Title: Instability and new phases of higher-dimensional rotating black holes

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Abstract: It has been conjectured that higher-dimensional rotating black holes become unstable at a sufficiently large value of the rotation, and that new black holes with pinched horizons appear at the threshold of the instability. We search numerically, and find, the stationary axisymmetric perturbations of Myers-Perry black holes with a single spin that mark the onset of the instability and the appearance of the new black hole phases. We also find new ultraspinning Gregory-Laflamme instabilities of rotating black strings and branes.

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- We need a phase diagram for black holes in higher dimensions: finding new non-linear solutions is very challenging, so we study stability and zero-modes.

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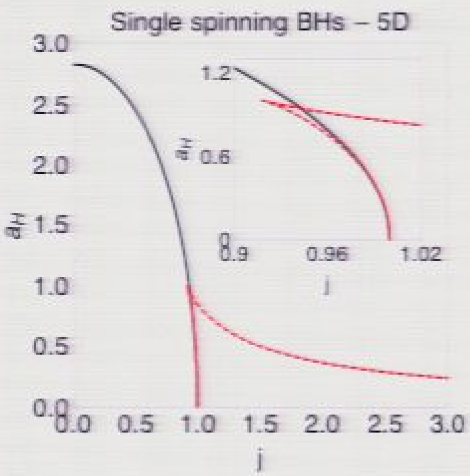

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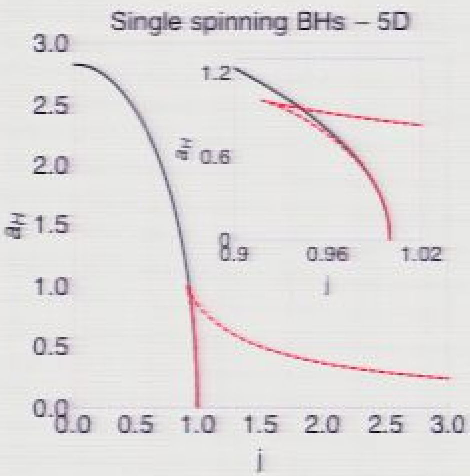

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- $d \geq 6$: the only explicit solution is MP - new static numerical black objects with spatial section horizon topology $S^2 \times S^{d-4}$ (Kleihaus *et al.*, conical singularity), many horizon topologies (Emparan *et al.*, blackfold approach). Stability?

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- Charged or AdS black strings have more complex line elements, but can be made stable.

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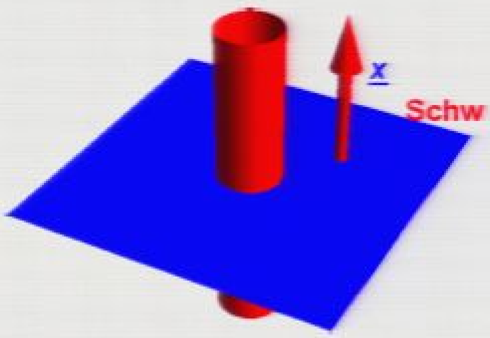

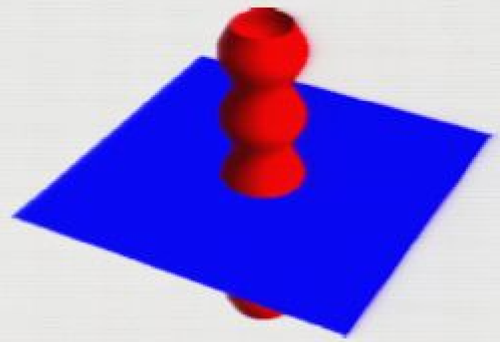

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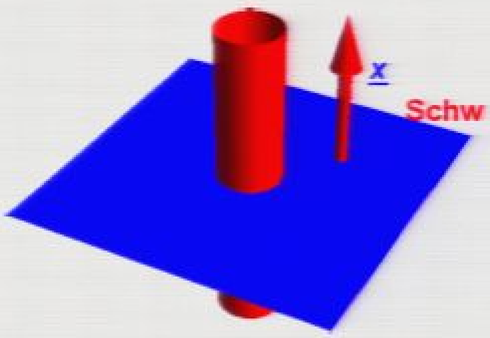

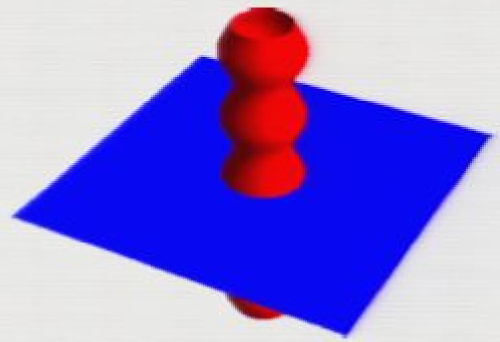

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$$H_{AB} = e^{i\underline{\mathbf{k}}\cdot\underline{\mathbf{x}}} \begin{bmatrix} h_{ab} & 0 \\ 0 & 0 \end{bmatrix}, \text{where } h_{ab} = e^{\Omega t}(spherical - wave)$$

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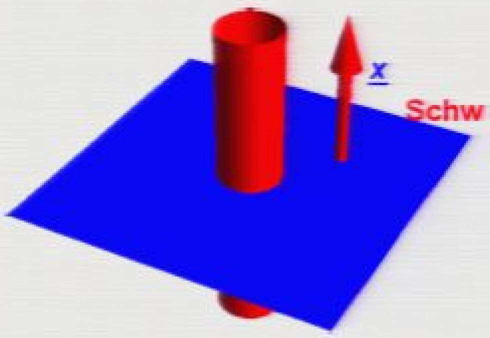

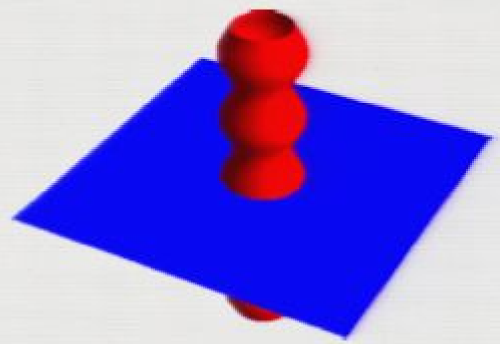

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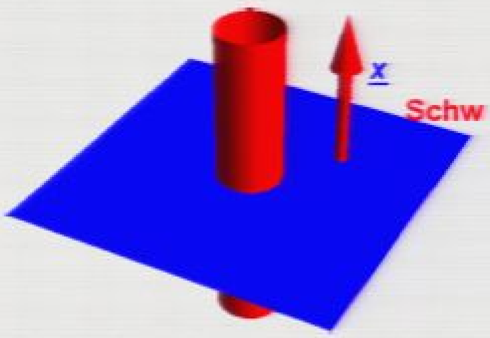

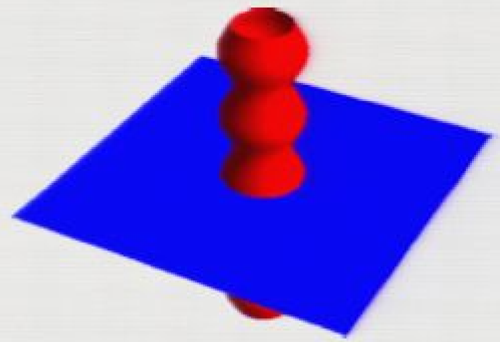

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- Einstein's equations: $\Delta_L H_{AB} = 0$ reduce to $\Delta_L h_{ab} = \lambda h_{ab}$, $\Delta_L P_{\alpha\beta} := -\Box P_{\alpha\beta} - 2R_\alpha{}^\gamma{}_\beta{}^\delta P_{\gamma\delta}$, and $\lambda = -\underline{\mathbf{k}} \cdot \underline{\mathbf{k}}$.

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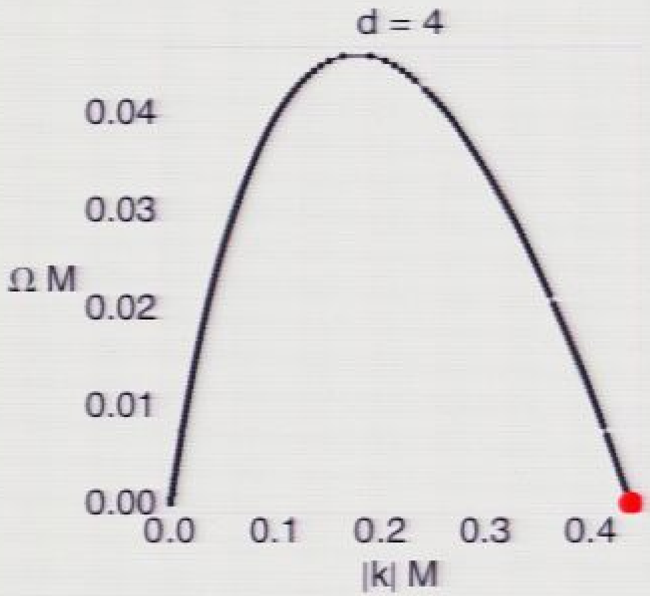

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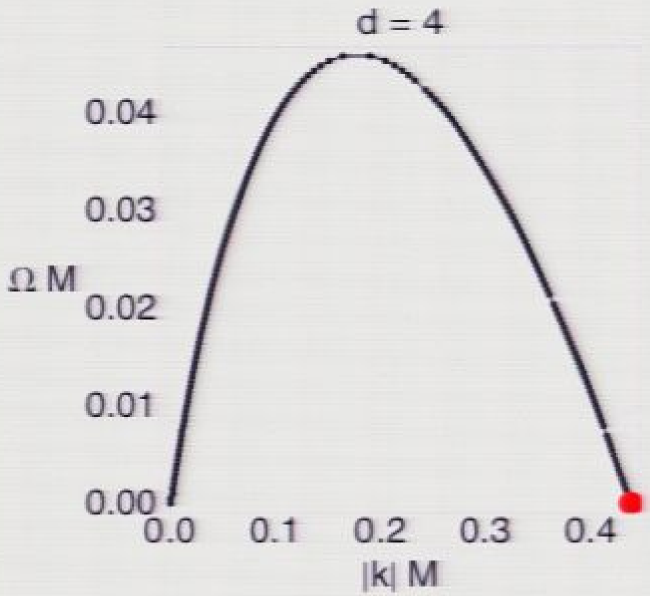

