

Title: The Feynman propagator on a causal set.

Date: Nov 13, 2009 11:00 AM

URL: <http://pirsa.org/09110035>

Abstract: How sure are you that spacetime is continuous? One approach to quantum gravity, causal set theory, models spacetime as a discrete structure: a causal set. This talk begins with a brief introduction to causal sets, then describes a new approach to modelling a quantum scalar field on a causal set. We obtain the Feynman propagator for the field by a novel procedure starting with the Pauli-Jordan commutation function. The candidate Feynman propagator is shown to agree with the continuum result. This model opens the door to physical predictions for scalar matter on a causal set.

The Fourier principle on a circle III  
Shur's Lemma

PROFESSOR  
TAL SHARON

Case of  $\mathbb{C}^2$

- $\mathbb{C}^2$
- $\mathbb{R}^2 \rightarrow \mathbb{C}$
- $\mathbb{R}^2 \rightarrow \mathbb{C}^2$

$\|f\|_{\infty} \leq C \|f\|_1$   
if  $f$  is radial



The Feynman propagator on a causal set  
Shah Jahan

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y$
- $x \prec y \wedge z \rightarrow x \wedge z \rightarrow y \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} \neq \emptyset$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$



The Fixpoint propagator on a causal set  
Shah Jahan

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Causal set  $(C, \prec)$

- $\cdot x \prec x$
- $\cdot x \prec y \wedge x \rightarrow z = y$
- $\cdot x \prec y \wedge z \rightarrow x \prec z$
- $\cdot \{z \in C \mid x \prec z \wedge z \prec y\} \prec \infty$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

The Fixpoint propagator on a causal set  
Shawn Johnson

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TEL 103 190401 1017

Causal set  $(C, \prec)$

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The Fixpoint propagator on a causal set  
Shawn Johnson

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Causal set  $(C, \prec)$

$\cdot x \prec x$

$\cdot x \prec y \wedge x \rightarrow x = y$

$\cdot x \prec y \wedge z \rightarrow x \prec z$

$\cdot \{ \{ z \in C \mid x \prec z \wedge z \prec y \} \} \prec \infty$

$x \prec y$  if  $x \prec y \wedge x \not\prec y$

The Fixpoint propagator on a causal set  
Shah Jahan

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PAL 103:180401,1017

Causal set  $(C, \prec)$

- $\cdot x \prec x$
- $\cdot x \prec y \wedge x \rightarrow x = y$
- $\cdot x \prec y \wedge z \rightarrow x \prec z$
- $\cdot \{ \{ z \in C \mid x \prec z \wedge z \prec y \} \} \prec \infty$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

The Fixpoint propagator on a causal set  
Steve Jahn

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PAL 103:180401,1017

Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z \rightarrow y \rightarrow x \rightarrow y$
- $x \prec y \wedge z \rightarrow x \rightarrow z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} \neq \emptyset$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$



The Fixpoint propagator on a causal set  
Steen Johnsen

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y \rightarrow x \rightarrow y \rightarrow x$
- $x \prec y \wedge z \rightarrow x \rightarrow z \rightarrow y \rightarrow z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} \prec \infty$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

The Fixpoint propagator on a causal set  
Shen Jitoh

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Causal set  $(C, \prec)$

$\cdot x \prec x$

$\cdot x \prec y \prec x \rightarrow$

$\cdot x \prec y \prec z$

$\cdot \{x \prec y\} \prec z$

$x \prec y$  if  $\exists z$

The Floyd propagator on a causal set  
Steve Jitlin

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z$
- $x \prec y \wedge z \rightarrow x$
- $\forall z \in C, x \prec z \rightarrow x \prec y$
- $x \prec y \wedge y \prec x$

$C$

The Fixpoint propagator on a causal set  
Shen Jitoh

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Causal set  $(C, \prec)$

$C = \{x, y, z\}$

- $x \prec x$
- $x \prec y \prec z \rightarrow x \prec z$
- $x \prec y \prec z \rightarrow x \prec z$
- $\{y, z\} \prec x$
- $x \prec y$  if  $x \prec y$

