Title: Asymptotic safety and deformed symmetry

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Abstract:

Asymptotic safety and deformed symmetry

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PI
Nov 2009

1) Prelude
2) An AS theory exists in d< 4
3) An AS exists in d=4 but it is unstable
4) The possibility reduced dimension or anomalous scaling
5) Deformed symmetry as a source of anomalous scaling
6) Why deformed symmetry?

ls. Nucl. Phys. B208 (1982) 439.
L. Crane and ls, Nucl. Phys. B267 (1986) 714-757
ls, in preparation

Prelude: sketch of the argument of the talk

Warning: this talk is based on new reflections on old results.

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/0809.0220, /0708.1737

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Deformation of Lorentz invariance is still allowed at Planck scales.

It has been shown that deformed symmetry can imply $d_S < 4$ (Dario)

Semiclassical QG arguments suggest Lorentz symmetry is deformed.

onsider Horava's hypothesis: that the critical point is haracterized by anisotrpic scaling with

$$E \sim p^3 M^{-2} c \qquad \rightarrow d_S = 2$$

lote that this already implies superluminal propagation in the ritical region:

$$v = \frac{dE}{dp} = 3c \left(\frac{E}{M} \right)^{\frac{2}{3}}$$

his then implies breaking *or* deformation of Lorentz invariance.

will show that this behavior is compatible with deformed Lorentz ıvariance. *(Magueijo)*

Vhichever it is should propagate to the matter sector, so we might vorry about bounds on breaking or deformation of Lorentz invariance n photons.

low does the transition from usual scaling occur?

$$E = pc \rightarrow \; E \sim p^3 M^{-2} c$$

Ve can posit a simple extrapolation (to be justified later by DSR)

$$\frac{E}{(1+\frac{E}{M})^{\frac{2}{3}}} = p$$

his implies at leading order

$$v = c(1+\frac{4}{3}\frac{E}{M})$$

' this applies to photons, with $M \sim M_p$, *this is already well ruled out if orentz symmetry is broken.*

I > 10^9 M_p from the crab nebula, 10^7 from some GRBs

arxiv.org/abs/0809.0220

LS Broken

$$\Delta H \sim \frac{1}{M_P} \nabla \cdot E \times B$$

$\Rightarrow$ birefringence

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γ-ray polarization constraints on Planck scale violations of special relativity

Luca Maccione, Stefano Liberati, Annalisa Celotti
SISSA/ISAS, via Beirut 2-4, 34014, Trieste and
INFN, Sezione di Trieste, via Valerio, 2, 34127 Trieste, Italy

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Pietro Ubertini
IASF-INAF, via Fosso del Cavaliere 100, Roma, Italy
(Dated: September 1, 2008)

Using recent polarimetric observations of the Crab Nebula in the hard X-ray band by INTEGRAL, we show that the absence of vacuum birefringence effects constrains $O(E/M)$ Lorentz violation in QED to the level $|\xi| < 9 \times 10^{-10}$ at 3σ CL, tightening by more than three orders of magnitude previous constraints. We show that planned X-ray polarimeters have the potential to probe $|\xi| \sim 10^{-16}$ by detecting polarization in active galaxies at red-shift ~ 1.

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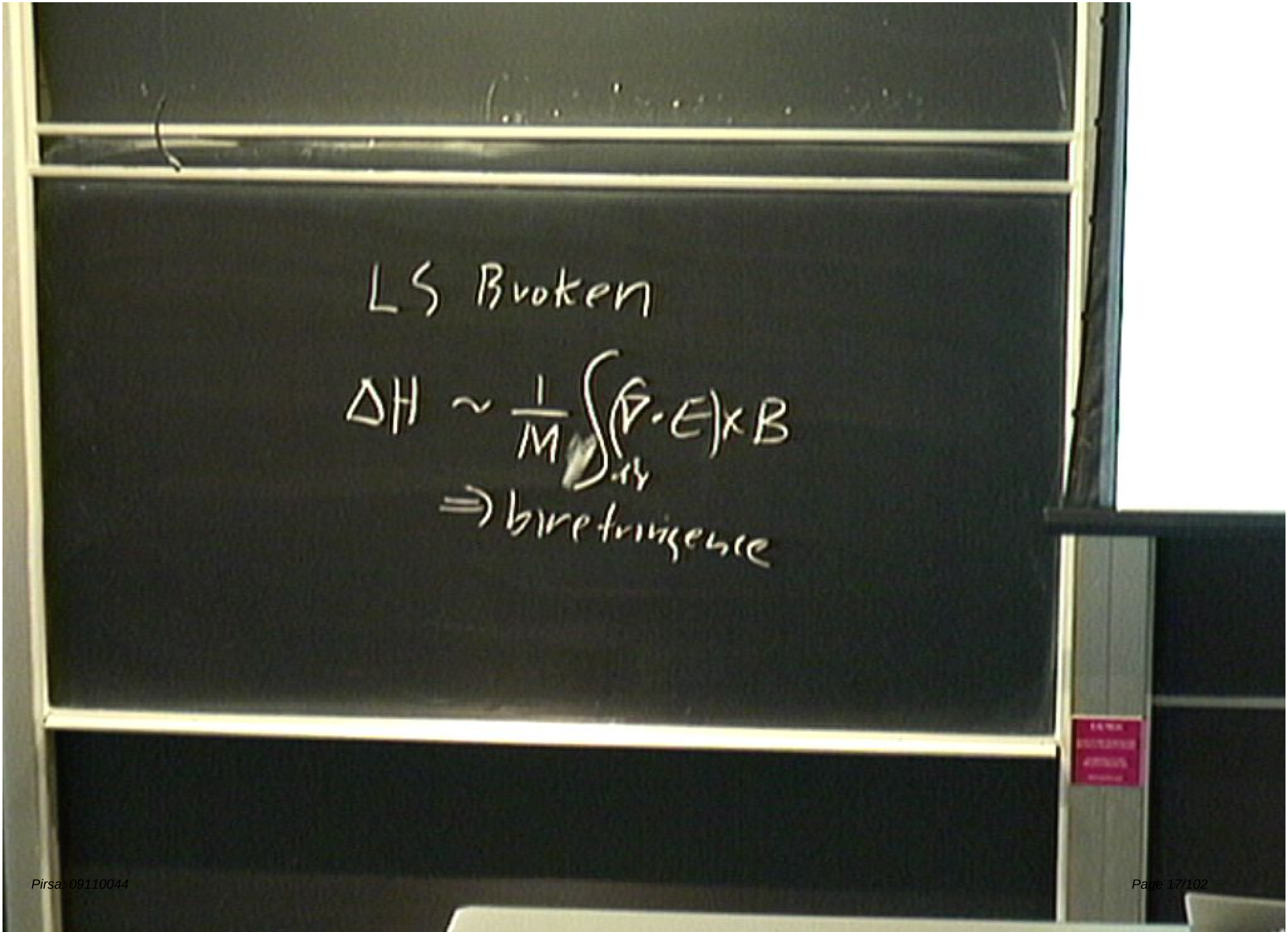
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⇒ birefringence

