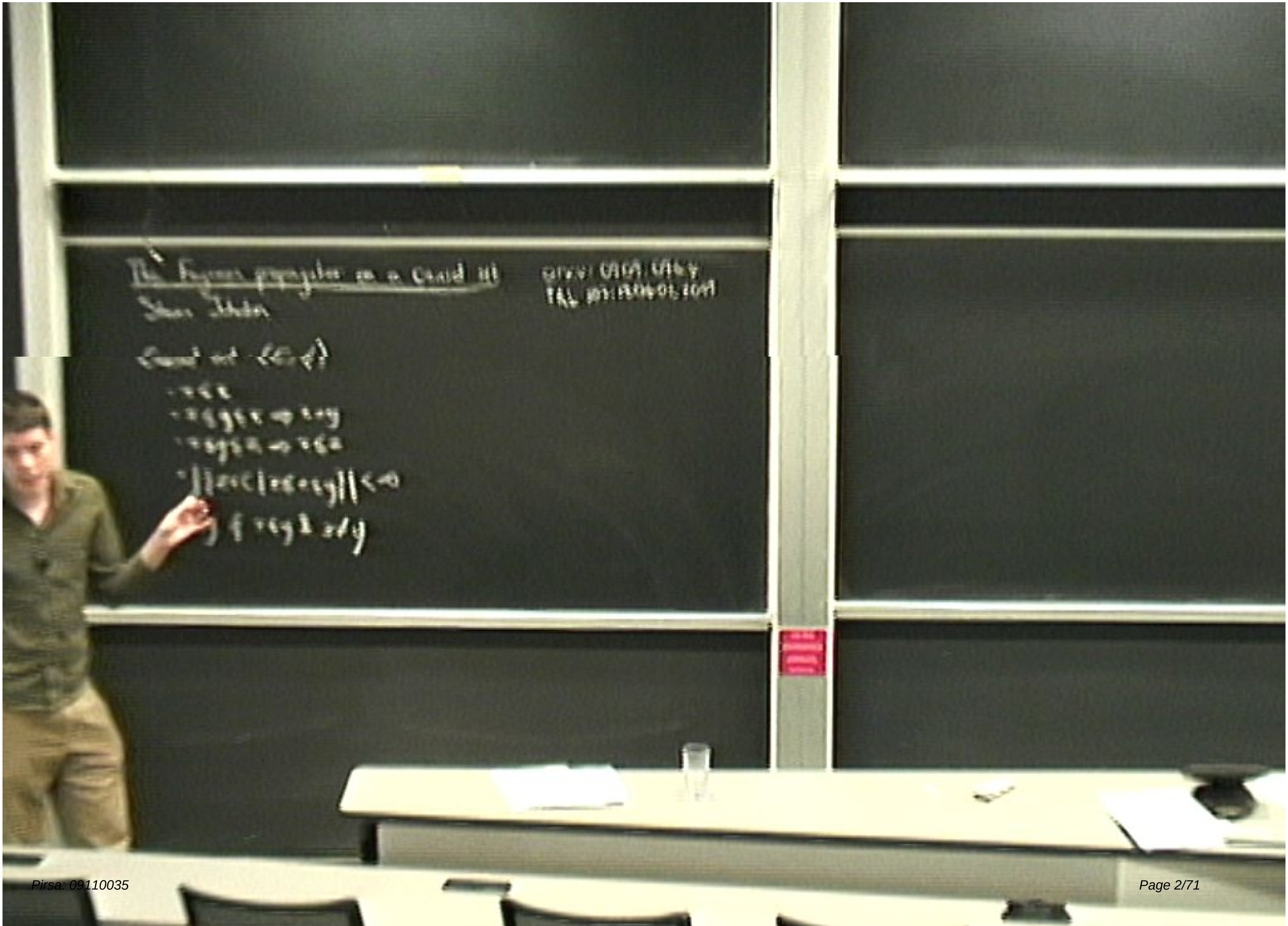
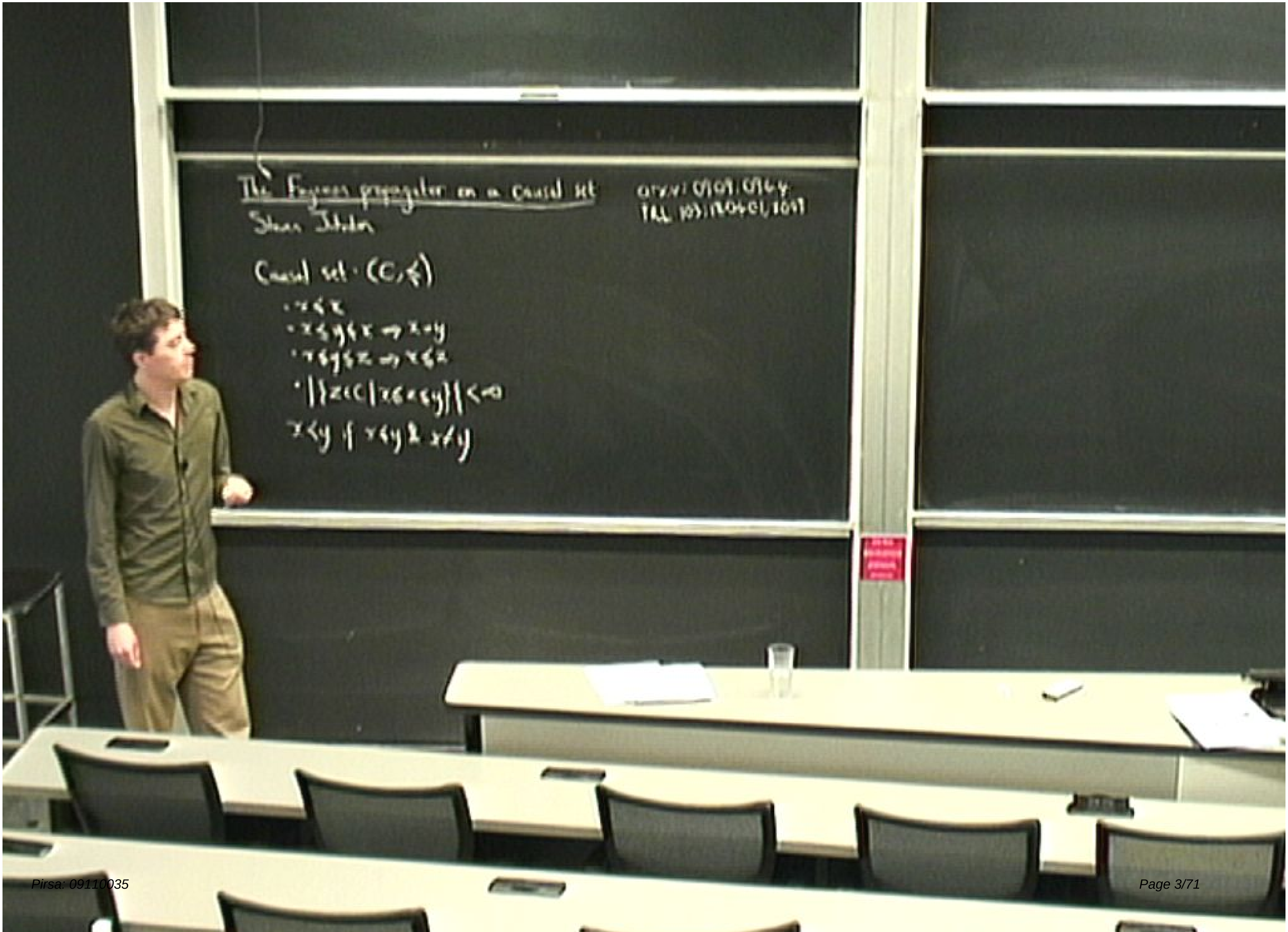
Title: The Feynman propagator on a causal set.

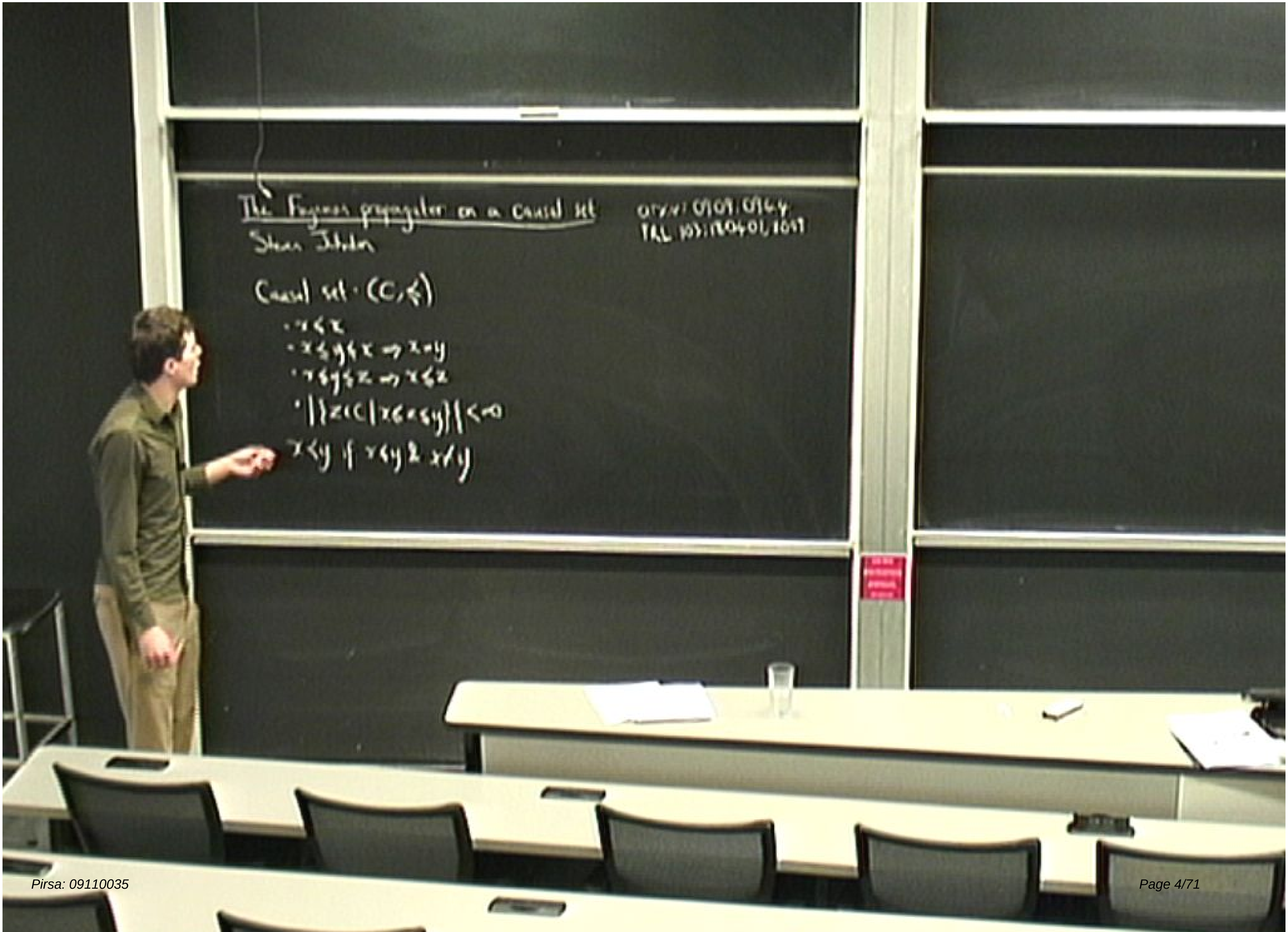
Date: Nov 13, 2009 11:00 AM

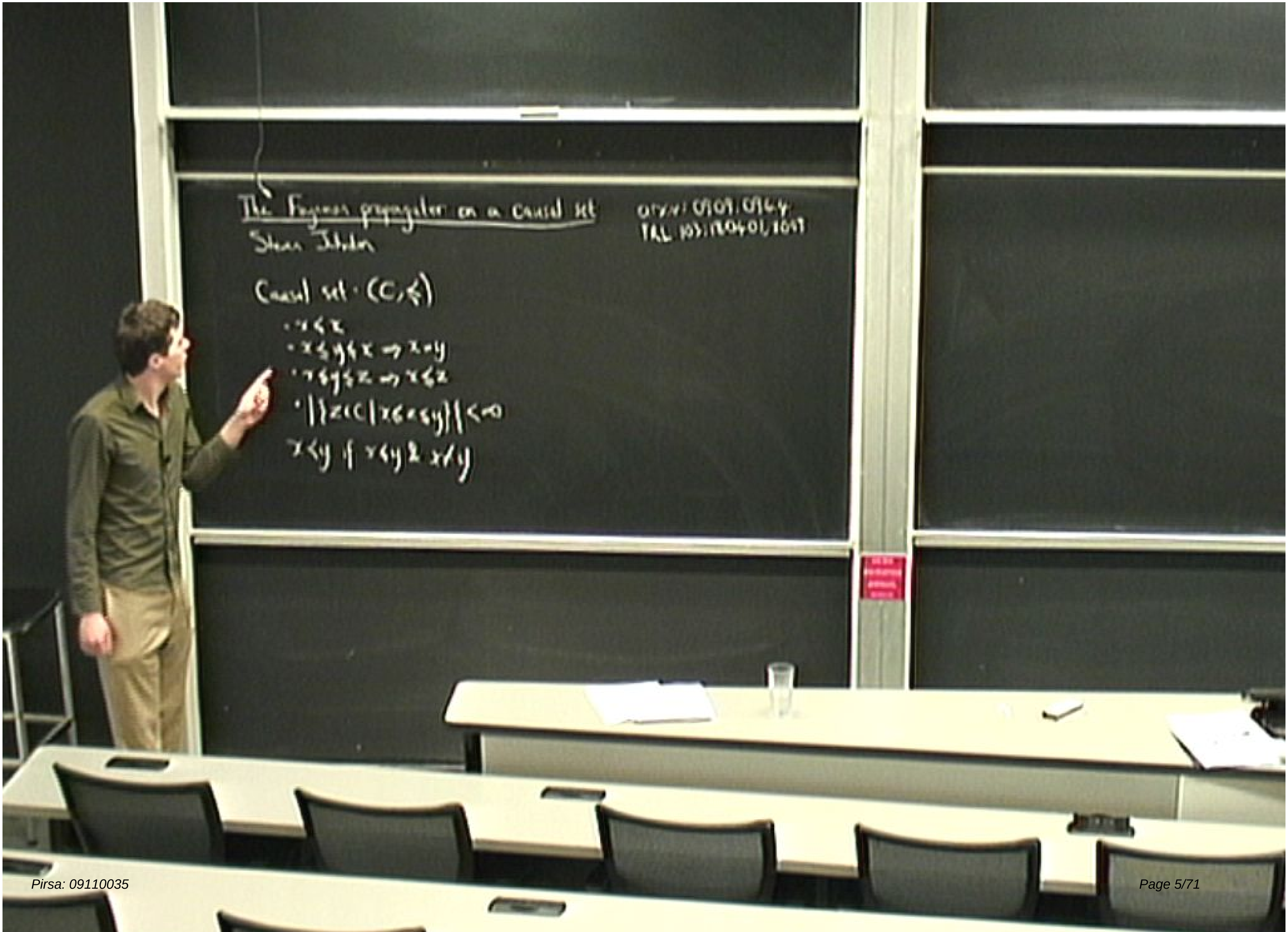
URL: http://pirsa.org/09110035

Abstract: How sure are you that spacetime is continuous? One approach to quantum gravity, causal set theory, models spacetime as a discrete structure: a causal set. This talk begins with a brief introduction to causal sets, then describes a new approach to modelling a quantum scalar field on a causal set. We obtain the Feynman propagator for the field by a novel procedure starting with the Pauli-Jordan commutation function. The candidate Feynman propagator is shown to agree with the continuum result. This model opens the door to physical predictions for scalar matter on a causal set.

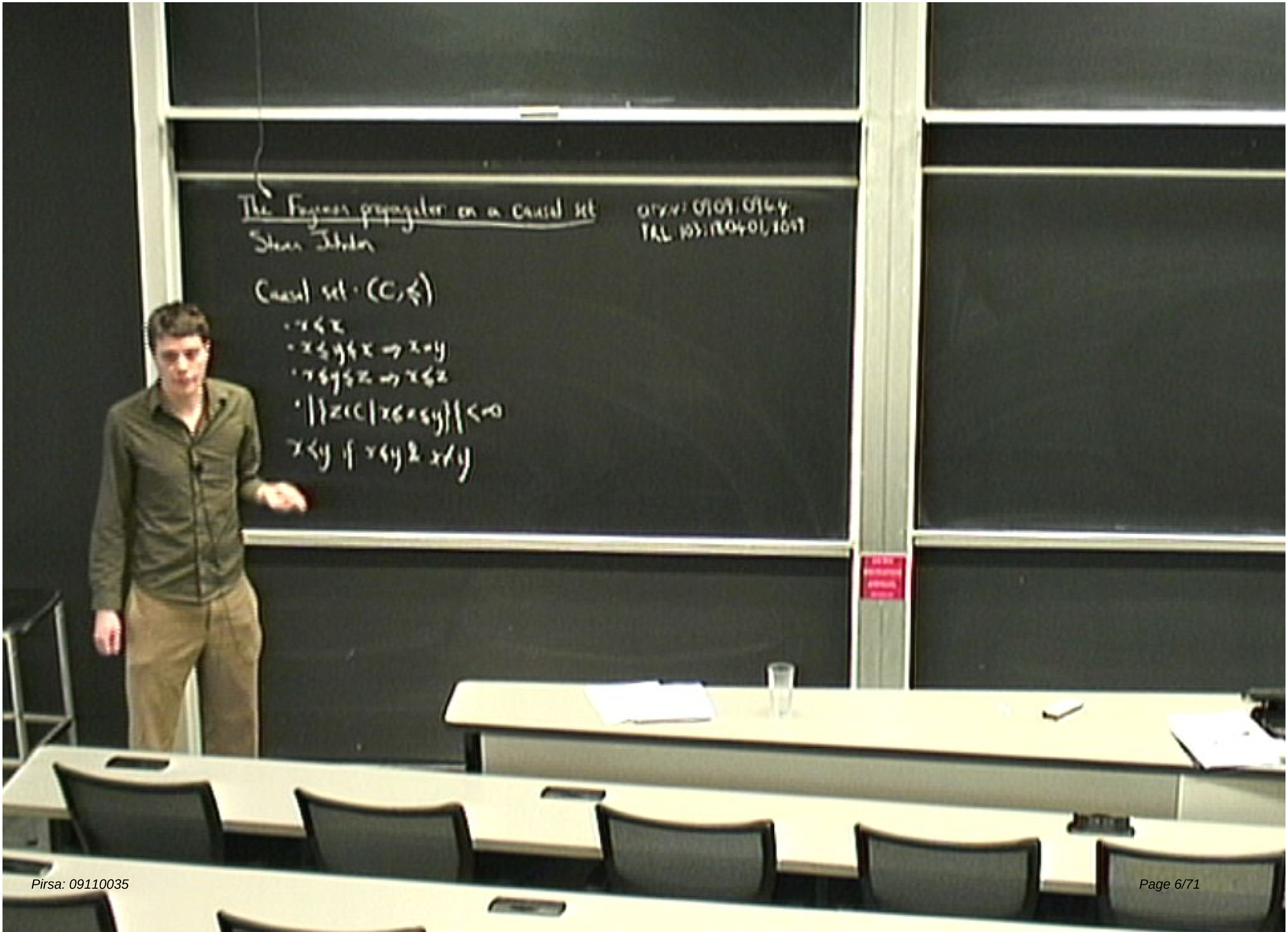



The Feynman propagator on a causal set
Causal set (C, ≼)
x≺y if x≼y & x≠y


The Feynman propagator on a causal set
Causal set: (C, ≤)



The Feynman propagator on a causal set
Steven Johnston
arXiv: 0909.0964
PRL 103:180401, 2009
Causal set: $(C, \preceq)$
• $x \preceq x$
• $x \preceq y \preceq x \Rightarrow x = y$
• $x \preceq y \preceq z \Rightarrow x \preceq z$
• $|\{z \in C \mid x \preceq z \preceq y\}| < \infty$
$x \prec y$ if $x \preceq y$ & $x \neq y$


