Title: Hydrodynamic long-time tails from AdS/CFT

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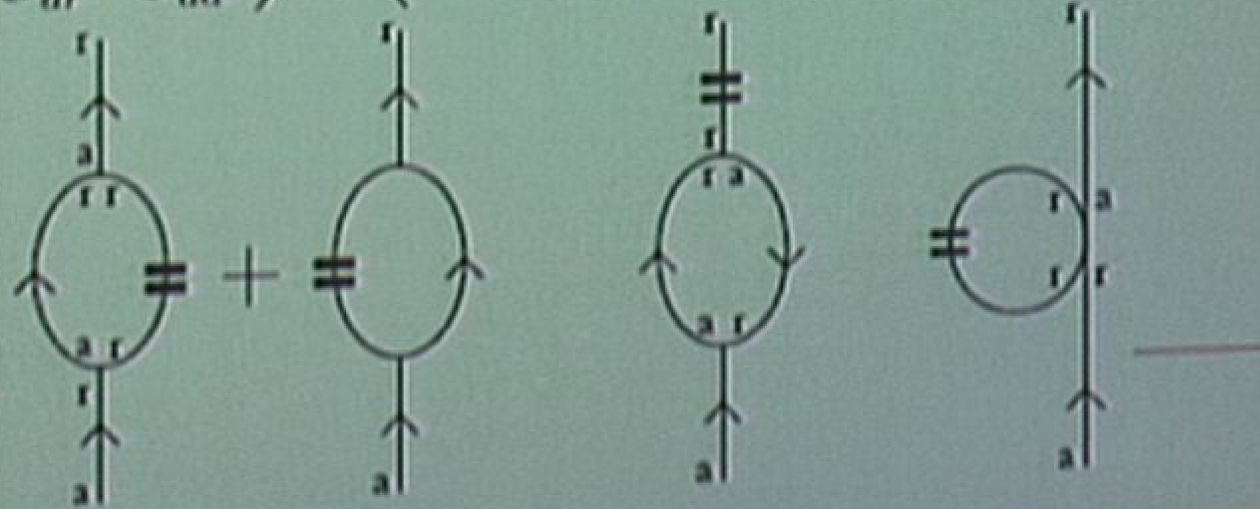
Abstract: For generic field theories at finite temperature, a power-law falloff of correlation functions of conserved currents at long times is a prediction of non-linear hydrodynamics. We demonstrate, through a one-loop computation in Einstein gravity in Anti de Sitter space, that this effect is reproduced by the dynamics of black hole horizons. The result is in agreement with the gauge-gravity correspondence.