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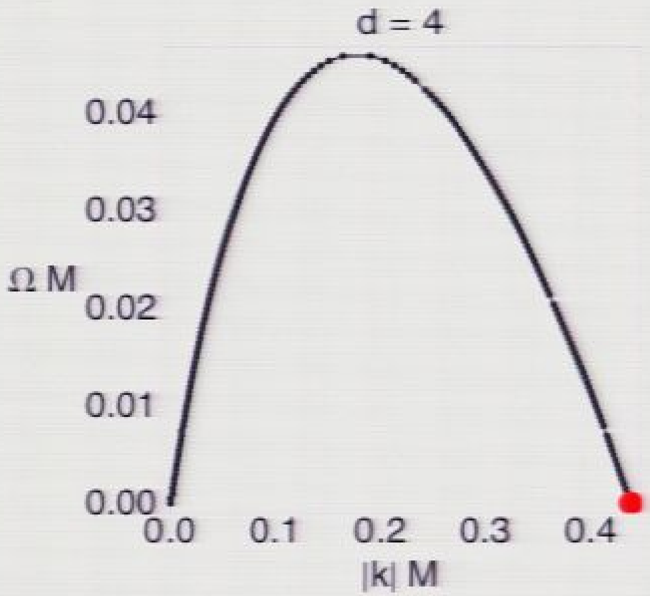

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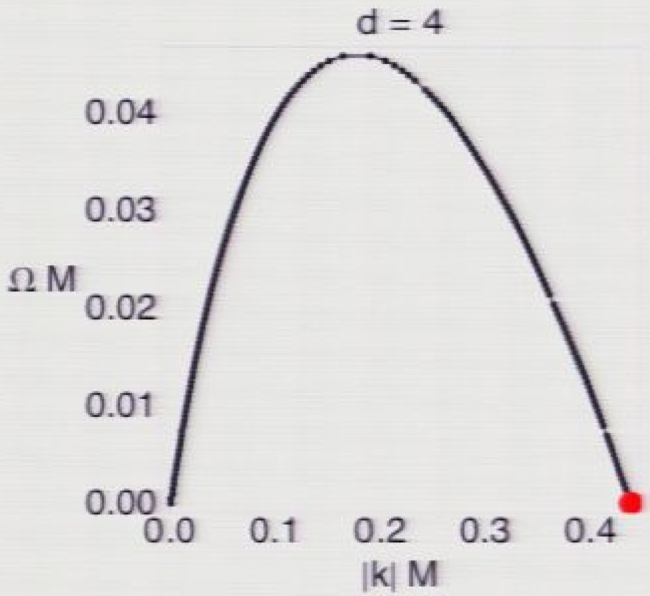

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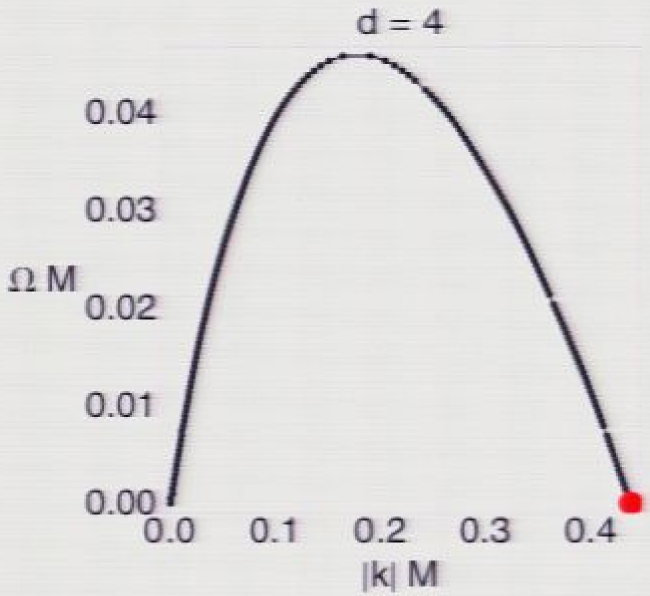

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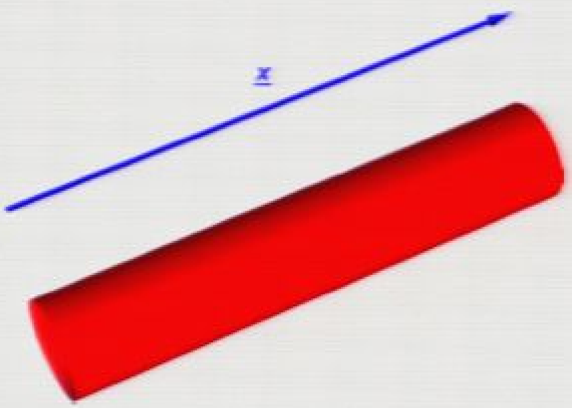

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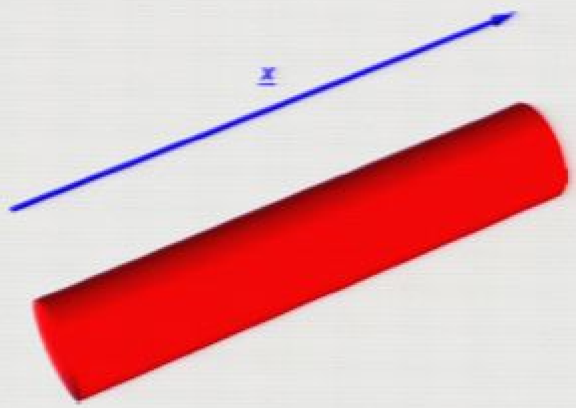

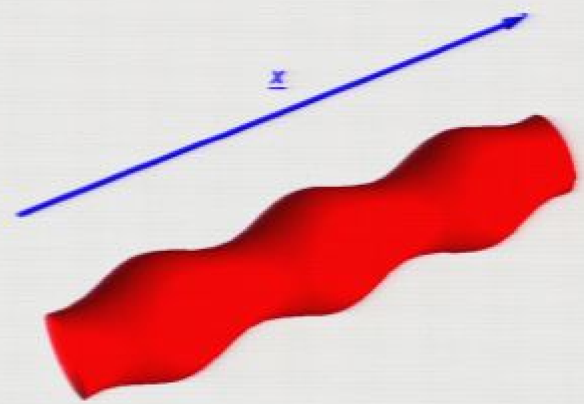

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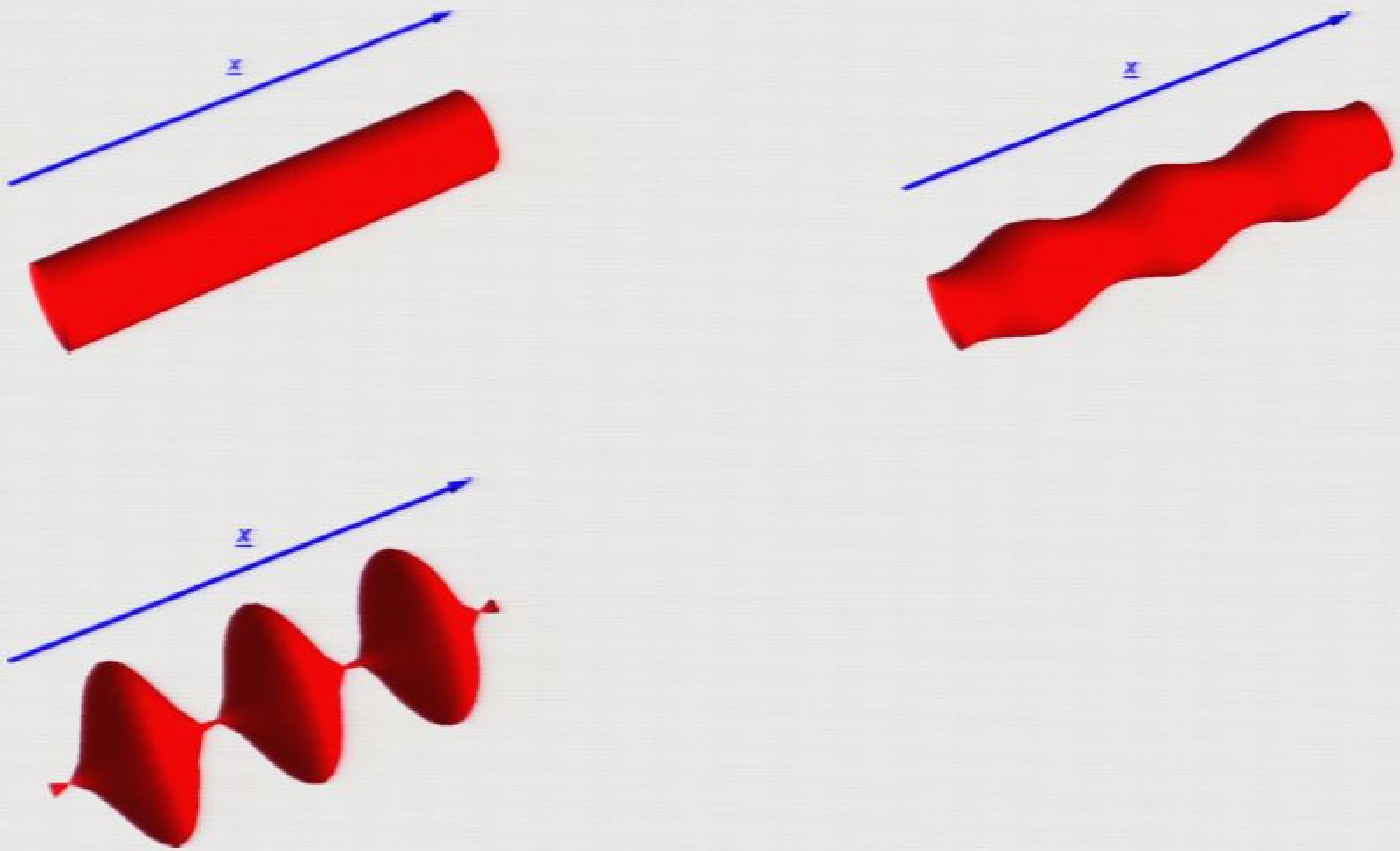

