Title: Horava gravity and strong coupling

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Abstract:

Strong coupling in Hořava gravity

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November 8, 2009

Outline

1. massive photons
2. massive photons: à la Stückelberg
3. massive gravitons
4. Hŏrava gravity

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massive photons

Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu$$

massive photons

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equations of motion

$$\begin{aligned} -\partial_\mu\partial^\mu A_\nu + \partial_\nu\partial_\mu A^\mu + m^2 A_\nu &= 0 \\ m^2\partial_\mu A^\mu &= 0 \end{aligned}$$

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so for $m \neq 0$

$$\begin{aligned} \partial_\mu A^\mu &= 0 \\ -\partial_\mu\partial^\mu A_\nu + m^2 A_\nu &= 0 \end{aligned}$$

massive photons

single mode

$$A^{\mu}(x) = a^{\mu} e^{ik.x}$$

dispersion relation

$$(k^0)^2 = m^2 + \underline{k}^2$$

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polarization vectors $\epsilon^\mu_{(\Lambda)}$

$$\epsilon^\mu_{(\Lambda)}\epsilon^\nu_{(\Lambda')}\eta_{\mu\nu} = \eta_{\Lambda\Lambda'}$$

$$\epsilon^\mu_{(\Lambda)}\epsilon^\nu_{(\Lambda')}\eta^{\Lambda\Lambda'} = \eta^{\mu\nu}$$

massive photons

e.g.

$$
\begin{aligned}
\epsilon_{(0)} &= (1, \underline{0}) \\
\epsilon_{(1)} &= (0, \underline{\hat{n}} \times \underline{k} / |\underline{\hat{n}} \times \underline{k}|) \\
\epsilon_{(2)} &= (0, \underline{k} \times (\underline{\hat{n}} \times \underline{k}) / |\underline{k} \times (\underline{\hat{n}} \times \underline{k})|) \\
\epsilon_{(3)} &= (0, \underline{\hat{k}}) \\
\underline{\hat{n}} &= (0, 0, 1)
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e.g.

$\epsilon_{(0)}$ timelike

$\epsilon_{(1)}$ transverse, $k_\mu \epsilon^\mu_{(1)} = 0$

$\epsilon_{(2)}$ transverse, $k_\mu \epsilon^\mu_{(2)} = 0$

$\epsilon_{(3)}$ longitudinal

massive photons

polarization of mode

$$a^{\mu} = \alpha^{\Lambda}\epsilon^{\mu}_{(\Lambda)}$$

$\partial_{\mu}A^{\mu} = 0, (m \neq 0)$

$$\alpha^{(0)} = \frac{|k|}{k^{0}}\alpha^{(3)}$$

massive photons

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$$a^\mu = \alpha^\Lambda \epsilon^\mu_{(\Lambda)}$$

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degrees of freedom

$$\textit{three physical modes} \begin{cases} \textit{two transverse} \\ \textit{one longitudinal} \end{cases}$$

massless photons

residual gauge freedom

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \qquad \partial_\mu \partial^\mu \Lambda = 0$$

This fixes $A^{(3)}$

$$\Lambda = \lambda e^{ik.x}$$
$$\alpha^{(3)} \to \alpha^{(3)} + i\lambda|\underline{k}|$$

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degrees of freedom

two physical transverse modes

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massive photons:Stückelberg technique

Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu$$

This has no gauge symmetry, but one appears at $m = 0$.

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redefine fields

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \frac{1}{m}\partial_\mu\phi \\ \mathcal{L} &\rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}A_\mu A^\mu - mA^\mu\partial_\mu\phi \end{aligned}$$

