

Title: Gravitational fixed points and asymptotic safety from perturbation theory

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Abstract: The fixed point structure of the renormalization flow in Einstein gravity and higher derivative gravity is investigated in terms of the background effective action. Using a covariant operator cutoff that keeps track of powerlike divergences and the transversal-traceless decomposition a construction is proposed that renders the *{it regularized}* one-loop effective action gauge independent on-shell. In combination with a 'Wilsonian' matching condition nontrivial strictly positive fixed points for the dimensionless Newton constant  $g$  and the cosmological constant  $\lambda$  can then be identified already in one loop perturbation theory. The renormalization flow is asymptotically safe with respect to the nontrivial fixed points in both cases. In Einstein gravity a residual gauge dependence of the fixed points is unavoidable while in higher derivative gravity both the fixed point and the flow equations are universal. Along this flow spectral positivity of the Hessians can be satisfied, evading the traditional positivity problems. Dependence on  $O(10)$  initial data is erased to accuracy  $10^{-5}$  after  $O(10)$  units of the renormalization mass scale and the flow settles on a  $\lambda(g)$  orbit.

Gravitational  
fixed points  
from PT

Niedermaier

for gravity

of gravity

of gravity

the role of PT

conclusions

# Gravitational fixed points and asymptotic safety from perturbation theory

Max Niedermaier

CNRS (Centre National de la Recherche Scientifique)

Asymptotic Safety – 30 years later, PI, Nov. 2009

# Outline

- 1 Refined PT for gravity theories
- 2 Results for Einstein-Hilbert gravity
- 3 Results for Higher Derivative gravity
- 4 The role of perturbation theory
- 5 Conclusions

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# Coupling redefinitions for $g_N, \lambda$

Dimensionless Newton and cosmological constants:

$$g_N := \mu^2 \kappa^2 = \mu^2 16\pi G_N, \quad \lambda := \mu^{-2} \Lambda.$$

Consider finite coupling redefinitions:

$$g_N = g'_N + \frac{\hbar}{(4\pi)^2} g'^2_N C'_1, \quad \lambda = \lambda' + \frac{\hbar}{(4\pi)^2} g'_N D'_1.$$

Beta functions transform **inhomogeneously!**

$$\mu \frac{d}{d\mu} g_N = \beta_g + \frac{\hbar}{(4\pi)^2} g_N^2 \left[ 2C_1 + 2g_N \frac{\partial C_1}{\partial g_N} - 2\lambda \frac{\partial C_1}{\partial \lambda} \right],$$

$$\mu \frac{d}{d\mu} \lambda = \beta_\lambda + \frac{\hbar}{(4\pi)^2} g_N \left[ 4D_1 + 2g_N \frac{\partial D_1}{\partial g_N} - 2\lambda \frac{\partial D_1}{\partial \lambda} \right],$$

Do they contain intrinsic information?

# Refined PT for gravity theories

Proposed answer: **Yes**, but need:

- (a) Regularization that keeps track of powerlike divergences in  $\Lambda_{UV}$ .
- (b) Nonminimal subtraction ansatz for  $\kappa^2, \Lambda$ .
- (c) 'Wilsonian' matching condition.
- (d) Background effective action  $\Gamma_1$  that is on-shell gauge independent in **regularized** form.

Plan: Develop this framework, then apply to Einstein-Hilbert (EH) and Higher Derivative (HD) gravity.

# Operator-heat kernel regularization

$\mathbf{A}$  symmetric, covariant differential operator of order  $2r$ .

$A(x, y; t) = \langle x | \exp(-t\mathbf{A}) | y \rangle$  its heat kernel.

$z \mapsto F_{k, \Lambda_{UV}}(z)$  regulator function depending on IR cutoff  $k$  and UV cutoff  $\Lambda_{UV}$ , with  $k < \mu \leq \Lambda_{UV}$ .

$t \mapsto \tilde{F}_{k, \Lambda_{UV}}(t)$  its inverse Laplace transform, normalized such that  $\tilde{F}_{0, \infty}(t) = -1/t$ . Minus  $n$ -th moment  $-q_n(\Lambda_{UV}^{2n} - k^{2n})$ .