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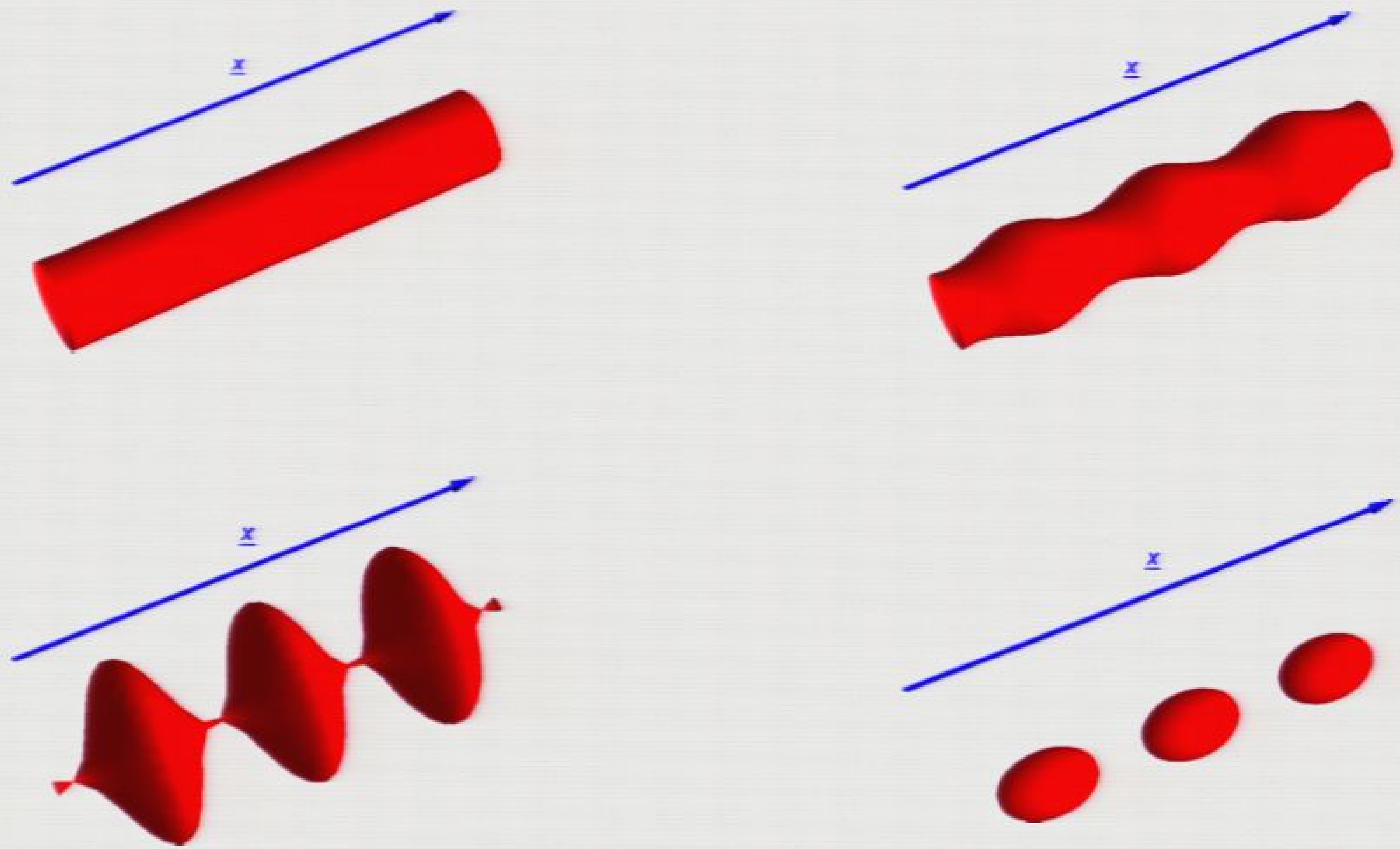

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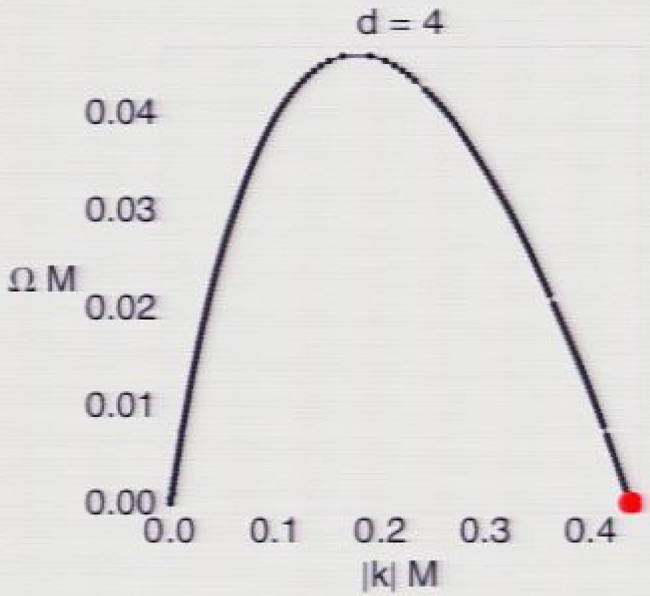

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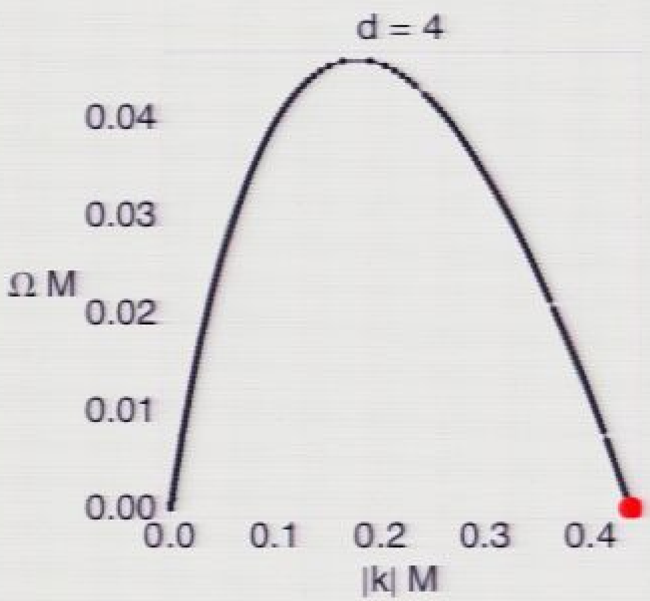

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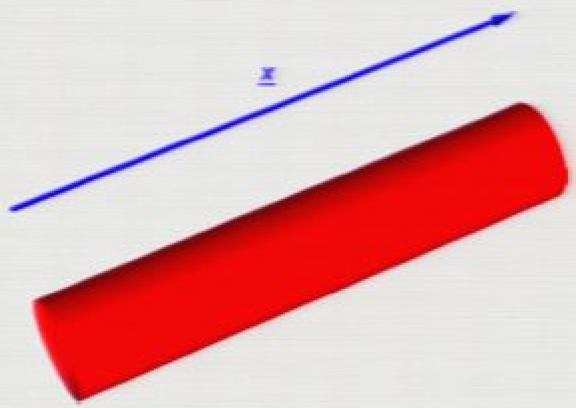