The Floyd-Warshall algorithm on a causal set

Shane Johnson

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Causal set  $(C, \prec)$

$C = \{v_i\}$

- $\cdot x \prec x$
- $\cdot x \prec y \wedge x \rightarrow y$
- $\cdot x \prec y \wedge z \rightarrow x \wedge z \rightarrow y$
- $\cdot \{x, y\} \prec z \iff x \prec z \wedge y \prec z$
- $x \prec y \iff x \prec y$

The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3\}$$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z \rightarrow y$
- $x \prec y \wedge z \rightarrow x \rightarrow y$
- $x \prec y \wedge z \rightarrow x \rightarrow y$
- $x \prec y$  if  $x \prec y$

The Floyd propagator on a causal set

Steve John

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Causal set  $(C, \prec)$

$$C = \{V_1, V_2, V_3, V_4\}$$

- $x \prec x$
- $x \prec y$
- $y \prec z$
- $x \prec z$
- $x \prec y \prec z$
- $x \prec z$

The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

- $x \prec x$
- $x \prec y \iff x \prec y$
- $x \prec y \iff z \prec x$
- $x \prec y \iff z \prec y$
- $x \prec y \iff x \prec z$
- $x \prec y \iff y \prec x$



The Floyd-Warshall algorithm on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$x \prec x$$

$$x \prec y \iff x \rightarrow y$$

$$x \prec y \iff z$$

$$x \prec z \iff x \prec y \iff y \prec z$$

$$x \prec y \iff y \prec x$$

The Floyd-Warshall algorithm on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

$C = \{v_1, v_2, v_3, v_4\}$

$v_i \prec v_j$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z$
- $x \prec y \wedge z \rightarrow x$
- $\forall z \in C, z \prec x$
- $x \prec y \wedge y \prec z$

The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2$$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y \rightarrow x$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\neg \exists z \in C (x \prec z \wedge z \prec y)$
- $x \prec y$  if  $x \prec y$  &  $x$



The Floyd-Warshall algorithm on a causal set

Shawn Johnston

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_3$$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y$
- $x \prec y \wedge z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\}$
- $x \prec y$  if  $x \prec y$  &  $x \rightarrow y$

The Floyd-Warshall algorithm on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3$$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} \neq \emptyset$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

The Fixpoint propagator on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge z \rightarrow x \prec z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y$  &  $x$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_2 \prec v_4, v_3 \prec v_4$$



The Floyd's propagator on a causal set

Shawn Johnston

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge z \rightarrow x \prec z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y \&$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$\prec \{v_1, v_2 \prec v_1, v_3 \prec v_2\}$$



The Fixpoint propagator on a causal set

Shen Jidun

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z\}$
- $x \prec y$  if  $x \prec y \wedge$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_2 \prec v_4, v_3 \prec v_4$$





The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$   
 $x \prec x$   
 $x \prec y \iff x \prec y$   
 $x \prec z \iff x \prec y \text{ and } y \prec z$   
 $x \prec y \text{ and } x \prec z \implies x \prec y \vee x \prec z$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4$

The Fixpoint propagator on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

$C = \{v_1, v_2, v_3, v_4\}$

$v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4$

$v_3 \prec v_4$

$x \prec z$

$x \prec y \wedge y \prec z \rightarrow x \prec z$

$x \prec y \wedge y \prec z \rightarrow x \prec z$

$\{x \in C \mid x \prec y\} \prec y$

$\{x \in C \mid x \prec y\} \prec y$



The Floyd-Warshall algorithm on a causal set

Shawn J. J. J.

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Causal set  $(C, \prec)$



$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_1 \prec v_4, v_3 \prec v_4$$

The Floyd's propagator on a causal set

Steve Jahn

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Causal set:  $(C, \prec)$

- 1
- 2
- 3

$x \prec y \iff x \prec y$   
 $y \prec x \iff x \prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_1 \prec v_4$$

$$v_3 \prec v_4$$

The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_1 \prec v_4$$

$$v_3 \prec v_4$$

$$x \rightarrow x=y$$

$$x \rightarrow x \neq y$$

$$x \prec y \iff x \neq y$$

$$x \prec y \wedge x \neq y$$

The Floyd-Warshall algorithm on a causal set

Steve John

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Causal set  $(C, \prec)$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_1 \prec v_4, v_3 \prec v_4$$

$x$   
 $y \prec x \rightarrow x = y$   
 $y \prec z \rightarrow x \prec z$   
 $\{y \prec z \mid x \prec z \wedge y \prec x\} \prec \infty$   
 $x \prec y \wedge x \prec y$



The Fixpoint propagator on a causal set

Shah Jahan

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\nexists z \in C \mid x \prec z$

$x \prec y$  if  $x \prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_2 \prec v_4, v_3 \prec v_4$$



The Floyd-Warshall algorithm on a causal set

Shawn J. J. J.