**Then:**  $\ln \mathbf{A}$  is replaced with integral operator

$$\ln \mathbf{A} \mapsto \ln_{k^r, \Lambda_{UV}^r} \mathbf{A} = F_{k^r, \Lambda_{UV}^r}(A)(x, y) := \int_0^\infty dt \tilde{F}_{k^r, \Lambda_{UV}^r}(t) A(x, y; t)$$

Gives unique expressions for regularized:

$$\mathbf{A}_{k, \Lambda_{UV}}^{-1}, \quad \text{Det}_{k, \Lambda_{UV}} \mathbf{A}, \quad \text{Gaussian integrals with kernel } \mathbf{A}.$$

# Divergent part of effective action

Small  $t$  expansion of  $A(x, y; t)$  gives

$$\text{Tr} F_{k^r, \Lambda_{UV}^r}(A) = -\frac{\Gamma(2/r)}{r(4\pi)^2} \int d^4x \sqrt{g} \\ \times \left[ \Lambda_{UV}^4 q_{2/r} E_0(x|A) + \Lambda_{UV}^2 q_{1/r} E_2(x|A) + 2r \ln \Lambda_{UV} E_4(x|A) \right] + O(\Lambda_{UV}^0)$$

with  $E_0, E_2, E_4$  heat kernel coefficients of  $A$ . Obtain

$$\Gamma_1^{\text{div}} = -\frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left\{ \Lambda_{UV}^4 \Upsilon_1 + \Lambda_{UV}^2 [\Upsilon_2 R + \mu^2 \Upsilon_3] \right. \\ \left. + \ln(\Lambda_{UV}/\mu) [\zeta_1 C^2 + \zeta_2 R^2 + \mu^2 \zeta_4 R + \mu^4 \zeta_5] \right\}$$

with  $\Upsilon_i, \zeta_i$  determined by  $E_n$  of constituent operators.

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with  $\Upsilon_i, \zeta_i$  determined by  $E_n$  of constituent operators.

# Nonminimal subtraction

Absorb  $\Gamma_1^{\text{div}}$  by coupling and field renormalizations. For  $\kappa^2$  and  $\tilde{\Lambda} = 2\Lambda/\kappa^2$  use:

$$\tilde{\Lambda}_0 = \mu^4 \frac{2\lambda}{g_N} \left\{ 1 + \frac{\hbar}{(4\pi)^2} \left[ a_0 + a_1 \ln(\Lambda_{\text{UV}}/\mu) + a_2 \left( \frac{\Lambda_{\text{UV}}}{\mu} \right)^2 + a_3 \left( \frac{\Lambda_{\text{UV}}}{\mu} \right)^4 \right] \right\},$$

$$\kappa_0^2 = \mu^{-2} g_N \left\{ 1 + \frac{\hbar}{(4\pi)^2} \left[ b_0 + b_1 \ln(\Lambda_{\text{UV}}/\mu) + b_2 \left( \frac{\Lambda_{\text{UV}}}{\mu} \right)^2 \right] \right\},$$

Cancellation condition  $\Delta S \stackrel{!}{=} -\Gamma_1^{\text{div}}$  fixes  $a_i, b_i, i \neq 0$ , in terms of  $\zeta$  and  $\Upsilon$  coefficients. Reparameterize  $a_0, b_0$  as

$$a_0 = g_N C_1 - \frac{g_N}{\lambda} D_1, \quad b_0 = -g_N C_1.$$

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# Matching condition

So far only  $\mu$ -independence of bare couplings was used.  
Impose in addition:

$$\kappa_0^2 \stackrel{!}{=} \Lambda_{\text{UV}}^{-2} g_N(\mu = \Lambda_{\text{UV}}) \quad \text{iff } b_0 + b_2 = 0,$$

$$\tilde{\Lambda}_0 \stackrel{!}{=} \Lambda_{\text{UV}}^4 \left( \frac{2\lambda}{g_N} \right) (\mu = \Lambda_{\text{UV}}) \quad \text{iff } a_0 + a_2 + a_3 = 0.$$