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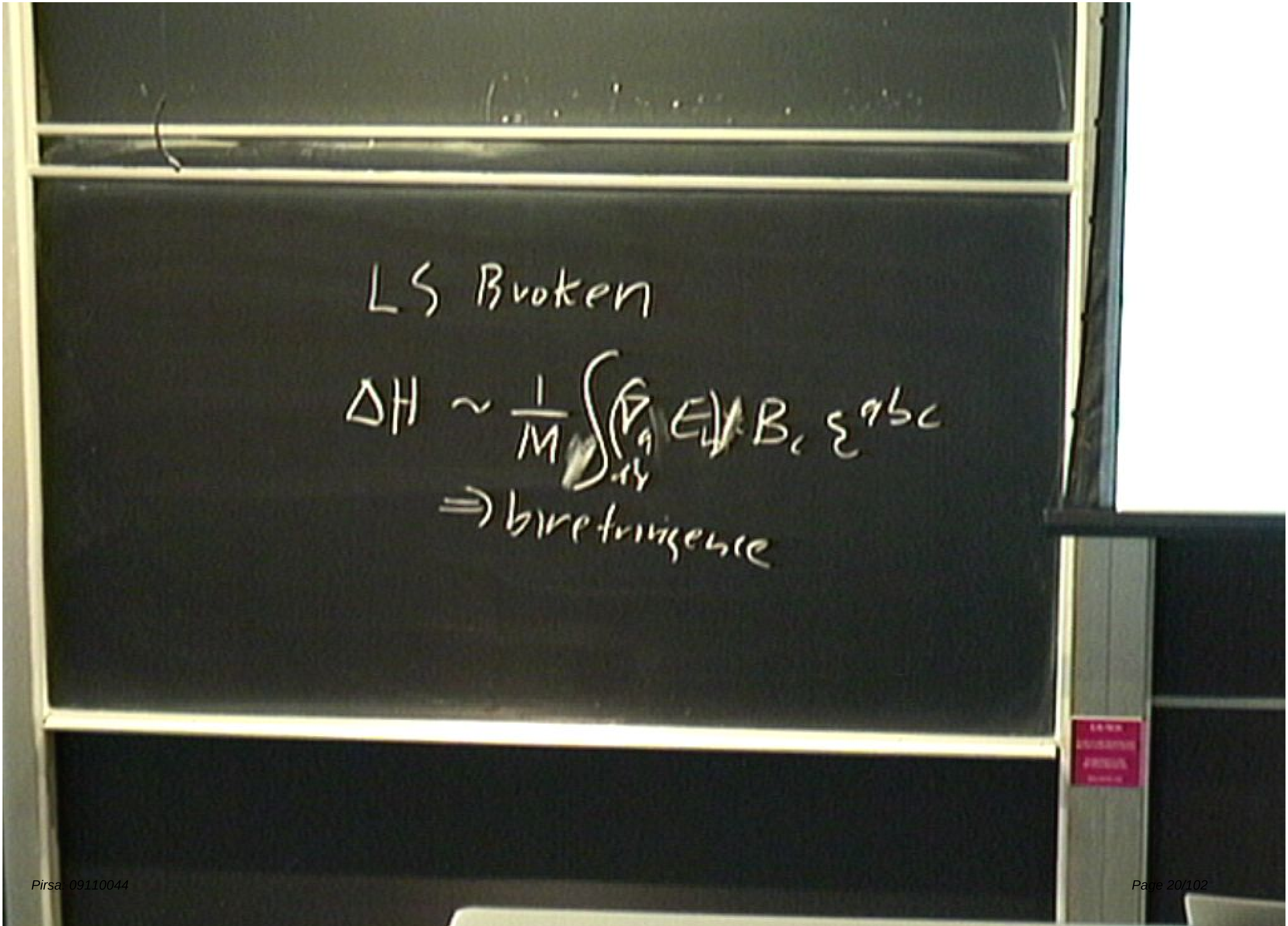
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f Lorentz symmetry is deformed it is the same order of magnitude f the bound set last week by FERMI

Nature: current issue, from the Fermi collaboration

A limit on the variation of the speed of light arising from quantum gravity effects

$$v = c\left(1 - \frac{E}{M_{QG}}\right)$$

$$M_{QG} > 1.2 M_{Pl}$$

A cornerstone of Einstein's special relativity is Lorentz invariance— the postulate that all observers measure exactly the same speed of light in vacuum, independent of photon-energy. While special relativity assumes that there is no fundamental length-scale associated with such invariance, there is a fundamental scale (the Planck scale, $l_{\rm Planck} \approx 1.62 \times 10^{-33}$ cm or $E_{\rm Planck} = M_{\rm Planck}c^2 \approx 1.22 \times 10^{19}$ GeV), at which quantum effects are expected to strongly affect the nature of space–time. There is great interest in the (not yet validated) idea that Lorentz invariance might break near the Planck scale. A key test of such violation of Lorentz invariance is a possible variation of photon speed with energy[1–7]. Even a tiny variation in photon speed, when accumulated over cosmological light-travel times, may be revealed by observing sharp features in γ-ray burst (GRB) light-curves[2]. Here we report the detection of emission up to ~31 GeV from the distant and short GRB 090510. We find no evidence for the violation of Lorentz invariance, and place a lower limit of $1.2E_{\rm Planck}$ on the scale of a linear energy dependence (or an inverse wavelength dependence), subject to reasonable assumptions about the emission (equivalently we have an upper limit of $l_{\rm Planck}/1.2$ on the length scale of the effect). Our results disfavour quantum-gravity theories[3,6,7] in which the quantum nature of space–time on a very

Vhy Asymptotic safety points to reduced scaling limension

. A fixed point for quantum gravity, Nucl Phys B208 (1982) 439.

