Title: Exploring the Theory Space of Asymptotically Safe Quantum Gravity

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Abstract:

Exploring the Theory Space of Asymptotically Safe Quantum Gravity

Frank Saueressig

D. Benedetti, P. Machado, F.S., Mod. Phys. Lett. A24 (2009) 2233

D. Benedetti, P. Machado, F.S., Nucl. Phys. B824 (2010) 168

E. Manrique, M. Reuter, F.S., in preparation

Asymptotic Safety - 30 years later

Perimeter Institute, November 6th, 2009

Classical gravity

Based on General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} + 8\pi G_N T_{\mu\nu}$$

- experiment:
  - Newtons constant: $G_N = 6.67 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$
  - cosmological constant: $\Lambda \approx 10^{-35}\ \mathrm{s}^{-2}$
  - good description of gravity between $10^{-2}\mathrm{cm}$ and $10^{28}\mathrm{cm}$

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conceptual and phenomenological puzzles:

- GR is classical $\Longleftrightarrow$ other forces QFT?
- structure of space-time at short distances?
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Theoretical guidance: Quantum Theory for Gravity

Quantizing General Relativity

perturbative quantization of General Relativity:

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  - infinite number of counterterms
  - General Relativity is perturbatively non-renormalizable

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a) Treat General Relativity as effective field theory:

- compute corrections in $E^2/M_{\rm Pl}^2 \ll 1$ (independent of UV-completion)
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- supersymmetry, extra dimensions, 10^{500} vacua, ...
- possibly: extension of QFT-framework

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Renormalization from a Wilsonian perspective

Central element: Renormalization Group (RG) flow of theory:

- coupling constants become scale dependent: $G_N \to G_N(k)$
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fundamental action	RG trajectory
Renormalizability	RG trajectory emanates from fixed point in UV
predictivity	UV critical surface of fixed point has finite dimension

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Non-trivial generalization of perturbative renormalization:

- perturbatively renormalizable theory $\Longleftrightarrow$ Gaussian Fixed Point
  - asymptotic freedom (QCD)
- non-perturbatively renormalizable theory $\Longleftrightarrow$ Non-Gaussian Fixed Point
  - asymptotic safety

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Non-trivial generalization of perturbative renormalization:

- non-perturbatively renormalizable theory $\Longleftrightarrow$ Non-Gaussian Fixed Point

Exciting perspective: Gravity is "non-perturbative renormalizable":

Weinbergs asymptotic safety conjecture (1979):

gravity in $d = 4$ has NGFP controlling its UV behavior

Outline

- Functional Renormalization Group Equations: a primer
- Exploring the Theory Space of pure Gravity:
  - The Einstein-Hilbert truncation
  - Including higher-derivative interactions
  - A first bi-metric setup
- Asymptotic safety vs. perturbative counterterms
  - Étude: Gravity coupled to scalar matter
- Conclusion

Functional Renormalization Group Equation for gravity

Requirement for testing asymptotic safety:

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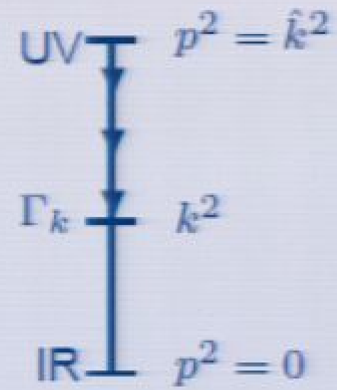
- Path-integral approach to Quantum Gravity:
  - do gravitational path-integral shell by shell
- implements Wilsons idea of renormalization:
  - $\Gamma_k \approx$ effective description of physics at scale k
  - FRGE: change of Γ_k with coarse-graining scale k:

$$k\partial_k \Gamma_k[\Phi, \bar{\Phi}] = \frac{1}{2}\mathrm{Tr}\left[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} k\partial_k \mathcal{R}_k\right]$$



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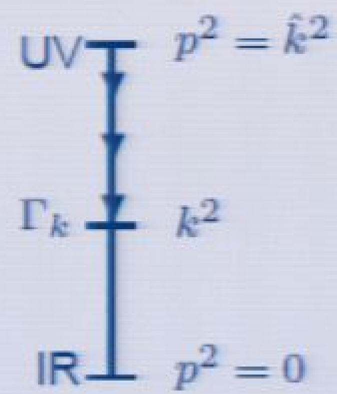

- versatile tool not limited to Quantum Gravity

Effective average action: Building blocks

Path-integral approach to Quantum gravity

- starting point: generic diffeomorphism invariant action

$$S^{\mathrm{grav}}[\gamma_{\mu\nu}]$$

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$$S^{\rm grav}[\gamma_{\mu\nu}]$$

- perform background gauge fixing $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$S^{\rm gf} = \frac{1}{2\alpha} \int d^4x \sqrt{\bar{g}} F_\mu Y^{\mu\nu} F_\nu$$

$$F_\mu = \bar{D}^\mu h_{\mu\nu} - \beta \bar{D}_\mu h \,, \quad Y^{\mu\nu} = \left[p_1 + p_2 \bar{D}^2\right] \bar{g}^{\mu\nu}$$

  - gauge choices:
    - harmonic gauge: $\beta = 1/2$,
    - geometric gauge: $\beta = 1/4$
  - add ghost term: $S^{\rm gh}[h, C, \bar{C}, \bar{b}, b; \bar{g}]$

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Path-integral approach to Quantum gravity