Shortcuts renormalization conditions for vertex functions.  
Fixes uniquely

$$C_1 = \Upsilon_2, \quad D_1 = \frac{1}{2}(\Upsilon_1 + \Upsilon_3) + \lambda \Upsilon_2.$$

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# On-shell independence of gauge condition

Usually only the logarithmically divergent parts of  $\Gamma_1$  are considered in PT, which are gauge-independent on-shell. **Fails** for other parts of regularized  $\Gamma_1$ . General theorems inapplicable, Vilkovisky-deWitt action no help, main culprit

$$\text{Det}\mathbf{AB} \neq \text{Det}\mathbf{A}\text{Det}\mathbf{B},$$

for regularized determinants.

Solution: Use

- transversal-traceless decomposition of metric fluctuations
- geometric measure on space of metrics.
- Hessians relative to (indefinite) deWitt metric.
- judicious resolution of  $\text{Det}\mathbf{AB} \neq \text{Det}\mathbf{A}\text{Det}\mathbf{B}$  ambiguities

to construct **regularized**  $\Gamma_1$  exactly gauge-independent on shell

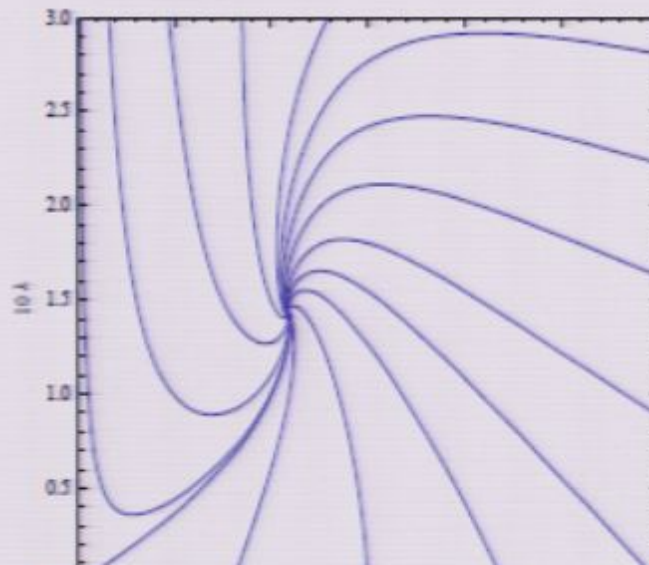
# Results for Einstein-Hilbert gravity

Nontrivial fixed point:

$$\frac{g_N^*}{(4\pi)^2} = -\frac{2}{\lambda_*(-\frac{2}{3}\zeta_1 + 4\zeta_2 + \zeta_4/\lambda) + 2\Upsilon_2} > 0 \text{ robustly}$$

$$\lambda_* = q_1 \frac{40}{87} \left[ -1 + \sqrt{1 + \frac{261}{160} \frac{q_2}{q_1^2}} \right] > 0 \text{ gauge-independent}$$

Flow equations **gauge dependent**, stability matrix has complex eigenvalues at  $g_N^*, \lambda_*$ , flow pattern spiralling.



# Higher derivative gravity

$$S = \int d^4x \sqrt{g} \left[ \Lambda - \frac{1}{\kappa^2} R + \frac{1}{2s} C^2 - \frac{\omega}{3s} R^2 \right]$$

$R$  : Ricci scalar,  $s, \omega$ , couplings

$C^2$  : square of Weyl tensor

- Perturbative renormalizability to **all loops** wrt  $1/p^4$  propagator expected [Stelle, 1977].
- 1-loop **log-flows** known explicitly;  $s$  is **asymptotically free** in PT [Fradkin-Tseytlin 1982; Avramidi-Barvinsky 1985; Berredo-Peixeto 2005].
- $\omega > 0$  classically,  $\omega < 0$  near UV fixed point  $\omega_* \approx -0.0228$  [dito].
- Unitarity issue studied in toy models [“Pais-Uhlenbeck”, “Lee-Wick”], results encouraging.

