Title: Quantum computational phases of matter: measurement-based quantum computing in the Haldane phase

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Abstract: A recent breakthrough in quantum computing has been the realization that quantum computation can proceed solely through single-qubit measurements on an appropriate quantum state. One exciting prospect is that the ground or low-temperature thermal state of an interacting quantum many-body system can serve as such a resource state for quantum computation. The system would simply need to be cooled sufficiently and then subjected to local measurements. It would be unfortunate, however, if the usefulness of a ground or low-temperature thermal state for quantum computation was critically dependent on the details of the system's Hamiltonian; if so, engineering such systems would be difficult or even impossible. A much more powerful result would be the existence of a robust ordered phase which is characterized by the ability to perform measurement-based quantum computation. I’ll discuss some recent results on the existence of such a computational phase of matter. I’ll first outline some positive results on a phase of a toy model that contains the cluster state. Then, in a realistic model of coupled spin-1 particles, I’ll demonstrate the existence of a computational phase. This result is obtained by using a local measurement sequence to “renormalize” the state to a computationally-universal fixed point. Together, these results reveal that the characterization of computational phases of matter has a rich, complex structure – one which is still poorly understood. Joint work with Gavin Brennen, Akimasa Miyake, and Joseph Renes.

Quantum computational phases of matter:

Measurement-based quantum computing in the Haldane phase

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Akimasa Miyake (Perimeter Institute)

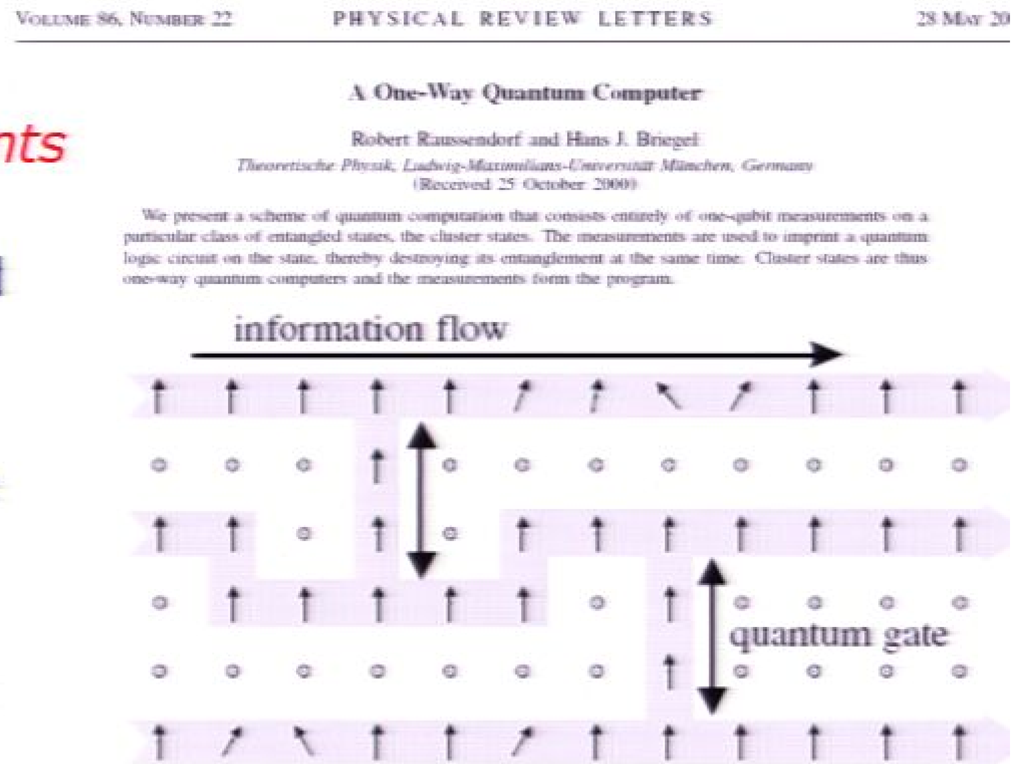
Joe Renes (TU Darmstadt)

Measurement-based quantum computing

- Quantum computing can proceed through *measurements* rather than unitary evolution
- Measurements are strong and incoherent: easier?

Uses a resource state such as the *cluster state*:

- a universal circuit board
- a lattice of spins in a specific *entangled* state


VOLUME 86, NUMBER 22 PHYSICAL REVIEW LETTERS 28 MAY 20

A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel

Theoretische Physik, Ludwig-Maximilians-Universität München, Germany

(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

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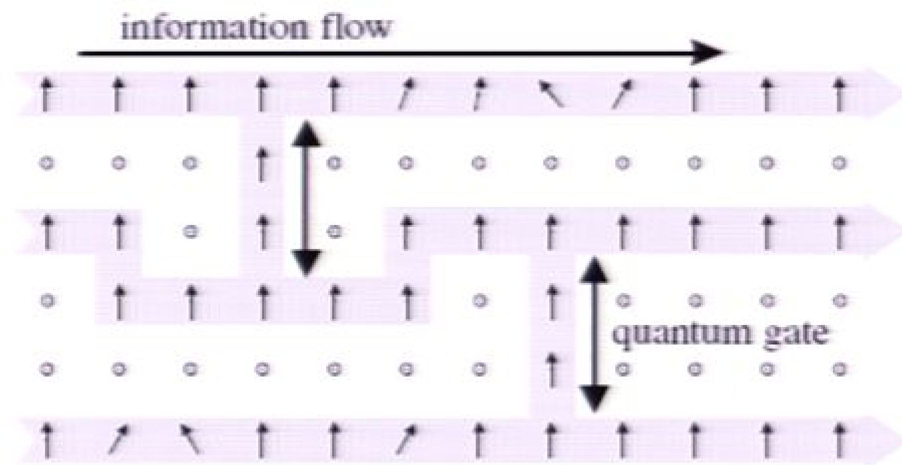

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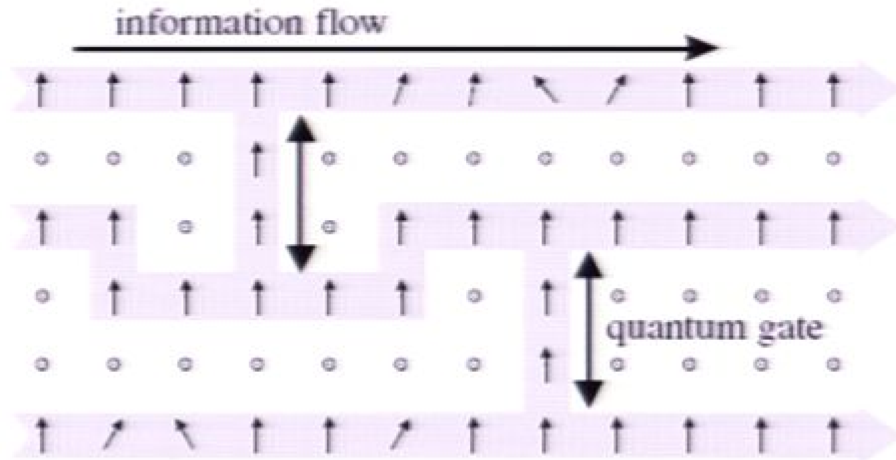

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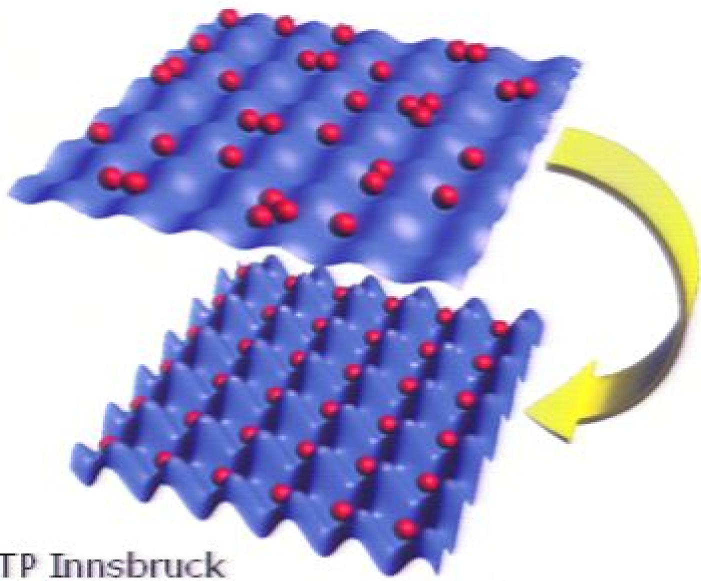
Computational properties → properties of states

Q: What makes the cluster state work?

Q: What properties of a state are necessary?

Resource states & Hamiltonians



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  - the ground state of a coupled quantum many-body system (Raussendorf *et al.*)

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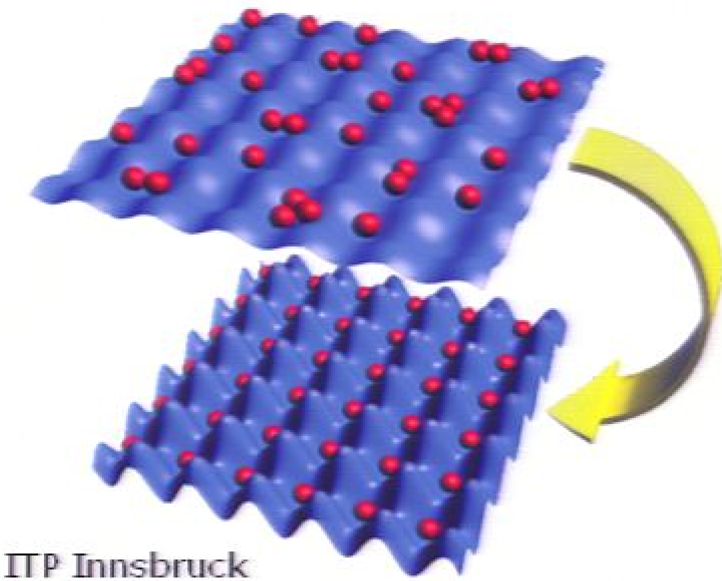

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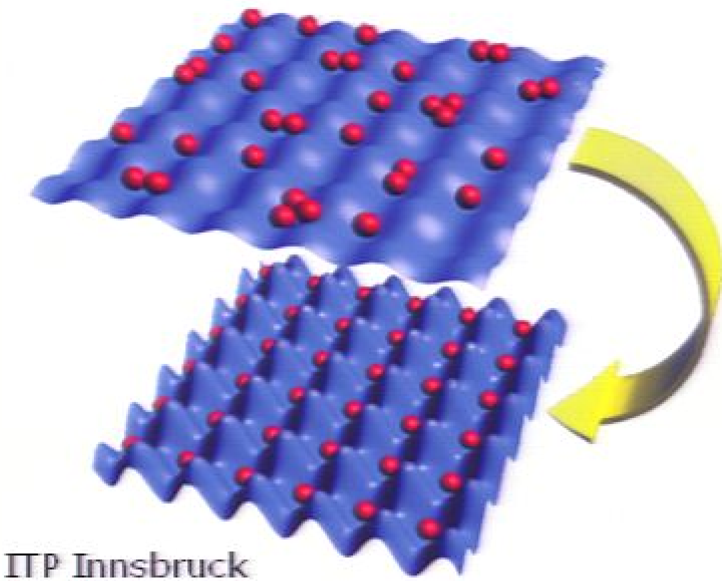

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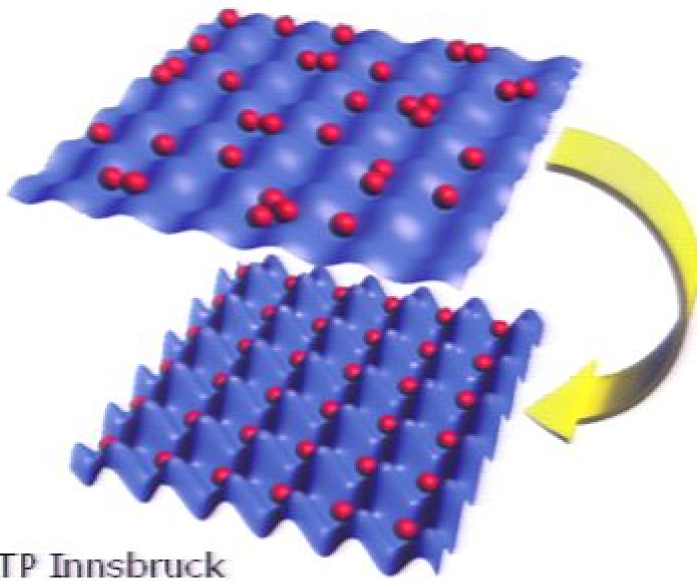

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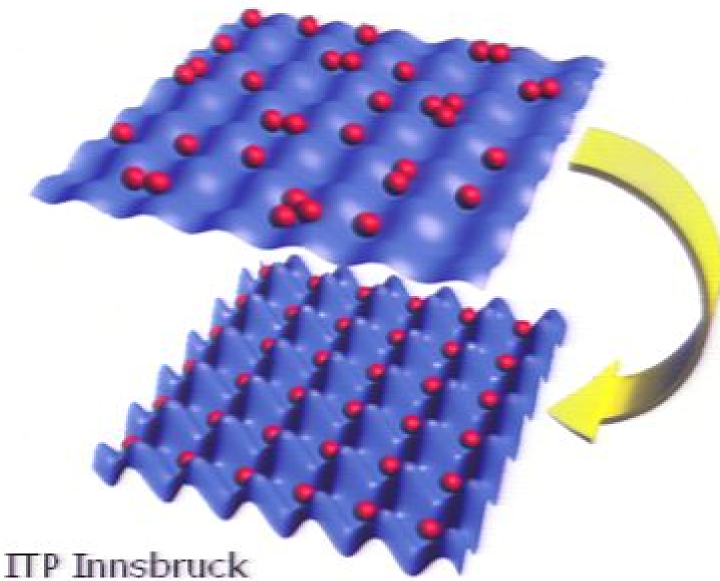