- action in the path integral:

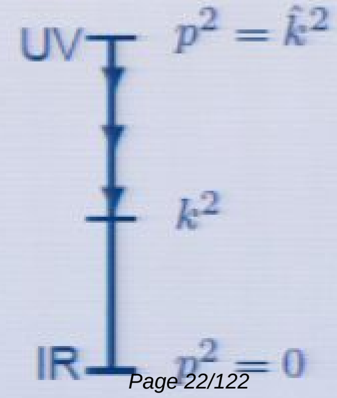
$$S^{\text{grav}}[\gamma] + S^{\text{gf}}[h;\bar{g}] + S^{\text{gh}}[h, C, \bar{C}, \bar{b}, b; \bar{g}]$$

- perform background gauge fixing $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- add: k-dependent IR cutoff:

$$\Delta_k S[h;\bar{g}] = \int d^4x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k[\bar{g}]^{\mu\nu\rho\sigma} h_{\rho\sigma}$$

  - $\mathcal{R}_k[\bar{g}] \propto \mathcal{Z}_k k^2 R^{(0)}$ = k-dependent mass term
  - discriminate between low/high- $\bar{D}^2$-eigenmodes

$$R^{(0)}(p^2/k^2) = \begin{cases} 1 & p^2 \ll k^2 \\ 0 & p^2 \gg k^2 \end{cases}$$



- high momentum modes: integrated out
- low momentum modes: suppressed by mass term

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k-dependent generating functional for connected Green functions

$$W_k[J] = \ln \int \mathcal{D}\Phi \exp\{S^{\mathrm{grav}}[\gamma] + S^{\mathrm{gf}}[h;\bar{g}] + S^{\mathrm{gh}}[h, C, \bar{C}, \bar{b}, b; \bar{g}] + \Delta_k S[h;\bar{g}] + S^{\mathrm{source}}[J]\}$$

- k-dependent effective action: $\tilde{\Gamma}_k[\Phi]$ = Legendre transform of $W_k[J]$
- Effective average action:

$$\Gamma_k[\Phi] \equiv \tilde{\Gamma}_k[\Phi] - \Delta_k S[\Phi]$$

Effective average action: Properties

- Definition:

$$\Gamma_k[\phi] = \tilde{\Gamma}_k[\phi] - \Delta_k S[\phi]$$

- k-dependence governed by Functional RG Equation (FRGE)

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

  - Formally: exact equation $\Leftrightarrow$ no approximations in derivation
  - independent of "fundamental theory" $\Leftrightarrow$ $S_{\hat{k}}$ enters as initial condition

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  - $k \to \infty$ $\simeq$ bare/classical action S
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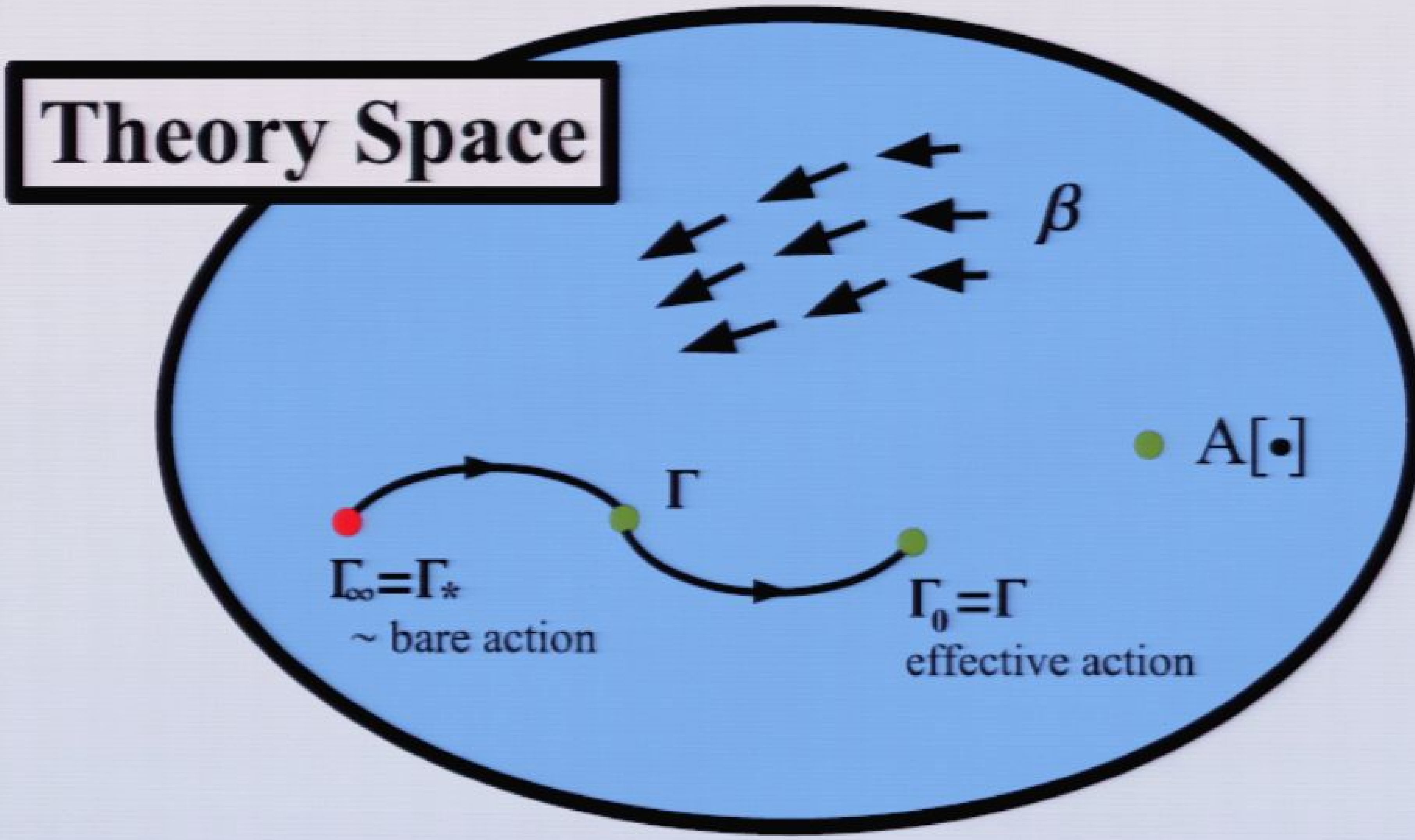
- Theory: specified by RG trajectory