The Feynman propagator on a causal set
Steven Johnston
arXiv: 0909.0944
PRL 103:180401, 2009
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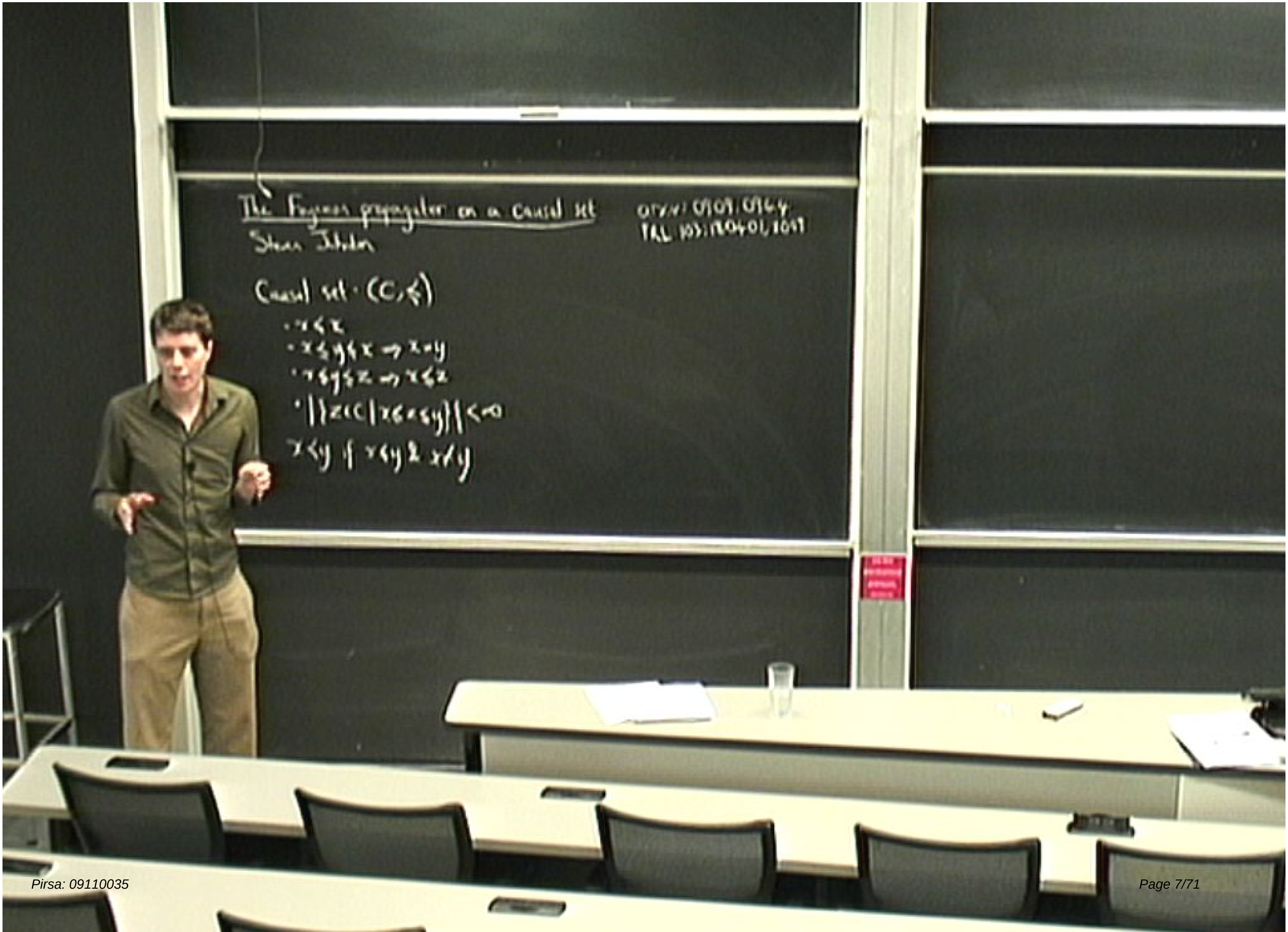
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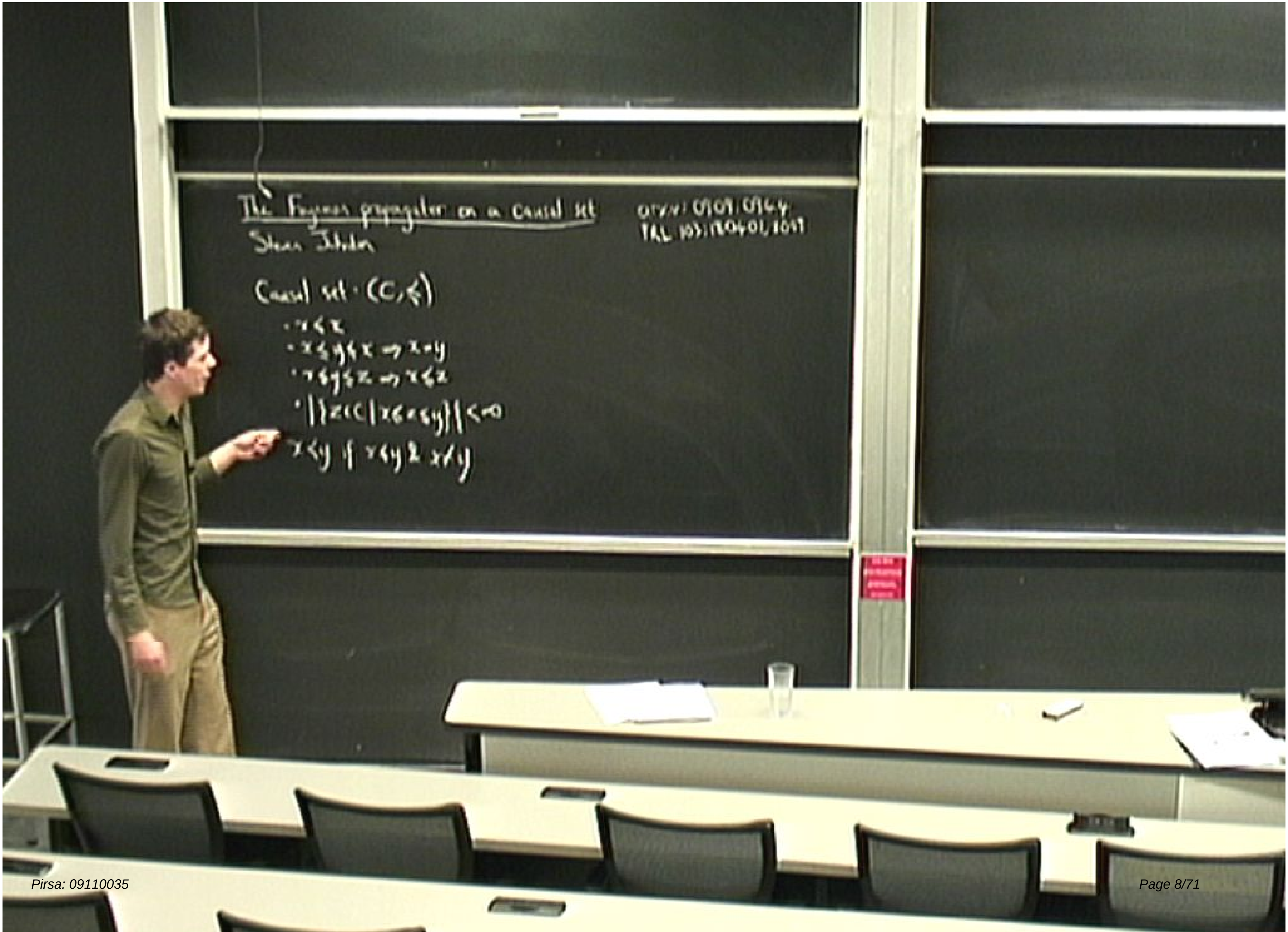
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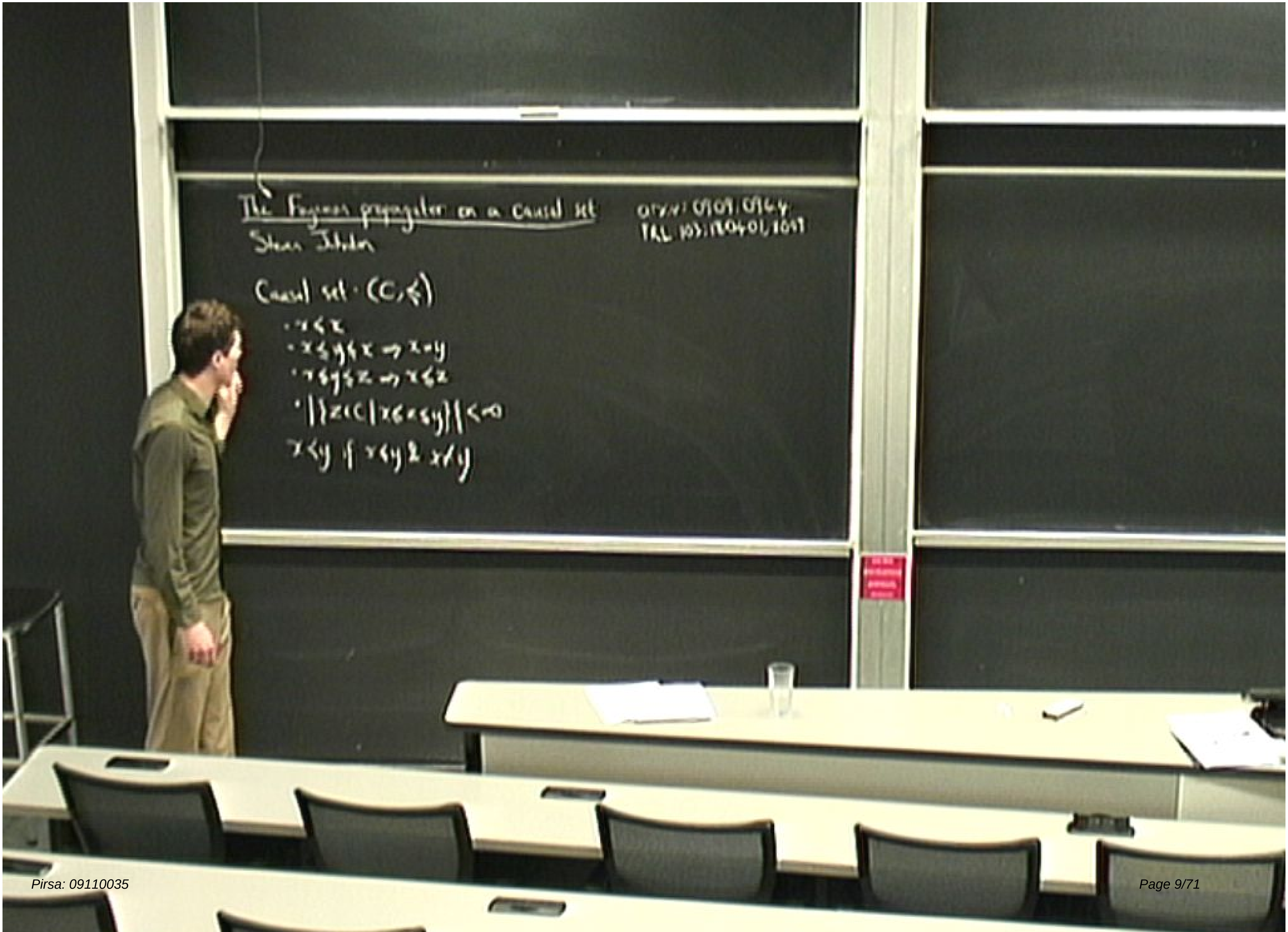
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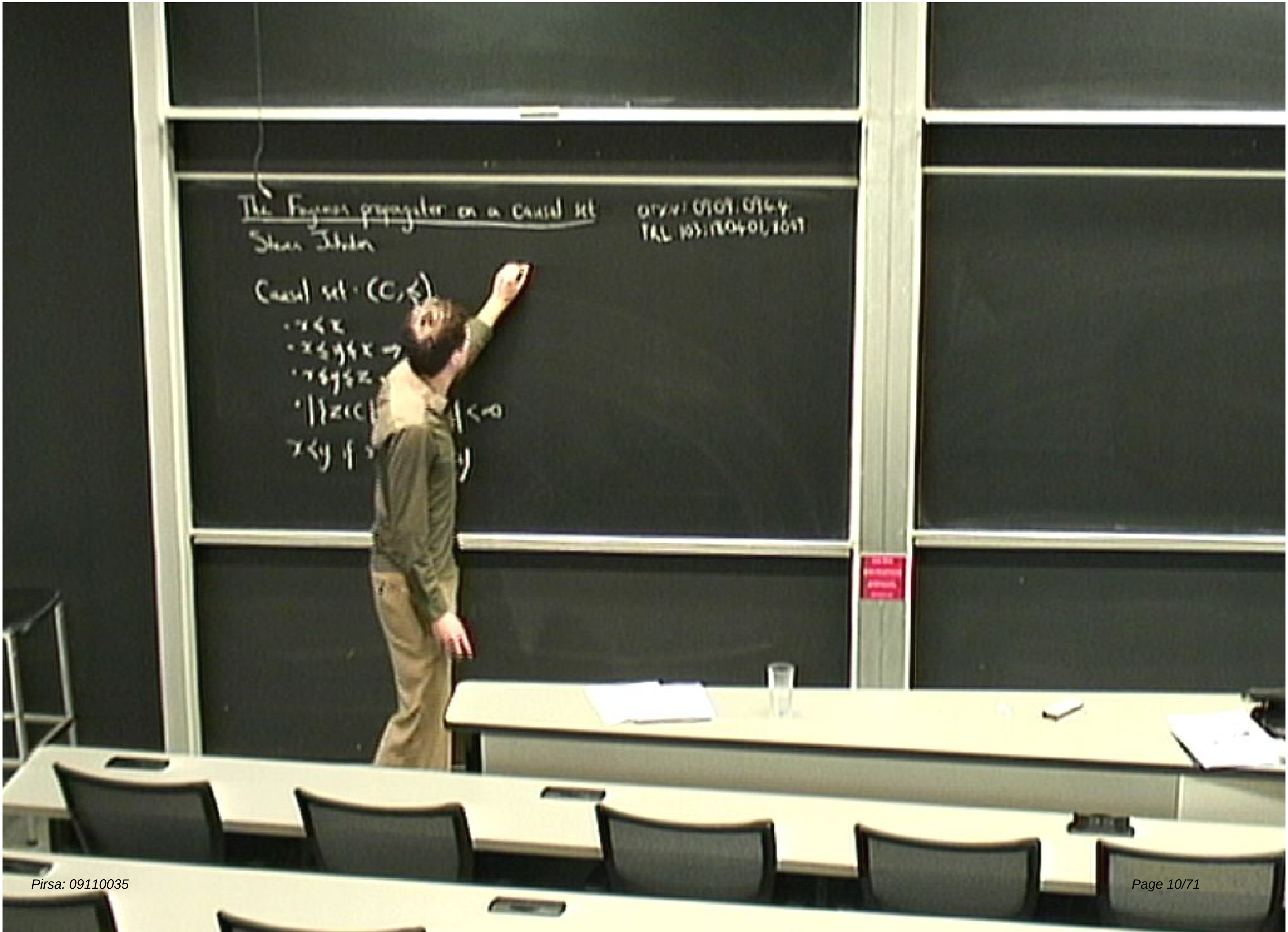
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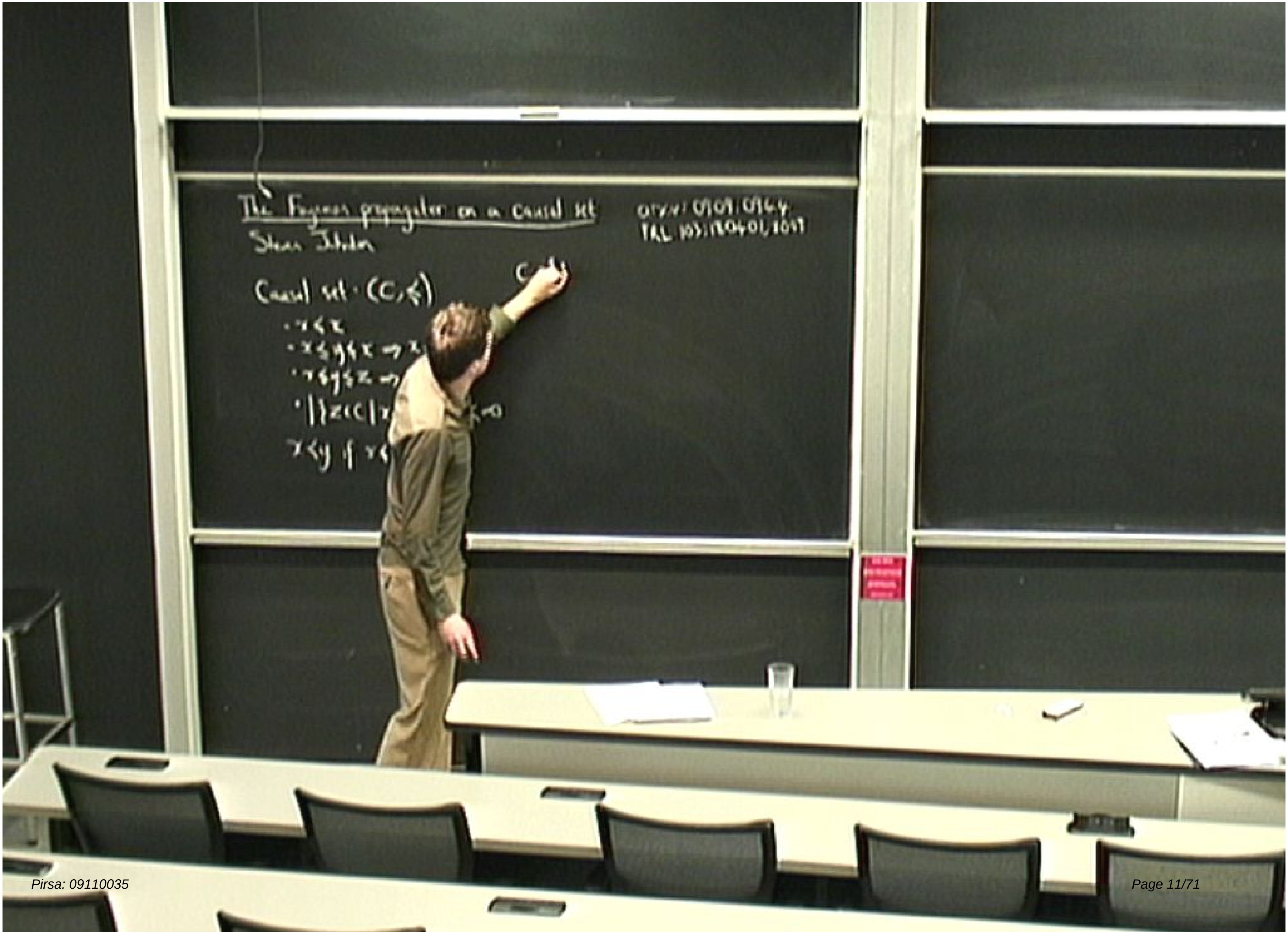
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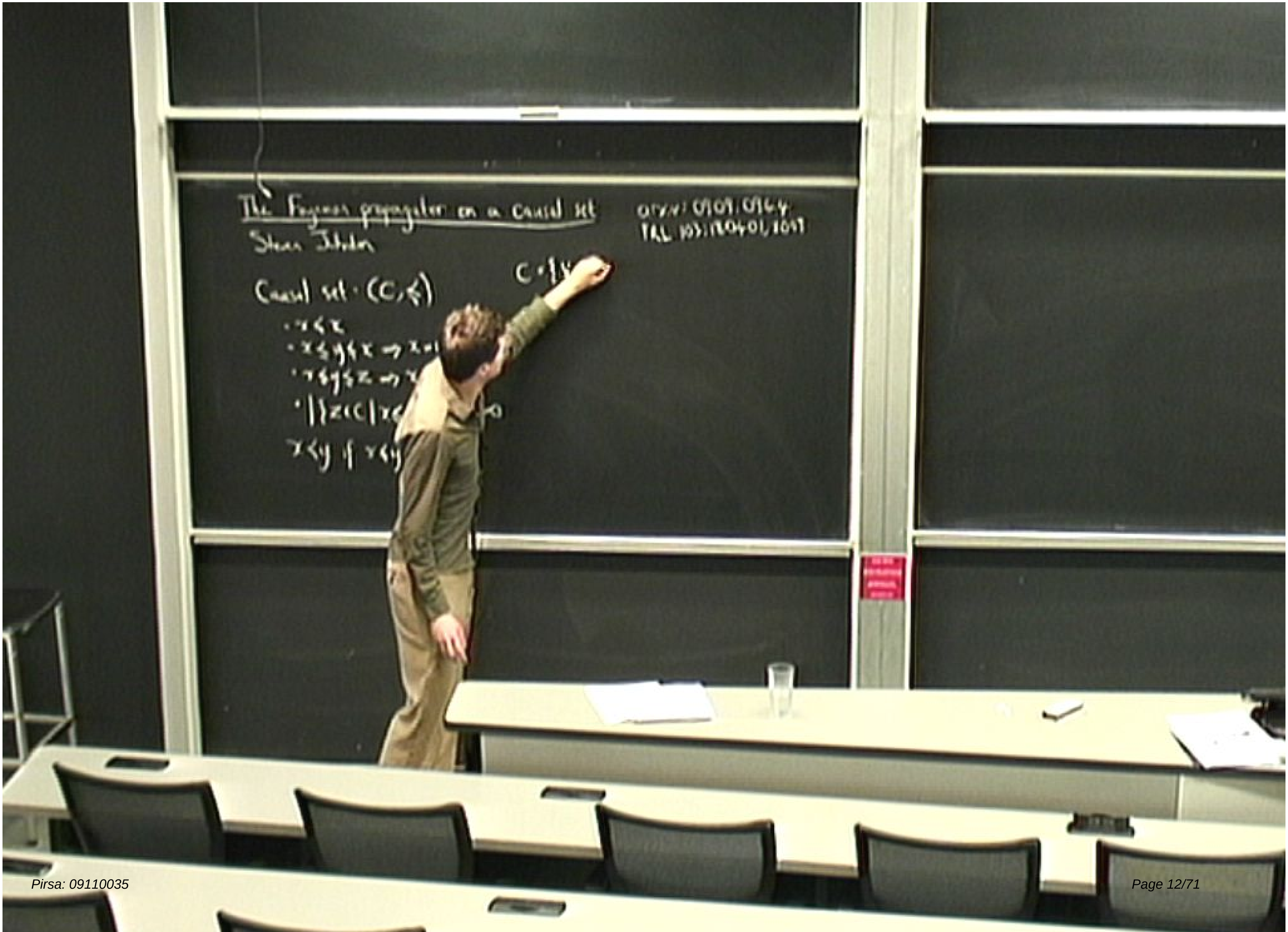
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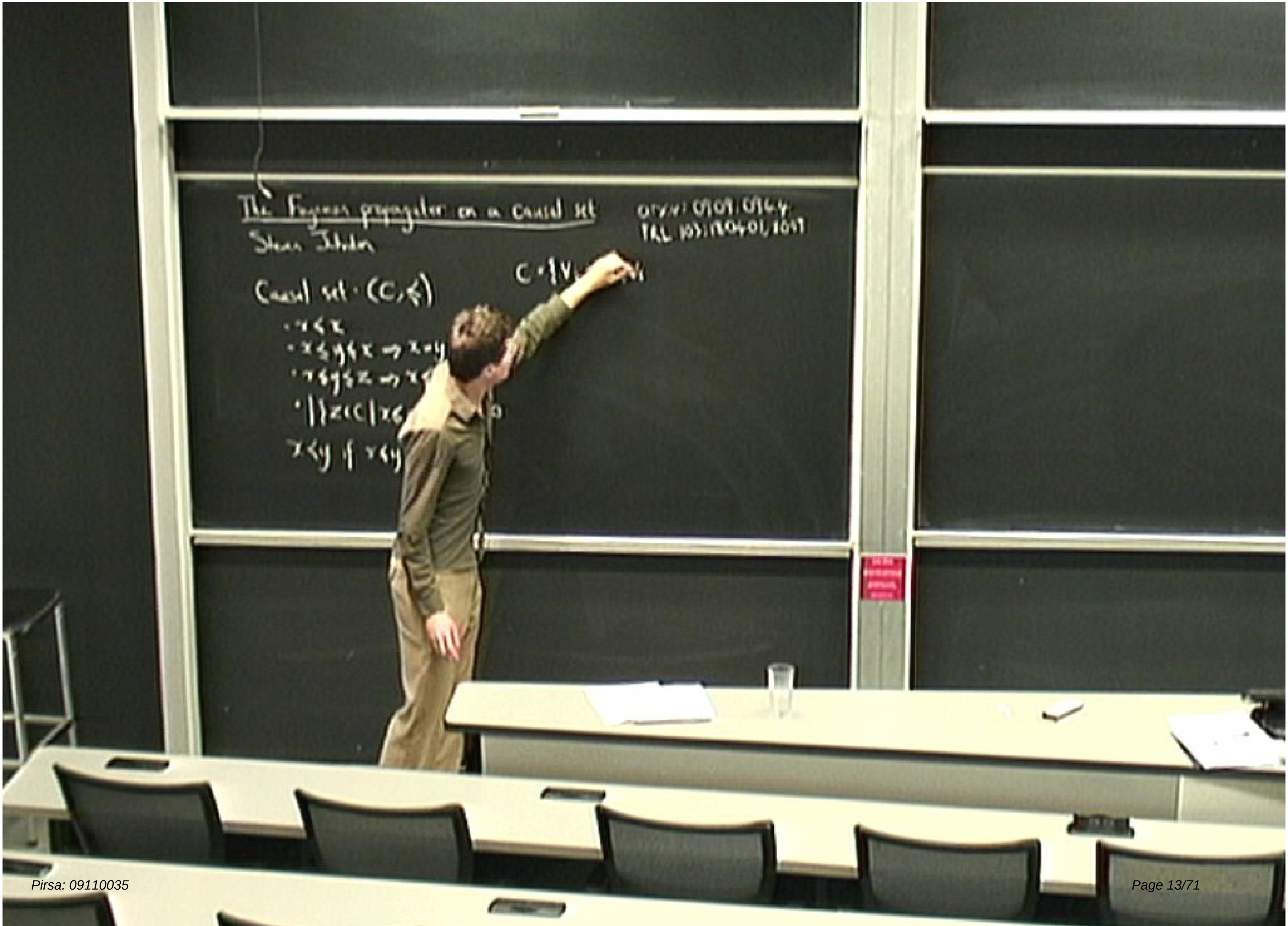
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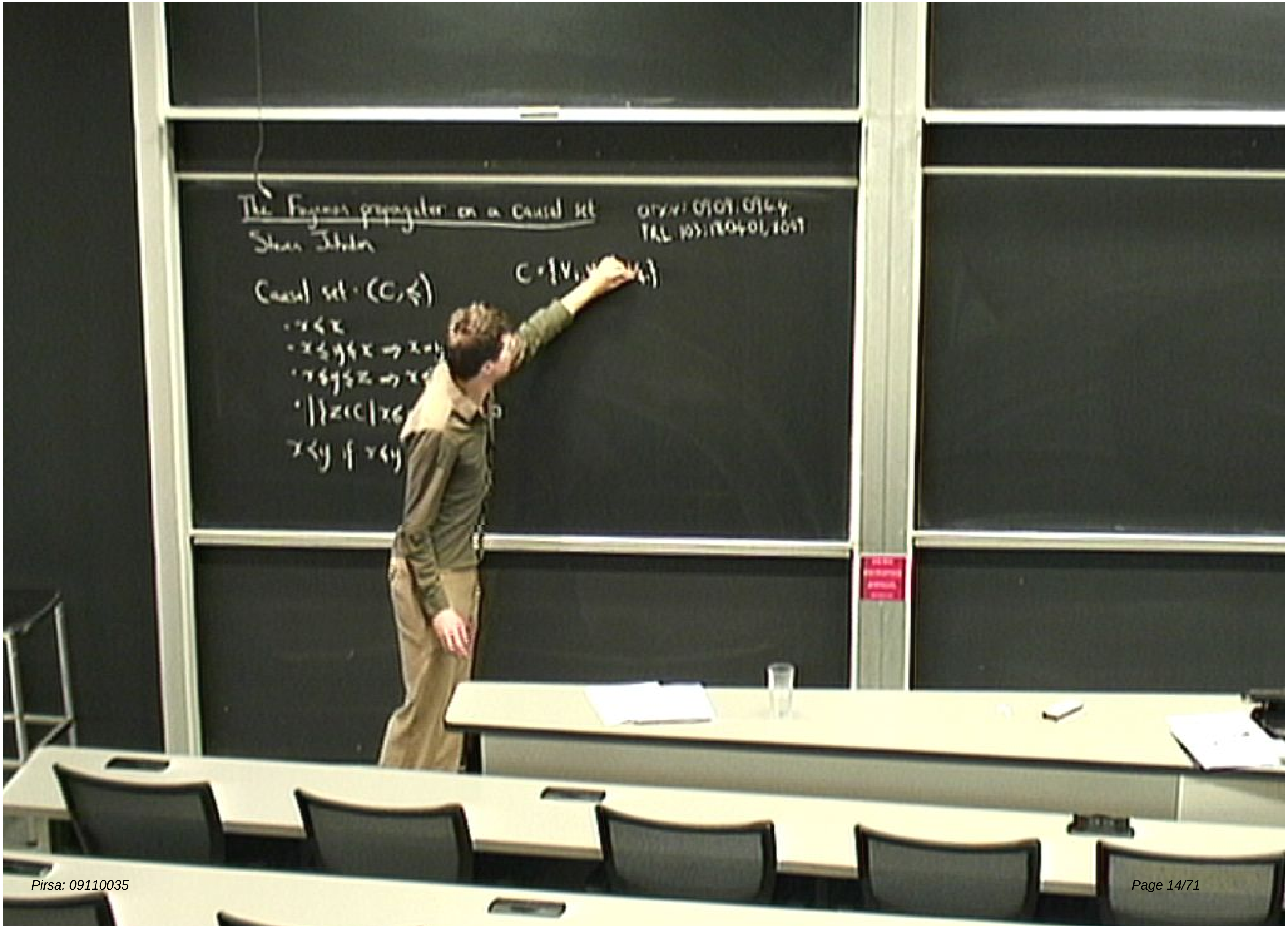
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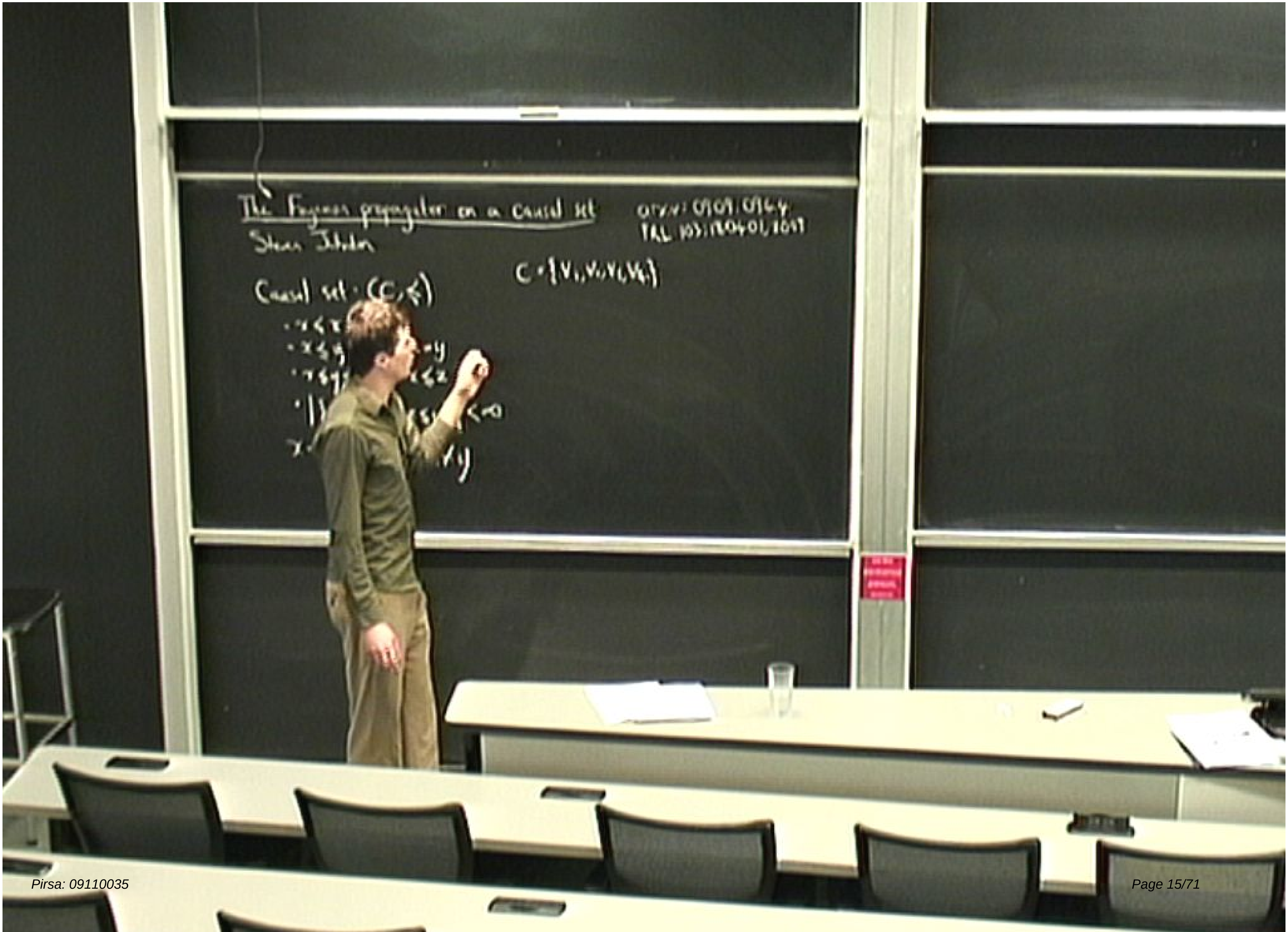
Causal set: (C, ≤)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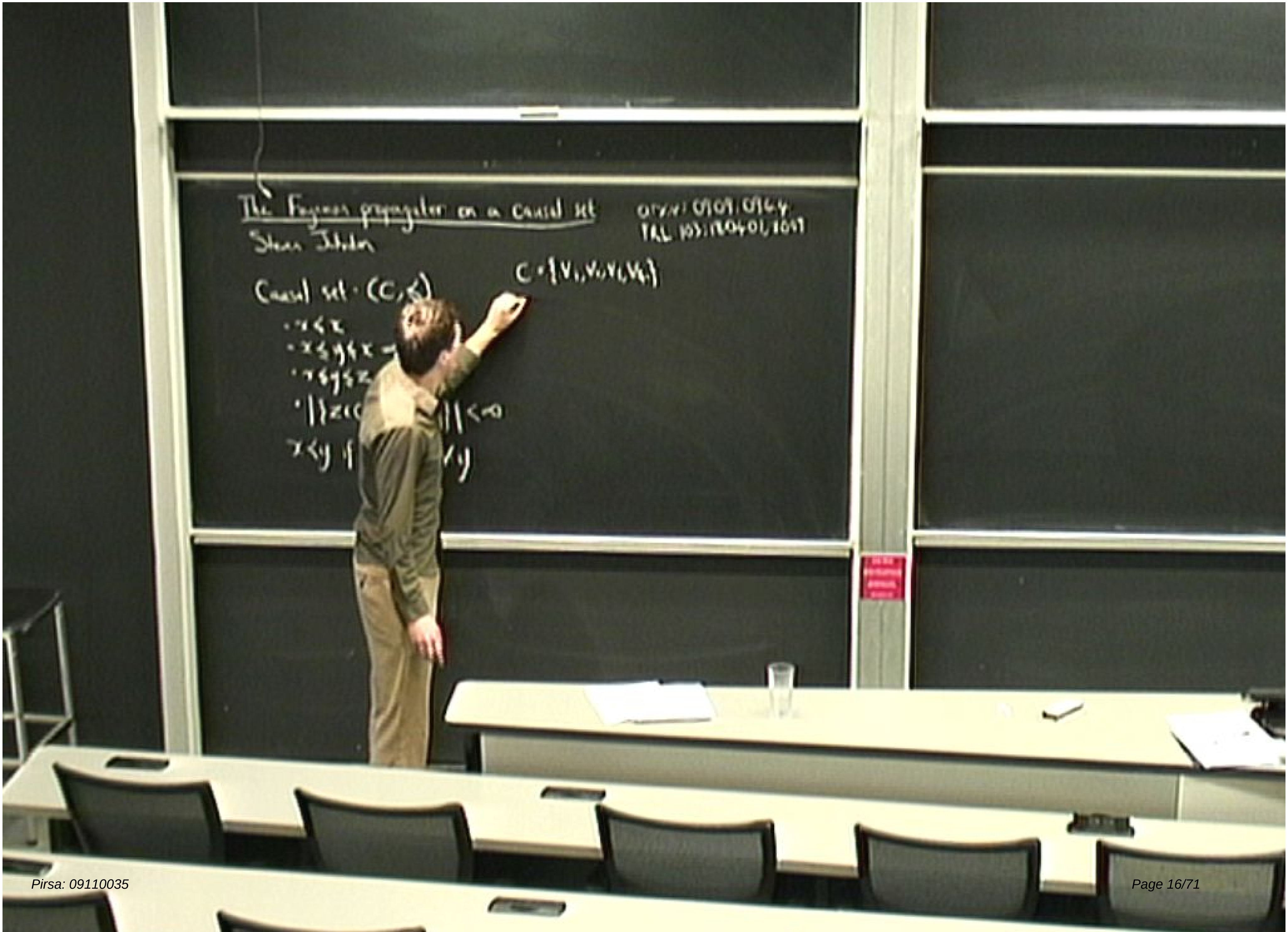
C = {v1, v2, v3, v4}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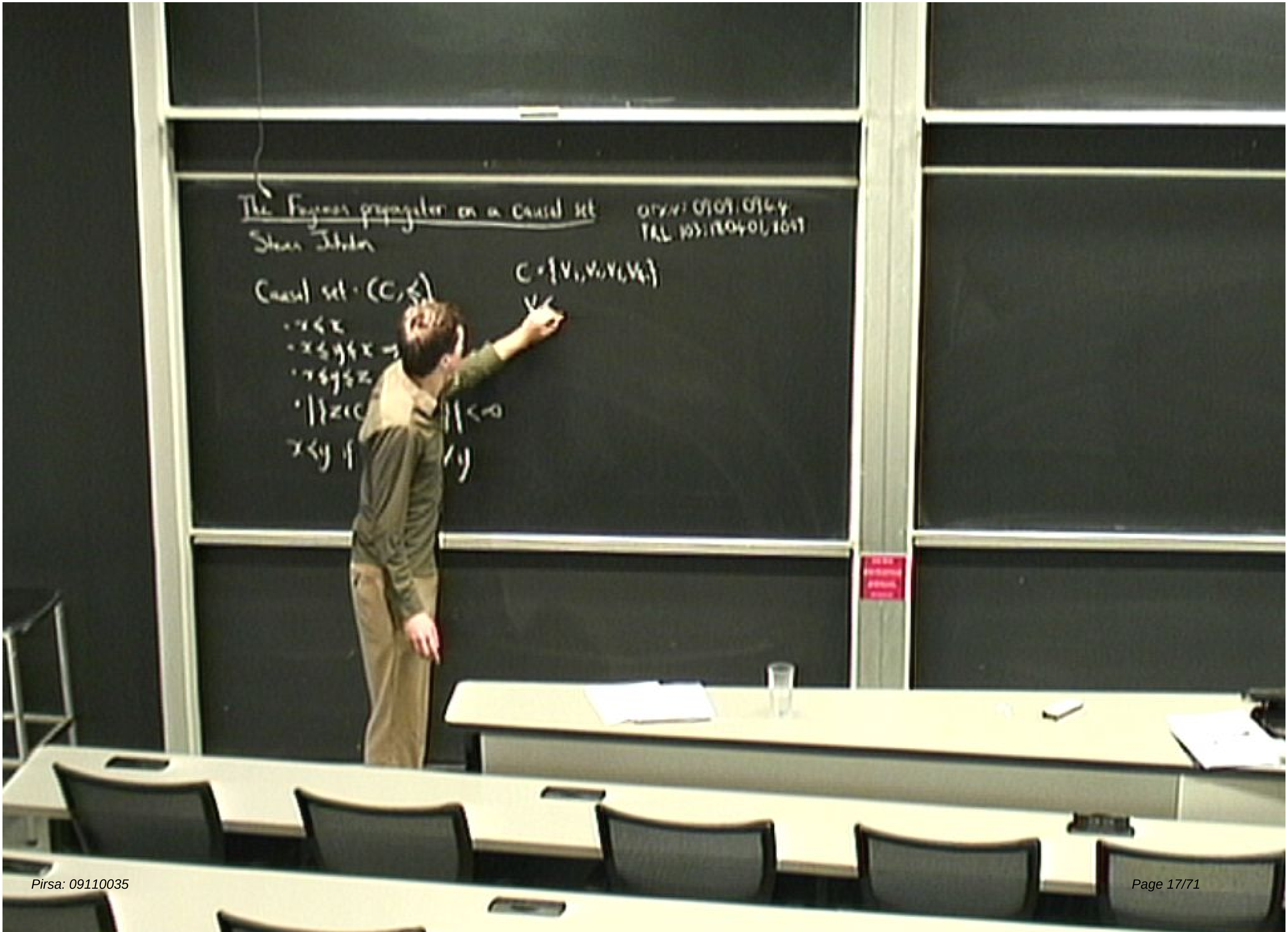