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new gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu\Lambda, \quad \phi \to \phi - m\Lambda$$

massive photons:Stückelberg technique

- $\phi = 0$ gauge recovers the original Lagrangian
- the first gauge symmetry leaves the vector with two transverse degrees of freedom
- the scalar mode (Stückelberg scalar), ϕ, is identified with the longitudinal mode of the massive vector.
- we now have a smooth massless limit

$$\mathcal{L}_{m\to 0} \to -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

where degrees of freedom do not simply vanish

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massive photons:strong coupling

coupling to matter

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu - j^\mu A_\mu$$

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Stückelberg redefinition

$$A_\mu \to A_\mu + \frac{1}{m}\partial_\mu \phi$$

to find

$$\mathcal{L} \to -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}A_\mu A^\mu - mA^\mu\partial_\mu\phi$$
$$-j^\mu A_\mu - \frac{1}{m}j^\mu\partial_\mu\phi$$

massive gravitons

Lagrangian

$$\mathcal{L} = M_P^2\left(-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\mu h\partial^\mu h\right) - \frac{M_P^2 m^2}{2}(h_{\mu\nu}h^{\mu\nu} - h^2) + h_{\mu\nu}T^{\mu\nu},$$

five degrees of freedom, compared to two for the massless limit.

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gauge symmetry (massless)

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu$$

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massive gravitons

imposing gauge symmetry

which we impose *via* the Stückelberg technique (Arkani-Hamed et al)to the massive case by introducing a scalar and a vector

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{1}{\sqrt{2}mM_P}(\partial_\mu A_\nu + \partial_\nu A_\mu) + \frac{1}{m^2 M_P}\partial_\mu \partial_\nu \phi$$

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$$\mathcal{L} = \mathcal{L}_{m=0} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - 2(h_{\mu\nu}\partial^\mu\partial^\nu\phi - h\partial^\mu\partial_\mu\phi) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

(for conserved matter sector). Gauge symmetries:

$$\delta h_{\mu\nu} = \partial\mu\zeta_\nu + \partial_\nu\zeta_\mu, \qquad \delta A_\mu = -\sqrt{2}m\zeta_\mu$$
$$\delta A_\mu = \partial_\mu\Lambda, \qquad \delta\phi = -\frac{m}{\sqrt{2}}\Lambda$$

degrees of freedom

- two degrees of freedom for $h_{\mu\nu}$
- two for A_μ
- one for ϕ

massive gravitons: massless limit

non-linear completion

The non-linear completion is what provides us with strong coupling. Terms such as

$$M_P^2 m^2 h^3$$

Stückelberged non-linear completion

$$\frac{(\partial^2 \phi)^3}{M_P m^4}$$

strong coupling

$$\Lambda_5 \sim \left(M_P m^4\right)^{1/5}$$

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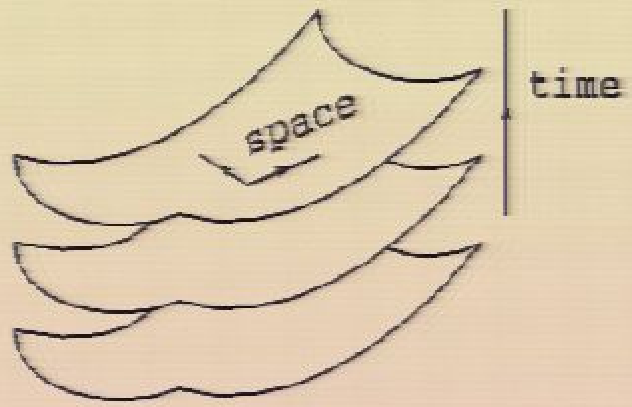
ADM split in gravity



$$ds^2 = -N^2(t,\underline{x})c^2dt^2 + [dx^i + N^i(t,\underline{x})dt][dx^j + N^j(t,\underline{x}dt]\gamma_{ij}(t,\underline{x})$$

1

1

$$[t] = -z, \quad [x] = -1, \quad [c] = z-1, \quad [\gamma_{ij}] = 0, \quad [N_i] = z-1, \quad [N] = 0$$

