Title: Prospects for Asymptotic Safety

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Abstract:

GRAVITATION AS AN EFFECTIVE FIELD THEORY

With an ultraviolet cut-off Λ,

$$I_\Lambda[g] = -\int d^4x\,\sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu} + \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots\Big]$$

[E.g., $g_1 \equiv 1/16\pi G\Lambda^2$]

Here Λ can be either

- A sharp UV cutoff (I_Λ includes loops with virtual momenta $> \Lambda$), or
- The momentum parameter in a regulator term (k), or
- A sliding renormalization scale(μ).

The Λ-dependence of the dimensionless couplings $g_n(\Lambda)$ is such that physics is independent of Λ.

$$\Lambda \frac{d}{d\Lambda} g_n(\Lambda) = \beta_n\Big(g(\Lambda)\Big)$$

In flat space theories there are essential couplings g_n and inessential couplings Z_a

$$\Lambda \frac{d}{d\Lambda} Z_a(\Lambda) = \beta_a\Big(Z(\Lambda), g(\Lambda)\Big)$$

$$\Lambda \frac{d}{d\Lambda} g_n(\Lambda) = \beta_n\Big(g(\Lambda)\Big)$$

In generally covariant theories, cut-off depends on metric, so all gravitational couplings are essential.

ASYMPTOTIC SAFETY

The theory is safe from couplings blowing up as Λ increases if $\beta(g_*) = 0$ and $g(\Lambda)$ is on a trajectory attracted to g_*.

Trajectories with $g \to g_*$ for $\Lambda \to \infty$ form the *ultraviolet critical surface*. The physical requirement that actual couplings lie on UV critical surface may play the same role for theories including gravitation as does renormalizability in QED or QCD.

The number of free parameters in the theory equals the dimensionality of the UV critical surface.

Even with an infinite number of couplings $g_n(\Lambda)$, it is not surprising to find a finite-dimensional critical surface.

a. Continuity: For an asymptotically free renormalizable theory, there is a Gaussian fixed point with a finite dimensional critical surface. If we vary some free parameter (e.g, spacetime dimensionality, numbers of fields), then the fixed point may become non-Gaussian, but we wouldn't expect it to disappear, and the dimensionality of the ultraviolet critical surface may increase or decrease, but we wouldn't expect it to become infinite.

b. Second-order phase transitions: A single IR-repulsive direction in coupling constant space is also a single UV-attractive direction.

Indications of Asymptotically Safe Gravitation

- Dimensional Continuation ($d = 2+\epsilon$)
  - SW 1979
  - Kawai, Kitazawa, & Ninomiya, 1993, 1996
  - Aida & Kitazawa, 1997 (2 loops)
  - Niedermaier 2003
- $1/N$ Expansion
  - Smolin 1982 ($R + C^2$)
  - Percacci, 2006
- Lattice Quantization
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- Truncated 'Exact" Renormalization Group
  - Wegener & Houghton, 1973
  - Polchinski, 1984
  - Wetterich, 1993

  (Exact renormalization group equations link all $g_n(\Lambda)$. Truncate equations by setting all but a finite number of $g_n(\Lambda)$ equal zero, ignore the non-zero value of the other $\beta_n(g)$.)

  - Reuter, 1998
  - Dou & Percacci, 1998
    (gravity + free matter)
  - Souma, 1999 $(R+\lambda)$
  - Lauscher & Reuter, 2001 $(R+\lambda)$
  - Reuter & Saueressig, 2002 $(R+\lambda)$
  - Lauscher & Reuter, 2002
    $(R+\lambda+R^2)$

- Reuter & Saueressig, 2002
- Percacci & Perini, 2002, 2003
  (constraints on free matter)
- Perini, 2004
- Litim, 2004
- Codello & Percacci, 2006
- Reuter & Saueressig, 2007
- Machado & Saueressig, 2007
- Litim, 2008

With only 2 non-zero couplings, UV critical surface is 2 dimensional.
With only 3 non-zero couplings, UV critical surface is 3 dimensional.
This was not encouraging.

Good News! Calculations with $N > 3$ UV attractive directions:

– Codello, Percacci, & Rahmede, 2008
$2 \leq N \leq 9\,(R^0,\ R^1,\ \ldots R^{N-1};$ also with matter)

– Benedetti, Machado, & Saueressig, 2009
$N = 4\ (R^0,\ R,\ R^2,\ C^2\ ;$ also with matter)

Both groups find a 3-dimensional UV critical surface in all cases.