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Nontrivial fixed point [M.N]:

$$\frac{g_N^*}{(4\pi)^2} = \frac{1}{0.9063q_{1/2} + 2.3224q_1} > 0,$$

$$\lambda_* = \frac{1.25q_1 + 237.617q_2}{2(0.9063q_{1/2} + 2.3224q_1)} > 0.$$

- Stability matrix has eigenvalues  $-2, -4$  at nontrivial fixed point.
- Flow is **gauge-independent!** Dependence on positive  $O(1)$  moments  $q_n$  expected, c.f. lattice.
- Memory of  $O(10)$  initial data erased to accuracy  $10^{-5}$  after  $O(10)$  units in  $\mu$  and settles on  $\lambda(g_N)$  orbit.
- **Spectral positivity** of Hessian (inverse propagator) satisfied for large  $\mu$ ! Traditional problem evaporates.

# Gauge-independent flow

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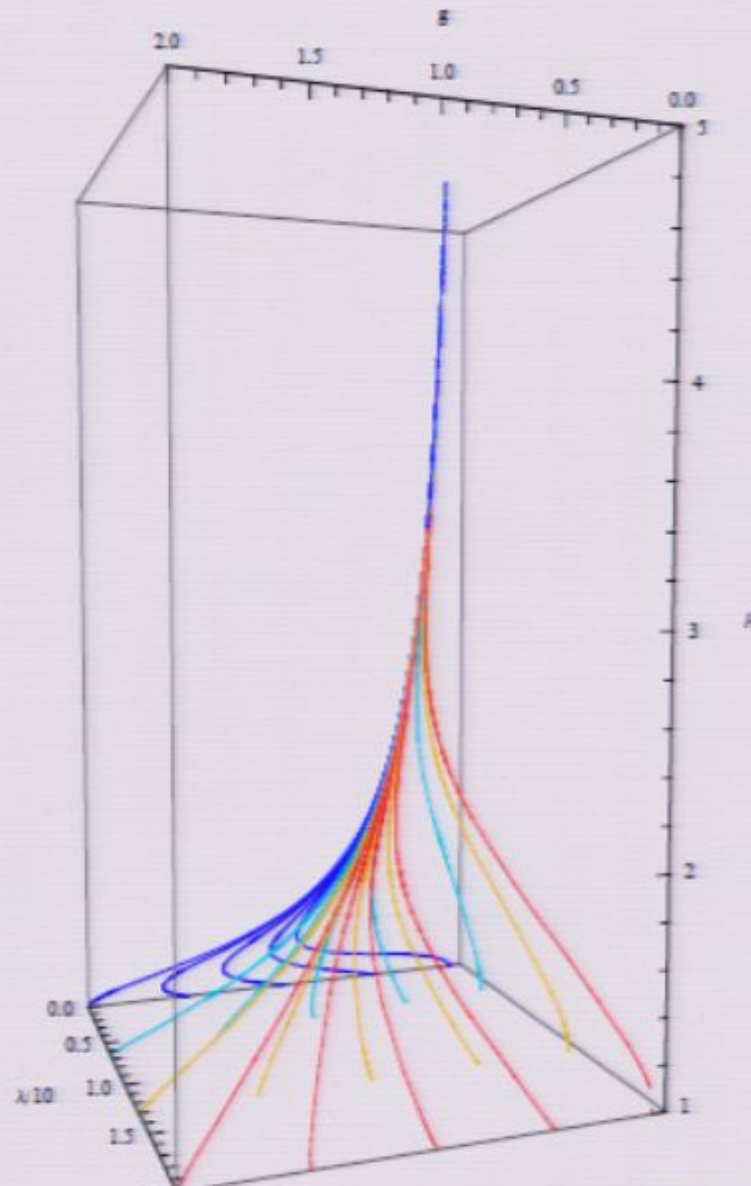
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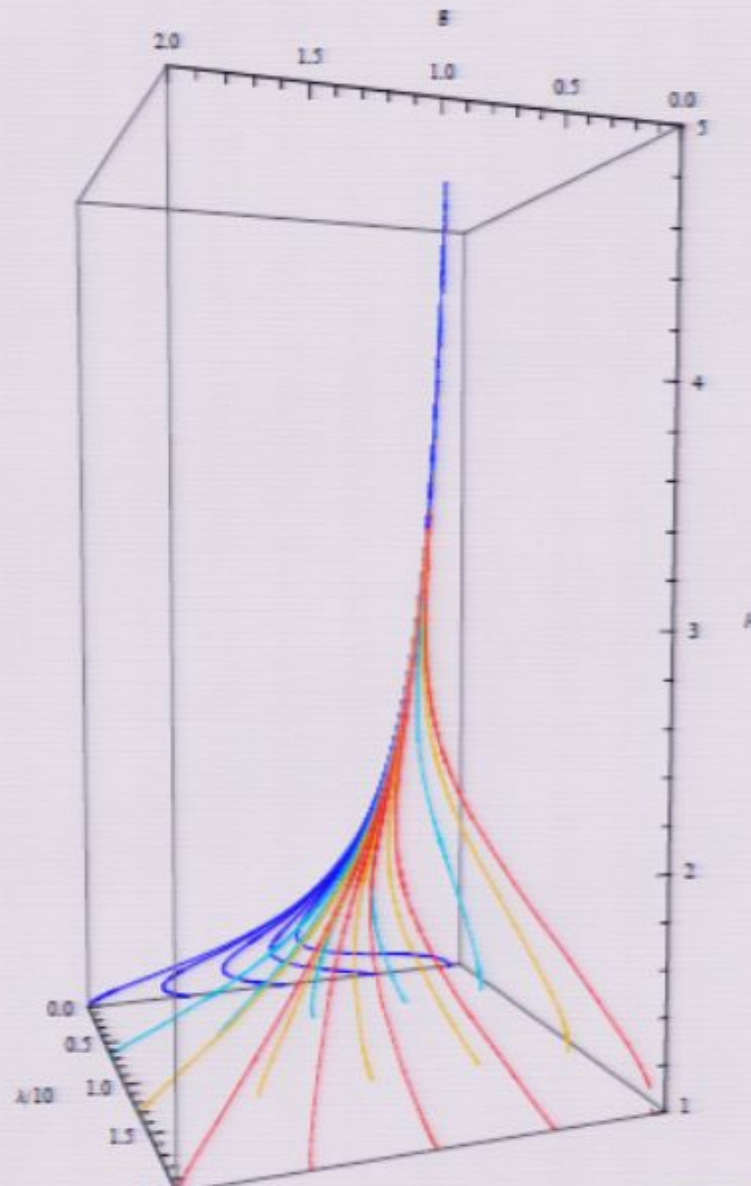
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# Why perturbation theory, not functional flow equation?

