

Title: The mass-inflation phenomenon in the asymptotic safety scenario

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Abstract:

# The mass-inflation phenomenon in the AS scenario

Alfio Bonanno – INAF, Catania

# Outline of the talk

- Motivation
- CH instability
- Mass-inflation singularity
- Flow equations around WF and AS FP
- An explicit QG modified solution

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- Motivation
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- Flow equations around WF and AS FP
- An explicit QG modified solution
- Extension of the ST beyond the CH
- Conclusions



“Gravitational collapse is the greatest crisis of physics of all time.”


J.A.Wheeler



use



# Gravitational collapse





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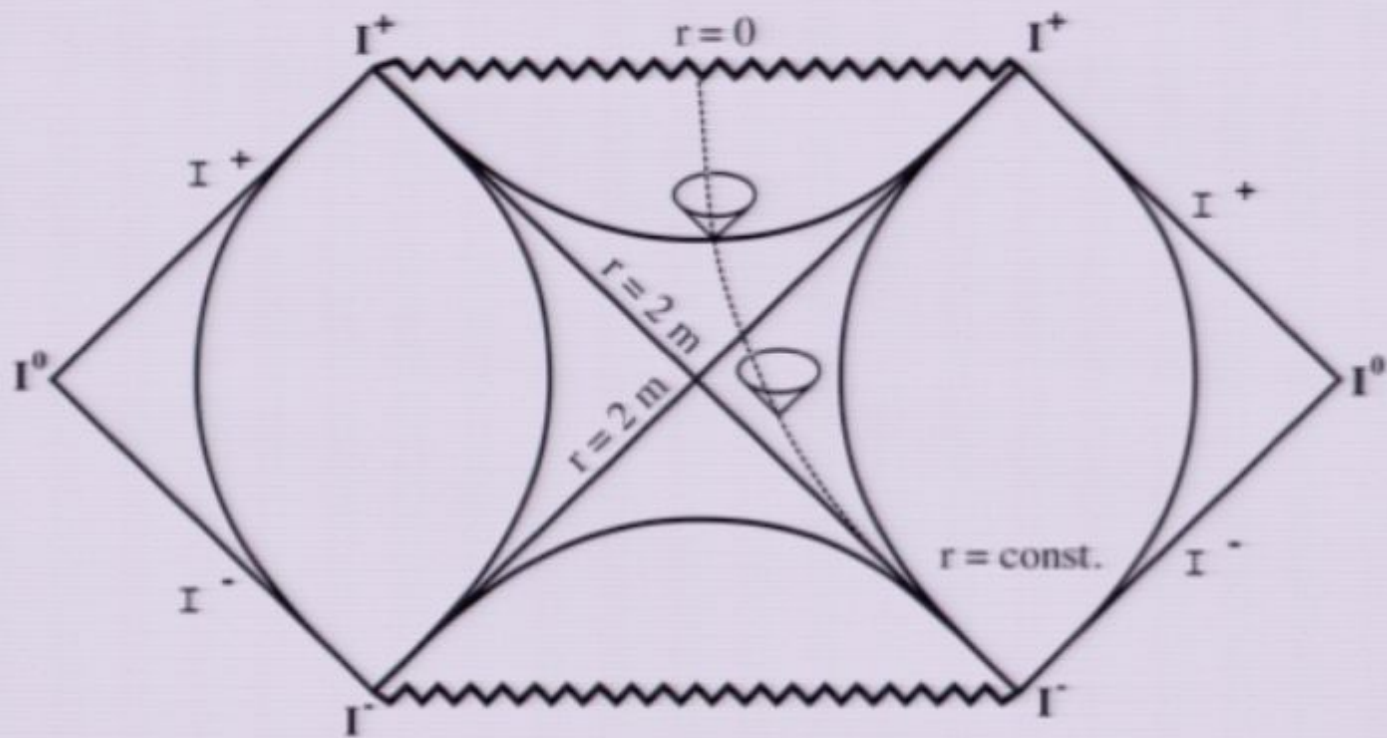
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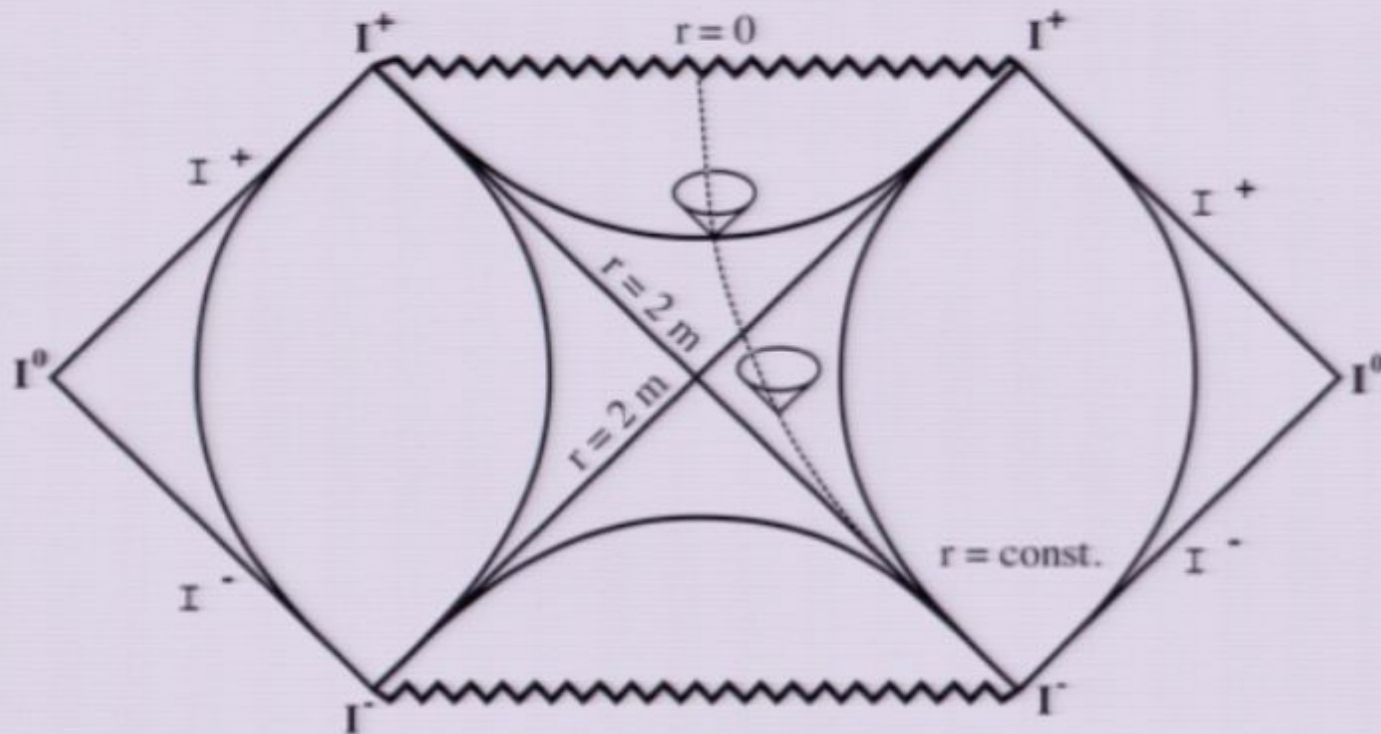
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- What is happening?