Crane, ls, Spacetime foam as the universal regulator, GRG 17 (1985) 1209.

Renormalizability of general relativity on a background of spacetime foam, Nuclear Physics B267 (1986) 714-757.

K.G. Wilson, Phys. Rev. D10 (1973) 2911

G. Parisi, Nucl. Phys. B100 (1975) 368; IHES/P/76/148 (1976)

S in quantum gravity for spacetime d<4 d=4-ε

heory: GR coupled to N fermions in a power series in 1/N

Parameters: G_N and λ

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$$\mathscr{L} = -\frac{1}{2\kappa^2} R + \sum_{i=1}^{N} \bar{\psi}_i \not{D} \psi_i - \tfrac{1}{2}\lambda \, .$$

rocedure:

) Cut the theory off at Euclidean $p^2 = \Lambda$.

) Scale the dimensional parameters by powers of Λ and N

$$G^{-1}_{\text{Newton}} = \frac{1}{\kappa^2} = cN\Lambda^{2-\varepsilon} \quad \lambda = g\Lambda^{4-\varepsilon} N$$

(c and g dimensionless functions of the ratio Λ/M_p)

) Compute the graviton propagator to leading order in 1/N

) Choose trajectories for c and g so the theory is finite as $\Lambda \longrightarrow \infty$

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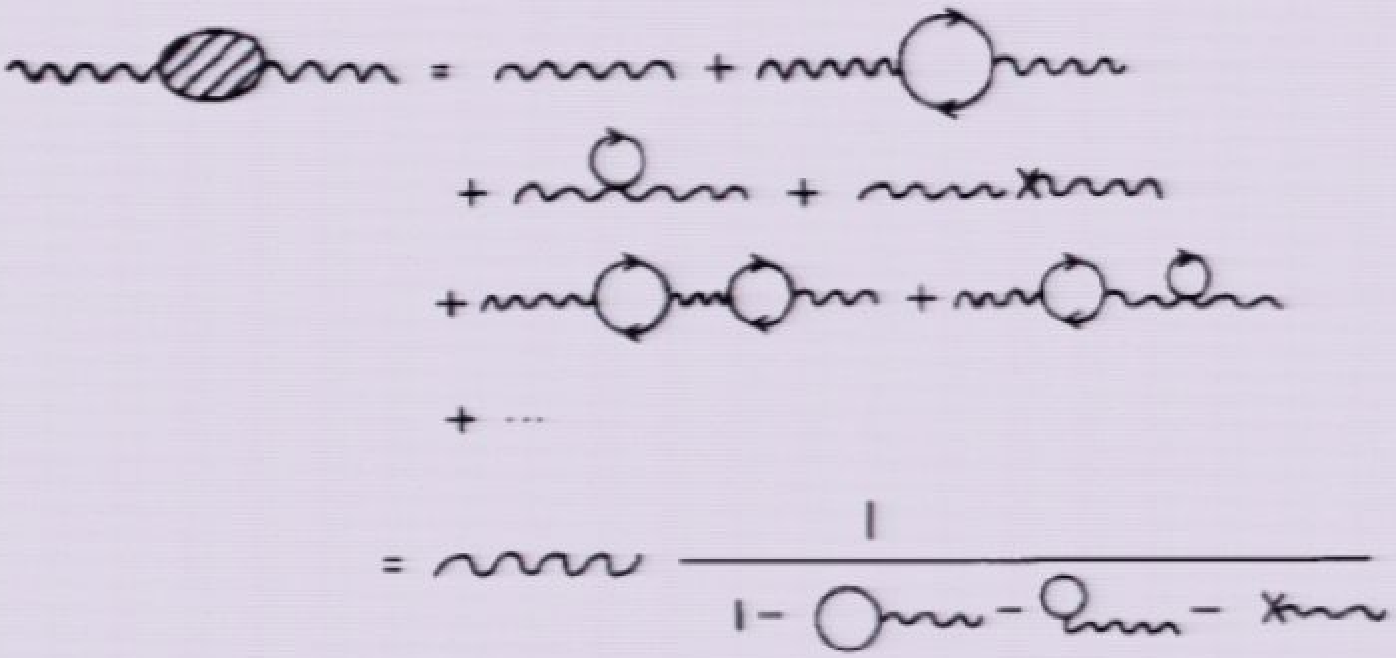
1/N is a small parameter, so c and g can be large.

he graviton propagator to leading order in 1/N at d=4-ε

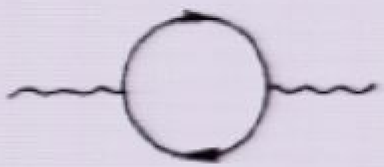


The spin two piece:

$$D^{1/N}_{d=4-\epsilon} = \frac{P^{(2)}_{\mu\nu\alpha\beta}}{cNp^2\Lambda^{2-\epsilon}[1-[cNp^2\Lambda^{2-\epsilon}]^{-1}[NF^{(2)}_{a}+NF^{(2)}_{b}+g\Lambda^{4-\epsilon}N]]}.$$

$F^{(2)}{}_a$ and $F^{(2)}{}_b$ are given by the spin two parts of the fermion loops:

Calculation of the fermion loops:



$$F_a^{(2)} = (1-\tfrac{1}{2}\varepsilon)\frac{\tfrac{1}{2}\pi d}{(2\pi)^d}\Gamma(1+\tfrac{1}{2}\varepsilon)\left[-\frac{\Lambda^{4-\varepsilon}}{2-\tfrac{1}{2}\varepsilon} - p^2\Lambda^{2-\varepsilon}\frac{1}{6}\left[1+\frac{2}{1-\tfrac{1}{2}\varepsilon}\right] + (p^2)^{2-\varepsilon/2}A(\varepsilon)\right]$$