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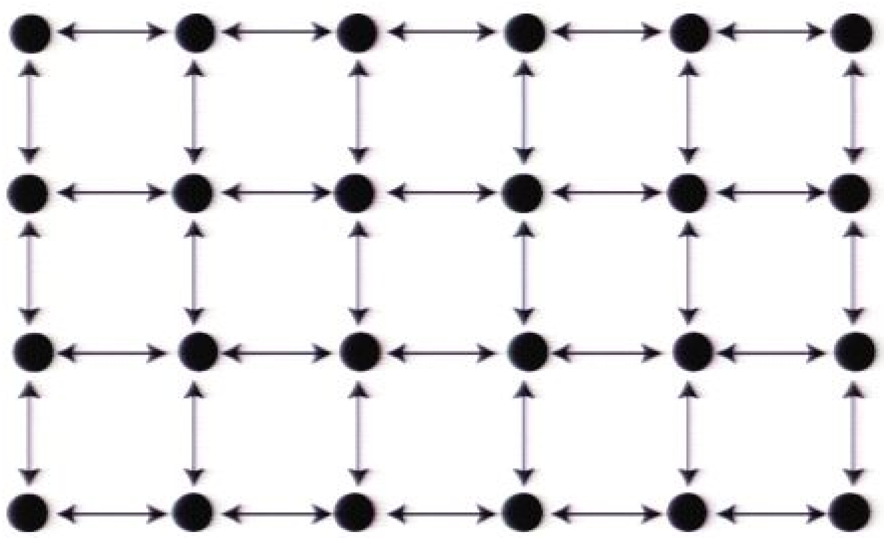
Central idea: identify suitable *order parameters* that characterize the ability to perform MBQC on a state

Identifying phases, part 1

MBQC gates and correlation functions

- With a cluster state, you can teleport a qubit (identity gate) between any two points on the lattice using local measurements on the qubits in between



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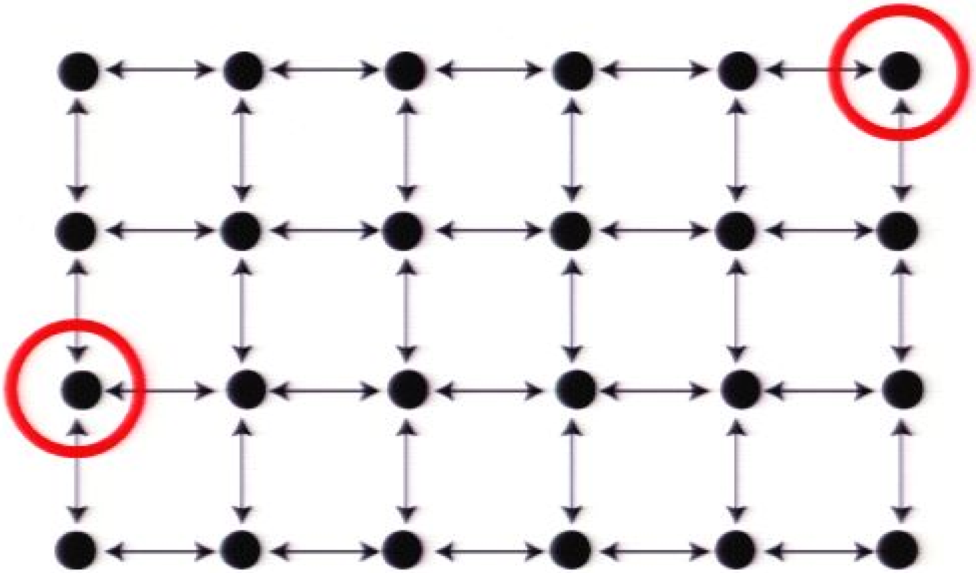

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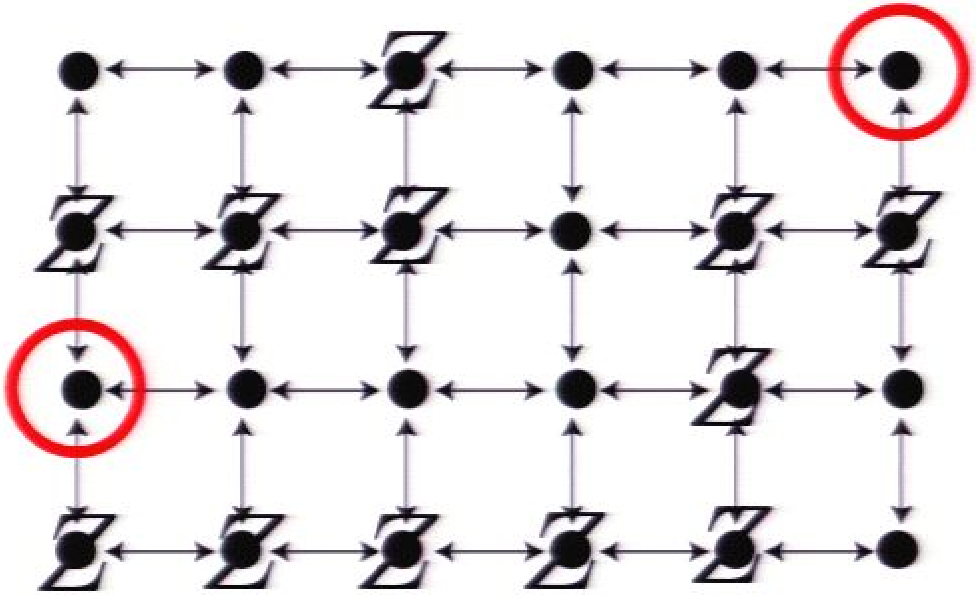

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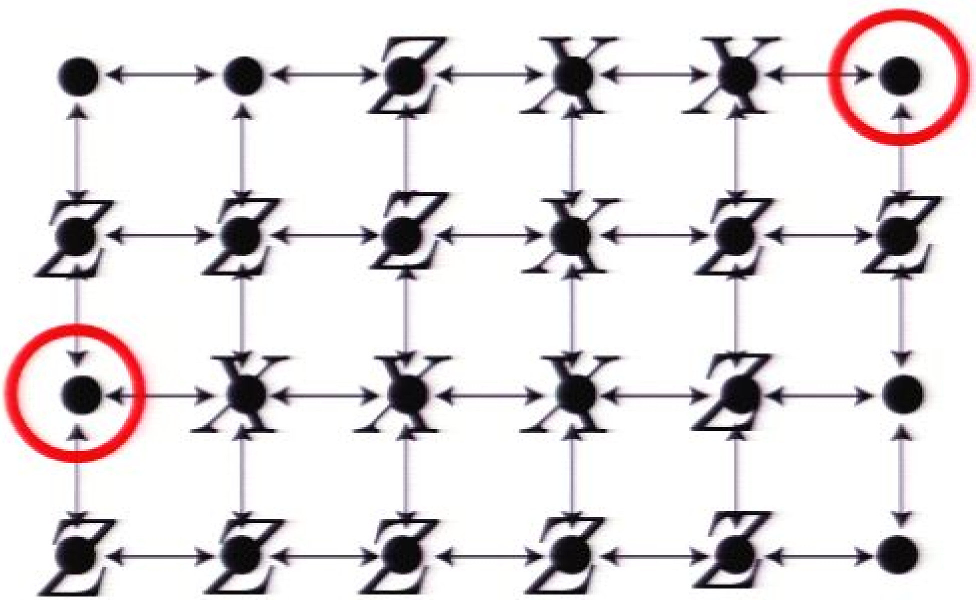

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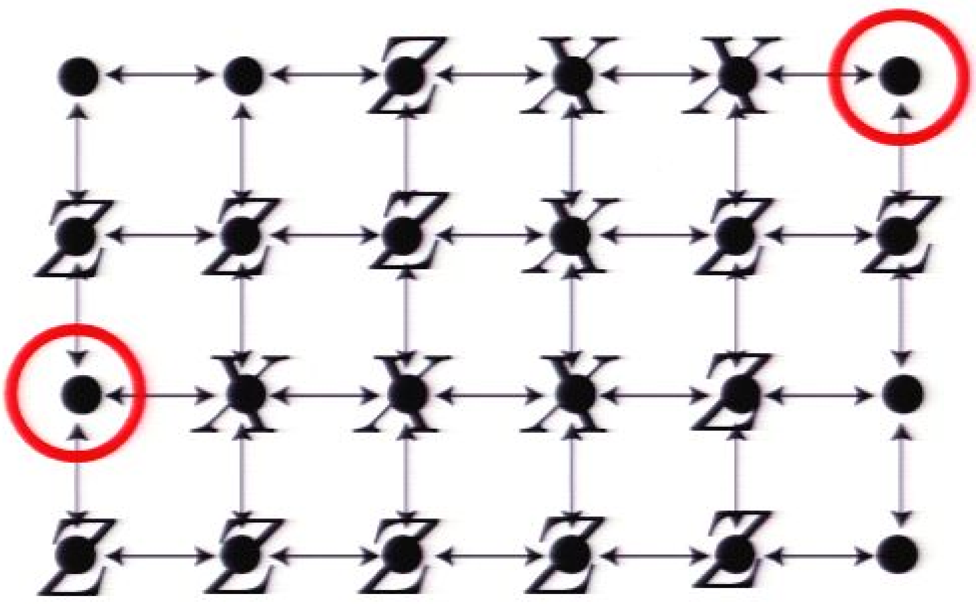
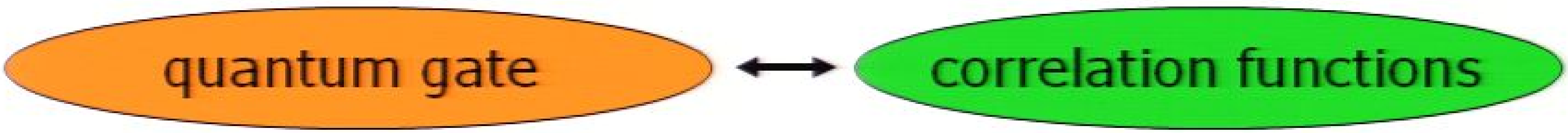


- Measurement outcomes determine a Pauli correction to gate
- Other gates are given by different measurement patterns
- Performance of a gate is quantified by the fidelity of the resulting entangled state

MBQC gates and correlation functions

Approach: Quantum gates as correlation functions

- Order parameters to characterise *phases* which are universal for MBQC



Chung, Bartlett, Doherty
Can. J. Phys. 87, 219 (2009)

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quantum gate ⟷ correlation functions

$$\rho_{\text{pm}} = \sum_J U_J P_J \rho_0 P_J U_J^\dagger$$

Measurement projector

Unitary correction

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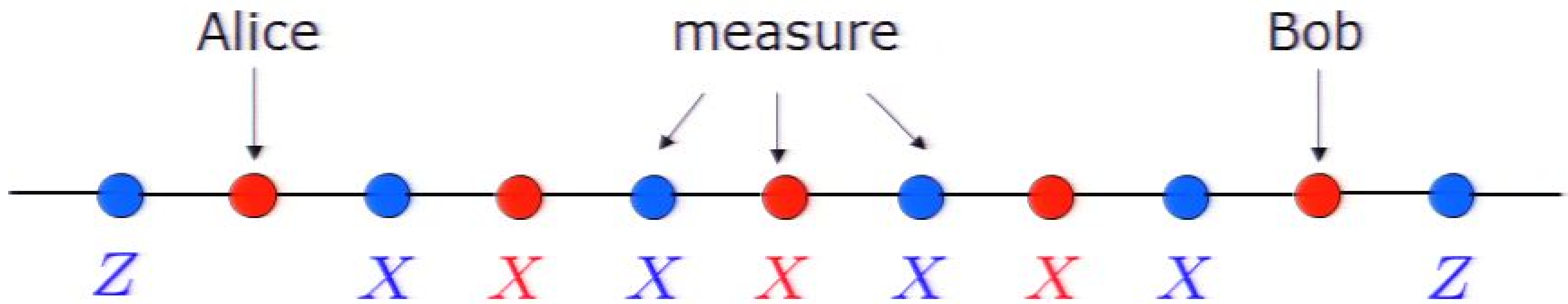
Example: Cluster-state gates



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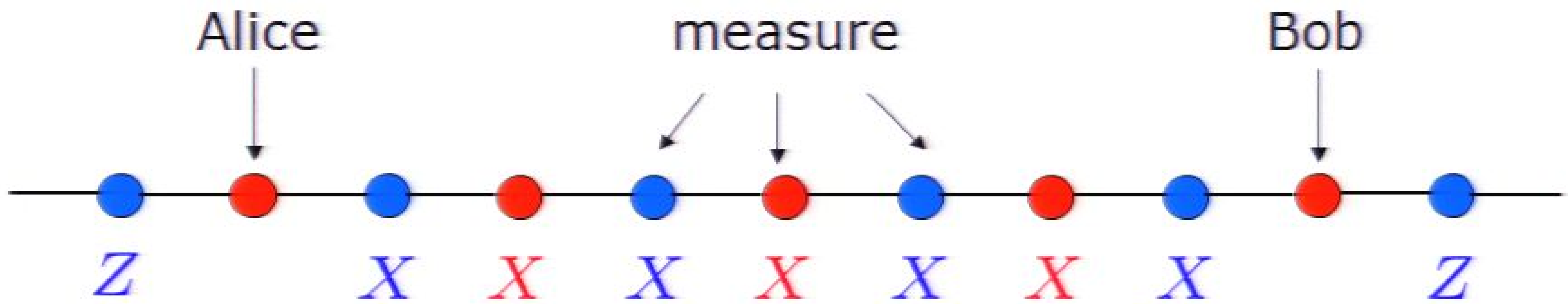