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ADM diffeomorphisms

spacetime diffeomorphisms

$$t \longrightarrow t - f(t, \underline{x}), \qquad x^i \longrightarrow x^i - \zeta^i(t, \underline{x})$$

induce the active diffeomorphisms (linear level)

$$\begin{aligned}
\gamma_{ij} &\rightarrow \gamma_{ij} + 2\nabla_{(i}\zeta_{j)} + f\dot{\gamma}_{ij} + N_i\partial_j f + N_j\partial_i f, \\
N_i &\rightarrow N_i + \partial_i(\zeta^j N_j) - 2\zeta^j\nabla_{[i}N_{j]} + \dot{\zeta}^j\gamma_{ij} + \dot{f}N_i + f\dot{N}_i + (-N^2 + \underline{N}^2)\partial_i f, \\
N &\rightarrow N + \zeta^j\partial_j N + \dot{f}N + f\dot{N} + NN^i\partial_i f.
\end{aligned}$$

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constructing an action

foliation-preserving diffeomorphisms, $f = f(t)$

$$\nabla_i,\ \gamma_{ij},\ K_{ij},\ R_{ijkl},\ (\nabla)^n R_{ijkl},\ \nabla_i \ln N, \ldots$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i).$$

$[K_{ij}] = z$

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constructing an action

kinetic, potential terms

$$T_{gr} = \frac{1}{\kappa} N \sqrt{\gamma} [K_{ij} K^{ij} - K^2]$$

$$V_{gr} = -\frac{c^2}{\kappa} N \sqrt{\gamma} \mathcal{R}$$

$[\kappa] = z - 3$

constructing an action

foliation-preserving diffeomorphisms, $f = f(t)$

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ADM diffeomorphisms

spacetime diffeomorphisms

$$t \longrightarrow t - f(t, \underline{x}), \qquad x^i \longrightarrow x^i - \zeta^i(t, \underline{x})$$

induce the active diffeomorphisms (linear level)

$$\gamma_{ij} \rightarrow \gamma_{ij} + 2\nabla_{(i}\zeta_{j)} + f\dot{\gamma}_{ij} + N_i\partial_j f + N_j\partial_i f,$$

$$N_i \rightarrow N_i + \partial_i(\zeta^j N_j) - 2\zeta^j\nabla_{[i}N_{j]} + \dot{\zeta}^j\gamma_{ij} + \dot{f}N_i + f\dot{N}_i + (-N^2 + \underline{N}^2)\partial_i f,$$

$$N \rightarrow N + \zeta^j\partial_j N + \dot{f}N + f\dot{N} + NN^i\partial_i f.$$

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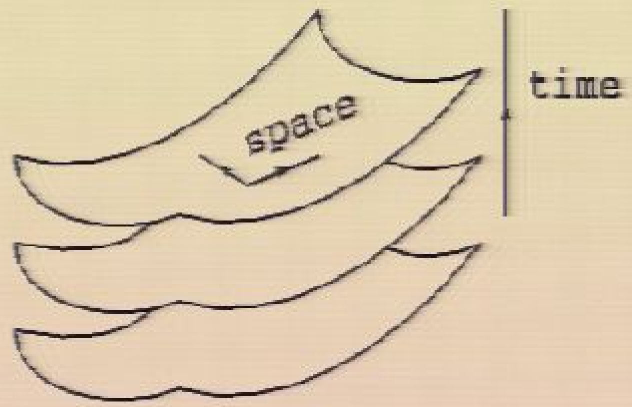
ADM split in gravity



$$ds^2 \;=\; -N^2(t,\underline{x})c^2dt^2 + [dx^i + N^i(t,\underline{x})dt][dx^j + N^j(t,\underline{x}dt]\gamma_{ij}(t,\underline{x})$$

1

1

$$[t] = -z, \quad [x] = -1, \quad [c] = z-1, \quad [\gamma_{ij}] = 0, \quad [N_i] = z-1, \quad [N] = 0$$

