Title: Cosmological hydrogen recombination: the effect of very high-n states and quadrupole transitions.

Date: Nov 24, 2009 02:00 PM

URL: http://pirsa.org/09110024

Abstract: Thanks to the ongoing Planck mission, a new window will be opened on the properties of the primordial density field, the cosmological parameters, and the physics of reionization. Much of Planck's new leverage on these quantities will come from temperature measurements at small angular scales and from polarization measurements. These both depend on the details of cosmological hydrogen recombination; use of the CMB as a probe of energies greater than 10^16 GeV compels us to get the ~eV scale atomic physics right.

One question that remains is how high in hydrogen principle quantum number we have to go to make sufficiently accurate predictions for Planck. Using sparse matrix methods to beat computational difficulties, I have modeled the influence of very high (up to and including n=200) excitation states of atomic hydrogen on the recombination history of the primordial plasma, resolving all angular momentum sub-states separately and including, for the first time, the effect of hydrogen quadrupole transitions. I will review the basic physics, explain the resulting plasma properties, discuss recombination histories, and close by discussing the effects on CMB observables.

COSMOLOGICAL HYDROGEN RECOMBINATION:
The effect of extremely high-n states and forbidden transitions

arXiv:0911.1359, submitted to Phys. Rev. D.

Daniel Grin

in collaboration with Christopher M. Hirata

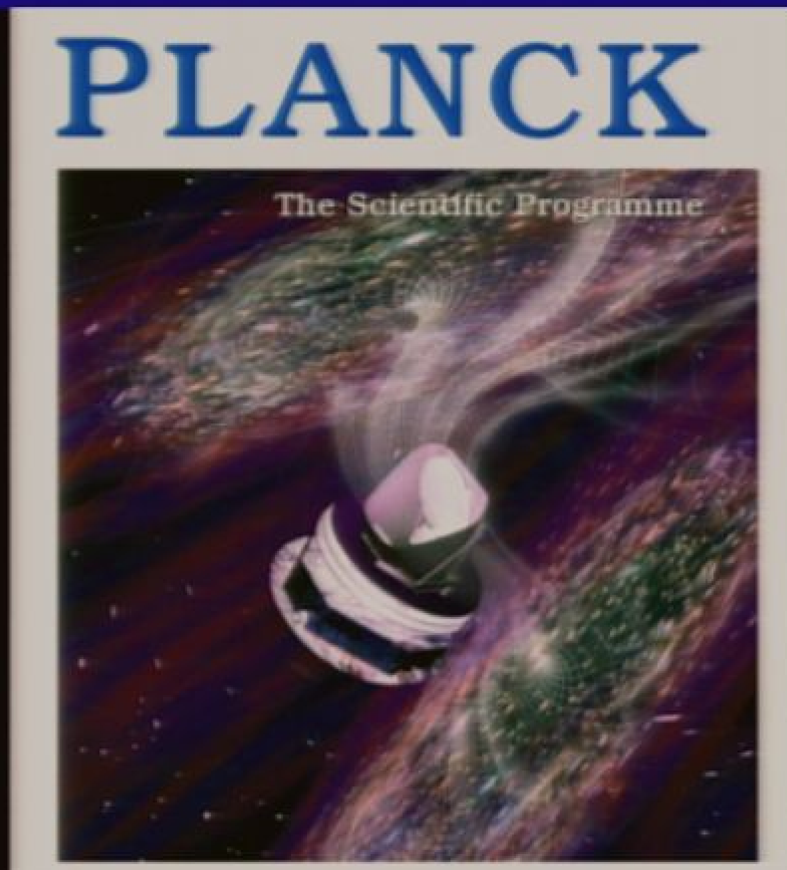
Perimeter Seminar

11/24/09

OUTLINE

- Motivation: CMB anisotropies and recombination spectra
- Recombination in a nutshell
- Breaking the Peebles/RecFAST mold
- RecSparse: a new tool for high-n states
- Forbidden transitions
- Results
- Ongoing/future work

WALK THE PLANCK

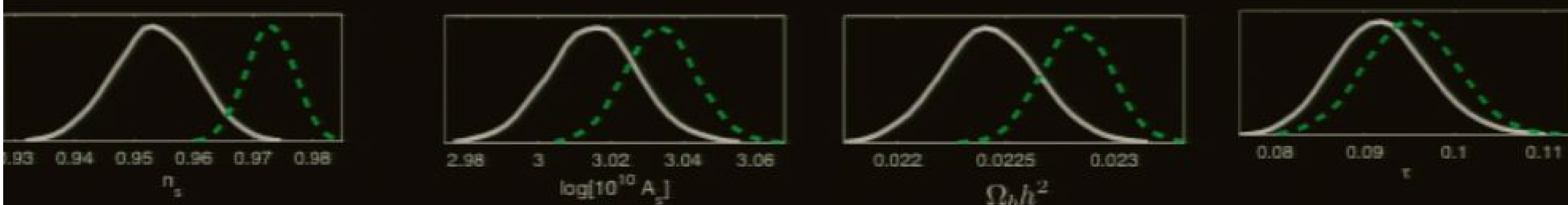


- Planck (launched May 2009) will make cosmic-variance limited CMB anisotropy measurements up to l~2500 (T), and l~1500 (E)
- Wong 2007 and Lewis 2006 show that $x_e(z)$ needs to be predicted to several parts in 10^4 accuracy for Planck data analysis

RECOMBINATION, INFLATION, AND REIONIZATION

$$P(k) = A_s \left(k\eta_0\right)^{n_s}$$

- Planck uncertainty forecasts using MCMC



- Cosmological parameter inferences will be off if recombination is improperly modeled (Wong/Moss/Scott 2007)
- Leverage on new physics comes from high l. Here the details of recombination matter!
- Inferences about inflation will be wrong if recombination is improperly modeled

$$\mathrm{n}_s = 1 - 4\epsilon + 2\eta \qquad \epsilon = \frac{m_{\rm pl}^2}{16\pi}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \qquad \mathrm{A}_s^2 = \left.\frac{32}{75}\frac{V}{m_{\rm pl}^4\epsilon}\right|_{k_{\rm pivot}=aH}$$

CAVEAT EMPTOR: $3 \lesssim \; ? \; \lesssim 16$

Need to do eV physics right to infer anything about $10^?$ GeV physics! 4

RECOMBINATION, INFLATION, AND REIONIZATION

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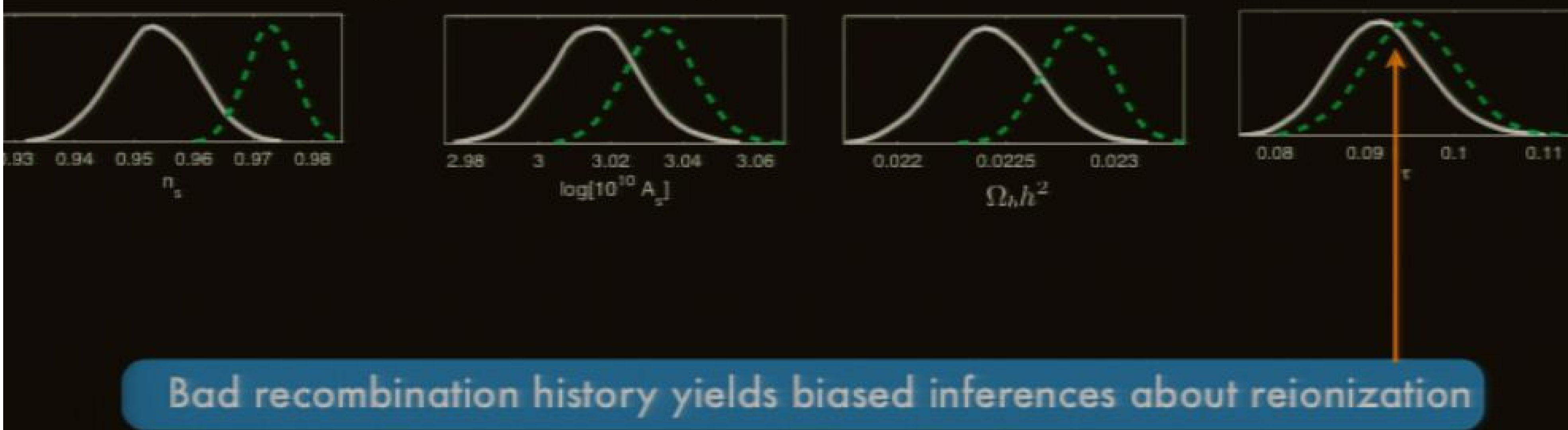

PHYSICAL RELEVANCE FOR CMB:
SMEARING AND MOVING THE SURFACE OF LAST SCATTERING (SLS)

- Photons kin. decouple when Thompson scattering freezes out

$$\gamma + e^- \Leftrightarrow \gamma + e^-$$

- Acoustic mode evolution influenced by visibility function

$$g = \dot{\tau} e^{-\tau} \qquad \tau(z) = \int_0^{\eta(z)} n_e \sigma_T a(\eta') d\eta'$$

- $z_{\rm dec} \simeq 1100$: Decoupling occurs during recombination

$$\boxed{C_l \to C_l e^{-2\tau(z)} \text{ if } l > \eta_{\rm dec}/\eta(z)}$$

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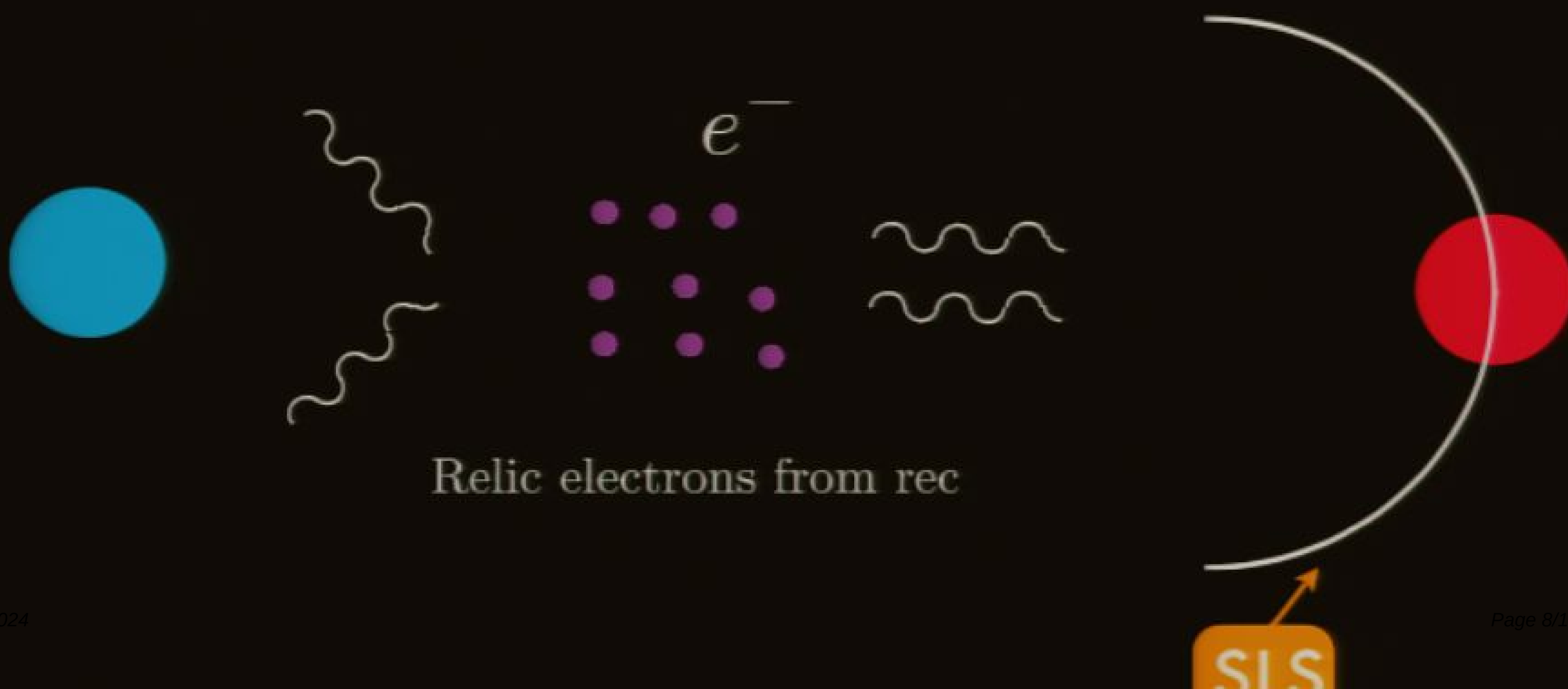
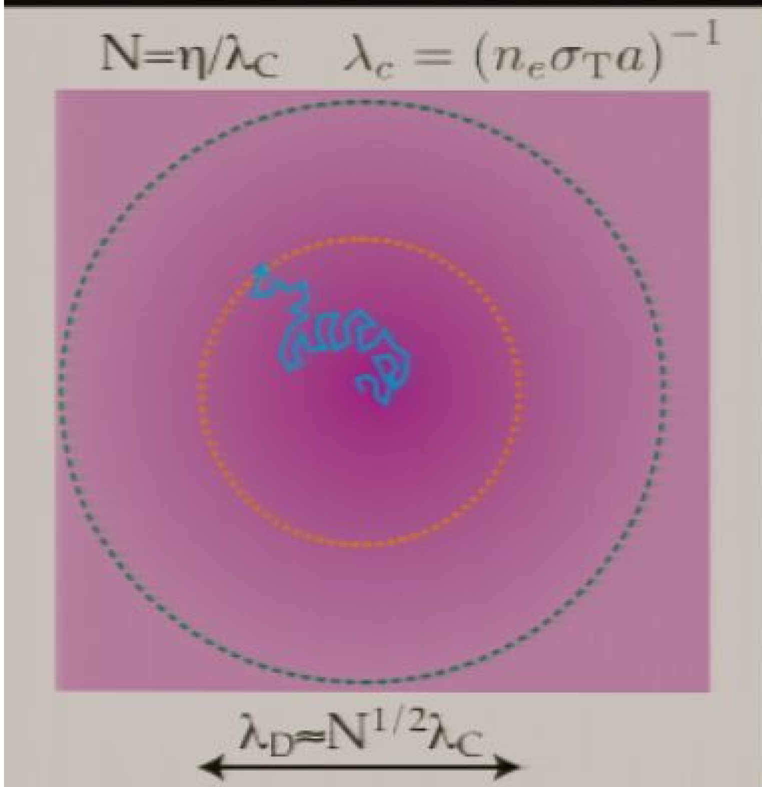
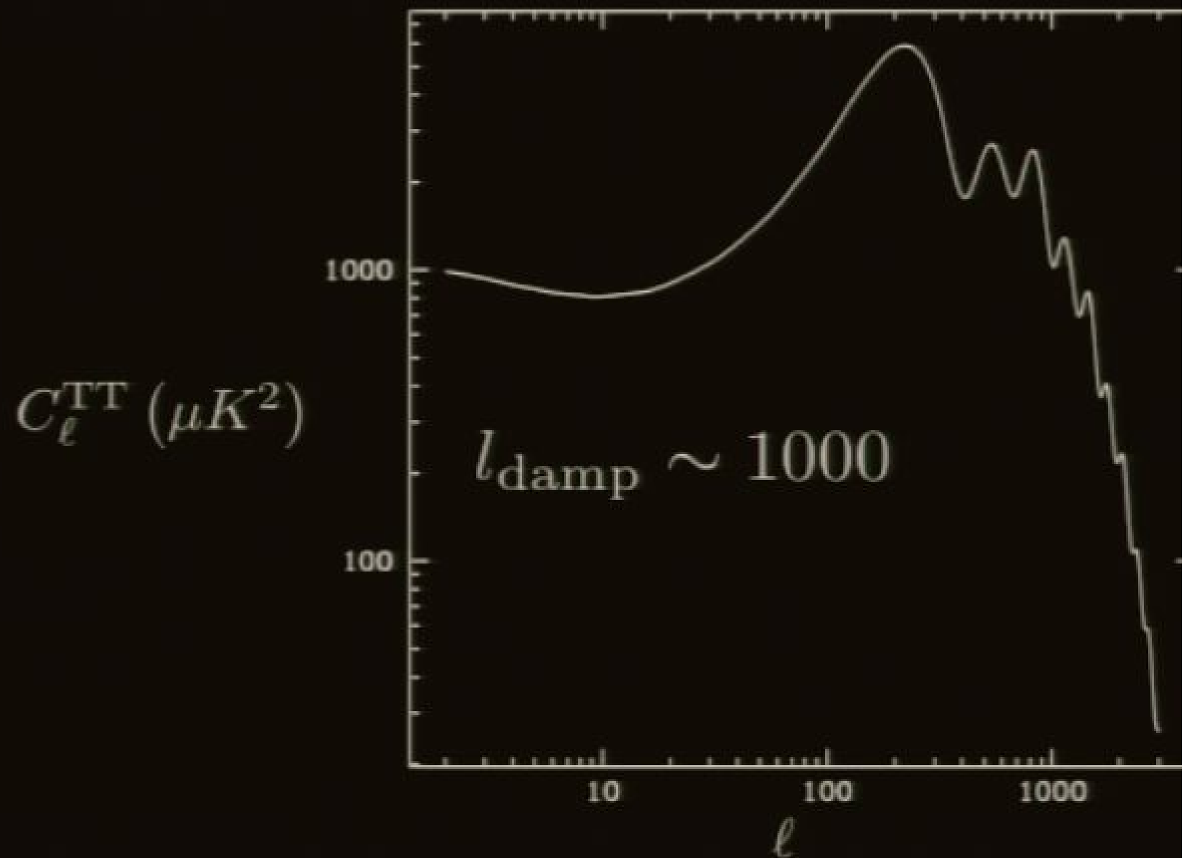


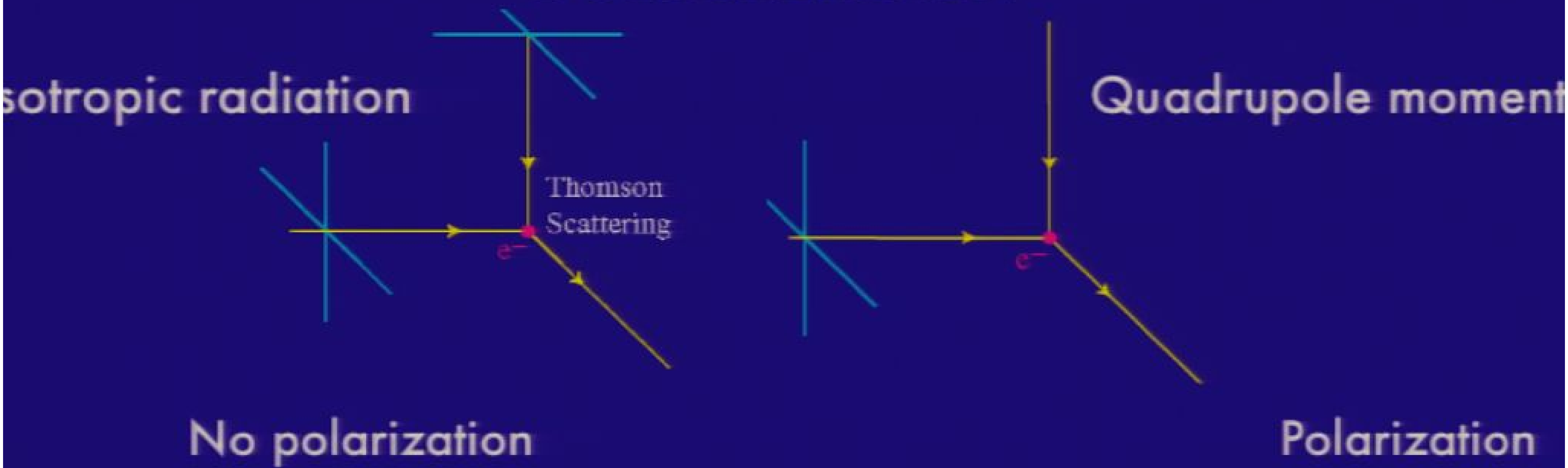
Physical relevance for CMB: The Silk Damping Tail