At optimal cutoff

$$c_0(\Lambda) = -\beta_0\Big(g(\Lambda)\Big) + 6\,\beta_1\Big(g(\Lambda)\Big)\,(\bar{H}/\Lambda)^2$$
$$-864\,\beta_{3a}\Big(g(\Lambda)\Big)\,(\bar{H}/\Lambda)^6$$
$$-216\,\beta_{3b}\Big(g(\Lambda)\Big)\,(\bar{H}/\Lambda)^6 + \dots$$

We get many e-foldings of inflation if couplings are near fixed point, where $\beta_n = 0$. Otherwise, c_0 is a sensitive ($\approx$ linear) function of free parameters. They may be anthropically required to take values for which $|c_0| \ll 1$ and hence $\mathcal{N} \gg 1$. Curvature must be small when galaxies form [Freivogel, Kleban, Martinez, Susskind 2006]. If there are many e-foldings from end of inflation to galaxy formation, then we need many e-foldings of flat-space inflation[Guth 1981].

Solution:

$$\delta H \propto \exp(\xi \bar{H} t)$$

where

$$c_0(\bar{H}, \Lambda) + c_1(\bar{H}, \Lambda)\,\xi + c_2(\bar{H}, \Lambda)\,\xi^2 + \ldots = 0$$

(All c_n with $n \geq 3$ would vanish if $\mathcal{F}_\Lambda$ were a function only of $R_{\mu\nu\rho\sigma}$.)

For $\xi > 0$, number of e-foldings is $\mathcal{N} \simeq 1/\xi$. If $|c_0(\Lambda)| \ll 1$ and $|c_n(\Lambda)| \approx 1$ for $n \geq 1$, then

$$\mathcal{N} \simeq |c_1(\Lambda)/c_0(\Lambda)| \gg 1$$

$$H(t) = \bar{H} + \delta H(t)$$

$$c_0(\bar{H}, \Lambda)\frac{\delta H}{\bar{H}} + c_1(\bar{H}, \Lambda)\frac{\delta \dot{H}}{\bar{H}^2} + c_2(\bar{H}, \Lambda)\frac{\delta \ddot{H}}{\bar{H}^3} + \ldots = 0$$

$$c_0(\bar{H}, \Lambda) = 12\left(\frac{\bar{H}}{\Lambda}\right)^2 g_1(\Lambda)$$

$$-5184\left(\frac{\bar{H}}{\Lambda}\right)^6 g_{3a}(\Lambda) - 1296\left(\frac{\bar{H}}{\Lambda}\right)^6 g_{3b}(\Lambda) + \ldots .$$

$$c_1(\bar{H}, \Lambda) = -216\, g_{2a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4 - 72\, g_{2b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4$$

$$+7776\, g_{3a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + 2160\, g_{3b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + \ldots .$$

$$c_2(\bar{H}, \Lambda) = -72\, g_{2a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4 - 24\, g_{2b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4$$

$$+2592\, g_{3a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + 720\, g_{3b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + \ldots .$$

$$I_\Lambda[g] = -\int d^4x \, \sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R$$
$$+ g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu}$$
$$+ \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \dots \Big]$$

gives the field equation

$$0 = \mathcal{N}_\Lambda(H, \dot H, \ddot H, \dots)$$
$$= -g_0(\Lambda) + 6\Lambda^{-2} g_1(\Lambda) H^2$$
$$-\Lambda^{-4} g_{2b}(\Lambda)\Big(72 H^2 \dot H - 12 \dot H^2 + 24 H \ddot H\Big)$$
$$+\Lambda^{-6} g_{3a}(\Lambda)\Big(-864\, H^6 + 7776 H^4 \dot H$$
$$+3240 H^2 \dot H^2 - 432\, \dot H^3 + 216\, H \ddot H (12 H^2 + 6 \dot H)\Big)$$
$$+\Lambda^{-6} g_{3b}(\Lambda)\Big(-216 H^6 + 2160 H^4 \dot H + 1008\, H^2 \dot H^2$$
$$-144 \dot H^3 + H \ddot H (720 H^2 + 432 \dot H)\Big) + \dots \, .$$

b. Truncation is plausible if we choose Λ so that $\Lambda \gg E$, but then there is no reason to neglect radiative corrections. Typically the tree approximation gives results that depend strongly on Λ, so radiative corrections are essential.

