Title: Comments on UV divergences in quantum gravity

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Abstract:

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Arkady Tseytlin

Einstein gravity is UV divergent in perturbation theory

•Pure gravity:

1-loop finite graviton S-matrix

2-loop logarithmic divergence C^3

[Goroff, Sagnotti 86; van de Ven 92]

•Gravity + matter:

S-matrix is divergent already at 1 loop

['t Hooft, Veltman 72]

Effective field theory approach

[Weinberg 76,79]

Add infinite number of counterterms, renormalize all ∞'s
Arbitrary constants predicted by a more fundamental theory
or effectively determined if a fixed point exists

All but first few of infinite set of parameters in a
non-renormalizable theory are suppressed and hence
can be neglected when computing low-energy effects

May include gravitons in loops but with an effective cutoff
low-energy physics is sensitive only to small number parameters:
expansion in $\frac{p^2}{M_{pl}^2} \ll 1$

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Examples:

- quantum corrections to Newton's law [Donoghu 95, 99]

$$\frac{1}{p^2}(1 + a_1 \frac{p^2}{M_{pl}^2} + a_2 \frac{p^2}{M_{pl}^2} \ln \frac{p^2}{M_{pl}^2} + ...)$$

- RG equations for $R\phi^2$ coupled Higgs field and inflation

mixing of scalars and gravitons, ...

[Bezrukov, Shaposhnikov; Barvinsky, Kamenshchik, Starobinsky; De Simone, Hertzberg, F. Wilczek 08-09]

Still, what about fundamental theory valid at all energies ?

Quantum gravity that is predictive at all scales –

principle that reduces ∞ of unknown parameters to

a finite number that can then be "measured"?

How to define quantum gravity theory?

Possible approaches:

I. Gravity as fundamental

II. Gravity must be coupled to matter + new symmetry

III. Gravity is emergent

I. Metric is fundamental, quantise metric
start with some regularised (non-local, e.g., lattice)
theory and try to define continuum limit
or start with R^2 theory and resolve unitarity issue

II. Only gravity + matter theory can be finite (supergravity)

III. Einstein gravity is large distance phenomenon,
"induced" from more fundamental theory (string theory,...)

option I is non-perturbative: hard

•definition of QFT is nontrivial: path integral + RG

asymptotic freedom, fixed point or finiteness

[fixed points and finiteness realised in susy 4d gauge theories]

•what about general coordinate invariance ?

•no fine tuning? predictability? calculability?

Narrow question:

how to compute quantum graviton S-matrix

at low energies ($p^2 \ll M_{pl}^2$) when perturbation theory

(no black hole production, etc) should apply

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Supergravity:

symmetry relating gravity to matter

remarkable improvement over Einstein gravity:

full S-matrix is 1-loop finite

also 2-loop finite (C^3 ruled out by susy)

Chance that $N = 8$ supergravity may be all-loop finite?!

Possible on-set of divergences (superspace counterterms, etc):

L=3 [Kallosh; Howe, Stelle, Townsend 1981]

L=5 [Howe, Stelle 06; Bossard, Howe, Stelle 09]

L=8 [Kallosh 1981]

L=9 [Green,Russo,Vanhove 06]

Recent dramatic progress: [Talk by John Carrasco]

$N = 8$ supergravity is finite at $L = 3$ and $L = 4$ loops

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using lessons from N=4 super Yang-Mills (generalised unitarity)
led to progress in computing amplitudes in N=8 supergravity

close relation between N=8 SG and N=4 SYM amplitudes
Gravity = gauge theory $\times$ gauge theory at tree level
string-theory inspired (Kawai-Lewellen-Tye) rules:
N=4 SYM results carry over to N=8 supergravity
[Bern, Carrasco, Dixon, Johansson, Kosower, Roiban 07-09]

important (unexpected) cancellations are generic to gravity
supersymmetry helps make theory finite
but is not the only ingredient for finiteness at $L = 3, 4$
Graviton scattering amplitudes are finite at least up to 5 loops!

