Title: Gravitational average action and asymptotic safety: past and future

Date: Nov 05, 2009 11:00 AM

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Abstract:

Asymptotic Safety

and the

Gravitational Average Action

— Past and Future —

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A crucial feature of this approach
which we got for free in the past
and have to pay for in the future :

Background independence

The requirement of "Background Independence"

- As in classical GR no special metric ("vacuum") should play a distinguished rôle.
- Quantization should not be based on any "a priori background" like $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$, etc.
- Would like to <u>explain</u>, rather than <u>postulate</u> structures of / on spacetime; for instance,
  $g_{\mu\nu}(x) = \langle \Psi | \hat{\gamma}_{\mu\nu}(x) | \Psi \rangle$

How to comply with this requirement?

Option 1: (⟶ LQG, Regge, CDT)

Construct operators $\hat{\gamma}_{\mu\nu}$, states $|\Psi\rangle$, ... <u>without</u> reference to <u>any</u> classical backgrd. spacetime.

Start, for instance, from state with $\langle \Psi_0 | \hat{\gamma}_{\mu\nu} | \Psi_0 \rangle = 0$

try to find $|\Psi_{s.c.}\rangle$ such that $\langle \Psi_{s.c.} | \hat{\gamma}_{\mu\nu} | \Psi_{s.c.} \rangle$ is smooth and non-degenerate.

Option 2: The "bi-metric" approach (EAA)

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Option 2: The "bi-metric" approach (EAA)

The bi-metric approach (I)

Employ arbitrarily chosen background $\bar{g}_{\mu\nu}(x)$ at intermediate steps of the quantization; verify at the end that all physical results are independent of $\bar{g}_{\mu\nu}(x)$.

background / quantum field split:

$$\boxed{\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}}$$

$\gamma_{\mu\nu}$, $h_{\mu\nu}$: integration variable (operator)

$h_{\mu\nu}$: non-linear fluctuation,
quantized on classical spacetime
with metric $\bar{g}_{\mu\nu}$

$$\langle h_{\mu\nu} \rangle \equiv \bar{h}_{\mu\nu}, \qquad g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + \bar{h}_{\mu\nu}[\bar{g}]$$

self-consistent background: $\bar{h}_{\mu\nu}[\bar{g}^{\text{self con.}}] = 0$

The metric $\bar{g}_{\mu\nu}$ is never fixed concretely:

Quantization of $h_{\mu\nu}$ <u>on all possible backgr.s at a time</u> !

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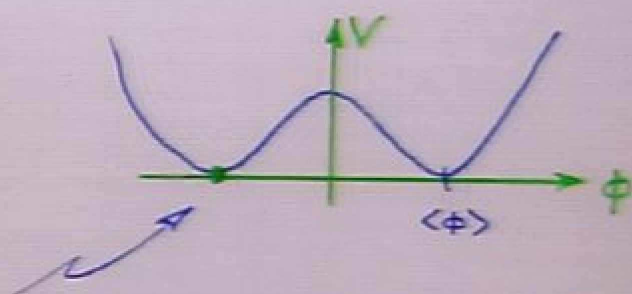
Options 1 and 2 have complementary advantages and disadvantages.

A loose analogy:

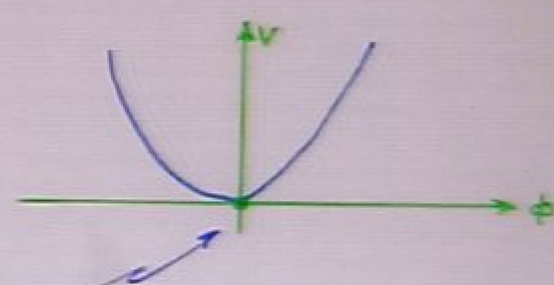
$\langle \gamma_{\mu\nu} \rangle \;\hat{=}\;$ condensate $\langle \phi \rangle$ of scalar theory

- **with SSB:**
  ($\to$ opt. 2)



$\hat{=}$ metric condensate $\langle \gamma_{\mu\nu} \rangle \neq 0$,
phase with broken diffeomorphism invariance

- **no SSB:**
  ($\to$ opt. 1)



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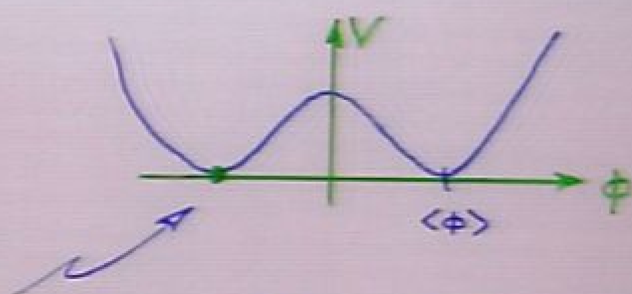

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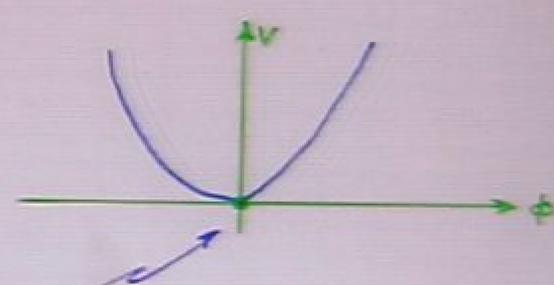

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The Wilsonian Renormalization Group

Severe conceptual problems:

- What is the meaning of "coarse graining" when the metric is dynamical?
- "Coarse" / "fine" with respect to what?
- Which metric should determine the proper size of a "block" of "spins"?
- Standard cutoff operators in FRGEs need metric to be defined.