- Non-perturbative aspects in  $\Gamma_k$  flow equation formalism for gravity are only 'skin-deep': gauge-fixing, Gribov problem not addressed, naive measure used, structure of  $\Gamma_k$  beyond local terms elusive, etc.
- UV finite initial data are postulated to exist but not constructed. Should include a fine-tuned dependence on  $\Lambda_{UV}$  such that:

$$\lim_{\Lambda_{UV} \rightarrow \infty} \Gamma_{k, \Lambda_{UV}} \stackrel{?}{=} S_{\text{ren}} \quad \text{for } k \text{ large,}$$

with  $S_{\text{ren}}$  determined by dofs aimed at.

- Reconstruction of bare action possible but physicswise and mathematically underdetermined.
- Convergence of (which?) truncations to exact result (in which sense?) elusive.

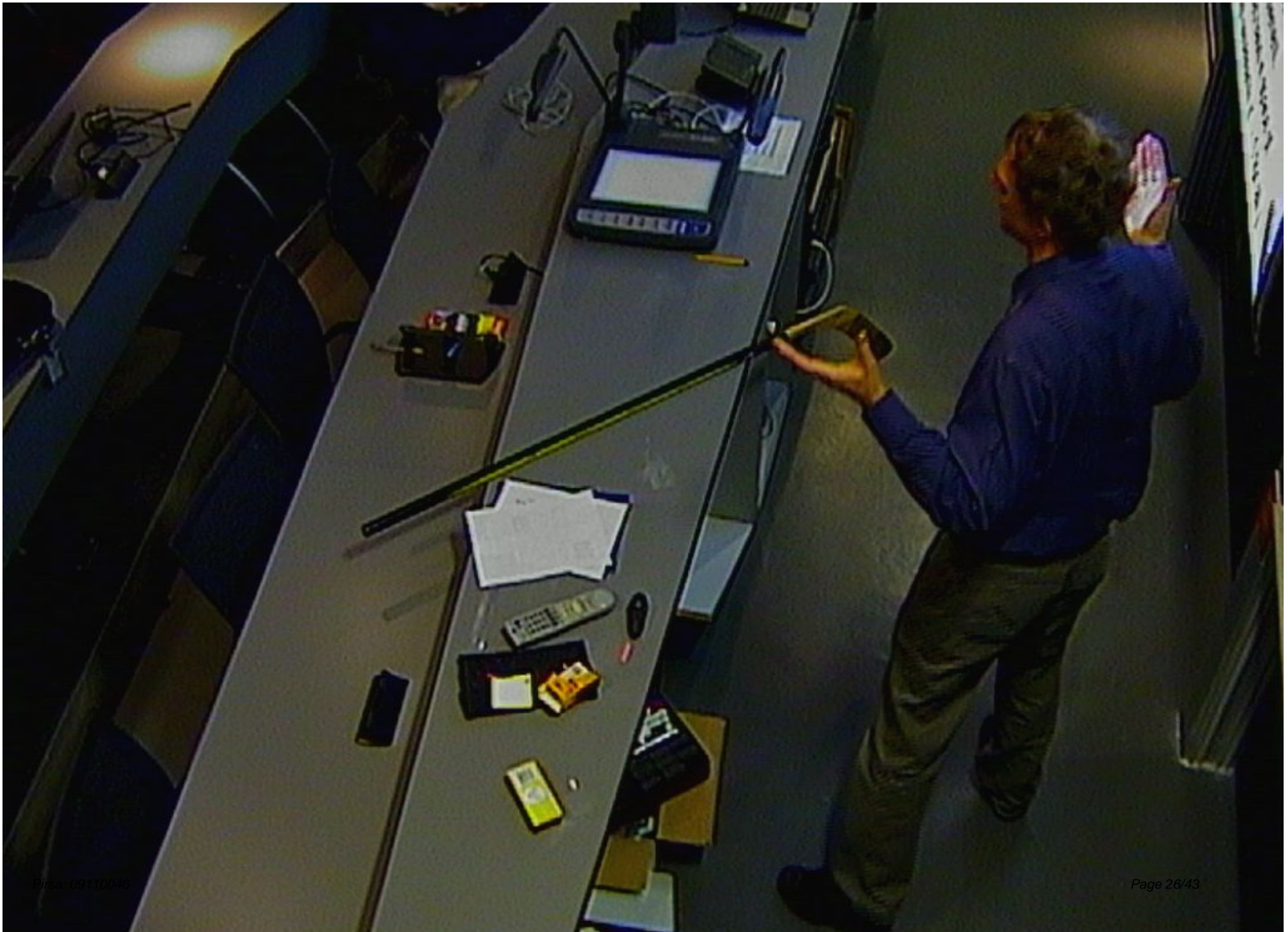


# PT addresses the UV problem

- PT produces series in loop counting parameter  $\hbar$ , UV cutoff can termwise be removed. PT wrt **asymptotically free** coupling should describe the UV regime correctly.
- Rationale extends to asymptotically **safe** theories:  
(Non-constant essential) coupling  $g_j$  is **asymptotically safe** iff

$$\sup_{\mu_0 \leq \mu \leq \infty} g_j(\mu) < \infty, \quad \lim_{\mu \rightarrow \infty} g_j(\mu) = g_j^* < \infty,$$

