Title: Mechanisms of Asymptotic Safety

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Abstract:

Mechanisms of Asymptotic Safety

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Gravity	& Astrid Eichhorn, Michael Scherer	arXiv:0907.1828, PRD in press
Extra-Dim YM		PRD **68**, 085015 (2003)
Fermionic SM	& Joerg Jaeckel, Christof Wetterich	PRD **69**, 105008 (2004)
Conformal VEV	& Stefan Rechenberger, Michael Scherer	arXiv:0901.2459, 0907.0327

Challenges, Riddles, Problems . . .

Challenges, Riddles, Problems ...

- quantum field theory $\longleftrightarrow$ gravity
  (non-renormalizability ?)
- triviality problems in the SM
  (Higgs sector & U(1) gauge)
- hierarchy problems
  (gauge hierarchy, cosmological coincidence)
- $\simeq$ fine-tuning problems
  (Higgs mass, cosmological constant)
- abundance of parameters in the SM
  (large mass hierarchies, origin of flavor physics)
- $D = 4 = D_{\text{RG, cr}}$

QFT $\leftrightarrow$ Gravity

(Goroff, Sagnotti'85'86; van de Ven'92)

▷ perturbative quantization fails

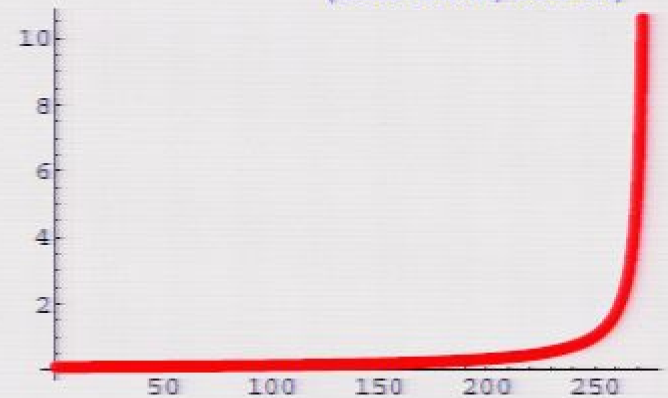
$$\Gamma^{\text{2-loop}}_{\text{div}} = \frac{1}{\epsilon}\frac{209}{2880}\frac{1}{(16\pi^2)^2}\int d^4x\,\sqrt{g}C_{\mu\nu\rho\sigma}C^{\rho\sigma\lambda\tau}C_{\lambda\tau}{}^{\mu\nu}$$

Triviality problem

▷ QED: perturbation theory predicts its own failure

(LANDAU'55)

(GELL-MANN, LOW'54)



$$\frac{1}{e_R^2} - \frac{1}{e_\Lambda^2} = \beta_0 \ln \frac{\Lambda}{m_R}, \quad \beta_0 = \frac{N_f}{6\pi^2}$$

▷ e_R^2 and m_R fixed:

$$\Longrightarrow \quad \Lambda_L \quad \simeq \quad m_R \exp\left(\frac{1}{\beta_0} e_R\right) \quad \simeq \quad 10^{272}\text{GeV (2 loop)}$$

Landau pole singularity

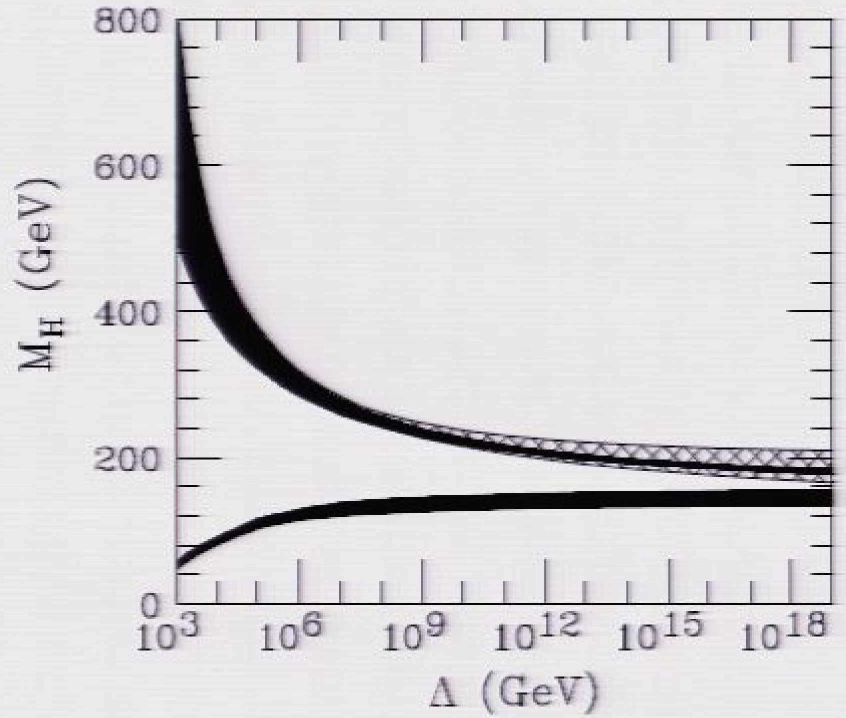
▷ $\lim\ (\Lambda/m_R) \to \infty$: $\quad \Longrightarrow \quad e_R \to 0$

Triviality

Triviality problem

$\Longrightarrow$ …scale of maximal UV extension

▷ triviality of the scalar Higgs sector:



(HAMBYE, RIESSELMANN'97)

$\Longrightarrow$ SM Higgs mass bounds from Landau pole position

Hierarchy problem $\Lambda_{UV} \ggg \Lambda_{EW}$

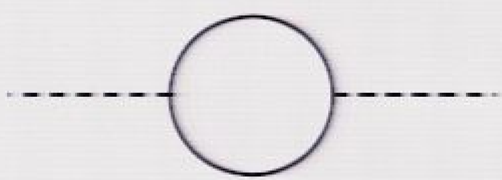
▷ renormalization of the scalar mass (e.g., $\Lambda_{UV} = 10^{16}$GeV)

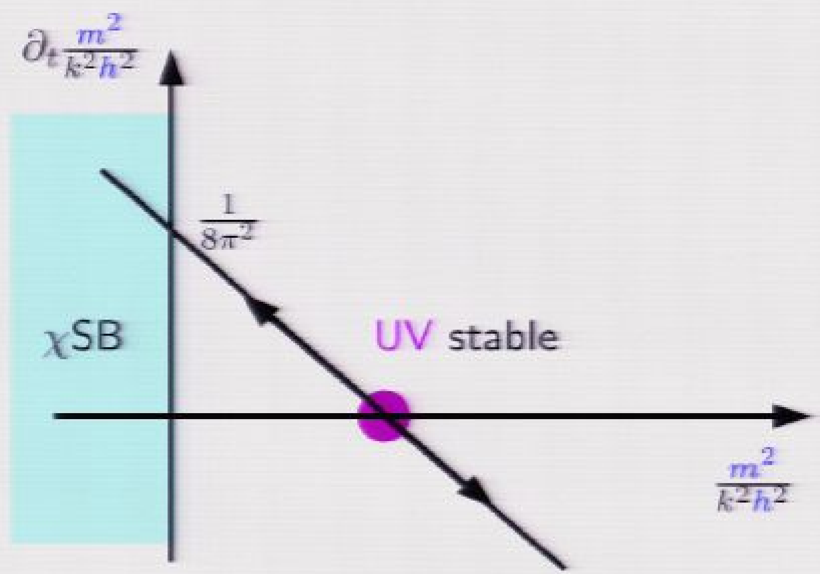
$$\underbrace{m_R^2}_{\sim 10^4 \mathrm{GeV}^2} \sim \underbrace{m_\Lambda{}^2}_{\sim 10^{32}\left(1+\ldots 10^{-28}\right)\mathrm{GeV}^2} - \underbrace{\delta m^2}_{\sim 10^{32}\mathrm{GeV}^2}$$

▷ RG viewpoint ($\partial_t = k\frac{d}{dk}$)

e.g., Yukawa theory:

$$\partial_t \frac{m^2}{k^2 h^2} = -2\,\frac{m^2}{k^2 h^2} + \frac{1}{8\pi^2}$$



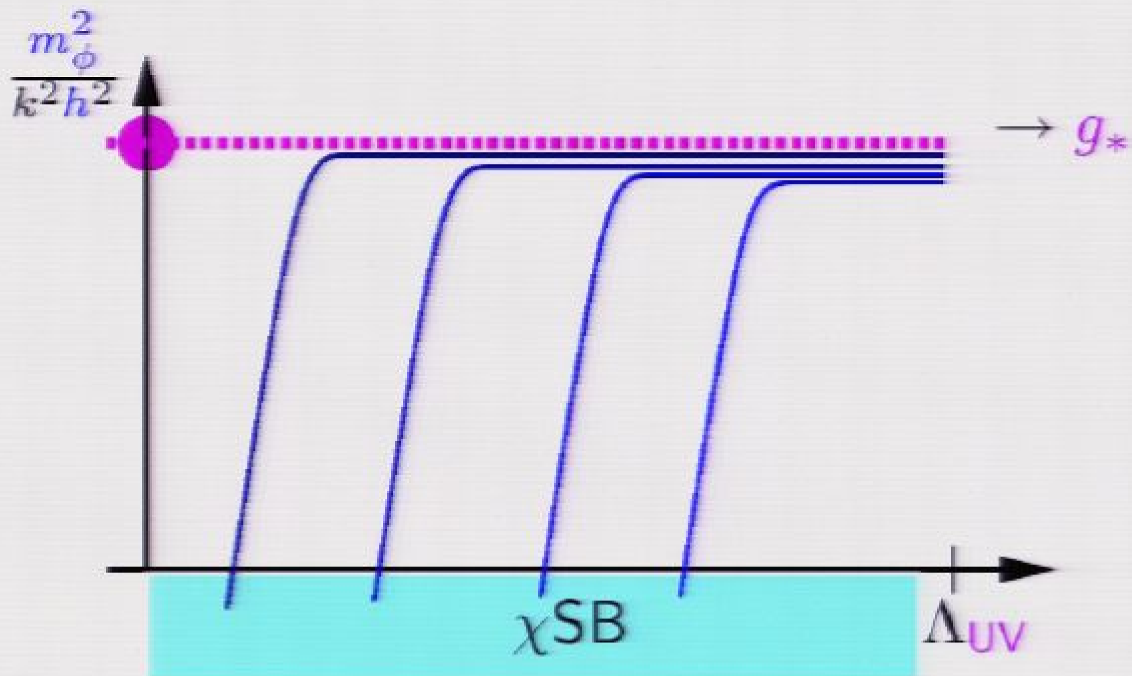


$\ldots \simeq$ Finetuning Problem

▷ "coupling": $g := \frac{m^2}{h^2 k^2}$, "β function": $\partial_t g = \beta(g)$

▷ critical exponent Θ

$$\Theta = -\frac{\partial \beta(g_*)}{\partial g} = 2$$



$\Longrightarrow$ $\Theta \sim$ measure for the required finetuning

e.g., cosmological constant near the Gaußian fixed point: $\Theta = 4$

Abundance of Parameters

Parameter	Input value	Free in fit	Results from global EW fits: *Standard fit*	*Complete fit*	*Complete fit w/o exp. input in line*
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1874 ± 0.0021	91.1876 ± 0.0021	$91.1974^{+0.0191}_{-0.0159}$
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4960 ± 0.0015	2.4956 ± 0.0015	$2.4952^{+0.0017}_{-0.0016}$
σ^0_{had} [nb]	41.540 ± 0.037	–	41.478 ± 0.014	41.478 ± 0.014	41.469 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.742 ± 0.018	20.741 ± 0.018	20.717 ± 0.027
$A^{0,\ell}_{FB}$	0.0171 ± 0.0010	–	0.01638 ± 0.0002	0.01624 ± 0.0002	$0.01617^{+0.0002}_{-0.0001}$
A_ℓ [(*)]	0.1499 ± 0.0018	–	0.1478 ± 0.0010	$0.1472^{+0.0009}_{-0.0008}$	–
A_c	0.670 ± 0.027	–	$0.6682^{+0.00045}_{-0.00011}$	$0.6679^{+0.00042}_{-0.00036}$	$0.6679^{+0.00041}_{-0.00036}$
A_b	0.923 ± 0.020	–	0.93469 ± 0.00010	$0.93463^{+0.00007}_{-0.00008}$	$0.93463^{+0.00007}_{-0.00009}$
$A^{0,c}_{FB}$	0.0707 ± 0.0035	–	$0.0741^{+0.0006}_{-0.0005}$	0.0737 ± 0.0005	0.0737 ± 0.0005
$A^{0,b}_{FB}$	0.0992 ± 0.0016	–	0.1036 ± 0.0007	$0.1032^{+0.0007}_{-0.0006}$	$0.1037^{+0.0004}_{-0.0005}$
R^0_c	0.1721 ± 0.0030	–	0.17225 ± 0.00006	0.17225 ± 0.00006	0.17225 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21578 ± 0.00005	0.21577 ± 0.00005	0.21577 ± 0.00005
$\sin^2\theta^\ell_{eff}(Q_{FB})$	0.2324 ± 0.0012	–	0.23142 ± 0.00013	$0.23151^{+0.00010}_{-0.00012}$	$0.23149^{+0.00013}_{-0.00010}$
M_H [GeV] [(o)]	Likelihood ratios	yes	$83^{+30[+75]}_{-23[-41]}$	$116^{+15.6[+36.5]}_{-1.3[-2.2]}$	$83^{+30[+75]}_{-23[-41]}$
M_W [GeV]	80.399 ± 0.023	–	$80.384^{+0.014}_{-0.015}$	$80.371^{+0.008}_{-0.011}$	$80.361^{+0.013}_{-0.012}$
Γ_W [GeV]	2.098 ± 0.048	–	$2.092^{+0.001}_{-0.002}$	2.092 ± 0.001	2.092 ± 0.001
$\overline{m}_c$ [GeV]	1.25 ± 0.09	yes	1.25 ± 0.09	1.25 ± 0.09	–
$\overline{m}_b$ [GeV]	4.20 ± 0.07	yes	4.20 ± 0.07	4.20 ± 0.07	–
m_t [GeV]	173.1 ± 1.3	yes	173.2 ± 1.2	173.6 ± 1.2	$179.5^{+8.8}_{-5.2}$
$\Delta\alpha^{(5)}_{had}(M^2_Z)$ [(†△)]	2768 ± 22	yes	2772 ± 22	2764^{+22}_{-21}	2733^{+57}_{-63}
$\alpha_s(M^2_Z)$	–	yes	$0.1192^{+0.0028}_{-0.0027}$	0.1193 ± 0.0028	0.1193 ± 0.0028
$\delta_{th} M_W$ [MeV]	$[-4, 4]_{theo}$	yes	4	4	–
$\delta_{th} \sin^2\theta^\ell_{eff}$ [(†)]	$[-4.7, 4.7]_{theo}$	yes	4.7	0.3	–
$\delta_{th} \rho^f_Z$ [(†)]	$[-2, 2]_{theo}$	yes	2	2	–
$\delta_{th} \kappa^f_Z$ [(†)]	$[-2, 2]_{theo}$	yes	2	2	–

[(*)] Average of LEP ($A_\ell = 0.1465 \pm 0.0033$) and SLD ($A_\ell = 0.1513 \pm 0.0021$) measurements. The *complete fit* w/o the LEP (SLD) measurement gives $A_\ell = 0.1473 \pm 0.0009$ ($A_\ell = 0.1465^{+0.0007}_{-0.0010}$). [(o)] In brackets the 2σ. [(†)] In units of 10^{-5}. [(△)] Rescaled due to α_s dependency.