- Inhomogeneities are damped for $\lambda < \lambda_D$

Physical relevance for CMB: Polarization



From Wayne Hu's website

- Need time to develop a quadrapole

$$\Theta_l(k\eta) \sim \frac{k\eta}{2\tau}\Theta_{l+1}(k\eta) \ll \Theta_{l+1}(k\eta) \text{ if } l \geq 2, \text{ in tight coupling regime}$$

- Need to scatter quadrapole to polarize CMB

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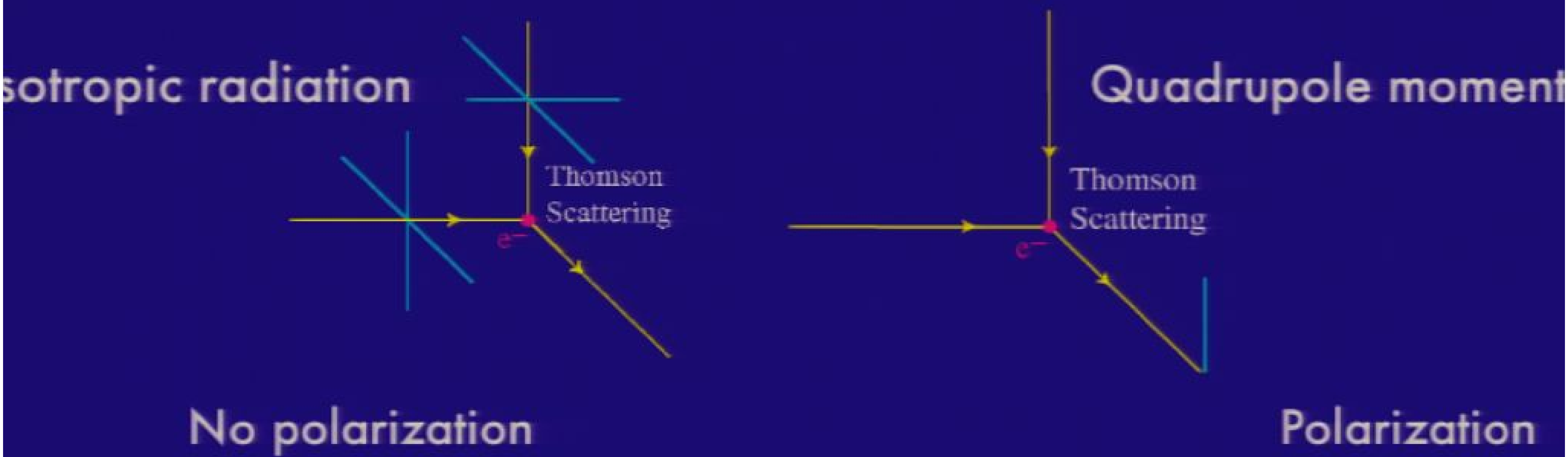

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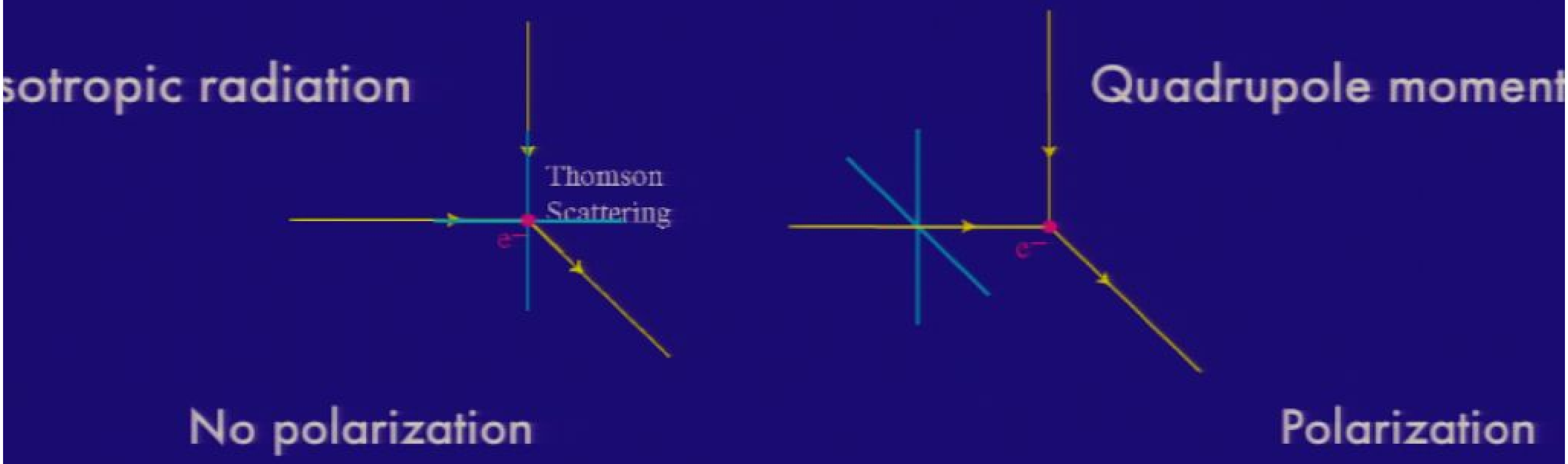

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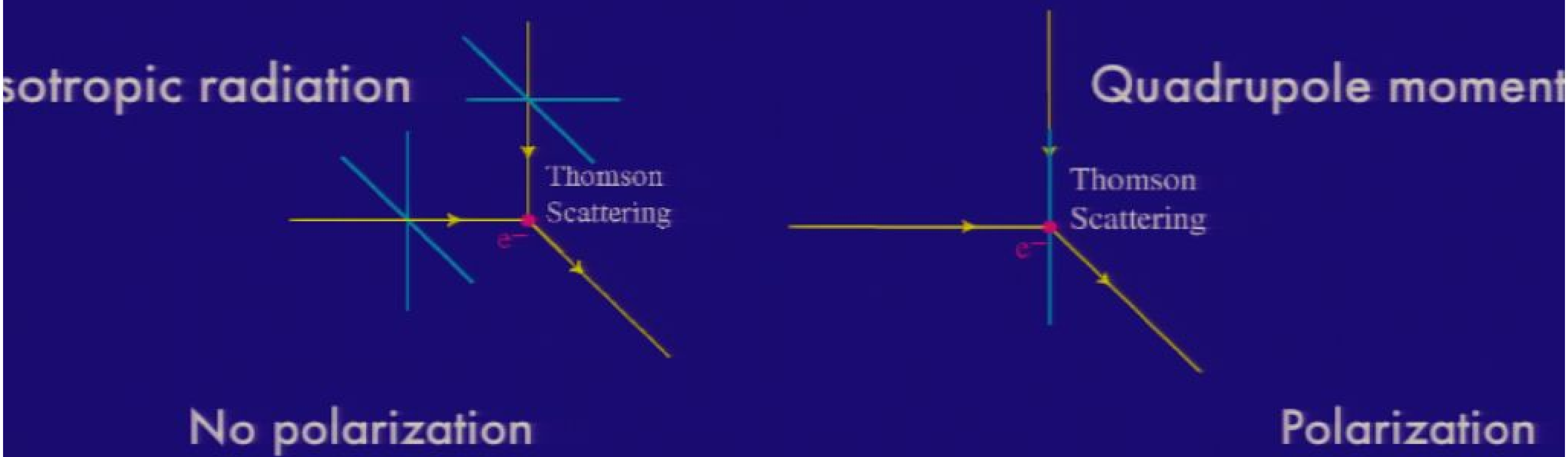

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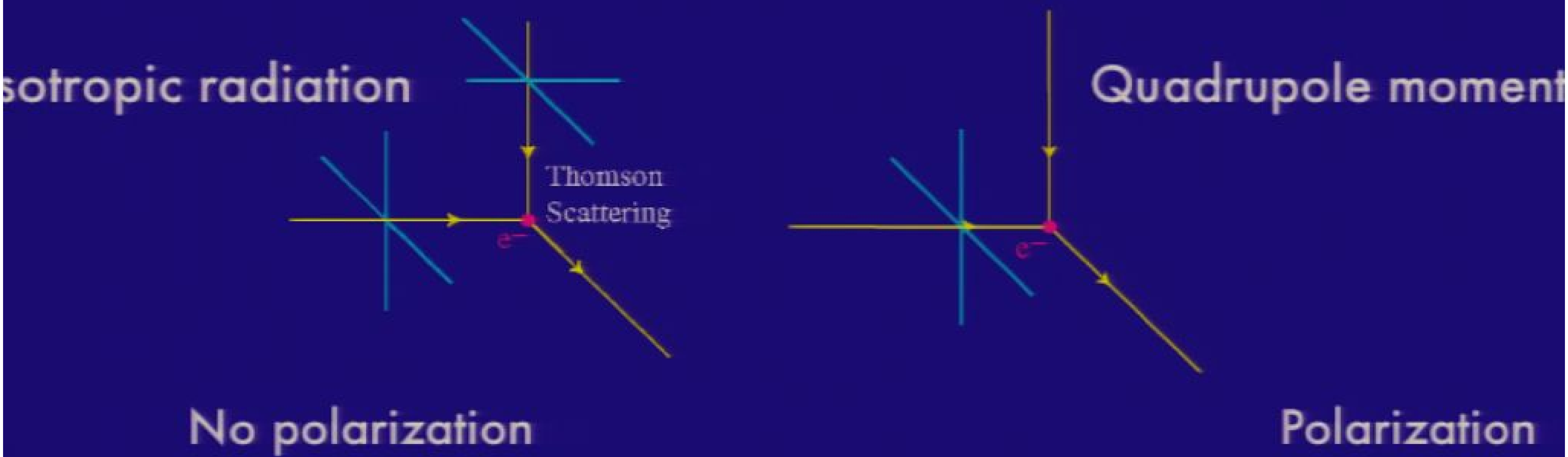

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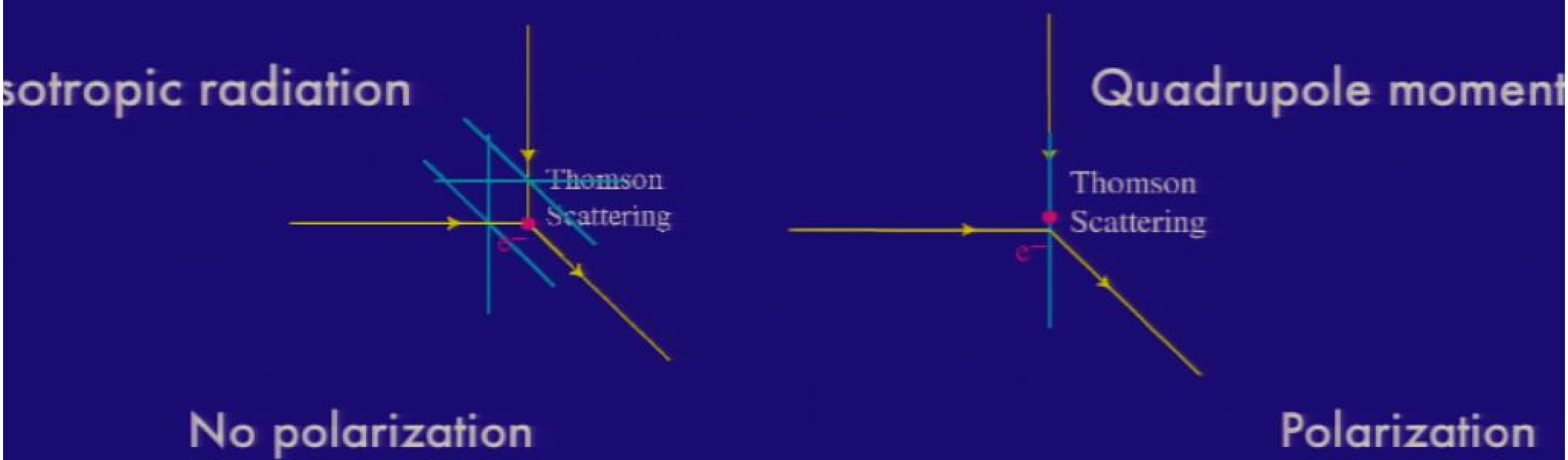

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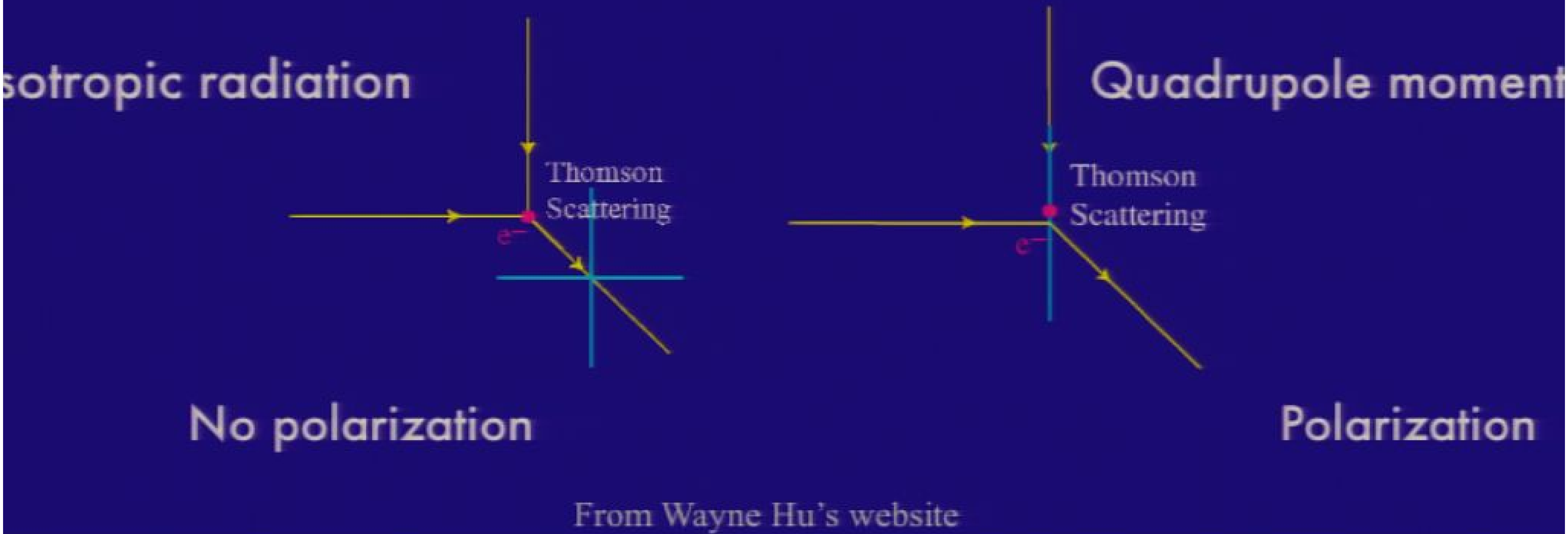

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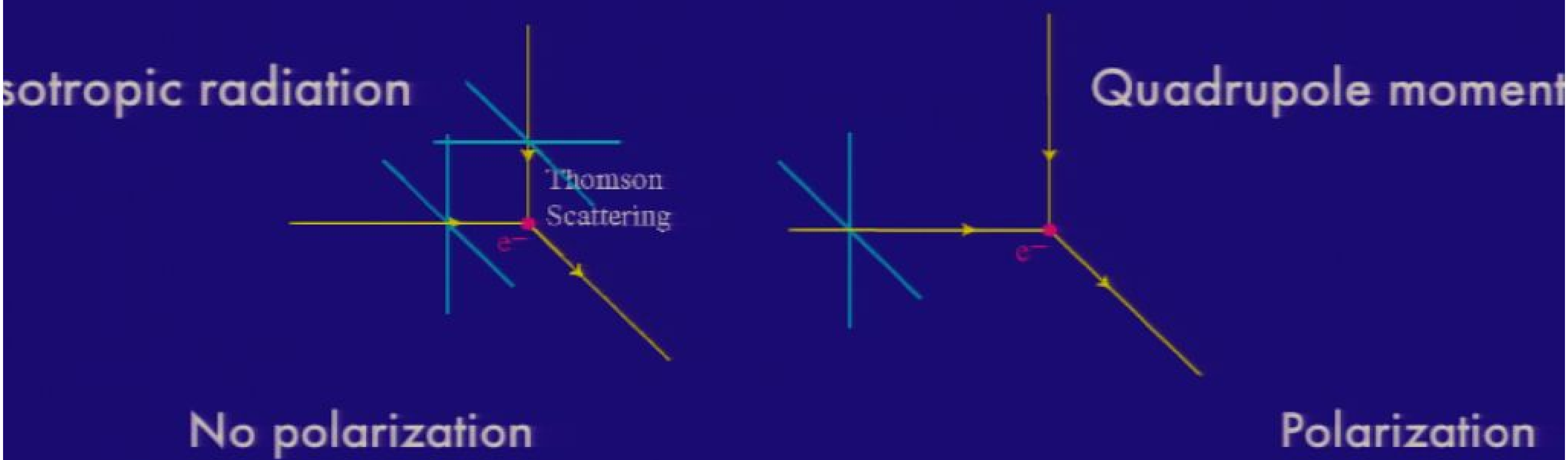

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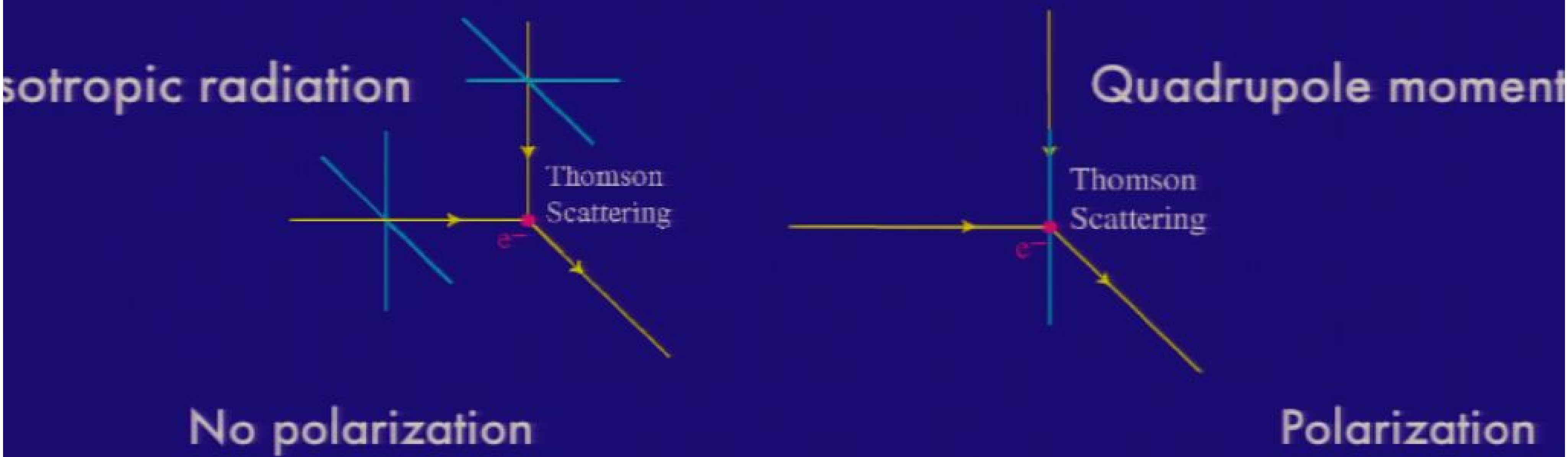

