mathcal{K}_\rho^\mu \mathcal{K}_{\nu\sigma} - \mathcal{K}_\sigma^\mu \mathcal{K}_{\nu\rho}$$

$\triangle$ Extended covariant action

$$\mathcal{L} = M_P^2 \left[\sqrt{-g} \left(-{}^{(4)}R + (1-\lambda)\mathcal{K} + \alpha P^{\mu\nu} \nabla_\mu u^\sigma \nabla_\nu u_\sigma \right) + O(1/M_P^2)\, V \right]$$

- Clear advantage of this formalism: GR+scalar field $\rho(x)$
- Most general action with dimension 2 operators ($[\rho] = L^{-1}$), invariant under Diff and reparametrizations of ρ!

Stückelberg formalism: EOM

Equations of motion:

- 10 coming from $\delta g_{\mu\nu}$ (4 constraints)
- 1 coming from $\delta\rho$: $\nabla_\mu J^\mu = 0$ with $u_\mu J^\mu = 0$.

Different gauge choices

- Unitary gauge: $\rho(x) = t$: non covariant results reproduced.
  - Only 3 residual tr: $\delta\rho = \xi_0 = 0$. Extra DOF in the metric!
  - The EOM for N turns into the secondary constraint

$$\nabla_\mu J^\mu = \nabla_i((1-\lambda)N\nabla^i K + \dots) = 0.$$

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⚠ The EOM for ρ is in general of forth order!

$$\mathcal{K}^2 \sim (\Box\rho + \dots)^2$$

Puzzle solved (at linear order) choosing the frame (background) $\bar{\rho} = t$: the EOM for $\zeta = \rho - \bar{\rho}$ are of first order in time.

△ NOT COVARIANT: the Cauchy problem has a preferred foliation!

Stückelberg formalism: instability and strong coupling

Phenomenologically we expect $|\delta\lambda| \ll 1$ (scalar+tensor $\delta\lambda \sim 10^{-7}$)
Thus the decoupling limit is physical: $\mathcal{L} = M_P^2\left[\sqrt{-g}R + (1-\lambda)F(\phi)\right]$

$$M_P \to \infty, \quad M_P(\lambda - 1)^{1/2} \sim 10^{15}\text{GeV fixed.}$$

The EOM for ζ (only piece with interactions) is $(\omega, p \gg \bar{K} \sim L^{-1})$

$$M_P^2(\lambda - 1)\left[2\partial^i \bar{K}\partial_i\dot{\zeta} + \bar{N}\Delta^2\zeta + 6\partial^i\bar{N}\partial_i\Delta\zeta + \partial_t(\Delta\zeta)^2\right] = 0.$$

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Diff invariance is naïvely restored by adding the DOF of the foliation $\rho(x)$:

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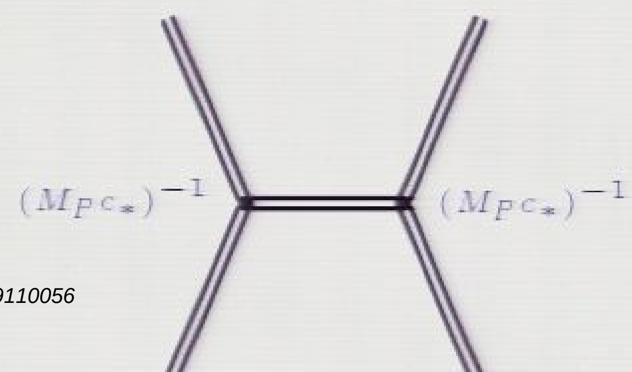
Projectable case: strong coupling

Well defined quadratic action around any smooth background:

$$\mathcal{L} = M_P^2 \left[-\left(\frac{\dot{\zeta}}{c_*}\right)^2 - \zeta\Delta\zeta + \frac{\zeta}{c_*^2}\left(\frac{\dot{\zeta}}{c_*}\right)^2 + O(\zeta^3) \right]$$

From stability we found $c_*^2 < 0$ and $|c_*| \leq 10^{-61}$ $(|c_*| \sim \sqrt{\lambda - 1})$. No need for the Stückelberg formalism to find a first strong coupling scale: Koyama, Arroja 0910.1998

$$\Lambda_p \sim M_P|c_*| \sim 10^{-34}\ \mathrm{eV}$$



This cut-off Λ_p is also found in the covariant formalism. 0906.3046

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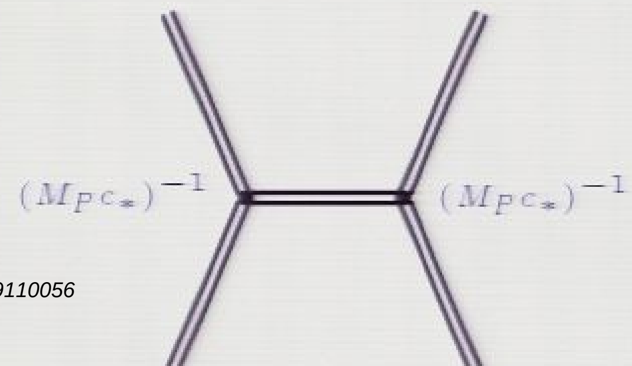

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Remaining "healthy" possibilities (no strong coupling or instabilities):

$$0 < \alpha < 2.$$

- Scalar-tensor theory (also in the IR)
  - Aether-like theory with $u_\mu \equiv \frac{\partial_\mu \rho}{\sqrt{g^{\sigma\nu}\partial_\sigma \rho \partial_\nu \rho}}$.
- Invariant under reparametrizations of the scalar.
- Breaking of Lorentz invariance controlled by the scalar.

First tests

△ Newton's law and light deviation (linear)

- For sources static in the preferred frame, same as in GR with

$$G_N = \frac{1}{8\pi M_P^2 (1 - \alpha/2)}$$

No constraints!

- For sources moving wrt the preferred frame, effects of order $\alpha, \delta\lambda$. PPN constraints are $O(10^{-7}) - O(10^{-4})$.

Foster and Jacobson 06; Mattingly 05

△ Homogeneous Cosmology: as GR with ($\partial_i N = 0$)

$$G_c = \frac{2}{8\pi M_P^2 (3\lambda - 1)}$$

- From BBN (^{4}He abundance): $G_N / G_c = 1 + O(10^{-2})$

► The cut-off scale is at worst $\Lambda_w \sim \alpha^{1/2} M_P \sim 10^{15}$ GeV.

Conclusions

- A "healthy" non-relativistic theory of quantum gravity is possible.
- For the model to remain weakly coupled, the massless modes in the UV must appear also in the IR.
- This is the case in the most general situation $0 < \alpha < \infty$: IR limit is GR with an additional scalar mode and a preferred frame.
- First phenomenological constraints $O(10^{-7})$ ($\Lambda_w \sim 10^{15}$ GeV).

Open issues

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- Recovery of the Lorentz invariance in the IR.
  (Perturbative or non perturbative mechanism to generate a mass-gap for ρ? symmetry? getting just 2 DOF?)
- More phenomenological test.
  (Cosmology, preferred frame, Čerenkov...).
- Exact solutions and black holes.
  (IR solution similar to the aether theories where there are some known solutions. Also, there are black hole solutions with no hair! BH Thermodynamics?)