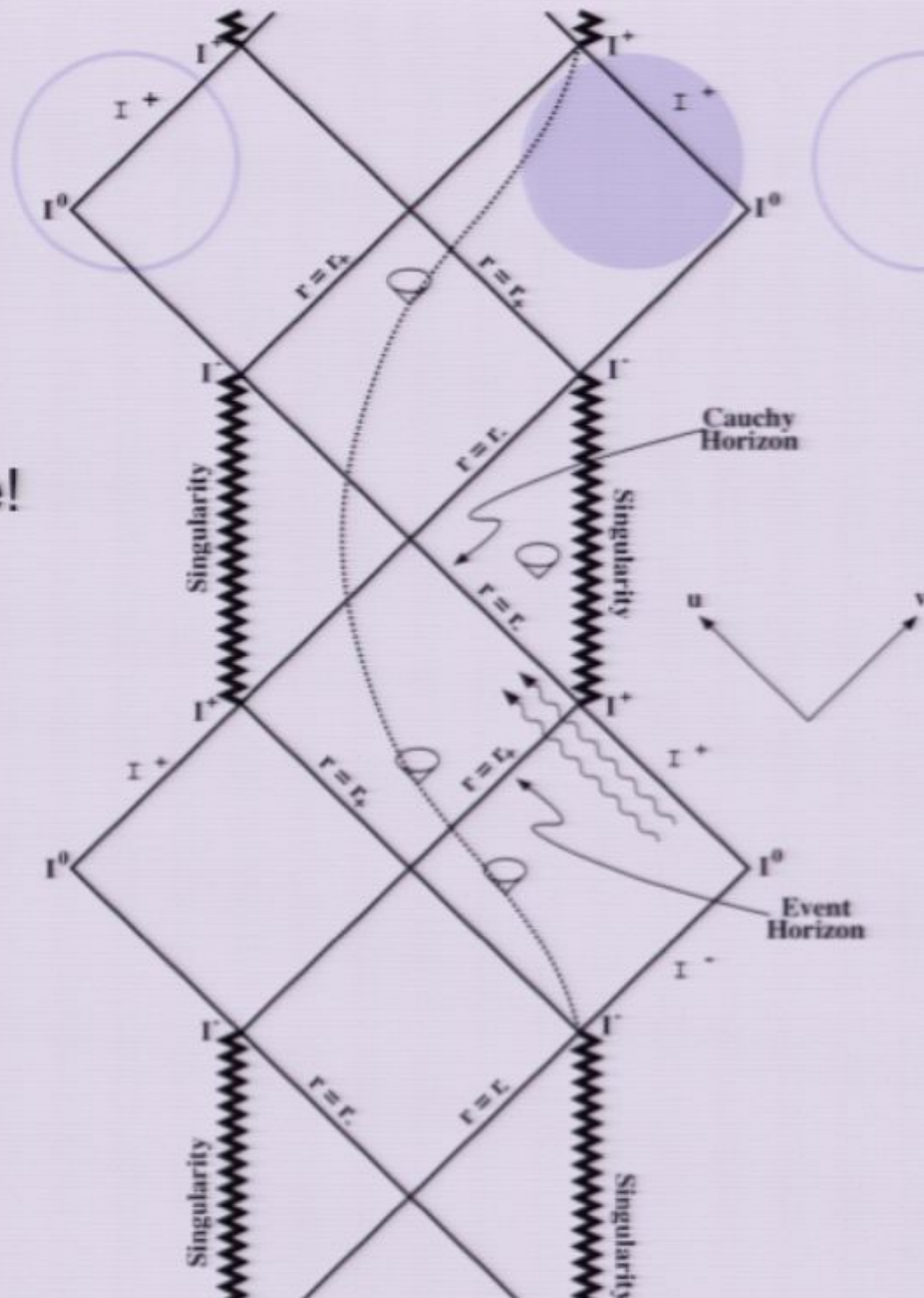




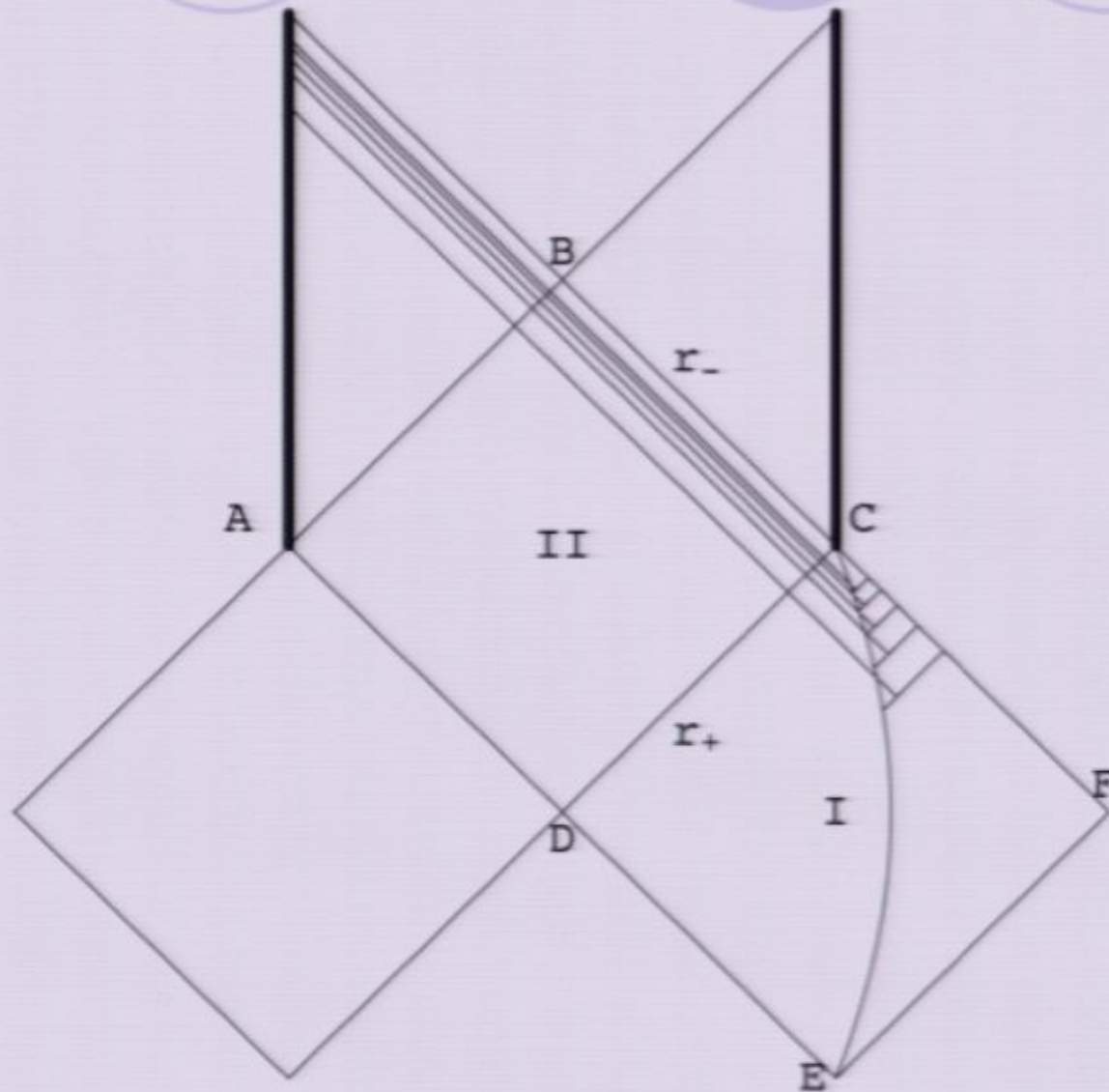
# Schwarzschild static BH



electric charge!

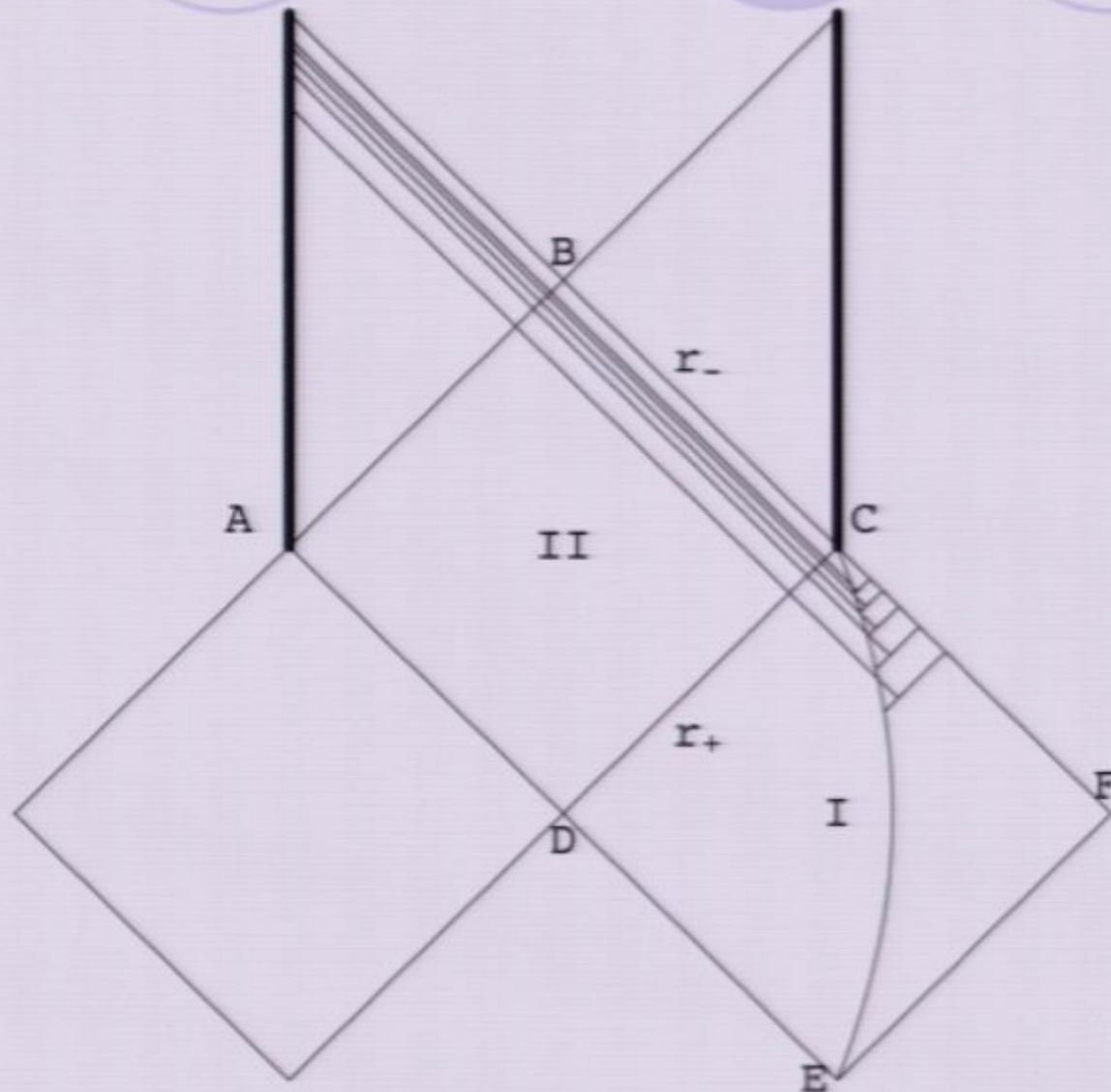


ears





# A Cauchy Horizon appears





Consider a geodesic of an observer which is crossing the CH

$$\dot{r}^2 = E^2 - f(r), \quad \dot{v} = [E - (E^2 - f(r))]^{1/2} / f(r)$$

$$\dot{r} \simeq -|E|, \quad \dot{v} \simeq -2|E|/f(r), \quad dr/dv \simeq \frac{1}{2}f(r)$$