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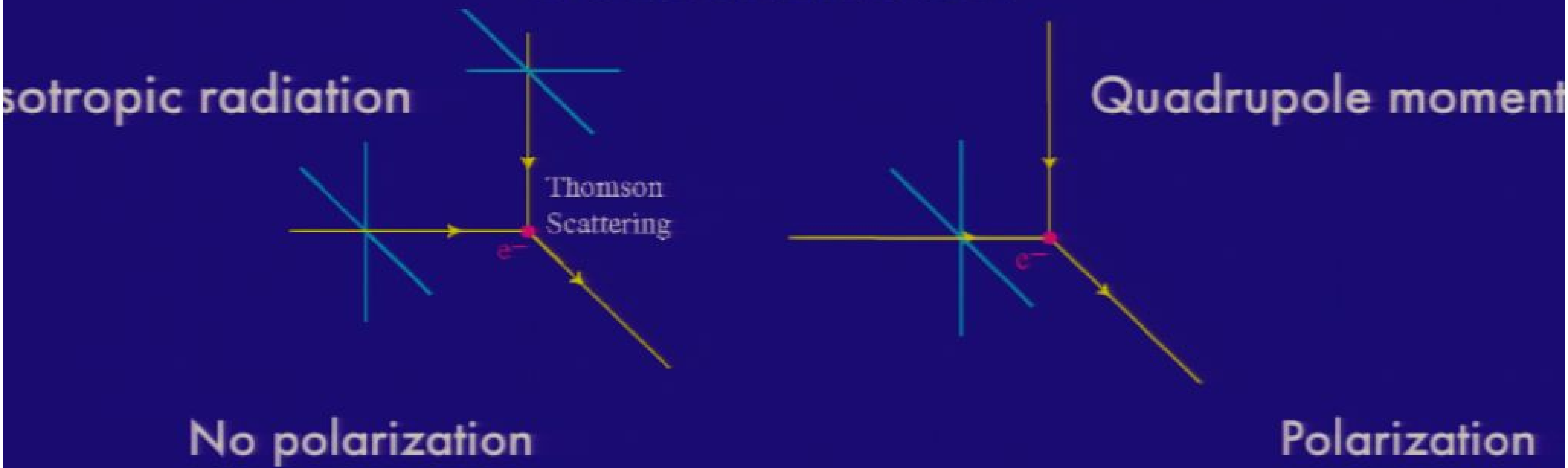

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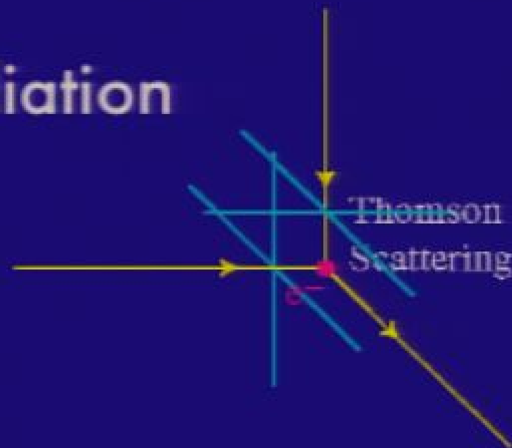
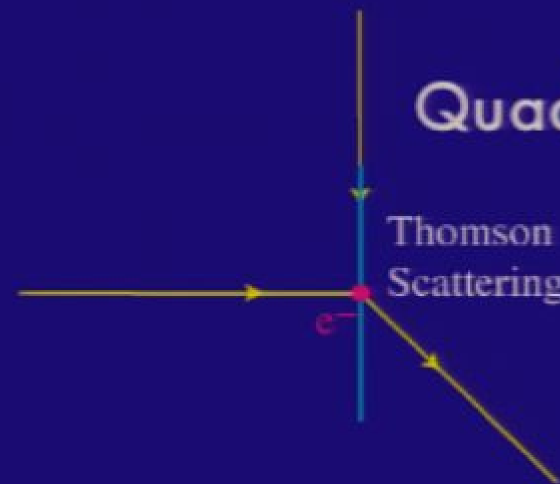
Physical relevance for CMB: Polarization

sotropic radiation

Quadrupole moment





No polarization

Polarization

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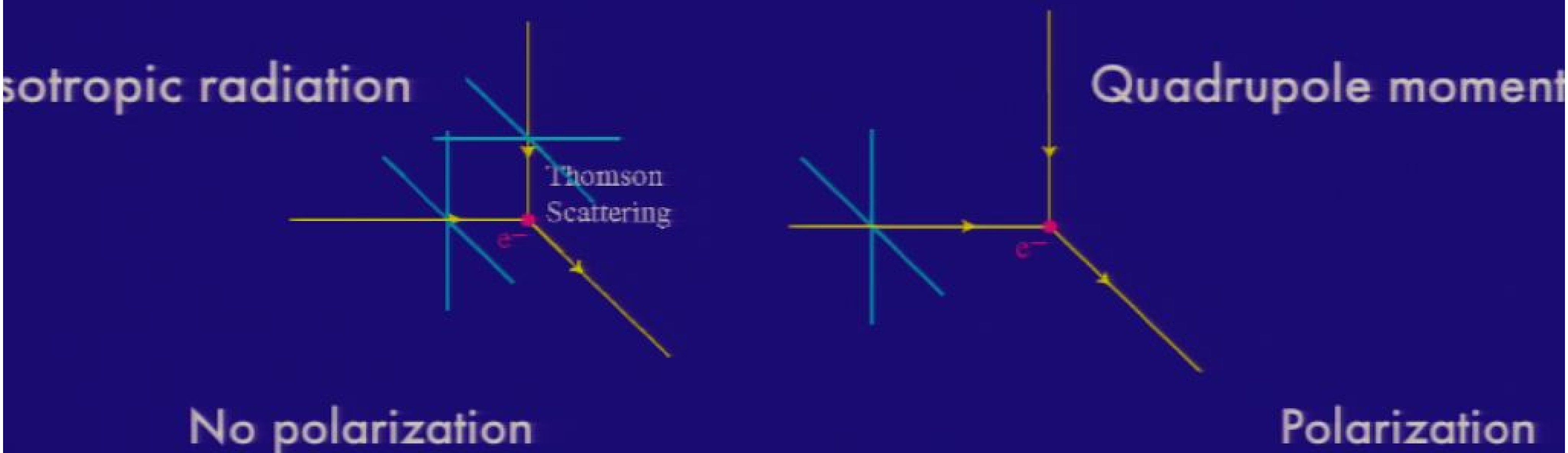

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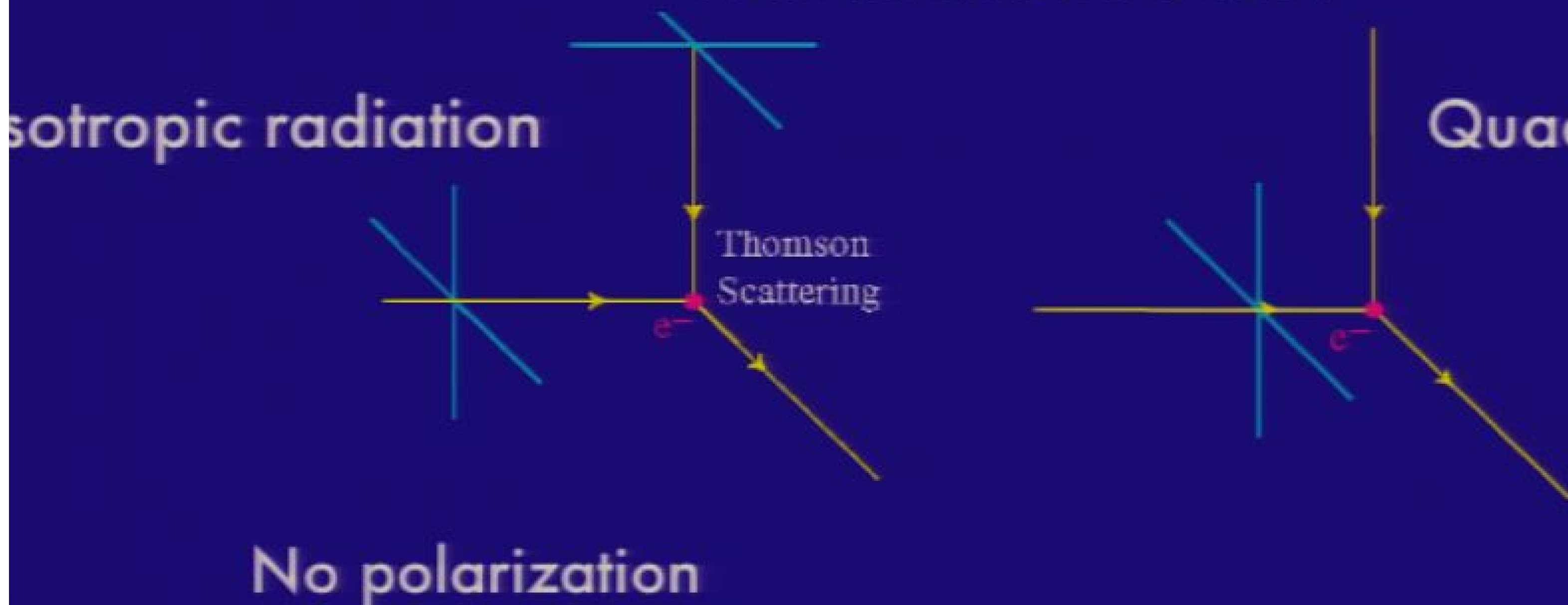

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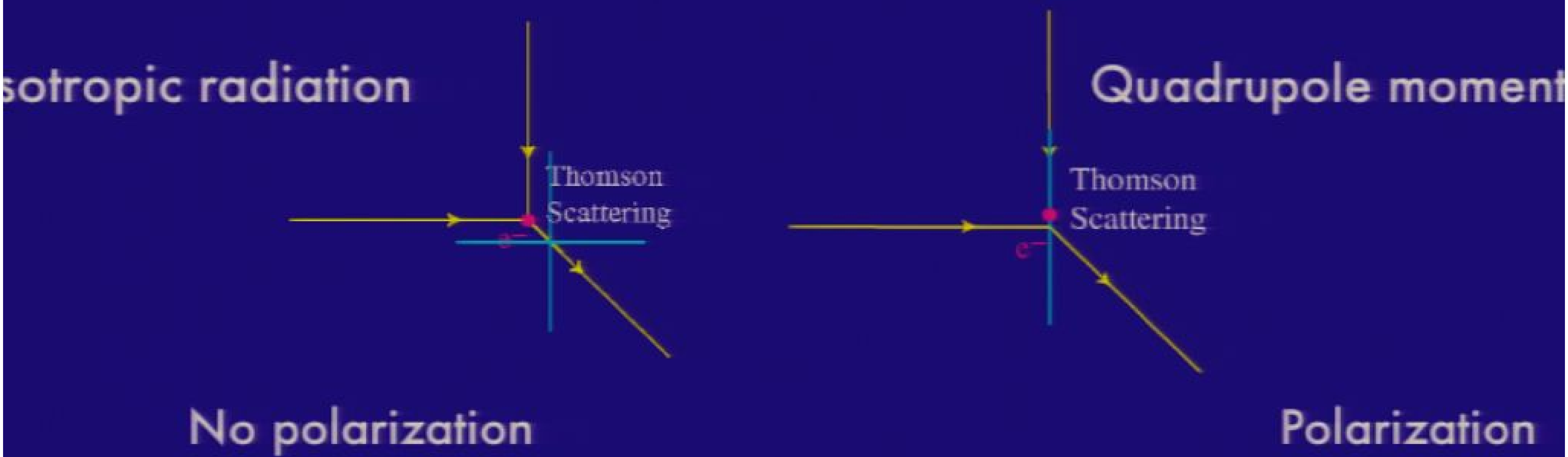

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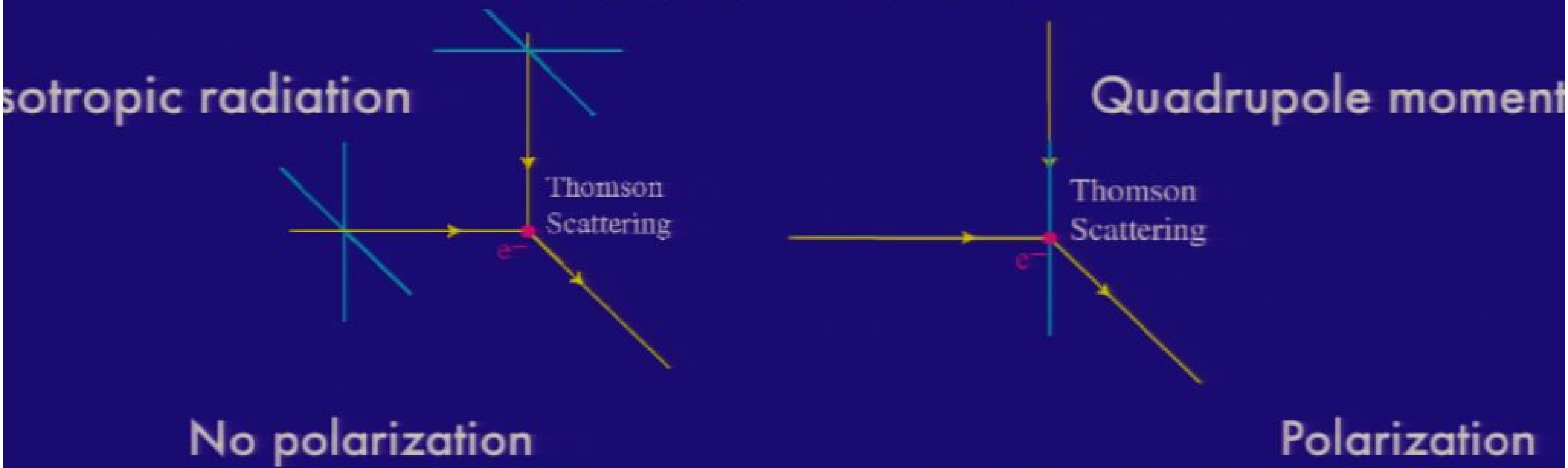

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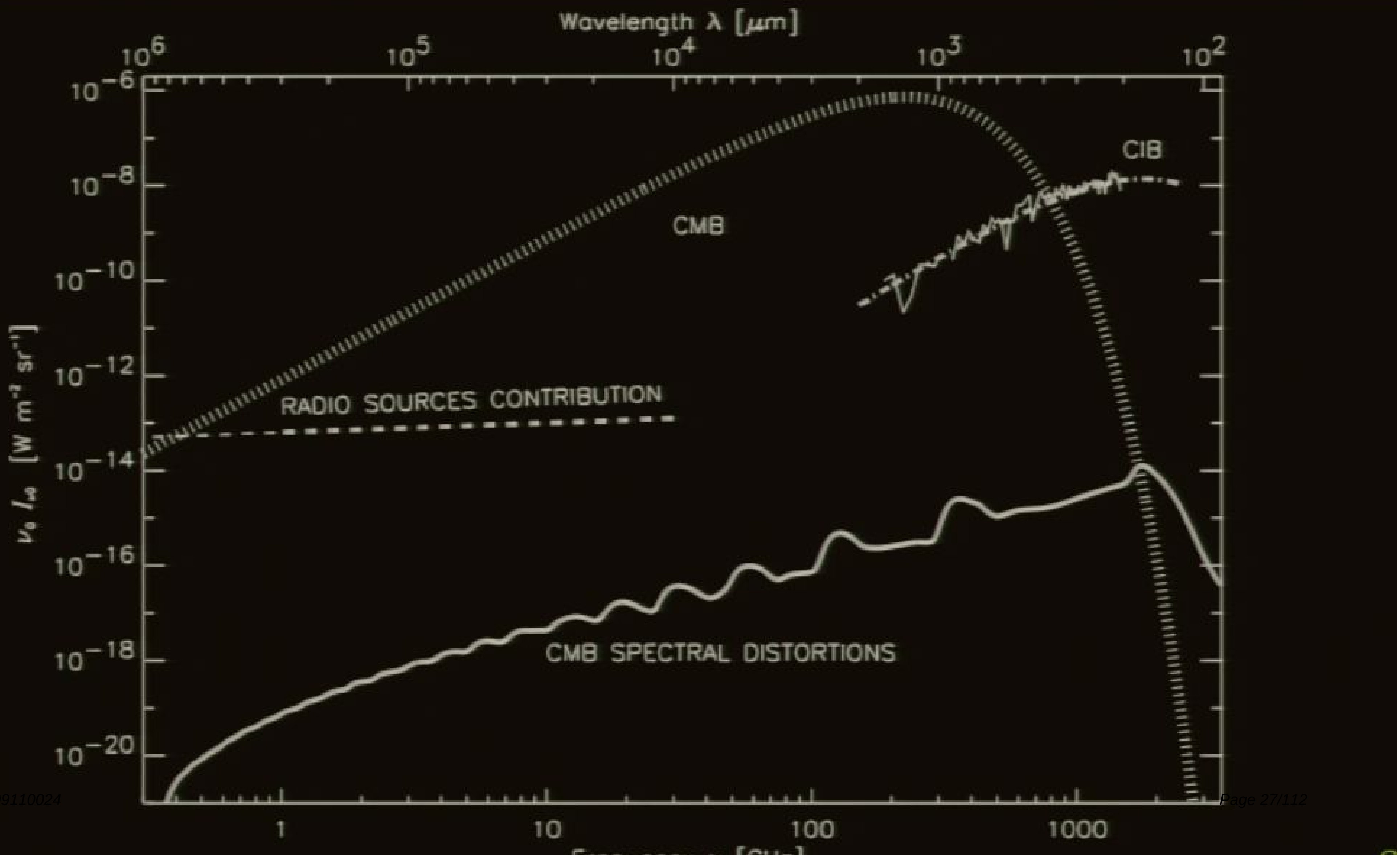
Physical relevance for CMB: Spectral distortions from recombination



SAHA EQUILIBRIUM IS INADEQUATE

$$p + e^- \leftrightarrow H^{(n)} + \gamma^{(nc)}$$

* Chemical equilibrium does reasonably well predicting "moment of recombination"

$$\frac{x_e^2}{1 - x_e} = \left(\frac{13.6}{T_{\mathrm{eV}}}\right)^{3/2} e^{35.9 - 13.6/T_{\mathrm{eV}}}$$

$x_e = 0.5$ when $T = T_{\mathrm{rec}} \simeq 0.3$ eV $z_{\mathrm{rec}} \simeq 1300$

* Further evolution falls prey to reaction freeze-out

$\Gamma < H$ when $T < T_{\mathrm{F}} \simeq 0.25$ eV

BOTTLENECKS/ESCAPE ROUTES

BOTTLENECKS

- Ground state recombinations are ineffective

$$\Gamma_{c\to 1s} = 10^{-1}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

- Resonance photons are re-captured, e.g. Lyman α

$$\Gamma_{2p\to 1s} = 10^{-2}\ \mathrm{s}^{-1} \gg H \simeq 10^{-12}\ \mathrm{s}^{-1}$$