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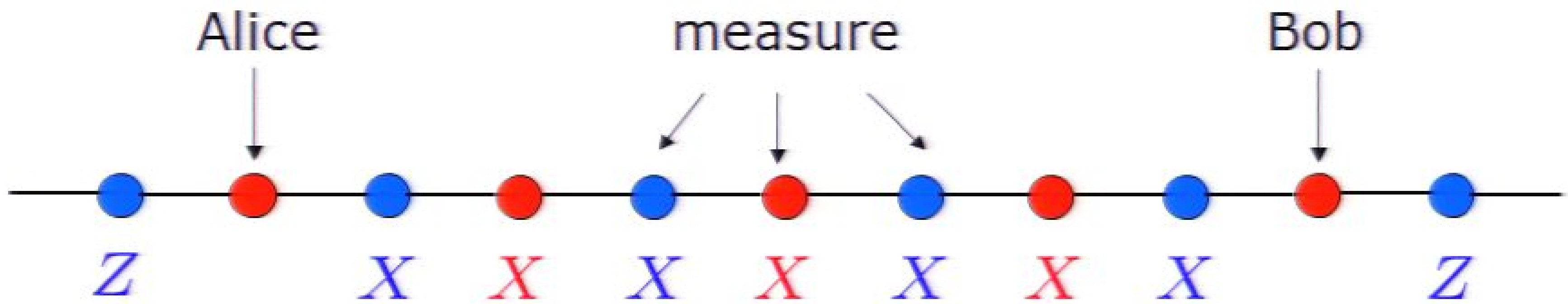

String-like order parameter
Quantifies fidelity of 1-qubit gates

$$\langle XX \rangle_{\text{pm}} = \langle Z_{2i-1} \left(\sum_{k=i}^{k=j-1} X_{2k} \right) Z_{2j-1} \rangle$$

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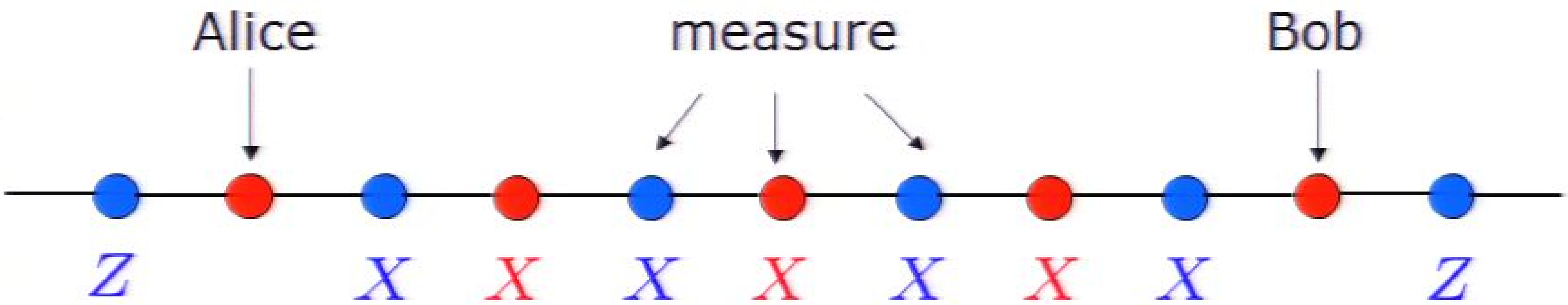

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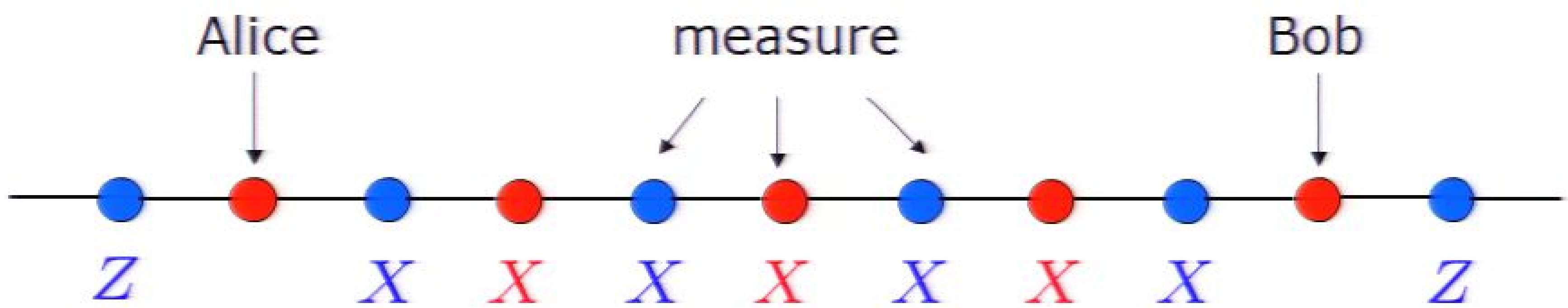

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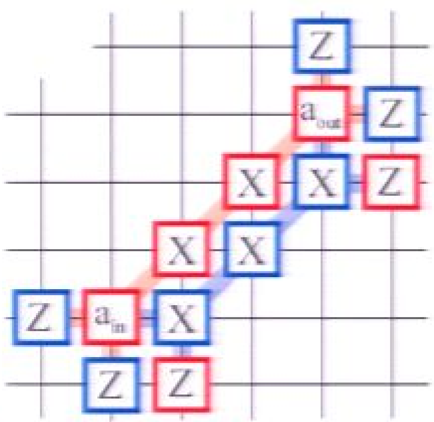

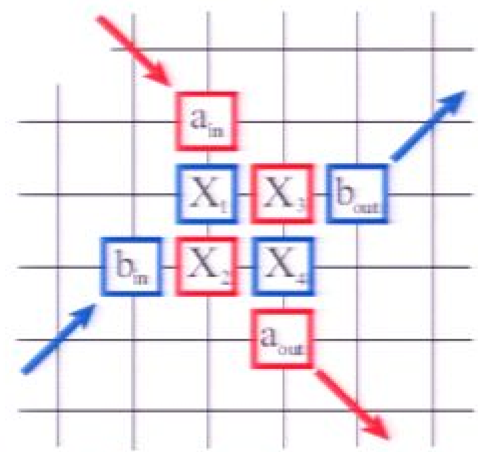

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Interacting strings order parameter
Quantifies CNOT gate fidelity

Cluster Hamiltonian with transverse field

$$H = -\sum_{\text{sites } i} K_i + BX_i$$

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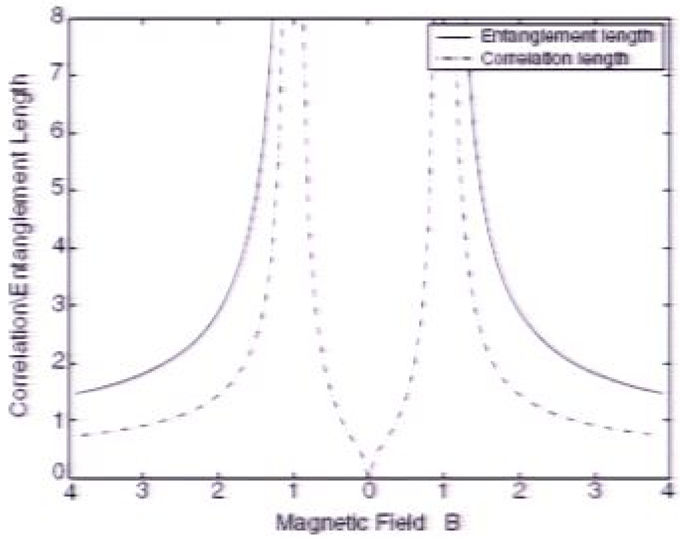

Pachos and Plenio
PRL 93, 056402 (2004)

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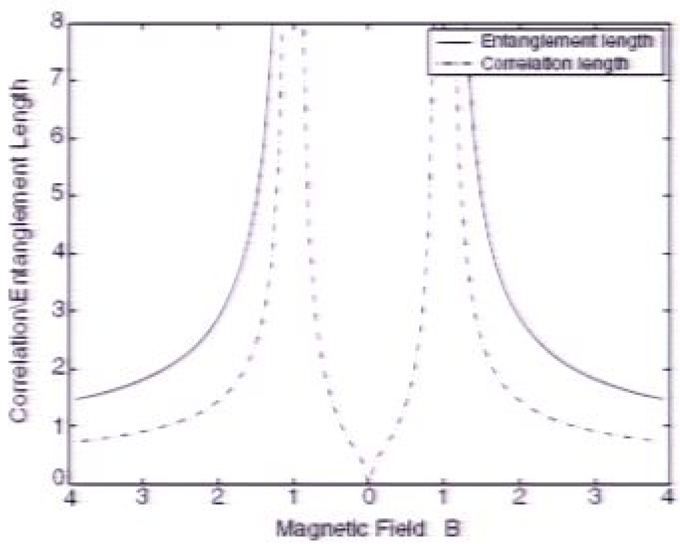

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2D: Universal gate set for MBQC is long-ranged in B<1 phase

Doherty and Bartlett
PRL 103, 020506 (2009)

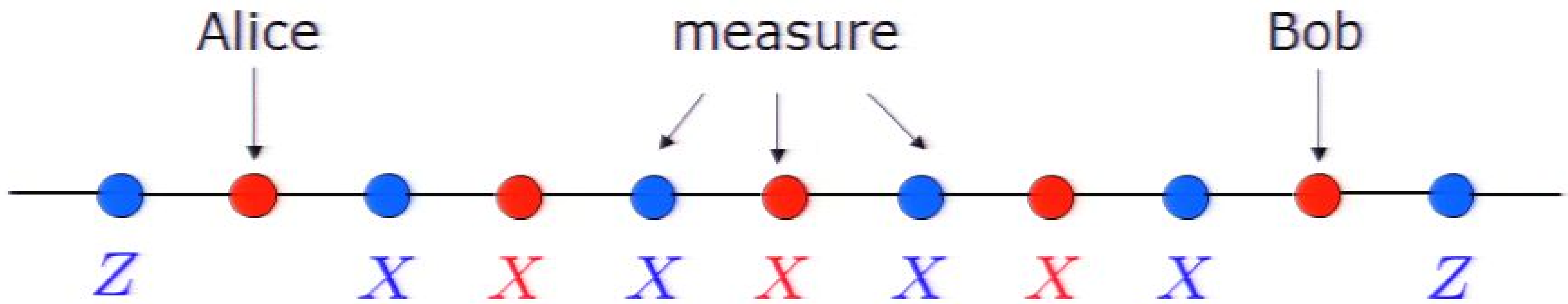
Conclusions 1.0

- Quantum gates – correlation functions – order parameters to identify **universal phases** for MBQC
- MBQC is a new type of long range "string" order

- **Problem:** the `cluster Hamiltonian' is a toy model that does not describe a realistic system

- Can we apply these results to a realistic spin system?

Example: Cluster-state gates



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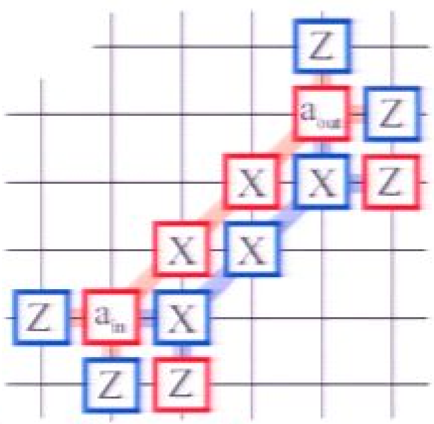

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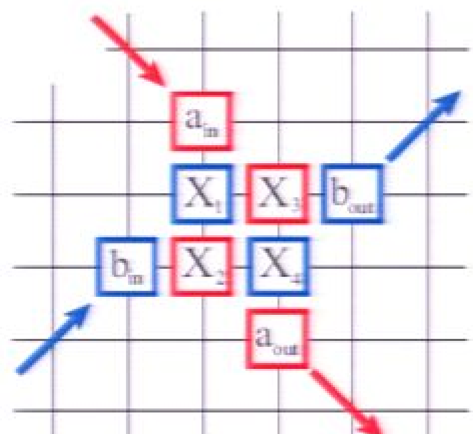

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Identifying phases, part 2

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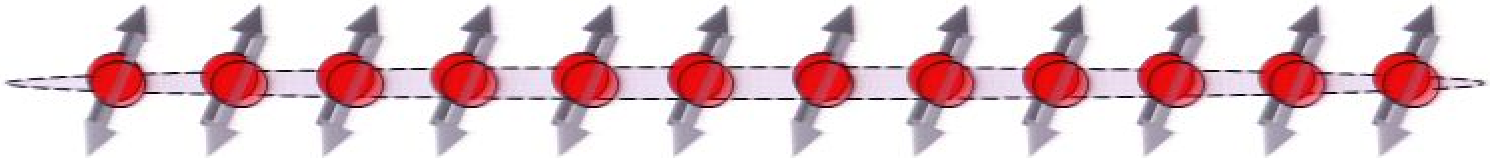

Rotationally-invariant spin chains

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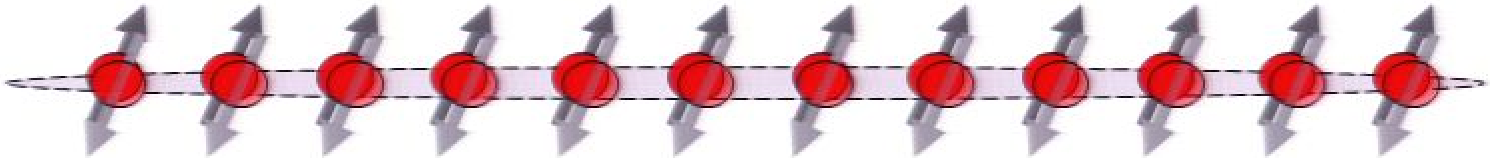

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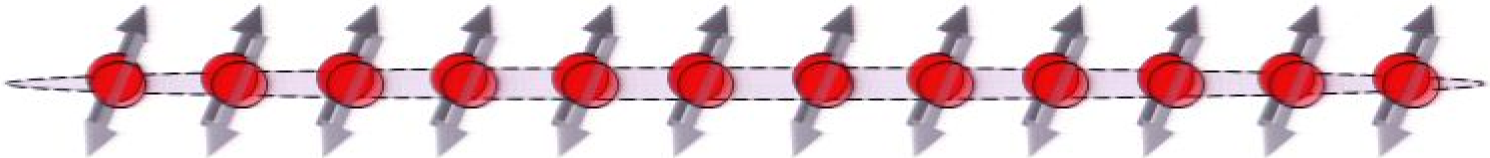