Thus

$$\dot{v} \simeq |E|e^{\kappa v}$$

Measured energy due to infalling radiation

$$\rho = T_{\alpha\beta}u^\alpha u^\beta = T_{vv}\dot{v}^2 = \frac{|E|^2}{4\pi r^2}L(v)e^{2\kappa v}$$

as  $v \rightarrow \infty$


# Internal evolution



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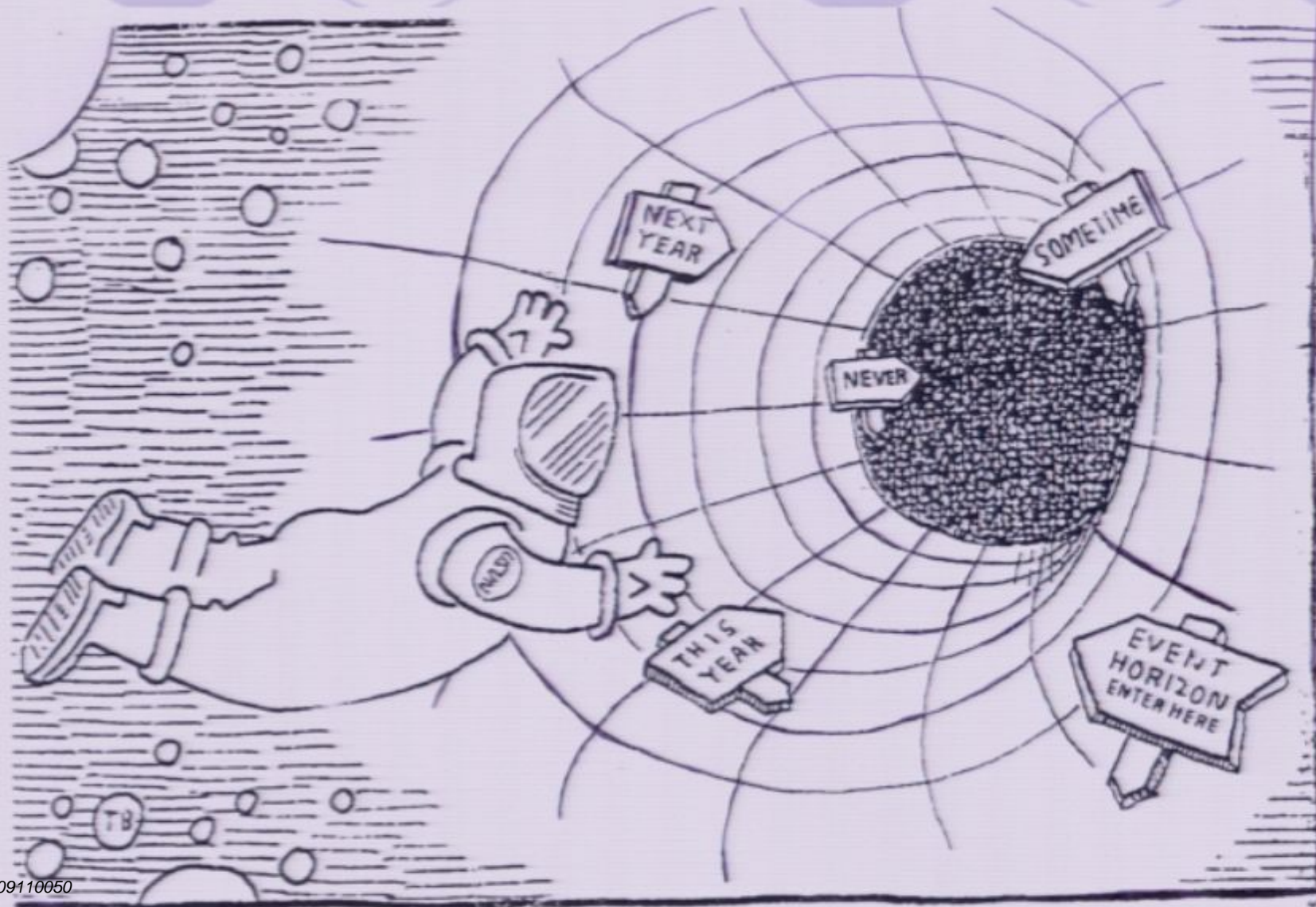
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- We should now integrate EFE with the known boundary conditions to obtain the internal structure of the BH
- Very difficult problem due to its non-linear nature!



# A journey into a BH





Singularity

CH

$v = \infty$

$\Sigma$

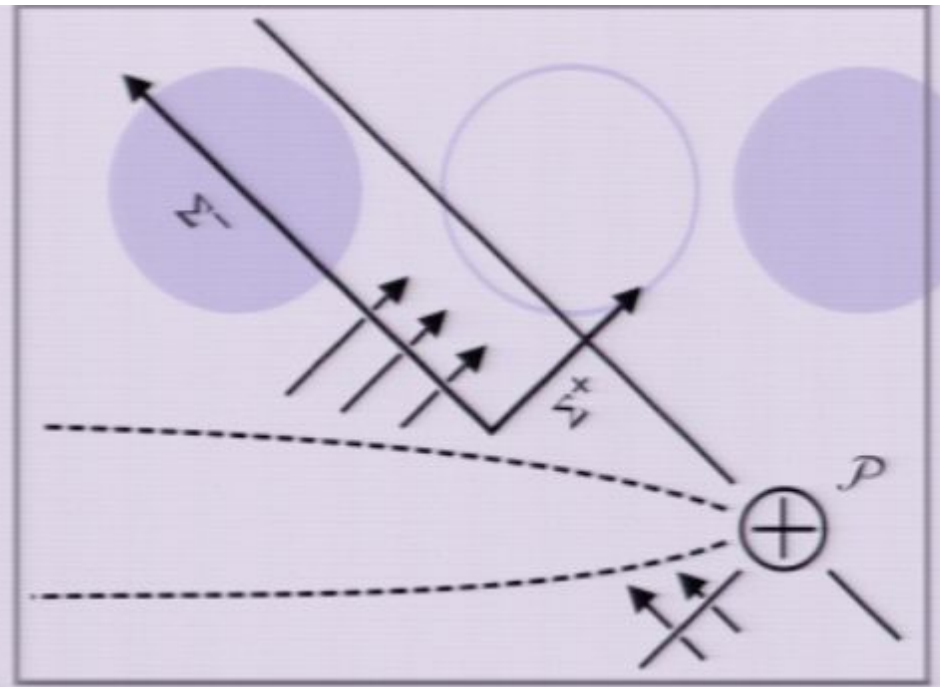
Potential Barrier

Influx:  $v^p$

Star

EH

$= 0$



$\mathcal{P}$

$\mathcal{I}^+$

$\oplus$


$\mathcal{I}^-$





Old and new work



The title 'Old and new work' is positioned on the left side of the slide. To its right, there are three circles of equal size arranged horizontally. The first circle is solid purple, the second is a white outline, and the third is solid purple. The text 'Old and new work' is in a large, black, sans-serif font.

