Title: Holography from CFT

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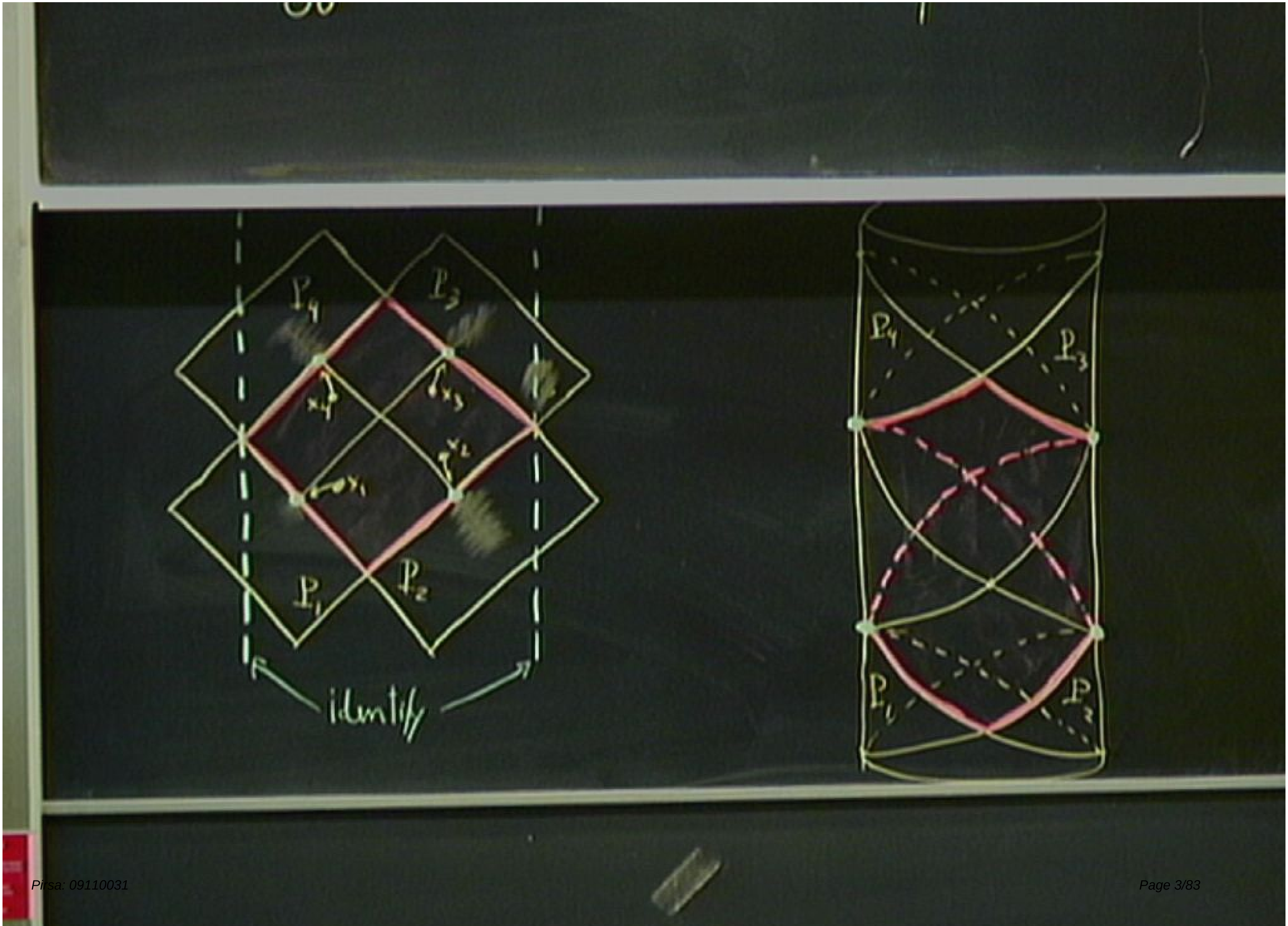
Abstract: The validity of a local AdS supergravity description of SYM at strong coupling requires the existence of a parametrically large gap in the spectrum of dimensions of the local operators, separating the low dimension operators dual to supergravity fields from the others. We shall give evidence that the reverse is also true: a large gap in the spectrum of dimensions implies a local bulk dual.

3. High Energy Scattering in AdS/CFT

- Regge Kinematics [Cornalba, Costa, P.]
- Regge theory [Cornalba 07]
- Strong coupling [Brower, Polchinski, Strassler, Tan] , [Cornalba, Costa, P.] [B S
- Weak coupling [Balitsky , ...]


P4
P3
x4
x3
x2
x1
P1
P2
identify
P4
P3
P1
P2

$$A(y_i) = \langle \mathcal{O}_1(y_1)\, \mathcal{O}_2(y_2)\, \mathcal{O}_1^*(y_3)\, \mathcal{O}_2^*(y_4) \rangle = \frac{\mathcal{A}(\sigma,\rho)}{y_{13}^{2\Delta_1}\; y_{24}^{2\Delta_2}}$$

$$A(x,\bar{x}) = \langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_1^*(x_3)\, \mathcal{O}_2^*(x_4) \rangle = \frac{\mathcal{A}(\sigma,\rho)}{x^{2\Delta_1}\; \bar{x}^{2\Delta_2}}$$

$$x = x_1 - x_3\,, \qquad \bar{x} = x_2 - x_4$$

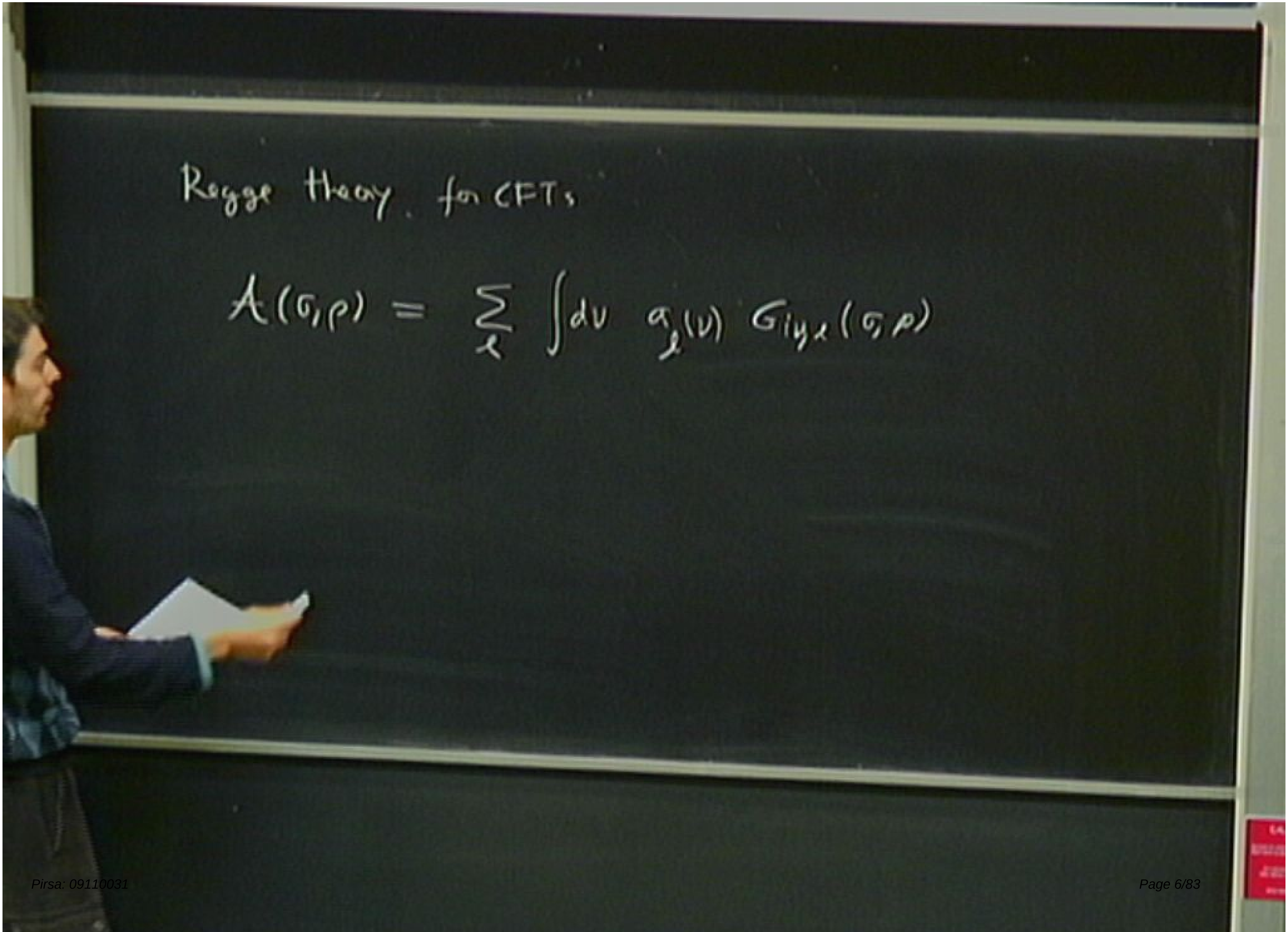
$$\sigma = x^2 \bar{x}^2\,, \qquad \cosh\rho = -\frac{x\cdot\bar{x}}{|x||\bar{x}|}$$

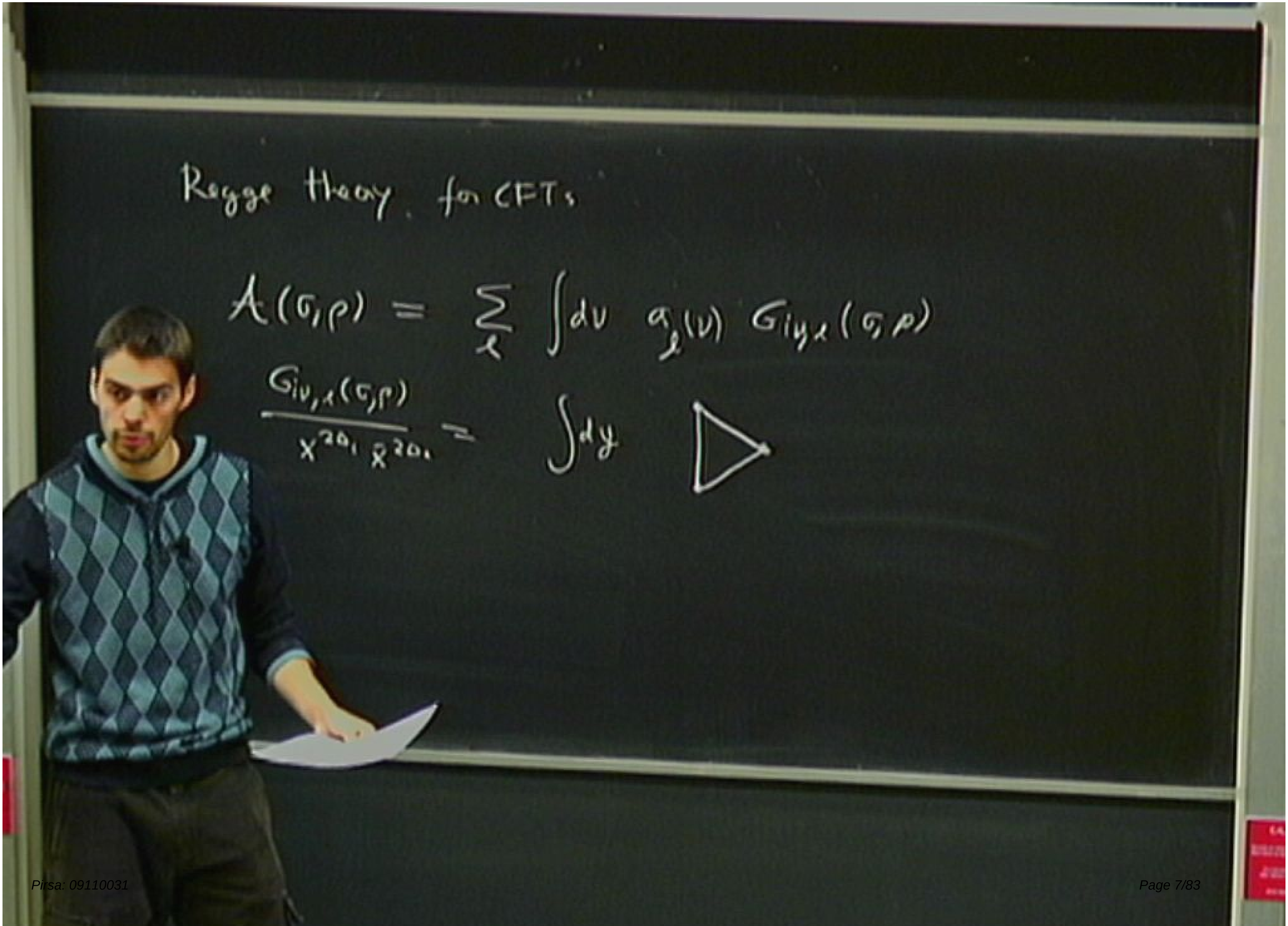
$$z\bar{z} = \frac{y_{13}^2\, y_{24}^2}{y_{12}^2\, y_{34}^2}\,, \qquad (1-z)(1-\bar{z}) = \frac{y_{14}^2\, y_{23}^2}{y_{12}^2\, y_{34}^2}$$

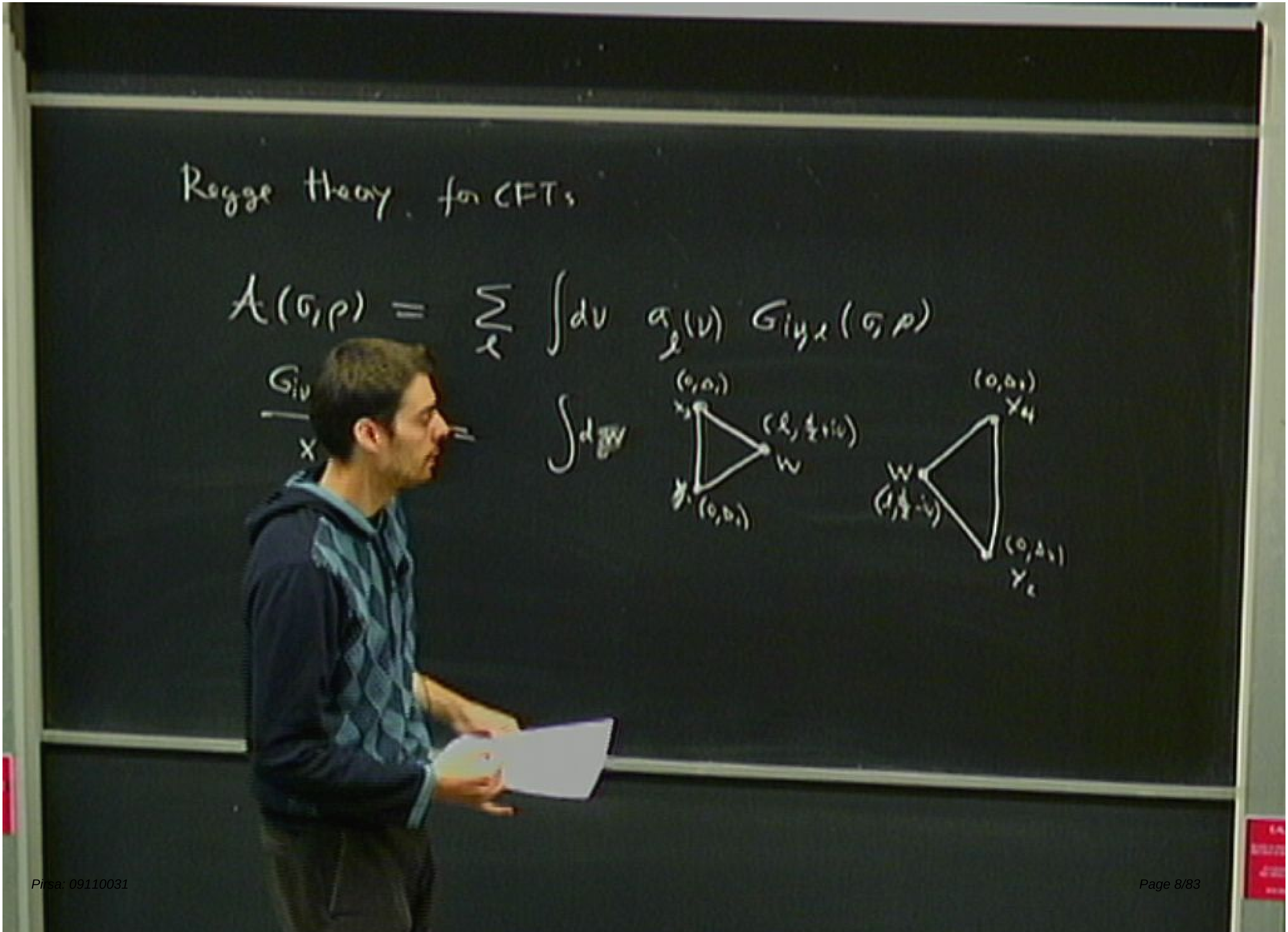
$$z = \sigma e^{\rho}\,, \qquad \bar{z} = \sigma e^{-\rho}$$

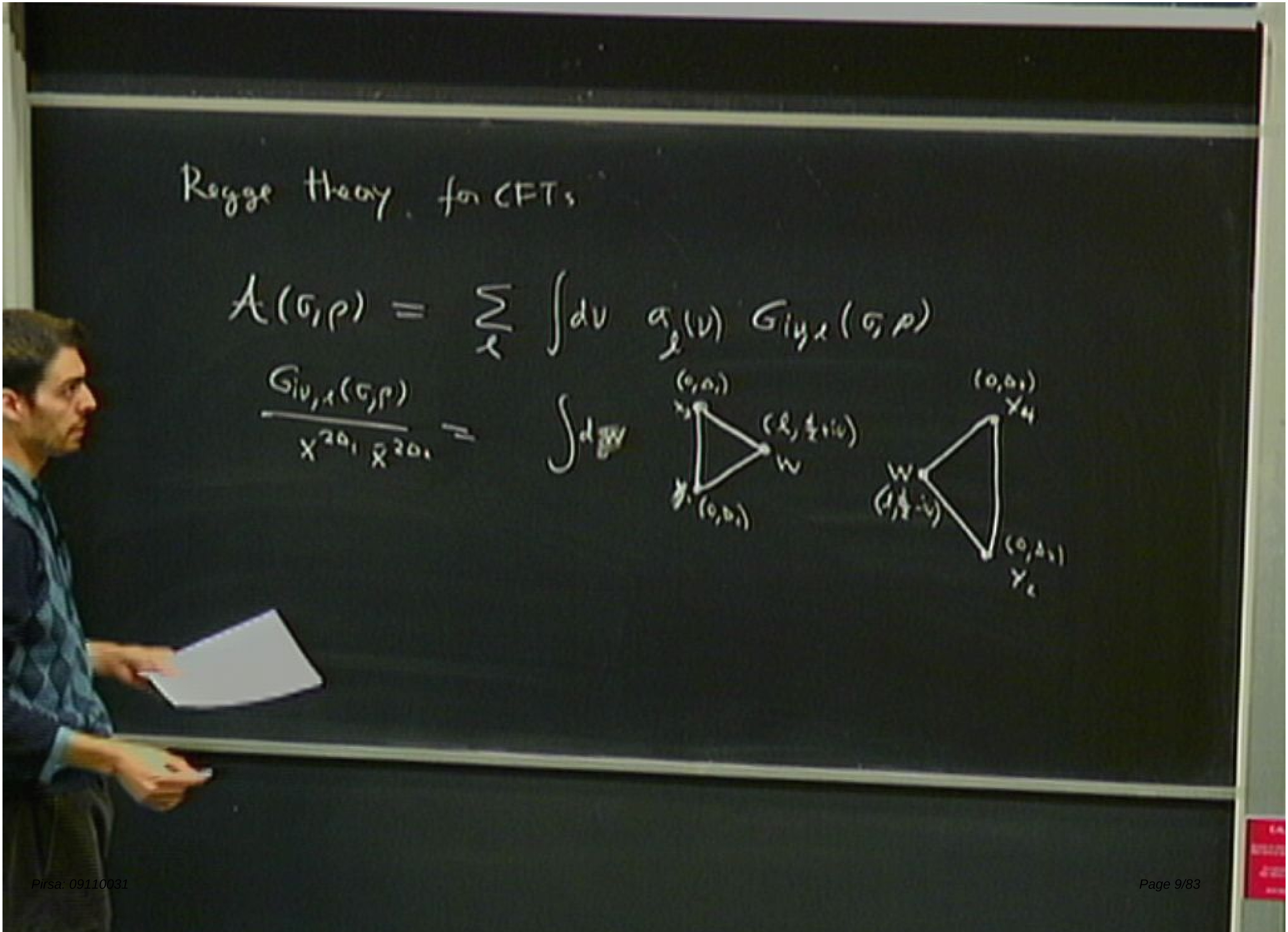
Regge limit: $\sigma \to 0$ with fixed ρ.

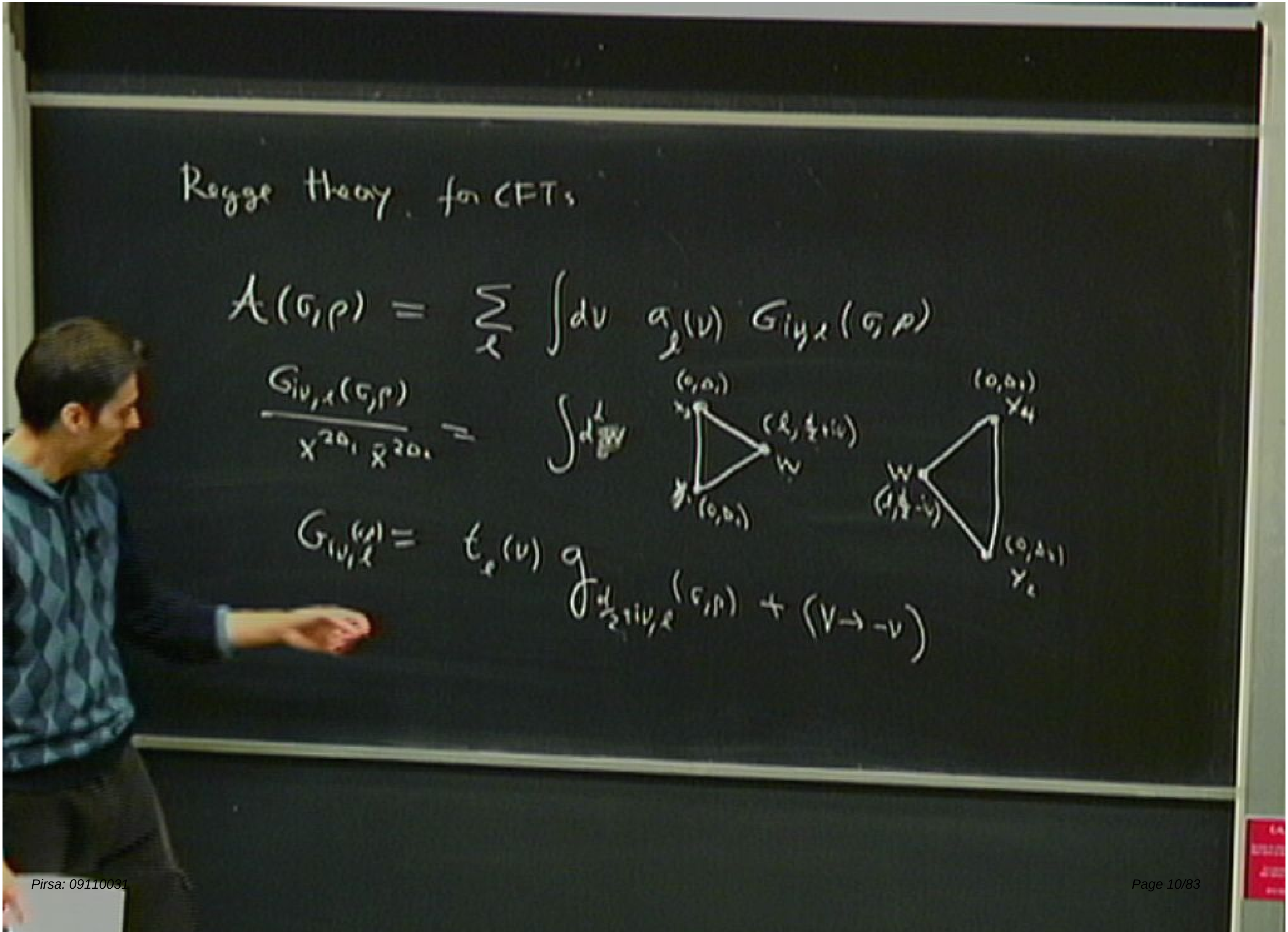
Regge theory for CFTs

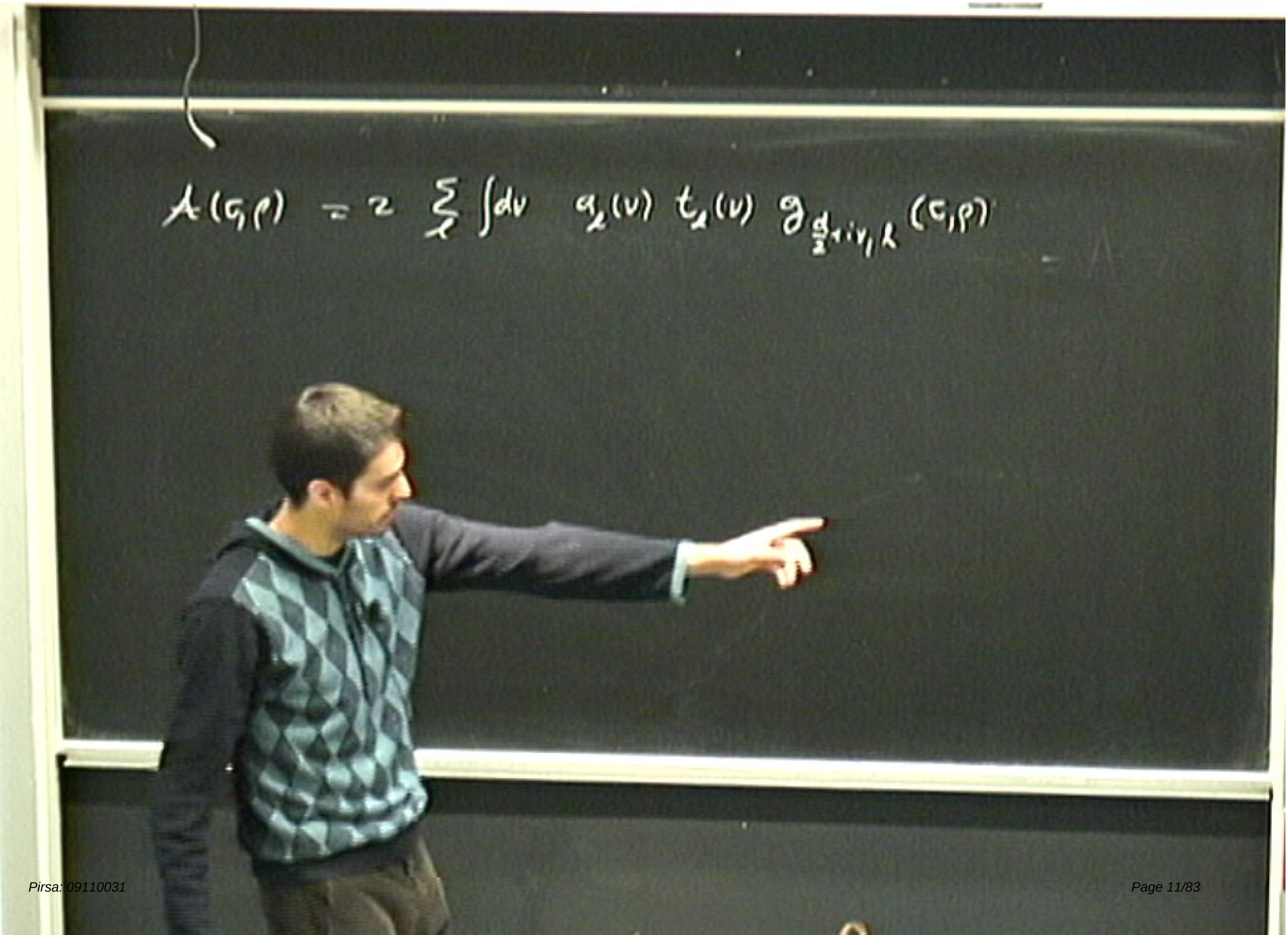

Regge theory for CFTs
$\mathcal{A}(\sigma,\rho) = \sum_{\ell} \int d\nu \; a_{\ell}(\nu) \, G_{\nu,\ell}(\sigma,\rho)$