SM parameters
from
EW precision data

[GFITTER]

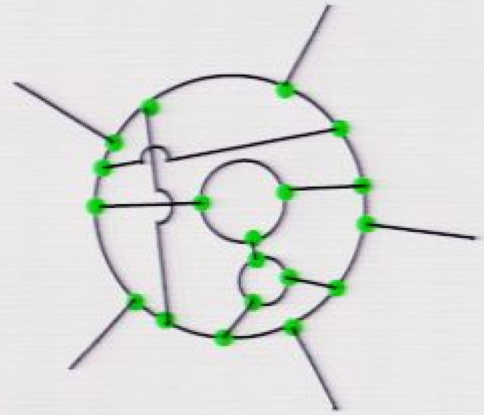
SUSY > 100

Strings > □

Spacetime Dimensionality

▷ (perturbative) QFT:

$$\delta(\gamma) = d - \sum_i n_{E_i}[\phi_i] + \sum_\alpha n_{V_\alpha}\delta(V_\alpha)$$



$\Longrightarrow$ RG critical dimension:

$$D_{\text{RG, cr}} = \begin{cases} 4 & \text{(gauge + matter, Yukawa/Higgs)} \\ 2 & \text{(gravity, pure fermionic matter)} \end{cases}$$

▷ (macroscopic) universe:

$$D = 4$$

"It is not known whether the fact that space time has just four dimensions is a mere coincidence or is logically connected with this property." (J. Zinn-Justin, in "QFT and Critical Phenomena")

"I know of only one promising approach to this problem . . . "

(S. Weinberg, in "Critical Phenomena for Field Theorists" (1976))

Asymptotic Safety

Necessity of Renormalizability

- IR physics well separated from UV physics

  (. . . cutoff Λ independence)

- # of physical parameters $\Delta < \infty$ . . . or countably ∞

  (. . . predictive power)

$\Longrightarrow$ realized by perturbative RG . . .

$\Longrightarrow$ . . . and by "Asymptotic Safety"

(Weinberg'76)

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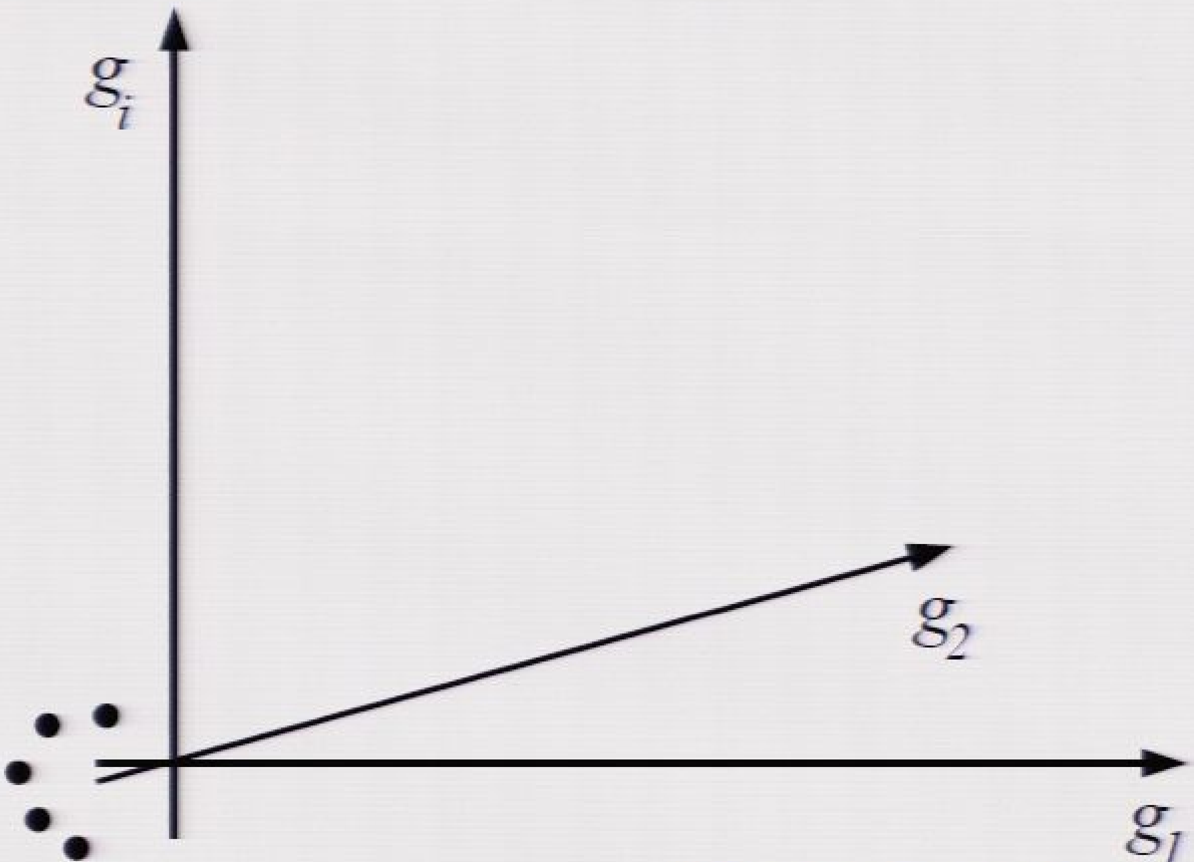
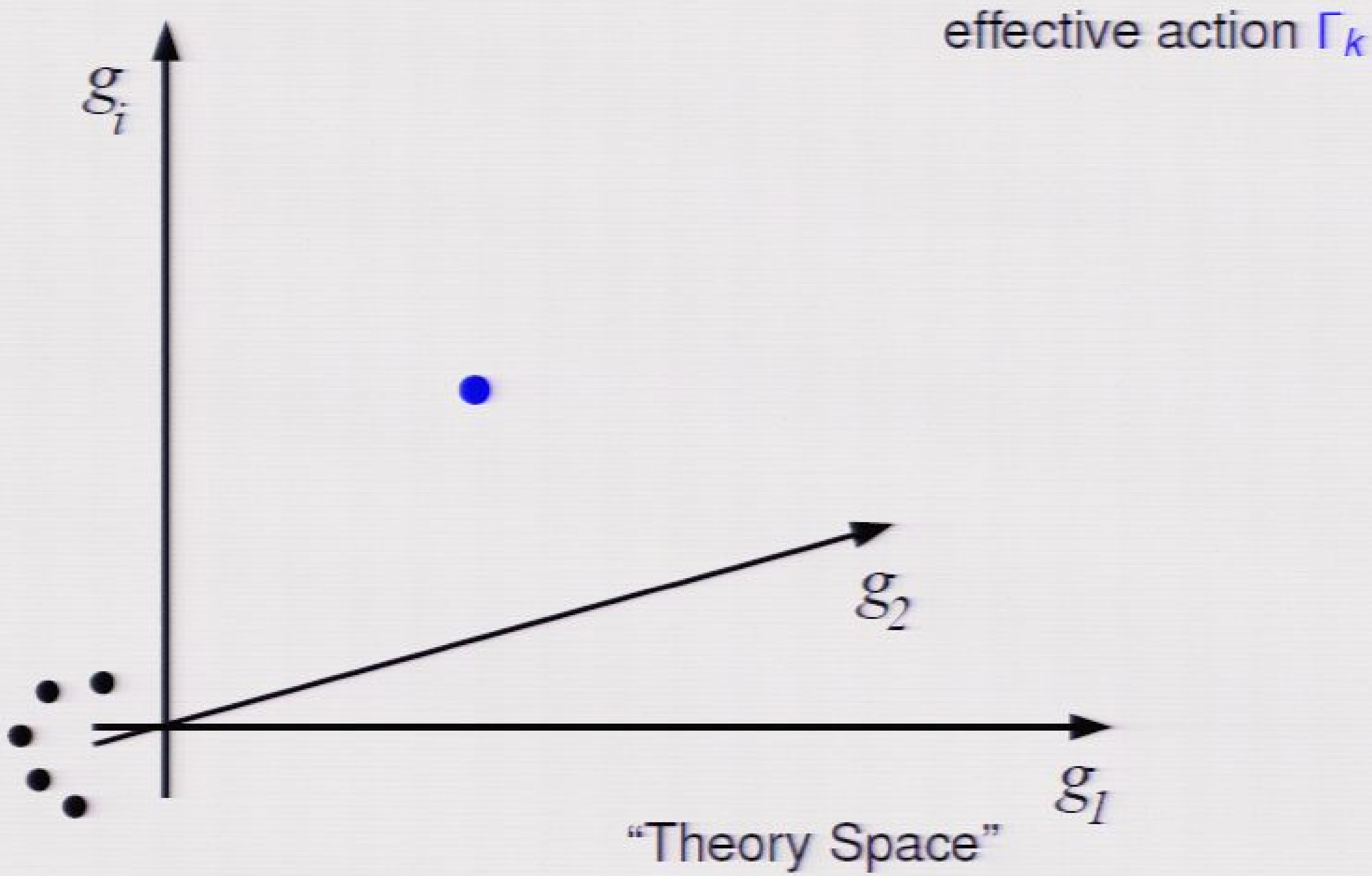
(WEINBERG '76)

Asymptotic Safety

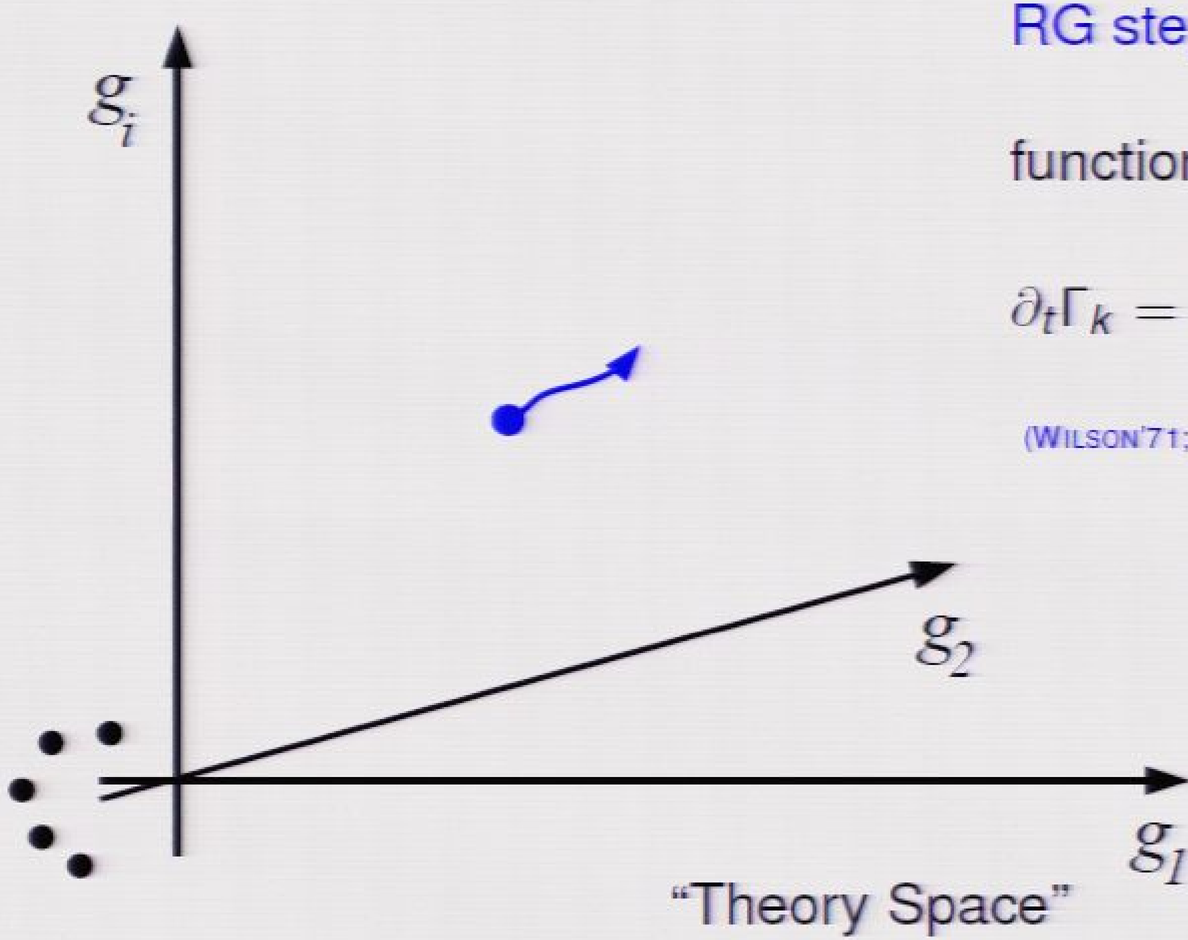


"Theory Space"

Asymptotic Safety



Asymptotic Safety



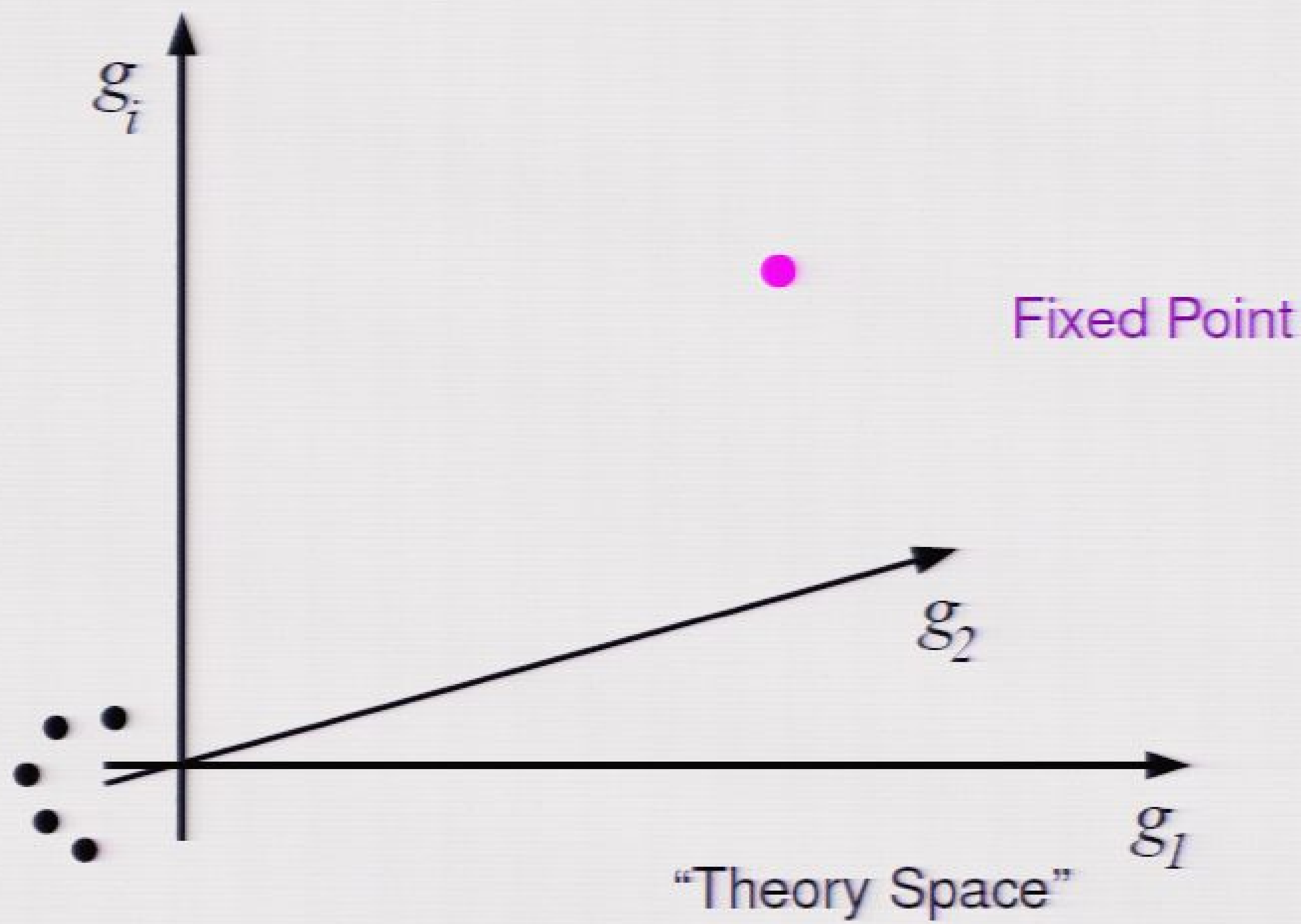
RG step

functional RG: (WETTERICH'93)