Compromise: Choose $\Lambda \approx E$. In QCD, Λ-dependence is logarithmic, and this is good enough. In theories of gravitation we need a way to identify an optimal Λ of order E.

3. How do we deal with radiative corrections? How do we choose Λ? Observable quantities are independent of Λ, but only if we include radiative corrections.

Consider a physical process with characteristic energy E. We have a hard choice:

a. Loop diagrams become negligible if we choose Λ so that $\Lambda \ll E$, but then truncation is dubious. The action is a sum of terms with increasing powers of $1/\Lambda$. A term in I_Λ proportional to $\Lambda^{-\alpha}$ gives a contribution to physical quantities containing a factor $(E/\Lambda)^\alpha$.

b. If Λ is a sliding renormalization scale, then even if the coupling constants $g_n(\Lambda)$ are defined as quantities that are directly observable at energy Λ, it is not necessary for them to approach finite definite values for $\Lambda \to \infty$. It is only necessary that they do not blow up at *finite* values of Λ. This condition does not require there to be a fixed point where $\beta_n(g_*) = 0$. For a single coupling

$$\ln \Lambda = \int^{g(\Lambda)} \frac{dg}{\beta(g)} \, .$$

For instance, if $\beta(g) > 0$ for finite $g > 0$, but $\beta(g) \to g^\alpha$ for $g \to \infty$ with $\alpha < 1$, then $g(\Lambda) \to \infty$ for $\Lambda \to \infty$, but this is not necessarily unphysical. With more than one coupling constant, there are many other possibilities.

PROBLEMS

1. Does the sequence of truncations converge?

UV attractive eigenvalues:

Codello, Percacci, & Rahmede, 2008
$(R^0, R^1, \ldots, R^{N-1})$
Benedetti, Machado, Sauressig 2009
(R^0, R^1, R^2, C^2)

$N=3$	$-1.38 \pm 2.32i$	-26.8
$N=4$	$-2.71 \pm 2.27i$	-2.07
$N=4$	$-2.33 \pm 0.76i$	-13.72
$N=5$	$-2.86 \pm 2.45i$	-1.55
$N=6$	$-2.53 \pm 2.69i$	-1.78
$N=7$	$-2.41 \pm 2.42i$	-1.50
$N=8$	$-2.51 \pm 2.44i$	-1.24
$N=9$	$-2.41 \pm 2.55i$	-1.40

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2. Is asymptotic safety necessary? Even if there is a zero of $\beta_n(g)$, how do we know the couplings have to lie on the ultraviolet critical surface?

a. If Λ is an ultraviolet cutoff or a parameter in a regulator term, then all observables must approach finite definite limits for $\Lambda \to \infty$, but the coupling constants $g_n(\Lambda)$ are not directly observable, so this does not necessarily require $g_n(\Lambda)$ to approach definite finite values at infinite Λ.

b. If Λ is a sliding renormalization scale, then even if the coupling constants $g_n(\Lambda)$ are defined as quantities that are directly observable at energy Λ, it is not necessary for them to approach finite definite values for $\Lambda \to \infty$. It is only necessary that they do not blow up at *finite* values of Λ. This condition does not require there to be a fixed point where $\beta_n(g_*) = 0$. For a single coupling

$$\ln \Lambda = \int^{g(\Lambda)} \frac{dg}{\beta(g)} .$$

For instance, if $\beta(g) > 0$ for finite $g > 0$, but $\beta(g) \to g^{\alpha}$ for $g \to \infty$ with $\alpha < 1$, then $g(\Lambda) \to \infty$ for $\Lambda \to \infty$, but this is not necessarily unphysical. With more than one coupling constant, there are many other possibilities.

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COSMOLOGICAL APPLICATIONS

Bonano & Reuter, 2002; Reuter & Saueressig, 2005

- Einstein-Hilbert truncation (g_0 & g_1)
- Matter with constant w
- Time-varying Λ

SW 2009

- No truncation (also no numerical results)
- No matter (yet)
- Constant Λ

Robertson–Walker solutions

For a line element

$$d\tau^2 = dt^2 - a^2(t)\, d\vec{x}^2 \,.$$

the action takes the form

$$I_\Lambda[g_{\rm RW}] = V\Lambda^4 \int dt\; a^3(t)\, \mathcal{F}_\Lambda\Big(H(t), \dot{H}(t), \ldots\Big) \,.$$

where

$$H(t) \equiv \dot{a}(t)/a(t)$$

Gravitational Field Equation:

$$0 = \mathcal{N}_\Lambda(H, \dot{H}, \ddot{H}, \ldots) \equiv \left(\frac{\delta I_\Lambda[g]}{\delta g_{00}}\right)_{\rm RW}$$

$$= \mathcal{F}_\Lambda - H\frac{\partial \mathcal{F}_\Lambda}{\partial H} + (-\dot{H} + 3H^2)\frac{\partial \mathcal{F}_\Lambda}{\partial \dot{H}}$$

$$+H\frac{d}{dt}\left(\frac{\partial \mathcal{F}_\Lambda}{\partial \dot{H}}\right) + \ldots$$