- Define “the coupling constants as coefficients in a power series expansion of the reaction rates themselves around some physical renormalization point” (S. Weinberg, 1979).
- Take one difference  $\delta g_1 = g_1 - g_1^*$  of order  $\hbar$  at  $\mu_0$  to define the expansion via RG improvement.
- ‘Gaussianity’ refers to preferred basis in space of interaction monomials not necessarily  $\sigma_j^* = 0$  for all  $j$

















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# Conclusions

- Refined perturbative framework for gravity allows investigation of asymptotic safety at one loop.
- Attractivity of classical gravity (used in anti-Wick rotation wrt indefinite deWitt metric) entails anti-screening  $\kappa^2 \sim g_N^*/\mu^2$  in UV regime.
- In HD gravity PT expansion is in asymptotically free  $s$  not in  $\kappa^2$ . Nontrivial positive gauge independent fixed point  $g_N^*, \lambda_*$  exists 'beyond reasonable doubt'.
- Traditional positivity problem in HD gravity absent.
- Next step: relate flow to asymptotic expansion of physical quantities.

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$p^{(1)}$

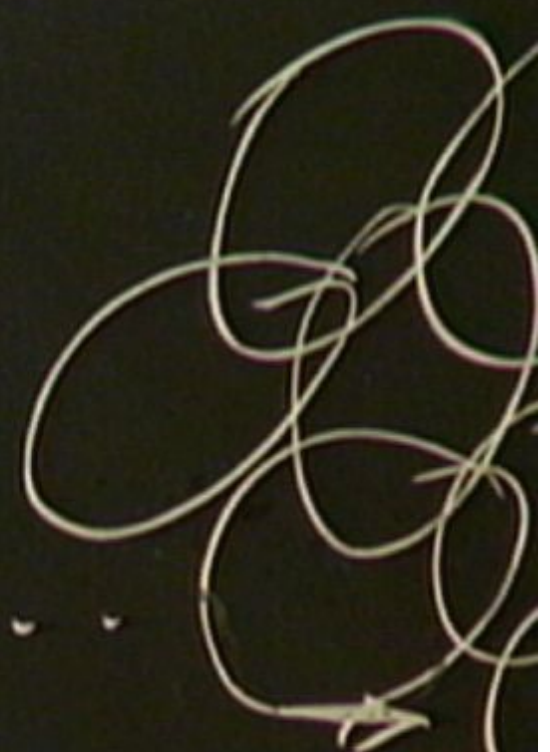
$Q^{(1)}$   $Q^{(0)}$   $p^{(0)}$   
 $p^{(1)}$

$W_0$

$o(s)$

$$H^{-1} = \frac{1}{\lambda_1} p^{(2)} \dots$$

$$\lambda_1 = p^4 + \frac{s}{k^2} (p^2 - 2L)$$



$p^2 \rightarrow$

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# PT addresses the UV problem

- PT produces series in loop counting parameter  $\hbar$ , UV cutoff can termwise be removed. PT wrt **asymptotically free** coupling should describe the UV regime correctly.
- Rationale extends to asymptotically **safe** theories:

(Non-constant essential) coupling  $g_j$  is **asymptotically safe** iff

$$\sup_{\mu_0 \leq \mu \leq \infty} g_j(\mu) < \infty, \quad \lim_{\mu \rightarrow \infty} g_j(\mu) = g_j^* < \infty,$$

- Define “the coupling constants as coefficients in a power series expansion of the reaction rates themselves around some physical renormalization point” (S. Weinberg, 1979).
- Take one difference  $\delta g_1 = g_1 - g_1^*$  of order  $\hbar$  at  $\mu_0$  to define the expansion via RG improvement.
- ‘Gaussianity’ refers to preferred basis in space of interaction monomials not necessarily  $\sigma_j^* = 0$  for all  $j$

Bookmarks

- Refined PT for gravity theories
- Results for Einstein-Hilbert gravity
- Results for Higher Derivative gravity
- The role of perturbation theory
- Conclusions

- Gravitational fixed points from PT
- Niedermaier
- PT for gravity
- EH gravity
- HD gravity
- The role of PT
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