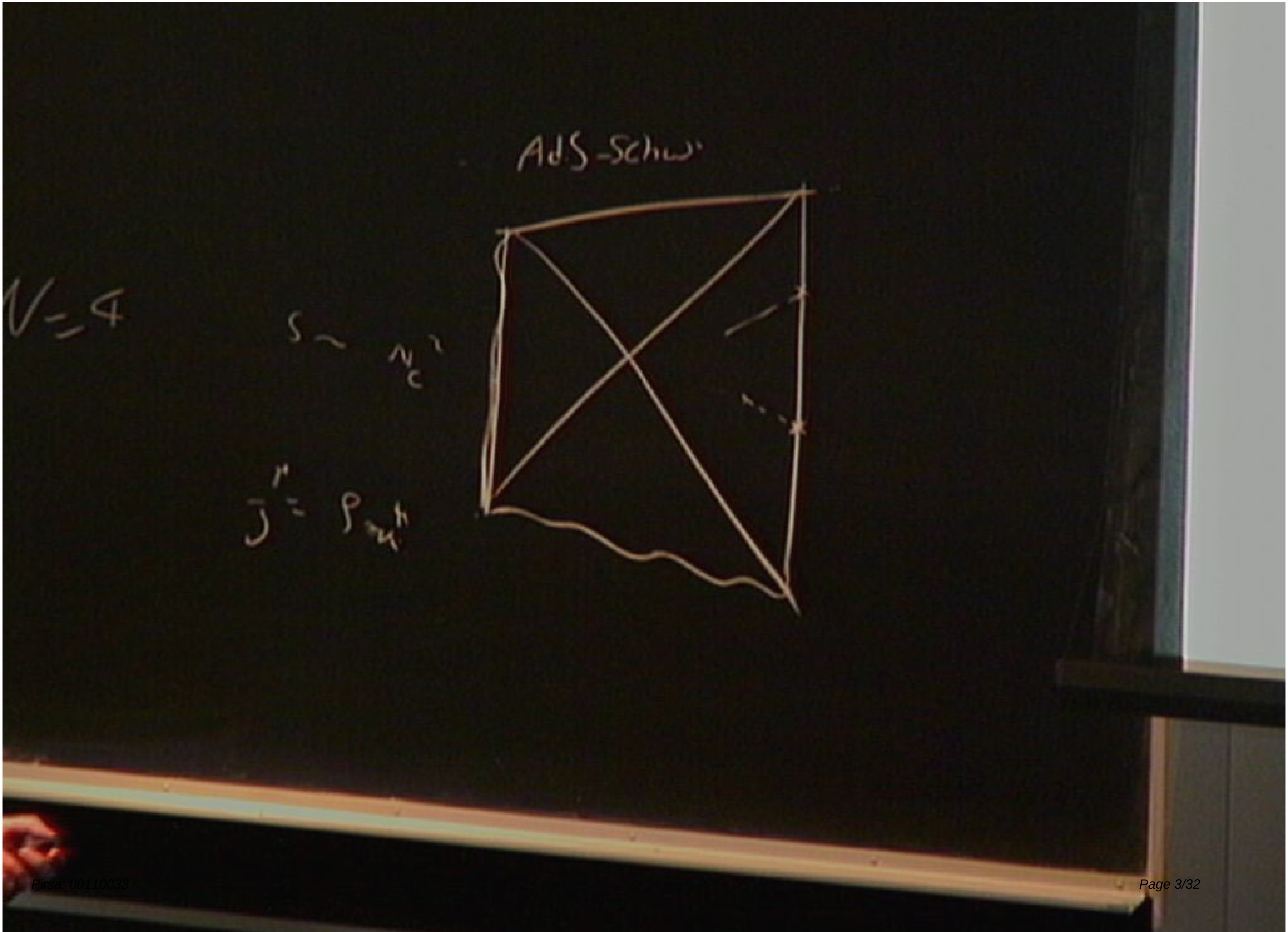
Diagrams

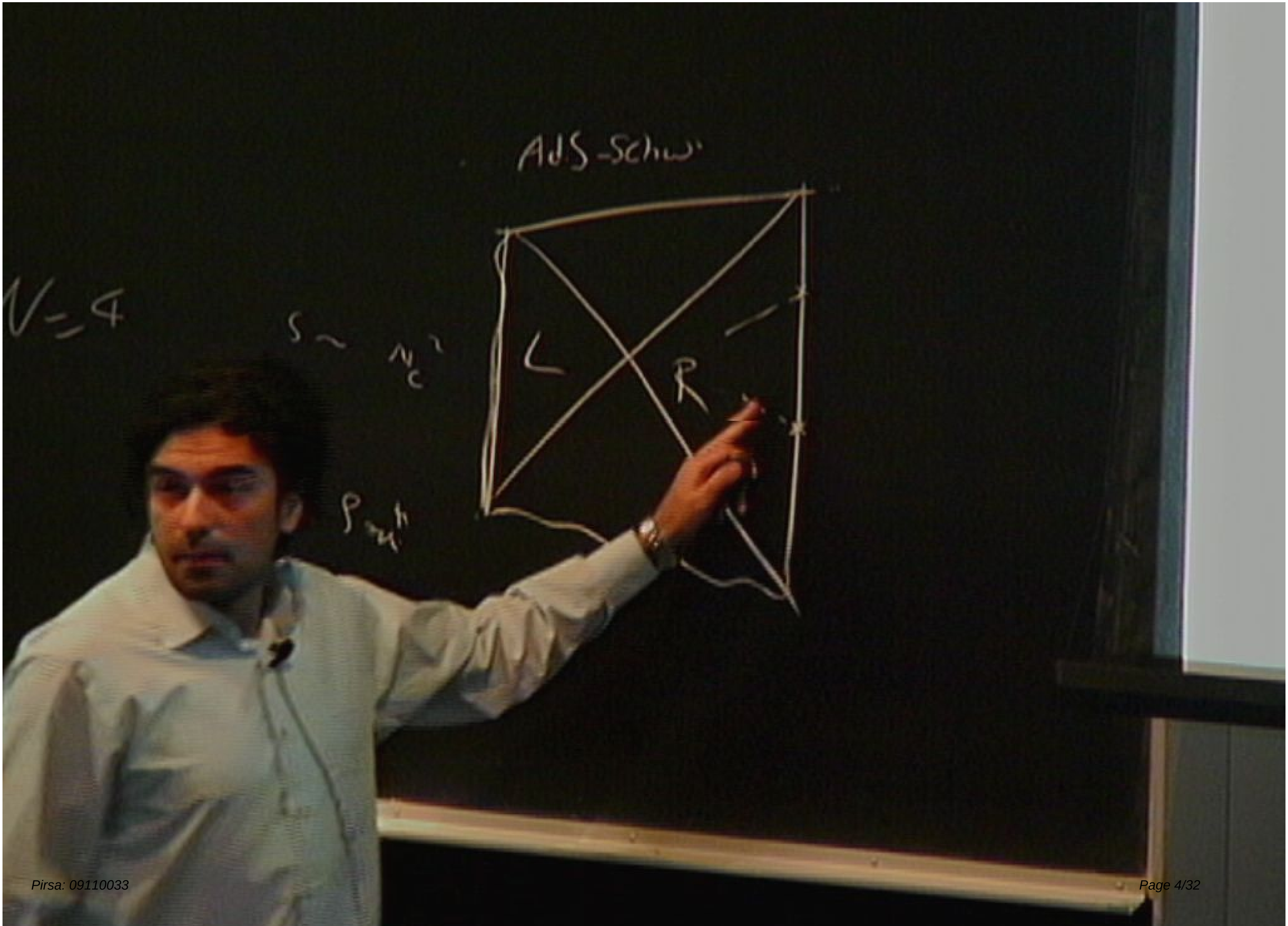
- Not every diagram is important at long times
- In Schwinger-Keldysh formalism in ra-basis

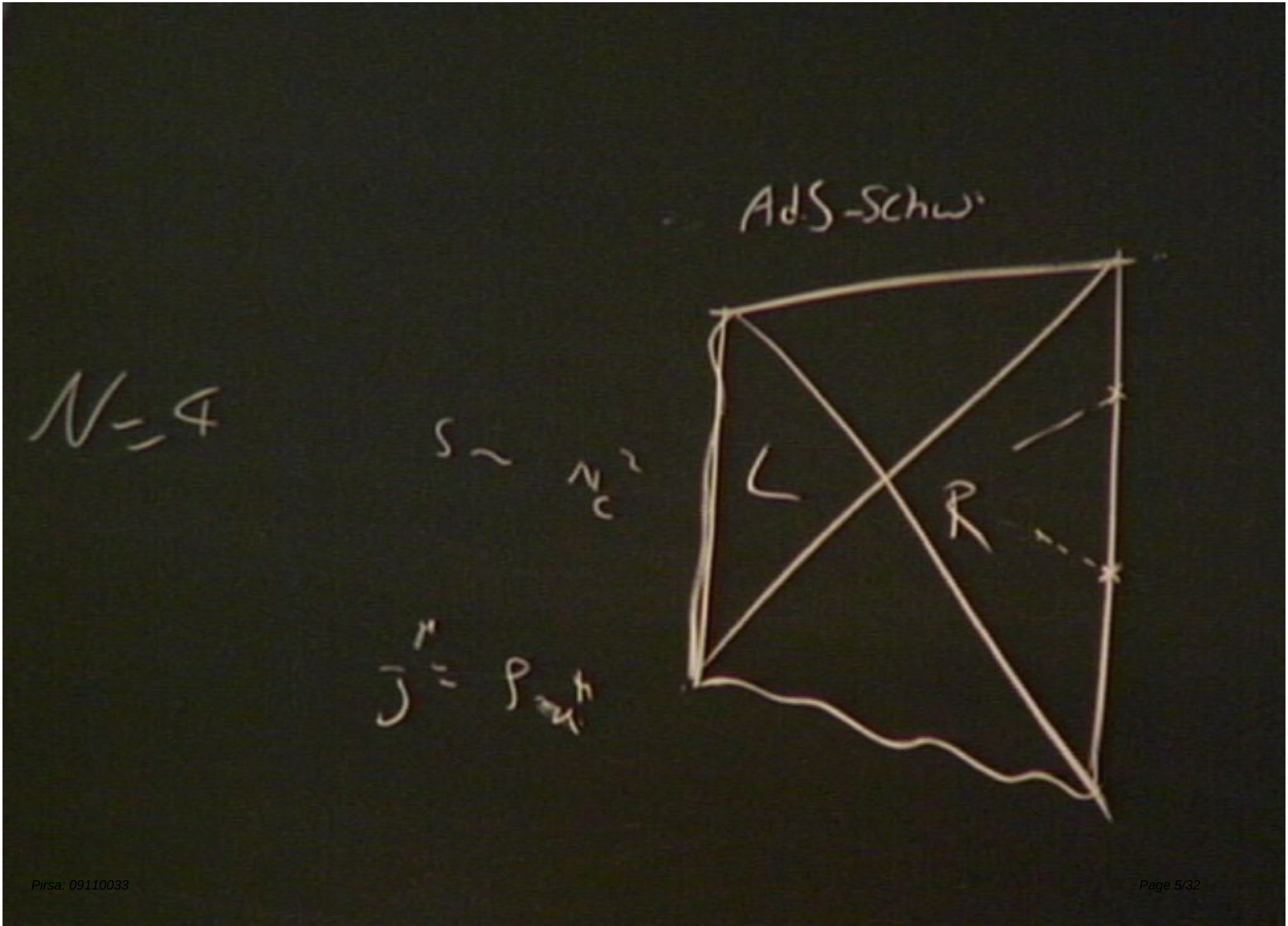
$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2), \quad \phi_a = \phi_1 - \phi_2.$$