Regge theory for CFTs
A(σ,ρ) = Σ_ℓ ∫dν a_ℓ(ν) G_iν,ℓ(σ,ρ)
G_iν,ℓ(σ,ρ) / x^2Δ1 x̄^2Δ1 = ∫dy


Regge theory for CFTs
A(σ,ρ) = Σ_ℓ ∫dν a_ℓ(ν) G_iν,ℓ(σ,ρ)
∫d^d w
(0,Δ1)
(0,Δ1)
W
W
(0,Δ2)
(0,Δ2)
x4
x2


Regge theory for CFTs
A(σ,ρ) = Σ_l ∫dν a_l(ν) G_{iν,l}(σ,ρ)
G_{iν,l}(σ,ρ) / x^{2Δ1} x^{2Δ2} = ∫dW
(0,Δ1)
(l, d/2+iν)
W
(0,Δ1)
(0,Δ2)
x4
W
(l, d/2−iν)
(0,Δ2)
x2


Regge theory for CFTs
A(σ,ρ) = Σ_ℓ ∫dν a_ℓ(ν) G_{iν,ℓ}(σ,ρ)
G_{iν,ℓ}(σ,ρ) / x^{2Δ1} x̄^{2Δ2} = ∫d^d W
(0,Δ1)
(ℓ, d/2+iν)
W
(0,Δ2)
(0,Δ1)
x4
W
(ℓ, d/2−iν)
(0,Δ1)
x2
G_{iν,ℓ}(σ,ρ) = t_ℓ(ν) g_{d/2+iν,ℓ}(σ,ρ) + (ν → −ν)


$A(q,p) = 2 \sum_{\ell} \int d\nu \; a_{\ell}(\nu) \; t_{\ell}(\nu) \; g_{\frac{d}{2}+i\nu,\ell}(q,p)$

$$\mathcal{A}(\sigma,\rho) = 2 \sum_{\ell} \int d\nu \;\; q_\ell(\nu)\; t_\ell(\nu)\; g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$$

$$\downarrow \qquad\qquad \searrow$$

$$\sum_{\Delta} \; g_{\Delta,\ell}(\sigma,\rho) \qquad \frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2}$$

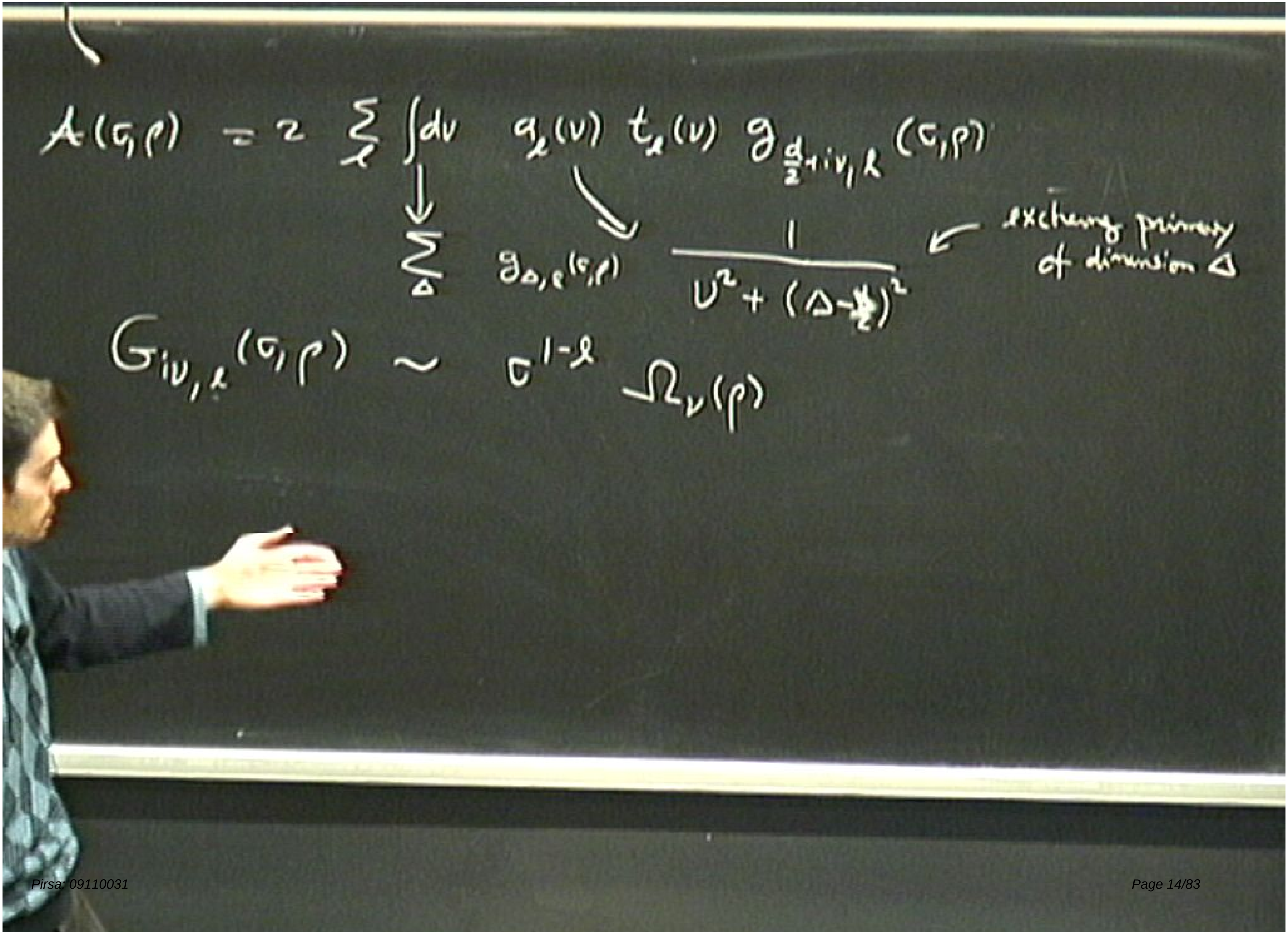
← exchange primary of dimension Δ

$$A(\sigma,\rho) = 2\sum_{\ell}\int d\nu \; q_\ell(\nu)\, t_\ell(\nu)\, g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$$

$$\sum_{\Delta} g_{\Delta,\ell}(\sigma,\rho) \qquad \frac{1}{\nu^2+(\Delta-\frac{d}{2})^2}$$

exchange primary of dimension Δ

$$G_{i\nu,\ell}(\sigma,\rho) \sim \sigma^{1-\ell}\,\Omega_\nu(\rho)$$


$A(\sigma,\rho) = 2 \sum_{\ell} \int d\nu \; a_{\ell}(\nu)\, t_{\ell}(\nu)\, g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$
$\sum_{\Delta} g_{\Delta,\ell}(\sigma,\rho)$
$\frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2}$
exchang primary
of dimension Δ
$G_{i\nu,\ell}(\sigma,\rho) \sim \sigma^{1-\ell}\, \Omega_{\nu}(\rho)$

$$\mathcal{A}(\sigma,\rho) = 2 \sum_{\ell} \int d\nu \quad a_\ell(\nu)\, t_\ell(\nu)\, g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$$

$$\downarrow \qquad\qquad \searrow$$

$$\sum_{\Delta} \quad g_{\Delta,\ell}(\sigma,\rho) \qquad \frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2} \quad \leftarrow$$

exchange primary of dimension Δ

$$G_{i\nu,\ell}(\sigma,\rho) \sim \sigma^{1-\ell}\, \Omega_\nu(\rho)$$

$$\mathcal{A}(\sigma,\rho) = \int d\nu \int \frac{d\ell}{2i \sin\pi\ell}\, e^{i\pi\ell} \quad a_\ell(\nu)\, G_{i\nu,\ell}(\sigma,\rho)$$

$$\mathcal{A}(\sigma,\rho) = 2 \sum_{\ell} \int d\nu \;\; a_\ell(\nu)\, t_\ell(\nu)\, g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$$

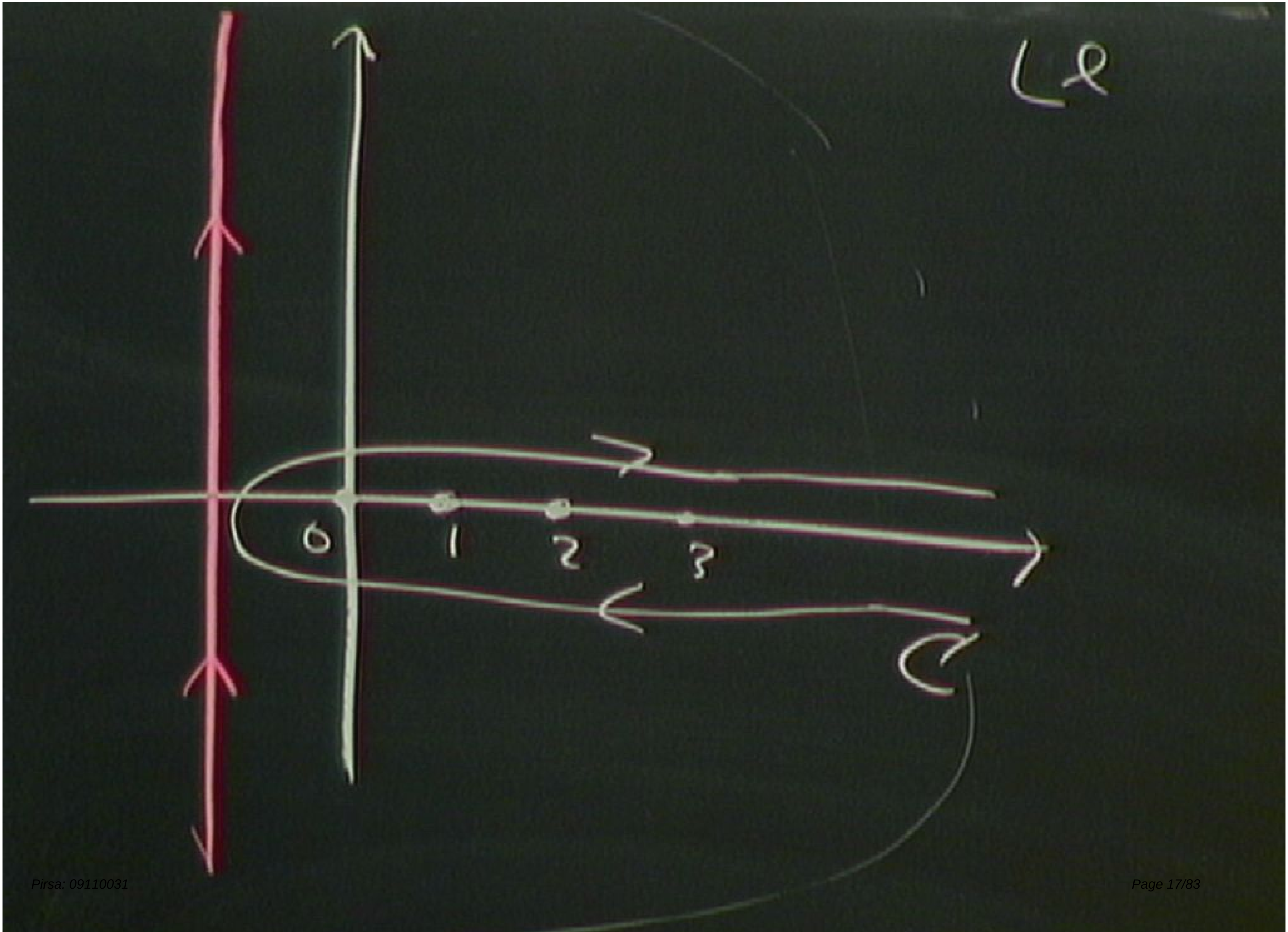
$$\downarrow \qquad \searrow$$

$$\sum_{\Delta} \; g_{\Delta,\ell}(\sigma,\rho) \;\; \frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2}$$

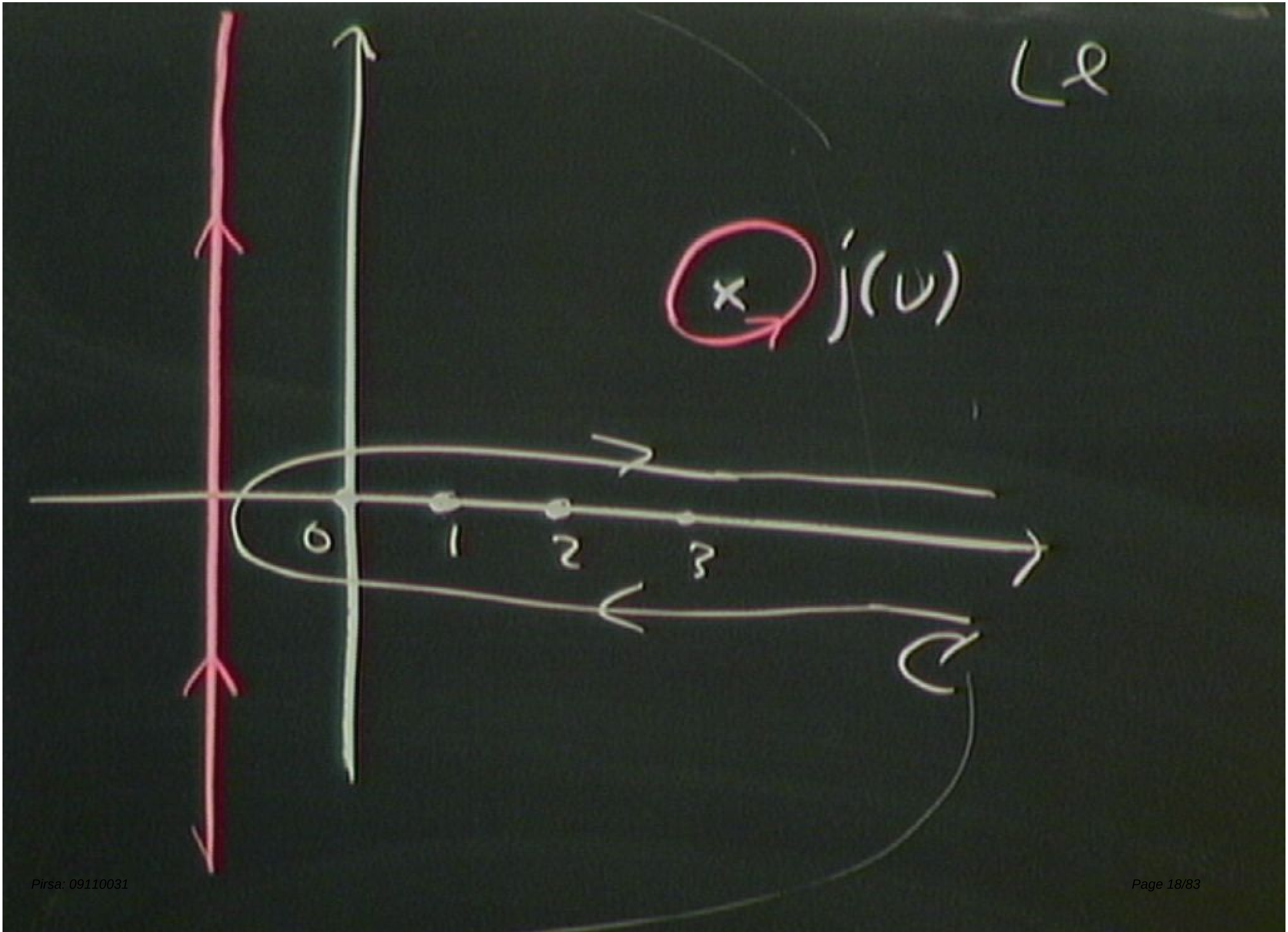
← exchange primary of dimension Δ

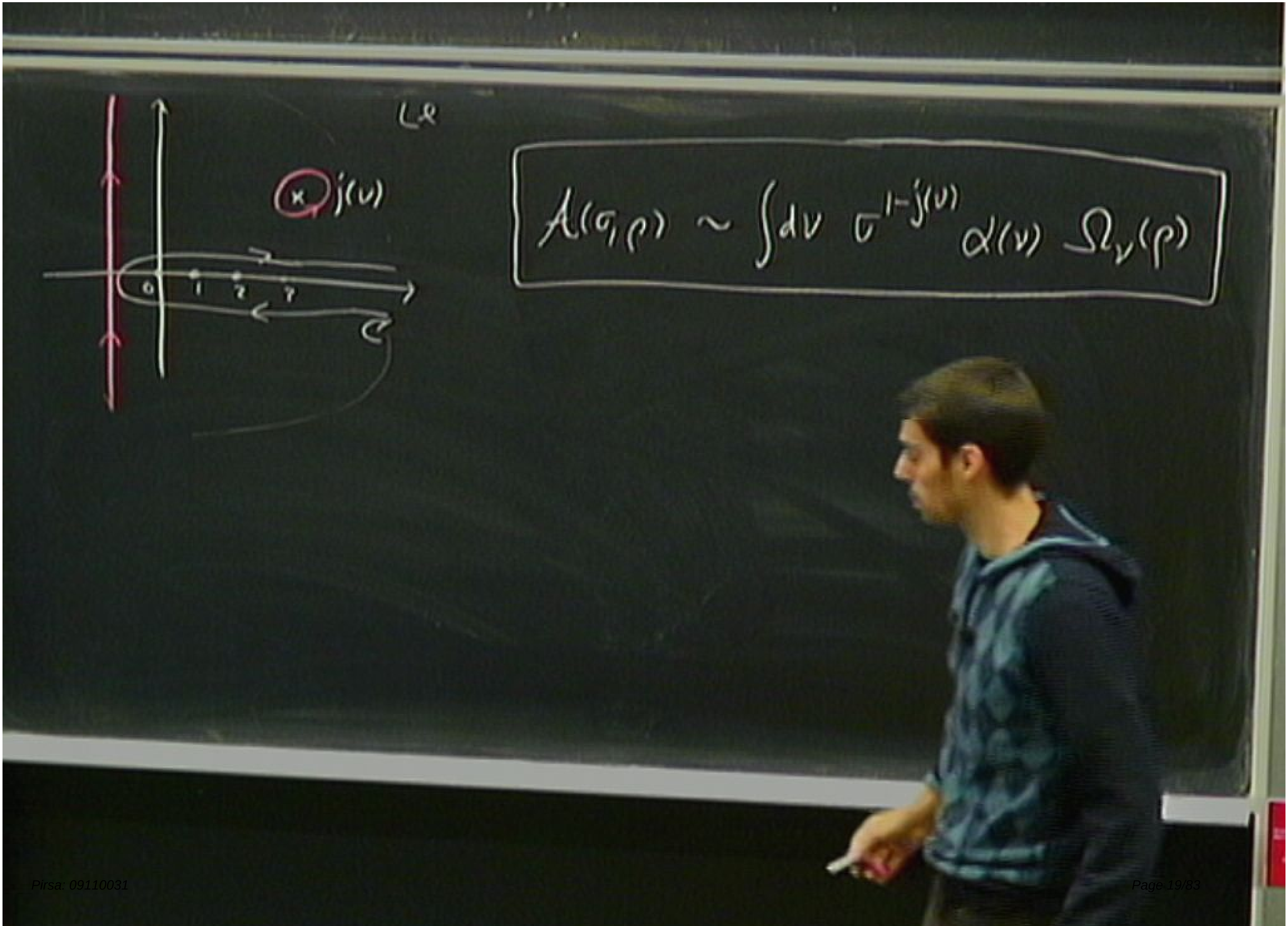
$$G_{i\nu,\ell}(\sigma,\rho) \sim \sigma^{1-\ell}\, \Omega_\nu(\rho)$$

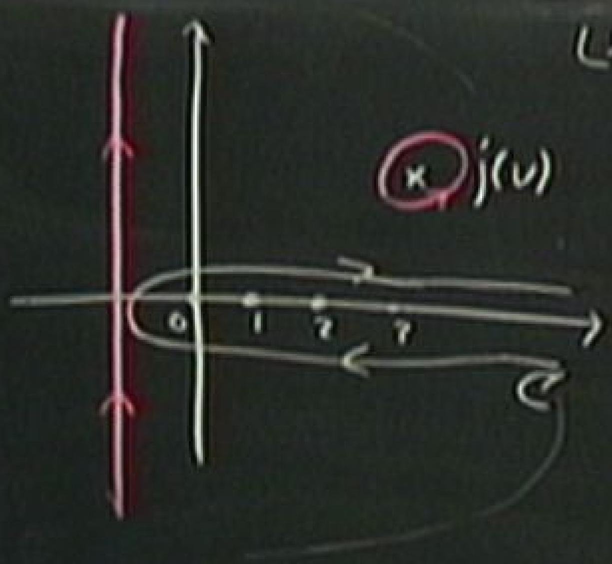
$$\mathcal{A}(\sigma,\rho) = \int d\nu \int_{\mathcal{C}} \frac{d\ell}{2i \sin\pi\ell}\, e^{i\pi\ell}\; a_\ell(\nu)\, G_{i\nu,\ell}(\sigma,\rho)$$



Lℓ
0
1
2
3
C


Lℓ
j(ν)
0
1
2
3
C


j(v)
0 1 2 3
A(q,ρ) ~ ∫dν σ^{1-j(ν)} α(ν) Ω_ν(ρ)



$$\mathcal{A}(\sigma,\rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

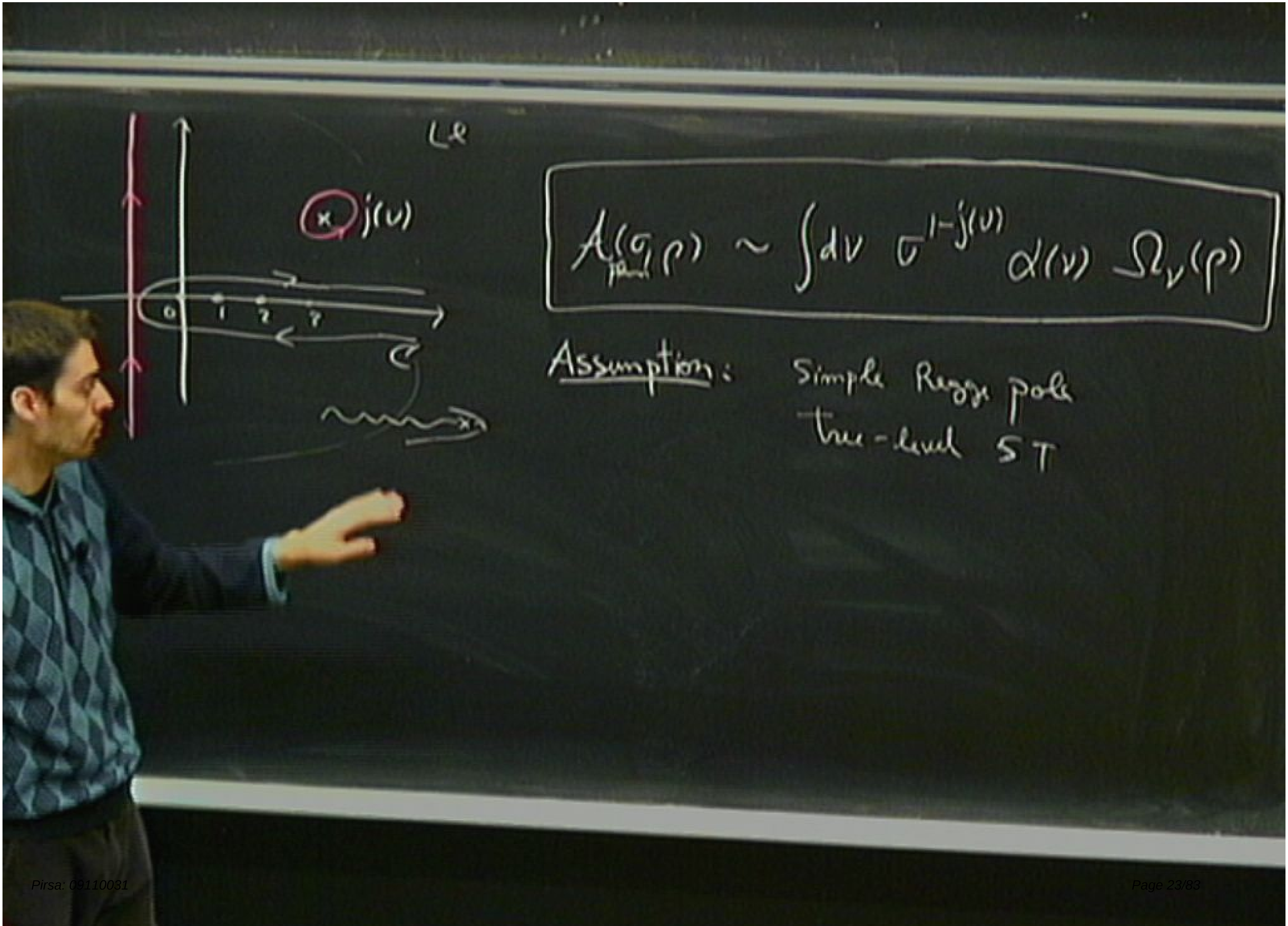
<u>Assumption</u>: Simple Regge pole

$$A_{\text{pomeron}}(\tau, \rho) \sim \int d\nu \; \tau^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

<u>Assumption</u>: Simple Regge pole

$$\mathcal{A}_{\text{planar}}(\sigma, \rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

Assumption: Simple Regge pole
tree-level ST


j(ν)
A(q,p) ~ ∫dν σ^{1-j(ν)} α(ν) Ω_ν(p)
Assumption: Simple Regge pole
tree-level ST

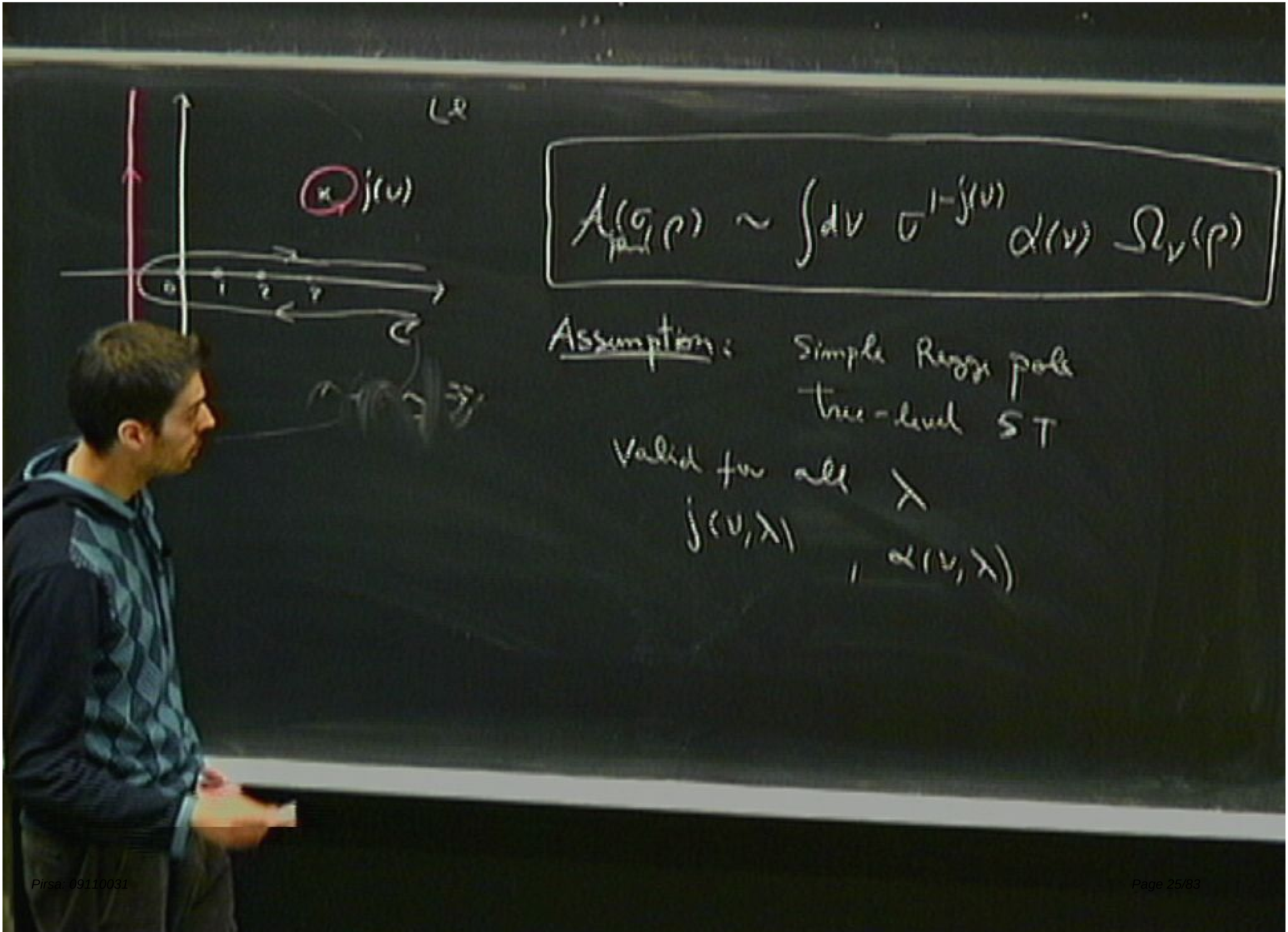
$$\mathcal{A}(\sigma,\rho) = 2 \sum_{\ell} \int d\nu \; a_\ell(\nu) \; t_\ell(\nu) \; g_{\frac{d}{2}+i\nu,\ell}(\sigma,\rho)$$