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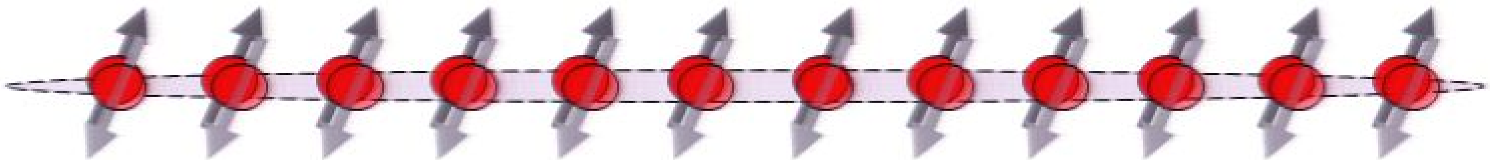

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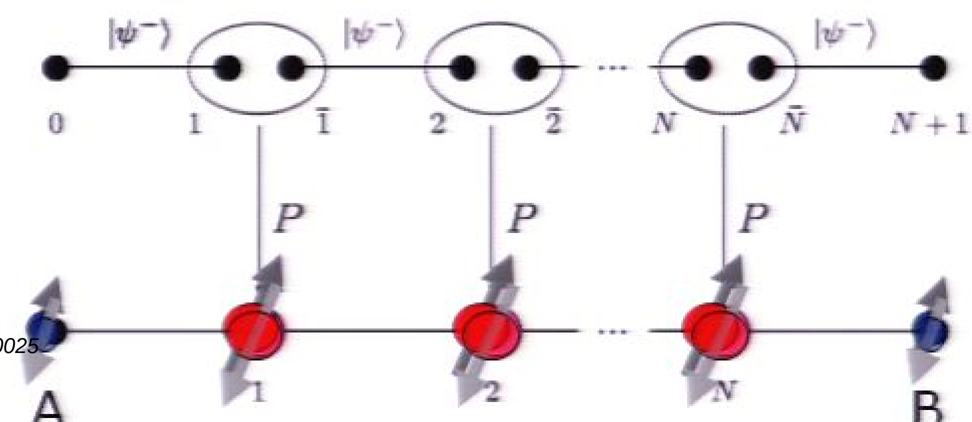
- **Ground state:** valence bond solid / projected entangled pair state (PEPS)



P = triplet projection (isometry)

AKLT as a Matrix Product State

- PEPS yields a Matrix Product State (MPS) description of the AKLT ground state

$$|\psi\rangle = \sum_{\{b_1 \ldots b_n\}} |b_1 \ldots b_n\rangle (\sigma_{b_n} \cdots \sigma_{b_2} \sigma_{b_1})_B |\Psi^-\rangle_{AB}$$

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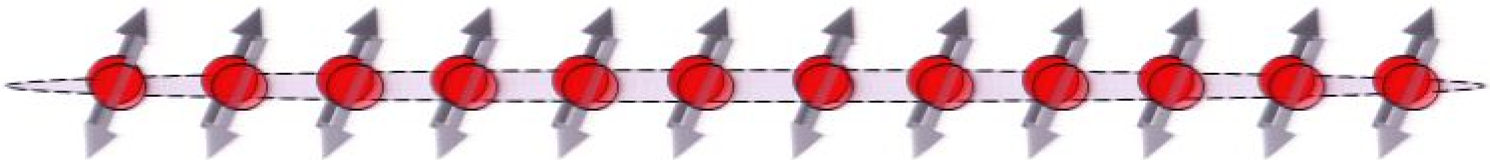

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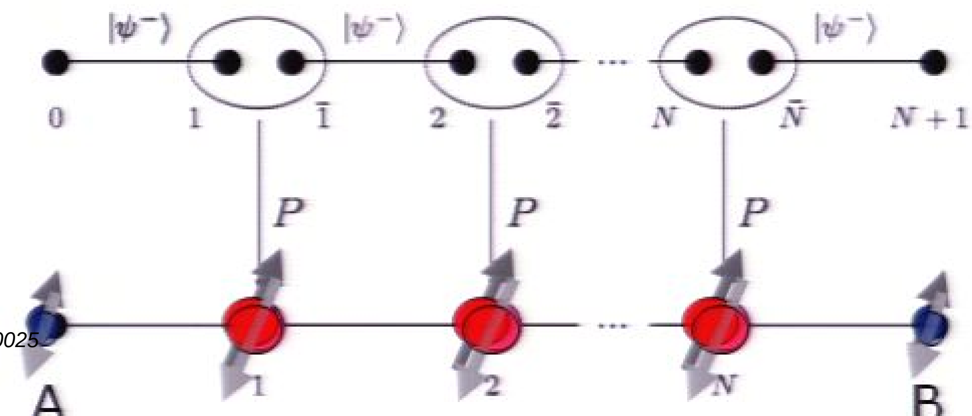

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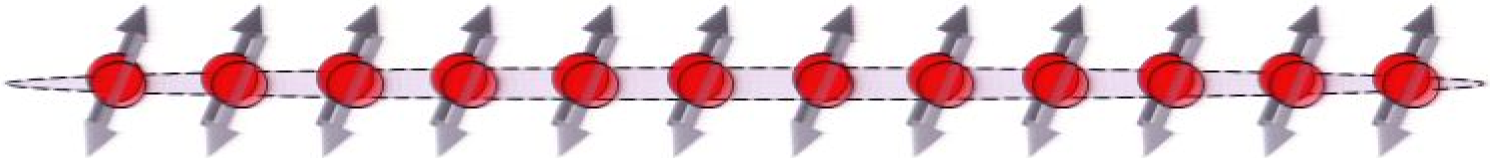

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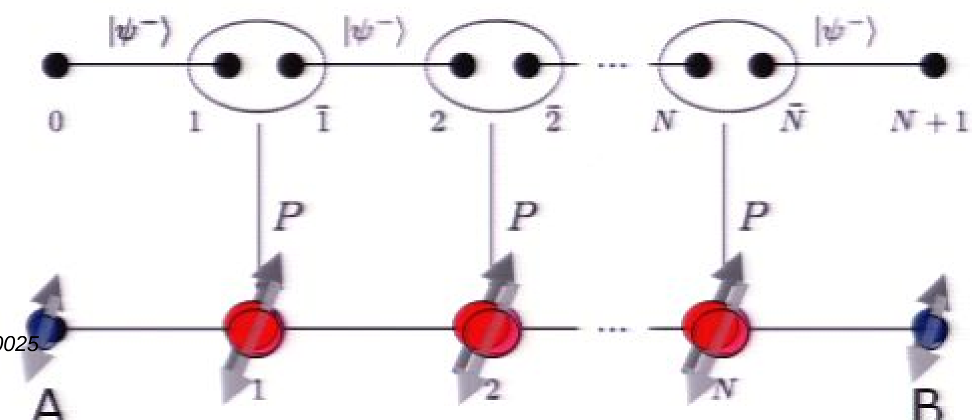

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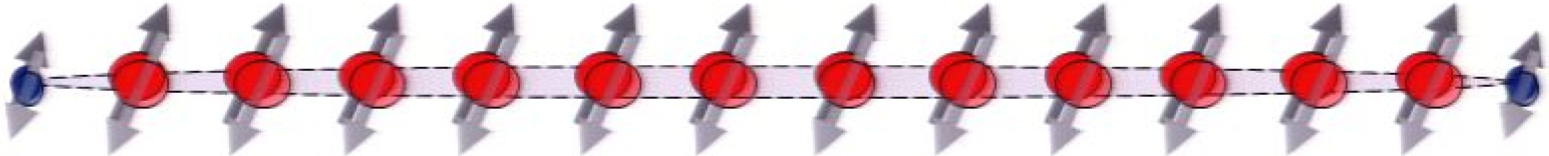

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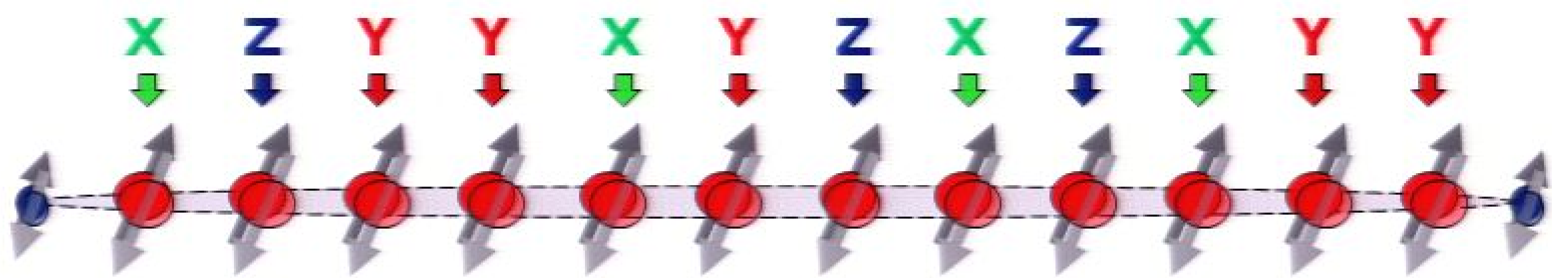

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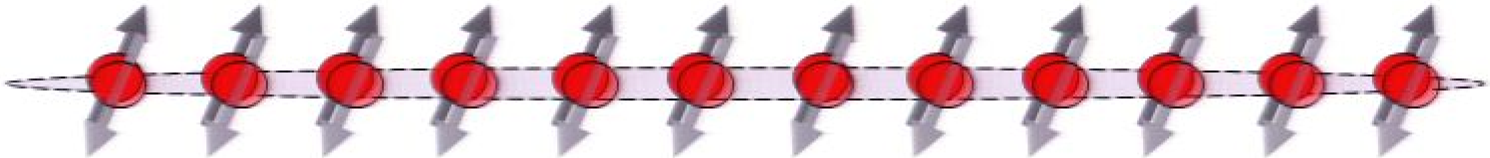

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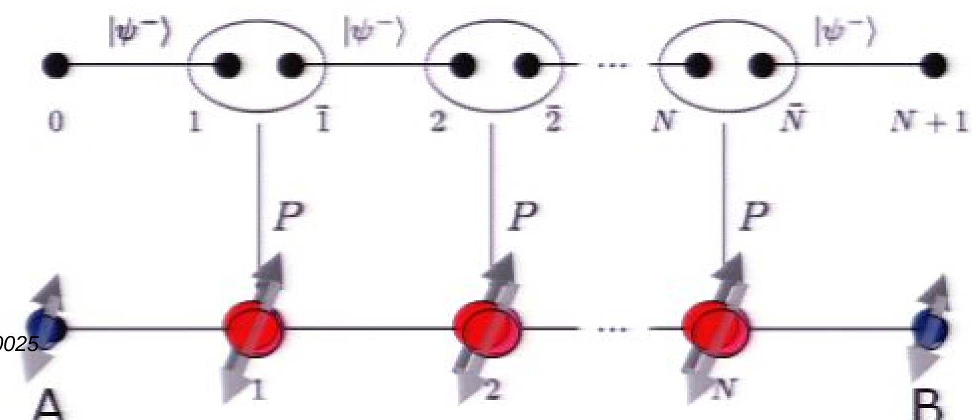

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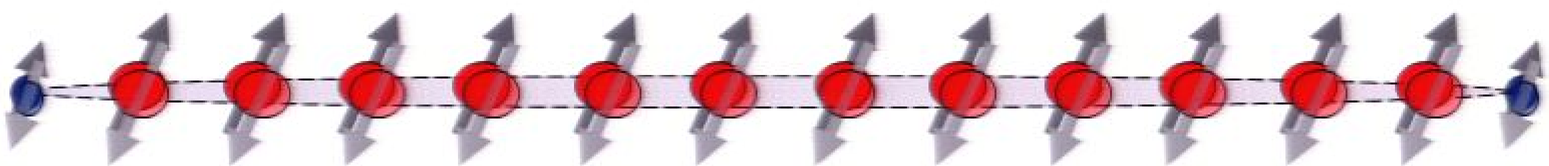

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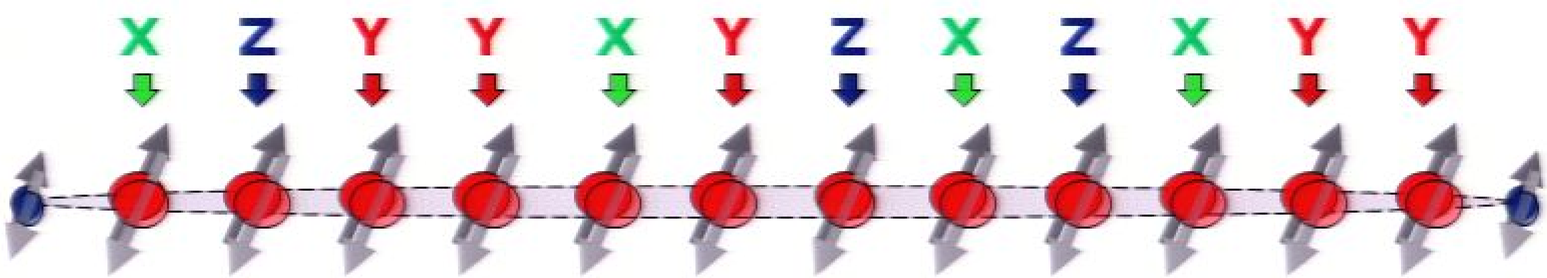

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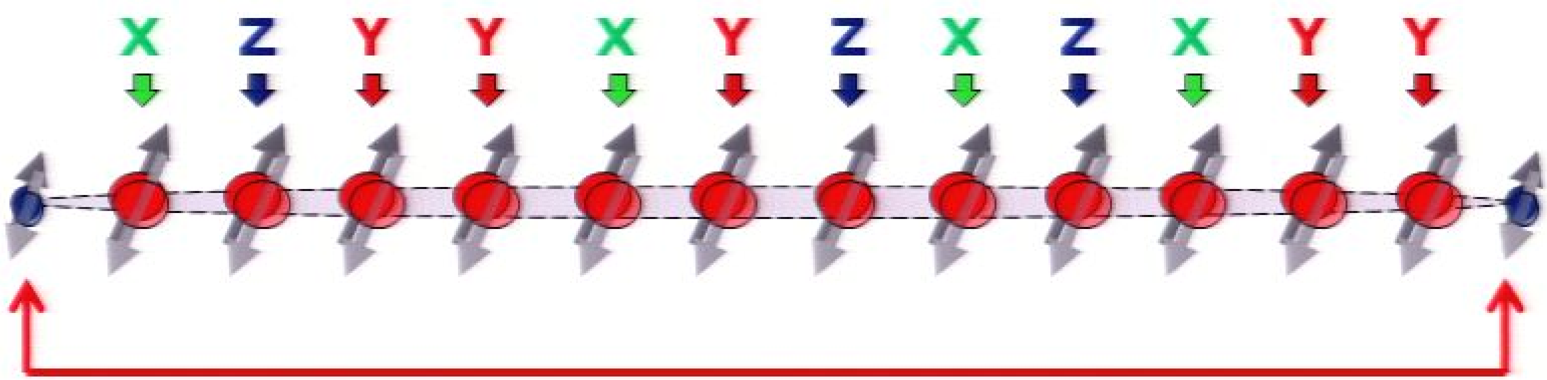

$$(X^{\#x,y} Z^{\#y,z})_B |\Psi^-\rangle_{AB}$$

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- Rotation is probabilistic, but heralded. (First outcome: try again.)