Option 1:

Scale must come from within the theory: clock / rod - subsystem, etc.

Option 2:

(unspecified) backgrd metric $\bar{g}_{\mu\nu}$ available;

use associated cov. Laplacian $\bar{D}^2 \equiv \bar{g}^{\mu\nu} \bar{D}_\mu \bar{D}_\nu$

to define coarse graining:

$$-\bar{D}^2 \varphi_\lambda = \lambda \varphi_\lambda \quad , \quad h(x) = \sum\!\!\!\!\!\!\int_{\substack{\lambda \\ \lambda < \lambda_{max}}} c_\lambda \, \varphi_\lambda(x)$$

Smallest length scale resolved: $\ell = \ell_\lambda \, [\bar{g}]$

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The bi-metric approach (II)

Formal covariant quantization:

$$\int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]} \longrightarrow$$

$$\int \mathcal{D}h_{\mu\nu}\, \mathcal{D}\bar{C}_\mu\, \mathcal{D}C^\mu\, \exp\left\{-S[\bar{g}+h] - S_{gf}[h;\bar{g}] - S_{gh}\right\}$$

(Ordinary) EA generates 1PI n-pt. fcts.

$\langle h_{\mu\nu}(x_1)\, h_{\rho\sigma}(x_2) \cdots \rangle_{\bar{g}}$ via

$$\langle h\, h \cdots \rangle_{\bar{g}} = \left(\frac{\delta}{\delta \bar{h}}\right)^n \Gamma[\bar{h};\bar{g}]\Big|_{\bar{h}=0}$$

Notation / Interpretation:

$$\Gamma[g,\bar{g}] = \Gamma[\bar{h};\bar{g}]\Big|_{\bar{h}=g-\bar{g}}$$

two complete metrics

"matter field" $h_{\mu\nu}$ is propagating on classical spacetime with non-dynamical metric $\bar{g}_{\mu\nu}$

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Choosing $S_{gf}[h;\bar{g}]$ appropriately ("background gauge"),
Γ can be made a diffeomorphism invariant functional
of its arguments:

$$\Gamma[g_{\mu\nu}+\delta_{\text{diff}}\, g_{\mu\nu},\ \bar{g}_{\mu\nu}+\delta_{\text{diff}}\,\bar{g}_{\mu\nu}] = \Gamma[g_{\mu\nu},\bar{g}_{\mu\nu}]$$

Derivative expansion:

$$\Gamma[g,\bar{g}] = \int \sqrt{g} + \sqrt{\bar{g}} + \sqrt{g}R + \sqrt{\bar{g}}\,\bar{R}$$
$$+ \sqrt{g}\,R_{\mu\nu}\bar{R}^{\mu\nu} + \sqrt{\bar{g}}\,\bar{g}^{\mu\nu}R_{\mu\nu} + \cdots$$

Note: $\Gamma[g,\bar{g}=\eta=\text{fixed}]$ is not strongly constrained!

The fctl. with the metrics put equal:

$$\bar{\Gamma}[\bar{g}] := \Gamma[g=\bar{g},\bar{g}] \equiv \Gamma[\bar{h}=0;\bar{g}]$$

is diff.-inv. and, via $\left(\frac{\delta}{\delta\bar{g}}\right)^n \bar{\Gamma}[\bar{g}]$, generates

the same on-shell S-matrix elements as

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Q: What is special about the spin-2 "matter" field $h_{\mu\nu}$ (as compared to tensor mesons, say)?

A: $\bar{g} + \bar{h}_{\mu\nu}$ is the fully quantum corrected metric expectation value $g_{\mu\nu}$ to which (according to the effective eqs. of motion) all matter couples.

Q: How is this reflected by Γ ?

A: At the classical level, $S[\gamma] = S[\bar{g}_{\mu\nu} + h_{\mu\nu}]$ is invariant under the split symmetry

$$\delta \bar{g}_{\mu\nu} = \varepsilon_{\mu\nu} \ , \quad \delta h_{\mu\nu} = - \varepsilon_{\mu\nu}$$

At the level of Γ this leads to the split Ward identities, symbolically:

$$\boxed{\Gamma[\bar{h}_{\mu\nu} - \varepsilon_{\mu\nu} ; \bar{g}_{\mu\nu} + \varepsilon_{\mu\nu}] = \Gamma[\bar{h}_{\mu\nu} ; \bar{g}]}$$

+ corrections due to the non-invariance of the gauge fixing term
(do not affect observable quantities!)