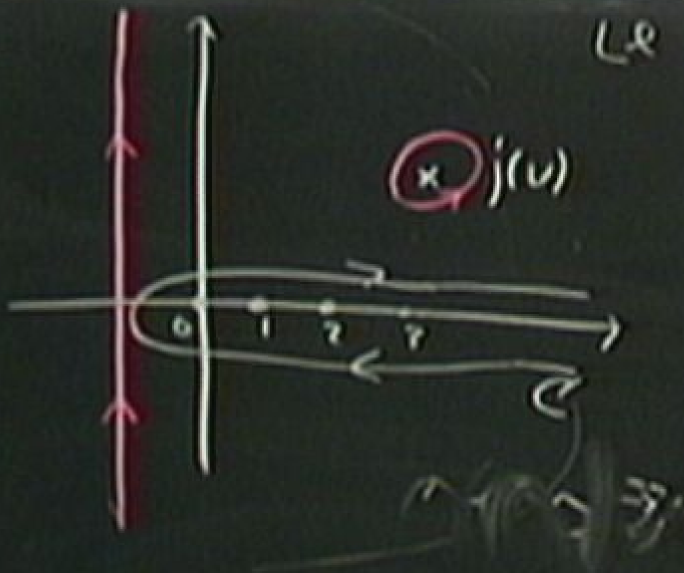
$$\sum_{\Delta} g_{\Delta,\ell}(\sigma,\rho) \qquad \frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2}$$

exchange primary of dimension Δ

$$G_{i\nu,\ell}(\sigma,\rho) \sim \sigma^{1-\ell} \; \Omega_\nu(\rho)$$

$$\mathcal{A}(\sigma,\rho) = \int d\nu \int_{C} \frac{d\ell}{2i \sin\pi\ell} \, e^{i\pi\ell} \; a_\ell(\nu) \; G_{i\nu,\ell}(\sigma,\rho)$$


j(ν)
A(σ,ρ) ~ ∫dν σ^{1-j(ν)} α(ν) Ω_ν(ρ)
Assumption: Simple Regge pole
tree-level ST
Valid for all λ
j(ν,λ) , α(ν,λ)



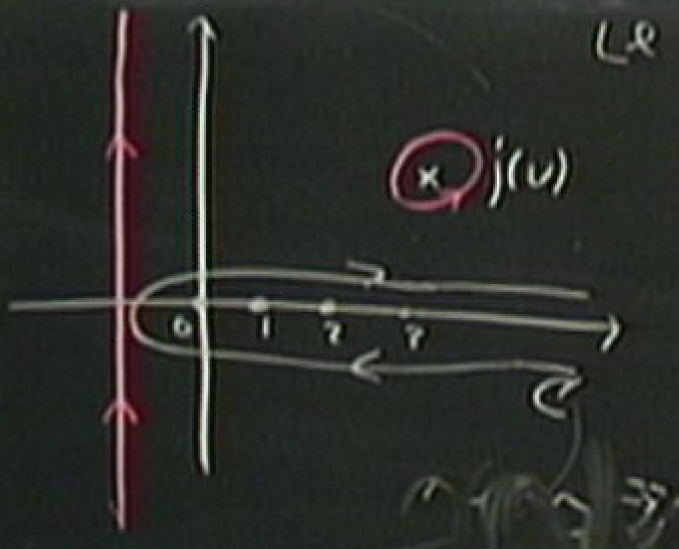
$$A(\sigma,\rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

Assumption: Simple Regge pole

tree-level ST

Valid for all λ

$j(\nu,\lambda)$, $\alpha(\nu,\lambda)$ → 3pt-function



ℓ $\mathrm{Tr}\left(F^{+\mu}(D^{+})^{\ell-2}F^{+}{}_{\mu}\right)$

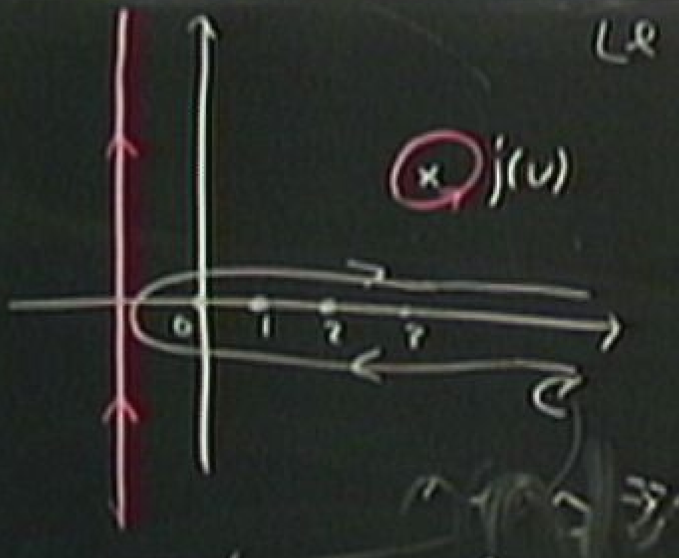
$$A_{\text{[?]}}(\sigma,\rho) \sim \int d\nu\; \sigma^{1-j(\nu)}\, \alpha(\nu)\, \Omega_\nu(\rho)$$

Assumption: Simple Regge pole

tree-level ST

Valid for all λ

$j(\nu,\lambda)$, $\alpha(\nu,\lambda)$ → 3pt-function



$$\mathcal{A}(\sigma,\rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

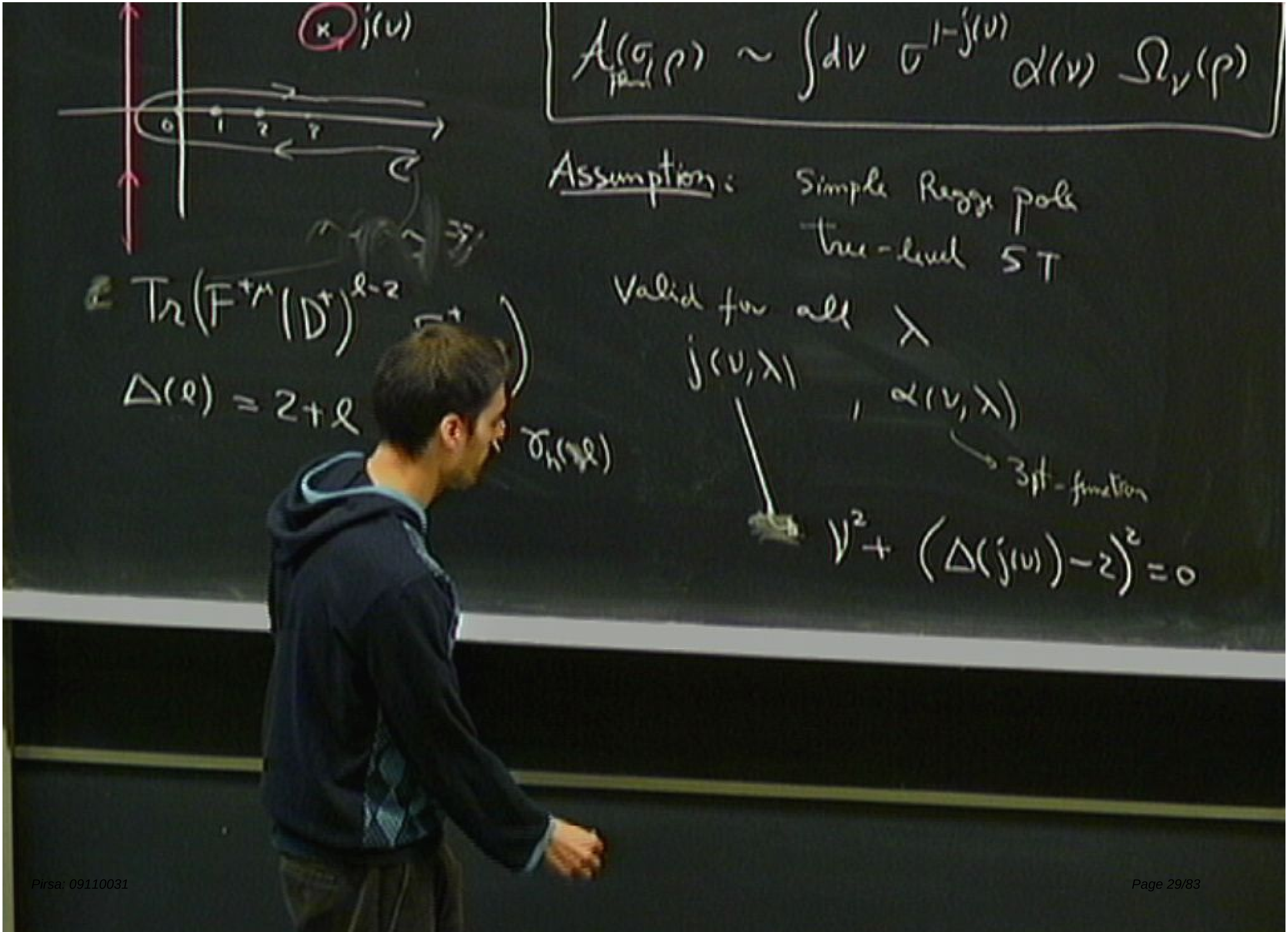
Assumption: Simple Regge pole + tree-level ST

Valid for all λ

$j(\nu,\lambda)$, $\alpha(\nu,\lambda)$ → 3pt-function

$\mathrm{Tr}\left(F^{+\mu}(D^+)^{\ell-2} F^{+}{}_{\mu}\right)$

$$\Delta(\ell) = 2+\ell + \sum_{n=1}^{\infty} \lambda^n \gamma_n(\ell)$$


j(ν)
Assumption: Simple Regge pole
tree-level ST
Valid for all λ
j(ν, λ) , α(ν, λ)
3pt-function
Δ(ℓ) = 2 + ℓ

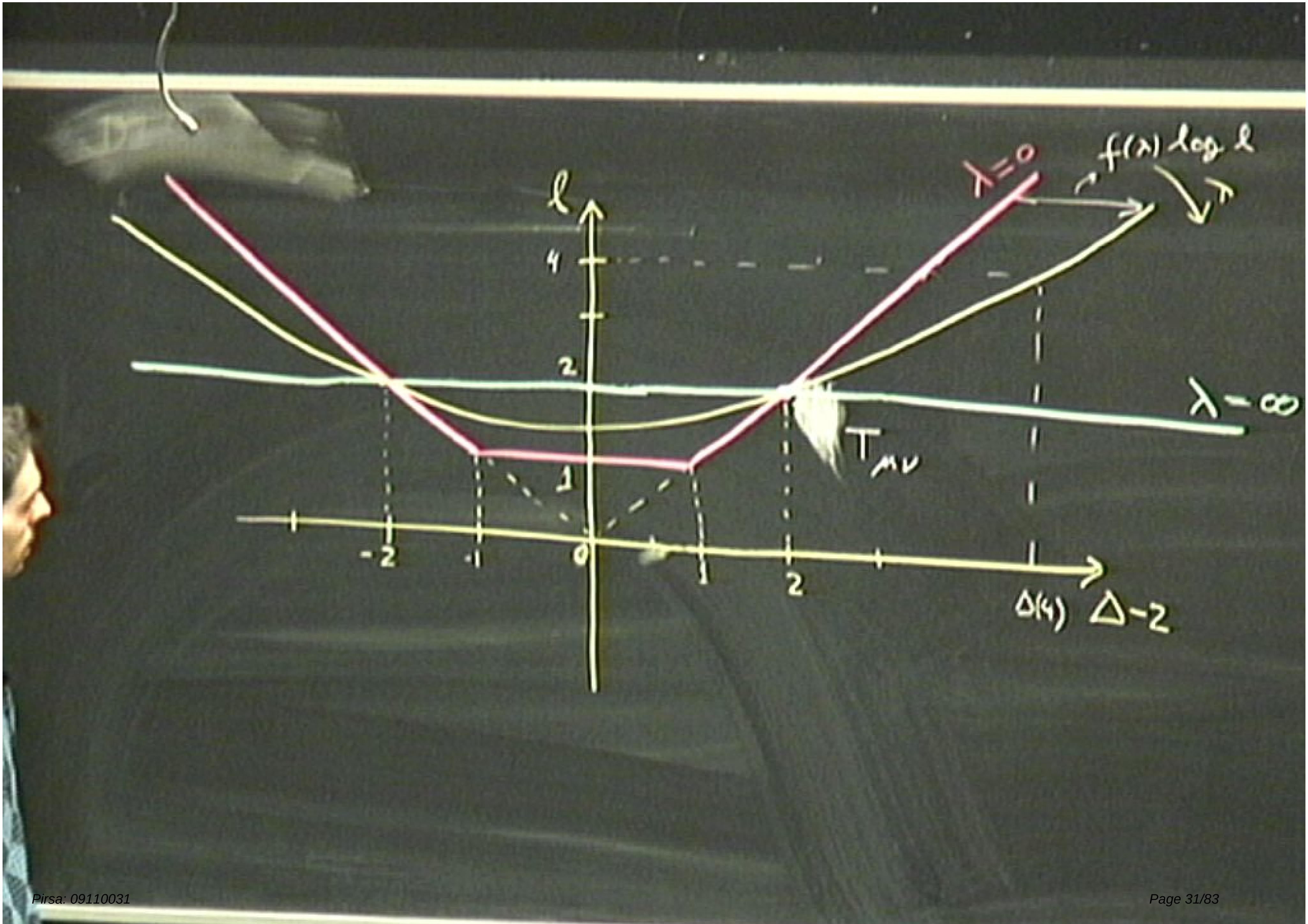
Assum

$$\mathrm{Tr}\left(F^{+\mu}(D^+)^{\ell-2}F^+_{\ \mu}\right)$$

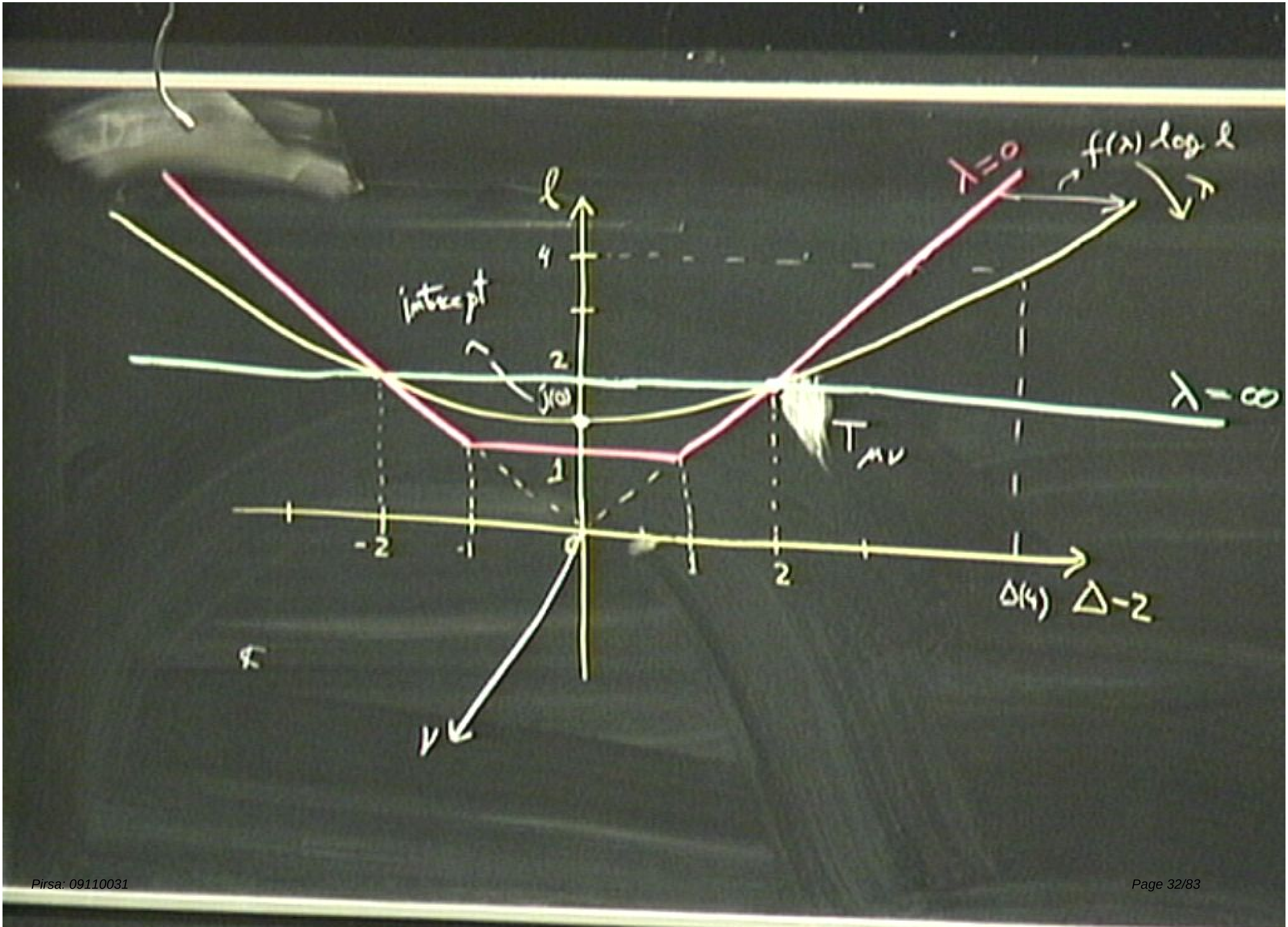
$$\Delta(\ell) = 2+\ell+\sum_{h=1}^{\infty}\lambda^h\,\gamma_h(\ell)$$