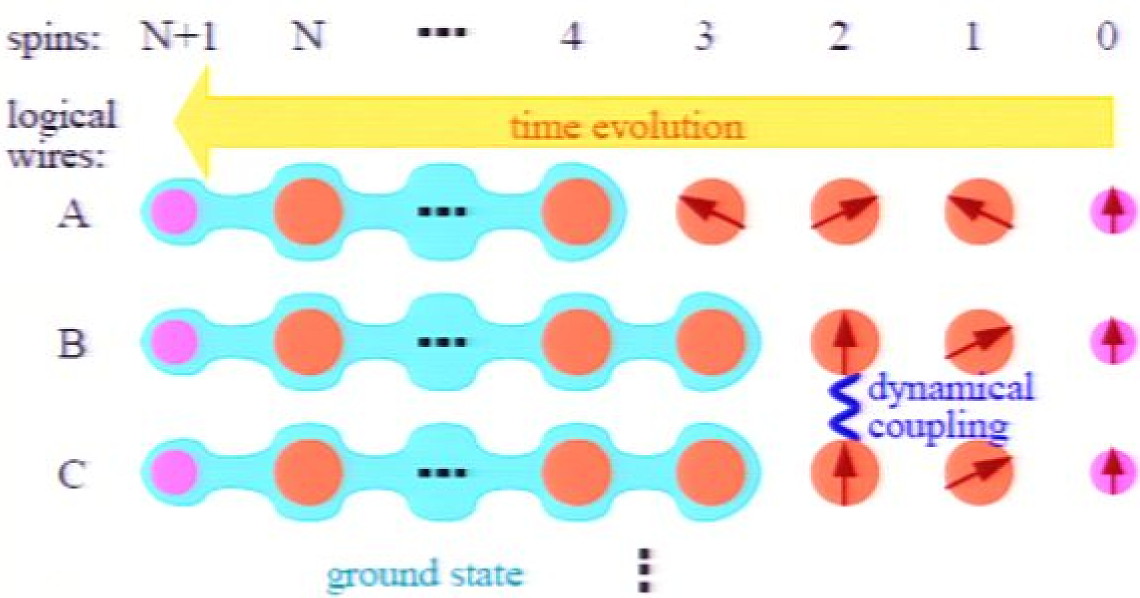
MBQC using AKLT states

PRL **101**, 010502 (2008) PHYSICAL REVIEW LETTERS week ending 4 JULY 2008

Measurement-Based Quantum Computer in the Gapped Ground State of a Two-Body Hamiltonian

Gavin K. Brennen[1,2] and Akimasa Miyake[2,3]
[1]*Department of Physics, Macquarie University, Sydney, NSW 2109, Australia*
[2]*Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Innsbruck, Austria*
[3]*Institute for Theoretical Physics, University of Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria*

(Received 12 March 2008; published 2 July 2008)



- 2-D spin lattice
- Individual chains as quantum comp wires
- Two-qubit logic (CPHASE) by dynamical coupling
- Gap protects against thermal noise

MBQC with AKLT

- AKLT state has two key properties:
  - **Simple:** algebraic properties of MPS make MBQC clear and intuitive
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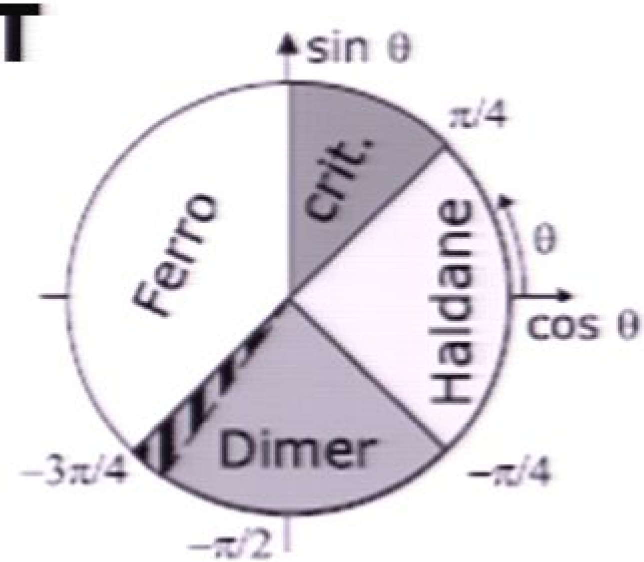

Romero-Isart, Eckert, Sanpera
PRA 75, 050303(R) (2007)

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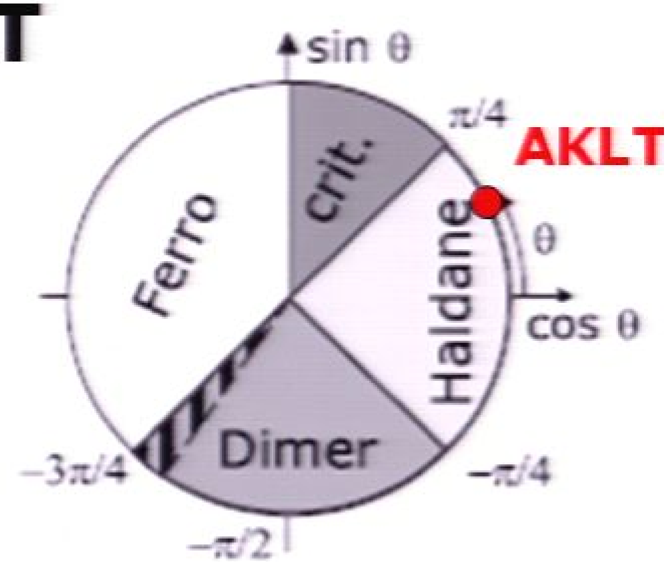

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- Haldane phase is gapped, so thermal protection would persist

- Is MBQC a **robust** feature of ground states in the entire Haldane phase?

MBQC gates and correlation functions

Approach: Quantum gates as correlation functions

- Order parameters to characterise *phases* which are universal for MBQC

quantum gate ⟷ correlation functions

Measurement projector

$$\rho = \sum_J U_J P_J \rho_0 P_J U_J^\dagger$$

Unitary correction

$$\langle AB \rangle = \sum_J \mathrm{Tr}[(AB) U_J P_J \rho_0 P_J U_J^\dagger] = \sum_J \mathrm{Tr}[(AB_J) P_J \rho_0 P_J]$$

Magic: $(AB_J)P_J = P_J A S B'$

$$B_J = U_J B U_J^\dagger$$

$$\langle AB \rangle = \mathrm{Tr}[(ASB')\rho_0]$$

Chung, Bartlett, Doherty
Can. J. Phys. 87, 219 (2009)

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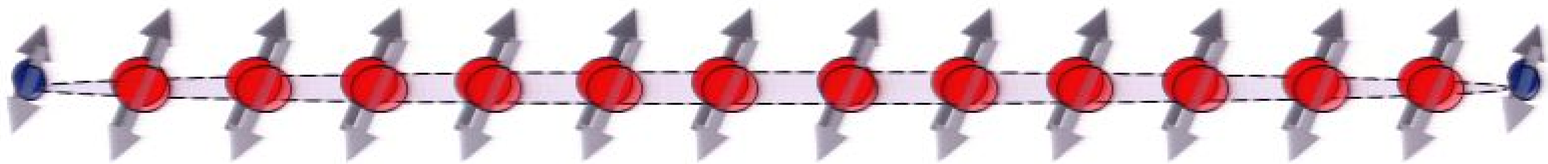

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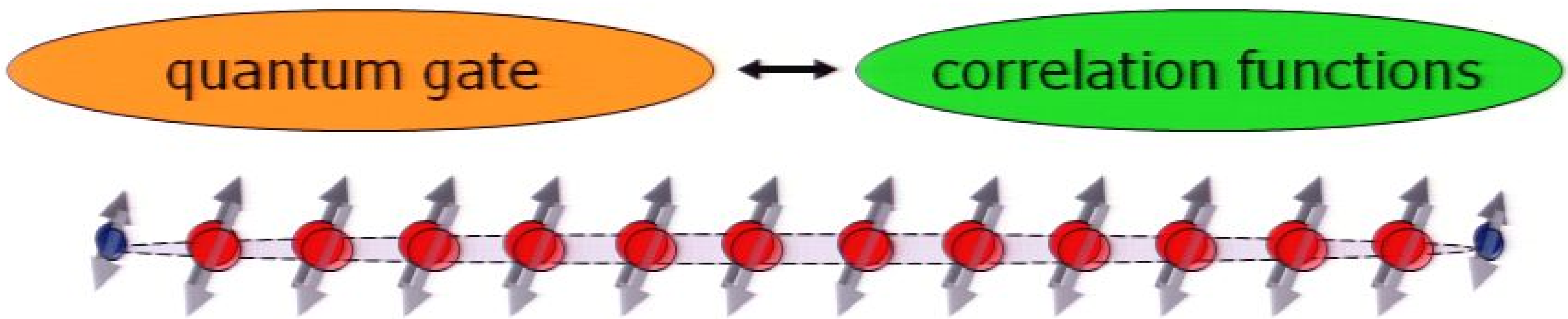

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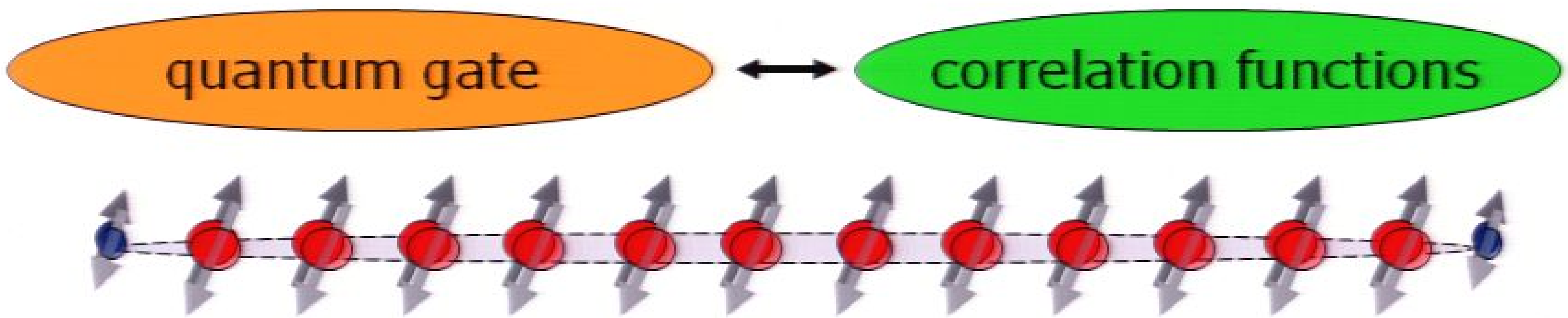

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Remarkable result: due to rotational-invariance of our Hamiltonian, these order parameters are 1 at any length, for any ground state in the Haldane phase

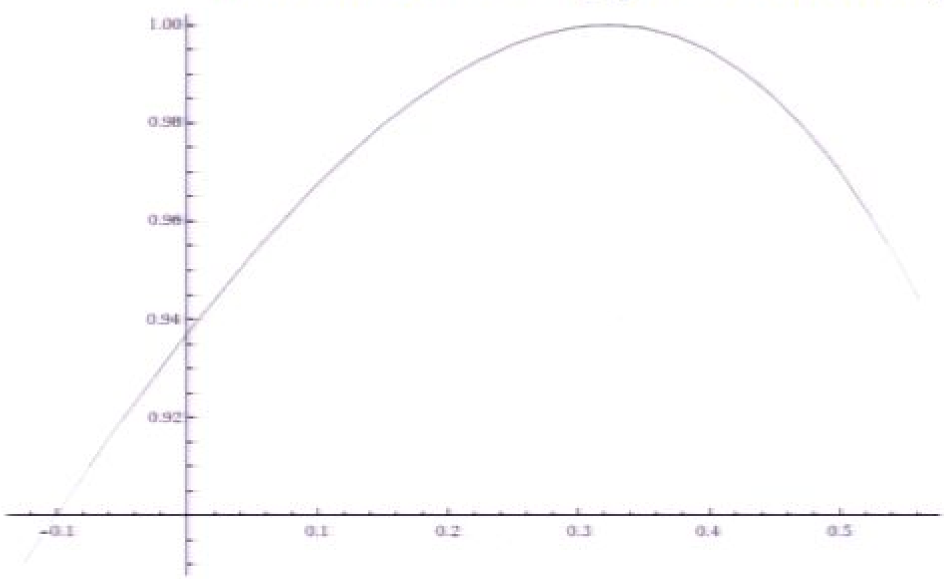
See *Popp et al., PRA 71, 042306 (2005)*

Order parameters exist to quantify the identity gate, and are long-ranged in the Haldane phase

Numerical results

- Find ground states of the open/fixed chain using DMRG
- Localizable entanglement = 1 throughout Haldane phase
- Rotation fidelity drops off slowly with theta

Rotated XX vs θ ($\pi/4$ Z-rotation)



Why are rotations failing?