Note: "Appropriate" fctls. $S_{gf}[h;\bar{g}]$, say $\int (\mathcal{F}_\mu(\bar{g}) h)^2$, depend on h and $\bar{g}$ separately, not only via $\bar{g} + h$.

The IR Cutoff

Suppress modes with (covariant momentum)$^2 < k^2$:

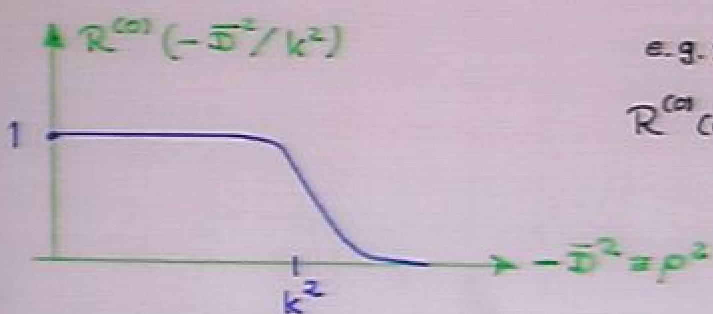
$$\int \mathcal{D}h_{\mu\nu}\,\mathcal{D}\bar{C}_\mu\,\mathcal{D}C^\mu \; e^{-S[\bar{g}+h]-S_{gf}-S_{gh}} \cdot e^{-\Delta_k S}$$

$$\Delta_k S = \tfrac{1}{2}\varkappa^2 \int d^dx \sqrt{\bar{g}}\; h_{\mu\nu}\, \mathcal{R}_k^{\mathrm{grav}}[\bar{g}]^{\mu\nu\rho\sigma}\, h_{\rho\sigma}$$
$$+\sqrt{2}\int d^dx \sqrt{\bar{g}}\; \bar{C}_\mu\, \mathcal{R}_k^{\mathrm{gh}}[\bar{g}]\, C^\mu$$

$$\boxed{\mathcal{R}_k[\bar{g}] = \mathcal{Z}_k\, k^2\, \underbrace{R^{(0)}\left(-\frac{\bar{D}^2}{k^2}\right)}_{\text{"shape function"}}}$$

ghosts : $\mathcal{Z}_k = Z_k^{\mathrm{ghost}}$

graviton : $\mathcal{Z}_k^{\mu\nu\rho\sigma} = Z_k^{\mathrm{grav}} \{ \bar{g}^{\mu\rho}\bar{g}^{\nu\sigma} + \cdots \}$



e.g.:

$$R^{(0)}(u) = \frac{u}{e^u - 1}$$

Low momentum modes ($p^2 \ll k^2$) suppressed by "mass" $\sim k^2$

high momentum modes ($p^2 \gg k^2$) integrated out

- couple $h_{\mu\nu}$, $\bar{C}_\mu$, C^μ to sources
- Legendre - transform the resulting $W_k[\text{sources}]$
- subtract $\Delta_k S[\bar{h}_{\mu\nu}, \bar{\xi}_\mu, \xi^\mu]$

$\Rightarrow$ **Effective Average Action**

$$\boxed{\Gamma_k[\bar{h}, \xi, \bar{\xi}; \bar{g}] = \Gamma_k[\overset{\bar{g}+\bar{h}}{g}, \bar{g}, \xi, \bar{\xi}]}$$

- generalization of standard eff. action $\Gamma_{k=0}$
- diffeomorphism inv. under transf. of <u>all</u> arguments
- satisfies FRGE:

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k[\bar{g}]\right)^{-1} k\partial_k \mathcal{R}_k[\bar{g}]\right]$$

$\Gamma_k^{(2)}$: Hessian w.r.t. $\bar{h}, \xi, \bar{\xi}$ at fixed $\bar{g}$

- Can set $\bar{h} = 0$, or $\bar{g} = g$, only <u>after</u> RG evolution !
- RG flow is necessarily on complicated theory space :

$$\left\{ \Gamma[g, \bar{g}, \xi, \bar{\xi}] \right\}$$

"extra" $\bar{g}_{\mu\nu}$ - dependence
(not combined with $\bar{h}_{\mu\nu}$!)

- $\Delta_k S \sim$

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- $\Delta_k S \sim \int h \, \mathcal{R}_k[\bar{g}] \, h$ additional source of "extra" $\bar{g}$ -dep.

The significance of the

Background Field Technique:

1st stage:

For a certain type of gauge fixing conditions, the eff. action Γ can be made a gauge invariant fctl. of its arguments: $S_{gf}[h;\bar{g}]$ invariant under simultaneous coord. transf. of $h_{\mu\nu}$ and $\bar{g}_{\mu\sigma}$.

DeWitt, 't Hooft, Abbott, ...

2nd stage:

Kadanoff–Wilson coarse graining can be performed in a gauge covariant way:
(effective) average action $\Gamma_k[A_\mu, \bar{A}_\mu]$.