$$k \mapsto \Gamma_k[\phi]$$

Theory space underlying the Functional Renormalization Group



Non-perturbative approximation: truncating Theory Space

- Caveat: FRGE cannot be solved exactly

$\Longleftrightarrow$ Gravity: need non-perturbative approximation scheme

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- Truncating the form of Γ_k:

$$\Gamma_k[\Phi] = \sum_{i=1}^{N} \bar{u}_i(k)\, \mathcal{O}_i[\Phi]$$

  - $\Longrightarrow$ substitute into FRGE
  - $\Longrightarrow$ projection of flow gives β-functions for running couplings

$$k\partial_k \bar{u}_i(k) = \beta_i(\bar{u}_i; k)$$

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- Testing the reliability:
  - within a given truncation:
    - cutoff-scheme dependence of physical quantities
  - stability of results under extensions of the truncation

Quantum effects and the gravitational effective average action

$$\Gamma_k[g, C, \bar{C}; \bar{g}] =$$

$$\Gamma_k^{\text{gravitational}}[g] + \widehat{\Gamma}_k^{\text{bi-metric}}[g - \bar{g}; \bar{g}] + \Gamma_k^{\text{ghost}}[g, C, \bar{C}; \bar{g}] + \cdots$$

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Interactions build from physical metric:

$$(16\pi G_k)^{-1} \int d^d x \sqrt{g}(-R + 2\Lambda_k) + \ldots$$

Reuter; Percacci, Dou, Perini; Suoma; Lauscher, Saueressig; Litim, Fischer; Bonanno; Forgacs, Niedermaier; Codello, Rahmede, Weyer, Daum, Schwindt; Nesti Benedetti, Machado; Manrique, Harst; Koslowski,Narain,Vacca,Zanusso,Zambelli

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Interactions vanishing for $g = \bar{g}$:

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Manrique, Reuter, Saueressig

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Manrique, Reuter, Saueressig

Quantum effects in the ghost sector:

- ghost wave-function renormalization
- k-dependent curvature ghost couplings

Eichhorn, Gies, Scherer

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Single-Metric truncations

Charting the theory space spanned by $\Gamma_k^{\text{grav}}[g]$

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma} C_{\rho\sigma}{}^{\kappa\lambda} C_{\kappa\lambda}{}^{\mu\nu}$	$R \Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$	$R_{\mu\nu} R^{\mu\nu}$	
R			
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Charting the theory space spanned by $\Gamma_k^{\text{grav}}[g]$

Einstein-Hilbert truncation

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R^2-truncation

$\vdots$

R^8	$\ldots$		
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Einstein-Hilbert truncation