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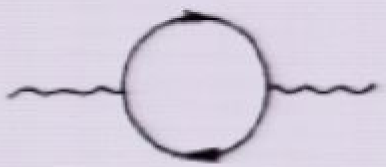

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Renormalizes Λ

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Renormalizes G_N

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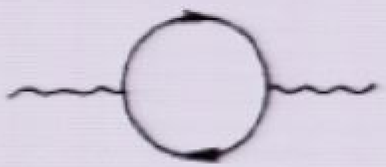

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p^d ie critical behavior at fixed point

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But A(ε) diverges as d→4, A(ε) ~ 1/ε

$$A(\varepsilon) = \left[\frac{1}{2(1-\varepsilon)} + \frac{2}{\varepsilon}\frac{(1-\tfrac{3}{2}\varepsilon)}{(1-\tfrac{1}{2}\varepsilon)}\right]\int_0^1 dy[y(1-y)]^{2-\varepsilon/2}.$$

There exists a non-trivial fixed point:

$$c^* = \tfrac{1}{6}(1-\tfrac{1}{2}\varepsilon)\left(1+\frac{2}{1-\frac{1}{2}\varepsilon}\right)\frac{\frac{1}{2}\pi d}{(2\pi)^d}\Gamma(1+\tfrac{1}{2}\varepsilon)\,,$$

$$g^* = \left[\frac{1-\frac{1}{2}\varepsilon}{2-\frac{1}{2}\varepsilon}+\frac{1}{4}-\frac{3}{8(4-\varepsilon)}\right]\frac{\frac{1}{2}\pi d}{(2\pi)^d}\Gamma(1+\tfrac{1}{2}\varepsilon)$$

and critical trajectories:

$$\frac{1}{c}\left(\frac{\Lambda}{M}\right) = \frac{1}{c^*} - \frac{M^{2-\varepsilon}}{\Lambda^{2-\varepsilon}}\frac{1}{c^*}\,,$$

$$g\left(\frac{\Lambda}{M}\right) = g^* - \frac{L^{4-\varepsilon}}{\Lambda^{4-\varepsilon}}d\,,$$

(d has input also from tadpole diagrams)

The spin 2 propagator in leading order in 1/N is after $\Lambda \longrightarrow \infty$

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One can show the theory is now renormalizable to higher order with only renormalizations of c and g required, so long as d < 4.

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$N\,M^{2-\varepsilon}$ *renormalized Newton's constant*

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theory diverges as d → 4

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There is unfortunately a ghost

ow do we control the divergence in the spin two propagator as d → 4?

$$D^{1/N}_{d=4-\epsilon} = \frac{P^{(2)}_{\mu\nu\alpha\beta}}{Np^2[M^{2-\epsilon} - (-p^2)^{1-\epsilon/2} A(\epsilon)]} \quad \sim 1/\varepsilon$$

s d→ 4 this is a divergent contribution to the propagator roportional to p^4. At finite Λ and d=4 the divergence is in ln(Λ/M)

o cancel it we must add a counterterm:

$$\Delta\mathcal{L} = \frac{N}{\alpha_b(M/\Lambda)} C^2_{\mu\nu\alpha}{}^{\beta}$$

(Weyl tensor squared)

Vhere α is on an asymptotically free RG trajectory

$$\alpha_b\left(\frac{M}{\Lambda}\right) = \frac{\alpha}{1 + (\alpha/480\pi^2)\ln(\Lambda^2/M^2)}$$

Vith the counterterm added we can again take $\Lambda \longrightarrow \infty$

he graviton propagator at leading order in 1/N at d=4 is now

$$D^{1/N}_{\mu\nu\alpha\beta} = \frac{P^{(2)}_{\mu\nu\alpha\beta}}{Np^2(M^2 - 1/\alpha p^2 - (1/480\pi^2)p^2 \ln[-p^2/M^2])}$$

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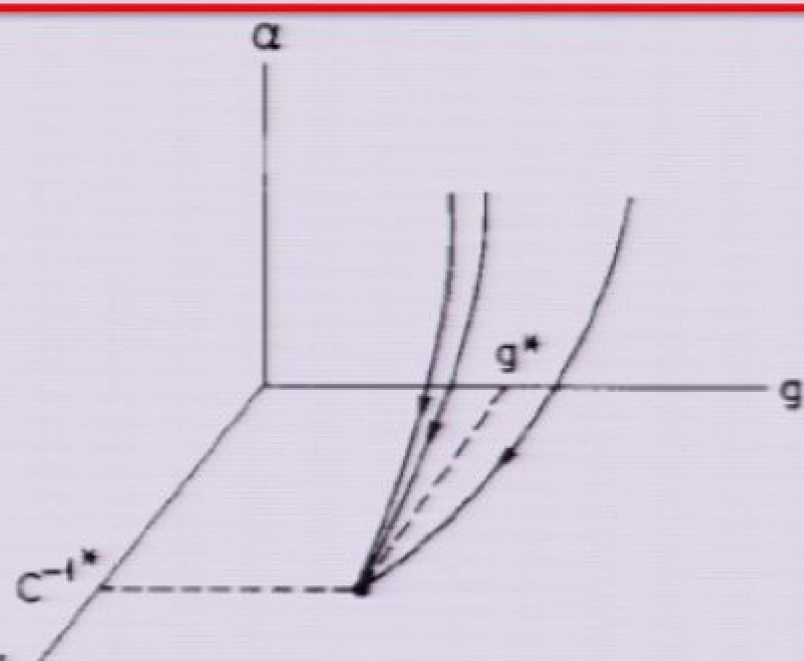
he the theory is still asymptotically safe:

$$(c^*, g^*, \alpha^*) = \left(\frac{1}{32\pi^2}, \frac{9}{128\pi^2}, 0\right)$$



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M_p~ NM renormalized Planck mass

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scaling behavior p^4

$$D^{1/N}_{\mu\nu\alpha\beta}(p^2) \xrightarrow[p^2 \to -\infty]{} \frac{1}{p^4 \ln(-p^2/M^2)} \sim \frac{\alpha_b(M^2/p^2)}{p^4}$$

Vith the counterterm added we can again take $\Lambda \longrightarrow \infty$

he graviton propagator at leading order in 1/N at d=4 is now

$$D^{1/N}_{\mu\nu\alpha\beta} = \frac{P^{(2)}_{\mu\nu\alpha\beta}}{Np^2(M^2 - 1/\alpha p^2 - (1/480\pi^2)p^2 \ln[-p^2/M^2])}$$

The ghost has become a Lee-Wick pole ie acausality

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With the addition of an R^2 counterterm for spin 0 the theory is now perturbatively renormalizable, as first showed by Stelle and by Tomboulis in the 1/N expansion.