M. R., C. Wetterich 1993

3rd stage:

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M. R., 1996

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What has been done:

Up to now, most effective average action (EAA)-based investigations of gravitational RG flows and the Asymptotic Safety conjecture focused on solving

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within (simple .... increasingly general) truncations of theory space, of the type

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What has been found:

According to all truncations considered, QEG is asymptotically safe, i.e. the RG flow of the effective average action has a non-trivial fixed point which seems suitable for taking the infinite cutoff limit there.

The RG flow of the EAA seems to realize the scenario proposed by S. Weinberg in 1979.

M.R.
Dou, Percacci
Souma
Lauscher, MR
MR, Saueressig
Percacci, Perini
Bonanno, MR
Codello, Percacci, Rahmede
Litim, Fischer
Machado, Saueressig

Benedetti, Machado, Saueressig
MR, Weyer
Manrique, MR
Eichhorn, Gies, Scherer
Zanusso, Zambelli, Vacca, Percacci
Daum, MR
Daum, Harst, MR
Niedermaier
Narain, Percacci
Koslowski

The Einstein-Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Gamma_k = -\frac{1}{16\pi G_k}\int d^dx \sqrt{g}\,\{R - 2\Lambda_k\}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant G_k , dimensionless: $g(k) = k^{d-2} G_k$

cosmological constant Λ_k , dimensionless: $\lambda(k) = \Lambda_k / k^2$

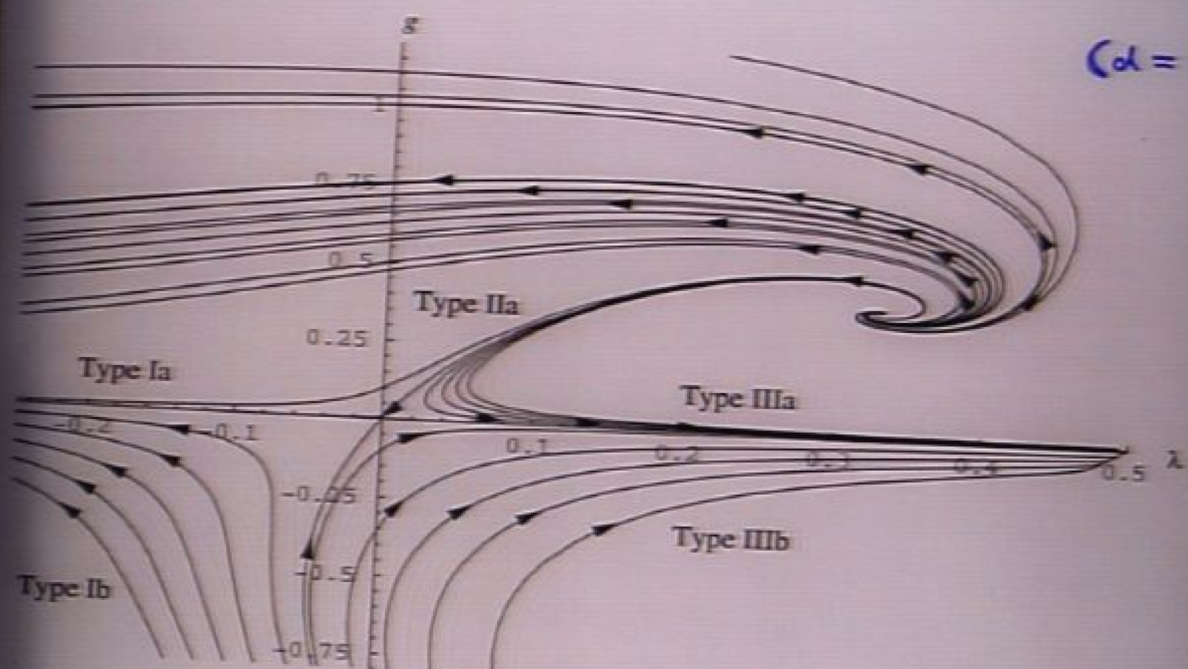
insert ansatz into flow equation, expand

$$\mathrm{Tr}[\cdots] = (\cdots)\int\sqrt{g} + (\cdots)\int\sqrt{g}R + \cdots$$

$$\rightsquigarrow \quad \boxed{\begin{aligned} k\partial_k g(k) &= \beta_g(g,\lambda) \\ k\partial_k \lambda(k) &= \beta_\lambda(g,\lambda) \end{aligned}}$$

RG - Flow in the Einstein - Hilbert Truncati

($d = 4$)



M.R., F. Saueressig, hep-th/0110054

Truncations:

Single metric $(\hat{\Gamma}=0)$ vs. bi-metric $(\hat{\Gamma}\neq 0)$

Ghost action kept classical:

$$\Gamma_k[g,\bar{g},\xi,\bar{\xi}] = \bar{\Gamma}_k[g] + \hat{\Gamma}_k[g,\bar{g}] + S_{gf}[g,\bar{g}] + S_{gh}[g,\bar{g},\xi,\bar{\xi}]$$

with

$$\bar{\Gamma}_k[g] \equiv \Gamma_k[g,g,0,0]$$

$$\hat{\Gamma}_k[g,g] = 0 \qquad (S_{gf}[g,g]=0)$$

Flow equation on truncated theory space:

$$k\frac{\partial}{\partial k}\left(\bar{\Gamma}_k[g] + \hat{\Gamma}_k[g,\bar{g}]\right) = \mathcal{J}[g,\bar{g}] \equiv$$

$$\equiv \frac{1}{2}\mathrm{Tr}\left[\left(\bar{\Gamma}^{(2)}[g] + \hat{\Gamma}^{(2)}[g,\bar{g}] + S_{gf}^{(2)}[\bar{g}] + \mathcal{R}_k[\bar{g}]\right)^{-1} \cdot k\frac{\partial}{\partial k}\mathcal{R}_k[\bar{g}]\right]$$

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$$\left.\left. + \mathcal{R}_k[\bar{g}]\right)^{-1} \cdot k\frac{\partial}{\partial k}\mathcal{R}_k[\bar{g}]\right]$$

+ ghost contribution

Coupled system of bi-metric truncations:

$$k\partial_k \bar{\Gamma}_k[g] = \mathcal{J}[g, g]$$

$$k\partial_k \hat{\Gamma}_k[g,\bar{g}] = \mathcal{J}[g,\bar{g}] - \mathcal{J}[g,g]$$

Single metric truncations:

Set $\hat{\Gamma}_k \equiv 0$, discard 2^{nd} flow equation,

assuming small $\mathcal{J}[g,\bar{g}] - \mathcal{J}[g,g] \propto$ "extra" $\bar{g}$-dep.,

retain:

$$k\partial_k \bar{\Gamma}_k[g] = \frac{1}{2}\mathrm{Tr}\left[\left(\bar{\Gamma}_k^{(2)}[g] + S_{gf}^{(2)}[g] + \right.\right.$$

$$\left.\left. + \mathcal{R}_k[g]\right)^{-1} \cdot k\partial_k \mathcal{R}_k[g]\right] + \cdots$$

Coarse graining operator becomes source of g-dependence on equal footing with $\Gamma^{(2)}$!

$\bar{g}$-contamination grows as $\mathcal{R}_k \sim k^2 \longrightarrow \infty$

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$$k\partial_k \bar{\Gamma}_k[g] = \frac{1}{2}\mathrm{Tr}\Big[\Big(\bar{\Gamma}_k^{(2)}[g] + S_{gf}^{(2)}[g] + $$

$$+ \mathcal{R}_k[g]\Big)^{-1} \cdot k\partial_k \mathcal{R}_k[g]\Big] + \cdots$$

Coarse graining operator becomes source of $\bar{g}$-dependence on equal footing with $\Gamma^{(2)}$!

$\bar{g}$-contamination grows as $\mathcal{R}_k \sim k^2 \longrightarrow \infty$

$$k\partial_k \hat{\Gamma}_k[g,\bar{g}] = \mathcal{J}[g,\bar{g}] - \mathcal{J}[g,g]$$

Single metric truncations:

Set $\hat{\Gamma}_k \equiv 0$, discard 2^{nd} flow equation, assuming small $\mathcal{J}[g,\bar{g}] - \mathcal{J}[g,g] \propto$ "extra" $\bar{g}$-dep., retain:

$$k\partial_k \bar{\Gamma}_k[g] = \frac{1}{2}\mathrm{Tr}\Big[\Big(\bar{\Gamma}_k^{(2)}[g] + S_{gf}^{(2)}[g] + \mathcal{R}_k[g]\Big)^{-1} \cdot k\partial_k \mathcal{R}_k[g]\Big] + \cdots$$

Coarse graining operator becomes source of g-dependence on equal footing with $\Gamma^{(2)}$!

$\bar{g}$-contamination grows as $\mathcal{R}_k \sim k^2 \longrightarrow \infty$.

Q: Is the field dependence of $\mathcal{R}_k[\bullet]$ essential for the structure of the RG flow ?

A: Yes !

Demonstration by a toy model with $\widehat{\Gamma} = 0$:

Conformally Reduced Einstein-Hilbert trunc.

M.R., H. Weyer, 2008

Q: What happens in the toy model when we include $\widehat{\Gamma} \neq 0$ - terms so that we can distinguish $g_{\mu\nu}$ from $\bar{g}_{\mu\nu}$?

A: Flow has intriguing new features ...