ℓ even

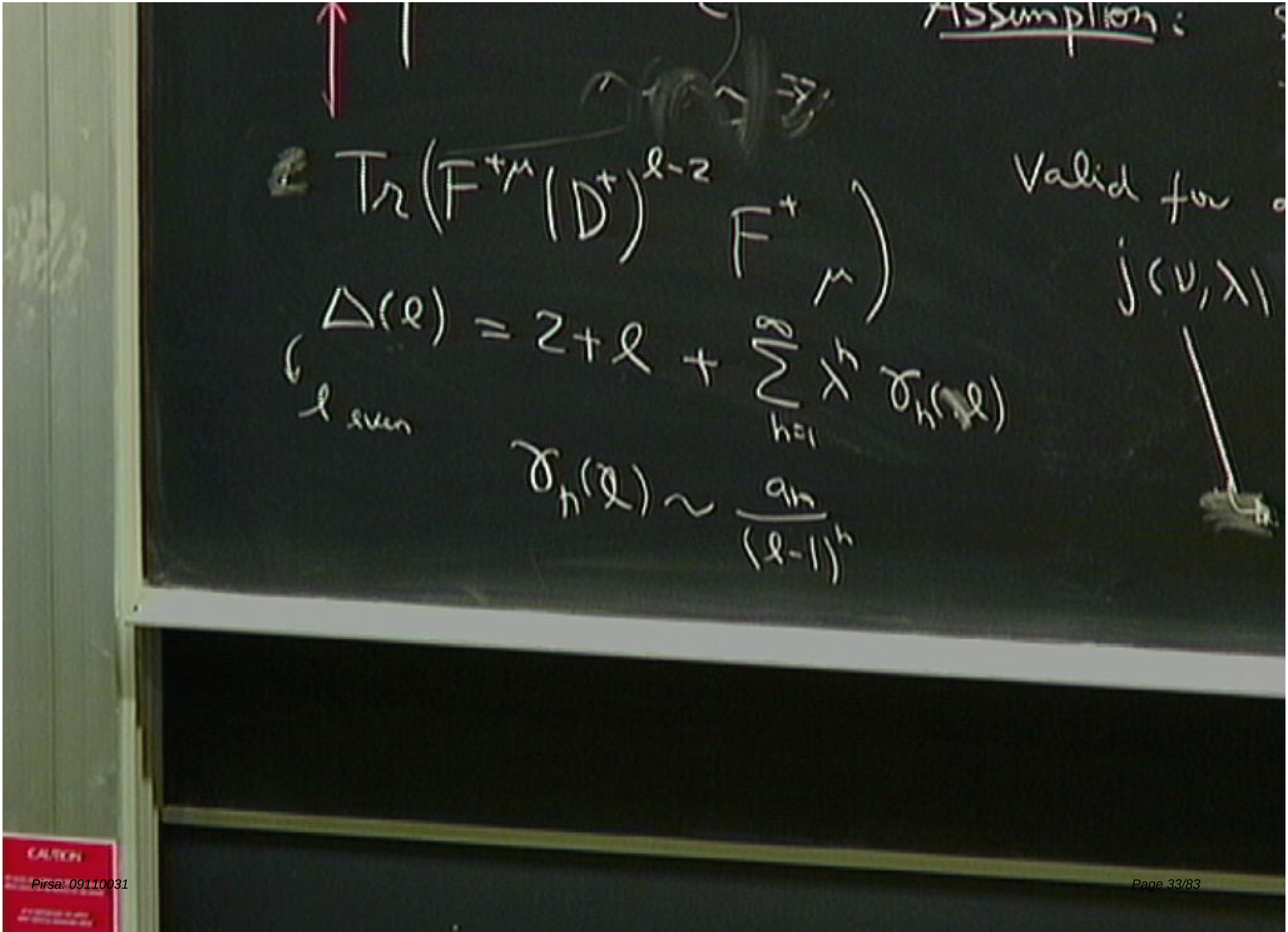
Vo

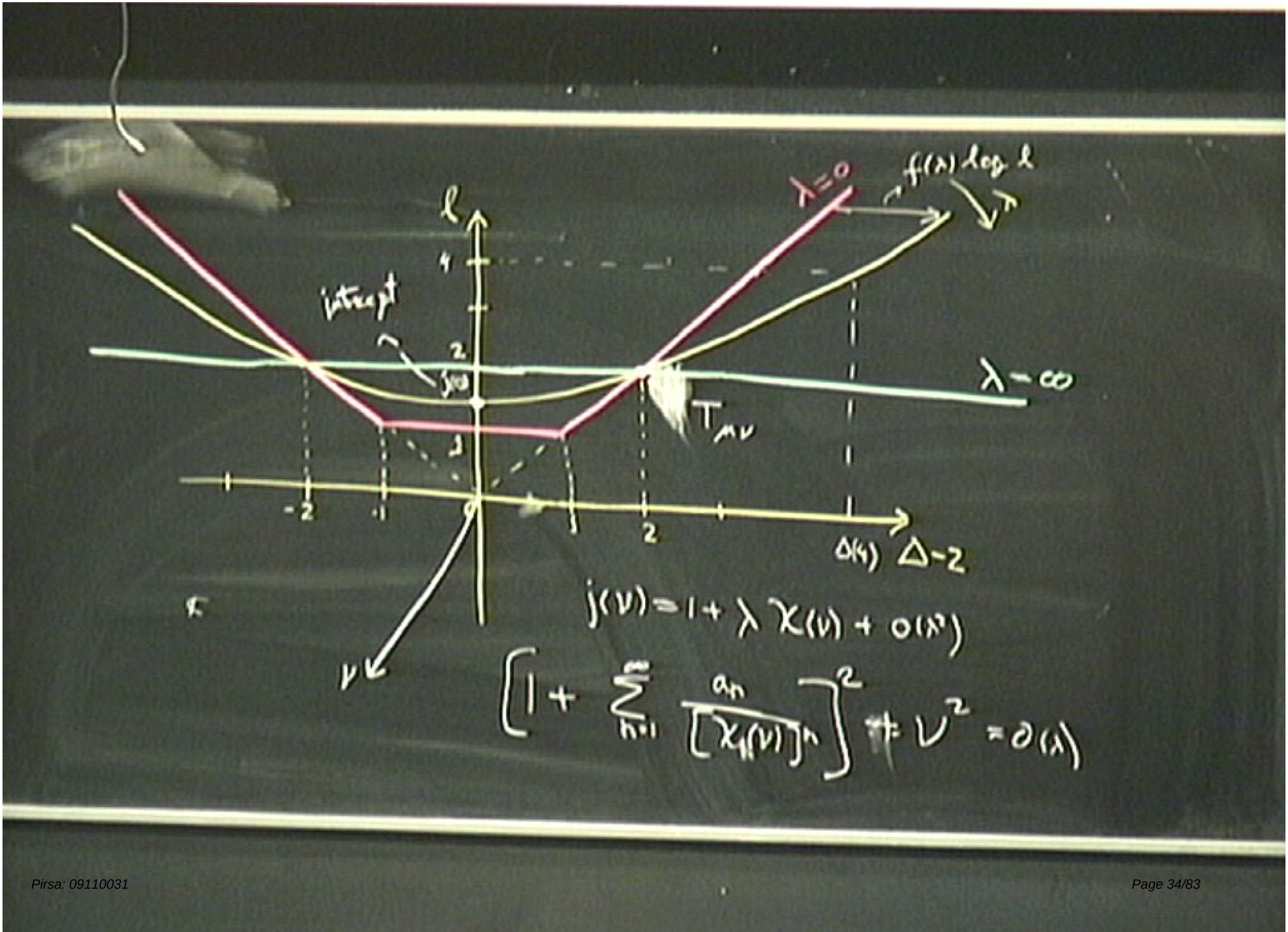


f(λ) log ℓ
λ=0
ℓ
4
2
1
λ→∞
$T_{\mu\nu}$
-2
-1
0
1
2
Δ(4)
Δ-2

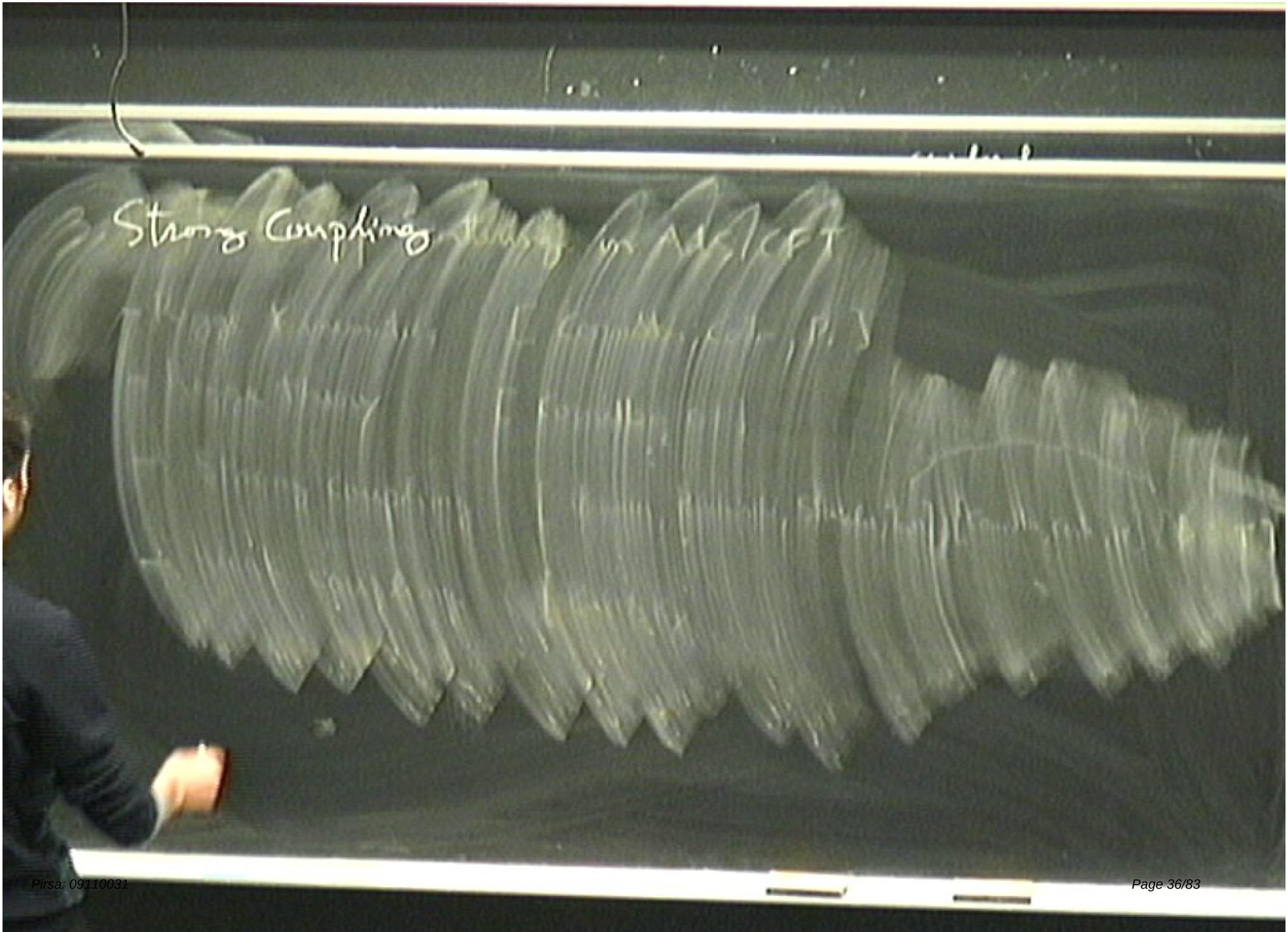


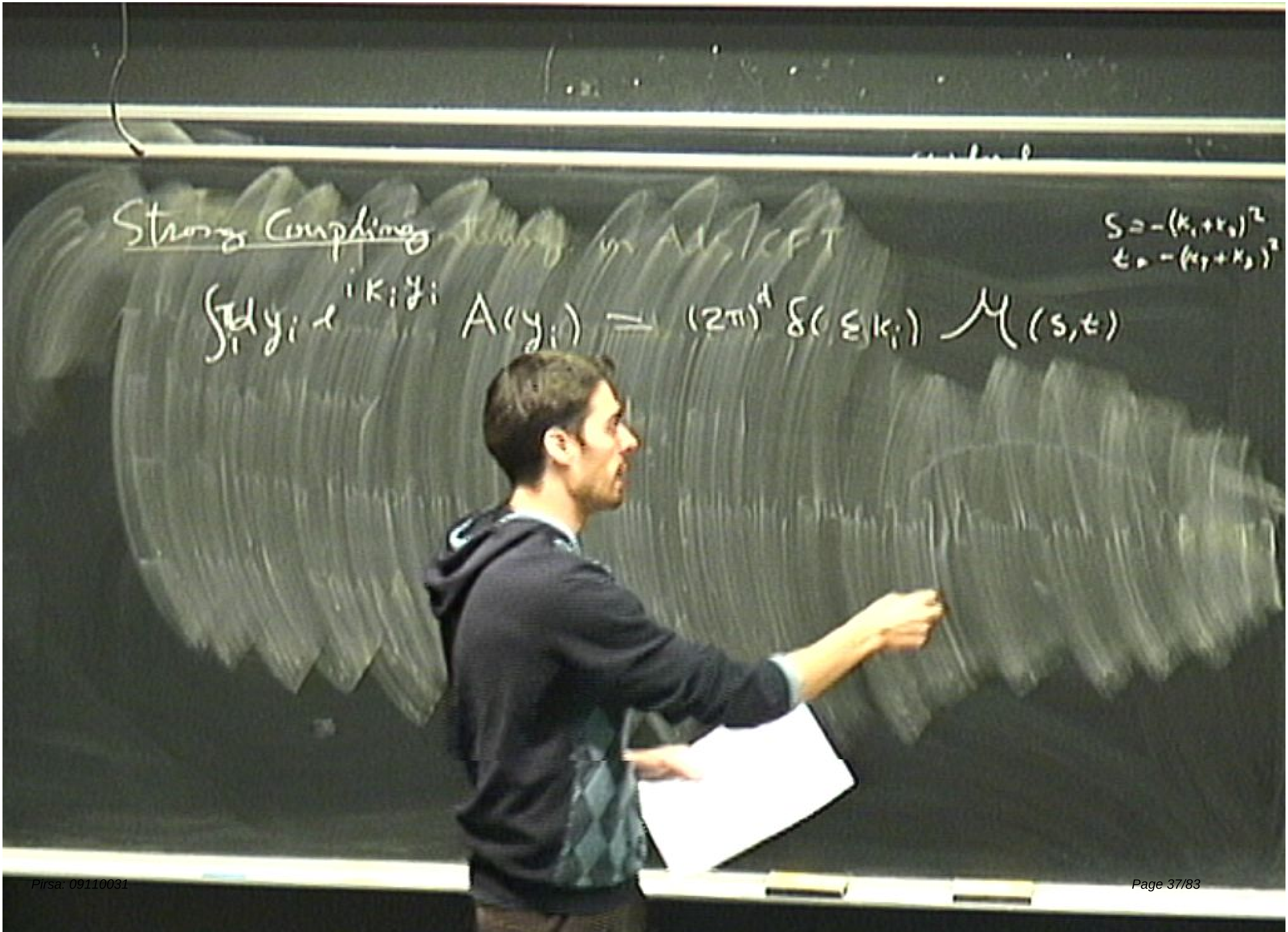
λ=0
f(λ) log ℓ
ℓ
4
intercept
2
j(0)
λ=∞
T_{μν}
1
-2
-1
0
1
2
Δ(4)
Δ-2
ν

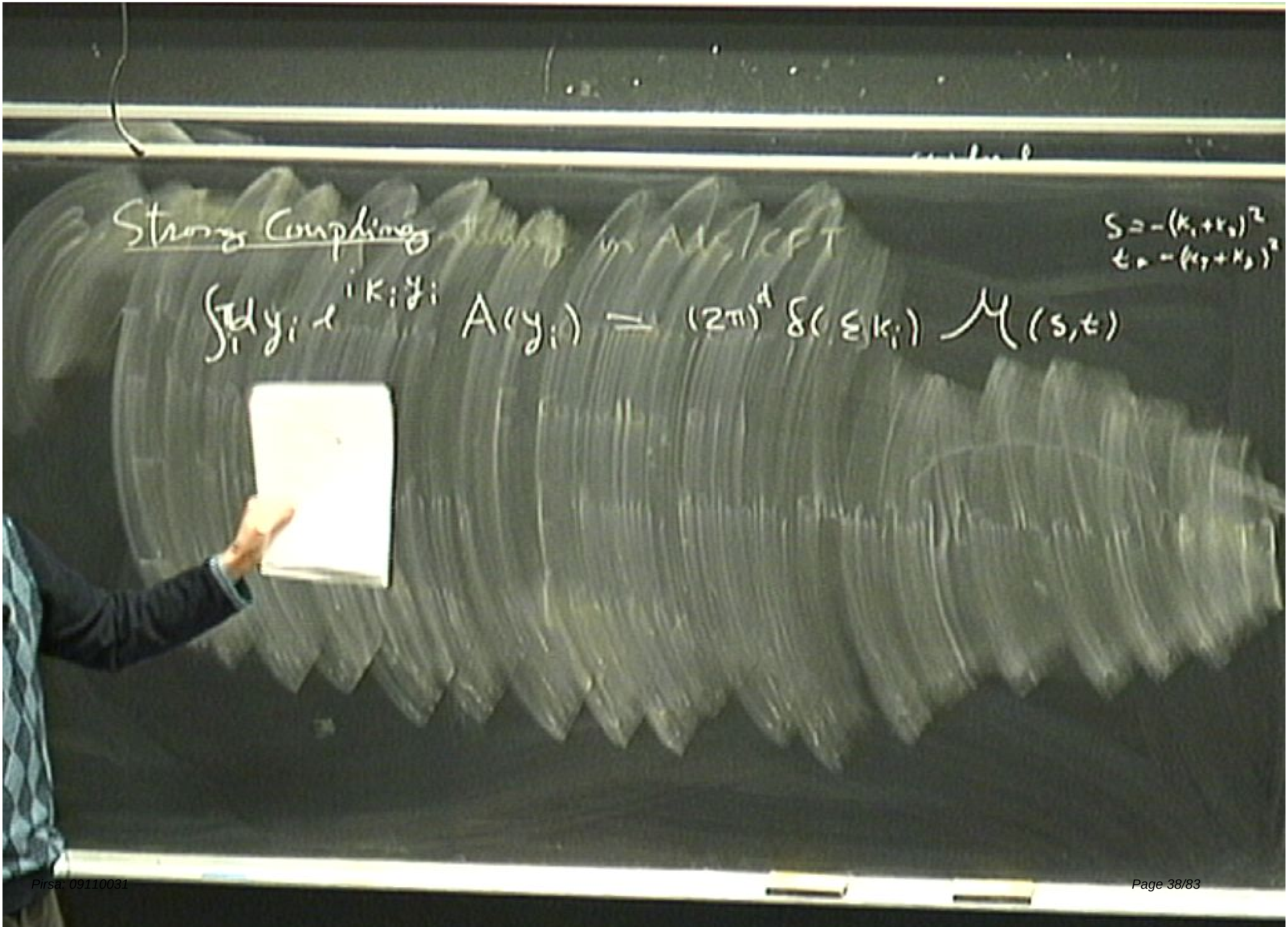

Assumption:
$Tr\left(F^{+\mu}(D^+)^{\ell-2} F^+_{\mu}\right)$
Valid for
$j(\nu,\lambda)$
$\Delta(\ell) = 2 + \ell + \sum_{n=1}^{\infty} \lambda^n \gamma_n(\ell)$
ℓ even
$\gamma_n(\ell) \sim \frac{a_n}{(\ell-1)^n}$


ℓ
λ=0
f(λ) log ℓ
intercept
j(0)
λ→∞
Tμν
-2
-1
0
1
2
Δ(4) Δ-2
ν
j(ν)=1+λ χ(ν) + O(λ²)
[1 + Σ_{n=1}^{∞} a_n / [χ_1(ν)]^n]² + ν² = O(λ)


Strong Coupling


Strong Coupling


Strong Coupling
$\int \prod_i dy_i \, e^{i k_i y_i} A(y_i) = (2\pi)^d \delta(\Sigma k_i) \mathcal{M}(s,t)$
$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$


Strong Coupling
$\int \prod_i dy_i\, e^{i k_i y_i} A(y_i) = (2\pi)^d \delta(\Sigma k_i)\, \mathcal{M}(s,t)$
$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$

Strong Coupling ... in AdS/CFT $d=4$

$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$

$$\int \prod_i dy_i \, e^{i k_i y_i} A(y_i) = \underbrace{(2\pi)^d \delta(\textstyle\sum_i k_i)}_{V} \mathcal{M}(s,t)$$

$$ds^2 = R^2 \frac{dr^2 + dv_\perp^2 - dv^+ dv^-}{r^2}$$

Strong Coupling

$d=4$

$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$

$$\int \prod_i dy_i \, e^{i k_i y_i} A(y_i) = (2\pi)^d \delta(\Sigma k_i) \, \mathcal{M}(s,t)$$

$$ds^2 = R^2 \, \frac{dr^2 + dv_\perp^2 - dv^+ dv^-}{r^2}$$

$$\Psi_i(r, v) = \int dy \, e^{i k_i \cdot y} K(y; r, v)$$

Strong Coupling in AdS/CFT $d=4$

$s = -(k_1+k_2)^2$

$t = -(k_1+k_3)^2$

$$\int \prod_i dy_i \, e^{i k_i y_i} A(y_i) = (2\pi)^d \delta\left(\textstyle\sum k_i\right) \mathcal{M}(s,t)$$

$$ds^2 = R^2 \, \frac{dr^2 + dv_\perp^2 - dv^+ dv^-}{r^2}$$

$$\Psi_i(r, v) = \int dy \, e^{i k_i \cdot y} K(y; r, v) = f_i(r)\, e^{i k_i \cdot v}$$

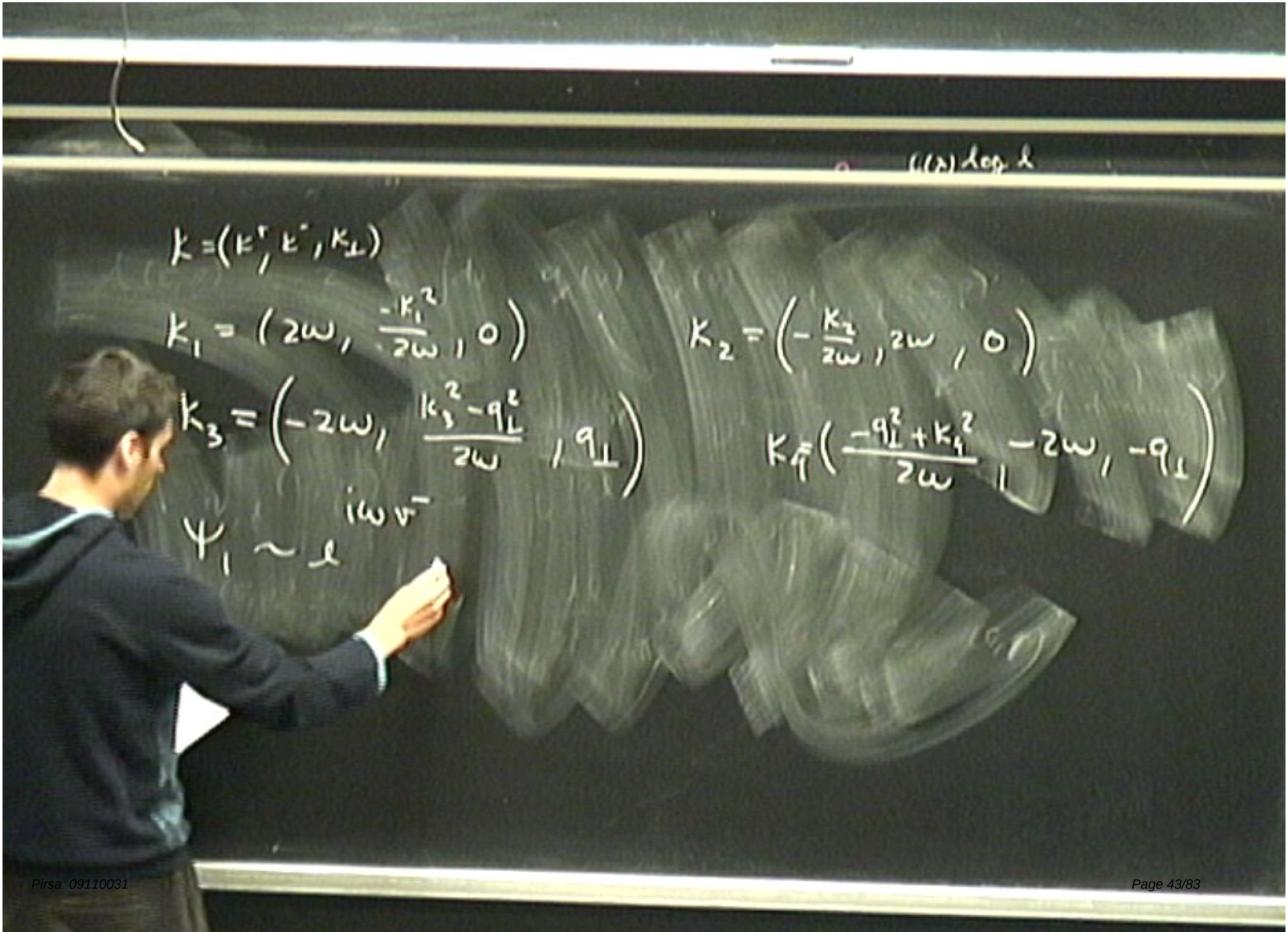
$$f_i = r^2 K_{\Delta_i - 1}\left(\sqrt{k_i^2 r^2}\right)$$

$f(z) \log \lambda$

$$k = (k^+, k^-, k_\perp)$$

$$k_1 = \left(2\omega, \frac{-k_1^2}{2\omega}, 0\right) \qquad k_2 = \left(-\frac{k_2}{2\omega}, 2\omega, 0\right)$$

$$k_3 = \left(-2\omega, \frac{k_3^2 - q_\perp^2}{2\omega}, q_\perp\right) \qquad k_4 = \left(\frac{-q_\perp^2 + k_4^2}{2\omega}, -2\omega, -q_\perp\right)$$


$k=(k^+, k^-, k_\perp)$
$k_1 = (2\omega, \frac{-k_1^2}{2\omega}, 0)$
$k_2 = (-\frac{k_2}{2\omega}, 2\omega, 0)$
$k_3 = (-2\omega, \frac{k_3^2 - q_\perp^2}{2\omega}, q_\perp)$
$k_4 = (\frac{-q_\perp^2 + k_4^2}{2\omega}, -2\omega, -q_\perp)$
$\Psi_1 \sim e^{i\omega v^-}$

$l(\lambda) \log l$

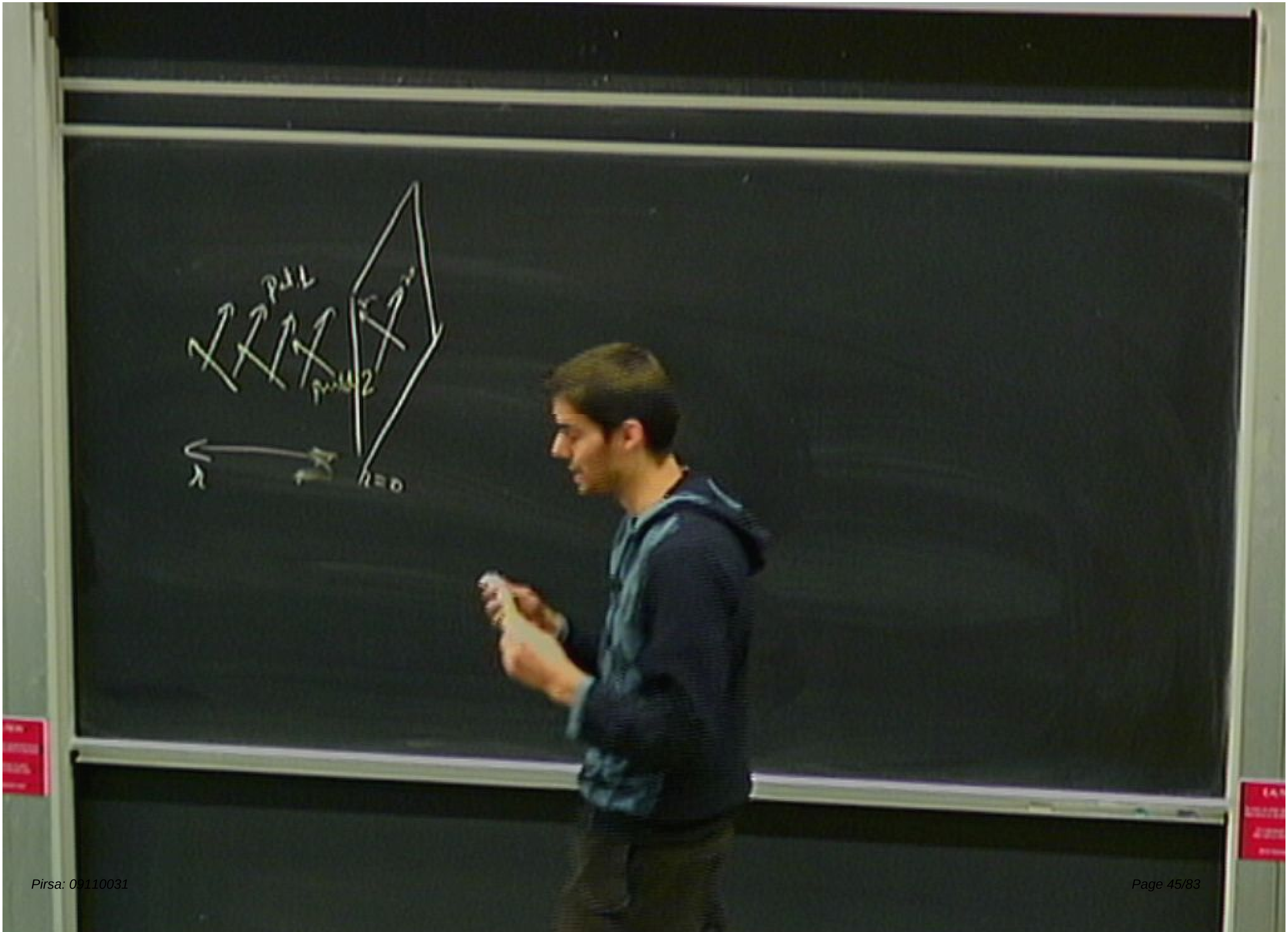
$k = (k^+, k^-, k_\perp)$

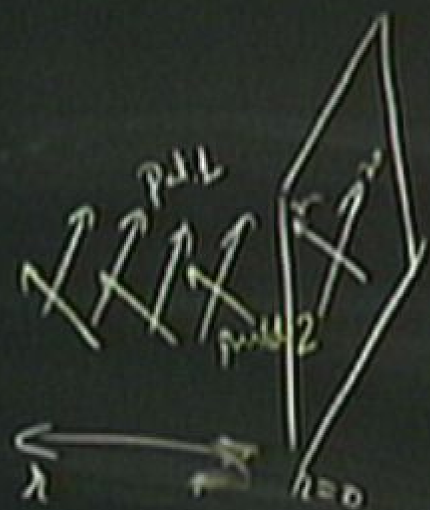
$k_1 = \left(2\omega, \frac{-k_1^2}{2\omega}, 0\right)$ $\quad k_2 = \left(-\frac{k_2}{2\omega}, 2\omega, 0\right)$

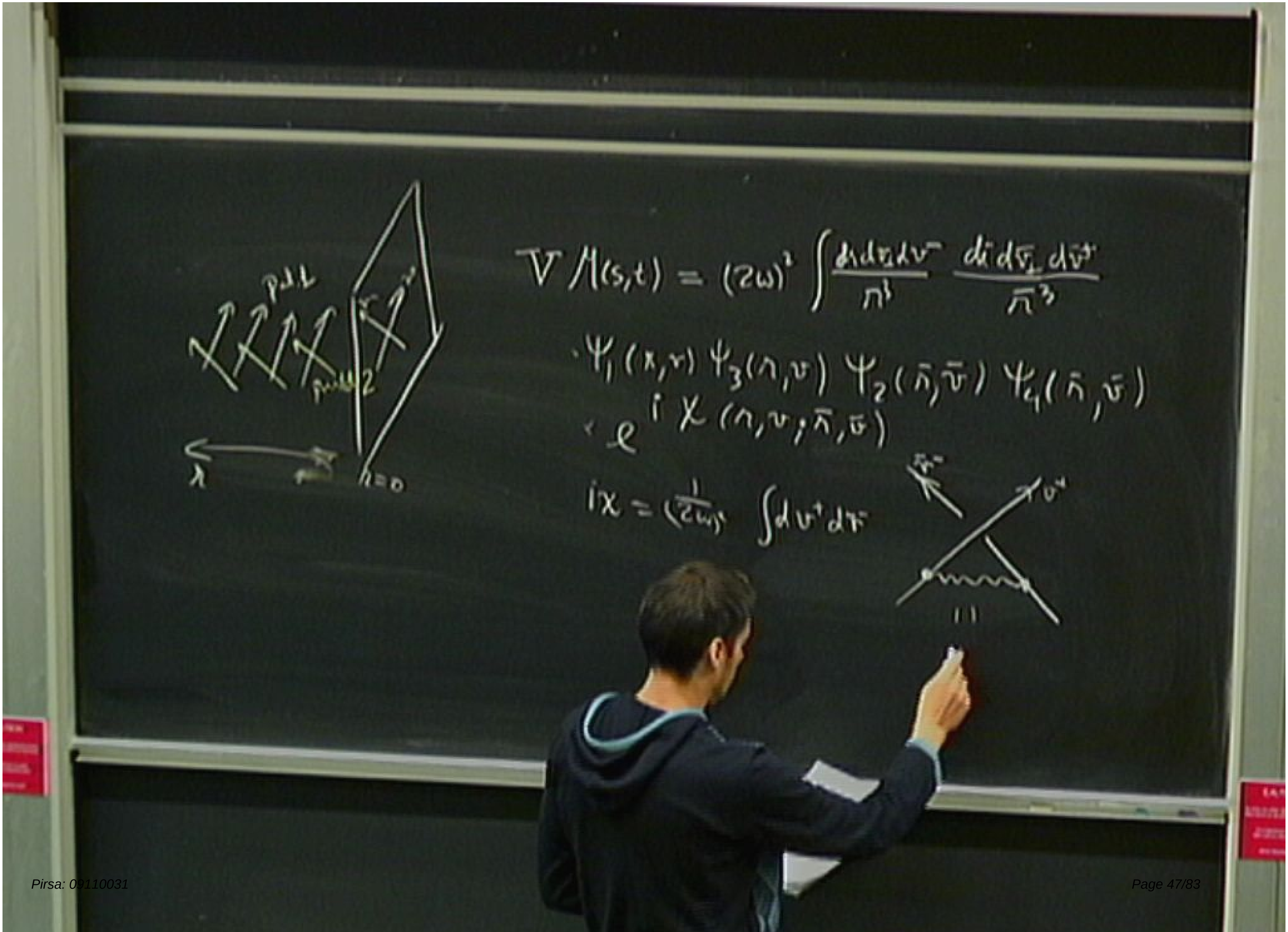
$k_3 = \left(-2\omega, \frac{k_3^2 - q_\perp^2}{2\omega}, q_\perp\right)$ $\quad k_4 = \left(\frac{-q_\perp^2 + k_4^2}{2\omega}, -2\omega, -q_\perp\right)$

$\psi_1 \sim e^{i\omega v^-} f_1(\lambda)$ $\quad \psi_2 \sim e^{-i\omega v^+} f_2(\lambda)$