The C^2 counterterm is ***absolutely necessary*** for the theory to exist at d=4 because it gives the right scaling to the spin 2 propagator

One AS completion of GR at d=4 is then the following theory:

$$\mathcal{L} = -\frac{1}{2\kappa^2}R + \sum_{1=1}^{N} \bar{\psi}_i D\Psi_i - \frac{1}{2}\lambda + \frac{N}{\alpha_b}C^2 + \frac{N}{\beta_b}R^2$$

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leither is fully acceptable, but the better bet is the theory with a lamiltonian bounded from below

similar situation holds for four fermi theory *(Parisi, Wilson)*

l fermions in d=4-ε $\mathscr{L} = \frac{1}{2}\bar{\psi}_i \partial\!\!\!/ \psi_i - \frac{G_F}{8N}(\bar{\psi}_i\psi_i)^2$;

cale the coupling with the cutoff: $G_F = g\Lambda^{\varepsilon-2}$,

he interaction
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n 1/N: a bosonic
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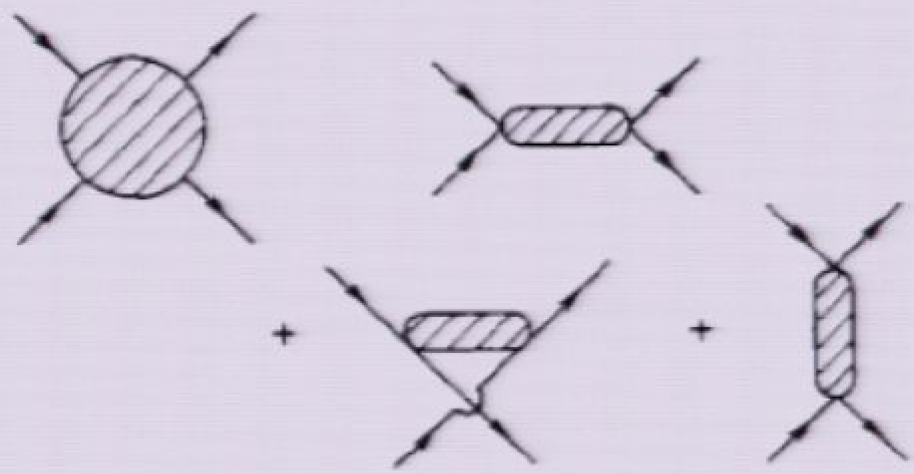

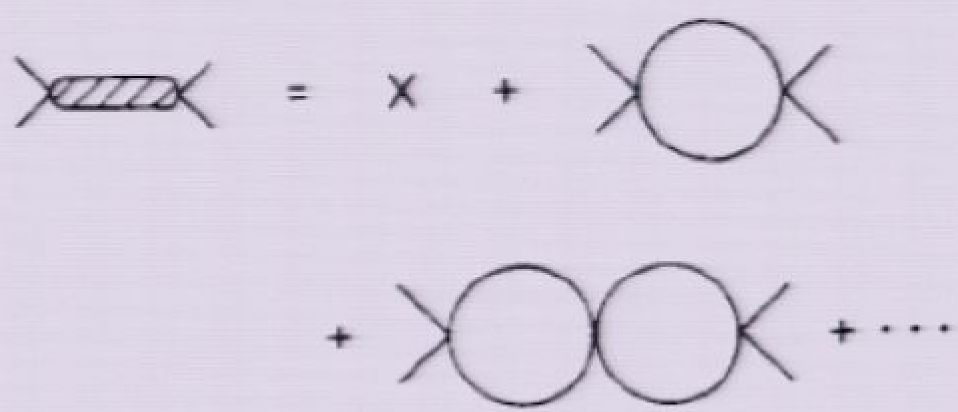

'he limit Λ → infinity for d<4 can be taken using a non-trivial fixed point

$$A = \frac{G_F}{1 - fG_F} \qquad f = \frac{4 \cdot \frac{1}{2}\pi d}{(2\pi)^d}\left[\frac{\Lambda^{2-\varepsilon}}{1-\frac{1}{2}\varepsilon} + (p^2)^{1-\varepsilon/2}B(\varepsilon)\right] \qquad G_F = g\Lambda^{\varepsilon-2},$$

$$g\left(\frac{M}{\Lambda}\right) = g^* + \frac{M^{2-\varepsilon}}{\Lambda^{2-\varepsilon}}g^{*2} \qquad \frac{1}{g^*} = \frac{4 \cdot \frac{1}{2}\pi d}{(2\pi)^d(1-\frac{1}{2}\varepsilon)}.$$

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$$A = \frac{1}{(-M^{2-\varepsilon} - (-p^2)^{1-\varepsilon/2}B(\varepsilon))}$$

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renormalization of G_F

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emergent intermediate boson

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B(ε) ~ 1/ε so divergent as d → 4

'o there is an AS theory for d< 4 but it is singular as d → 4

$$f_{d\to 4} = \frac{4 \cdot \frac{1}{2}\pi d}{(2\pi)^d}\left[\frac{1}{2}\Lambda^2 + p^2 + \frac{7}{6} p^2 \ln \frac{p^2}{\Lambda^2}\right]$$

here is an AS completion in d=4, but the boson requires a counterterm nd so becomes fundamental

ntroduce an auxilliary field: $\mathcal{L} = \frac{1}{2}\bar{\psi}_i \partial\psi_i - \frac{1}{2}\sigma\bar{\psi}_i\psi_i + \frac{1}{2G_F}\sigma^2$

he limit d→ 4 requires a counterterm: $\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 \ln\frac{\Lambda^2}{\mu^2}.$

fter which the renormalized amplitude is:

$$A \sim \frac{1}{p^2 - M^2 - (7/24\pi^2)p^2 ln(-p^2/M^2)}$$

o in this case there is an AS completion in d=4 and it is stable, unitary, renormalizable theory: ie the four fermion interaction softened by an intermediate boson.

his is what doesn't exist for gravity in d=4 which is why an AS ompletion of gravity requires at least a reduced scaling dimension.