E. Manrique, M.R., 2009

Conformally Reduced QEG

- quantize only the conformal factor:

$$\underbrace{\gamma_{\mu\nu}}_{\text{integration variable}} = \chi^2 \underbrace{\hat{g}_{\mu\nu}}_{\substack{\text{class reference metric,} \\ \neq \text{ backgrd. metric !}}}$$

- treat scalar-like theory $\int \mathcal{D}\chi\, e^{-S[\chi]}$ in the same way as full QEG:
  effective average action ⊕ backgrd. field method

- introduce background conf. factor:

$$\bar{g}_{\mu\nu} = \chi_B^2 \, \hat{g}_{\mu\nu}$$

- decompose quantum field:

$$\chi = \chi_B + \underbrace{f}_{\text{"fluctuation"}}$$

- expectation values:

$$\phi \equiv \langle \chi \rangle = \chi_B + \bar{f} \,, \quad \bar{f} \equiv \langle f \rangle$$

$$g_{\mu\nu} = \langle \gamma_{\mu\nu} \rangle = \langle \chi^2 \rangle \, \hat{g}_{\mu\nu} = \langle (\chi_B + f)^2 \rangle \, \hat{g}_{\mu\nu}$$

• Conformally Reduced Einstein - Hilbert truncation:

$$\Gamma_k[\bar{f};\chi_B] \equiv \Gamma_k[\phi,\chi_B] =$$

$\phi = \chi_B + \bar{f}$

$$= -\frac{1}{16\pi G_k}\int d^4x\sqrt{g}\,\big(R(g) - 2\Lambda_k\big)\Big|_{g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}}$$

$$= -\frac{3}{4\pi G_k}\int d^4x\sqrt{\hat{g}}\,\Big\{-\tfrac{1}{2}\phi\hat{\Box}\phi + \tfrac{1}{12}\hat{R}\phi^2 - \frac{\Lambda_k}{6}\phi^4\Big\}$$

• How to define $\mathcal{R}_k$?

(i) k is cutoff in the spectrum of $\hat{\Box}$

IR modes have inv. propagator $\sim -\hat{\Box} + k^2$.

$$\mathcal{R}_k \sim k^2 R^{(0)}\Big(\frac{-\hat{\Box}}{k^2}\Big)$$ is field independent

$\Rightarrow$ familiar RG flow of Symanzik's $(-\phi^4)$ matter theory

(ii) k is cutoff in the spectrum of $\bar{\Box} = \hat{\Box}\chi_B^{-2} + \cdots$,

as required by "background independence".

IR modes have inv. propagator $\sim -\bar{\Box} + k^2$

$$\mathcal{R}_k[\bar{g} = \chi_B^2\hat{g}] \sim \chi_B^2\, k^2 R^{(0)}\Big(\frac{-\hat{\Box}}{\chi_B^2 k^2}\Big) \quad ; \quad \chi_B \to \phi$$

$\Rightarrow$ RG flow similar to full QEG !!!

$$\Big|_{g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}}$$

$$= -\frac{3}{4\pi G_k} \int d^4x \sqrt{\hat{g}} \left\{ -\tfrac{1}{2}\phi \hat{\Box} \phi + \tfrac{1}{12} \hat{R} \phi^2 - \frac{\Lambda_k}{6} \phi^4 \right\}$$

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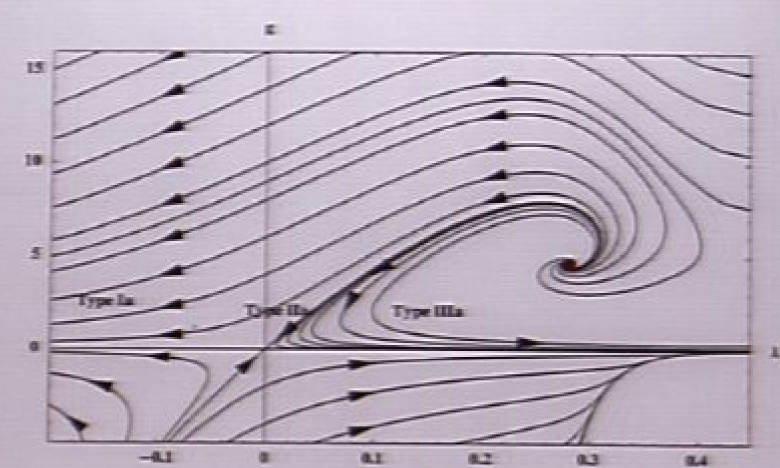
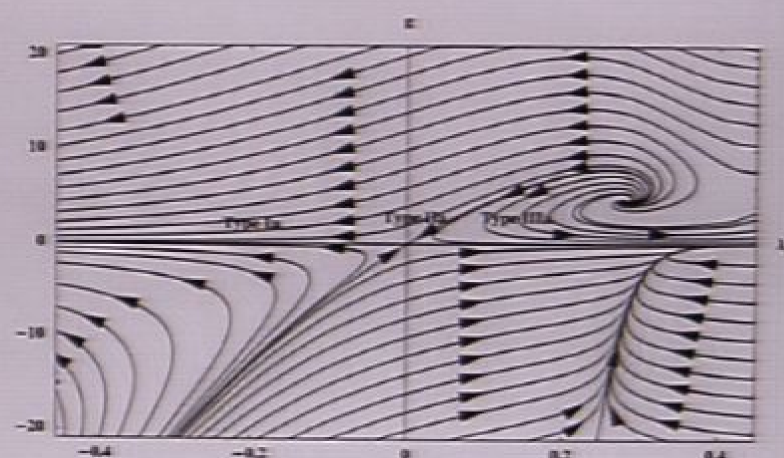
$\Rightarrow$ RG flow similar to full QEG !!!