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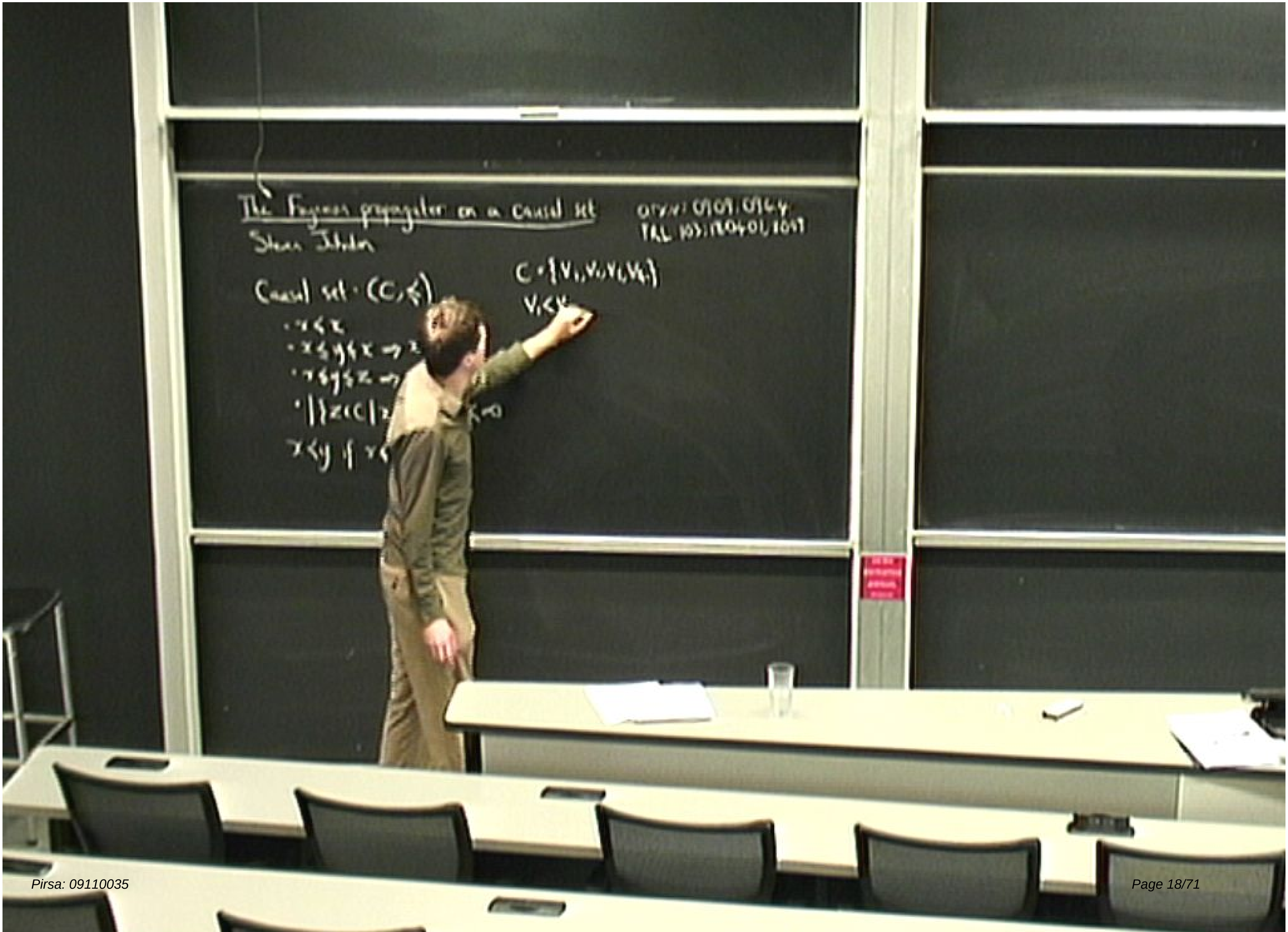
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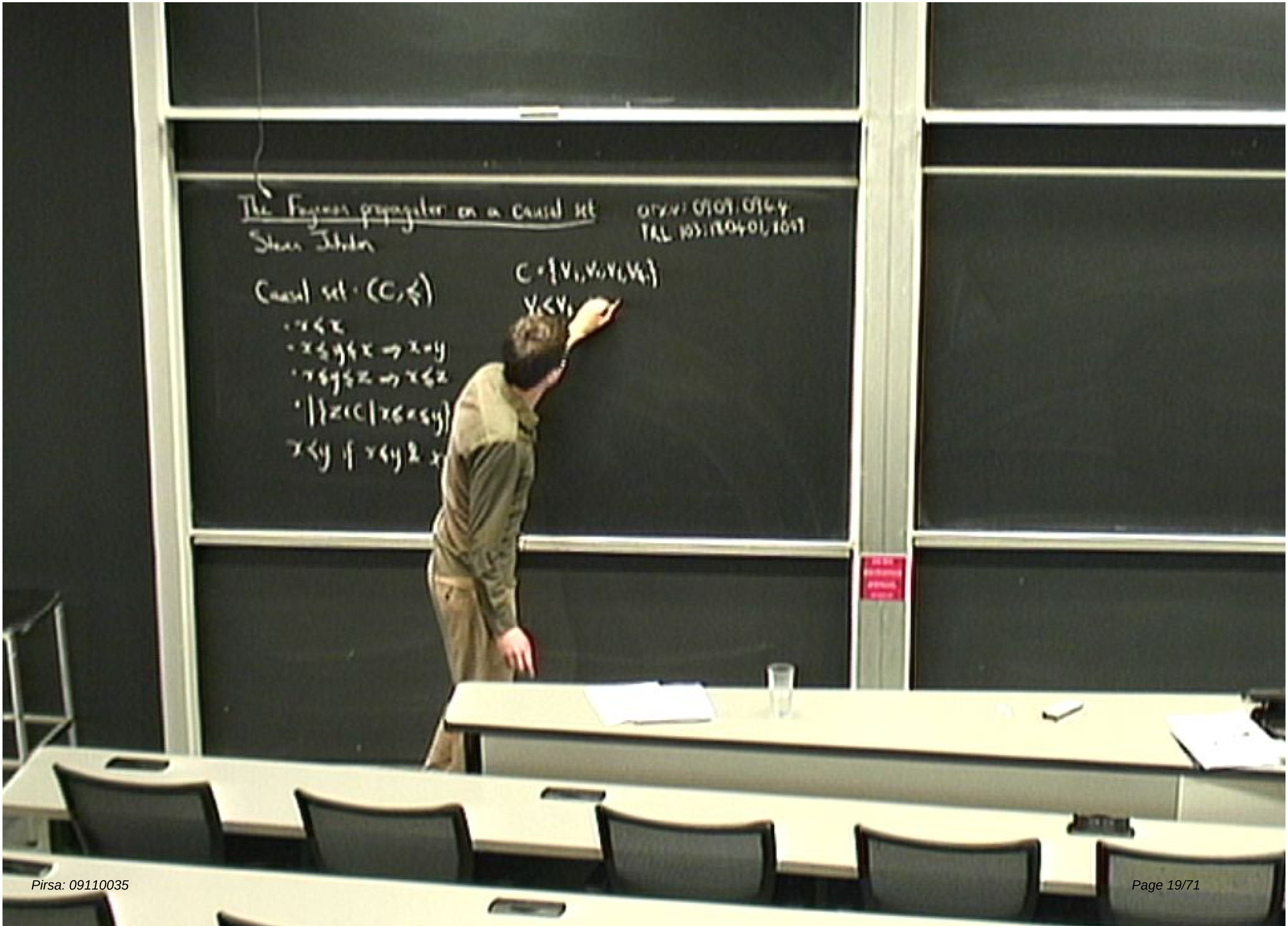
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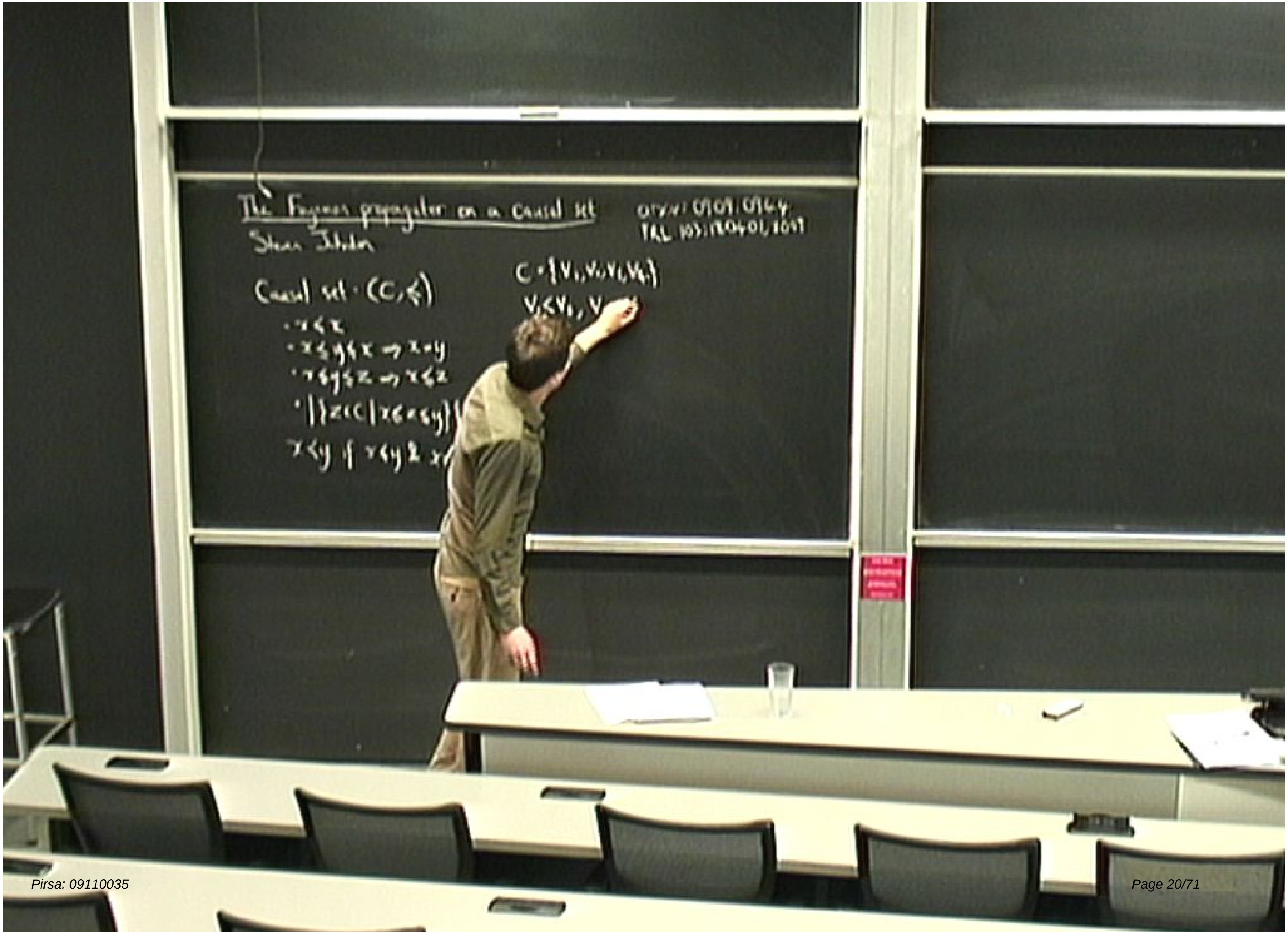
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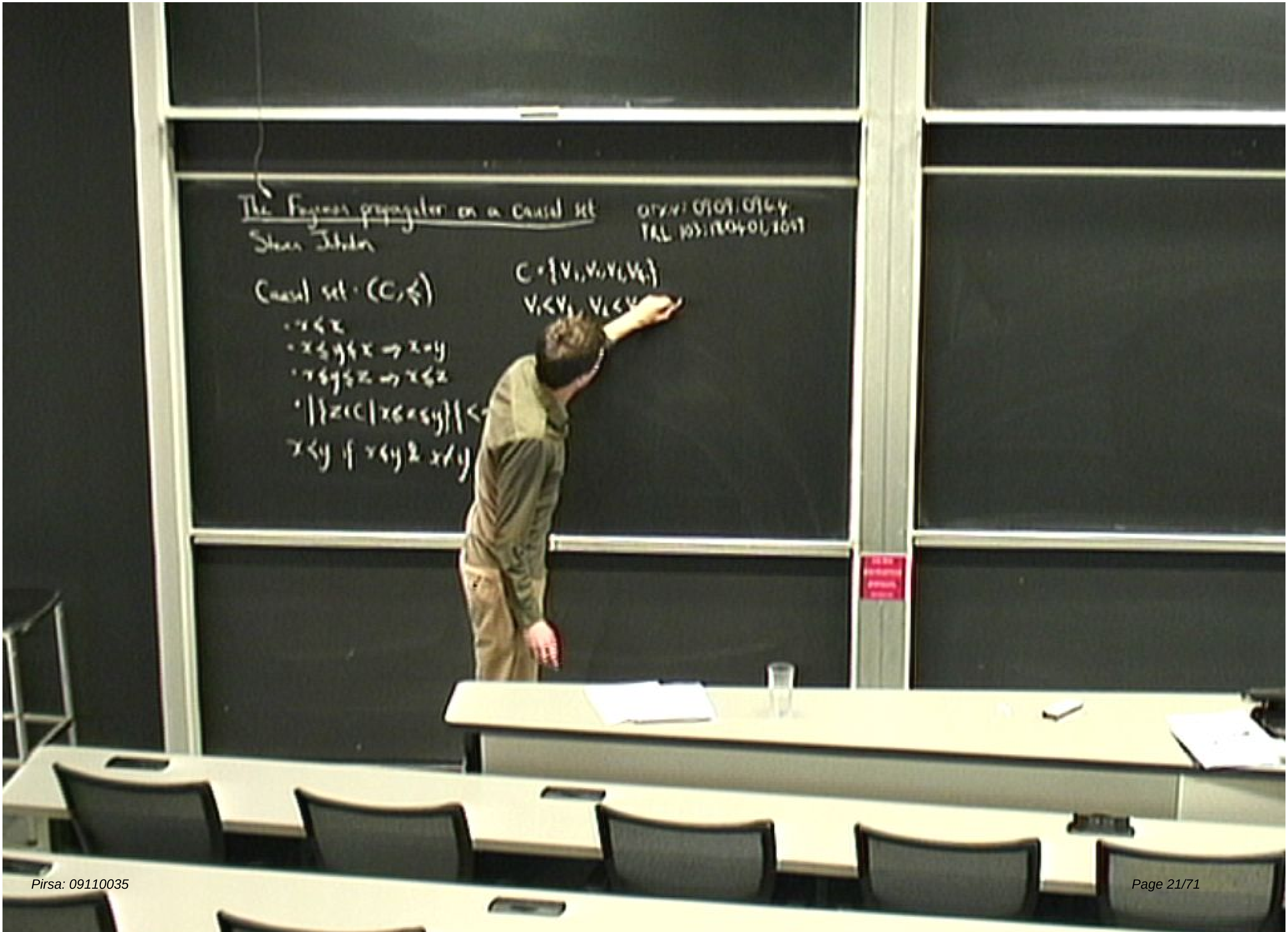
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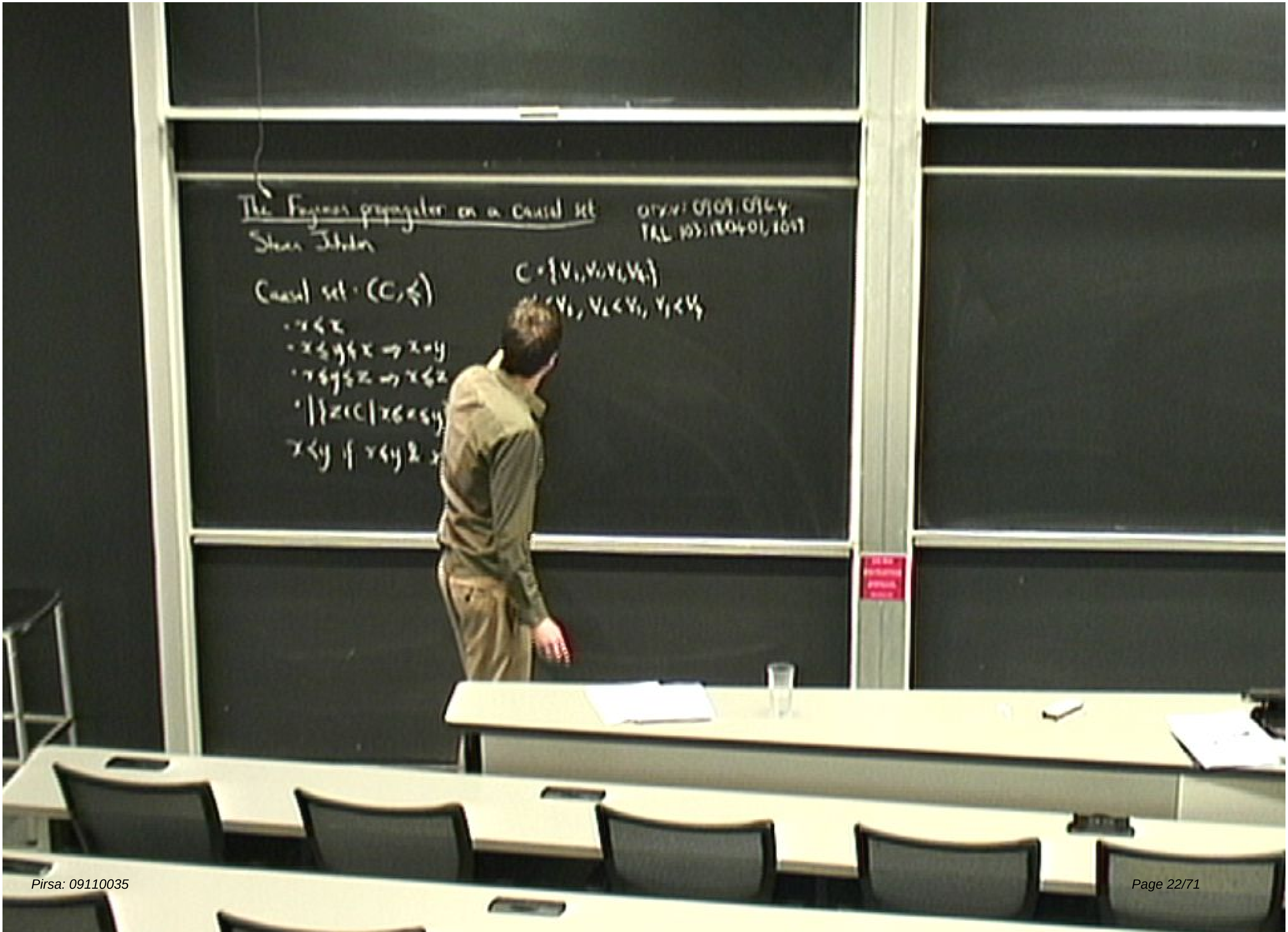