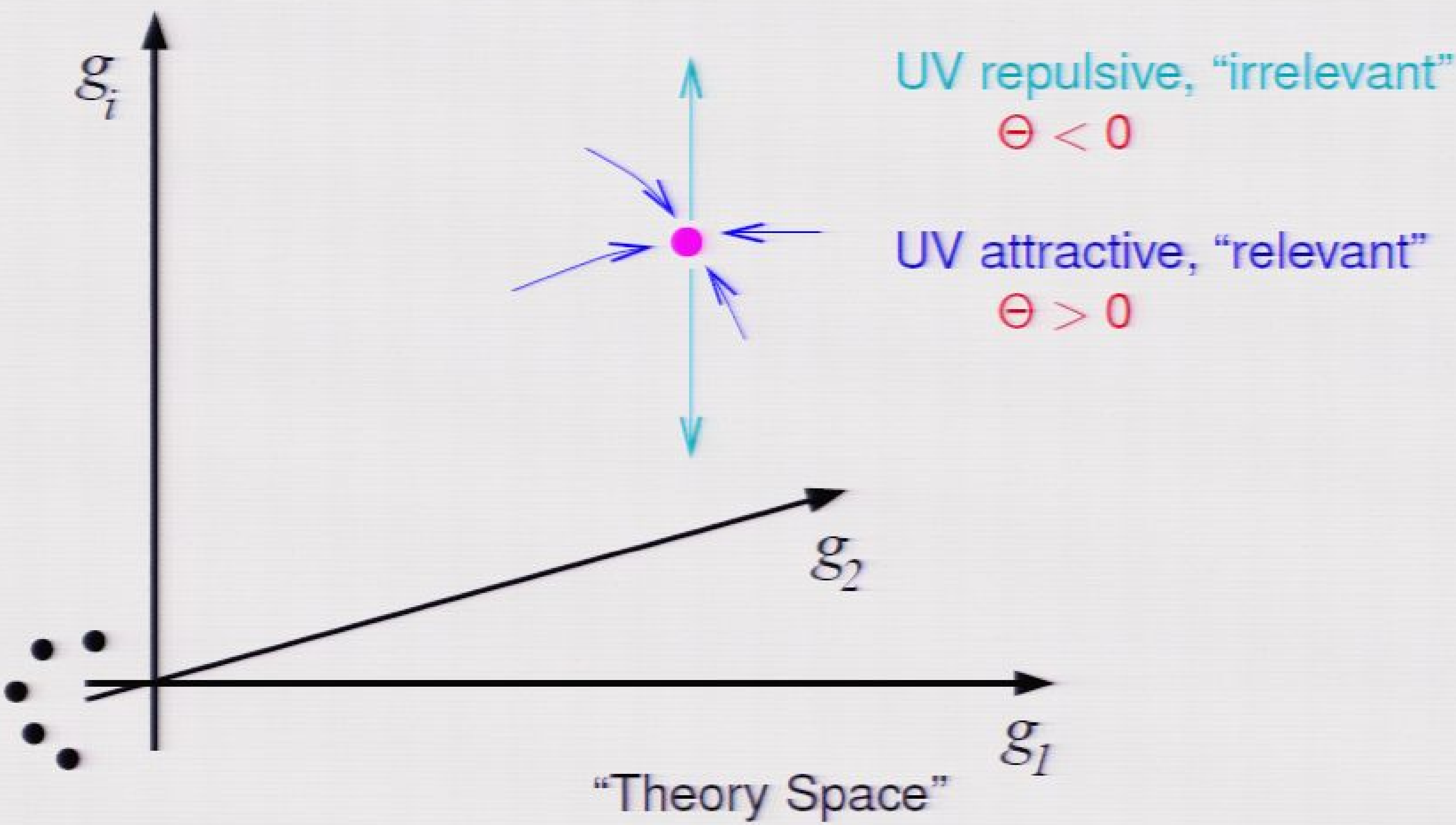
$$\partial_t \Gamma_k = \frac{1}{2} \, \mathrm{Tr} \, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

(WILSON'71; WEGNER, HOUGHTON'73; POLCHINSKI'84)

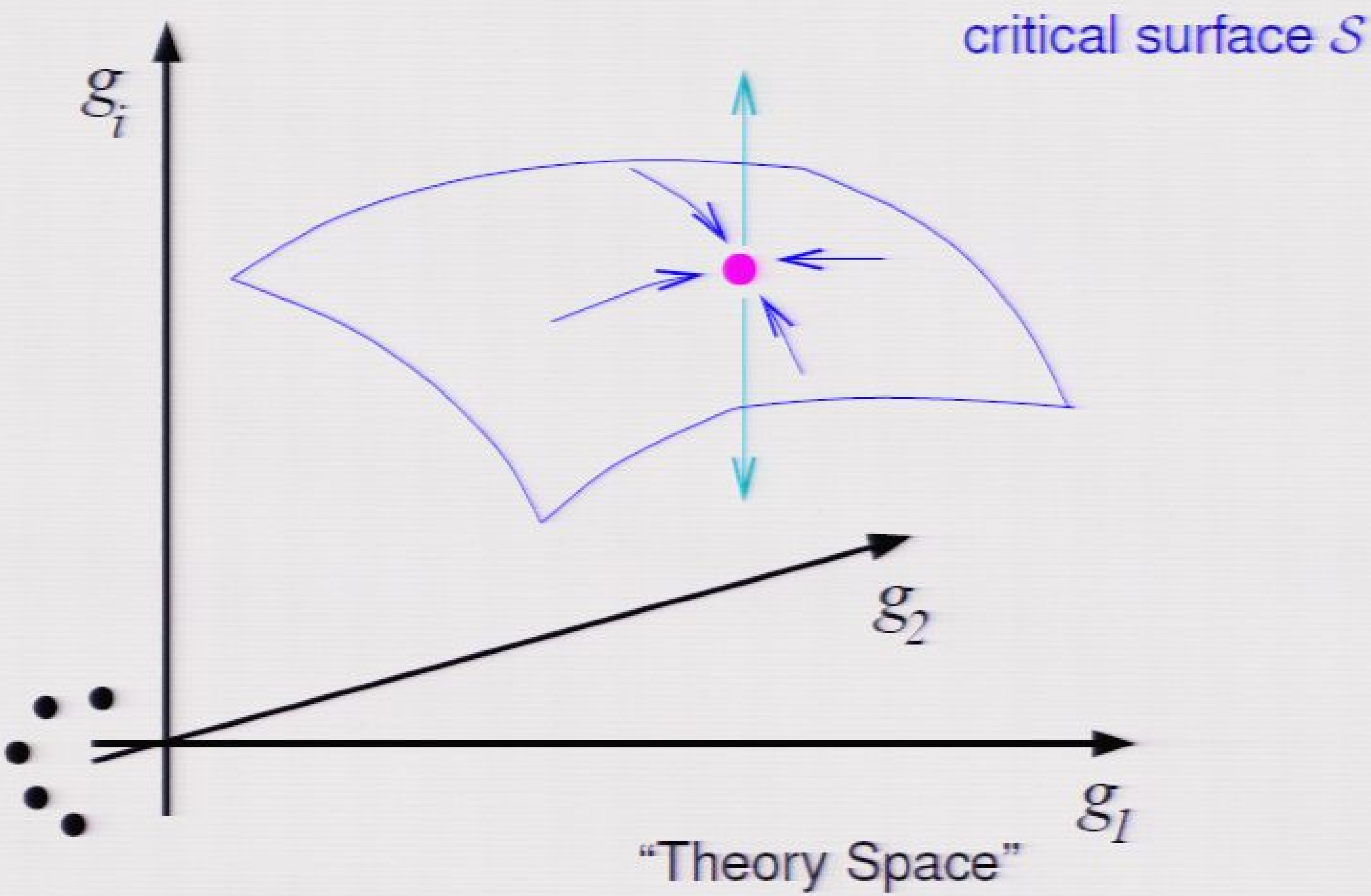
Asymptotic Safety



Asymptotic Safety

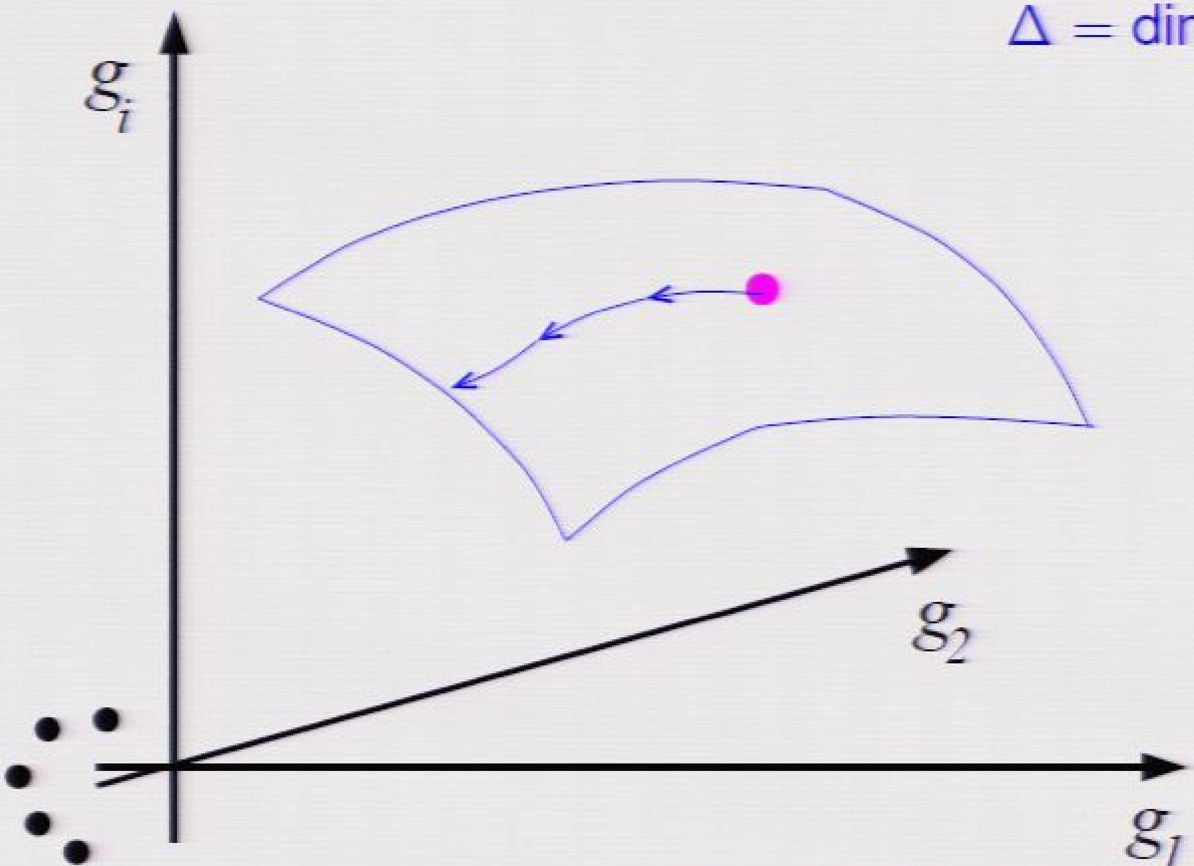


Asymptotic Safety



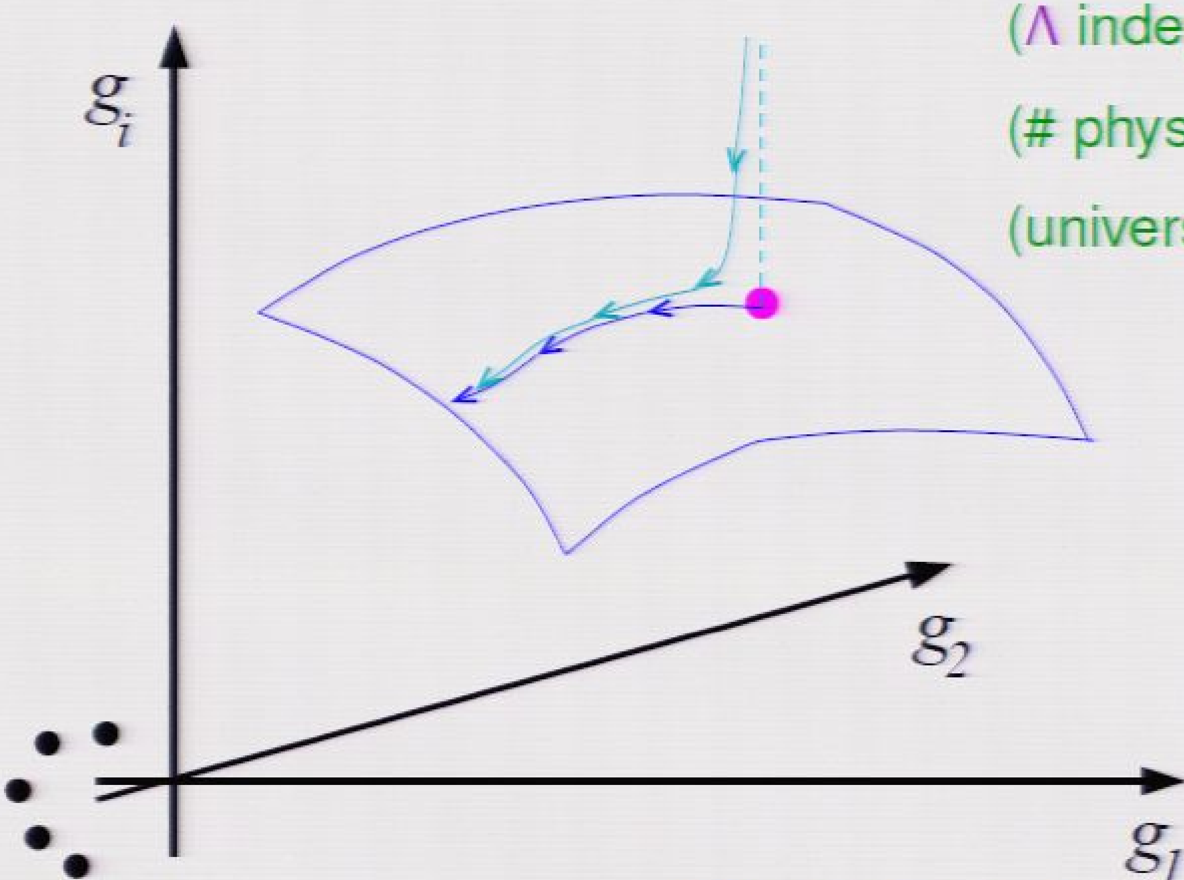
Asymptotic Safety

$\Delta = \dim \mathcal{S} = \#\ \Theta\text{'s}>0$



"Theory Space"