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N=8 supergravity S-matrix has remarkable features
if amplitudes are determined by their leading singularities
then UV finiteness follows
[Arkani-Hamed, Cachazo, Kaplan 08]

Even if N=8 supergravity is UV finite perturbatively
does it define quantum gravity nonperturbatively?
too early to worry but .... perturbation theory not convergent...
black hole production in scattering → pert. theo. ill-defined?
add massless black holes as asymptotic states?
N=8 supergravity is nonperturbatively incomplete?
Limit of string theory?
Cannot decouple maximal supergravity from closed string
[Green, Ooguri, Schwarz 07]

R^2 gravity :

$L = \lambda + M^2 R + aC^2 + bR^2$

analog of YM theory in d=6

[cf. $L = m^2F^2 + aFFF + bFD^2F$]

natural continuum limit of a euclidean lattice model

1-loop renormalization:

asymptotic freedom in all essential couplings

[Julve, Tonin '78; Fradkin, AT '82]

RG equations with matter: asymptotic freedom in $1/a$

"drives" AF in other couplings (Yukawa and masses)

possibility of fixed points

But: unitarity of S-matrix? which are asymptotic states?

need non-perturbative formulation... "confinement" of ghosts?

few attempts, no conclusive evidence

Comments:

•quartic divergences are sensitive to measure or defn. of theory

•$M^2 R$ analogous to mass term in renormalizable theory:
quadratic divergences should be treated in a similar way as in mass renormalization (cf. RG eq. for masses)
dim. reg. is perfect regularization, esp. beyond 1 loop

•asymptotic freedom beyond 1 loop?
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need to know 2-loop β to believe in AF (cf. YM theory)
$aC^2 + bR^2$ – 2-coupling theory:
2-loop β-function of a may depend on b
(and also on gauge coupling if YM term is added; and vice versa!)
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•Compute 2-loop renormalization: very interesting problem
hard but should be tractable with modern technology
[e.g. generalize standard 2-loop algorithm to 4-derivative theories]

Living with Ghosts

S.W. Hawking, T. Hertog

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Abstract: Perturbation theory for gravity in dimensions greater than two requires higher derivatives in the free action. Higher derivatives seem to lead to ghosts, states with negative norm. We consider a fourth order scalar field theory and show that the problem with ghosts arises because in the canonical treatment, ϕ and $\Box\phi$ are regarded as two independent variables. Instead, we base quantum theory on a path integral, evaluated in Euclidean space and then Wick rotated to Lorentzian space. The path integral requires that quantum states be specified by the values of ϕ and $\phi_{,\tau}$. To calculate probabilities for observations, one has to trace out over $\phi_{,\tau}$ on the final surface. Hence one loses unitarity, but one can never produce a negative norm state or get a negative probability. It is shown that transition probabilities tend toward those of the second order theory, as the coefficient of the fourth order term in the action tends to zero. Hence unitarity is restored at the low energies that now occur in the universe.

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Conformal Supergravity:

demand Weyl symmetry as exact quantum symmetry

R term as a result of its spontaneous breaking

$L = aC^2 + \phi(-D^2 + \frac{R}{6})\phi$

$< \phi >= M_{pl}$

advantage: well-defined euclidean path integral

But consistency at quantum level requires UV finiteness –

no conformal anomaly

Conformal supergravity is the answer:

power counting renormalizable

and for $N \geq 2$ finite beyond 1 loop

(supergraph counting rules as in SYM case)

it remains only to cancel 1-loop β-function

UV finite Weyl-invariant theory exists and is unique:
[Fradkin, AT '82]
N=4 conformal supergravity + four N=4 SYM multiplets
[e.g. N=4 conformal SG coupled to $SU(2) \times U(1)$ SYM]

global superconformal symmetry of N=4 SYM here is gauged

N=4 conformal SG as induced theory from N=4 SYM
relation to AdS/CFT

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Superstring Theory

inevitably and automatically includes gravity:

graviton as massless spin 2 excitation about flat space

low energy effective field theory

$L = R + c_1\alpha' R^2 + c_2\alpha'^2 R^3 + \ldots$

with definite coefficients

quantum string corrections ($g_s = e^\phi$): UV finite

α' as built-in cutoff: soft UV behaviour

[cf. $L_{str.f.th.} = \phi e^{-\alpha' \partial^2} \partial^2 \phi + \ldots$]

superstring graviton S-matrix is well-defined:

perturbatively complete theory

Explicit results:

1-loop 4-graviton amplitude [Green, Schwarz, Brink 82]

2-loop 4-graviton amplitude [Jengo, Zhu 86; D'Hoker, Phong 05]

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Pattern of UV finiteness:

compare to Einstein gravity in $d = 10$:

$$L_\infty = \Lambda^{10} + \Lambda^8 R^2 + ... + \Lambda^2 R^4 + \log\Lambda\,(R^5 + R^2 D^2 R^2 + ..)$$

1-loop superstring: $\Lambda^2 \sim \frac{1}{\alpha'}$ as a cutoff

$$L_{eff} = \frac{1}{\alpha'} R^4 + R^2 \ln(\alpha' D^2) D^2 R^2 + ...$$

price for finiteness: functional theory

complicated form of loop (modular) integrals

but in fact organized simpler than QFT pert. theory:

reduces all field theory interaction vertices to a single

string interaction: splitting or joining of strings

economical organization of Feynmann diagrams:

[e.g. B.Kors and M.G.Schmidt, "Two-loop Feynman diagrams

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relation between string and field theory S-matrix in $\alpha' \to 0$ limit

[e.g. Stieberger 09, ...]