polynomial $f(R)$-truncation

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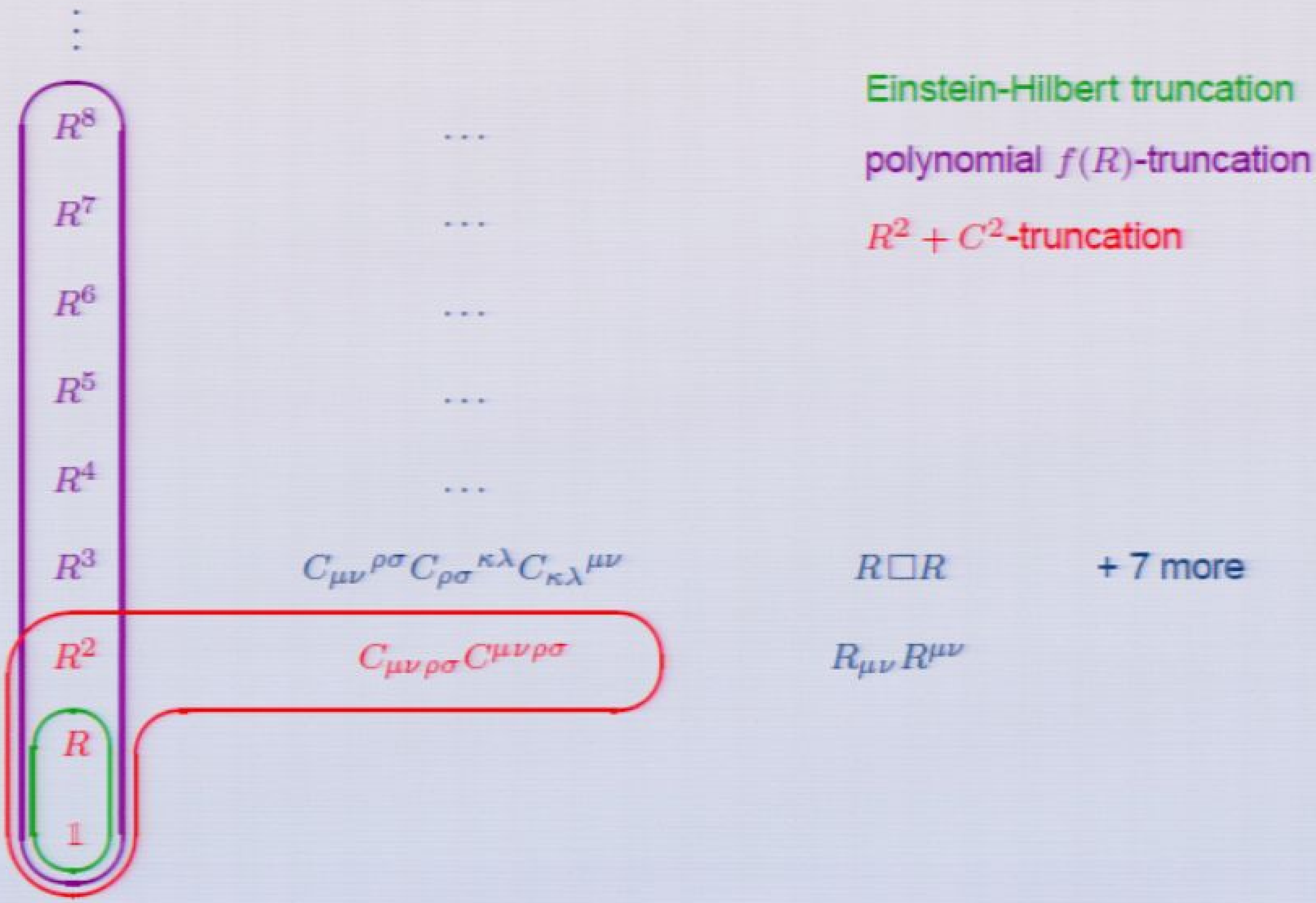

Einstein-Hilbert truncation: setup

- Projects RG-flow onto two "running" couplings: $G(k), \Lambda(k)$

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- result: non-perturbative β-functions for dimensionless couplings

$$g_k := k^2 G_k \,, \quad \lambda_k := \Lambda_k k^{-2}$$

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- Explicit example: $\mathcal{R}_k$ = sharp cutoff, $\alpha = 16\pi G_k$, $\beta = 1/2$

$$k\,\partial_k g_k = (\eta_N + 2) g_k \,,$$

$$k\,\partial_k\,\lambda_k = -\,(2 - \eta_N)\,\lambda_k - \tfrac{g_k}{\pi}\left[5\ln(1 - 2\,\lambda_k) - 2\zeta(3) + \tfrac{5}{2}\eta_N\right]$$

"non-perturbative" anomalous dimension of Newton's constant:

$$\eta_N = -\frac{2g_k}{6\pi + 5g_k}\left[\frac{18}{1 - 2\lambda_k} + 5\ln(1 - 2\lambda_k) - \zeta(2) + 6\right]$$

Einstein-Hilbert truncation: Fixed Point structure

- β-functions for dimensionless couplings: $g_k := k^2 G_k$, $\lambda_k := \Lambda_k k^{-2}$

$$k\,\partial_k g_k = \beta_g(g_k, \lambda_k)\,, \quad k\,\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k)$$

microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

$$\beta_g(g^*, \lambda^*) = 0\,, \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

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  - saddle point in the g-λ-plane
- Non-Gaussian Fixed Point ($\eta_N^* = -2$):
  - at $g^* > 0, \lambda^* > 0 \Longleftrightarrow$ "interacting" theory
  - UV attractive in g_k, λ_k

Einstein-Hilbert truncation: Fixed Point structure

- β-functions for dimensionless couplings: $g_k := k^2 G_k\,,\ \lambda_k := \Lambda_k k^{-2}$

$$k\,\partial_k g_k = \beta_g(g_k, \lambda_k)\,, \quad k\,\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k)$$

