Title: The optical properties of inhomogeneous space-times before and after averaging

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Abstract: Underlying the standard cosmological model is the assumption that it is possible to coarse-grain the energy density of the Universe, and that the dynamical and optical properies of space-time should be well modelled by the result. However, even if the average coarse-grained geometry does have the same dynamical properties as the fine-grained system it is intended to imitate, there are good reasons to suspect that the optical properties may be different. To investigate this we consider a simple model of the Universe in which the matter content is in the form of uniformly distributed discrete islands, rather than a continuous fluid. It is found that in the appropriate limits the resulting large-scale dynamics of the model approach those of an FRW universe, while the optical properties do not. We find the angular diameter distance, luminosity distance and redshifts that would be measured by observers inside such a space-time, and use preliminary results to show that the effect on estimates of the cosmological constant can be of the order of 10%.

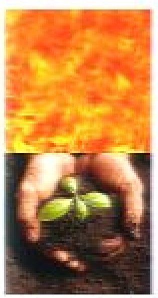
The Optical Properties of Inhomogeneous Space-Times Before and After Averaging

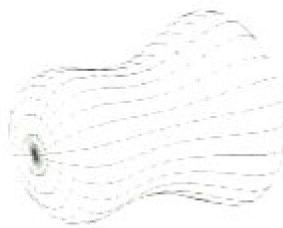
Perimeter Institute
3rd November 2009

Timothy Clifton (Oxford,UK)

The Standard Cosmology

The Standard Cosmology





Choose some matter fields
and
Choose some space-time
symmetries

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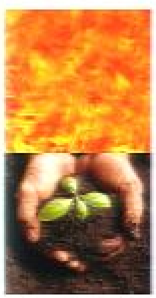

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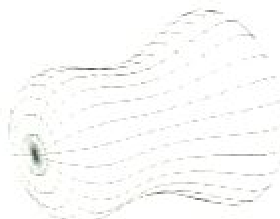

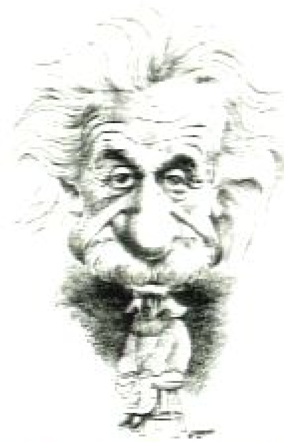

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Solve Einstein's
equations

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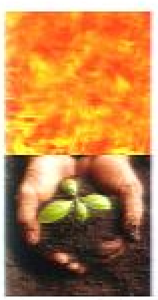

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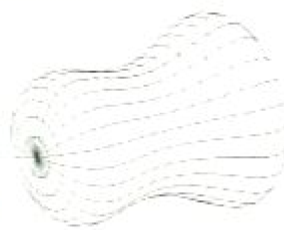

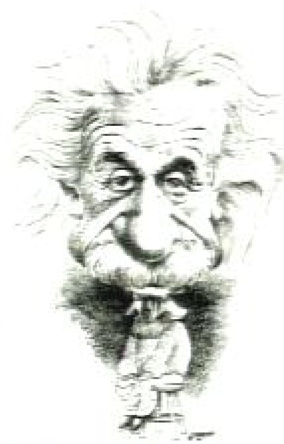

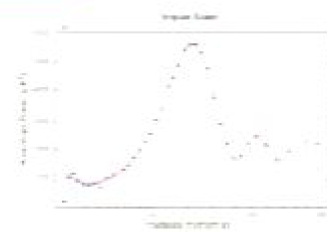

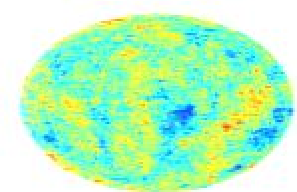

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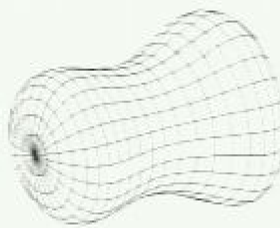

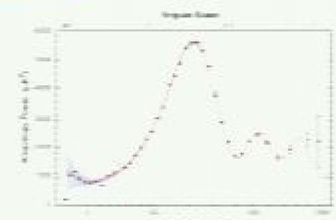

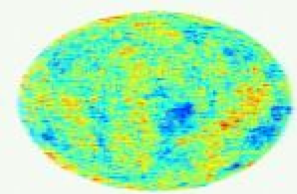

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DARK ENERGY!

Friedmann-Robertson-Walker

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- The Copernican Principle

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  - Statistical isotropy
    (Rasanen)

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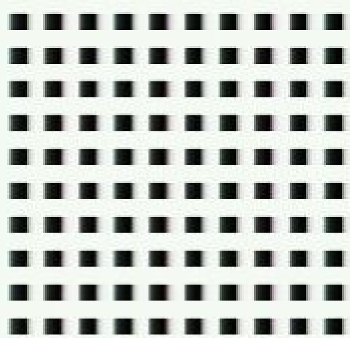

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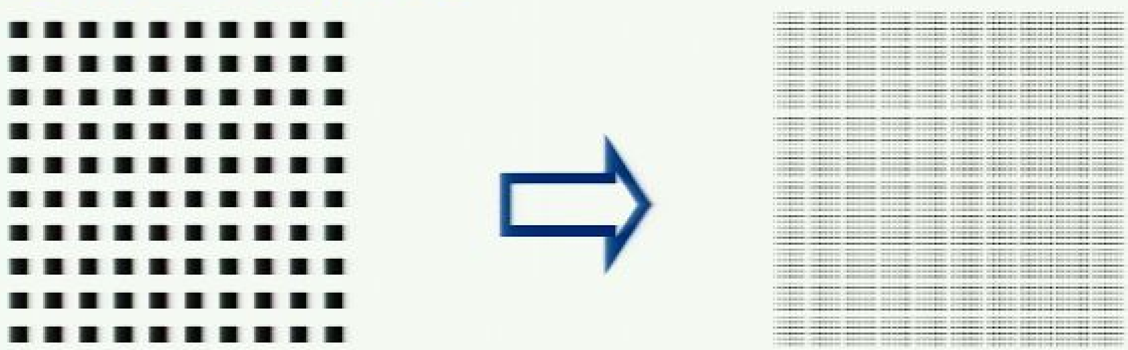

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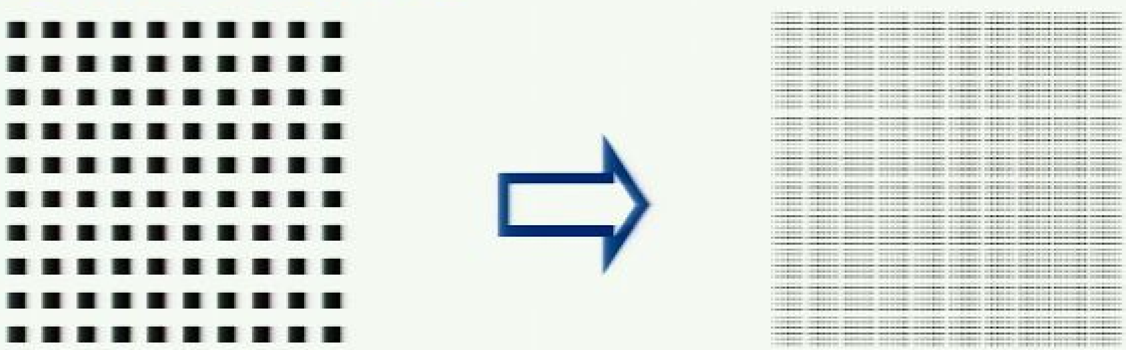
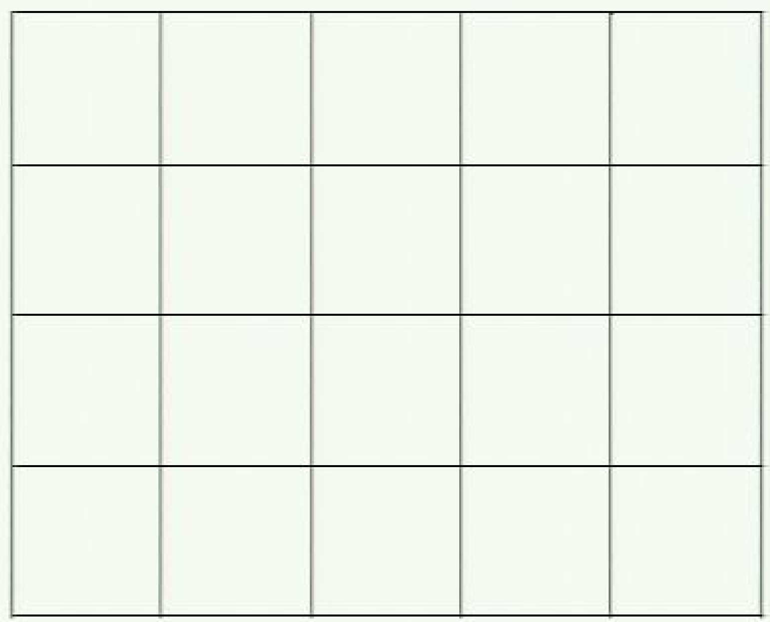


Does the smoothed geometry have the same properties as the unsmoothed space-time?

(averaging problem, back-reaction problem, lensing effects, etc.)

The Lindquist-Wheeler model

Lattice model of the Universe, inspired by the Wigner-Seitz construction of electromagnetism.



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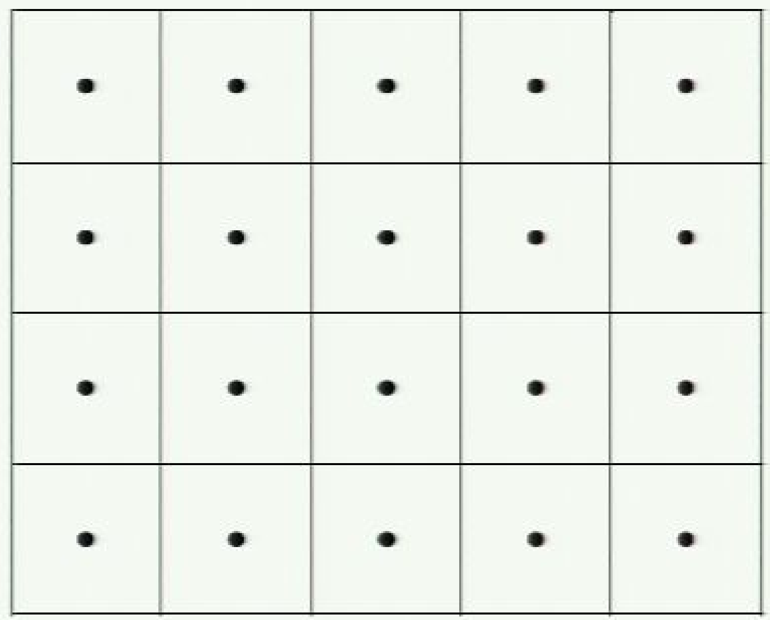

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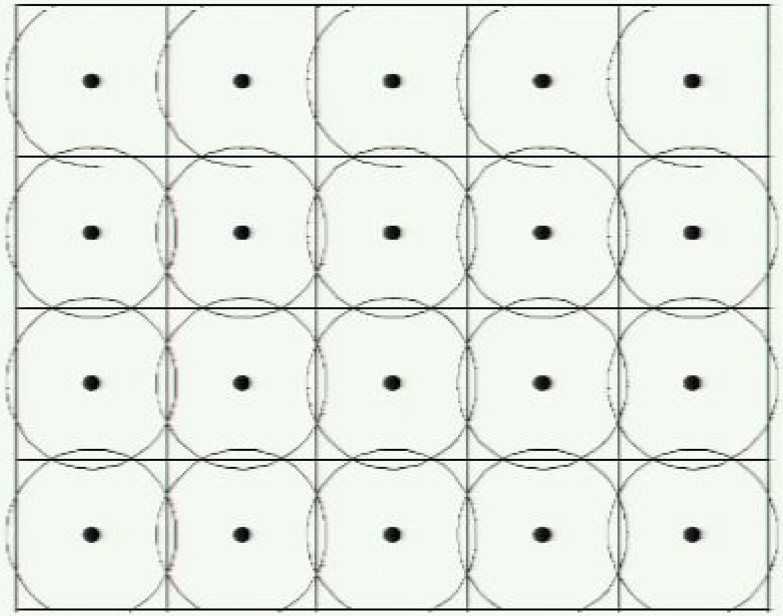

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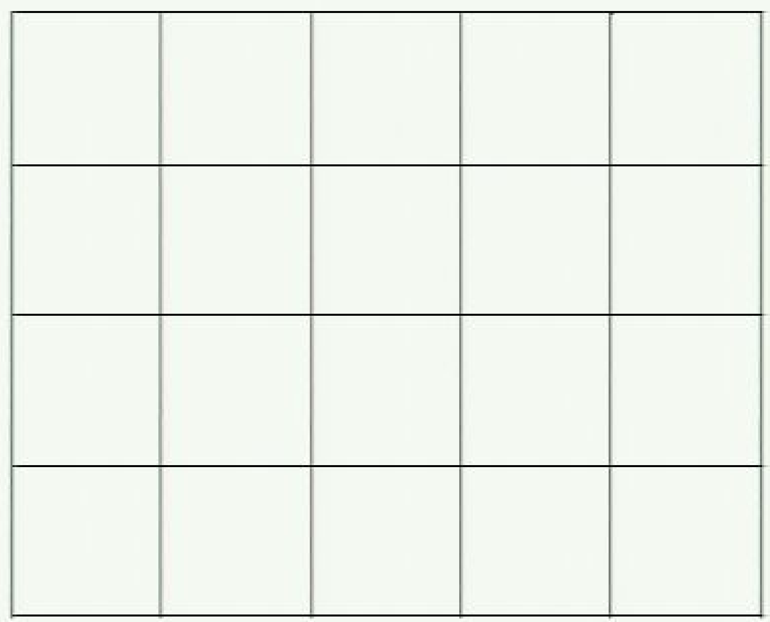

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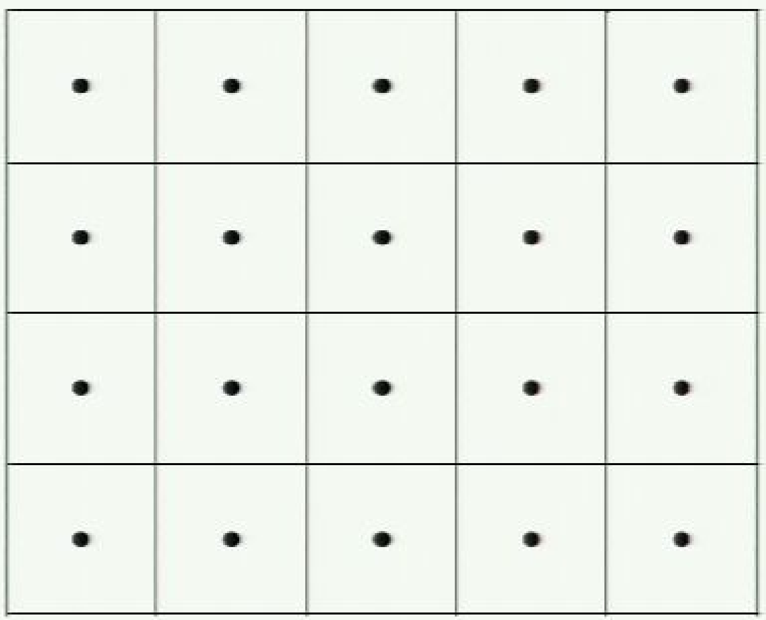

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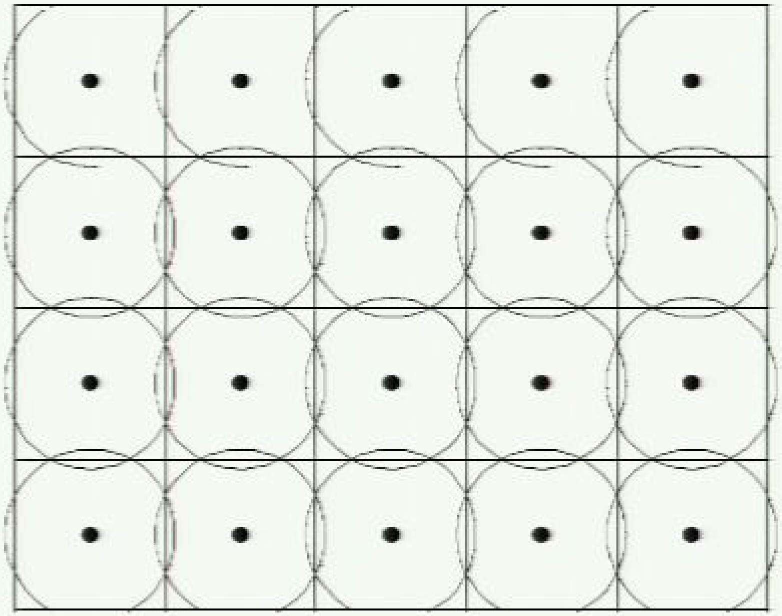

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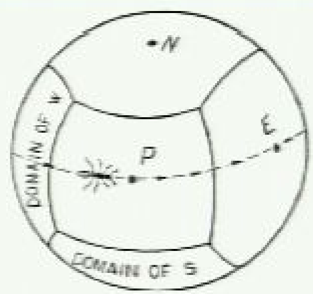

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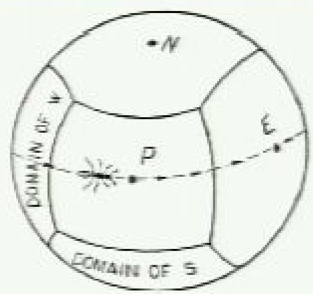