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\}$
- $x \prec y$  if  $x \prec y$  &  $x \prec z$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$\{v_1, v_2, v_3, v_4\}$$





The Feynman propagator on a causal set

Steve Jitlin

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \iff x \prec z \prec y$
- $x \prec y \iff x \prec z \prec y$
- $x \prec y \iff x \prec z \prec y$
- $x \prec y \iff x \prec z \prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4$$

$$v_3 \prec v_4$$



The Fixpoint propagator on a causal set

Shen Jitoh

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z$
- $x \prec y \wedge z \rightarrow x$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y \iff x \prec z \wedge z \prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_2 \prec v_4, v_3 \prec v_4$$



The Fixpoint propagator on a causal set

Shawn Jahnke

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z \rightarrow y$
- $x \prec y \wedge z \rightarrow x \rightarrow y$
- $\neg \exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4$



The Floyd-Warshall algorithm on a causal set

Shah Jahan

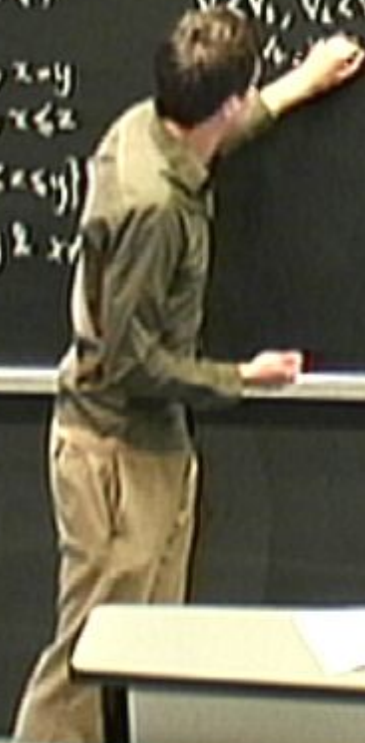
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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\}$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4$$



The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \prec z \Rightarrow x \prec y \wedge z$
- $x \prec y \wedge y \prec z \Rightarrow x \prec z$
- $x \prec y \wedge y \prec x \Rightarrow x = y$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4, v_3 \prec v_4$

The Floyd-Warshall algorithm on a causal set

Shah Jahan

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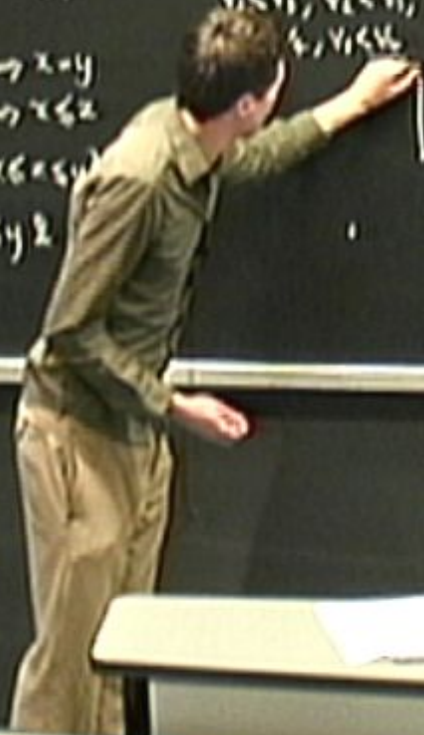
Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y \rightarrow x$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y \wedge \neg y \prec x$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4$$