- Re-express the Hamiltonian as

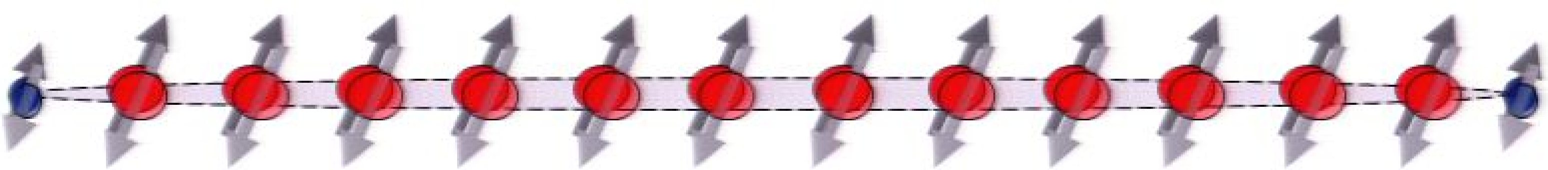
$$H(\theta_{\mathrm{AKLT}}+\varepsilon)=H_{\mathrm{AKLT}}+\tfrac{5}{6}\varepsilon\sum_{j=1}^{n}\mathrm{SWAP}_{j,j+1}$$

- Ansatz:

$$|\mathrm{gs}(\theta)\rangle \simeq \alpha|\mathrm{AKLT}\rangle+\sum_{j=1}^{n}\beta_j\mathrm{SWAP}_{j,j+1}|\mathrm{AKLT}\rangle$$

- Identity gate: SWAP-invariant, only total number of X, Y, Z's matter

- Rotation gates: buffer them against SWAP errors



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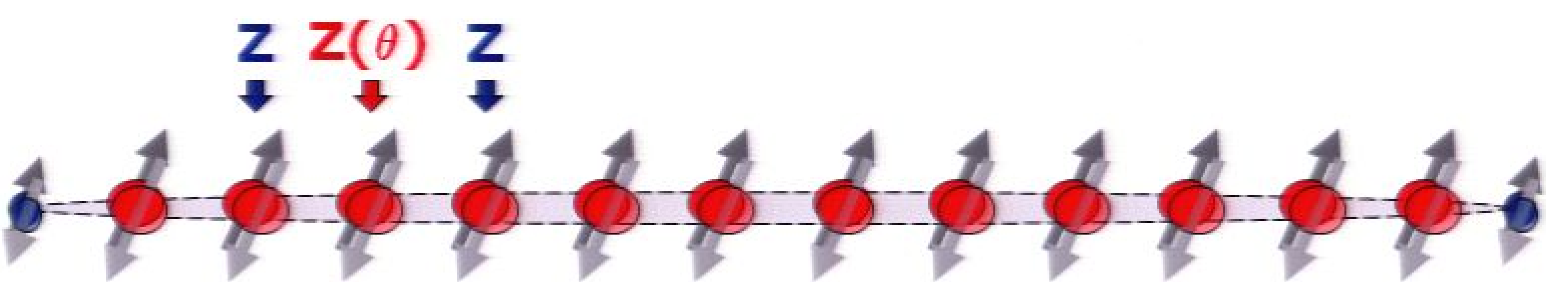

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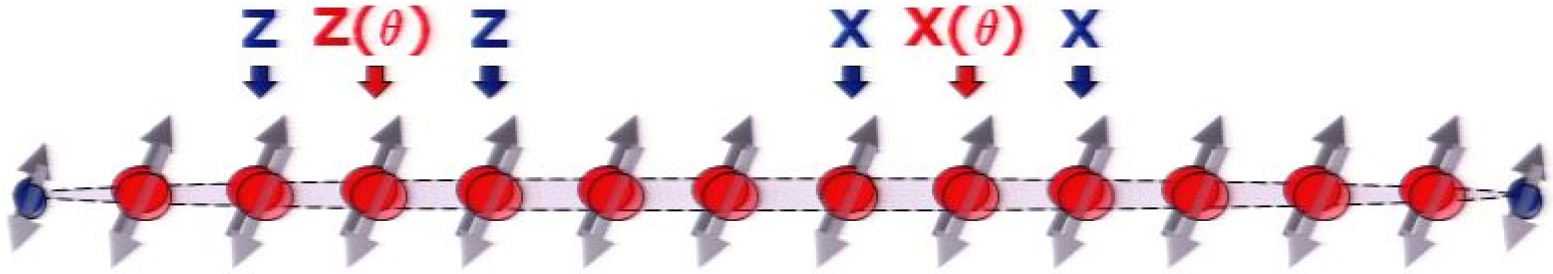
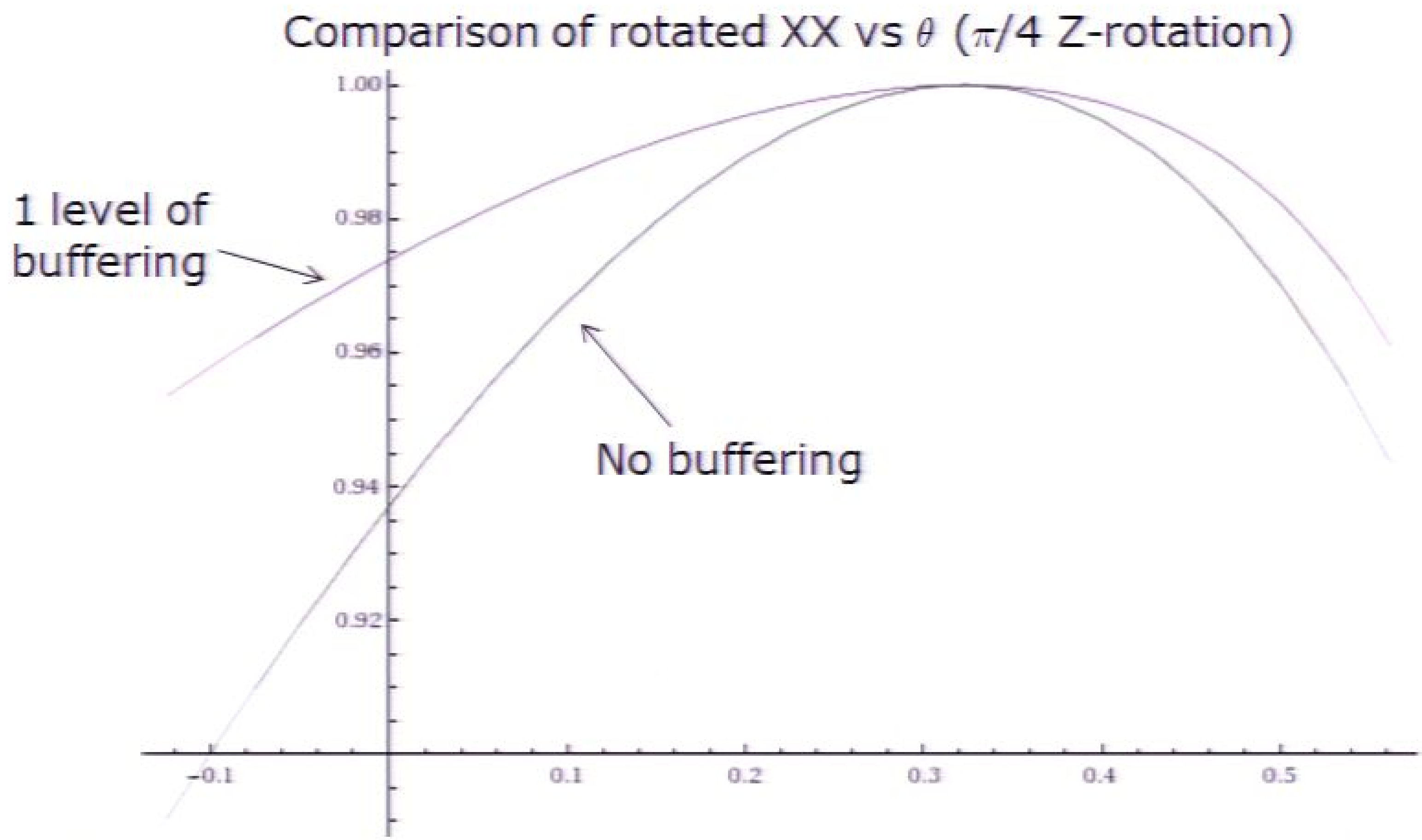


Buffered rotations



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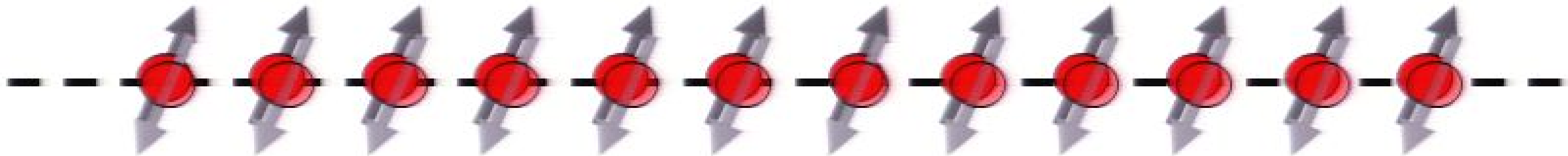

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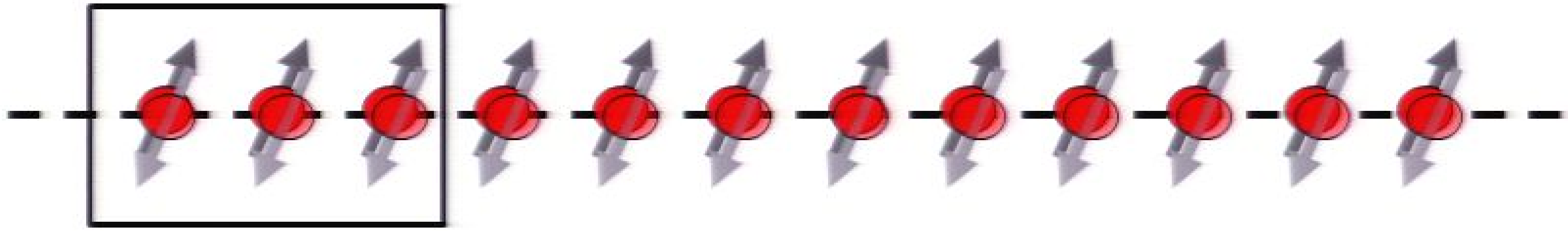

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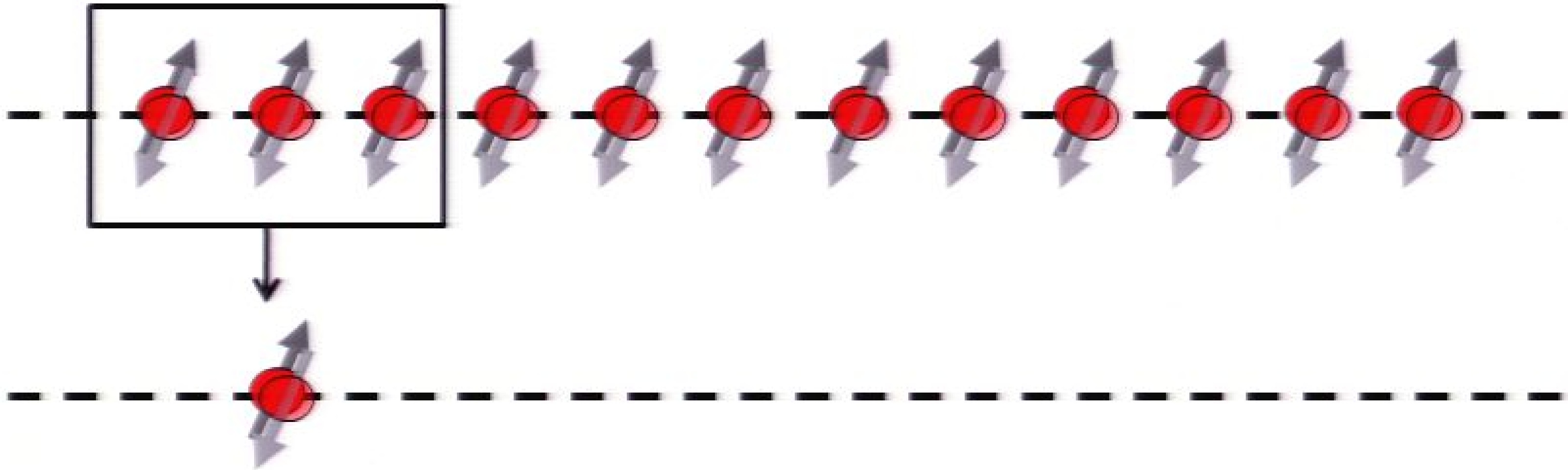