# Old and new work







Singularity

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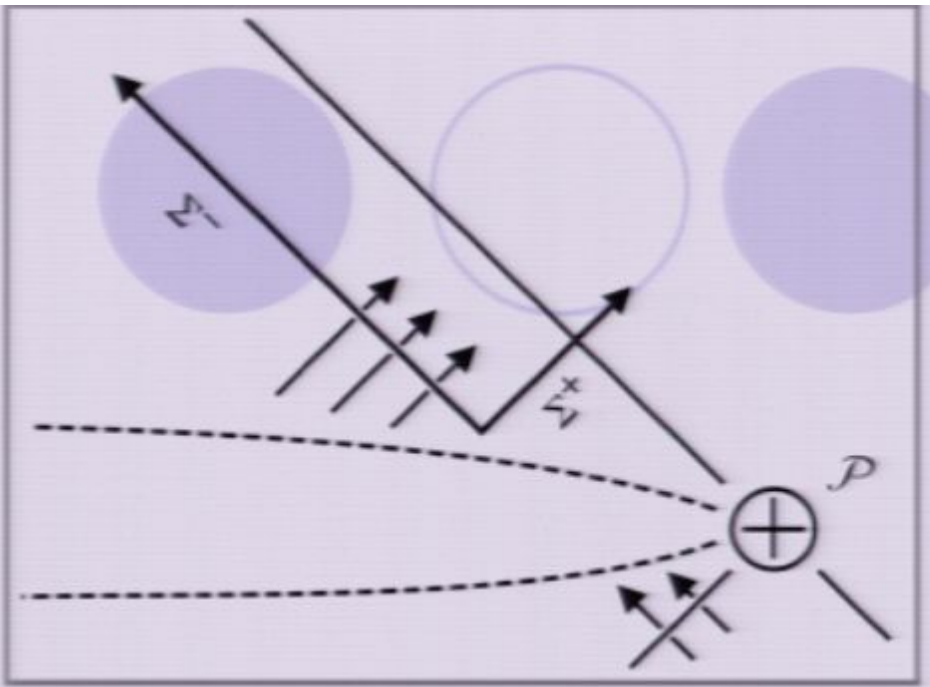
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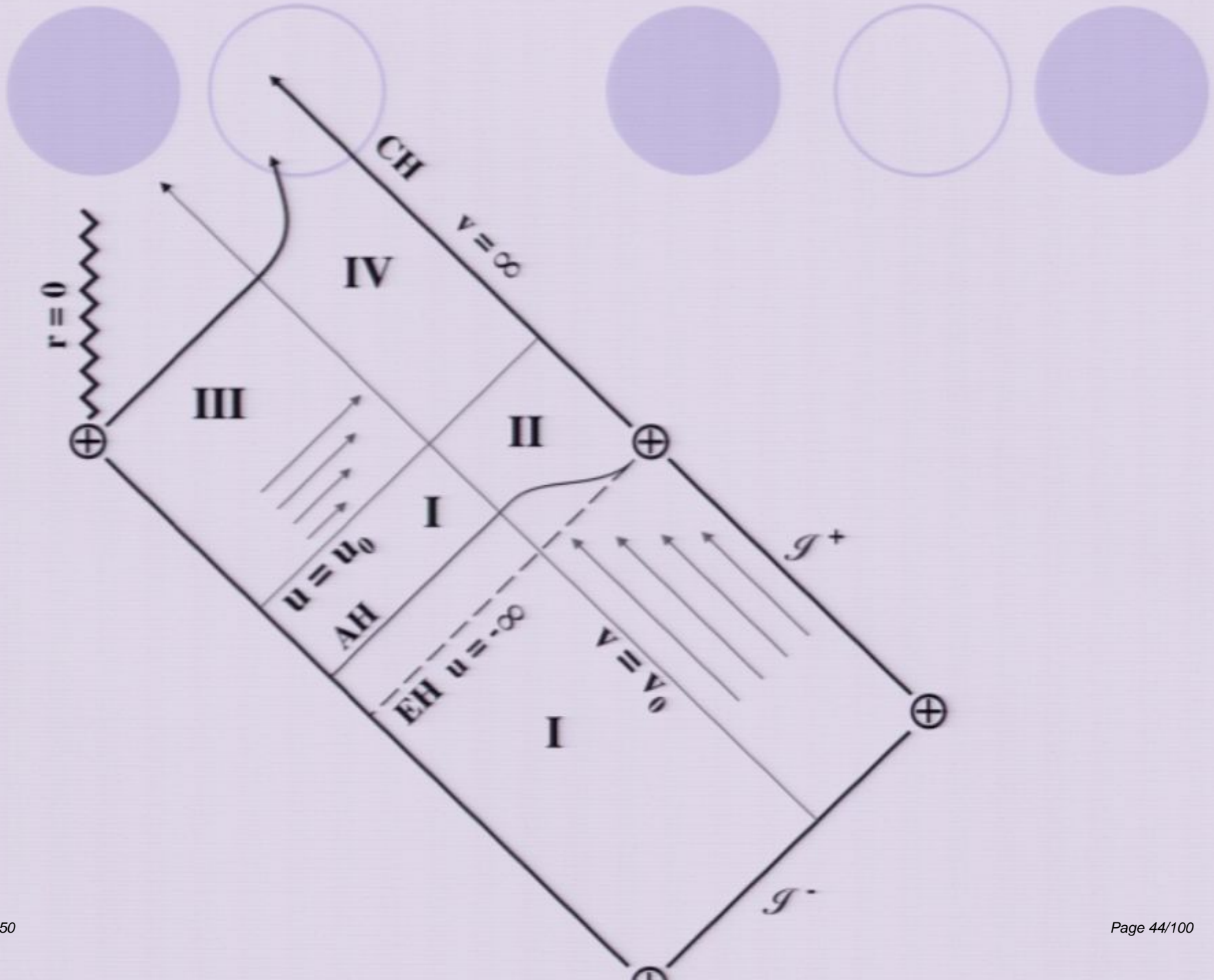
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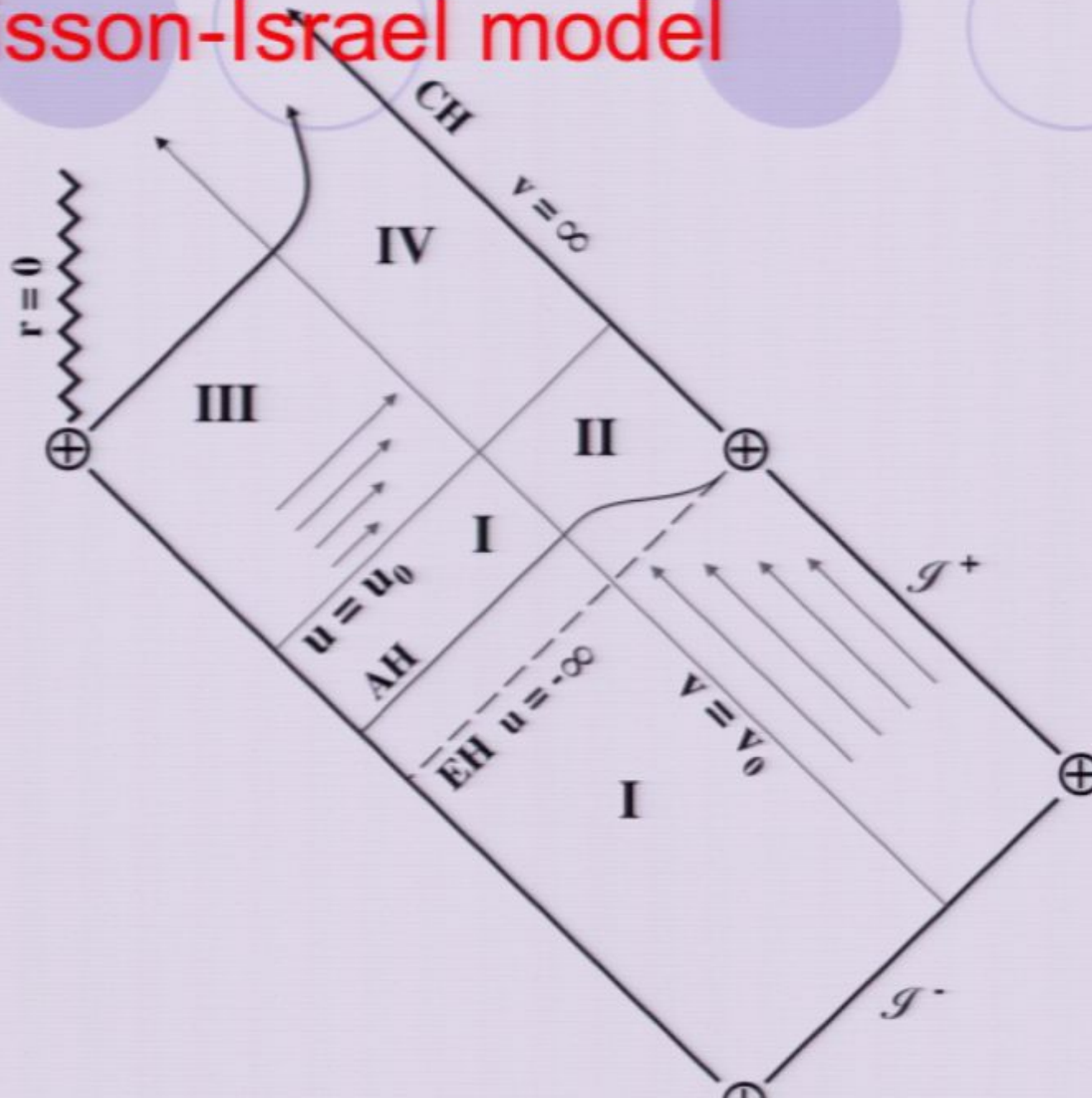
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- ... **Renewed interest in this problem**





# Poisson-Israel model



In the original mass inflation analysis by Poisson-Israel, a null crossflow stress tensor was used to model the gravitational radiation. The stress tensor for null crossflowing radiation can be written as

$$T_{\alpha\beta} = \frac{L_{in}(V)}{4\pi r^2} \partial_\alpha V \partial_\beta V + \frac{L_{out}(U)}{4\pi r^2} \partial_\alpha U \partial_\beta U \quad (1)$$

which satisfies the conservation equations and has  $P = T = 0$ . The conservation equations force  $L_{in}$  ( $L_{out}$ ) to be a function only of  $V$  ( $U$ ).

In the Kruskal coordinate  $V$ , the Price power-law tail has the form

$$L_{in}(V) = \frac{dm_{in}}{dv} \left( \frac{dv}{dV} \right)^2 = \frac{\beta}{(-\kappa_- V)^2} (-\ln(-\kappa_- V))^{-p}.$$

As the Cauchy horizon is approached, in the limit  $V \rightarrow 0_-$ ,  $L_{in}$  diverges and the source term in the wave equation for  $m$  diverges as well.