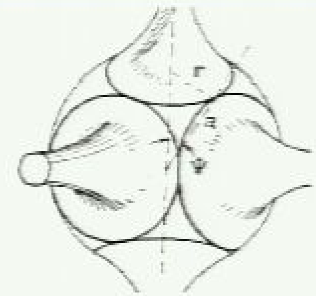

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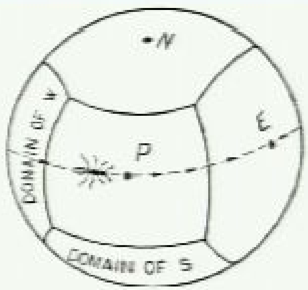

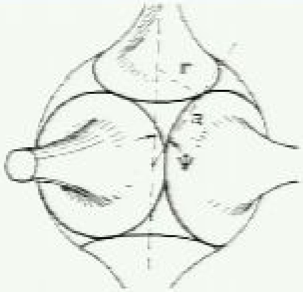

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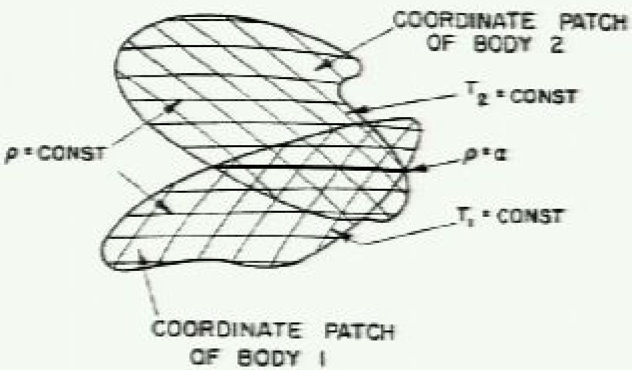

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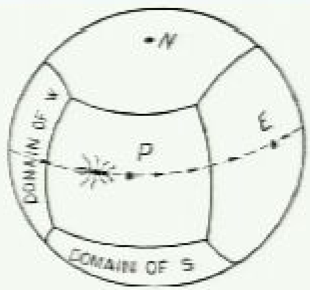

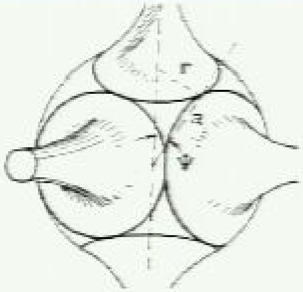

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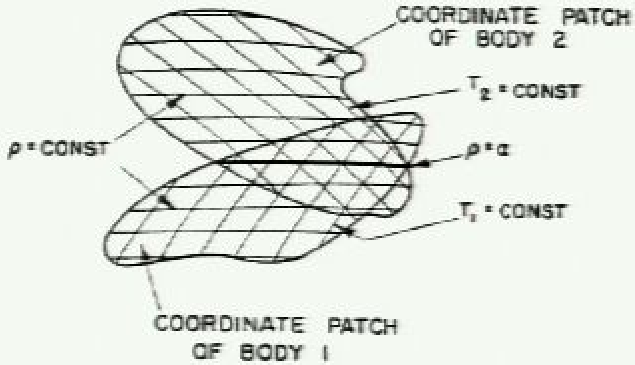

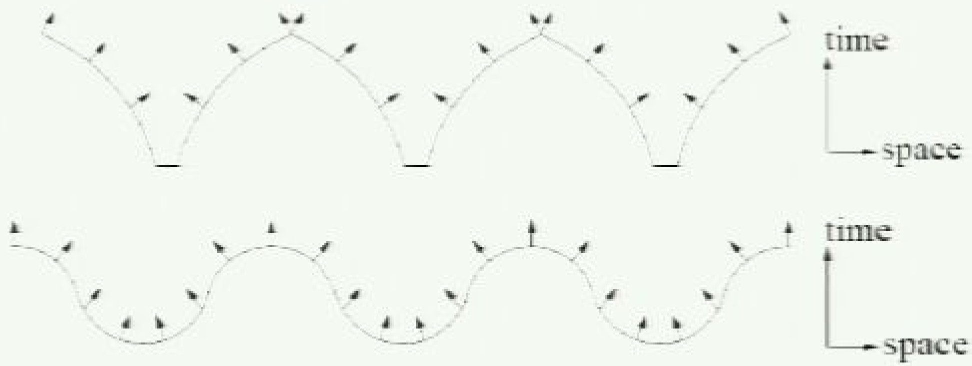

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- Dynamics governed by the Friedmann equation:

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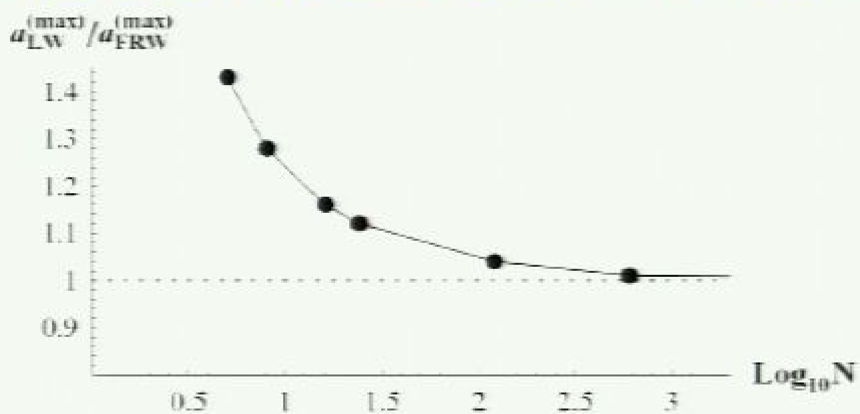
- Scale of expansion approaches FRW as N→∞:



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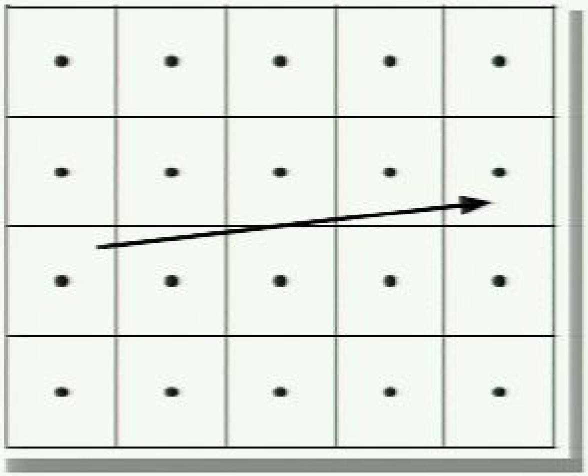
• To determine this we will need to consider bundles of null geodesics in the lattice model:



Cosmological time in a lattice

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- Inside each cell we have Schwarzschild space-time:

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

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- The boundary then contracts with 4-velocity $u^a = \left(1; \sqrt{(E-1)+\frac{2m}{r}}, 0, 0\right)$

  and is orthogonal to the surfaces of constant time.

  i.e. if $n^a = (0; n^r, n^\theta, n^\phi)$ then $u^a n_a = 0$

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Deviation from the mean

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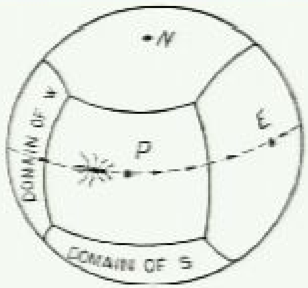

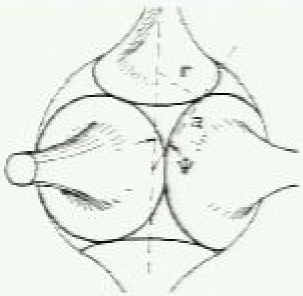

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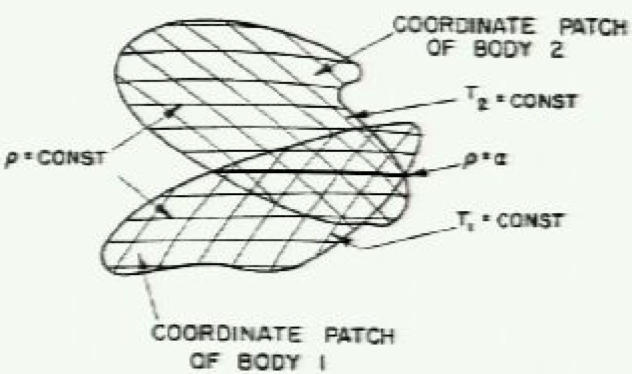

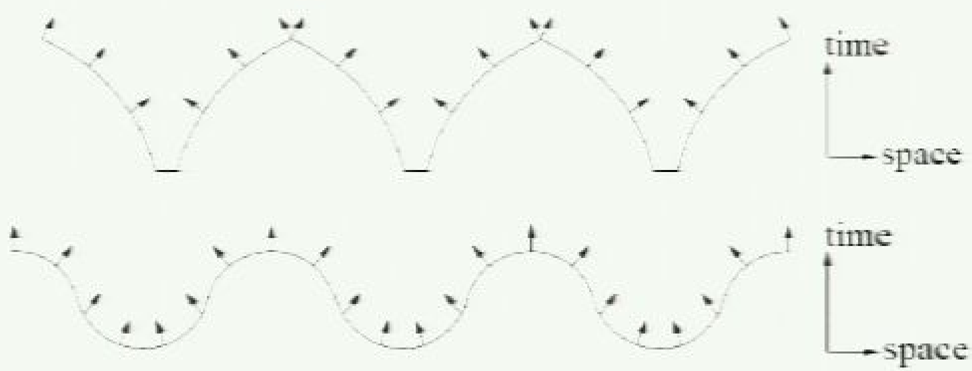

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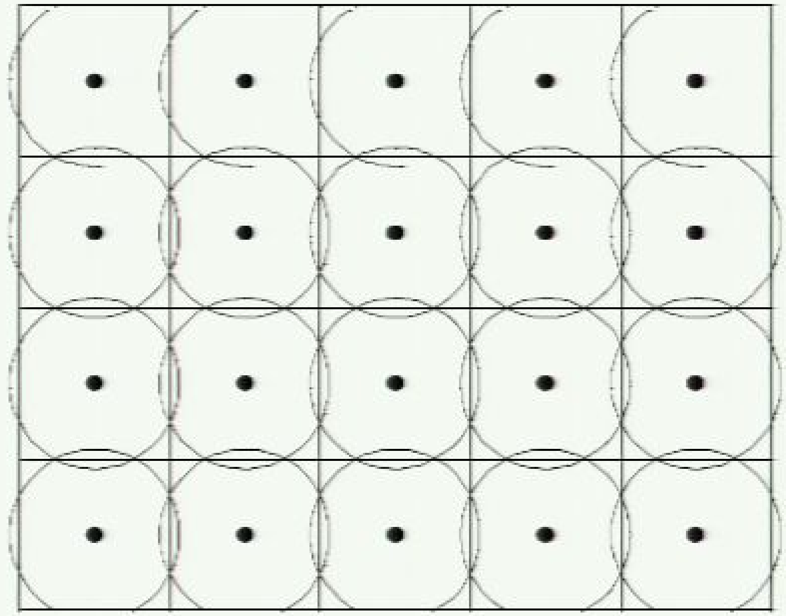

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Null trajectories in a lattice

- Null geodesic equations can be read off the metric directly.
- At the boundaries it is convenient to decompose the 4-vector tangent to the null trajectory, so that

$$k^a = \frac{dx^a}{d\lambda} = \left(\dot{t}, \dot{r}, \dot{\theta}, \dot{\phi}\right) = (-u^b k_b)(u^a + n^a) = \dot{t}\left(1, \sqrt{(E-1) + \frac{2m}{r}} + n^r, n^\theta, n^\phi\right)$$

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  - Method II: Account for imperfect tangency by allowing a small correction along each null trajectory.

Cosmological redshifts in a lattice

- The frequency of a photon is measured as $-u^a k_a = \dot{\tau}$, so that the redshift is given by:

$$1 + z = \frac{\dot{\tau}|_e}{\dot{\tau}|_o}$$

Cosmological time in a lattice

- Inside each cell we have Schwarzschild space-time:

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \frac{dr^2}{(1-\frac{2m}{r})} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

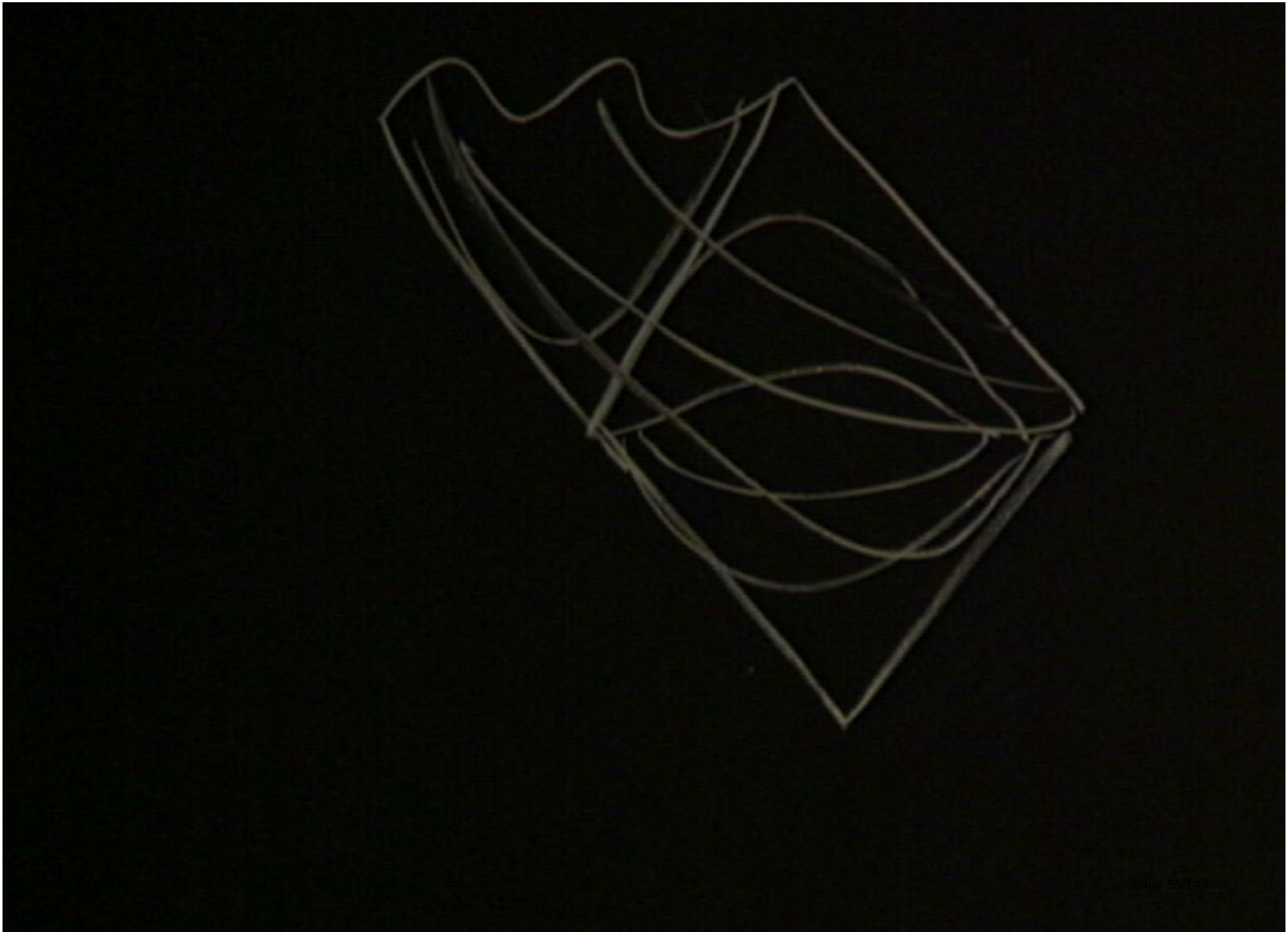
- Transforming time coordinates according to $d\tau = \sqrt{E}dt - \frac{\sqrt{E-(1-\frac{2m}{r})}}{(1-\frac{2m}{r})}dr$ results in:

$$ds^2 = -\frac{1}{E}\left(1-\frac{2m}{r}\right)d\tau^2 - \frac{2}{E}\sqrt{E-\left(1-\frac{2m}{r}\right)}d\tau dr + \frac{dr^2}{E} + r^2 d\Omega^2$$

- The boundary then contracts with 4-velocity $u^a = \left(1; \sqrt{(E-1)+\frac{2m}{r}}, 0, 0\right)$

  and is orthogonal to the surfaces of constant time.

  i.e. if $n^a = (0; n^r, n^\theta, n^\phi)$ then $u^a n_a = 0$



Null trajectories in a lattice

- Null geodesic equations can be read off the metric directly.
- At the boundaries it is convenient to decompose the 4-vector tangent to the null trajectory, so that

$$k^a = \frac{dx^a}{d\lambda} = \left(\dot{t}, \dot{r}, \dot{\theta}, \dot{\phi}\right) = (-u^b k_b)(u^a + n^a) = \dot{t}\left(1, \sqrt{(E-1) + \frac{2m}{r}} + n^r, n^\theta, n^\phi\right)$$