$$v_1 \prec v_4$$



The Fixpoint propagator on a causal set

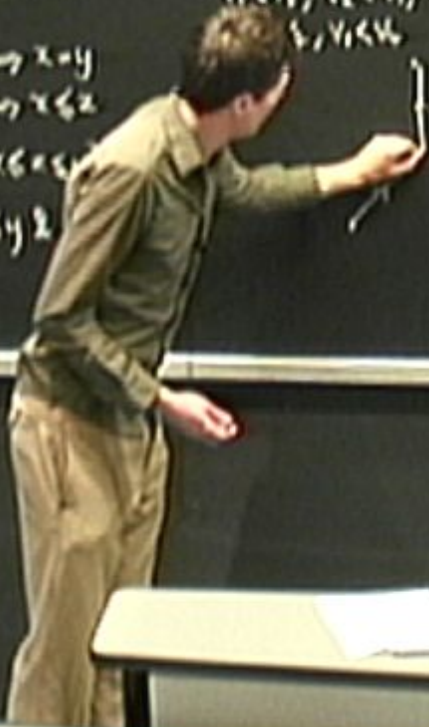
Shen Jitoh

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y \wedge$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4$   
 $v_1 \prec v_4$



The Fixpoint propagator on a causal set

Shah Jahan

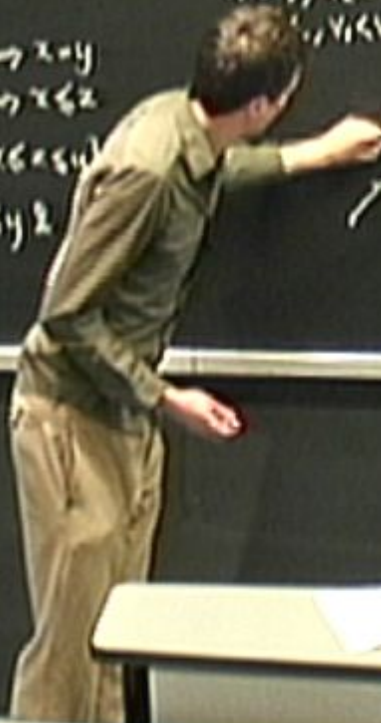
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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow x = y$
- $x \prec y \wedge z \rightarrow x \prec z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y \text{ if } x \prec y \wedge$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$$





The Floyd-Warshall algorithm on a causal set

Shawn Johnson

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y \rightarrow z \rightarrow x$
- $x \prec y \wedge z \rightarrow x \rightarrow y \rightarrow z$
- $\exists z \in C \mid x \prec z \wedge z \prec y$
- $x \prec y$  if  $x \prec y \wedge \neg y \prec x$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_2 \prec v_3, v_3 \prec v_4, v_1 \prec v_4$$



The Floyd propagator on a causal set

Shen Jidun

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \prec z \rightarrow x \prec z$
- $x \prec y \prec z \rightarrow x \prec z$
- $x \prec y \prec z \rightarrow x \prec z$
- $x \prec y \prec z \rightarrow x \prec z$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4, v_3 \prec v_4$



The Floyd-Warshall algorithm on a causal set

Shawn Jahnke

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \prec z \rightarrow x \prec y \wedge z$
- $x \prec y \wedge y \prec z \rightarrow x \prec z$
- $\{z \in C \mid x \prec z \wedge z \prec y\}$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec z$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$$



The Foyner propagator on a causal set

Shen Jidun

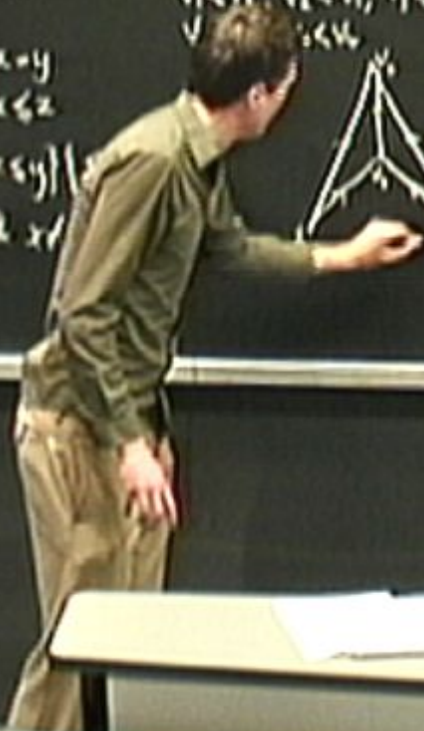
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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow z = y$
- $x \prec y \wedge z \rightarrow x \rightarrow z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} = \emptyset$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec z \prec y$

$$C = \{v_1, v_2, v_3, v_4\}$$

$$v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$$



The Feynman propagator on a causal set

Shawn Johnston

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Causal set  $(C, \prec)$

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- $x \prec y \iff x \prec z$
- $x \prec y \iff x \prec z$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4, v_3 \prec v_4$