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constructing an action

kinetic, potential terms

$$T_{gr} = \frac{1}{\kappa} N\sqrt{\gamma}[K_{ij}K^{ij} - K^2]$$

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constructing an action

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$[\kappa] = z - 3$

general form

$$T = T_{gr} + \frac{1}{\kappa} N\sqrt{\gamma}(1-\lambda)K^2$$

$$V = V_{gr} + V_{uv}(R_{ijkl}, \nabla^n R_{ijkl}, \nabla_i \ln N \ldots)$$

($\lambda = 1$ required for general relativity)

matter

action

$$S = S_H + S_m$$

introduce

$$\rho = -\frac{1}{\sqrt{g}}\frac{\delta S_m}{\delta N} \qquad \left(= \frac{1}{N^2} T_{00}\right)$$

$$v^i = -\frac{1}{\sqrt{g}}\frac{\delta S_m}{\delta N_i} \qquad \left(= \frac{N}{2} T^{i0}\right)$$

$$\tau^{ij} = -\frac{1}{\sqrt{g}}\frac{\delta S_m}{\delta \gamma_{ij}} \qquad \left(= -\frac{N}{2} T^{ij}\right)$$

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matter and conservation laws

full diffeomorphism invariance

$$\delta S = \int d^4x\, T^{\mu\nu}\delta g_{\mu\nu} = \int d^4x\, T^{\mu\nu}(\delta g_{\mu\nu} - \nabla_\mu \zeta_\nu - \nabla_\nu \zeta_\mu)$$

$$\Rightarrow 0 = \int d^4x\, \zeta_\nu \nabla_\mu T^{\mu\nu}$$

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$$\delta S = \int d^4x\, T^{\mu\nu}\delta g_{\mu\nu} = \int d^4x\, T^{\mu\nu}(\delta g_{\mu\nu} - \nabla_\mu \zeta_\nu - \nabla_\nu \zeta_\mu)$$
$$\Rightarrow 0 = \int d^4x\, \zeta_\nu \nabla_\mu T^{\mu\nu}$$

foliation-preserving diffeomorphism invariance

$$\int d^3x \sqrt{\gamma}\left[\dot{\gamma}_{ij}\tau^{ij} - N\frac{(\rho\sqrt{\gamma})^{\cdot}}{\sqrt{\gamma}} - N_i\frac{(v^i\sqrt{\gamma})^{\cdot}}{\sqrt{\gamma}}\right] = 0,$$
$$2\nabla^i\tau_{ij} - \rho\partial_j N + \frac{(v^i\sqrt{\gamma})^{\cdot}}{\sqrt{\gamma}} + N_j\nabla_i v^i + 2v^i\nabla_{[i}N_{j]} = 0.$$

defining a background

- the background has vanishing momentum conjugate to γ_{ij}, so $\bar{K}_{ij} = 0$.
- matter sources are switched off, $\bar{\rho} = 0$, $\bar{v}^i = 0$, $\bar{\tau}^{ij} = 0$.
- choose a gauge where $\bar{N}_i = 0$

conservation law

$$\int d^3x \sqrt{\bar{\gamma}} \left[\bar{N} \dot{\rho} \right] = 0$$

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perturbation equations

action

$$S_{(H)} = S_{(gr)} + S_m + \int d^4x\, N\sqrt{\gamma}\left\{\frac{1}{\kappa}(1-\lambda)K^2 - V_{uv}(R_{ijkl}, \nabla^n R_{ijkl}, \nabla \ln N \ldots)\right\}$$

quadratic action

$$S^{(2)}_{(H)} = S^{(2)}_{(gr)} + S^{(2)}_m + S^{(2)}_{uv}$$

by perturbing about the background

$$N = \bar{N} + n, \qquad \underline{N} = \underline{\bar{N}} + \underline{n}, \qquad \gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}$$

perturbation equations

quadratic action

$$S_m^{(2)} = -\int dt d^3x \sqrt{\bar{\gamma}}[n\rho + n_i v^i + h_{ij}\tau^{ij}]$$

$$S_{uv}^{(2)} = \int dt d^3x \sqrt{\bar{\gamma}}\bar{N}\left\{\frac{1}{\kappa}(1-\lambda)(\delta K)^2 - V_{uv}^{(2)}(R_{ijkl}\ldots)\right\}$$