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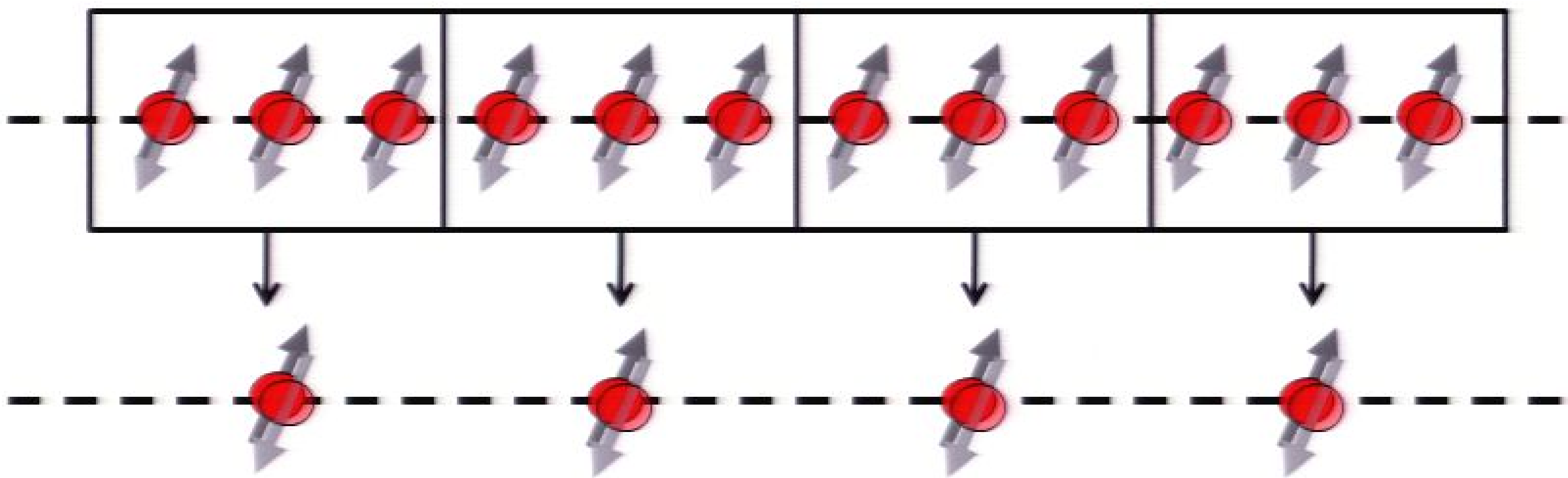

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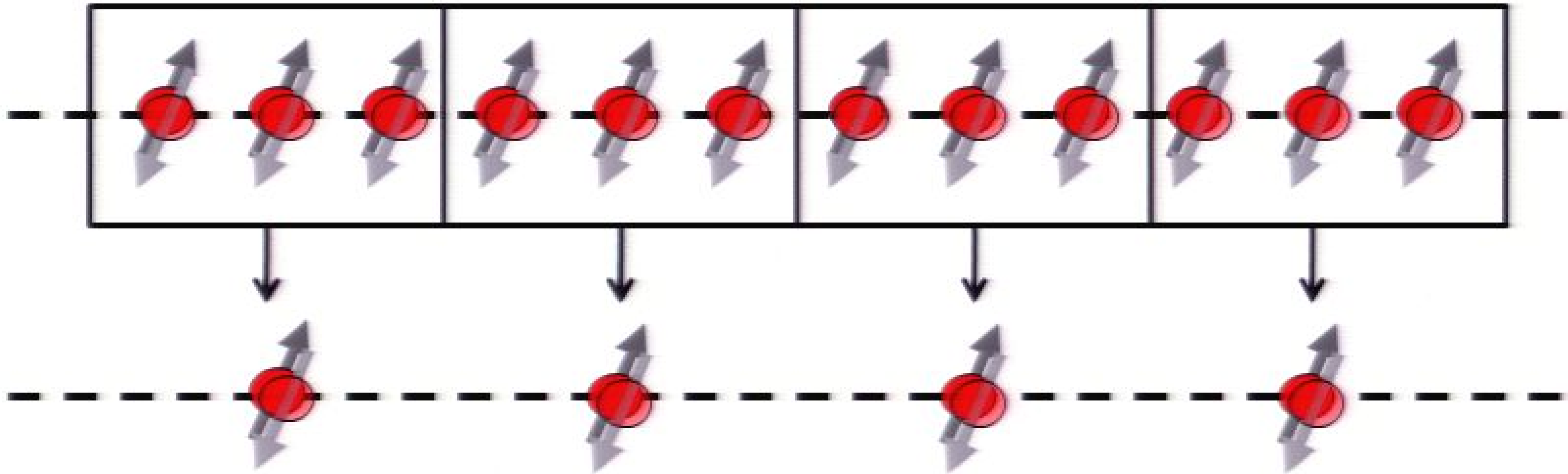

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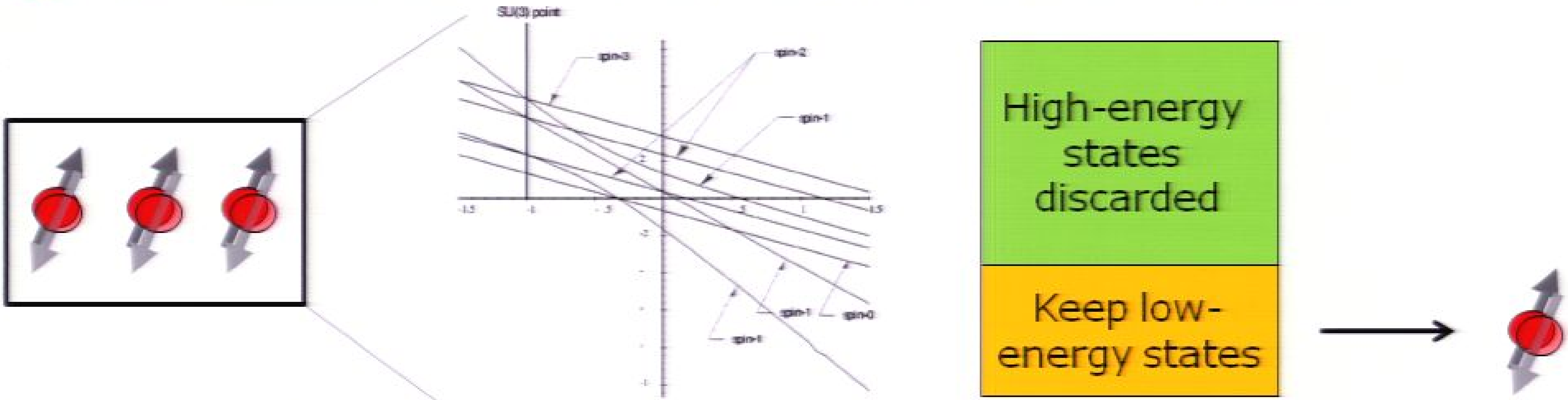

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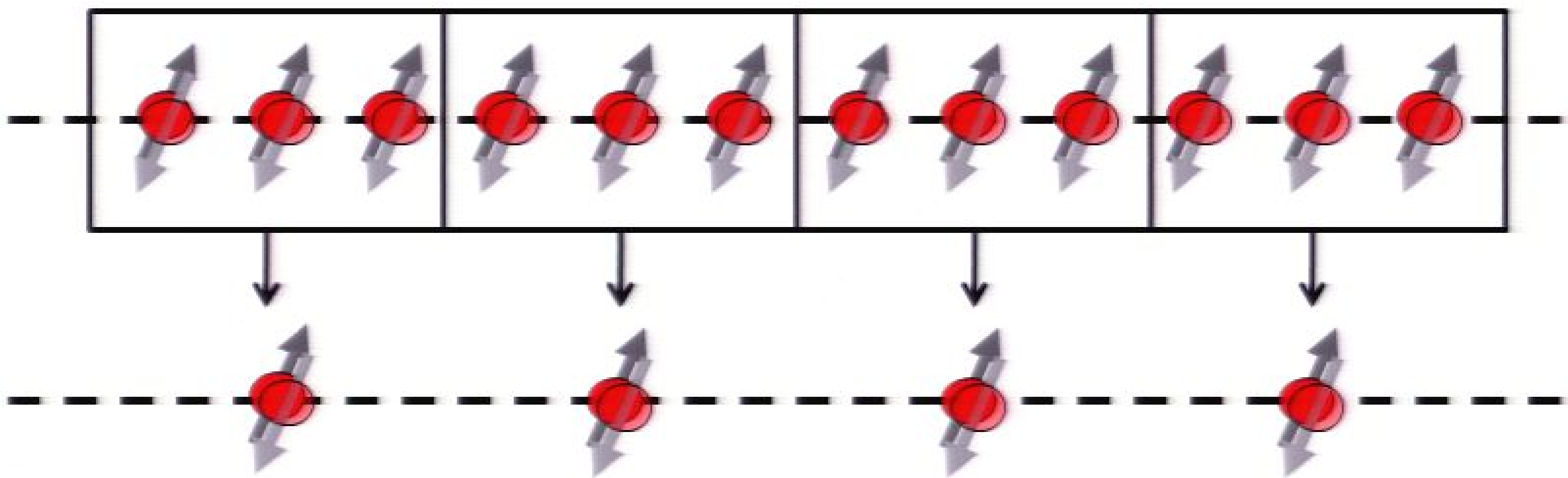

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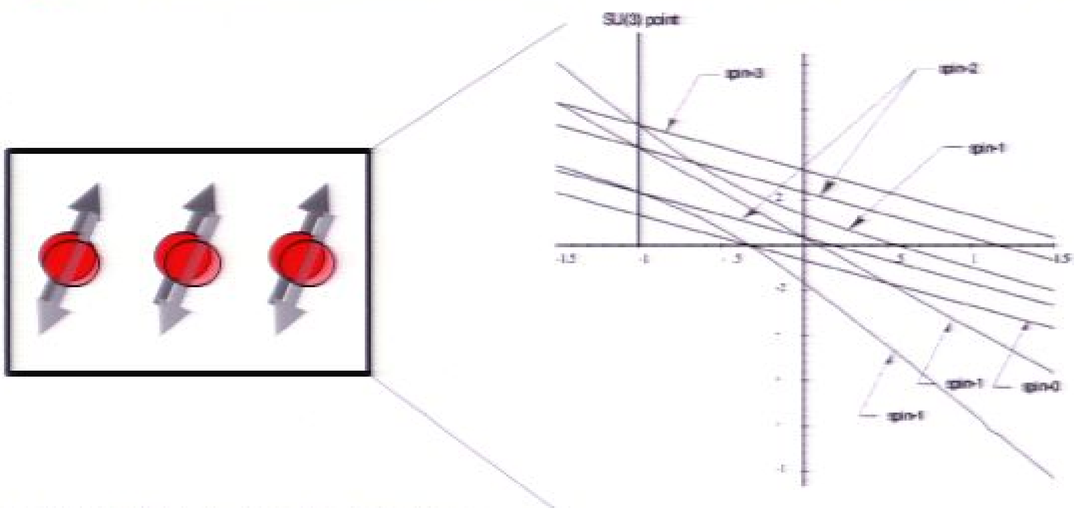

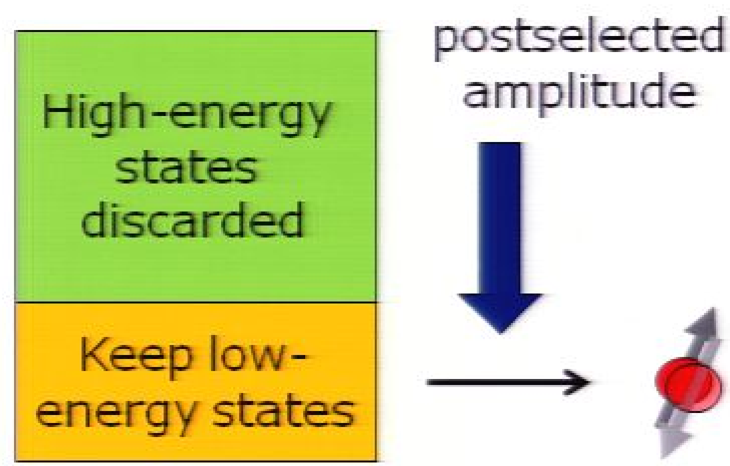

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Conclusions 2.0

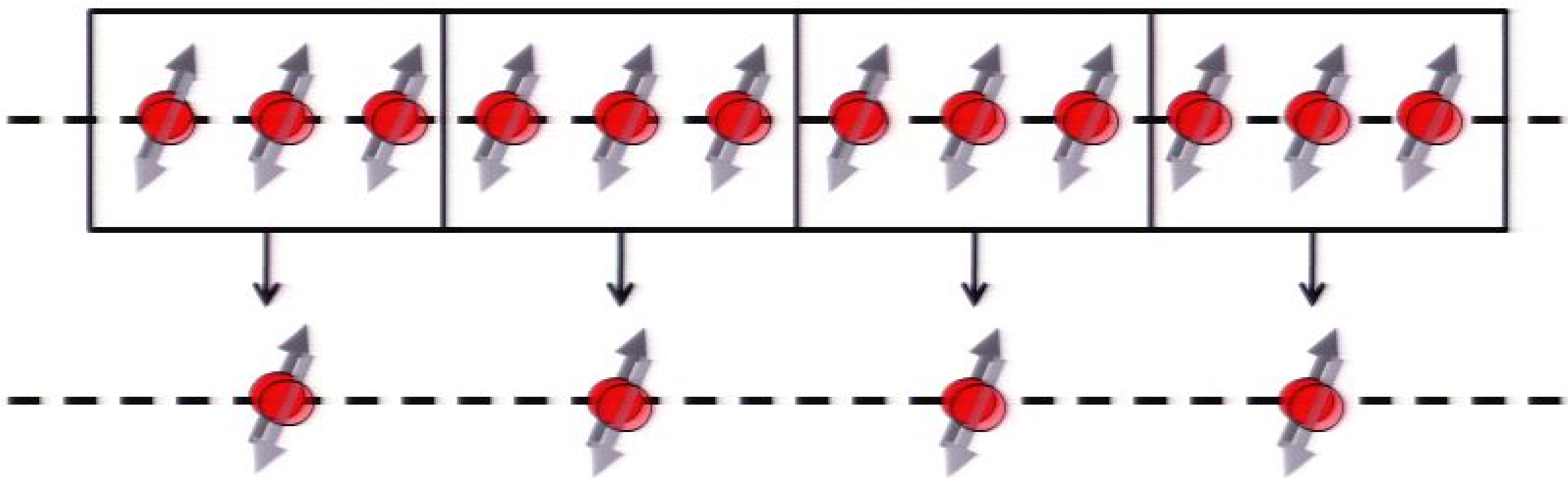
- Quantum gates – correlation functions – order parameters to identify **universal phases** for MBQC
- MBQC is a new type of long range "string" order