The CREH flow: $Y_k(\varphi) = -\frac{1}{6}\lambda_k \varphi^4$



(a)



(b)

Figure 1: The figures show the RG flow on the (g, λ)-plane which is obtained from the CREH truncation with $\eta_N^{(\text{kin})}$. The arrows point in the direction of decreasing k.

M. R., H. Weyer, arXiv: 0801.3287

A bi-metric truncation

E. Manrique, M.R., 2009

$$\Gamma_k[g,\bar{g}] \equiv \Gamma_k[\phi, \chi_B] =$$

$$= -\frac{1}{16\pi G_k}\int d^4x \sqrt{g}\,\big(R(g) - 2\Lambda_k\big)$$

$$-\frac{1}{16\pi G_k^B}\int d^4x \sqrt{\bar{g}}\,\big(R(\bar{g}) - 2\Lambda_k^B\big)$$

$$-\frac{3}{8\pi}\frac{M_k}{G_k}\int d^4x \left(\sqrt{g}\sqrt{\bar{g}}\right)^{1/2} \Bigg|\; \begin{array}{l} g_{\mu\nu} = \phi^2 \delta_{\mu\nu} \\ \bar{g}_{\mu\nu} = \chi_B^2 \delta_{\mu\nu} \end{array}$$

$$= \frac{3}{8\pi}\int d^4x \Big\{ \frac{1}{G_k}\phi \hat{\Box}\phi + \frac{1}{3}\frac{\Lambda_k}{G_k}\phi^4$$

$$+ \frac{1}{G_k^B}\chi_B \hat{\Box}\chi_B + \frac{1}{3}\frac{\Lambda_k^B}{G_k^B}\chi_B^4$$

$$- \frac{1}{G_k} M_k \chi_B^2 \phi^2 \Big\}$$

A bi-metric truncation

E. Manrique, M.R., 2009

$$\Gamma_k[g,\bar{g}] \equiv \Gamma_k[\phi, \chi_B] =$$

$$= -\frac{1}{16\pi G_k}\int d^4x \sqrt{g}\left(R(g) - 2\Lambda_k\right)$$

$$-\frac{1}{16\pi G_k^B}\int d^4x \sqrt{\bar{g}}\left(R(\bar{g}) - 2\Lambda_k^B\right)$$

$$-\frac{3}{8\pi}\frac{M_k}{G_k}\int d^4x \left(\sqrt{g}\sqrt{\bar{g}}\right)^{1/2}$$

$$g_{\mu\nu} = \phi^2 \delta_{\mu\nu}$$

$$\bar{g}_{\mu\nu} = \chi_B^2 \delta_{\mu\nu}$$

$$= \frac{3}{8\pi}\int d^4x \left\{ \frac{1}{G_k}\phi\hat{\Box}\phi + \frac{1}{3}\frac{\Lambda_k}{G_k}\phi^4 + \frac{1}{G_k^B}\chi_B\hat{\Box}\chi_B + \frac{1}{3}\frac{\Lambda_k^B}{G_k^B}\chi_B^4 - \frac{1}{G_k}M_k\chi_B^2\phi^2 \right\}$$

$\mathcal{R}_k[\chi_B]$: Employ "background independent" choice, as in (ii)

$\Rightarrow$ flow on 5-dimensional theory space:

$$\{ (g, \lambda, m, g^B, \lambda^B) \}$$

$$\hat{=} \quad (G_k, \Lambda_k, \underbrace{M_k, G_k^B, \Lambda_k^B}_{})$$

Vanish at $k=0$:
no "extra" χ_B-dependence,
no violation of split sym.

Λ_k: The genuine cosmological constant!
$\Lambda_{k=0}$ appears in the effective field equation (Einstein eq.) $\delta\Gamma / \delta g_{\mu\nu} = 0$; determines curvature of space time.

Λ_k^B: Background analogue of the cos. const.;
vanishes at $k=0$;
does not enter the effective Einstein eq.:

$$\frac{\delta}{\delta g_{\mu\nu}} \int \sqrt{g}\, \Lambda_k^B \equiv 0$$

Solution of the flow eqs. near the GFP

$(\eta_N, \eta_N^B, m \ll 1)$

$$\boxed{\begin{array}{ll} \Lambda_k \sim \frac{1}{\log(k)} \quad ; & \Lambda_k^B = \frac{\bar{G}^B}{16\pi} k^4 \\ M_k = -\frac{1}{12\pi} \bar{G}\lambda_* k^2 \quad ; & G_k, G_k^B \approx \text{const} \end{array}}$$

Familiar from the scalar perspective (ϕ^4-cplg. $\sim \log(k)$, $(\text{mass})^2 \sim k^2$)

but surprising from the gravity point of view:

The genuine cosmological constant is almost scale independent, while the expected k^4-dependence (sum over zero-point energies, "$\sum \frac{1}{2}\hbar\omega$") occurs in the running of the background cosmological constant Λ_k^B !