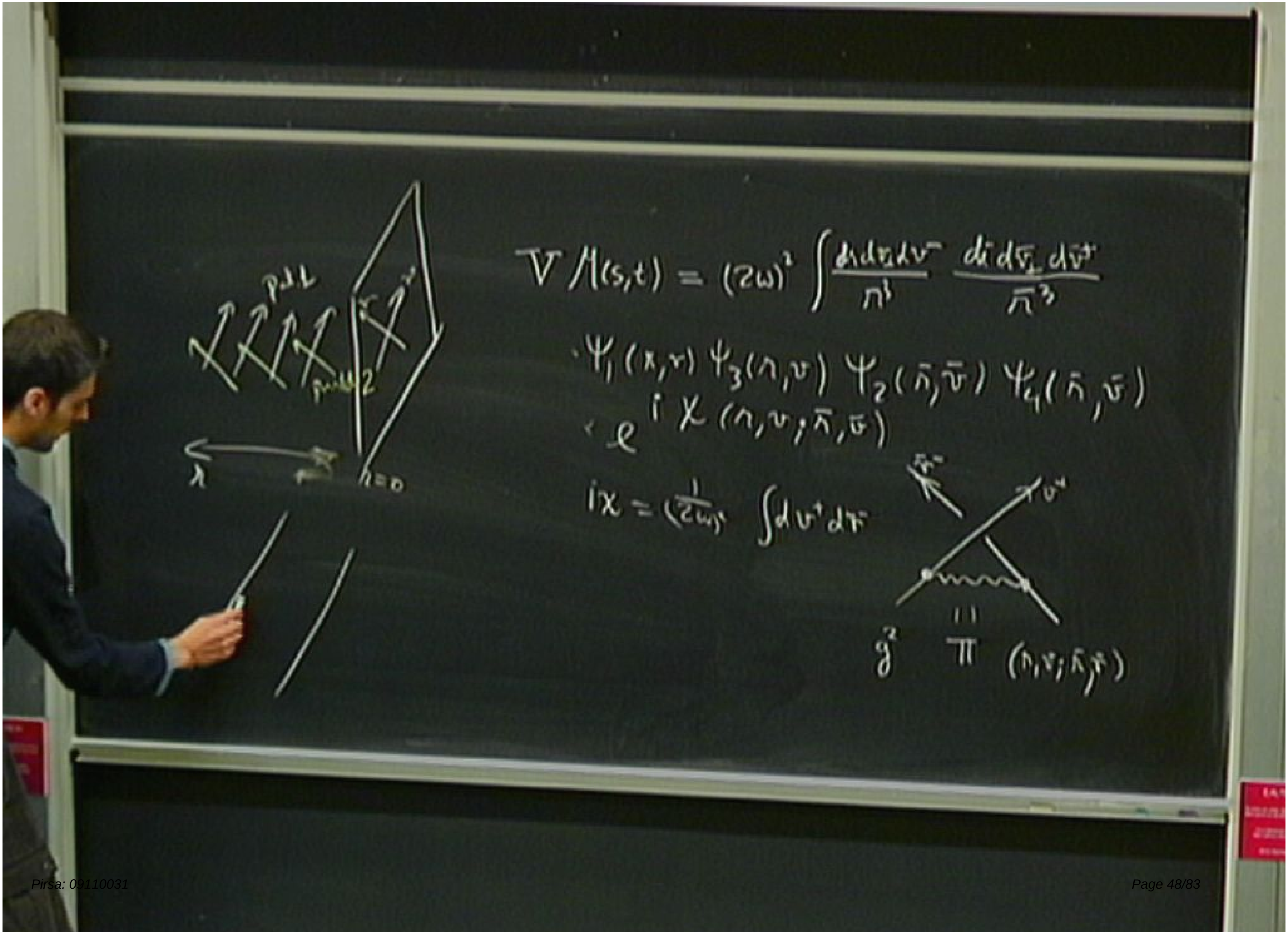
$\psi_3 \sim e^{i\omega v^-} e^{iq_\perp \cdot v_\perp} f_3(\lambda)$ $\quad \psi_4 \sim e^{i\omega v^+} e^{-iq_\perp v_\perp} f_4(\lambda)$



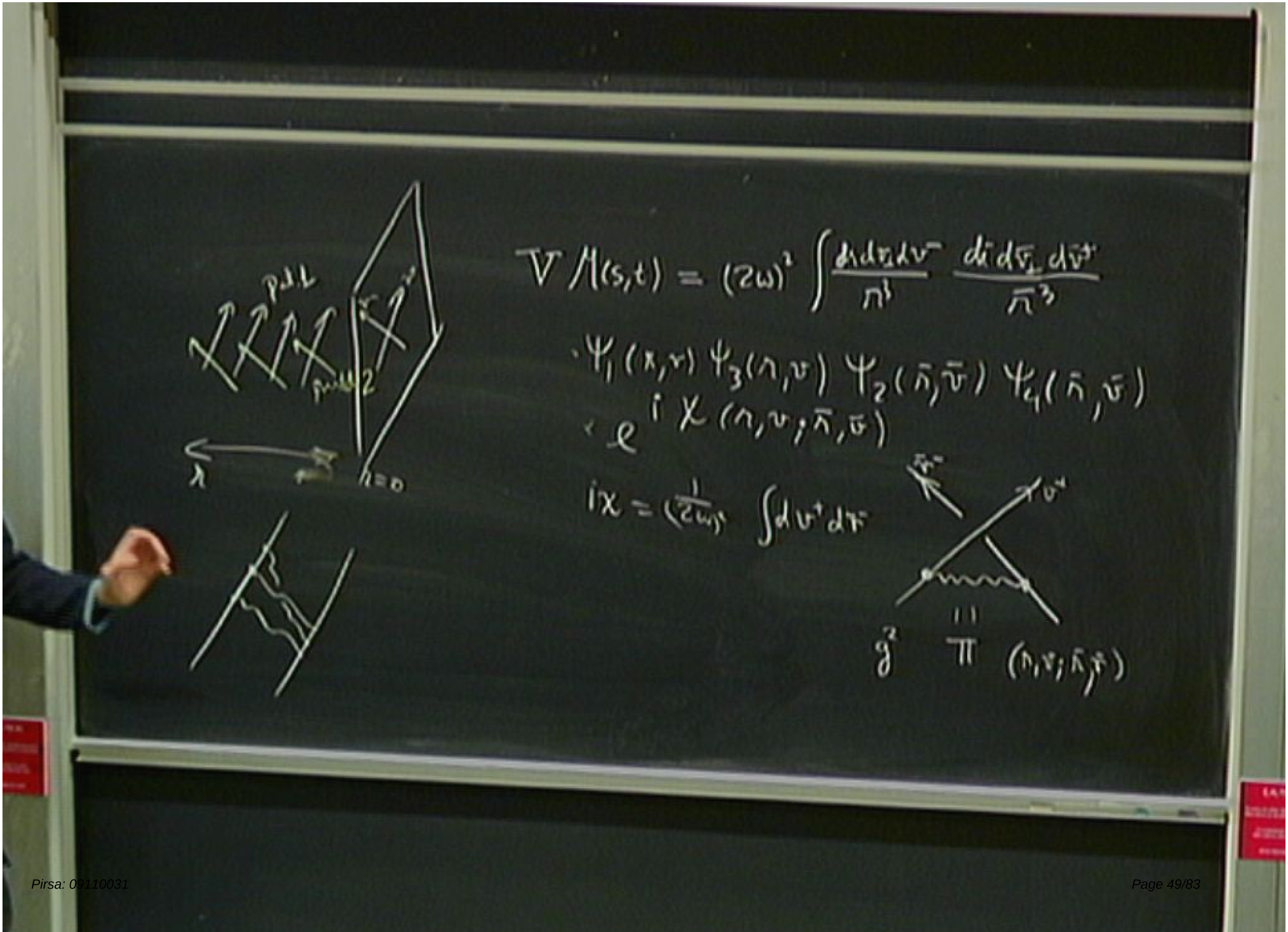


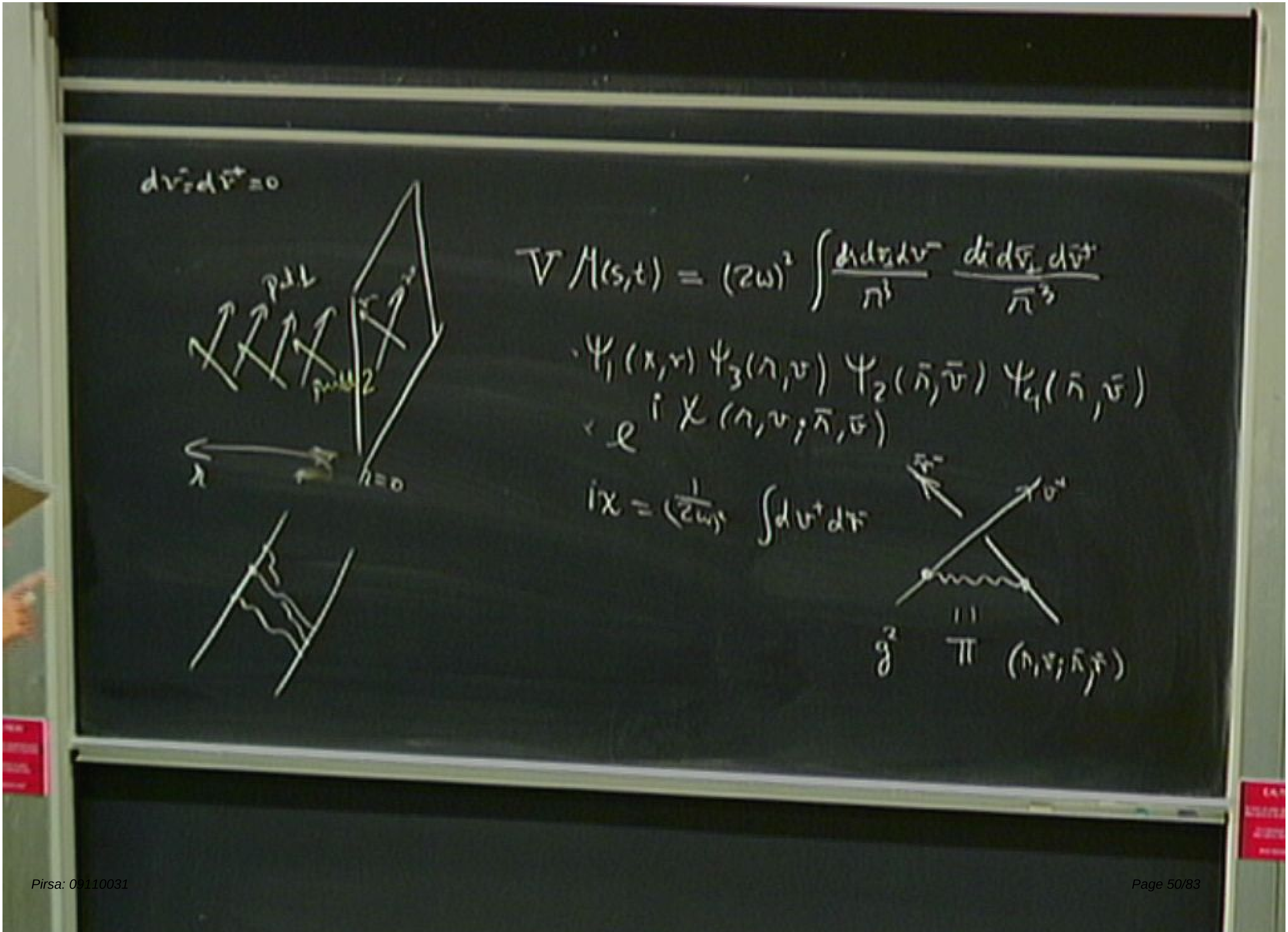


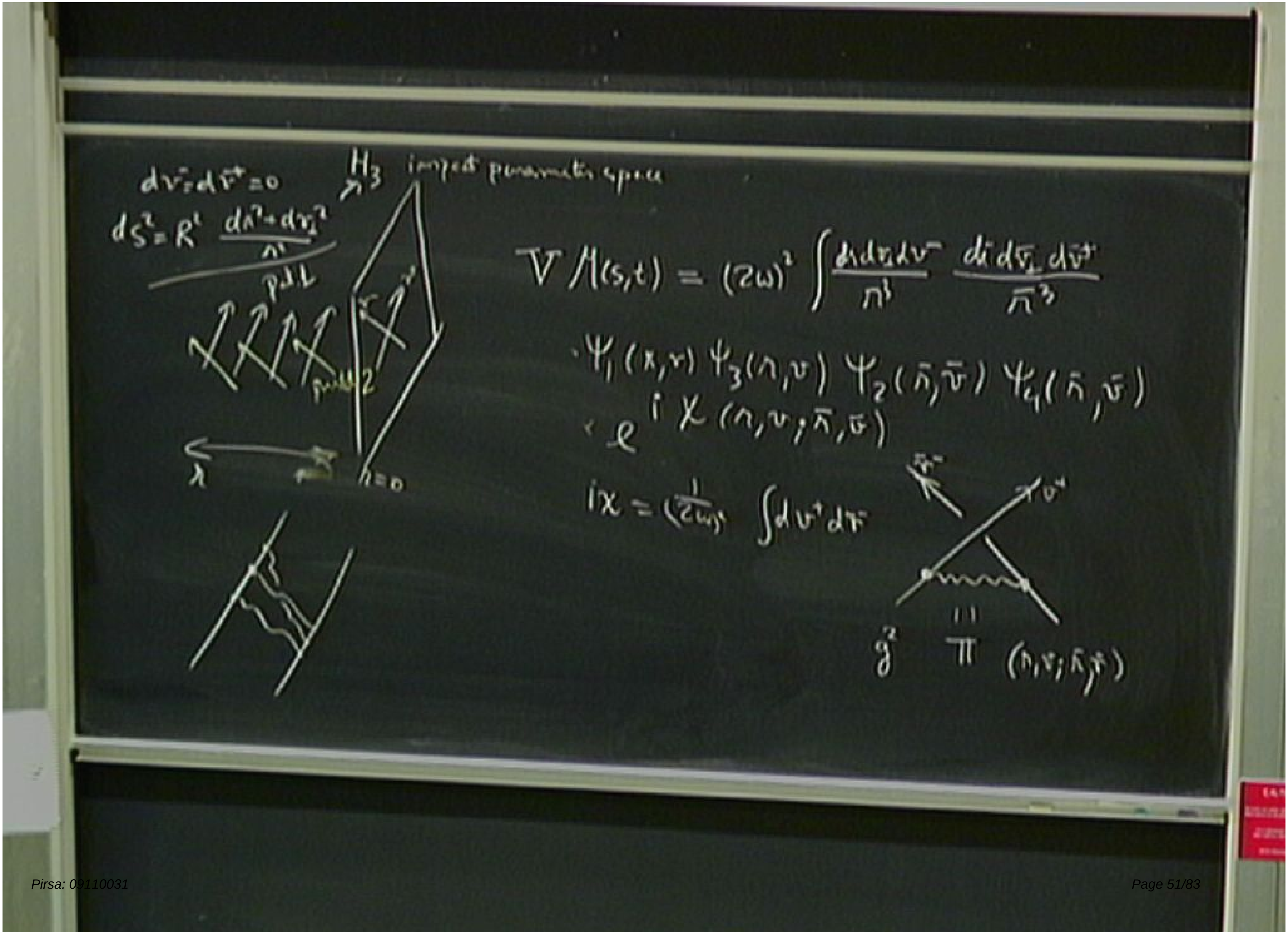
T A(s,t) = (2ω)² ∫ dλ dv₁ dv⁻ / π³ · dλ̄ dv̄₁ dv̄⁺ / π̄³
· ψ₁(π, v) ψ₃(π, v) ψ₂(π̄, v̄) ψ₄(π̄, v̄)
· e^{iχ(π, v; π̄, v̄)}
iχ = 1/(2ω)² ∫ dv⁺ dv⁻
λ
λ=0

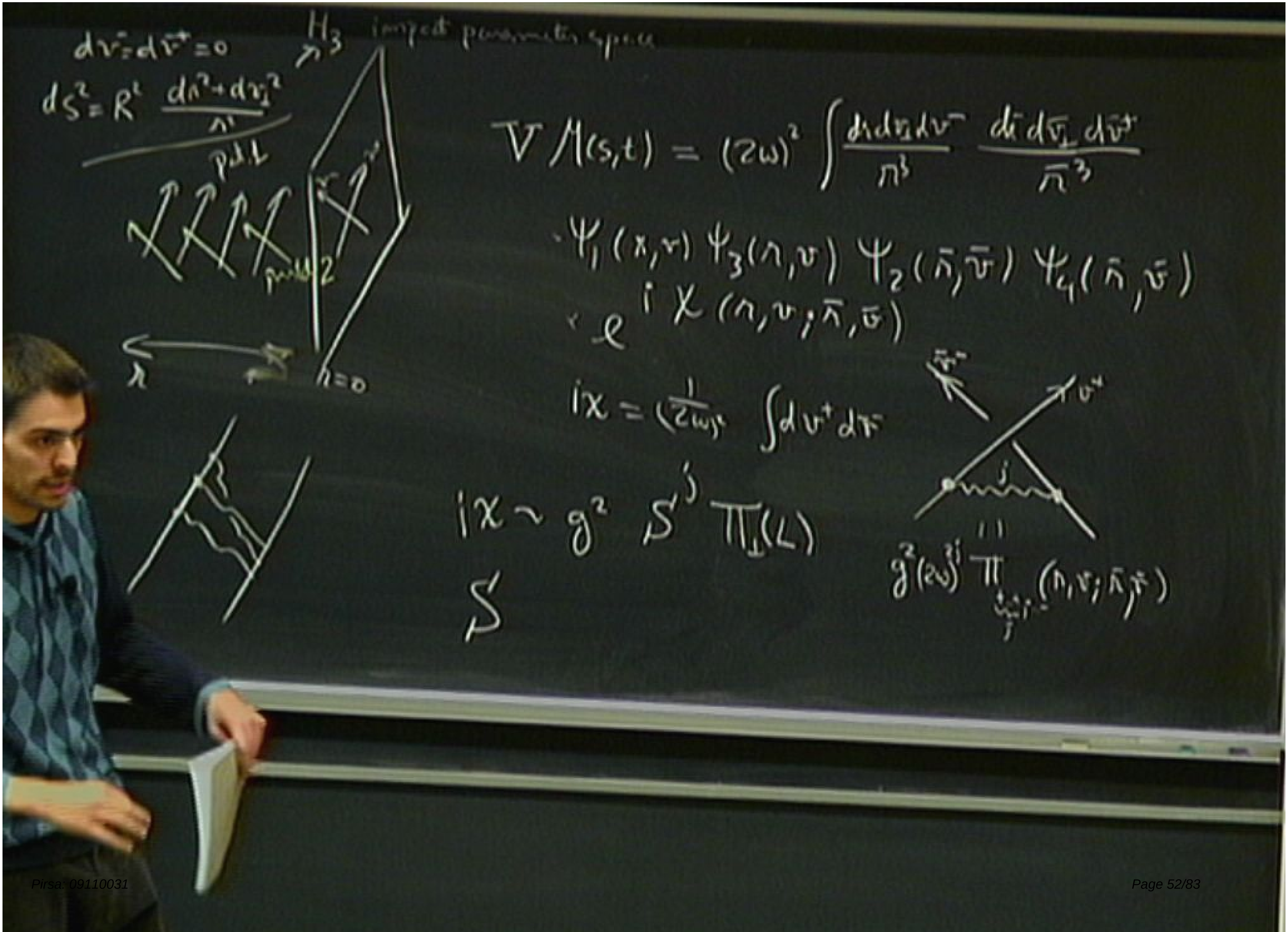


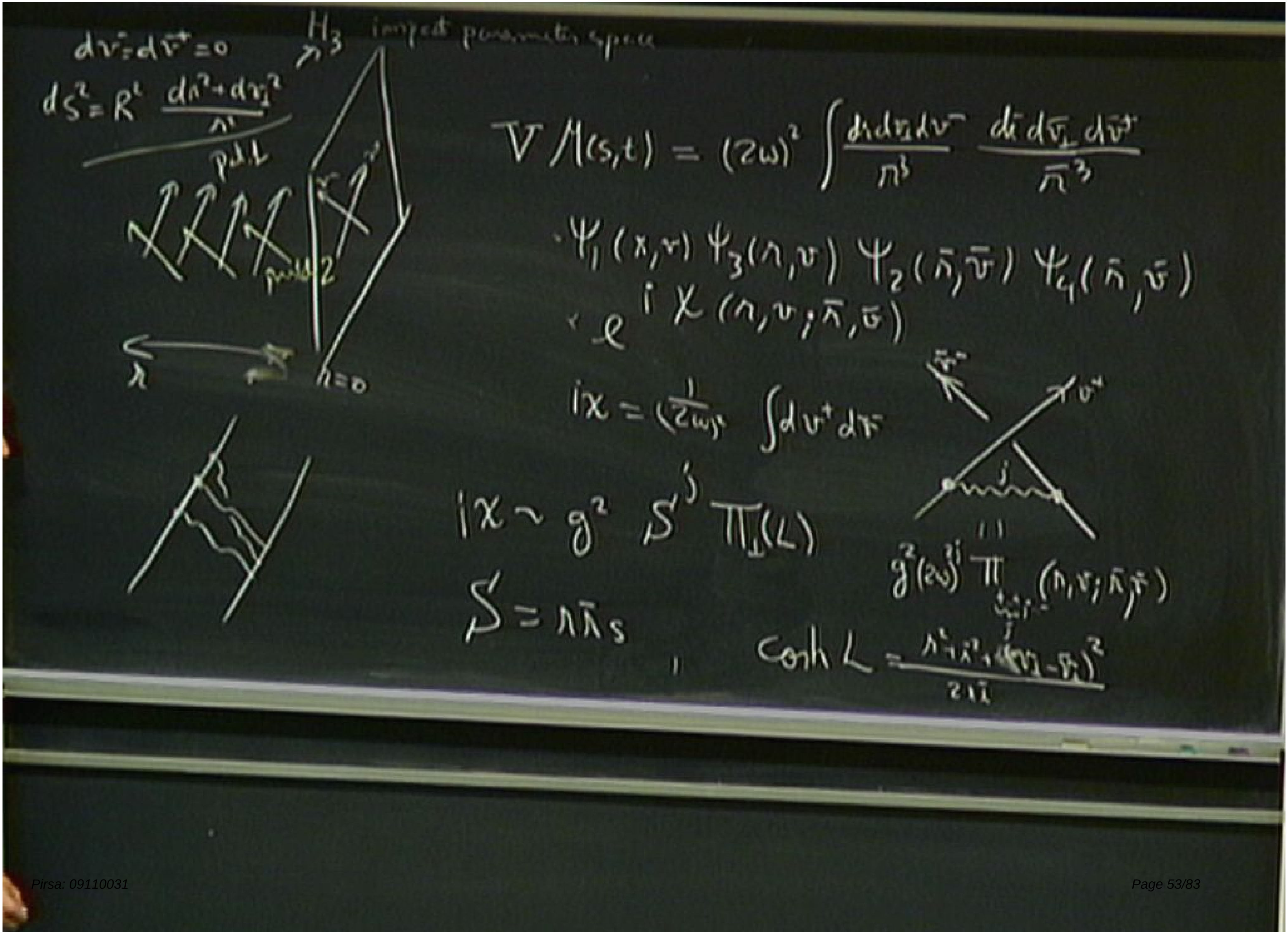
V A(s,t) = (2ω)² ∫ di dr₁ dv⁻ / π³ · di dr̄₁ dv̄⁺ / π̄³
· ψ₁(n,v) ψ₃(n,v) ψ₂(n̄,v̄) ψ₄(n̄,v̄)
· e^{iχ(n,v;n̄,v̄)}
iχ = (1/(2ω)²) ∫ dv⁺ dv⁻
g² Π (n,v;n̄,v̄)
λ
n=0


p.d.L
V A(s,t) = (2ω)² ∫ dλ d²v⊥ dv⁻ / π³ · dλ̄ d²v̄⊥ dv̄⁺ / π̄³
· ψ₁(λ,v) ψ₃(λ,v) ψ₂(λ̄,v̄) ψ₄(λ̄,v̄)
· e^{iχ(λ,v; λ̄,v̄)}
iχ = 1/(2ω)² ∫ dv⁺ dv̄⁻
λ
λ=0
v⁺
g²
π (λ,v; λ̄,v̄)

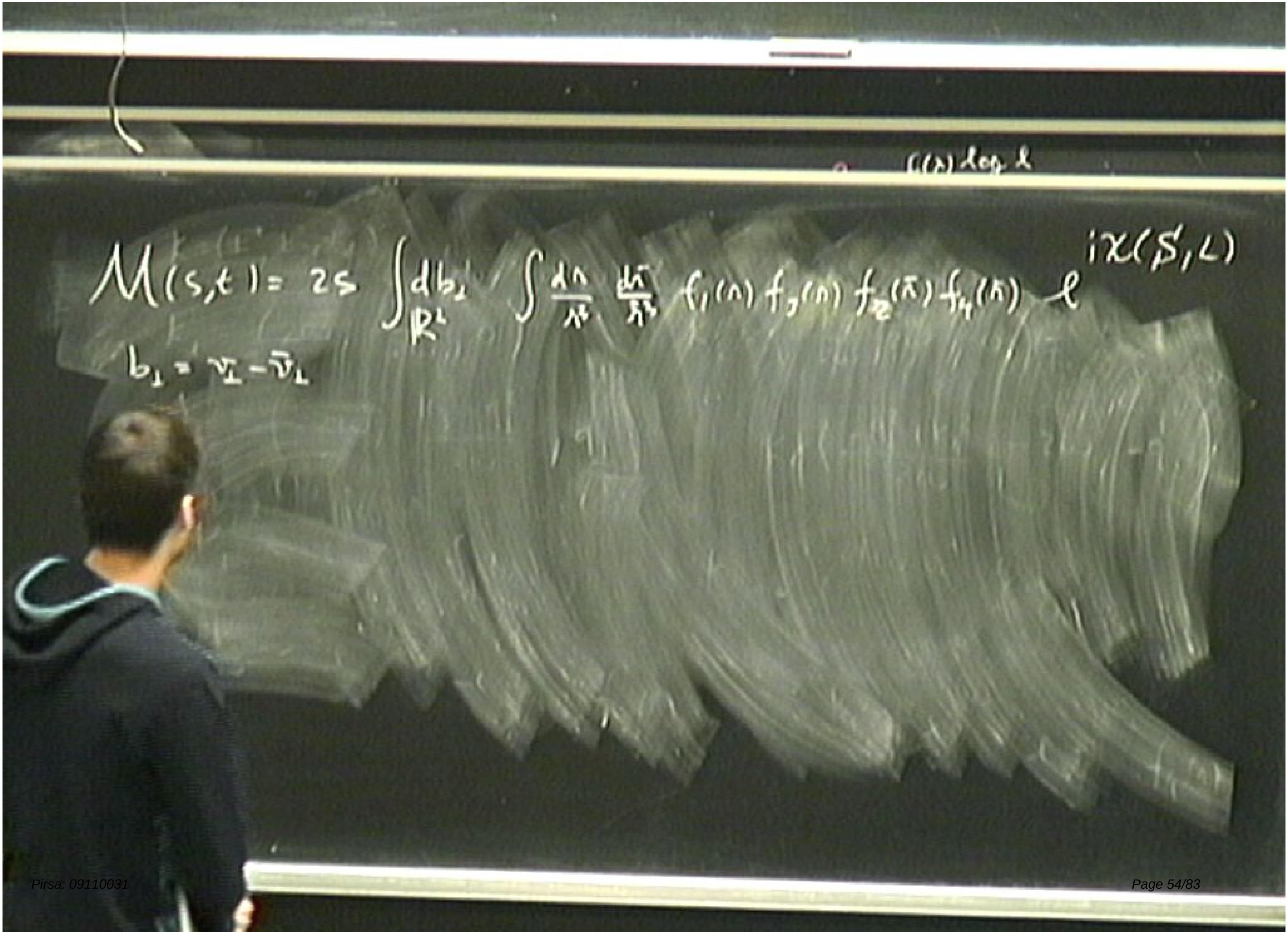

$d v^- d \bar{v}^+ = 0$
p.d.L
λ
$\lambda = 0$
$T\,A(s,t) = (2\omega)^2 \int \frac{d\lambda\, d\vec{v}_\perp\, dv^-}{\pi^3} \frac{d\bar{\lambda}\, d\bar{v}_\perp\, d\bar{v}^+}{\pi^3}$
$\cdot\, \psi_1(\lambda, v)\, \psi_3(\lambda, v)\, \psi_2(\bar{\lambda}, \bar{v})\, \psi_4(\bar{\lambda}, \bar{v})$
$\times\, e^{i \chi(\lambda, v; \bar{\lambda}, \bar{v})}$
$i\chi = \frac{1}{(2\omega)^2} \int dv^+ d\bar{v}^-$
$\bar{v}^-$
v^+
$g^2 \quad \Pi \quad (\lambda, v; \bar{\lambda}, \bar{v})$