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perturbation equations

full 4D diffeomorphism

$$t \longrightarrow t - f(t, \underline{x}), \qquad x^i \longrightarrow x^i - \zeta^i(t, \underline{x})$$

is

$$\begin{aligned}
\gamma_{ij} &\rightarrow \gamma_{ij} + 2\nabla_{(i}\zeta_{j)} + f\dot{\gamma}_{ij} + N_i\partial_j f + N_j\partial_i f, \\
N_i &\rightarrow N_i + \partial_i(\zeta^j N_j) - 2\zeta^j \nabla_{[i}N_{j]} + \dot{\zeta}^j\gamma_{ij} + \dot{f}N_i + f\dot{N}_i + (-N^2 + \underline{N}^2)\partial_i f, \\
N &\rightarrow N + \zeta^j\partial_j N + \dot{f}N + f\dot{N} + NN^i\partial_i f.
\end{aligned}$$

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\end{aligned}$$

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perturbation equations

full 4D diffeomorphism in the background

$$
\begin{aligned}
h_{ij} &\rightarrow h_{ij} + 2\nabla_{(i}\zeta_{j)}, \\
n_i &\rightarrow n_i + \dot{\zeta}^j \bar{\gamma}_{ij} - \bar{N}^2 \partial_i f, \\
n &\rightarrow n + \zeta^j \partial_j \bar{N} + \dot{f}\bar{N} + f\dot{\bar{N}}.
\end{aligned}
$$

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Stückelberg the perturbation equations

restore full diffeomorphism: ξ, ϕ

$$
\begin{aligned}
h_{ij} &\rightarrow h_{ij} + 2\nabla_{(i}\xi_{j)}, \\
n_i &\rightarrow n_i + \dot{\xi}^j \bar{\gamma}_{ij} - \bar{N}^2 c^2 \partial_i \phi, \\
n &\rightarrow n + \xi^j \partial_j \bar{N} + \dot{\phi}\bar{N} + \phi\dot{\bar{N}}.
\end{aligned}
$$

restore full diffeomorphism

- $S_{gr}^{(2)}$ is invariant
- ξ^i is a 3d diffeomorphism
- by construction $S_H^{(2)}$ invariant under ξ^i, so ξ^i does not appear as a degree of freedom

Stückelberg the perturbation equations

Stückelberg scalar Lagrangian

$$\mathcal{L}^{(2)} \rightarrow \mathcal{L}^{(2)} + \frac{c^2}{\kappa}(1-\lambda)\left(\frac{2}{\bar{N}}\nabla^i(\bar{N}^2\partial_i\phi)\delta K + \frac{c^2}{\bar{N}^2}(\nabla^i(\bar{N}^2\partial_i\phi))^2\right) - \left(\dot{\rho} + \nabla_i(\bar{N}^2 v^i)/\bar{N}\right)\phi + \textit{non linear terms}$$

canonically normalize

$$\phi \rightarrow \sqrt{\frac{\kappa}{c^3(1-\lambda)}}\phi$$

Stückelberg the perturbation equations

Stückelberg scalar Lagrangian

$$S_\phi = \int c\,dt\,d^3x\sqrt{\gamma}N\left\{\frac{1}{\bar{N}^2}(\nabla^i(\bar{N}^2\partial_i\phi))^2 + \frac{2}{\bar{N}}\sqrt{\frac{1-\lambda}{\kappa c}}\nabla^i(\bar{N}^2\partial_i\phi)\delta K - \sqrt{\frac{\kappa}{c^3(1-\lambda)}}\left(\frac{1}{c}\dot{\rho} + \nabla_i(\bar{N}^2 c v^i)/\bar{N}\right)\phi\right\}$$

$+$*non linear terms*

strong coupling

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GR limit ($\lambda \to 1$) strongly coupled on all scales ($[\Lambda_{naive}] = z = [t^{-1}]$)

Stückelberg the perturbation equations

higher order terms

$$
\begin{aligned}
S_{uv} &= \int d^4x\, N\sqrt{\gamma}\frac{1}{\kappa}(1-\lambda)K^2 + \dots \\
K_{ij} &= \frac{1}{2N}\left(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \\
N &= \bar{N} + n, \qquad \underline{N} = \underline{\bar{N}} + \underline{n}, \qquad \gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}
\end{aligned}
$$