The Feynman propagator on a causal set

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Causal set  $(C, \prec)$

- $x \prec x$
- $x \prec y \wedge x \rightarrow y \rightarrow z \rightarrow x \rightarrow y$
- $x \prec y \wedge z \rightarrow x \rightarrow z$
- $\{z \in C \mid x \prec z \wedge z \prec y\} \neq \emptyset$
- $x \prec y$  if  $x \prec y$  &  $x \not\prec y$

$C = \{v_1, v_2, v_3, v_4\}$   
 $v_1 \prec v_2, v_1 \prec v_3, v_1 \prec v_4$   
 $v_2 \prec v_4, v_3 \prec v_4$



A link is  $u < v$  st.  $\nexists w \in C$   
with  $u < w < v$

For a causal set with  $p$  elements  $v_1, \dots, v_p$

$$C_{ij} = \begin{cases} 1 & \text{if } v_i < v_j \\ 0 & \text{otherwise} \end{cases}$$

$$L_{ij} = \begin{cases} 1 & \text{if } v_i < v_j \text{ is a link} \\ 0 & \text{otherwise} \end{cases}$$

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A linear extension of  $(C, <)$   
is a total order  $(C, \leq)$   
st.  $u < v \Rightarrow u \leq v$



A link is  $u \prec v$  st.  $\nexists w \in C$   
with  $u \prec w \prec v$

For a causal set with  $p$  elements  $v_1, \dots, v_p$

$$C_{ij} = \begin{cases} 1 & \text{if } v_i \prec v_j \\ 0 & \text{otherwise} \end{cases}$$

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A linear extension of  $(C, \prec)$   
is a total order  $(C, \leq)$   
st.  $u \prec v \Rightarrow u \leq v$

Sprinkling  
Embed  
#pts. in  $\Delta$



QFT Propagators  $(+, -, -)$   $S = \begin{cases} \sqrt{x^2 - m^2} & \text{for } (x^0)^2 > x^2 \\ -i\sqrt{x^2 - m^2} & \text{for } (x^0)^2 < x^2 \end{cases}$

$$(\square + m^2) G(x) = \delta^d(x)$$

Retarded:  $G_R^{(2)}(x) = \theta(x^0) \theta(x^2) \frac{1}{2} J_0(m|x^0|)$

Advanced:  $G_A^{(2)}(x) = G_R^{(2)}(-x)$

Feynman:  $G_F^{(2)}(x) = \frac{1}{4} H_0^{(1)}(m|x^0|)$



QFT Propagators  $(+, -, -)$

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$$\theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Pauli-Jordan  
function

Retarded:  $G_R^{(2)}(x) = \theta(x^0) \theta(x^2) \frac{1}{2} J_0(m|x|)$

Advanced:  $G_A^{(2)}(x) = G_R^{(2)}(-x)$

Feynman:  $G_F^{(2)}(x) = \frac{1}{4} H_0^{(2)}(m|x|)$

$$\left(\frac{G_F^{(2)}}{G_R^{(2)}}\right)(x) = \theta(x^0) \theta(x^2) \left( \frac{\delta(x^2)}{2\pi} - \frac{m}{4\pi S} J_1(m|x|) \right)$$

$$G_F^{(2)}(x) = \frac{\delta(x^2)}{4\pi} - \frac{m}{2\pi S} H_1^{(2)}(m|x|)$$

$\Delta(x)$



QFT Propagators  $(\eta, -)$   $S = \begin{cases} \sqrt{b^2 - x^2} & \text{for } (x^0)^2 > x^2 \\ -i\sqrt{x^2 - (x^0)^2} & \text{for } (x^0)^2 < x^2 \end{cases}$

$$(\square + m^2) G(x) = \delta^4(x)$$

Retarded:  $G_R^{(m)}(x) = \Theta(x^0) \Theta(x^2) \frac{1}{2} J_0(mx)$

$$\Theta(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Pauli-Jordan  
function