$$G = \begin{pmatrix} G_{rr} & G_{ra} \\ G_{ar} & G_{aa} \end{pmatrix} = \begin{pmatrix} -i(G_{\rm R}-G_{\rm A})\left(\frac{1}{2}+n_{\rm B}(\omega)\right) & -iG_{\rm R} \\ -iG_{\rm A} & 0 \end{pmatrix}$$




AdS-Schw.
N = 4
S ~ N_c^2


AdS-Schw
L
R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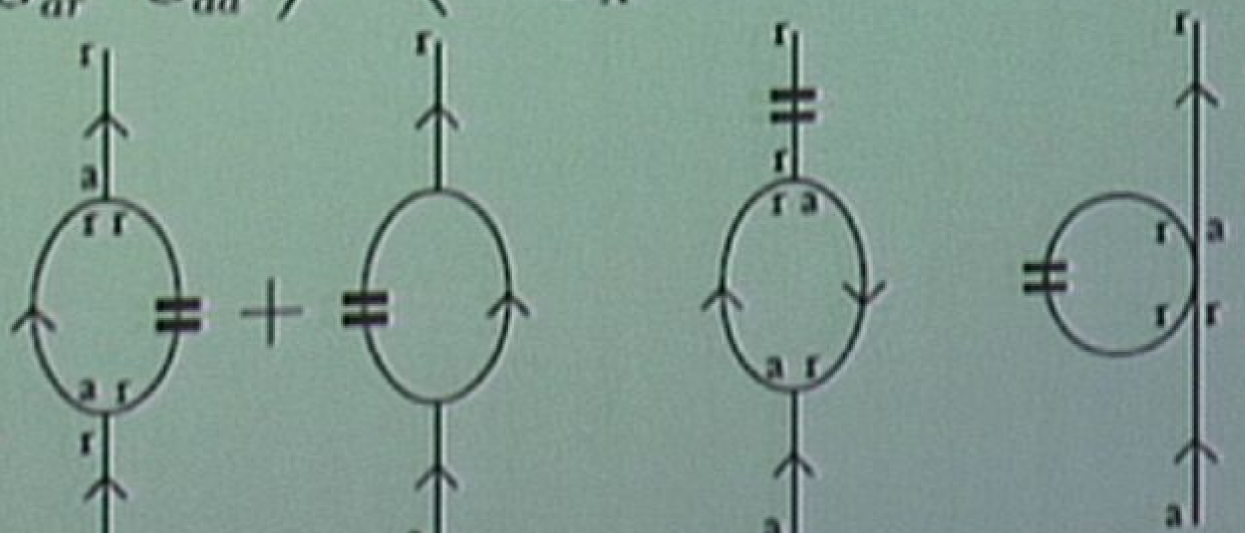


The origin of the boundary KMS conditions

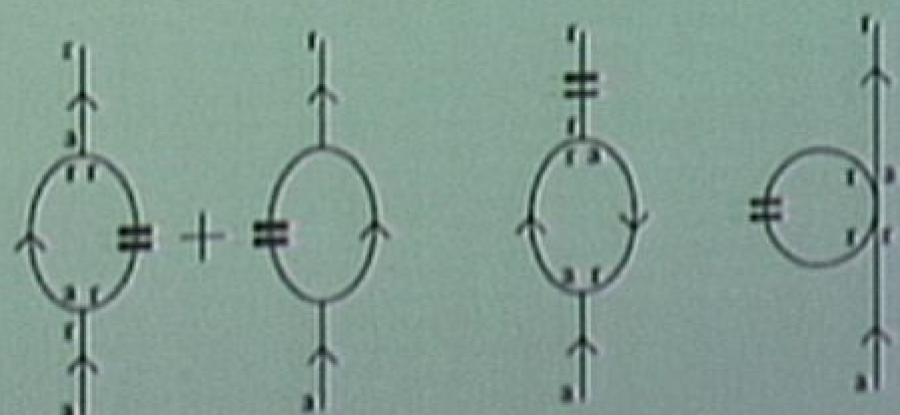
- In QFT in curved s pace limit of QG: BH is in Therma equilibrium with the environment to all orders in perturbation theory [Gibbons-Perry '76]

$$G_{12}(r,r';p) = -in_{\mathrm{B}}(p)(G_{\mathrm{R}}(r,r';p) - G_A(r,r';p)), \quad \text{etc.}$$

- Fluctuations around H-H state are thermal to all or in perturbation theory
- The KMS conditions are obeyed in the bulk
- So in an AdS/CFT setting, the boundary KMS conditions are inherited from the bulk

Dominant fields in the hydrodynamic regime: Generalities

- Both perturbations in the loop: long-lived and diffusive
- Charge fluctuations dominate over current perturbations $A_t/A_z \sim k/\omega$
- h_{tx} dominates over h_{zx} : momentum fluctuation dominates over stress
- Spin-two fluctuations are suppressed by viscous effects



Gravity Vertices j^x

- Third variation of the Yang-Mills action

$$V_{A-A-g} = \delta_g \delta_A \delta_A S_{YM} = -\int d^{d+1}x\, \delta A_x\, \partial_M [\delta_g \delta_A (\frac{\sqrt{-g} F^{xM}}{g^2_{d+1}})]$$

$$= \int d^{d+1}x\, \frac{\delta A_x}{g^2_{d+1}} \partial_M [\sqrt{-g}(\delta F^{\mu M} h^x{}_\mu + \delta F^{x\mu} h^M{}_\mu - \frac{1}{2} \delta F^{xM} h^\sigma{}_\sigma)].$$

- Terms in the second sum: k-suppressed or zero by gauge choice

$$V_{A-A-g} = \int d^{d+1}x\, \delta A_x \frac{\sqrt{-g}}{g^2_{d+1}} \left[\delta F^{\mu r} \partial_r h^x{}_\mu - \frac{1}{2} \delta F^{xr} \partial_r h^\sigma{}_\sigma\right]$$

- The last term mixes a sound mode and diffusive mode: no long-time tail here

$$V_{A-A-g} = \int d^{d+1}x\, \delta A_x \frac{\sqrt{-g}\delta F^{tr}}{g^2_{d+1}} \partial_r h^x{}_t.$$