$dv^- dv^+ = 0$
$ds^2 = R^2 \frac{dr^2 + dx_\perp^2}{r^2}$
H_3 impact parameter space
$r = 0$
$\mathcal{A}(s,t) = (2\omega)^2 \int \frac{dr\, dx_\perp dv^-}{r^3} \frac{d\bar{r}\, d\bar{x}_\perp d\bar{v}^+}{\bar{r}^3}$
$\cdot \Psi_1(r,v)\, \Psi_3(r,v)\, \Psi_2(\bar{r},\bar{v})\, \Psi_4(\bar{r},\bar{v})$
$\cdot e^{i\chi(r,v;\bar{r},\bar{v})}$
$i\chi = \frac{1}{(2\omega)^2} \int dv^+ dv^-$
$g^2\ \Pi\ (r,v;\bar{r},\bar{v})$


dv⁻ = dv̄⁺ = 0
ds² = R² (dr² + dv⊥²)/r²
H₃ impact parameter space
p.d. 1
p.d. 2
r = 0
V A(s,t) = (2ω)² ∫ (dr dv⊥ dv⁻)/r³ (dr̄ dv̄⊥ dv̄⁺)/r̄³
· ψ₁(r,v) ψ₃(r,v) ψ₂(r̄,v̄) ψ₄(r̄,v̄)
· e^{iχ(r,v;r̄,v̄)}
iχ = 1/(2ω)² ∫ dv⁺ dv̄⁻
iχ ~ g² s^j Π⊥(L)
g²(2ω)^j Π(r,v;r̄,v̄)



dv⁻ = dv̄⁺ = 0
ds² = R² (dr² + dx⊥²)/r²
H₃ impact parameter space
V A(s,t) = (2ω)² ∫ dr dx⊥ dv⁻ / r³ · dr̄ dx̄⊥ dv̄⁺ / r̄³
· ψ₁(x,v) ψ₃(r,v) ψ₂(r̄,v̄) ψ₄(r̄,v̄)
· e^{iχ(r,v; r̄,v̄)}
iχ = 1/(2ω)² ∫ dv⁺ dv̄⁻
iχ ~ g² S^j Π⊥(L)
S = r r̄ s
cosh L = (r² + r̄² + (x⊥ − x̄⊥)²) / (2 r r̄)
g²(2ω)^j Π(r,v; r̄,v̄)
r = 0


M(s,t) = 2s ∫_{R^2} d b_⊥ ∫ dλ/λ^3 dλ̄/λ̄^3 f_1(λ) f_3(λ) f_2(λ̄) f_4(λ̄) e^{iχ(S,L)}
b_⊥ = x_⊥ − x̄_⊥

$f(\lambda)\log\lambda$

$$\mathcal{M}(s,t) = 2s \int_{\mathbb{R}^2} d b_\perp \int \frac{d\lambda}{\lambda^3} \frac{d\bar\lambda}{\bar\lambda^3} f_1(\lambda) f_3(\lambda) f_2(\bar\lambda) f_4(\bar\lambda)\, e^{i\chi(S,L)}$$

$$b_\perp = x_\perp - \bar x_\perp$$

$f(\lambda)\log\lambda$

$$\mathcal{M}(s,t) = 2s \int_{\mathbb{R}^2} d b_\perp \int \frac{d\lambda}{\lambda^3}\frac{d\bar\lambda}{\bar\lambda^3}\, f_1(\lambda)\, f_3(\lambda)\, f_2(\bar\lambda)\, f_4(\bar\lambda)\, e^{i\chi(S,L)}$$

$$b_\perp = x_\perp - \bar{x}_\perp$$

$$i\chi = \frac{4\pi i G_N}{R^3}\, S\, \Pi_\perp(L)$$

$$\rightarrow (\Box_{H_3} - 3)\Pi = -\delta$$

$$A(x,\bar{x}) = N \int dp\, d\bar{p}\; e^{ip\cdot x + i\bar{p}\cdot\bar{x}} \frac{e^{i\chi(\mathcal{S},L)}}{(p^2)^{2-\Delta_1}(\bar{p}^2)^{2-\Delta_2}}$$

Regge limit

$\mathcal{S}\to\infty$

L fixed

$$\mathcal{S}^2 = p^2 \bar{p}^2 , \qquad \cosh L = -\frac{p\cdot\bar{p}}{|p||\bar{p}|}$$

$$(2\pi)^d\, \delta\Big(\sum_{i=1}^{4} k_i\Big)\, \mathcal{M}(s,t) = \int \prod_{i=1}^{4} dy_i\, e^{ik_i\cdot y_i}\, A(y_i)$$

$s = -(k_1+k_2)^2$

$t = -(k_1+k_3)^2$

$$\mathcal{M}(s,t) = 2s \int_{\mathbb{R}^2} d^2 b_\perp\, e^{iq_\perp\cdot b_\perp} \int_0^\infty \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3}\, f_1(r)\, f_3(r)\, f_2(\bar{r})\, f_4(\bar{r})\, e^{i\chi(\mathcal{S},L)}$$

$$q_\perp^2 = -t , \qquad \mathcal{S} = r\bar{r}s , \qquad \cosh L = \frac{r^2+\bar{r}^2+b_\perp^2}{2r\bar{r}}$$

$$A(x,\bar{x}) = N \int_{M^4} dp\, d\bar{p}\; e^{ip\cdot x + i\bar{p}\cdot\bar{x}} \frac{e^{i\chi(S,L)}}{(p^2)^{2-\Delta_1}(\bar{p}^2)^{2-\Delta_2}}$$

Regge limit
$S \to \infty$
L fixed

$$S^2 = p^2 \bar{p}^2 \,, \qquad \cosh L = -\frac{p\cdot\bar{p}}{|p||\bar{p}|}$$

$$(2\pi)^d\, \delta\Big(\sum_{i=1}^{4} k_i\Big)\, \mathcal{M}(s,t) = \int \prod_{i=1}^{4} dy_i\, e^{ik_i\cdot y_i} A(y_i)$$

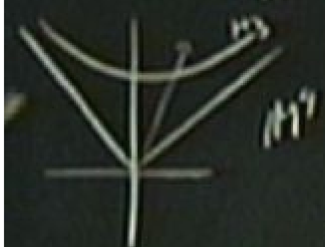
$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$

$$\mathcal{M}(s,t) = 2s \int_{\mathbb{R}^2} d^2 b_\perp\, e^{iq_\perp\cdot b_\perp} \int_0^\infty \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3}\, f_1(r) f_3(r) f_2(\bar{r}) f_4(\bar{r})\, e^{i\chi(S,L)}$$

$$q_\perp^2 = -t \,, \qquad S = r\bar{r}s \,, \qquad \cosh L = \frac{r^2 + \bar{r}^2 + b_\perp^2}{2r\bar{r}}$$

$$A(x,\bar{x}) = N \int_{M^4} dp\, d\bar{p}\; e^{ip\cdot x + i\bar{p}\cdot\bar{x}} \frac{e^{i\chi(S,L)}}{(p^2)^{2-\Delta_1}(\bar{p}^2)^{2-\Delta_2}}$$

Reggo limit
$S \to \infty$
L fixed



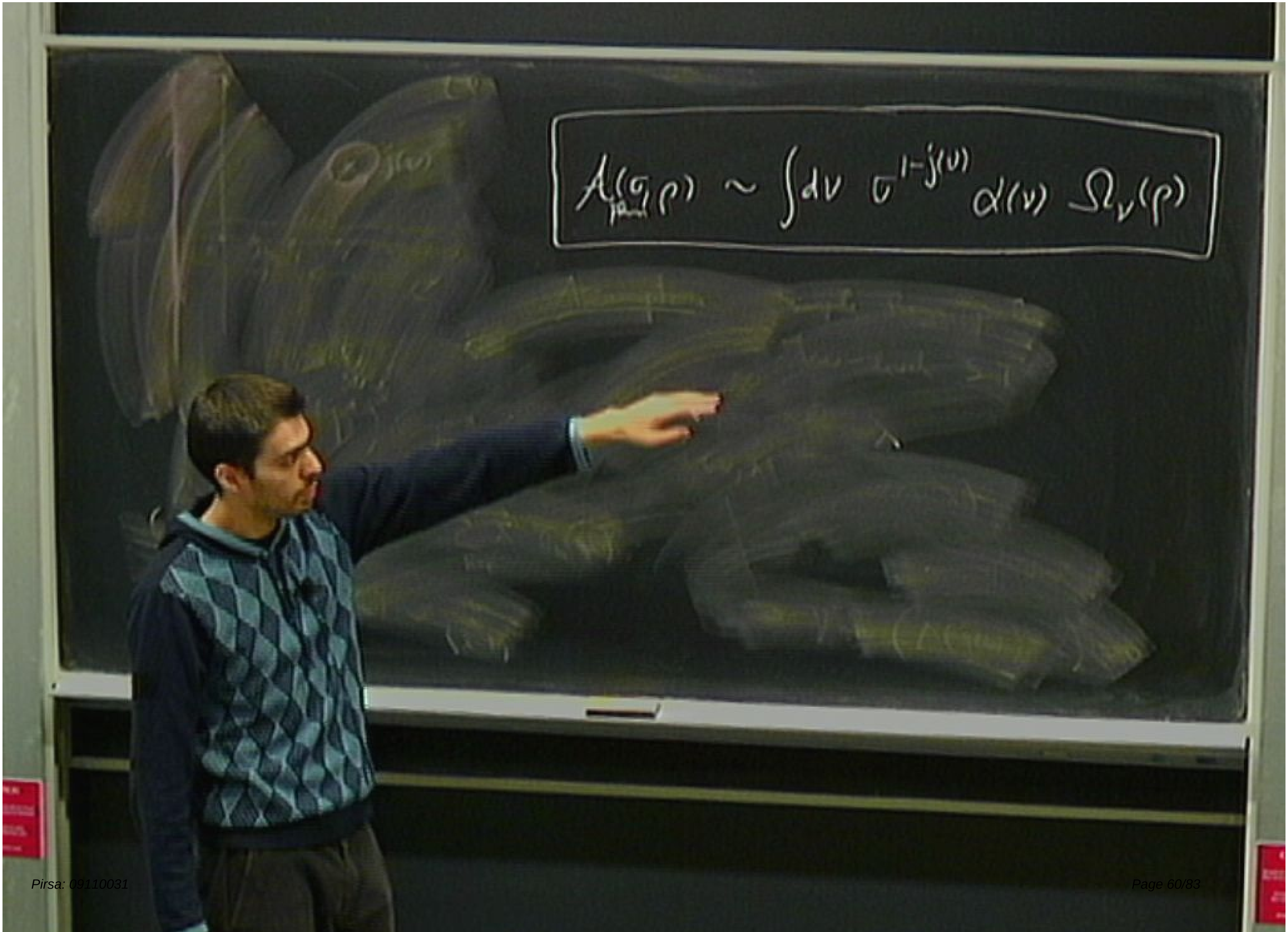
$$S^2 = p^2 \bar{p}^2 \,, \qquad \cosh L = -\frac{p\cdot\bar{p}}{|p||\bar{p}|}$$

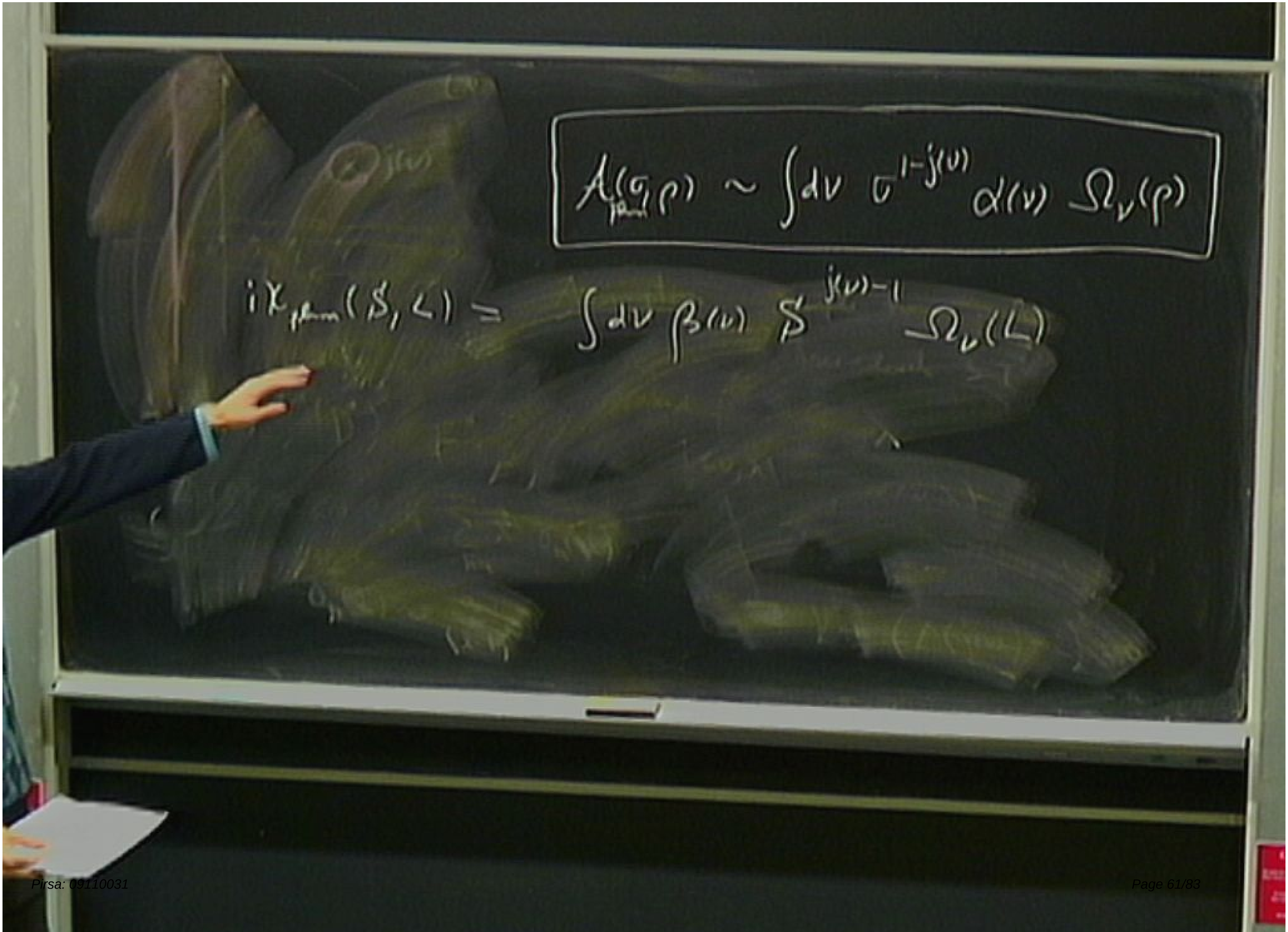
$$(2\pi)^d\, \delta\Big(\sum_{i=1}^{4} k_i\Big)\, \mathcal{M}(s,t) = \int \prod_{i=1}^{4} dy_i\, e^{ik_i\cdot y_i}\, A(y_i)$$

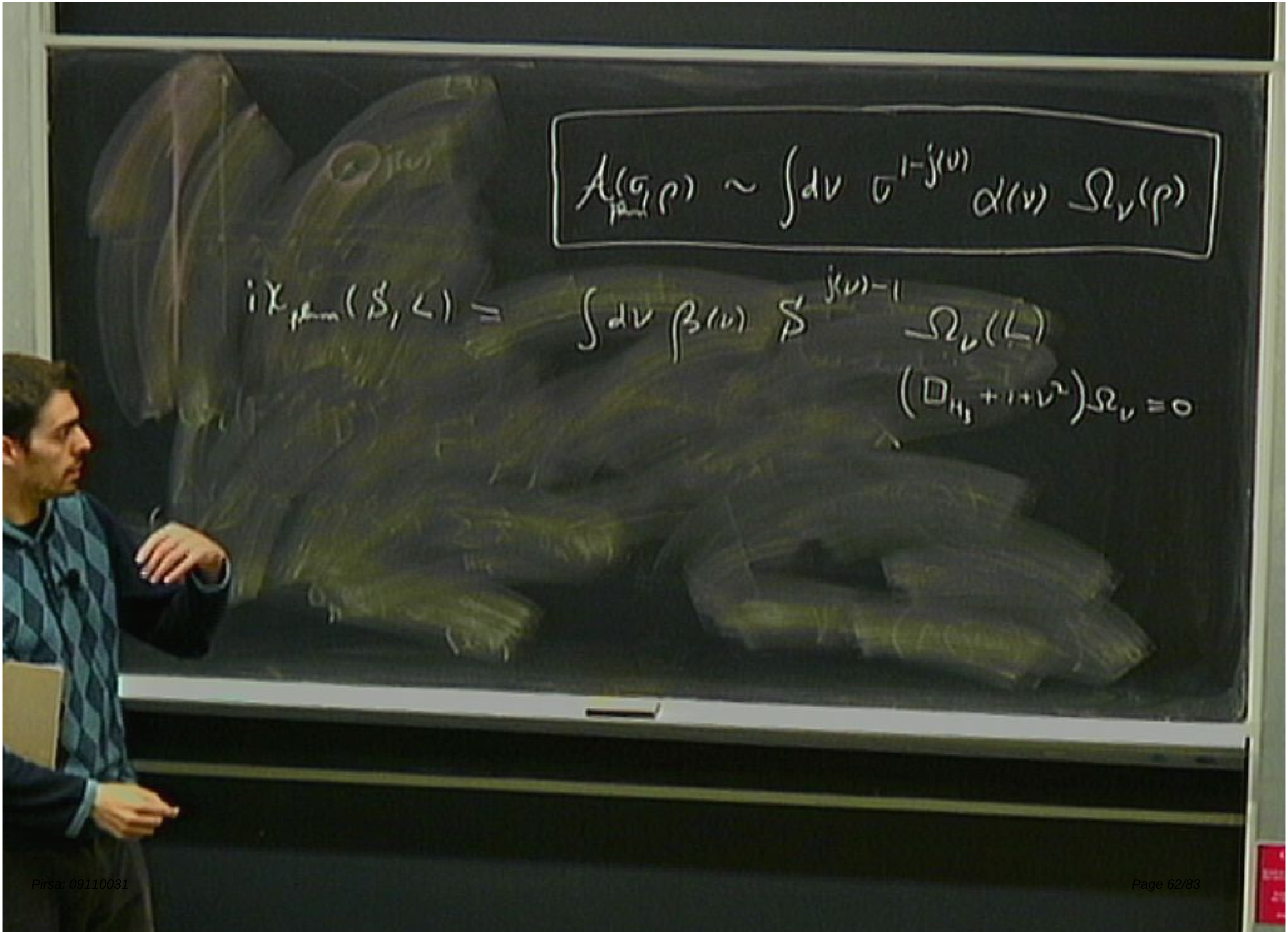
$s = -(k_1+k_2)^2$
$t = -(k_1+k_3)^2$

$$\mathcal{M}(s,t) = 2s \int_{\mathbb{R}^2} d^2b_\perp\, e^{iq_\perp\cdot b_\perp} \overbrace{\int_0^\infty \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3}\, f_1(r) f_3(r) f_2(\bar{r}) f_4(\bar{r})\, e^{i\chi(S,L)}}^{e^{2i\delta(s,b_\perp)}}$$

$$q_\perp^2 = -t \,, \qquad S = r\bar{r}s \,, \qquad \cosh L = \frac{r^2+\bar{r}^2+b_\perp^2}{2r\bar{r}}$$


$A(\sigma, \rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \alpha(\nu) \, \Omega_\nu(\rho)$


$\mathcal{A}_{pom}(s,\rho) \sim \int d\nu \; s^{1-j(\nu)} \alpha(\nu) \; \Omega_\nu(\rho)$
$i\chi_{pom}(S,L) = \int d\nu \; \beta(\nu) \; S^{j(\nu)-1} \; \Omega_\nu(L)$


$\mathcal{A}_{pom}(\sigma,\rho) \sim \int d\nu\, \sigma^{1-j(\nu)}\, \alpha(\nu)\, \Omega_\nu(\rho)$
$i\chi_{pom}(S,L) = \int d\nu\, \beta(\nu)\, S^{j(\nu)-1}\, \Omega_\nu(L)$
$(\Box_{H_3}+1+\nu^2)\Omega_\nu = 0$

$$\mathcal{A}_{pom}(s,\rho) \sim \int d\nu \; s^{1-j(\nu)} \, \alpha(\nu) \; \Omega_\nu(\rho)$$

$$i\chi_{pom}(S,L) = \int d\nu \; \beta(\nu) \; S^{j(\nu)-1} \; \Omega_\nu(L)$$

$$\left(\Box_{H_3} + 1 + \nu^2\right)\Omega_\nu = 0$$

$$A_{\text{pom}}(\sigma, \rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \alpha(\nu) \, \Omega_\nu(\rho)$$

large λ: $i\chi_{\text{pom}}(S, L) = \int d\nu \; \beta(\nu) \, S^{j(\nu)-1} \, \Omega_\nu(L)$

$j(\nu) = 2$

$$(\Box_{H_3} + 1 + \nu^2)\Omega_\nu = 0$$

$$\mathcal{A}_{pom}(\sigma,\rho) \sim \int d\nu \, \sigma^{1-j(\nu)} \, \alpha(\nu) \, \Omega_\nu(\rho)$$

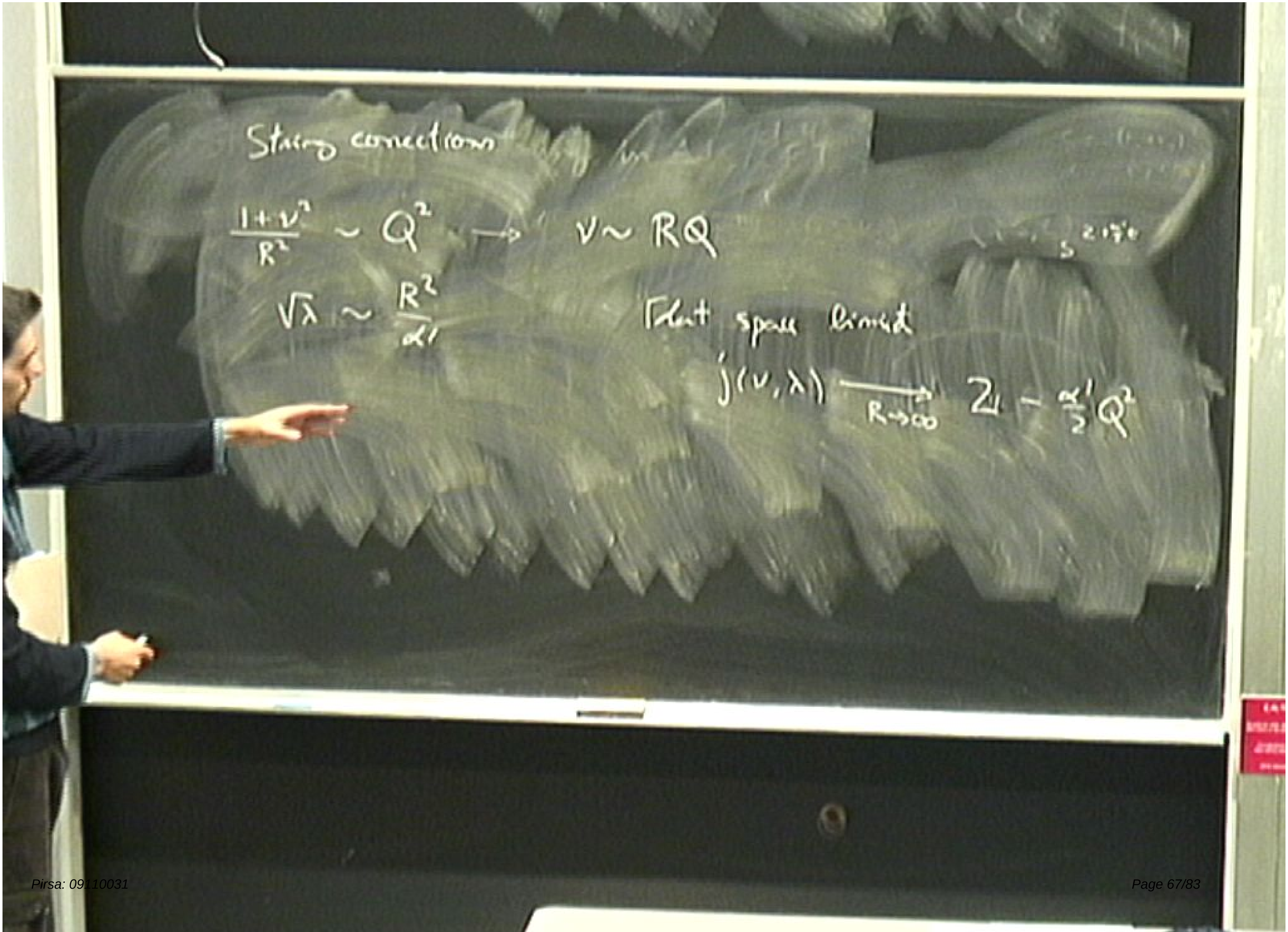
large λ:

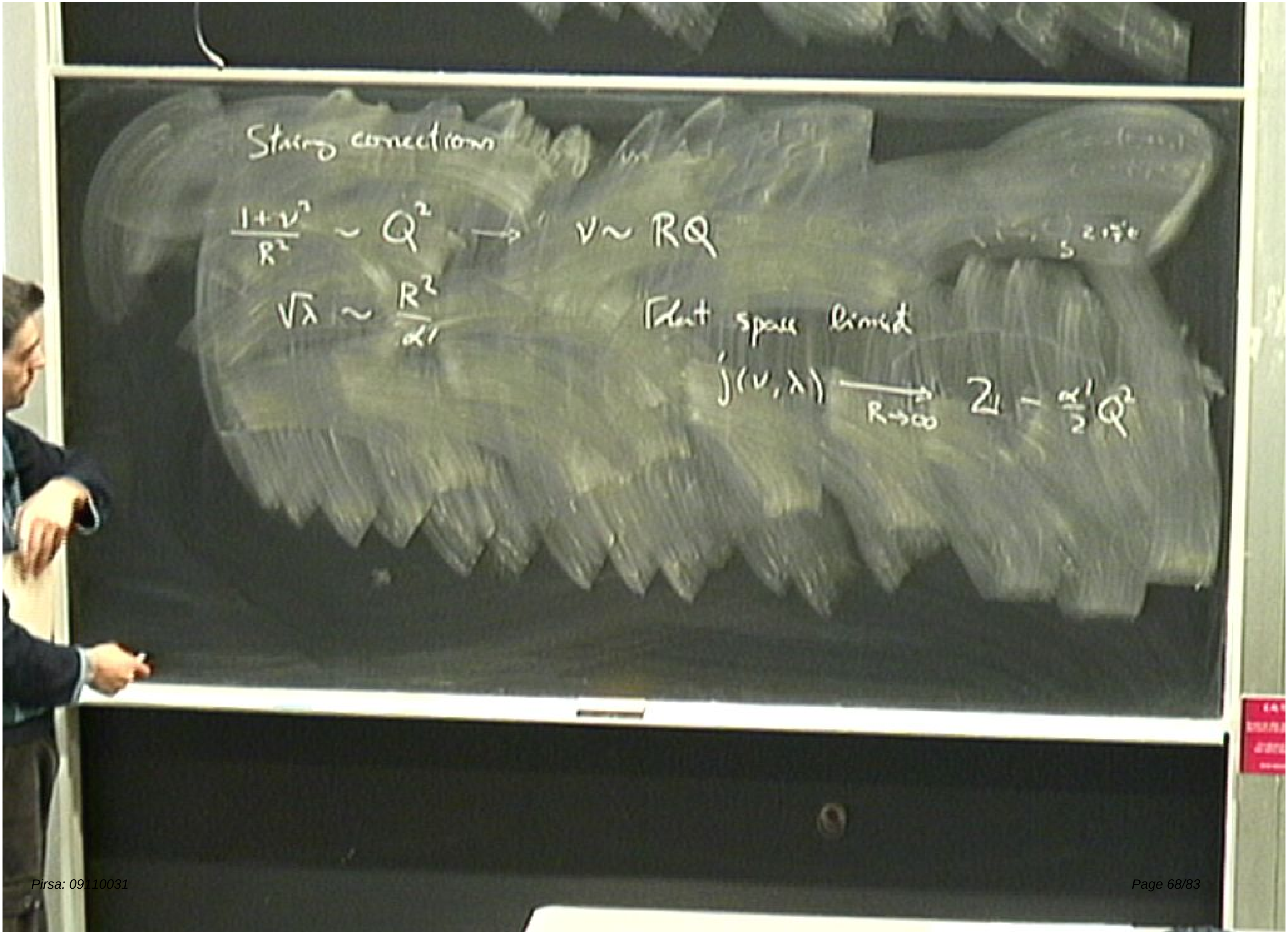
$$i\chi_{pom}(S, L) = \int d\nu \, \beta(\nu) \, S^{j(\nu)-1} \, \Omega_\nu(L)$$