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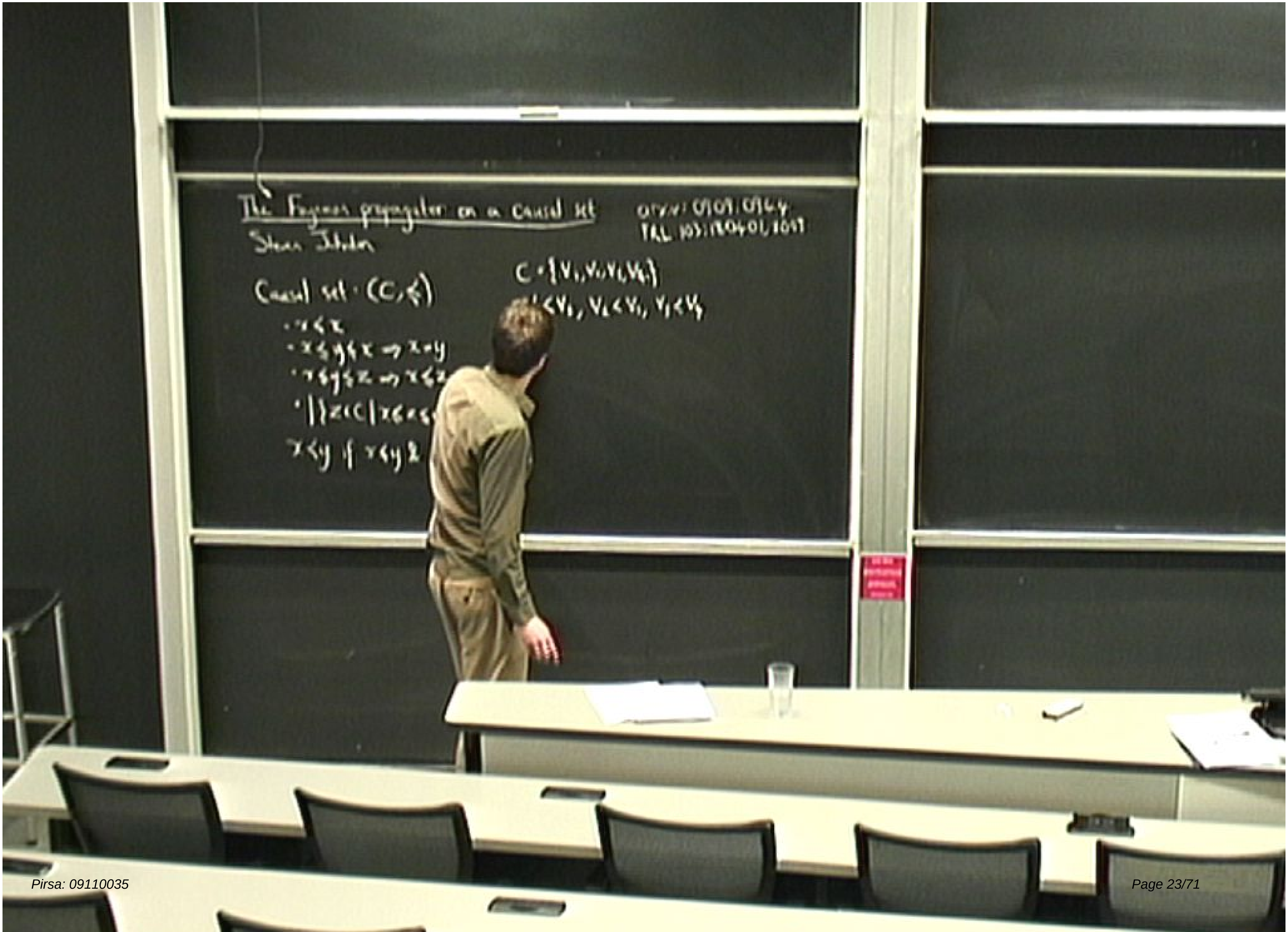
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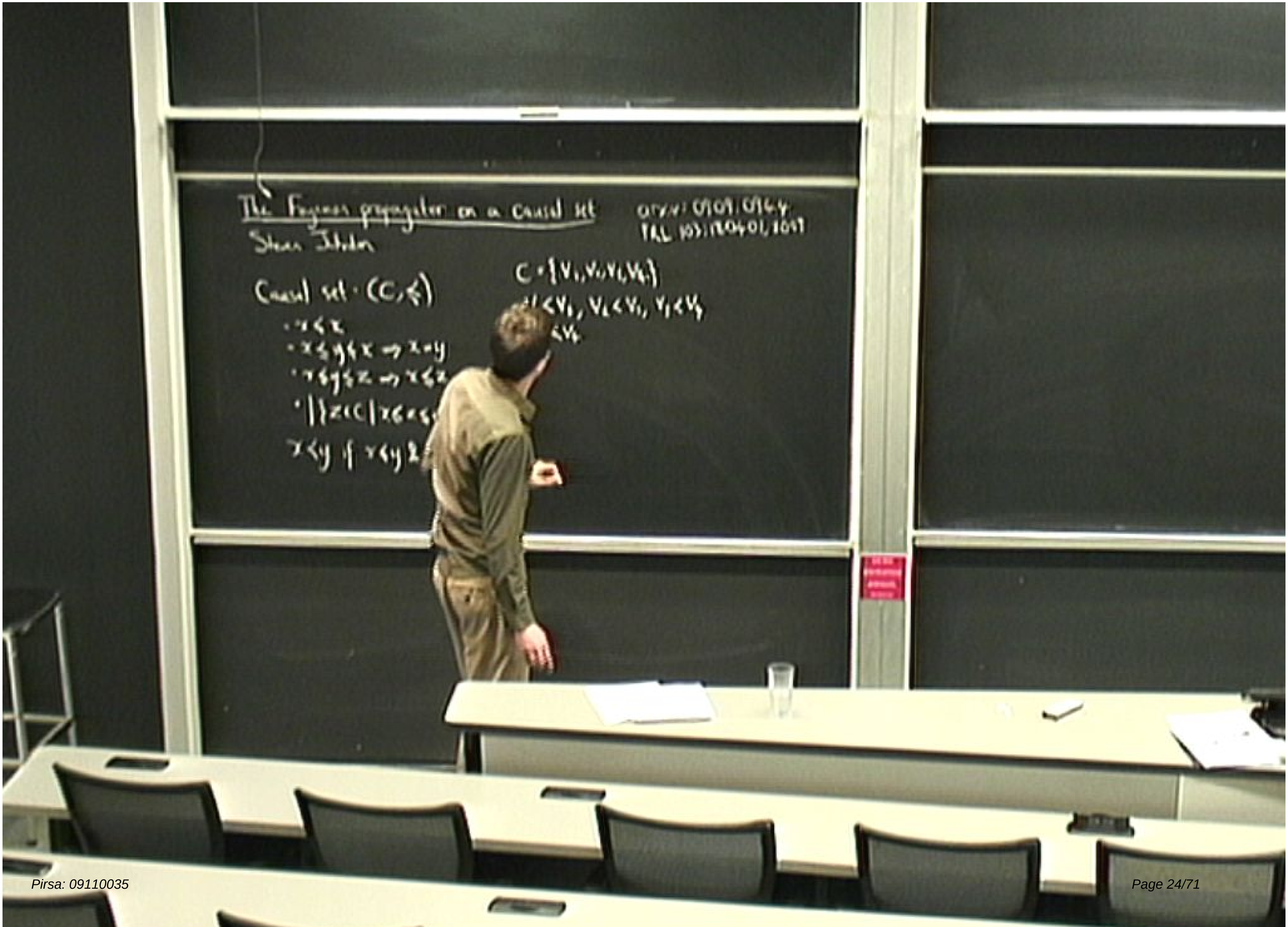
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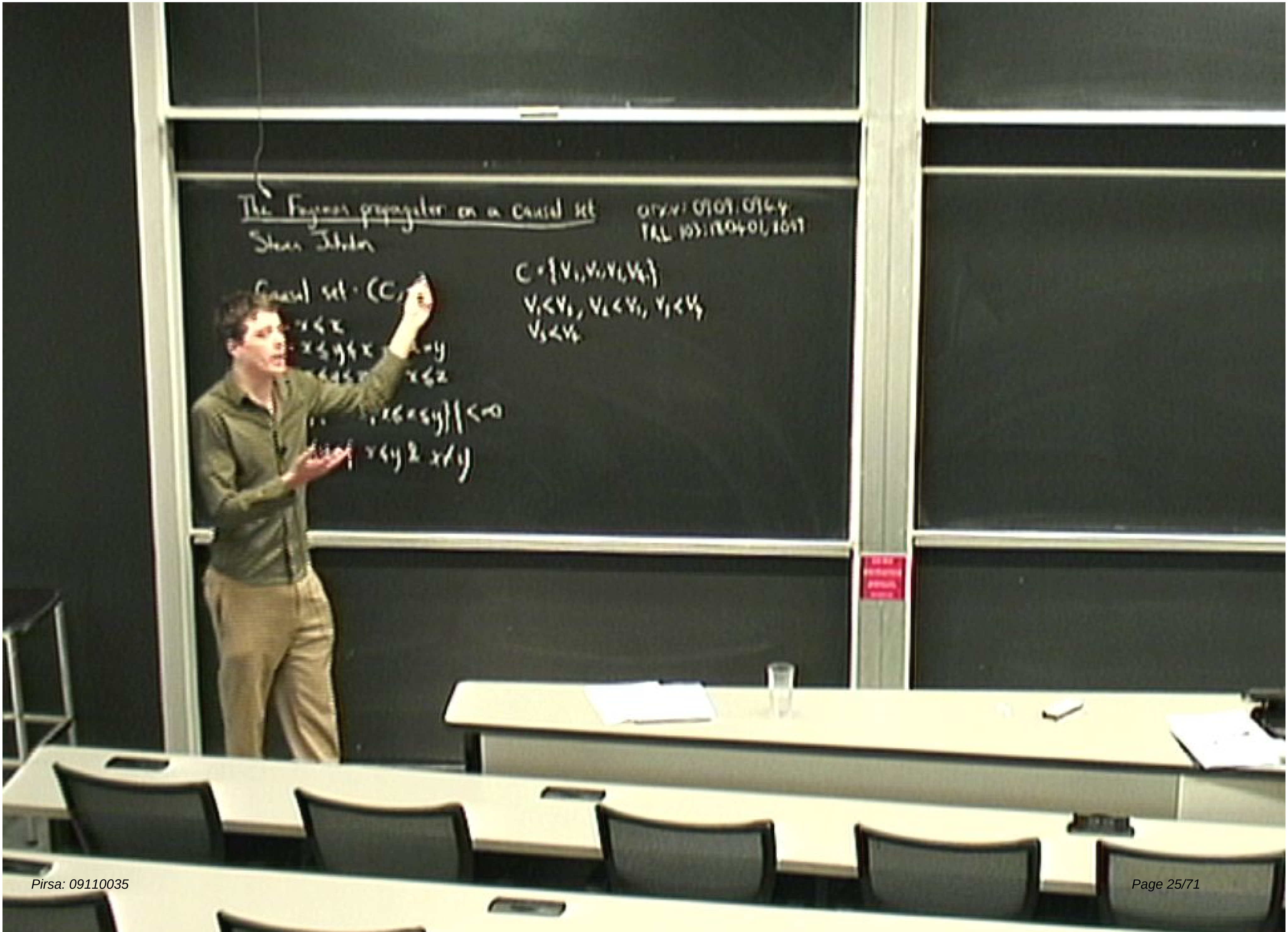
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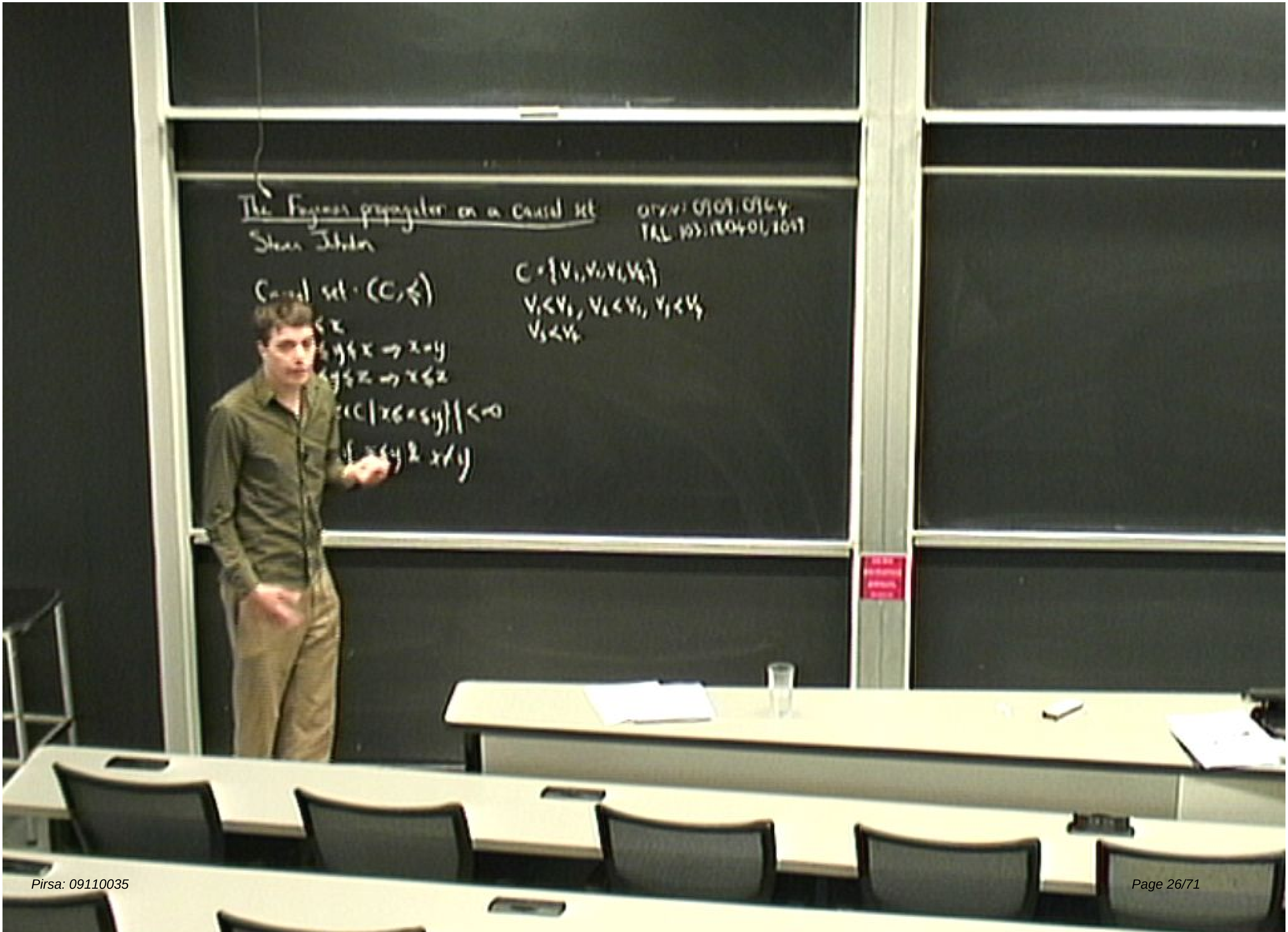
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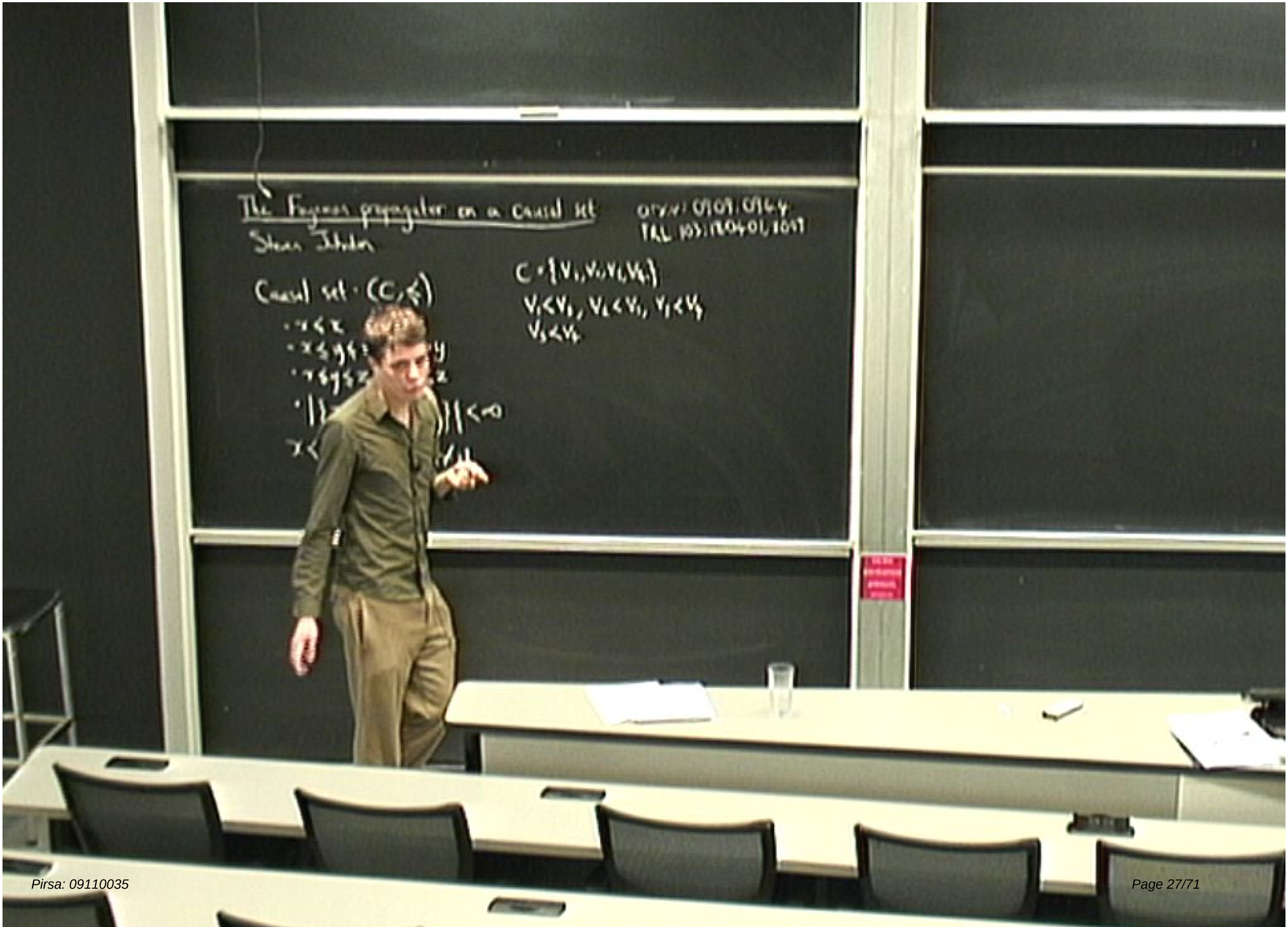
Causal set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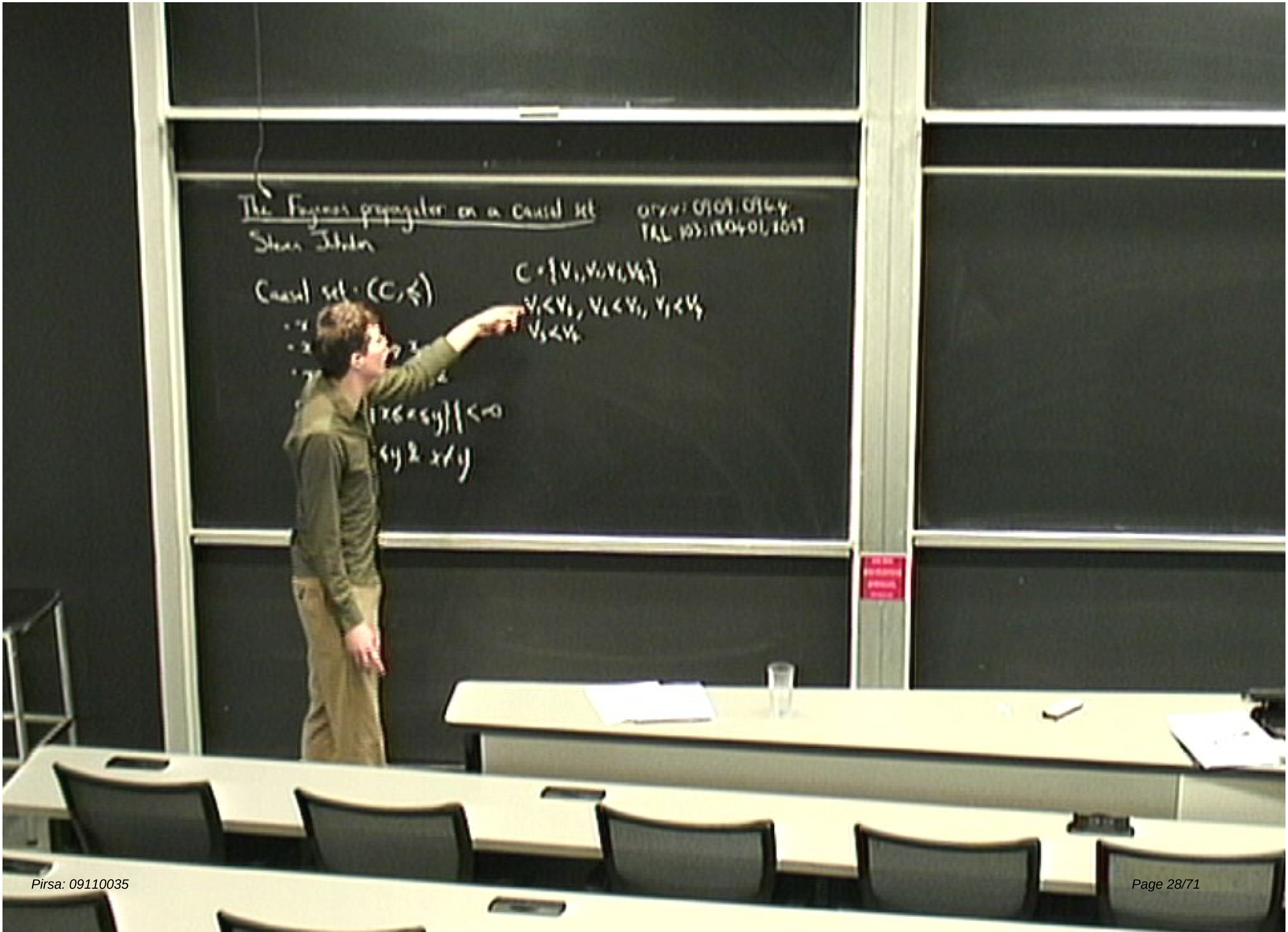
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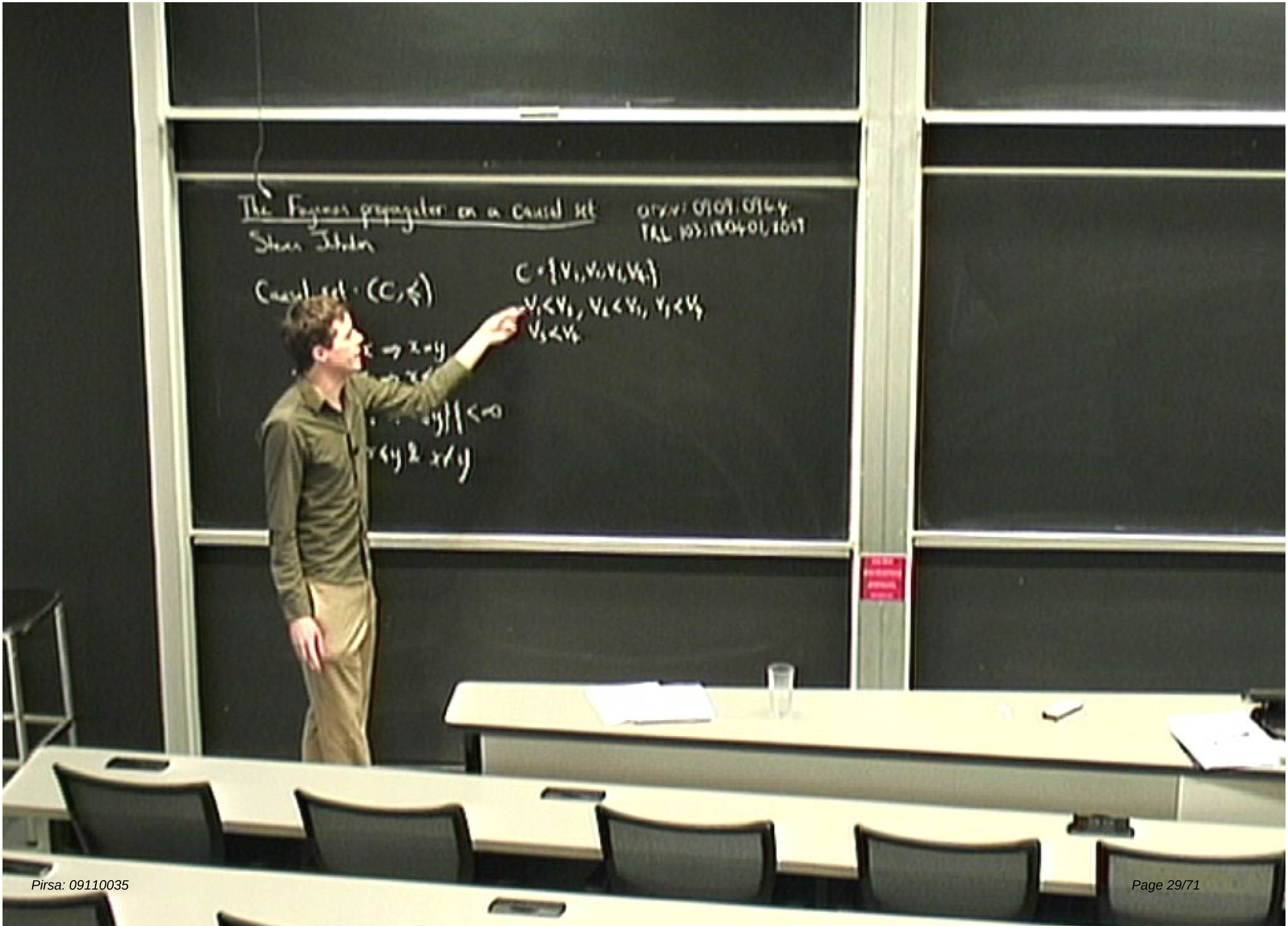