The integral solution for the mass function is

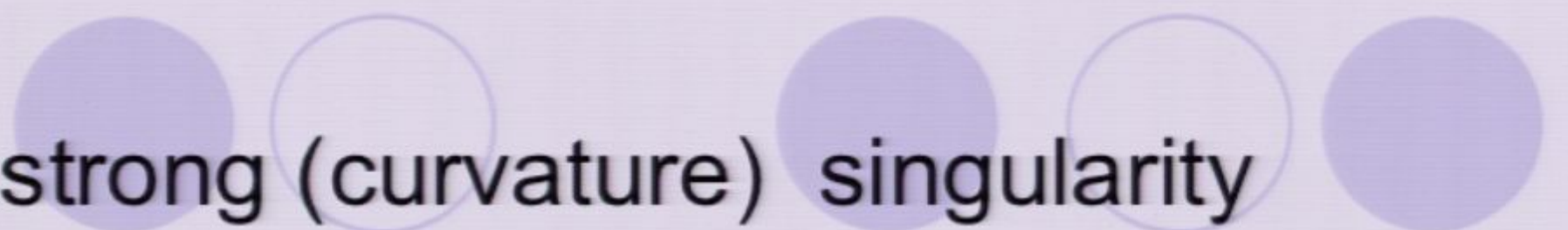
$$m(U, V) = \int_{U_1}^U \int_{V_1}^V r'^{-1} e^{-\lambda'} L_{in}(V') L_{out}(U') dU' dV' \\ + m_{in}(V) + m_{out}(U) - m_1$$

The gravitational wave tail influx is turned on at advanced time  $V_1$  and the outflux is assumed to be switched on at the advanced time  $U_1$ , which is behind the event horizon. The divergence of  $L_{in}(V') dV'$  leads to mass inflation with the mass function behaving as

$$m \sim \frac{1}{(-V) \ln(-V)^p}, \quad V \rightarrow 0_-$$

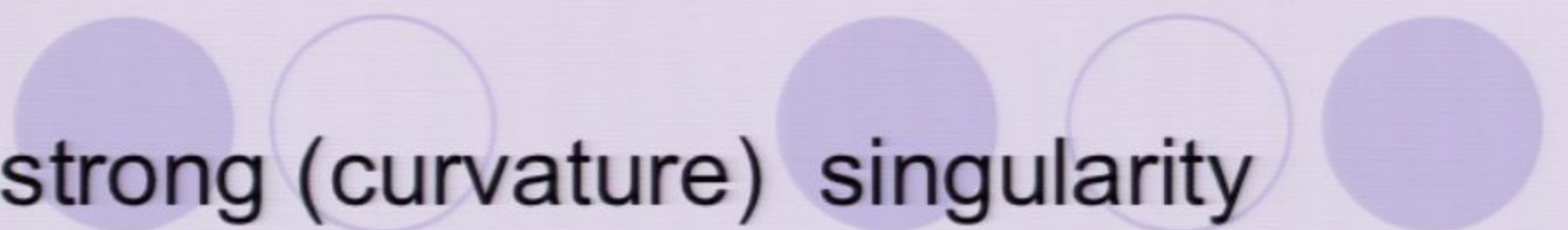
thus  $R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \sim m^2 \rightarrow \infty$





A strong (curvature) singularity  
develops at Cauchy Horizon!





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- Spacetime just “ends” there!



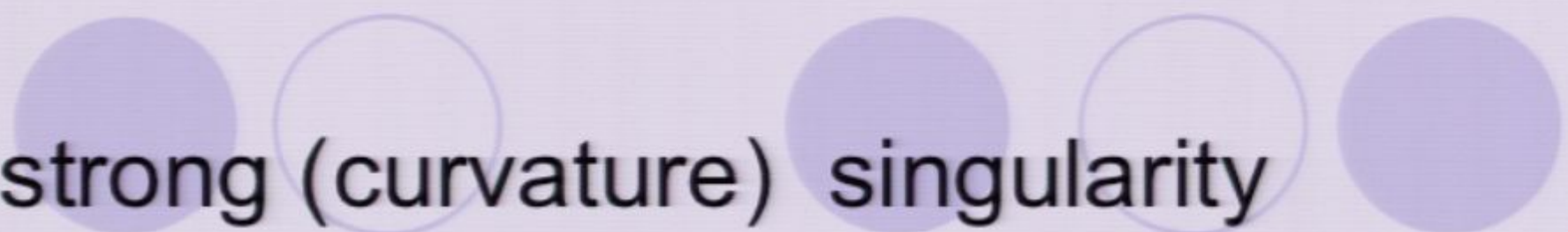
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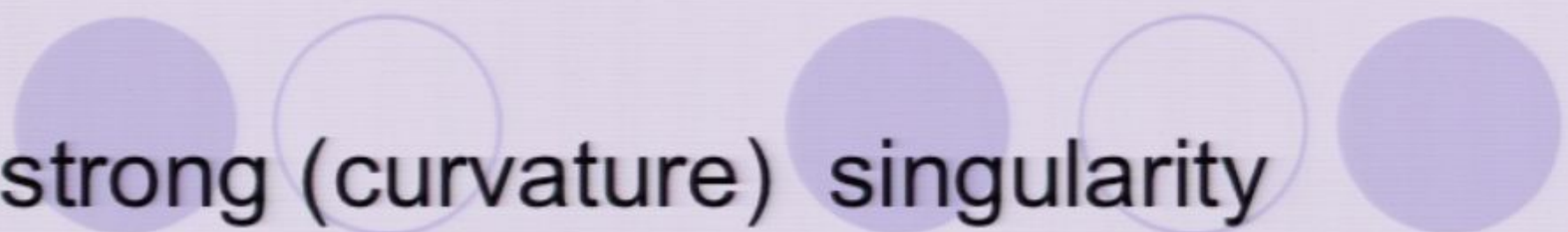
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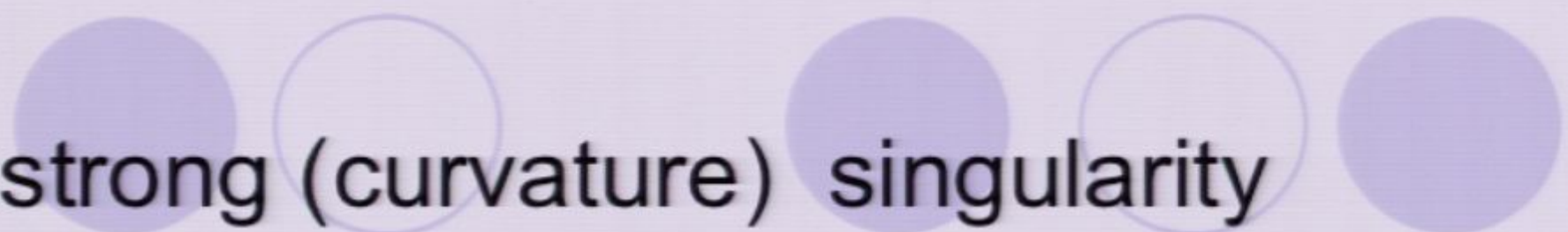
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- QG effects “cure” the singularity



A strong (curvature) singularity  
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- Spacetime just “ends” there!
- QG effects “cure” the singularity
- Spacetime can still be classically extended as we do in fluid mechanics when shock develops

# Quantum Effect and recent works

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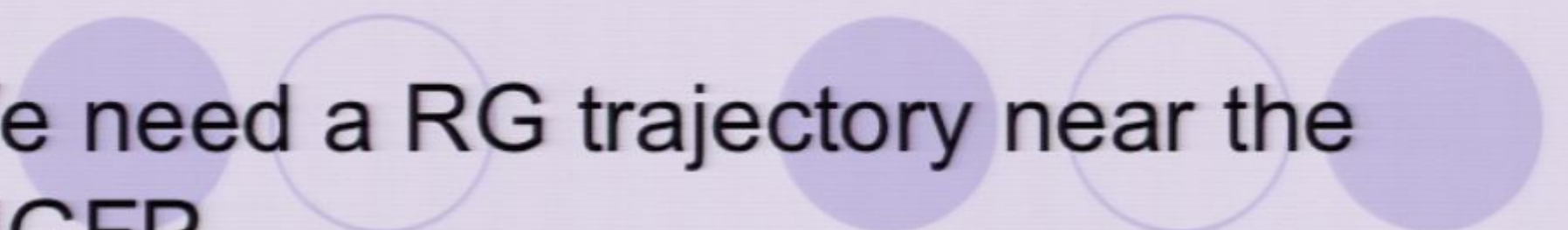


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- Encode the running of  $G$  into the Einstein Equations

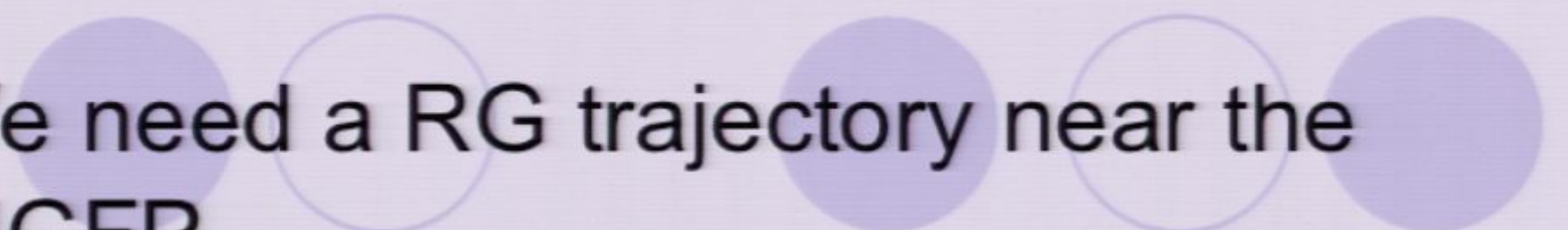
# RG improvement of dynamical eqs:

**Le Chatelier-Braun Principle (1884, 1888):** “Every physical system in stable equilibrium under the influence of an external force (a change in an environmental property A) which tends to alter an intensive characteristic B of the system (temperature, pressure, concentration, number density of molecules, etc.) every where or just in some parts. Can only experience interior changes--the secondary effect---in some other parameter of state C of the system, usually extensive (entropy, volume, number of particles of a specific kind, etc. ) producing a current (or flow) that causes a feedback effect B of opposite sign to that resulting from the exterior force.