ESCAPE ROUTES (e.g. n=2)

- Two-photon processes

$$H^{2s} \to H^{1s} + \gamma + \gamma \quad \boldsymbol{\Lambda}_{\mathbf{2s\to 1s}} = \mathbf{8.22}\ \mathrm{s}^{-1}$$

- Redshifting off resonance

$$R \sim \left(n_{\mathrm{H}}\lambda_\alpha^3\right)^{-1} H$$

THE PEEBLES PUNCHLINE

- Only n=2 bottlenecks are treated
- Net Rate is suppressed by bottleneck vs. escape factor

$$-\frac{dx_e}{dt} = \mathcal{S} \sum_{n,l>1s} \alpha_{nl}(T) \left\{ n x_e^2 - x_{1s} f(T) \right\}$$

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Ionization rate

11

The Peebles Punchline

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$$-\frac{dx_e}{dt} = S \sum_{n,l>1s} \alpha_{nl}(T)\left\{ n x_e^2 - x_{1s} f(T) \right\}$$

The peebles model

* Net Rate is suppressed by bottleneck vs. escape factor

$$\mathcal{S} = \frac{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + \Lambda_{2s\to 1s}}{\frac{8\pi}{\lambda_\alpha^3 n_{1s}} H + (\Lambda_{2s\to 1s} + \beta_c)}$$

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Redshifting term

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β_c → Ionization Term

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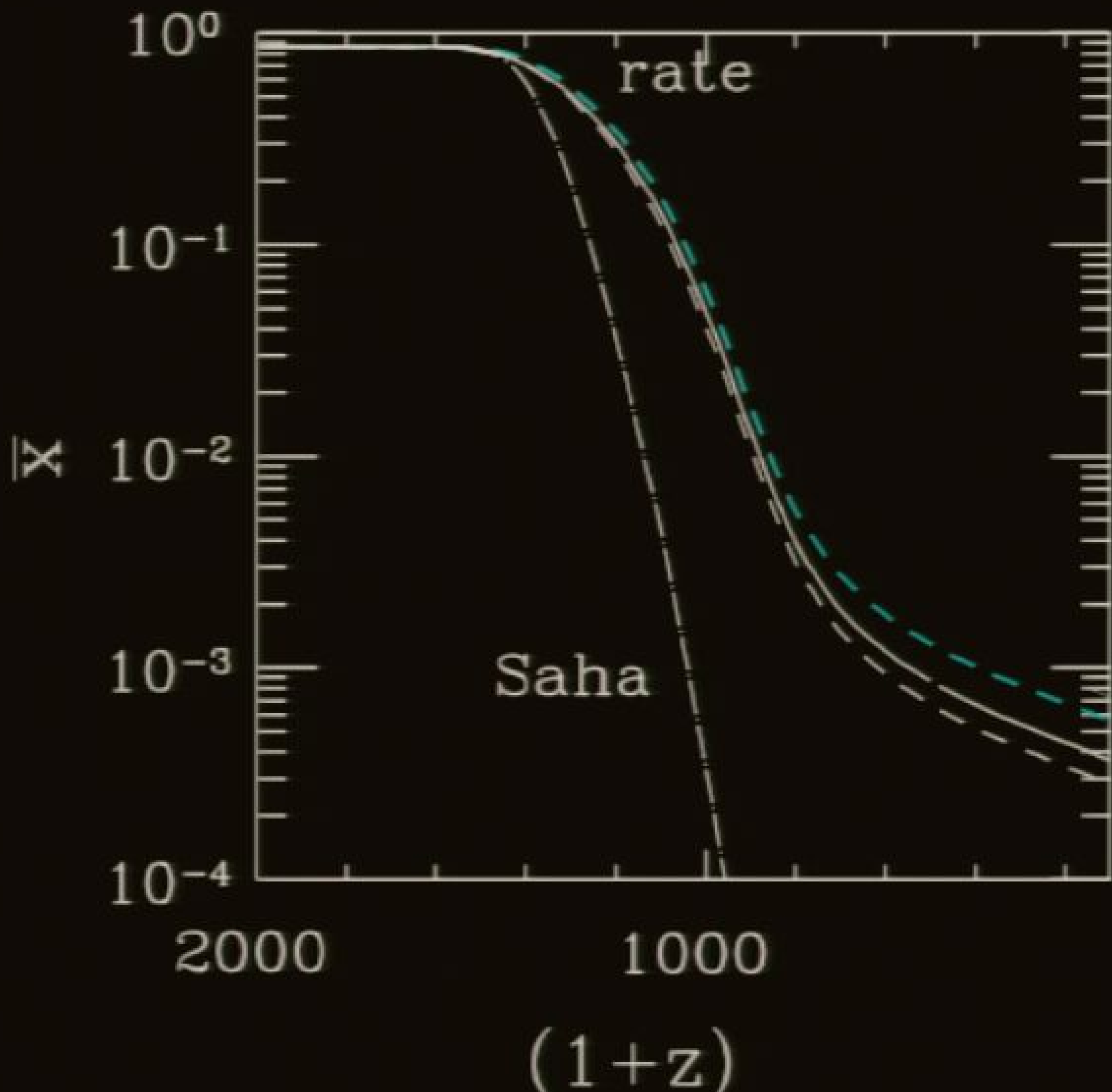
Ionization Term

$$\frac{\text{redshift term}}{2\gamma \text{ term}} \simeq 0.02 \frac{\Omega_m^{1/2}}{(1 - x_e[z])\left(\frac{1+z}{1100}\right)^{3/2}}$$

2γ process dominates until late times ($z \lesssim 850$)

THE PEEBLES MODEL

- Peebles 1967: State of the Art for 30 years!



EQUILIBRIUM ASSUMPTIONS

* Radiative/collisional eq. between different l

$$\mathcal{N}_{nl} = \mathcal{N}_n \frac{(2l+1)}{n^2}$$

* Radiative eq. between different n-states

$$\mathcal{N}_n = \sum_l \mathcal{N}_{nl} = \mathcal{N}_2 e^{-(E_n - E_2)/T}$$

Non-eq rate equations

* Matter in eq. with radiation due to Thompson scattering

$$T_m = T_\gamma \text{ since } \frac{\sigma_{\rm T} a T_\gamma^4 c}{m_e c^2} < H(T)$$

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Seager/Scott/Sasselov 2000/RECFAST!

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BREAKING EQUILIBRIUM

- Chluba et al. (2005,6) follow l, n separately, get to $n_{\max} = 100$
- 0.1 %-level corrections to CMB anisotropies at $n_{\max} = 100$
- ~~Equilibrium~~ between *l states:* $\Delta l = \pm 1$ bottleneck
- Beyond this, testing convergence with $n_{\max}$ is hard!

$$t_{\text{compute}} \sim \mathcal{O}\,(\text{years}) \;\; \text{for } \mathrm{n}_{\max} = 300$$

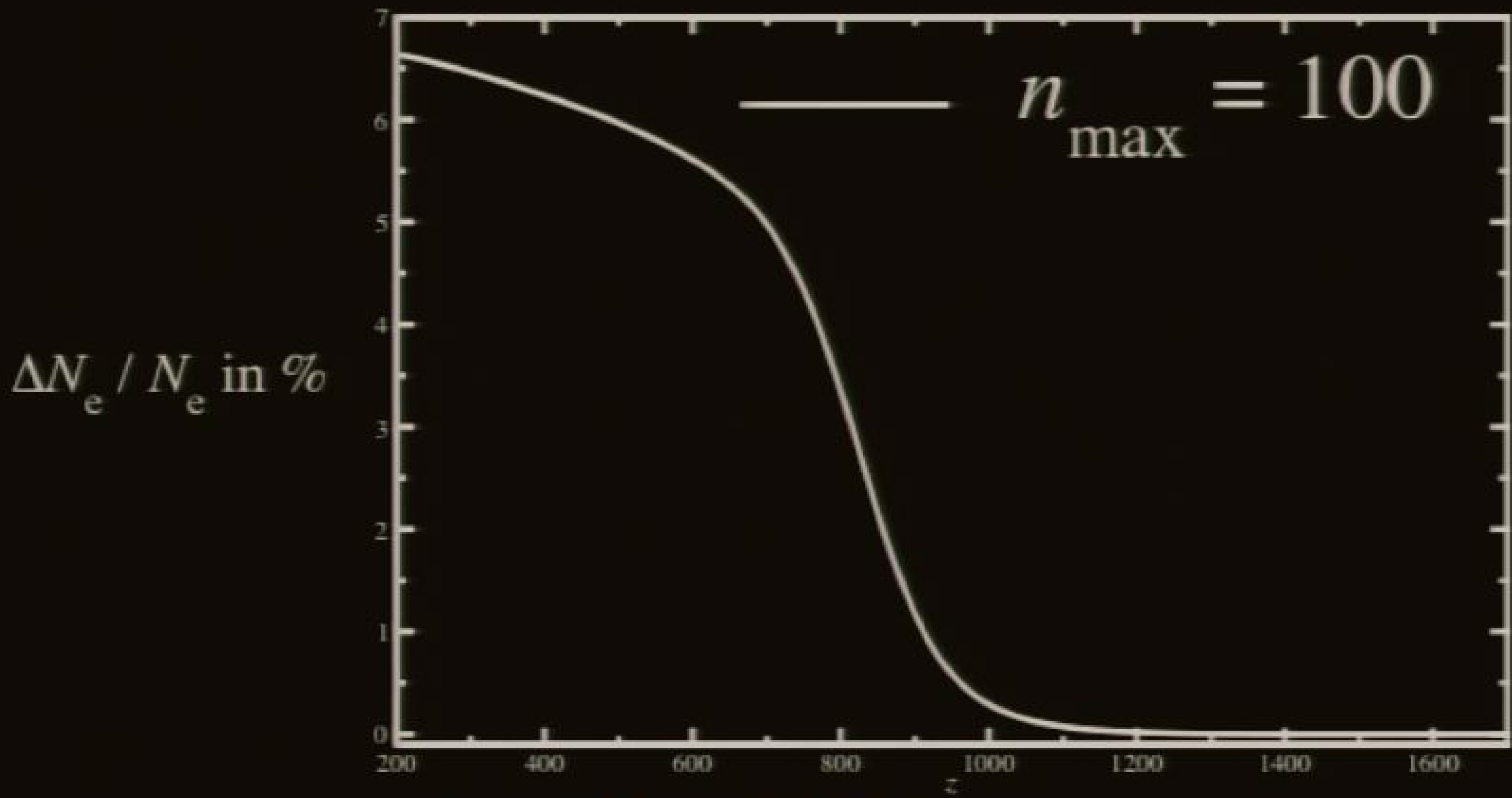
How to proceed if we want $\mathcal{O}\,(1) \times 10^{-4}$ accuracy in C_ℓ?

THESE ARE REAL STATES

- Still inside plasma shielding length for n<100000
- $r \sim a_0 n^2$ is as large as 2μm for $n_{\max} = 200$
- $\dfrac{\Delta E|_{\text{thermal}}}{E} < \dfrac{2}{n^3}$
- Similarly high n are seen in emission line nebulae

THE EFFECT OF RESOLVING l- SUBSTATES

Resolved l vs unresolved l



From Chluba et al. 2006

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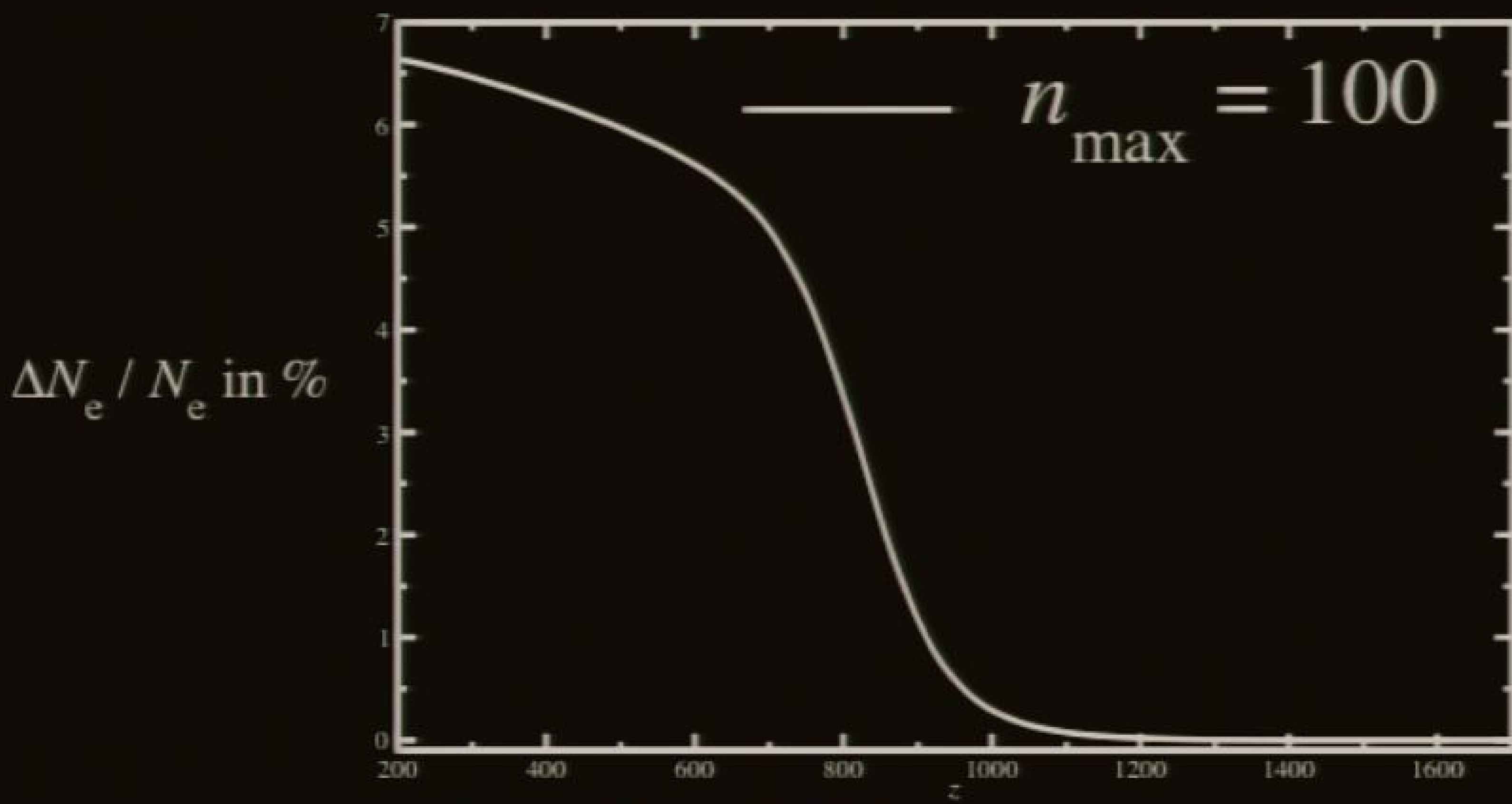

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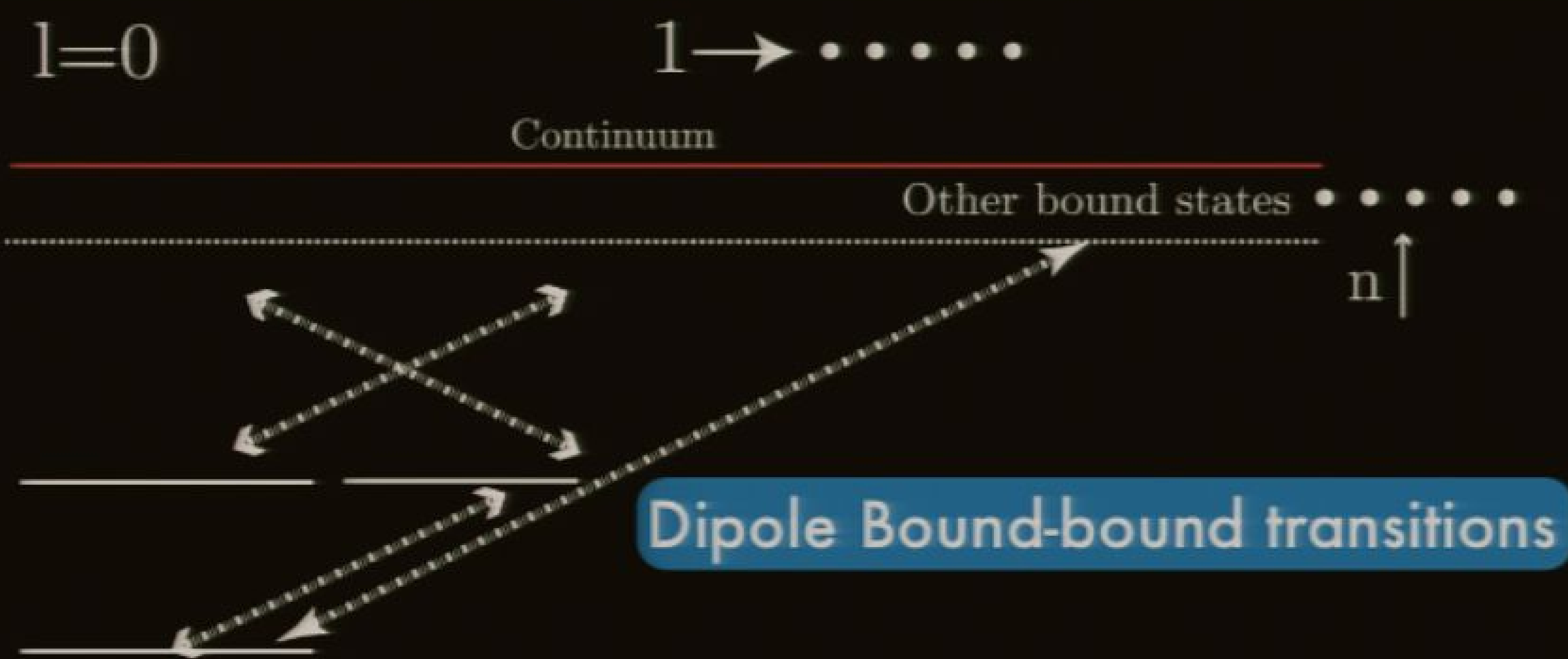
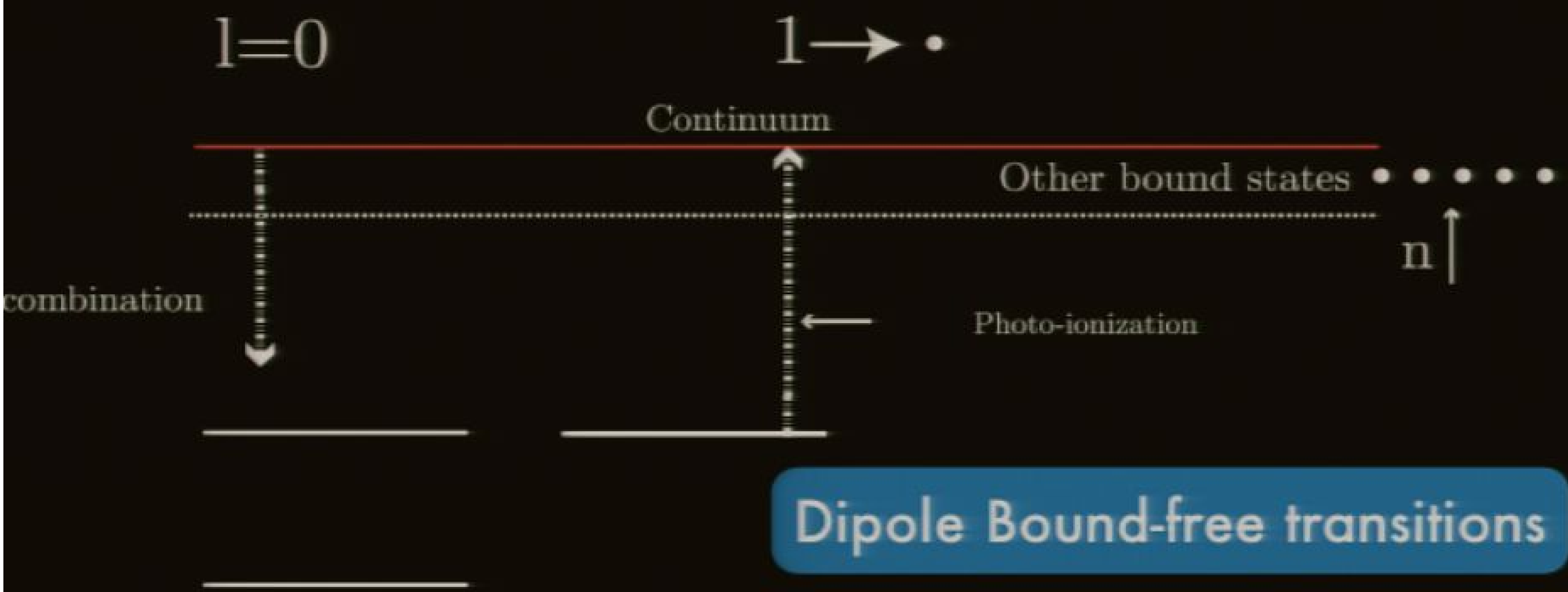
RecSparse and the multi-level atom