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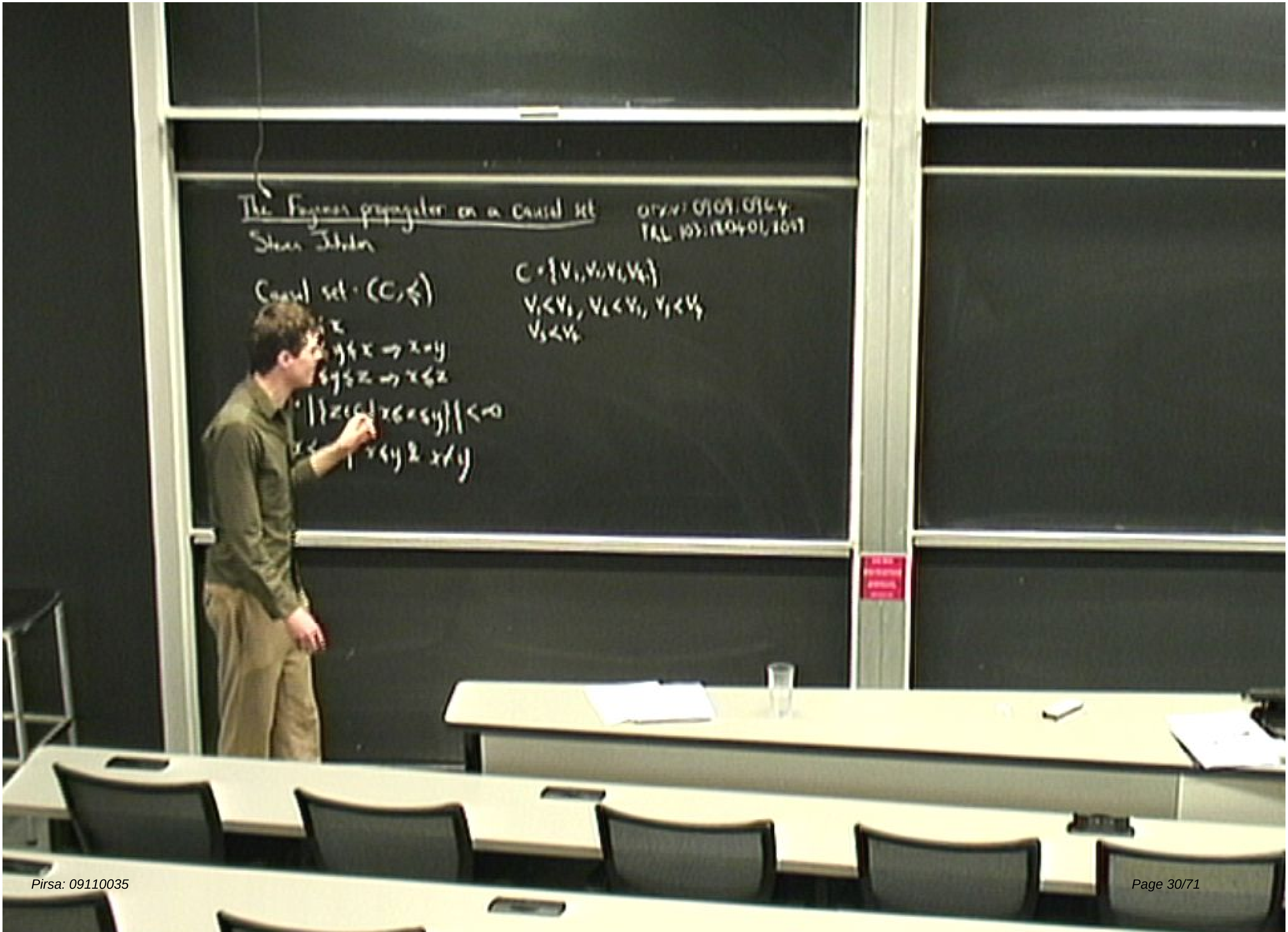
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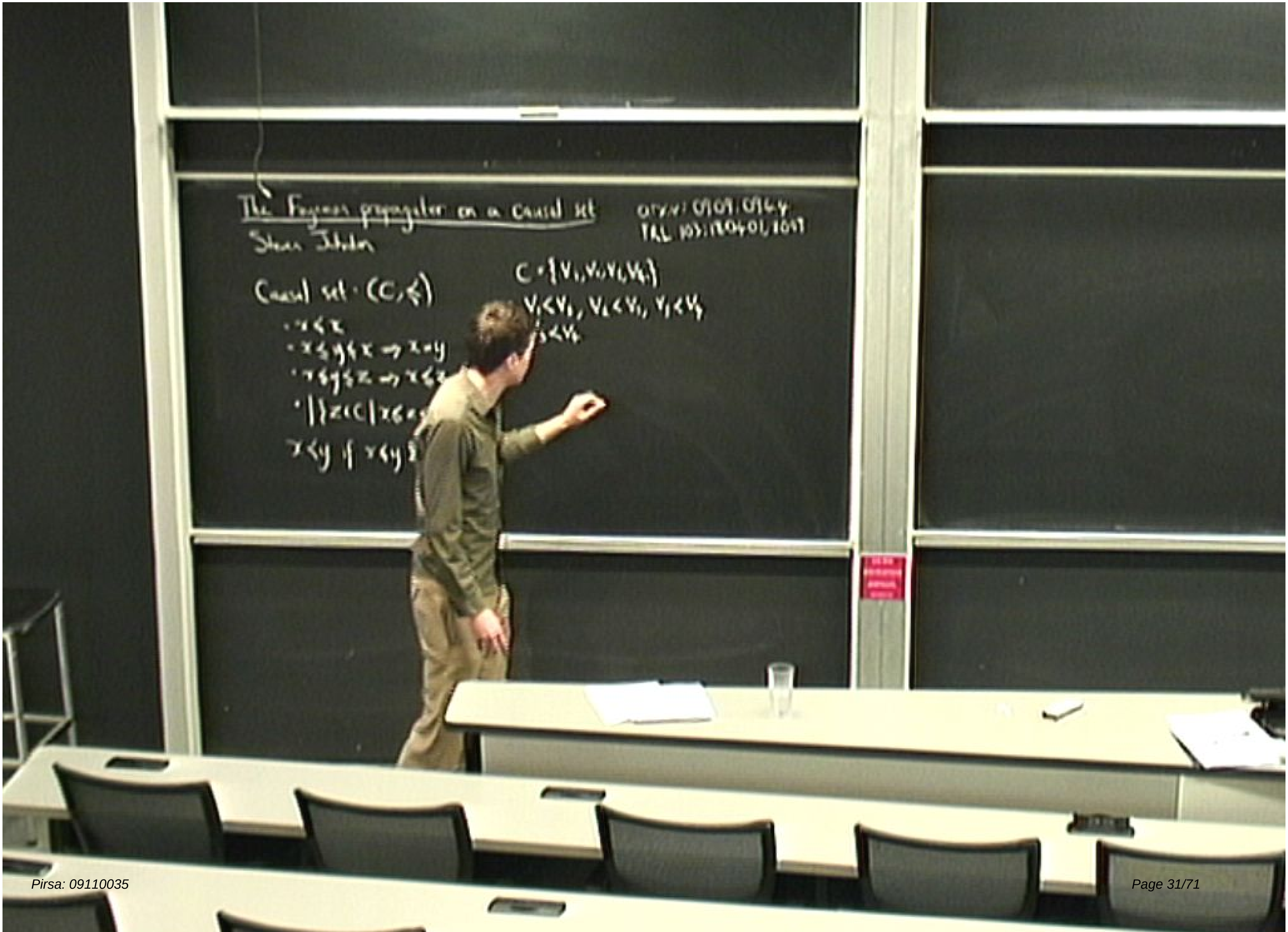
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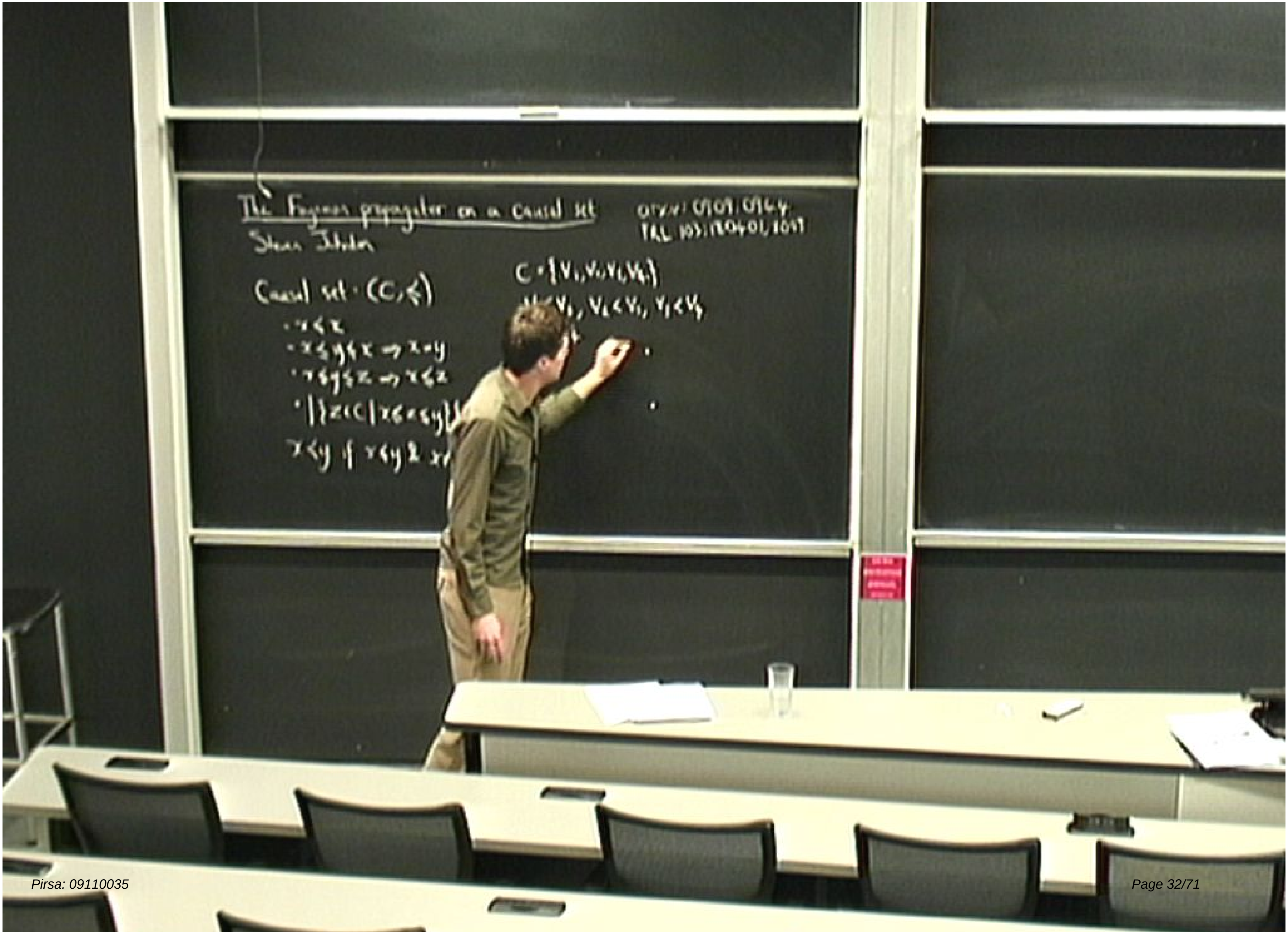
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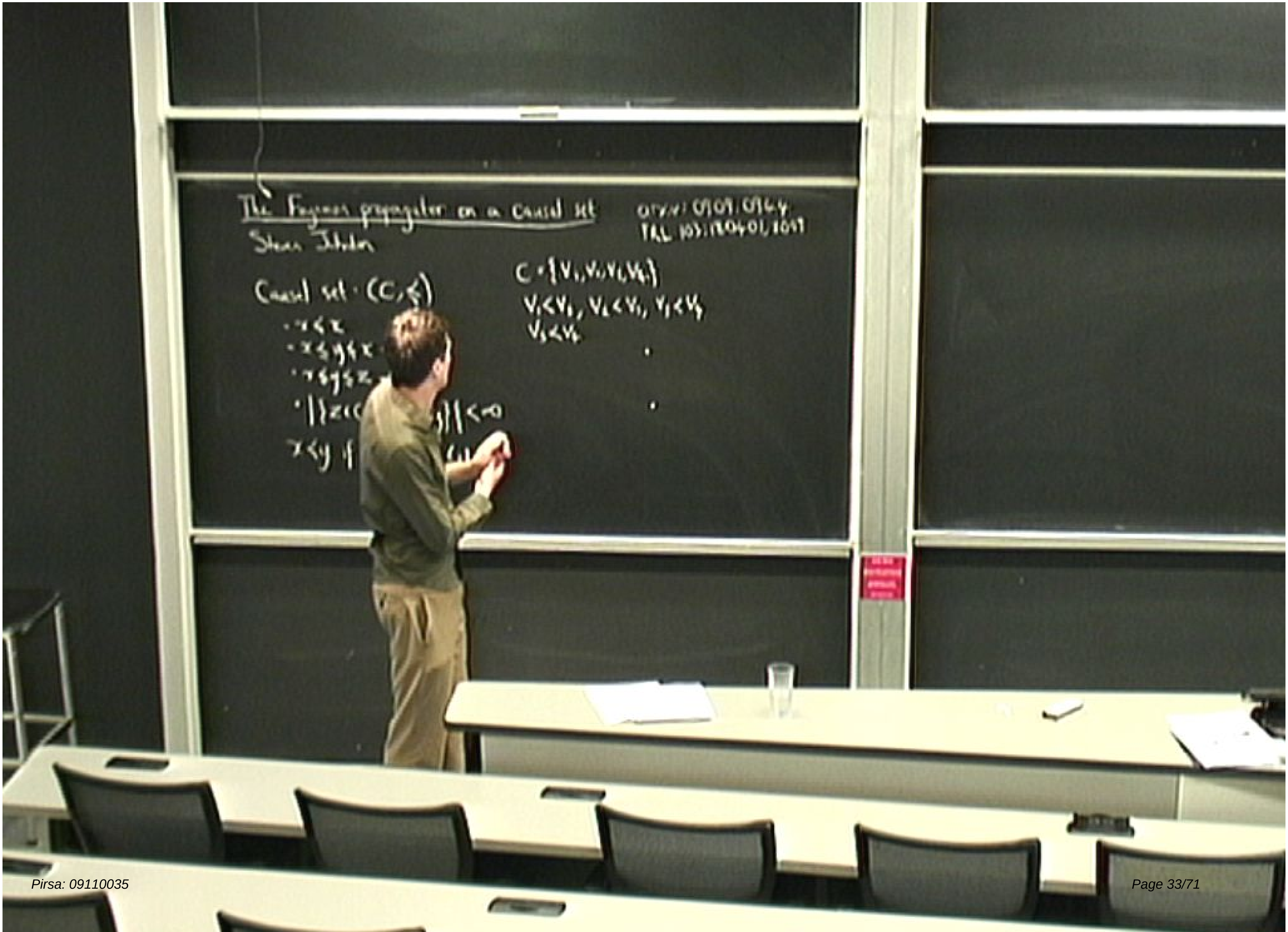
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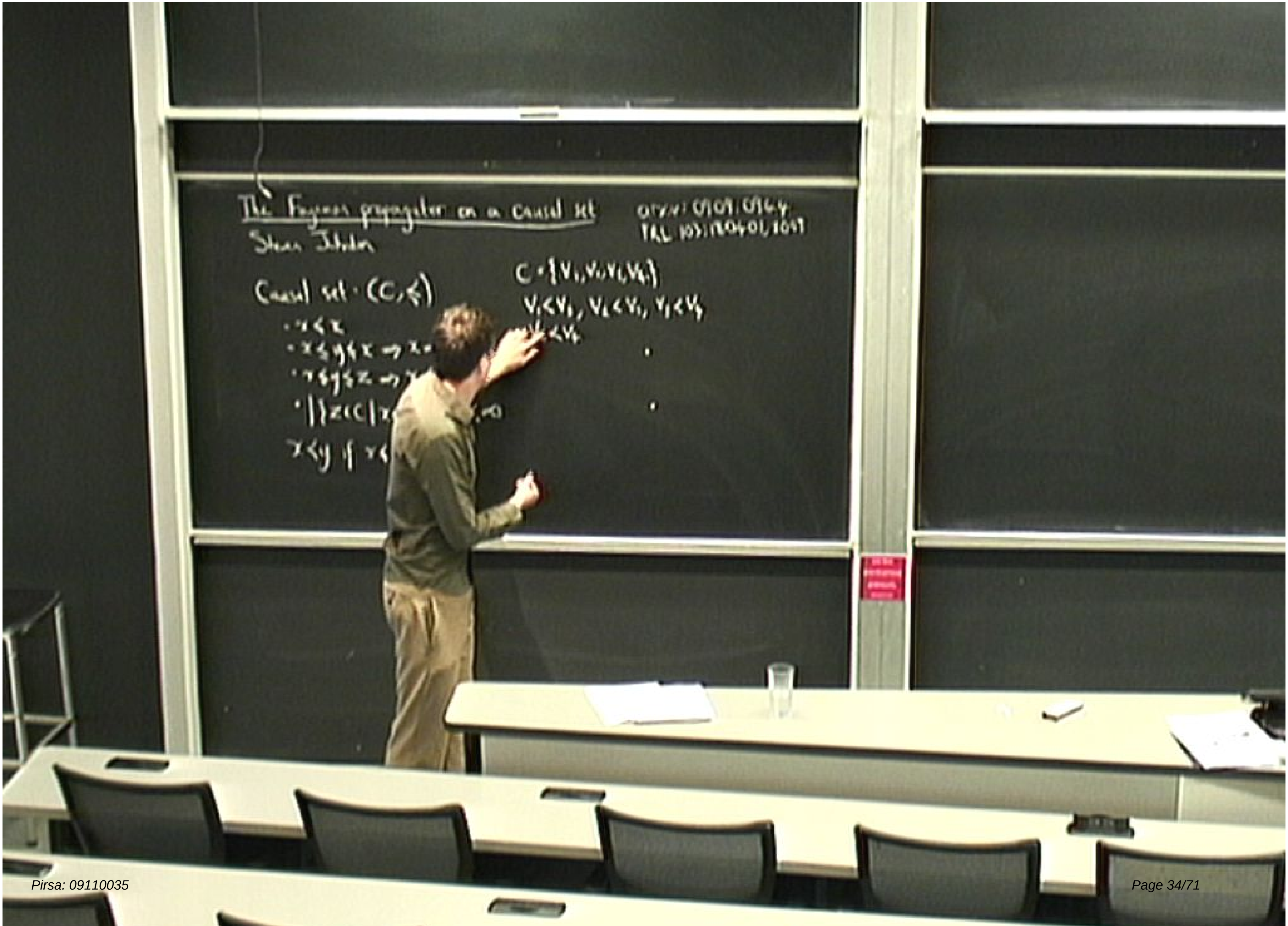
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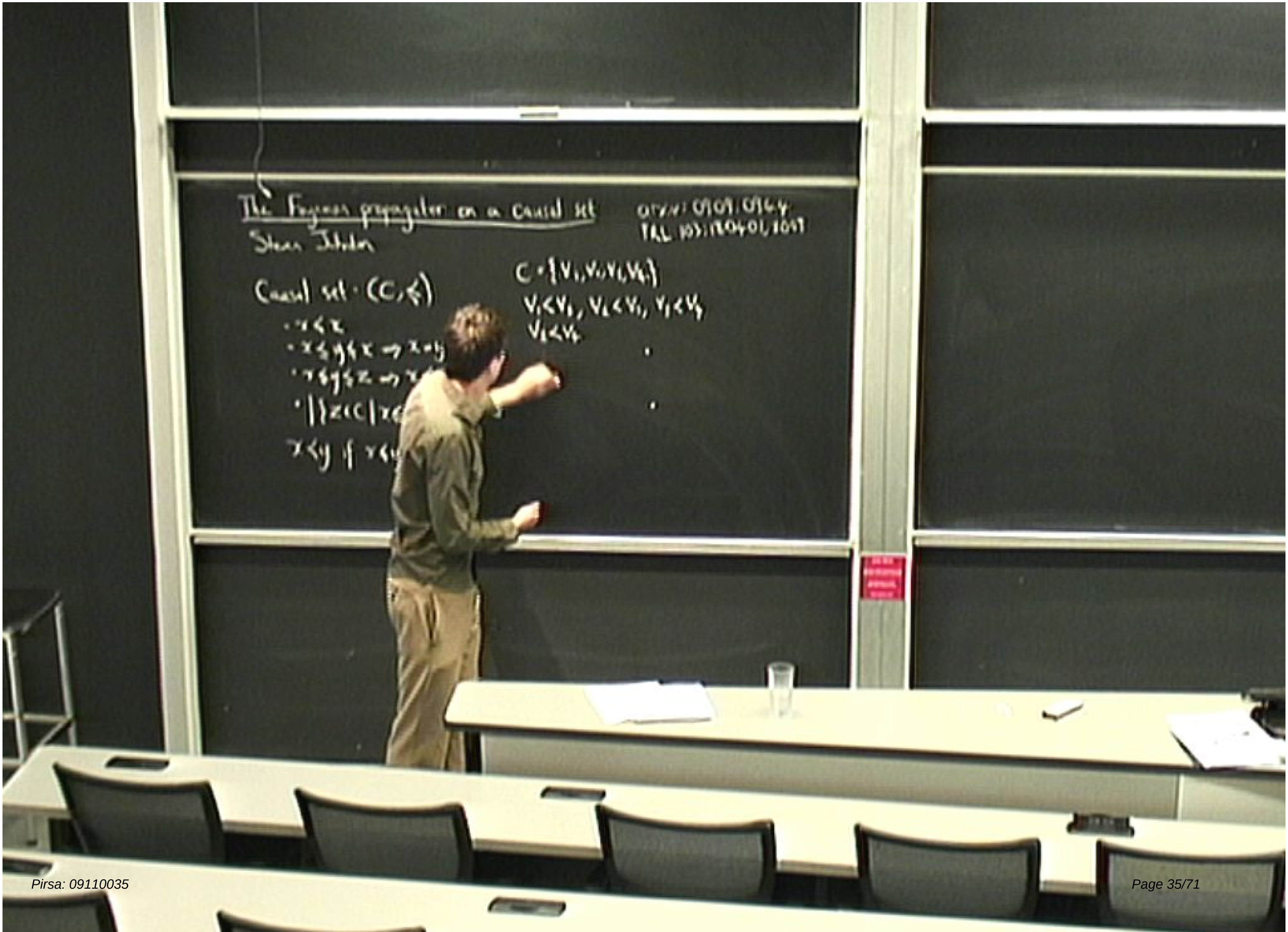
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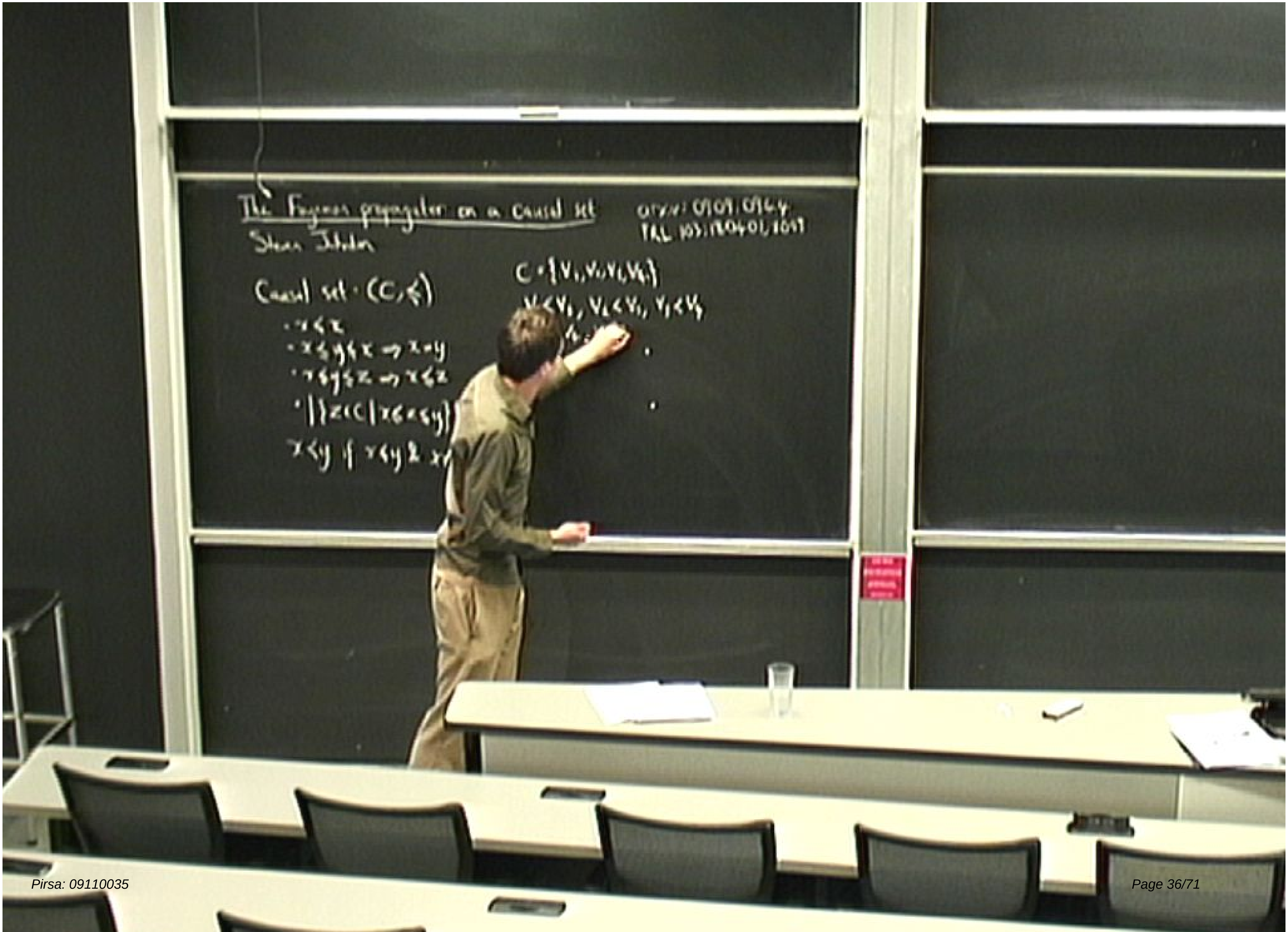
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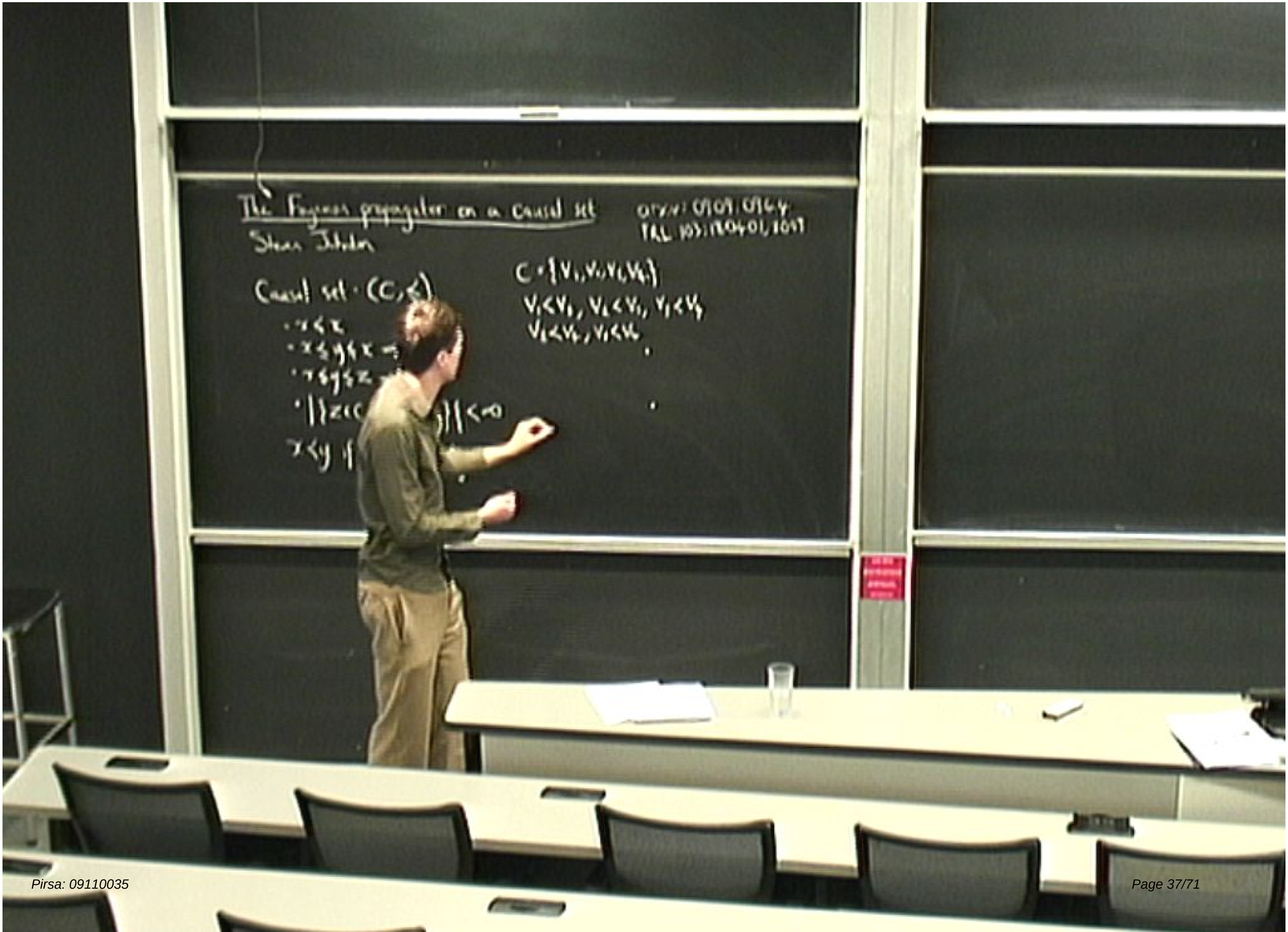