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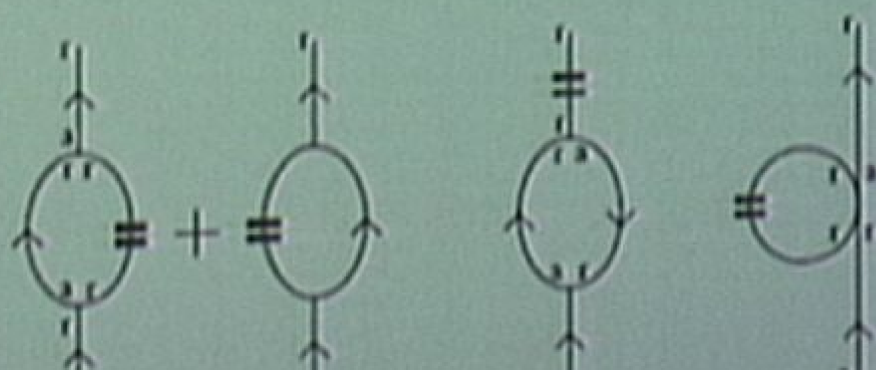

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$\omega = \omega(k) \to \cdot$

$k \to \cdot$

$\mathcal{N} = 4$

$s \sim N_c^2$

$T^{\mu\nu}$

$\sim (k_{\mu\nu} - \frac{1}{2} k h_{\mu\nu}) \sim \pi_{\mu\nu}$

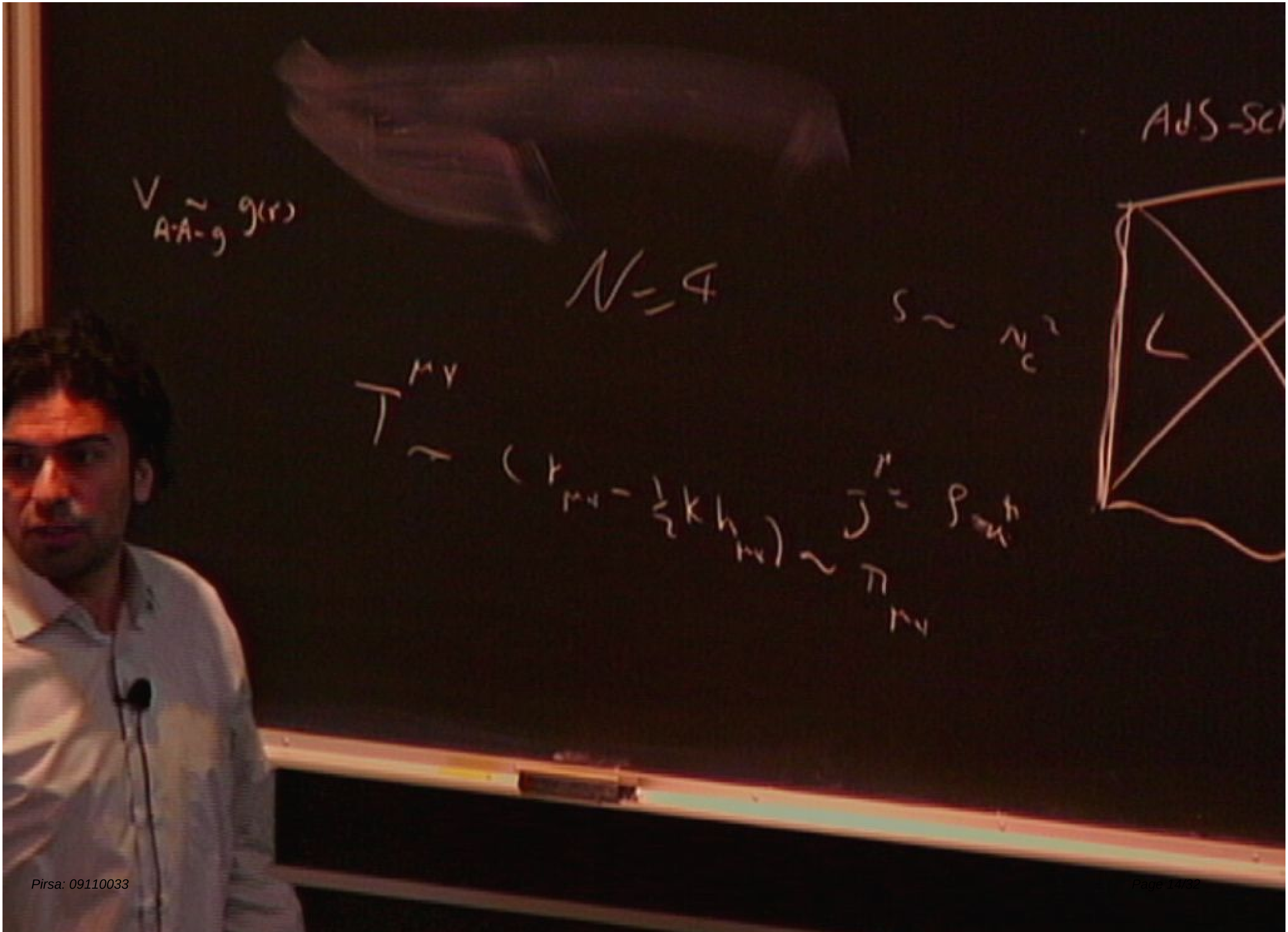
$\bar{J}^{\mu} = \rho u^{\mu}$

Gravity Vertices: j^x

- Rewriting in terms of the bulk local currents

$$V_{A-A-g} = \int d^{d-1}x\, \delta j^t(\tau)\, \delta t^t{}_x(\tau) \int \frac{dr\, 16\pi G g^{xx}}{\sqrt{-g}|g^{tt}|} + \mathcal{O}(1/\tau)$$
$$= \int d^{d-1}x \frac{\delta j^t(\tau)\, \delta t^t{}_x(\tau)}{\epsilon + p},$$

- Gauge-gauge-gauge vertices are also sub-leading : $\delta A^x \partial_r \delta A^t$ and $\delta A^t \partial_r \delta A^x$. involve current fluctuation


AdS-Sc
N=4

Gravity Vertices: t^{xy}

- Third variation of the Einstein-Hilbert action

$$V_{g-g-g} = \delta_g \delta_g \delta_g S_{EH} = \frac{1}{16\pi G}\delta^3 \int d^{d+1}x\sqrt{-g}(R + d(d-1)/\ell^2)$$

$$= -\frac{1}{8\pi G}\int d^{d+1}x h_{xy}\delta^2 \sqrt{-g} g^{x\mu} g^{y\nu}\left[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{d(d-1)}{2\ell^2}g_{\mu\nu}\right].$$