Stückelberg fields

$$
\begin{aligned}
n &\rightarrow n + \xi^j \partial_j \bar{N} + \dot{\phi}\bar{N} + \phi\dot{\bar{N}} \\
n_i &\rightarrow n_i + \dot{\xi}^j \bar{\gamma}_{ij} - \bar{N}^2 c^2 \partial_i \phi, \\
h_{ij} &\rightarrow h_{ij} + 2\nabla_{(i}\xi_{j)},
\end{aligned}
$$

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S_{uv} &= \int d^4x \, N\sqrt{\gamma}\frac{1}{\kappa}(1-\lambda)K^2 + \ldots \\
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Stückelberg the perturbation equations

typical terms

$$(1-\lambda)(\partial_t)^{\#}(\partial_x)^{\#}(n)^{\#}(h_{ij})^{\#}(n_i)^{\#}$$

which Stückelberg to

$$(1-\lambda)(\partial_t)^{\#}(\partial_x)^{\#}(n)^{\#}(h_{ij})^{\#}(n_i)^{\#}(\phi)^{\#}$$

canonically normalize

$\phi \to \phi/\sqrt{1-\lambda}$

$$\frac{(\partial_t)^{\#}(\partial_x)^{\#}(n)^{\#}(h_{ij})^{\#}(n_i)^{\#}(\phi)^{\#}}{(1-\lambda)^{\#}}$$

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Stückelberg the perturbation equations

higher order terms

$$S_{uv} = \int d^4x\, N\sqrt{\gamma}\frac{1}{\kappa}(1-\lambda)K^2 + \dots$$

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Stückelberg the perturbation equations

Stückelberg scalar Lagrangian

$$S_\phi = \int c\,dt\,d^3x\sqrt{\gamma}N\left\{\frac{1}{\bar{N}^2}(\nabla^i(\bar{N}^2\partial_i\phi))^2 + \frac{2}{\bar{N}}\sqrt{\frac{1-\lambda}{\kappa c}}\nabla^i(\bar{N}^2\partial_i\phi)\delta K - \sqrt{\frac{\kappa}{c^3(1-\lambda)}}\left(\frac{1}{c}\dot{\rho} + \nabla_i(\bar{N}^2 c v^i)/\bar{N}\right)\phi\right\}$$

$$+\textit{non linear terms}$$

strong coupling

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canonically normalize

$$\phi \rightarrow \sqrt{\frac{\kappa}{c^3(1-\lambda)}}\phi$$

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perturbation equations

full 4D diffeomorphism in the background

$$
\begin{aligned}
h_{ij} &\rightarrow h_{ij} + 2\nabla_{(i}\zeta_{j)}, \\
n_i &\rightarrow n_i + \dot{\zeta}^j \bar{\gamma}_{ij} - \bar{N}^2 \partial_i f, \\
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The University of Nottingham

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\end{aligned}$$

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Stückelberg the perturbation equations

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Stückelberg the perturbation equations