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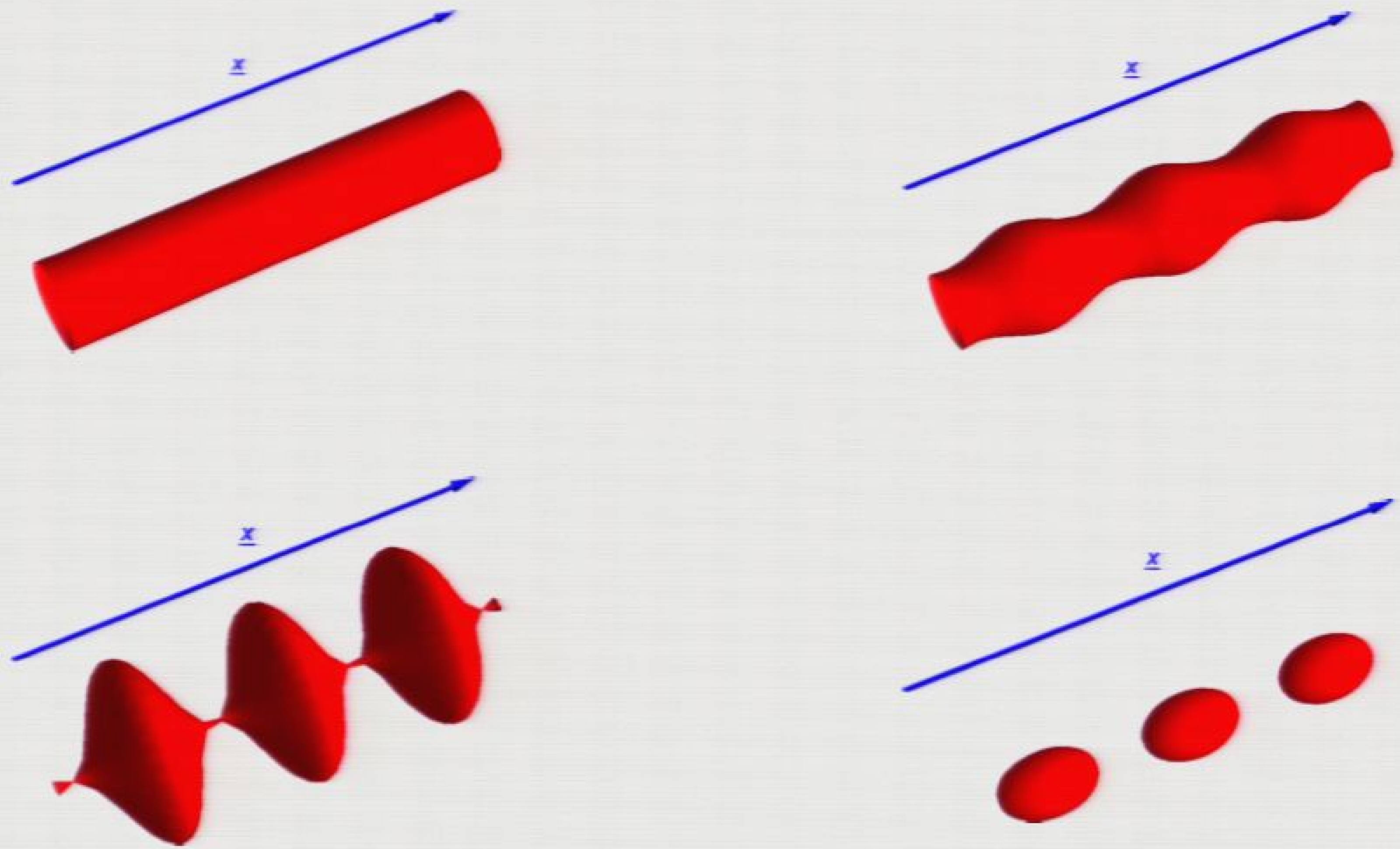

Single spinning Myers-Perry (MP) (Myers and Perry '86)

Single spinning MP line element: Kerr like $\{t, r, \theta, \phi\} \times S^{d-4}$

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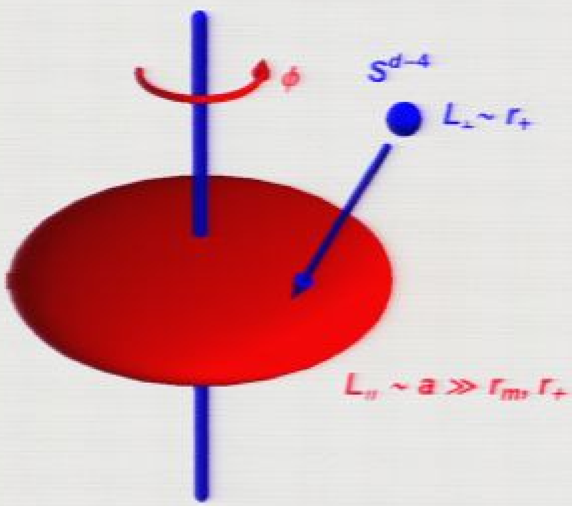
- $d = 4$: Kerr Black hole, extremality bound $|a| \leq r_m/2$.
- $d = 5$: Extremality bound, $|a| < r_m$, and naked singularity at $|a| = r_m$.

Ultra-spinning instability (Emparan, Myers '03)

- For $d \geq 6$, no upper bound on $|a|$, so spin it up!



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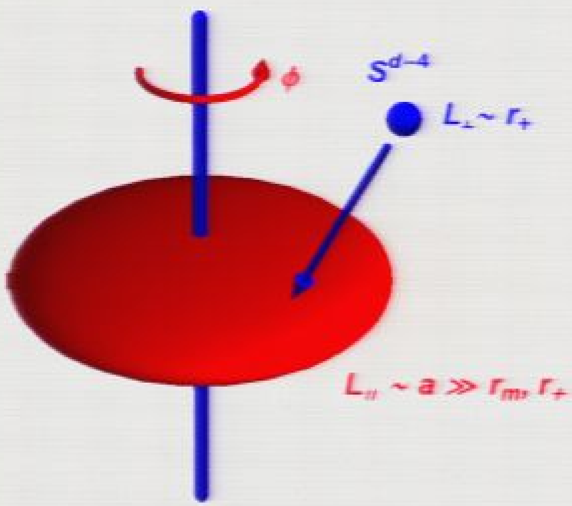

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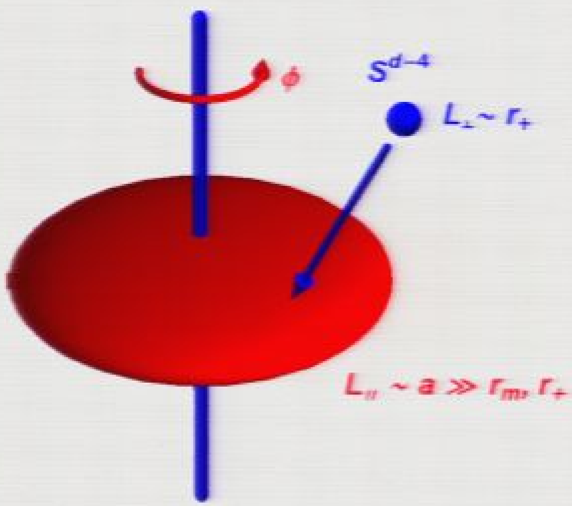

- $L_{\|}$ is the $\{\theta, \phi\}$ pancake scale
  $L_\perp$ is transverse S^{d-4} scale.
- $L_{\|} \gg L_\perp$: the S^{d-4} sphere 'sees' the pancake as a two-plane - locally $(BH)_{d-2} \times \mathbb{R}^2$ *i. e.* a string $\leftrightarrow$ GL unstable!!

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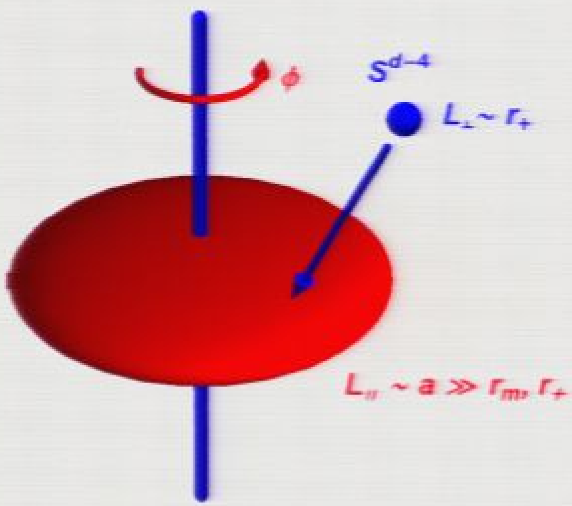


- $L_\parallel$ is the $\{\theta, \phi\}$ pancake scale
  $L_\perp$ is transverse S^{d-4} scale.
- $L_\parallel \gg L_\perp$: the S^{d-4} sphere 'sees' the pancake as a two-plane - locally $(BH)_{d-2} \times \mathbb{R}^2$ *i. e.* a string $\leftrightarrow$ GL unstable!!

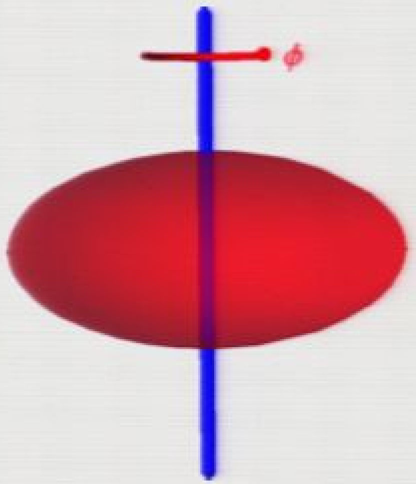
- Zooming near the pole $\theta = 0$, taking $|a| \to +\infty$ and keeping $\hat{r}_m^{d-3} = r_m^{d-3}/a^2$ fixed:

$$ds^2 \simeq -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-4} + \underline{d\sigma^2 + \sigma^2 d\phi^2},$$

where

$$f(r) = 1 - \frac{\hat{r}_m^{d-5}}{r^{d-5}} \text{ and } \sigma = a \sin\theta.$$

Ultra-spinning instability - What to expect? (Emparan, Myers '03)



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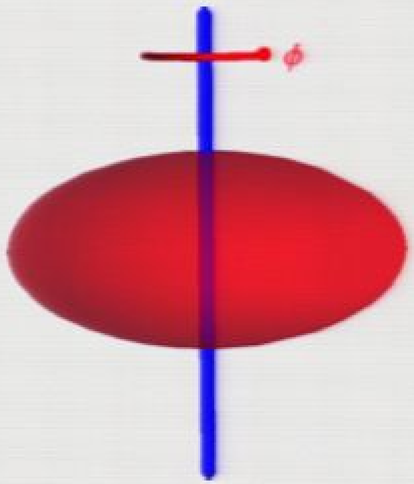

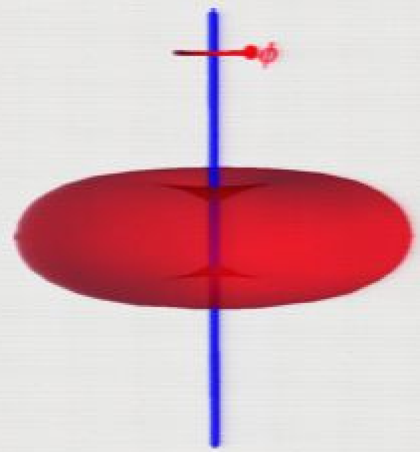

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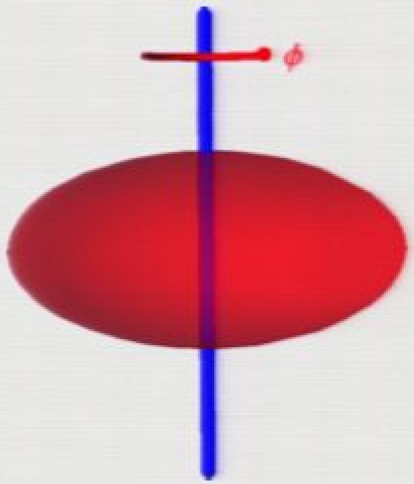