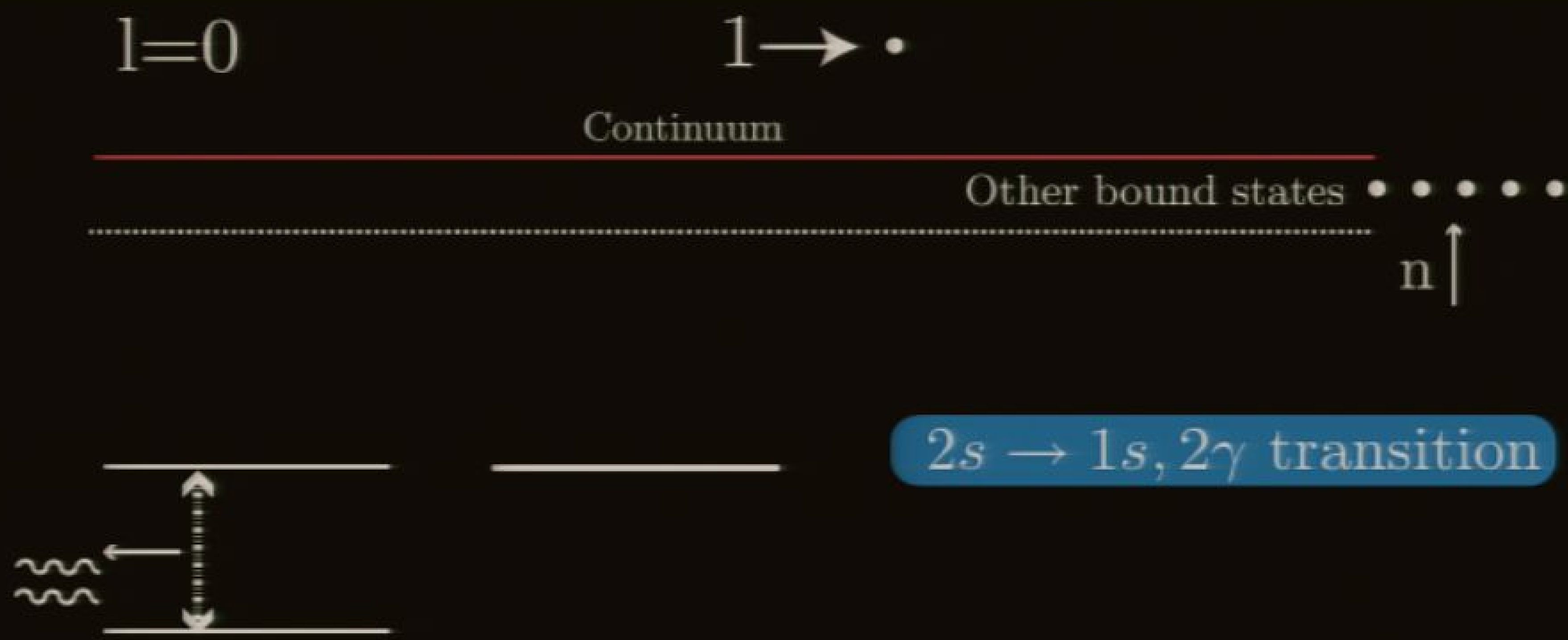
- We implement a multi-level atom computation in a new code, RecSparse!
- Boltzmann eq. solved for $T_m(T_\gamma)$
- Spontaneous/stimulated emission/absorption included

RecSparse and the multi-level atom



- We implement a multi-level atom computation in a new code, RecSparse!
- Boltzmann eq. solved for $T_m\ (T_\gamma)$
- Spontaneous/stimulated emission/absorption included

RecSparse and the multi-level atom



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RecSparse and the multi-level atom

- Free electron fraction evolved according to

$$
\begin{aligned}
\dot{x}_e &= -\dot{x}_{1s} \\
&= -\Lambda_{2s\to 1s}\left(x_{2s} - x_{1s} e^{-E_{2s\to 1s}/T_\gamma}\right) + \sum_{n,l>1s} A_{n1}^{l\,0} P_{n1}^{l0}\left\{g(T,n,l)\right\}
\end{aligned}
$$

2s-1s decay rate

RecSparse and the multi-level atom

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\end{aligned}
$$

Lyman series current to ground state

RecSparse and the multi-level atom

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Einstein coeff.

RecSparse and the multi-level atom

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Escape probability

RADIATION FIELD: BLACK BODY +

- Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

$$P^{l,l'}_{n,n'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$\boxed{\tau_s \propto \frac{n_{\mathrm{H}} x^l_n A^{ll'}_{nn'}}{H\left(z\right)} \qquad n' > n}$$

- RecSparse includes radiative feedback

- Ongoing work in field focuses on corrections to simple radiative transfer picture

- Ultimate goal is to combine all new atomic physics effect in one fast recombination code

RADIATION FIELD: BLACK BODY +

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Resonant absorber density

$$\tau_s \propto \frac{n_{\mathrm{H}} x_n^l A_{nn'}^{ll'}}{H(z)} \qquad n' > n$$

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RADIATION FIELD: BLACK BODY +

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Einstein coefficient

$$\tau_s \propto \frac{n_{\mathrm{H}} x_n^l \boxed{A_{nn'}^{ll'}}}{H(z)} \qquad n' > n$$

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RADIATION FIELD: BLACK BODY +

- Escape probability treated in Sobolev approx: depends on steady-state and incoherent scattering approximations

Cosmological expansion

$$\tau_s \propto \frac{n_{\mathrm{H}} x_n^l A_{nn'}^{ll'}}{\boxed{H(z)}} \qquad n' > n$$

- RecSparse includes radiative feedback
- Ongoing work in field focuses on corrections to simple radiative transfer picture
- Ultimate goal is to combine all new atomic physics effect in one fast recombination code

OTHER CORRECTIONS TO RECOMBINATION

- Deviations from steady-state approx (Chluba/Sunyaev 2008)
- Coherent scattering (Forbes and Hirata 2009, Switzer/Hirata 2007)
- Atomic recoil (Forbes and Hirata 2009, Dubrovich and Grachev 2008)
- Diffusion near resonance lines
- Line overlap (Ali-Haimoud, Grin, Hirata in progress)
- Feedback from hydrogen/helium (Chluba/Sunyaev 2007)
- Higher-n two-photon processes (Chluba/Sunyaev 2007, Hirata 2008) in hydrogen and Helium (Switzer/Hirata 2007)
- Deuterium
- Additional effects in Helium (Switzer/Hirata 2007)

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Steady-state for excited levels

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$$\vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \vec{x}_l \\ \cdots \\ \vec{x}_{l_{\max}} \end{pmatrix}$$

STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

$$\vec{x}_l = \begin{pmatrix} x_{l,l+1} \\ \cdots \\ x_{l,n_{\max}} \end{pmatrix} \longleftarrow \qquad \vec{x} = \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \boxed{\vec{x}_l} \\ \cdots \\ \vec{x}_{l_{\max}} \end{pmatrix}$$

STEADY-STATE FOR EXCITED LEVELS

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$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For state l, includes BB transitions out of l to all other l”, photo-ionization, 2γ transitions to ground state

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For state l, includes BB transitions into l from all other l'

Steady-state for excited levels

* Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

Includes recombination to l,
1 and 2γ transitions from ground state

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\frac{d\vec{x}}{dt} = \mathbf{R}\vec{x} + \vec{s}$$

For n>1, $\mathbf{R}$, $\vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

STEADY-STATE FOR EXCITED LEVELS

* Evolution equations may be re-written in matrix form

$$\boxed{\frac{d\vec{x}}{dt}} = \mathbf{R}\vec{x} + \vec{s}$$

$_{\rm rec}^{-1} \sim 10^{-12}\ \mathrm{s}^{-1}$

For n>1, $\mathbf{R}, \vec{s} \geq 1\ \mathrm{s}^{-1}$ e.g. Lyman-α

STEADY-STATE FOR EXCITED LEVELS

- Evolution equations may be re-written in matrix form

$$\cancel{\frac{d\vec{x}}{dt}} = \mathbf{R}\vec{x} + \vec{s} \qquad \boxed{\text{LHS} \ll \text{RHS}} \qquad \boxed{\vec{x} \simeq -\mathbf{R}^{-1}\vec{s}}$$

$\boxed{{}^{-1}_{\mathrm{rec}} \sim 10^{-12}\ \mathrm{s}^{-1}}$ $\boxed{\text{For n>1, } \mathbf{R}, \vec{s} \geq 1\ \mathrm{s}^{-1} \text{ e.g. Lyman-}\alpha}$

Rapid matrix inversion: Sparsity to the rescue

- Matrix is $\sim n_{max}^2 \times n_{max}^2$
- Brute force would require A$n_{max}^6 \sim 10^5$ s for $n_{\max} = 200$ for a single time step
- Dipole selection rules: $\Delta l = \pm 1$

  $\mathrm{M}_{l,\mathbf{l-1}}\vec{x}_{\mathbf{l-1}} + \mathbf{M}_{l,l}\vec{x}_l + \mathbf{M}_{l,\mathbf{l+1}}\vec{x}_{\mathbf{l+1}} = \vec{s}_l$

$$\begin{pmatrix} \blacksquare & \blacksquare & 0 & 0 & 0 & \\ \blacksquare & \blacksquare & \blacksquare & 0 & 0 & \\ 0 & \blacksquare & \blacksquare & \blacksquare & 0 & \\ & & & \ddots & & \\ & & & & & \blacksquare \end{pmatrix} \begin{pmatrix} \vec{x}_0 \\ \vec{x}_1 \\ \cdots \\ \vec{x}_{n_{\max}-1} \end{pmatrix} = \vec{s}_l$$

- Physics imposes sparseness on the problem. Solved in closed form to yield algebraic $\vec{x}_{l_{\max}}$, then $\vec{x}_l$ in terms of $\vec{x}_{l+1}$

Rapid matrix inversion: Sparsity to the rescue

- Einstein coefficients to states with $n > n_{\max}$ are set $A = 0$: more later!
- RecSparse generates rec. history with computation time ~ $n_{max}^{2.5}$: Huge improvement!
- Case of $n_{\max} = 100$ runs in less than a day, $n_{\max} = 200$ takes ~ 4 days.

FORBIDDEN TRANSITIONS AND RECOMBINATION

- Higher-n 2γ transitions in H important at 7-σ for Planck (TT/EE) data analysis (Hirata 2008, Kholupenko 2006)
- Some forbidden transitions are important in Helium recombination (Dubrovich 2005, Lewis 2006) and would bias cosmological parameter estimation.
- *Are other forbidden transitions in hydrogen important, particularly for Planck data analysis? How about electric quadrupole (E2) transitions?*

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Ground-state electric quadrupole (E2) lines are optically thick!

$$P^{l,l'}_{n,n'} = \frac{1 - e^{-\tau_s}}{\tau_s}$$

$$R \propto AP \propto A/\tau \text{ if } \tau \gg 1$$

$$\tau \propto A \rightarrow R \rightarrow A/A \rightarrow \text{const}$$

- Coupling to ground state will overwhelmingly dominate:

$$\frac{A^{\text{quad}}_{n,2\rightarrow 1,0}}{A^{\text{quad}}_{n,2\rightarrow m,0}} \propto \frac{\omega^5_{n1}}{\omega^5_{nm}} \geq 10^3 \text{ if } m \geq 2$$

QUADRUPOLE TRANSITIONS AND RECOMBINATION

- Lyman lines are optically thick, so $nd \to 1s$ immediately followed by $1s \to np$, so this can be treated as an effective $d \to p$ process with rate $A_{nd\to 1s} x_{nd}$.