- The condition for matching trajectories across boundaries is then that an observer on the boundary should measure the same redshift and direction (in their rest space) for the photon on either side of the boundary.
    - Method I: Assume imperfect tangency cancels on average, and take $\dot{t}|_{\rm out} \simeq \dot{t}|_{\rm in}$.
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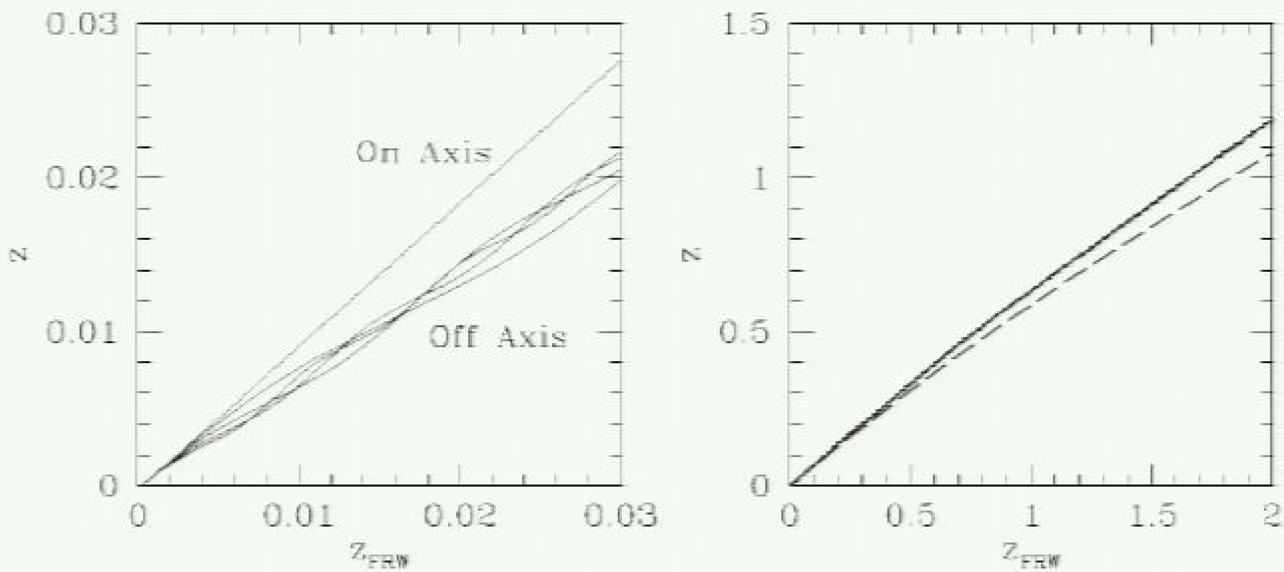

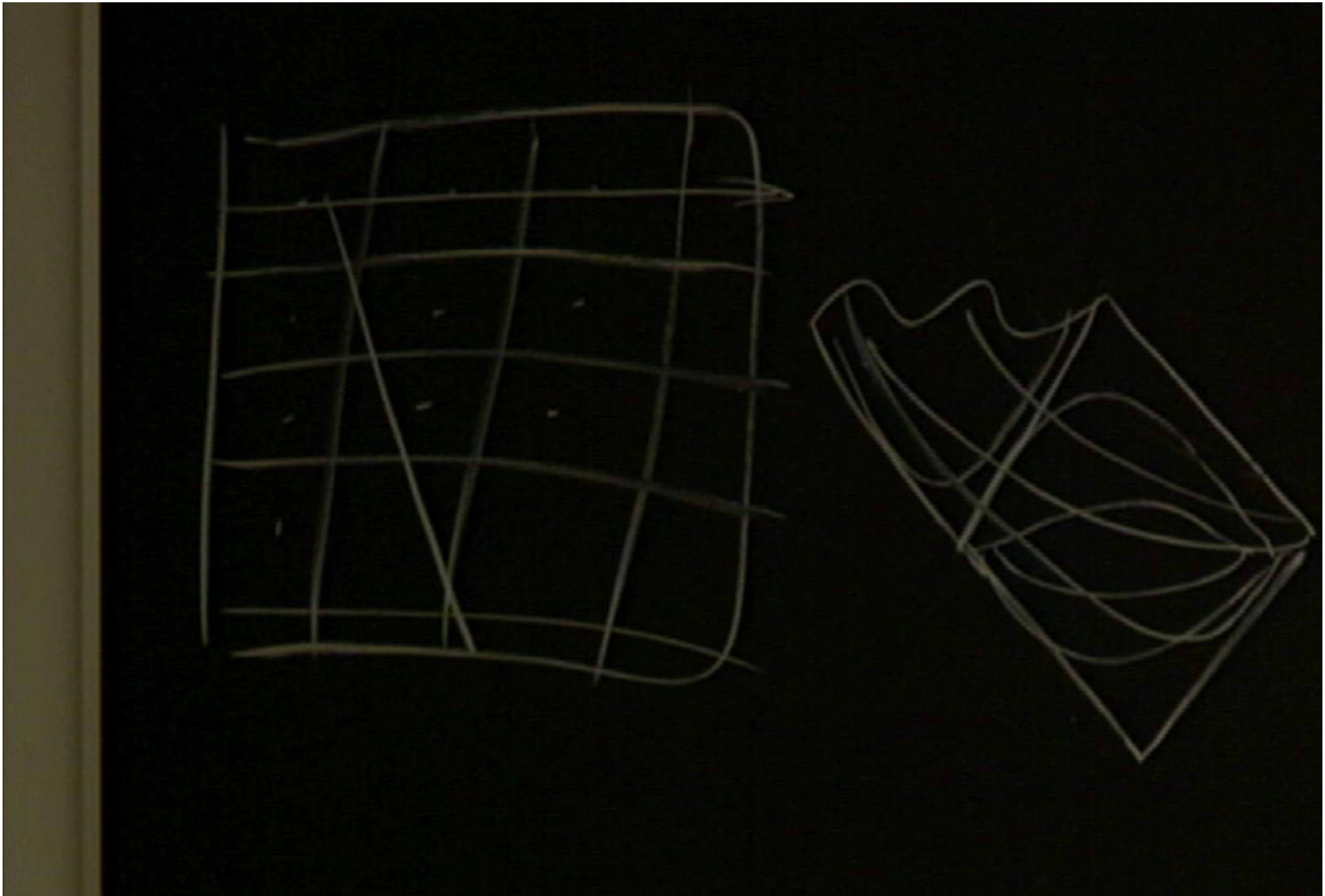

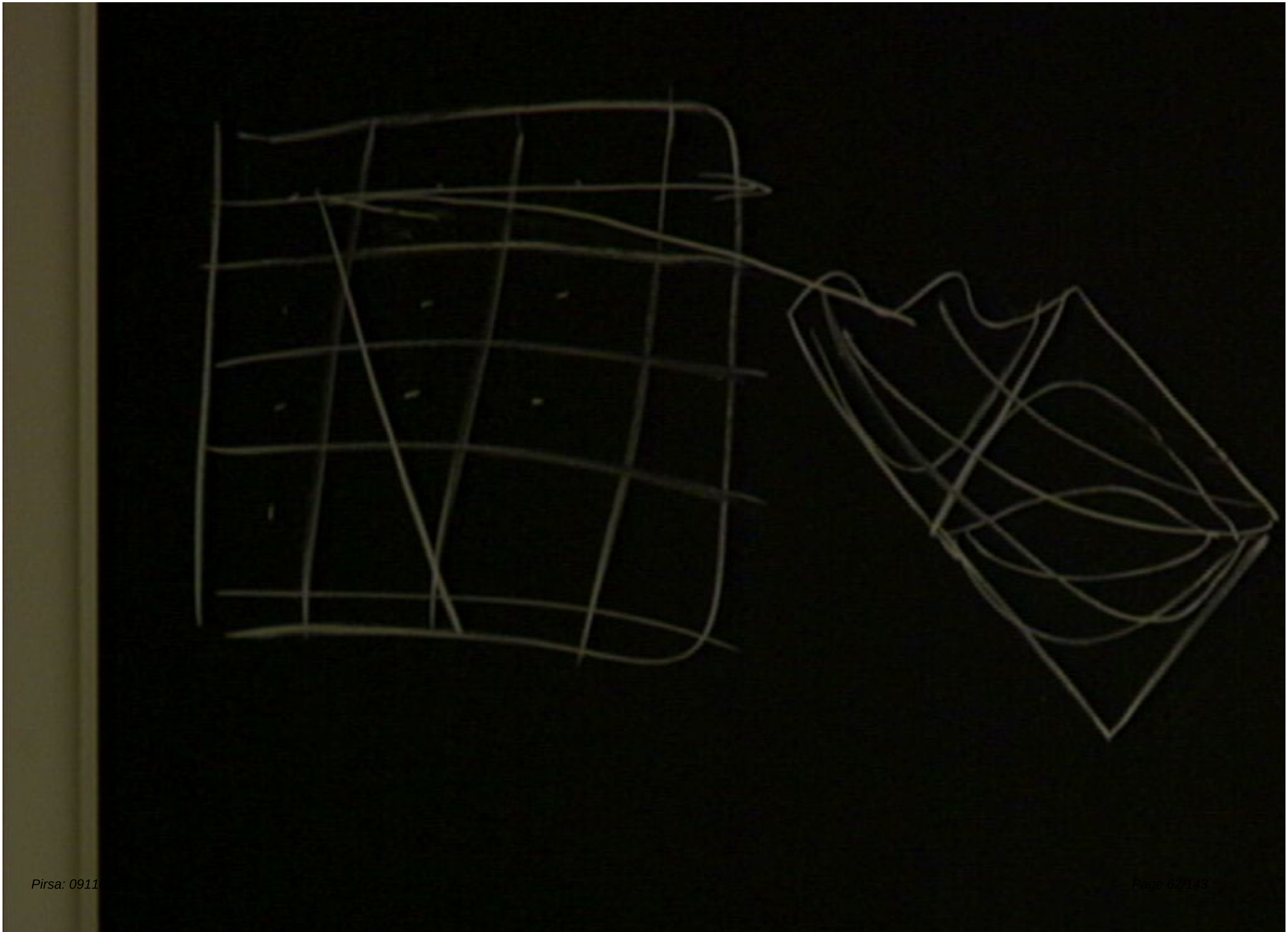

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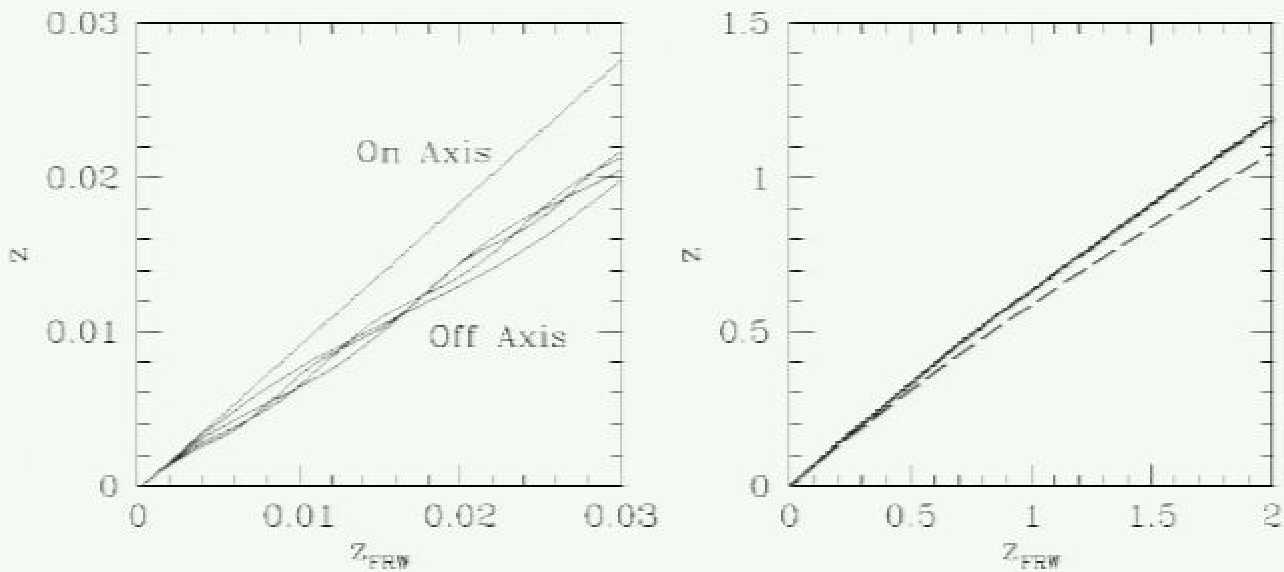

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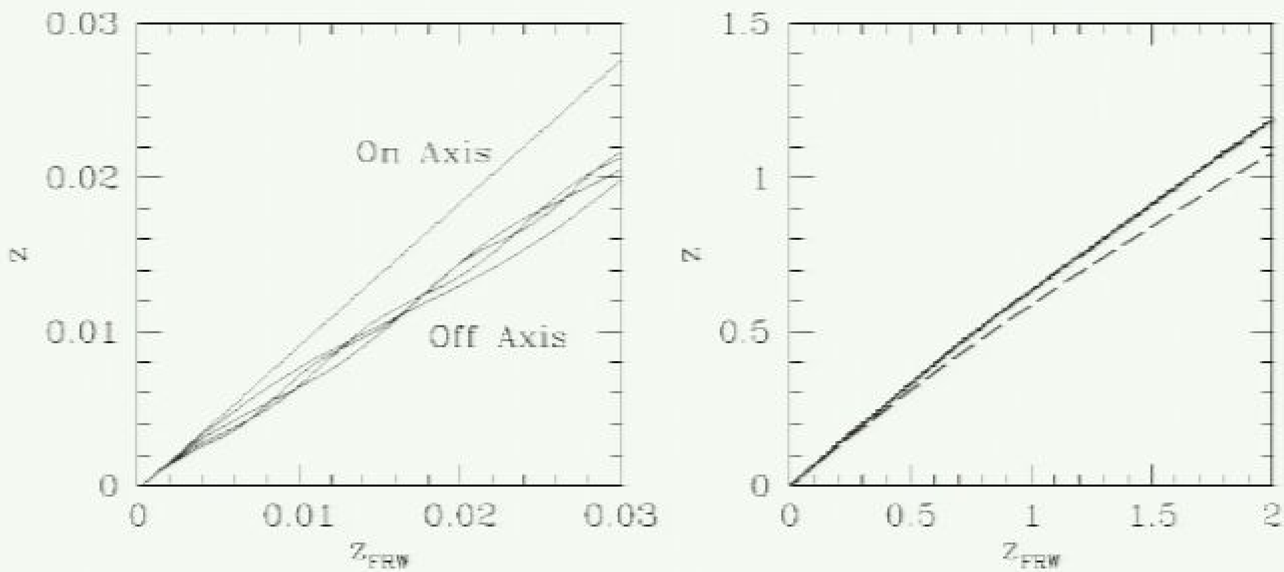

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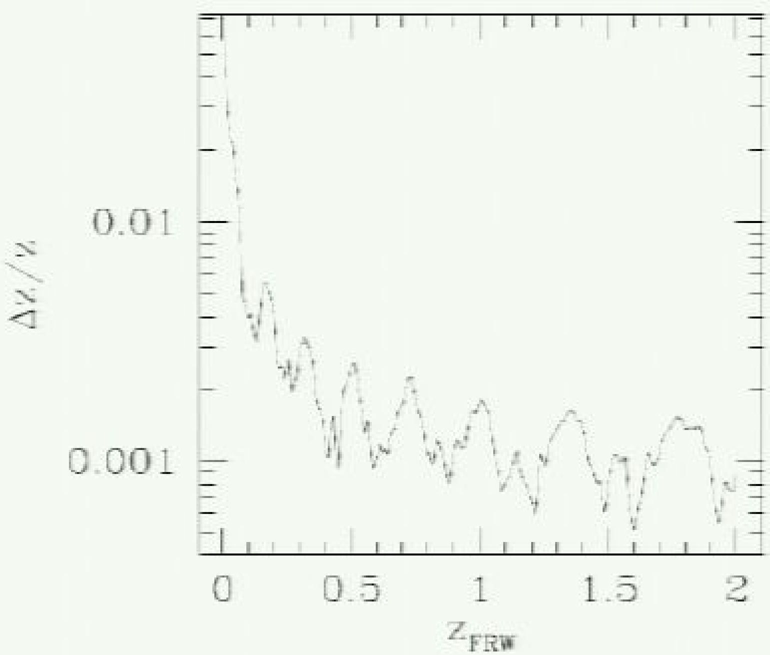

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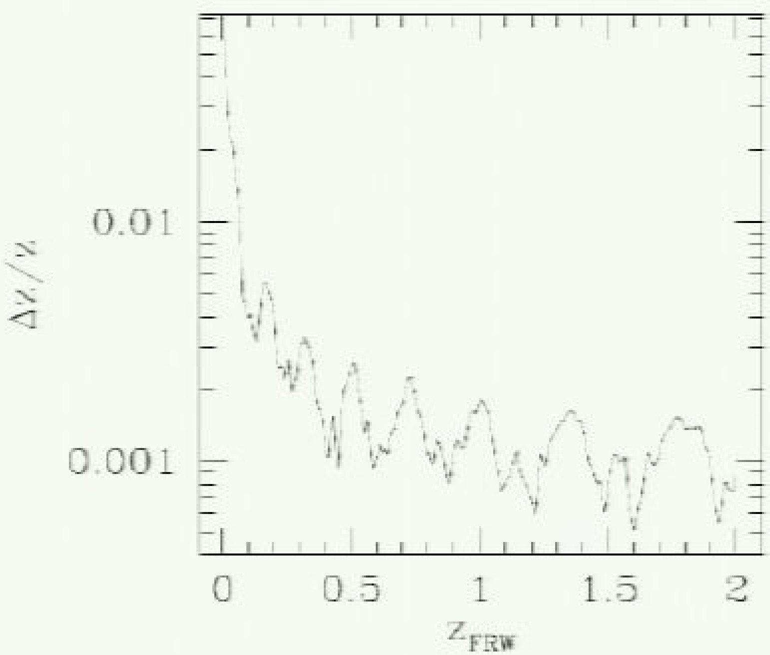

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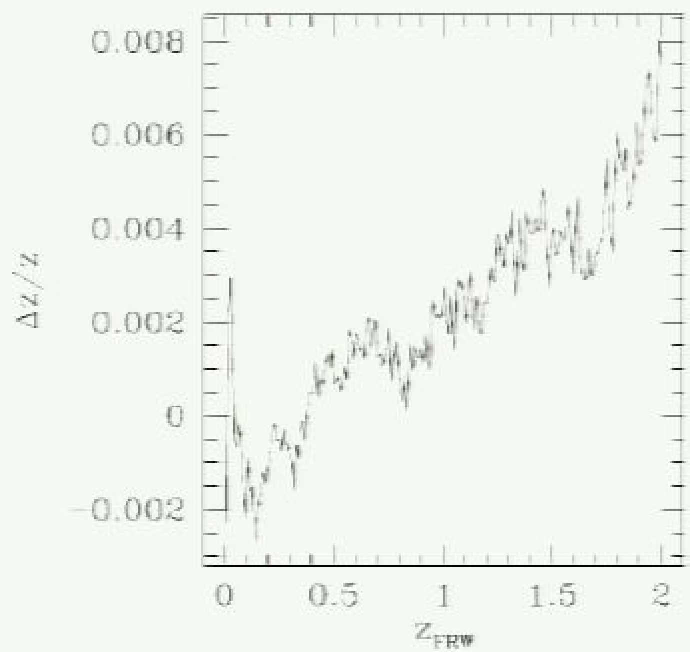