$$\Delta(x) = G_R(x) - G_A(x)$$

Advanced:  $G_A^{(m)}(x) = G_R^{(m)}(-x)$

Feynman:  $G_F^{(m)}(x) = \frac{1}{4} H_0^{(1)}(mx)$

$$G_R^{(m)}(x) = \Theta(x^0) \Theta(x^2) \left( \frac{b(x^0)}{2\pi} - \frac{m}{4\pi^2} J_1(mx) \right)$$

$$G_A^{(m)}(x) = \frac{b(x^0)}{4\pi} - \frac{m}{2\pi^2} H_1^{(1)}(mx)$$

Fields  $\hat{\phi}(x)$  operators on Fock space  $F$

1.  $(\square + m^2) \hat{\phi}(x) = 0$

2.  $\hat{\phi}(x) = \hat{\phi}^\dagger(x)$

3.  $[\hat{\phi}(x), \hat{\phi}(y)] = i\Delta(y-x)$

Also a Poincaré invariant vacuum state  $|0\rangle \in F$

s.t.

$$G_F(x, y) = i \langle 0 | T \phi(x) \phi(y) | 0 \rangle$$

## QFT on a causal set

Fix  $(C, \leq)$  generated by sprinkling  
into a finite causal interval in  $M^d$

Let  $\rho$  be the sprinkling density  
 $p$  be # points in  $C$ .

## QFT on a causal set

Fix  $(C, \leq)$  generated by sprinkling  
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Let  $\rho$  be the sprinkling density

$p$  be # points in  $C$ .

$$K_R^{(1)} = aC(I - abc)^{-1}, \quad a = \frac{1}{2} \quad b = \frac{-\beta^2}{r}$$

$$K_R^{(1)} = aL(I - aBL)^{-1}, \quad a = \frac{\sqrt{p}}{2\pi\sqrt{6}} \quad b = \frac{-\beta^2}{p}$$

## QFT on a causal set

AMIV:0806.3043

Fix  $(C, \leq)$  generated by sprinkling  
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Let  $\rho$  be the sprinkling density

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$$K_R^{(1)} = aC(I - aC)^{-1}, \quad a = \frac{1}{2} \quad b = -\frac{1}{\rho^2}$$

$$K_R^{(1)} = aL(I - aL)^{-1}, \quad a = \frac{\sqrt{\rho}}{2\pi\sqrt{6}} \quad b = -\frac{1}{\rho^2}$$



a causal set

↳ generated by sprinkling  
finite causal interval in  $M^d$   
be the sprinkling density  
be # points in  $C$ .

arXiv:0806.3043

$$a_C(I-abc)^{-1}, \quad a = \frac{1}{2} \quad b = \frac{-3\pi}{r}$$

$$a_L(I-abL)^{-1}, \quad a = \frac{\sqrt{p}}{2\pi\sqrt{6}} \quad b = \frac{-3\pi^2}{p}$$