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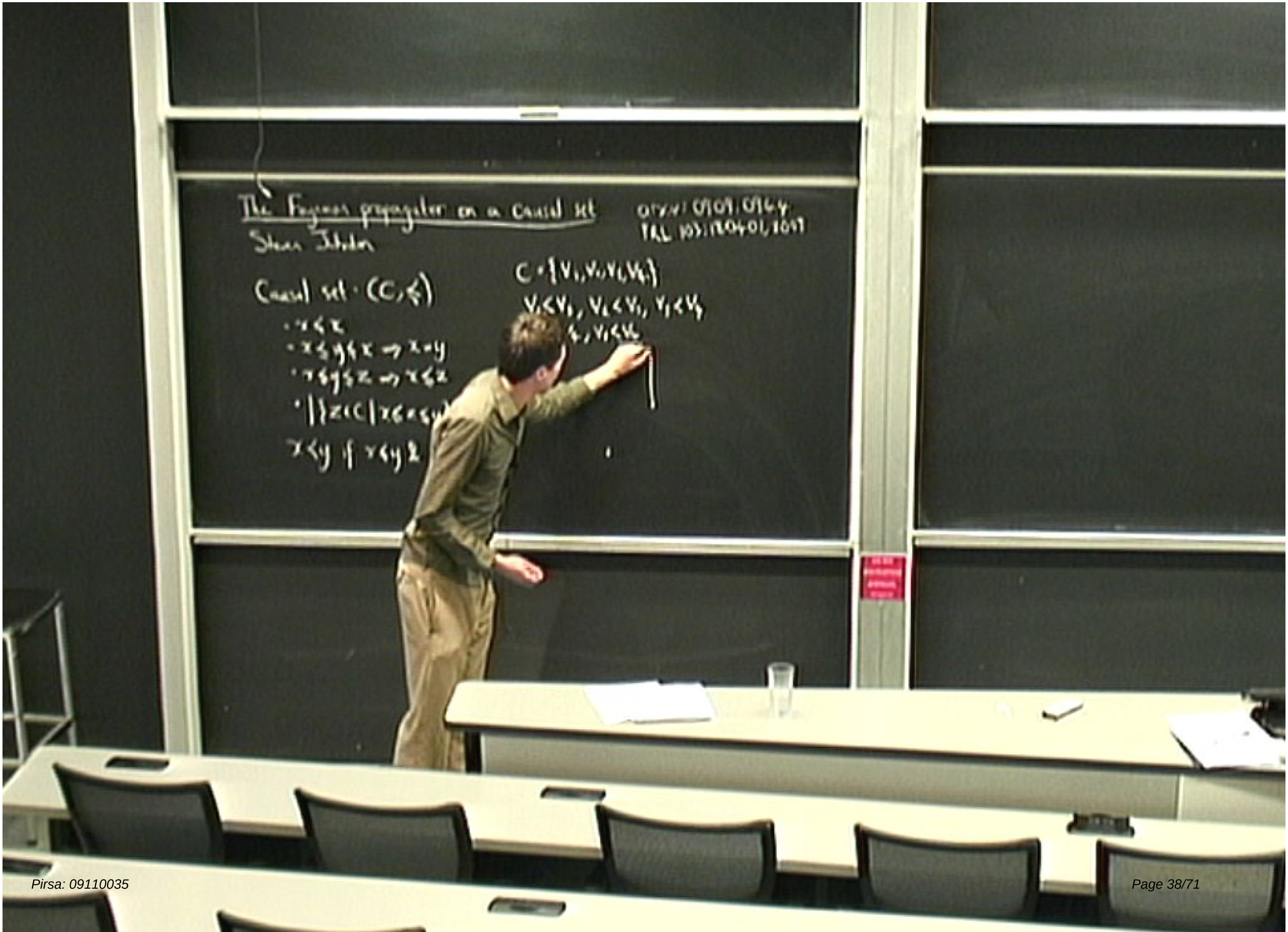
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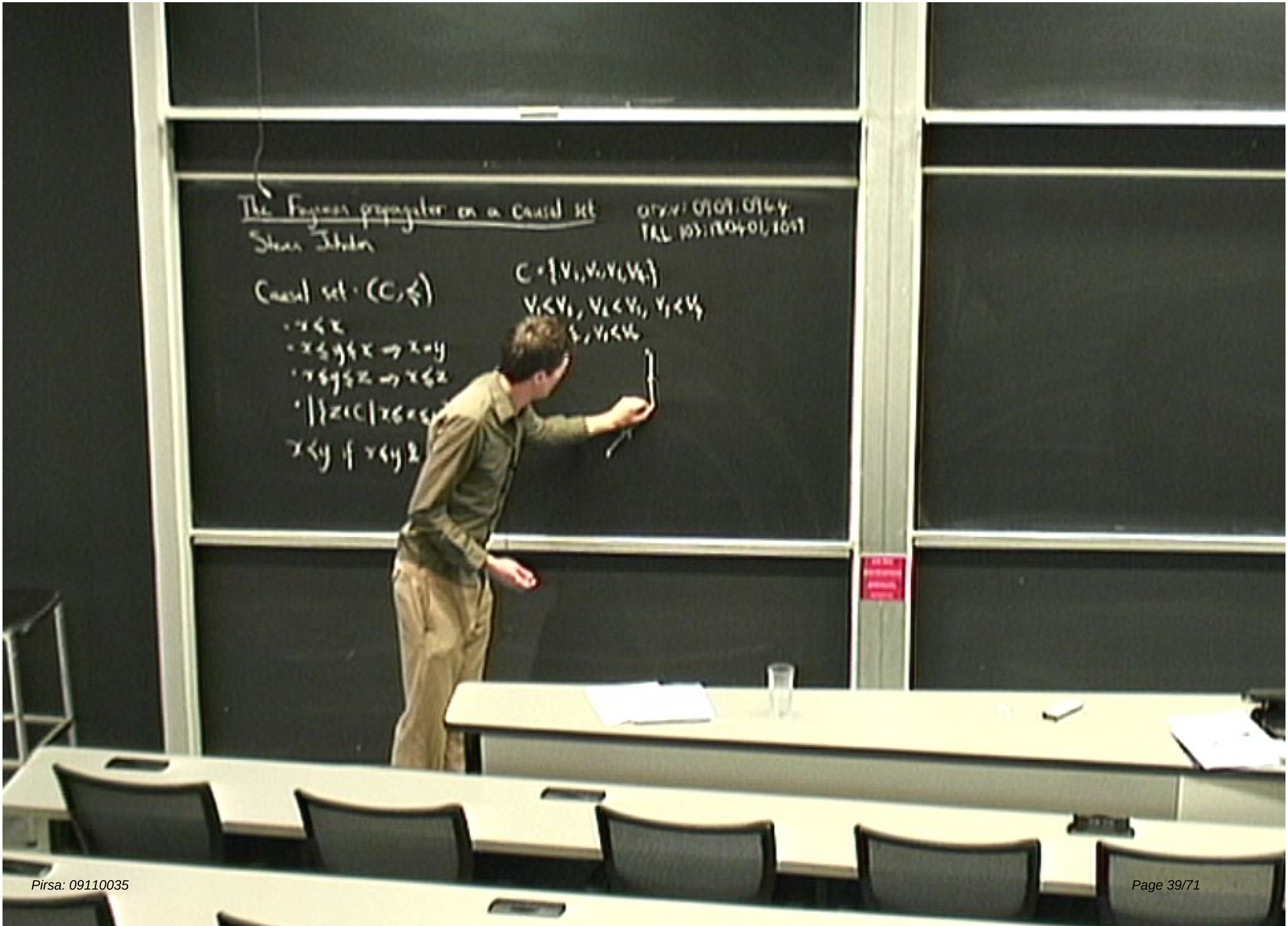
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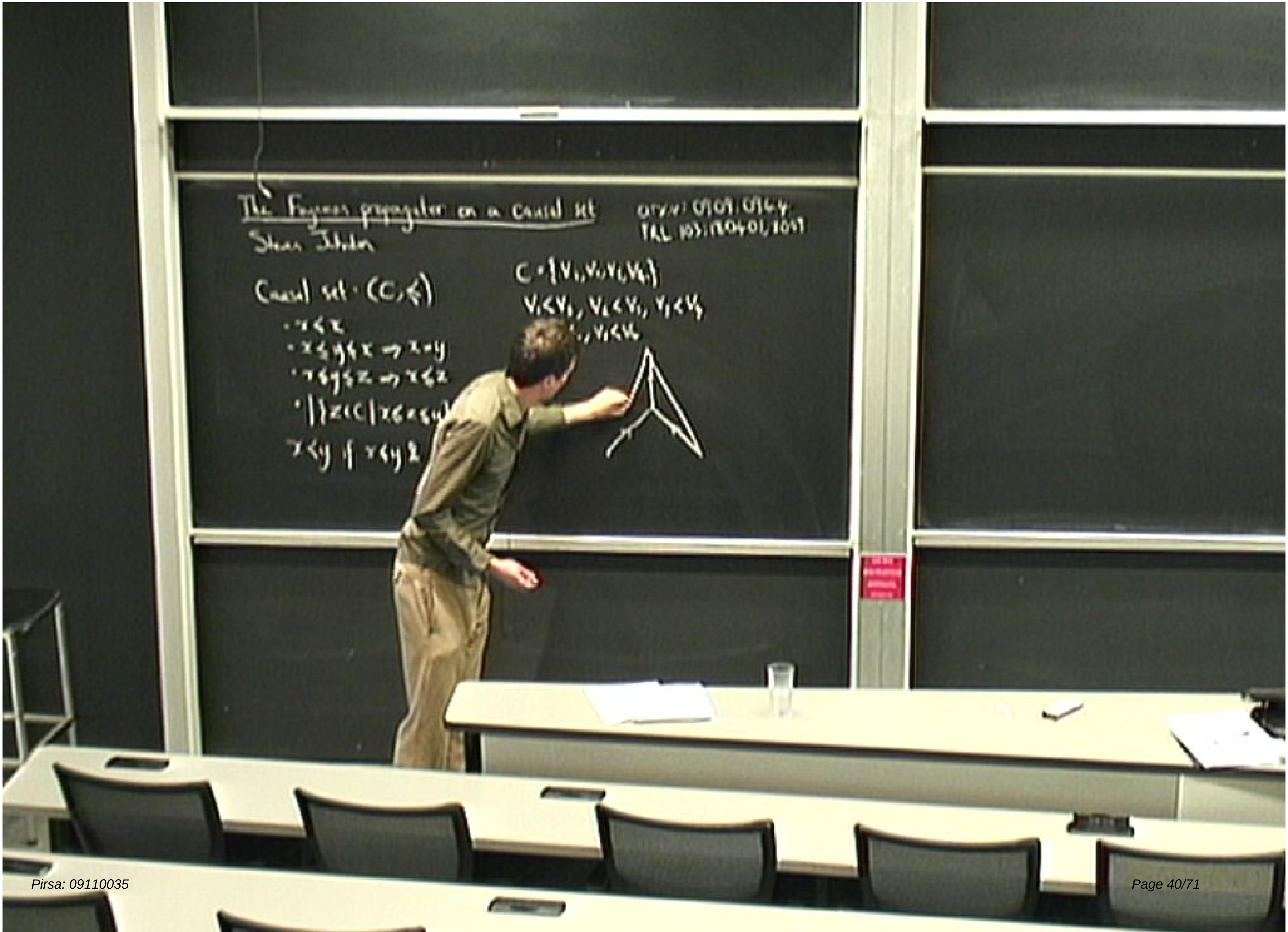
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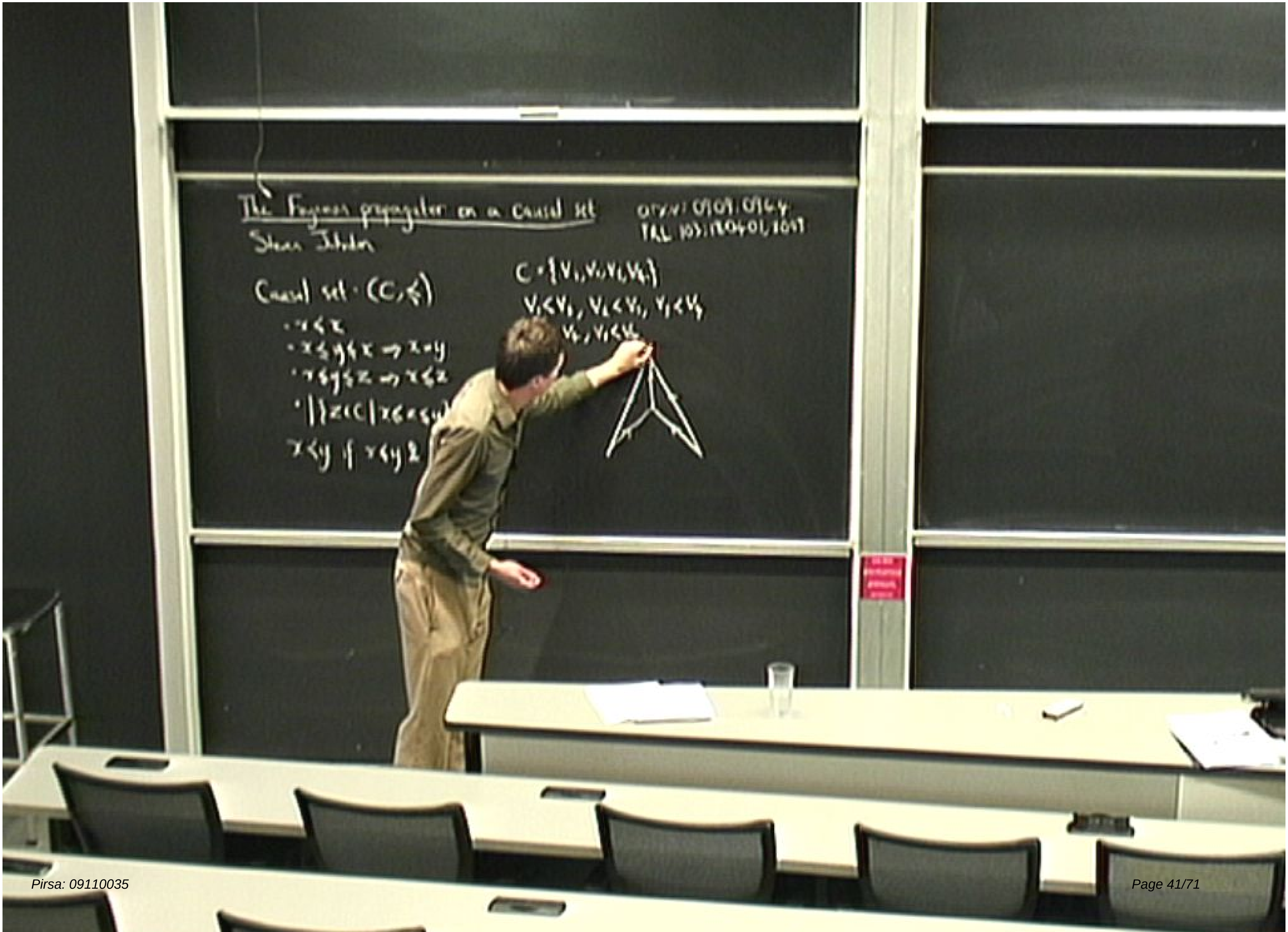
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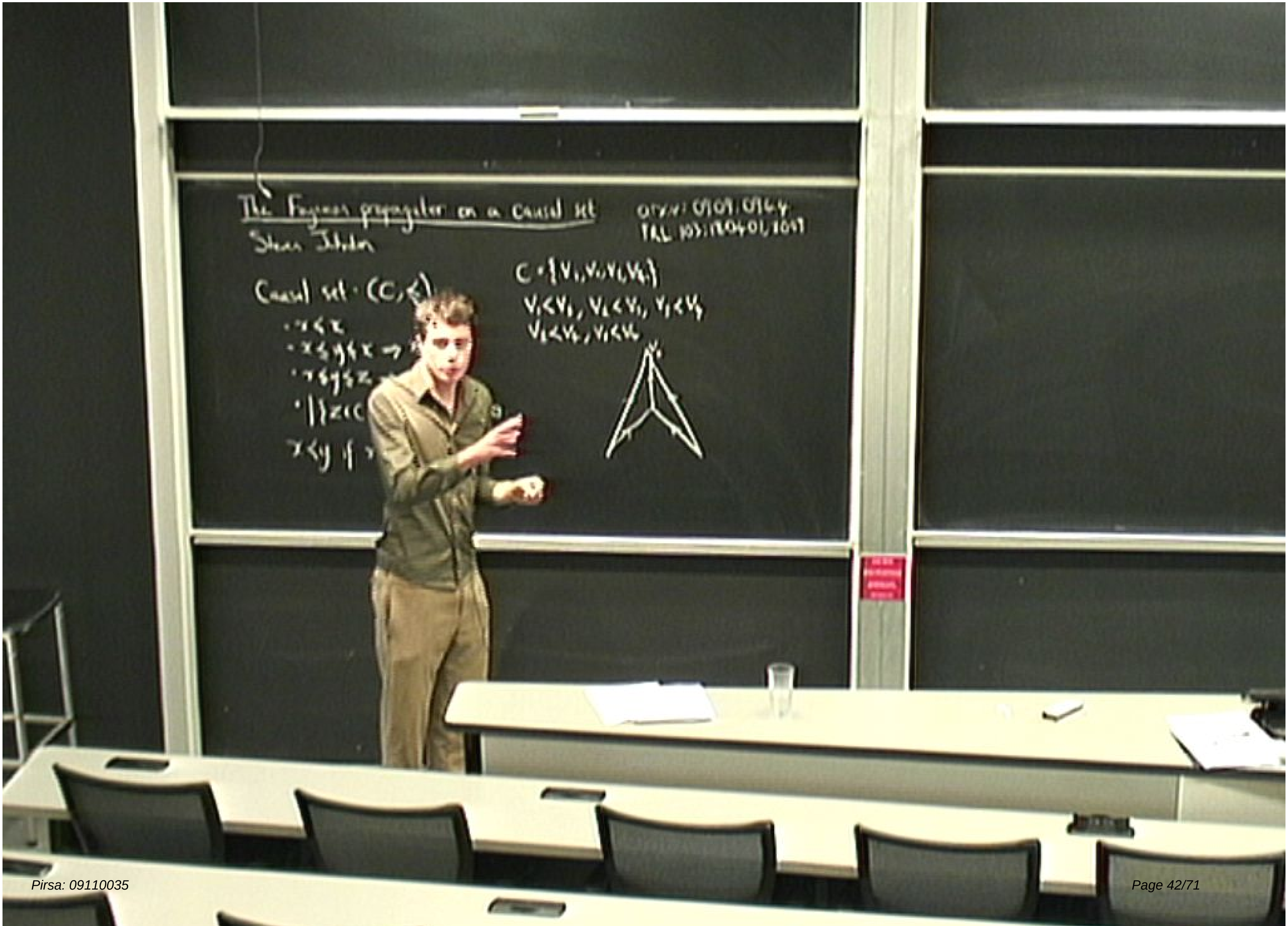
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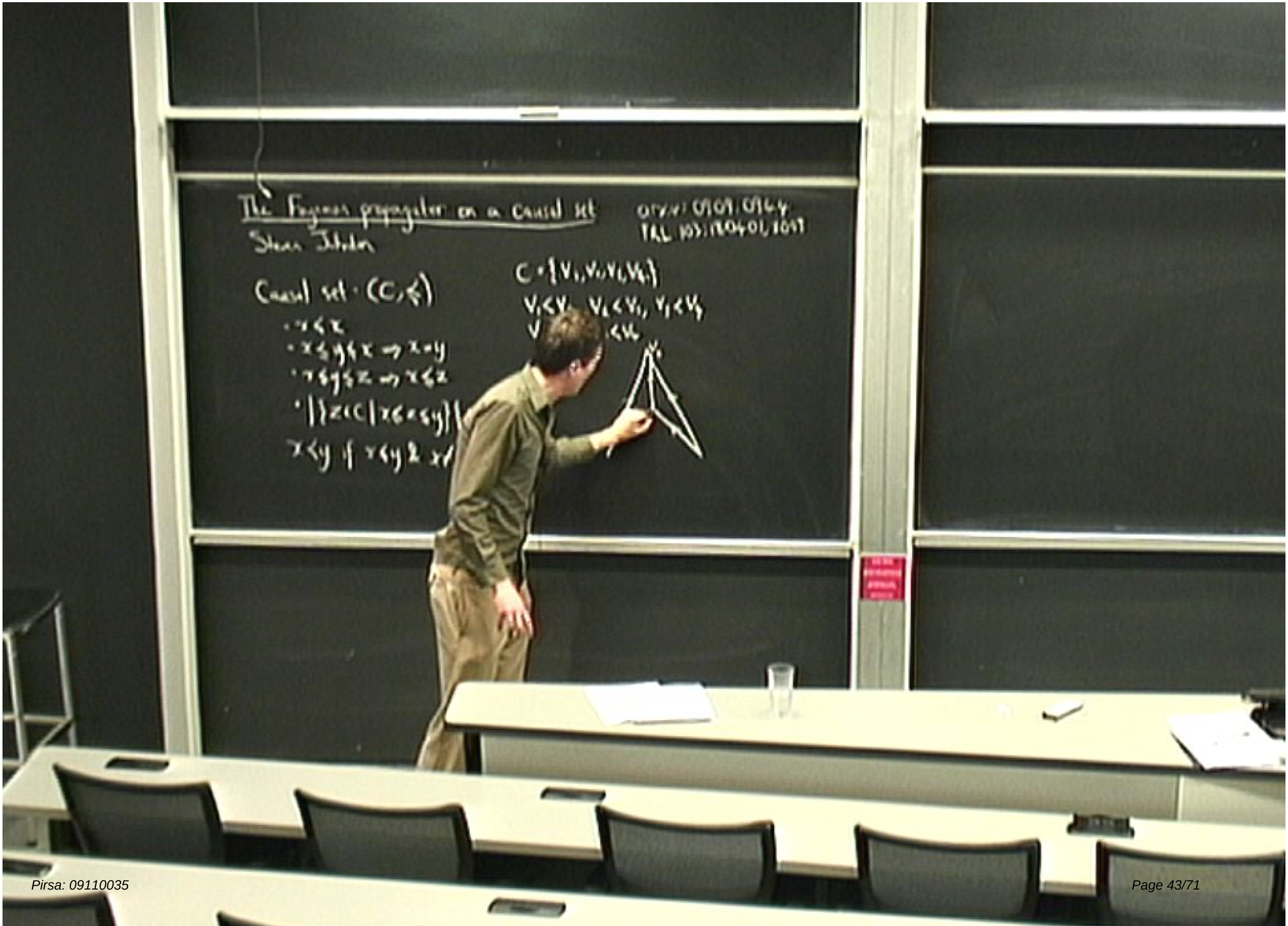
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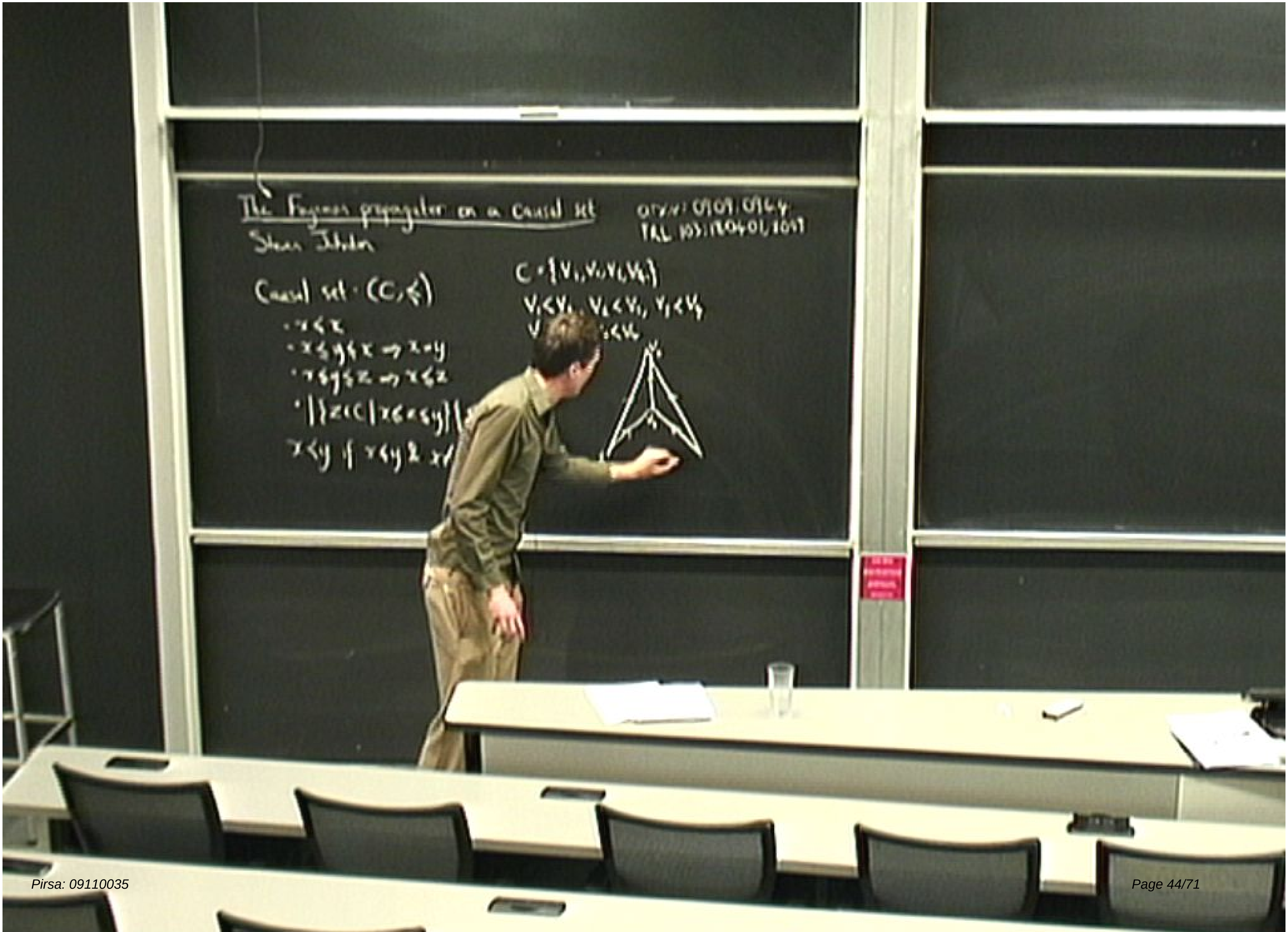
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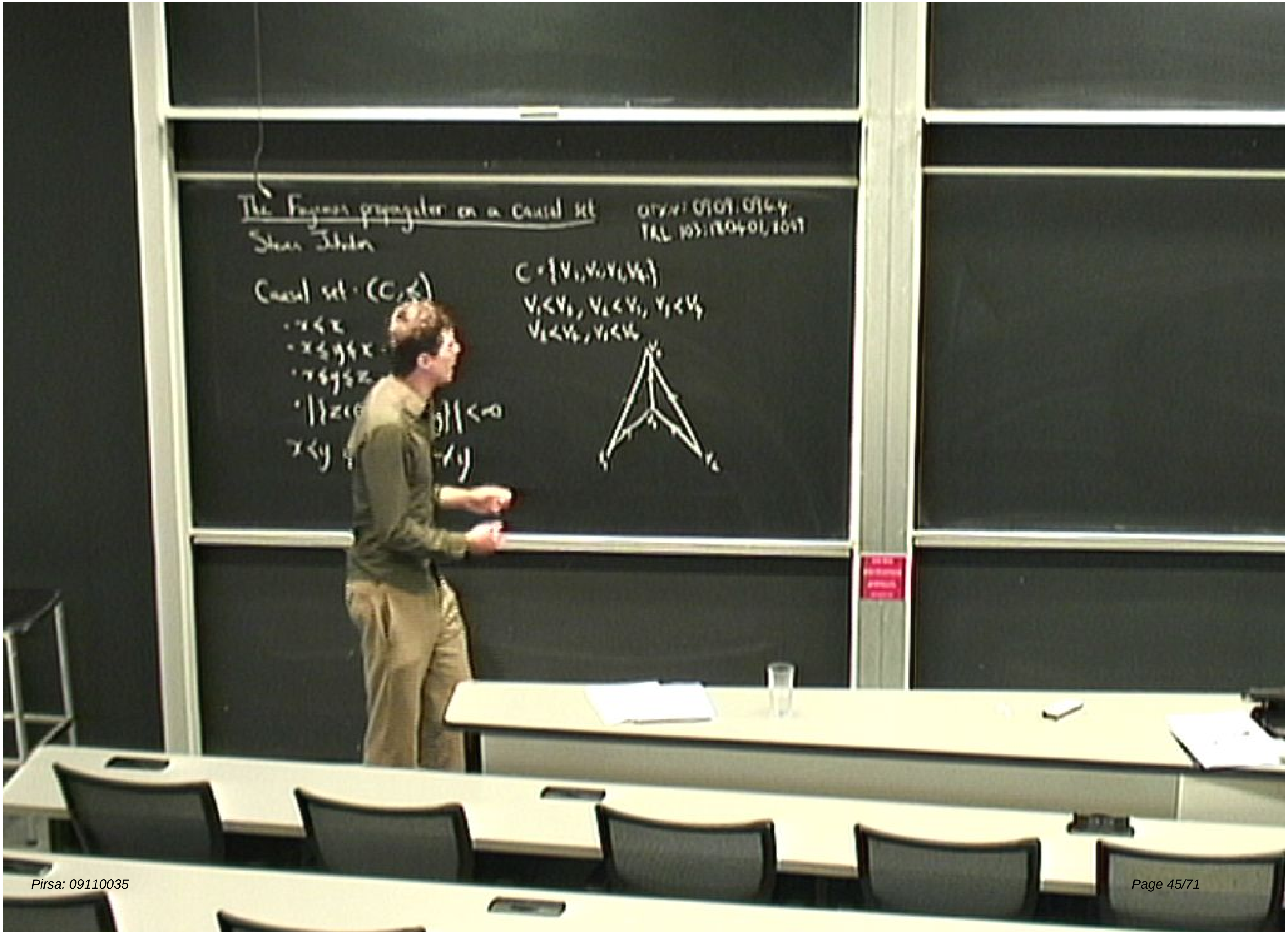
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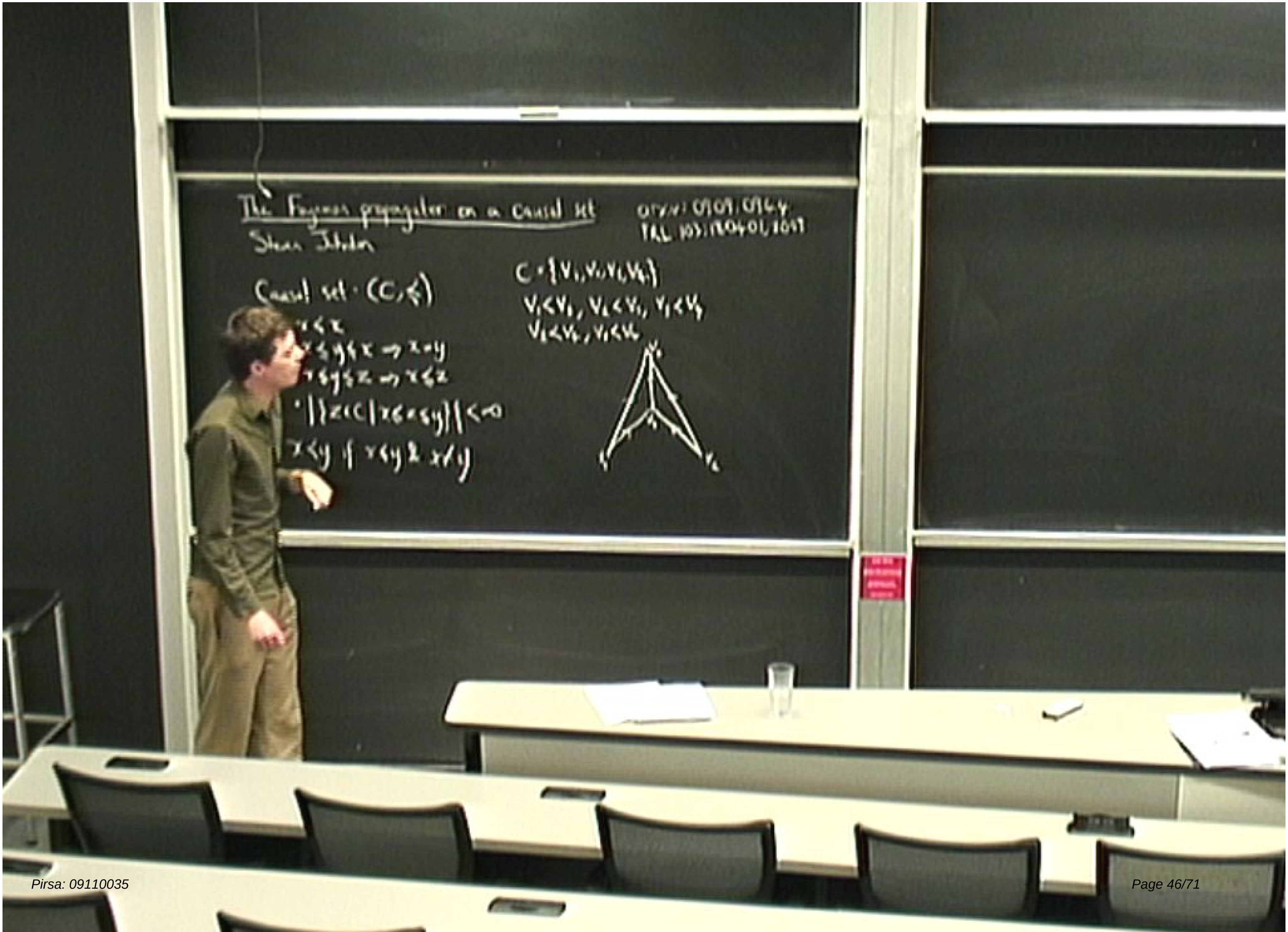
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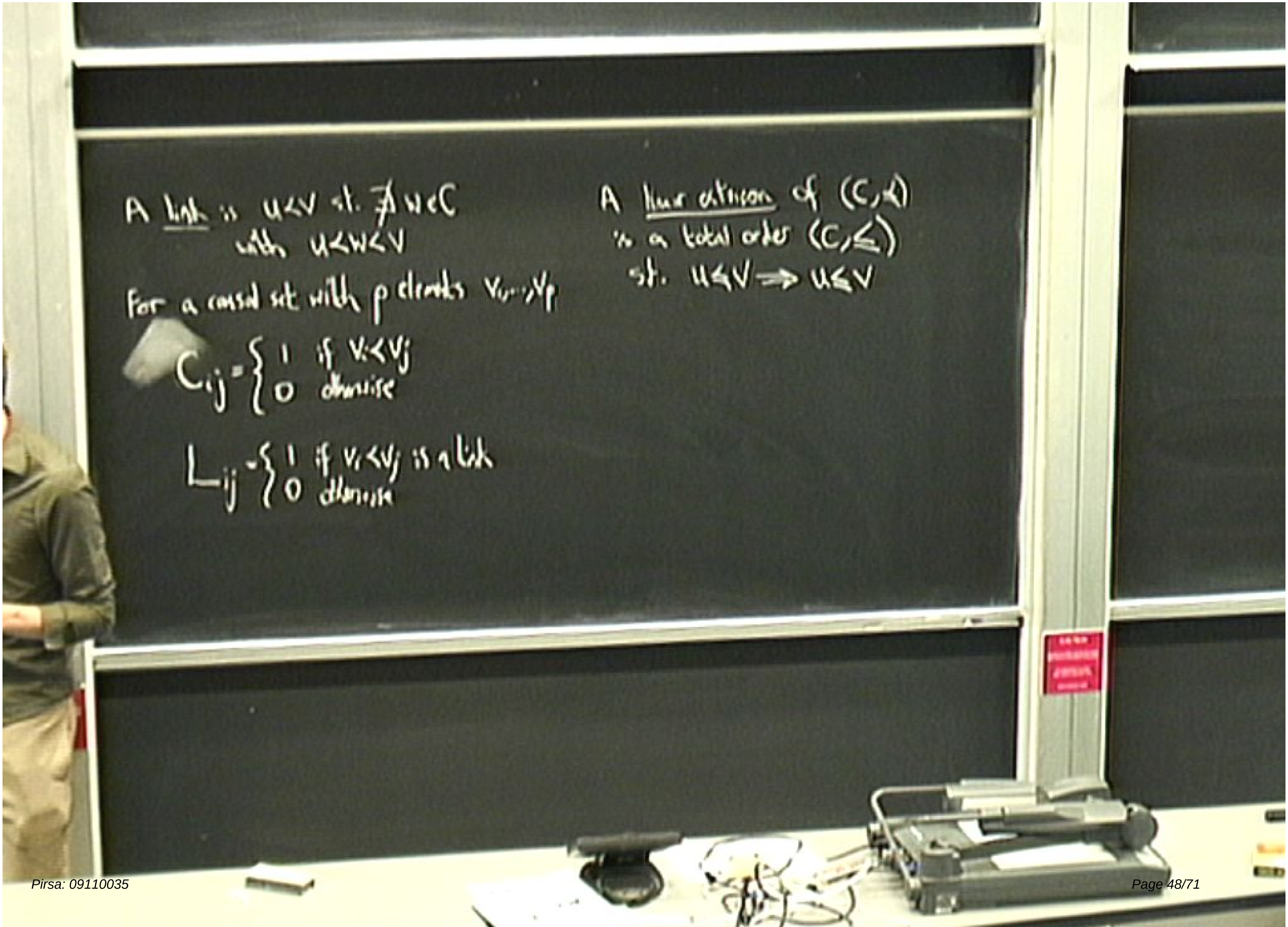
x < y if x ≤ y & x ≠ y