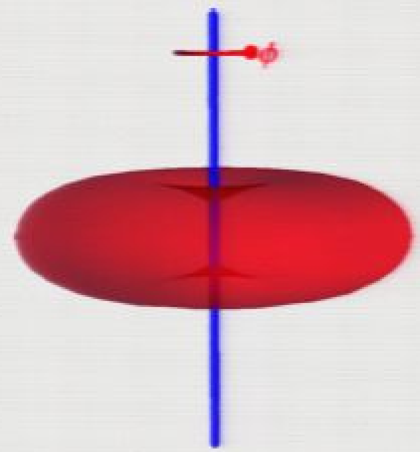

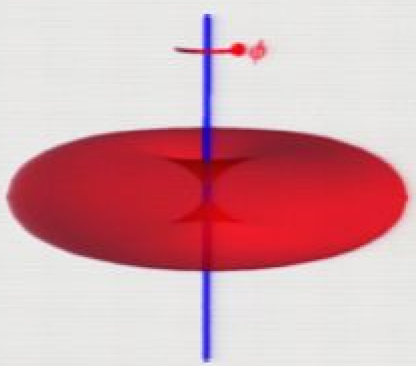

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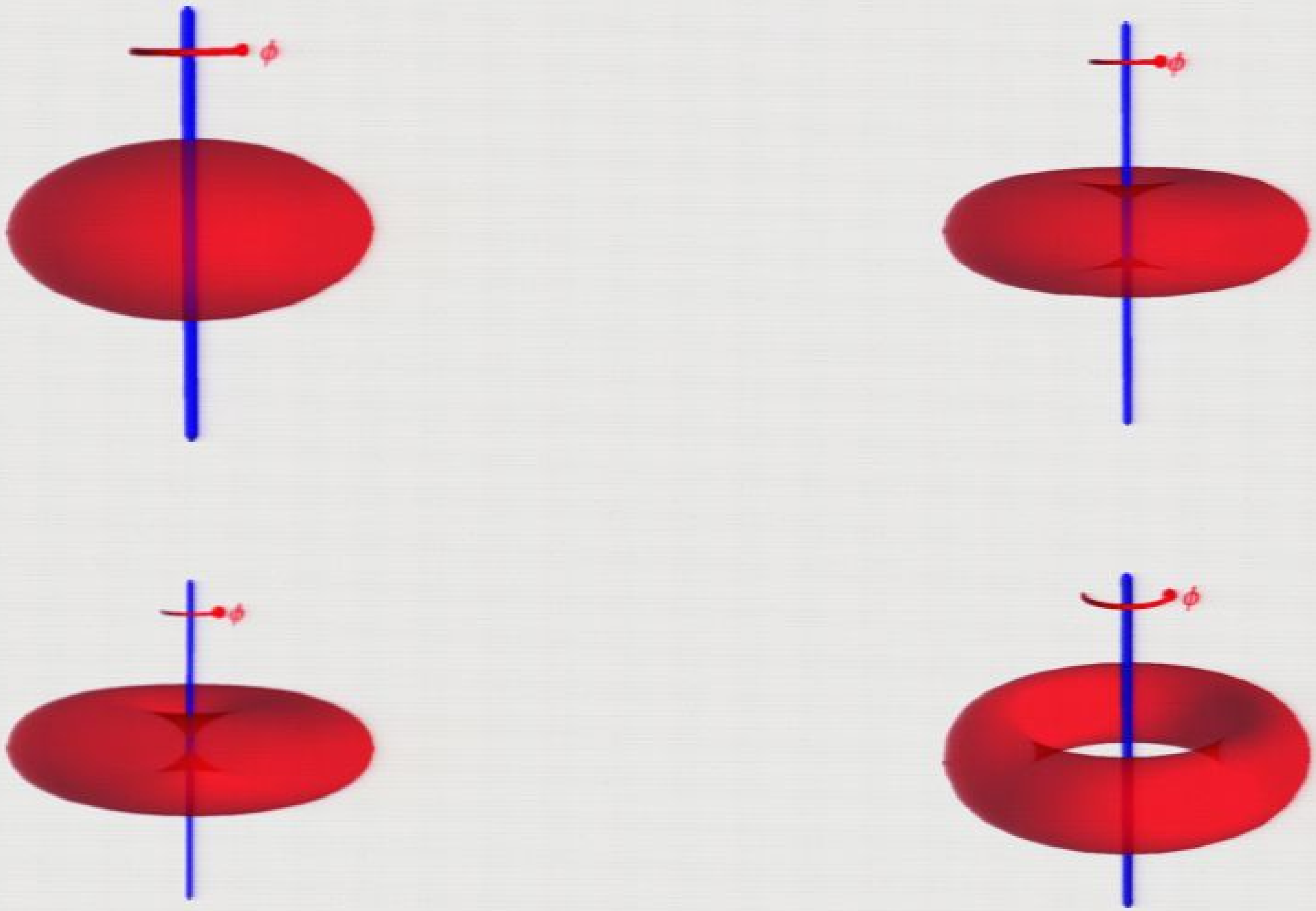

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'Spectral power' & Single spinning MP (Dias, Figueras, Monteiro, JES and Emparan '09)

- We want to study:

$$\Delta_L h_{AB} = 0,$$

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- Search for solutions which preserve the $\mathbb{R}_t \times U(1)_\phi \times SO(d-3)_{\Omega_{d-4}}$

$$h_{AB} = \left[\begin{array}{cccc|c} h_{tt} & 0 & 0 & h_{t\phi} & 0 \\ 0 & h_{rr} & h_{r\theta} & 0 & 0 \\ 0 & h_{r\theta} & h_{\theta\theta} & 0 & 0 \\ h_{t\phi} & 0 & 0 & h_{\phi\phi} & 0 \\ \hline 0 & 0 & 0 & 0 & h_\Omega g_{\Omega_{d-4}} \end{array}\right]$$

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  - At infinity:

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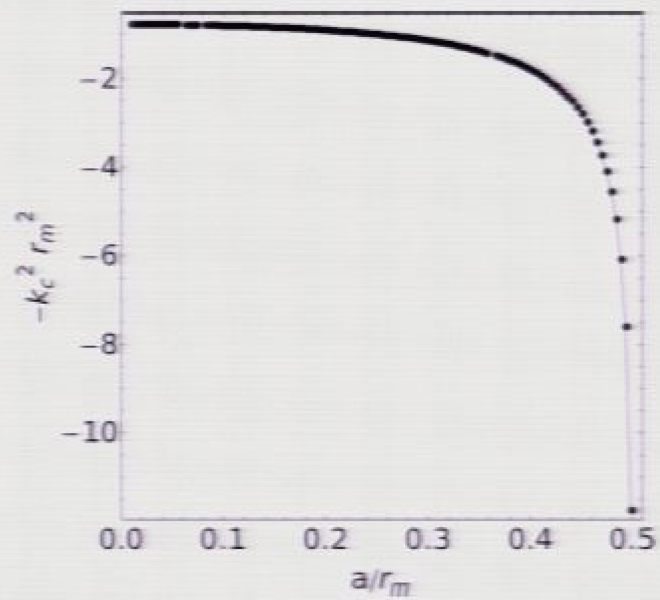

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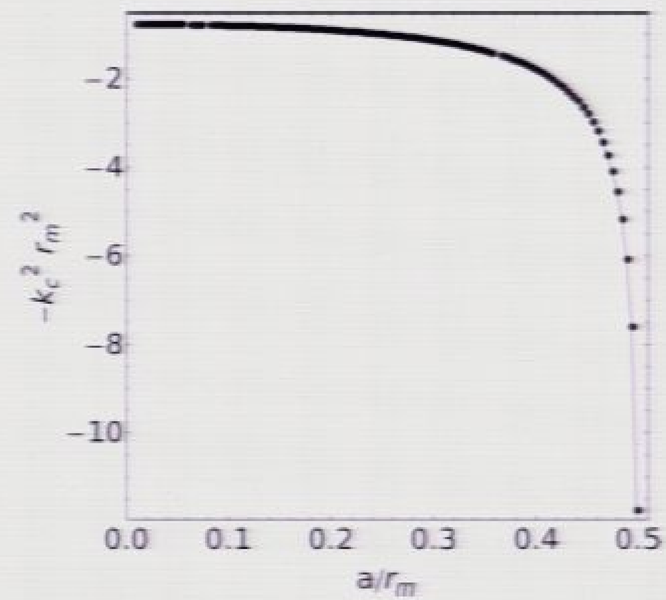

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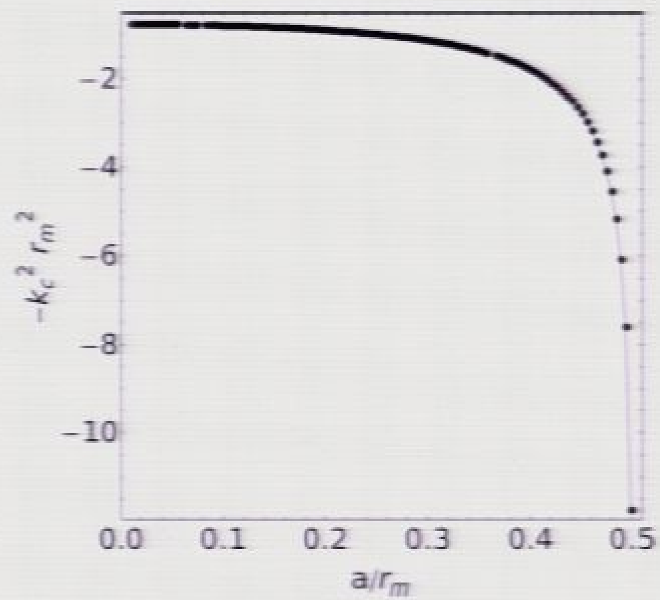