$$K_A = K_A^T$$

$$\Delta = K_R - K_A$$

$i\Delta$  is Hermitian & antisymmetric

$$\text{rank}(i\Delta) = 2S$$

$$i\Delta u_i = \lambda_i u_i, \quad i\Delta v_i = -\lambda_i v_i$$

( $\lambda_i > 0$ )  
 $i = 1, 2, \dots, S$

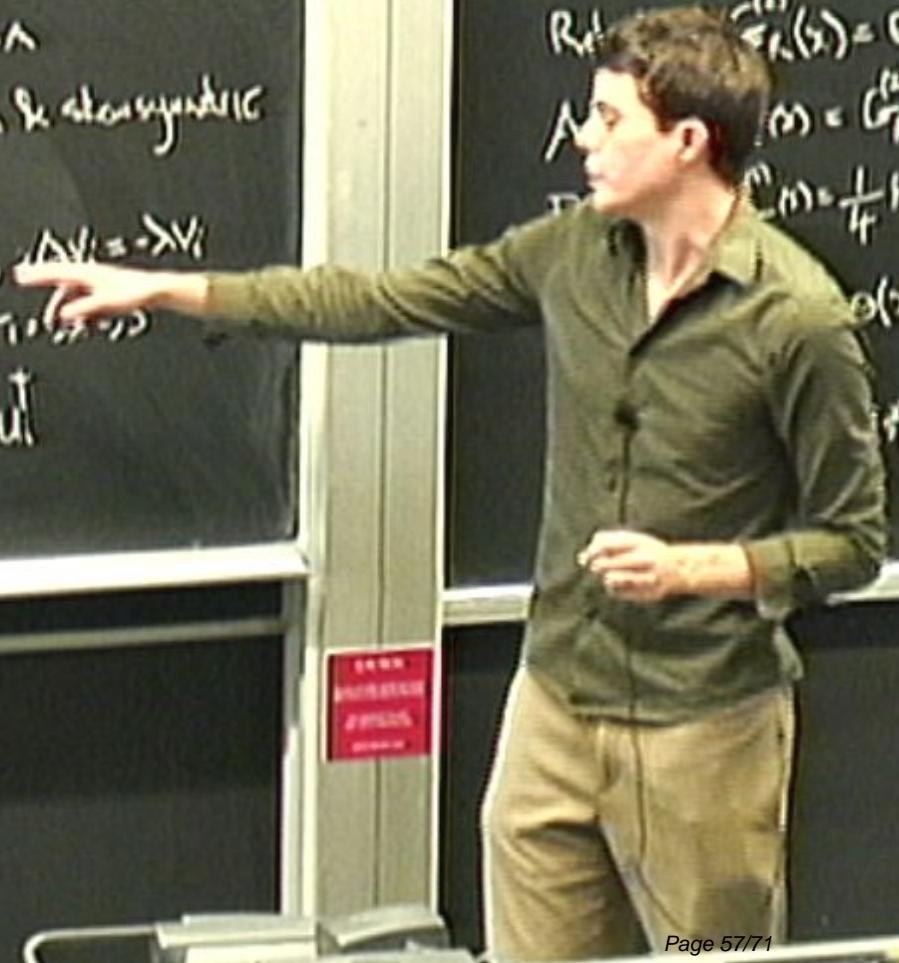
$$Q = \sum_{i=1}^S \lambda_i u_i u_i^\dagger$$

QFT Propagator

$$(\square + m^2)G(x)$$

$$R(x) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2 + m^2} = 0$$

$$A(x) = \int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2 + m^2} = \frac{e^{-m|x|}}{4}$$



## QFT on a causal set

Fix  $(C, \leq)$  generated by sprinkling  
into a finite causal interval in  $M^d$

Let  $\rho$  be the sprinkling density  
 $p$  be # points in  $C$ .

$$K_R^{(A)} = aC(I - aC)^{-1}, \quad a = \frac{1}{2} \quad b = \frac{-\beta^d}{\rho^d}$$

$$K_R^{(A)} = aL(I - aL)^{-1}, \quad a = \frac{\sqrt{\rho}}{2\pi\sqrt{6}} \quad b = \frac{-\beta^d}{\rho^d}$$

arXiv:0806.3043

$$K_A = K_R^T$$

$$\Delta = K_R - K_A$$

$i\Delta$  is Hermitian & skew-symmetric

$$\text{Rank}(i\Delta) = 2S$$

$$i\Delta u_i = \lambda_i u_i, \quad i\Delta v_i = -\lambda_i v_i$$

$$(\lambda_i > 0)$$

$$i = 1, \dots, S$$

$$Q = \sum_{i=1}^S \lambda_i u_i u_i^\dagger$$

Then  $i\Delta = Q - Q^T$   
 $= Q - Q^T$

## Fields

For every  $\psi_k \in \mathbb{C}$ ,  $\hat{\phi}_k$  operator on  $H$

1.  $\hat{\phi}_k = \hat{\phi}_k^\dagger$

2.  $[\hat{\phi}_k, \hat{\phi}_j] = i\Delta_{kj}$

3.  $i\Delta W = 0 \Rightarrow \sum_{k=1}^n W_k \hat{\phi}_k = 0$

# Fields

For every  $\lambda \in \mathbb{C}$ ,  $\hat{\phi}_\lambda$  operator on  $H$

1.  $\hat{\phi}_\lambda = \hat{\phi}_\lambda^\dagger$

2.  $[\hat{\phi}_\lambda, \hat{\phi}_\mu] = i\Delta_{\lambda\mu}$

3.  $i\Delta W = 0 \Rightarrow \sum_{\lambda=1}^S W_\lambda \hat{\phi}_\lambda = 0$

$$a_i = \sum_{\lambda=1}^S (V_\lambda)_i \hat{\phi}_\lambda \quad a_i^\dagger = \sum_{\lambda=1}^S (W_\lambda)_i \hat{\phi}_\lambda$$

$$[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$$

$$[a_i, a_j^\dagger] = \lambda_i \delta_{ij}$$

$D_\lambda \xi_\mu = |\varphi\rangle \in H$  by  $a_i |\varphi\rangle = 0 \quad \forall i$   
 $\langle \varphi | \varphi \rangle = 1$