We need a RG trajectory near the  
NGFP

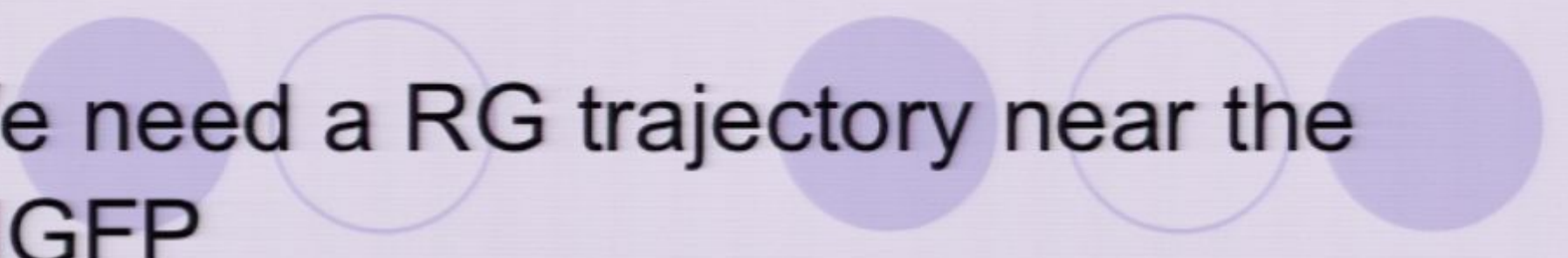




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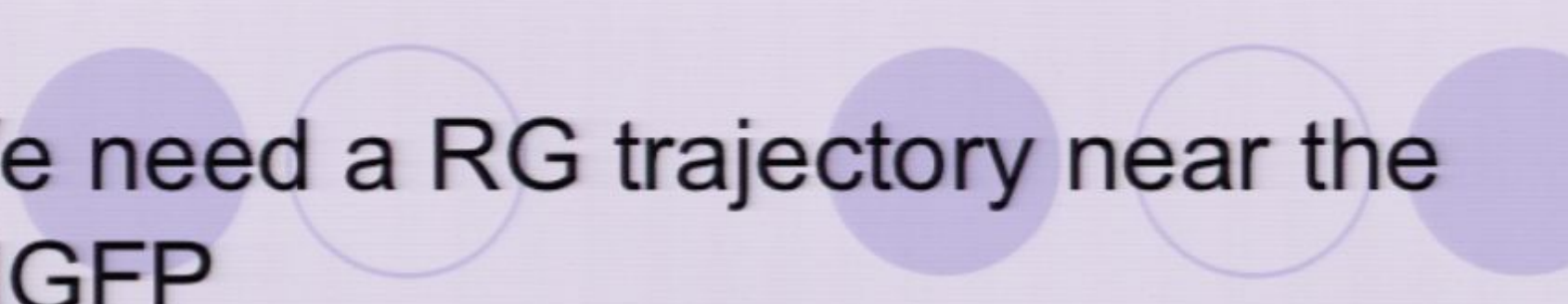
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# We need a RG trajectory near the NGFP

- Insert this trajectory into the FE
- Only valid near the CH below the inner potential barrier
- Compute the new mass function

# Proper-time flow equation for gravity

A.B. & M.Reuter, JHEP, 2005

$$\partial_t \hat{S}_k[g, \bar{g}] = -\frac{1}{2} \text{Tr} \int_0^\infty \frac{ds}{s} \partial_t f_k^m(s) [\exp(-s \hat{S}_k^{(2)}) - 2 \exp(-s S_{\text{gh}}^{(2)})]$$

$$f_k^m(s) = \frac{\Gamma(m+1, \mathcal{Z} s k^2) - \Gamma(m+1, \mathcal{Z} s \Lambda^2)}{\Gamma(m+1)}$$

This is not “exact” at the level of the general functional equation, but local truncations work **VERY** well!



$m$	$\eta$	$m$	$\eta$
1	0.0653	11	0.0365
2	0.0507	12	0.0362
3	0.0452	13	0.0360
4	0.0423	14	0.0358
5	0.0405	15	0.0356
6	0.0393	16	0.0354
7	0.0385	17	0.0353
8	0.0378	20	0.0350
9	0.0373	30	0.0343
10	0.0369	40	0.0340

The anomalous dimension  $\eta$  at the Wilson-Fisher fixed point. Note: R. Guida and J. Zinn-Justin, (1998) find  $\eta = 0.0335$  from seven loop pt in  $D = 3$



A.B. & D.Zappala', (2001), Phys.Lett.B.

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$$\partial_t g = \beta_g(g, \lambda) \equiv [d - 2 + \eta_N]g \quad (1a)$$

$$\partial_t \lambda = \beta_\lambda(g, \lambda) \quad (1b)$$

The anomalous dimension  $\eta_N \equiv -\partial_t \ln Z_{Nk}$  is given by

$$\eta_N = 8(4\pi)^{1-\frac{d}{2}} \left[ \frac{d(7-5d)}{24} (1-2\lambda)^{\frac{d}{2}-m-2} - \frac{d+6}{6} \right] g \frac{\Gamma(m+2-\frac{d}{2})}{\Gamma(m+1)}$$

and the beta-function of  $\lambda$  reads

$$\beta_\lambda = -(2-\eta_N)\lambda + 4(4\pi)^{1-\frac{d}{2}} \left[ \frac{d(d+1)}{4} (1-2\lambda)^{\frac{d}{2}-m-1} - d \right] g \frac{\Gamma(m+1-\frac{d}{2})}{\Gamma(m+1)}$$