Asymptotic Safety



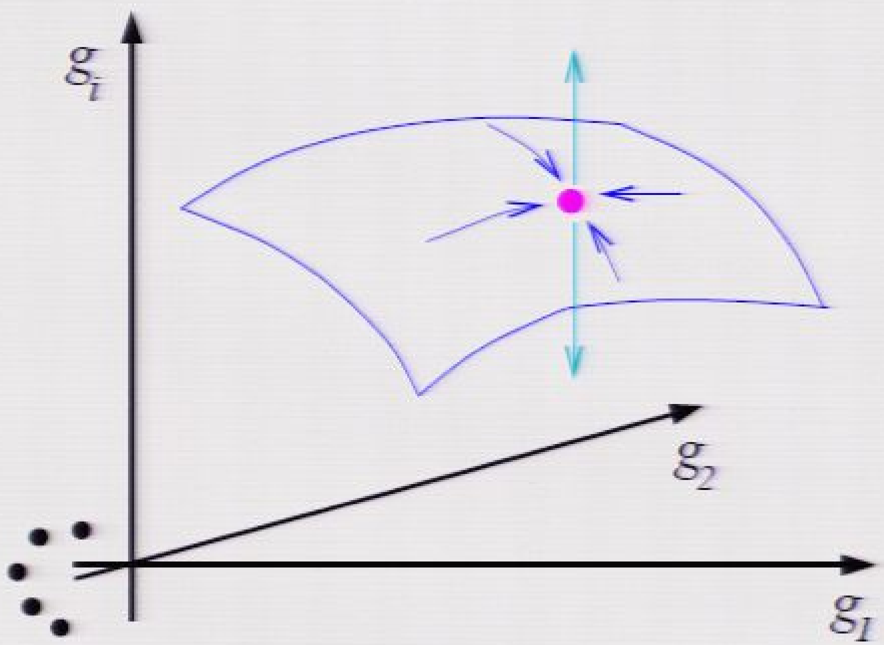
"Theory Space"

(Λ independence ✓)

(# phys. parameters $< \infty$ ✓)

(universality & predictivity ✓)

Asymptotic Safety



"Theory Space"

- ▷ FP regime:

$$\partial_t g_i = B_i{}^j \, (g_j - g_{*j}) + \ldots$$

- ▷ stability matrix

$$B_i{}^j = \frac{\partial \beta_i(g_*)}{\partial g_j}$$

- ▷ critical exponents:

$$\{\Theta\} = \mathrm{spect}(-B_i{}^j)$$

Mechanisms of Asymptotic Safety I:

Dimensional Balancing

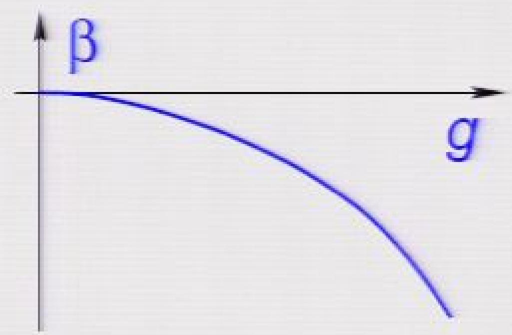
Non-Gaußian Fixed Points

▷ coupling $\bar{g}$ with canonical dimension $\delta_{\bar{g}}(D)$ in spacetime dimension D:

$$[\bar{g}] = \delta_{\bar{g}}(D)$$

▷ critical RG dimension:

$$\delta_{\bar{g}}(D_{\text{RG, cr}}) = 0$$



▷ perturbative β function in $D_{\text{RG, crit}}$, e.g.:

$$\beta_{\bar{g}} = b_0\,\bar{g}^2 + \dots$$

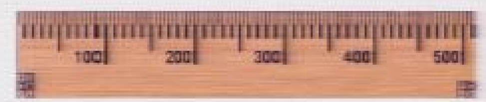
$\Longrightarrow$ if $b_0 < 0$: theory is asymptotically free (and safe)

Non-Gaußian Fixed Points

▷ away from $D_{\mathrm{RG,\,cr}}$ (+ analyticity in D):

$$\beta_{\bar{g}} = \frac{b_0(D)}{k^{\delta_{\bar{g}}(D)}}\,\bar{g}^2 + \dots$$

▷ dimensionless coupling in units of a given scale k



$$g = \frac{\bar{g}}{k^{\delta(\bar{g};D)}}$$

▷ RG flow of dimensionless coupling:

$$k\frac{d}{dk}g \equiv \beta_g = -\delta_{\bar{g}}(D)g + b_0(D)\,g^2$$

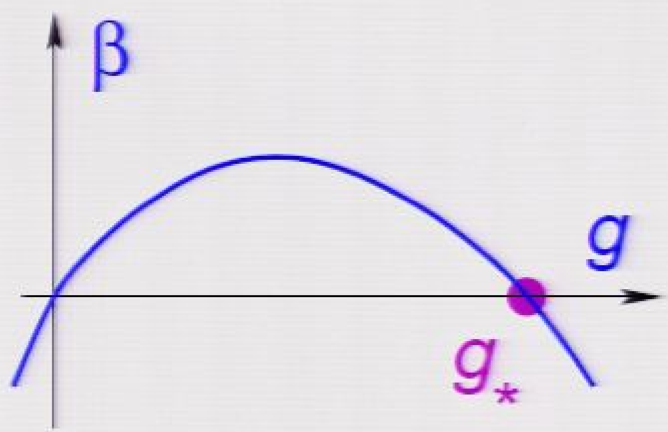
Non-Gaußian Fixed Points

▷ RG flow of dimensionless coupling:

$$k\frac{d}{dk}g \equiv \beta_g = \underbrace{-\delta_{\bar{g}}(D)g}_{\text{dimensional running}} \quad \underbrace{+b_0(D)\,g^2}_{\text{fluctuation-induced running}}$$

$\Longrightarrow$ NGFP g_* for: $\quad \mathrm{sign}(\delta_{\bar{g}}(D)) = \mathrm{sign}(b_0(D))$

$\Longrightarrow$ $g_* > 0$ for: $\quad \delta_{\bar{g}}(D), b_0(D) < 0$



Example: Quantum Einstein Gravity

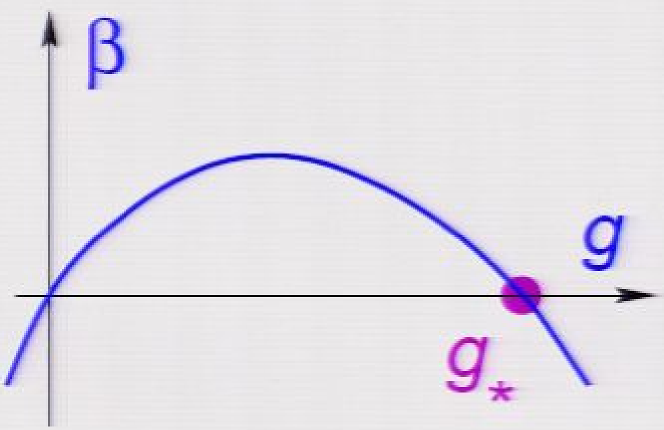
▷ effective action in Einstein-Hilbert truncation

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^D x \sqrt{g}(-R + 2\bar{\lambda}_k)$$

(REUTER'96)

▷ running dim'less Newton's constant in $D = 4$: $g = k^2 G_k$

$$\partial_t g = 2\,g - \frac{5\pi}{18} g^2 + \mathcal{O}(g^3), \quad \bar{\lambda}_k = 0$$



(DOU, PERCACCI'97)

(SOUMA'99)

(LAUSCHER, REUTER'01'02)

(REUTER, SAUERESSIG'01)

(NIEDERMAIER'02)

(LITIM'03)

(CODELLO, PERCACCI'06)

(CODELLO, PERCACCI, RAHMEDE'07'08)

(MACHADO, SAUERESSIG'07)

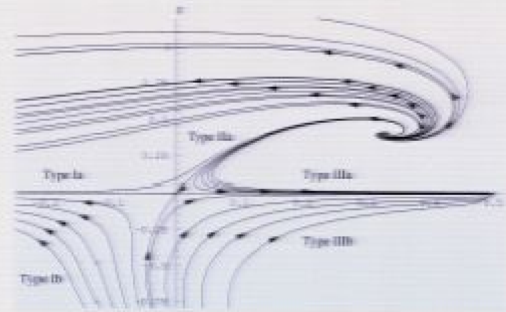
(BENEDETTI, MACHADO, SAUERESSIG'09)

Example: Quantum Einstein Gravity

(Reuter,Saueressig'01)

▷ larger "theory space":

$\vdots$			
R^8	$\dots$		
R^7	$\dots$		
R^6	$\dots$		
R^5	$\dots$		
R^4	$\dots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\kappa\lambda}C_{\kappa\lambda}{}^{\mu\nu}$	$R\Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$	
R			
$\mathbb{1}$			



critical exponents:

(e.g. f(R) truncation)

$$\text{Re}\ \Theta_{1,2} \simeq 2.4$$

$$\Theta_3 \simeq 1.5$$

$$\Theta_{>3} \lesssim -4$$

▷ many tests of robustness

- larger truncations
- regulator/gauge dependencies, etc.

(Codello,Percacci,Rahmede'07'08; Machado,Saueressig'07)

Example: Quantum Einstein Gravity

▷ ghost sector:

$$\langle \mathcal{O}[g] \rangle = \frac{1}{\mathcal{N}} \int \mathcal{D}g \, \delta[F] |\det(-\mathcal{M})| \, \mathcal{O}[g] \, e^{-S}$$

▷ gauge fixing and Faddeev-Popov operator (background gauge):

$$F_\mu[\bar{g}, h] = \sqrt{\frac{1}{16\pi G_N}} \left(\bar{D}^\nu h_{\mu\nu} - \frac{1+\rho}{d} \bar{D}_\mu h^\nu{}_\nu \right)$$

$$\mathcal{M}^\mu{}_\nu = \bar{g}^{\mu\rho} \bar{g}^{\sigma\lambda} \bar{D}_\lambda (g_{\rho\nu} D_\sigma + g_{\sigma\nu} D_\rho) - 2 \frac{1+\rho}{d} \bar{g}^{\rho\sigma} \bar{g}^{\mu\lambda} \bar{D}_\lambda g_{\sigma\nu} D_\rho$$

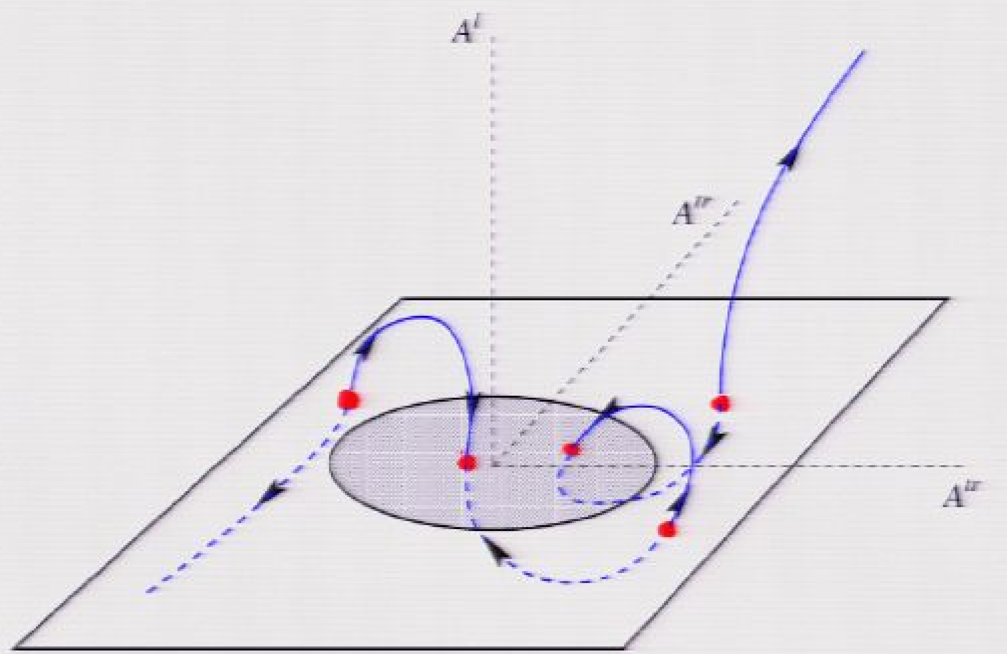
Example: Quantum Einstein Gravity

▷ ghost sector:

$$\langle \mathcal{O}[g] \rangle = \frac{1}{\mathcal{N}} \int \mathcal{D}g \, \delta[F] |\det(-\mathcal{M})| \, \mathcal{O}[g] \, e^{-S}$$

▷ gauge theories: Gribov problem (GRIBOV'78)

[J.PAWLOWSKI@ERG08]



exists also in gravity (DAS,KAKU'79)

Example: Quantum Einstein Gravity

▷ constrained integration domain to first Gribov horizon Ω: (GRIBOV'78)

$$\langle \mathcal{O}[g] \rangle = \frac{1}{\mathcal{N}} \int_\Omega \mathcal{D}g \, \delta[F] |\det(-\mathcal{M})| \, \mathcal{O}[g] \, e^{-S}$$

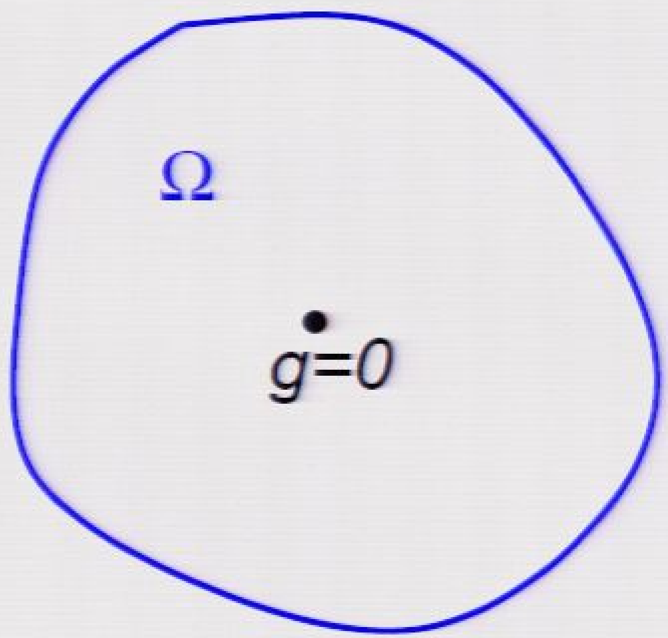
▷ Gribov region

$$\Omega = \{ g | \, F_\mu[g] = 0, \, -\mathcal{M} \geq 0 \}$$

▷ $\partial\Omega$ can dominate at strong coupling due to entropy $\int \mathcal{D}g$ (ZWANZIGER'04)

▷ ghost enhancement

$$\det(-\mathcal{M})|_\Omega \to 0 \quad \to \frac{1}{\mathcal{M}} \to \infty$$



Example: Quantum Einstein Gravity

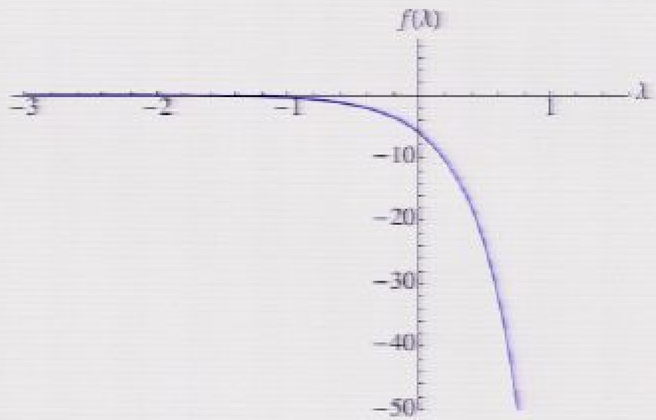
▷ first test: ghost-curvature coupling (EICHHORN, GIES, SCHERER '09)

$$\Gamma_{\mathrm{R\,gh}} = \bar{\zeta} \int d^4x \sqrt{\gamma}\, \bar{c}^{\mu} R c_{\mu}$$