Can we find a non-perturbative mechanism to reduce the scaling dimension below d=4 at high energies?

That is, find a non-perturbative mechanism so that in the critical region the theory scales as if d<4, ie for finite ε.

$$\int \frac{d^4 p}{(2\pi)^4} \to \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} M_p^{\epsilon}$$

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This way, the theory can be well defined to all orders in 1/N without the need for the Weyl^2 counter-term which destroys the stability of the theory.

Can we find a non-perturbative mechanism to reduce the scaling dimension below d=4 at high energies?

First try: (1985) Suppose that at short distances there is a scale invariant gas of virtual black holes. Propagation only coherent on fractal set outside of all horizons. Reduces scaling dimension below d=4.

Nuclear Physics B267 (1986) 714–757

RENORMALIZABILITY OF GENERAL RELATIVITY ON A BACKGROUND OF SPACETIME FOAM

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But what about Lorentz invariance?

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- *The vacuum is described by a gas of Wilson loops of the spacetime connection.*
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In fact, there is a physical cutoff, so spin foam amplitudes are uv finite, so the connection to AS was forgotten.

ome lessons and opportunities for AS from LQG

There is a new fixed point. It is a topological quantum field theory

$$S = \int_{\mathcal{M}^4} B^{IJ} \wedge F_{IJ}$$

SO(1,4) gauge theory, I,J=0,...4

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GR can be understood as a cubic interaction added to this TQFT

Because of that there is a well defined path integrals (spin foams)

- Some are uv finite but ir divergences have to be dealt with.
- They can be expressed in terms of matrix models (group field theory)
- RG becomes a Hopf algebra (Connes-Kreimer, Markopoulou)

There is a lot of scope for RG methods to apply to spin foam models.

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Can we find a non-perturbative mechanism to reduce the scaling dimension below d=4 at high energies?

Recently several research programs have found evidence for d_S ~ 2 at high energies:

- ***Casual dynamical triangulations***
- ***Modern RG approaches:***

There is also interest in the proposal by Horava on anisoptropic scaling:

$$E \sim p^z M_p^{1-z}$$

which has the effect of reducing the scaling dimension

$$d_S = 1 + \frac{3}{z}$$

here is a key question that must be asked of any scenario that claims the graviton propagator scales anomalously at high energies.

Any anomalous scaling introduces an energy scale, M_p, which marks the threshold above which we observe the new physics

- **Reduced dimension:**

$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} M_p^{\epsilon} \quad D(p^2)_{\mu\nu\alpha\beta} \sim \frac{1}{N} \frac{1}{(-p^2)^{2-\frac{1}{2}\epsilon}} \frac{1}{M_p^{\epsilon}}$$

- **Anisotropic scaling:**

$$E \sim p^{\Gamma} M_p^{1-\Gamma}$$

- **Cutoff** $p < M_p$

Does this imply the breaking or deformation of the lorentz transformations, as applied by observers who live at d=4? ie what is the symmetry of the scaling region at the fixed point?

et us put this in phenomenological terms. The scaling at the fixed oint should determine the propagation and scattering of gravitons nd other particles at transplankian energies and momenta. What ymmetry group governs those interactions?

o ask this it is sufficient to consider the theory in the limit

$$\hbar \rightarrow 0, \qquad G_N \rightarrow 0 \qquad M_p = \sqrt{\frac{\hbar}{G_N}}$$

ut with their ratio, M_p, held fixed

his is an experimental regime with two constants, c and M_p.

Vhat is the symmetry group that governs their phenomenology?

his should be determined by the physics at the non-trivial xed point.

he classical Planck energy regime:

$$\hbar \to 0, \qquad G_N \to 0$$

$$l_p = \sqrt{\hbar G_N} \to 0 \qquad M_p = \sqrt{\frac{\hbar}{G_N}}$$

o observations of this regime teach us something about he scaling at the non-trivial fixed point?

hree general possibilities for the symmetry in this regime:

- Lorentz invariance
- Broken lorentz invariance
- Deformed lorentz invariance (DSR)

Principles of deformed special relativity (DSR):

1) Relativity of inertial frames
2) The constancy of c, a velocity
3) The constancy of an energy E_{planck}
7) c is the universal speed of photons for $E<<E_{planck.}$

Amelino-Camelia , Magueijo and ls

Principles of deformed special relativity (DSR):

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Consequences:

- Modified energy-momentum relations
- Anisotropic scaling, $E \sim p^z$
- Momentum space has constant curvature given by E_{planck}
- Energy-momentum conservation becomes non-linear (Coproduct)

Mathematical realizations:

1) Deformed poincare algebra is a hopf algebra
   Acts on a spacetime geometry which is non-commutative.

2) Energy dependent metric: g_{ab} (E), ie the metric is a coupling constant of matter fields and hence runs under the RG.

Models of DSR:

- DSR is realized precisely in 2+1 gravity with matter
  hep-th/0307085
- QFT on kappa-minkowki
- Rainbow metric
- Energy dependent h and c

Energy dependent metric: g_{ab} (E)

- *The metric is a coupling constant of matter fields and hence runs under the RG.*

- DSR is the statement that an energy dependent metric has a symmetry group which is modified but not broken by the energy dependence.