A link is $u \prec v$ s.t. $\nexists\, w \in C$ with $u \prec w \prec v$

For a causal set with p elements $v_1, \ldots, v_p$

$$C_{ij} = \begin{cases} 1 & \text{if } v_i \prec v_j \\ 0 & \text{otherwise} \end{cases}$$

$$L_{ij} = \begin{cases} 1 & \text{if } v_i \prec v_j \text{ is a link} \\ 0 & \text{otherwise} \end{cases}$$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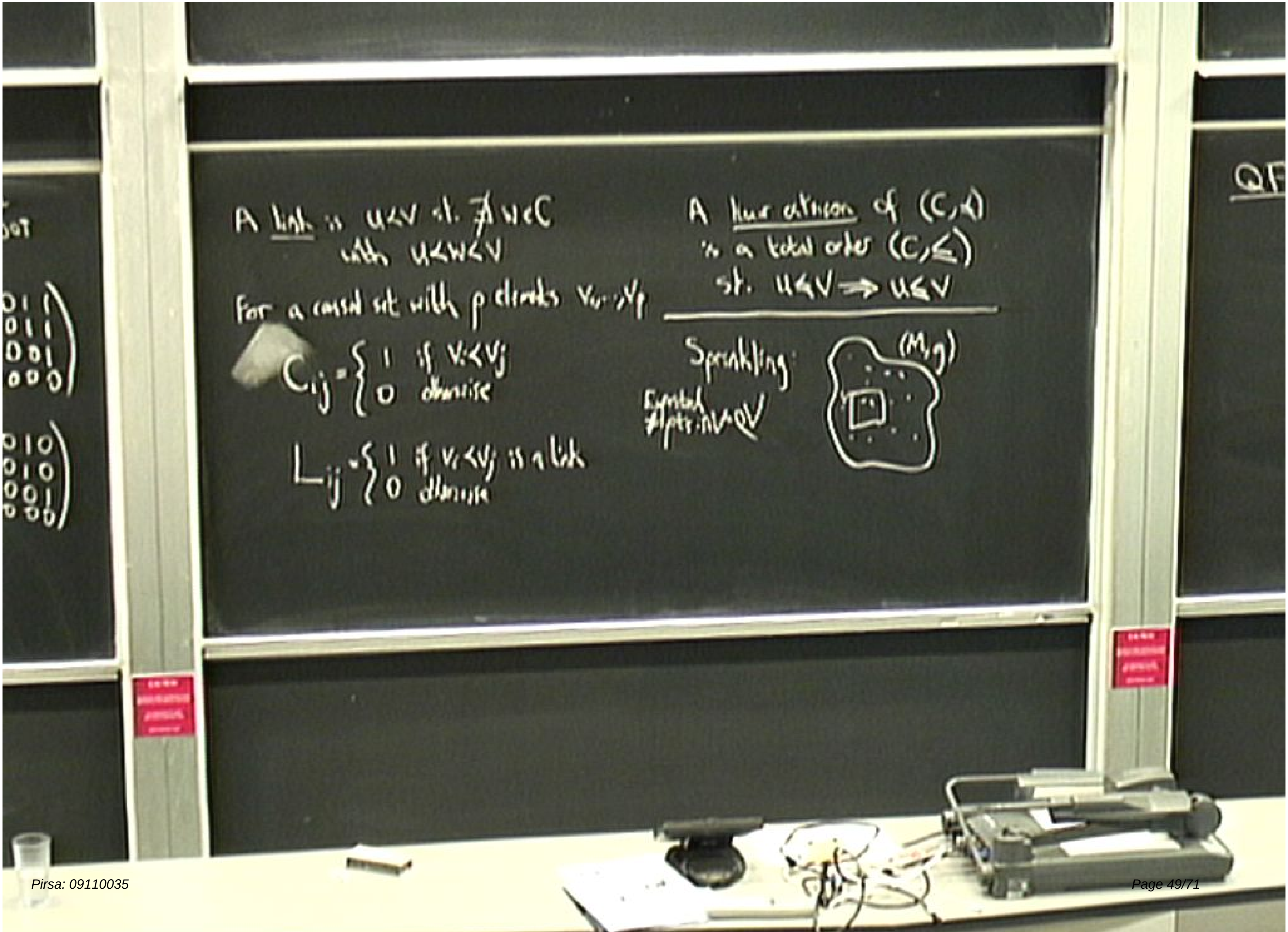
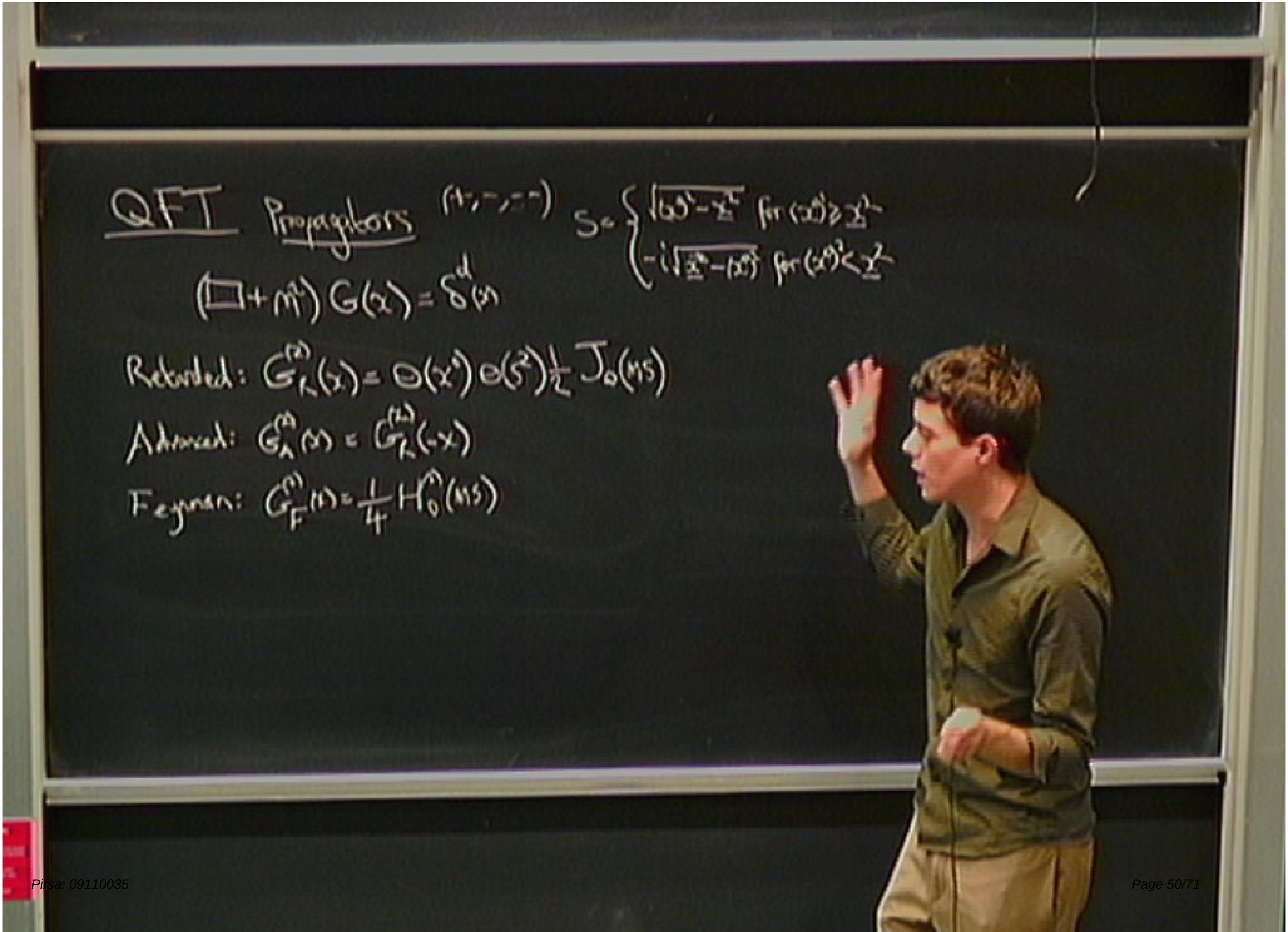
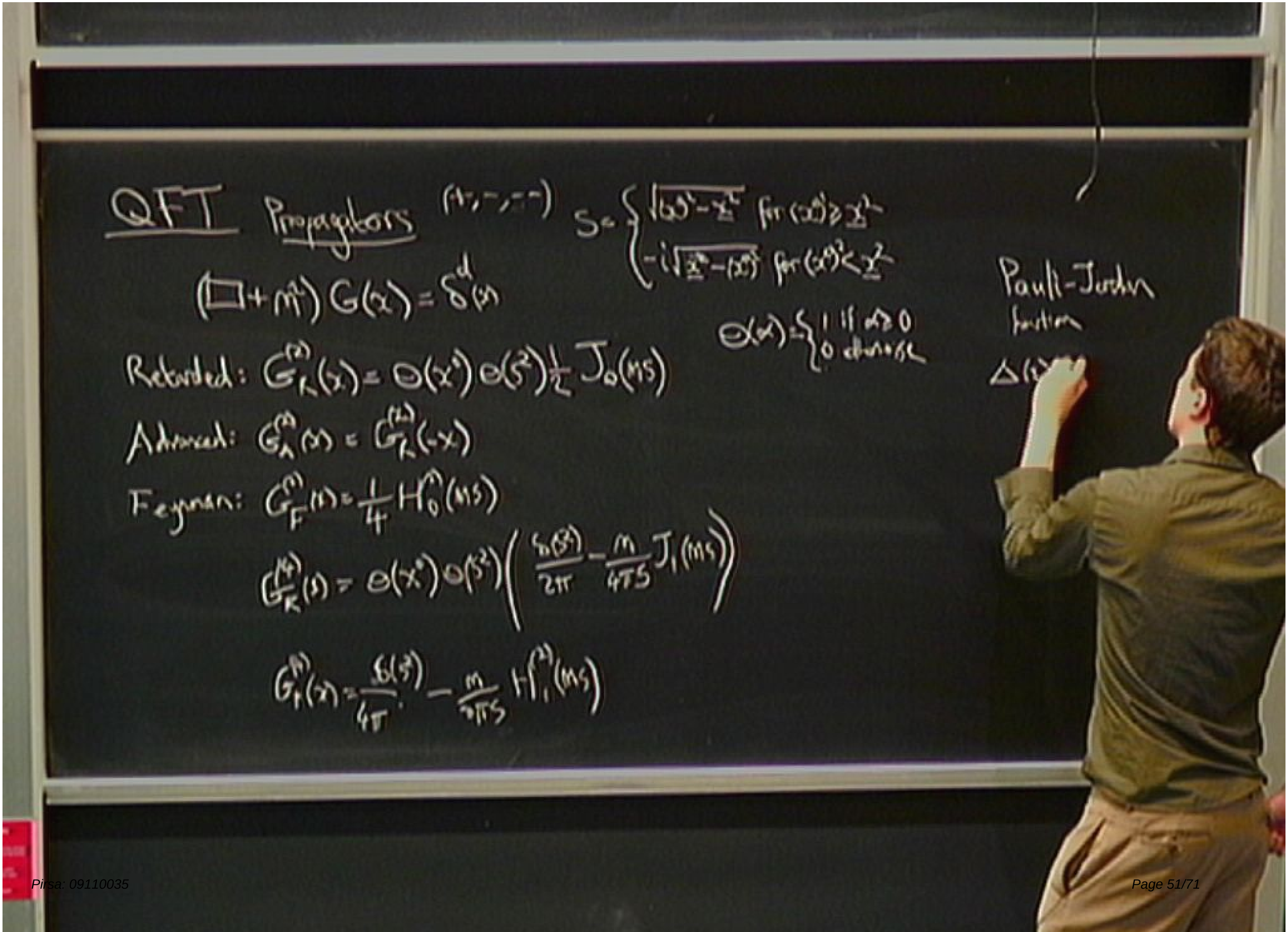
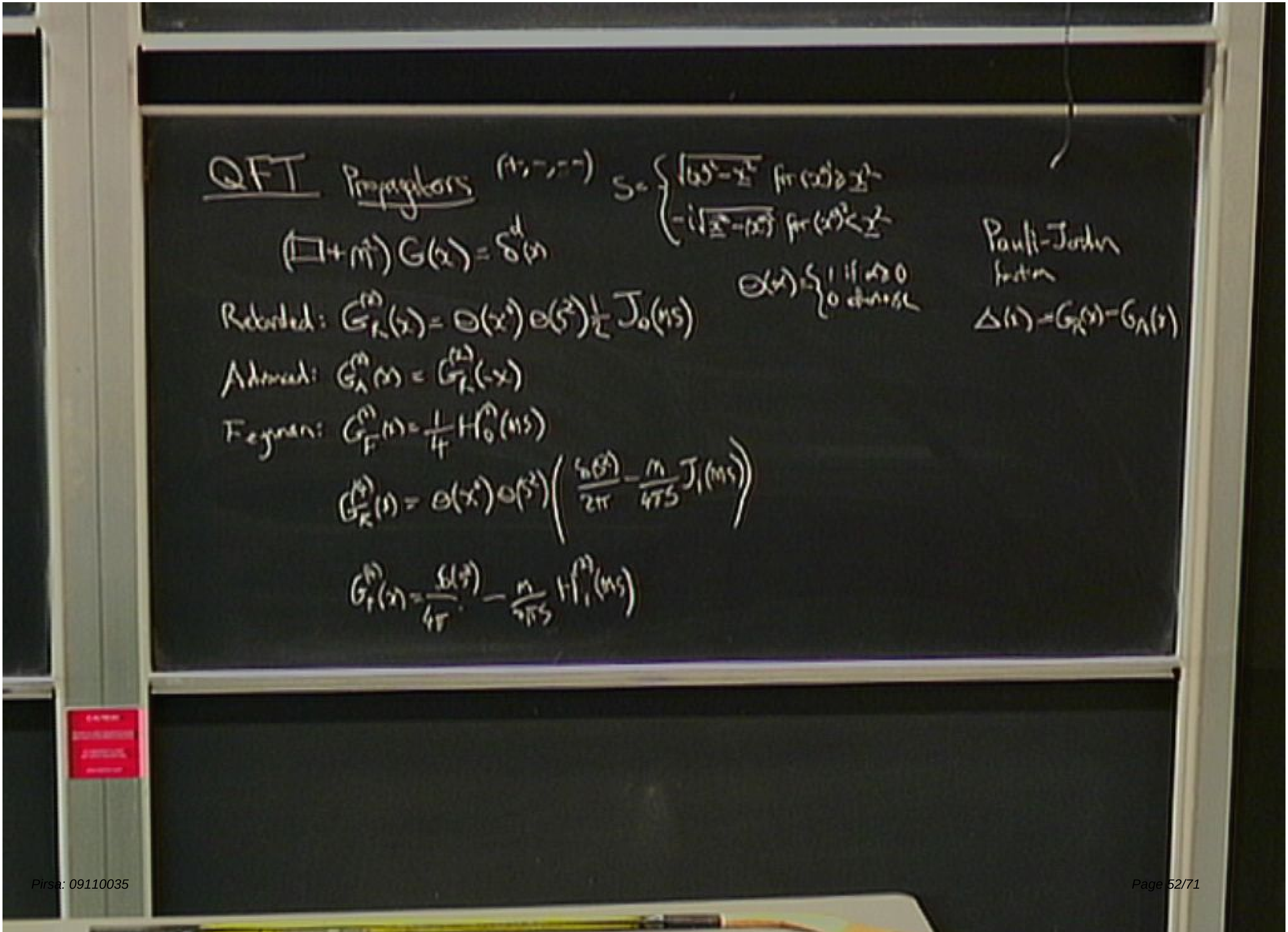
A linear extension of $(C, \prec)$ is a total order $(C, \leq)$ s.t. $u \prec v \Rightarrow u \leq v$


A link is u<v s.t. ∄ w∈C
with u<w<v
For a causal set with p elements v1,...,vp
Cij = { 1 if vi<vj
0 otherwise
Lij = { 1 if vi<vj is a link
0 otherwise
A linear extension of (C,≺)
is a total order (C,≤)
s.t. u≺v ⇒ u≤v
Sprinkling:
(M,g)
Expected
#pts in V = ρV



QFT Propagators (+,−,−,−)
$(\Box + m^2)\, G(x) = \delta^d(x)$
$s = \sqrt{(x^0)^2 - \underline{x}^2}$ for $(x^0)^2 \geq \underline{x}^2$
$-i\sqrt{\underline{x}^2 - (x^0)^2}$ for $(x^0)^2 < \underline{x}^2$
Retarded: $G_R^{(2)}(x) = \Theta(x^0)\,\Theta(s^2)\,\frac{1}{2}\,J_0(ms)$
Advanced: $G_A^{(2)}(x) = G_R^{(2)}(-x)$
Feynman: $G_F^{(2)}(x) = \frac{1}{4}\,H_0^{(2)}(ms)$


QFT Propagators (+,−,−,−)
(□+m²) G(x) = δ^d(x)
Retarded: G_R^(2)(x) = Θ(x⁰) Θ(s²) ½ J₀(ms)
Advanced: G_A^(2)(x) = G_R^(2)(−x)
Feynman: G_F^(2)(x) = ¼ H₀^(2)(ms)
G_R^(4)(s) = Θ(x⁰) Θ(s²) (δ(s²)/2π − m/4πs J₁(ms))
G_F^(4)(x) = δ(s²)/4π − m/8πs H₁^(2)(ms)
s = √((x⁰)² − x²) for (x⁰)² ≥ x²; −i √(x² − (x⁰)²) for (x⁰)² < x²
Θ(x) = 1 if x ≥ 0; 0 otherwise
Pauli-Jordan function
Δ(x)


QFT Propagators (+,−,−,−)
$(\Box + m^2) G(x) = \delta^d(x)$
Retarded: $G_R^{(m)}(x) = \Theta(x^0)\Theta(s^2)\frac{1}{2}J_0(ms)$
Advanced: $G_A^{(m)}(x) = G_R^{(m)}(-x)$
Feynman: $G_F^{(m)}(x) = \frac{i}{4}H_0^{(1)}(ms)$
Pauli-Jordan function
$\Delta(x) = G_R(x) - G_A(x)$

Fields $\hat{\Phi}(x)$ operators on Fock space F

1. $(\Box + m^2)\hat{\Phi}(x) = 0$
2. $\hat{\Phi}(x) = \hat{\Phi}^\dagger(x)$
3. $[\hat{\Phi}(x), \hat{\Phi}(y)] = i\Delta(y-x)$

Also a Poincaré invariant vacuum state $|0\rangle \in F$ s.t.

$$G_F(x-y) = i\langle 0| T\phi(x)\phi(y)|0\rangle$$

QFT on a causal set

Fix $(C, \preceq)$ generated by sprinkling into a finite causal interval in $\mathbb{M}^d$

Let ϱ be the sprinkling density

ρ be # points in C.

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$$K_R^{(2)} = aC(I - abC)^{-1}, \quad a = \frac{1}{2} \quad b = -\frac{m^2}{\rho}$$

$$K_R^{(4)} = aL(I - abL)^{-1}, \quad a = \frac{\sqrt{\rho}}{2\pi\sqrt{6}} \quad b = -\frac{m^2}{\rho}$$

QFT on a causal set

arxiv:0806.3083

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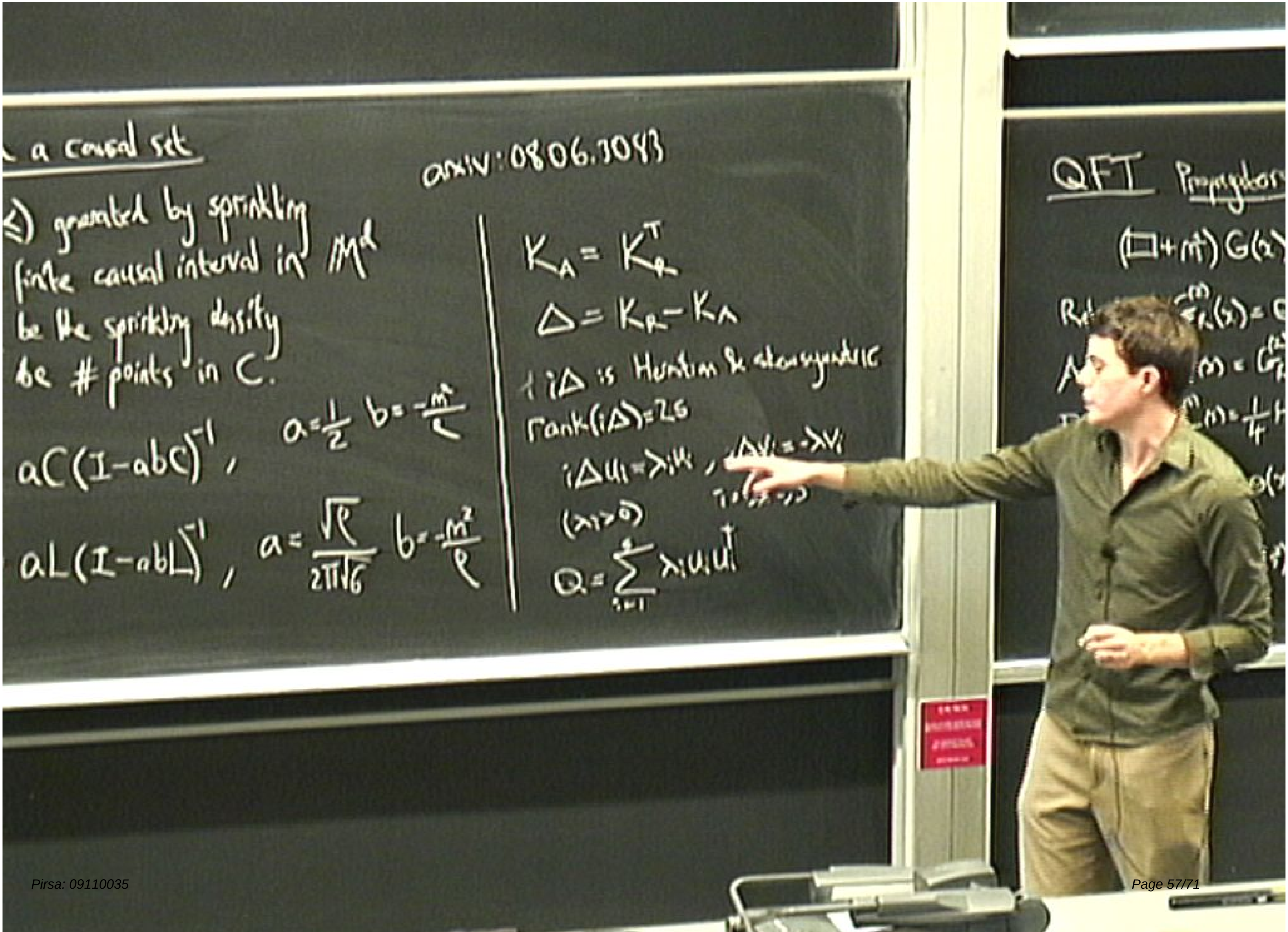
a causal set
arxiv:0806.3083
generated by sprinkling
finite causal interval in M^d
be the sprinkling density
be # points in C.
aC(I-abC)^-1, a=1/2 b=-m^2/ρ
aL(I-abL)^-1, a=√ρ/(2π√6) b=-m^2/ρ
K_A = K_R^T
Δ = K_R - K_A
iΔ is Hermitian
rank(iΔ)=2s
iΔu_i=λ_iu_i
(λ_i>0)
Q=Σ λ_iu_iu_i^†
QFT Propagators
(□+m^2)G(x)