- Same sparsity pattern of rate matrix, similar to l-changing collisions

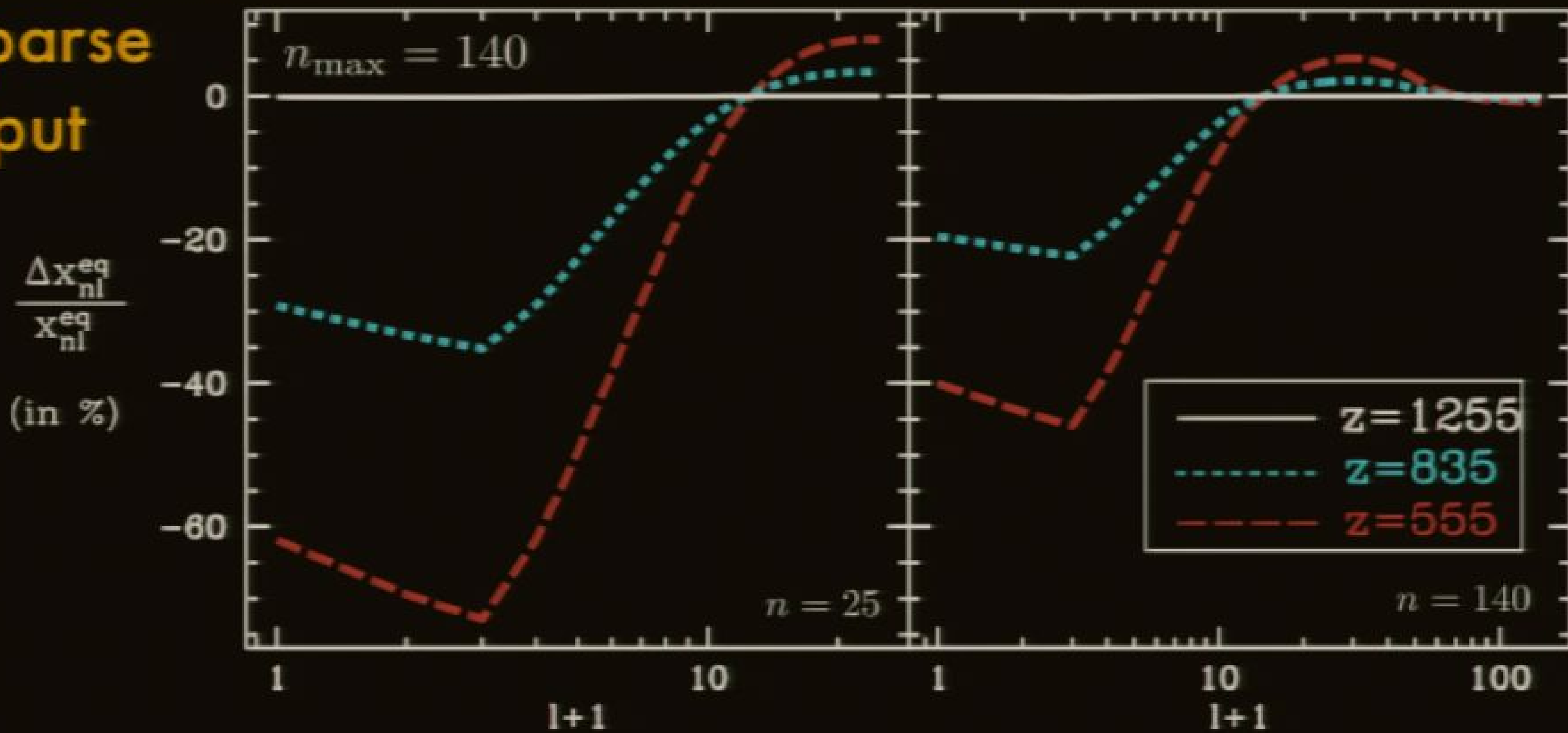
- Detailed balance yields net rate

$$\boxed{R^{\text{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)}$$

RESULTS: STATE OF THE GAS

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

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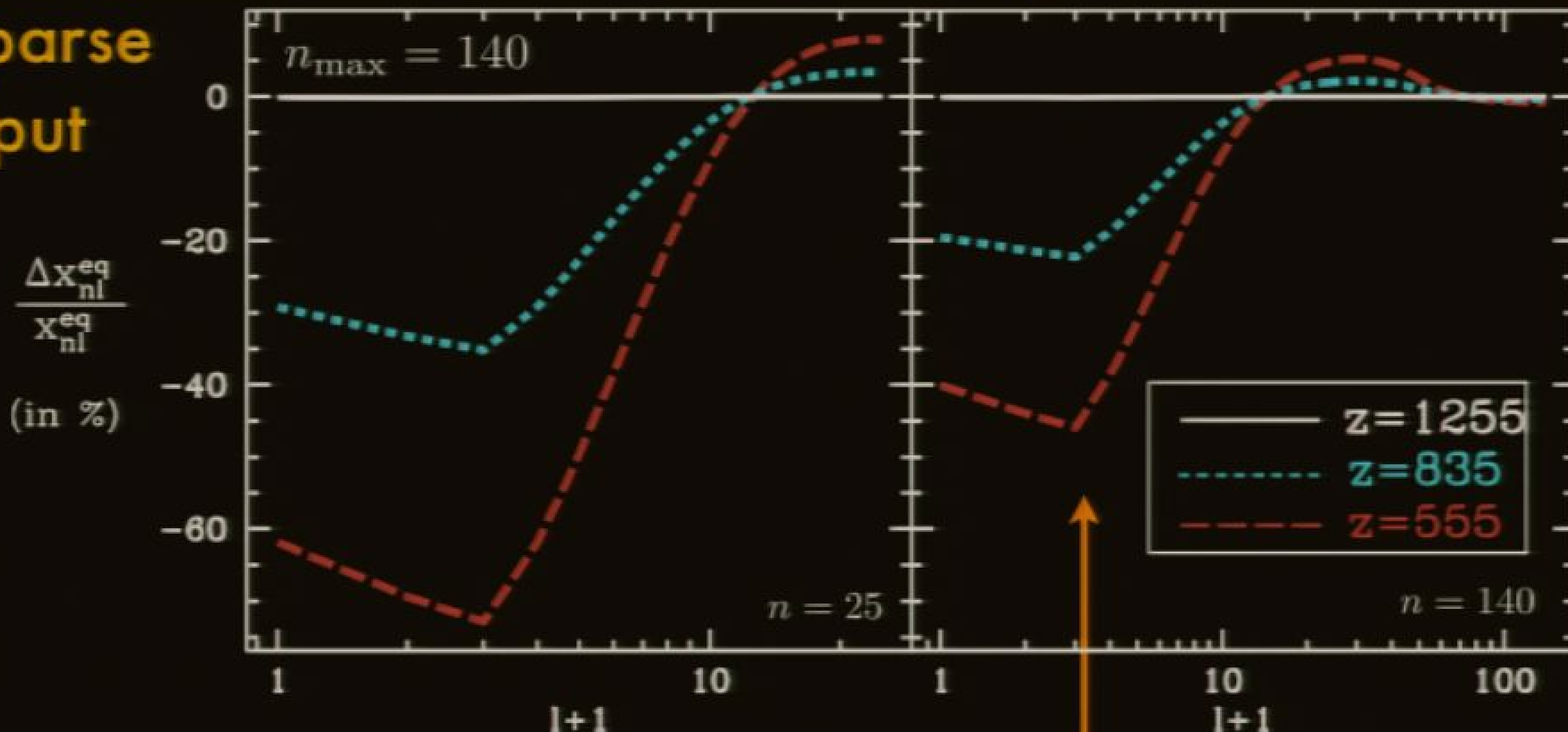
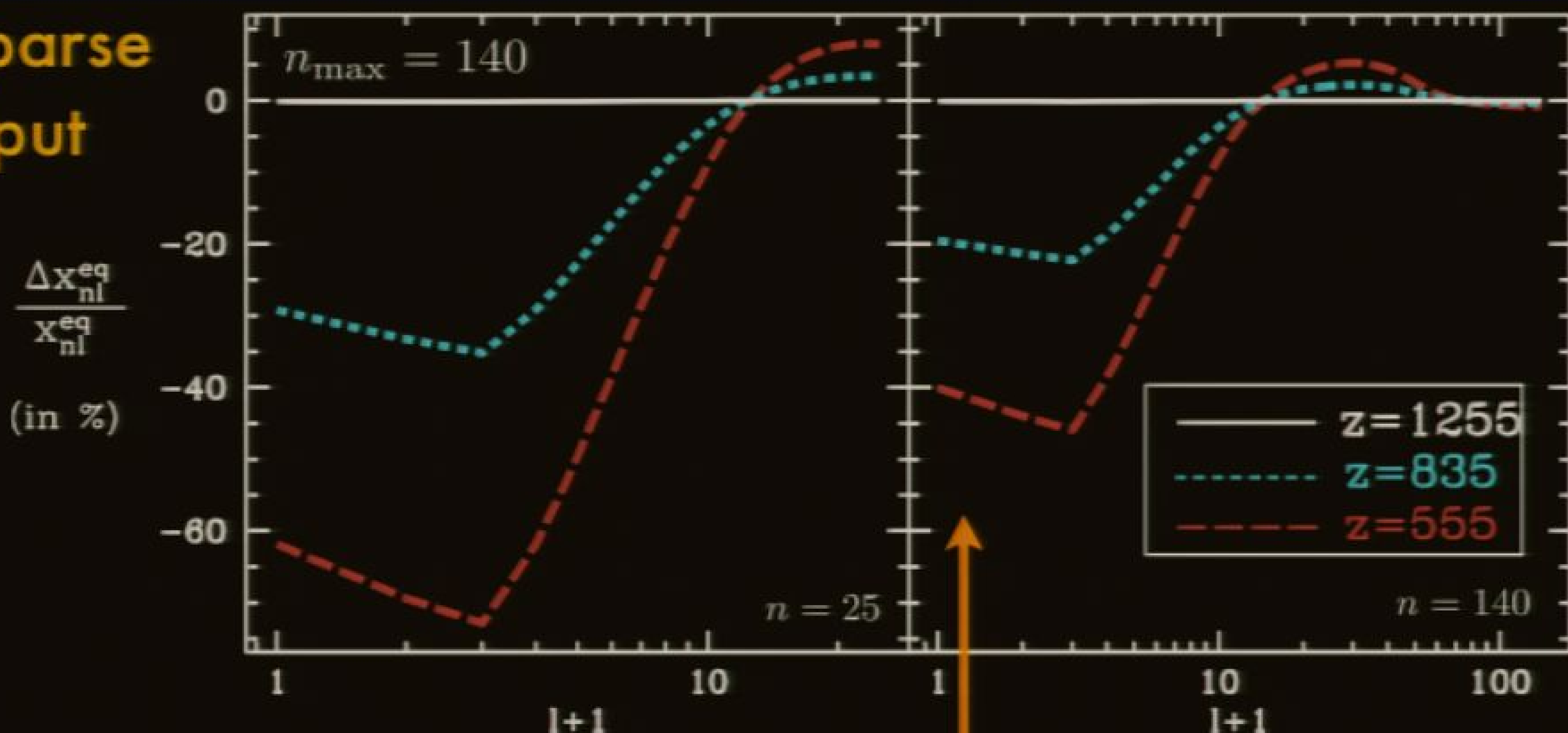


Lower l states can easily cascade down, and are relatively under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

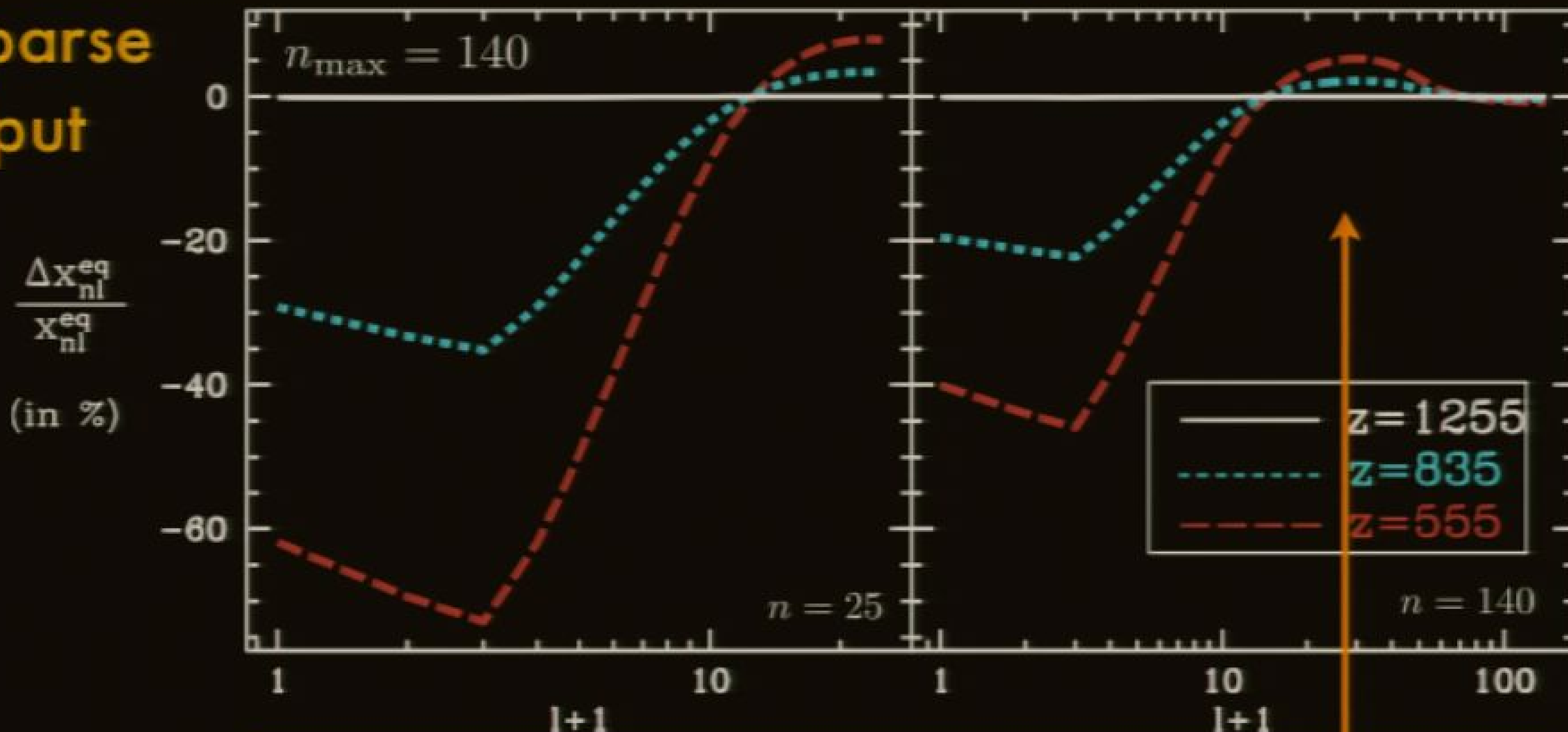
RecSparse output



=0 can't cascade down, so s states are not as under-populated

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

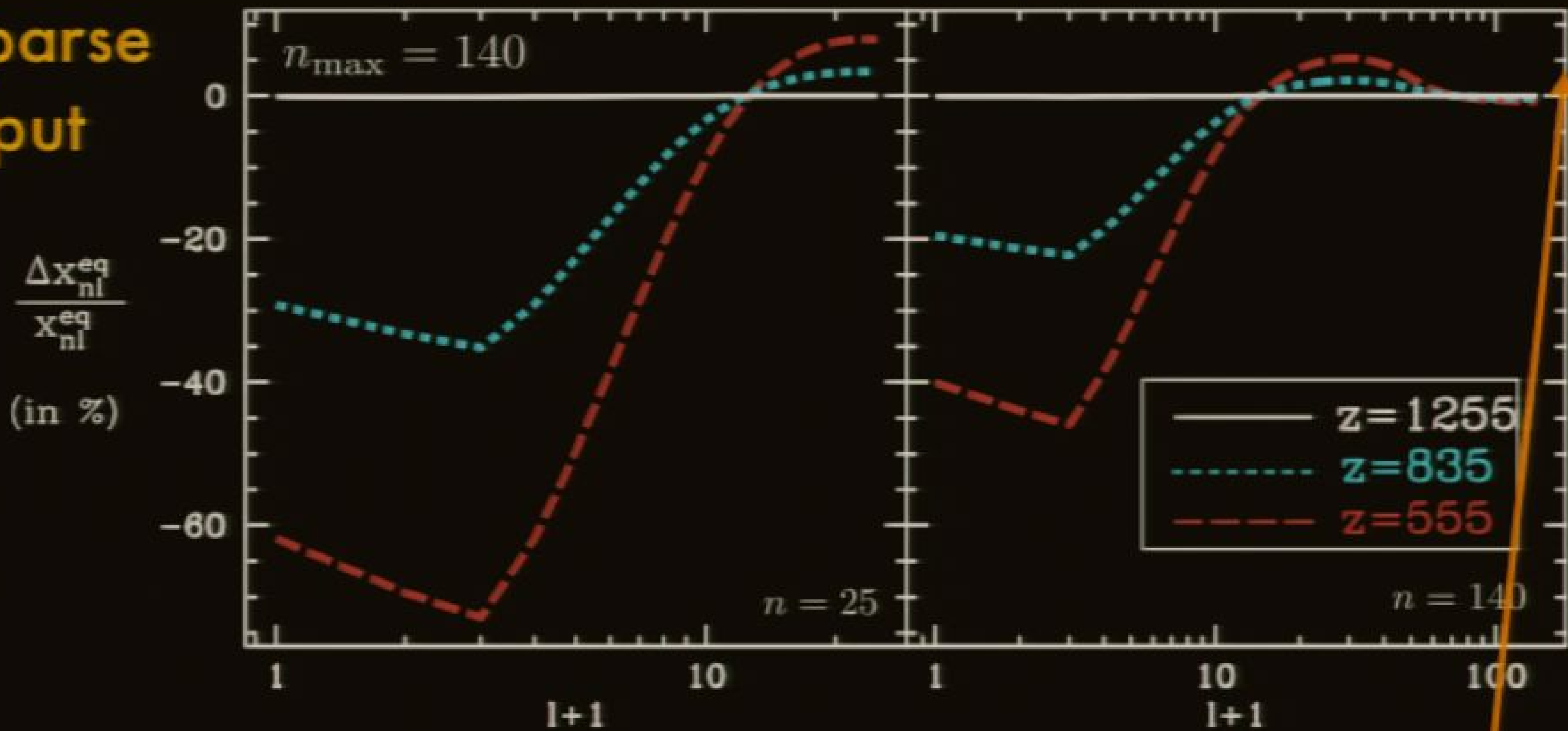
RecSparse output



Higher l are bottlenecked by $\Delta l = \pm 1$ (over-pop)

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

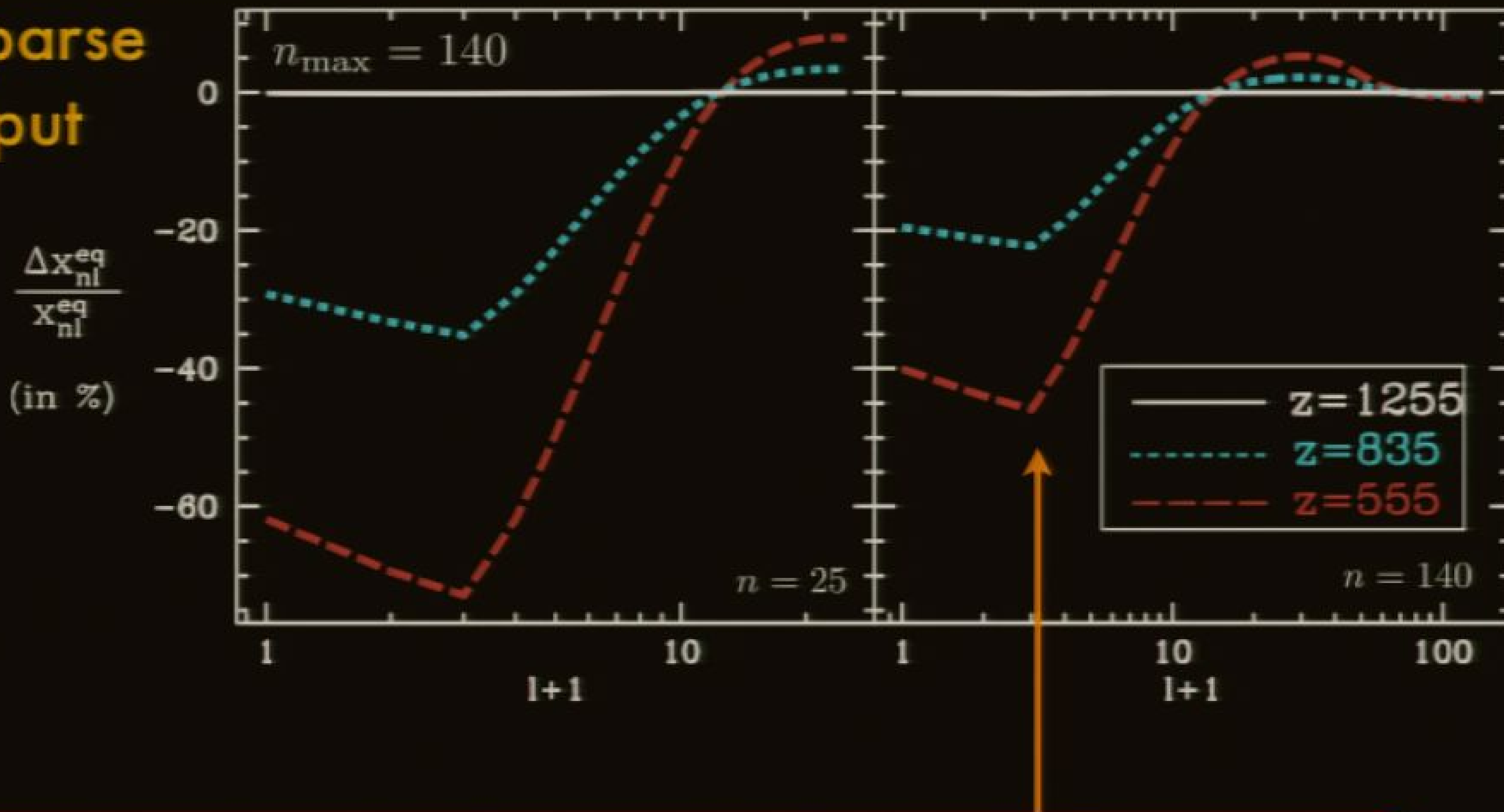
RecSparse output



Highest l states recombine inefficiently, and are under-populated

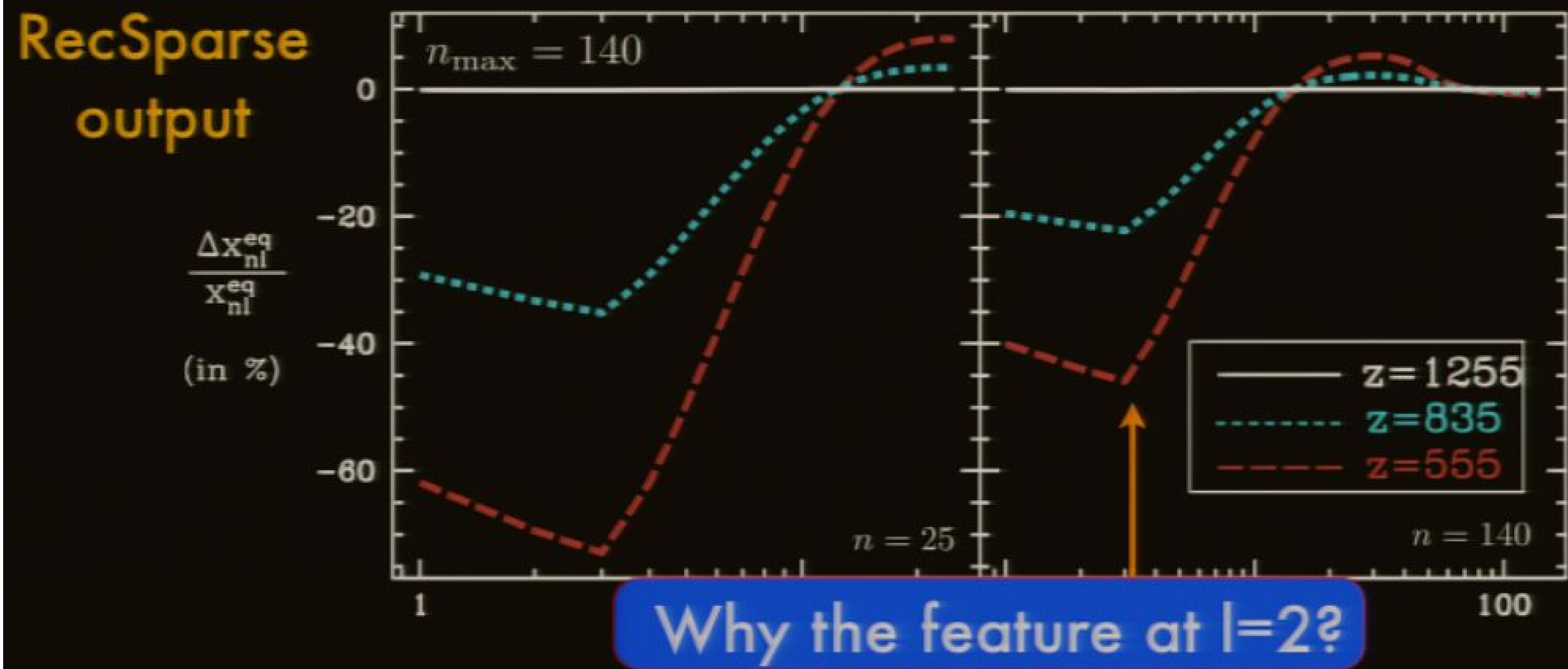
DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