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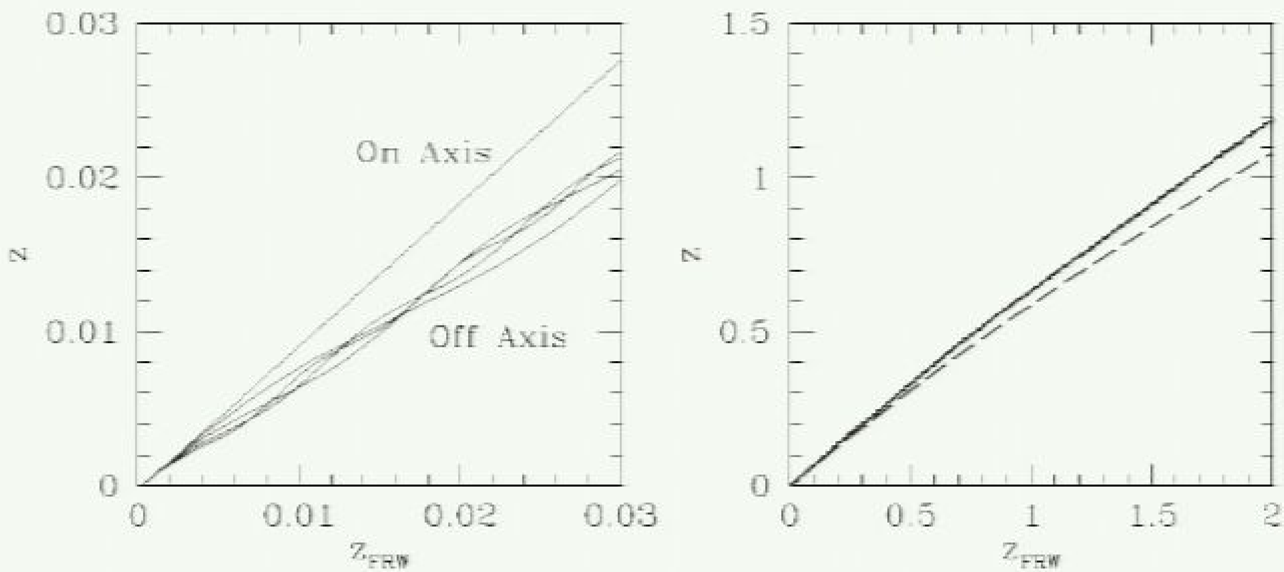

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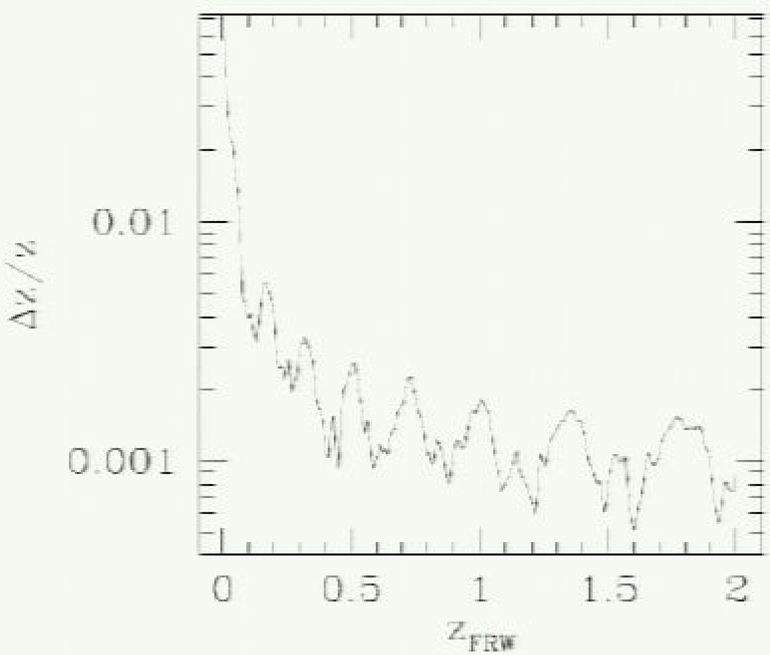

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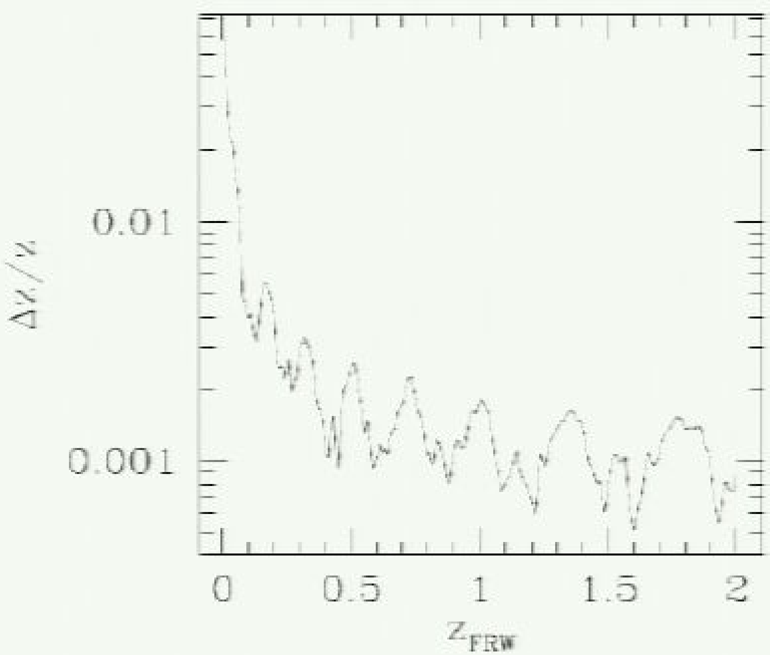

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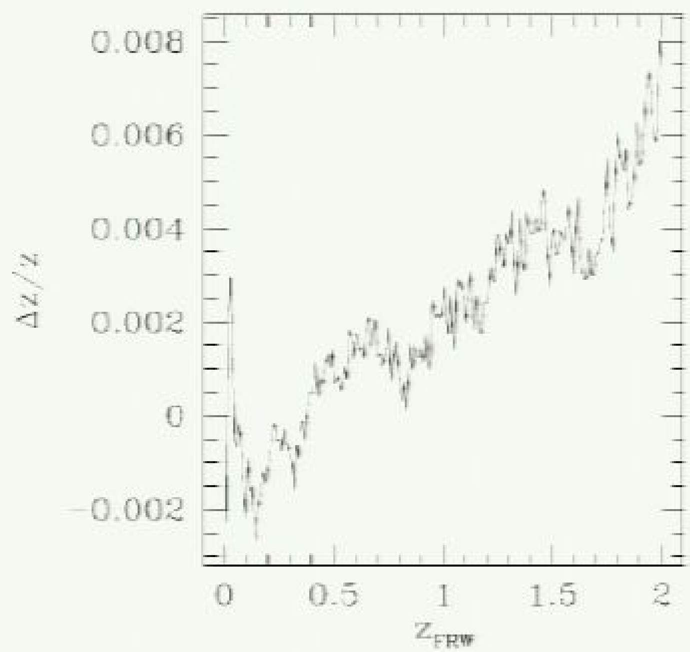

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• Now $v \equiv \sqrt{2m/r}$ is the velocity of the observer. Expanding in this quantity gives the following expression at small z:

$$(1+\delta z_i) = \frac{\dot{\tau}|_{\rm in}}{\dot{\tau}|_{\rm out}} \simeq \frac{(1-v_{in}(\alpha_{in}-v_{in}))}{(1-v_{out}(\alpha_{out}-v_{out})} \simeq 1+v(\alpha_{out}-\alpha_{in})$$

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- Along a principle axis we find $\langle\gamma\rangle$ =1, and for random trajectories crossing many spheres we find $\langle\gamma\rangle$ =2/3. Comparing to the results of the numerical integration gives $\langle\gamma\rangle$ =7/10.

Optics in a lattice

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- The Sachs optical equations are:

$$\begin{aligned}\frac{d\tilde{\theta}}{d\lambda} + \tilde{\theta}^2 - \omega^2 + \sigma^*\sigma &= -\frac{1}{2}R_{ab}k^a k^b \\ \frac{d\omega}{d\lambda} + 2\omega\tilde{\theta} &= 0 \\ \frac{d\sigma}{d\lambda} + 2\sigma\tilde{\theta} &= C_{abcd}(t^*)^a k^b (t^*)^c k^d\end{aligned}$$

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- Angular diameter distance and luminosity distance are then given by:

$$r_A \propto \exp\left\{\int_e^o \tilde{\theta}\, d\lambda\right\} \quad \text{and} \quad r_L=(1+z)^2 r_A\,.$$

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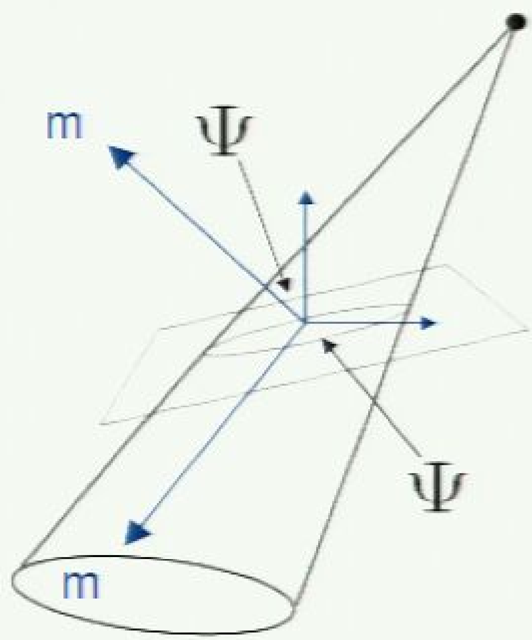

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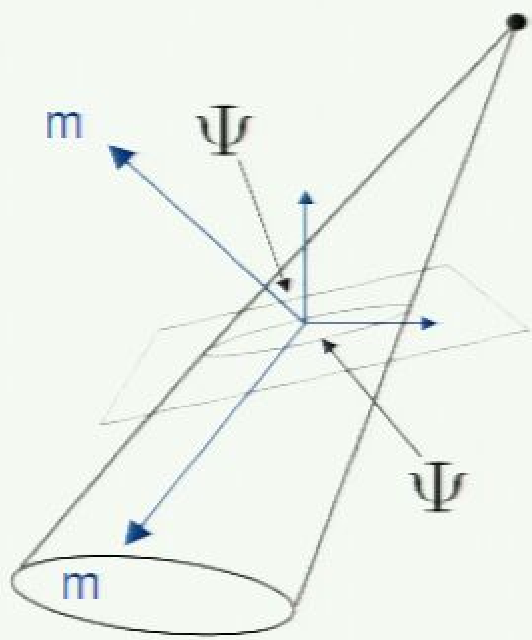
- The magnitude of the shear scalar therefore takes an (approximately) random walk as it travels through many cells.



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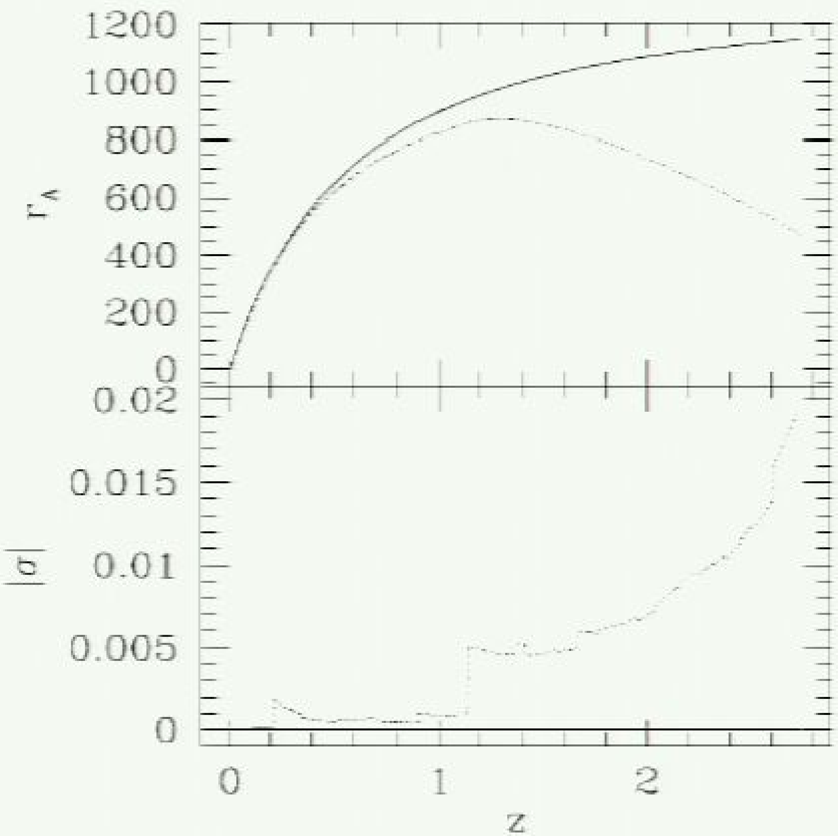

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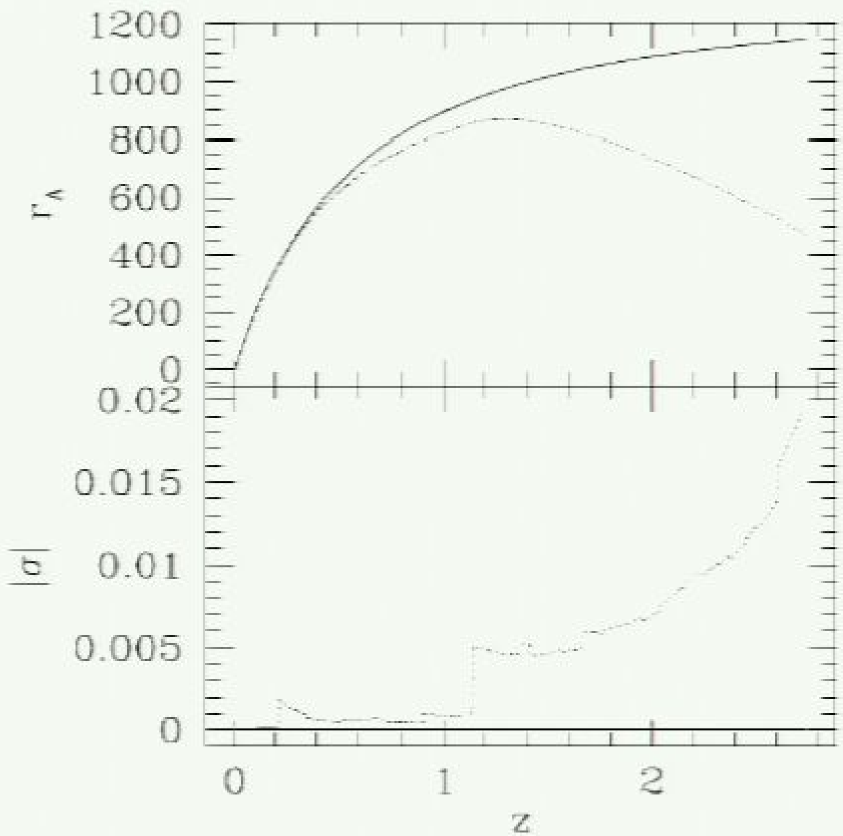

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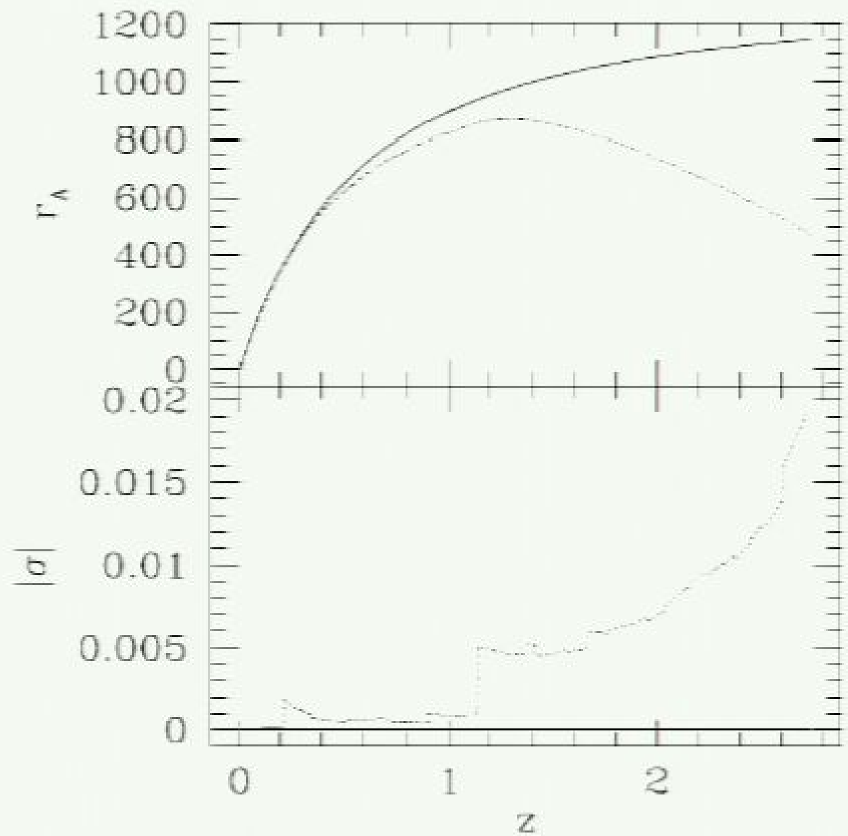