▷ $[\bar{\zeta}] = 0$, power-counting marginal

▷ RG flow at NGFP (Landau-DeWitt gauge):

$$\partial_t \bar{\zeta} = +\frac{25 g_*}{6\pi} f(\lambda_*)\, \bar{\zeta} \simeq -\underbrace{1.404}_{=\Theta}\, \bar{\zeta}$$



▷ ghost-curvature coupling is asymptotically free and RG relevant

. . . but subject to gauge constraints

$\Longrightarrow$ asymptotic safety remains robust

Example: Extra-dimensional Yang-Mills theory

"To see how this works in practice,
let us consider the theory . . . in five dimensions." (WEINBERG'76)

▷ $D > 4$: $[\bar{g}] = (4 - D)/2$:

$$S = \int_x d^D x \, \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \bar{g} f^{abc} A^b_\mu A^c_\nu$$

▷ $D - 4 = \epsilon$ expansion, dim'less coupling: $g^2 \sim k^{D-4} \bar{g}^2$ (PESKIN'80)

$$\partial_t g^2 \equiv \beta_{g^2} = (D - 4) g^2 - \frac{22N}{3} \frac{g^4}{16\pi^2} + \dots$$

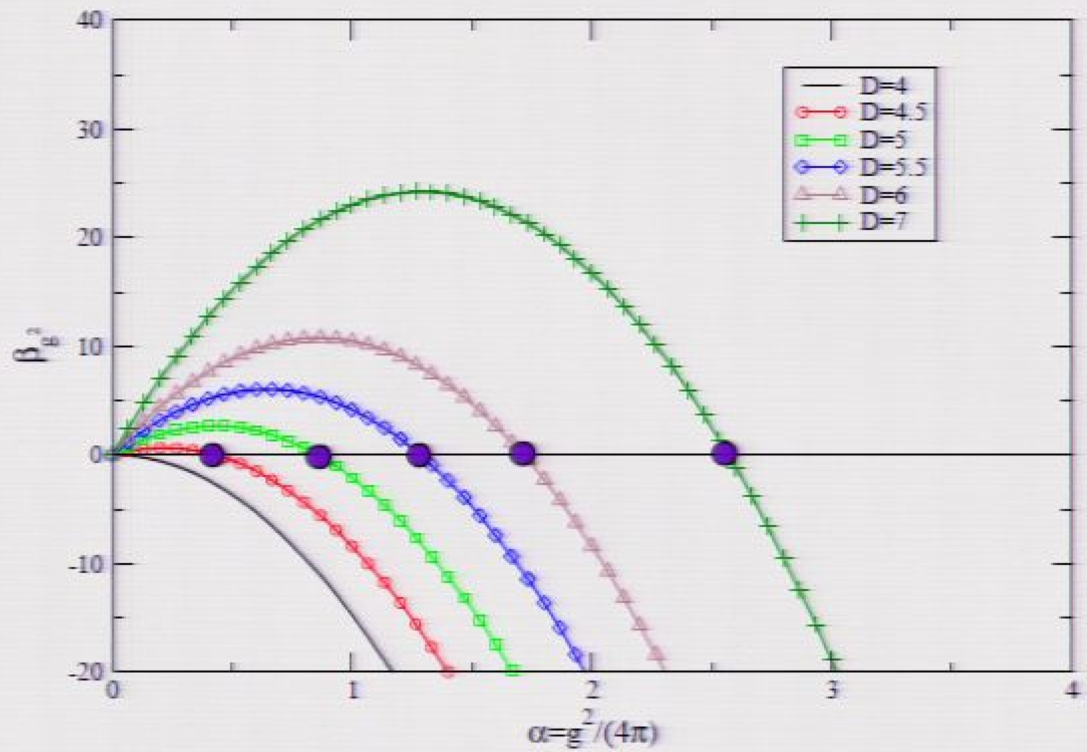
▷ UV fixed point:

$$g_*^2 = (24\pi^2 / 11N)\epsilon$$

for all ϵ . . . ?

Example: Extra-dimensional Yang-Mills theory

▷ naive ϵ expansion:



$\Longrightarrow$ nonperturbative problem for $\epsilon \gtrsim 1$

$\Longrightarrow$ mass-dependent RG scheme required for threshold behavior

(CAVE: mass gap)

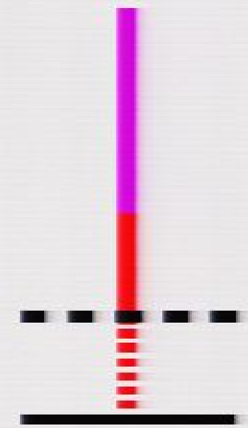
Example: Extra-dimensional Yang-Mills theory

▷ $D = 4$: Yang-Mills mass gap M

$\Longrightarrow$ threshold behavior for $k^2 \ll M$

$\Longrightarrow$ freeze-out of couplings in the IR

$\Longrightarrow$ IR fixed point expected



▷ combined evidence for threshold/decoupling behavior from

- FRG (background gauge)

  (REUTER,WETTERICH'97;GIES'02)

- DSE (Landau gauge)

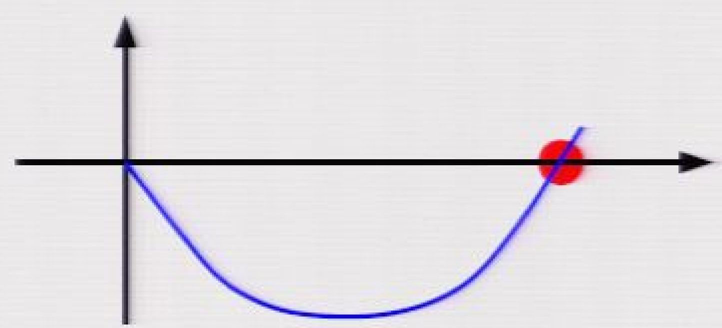
  (V.SMEKAL,HAUCK,ALKOFER'97,...)

- FRG (Landau gauge)

  (PAWLOWSKI ET AL.'05,...)

- lattice (Laudau gauge)

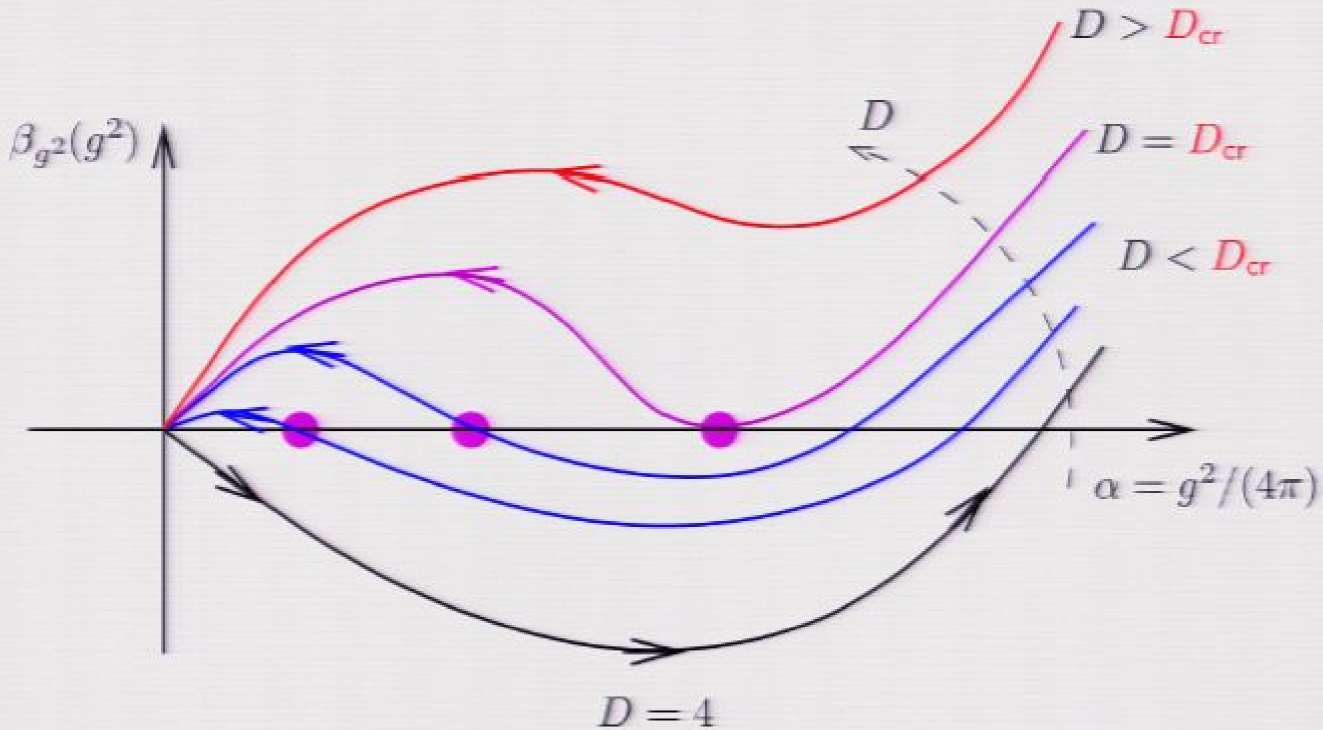
  (CUCCHIERI,MENDEZ'07,...)



Example: Extra-dimensional Yang-Mills theory

▷ conjecture from $D = 4$ IR behavior + D-analyticity: (GIES'03)

existence of D_{cr}



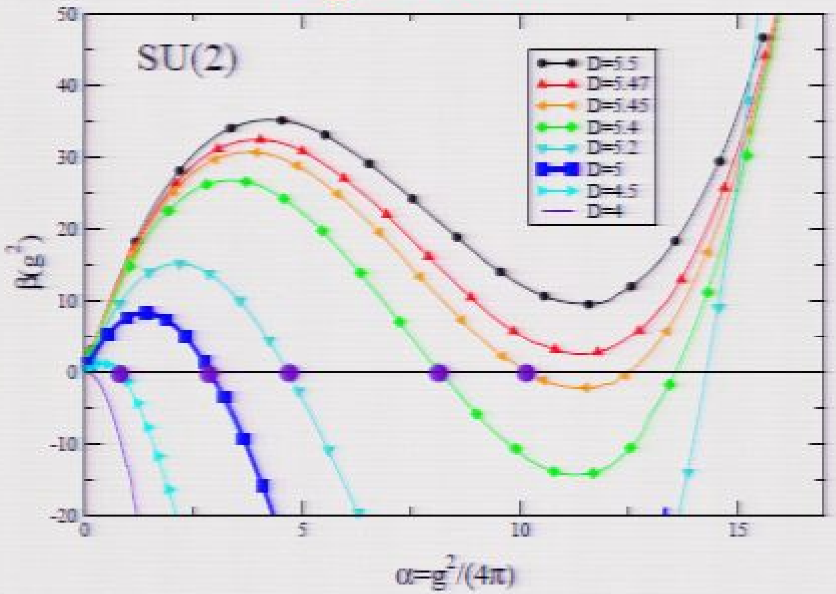
$\Longrightarrow$ no asymptotic safety for $D > D_{cr}$

Example: Extra-dimensional Yang-Mills theory

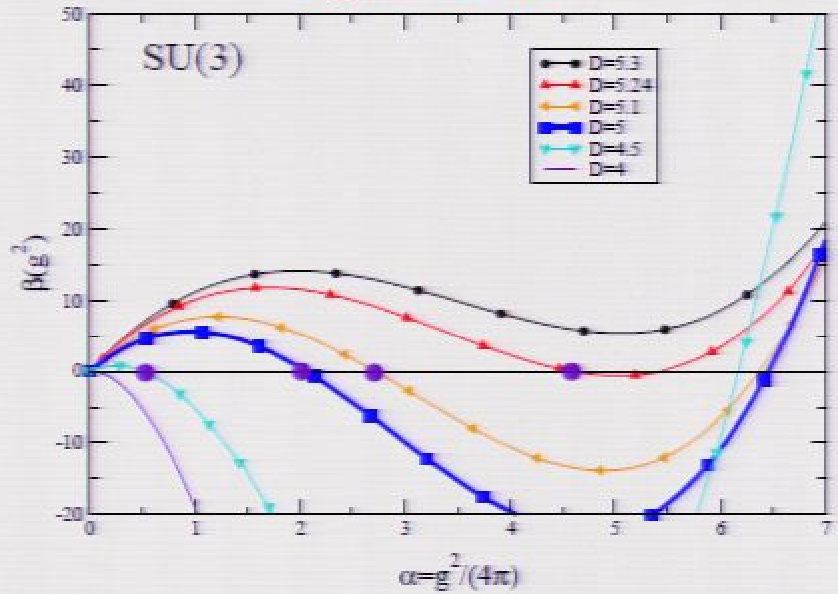
▷ functional RG calculation: (GIES'03)

$$\text{gluonic truncation:} \quad \Gamma_k = \int d^D x W_k(F^2), \quad F^2 = F^a_{\mu\nu} F^a_{\mu\nu}$$

$D_{cr} \simeq 5.46$



$D_{cr} \simeq 5.26$



▷ SU(5): $D_{cr} \simeq 5$

…no evidence for D_{cr} in gravity (FISCHER, LITIM'06)

Mechanisms of Asymptotic Safety II:
Conversion of Degrees of Freedom

Example: Fermionic Systems

▷ for instance, Nambu–Jona-Lasinio / Gross-Neveu in 3 dimensions:

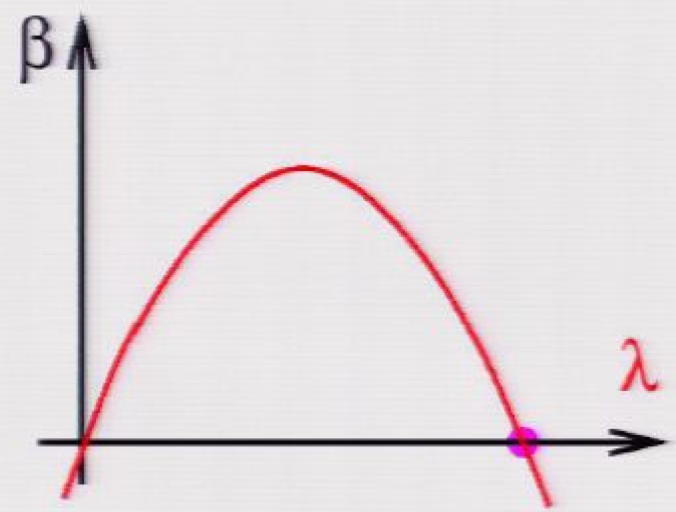
$$\Gamma_k = \int d^3x \bar{\psi} i\partial\!\!\!/ \psi + \frac{1}{2}\bar{\lambda}(\bar{\psi}\psi)^2 + \ldots \quad , \quad [\bar{\lambda}] = -1$$

▷ dim'less coupling $\lambda = k\,\bar{\lambda}$

$$\partial_t \lambda = \lambda - c\,\lambda^2$$

▷ UV fixed point $\lambda_* = 1/c$

▷ critical exponent $\Theta = 1$



→ "χSB"

⟹ asymptotically safe

(Gawedzki, Kupiainen'85; Rosenstein, Warr, Park'89; de Calan et al.'91)

Example: Fermionic Systems

▷ for instance, Nambu–Jona-Lasinio / Gross-Neveu in D dimensions:

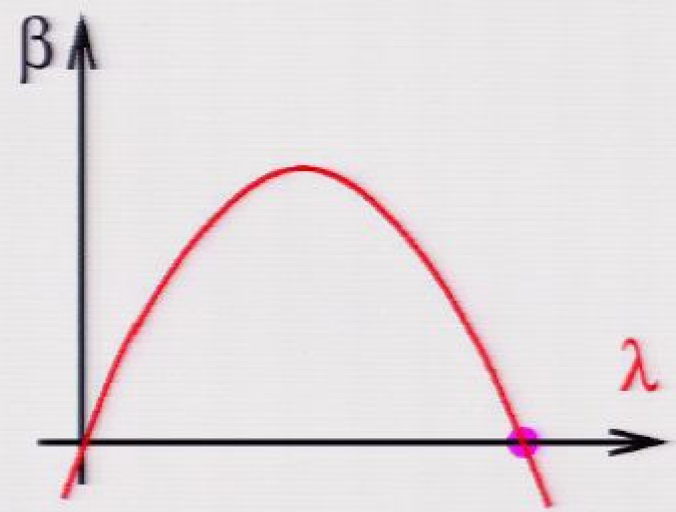
$$\Gamma_k = \int d^D x \bar{\psi} \mathrm{i}\partial\!\!\!/ \psi + \frac{1}{2}\bar{\lambda}(\bar{\psi}\psi)^2 + \dots \quad , \quad [\bar{\lambda}] = 2 - D$$

▷ dim'less coupling $\lambda = k^{D-2}\,\bar{\lambda}$

$$\partial_t \lambda = (D-2)\lambda - c\,\lambda^2$$

▷ UV fixed point $\lambda_* = (D-2)/c$

▷ critical exponent $\Theta = D - 2$



$\longrightarrow$ "χSB"

$\Longrightarrow$ asymptotically safe for all D?