# Beta-functions

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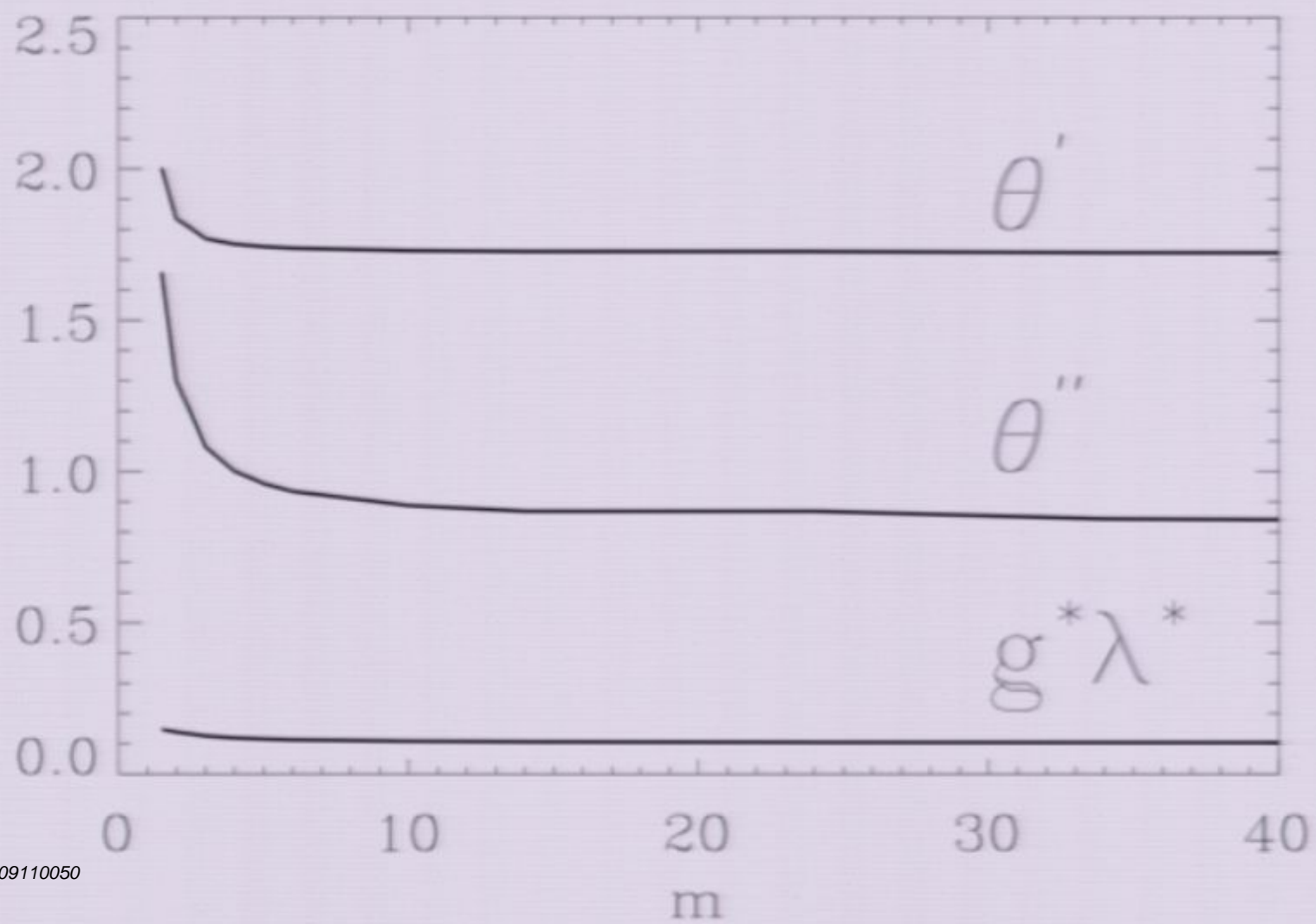
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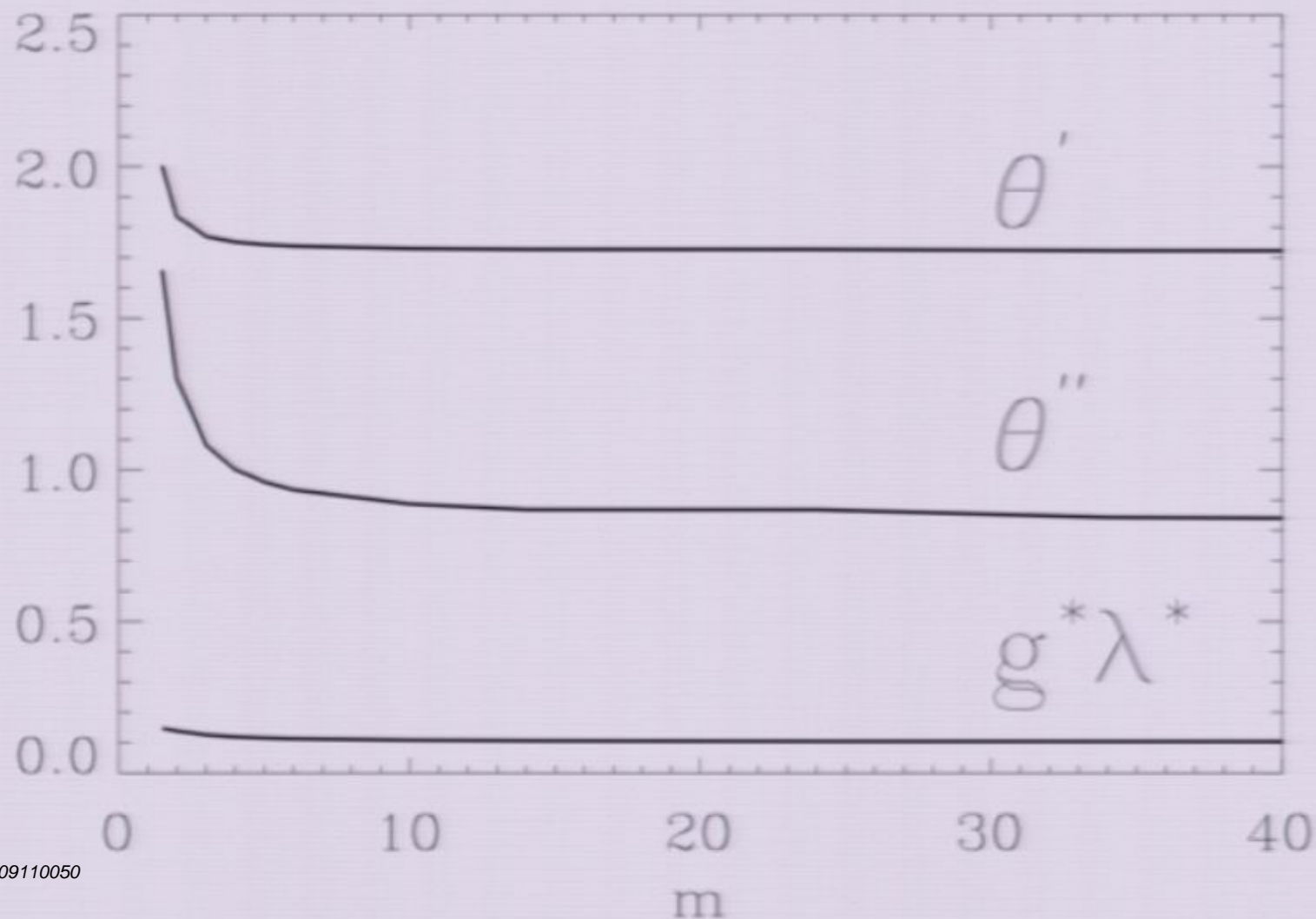
# Critical exponents

$m$	$g_*$	$\lambda_*$	$\lambda_* g_*$	$\theta'$	$\theta''$
3/2	0.763	0.192	0.147	2.000	1.658
2	1.663	0.118	0.138	1.834	1.230
3	1.890	0.066	0.125	1.769	1.081
4	2.589	0.046	0.119	1.750	1.001
5	3.281	0.035	0.115	1.742	0.959
6	3.970	0.028	0.113	1.737	0.934
10	6.718	0.016	0.108	1.729	0.886
40	27.271	0.0038	0.103	1.722	0.840





# Critical exponents



# Explicit solution for g-running

$$\eta = -\frac{4}{3}2^{-d}\pi^{1-\frac{d}{2}}(-3d + 5d^2 + 24)$$

$$g(t) = -\frac{d-2}{\eta - (d-2) C_0 e^{-(d-2)t}}$$

Assume cutoff-id:

$$k^2 = |\psi_2|$$

Coulombian component of the Weyl curvature!

## oved FE

Use coordinates  $x^a$  ( $a, b = 0, 1$ ) on the radial two-spaces  $(\theta, \phi) = \text{const}$  and the function  $r(x^a)$  that measures the area of those two-spheres whose line element is  $r^2 d\Omega^2$ . The metric element is then

$$ds^2 = g_{ab} dx^a dx^b + r^2 d\Omega^2$$

By defining the scalar fields  $f(x^a)$ ,  $m(x^a)$  and

$$-2\kappa(x^a) = \frac{\partial f}{\partial r}, \quad f = 1 - \frac{2M}{r} + \frac{e^2}{r^2}$$



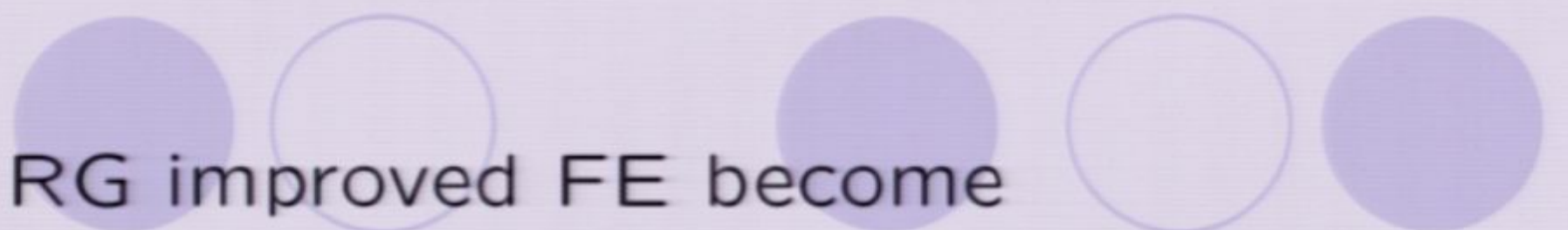
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The RG improved FE become

$$r_{;ab} + \kappa g_{ab} = -4\pi G_k r (T_{ab} - g_{ab} T)$$

$$R - 2\partial_r \kappa = 8\pi G_k (T - 2P)$$

where the static electro-magnetic field is generated by a charge of strength  $e$  and  $T_{ab}$  is the stress-energy tensor of the matter field whose two-dimensional trace is  $T$  and tangential pressure is  $P$ .

# Dynamical equation for M

Wave-equation for the mass function

$$\square M = -16\pi^2 r^3 G_k^2 T_{ab} T^{ab} + 8\pi G_k f(P - T) \\ + 4\pi r^2 G_k \kappa T - 4\pi r^2 G_k r_{,a} T^{,a}$$

where

$$G_k = \frac{G_N}{1 + cM(U, V)}$$

# Asymptotic solution valid near CH

QG correction

$$M \sim \frac{1}{(-V)^{1/3} \ln(-V)^p}$$

Classical behavior

$$M \sim \frac{1}{(-V) \ln(-V)^p}$$

As a consequence :

$$R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \sim M^2 \quad (1)$$

is now integrable along  $V$  !