- Two derivative action

$$V_{g-g-g} = -\frac{1}{8\pi G}\int d^{d+1}x\sqrt{-g}h^{xy}\delta^2\left[2K_{\sigma x}K^{\sigma}{}_{y} - K^{\sigma}_{\sigma}K_{xy} - \frac{1}{2}\partial_r^2 g_{xy}\right]$$

- No long-lived spin-2:

$$V_{g-g-g} = \frac{1}{4\pi G}\int d^{d+1}x\sqrt{-g}h^{xy}|g_{tt}|\delta K^{t}{}_{x}\delta K^{t}{}_{y}$$

$V_{AdA\text{-}g} \sim g(r)\, j^t\, t_x$

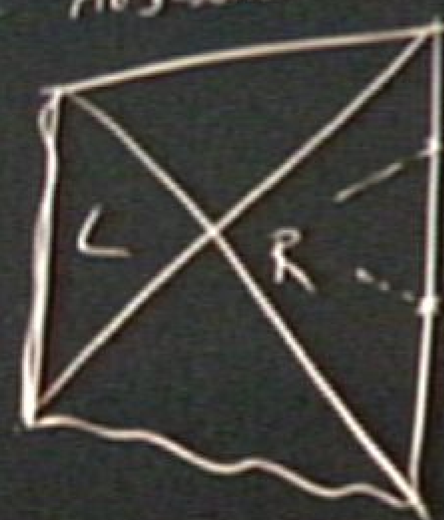
$\mathcal{N} = 4$

$S \sim N_c^2$

$T^{\mu\nu} \sim (\gamma_{\mu\nu} - \frac{1}{2} k h_{\mu\nu}) \sim \pi_{\mu\nu}$

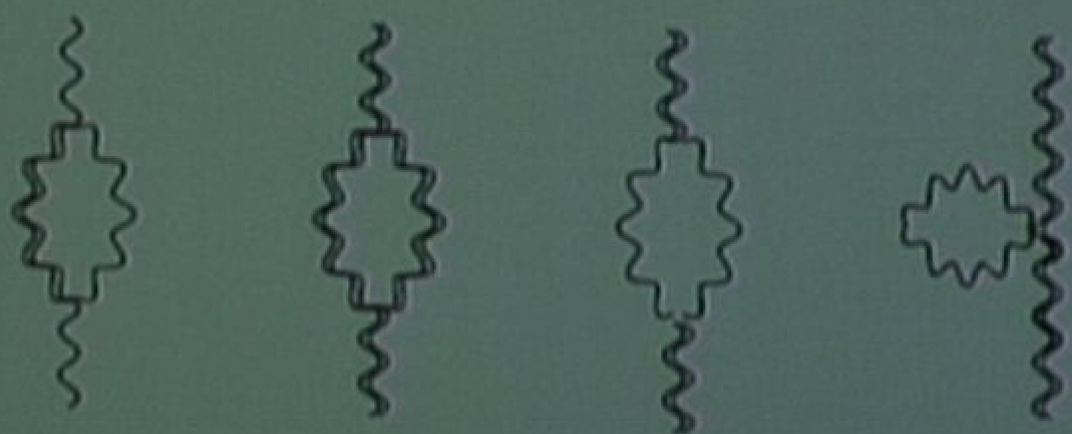
$j^\mu = \rho_{\mu\nu}{}^h$

AdS-Schw



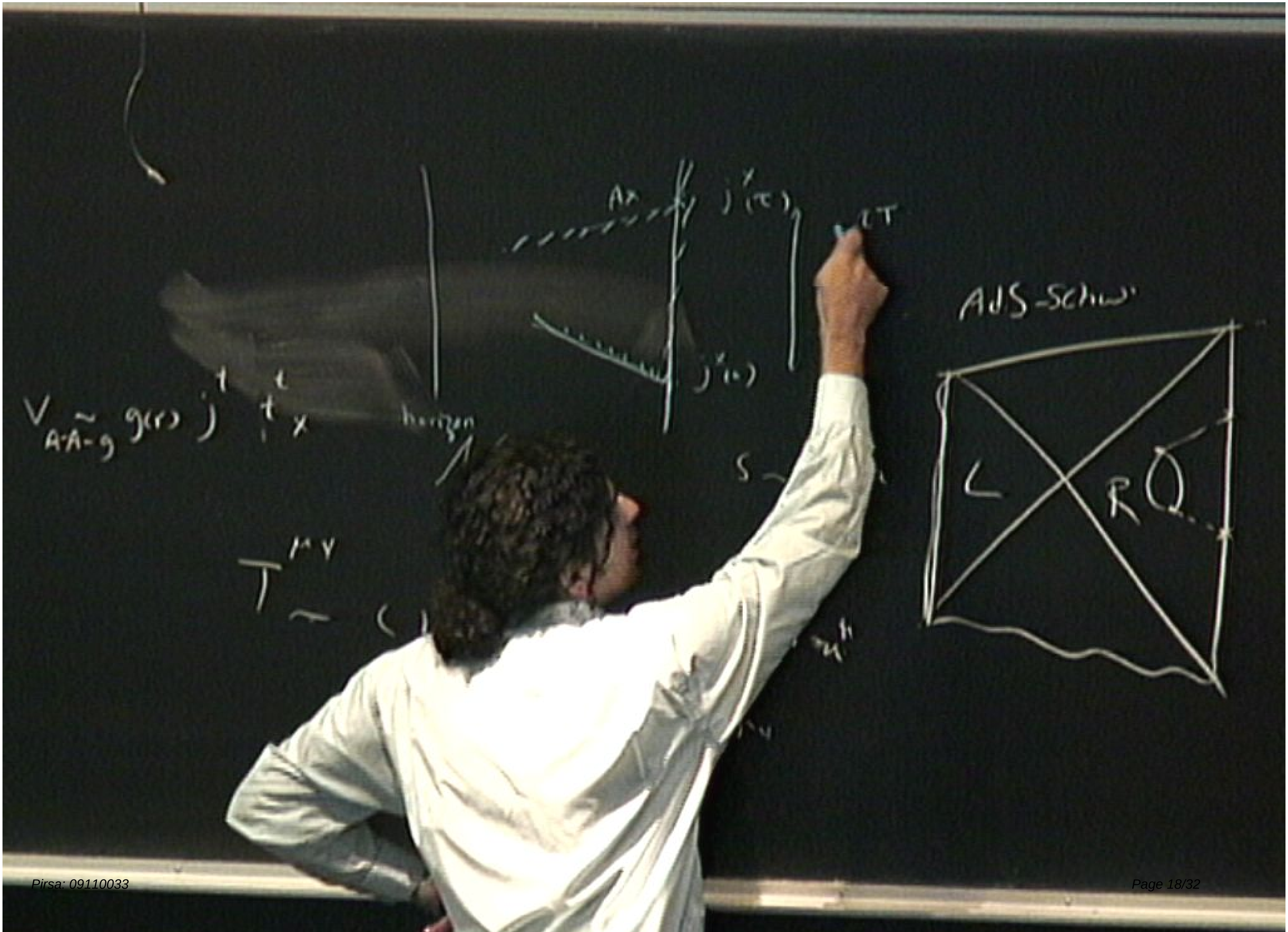
Diagrams

- At one loop



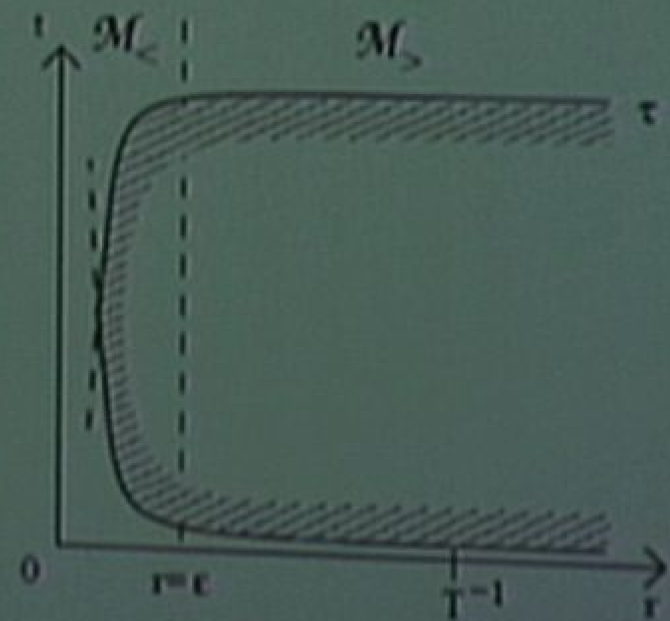
(a) (b) (c)

Feynman diagrams contributing to (a) current correlator (b) stress tensor correlator. Four-point vertices (c) will be found to have negligible effects. Wavy lines are bulk Yang-Mills fields and double lines are gravitons.


AdS-Schw
horizon
L
R

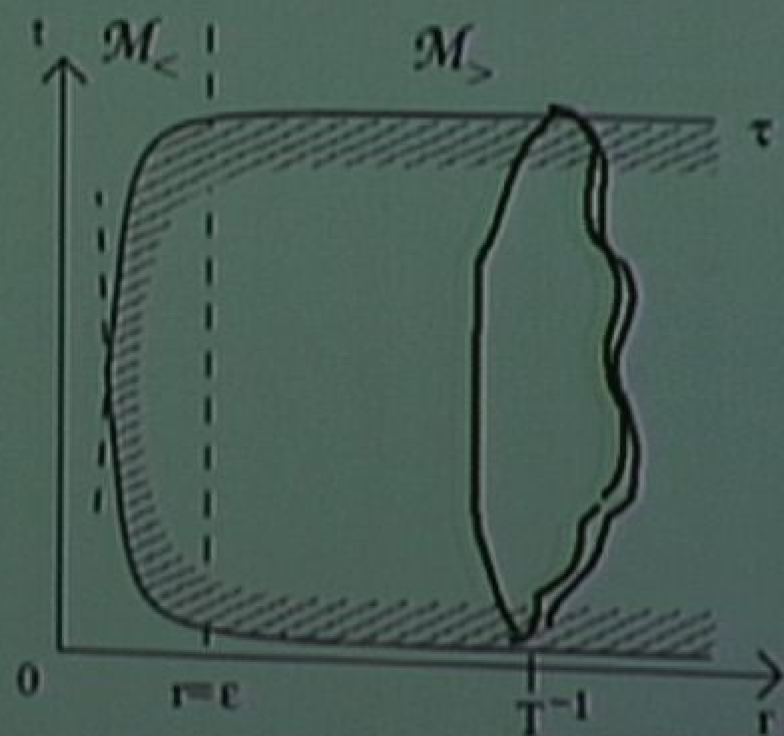
External Wavefunction

- A_x relaxes microscopically. A_x dissipate on time scale T^{-1}