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[e.g. Stieberger 09, ...]

Standard mantra:
String theory is a very fertile framework
for a consistent theory of quantum gravity
but still rather preliminary stage of our understanding
open issues: non-perturbative formulation of the theory
and uncovering its symmetries

Remarkable recent progress: gauge-string duality
AdS/CFT: uncovers hidden simplicity in string theory
unexpected coonections: all-loop finiteness of string theory
since 1/N expansion of SYM gauge theory is finite
non-perturbative definition of string theory is gauge theory?

Gauge-string duality

example of quantum-consistent string theory in curved space
type IIB supergravity seen at level of fluctuations
quantum gravity phenomena contained as well in certain regime

gauge theory at large N_c and large coupling $\lambda = g^2 N_c$:
(classical) gravity "emerges"

string theory is "hidden" in gauge theory
thus is not an invented or artificial concept
string variables are non-local, locality is emergent

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Future?

- finiteness of N=8 supergravity to all orders? ...
- proving AS conjecture rigorously and turning it

into a practical formalism for QG?...

- other non-perturbative approaches to QG

...making sense of R^2 theory ...?

- string theory seems best bet

for UV completion of Einstein gravity...

aside from being candidate theory of quantum gravity
string theory is very useful for gauge theory:
describes "safest" of all theories: $N = 4$ SYM

Comments on gauge-string duality

General aims:

•understand quantum gauge theories at any coupling
[applications to both perturbative and non-perturbative issues]
•understand string theories in non-trivial backgrounds
[e.g. RR ones for flux compactifications]

AdS/CFT duality:

•relates the two questions suggesting solving them together
rather than separately is best strategy
•relates simplest most symmetric theories
use of symmetries on both sides to make progress

Integrability:

Existence of powerful hidden symmetries
allowing to solve problem "in principle"

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string theory is very useful for gauge theory:
describes "safest" of all theories: $N = 4$ SYM

Comments on gauge-string duality

General aims:

- understand quantum gauge theories at any coupling
[applications to both perturbative and non-perturbative issues]
- understand string theories in non-trivial backgrounds
[e.g. RR ones for flux compactifications]

AdS/CFT duality:

- relates the two questions suggesting solving them together rather than separately is best strategy
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Integrability:

Existence of powerful hidden symmetries
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[or spectrum of gauge-theory energies on $R \times S^3$]

• correlation functions of these operators (to which extent?)

Recent remarkable progress:

Spectrum of states

I. Spectrum of "long" operators = "semiclassical" string states
determined by Asymptotic Bethe Ansatz (2002-2007)

- its final (BES) form found after intricate superposition of information from perturbative gauge theory (spin chain, BA,...) and perturbative string theory (classical and 1-loop phase,...), symmetries (S-matrix), assumption of exact integrability
- consequences checked against all available gauge and string data

Key example:
cusp anomalous dimension $\mathrm{Tr}(\Phi D^S \Phi)$

$$f(\lambda \ll 1) = \frac{\lambda}{2\pi^2}\Big[1 - \frac{\lambda}{48} + \frac{11\lambda^2}{2^8 \cdot 45} - (\frac{73}{630} + \frac{4\zeta^2(3)}{\pi^6})\frac{\lambda^3}{2^7} + ...\Big]$$

$$f(\lambda \gg 1) = \frac{\sqrt{\lambda}}{\pi}[1 - \frac{3\ln 2}{\sqrt{\lambda}} - \frac{K}{(\sqrt{\lambda})^2} - ...]$$

II. Spectrum of "short" operators = all quantum string states

Thermodynamic Bethe Ansatz (2005-2009)