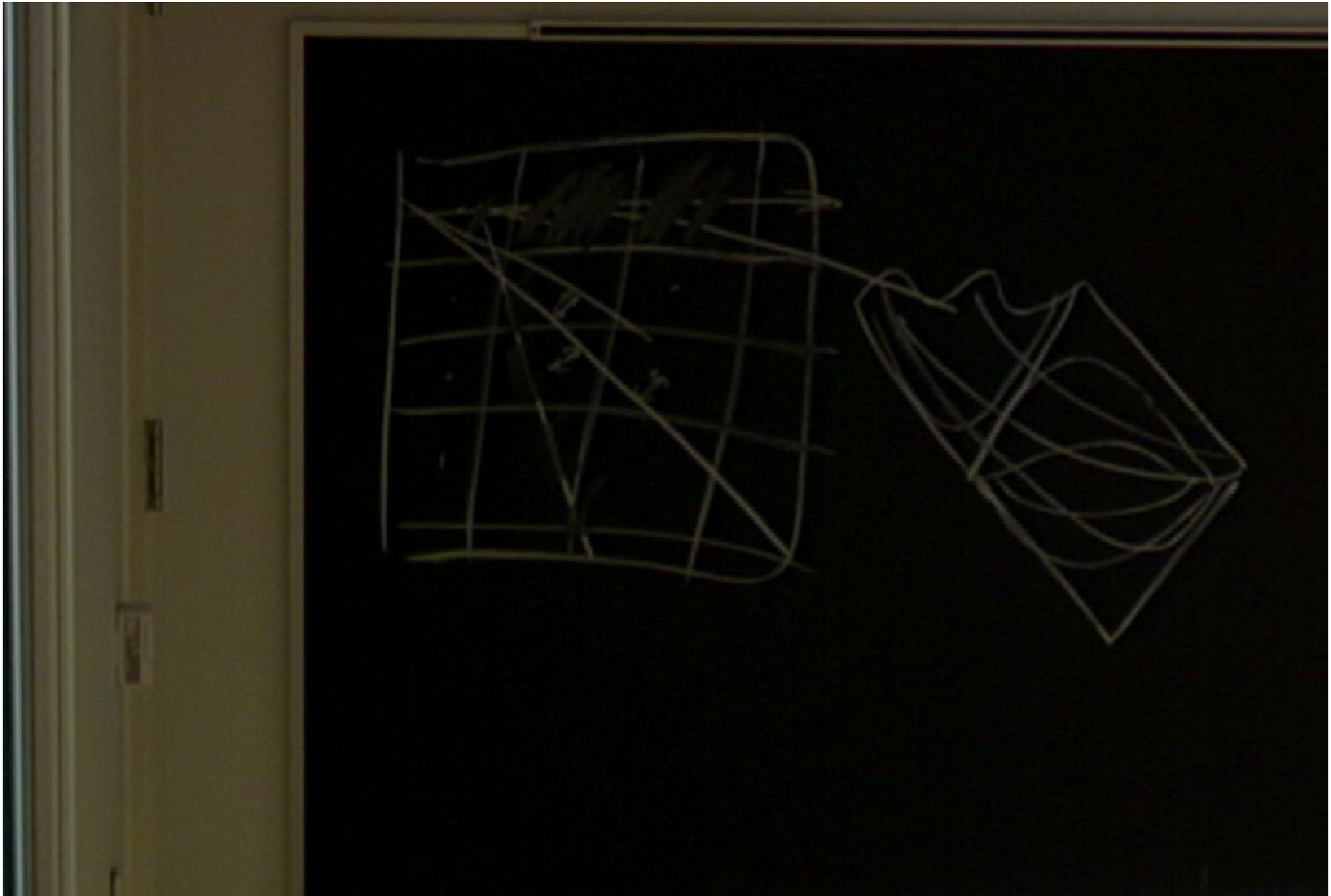

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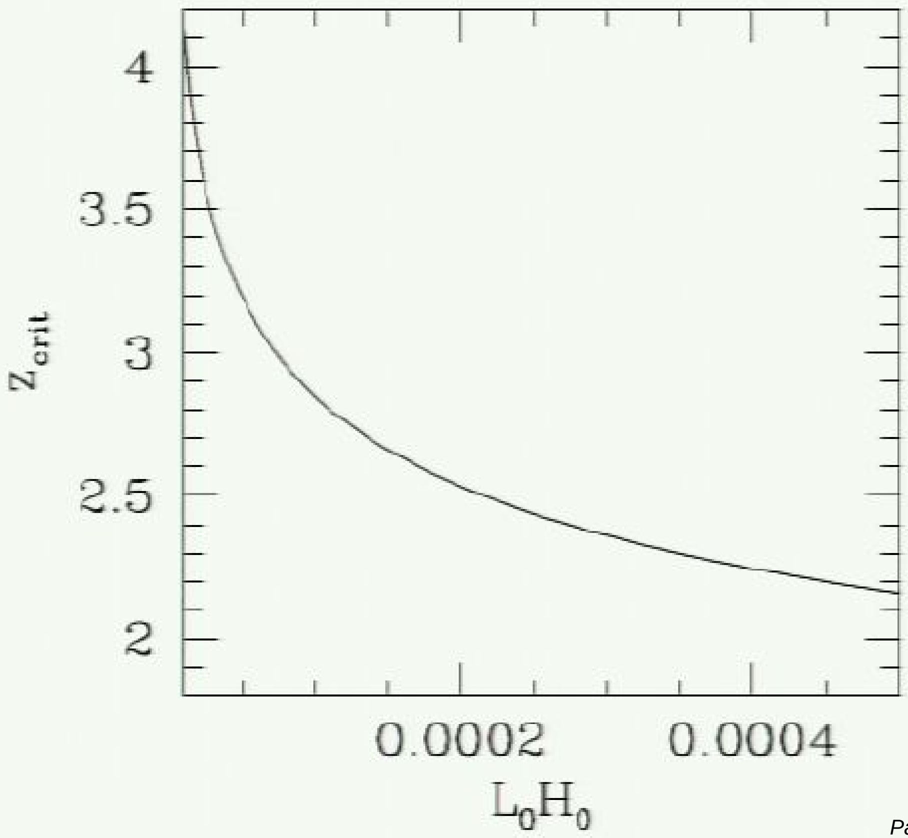

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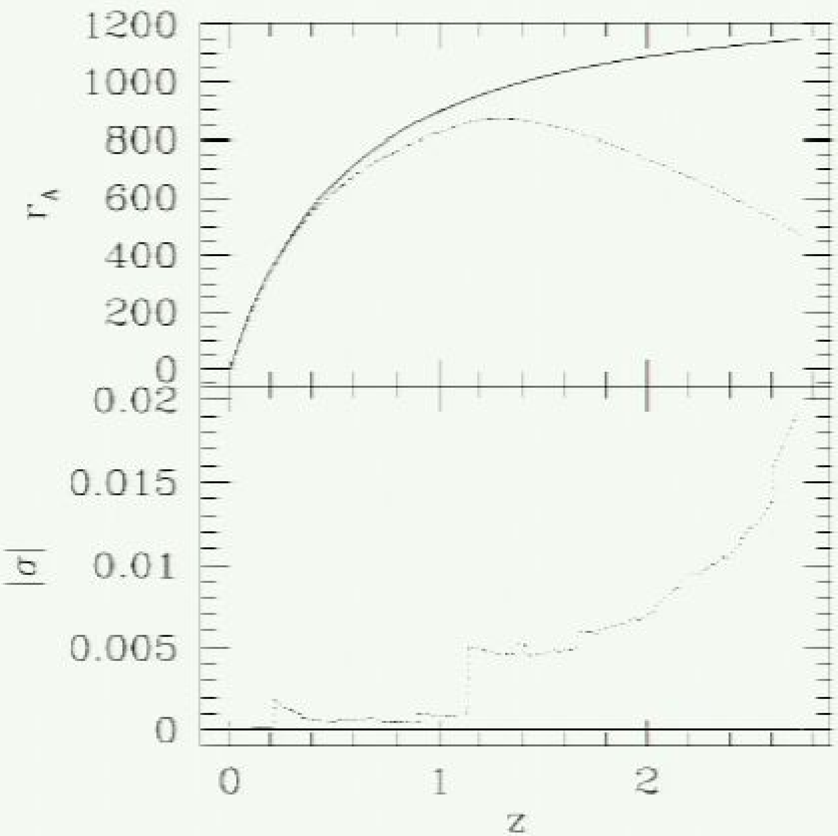

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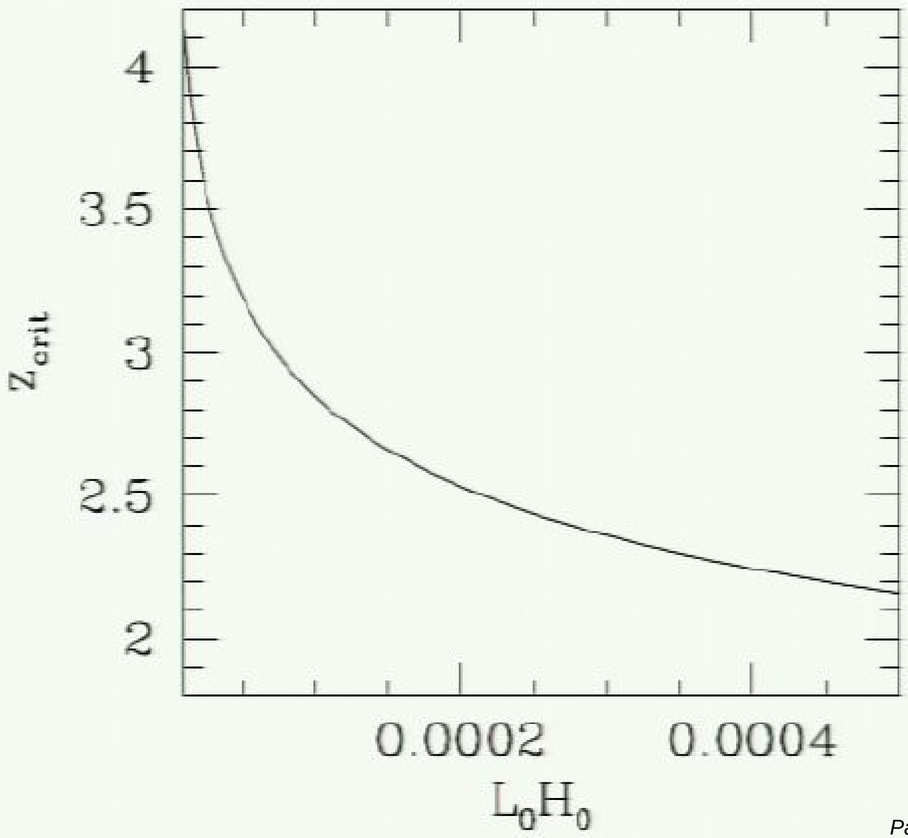

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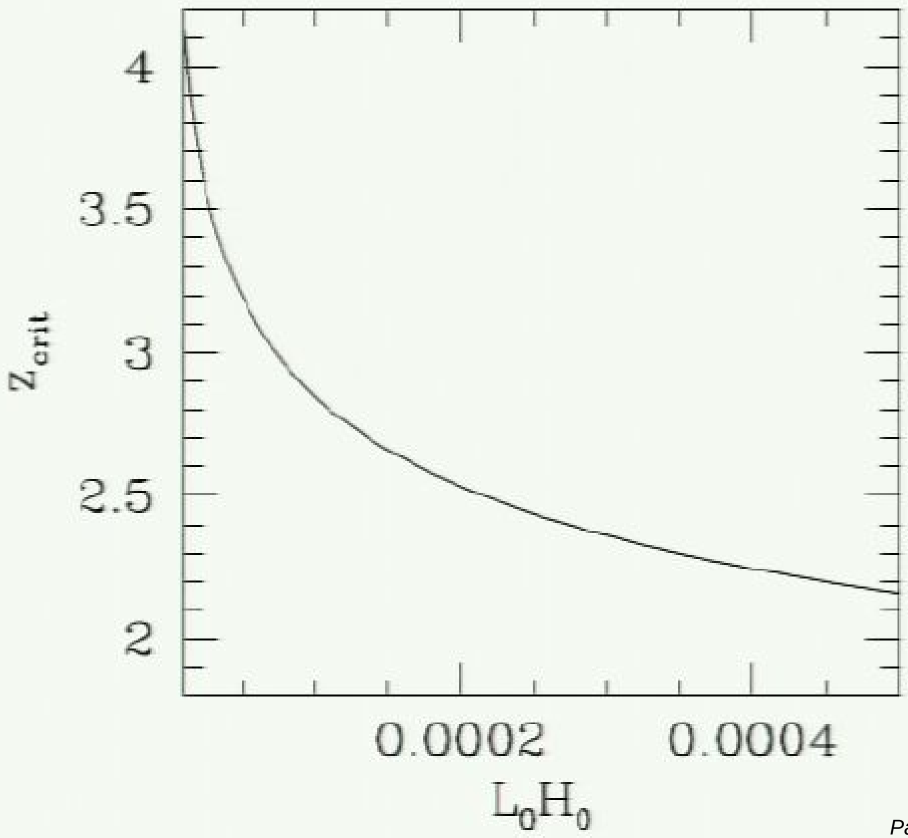

Observables in a lattice

- For a typical trajectory, the low z luminosity distance is given by:

$$r_L = (1+z)^2 - (1+z)^{1-\frac{3}{2\langle\gamma\rangle}} \sim z + \left(1 - \frac{3}{4\langle\gamma\rangle}\right) z^2 + \frac{(3-2\langle\gamma\rangle)}{8\langle\gamma\rangle^2} z^3 + O(z^4)$$

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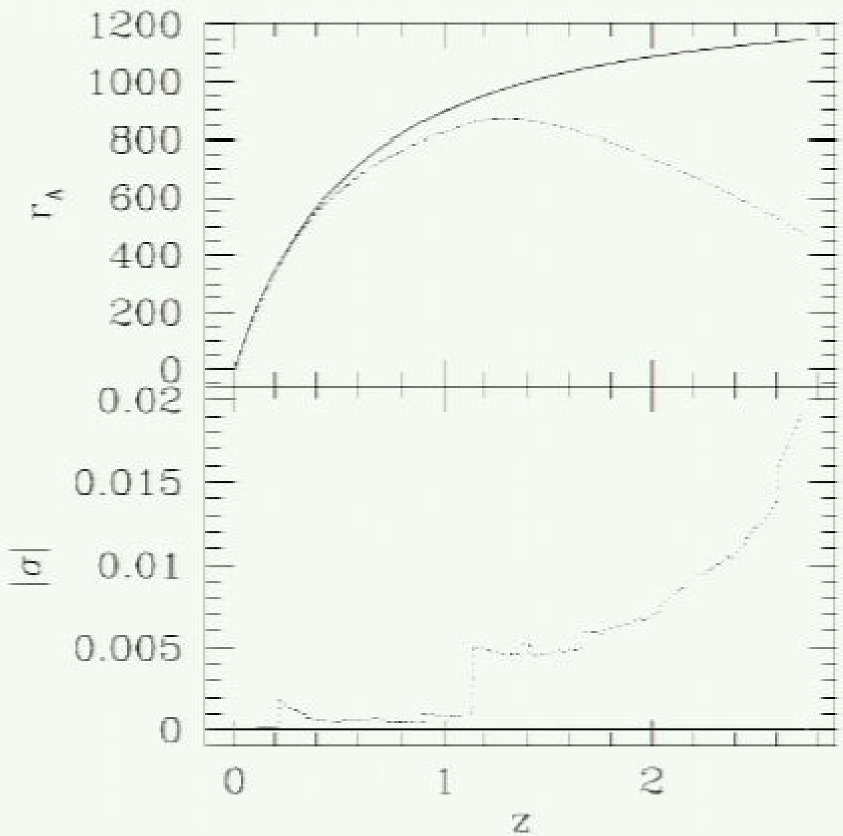

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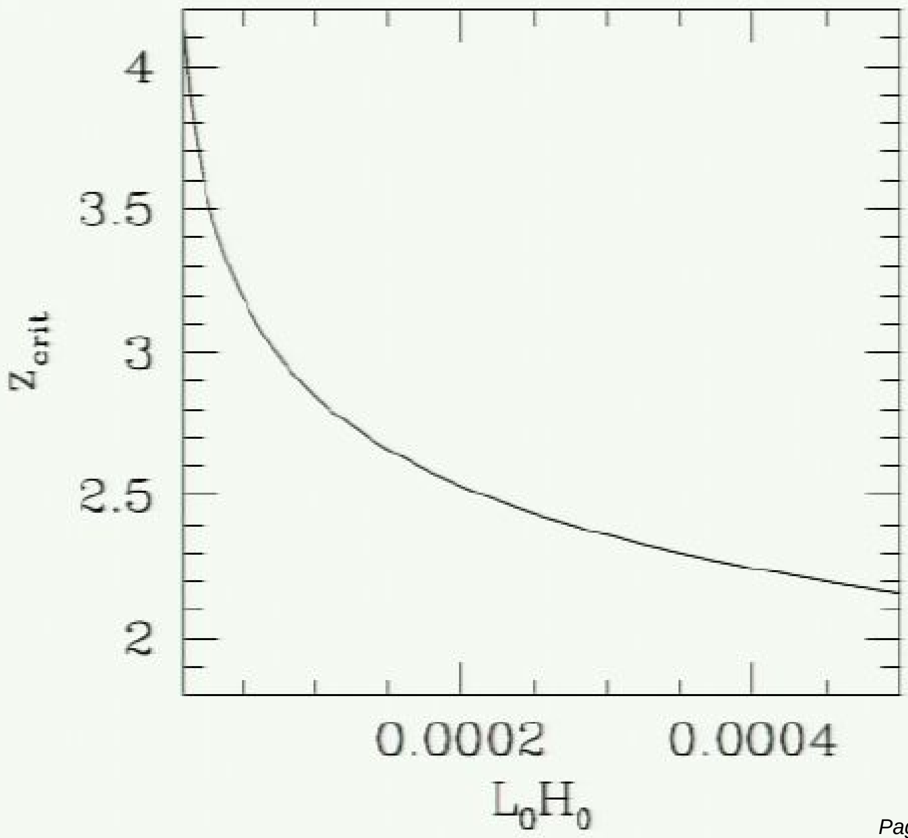