- The field profile falls into the horizon along a null ray $A_x^{\text{ret}}(r,t) \xrightarrow{r} A_x^{\text{ret}}(t - r_*(r))$



Emergence of Hydrodynamics from Anti de Sitter Space

- $r-t$ plane : the causal diamond: insertions at 0 and τ



Emergence of non-linear Hydro from Anti de Sitter Space

- Tip of the half-almond $r_0(\tau) \approx \frac{\sqrt{2}}{2\pi T} e^{-\pi T \tau}$.
- The shaded region $T^{-1} \ll t$
- Not all of the half-almond contributes: the external disturbances relax on microscopic scale T^{-1}
- Vertices near the tip would correspond to a scale of non-locality in the boundary of the order $\sim \tau$
- In contrast, in hydro the vertices are instantaneous on the microscopic time scale T^{-1}
- A minimal choice for ϵ is $\epsilon(\tau) \sim \frac{1}{T} e^{-\pi T \tau/2}$.

- Separation in $\mathcal{M}_>$ is $\sim \tau$.
- The Membrane picture for hydrodynamic is lost from the point of view of the boundary observer

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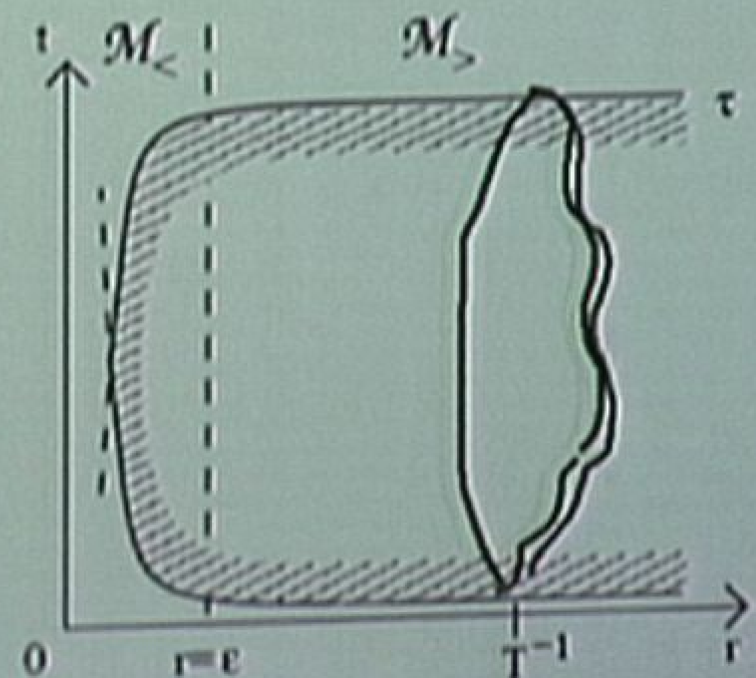