- For the Kerr black hole the negative mode is finite at extremality.
- In 5D the negative mode diverges near extremality as

$$k_c^2 \sim (r_m - a)^{-2}.$$

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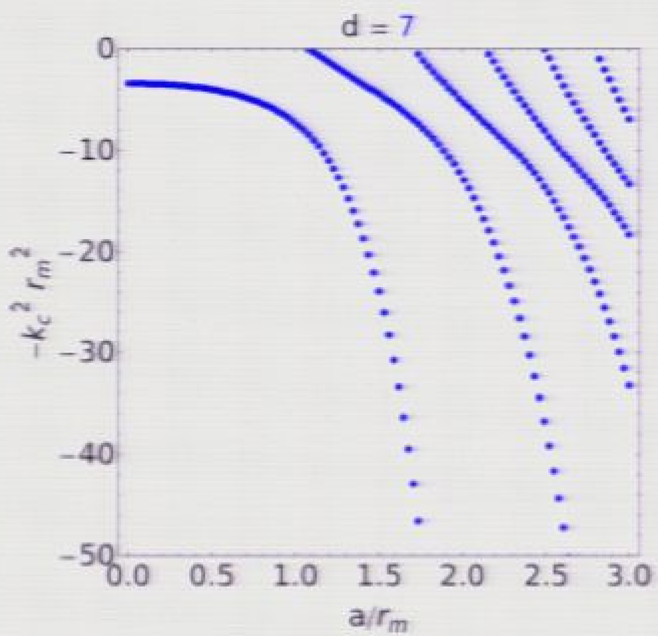

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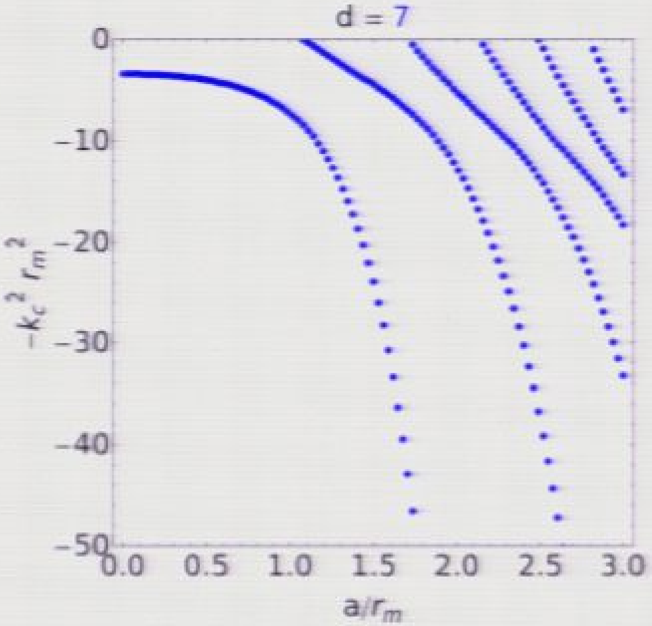

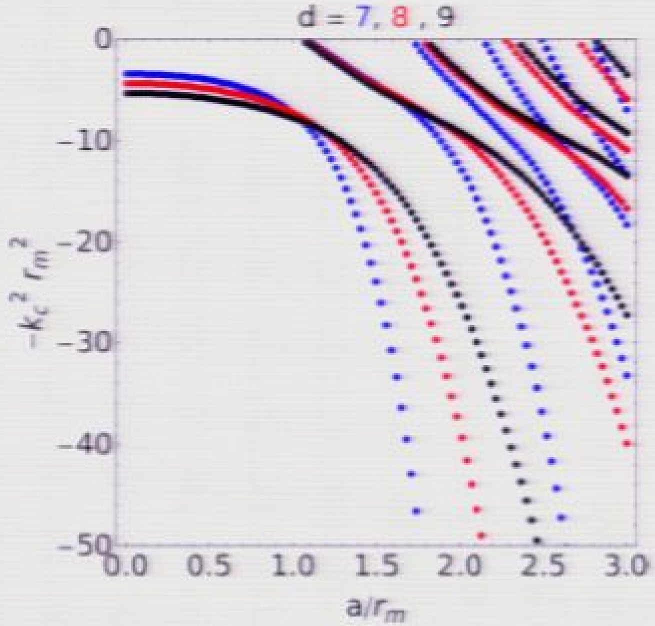

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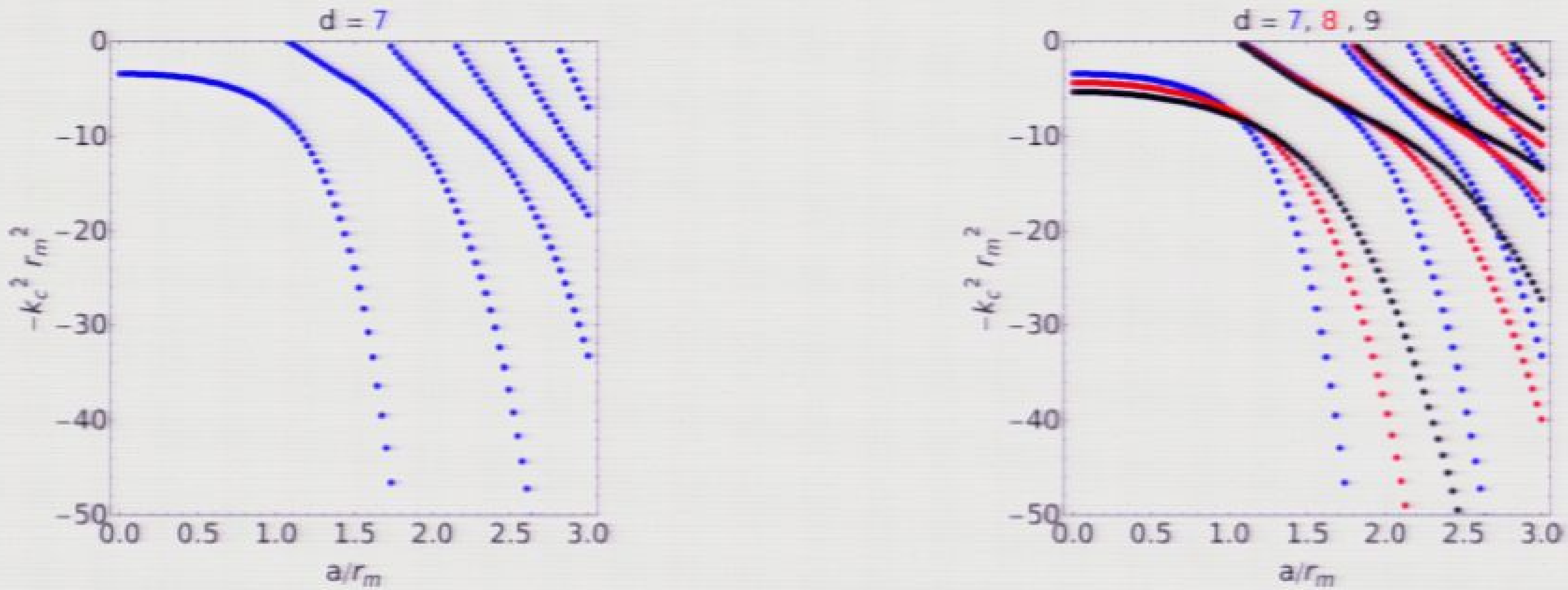

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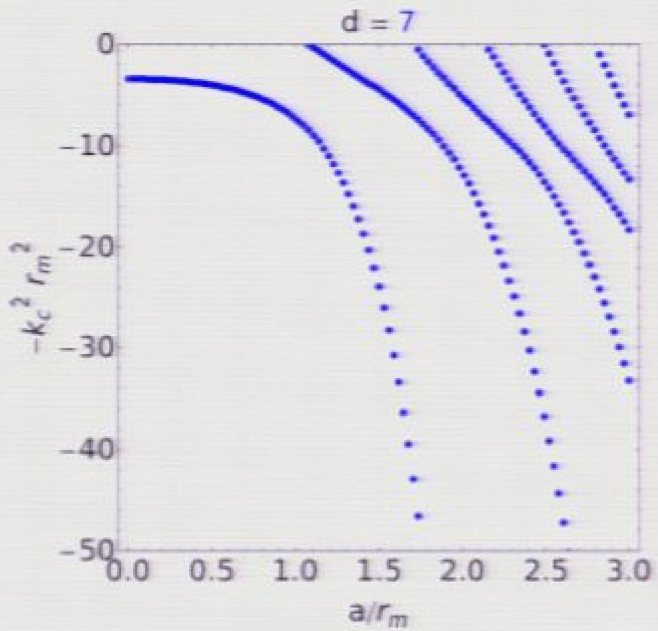