QFT on a causal set

arxiv:0806.3083

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$$K_A = K_R^T$$

$$\Delta = K_R - K_A$$

$i\Delta$ is Hermitian & skew-symmetric

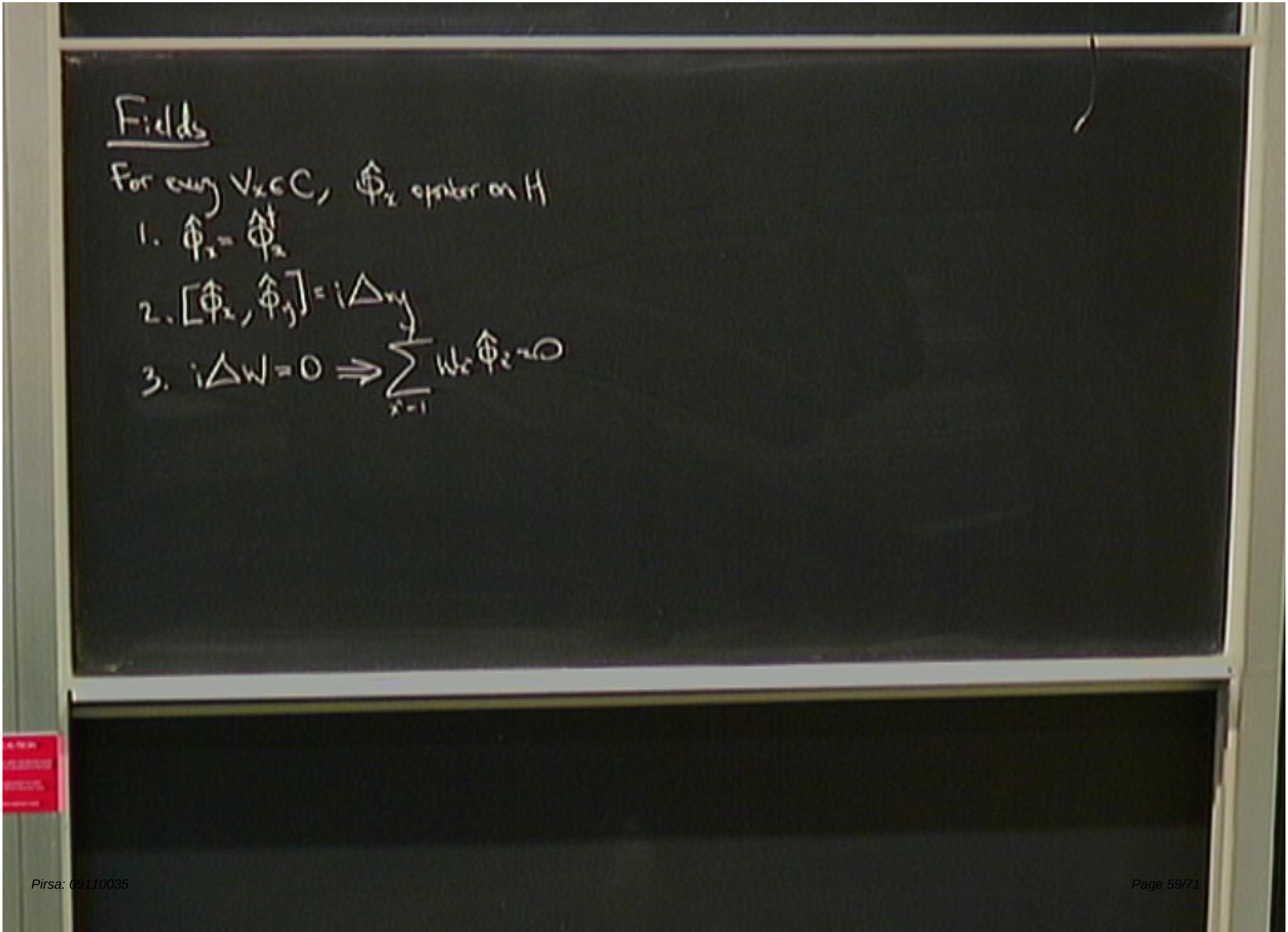
$\operatorname{rank}(i\Delta) = 2s$

$i\Delta u_i = \lambda_i u_i \,, \quad i\Delta v_i = -\lambda v_i$

$(\lambda_i > 0) \qquad i = 1, \ldots, s$

$$Q = \sum_{i=1}^{s} \lambda_i u_i u_i^\dagger$$

Then $i\Delta = Q - Q^T$
$= Q - \bar{Q}$


Fields
For every $v_x \in C$, $\hat{\Phi}_x$ operator on H
1. $\hat{\Phi}_x = \hat{\Phi}_x^\dagger$
2. $[\hat{\Phi}_x, \hat{\Phi}_y] = i\Delta_{xy}$
3. $i\Delta W = 0 \Rightarrow \sum_{x=1} W_x \hat{\Phi}_x = 0$

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$$a_i = \sum_{x=1} (v_i)_x \hat{\Phi}_x \qquad a_i^\dagger = \sum_{x=1} (u_i)_x \hat{\Phi}_x$$

$i = 1, \ldots, s$

$$[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$$

$$[a_i, a_j^\dagger] = \lambda_i \delta_{ij}$$

Define $|\Omega\rangle \in H$ by $a_i|\Omega\rangle = 0 \ \forall i$, $\langle 0|\Omega\rangle = 1$

Fields

For every $v_x \in C$, $\hat{\Phi}_x$ operator on H

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3. $i\Delta W = 0 \Rightarrow \sum_{x=1}^{r} W_x \hat{\Phi}_x = 0$

$$a_i = \sum_{x=1}^{r} (V_i)_x \hat{\Phi}_x \qquad a_i^\dagger = \sum_{x=1}^{r} (U_i)_x \hat{\Phi}_x$$

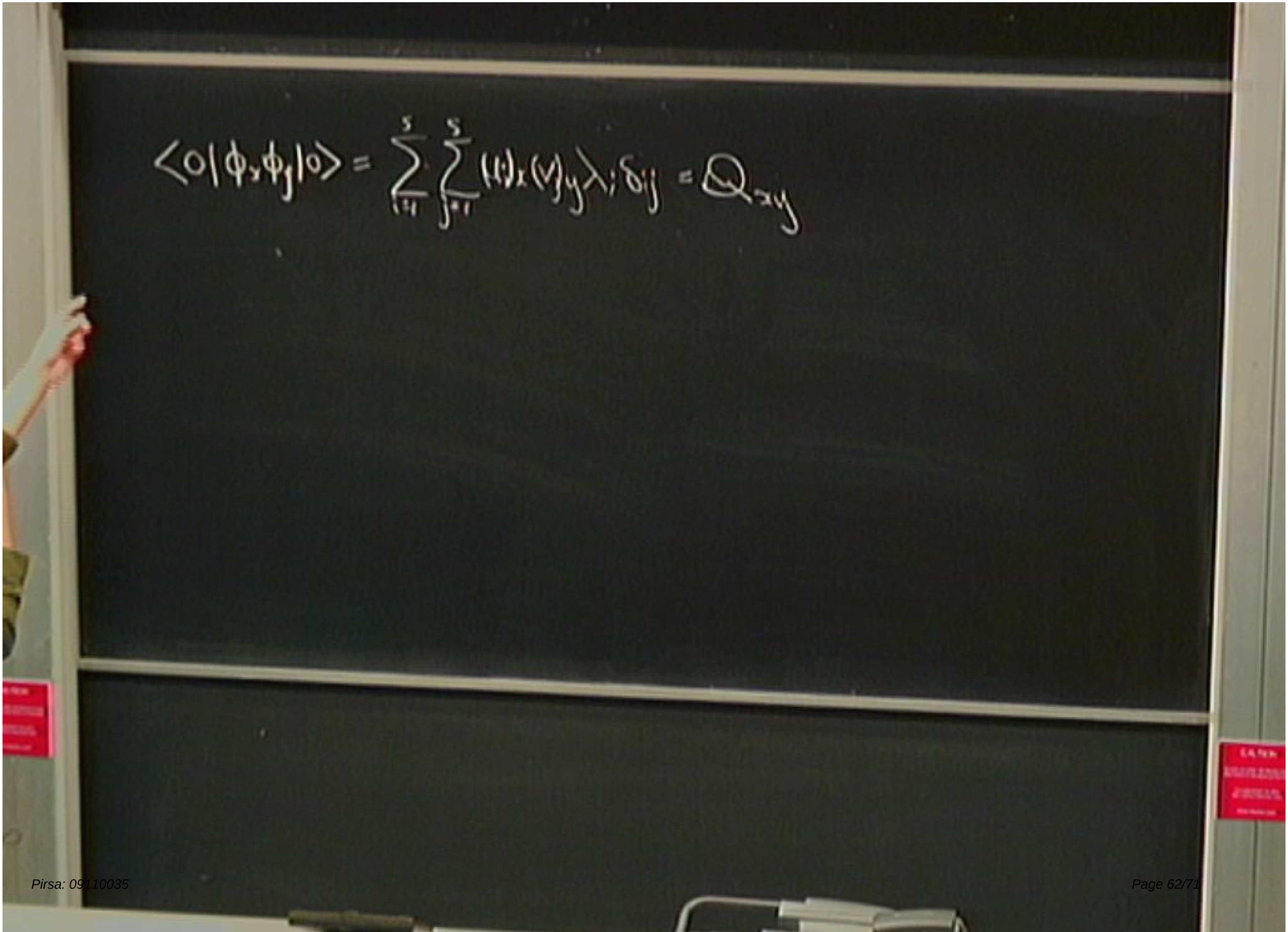
$i = 1, \dots, s$

$$[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$$

$$[a_i, a_j^\dagger] = \lambda_i \delta_{ij}$$

Define $|\Omega\rangle \in H$ by $a_i|\Omega\rangle = 0 \ \forall i$, $\langle 0|\Omega\rangle = 1$

$$\hat{\phi}_x = \sum_{i=1}^{s} (U_i)_x a_i + (V_i)_x a_i^\dagger$$


$\langle 0|\phi_x\phi_y|0\rangle = \sum_{i=1}^{s}\sum_{j=1}^{s}(u_i)_x(v_j)_y\lambda_i\delta_{ij} = Q_{xy}$

$$\langle 0|\phi_x \phi_y|0\rangle = \sum_{i=1}^{s}\sum_{j=1}^{s} (u_i)_x (v_j)_y \lambda_i \delta_{ij} = Q_{xy}$$

$$\langle 0|[\phi_x, \phi_y]|0\rangle = Q_{xy} - Q_{yx} = i\Delta_{xy}$$

$$(K_F)_{xy} = i\langle 0|T\,\phi_x \phi_y|0\rangle$$

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$$(K_F)_{xy} = i\langle 0|T\phi_x \phi_y|0\rangle$$

$\bar{A}$ is causal matrix for a linear ext

$$= i\left(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{xy}\right)$$

$$\langle 0|\phi_x \phi_y|0\rangle = \sum_{i=1}^{s}\sum_{j=1}^{s} (v_i)_x (v_j)_y \lambda_i \delta_{ij} = Q_{xy}$$

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$\bar{A}$ is causal matrix for a linear ext

$$= i(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{xy})$$

$$\boxed{K_F = K_R + iQ}$$

$$\langle 0|\phi_x\phi_y|0\rangle = \sum_{i=1}^{s}\sum_{j=1}^{s} (U_i)_x (V_j)_y \lambda_i \delta_{ij} = Q_{xy}$$

$$\langle 0|[\phi_x,\phi_y]|0\rangle = Q_{xy} - Q_{yx} = i\Delta_{xy}$$

$$(K_F)_{xy} = i\langle 0|T\phi_x\phi_y|0\rangle$$

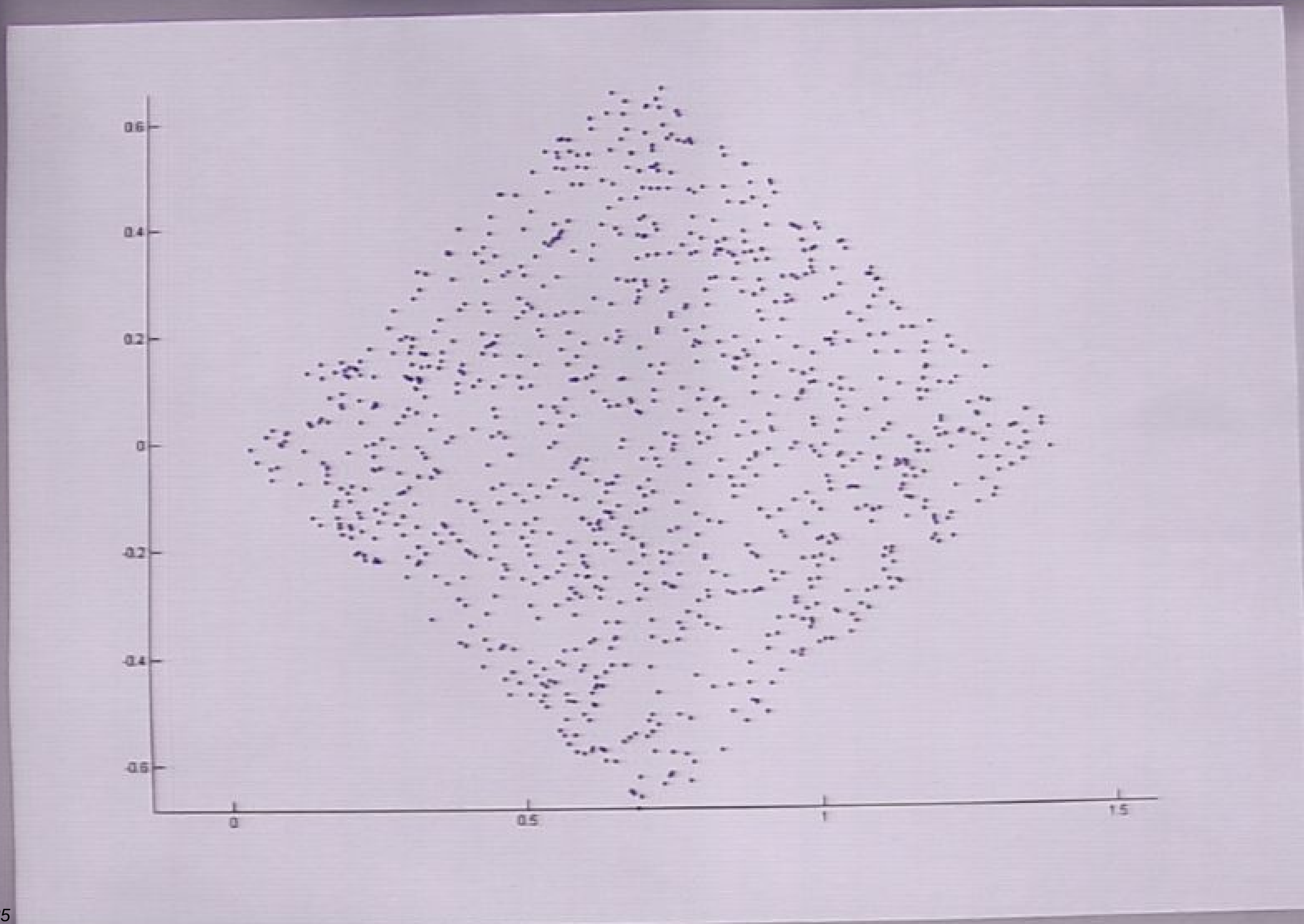
$$= i(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{xy})$$

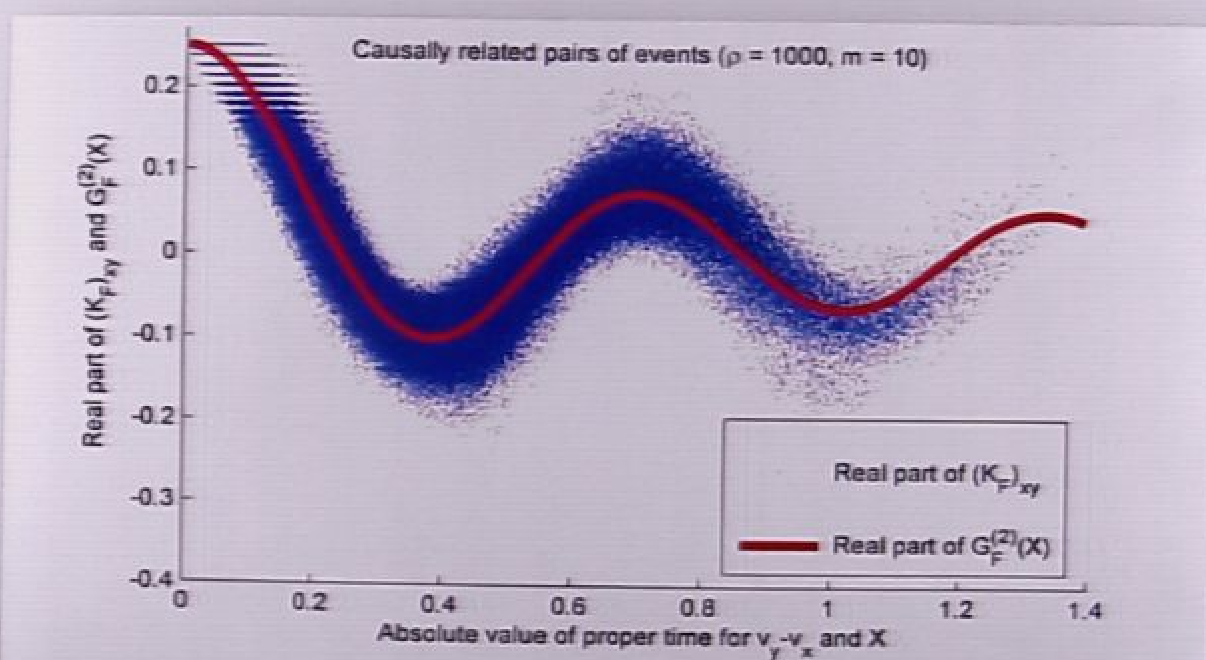
$\bar{A}$ is causal matrix for a linear ext

$$\boxed{K_F = K_R + iQ}$$

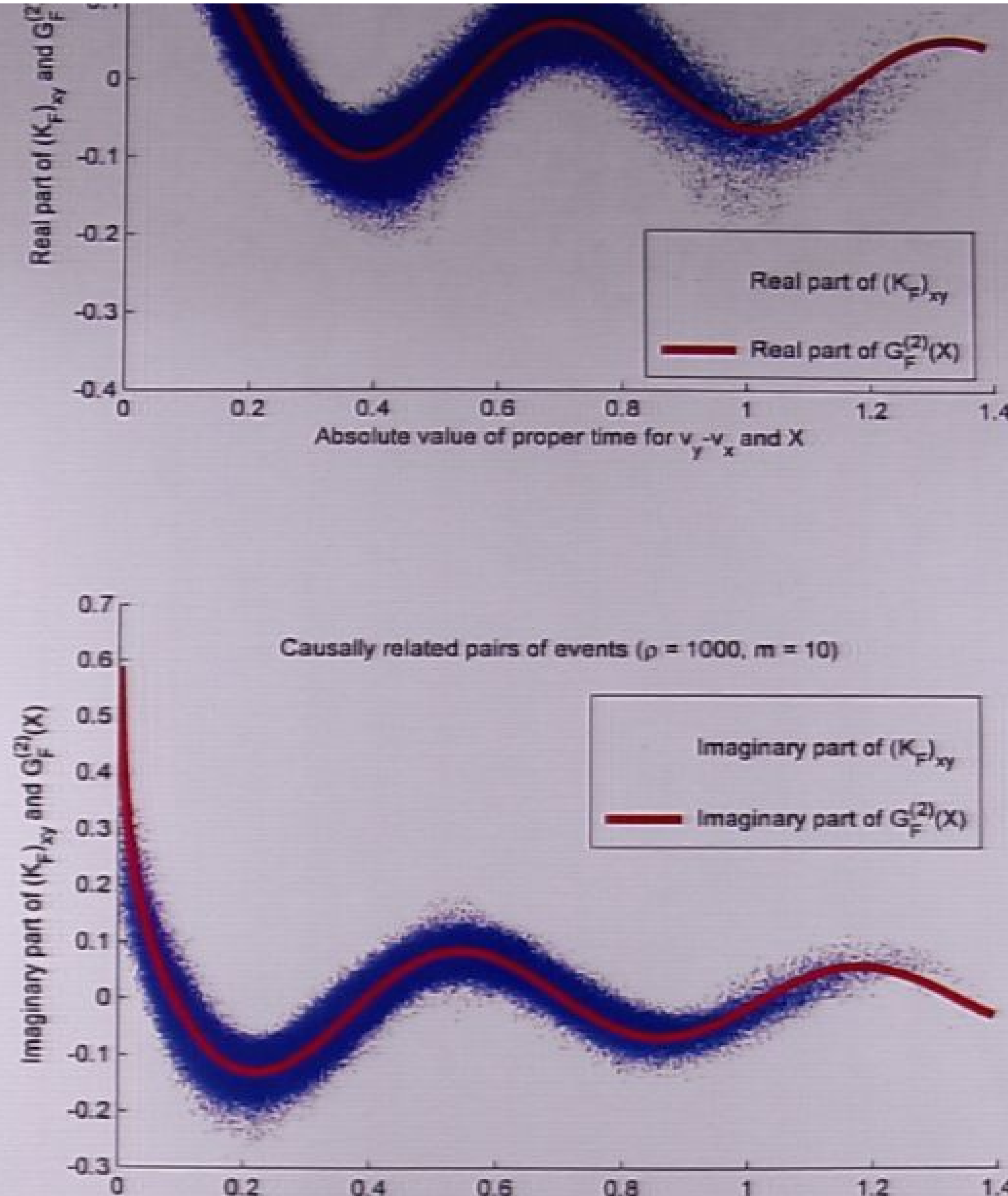
$$\mathrm{Re}(K_F) = \frac{K_R + K_A}{2}$$

$$\mathrm{Im}(K_F) = \mathrm{Re}(Q)$$


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.5
1
1.5



Causally related pairs of events (ρ = 1000, m = 10)
Real part of $(K_F)_{xy}$ and $G_F^{(2)}(X)$
Absolute value of proper time for v_y-v_x and X
Real part of $(K_F)_{xy}$
Real part of $G_F^{(2)}(X)$
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4


Real part of $(K_F)_{xy}$ and $G_F^{(2)}$
Real part of $(K_F)_{xy}$
Real part of $G_F^{(2)}(X)$
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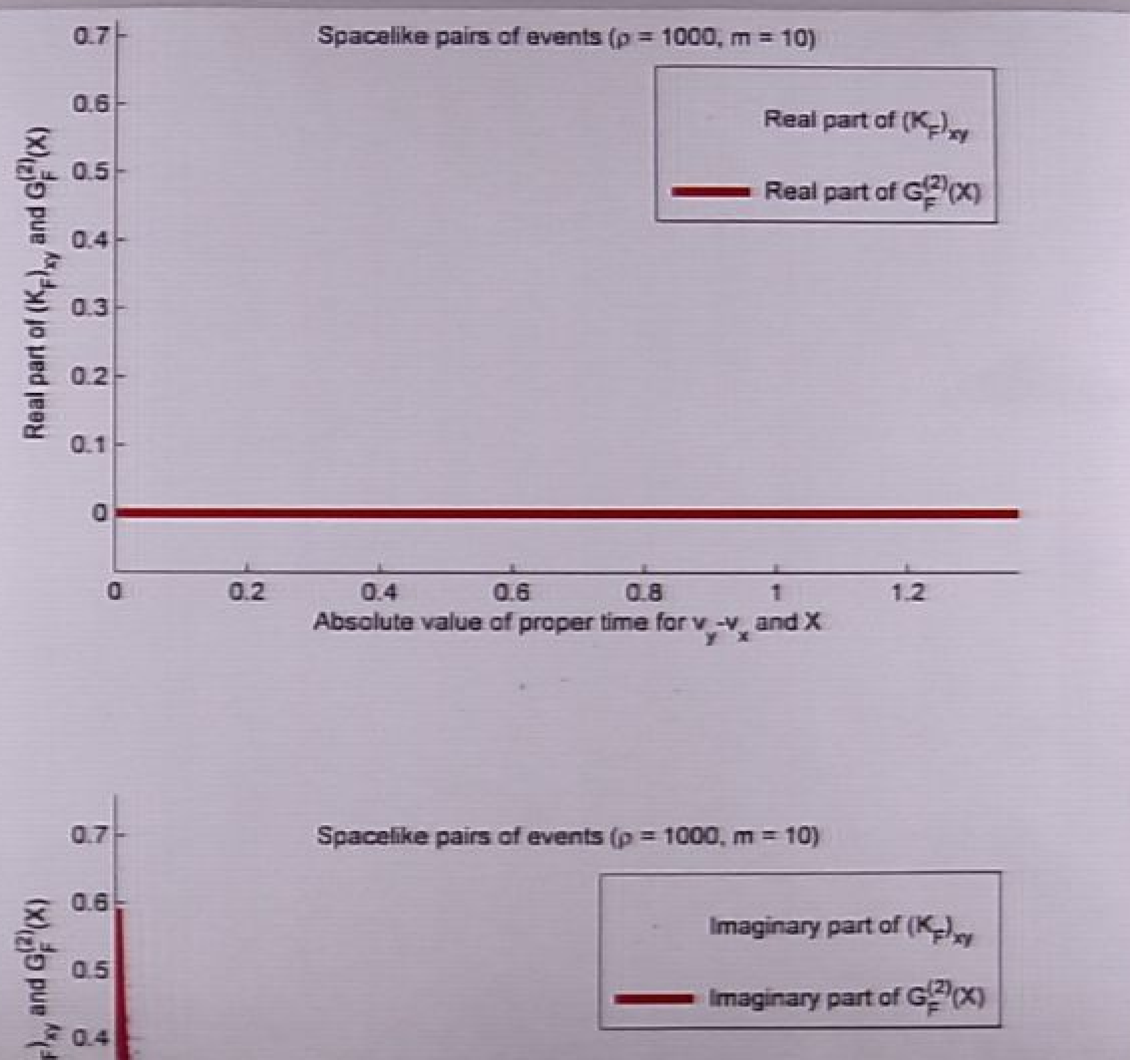
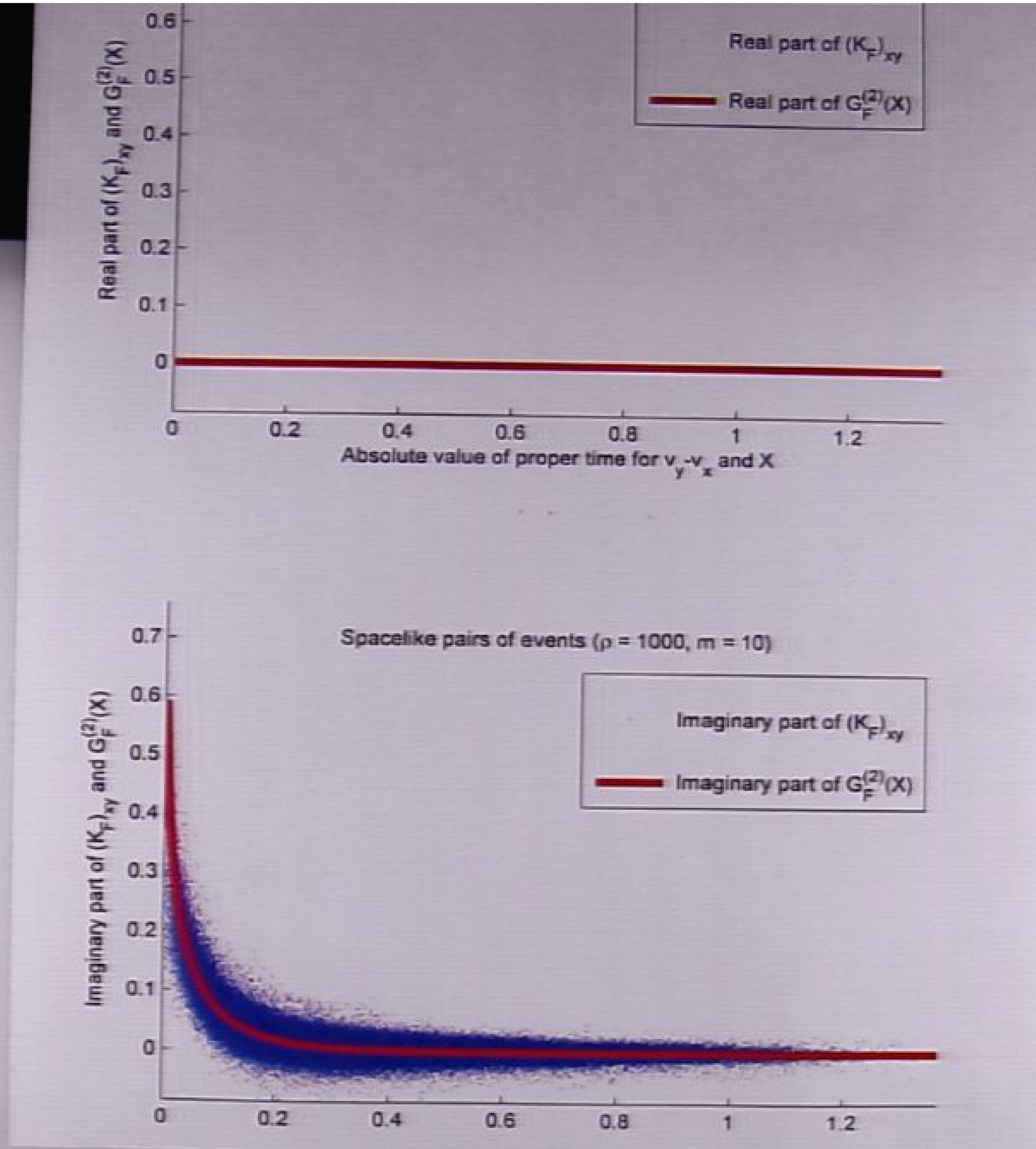
Imaginary part of $(K_F)_{xy}$
Imaginary part of $G_F^{(2)}(X)$


Spacelike pairs of events (ρ = 1000, m = 10)
Real part of $(K_F)_{xy}$
Real part of $G_F^{(2)}(X)$
Real part of $(K_F)_{xy}$ and $G_F^{(2)}(X)$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
Absolute value of proper time for v_y-v_x and X
Spacelike pairs of events (ρ = 1000, m = 10)
Imaginary part of $(K_F)_{xy}$
Imaginary part of $G_F^{(2)}(X)$
0.7
0.6
0.5
0.4


Real part of $(K_F)_{xy}$
Real part of $G_F^{(2)}(X)$
Real part of $(K_F)_{xy}$ and $G_F^{(2)}(X)$
Absolute value of proper time for v_y-v_x and X
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1
1.2
Spacelike pairs of events (ρ = 1000, m = 10)
Imaginary part of $(K_F)_{xy}$
Imaginary part of $G_F^{(2)}(X)$
Imaginary part of $(K_F)_{xy}$ and $G_F^{(2)}(X)$
0.7