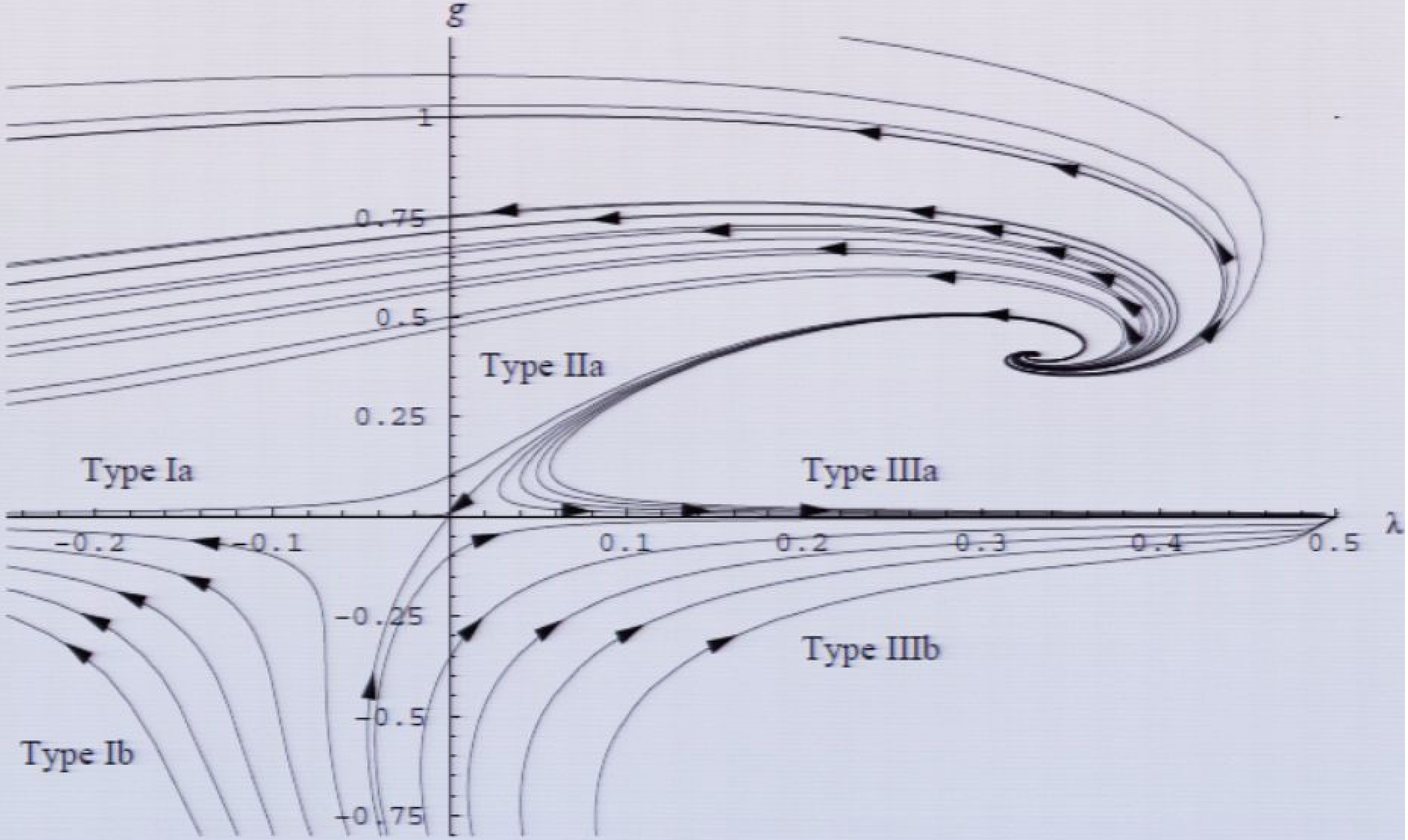
microscopic theory $\Longleftrightarrow$ fixed points of the β-functions

$$\beta_g(g^*, \lambda^*) = 0\,, \qquad \beta_\lambda(g^*, \lambda^*) = 0$$

- Gaussian Fixed Point:
  - at $g^* = 0, \lambda^* = 0 \Longleftrightarrow$ free theory
  - saddle point in the g-λ-plane
- Non-Gaussian Fixed Point ($\eta_N^* = -2$):
  - at $g^* > 0, \lambda^* > 0 \Longleftrightarrow$ "interacting" theory
  - UV attractive in g_k, λ_k

Asymptotic safety conjecture: NGFP provides UV completion for gravity

Einstein-Hilbert-truncation: the phase diagram



Einstein-Hilbert truncation: Stability properties

Ref.	g^*	λ^*	$g^*\lambda^*$	$\theta' \pm i\theta''$	α	β	cutoff
BMS	0.902	0.109	0.099	$2.52 \pm 1.78i$	0	1/4	II, opt
RS	0.403	0.330	0.133	$1.94 \pm 3.15i$	1	1/2	I, sharp
LR	0.272	0.348	0.095	$1.55 \pm 3.84i$	1	1/2	I, exp
	0.344	0.339	0.117	$1.86 \pm 4.08i$	0	1/2	I, exp
L	1.17	0.25	0.295	$1.67 \pm 4.31i$	0	1/2	I, opt
CPR	0.707	0.193	0.137	$1.48 \pm 3.04i$	1	1/2	I, opt
	0.556	0.092	0.051	$2.43 \pm 1.27i$	1	1/2	II, opt
	0.332	0.274	0.091	$1.75 \pm 2.07i$	1	1/2	III, opt

BMS: Benedetti, Machado, Saueressig, 2009.

RS: Reuter, Saueressig, 2002.

LR: Lauscher, Reuter, 2002.

L: Litim, 2004.

CPR: Codello, Percacci, Rahmede, 2009.

Einstein-Hilbert truncation: Open Questions

- Existence of NGFP in extended truncations?
- Dimension of UV-critical surface?
- What about . . . the Goroff-Sagnotti Counterterm?