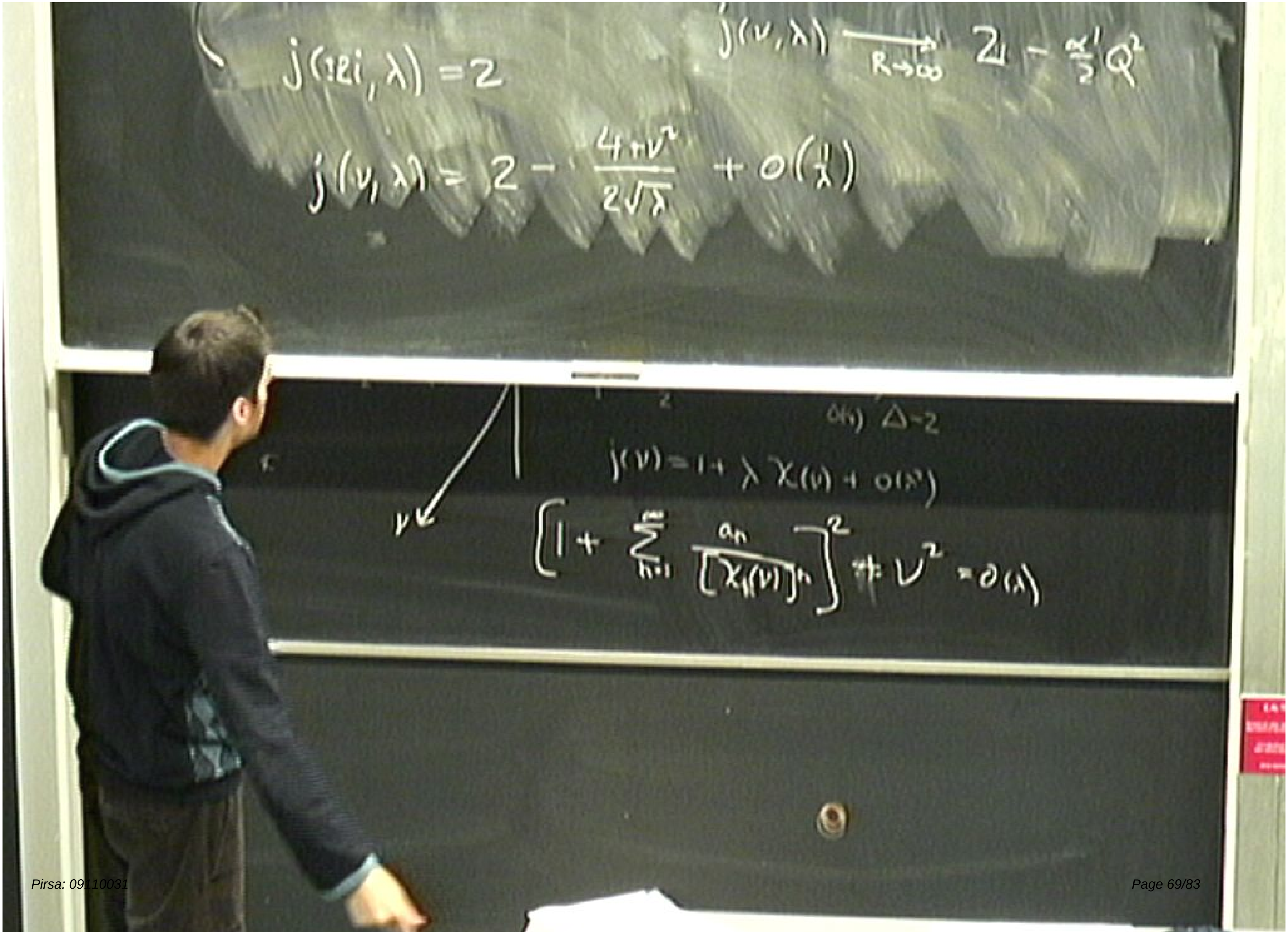
$$j(\nu) = 2$$

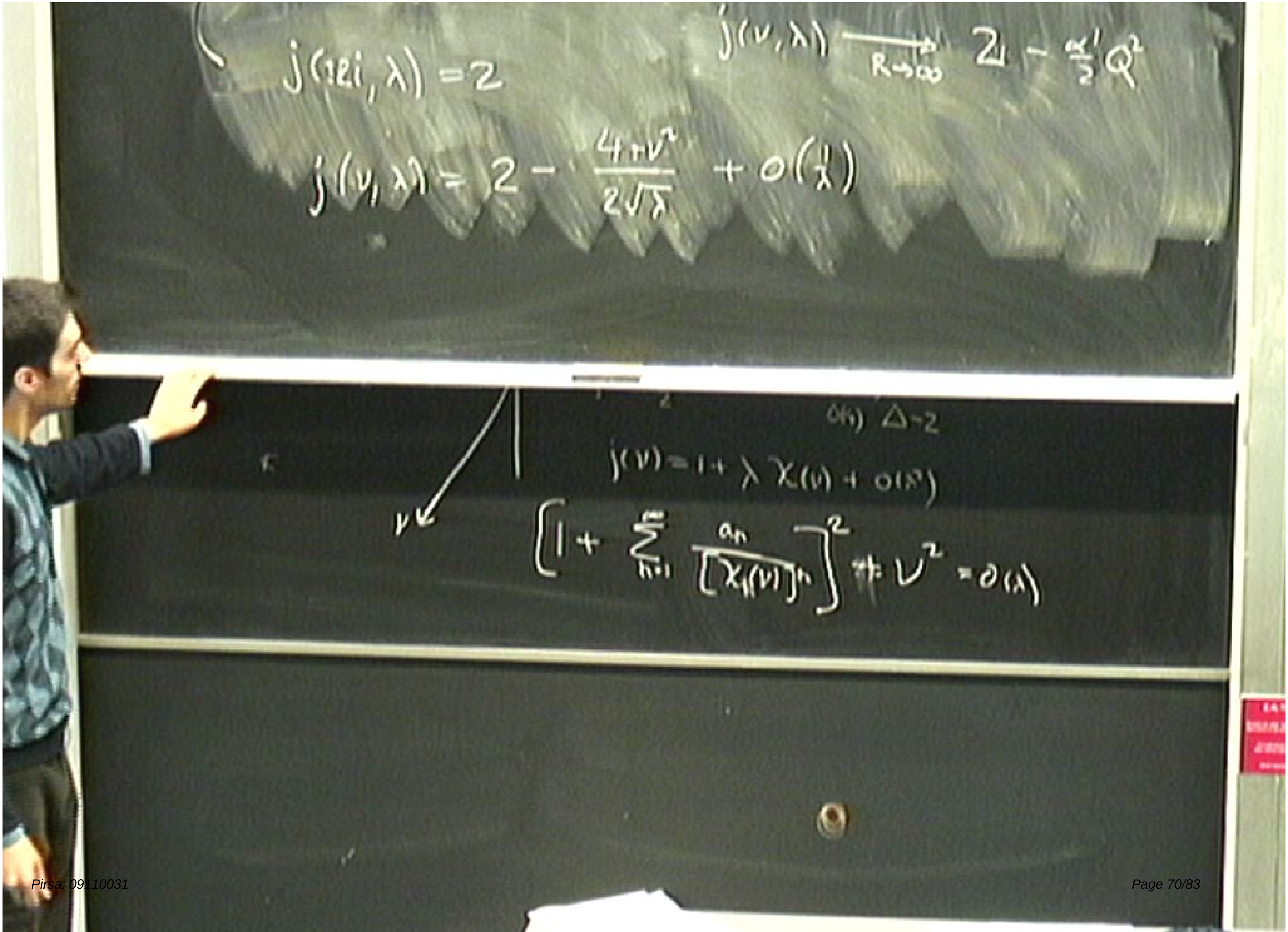
$$\left(\Box_{H_3} + 1 + \nu^2\right)\Omega_\nu = 0$$

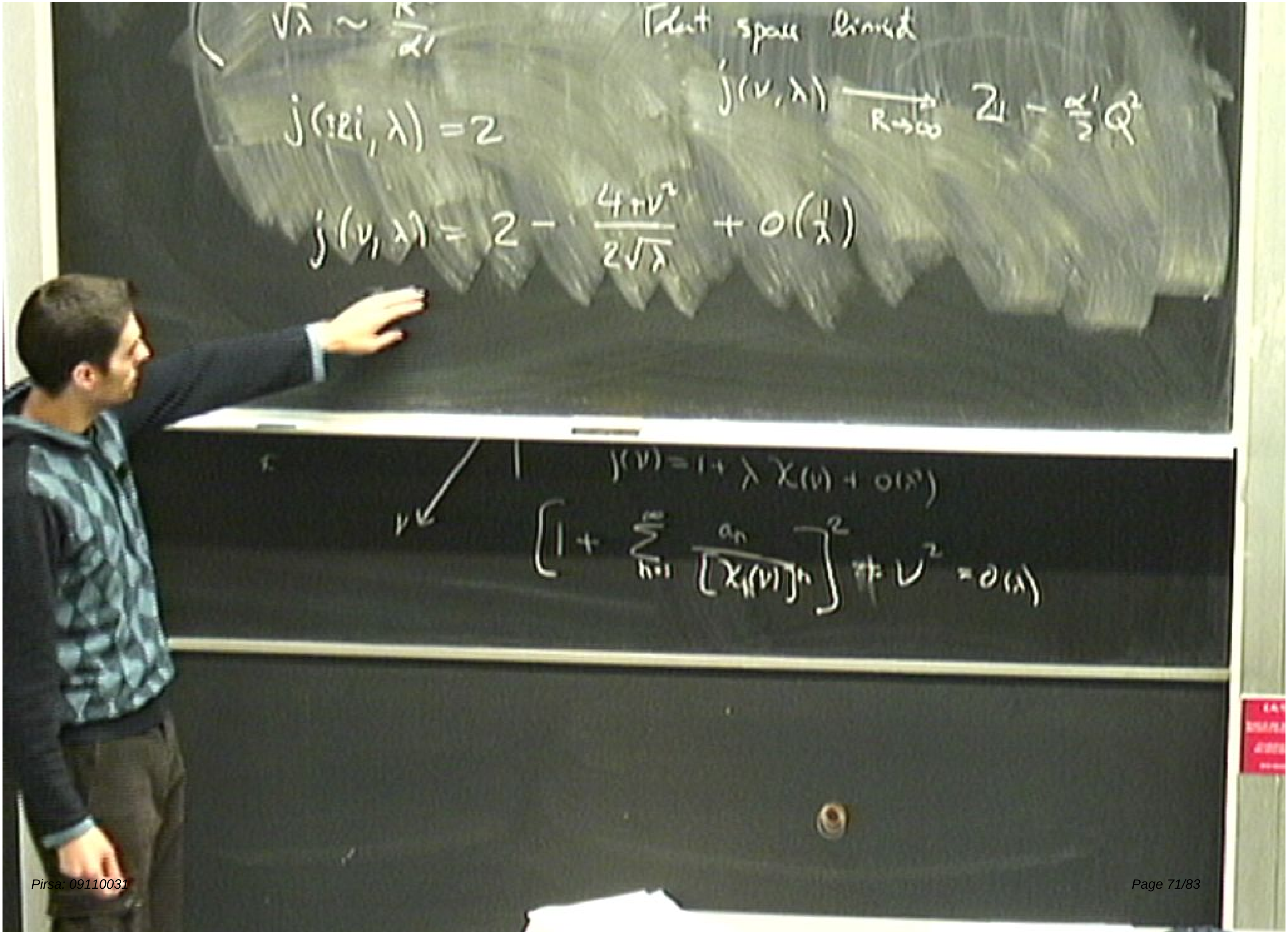

String connections

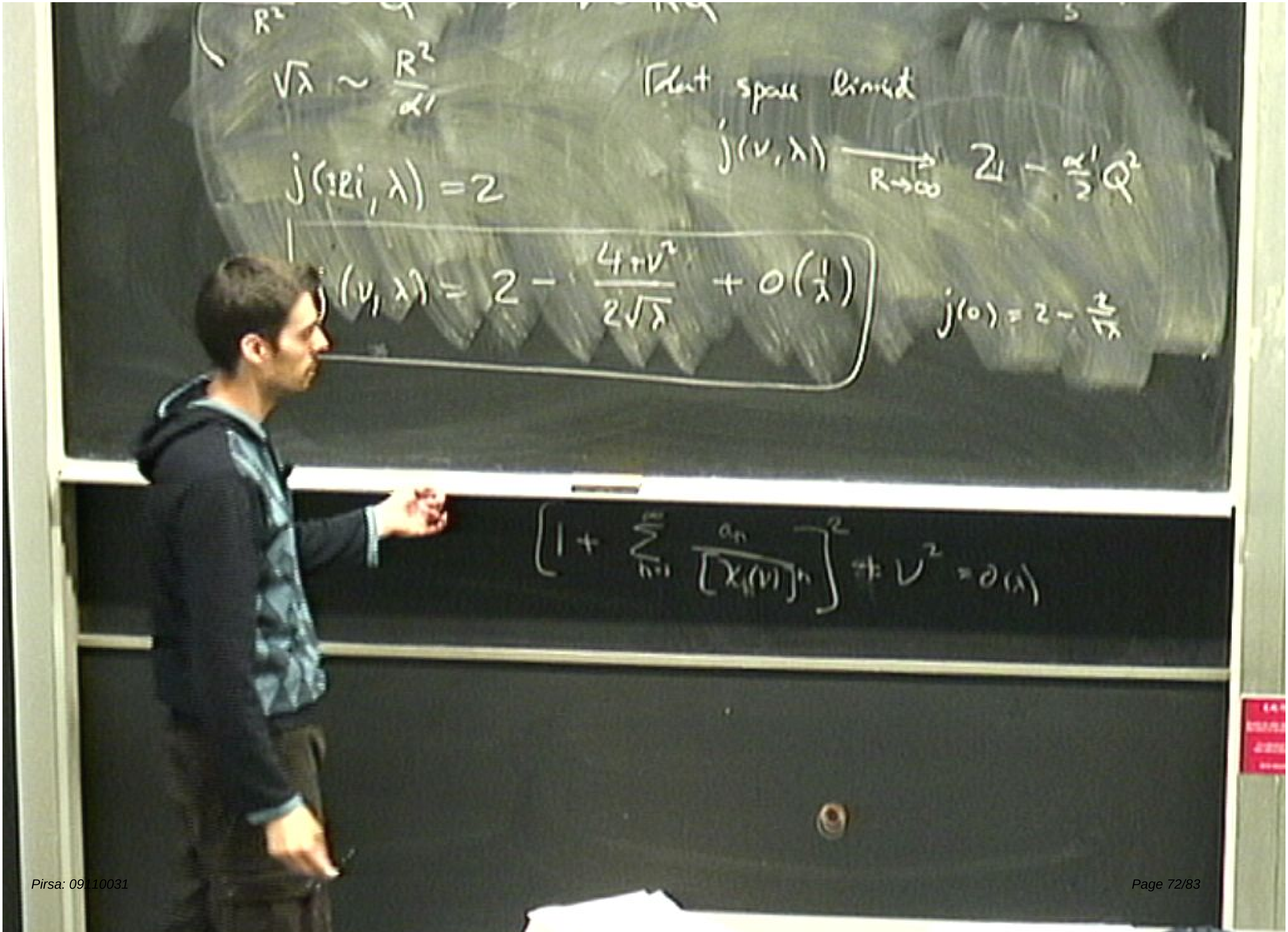

String corrections
$\frac{1+\nu^2}{R^2} \sim Q^2 \longrightarrow \nu \sim RQ$
$\sqrt{\lambda} \sim \frac{R^2}{\alpha'}$
Flat space limit
$j(\nu,\lambda) \xrightarrow[R\to\infty]{} 2 - \frac{\alpha'}{2}Q^2$


String corrections
(1+ν²)/R² ~ Q² → ν ~ RQ
√λ ~ R²/α'
Flat space limit
j(ν, λ) → R→∞ 2 − α'/2 Q²

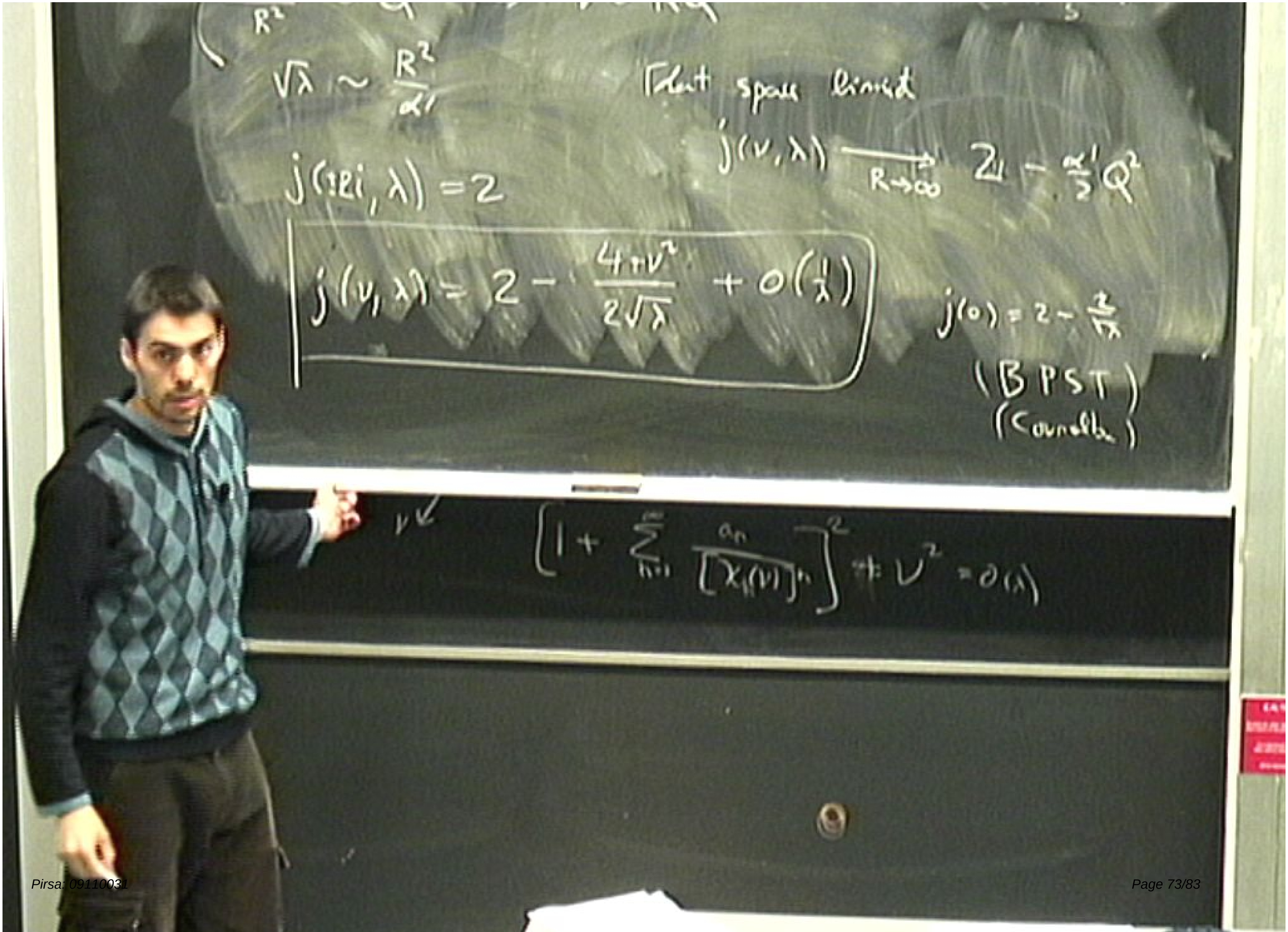

j(±2i, λ) = 2
j(ν, λ) → R→∞ 2 − α'/2 Q²
j(ν, λ) = 2 − (4+ν²)/(2√λ) + O(1/λ)
Δ−2
j(ν) = 1 + λ χ(ν) + O(λ²)
[1 + Σ a_n/[χ(ν)]^n]² + ν² = O(λ)


j(±2i, λ) = 2
j(ν, λ) → R→∞ 2 − α′/2 Q²
j(ν, λ) = 2 − (4 + ν²)/(2√λ) + O(1/λ)
j(ν) = 1 + λ χ(ν) + O(λ²)


Flat space limit
j(ν, λ) → 2 − α'/2 Q²
R→∞
j(±2i, λ) = 2
j(ν, λ) = 2 − (4+ν²)/(2√λ) + O(1/λ)
j(ν) = 1 + λ χ(ν) + O(λ²)



√λ ~ R²/α'
Flat space limit
j(ν, λ) → 2 − α'/2 Q²
R→∞
j(±2i, λ) = 2
j(ν, λ) = 2 − (4+ν²)/(2√λ) + O(1/λ)


√λ ~ R²/α'
Flat space limit
j(ν, λ) → 2 − (α'/2) Q²
R→∞
j(±2i, λ) = 2
j(ν, λ) = 2 − (4+ν²)/(2√λ) + O(1/λ)
j(0) = 2 − 2/√λ
(BPST)
(Cornalba)

$$\mathcal{A}_{pom}(\sigma, \rho) \sim \int d\nu \; \sigma^{1-j(\nu)} \, \alpha(\nu) \; \Omega_\nu(\rho)$$

large λ:

$$i\chi_{pom}(S, L) = \int d\nu \; \beta(\nu) \; S^{j(\nu)-1} \; \Omega_\nu(L)$$

$$(\Box_{H_3} + 1 + \nu^2)\Omega_\nu = 0$$

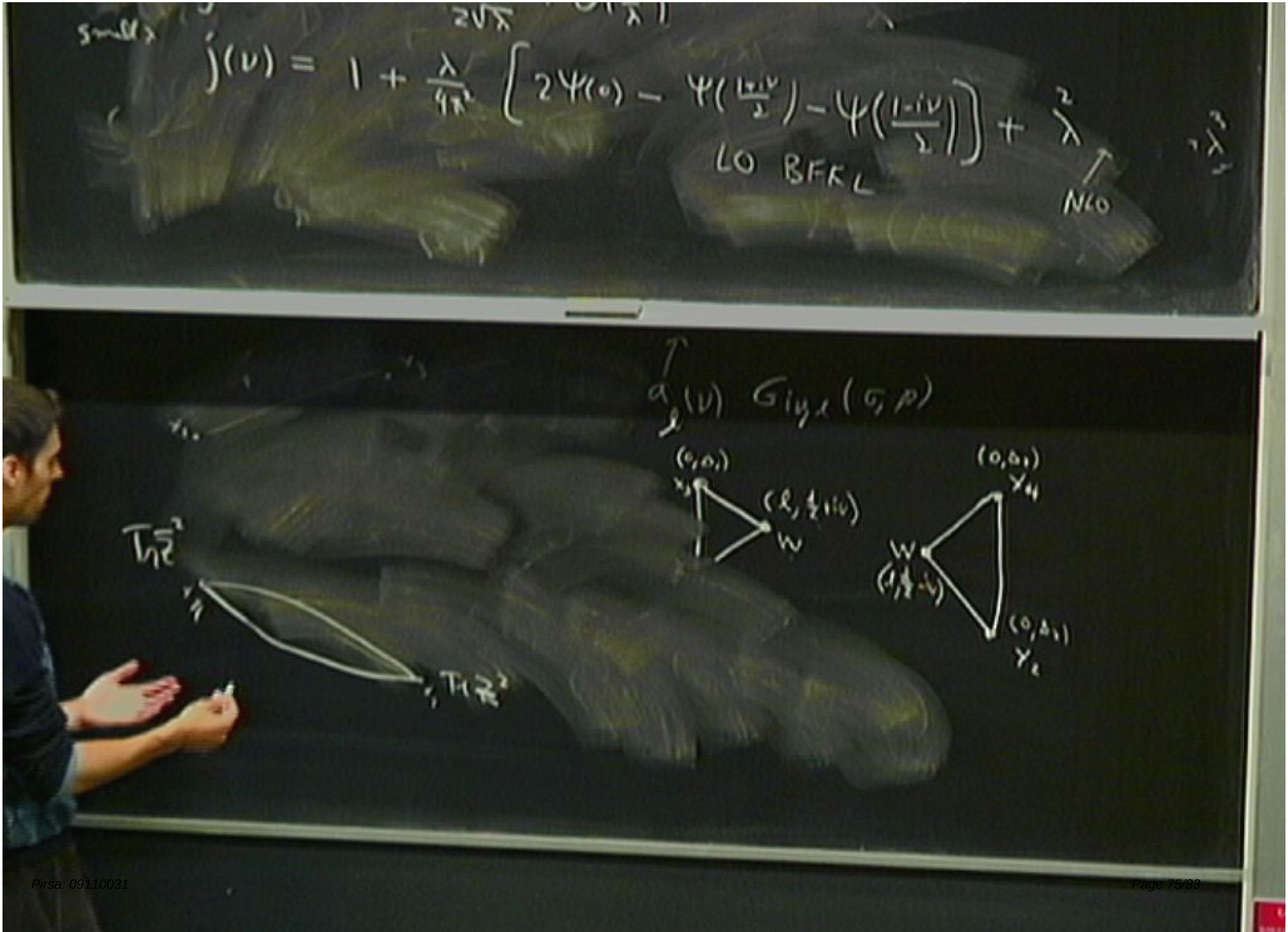
$$j(\nu) = 2 - \frac{4+\nu^2}{2\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)$$

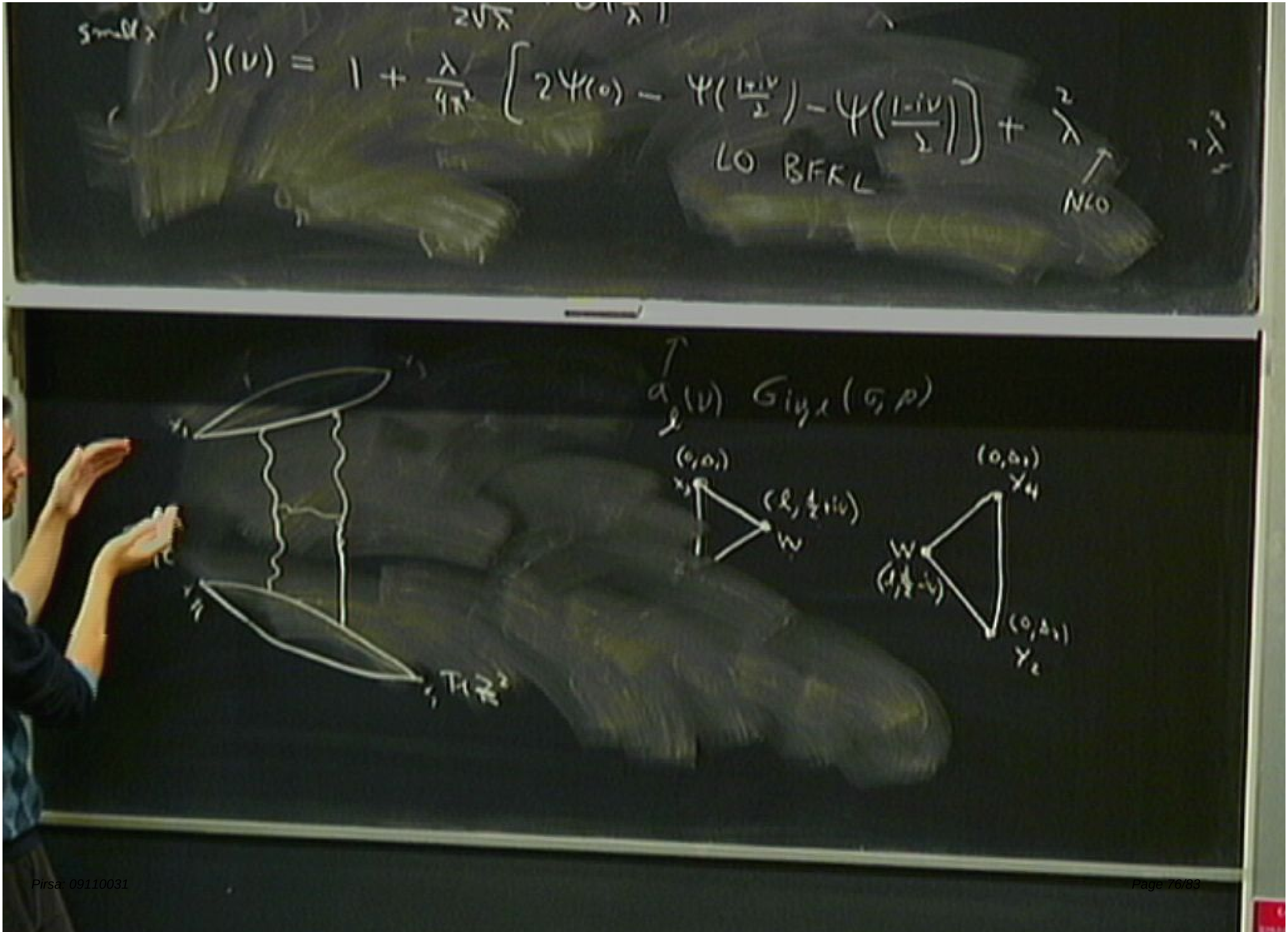
small λ:

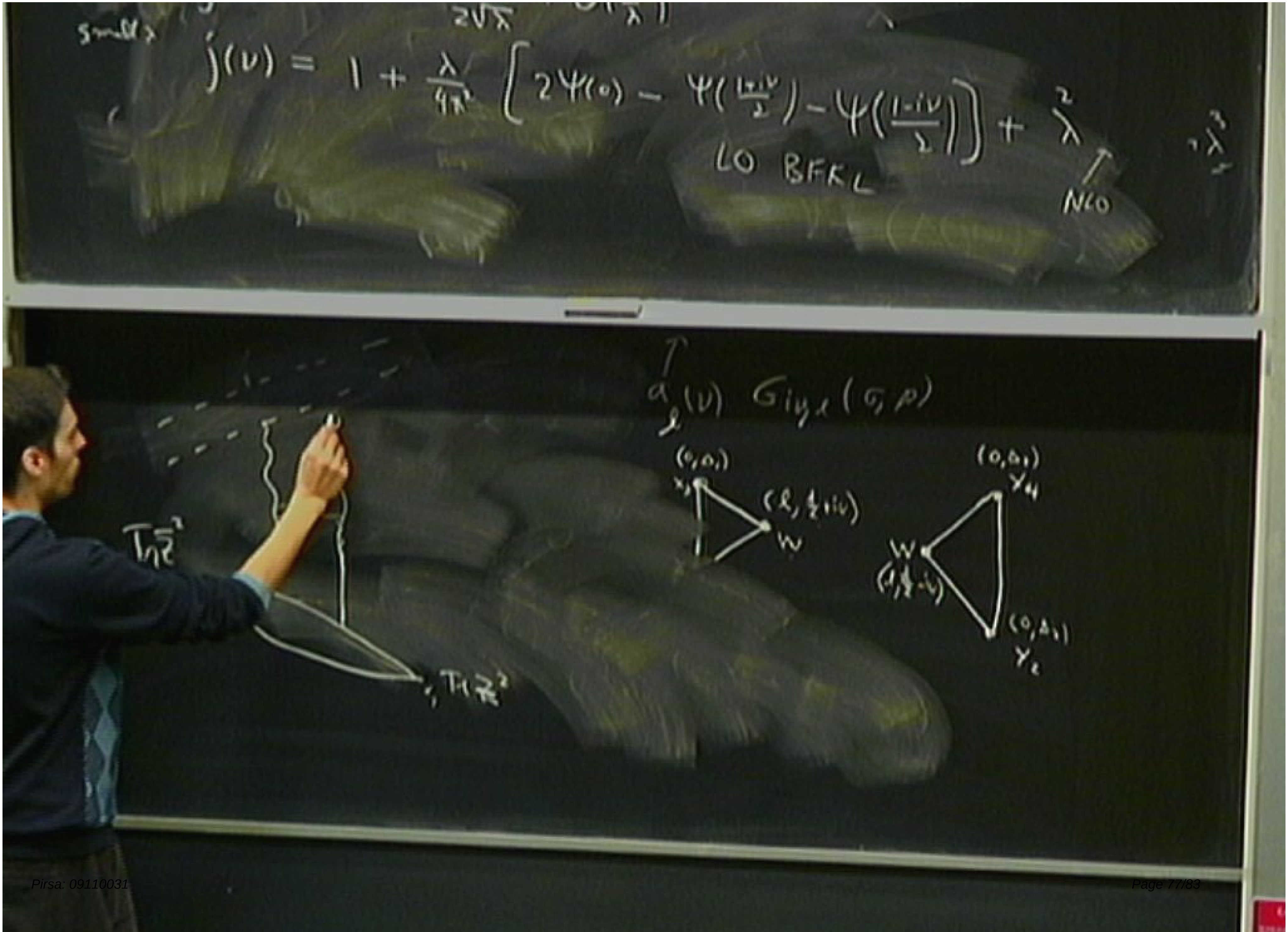
$$j(\nu) = 1 + \frac{\lambda}{4\pi^2}\left[2\psi(1) - \psi\left(\frac{1+i\nu}{2}\right) - \psi\left(\frac{1-i\nu}{2}\right)\right] + \lambda^2 + \lambda^3$$

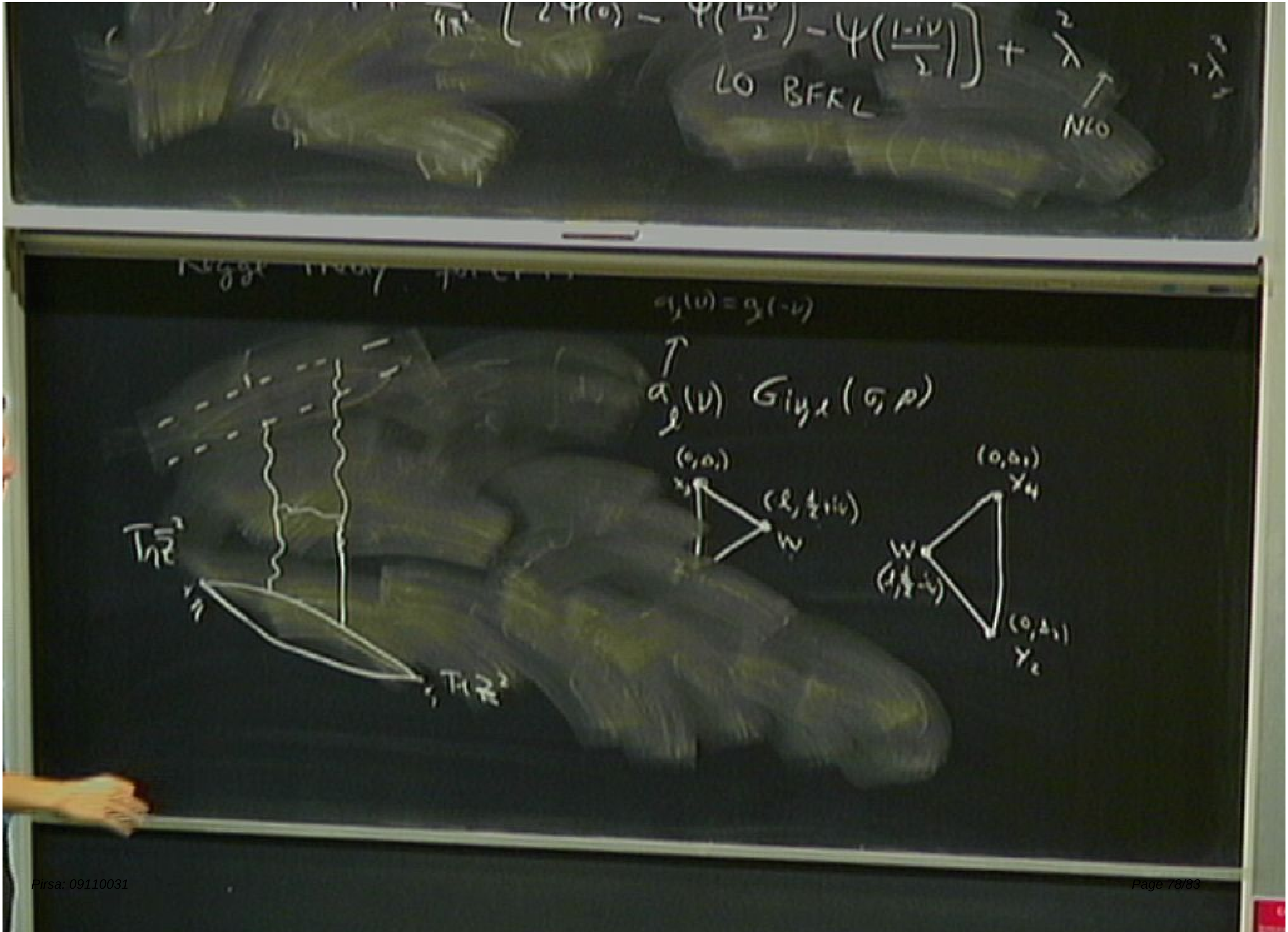
LO BFKL

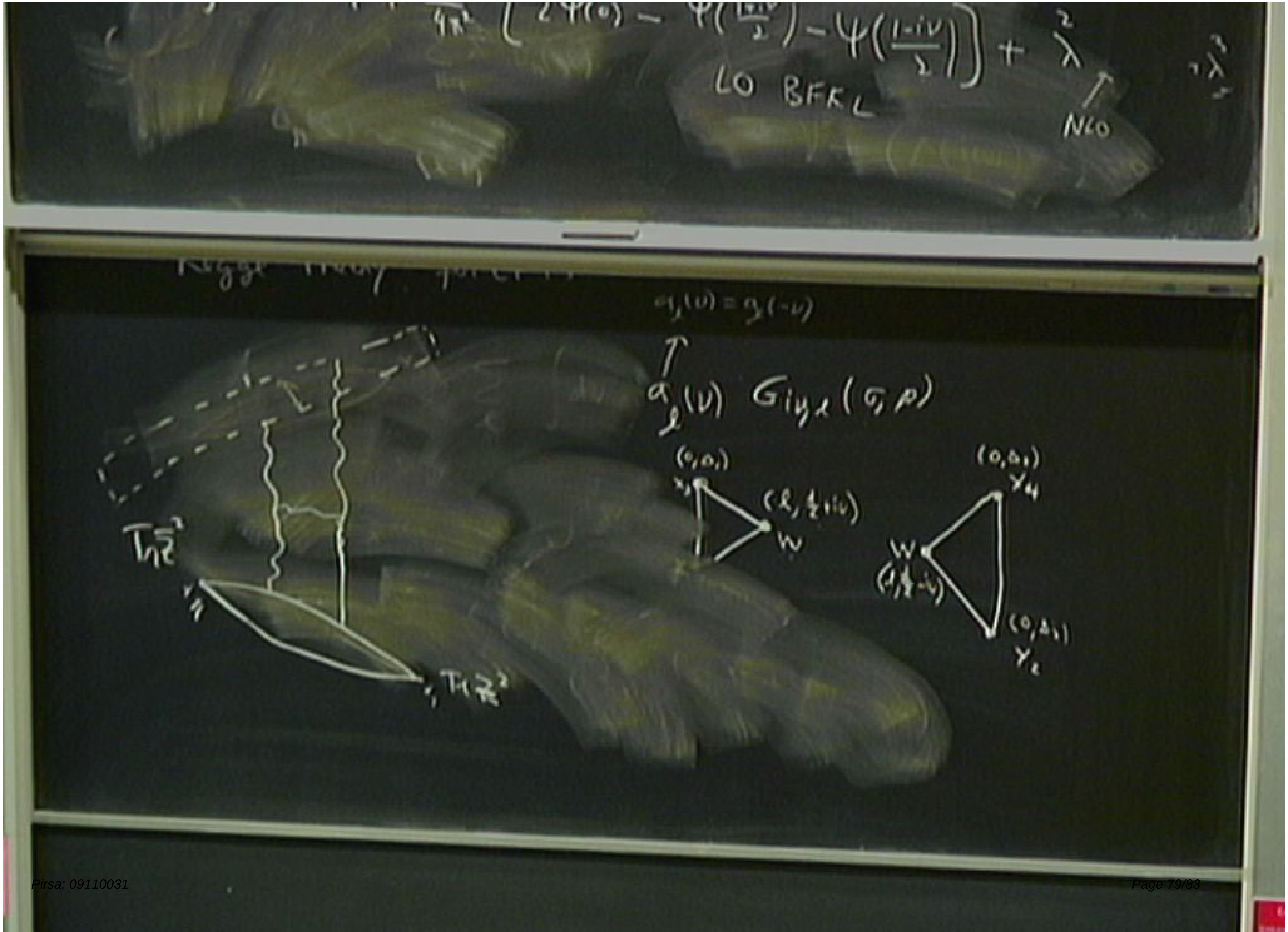
NLO

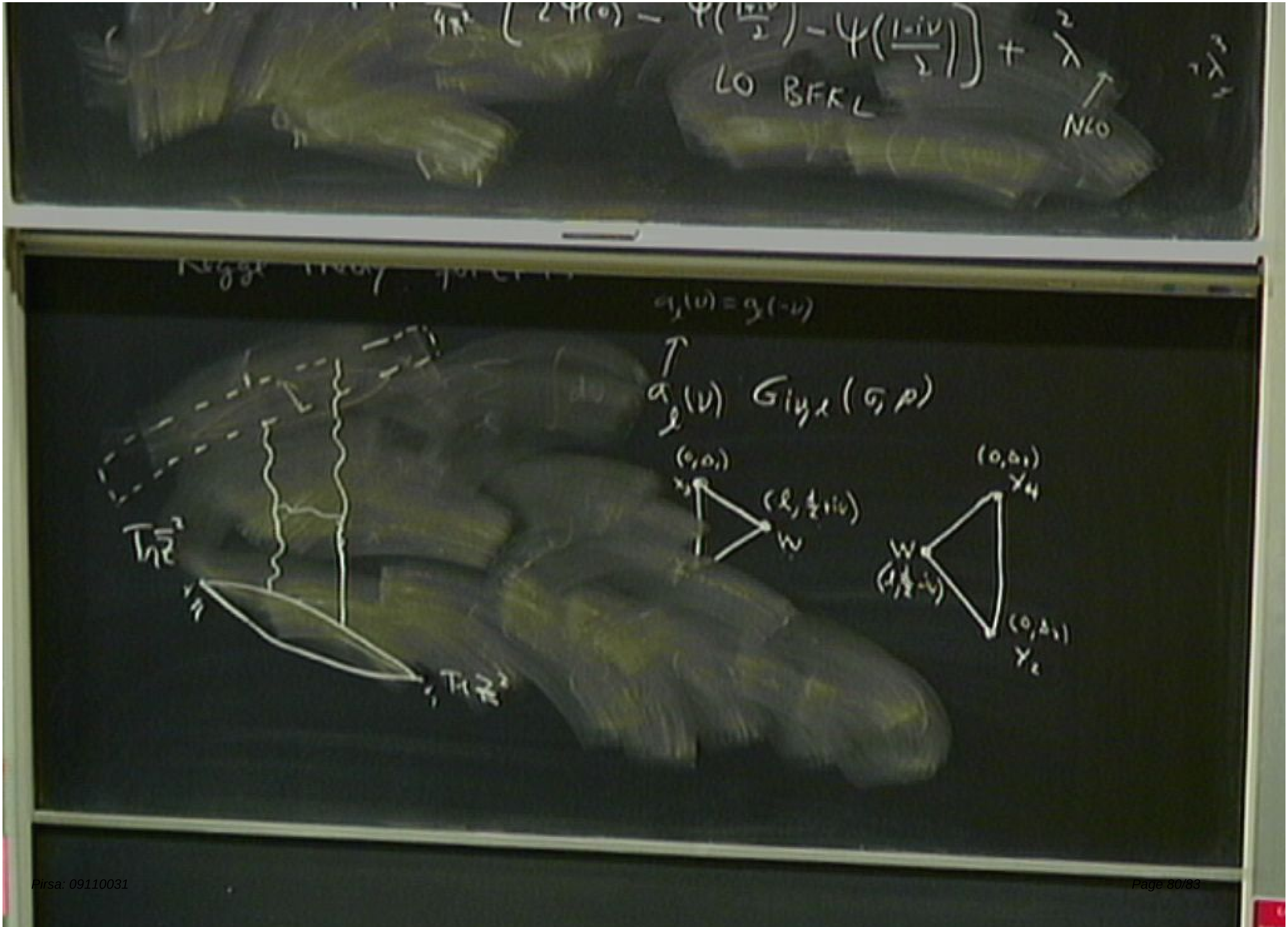

small λ
LO BFKL
NLO
(0,Δ1)
W
(0,Δ2)
X4
X2


small λ
j(ν) = 1 + λ/4π² [2ψ(1) − ψ((1+iν)/2) − ψ((1−iν)/2)] + λ² + λ³
LO BFKL
NLO
(0,Δ₁)
(ℓ, d/2+iν)
W
(0,Δ₂)
(d, d/2−iν)
(0,Δ₁)

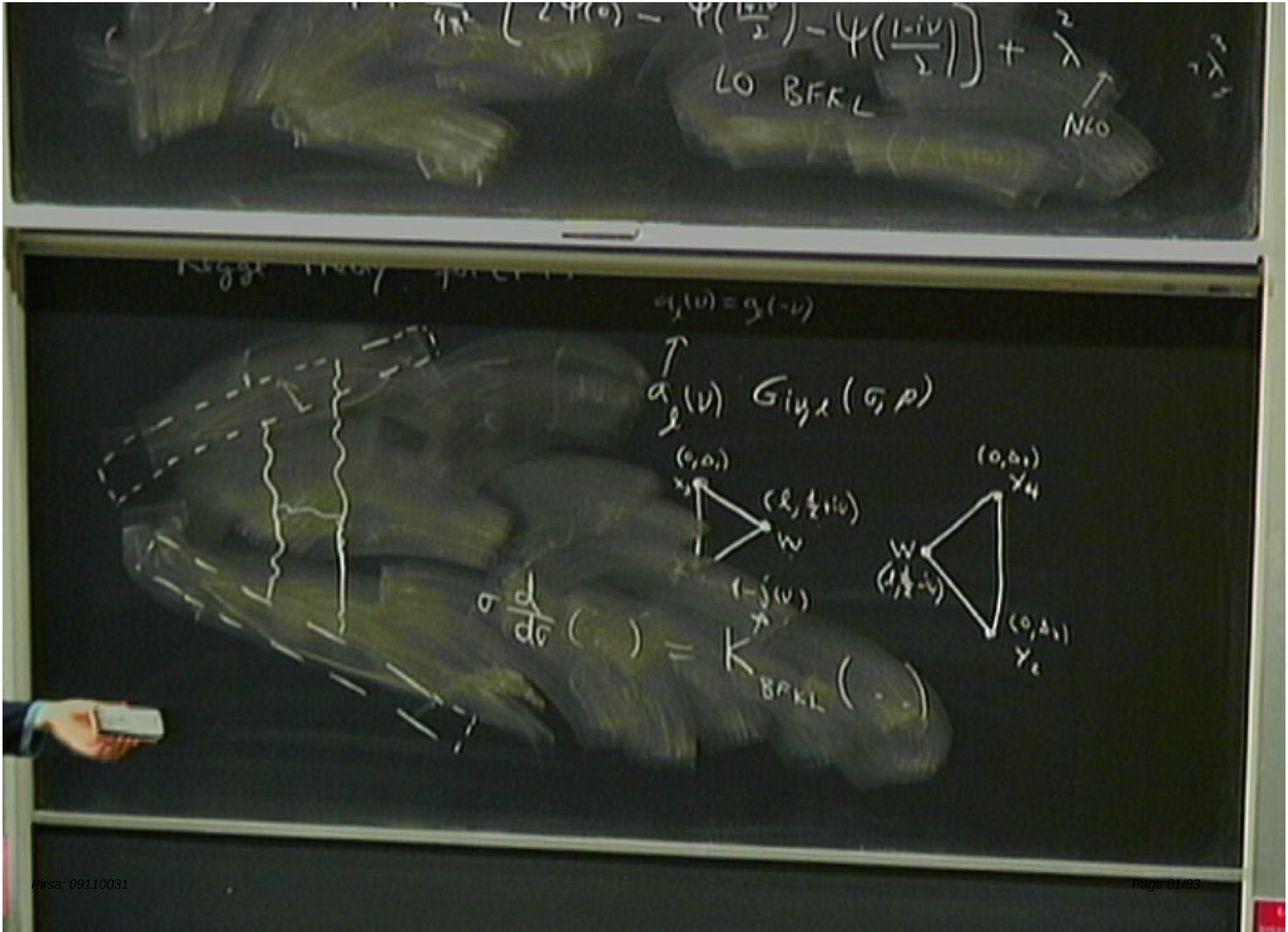

small λ
j(ν) = 1 + λ/4π² [2ψ(0) − ψ((1+iν)/2) − ψ((1−iν)/2)] + λ² + λ³
LO BFKL
NLO
W
(0,Δ₁)
(0,Δ₂)

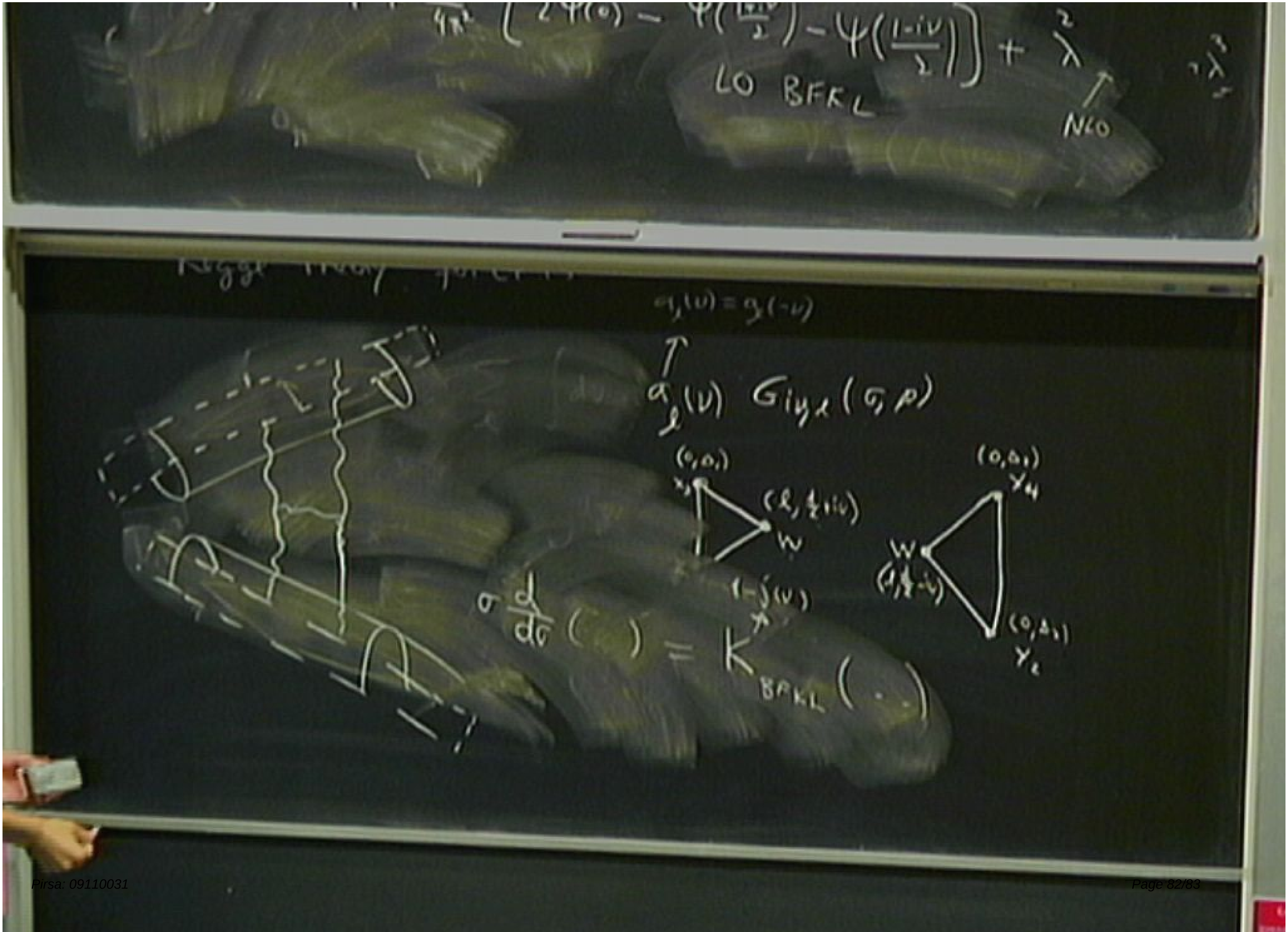

LO BFKL
NLO
W
W

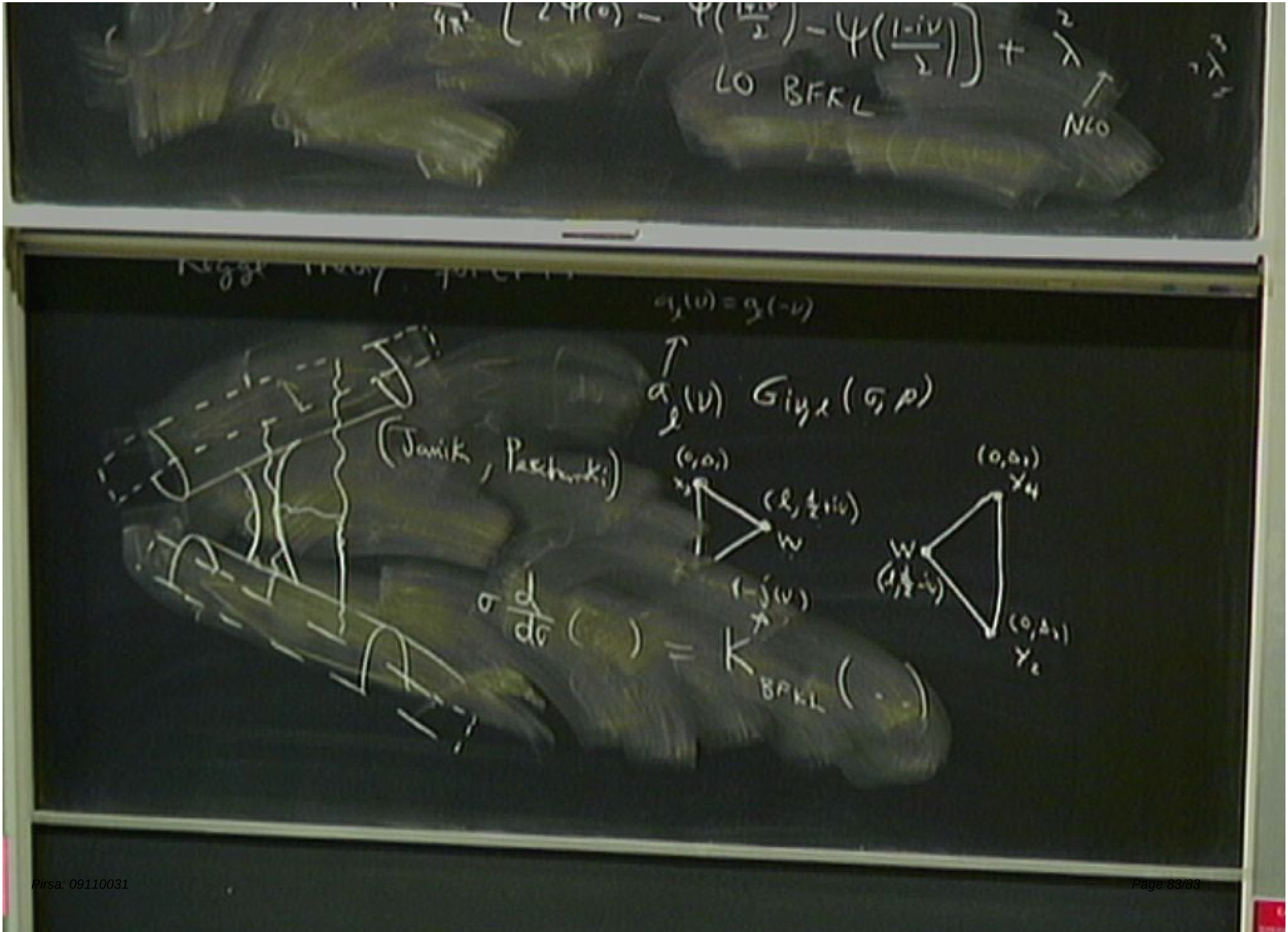

LO BFKL
NLO
W
(0,Δ1)
(0,Δ4)
(0,Δ2)
x4
x2



LO BFKL
NLO


LO BFKL
NLO
BFKL


LO BFKL
NLO
W
W
$\sigma \frac{d}{d\sigma}(\ ) = K_{BFKL}(\ )$


LO BFKL
NLO
Janik, Pastukovski
W
BFKL