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- But the tidal acceleration integrated along the infalling geodesic does not diverge at CH

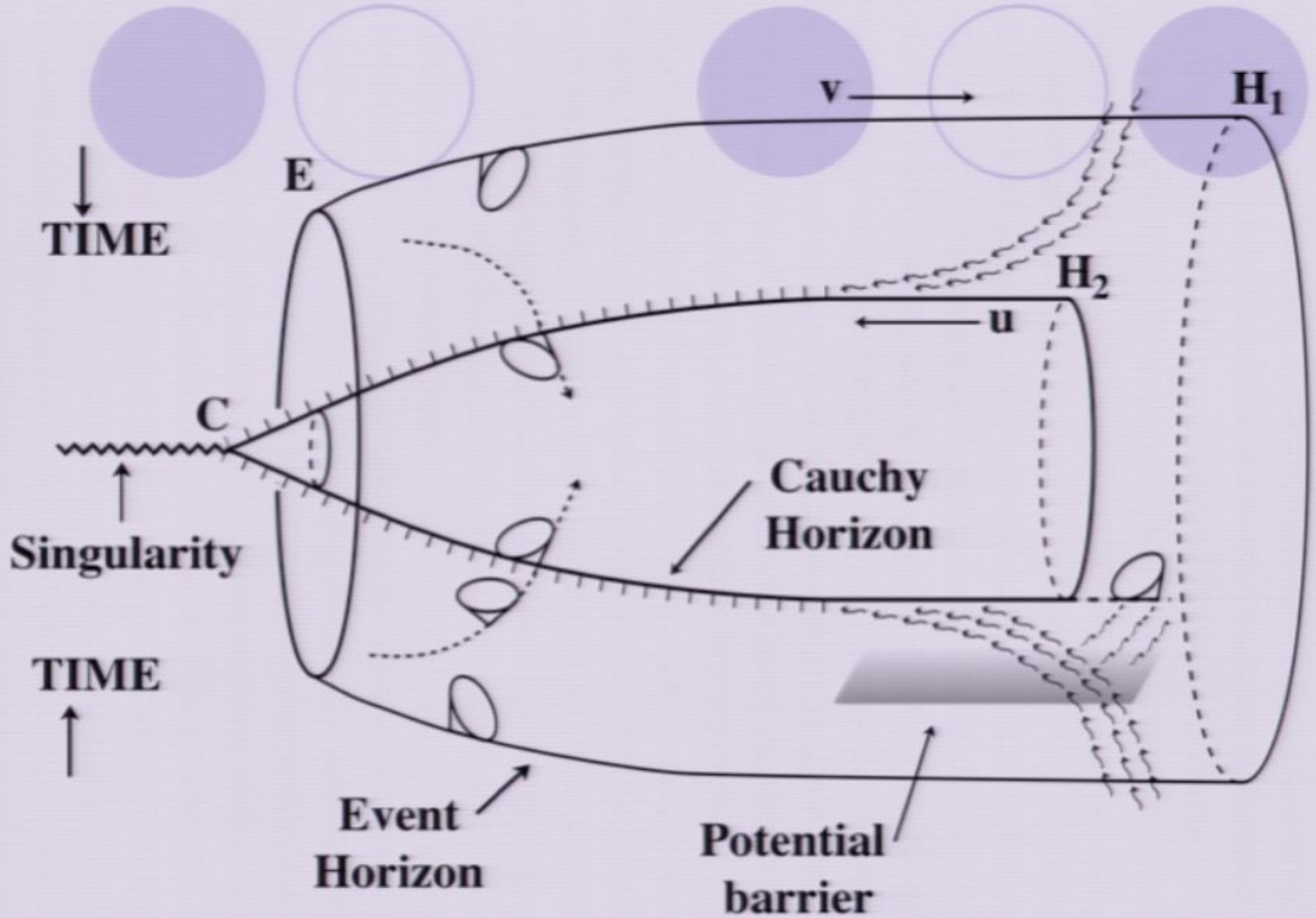
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- Do we predict a  $C^1$  (unique) continuation of the classical spacetime along the CH?





In terms of lightlike coordinates  $U, V$  the minimally-coupled wave equation is

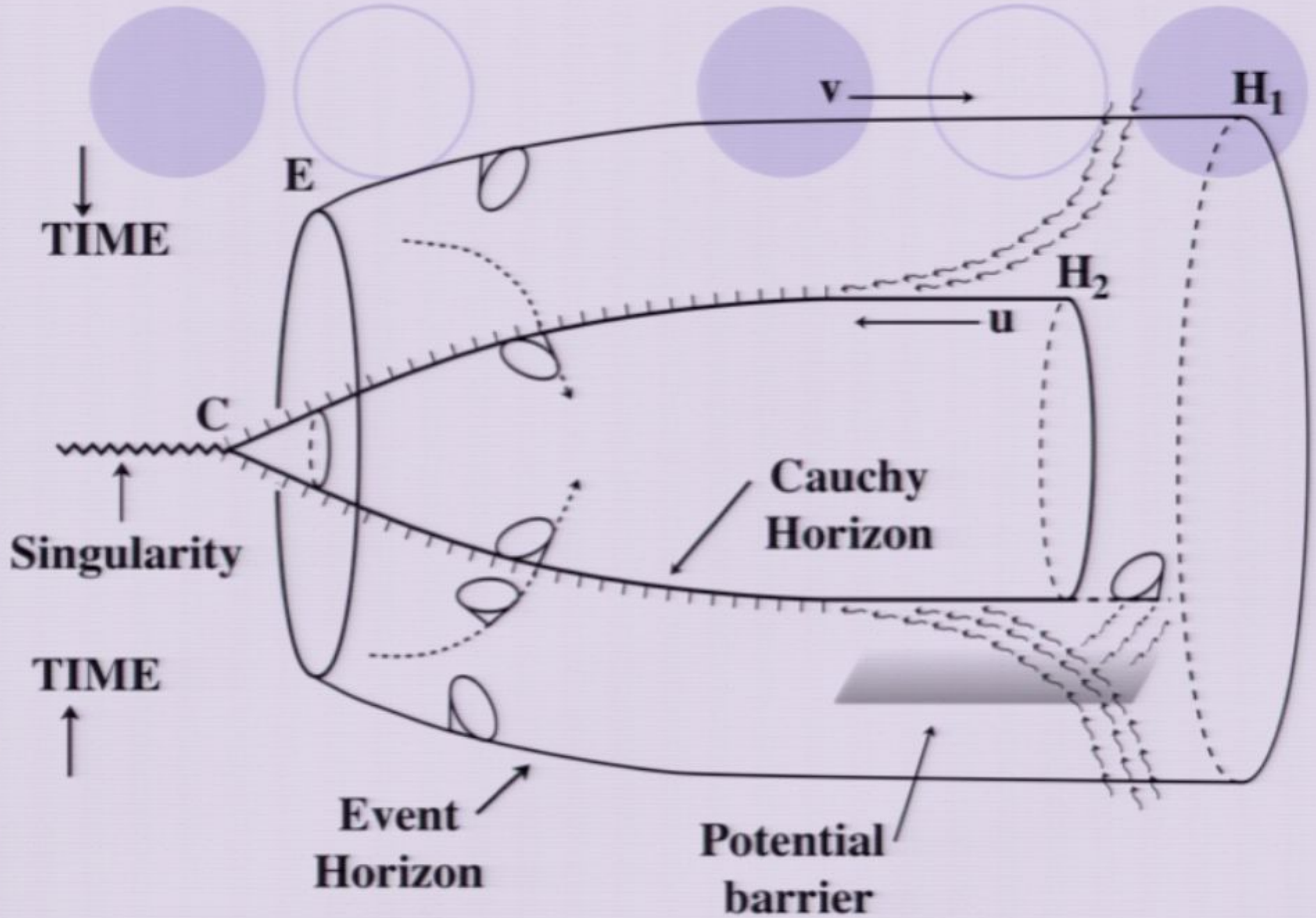
$$r\varphi_{UV} + r_U\varphi_V + r_V\varphi_U = 0$$

for a spherisymmetric massless field  $\varphi(U, V)$ .

The Einstein equations now appear as

$$\begin{aligned} m_U &= -4\pi r^2 e^{-2\sigma} \varphi_U^2 r_V, \\ r_{UU} - 2\sigma_U r_U &= -4\pi r \varphi_U^2, \\ (r^2)_{UV} &= -e^{2\sigma} \left(1 - e^2/r^2\right), \\ \sigma_{UV} &= (e^{2\sigma}/r^3)(m - e^2/r) - 4\pi \varphi_U \varphi_V \end{aligned}$$







# Minimally coupled field

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Define functions  $a(U), b(V)$  by setting their derivatives  $\dot{a}, \dot{b}$  equal respectively to  $\varphi_U|_b, \varphi_V|_b$ , the values on the underside of the inner potential barrier. Define further functions  $A(U), B(V)$  by  $\ddot{A} = 4\pi r_0^2 \dot{a}^2, \ddot{B} = 4\pi r_0^2 \dot{b}^2$ , with the boundary conditions  $A(-\infty) = B(0) = 0$ . Then

$$\begin{aligned}\varphi &= a(U) + b(V) \\ &\quad + r_0^{-2} \{A(U)b(V) + a(U)B(V)\}, \\ r^2 &= r_s^2(U, V) - 2A(U) - 2B(V), \\ \sigma &= \sigma_s(U, V) + r_0^{-4} A(U)B(V), \\ m &= m_0 + (\kappa_0^2/r_0) \dot{A}(U) \dot{B}(V)\end{aligned}$$

Subscript  $s$  refers to the static RN solution (mass  $m_0$ , inner-horizon radius  $r_0$ ) which forms the final exterior state. The general conditions for the validity of the approximation,

$$\dot{A}^2 \ll \ddot{A}, \quad \dot{B}^2 \ll \ddot{B},$$

are satisfied in the situation of interest to us:

$$A \sim [\ln(-U)]^{-(p-1)}, \quad B \sim |-\ln(-V)|^{-(p-1)} \quad (U \rightarrow -\infty, V \rightarrow -0).$$

These expressions confirm that the metric components  $e^{2\sigma}$ ,  $r^2$  (though not their derivatives) are regular and approach the RN values toward the past end of CH.



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- Integrated tidal forces are finite
- Possible C0 extension (Amos Ori conjecture) is reinforced
- What happens beyond EH truncation?