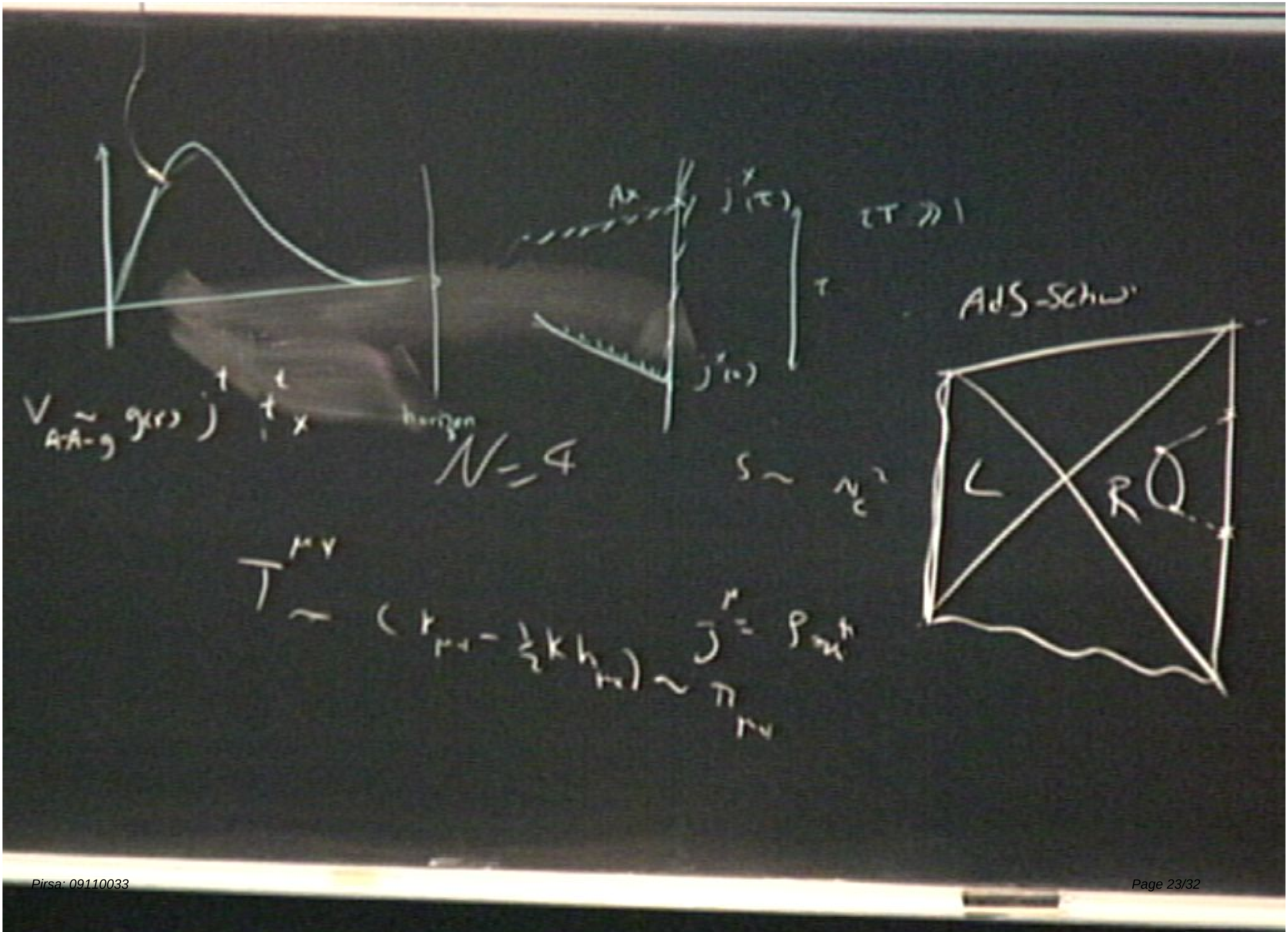
horizon
N=4
AdS-Schw
L
R

Bulk-bulk correlators

- The full bulk-bulk propagator

$$G^{tt}_{\mathrm{R}}(r,r';p) + \frac{\delta(r-r')}{C(r)g^{zz}} = i\sigma k^2 \frac{G(r)F(r')\theta(r-r') + F(r)G(r')\theta(r'-r)}{2(\omega + i\frac{\sigma}{\Xi}k^2)}$$

$$= \frac{i\sigma k^2}{\omega + i\frac{\sigma}{\Xi}k^2} + \mathcal{O}(\omega \log \frac{1}{r}, k^2).$$

- It loses its radial dependence up to corrections which are only large in $\mathcal{M}_<$

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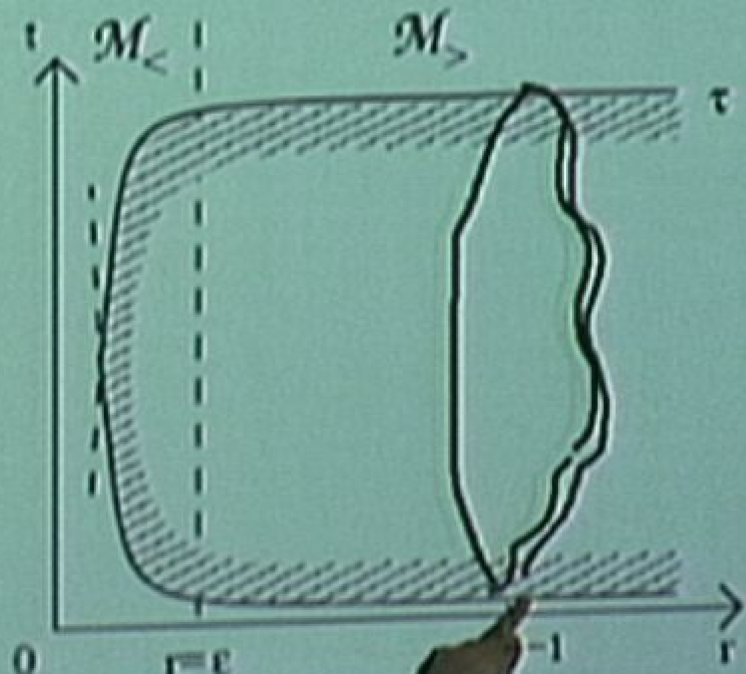


Bulk Variable

- The bulk computation is best organized in momentum variables rather than in field space.

$$t^{\mu}{}_{\nu} = 2\frac{\delta S_{\mathrm{bulk}}}{\delta \gamma_{\mu\nu}} = \frac{\sqrt{-\gamma}}{8\pi G_N}(\delta^{\mu}{}_{\nu}K - K^{\mu}{}_{\nu}),$$

$$j^{\mu} = \frac{\delta S_{\mathrm{bulk}}}{\delta A_{\mu}} = \frac{\sqrt{-\gamma}}{g^2_{d+1}}F^{\mu r},$$