Charting the theory space of gravity

Einstein-Hilbert truncation

polynomial $f(R)$-truncation

$\vdots$

R^8	$\cdots$		
R^7	$\cdots$		
R^6	$\cdots$		
R^5	$\cdots$		
R^4	$\cdots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\kappa\lambda}C_{\kappa\lambda}{}^{\mu\nu}$	$R\Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$	
R			
$\mathbb{1}$			

Renormalization group flow of $f(R)$-gravity

A. Codello, R. Percacci, C. Rahmede, Int. J. Mod. Phys. A23 (2008) 143

P. Machado, F. Saueressig, Phys. Rev. D77 (2008) 124045

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Flow equation for $f(R)$-gravity:

$$\Gamma_k^{\text{grav}}[g] = \int d^4x \sqrt{g}\, f_k(R)$$

- complicated partial differential equation governing k-dependence of $f_k(R)$

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UV properties of RG flow:

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} \bar{u}_n R^n + \ldots$
- expand flow equation $\Longrightarrow$ β-functions for $g_n = \bar{u}_n k^{2i-4}$

$$k\partial_k g_n = \beta_{g_n}(g_0, g_1, \ldots)\,, \quad n = 0, \ldots, N$$

- reduces search for NGFP to algebraic problem

Renormalization group flow of $f(R)$-gravity

- Polynomial expansion: $f_k(R) = \sum_{n=0}^{N} g_n \, k^4 \, (R/k^2)^n + \ldots$

$$k\partial_k g_n = \beta_{g_n}(g_0, g_1, \ldots)\,, \quad n = 0, \ldots, N$$

- NGFP can be traced through extensions of truncation subspace

N	g_0^*	g_1^*	g_2^*	g_3^*	g_4^*	g_5^*	g_6^*
1	0.00523	-0.0202					
2	0.00333	-0.0125	0.00149				
3	0.00518	-0.0196	0.00070	-0.0104			
4	0.00505	-0.0206	0.00026	-0.0120	-0.0101		
5	0.00506	-0.0206	0.00023	-0.0105	-0.0096	-0.00455	
6	0.00504	-0.0208	0.00012	-0.0110	-0.0109	-0.00473	0.00238

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- linearized RG flow at NGFP $\Longrightarrow$ three UV relevant directions

N	Re $\theta_{0,1}$	Im $\theta_{0,1}$	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.38	2.17					
2	1.26	2.44	27.0				
3	2.67	2.26	2.07	-4.42			
4	2.83	2.42	1.54	-4.28	-5.09		
5	2.57	2.67	1.73	-4.40	-3.97 + $4.57i$	-3.97 - $4.57i$	
6	2.39	2.38	1.51	-4.16	-4.67 + $6.08i$	-4.67 - $6.08i$	-8.67

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NGFP is stable under extension of truncation subspace

good evidence: fundamental theory has finite number of relevant parameters

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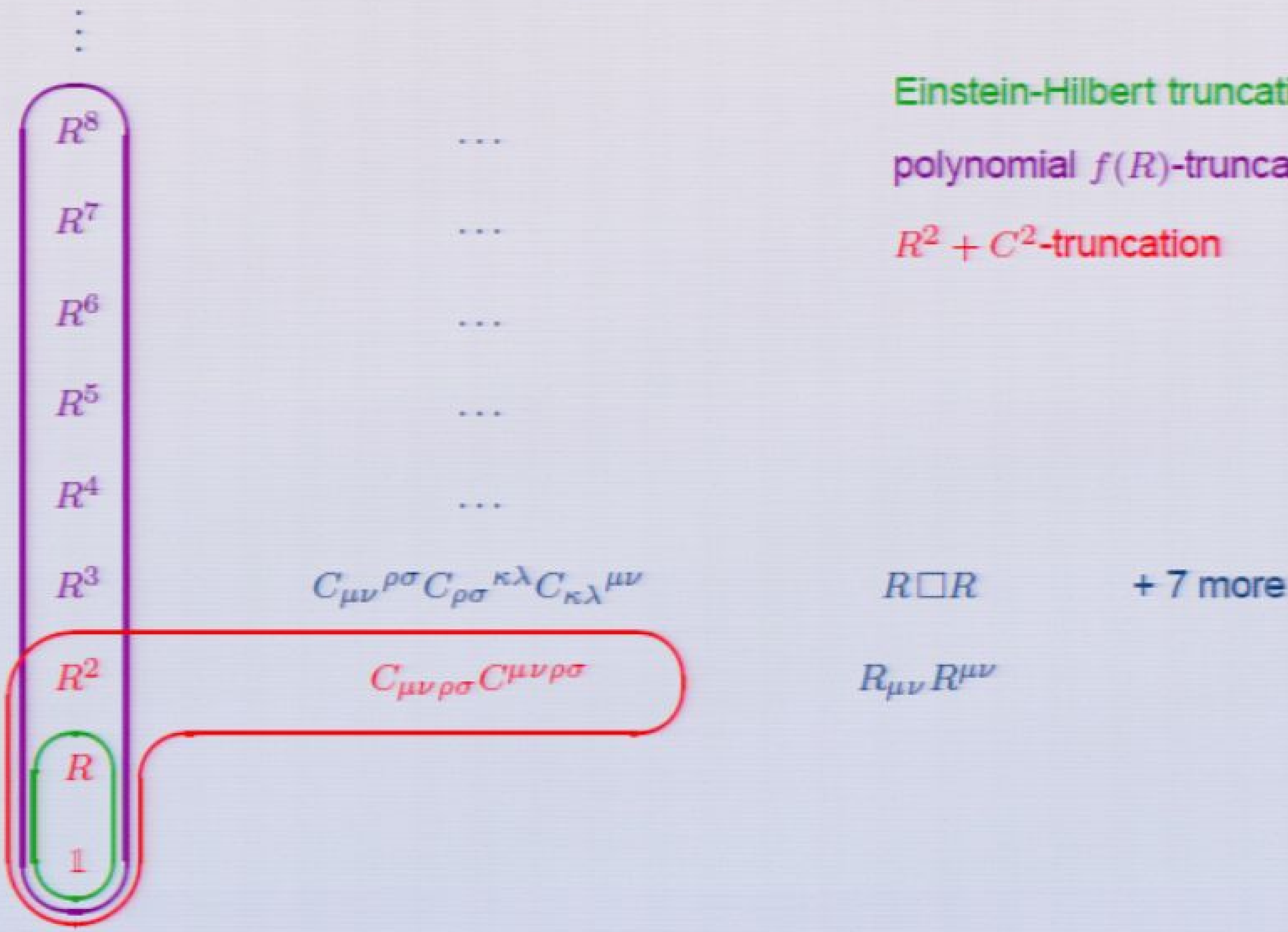
$R^2 + C^2$-truncation