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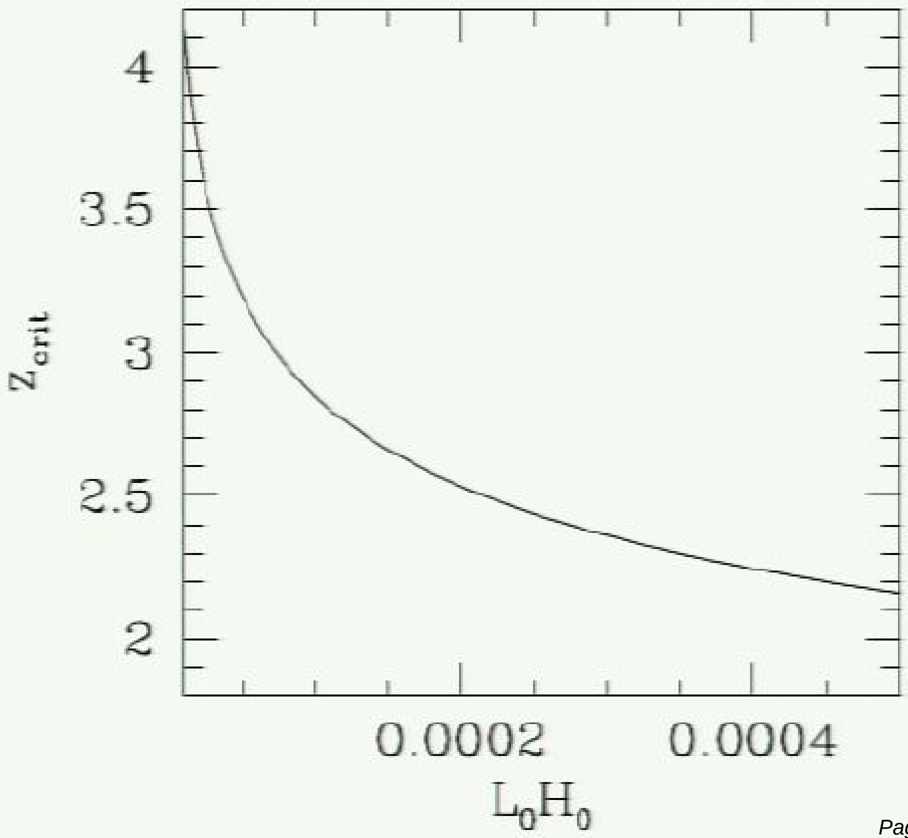

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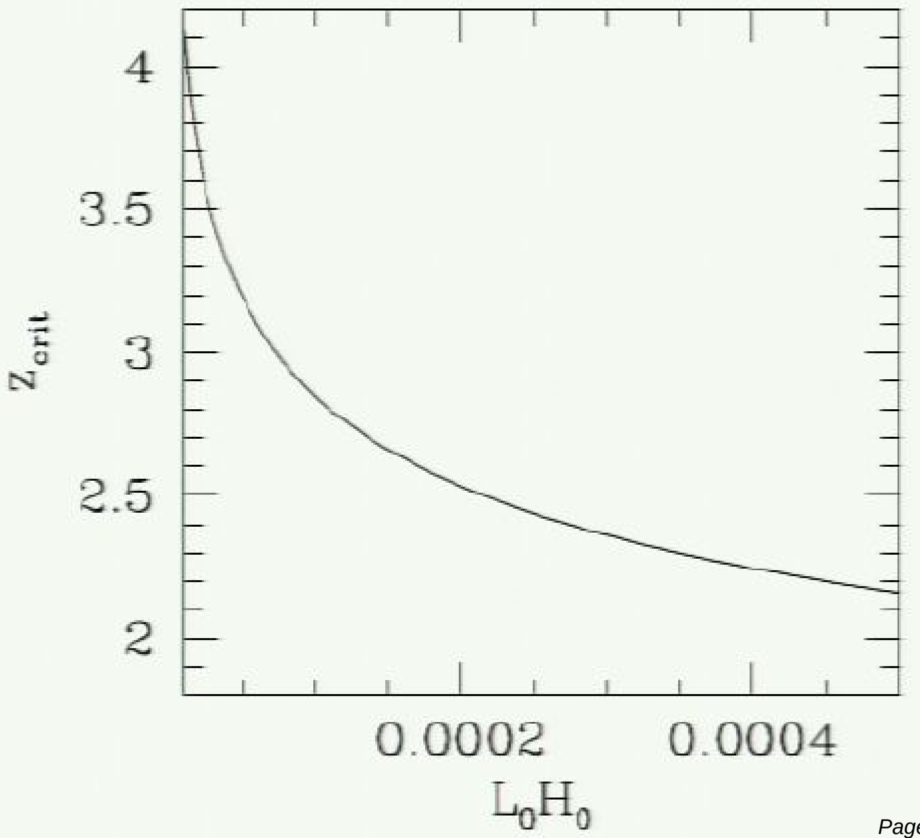
- Below z=1 it is a good approximation to neglect shear entirely, for a typical trajectory.



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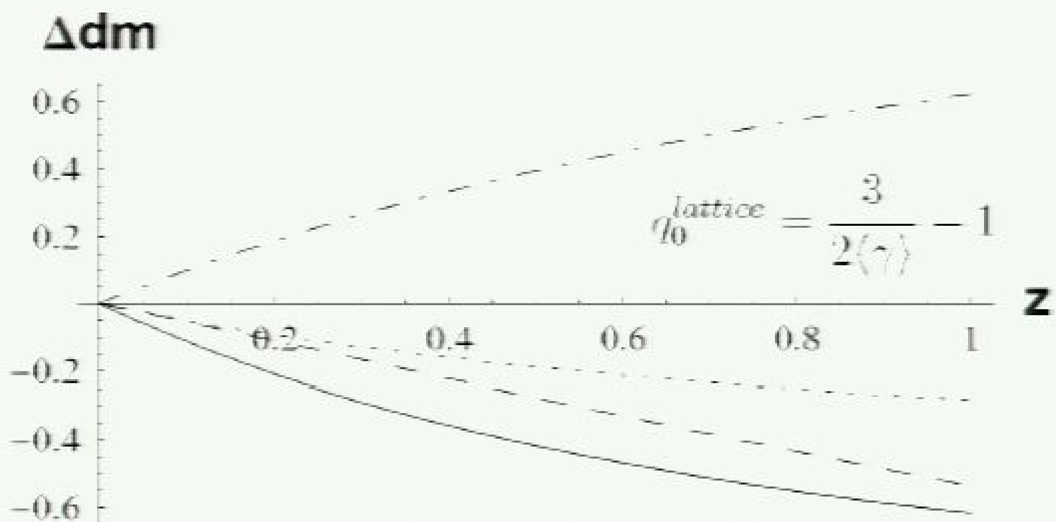

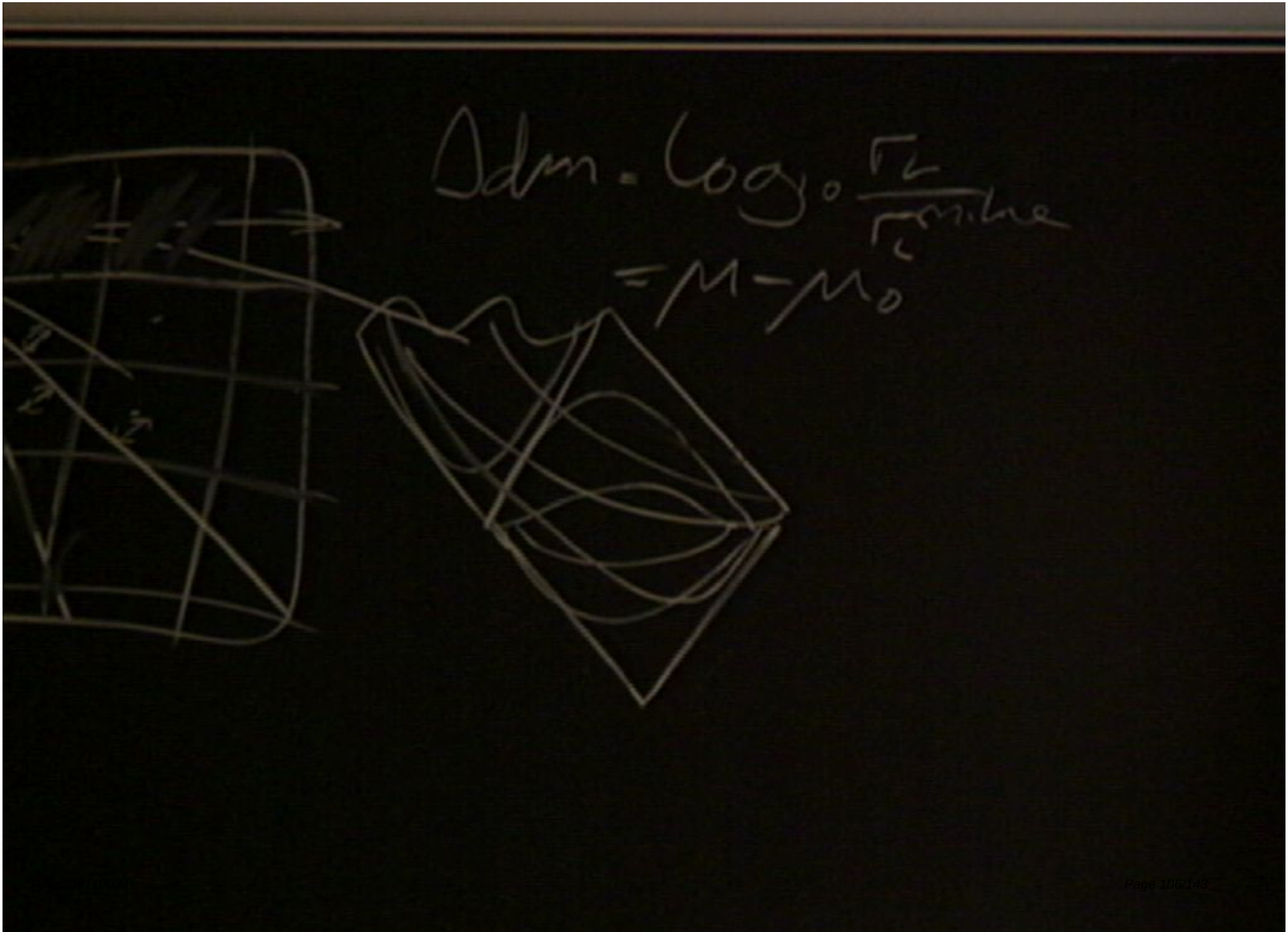
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where:

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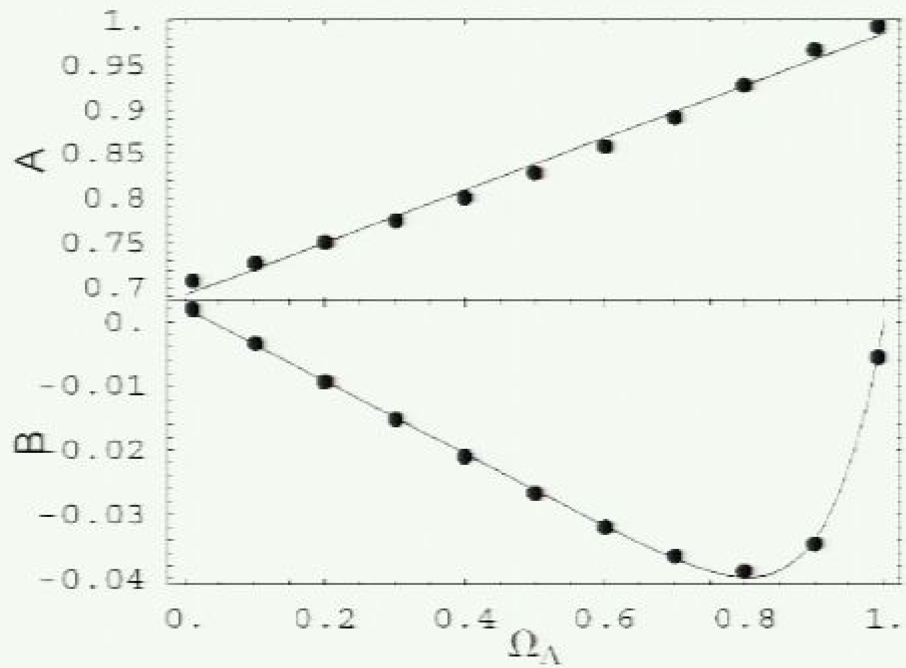

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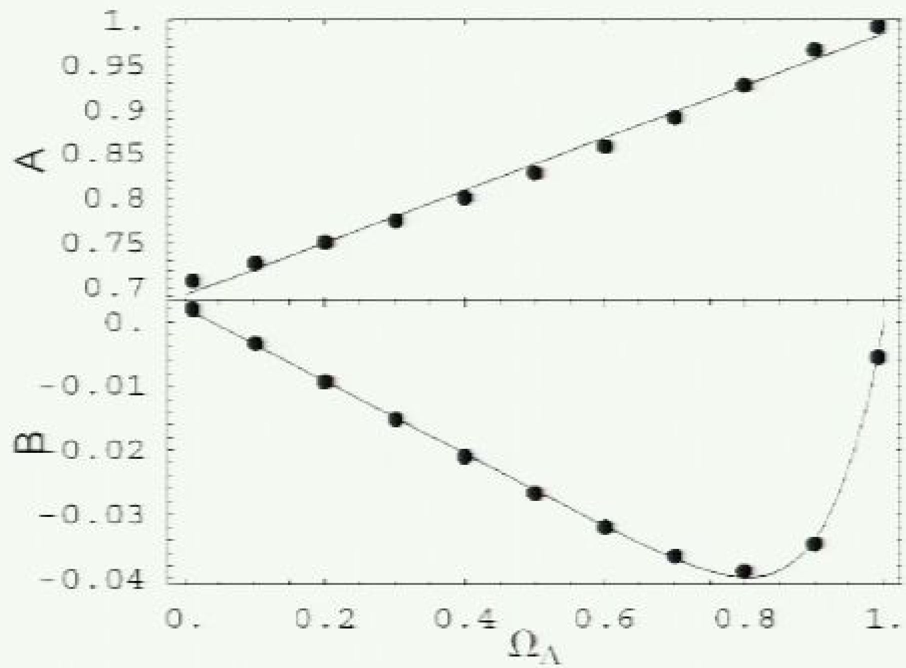

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- The luminosity distance is then given by:

$$r_L \propto (1+z)^2 \lambda \propto (1+z)^2 \left[\frac{f(z)}{(1+z)^{\frac{3+2\langle\gamma\rangle}{2\langle\gamma\rangle}}} - f(0) \right]$$

where: $f(z) = {}_2F_1 \left(\frac{3+2\langle\gamma\rangle}{6}, \frac{1}{2}; \frac{9+2\langle\gamma\rangle}{6}; -\frac{\Omega_\Lambda}{\Omega_m}(1+z)^{-\frac{3}{\langle\gamma\rangle}} \right)$

and: $z(1-z)f''(z) + [c-(a+b+1)z]f'(z) = abf$

Comparison with the SNLS

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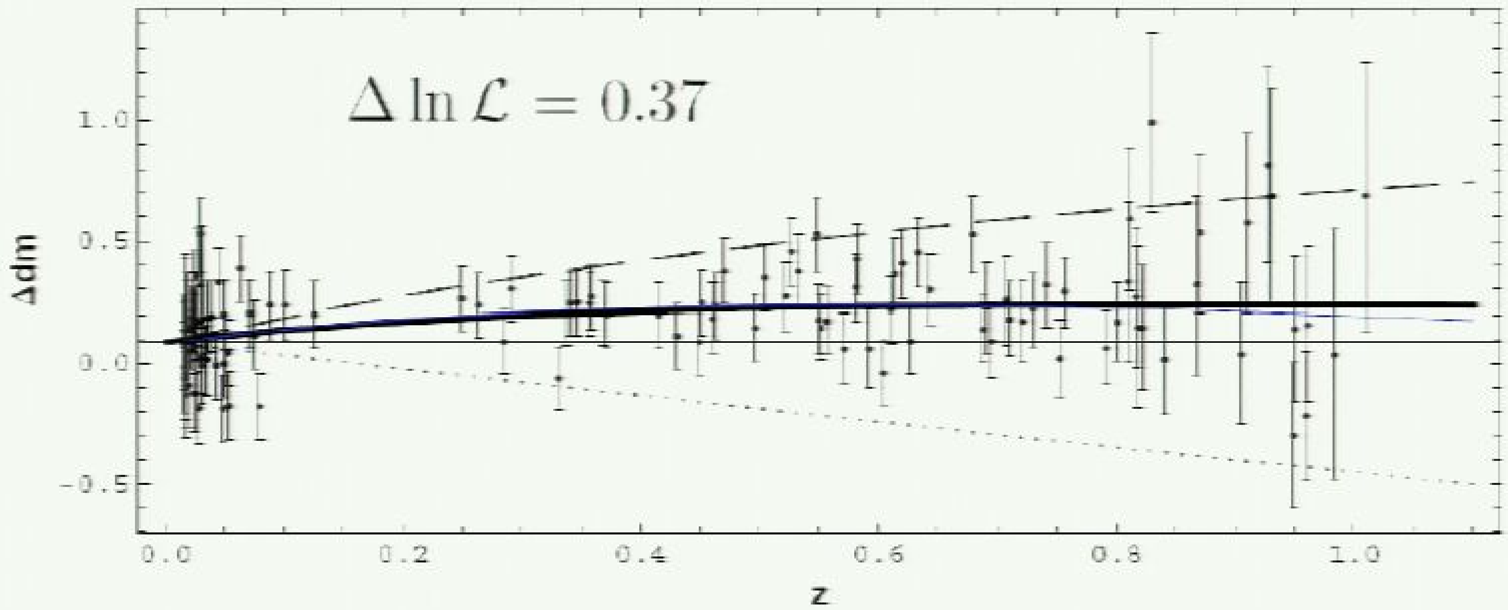
- Comparison with the 115 Sne of Astier et al gives:



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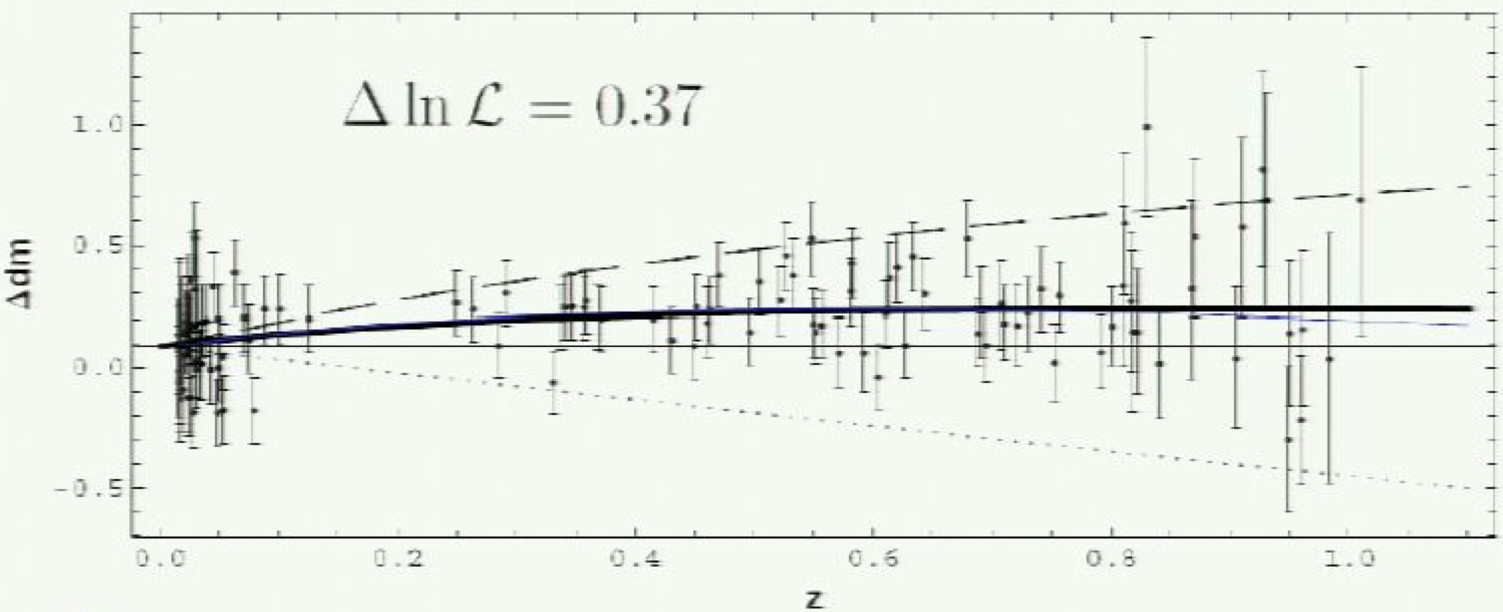

$$\Rightarrow \quad \Omega_\Lambda^{FRW} = 0.74 \pm 0.04 \quad \text{and} \quad \Omega_\Lambda = 0.66 \pm 0.04$$

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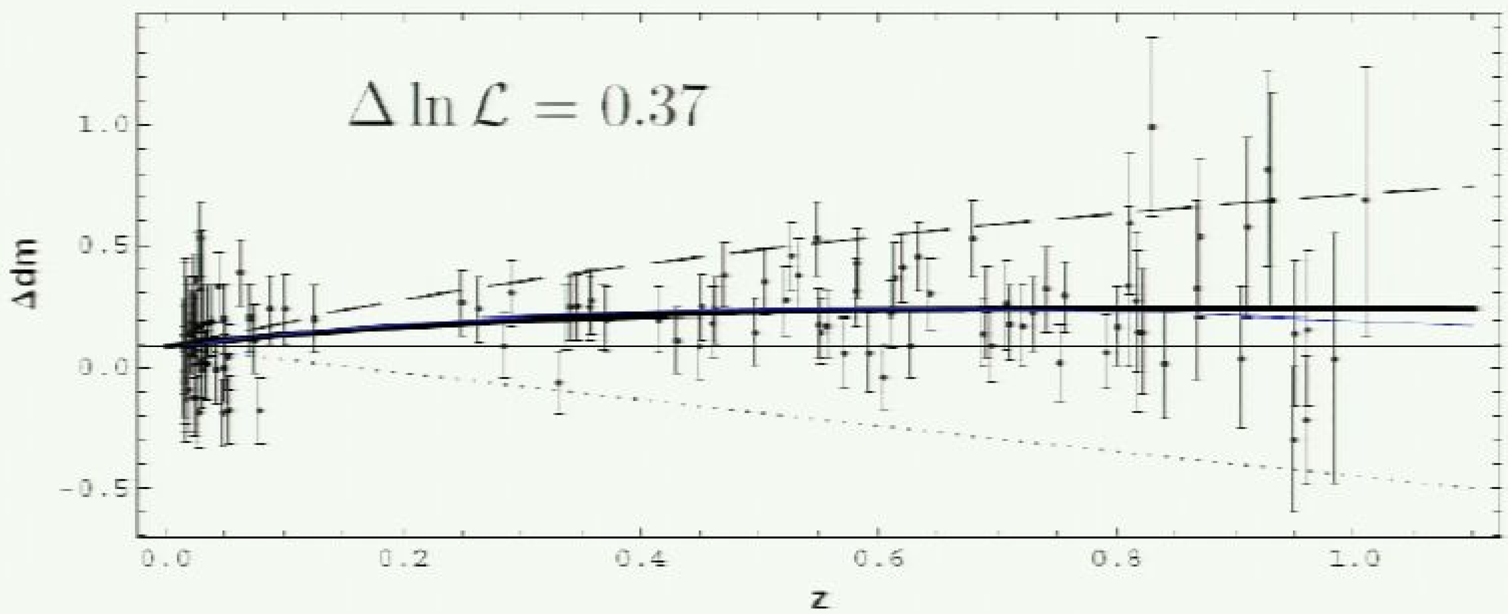

Observables in a lattice

- For a typical trajectory, the low z luminosity distance is given by:

$$r_L = (1+z)^2 - (1+z)^{1-\frac{3}{2\langle\gamma\rangle}} \sim z + \left(1 - \frac{3}{4\langle\gamma\rangle}\right) z^2 + \frac{(3-2\langle\gamma\rangle)}{8\langle\gamma\rangle^2} z^3 + O(z^4)$$

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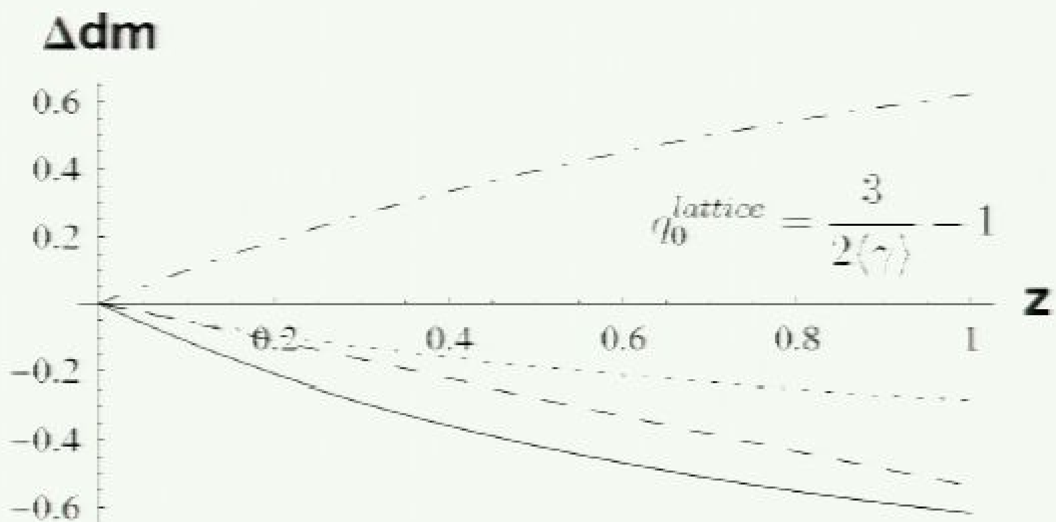

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where:

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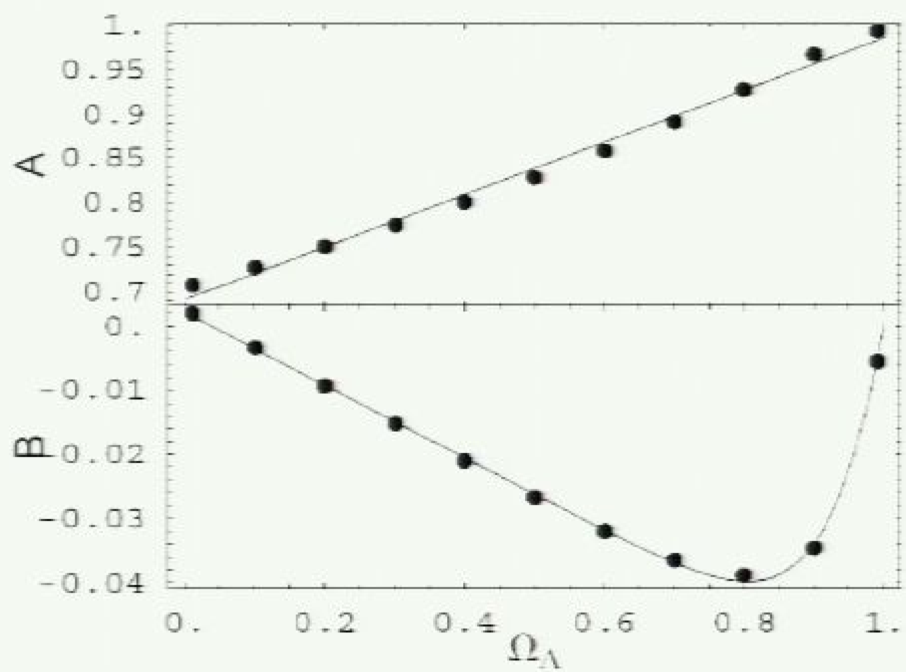

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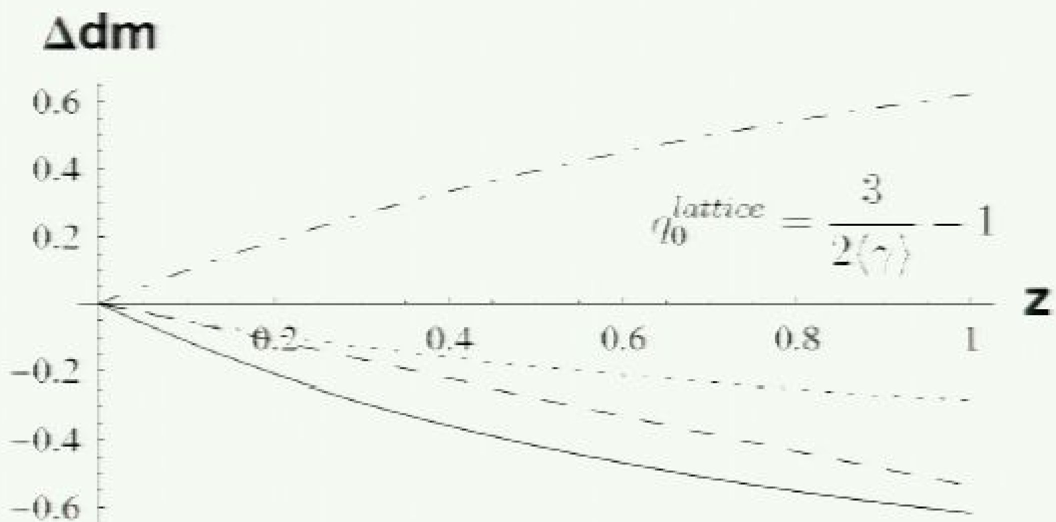

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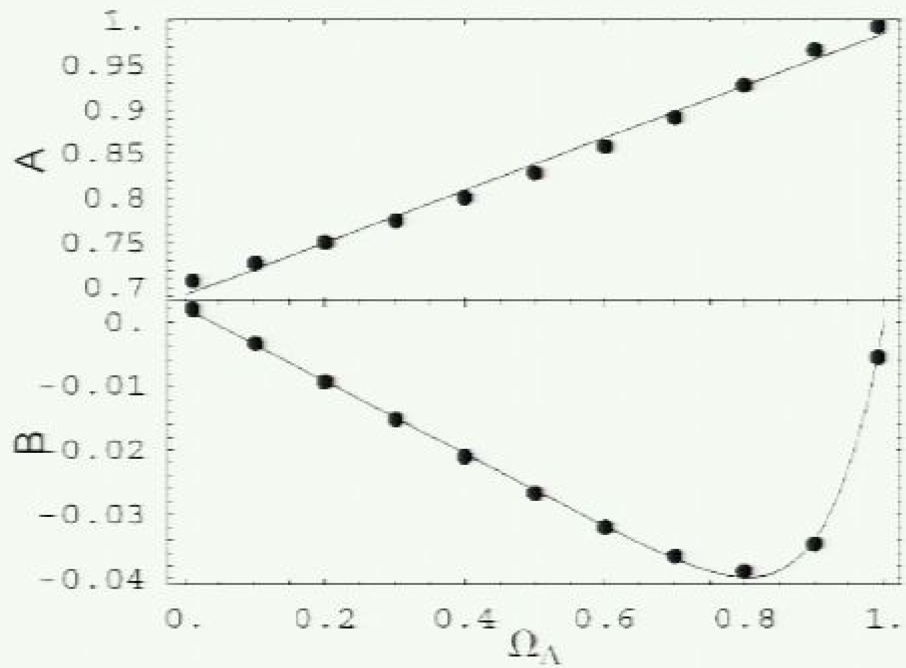

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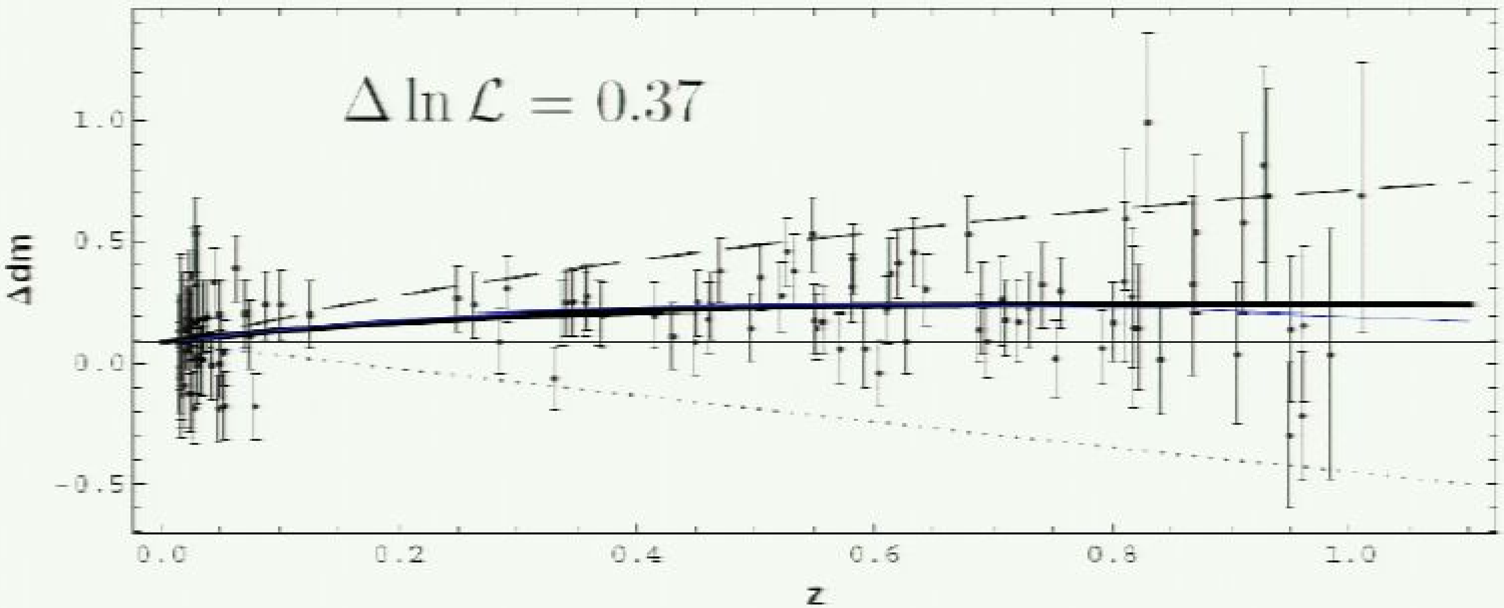