(No higher terms if $\mathcal{F}_\Lambda$ function of $R_{\mu\nu\rho\sigma}$.)

$$I_\Lambda[g] = -\int d^4x\,\sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R$$
$$+ g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu}$$
$$+ \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots\Big]$$

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$$+\Lambda^{-6} g_{3a}(\Lambda)\Big(-864 H^6 + 7776 H^4 \dot{H}$$
$$+3240 H^2 \dot{H}^2 - 432 \dot{H}^3 + 216 H \ddot{H}(12 H^2 + 6\dot{H})\Big)$$
$$+\Lambda^{-6} g_{3b}(\Lambda)\Big(-216 H^6 + 2160 H^4 \dot{H} + 1008 H^2 \dot{H}^2$$
$$-144 \dot{H}^3 + H \ddot{H}(720 H^2 + 432 \dot{H})\Big) + \ldots\,.$$

De Sitter Solution

$$a(t) \propto \exp(\bar{H}t)\,. \quad \bar{H} \text{ constant}$$

where

$$\begin{aligned} 0 = N_\Lambda(\bar{H}) &\equiv \mathcal{N}_\Lambda(\bar{H}, 0, 0, \dots) \\ &= -g_0(\Lambda) + 6\, g_1(\Lambda)\, (\bar{H}/\Lambda)^2 - 864\, g_{3a}(\Lambda)\, (\bar{H}/\Lambda)^6 \\ &\quad -216\, g_{3b}(\Lambda)\, (\bar{H}/\Lambda)^6 + \dots \end{aligned}$$

A hard choice:

- If $\Lambda \ll \bar{H}$ then radiative corrections are small, but series diverges badly.
- If $\Lambda \gg \bar{H}$ then series may be dominated by low terms, but radiative corrections are important, as shown by Λ dependence of $\bar{H}$. (Near fixed point, $\bar{H} \propto \Lambda$.)

Choose $\Lambda \approx \bar{H}$, but at a local minimum of radiative corrections. Write true expansion rate as

$$H_{\text{true}} = \bar{H}(\Lambda) + \Delta H(\Lambda)$$

Choose optimal cutoff Λ so

$$\frac{d}{d\Lambda}\Delta H(\Lambda) = 0$$

But H_{true} is independent of Λ, so at optimal cutoff

$$\frac{d}{d\Lambda}\bar{H}(\Lambda) = 0$$

By definition, for all Λ,

$$N_\Lambda\left(\bar{H}(\Lambda)\right) = 0$$

so at optimal cutoff

$$\frac{\partial}{\partial\Lambda}N_\Lambda\left(\bar{H}\right) = 0$$

This gives two equations for $\bar{H}$ & Λ:

$$
\begin{aligned}
0 = & -g_0(\Lambda) + 6\, g_1(\Lambda)\, (\bar{H}/\Lambda)^2 - 864\, g_{3a}(\Lambda)\, (\bar{H}/\Lambda)^6 \\
& -216\, g_{3b}(\Lambda)\, (\bar{H}/\Lambda)^6 + \dots
\end{aligned}
$$

and

$$
\begin{aligned}
0 = & -12 \left(\frac{\bar{H}}{\Lambda}\right)^2 g_1(\Lambda) + 5184 \left(\frac{\bar{H}}{\Lambda}\right)^6 g_{3a}(\Lambda) \\
& +1296 \left(\frac{\bar{H}}{\Lambda}\right)^6 g_{3b}(\Lambda) + \dots \\
& -\beta_0\Big(g(\Lambda)\Big) + 6\, \beta_1\Big(g(\Lambda)\Big)\, (\bar{H}/\Lambda)^2 \\
& -864\, \beta_{3a}\Big(g(\Lambda)\Big)\, (\bar{H}/\Lambda)^6 \\
& -216\, \beta_{3b}\Big(g(\Lambda)\Big)\, (\bar{H}/\Lambda)^6 + \dots .
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We expect solutions with

$$\bar{H} \approx \Lambda \approx M$$

($M \equiv$ scale at which $g_n(\Lambda)$ approach fixed point.)