Example: Fermionic Systems

▷ Towards the standard model ... ? (HG,JAECKEL,WETTERICH'04)

▷ $\mathrm{U}(1) \times \mathrm{SU}(N_c)$ gauge symmetry
+ chiral $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ flavor symmetry in $D = 4$

$$\Gamma_k = \int \bar{\psi}\,(\mathrm{i}Z_\psi \partial\!\!\!/ + Z_1 \bar{g} A\!\!\!/ + Z_1^B \bar{e} B\!\!\!/)\,\psi + \frac{Z_F}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{Z_B}{4} B^{\mu\nu} B_{\mu\nu}$$
$$+\frac{1}{2}\Big[\bar{\lambda}_-\,(\mathrm{V–A}) + \bar{\lambda}_+\,(\mathrm{V+A}) + \bar{\lambda}_\sigma\,(\mathrm{S–P}) + \bar{\lambda}_{\mathrm{VA}}\,[2(\mathrm{V–A})^{\mathrm{adj}} + (1/N_c)(\mathrm{V–A})]\Big]$$

▷ pointlike four-fermion interactions

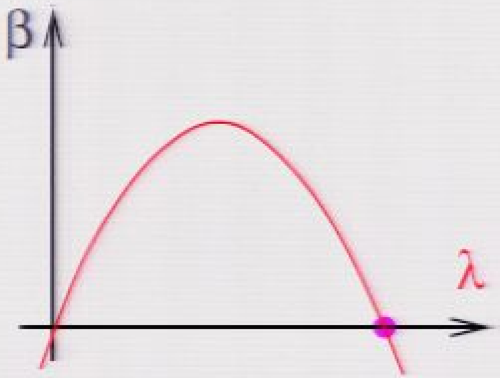
$$\begin{aligned}
(\mathrm{V–A}) &= (\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{V+A}) &= (\bar{\psi}\gamma_\mu\psi)^2 - (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \\
(\mathrm{S–P}) &= (\bar{\psi}^a\psi^b)^2 - (\bar{\psi}^a\gamma_5\psi^b)^2 \equiv (\bar{\psi}_i^a\psi_i^b)^2 - (\bar{\psi}_i^a\gamma_5\psi_i^b)^2 \\
(\mathrm{V–A})^{\mathrm{adj}} &= (\bar{\psi}\gamma_\mu T^z\psi)^2 + (\bar{\psi}\gamma_\mu\gamma_5 T^z\psi)^2
\end{aligned}$$

cf. top-quark condensation (MIRANSKI,TABANASHI,YAMAWAKI'89; BARDEEN,HILL,LINDNER'90)

Example: Fermionic Systems

▷ Fixed-Point Structure (e.g., for $e^2, g^2 \to 0$):

$$\partial_t \lambda_i = (d-2)\lambda_i + \lambda_k A_i^{kl} \lambda_l$$

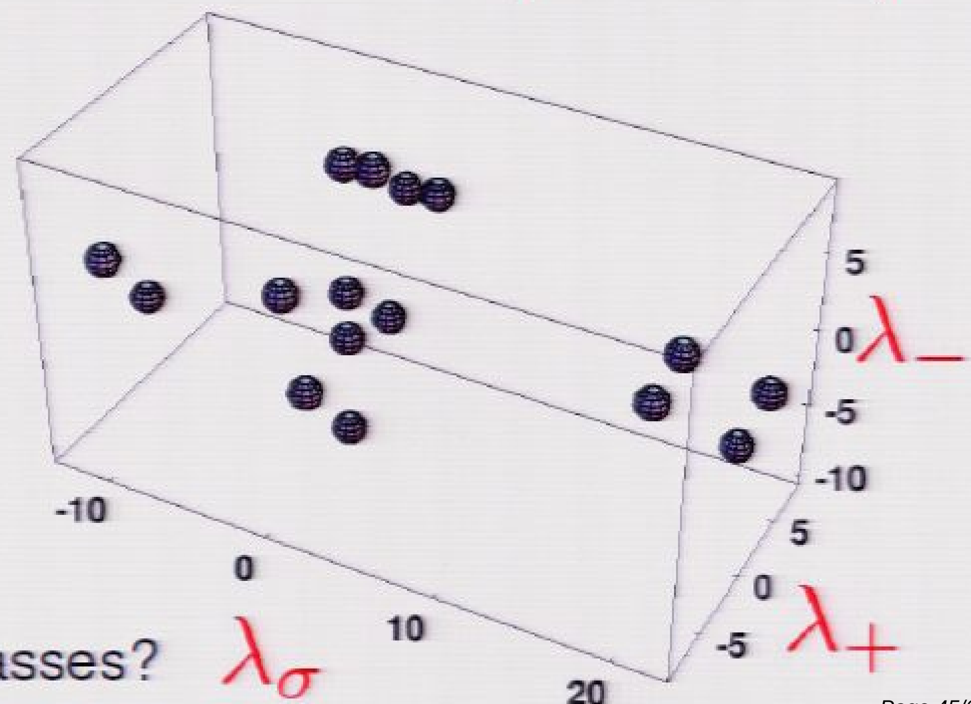


▷ 2 FPs per λ

$\Longrightarrow \quad 2^4 = 16$ FP

▷ in general: 2^n FP's

for $n = \#$ of λ's



$\Longrightarrow$ new SM UV universality classes?

Example: Fermionic Systems

BUT:

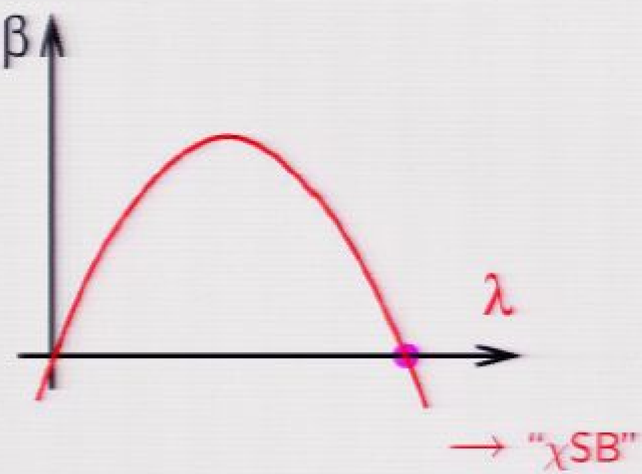
▷ conversion

$$\Gamma_{\text{int}} = \frac{1}{2}\bar{\lambda}\int d^D x(\bar{\psi}\psi)^2$$

Gaußian FP

RG irrelevant

$\Theta = 2 - D < 0$



Non-Gaußian FP

RG relevant

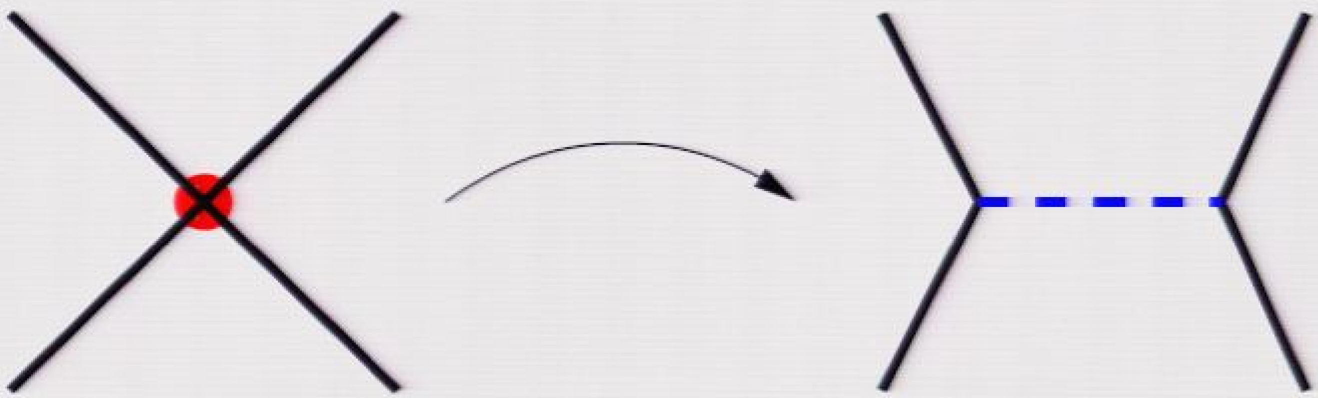
$\Theta = D - 2 > 0$

Example: Fermionic Systems

▷ Γ_{int} can acquire marginal "nonlocal" subcomponents at the NGFP:

$$\Gamma_{\text{int}} = \frac{1}{2}\bar{\lambda}\int d^D x (\bar{\psi}\psi)^2 \quad \rightarrow \quad \frac{1}{4}\int \frac{d^D p}{(2\pi)^D}(\bar{\psi}\psi)(-p)\frac{\bar{h}^2}{\bar{m}^2 + p^2}(\bar{\psi}\psi)(p)$$

▷ Hubbard-Stratonovich transformation: (STRATONOVICH'57, HUBBARD'59)



$$\frac{1}{2}(\bar{\psi}\psi)\bar{\lambda}(p)(\bar{\psi}\psi) \quad \rightarrow \quad \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + ih\bar{\psi}\psi\phi$$

Example: Fermionic Systems

▷ conversion of DoF: (ZINN-JUSTIN'91)

(HASENFRATZ, HASENFRATZ, JANSEN, KUTI, SHEN'91)

fermionic ψ^4 systems $\hat{=}$ Yukawa systems

$$\frac{1}{2}(\bar{\psi}\psi)\bar{\lambda}(p)(\bar{\psi}\psi) \quad \rightarrow \quad \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + ih\bar{\psi}\psi\phi$$

▷ D<4:

non-Gaußian FP	Gaußian Yukawa FP
pert. non-renormalizable	pert. super-renormalizable

▷ D=4:

non-Gaußian FP potentially destabilized by triviality of Yukawa & ϕ^4

e.g., for NJL, see (KIM, KOCIC, KOGUT'94)

for Z_2-Yukawa, see (GIES, SCHERER'09)

Mechanisms of Asymptotic Safety III:

Conformal Vacuum Expectation Values

Example: Yukawa Systems

▷ nonperturbative features of the functional RG

$$\partial_t \Gamma_k = \frac{1}{2} \,\mathrm{Tr}\, \partial_t R_k (\Gamma_k^{(2)} + R_k)^{-1}$$

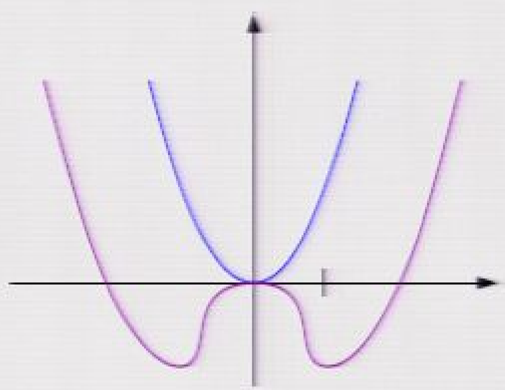
large coupling

$$g \gg 1$$

threshold behavior

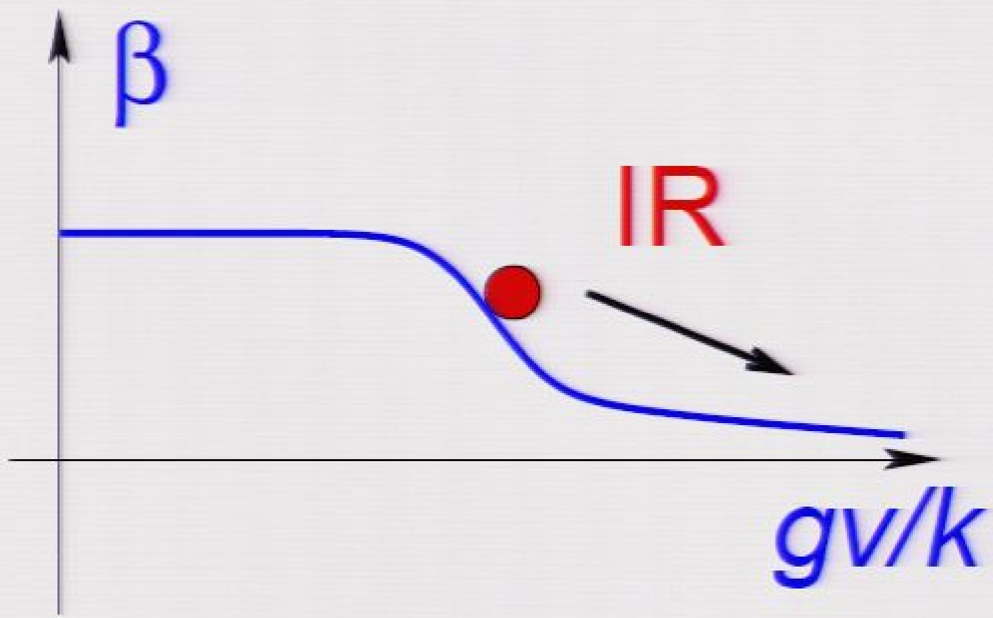
e.g.,

$$(\Gamma_k^{(2)} + R_k)^{-1} \to \frac{1}{p^2 + (gv)^2}$$



Example: Yukawa Systems

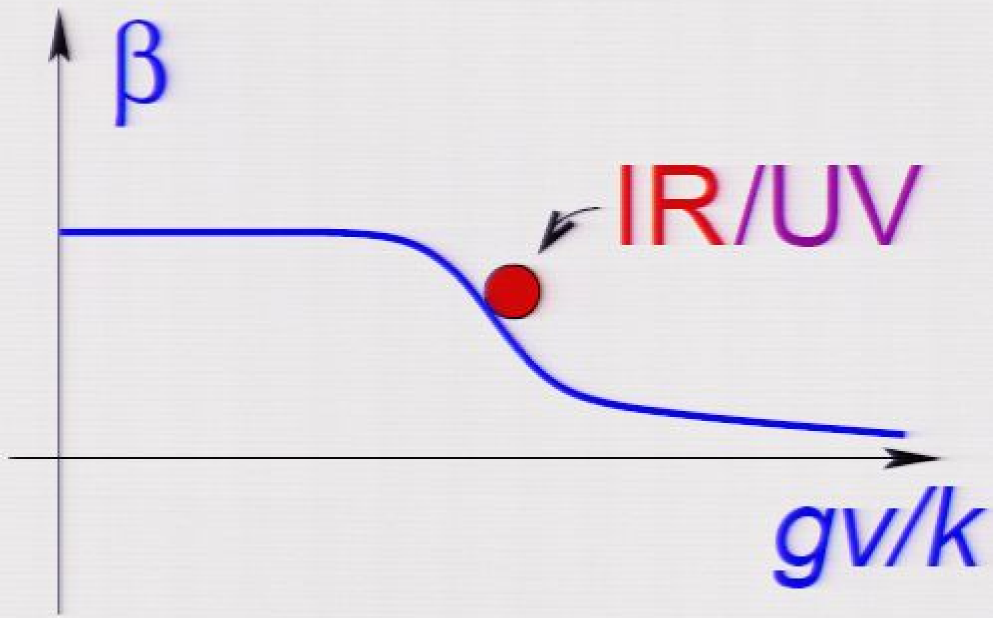
▷ standard threshold behavior



$\Longrightarrow$ decoupling of massive modes in the IR for $k \to 0$

Example: Yukawa Systems

▷ if $v \sim k$: conformal threshold behavior (GIES, SCHERER'09)