$\Longrightarrow$ No k^4-contribution to $\rho_{vac} = \Lambda / 8\pi G$ entering Einstein's eq., in this truncation: "$\sum \frac{1}{2}\hbar\omega$ does not gravitate".

In the corresponding single-metric truncation the two cosmological constants are not disentangled:

$$\Lambda_k \sim k^4 \quad ; \quad G_k \approx \text{const}$$

- $M_k \varkappa_8^2 \phi^2 \sim k^2$ does enter the effective field eq. and the eq. for self-consistent backgrounds.
- In more general bi-metric truncations a k^4-dependence, too, can occur in the running of dynamical (ϕ-) terms.
- The cosmological constant(s) problem should be reconsidered, nevertheless.

Fixed Points

The full 5-dimensional RG flow has a NGFP suitable for the Asymptotic Safety construction.
In comparison to the $\hat{\Gamma} = 0$ - truncation the numerical changes are not small, though.

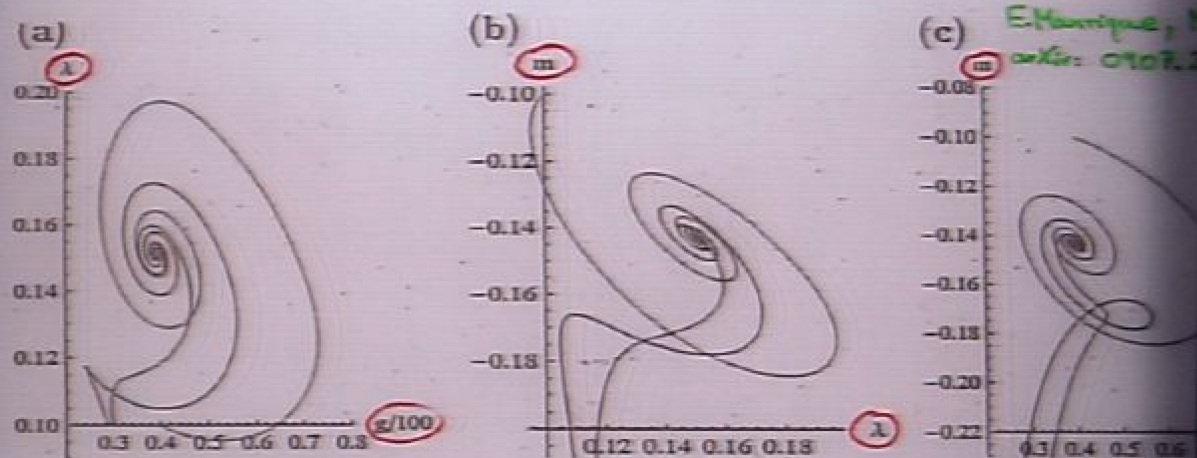


Figure 2: The plots show three different projections of the same RG trajectories onto the g-λ–plane (a), the λ-m–plane (b), and g-m–plane (c), respectively.

Fixed Point	FP in the Subsystem	g_*	λ_*	m_*	g_*^B	λ_*^B
G-G-FP	GFP	0	0	0	0	0
G-NG-FP	GFP	0	0	0	-24π	$-\frac{3}{2}$
NG-G-FP	NGFP	39.3	0.151	$-\frac{1}{7}$	0	0
NG-NG-FP	NGFP	39.3	0.151	$-\frac{1}{7}$	-25.2	-0.779

with single metric truncati

Table 1: The table displays the coordinates of the fixed points in the 5-dimen flow. The related fixed point in the subsystem, and the $g\lambda$ products are also gi

Conclusion

- It <u>is</u> important to disentangle the $g_{\mu\nu}$- and $\bar{g}_{\mu\nu}$-contributions to Γ_k !
- $\hat{\Gamma}_k$ is probably more important than a further refinement of $\bar{\Gamma}_k$.
- The next generation of "precision truncations" should be of the bi-metric type in order to avoid the $\bar{g}_{\mu\nu}$-contamination.

- <u>The new picture:</u>

  $h_{\mu\nu}$ is a "matter field" on a freely variable classical spacetime, related to $\bar{g}_{\mu\nu}$ only very indirectly since the split symmetry is broken by the coarse graining.

- A bi-metric truncation in full quantum gravity:

  $\longrightarrow$ Frank Saueressig's talk

... but there is much more to be done :

- The Reconstruction Problem:

  (re-) construct regularized path integral from a given trajectory $\Gamma_k \rightsquigarrow \Gamma_*$

  $\longrightarrow$ bare action S_Λ

  $\longrightarrow$ Hamiltonian description, ...

  $\longrightarrow$ Elisa Manrique's talk

- Different field variables, e.g. $(e_\mu^{\ a}, \omega_\mu^{\ ab})$

  Scale dependence of the Holst action, running Immirzi parameter ?

  with J.-E. Daum

  ⋮

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