The $R^2 + C^2$-Truncation

Truncation ansatz

$$\Gamma_k^{\text{grav}}[g] = \int d^4x \sqrt{g} \left[\bar{u}_0 + \bar{u}_1 R + \bar{u}_2 R^2 + \bar{u}_3 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \right]$$

- C^2-coupling: characteristic behavior of Higher-Derivative gravity

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computing β-functions: Novel projection technique for traces

- $f(R)$-computation: maximally symmetric background metric (S^d)
  - insufficient to disentangle running of $\bar{u}_2$ and $\bar{u}_3$!
- evaluate traces on generic Einstein background
  - background sufficiently general to distinguish R^2 and C^2 interaction!
  - all differential operators are Lichnerowicz:

$$\Delta_{2,L}\phi_{\mu\nu} = -D^2\phi_{\mu\nu} - 2R_\mu{}^\alpha{}_\nu{}^\beta\phi_{\alpha\beta}$$

$$\Delta_{1,L}\phi_\mu = -D^2\phi_\mu - R_{\mu\nu}\phi^\nu$$

$$\Delta_{0,L}\phi = -D^2\phi$$

  - Trace-evaluation: standard heat-kernel techniques

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Structure of the flow equation:

$$k\partial_k \Gamma_k[\Phi] = \mathcal{S}_{2\mathrm{T}} + \mathcal{S}_{\mathrm{hh}} - \mathcal{S}_{1\mathrm{T}} - \mathcal{S}_0$$

- gravity contribution:

$$\mathcal{S}_{2\mathrm{T}} = \tfrac{1}{2} \mathrm{Tr}_{2\mathrm{T}} \left[\frac{\partial_t \left\{ 2\bar{u}_3 (P_{2,k}^2 - \Delta_{2,L}^2) - (\bar{u}_1 + \bar{u}_\flat R) R_{2,k} \right\}}{2\bar{u}_3 P_{2,k}^2 - (\bar{u}_1 + \bar{u}_\flat R) P_{2,k} - \frac{1}{2} \bar{u}_1 R - \bar{u}_0} \right],$$

$$\mathcal{S}_{\mathrm{hh}} = \tfrac{1}{2} \mathrm{Tr}_0 \left[\frac{\partial_t \left\{ 6\bar{u}_2 (P_{0,k}^2 - \Delta_{0,L}^2) + (\bar{u}_1 - 2\bar{u}_2 R) R_{0,k} \right\}}{6\bar{u}_2 P_{0,k}^2 + (\bar{u}_1 - 2\bar{u}_2 R) P_{0,k} + \frac{2}{3} \bar{u}_0} \right]$$

- universal contribution (auxiliary and ghost fields):

$$\mathcal{S}_{1\mathrm{T}} = \tfrac{1}{2} \mathrm{Tr}_{1\mathrm{T}} \left[\frac{\partial_t R_{1,k}}{P_{1,k}} \right], \quad \mathcal{S}_0 = \tfrac{3}{2} \mathrm{Tr}_0 \left[\frac{\partial_t R_{0,k}}{3P_{0,k} - R} \right]$$

NGFP in the $R^2 + C^2$-Truncation

Truncation	g^*	λ^*	u_2	u_3	u_4	$\lambda^* g^*$	cutoff
$R^2 + C^2$	1.960	0.218	0.008	-0.0050	–	0.427	non-pert.
LR II	0.292	0.330	0.005	–	–	0.096	non-pert.
CP	1.389	0.221	$*$	$*$	$*$	0.307	one-loop

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- Stability properties: (lift degeneracy of marginal couplings)

$$\theta_0 = 2.51\,, \quad \theta_1 = 1.69\,, \quad \theta_2 = 8.40\,, \quad \theta_3 = -2.11$$

  - 3 UV-attractive + 1 UV-repulsive eigendirection

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- RG trajectory captured by NGFP in UV $\Longleftrightarrow$ condition between couplings
  - linear regime around NGFP:

$$g_3 = -0.116 + 0.745 g_0 - 2.441 g_1 + 11.06 g_2$$

flow towards IR ⇔ relation between couplings in effective field theory

Bimetric truncations

Bimetric truncations

Bimetric truncation: General philosophy

- General ansatz for Γ_k:

$$\Gamma_k \approx \Gamma_k^{\text{grav}}[g] + \widehat{\Gamma}_k[g,\bar{g}] + S^{\text{gf}} + S^{\text{ghost}}, \qquad \widehat{\Gamma}_k\Big|_{g=\bar{g}} = 0$$

- Single-metric truncation: $\widehat{\Gamma}_k[g,\bar{g}] \approx 0$

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A priori: unclear if this is the case