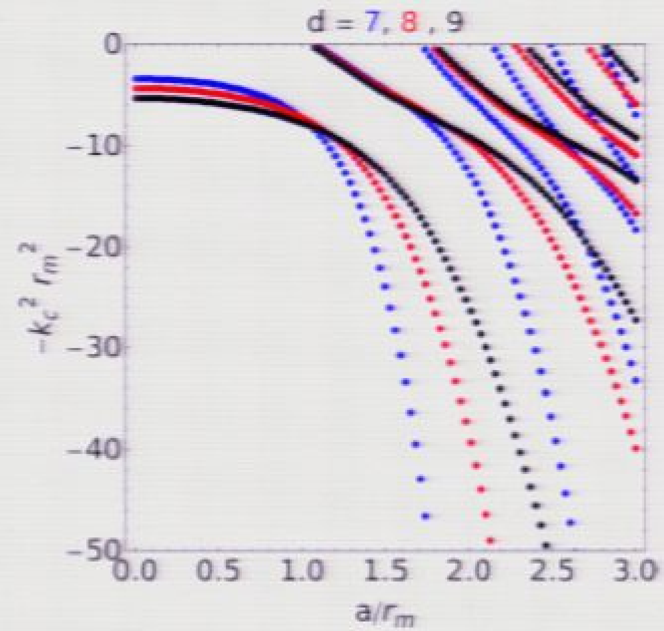

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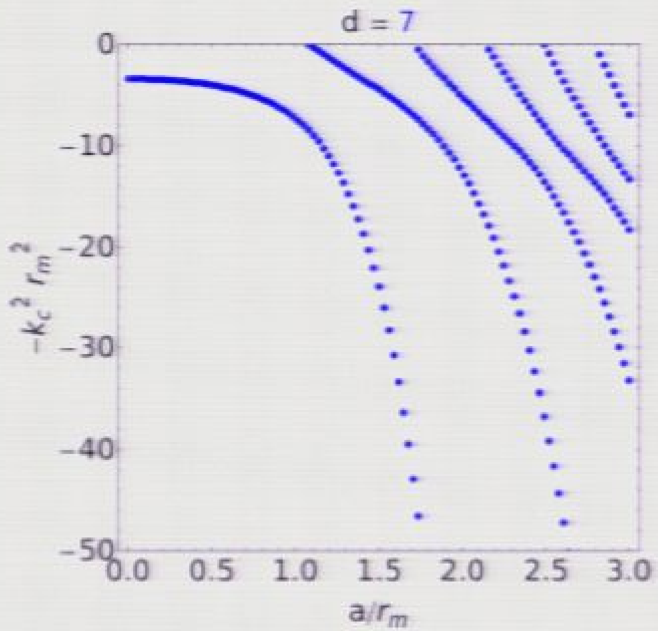

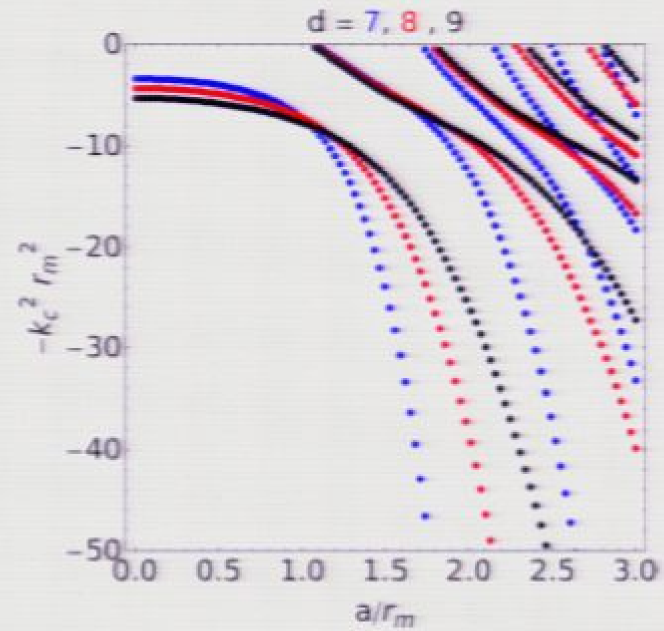

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- Are these the instabilities we expect? If so, why so many? Number the curves from left to right as $\ell = 0, 1, 2 \ldots$

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  - $(a/r_+) < \alpha_\star$: T_H and S decrease like in Kerr
    $(a/r_+) > \alpha_\star$: rapidly $T_H \sim r_+^{-1}$ and $S \sim a^2 r_+^{d-4}$ (membrane)

The first threshold mode $\ell = 1$

- Myers and Emparan provided heuristic arguments for the instability appearance:
  - The black hole temperature is given by

$$T_H = \frac{1}{4\pi}\left(\frac{2r_+}{r_+^2 + a^2} + \frac{d-5}{r_+}\right)$$

  - The temperature reaches a minimum for

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- The $\ell = 1$ mode appears in all dimensions precisely for $a/r_+ = \alpha_\star$!

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  - Embed the two dimensional horizon (the S^{d-4} is suppressed) in four-dimensional Euclidean space (Frolov '04): covers the entire horizon for all values of the rotation a/r_m, but has a cone at the pole.

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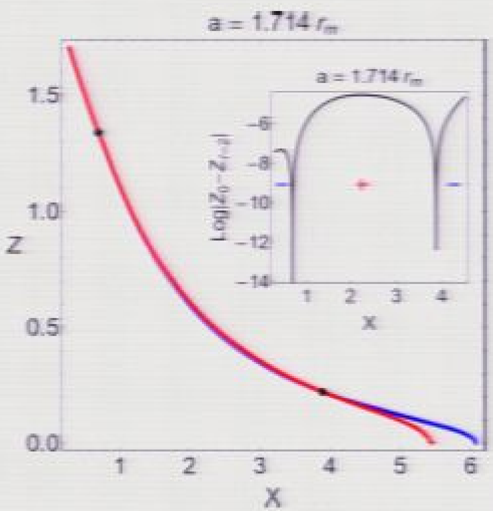

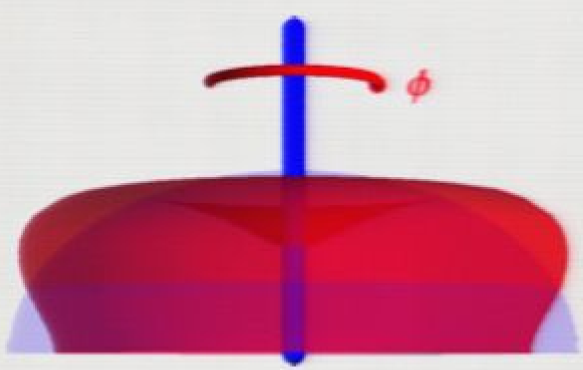

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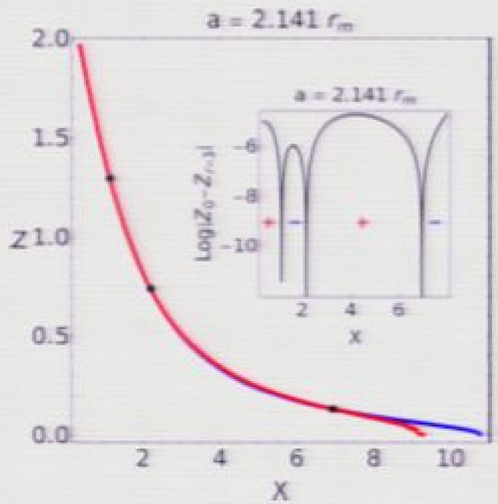