- Consider them as extending in the bulk: local density of the currents

$$t^{\mu}_{\nu} = t^{\mu}_{\nu}(r), \quad j^{\mu} = j^{\mu}(r)$$

Radial Flow equation for the Hydrodynamic Variable

- Bulk Hamiltonian (Einstein and Yang-Mills) : the radial evolution of the momenta. It can be organized written as a derivative expansion

$$\partial_r(\delta t^\mu{}_\nu) = \frac{1}{8\pi G}\delta(\sqrt{-g}^{(d)}R^\mu{}_\nu) = 0 + \mathcal{O}(\omega^2, q^2) \quad \mu \neq \nu,$$

$$\partial_r(\delta j^\mu) = \frac{1}{g^2_{d+1}}\delta(\partial_\nu\sqrt{-g}F^{\nu\mu}) = 0 + \mathcal{O}(\omega^2, q^2),$$

$$\frac{1}{\sqrt{g}}\partial_r(\sqrt{g}K^\mu{}_\nu) = {}^{(d)}R^\mu{}_\nu + \frac{d}{\ell^2}\delta^\mu{}_\nu.$$

- Holds up to exponentially close to the horizon due to oscillations

$$r \sim e^{-2\pi T/\omega}$$

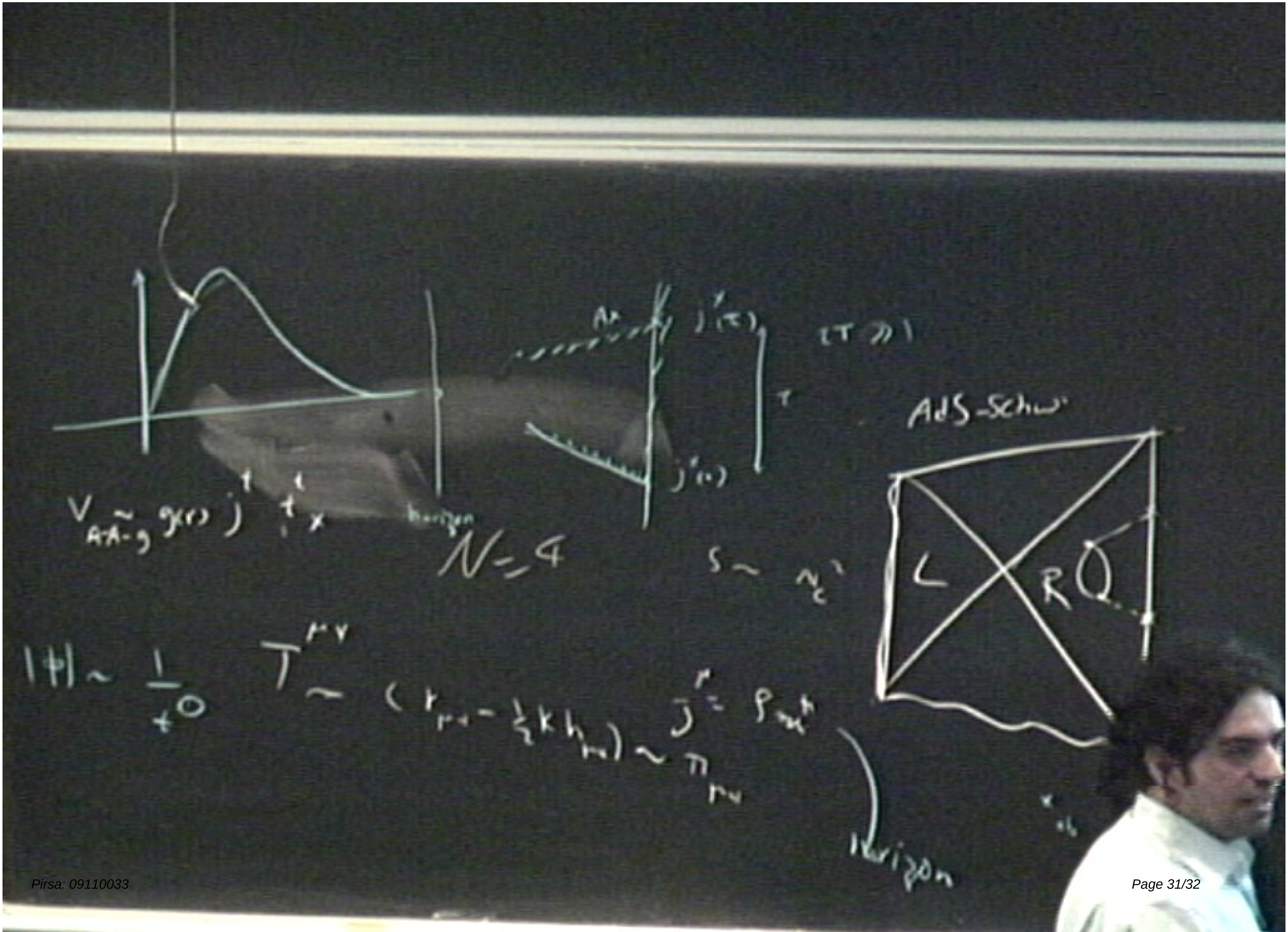
Relation to Long-time tails in Black hole Physics

- Evolution of probe fields on black hole spacetimes
- Late-time power-law tails in flat space $\Phi \sim t^{-n}$
- In AdS space they do not exist. Dirichlet boundary condition kills it
- We have argue that they do exist even in AdS but at non-linear level
- The argument comes from an unlikely source: hydrodynamics!

$$|\phi| \sim \frac{1}{t^{0}}$$

Outlook

- Emergence of non-linear hydrodynamics from anti de Sitter Space
- Counter-intuitive: late-time tails must have come from deep IR: fluctuations are tide to the horizon but the scale of vertices are microscopic since they are local in time
- The role of the asymptotic region not clear. This analysis must be true in flat space as- well
- D=3 and below is very interesting. Work in progress
- Mode-coupling theory of critical systems from gravity


AdS-Schw
horizon
$\mathcal{N}=4$
L
R
Horizon

Long-Time Tails in Hydrodynamic Perturbation Theory

- Hydrodynamic diagrams
- In hydrodynamics perturbation theory
- $$\int d^{d-1}x \, \langle j^a(\tau,x) j^b(0)\rangle \simeq \int d^{d-1}x \, \langle \rho(\tau,x) u^a(t,x)\, \rho(0) u^b(0)\rangle$$

$$\langle \rho(\tau,x)u^a(\tau,x)\,\rho(0)u^b(0)\rangle = \langle\rho(\tau,x)\rho(0)\rangle \, \langle u^a(\tau,x)u^b(0)\rangle$$

$$\int d^{d-1}x \, \langle j^a(\tau,x)j^b(0)\rangle = \frac{T\Xi\delta^{ab}}{\epsilon+p}\frac{d-2}{d-1}\frac{1}{[4\pi(D+\gamma_\eta)|\tau|]^{\frac{d-1}{2}}}$$

- Long-Time tails are suppressed by entropy density