Bimetric truncation: Motivation

Effective average action $\Gamma_k[g, \xi, \bar{\xi}, \bar{g}]$ has "bi-metric" structure:

$$S^{\text{gf}}[h; \bar{g}], \qquad S^{\text{ghost}}[h; \bar{g}], \qquad \Delta_k S[h; \bar{g}] \propto \int d^4x \sqrt{\bar{g}} h_{\mu\nu} \mathcal{R}_k[\bar{g}]^{\mu\nu\rho\sigma} h_{\rho\sigma}$$

- involve metric so that $\bar{g}$ does not combine as $g = h + \bar{g}$

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Infrared cutoff $\Delta_k S[h;\bar{g}]$:

$$\mathcal{R}_k[\bar{g}] \propto k^2 R^{(0)}(p^2/k^2)\,, \qquad \lim_{k^2 \to \infty} R^{(0)}(p^2/k^2) = 1$$

- quadratically divergent in UV
- source of strong $g \neq \bar{g}$ effects?

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Bimetric truncation: Example

Ansatz for the Double-Einstein-Hilbert truncation:

$$\Gamma_k^{\mathrm{grav}} + \widehat{\Gamma}_k = (16\pi G_k)^{-1} \int d^4x \sqrt{g}\,(-R + 2\Lambda_k)$$
$$+ (16\pi G_k^B)^{-1} \int d^4x \sqrt{\bar{g}}\,(-\bar{R} + 2\Lambda_k^B) + M_k \int d^4x \,(\sqrt{g}\sqrt{\bar{g}})^{1/2}$$

- similar structure as bi-gravity theories

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- similar structure as bi-gravity theories

Novel projection scheme:

- Caveat: compute $\Gamma^{(2)}$ and set $g=\bar g$ no longer works!
- Solution: $g, \bar g$ conformally related and approximately same:

$$g_{\mu\nu} = (1+\epsilon)\bar g_{\mu\nu}$$

  - projection: expansion in R, ϵ!
- suffices to disentangle β-functions of physical and background-couplings!

Bimetric truncation: The Fixed Point structure

Fixed Points of the Double-Einstein-Hilbert truncation:

Fixed Point	g_*	λ_*	m_*	g_*^B	λ_*^B	$g_*\lambda_*$	$g_*^B\lambda_*^B$
G-G-FP	0	0	$\frac{31}{192\pi^2}$	0	0	0	0
G-NG-FP	0	0	$\frac{31}{192\pi^2}$	$\frac{144\pi}{205}$	$-\frac{24}{41}$	0	-1.29
NG-G-FP	0.41	0.59	-0.07	0	0	0.24	0
NG-NG-FP	0.41	0.59	-0.07	-0.57	-0.40	0.24	0.23
Single-Metric EH	0.99	0.13	–	–	–	0.13	–

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Stability coefficients NG-NG-FP:

Fixed Point	$\theta_{0,1}$	θ_3	θ_4	θ_5
NG-NG-FP	3.90 ± 2.75	-26.5	4	2
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First bimetric truncation: supports NGFP

Properties: surprisingly similar to single-metric setup

Perturbative counterterms

Renormalizing the non-renormalizable: $R^2 + C^2$+scalar field

Prototype of gravitational theory: perturbatively non-renormalizable at one-loop

$$\Gamma_k[\Phi] = \Gamma_k^{\rm grav}[g] + \Gamma^{\rm sf}[g,\phi] + S^{\rm gf}[g;\bar{g}] + S^{\rm gh}[g,\bar{g},C,b]$$

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(Percacci, Perini, Phys.Rev.D67 (2003) 081503)

  - Truncation has NGFP = as pure gravity

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Influence of perturbative counterterms on asymptotic safety?

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$$k\partial_k\Gamma_k[\Phi] = \mathcal{S}_{2T} + \mathcal{S}_{hh} - \mathcal{S}_{1T} - \mathcal{S}_0 + n_s\mathcal{S}_{sf}$$

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$$\mathcal{S}_{sf} = \frac{1}{2}{\rm Tr}_0\left[\frac{\partial_t R_{0,k}}{P_{0,k}}\right]$$

- computation of β-functions: completely analogous to pure gravity!

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NGFP for gravity coupled to scalar matter

pure gravity:

Truncation	g^*	λ^*	u_2	u_3	$g^*\lambda^*$
Einstein-Hilbert	0.902	0.109	–	–	0.099
$R^2 + C^2$	1.960	0.218	0.008	−0.0050	0.427

gravity minimally coupled to free scalar (includes perturbative divergences!):

Truncation	g^*	λ^*	u_2	u_3	$g^*\lambda^*$
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gravity-matter NGFP persists upon inclusion of perturbative counterterms

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Einstein-Hilbert	0.860	0.131	–	–	0.112
$R^2 + C^2$+scalar	2.280	0.251	0.010	−0.0043	0.571

gravity-matter NGFP persists upon inclusion of perturbative counterterms

NGFP for gravity coupled to scalar matter

pure gravity:

Truncation	g^*	λ^*	u_2	u_3	$g^*\lambda^*$
Einstein-Hilbert	0.902	0.109	–	–	0.099
$R^2 + C^2$	1.960	0.218	0.008	−0.0050	0.427

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Stability properties of the NGFP

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Truncation	θ_0	θ_1	θ_2	θ_3
Einstein-Hilbert	$2.52 + 1.78i$	$2.52 - 1.78i$	–	–
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