Energy dependent frame fields

$$e^a_\mu(E/M_p) = [e^0_\mu f(E/M_p), e^i_\mu g(E/M_p)]$$

Implies modified energy-momentum relations

$$g^{\mu\nu}(E/M) p_\mu p_\nu = m^2 c^4$$

SR modified energy momentum relations:

$$E^2 f^2(\frac{E}{M}) = p^2 g^2(\frac{E}{M})c^2 + \mu c^4$$

eformed Lorentz transformations:

$(Ef, \vec{p}g)$ transforms as a regular Lorentzian 4-vector

class of examples, for massless particles

$$E\frac{f}{g} = \frac{E}{(1+\beta\frac{E}{M})^{\alpha}} = p$$

nplies anisotropic scaling:

$$E^{1-\alpha}(\frac{M}{\beta})^{-\alpha} \sim p$$

'arieties of asymptotic behavior for DSR

$$\frac{E}{(1+\beta\frac{E}{M})^{\alpha}} = p$$

he leading order modification of the speed of light is

$$v = \frac{dE}{dp} = 1 + 2\alpha\beta\frac{E}{M} + \ldots$$

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he leading order modification of the speed of light is

$$v = \frac{dE}{dp} = 1 + 2\alpha\beta\frac{E}{M} + ...$$

$\alpha>0$, $\beta>0$: both E and p are unbounded: $E^{1-\alpha}(\frac{M}{\beta})^{-\alpha} \sim p$

- Anisotropic scaling
- Superluminal speed of light
- Lifshitz: $z = 3 \rightarrow \alpha = \frac{2}{3}$

'arieties of asymptotic behavior for DSR

1> α >0, β< 0: E bounded, p unbounded:

$$\frac{E}{(1-|\beta|\frac{E}{M})^{\alpha}} = p$$

This also has a kind of inverse scaling.

$$\epsilon = \frac{M}{|\beta|} - E \qquad p \sim \frac{M^{1+\alpha}}{\epsilon^{\alpha}}$$

This case has a subluminal speed of light v <1

$$v = 1 - 2\alpha|\beta|\frac{E}{M} + ...$$

'arieties of asymptotic behavior for DSR

$\alpha = 1, \beta > 0$: E unbounded, p bounded:

$$\frac{E}{(1+\beta\frac{E}{M})} = p \qquad \frac{p}{(1-\beta\frac{p}{M})} = E$$

This also has a kind of inverse scaling and is superluminal

$$E \sim \frac{M^2}{\beta^{-1}M - p}$$

arieties of asymptotic behavior for DSR

$\alpha < 0, \beta < 0$: E and p both bounded:

$$E^2(1 - \frac{E}{M})^2 = p^2 + \mu^2$$

Could this eliminate the ghost? ie no pole if $\mu > M$

SR, anisotropic scaling and observation.

orava's hypothesis:

$$E \sim p^3 M^{-2} c$$

his is the asymptotic behavior of a version of DSR with

$$\frac{E}{(1+\frac{E}{M})^{\frac{2}{3}}} = p$$

his implies at leading order

$$v = c(1 + \frac{4}{3}\frac{E}{M})$$

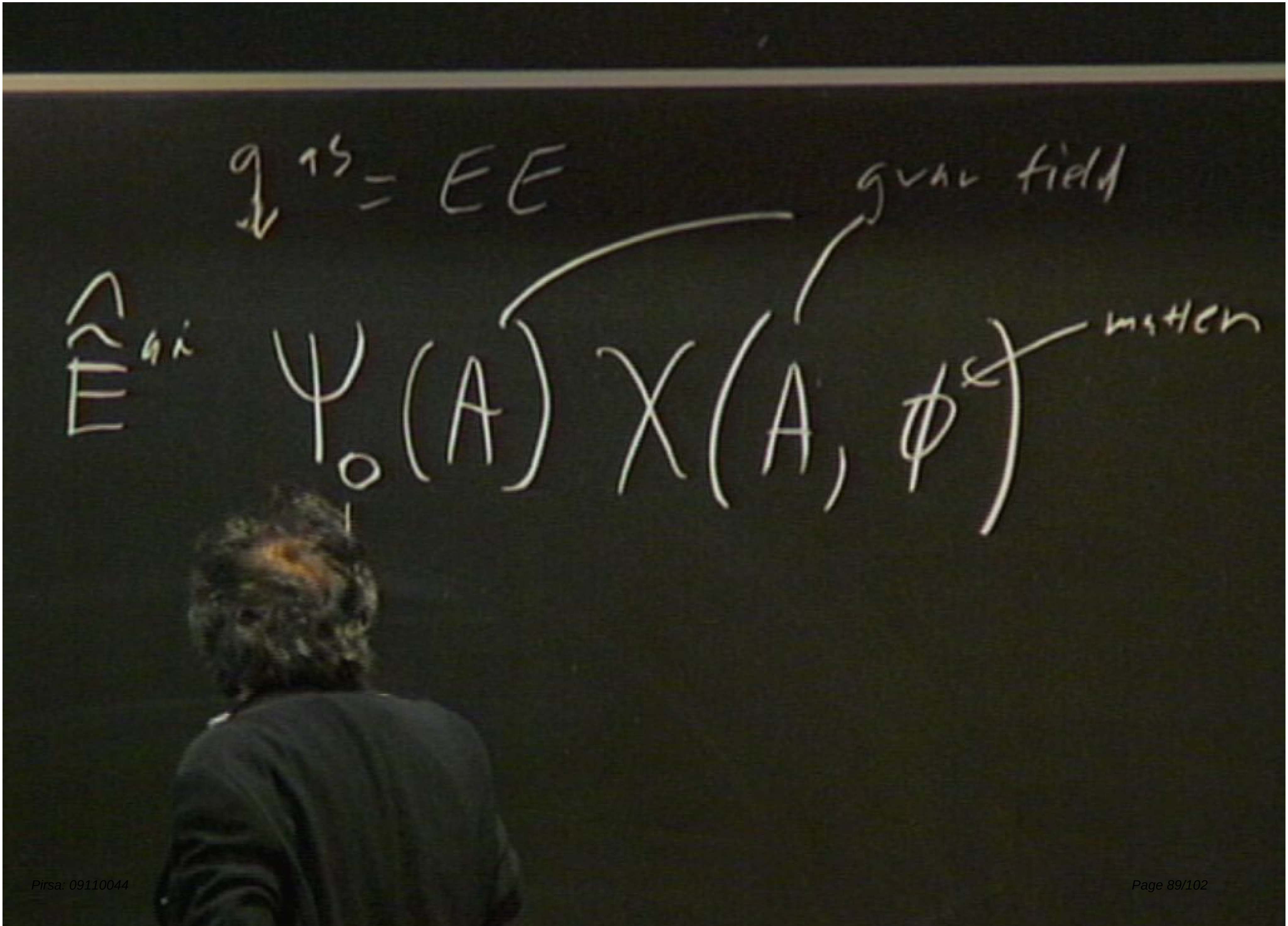
his is the same order of magnitude of the bound set by FERMI.