$$g_n(\Lambda) \to g_{n*} + \sum_i u_{in} \left(\frac{\Lambda}{M}\right)^{\lambda_i}$$

where

$$\sum_m B_{nm}\, u_{im} = \lambda_i\, u_{in}\,, \quad B_{nm} \equiv \left(\frac{\partial \beta_n(g)}{\partial g_m}\right)_*$$

and

$$\max(u_{in}) = O(1)$$

Free parameters are M and relative normalization of eigenvectors u_{in}

$$H(t) = \bar{H} + \delta H(t)$$

$$c_0(\bar{H},\Lambda)\frac{\delta H}{\bar{H}} + c_1(\bar{H},\Lambda)\frac{\delta \dot{H}}{\bar{H}^2} + c_2(\bar{H},\Lambda)\frac{\delta \ddot{H}}{\bar{H}^3} + \ldots = 0$$

$$c_0(\bar{H},\Lambda) = 12\left(\frac{\bar{H}}{\Lambda}\right)^2 g_1(\Lambda)$$

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$$+7776\, g_{3a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + 2160\, g_{3b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + \ldots ,$$

$$c_2(\bar{H},\Lambda) = -72\, g_{2a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4 - 24\, g_{2b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4$$

$$+2592\, g_{3a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + 720\, g_{3b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^6 + \ldots .$$

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$$c_0(\bar{H}, \Lambda)\frac{\delta H}{\bar{H}} + c_1(\bar{H}, \Lambda)\frac{\delta \dot{H}}{\bar{H}^2} + c_2(\bar{H}, \Lambda)\frac{\delta \ddot{H}}{\bar{H}^3} + \ldots = 0$$

$$c_0(\bar{H}, \Lambda) = 12\left(\frac{\bar{H}}{\Lambda}\right)^2 g_1(\Lambda)$$

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$$c_1(\bar{H}, \Lambda) = -216\, g_{2a}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4 - 72\, g_{2b}(\Lambda)\left(\frac{\bar{H}}{\Lambda}\right)^4$$

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De Sitter Solution

$$a(t) \propto \exp(\bar{H}t)\ . \quad \bar{H} \text{ constant}$$

where

$$\begin{aligned} 0 &= N_\Lambda(\bar{H}) \equiv \mathcal{N}_\Lambda(\bar{H}, 0, 0, \dots) \\ &= -g_0(\Lambda) + 6\, g_1(\Lambda)\, (\bar{H}/\Lambda)^2 - 864\, g_{3a}(\Lambda)\, (\bar{H}/\Lambda)^6 \\ &\quad -216\, g_{3b}(\Lambda)\, (\bar{H}/\Lambda)^6 + \dots \end{aligned}$$

A hard choice:

- If $\Lambda \ll \bar{H}$ then radiative corrections are small, but series diverges badly.
- If $\Lambda \gg \bar{H}$ then series may be dominated by low terms, but radiative corrections are important, as shown by Λ dependence of $\bar{H}$. (Near fixed point, $\bar{H} \propto \Lambda$.)

$$I_\Lambda[g] = -\int d^4x\, \sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R$$
$$+ g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu}$$
$$+ \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots\Big]$$

gives the field equation

$$\begin{aligned}
0 &= \mathcal{N}_\Lambda(H, \dot{H}, \ddot{H}, \ldots) \\
&= -g_0(\Lambda) + 6\Lambda^{-2} g_1(\Lambda) H^2 \\
&-\Lambda^{-4} g_{2b}(\Lambda)\Big(72 H^2 \dot{H} - 12 \dot{H}^2 + 24 H \ddot{H}\Big) \\
&+\Lambda^{-6} g_{3a}(\Lambda)\Big(-864 H^6 + 7776 H^4 \dot{H} \\
&+3240 H^2 \dot{H}^2 - 432 \dot{H}^3 + 216 H \ddot{H}(12 H^2 + 6\dot{H})\Big) \\
&+\Lambda^{-6} g_{3b}(\Lambda)\Big(-216 H^6 + 2160 H^4 \dot{H} + 1008 H^2 \dot{H}^2 \\
&-144 \dot{H}^3 + H \ddot{H}(720 H^2 + 432 \dot{H})\Big) + \ldots .
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Swipe your finger slower

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