- reconstructed from ABA using solely methods/intuition of 2-d integrable QFT, i.e. string-theory side (how to incorporate wrapping terms directly on gauge-theory side?)
- highly non-trivial construction – lack of 2-d Lorentz invariance in the standard "BMN-vacuum-adapted" l.c. gauge
- in few cases ABA "improved" by Luscher corrections is enough: 4 and 5-loop Konishi dimension, 4-loop minimal twist op. dim
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anomalous dimension of Konishi operator $\mathrm{Tr}(\bar{\Phi}_i \Phi_i)$

$$\gamma(\lambda \ll 1) = \frac{12\lambda}{(4\pi)^2}\Big[1 - \frac{4\lambda}{(4\pi)^2} + \frac{28\lambda^2}{(4\pi)^4} - [208 - 48\zeta(3) + 120\zeta(5)]\frac{\lambda^3}{(4\pi)^6} + 8[158 + 72\zeta(3) - 54\zeta^2(3) - 90\zeta(5) + 315\zeta(7)]\frac{\lambda^4}{(4\pi)^8} + ...\Big]$$

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Suppose sum up $\lambda \ll 1$ expansion and re-expand at $\lambda \gg 1$

values of $\mathrm{b}_0, b_1, b_2, ...$?

directly from string theory ?

from TBA/Y-system that should be describing string spectrum ?

Many open questions:

Analytic form of strong-coupling expansion from TBA/Y-system?

Matching onto string spectrum in near-flat-space expansion?

No level crossing?

Strong-coupling expansion is Borel (non)summable?

Exponential corrections $e^{-a\sqrt{\lambda}}$ like in cusp anomaly case?

...

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Future?

- finiteness of N=8 supergravity to all orders? ...
- proving AS conjecture rigorously and turning it into a practical formalism for QG?...
- other non-perturbative approaches to QG ...making sense of R^2 theory ...?
- string theory seems best bet for UV completion of Einstein gravity...

aside from being candidate theory of quantum gravity string theory is very useful for gauge theory: describes "safest" of all theories: $N = 4$ SYM

Standard mantra:

String theory is a very fertile framework

for a consistent theory of quantum gravity

but still rather preliminary stage of our understanding

open issues: non-perturbative formulation of the theory

and uncovering its symmetries

Remarkable recent progress: gauge-string duality

AdS/CFT: uncovers hidden simplicity in string theory

unexpected coonections: all-loop finiteness of string theory

since 1/N expansion of SYM gauge theory is finite

non-perturbative definition of string theory is gauge theory?

Pattern of UV finiteness:

compare to Einstein gravity in $d = 10$:

$L_\infty = \Lambda^{10} + \Lambda^8 R^2 + ... + \Lambda^2 R^4 + \log\Lambda\,(R^5 + R^2 D^2 R^2 + ..)$

1-loop superstring: $\Lambda^2 \sim \frac{1}{\alpha'}$ as a cutoff

$$L_{eff} = \frac{1}{\alpha'} R^4 + R^2 \ln(\alpha' D^2) D^2 R^2 + ...$$

price for finiteness: functional theory

complicated form of loop (modular) integrals

but in fact organized simpler than QFT pert. theory:

reduces all field theory interaction vertices to a single

string interaction: splitting or joining of strings

economical organization of Feynmann diagrams:

[e.g. B.Kors and M.G.Schmidt, "Two-loop Feynman diagrams

in Yang-Mills theory from bosonic string amplitudes" '00]

relation between string and field theory S-matrix in $\alpha' \to 0$ limit

[e.g. Stieberger 09, ...]

Superstring Theory

inevitably and automatically includes gravity:

graviton as massless spin 2 excitation about flat space

low energy effective field theory

$L = R + c_1\alpha' R^2 + c_2\alpha'^2 R^3 + ...$

with definite coefficients

quantum string corrections ($g_s = e^\phi$): UV finite

α' as built-in cutoff: soft UV behaviour

[cf. $L_{str.f.th.} = \phi e^{-\alpha'\partial^2}\partial^2\phi + ...$]

superstring graviton S-matrix is well-defined:

perturbatively complete theory

Explicit results:

1-loop 4-graviton amplitude [Green, Schwarz, Brink 82]

2-loop 4-graviton amplitude [Jengo, Zhu 86; D'Hoker, Phong 05]

UV finite Weyl-invariant theory exists and is unique:
[Fradkin, AT '82]
N=4 conformal supergravity + four N=4 SYM multiplets
[e.g. N=4 conformal SG coupled to $SU(2) \times U(1)$ SYM]

global superconformal symmetry of N=4 SYM here is gauged

N=4 conformal SG as induced theory from N=4 SYM
relation to AdS/CFT

But higher derivatives and unitarity issue remains...

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