## Fields

For every  $\lambda \in \mathbb{C}$ ,  $\hat{\phi}_\lambda$  operator on  $\mathcal{H}$

1.  $\hat{\phi}_\lambda = \hat{\phi}_\lambda^\dagger$

2.  $[\hat{\phi}_\lambda, \hat{\phi}_\mu] = i\Delta_{\lambda\mu}$

3.  $i\Delta W = 0 \Rightarrow \sum_{\lambda=1}^S W_\lambda \hat{\phi}_\lambda = 0$

$$a_i = \sum_{\lambda=1}^S (V)_\lambda \hat{\phi}_\lambda \quad a_i^\dagger = \sum_{\lambda=1}^S (W)_\lambda \hat{\phi}_\lambda$$

$$[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$$

$$[a_i, a_j^\dagger] = \lambda_i \delta_{ij}$$

$\forall \lambda \in \mathbb{R} \quad |\varphi\rangle \in \mathcal{H}$  by  $a_i |\varphi\rangle = 0 \quad \forall i$   
 $\langle \varphi | \varphi \rangle = 1$

$$\phi_\lambda = \sum_{i=1}^S (W)_i a_i + (V)_i a_i^\dagger$$

$$\langle 0 | \phi_x \phi_y | 0 \rangle = \sum_{i=1}^s \sum_{j=1}^s (W_x)_i (V_y)_j \lambda_i \delta_{ij} = Q_{xy}$$

$$\langle 0 | \phi_x \phi_y | 0 \rangle = \sum_{i=1}^s \sum_{j=1}^s (U_x)_{ij} (V_y)_{ij} \lambda_i \delta_{ij} = Q_{xy}$$

$$\langle 0 | [\phi_x, \dot{\phi}_y] | 0 \rangle = Q_{xy} - Q_{yx} = i\Delta_{xy}$$

$$(K_F)_{xy} = i \langle 0 | \mathbb{T} \phi_x \phi_y | 0 \rangle$$

$$\langle 0 | \phi_x \phi_y | 0 \rangle = \sum_{i=1}^s \sum_{j=1}^s (W_x(V)_y \lambda_i) \delta_{ij} = Q_{xy}$$

$$\langle 0 | [\phi_x, \dot{\phi}_y] | 0 \rangle = Q_{xy} - Q_{yx} = i\Delta_{xy}$$

$$(K_F)_{xy} = i \langle 0 | T \phi_x \phi_y | 0 \rangle \quad \bar{A} \text{ is causal matrix for a linear ext}$$

$$i(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{xy})$$



$$\langle 0 | \phi_x \phi_y | 0 \rangle = \sum_{i=1}^s \sum_{j=1}^s (W_x(V)_y)_i \lambda_i \delta_{ij} = Q_{xy}$$

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$$(K_F)_{xy} = i \langle 0 | T \phi_x \phi_y | 0 \rangle \quad \bar{A} \text{ is causal matrix for a linear ext}$$

$$= i(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{yy})$$

$$\boxed{K_F = K_R + iQ}$$

$$\langle 0 | \phi_x \phi_y | 0 \rangle = \sum_{i=1}^s \sum_{j=1}^s (W_x(V)_y)_i \lambda_i \delta_{ij} = Q_{xy}$$

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$$(K_F)_{xy} = i \langle 0 | T \phi_x \phi_y | 0 \rangle \quad \bar{A} \text{ is causal matrix for a linear ext}$$

$$= i(\bar{A}_{xy} Q_{yx} + \bar{A}_{yx} Q_{xy} + \delta_{xy} Q_{xy})$$

$$\operatorname{Re}(K_F) = \frac{K_R + K_A}{2}$$

$$\operatorname{Im}(K_F) = \operatorname{Re}(Q)$$

$$\boxed{K_F = K_R + iQ}$$