RecSparse output



l-substates are highly out of Boltzmann eqb'm at late times

DEVIATIONS FROM BOLTZMANN EQ: l-SUBSTATES

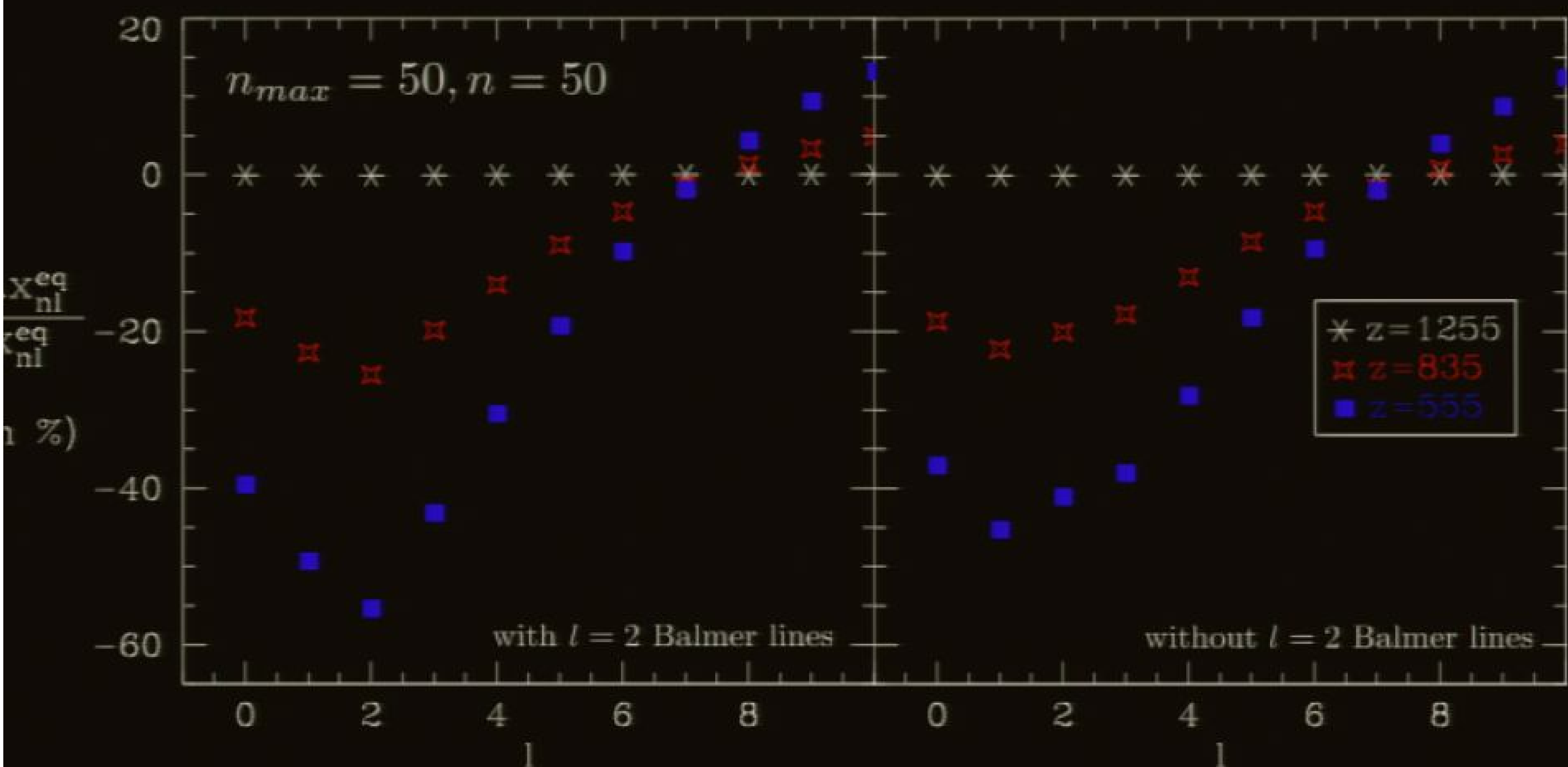


WHAT IS THE ORIGIN OF THE l=2 DIP?

$$A_{\mathrm{nd}\to 2\mathrm{p}} > A_{\mathrm{np}\to 2\mathrm{s}} > A_{\mathrm{ns}\to 2\mathrm{p}}$$

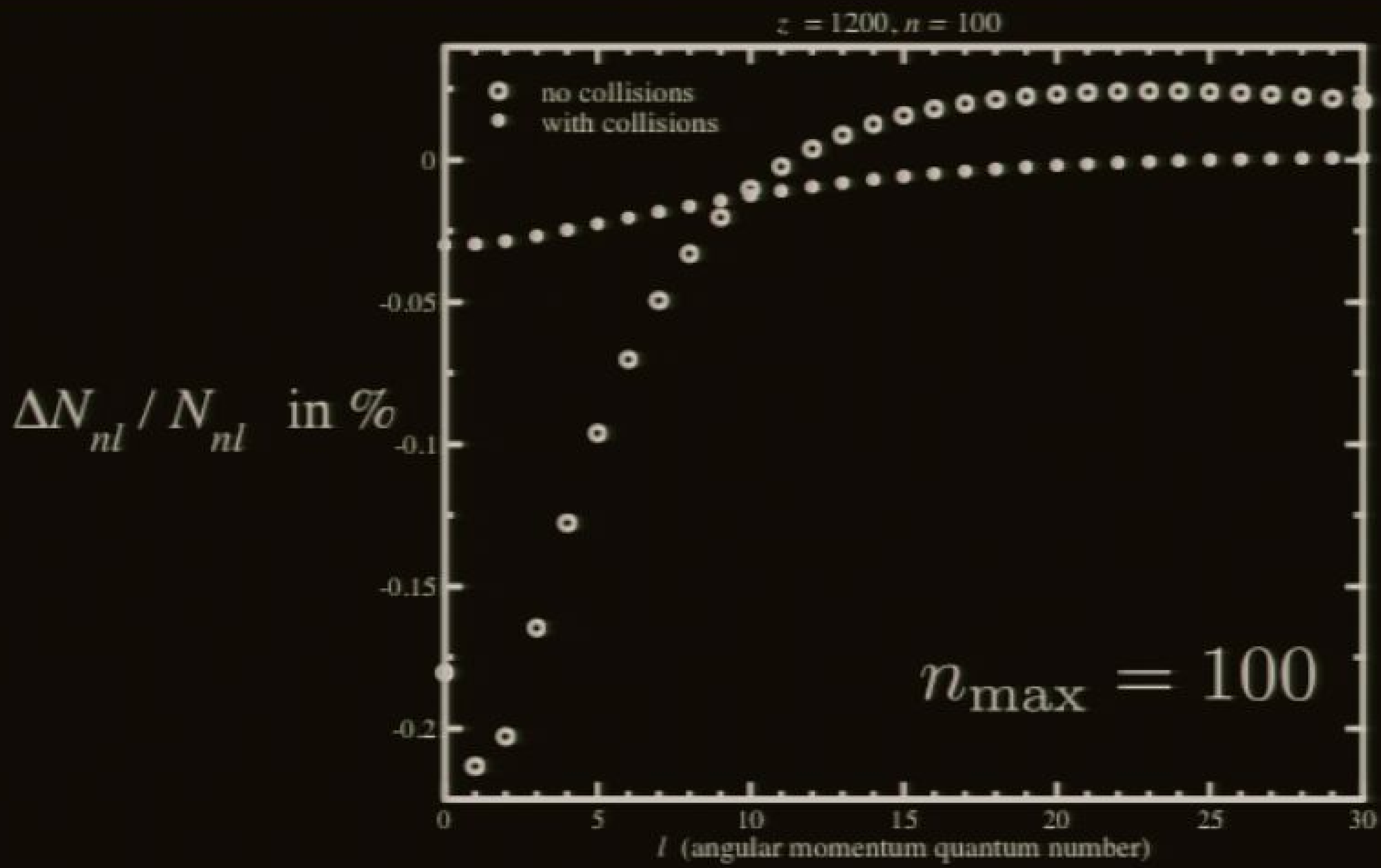
* l=2 depopulates more rapidly than l=1 for higher (n>2) excited states
* We can test if this explains the dip at l=2 by running the code with these Balmer transitions the blip should move to l=1

L-SUBSTATE POPULATIONS, BALMER LINES OFF



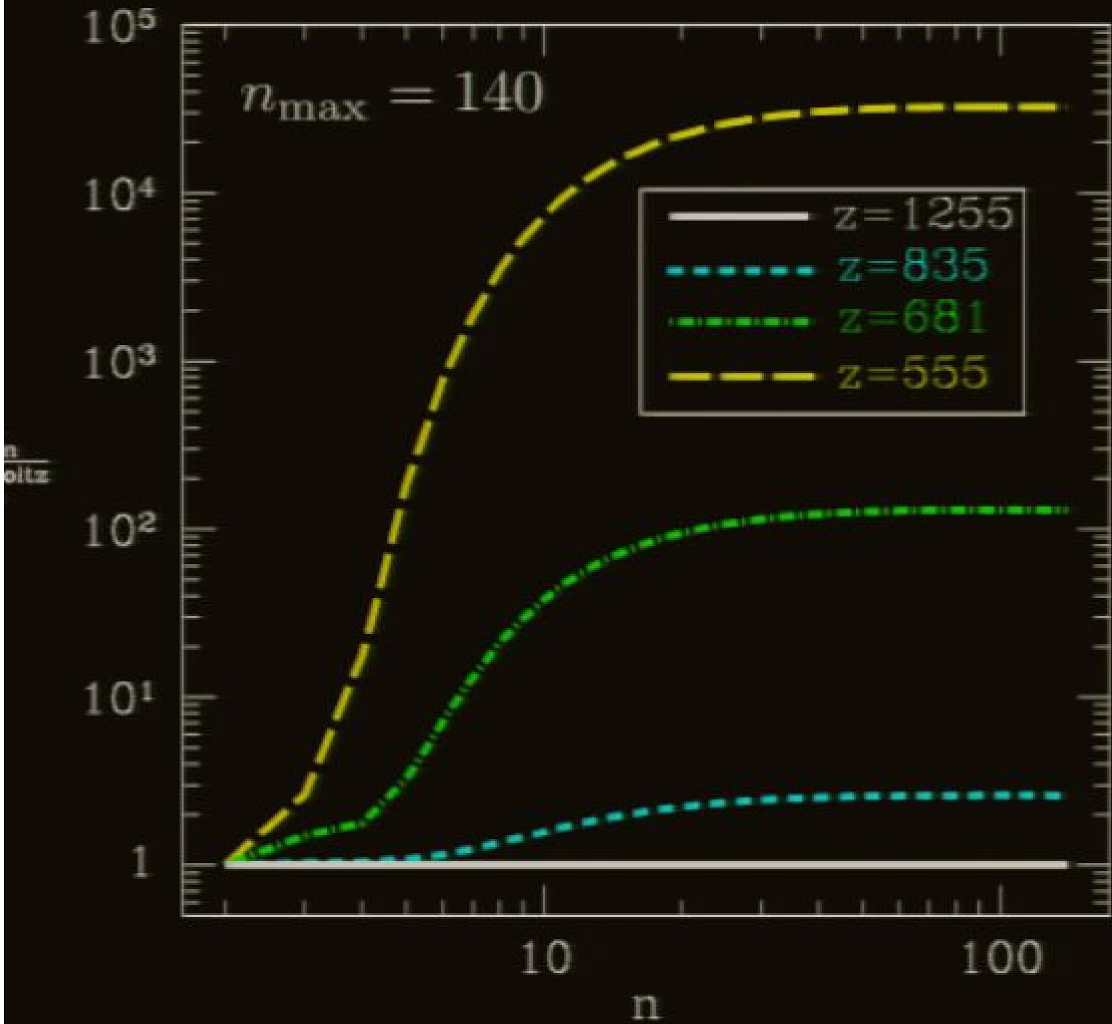
Dip moves as expected when Balmer lines are off!

ATOMIC COLLISIONS



- l-changing collisions bring l-substates closer to statistical equilibrium (SE) (Chluba, Rubino Martin, Sunyaev 2006)
- Theoretical collision rates unknown to factors of 2!

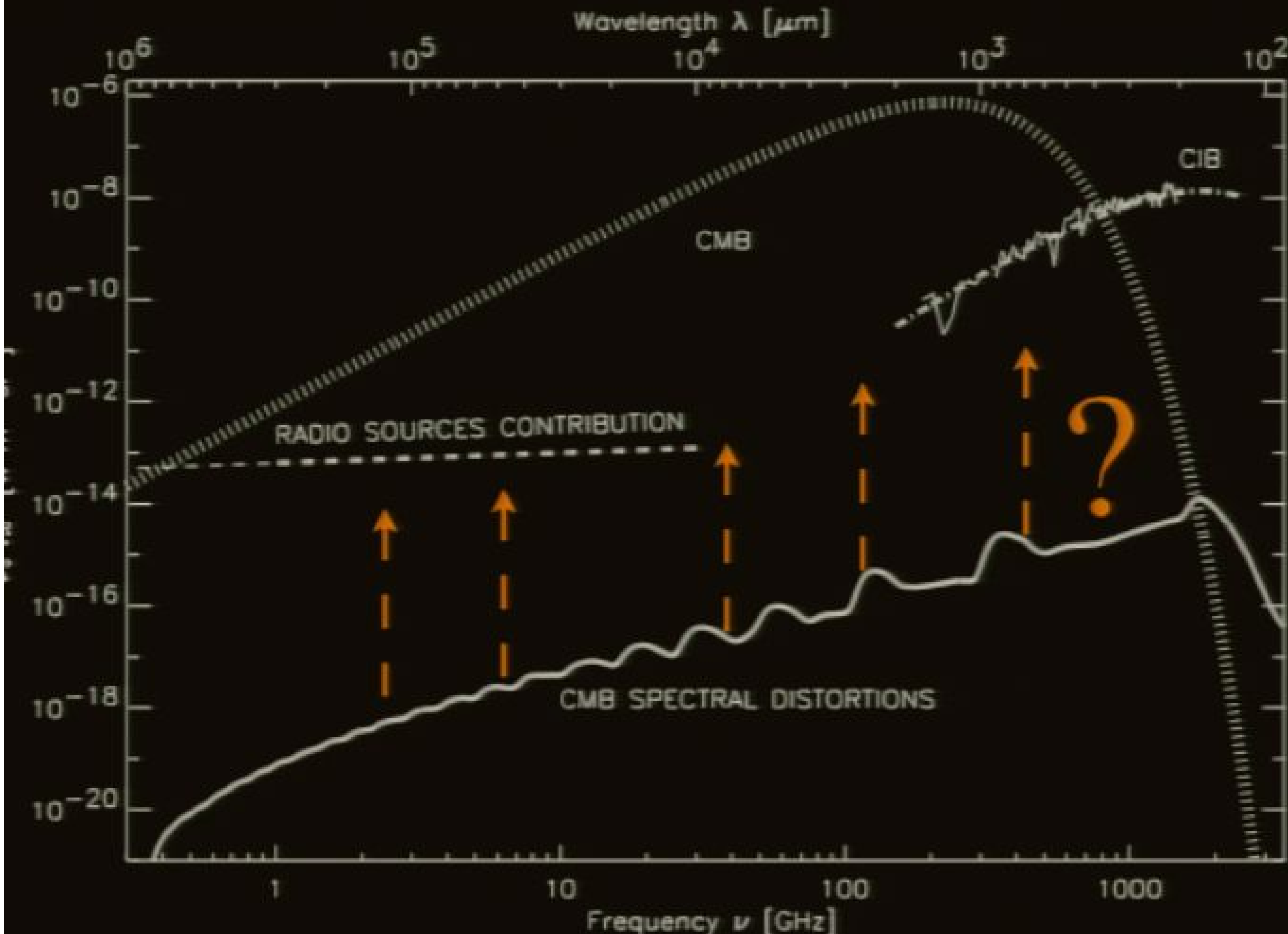
DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



$$\alpha_n n_e > \sum_{n'l}^{n'<n} A_{nn'}^{ll\pm1}$$

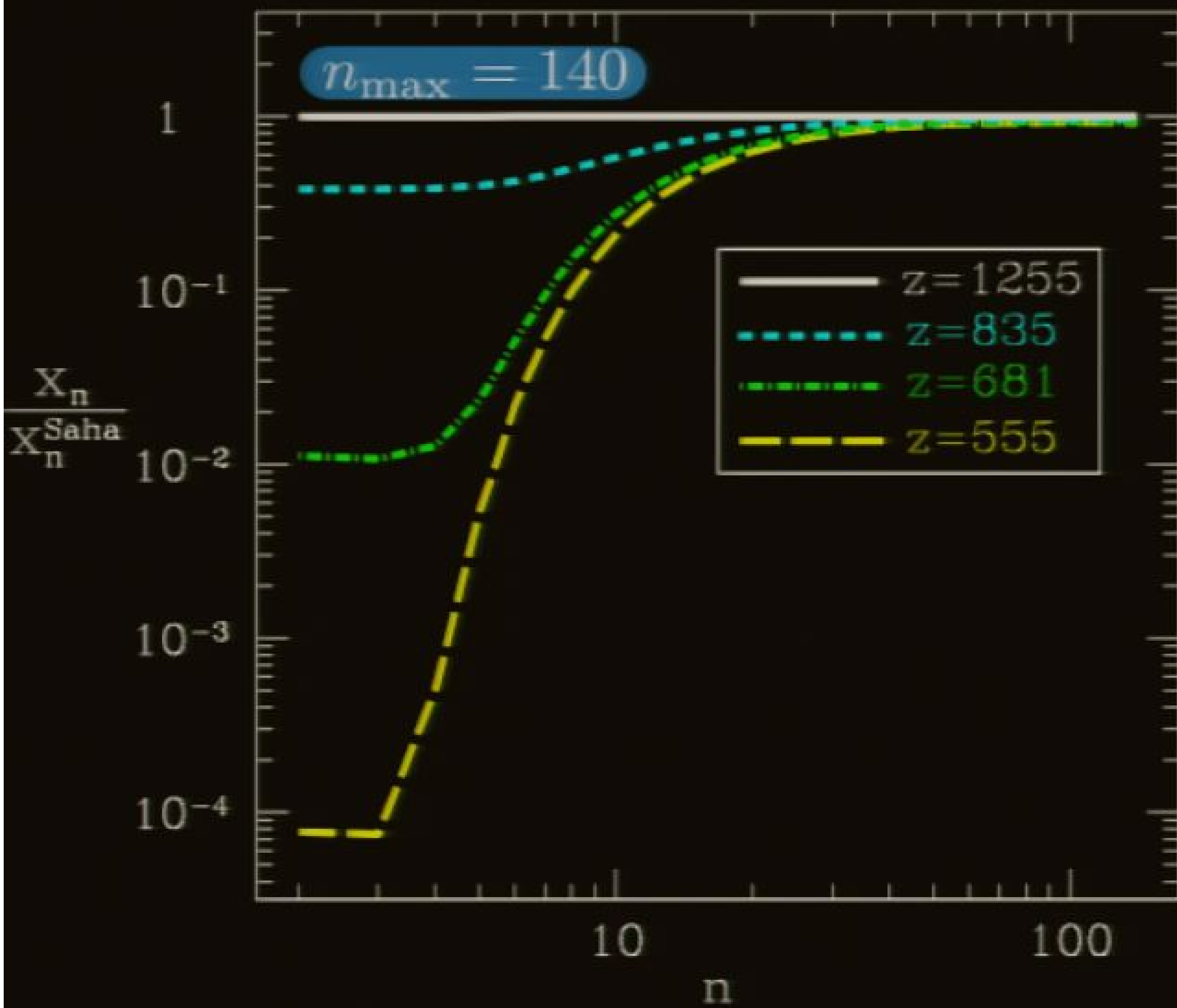
- No inversion relative to n=2 (just over-population)
- Population inversion seen between some excited states: Does radiation stay coherent? Does recombination mase?