Arguments for DSR from the semi-classical limit of quantum gravity

- **Not rigorous**
- **Two**

ls hep-th/0501091, Nucl.Phys. B742 (2006) 142-157.
ls arXiv:0808.3765

Why should the metric become energy dependent?



$q^{ab} = EE$
gauge field
$\hat{E}^{ai}\,\Psi_0(A)\,\chi(A, \phi)$
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$\downarrow$

gauge field

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matter

$$\hat{E}^{ai}\, e^{i S^{HJ}(A)} = E_0^{ai}(1 + \omega)$$

Variables: $A_a^i, \quad \tilde{E}_i^a, \quad qq^{ab} = \tilde{E}_i^a \tilde{E}_j^b \delta^{ij}$

Poisson brackets: $\{A_a^i(x), \tilde{E}_j^b(y)\} = G\delta_a^b \delta_j^i \delta^3(x,y)$

Connection rep: $\Psi(A,\phi) \quad \hat{\tilde{E}}_i^a(x) = G\hbar \frac{\delta}{\delta A_a^i(x)}$

ϕ matter fields

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Connection rep: $\Psi(A, \phi)$ $\qquad \hat{\tilde{E}}_i^a(x) = -\imath G\hbar \frac{\delta}{\delta A_a^i(x)}$

Semiclassical states: $\Psi(A, \phi) = e^{iS(A)} \xi(A, \phi)$ S: Hamilton-Jacobi function

classical solution: $\tilde{E}_0^{ai}(x) = \frac{\delta S}{\delta A_{ai}(x)}$

S(A) is a time coordinate on configuration space and on solutions
S=μ T where T is a coordinate on the spacetime

So an energy eigenstate $\xi[T, \phi] = e^{-\imath\omega T} \xi_\omega[\phi]$

Semiclassical states: $$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution

a_{ai}- fluctuations of metric, we ignore them.

$$\begin{aligned}\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] &= -\imath\hbar\rho\frac{\delta\xi[\mathcal{A},\phi]}{\delta\mathcal{A}^i_a(x)} \\ &= \left(-\tilde{E}^{0a}_i\frac{\imath\hbar\rho}{M}\frac{\delta}{\delta\mathcal{S}(x)} - \imath\hbar\rho\frac{\delta}{\delta a_{ai}(x)}\right)\xi[\mathcal{S},a_{ai},\phi]\end{aligned}$$

Semiclassical states: $$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution

$$\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] = \tilde{E}^{0a}_i \frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi]$$

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution:

$$\hat{\tilde{E}}_i^a(x)\xi[\mathcal{A},\phi] = \tilde{E}_i^{0a} \frac{-\imath\hbar\rho}{\mu M} \frac{\partial}{\partial T} \xi[T,\phi] = -\tilde{E}_i^{0a} \alpha l_{Pl} \omega e^{-\imath T\omega} \xi_\omega[\phi]$$

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

emiclassical states: $$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution: $$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

$$\hat{\tilde{E}}_i^a(x)\xi[\mathcal{A},\phi] = \tilde{E}_i^{0a} \frac{-\imath\hbar\rho}{\mu M} \frac{\partial}{\partial T} \xi[T,\phi] = -\tilde{E}_i^{0a} \alpha l_{Pl} \omega e^{-\imath T\omega} \xi_\omega[\phi]$$

Putting everything together

$$\hat{\tilde{E}}_i^a(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}_i^{0a}\left(1 - \alpha l_{Pl}\omega\right)\xi[T,\phi]$$

Classical term

emiclassical states: $$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

ecompose E operator around a solution: $$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

$$\hat{\tilde{E}}_i^a(x)\xi[\mathcal{A},\phi] = \tilde{E}_i^{0a} \frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi] = -\tilde{E}_i^{0a}\alpha l_{Pl}\omega e^{-\imath T\omega}\xi_\omega[\phi]$$

utting everything together

$$\hat{\tilde{E}}_i^a(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}_i^{0a}\left(1 + \alpha l_{Pl}\omega\right)\xi[T,\phi]$$

Energy dependent correction

emiclassical states: $\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$

Decompose E operator around a solution: $\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$

$$\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] = \tilde{E}^{0a}_i \frac{-\imath\hbar\rho}{\mu M} \frac{\partial}{\partial T} \xi[T,\phi] = -\tilde{E}^{0a}_i \alpha l_{Pl} \omega e^{-\imath T\omega} \xi_\omega[\phi]$$

utting everything together

$$\hat{\tilde{E}}^a_i(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}^{0a}_i (1-\alpha l_{Pl}\omega)\,\xi[T,\phi]$$

o the spacetime metric has become energy dependent

$$g \rightarrow g(\omega) = -dT \otimes dT + \sum_i e_i \otimes e_i (1-\alpha l_{Pl}\omega)$$

nd there is a modified dispersion relation to leading order:

$$m^2 = -g(\omega)^{\mu\nu} k_\mu k_n u = \omega^2 - \frac{k_i^2}{(1-\alpha l_{Pl}\omega)} + O[(l_{Pl}\omega)^2]$$

Asymptotic safety suggests a reduced scaling dimension in the critical region to avoid the catastrophe of the Weyl^2 counterterm.

A key question is then what happens to the space-time symmetries in the scaling region.

One attractive possibility is anisotrpic scaling, such as $E \sim p^{3.}$

This is consistent with breaking or deformation of lorentz symmetry

The symmetry will characterize the scaling of matter at high energy

Anisotropic scaling implies variation of the speed of light with energy.

The leading order transition to this is potentially observable now and is disfavored if Lorentz invariance is broken, rather than deformed.

So we can hypothesize that the non-trivial fixed point is characterized by deformed lorentz symmetry. This is just the statement that at the semi-classical level we should treat the metric as coupling constants that run under the RG.