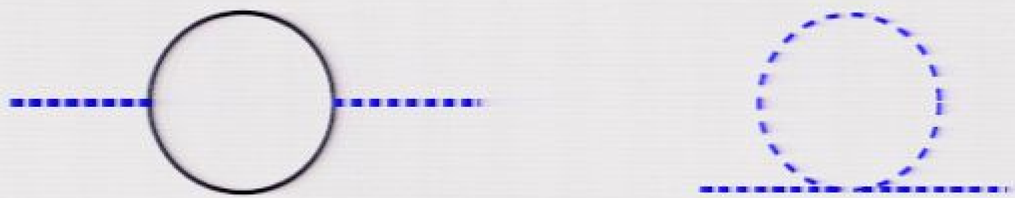


$\Longrightarrow$ can induce conformal FP behavior in both IR/UV

Example: Yukawa Systems

▷ running of VEV in Yukawa systems:

$$\partial_t \frac{v^2}{k^2} = -2\,\frac{v^2}{k^2} - \text{fermion fluctuations} + \text{boson fluctuations}$$

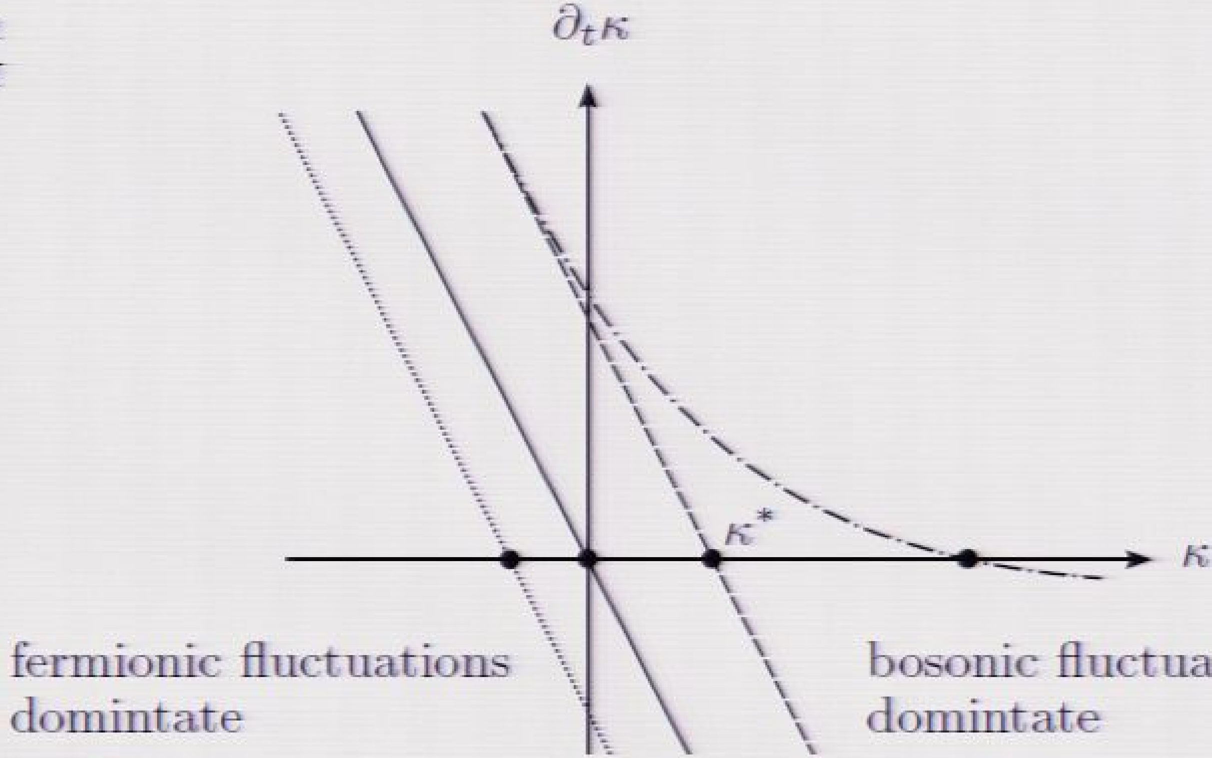


Example: Yukawa Systems

▷ running of VEV in Yukawa systems:

$$\partial_t \frac{v^2}{k^2} = -2\frac{v^2}{k^2} - \text{fermion fluctuations} + \text{boson fluctuations}$$

▷ $\kappa \sim \frac{v^2}{k^2}$



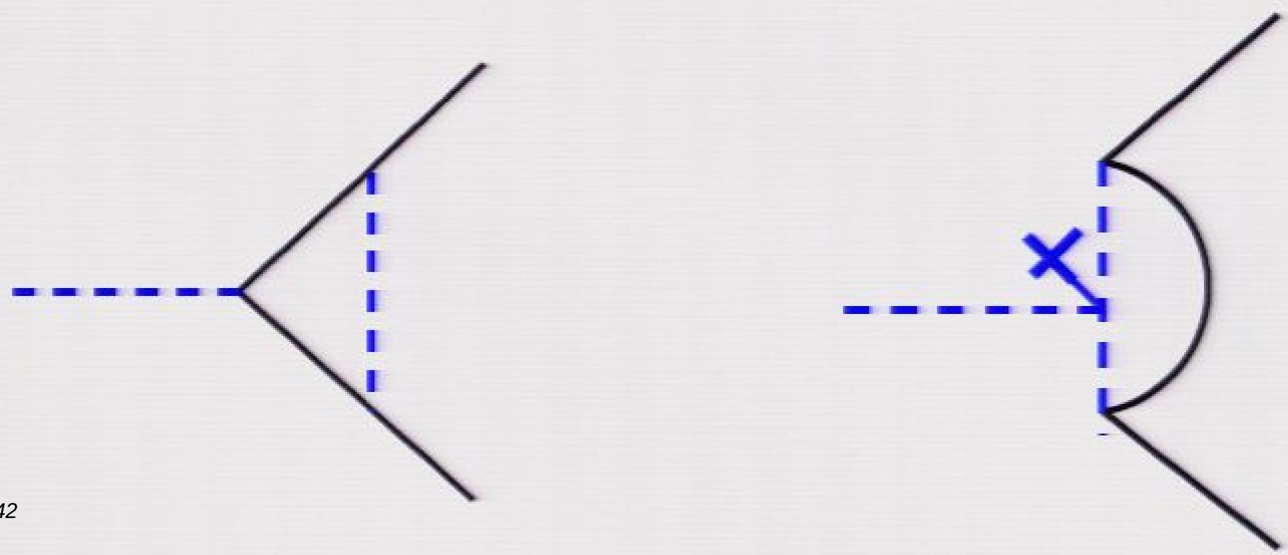
Example: Z_2 invariant Yukawa System

- ▷ Z_2 invariant Yukawa system at NLO derivative expansion

$$\Gamma_k = \int d^4x \left(\frac{Z_{\phi,k}}{2} (\partial_\mu \phi)^2 + U_k(\rho) + Z_{\psi,k} \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi + \mathrm{i} \bar{h}_k \phi \bar{\psi} \psi \right)$$

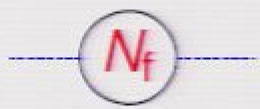
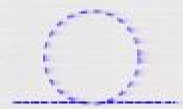
- ▷ no non-Gaußian FP in symmetric regime! (GIES,SCHERER'09)

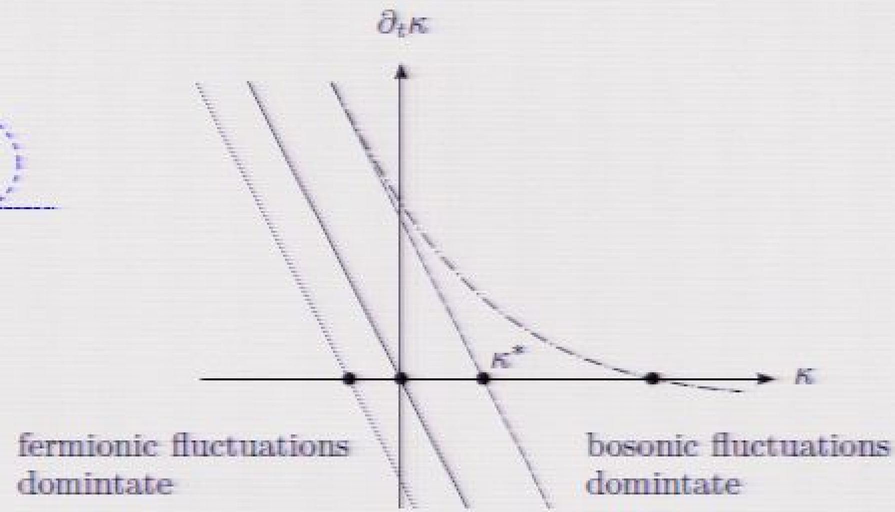
- ▷ new effective couplings in broken regime, e.g.,



Example: Z_2 invariant Yukawa System

▷ existence of NGFP depends on N_f:

$$\partial_t \frac{v^2}{k^2} = -2\,\frac{v^2}{k^2} +$$  $$+$$ 



▷ Conformal-VEV behavior and NGFP for: (GIES, SCHERER '09)

$$N_f < N_{f,\mathrm{cr}} \simeq 0.3$$

Example: Z_2 invariant Yukawa System

▷ "proof of principle": fixed-point properties for, e.g., $N_f = 1/10$:

NLO: $\kappa^* = 0.00163, \quad \lambda_2^* = 42.77, \quad h^{*2} = 191.22,$

$$\eta_\phi^* = 0.086, \quad \eta_\psi^* = 0.565 \qquad (\lesssim 1\,!)$$

▷ critical exponents:

$$\Theta_{1,2} = 1.619 \mp 0.280i, \quad \Theta_3 = -3.680$$

$\Longrightarrow$ 2 relevant directions

Example: Z_2 invariant Yukawa System

▷ cf. Yukawa system near Gaußian FP:

3 physical parameters : v, m_{top}, m_{Higgs}

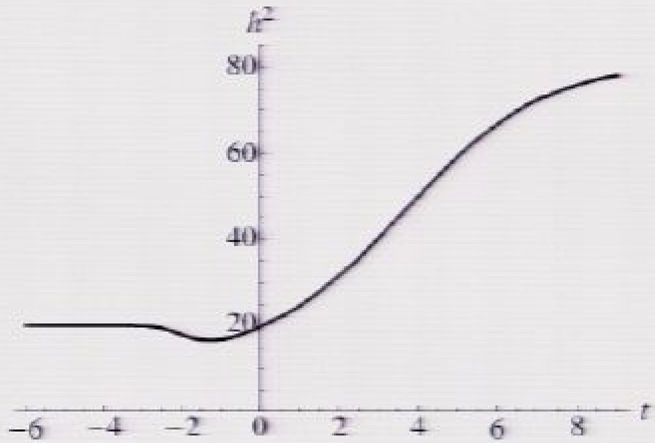
▷ Yukawa system near non-Gaußian FP:

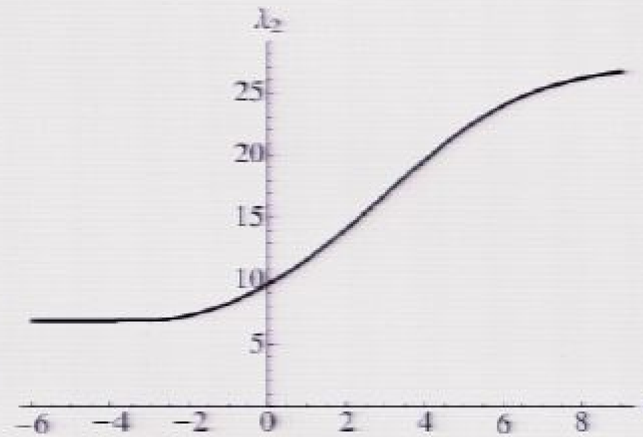
2 physical parameters : v, m_{top}

$\Longrightarrow$ m_{Higgs} is a prediction

Example: Z_2 invariant Yukawa System

▷ typical flow, $N_f = 1/10$, $m_{top} = 20v$:





▷ predicted Higgs mass:

$$m_{\text{Higgs}} = \sqrt{6.845}\, v \simeq 644\,\text{GeV}$$

▷ top quark mass is dynamically bounded from below

finite RG time for crossover

▷ hierarchy/fine-tuning problem slightly less severe:

$$\Theta_{\max} \simeq 1.619 < 2\,(= \Theta_{\text{SM}})$$

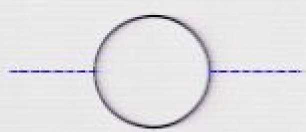
Example: chiral Yukawa System

▷ chiral $U(N_L)_L \otimes U(1)_R$ Yukawa system: (Gies, Rechenberger, Scherer'09)

$$\Gamma_k = \int d^d x \Big\{ i(Z_{L,k}\bar{\psi}_L^a \partial\!\!\!/ \psi_L^a + Z_{R,k}\bar{\psi}_R \partial\!\!\!/ \psi_R) + Z_{\phi,k}(\partial_\mu \phi^{a\dagger})(\partial^\mu \phi^a) + U_k(\phi^{a\dagger}\phi^a) + \bar{h}_k \bar{\psi}_R \phi^{a\dagger} \psi_L^a - \bar{h}_k \bar{\psi}_L^a \phi^a \psi_R \Big\}.$$