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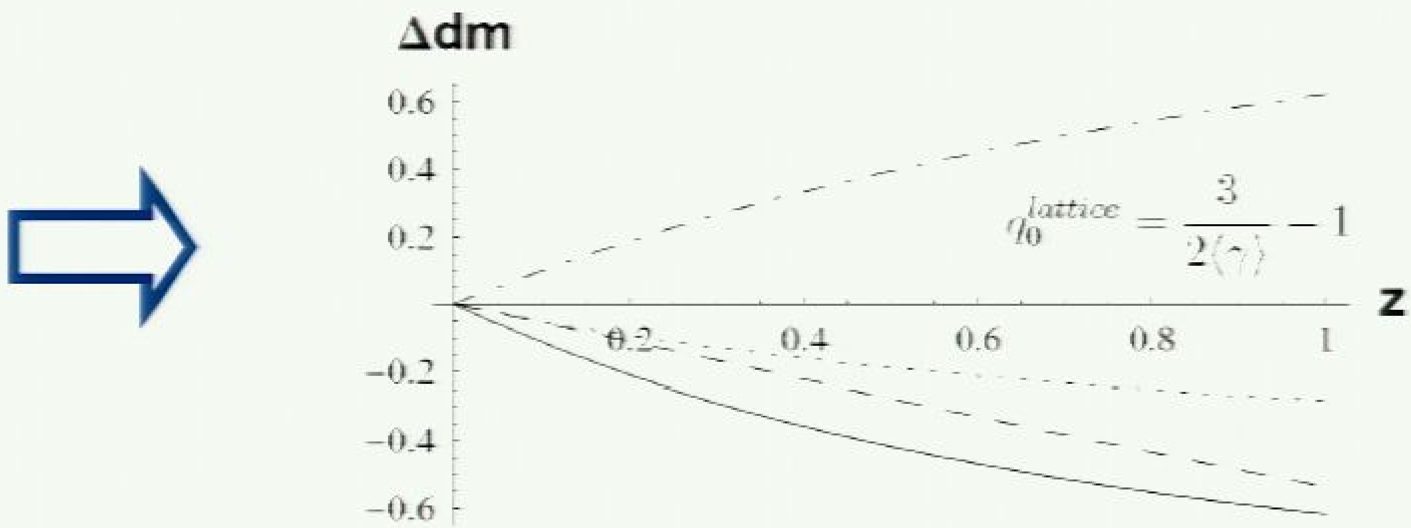

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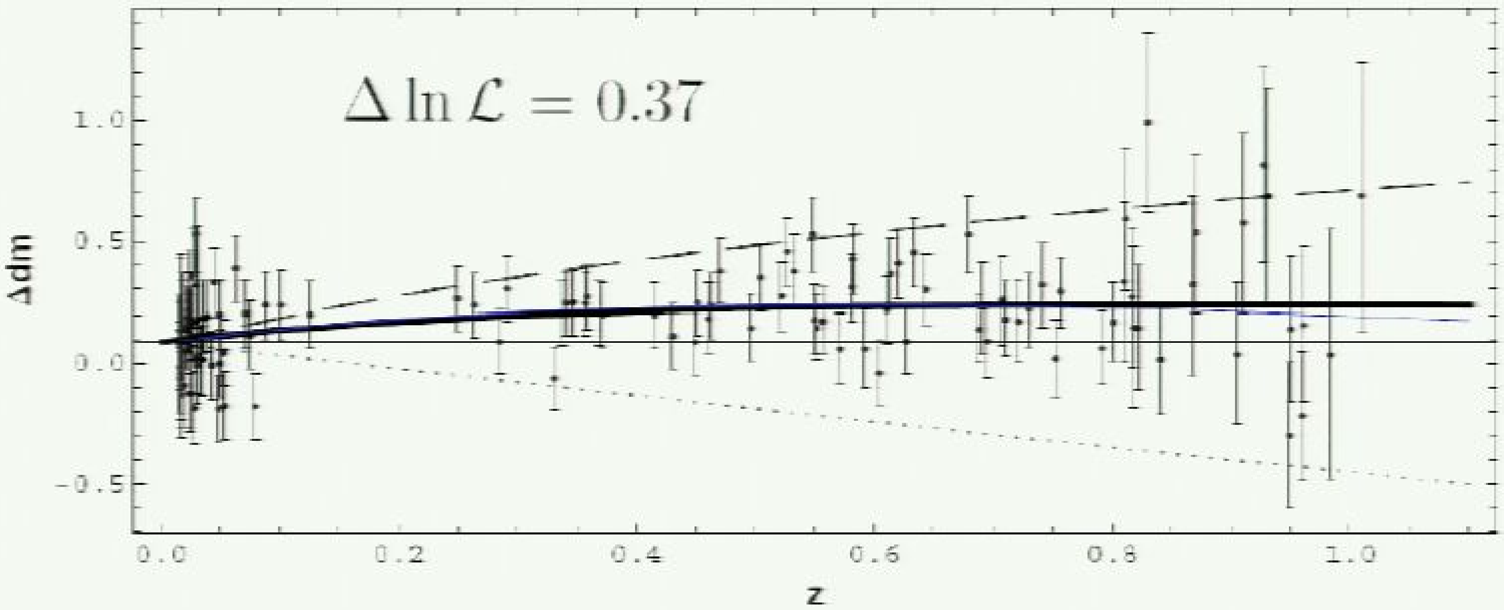

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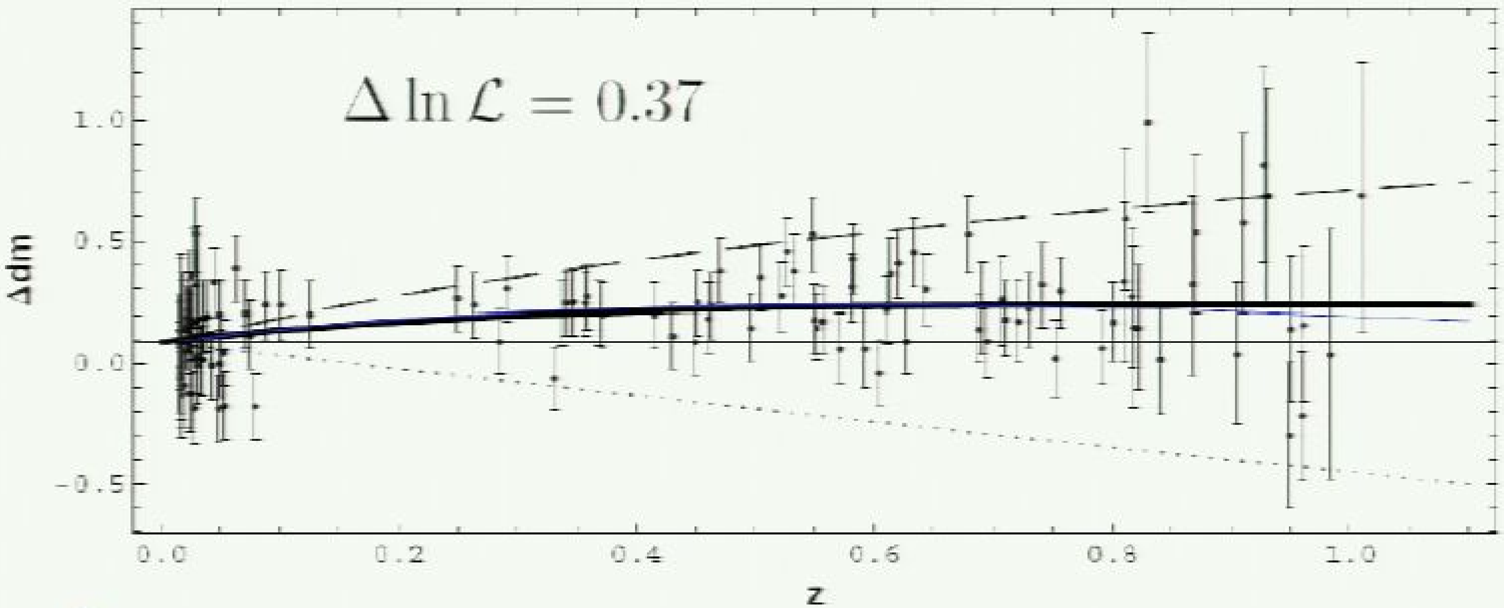

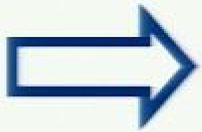 $\Omega_\Lambda^{FRW} = 0.74 \pm 0.04$ and $\Omega_\Lambda = 0.66 \pm 0.04$

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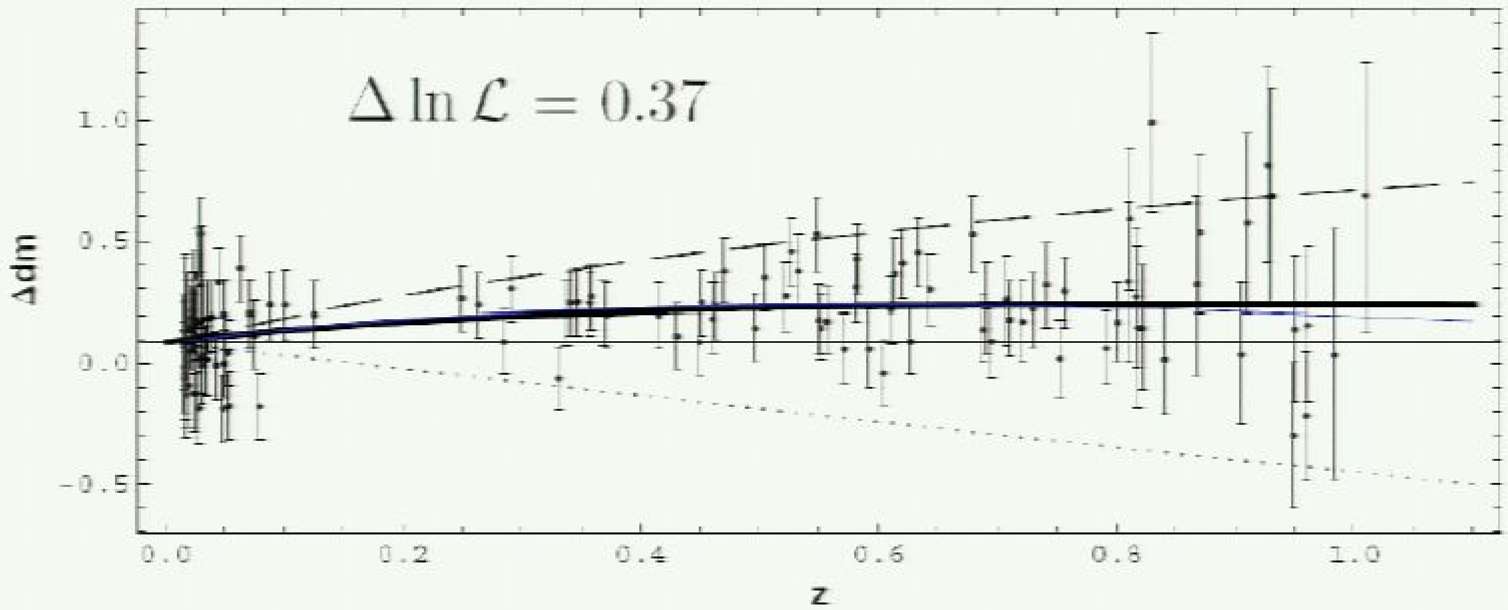

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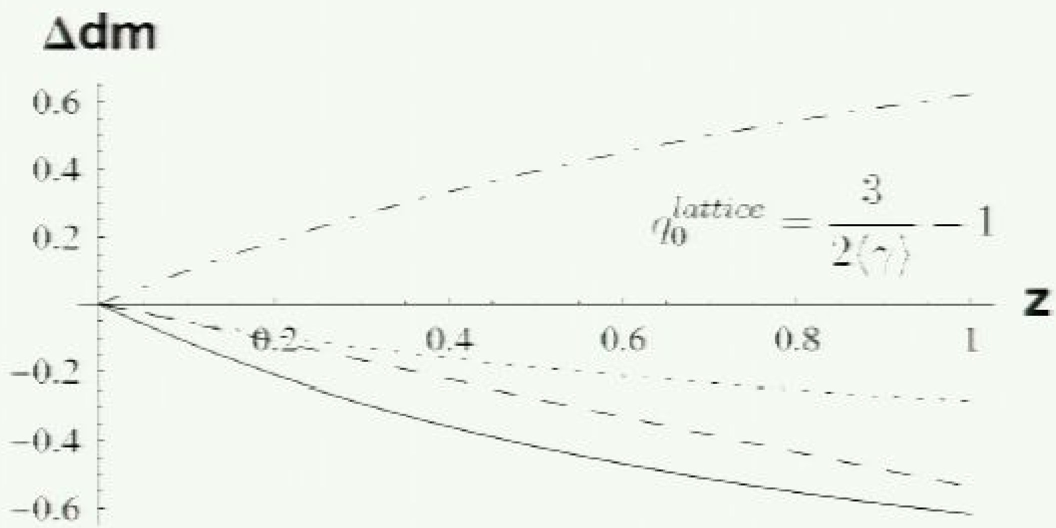

Including Lambda

- The driving terms in the optical equations are the same as before:

$$R_{ab}k^a k^b = 0 \quad \text{and} \quad C = \frac{3mJ^2}{r^5} e^{i\Psi}$$

- The luminosity distance is then given by:

$$r_L \propto (1+z)^2 \lambda \propto (1+z)^2 \left[\frac{f(z)}{(1+z)^{\frac{3+2\langle\gamma\rangle}{2\langle\gamma\rangle}}} - f(0) \right]$$

where: $f(z) = {}_2F_1\left(\frac{3+2\langle\gamma\rangle}{6}, \frac{1}{2}; \frac{9+2\langle\gamma\rangle}{6}; -\frac{\Omega_\Lambda}{\Omega_m}(1+z)^{-\frac{3}{\langle\gamma\rangle}} \right)$

and: $z(1-z)f''(z) + [c-(a+b+1)z]f'(z) = abf$

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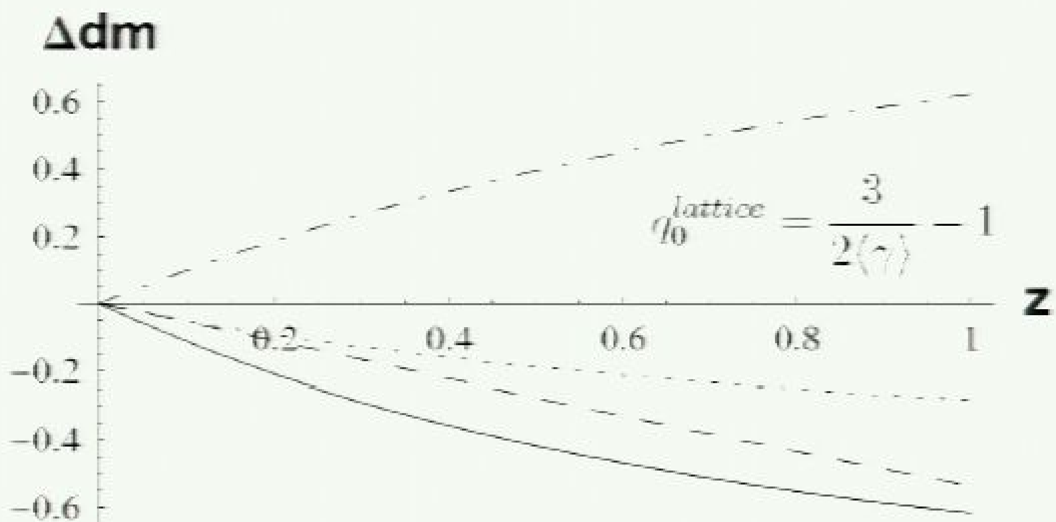

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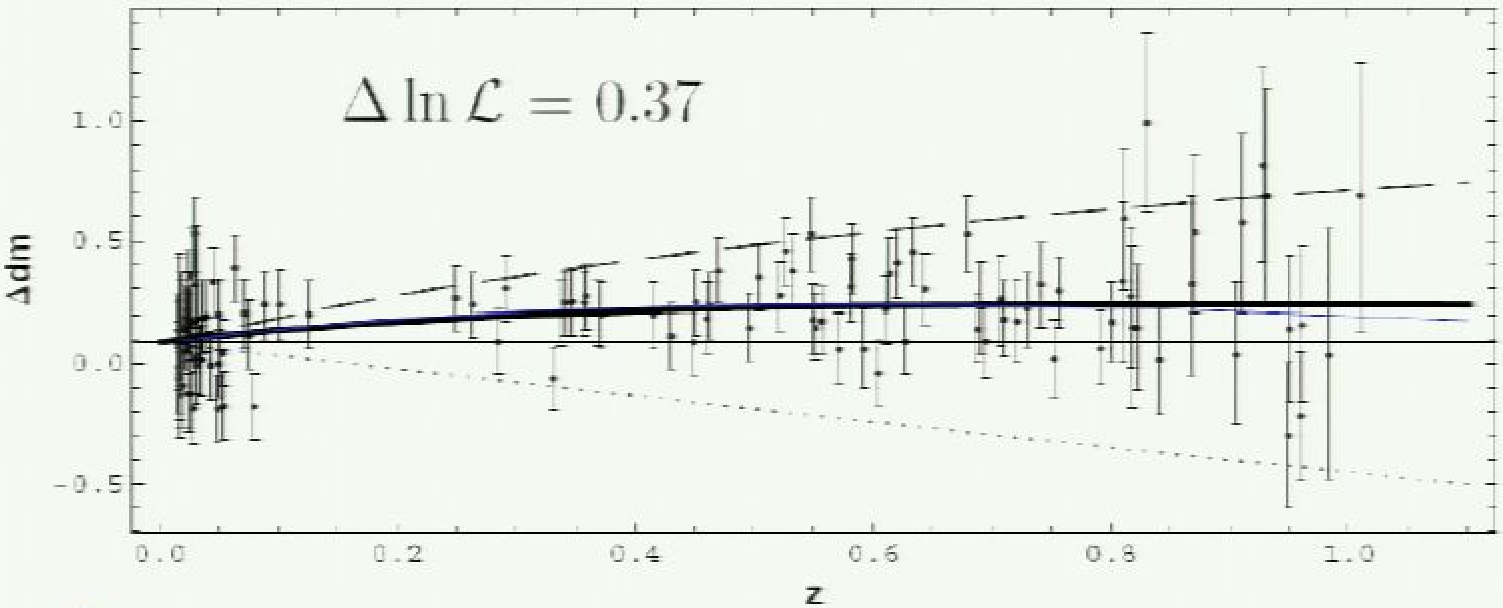

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