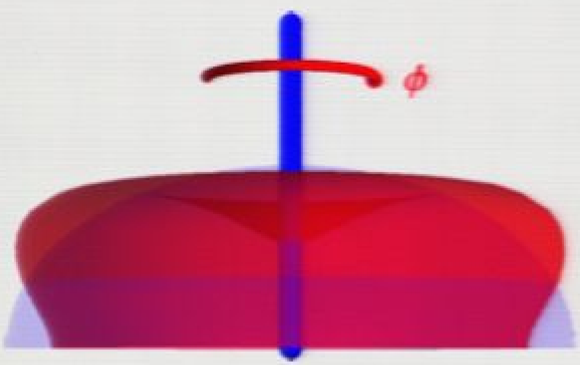

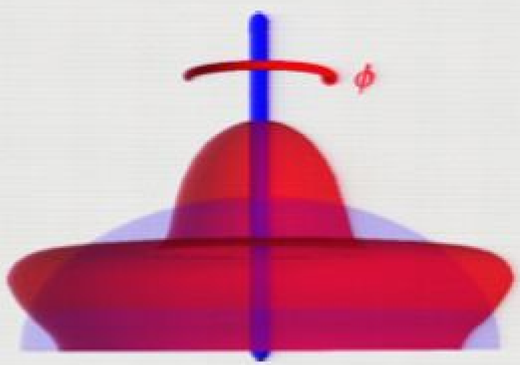

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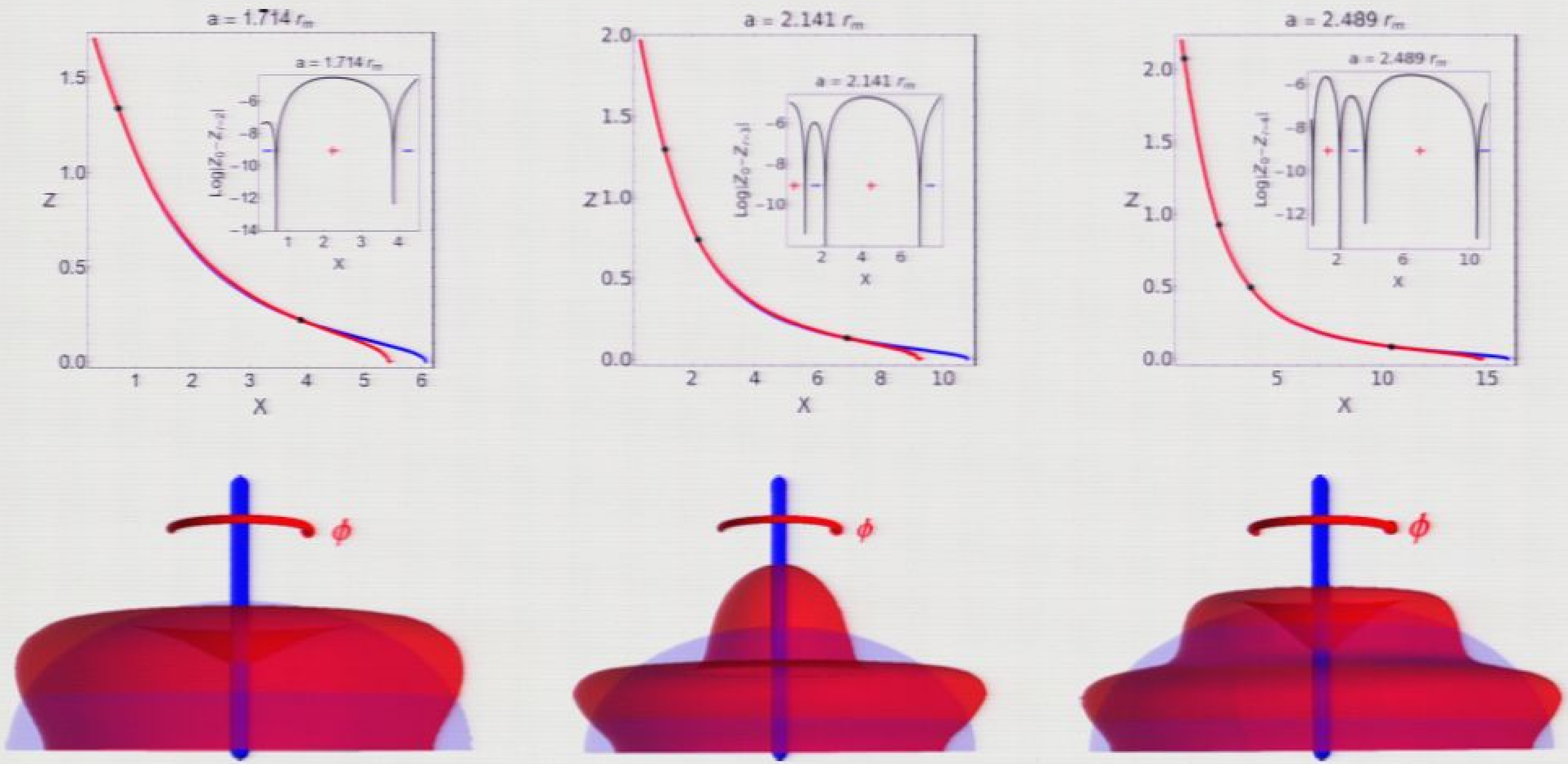

Equal angular momenta MP solution in odd dimensions

- In $d = 2N+3$, the equal angular momenta MP solution can be written as a co-dimension 1 manifold:

$$ds^2 = -f(r)^2 dt^2 + g(r)^2 dr^2 + h(r)^2 [d\psi + \mathbb{A}_\mu dx^\mu - \Omega(r) dt]^2 + r^2 ds^2_{\mathbb{CP}^N},$$

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- Perturbations expanded in charged Harmonics on $\mathbb{CP}^N \rightarrow$ co-dimension two problem (t and r dependence): non-trivial due to coupling of $\mathbb{A}$ with ψ and t. (Kunduri, Lucietti, Reall '06 & Martin, Reall '09)

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- For $N \geq 2$, the following inequality holds:

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- If modes with $\Omega > 0$ are detected, then we undoubtedly have an unstable asym. flat black hole with compact horizon.
- Harmonic expansion on $\mathbb{CP}^N$ is used to study which symmetries are broken by the perturbations. For $N = 3$ $(d = 9)$, the $\ell = 2$ harmonic breaks all the $\mathbb{CP}^3$ symmetries: perturbative black hole saturates generalisation of Hawking's rigidity theorem to higher d (Hollands, Ishibashi and Wald '06, and Isenberg and Moncrief '08).

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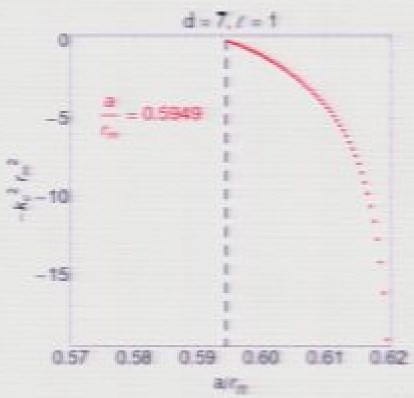

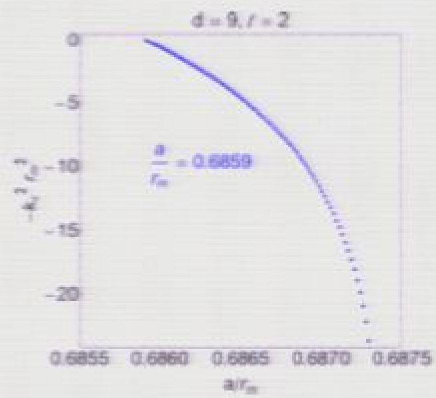

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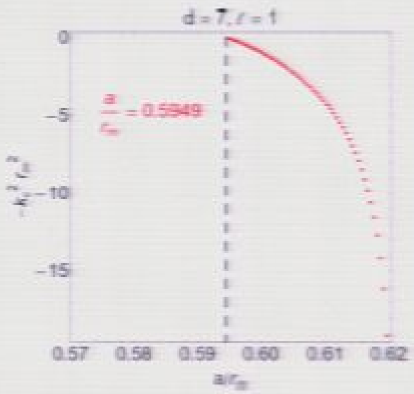

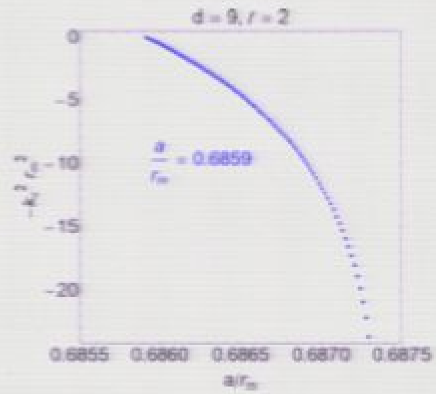

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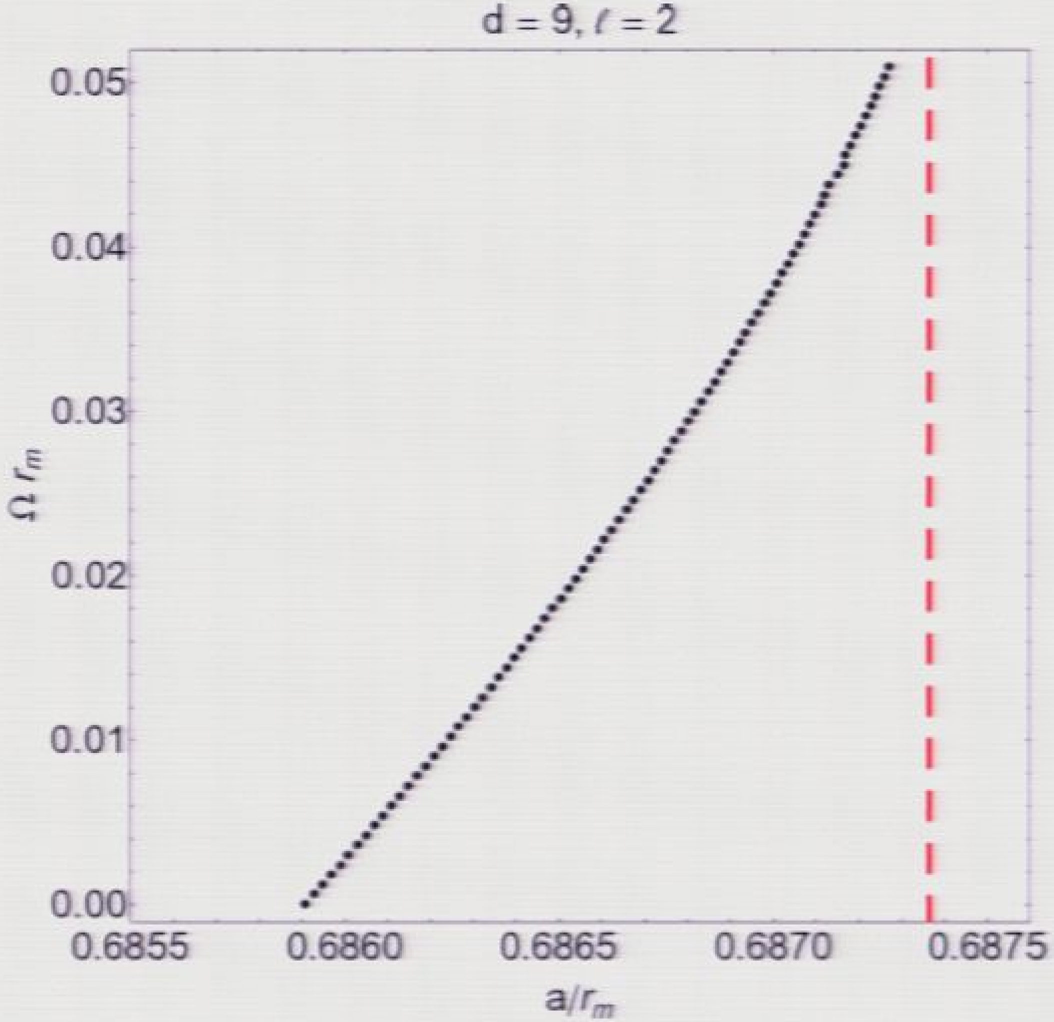
- Dispersion relation $\Omega \neq 0$:
  - $\ell = 1$ is NOT an instability of the MP, but $\ell = 2$ IS:

Dispersion relation of the equal angular momenta MP

- For $k = 0$, one finds



Discussion & Conclusions

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- Future directions:
  - Consider the time dependence in the single spinning MP solution (PDEs).
  - Break transverse sphere in the single spinning MP solution (saturate Hawking's rigidity theorem).
  - Consider a background MP with several angular momenta turned on.