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canonically normalize

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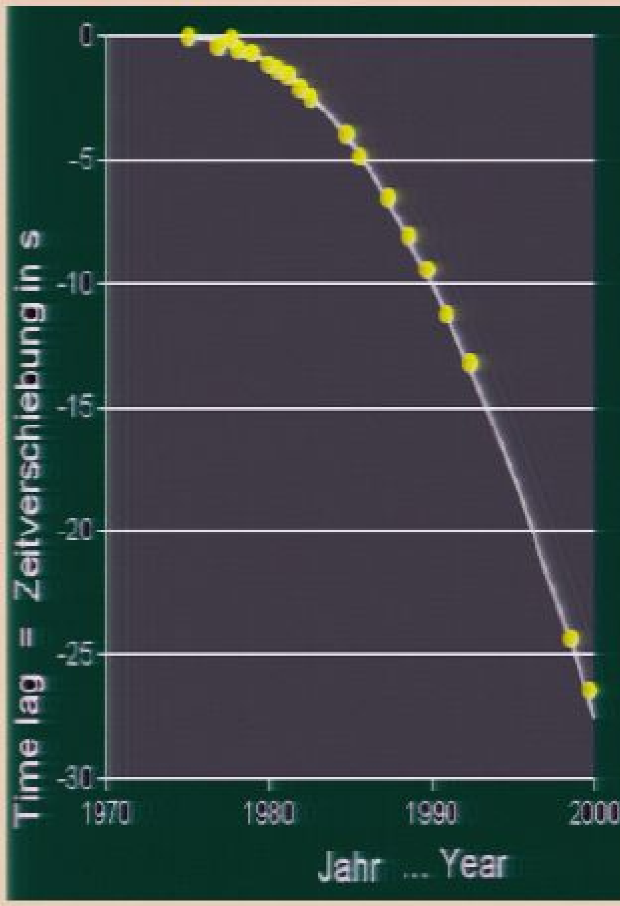
Stückelberg the perturbation equations

pulsar timing data



relativistic Hŏrava gravity

ADM split, Germani et al, Blas et al

$$\begin{aligned}
\gamma_{ij} &\rightarrow h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \\
K_{ij} &\rightarrow \mathcal{K}_{\mu\nu} = h^\alpha_\mu h^\beta_\nu \nabla_{(\alpha} n_{\beta)} \\
R_{ijkl} &\rightarrow \mathcal{R}_{\lambda\mu\nu\rho} = R_{\alpha\beta\gamma\delta} h^\alpha_\lambda h^\beta_\mu h^\gamma_\nu h^\delta_\rho + 2K_{\mu[\nu} K_{\rho]\lambda}
\end{aligned}$$

slices and the Stückelberg scalar

$$n_\mu = \frac{\partial_\mu \phi}{\sqrt{-\partial_\mu \phi \partial^\mu \phi}}$$

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relativistic Hořava gravity

gauge choice

aligning the time co-ordinate with ϕ ($\phi = t$) recovers the original Hořava action

gauge choice

$$
\begin{aligned}
n_0 &= N \\
h_{ij} &= \gamma_{ij} \\
\mathcal{K}_{ij} &= K_{ij} \\
&\dots
\end{aligned}
$$

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relativistic Hŏrava gravity

gauge choice

aligning the time co-ordinate with ϕ ($\phi = t$) recovers the original Hŏrava action

gauge choice

$$\begin{aligned} n_0 &= N \\ h_{ij} &= \gamma_{ij} \\ \mathcal{K}_{ij} &= K_{ij} \\ &\dots \end{aligned}$$

Lagrangian

$$\mathcal{L} \sim R + (1-\lambda)\mathcal{K}^2 + h^{\mu\nu} h^{\rho\sigma} \mathcal{R}_{\mu\rho} \mathcal{R}_{\nu\sigma} + \dots$$

strong coupling about different backgrounds

Blas et al

$$\phi = \bar{\phi} + \chi$$

In terms of canonically normalized fields

$$S_\chi = \int d^4x \left\{ v^i \dot{\chi} \partial_i \chi + (\nabla^2 \chi)^2 + \frac{L^9}{\Lambda_{naive}} \dot{\chi} (\nabla^2 \chi)^2 \right\}$$

- v^i - unit vector along spatial gradient
- L - lengthscale of background, $\bar{R}_{ij} \sim 1/L^2$, $\bar{K}_{ij} \sim 1/L^3$.
- χ is dynamical
- strong coupling energy scale is $(\Lambda)^4 \sim \left(\frac{\Lambda_{naive}}{L}\right)^3$

Summary

- more derivatives in the action gives the possibility of a renormalizable theory of gravity
- maintaining Lorentz invariance leads to ghosts
- losing Lorentz invariance leads to extra degrees of freedom
- Hŏrava gravity has a single extra degree of freedom
- that extra degree of freedom is strongly coupled in the vacuum
- change your degrees of freedom, c.f. QCD
- Vainshtein effect

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