▷ N_L scaling:

$$\partial_t \frac{v^2}{k^2} = -2\,\frac{v^2}{k^2} + $$



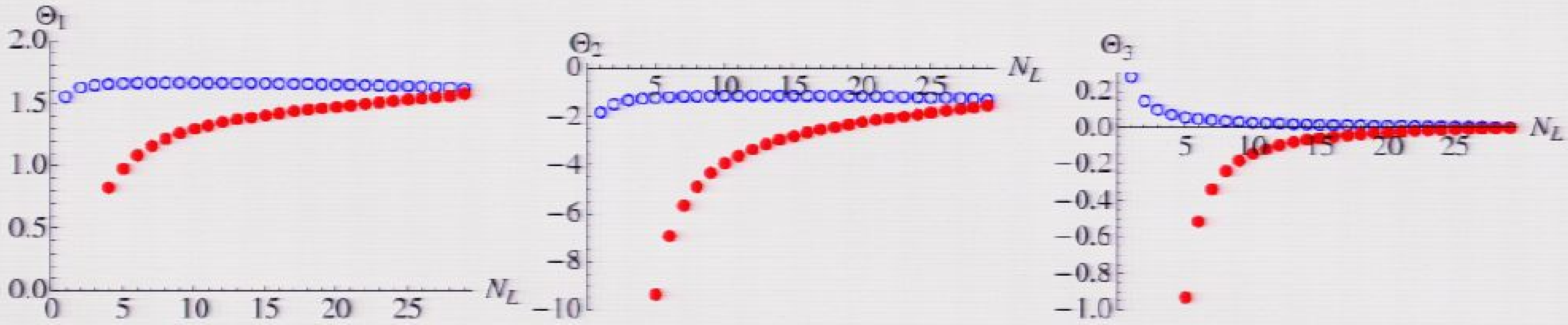
$+$

▷ LO derivative expansion: conformal-VEV & NGFP for

$$1 \leq N_L \leq 57$$

Example: chiral Yukawa System

▷ fixed-point properties at LO: (GIES, RECHENBERGER, SCHERER'09)



$\Longrightarrow$ only 1 relevant direction $\hat{=}$ 1 physical parameter

▷ predictions of top and Higgs mass for $N_L = 10$:

$$m_{top} = 5.78v \simeq 1422\,\text{GeV}, \quad m_{Higgs} = 0.97v \simeq 239\,\text{GeV}$$

▷ BUT: FP destabilized by massless Goldstone & fermion DoF at NLO

→ different in the full standard model

Conclusion

- several mechanisms of asymptotic safety available
  - Dimensional Balancing
  - Conversion of Degrees of Freedom
  - Conformal Vacuum Expectation Values

- asymptotic safety has to potential to solve
  - triviality problems
  - hierarchy/fine-tuning problems
  - issue of abundant parameters

all within QFT

- provides a new/enlarged view on the question why

$$D_{\mathrm{RG,cr}} = 4 = D$$

Example: Fermionic Systems

▷ for instance, Nambu–Jona-Lasinio / Gross-Neveu in 3 dimensions:

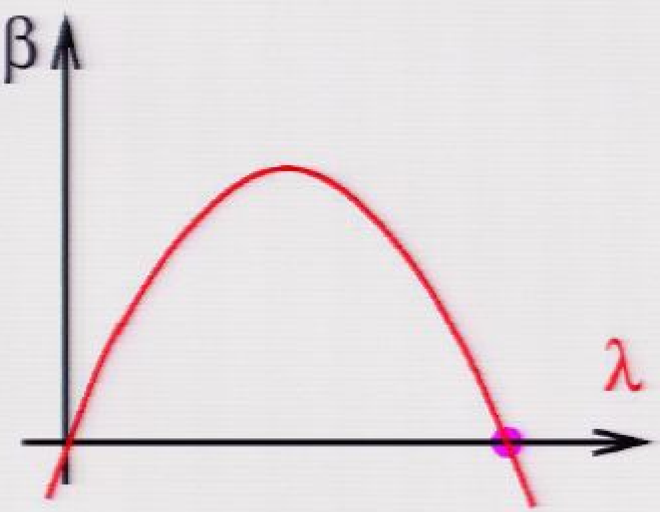
$$\Gamma_k = \int d^3x \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2} \bar{\lambda} (\bar{\psi}\psi)^2 + \dots \quad , \quad [\bar{\lambda}] = -1$$

▷ dim'less coupling $\lambda = k\,\bar{\lambda}$

$$\partial_t \lambda = \lambda - c\,\lambda^2$$

▷ UV fixed point $\lambda_* = 1/c$

▷ critical exponent $\Theta = 1$



$\longrightarrow$ "χSB"

$\Longrightarrow$ asymptotically safe

(Gawedzki, Kupiainen'85; Rosenstein, Warr, Park'89; de Calan et al.'91)

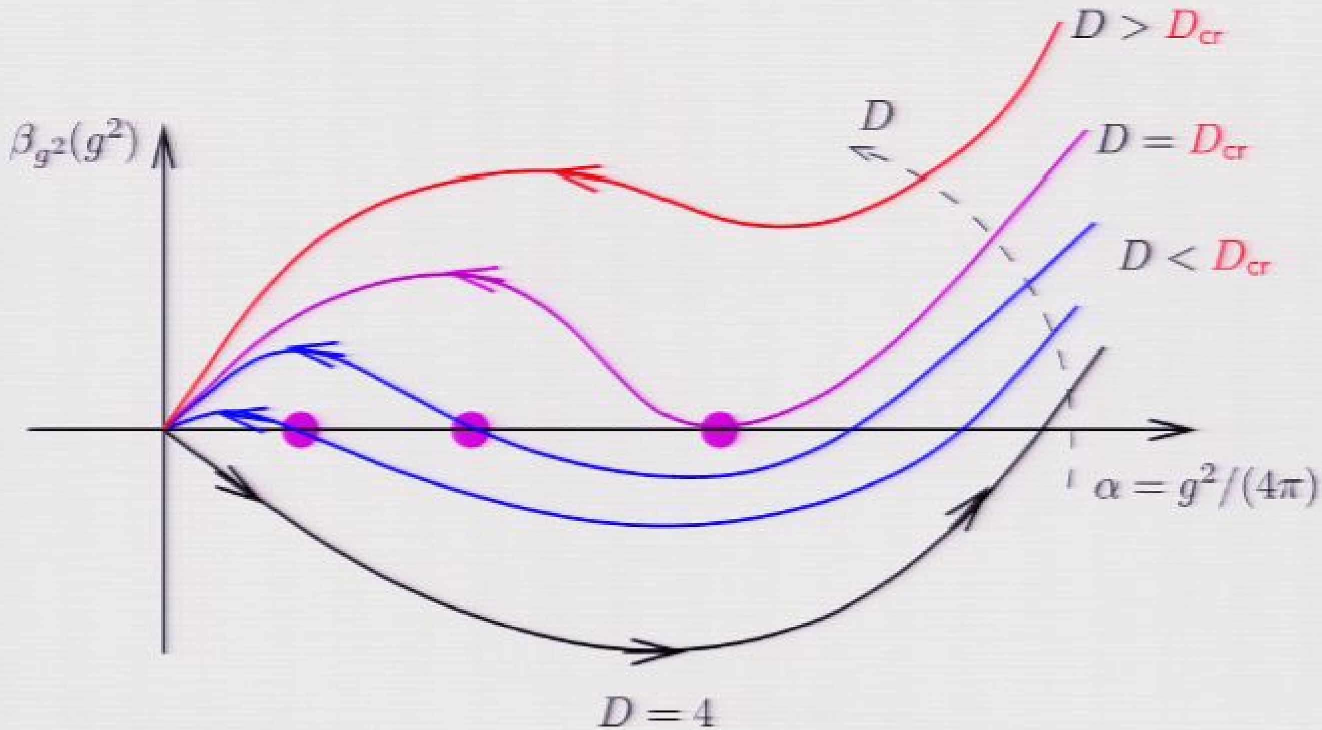
Mechanisms of Asymptotic Safety II:

Conversion of Degrees of Freedom

Example: Extra-dimensional Yang-Mills theory

▷ conjecture from $D = 4$ IR behavior + D-analyticity: (GIES'03)

existence of $D_{\rm cr}$



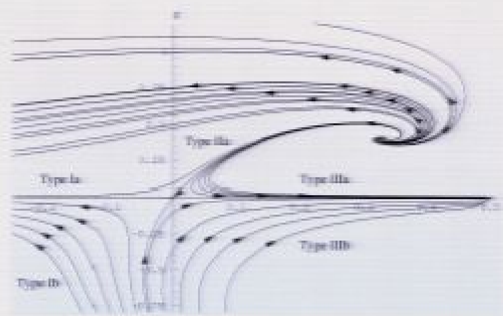
$\Longrightarrow$ no asymptotic safety for $D > D_{\rm cr}$

Example: Quantum Einstein Gravity

(Reuter, Saueressig'01)

▷ larger "theory space":

$\vdots$			
R^8	$\cdots$		
R^7	$\cdots$		
R^6	$\cdots$		
R^5	$\cdots$		
R^4	$\cdots$		
R^3	$C_{\mu\nu}{}^{\rho\sigma}C_{\rho\sigma}{}^{\kappa\lambda}C_{\kappa\lambda}{}^{\mu\nu}$	$R\Box R$	+ 7 more
R^2	$C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$	$R_{\mu\nu}R^{\mu\nu}$	
R			
$\mathbb{1}$			



critical exponents:

(e.g. f(R) truncation)

$$\mathrm{Re}\,\Theta_{1,2} \simeq 2.4$$

$$\Theta_3 \simeq 1.5$$

$$\Theta_{>3} \lesssim -4$$

▷ many tests of robustness

- larger truncations
- regulator/gauge dependencies, etc.

(Codello, Percacci, Rahmede'07'08; Machado, Saueressig'07)

Hierarchy problem $\Lambda_{UV} \ggg \Lambda_{EW}$

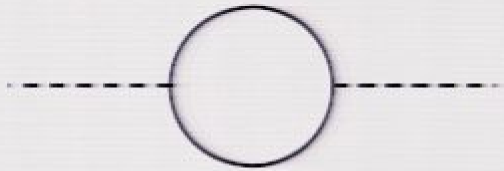
▷ renormalization of the scalar mass (e.g., $\Lambda_{UV} = 10^{16}$GeV)

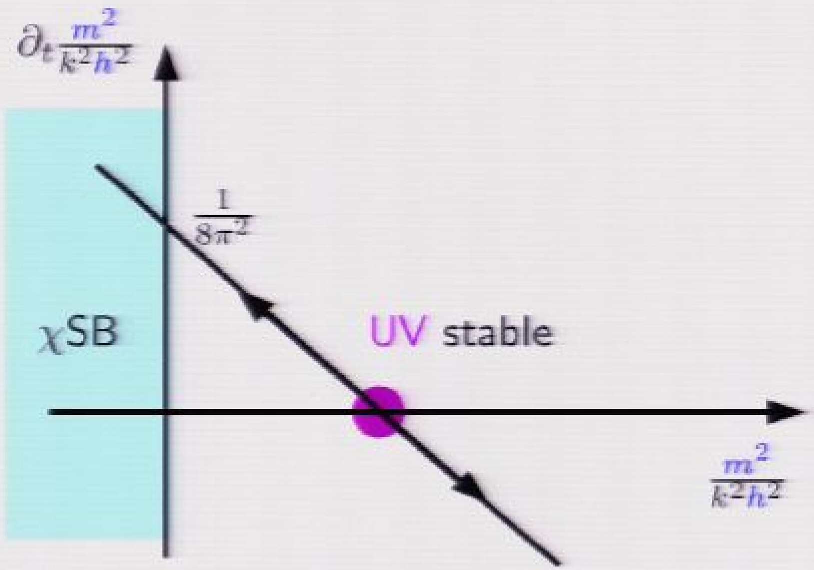
$$\underbrace{m_R^2}_{\sim 10^4 \mathrm{GeV}^2} \sim \underbrace{m_\Lambda{}^2}_{\sim 10^{32}\left(1+\ldots 10^{-28}\right)\mathrm{GeV}^2} - \underbrace{\delta m^2}_{\sim 10^{32}\mathrm{GeV}^2}$$

▷ RG viewpoint ($\partial_t = k\frac{d}{dk}$)

e.g., Yukawa theory:

$$\partial_t \frac{m^2}{k^2 h^2} = -2\,\frac{m^2}{k^2 h^2} + \frac{1}{8\pi^2}$$

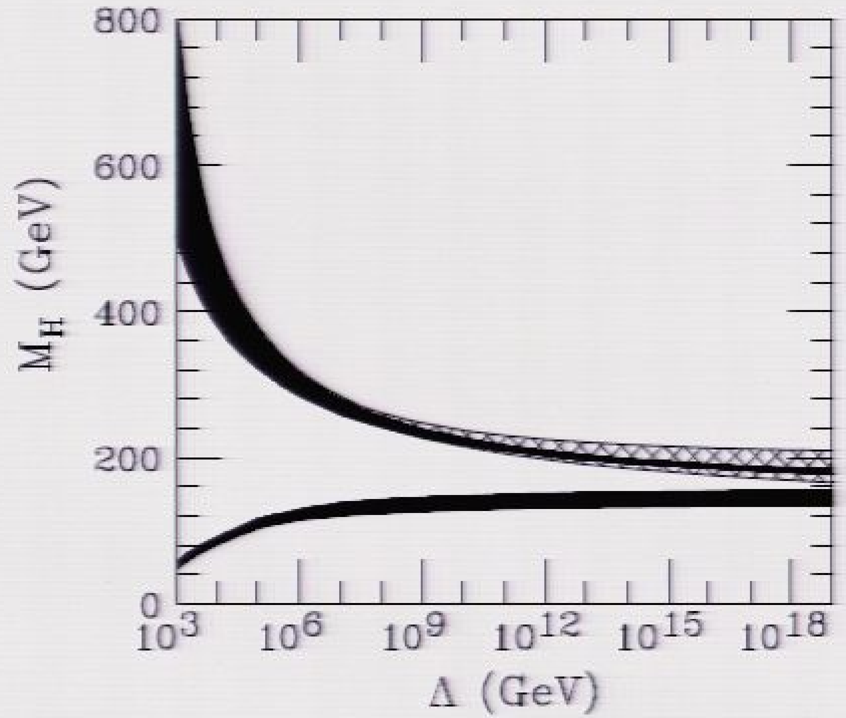




Triviality problem

$\Longrightarrow$ ...scale of maximal UV extension

▷ triviality of the scalar Higgs sector:



(Hambye, Riesselmann'97)

$\Longrightarrow$ SM Higgs mass bounds from Landau pole position

Triviality problem

$\Longrightarrow$ …scale of maximal UV extension

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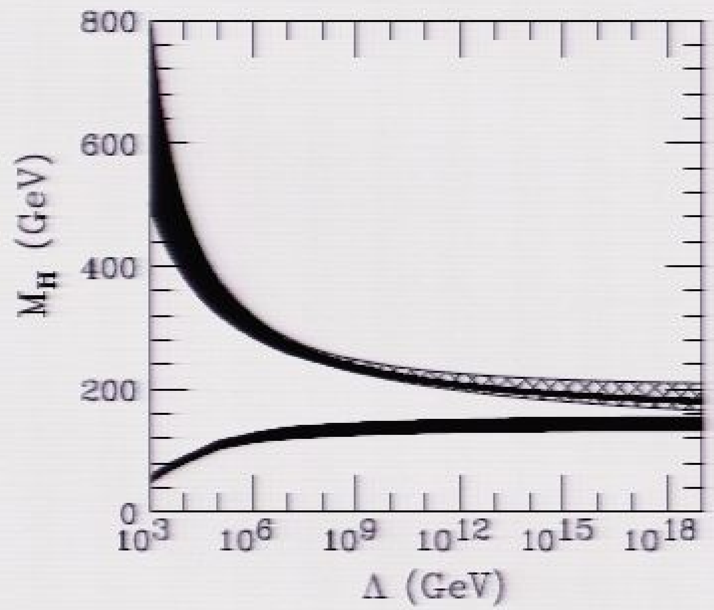

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