- Local filtering can **renormalize** the system – **general?**
- Short-ranged "errors" removed (probabilistically)
- Long-ranged MBQC order retained

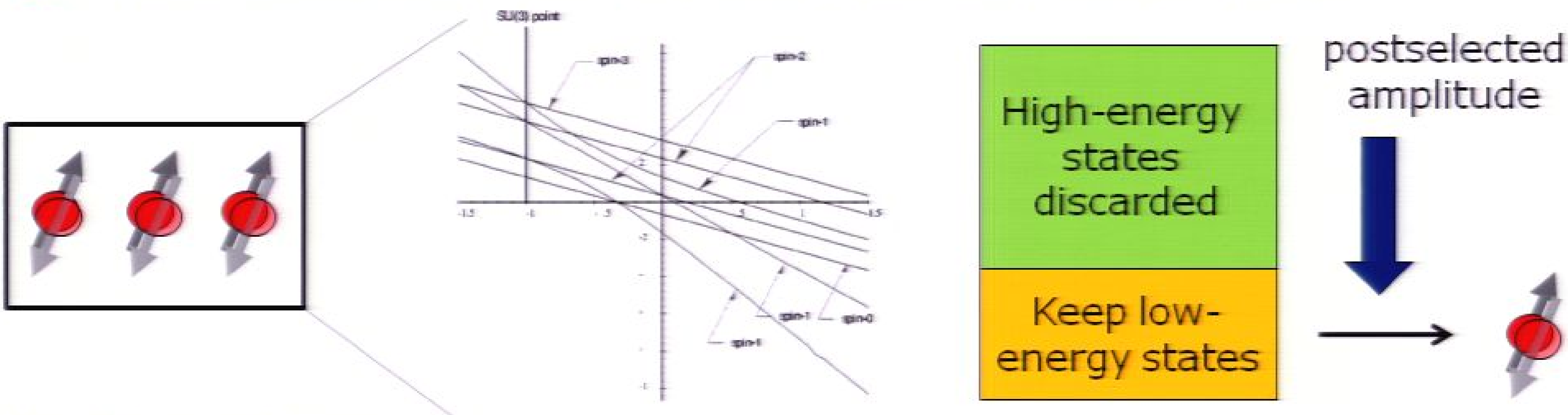
Outlook

- Finite temperature
- Higher dimensions
- Adiabatic gate teleportation vs MBQC

Buffering as renormalization?

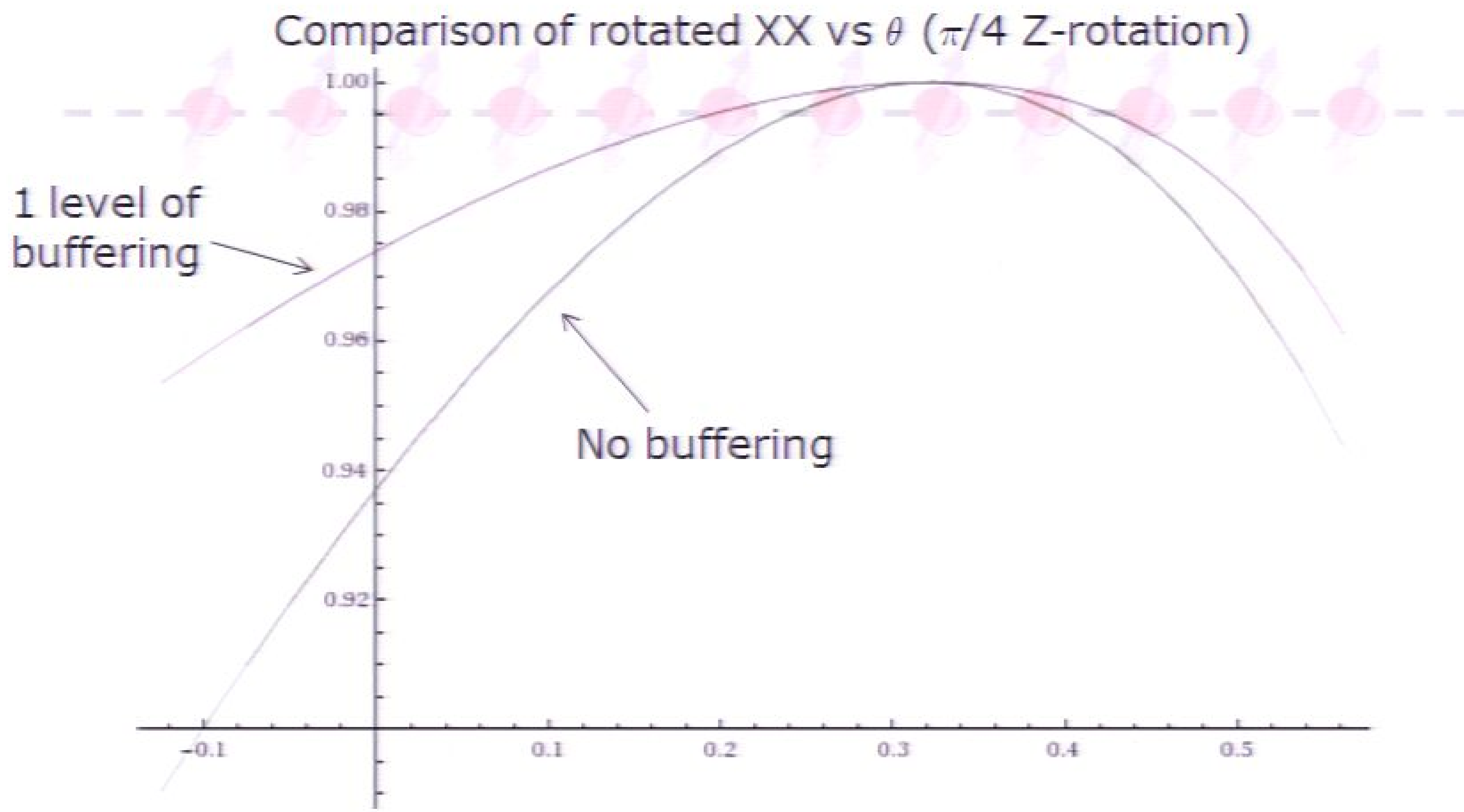


- Renormalization – AKLT as fixed point for Haldane phase*



* actually much more complicated. See *Weinstein PRB 63, 174421 (2001)*

Buffered rotations



Why are rotations failing?

- Re-express the Hamiltonian as

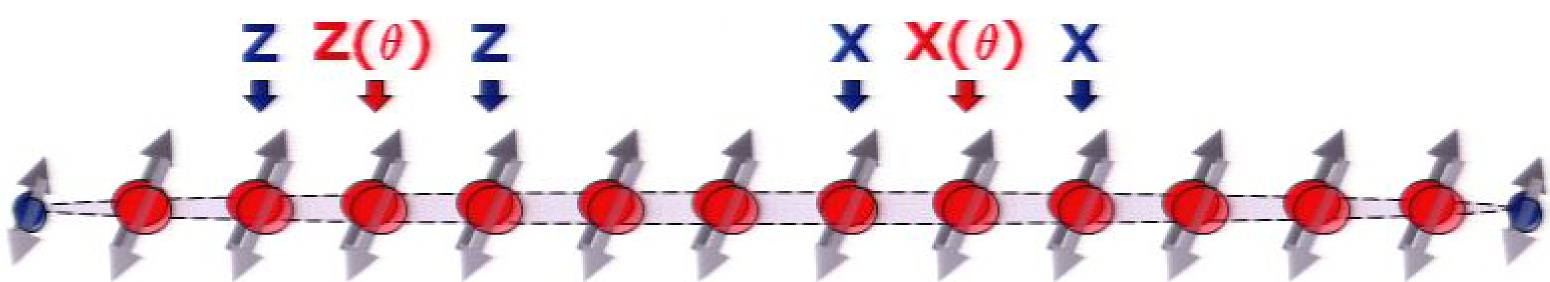
$$H(\theta_{\mathrm{AKLT}}+\varepsilon) = H_{\mathrm{AKLT}} + \tfrac{5}{6}\varepsilon \sum_{j=1}^{n} \mathrm{SWAP}_{j,j+1}$$

- Ansatz:

$$|\mathrm{gs}(\theta)\rangle \simeq \alpha|\mathrm{AKLT}\rangle + \sum_{j=1}^{n} \beta_j \mathrm{SWAP}_{j,j+1}|\mathrm{AKLT}\rangle$$

- Identity gate: SWAP-invariant, only total number of X, Y, Z's matter

- Rotation gates: buffer them against SWAP errors



MBQC gates and correlation functions

Approach: Quantum gates as correlation functions

- Order parameters to characterise *phases* which are universal for MBQC

quantum gate ⟷ correlation functions

Measurement projector

$$\rho = \sum_J U_J P_J \rho_0 P_J U_J^\dagger$$

Unitary correction

$$\langle AB \rangle = \sum_J \mathrm{Tr}[(AB) U_J P_J \rho_0 P_J U_J^\dagger] = \sum_J \mathrm{Tr}[(AB_J) P_J \rho_0 P_J]$$

Magic: $(AB_J)P_J = P_J A S B'$

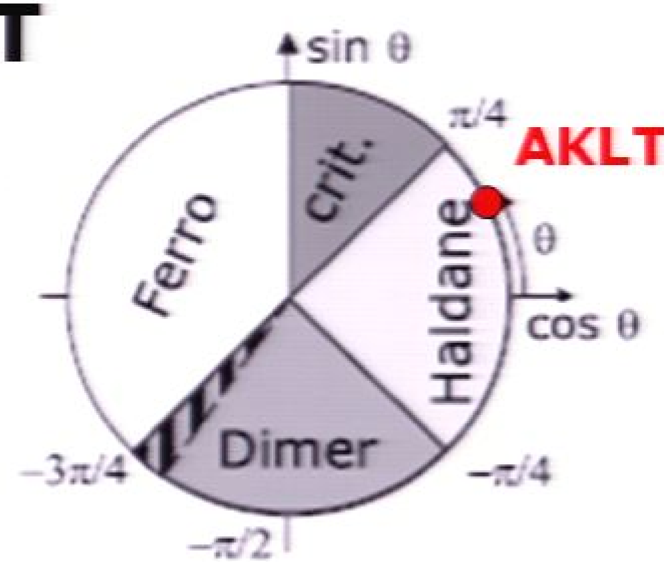
$$B_J = U_J B U_J^\dagger$$

$$\langle AB \rangle = \mathrm{Tr}[(ASB')\rho_0]$$

Chung, Bartlett, Doherty
Can. J. Phys. 87, 219 (2009)

MBQC with AKLT

- AKLT state has two key properties:
  - **Simple:** algebraic properties of MPS make MBQC clear and intuitive
  - **Realistic:** some physical systems are well described by it



Romero-Isart, Eckert, Sanpera
PRA 75, 050303(R) (2007)

- The AKLT point is part of an ordered phase, that also contains the 'Heisenberg point'

$$H = \sum_i \cos\theta(\vec{S}_i \cdot \vec{S}_{i+1}) + \sin\theta(\vec{S}_i \cdot \vec{S}_{i+1})^2 \qquad \tan\theta_{\text{AKLT}} = 1/3$$

- Haldane phase is gapped, so thermal protection would persist

- Is MBQC a **robust** feature of ground states in the entire Haldane phase?

MBQC with AKLT

Single-qubit rotations on the AKLT state

- PEPS yields a Matrix Product State (MPS) description of the AKLT ground state

$$|\psi\rangle = \sum_{\{b_1 \ldots b_n\}} |b_1 \ldots b_n\rangle (\sigma_{b_n} \cdots \sigma_{b_2} \sigma_{b_1})_B |\Psi^-\rangle_{AB}$$

$$|x_\theta\rangle = |x\rangle \qquad X$$

$$|y_\theta\rangle = \cos\theta/2|y\rangle + i\sin\theta/2|z\rangle \qquad XZR_x(\theta)$$

$$|z_\theta\rangle = i\sin\theta/2|y\rangle + \cos\theta/2|z\rangle \qquad ZR_x(\theta)$$

- Rotation is probabilistic, but heralded. (First outcome: try again.)

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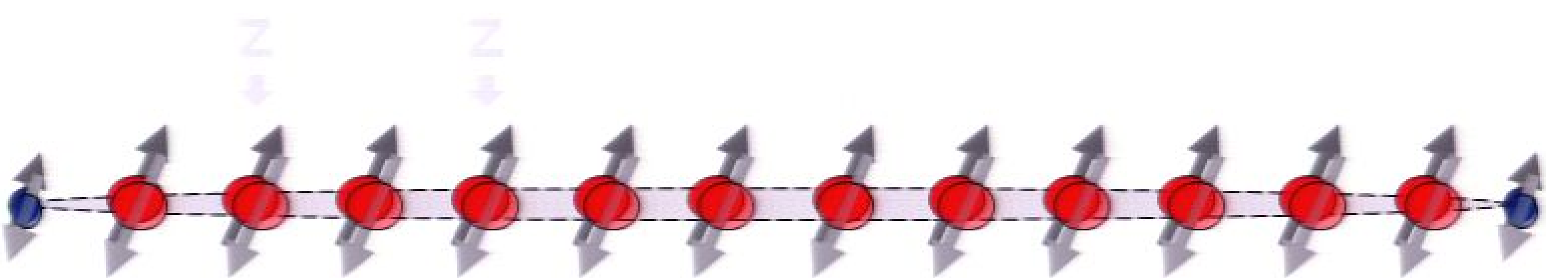

Why are rotations failing?

- Re-express the Hamiltonian as

$$H(\theta_{\text{AKLT}} + \varepsilon) = H_{\text{AKLT}} + \tfrac{5}{6}\varepsilon \sum_{j=1}^{n} \text{SWAP}_{j,j+1}$$

- Ansatz:

$$|\text{gs}(\theta)\rangle \simeq \alpha|\text{AKLT}\rangle + \sum_{j=1}^{n} \beta_j \text{SWAP}_{j,j+1}|\text{AKLT}\rangle$$

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