DEVIATIONS FROM BOLTZMANN EQUILIBRIUM: DIFFERENT n-SHELLS



Masing could make spectral distortions detectable!

DEVIATIONS FROM SAHA EQUILIBRIUM

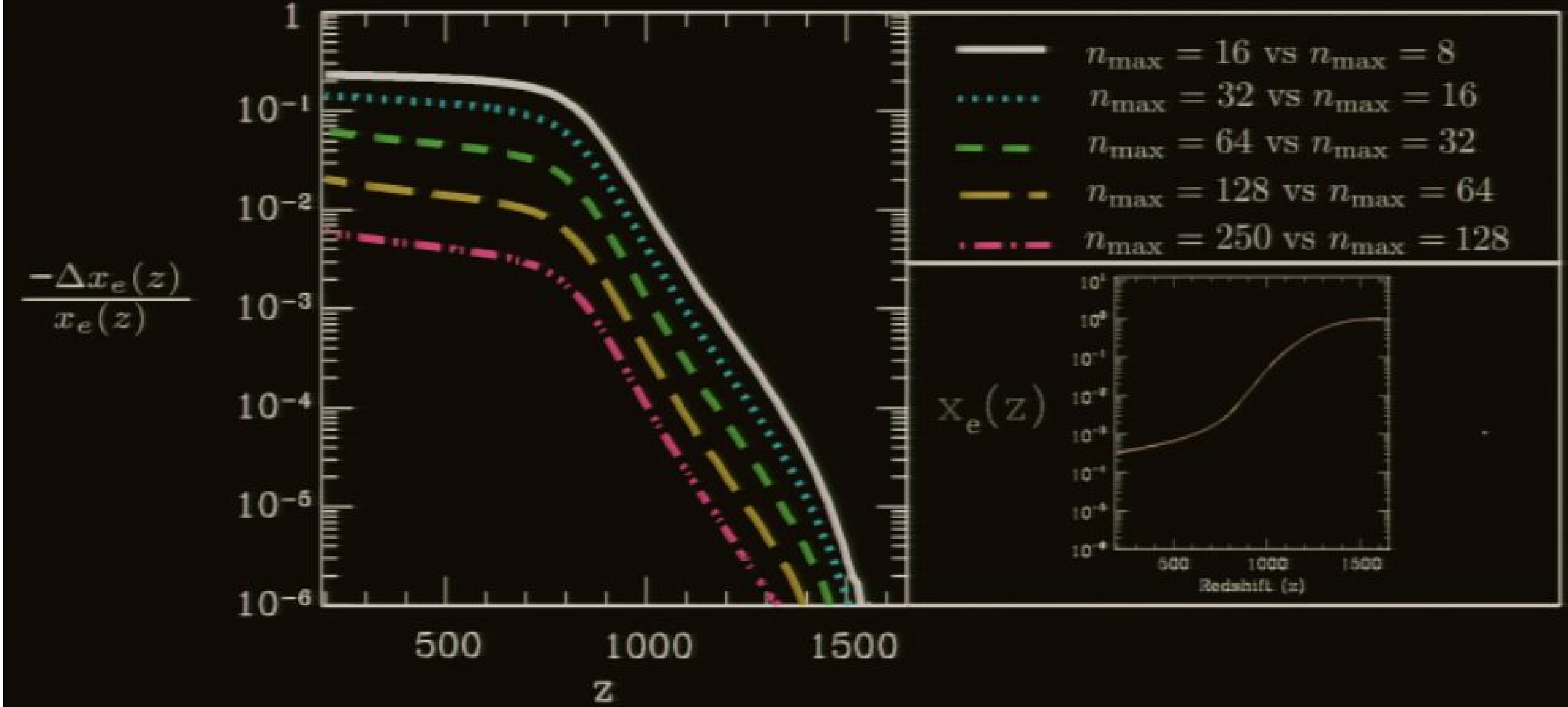


HUGE DEVIATIONS FROM SAHA EQ!

* Effect of states with $n > n_{max}$ could be approximated using asymptotic Einstein coeffs. and Saha eq, but Saha is elusive at high n/late times.

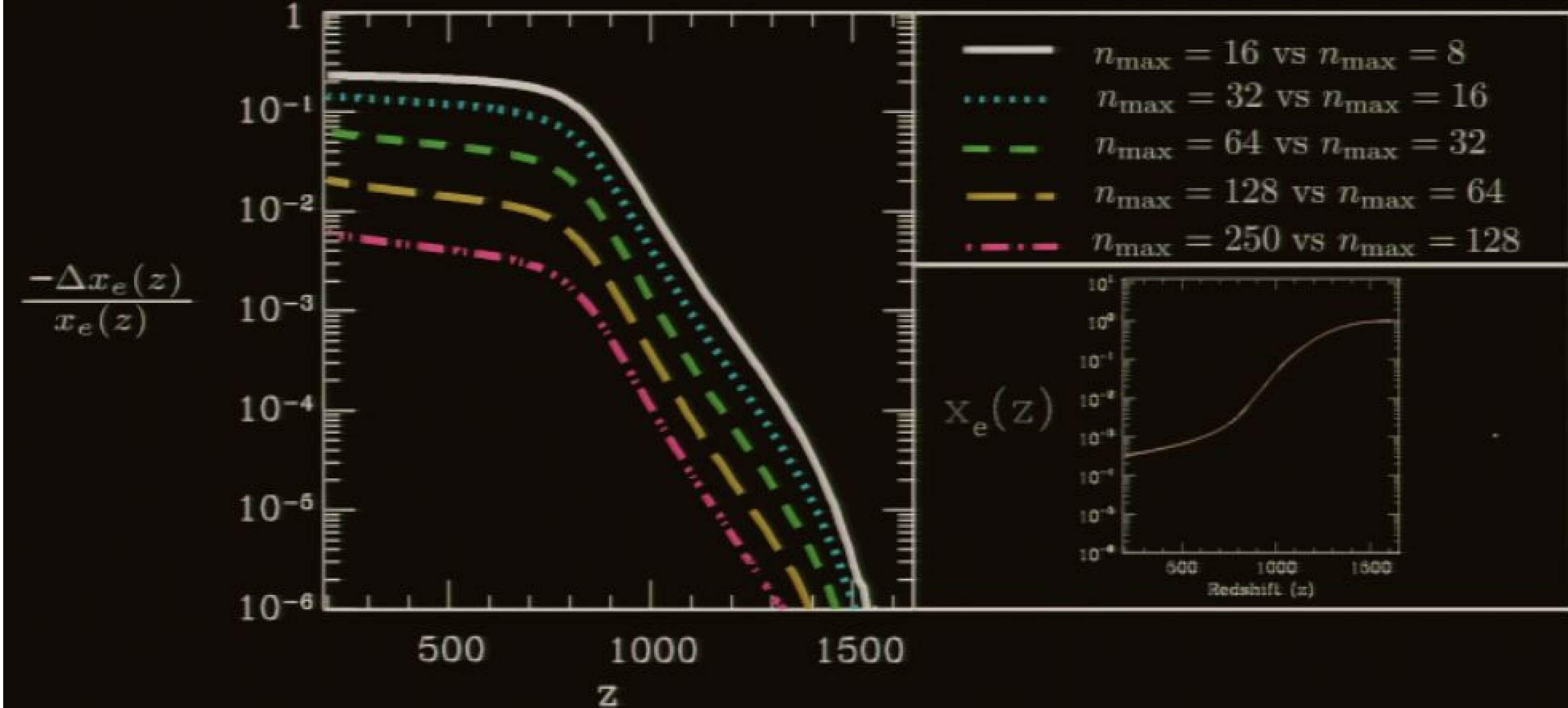

RESULTS: RECOMBINATION HISTORIES
34

ESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



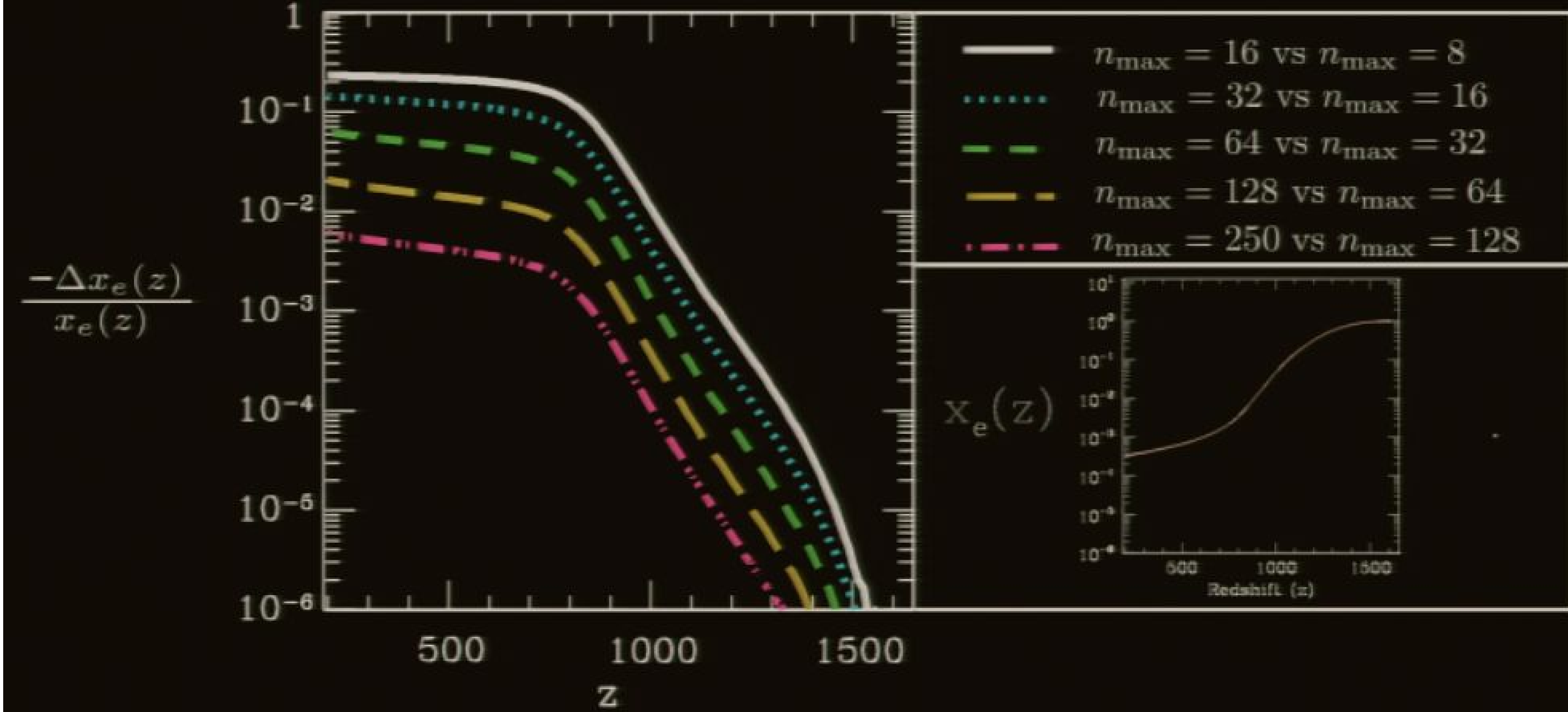
- $x_e(z)$ falls with increasing $n_{\max} = 10 \rightarrow 250$, as expected.
- Rec Rate>downward BB Rate> Ionization, upward BB rate
- For $n_{max} = 100$, code computes in only 2 hours

ESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



- Relative convergence is not the same thing as absolute convergence: Want to see Saha asymptote and impose well-motivated cutoff! Collisions could help
- These are lower limits to the actual error
- $n_{\max}$=300 just completed

RESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



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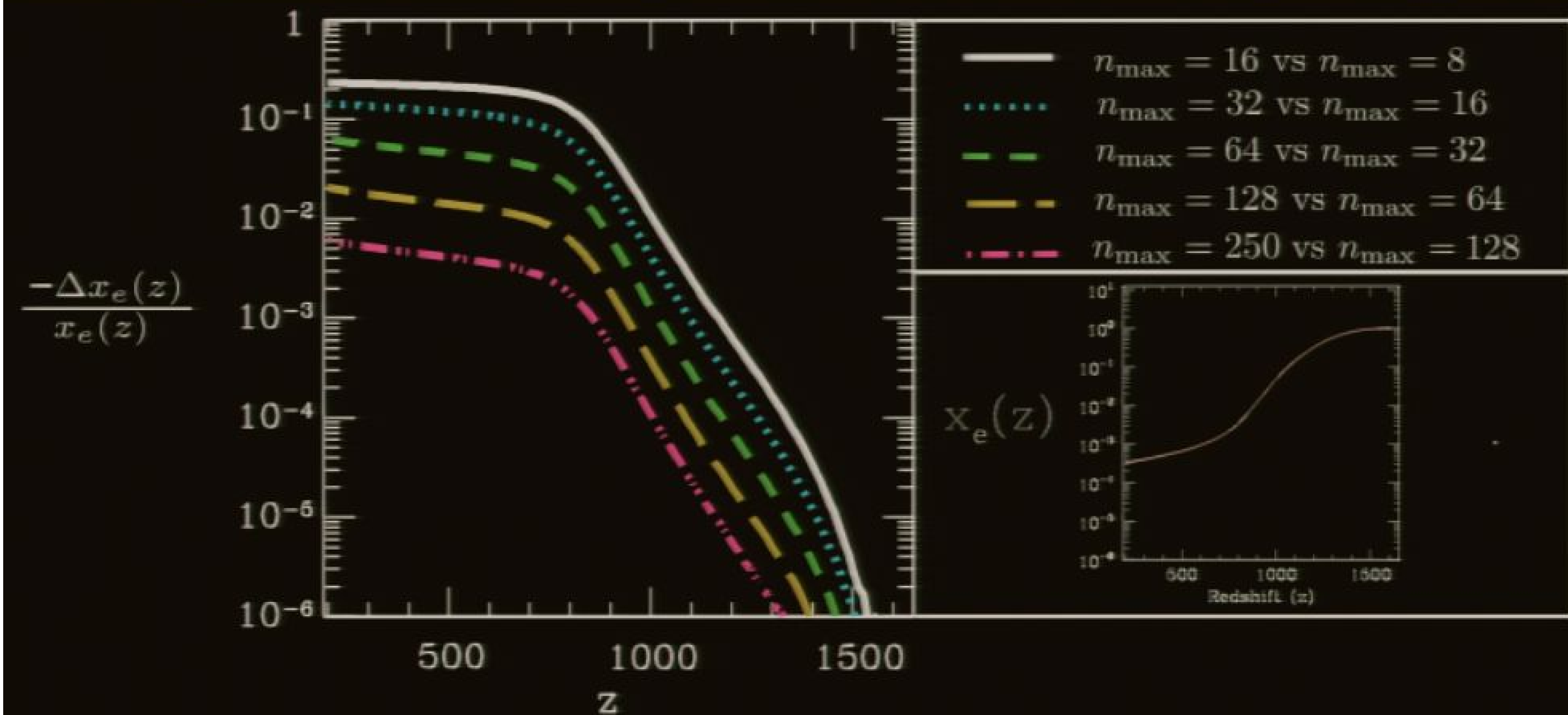

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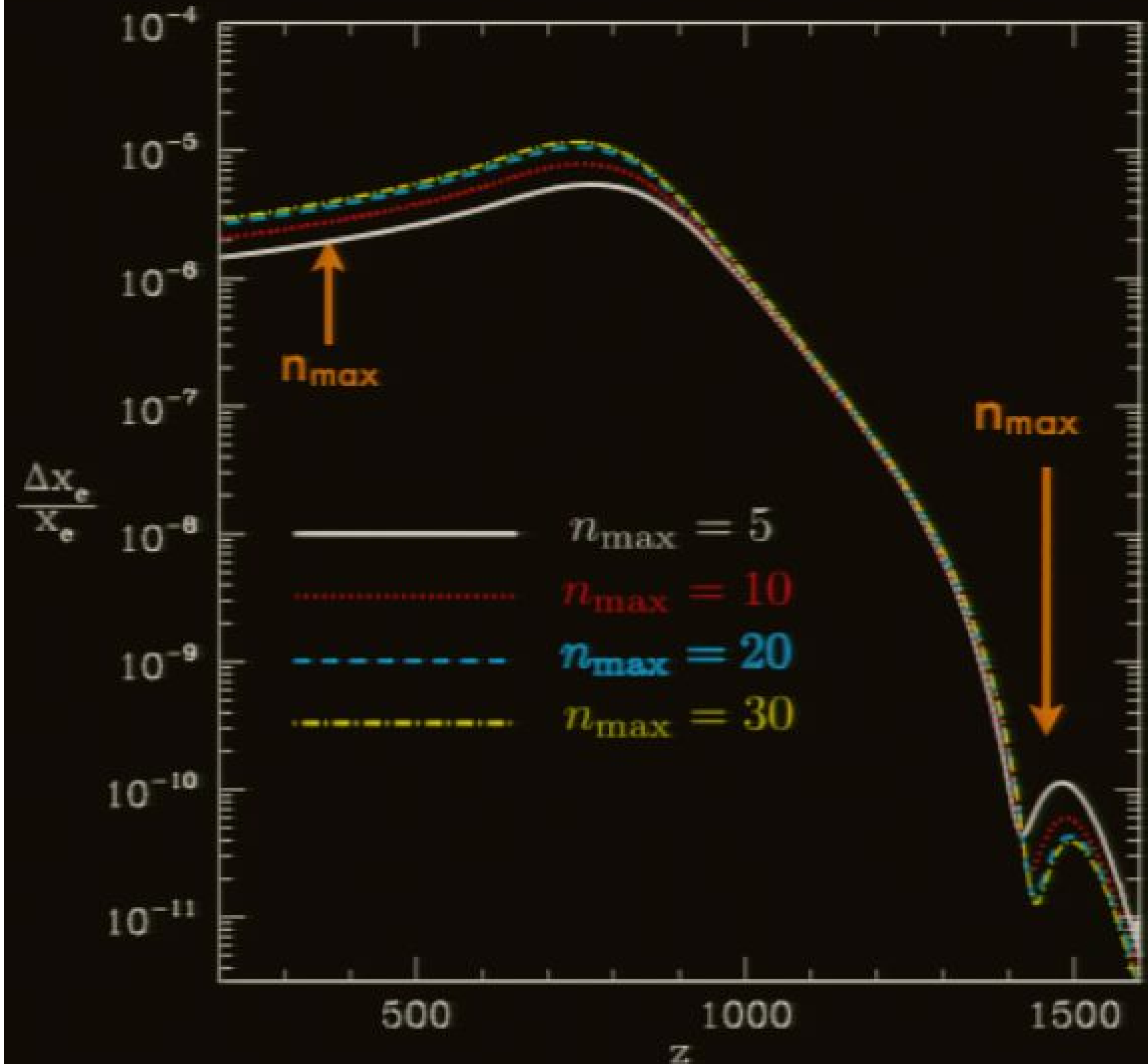
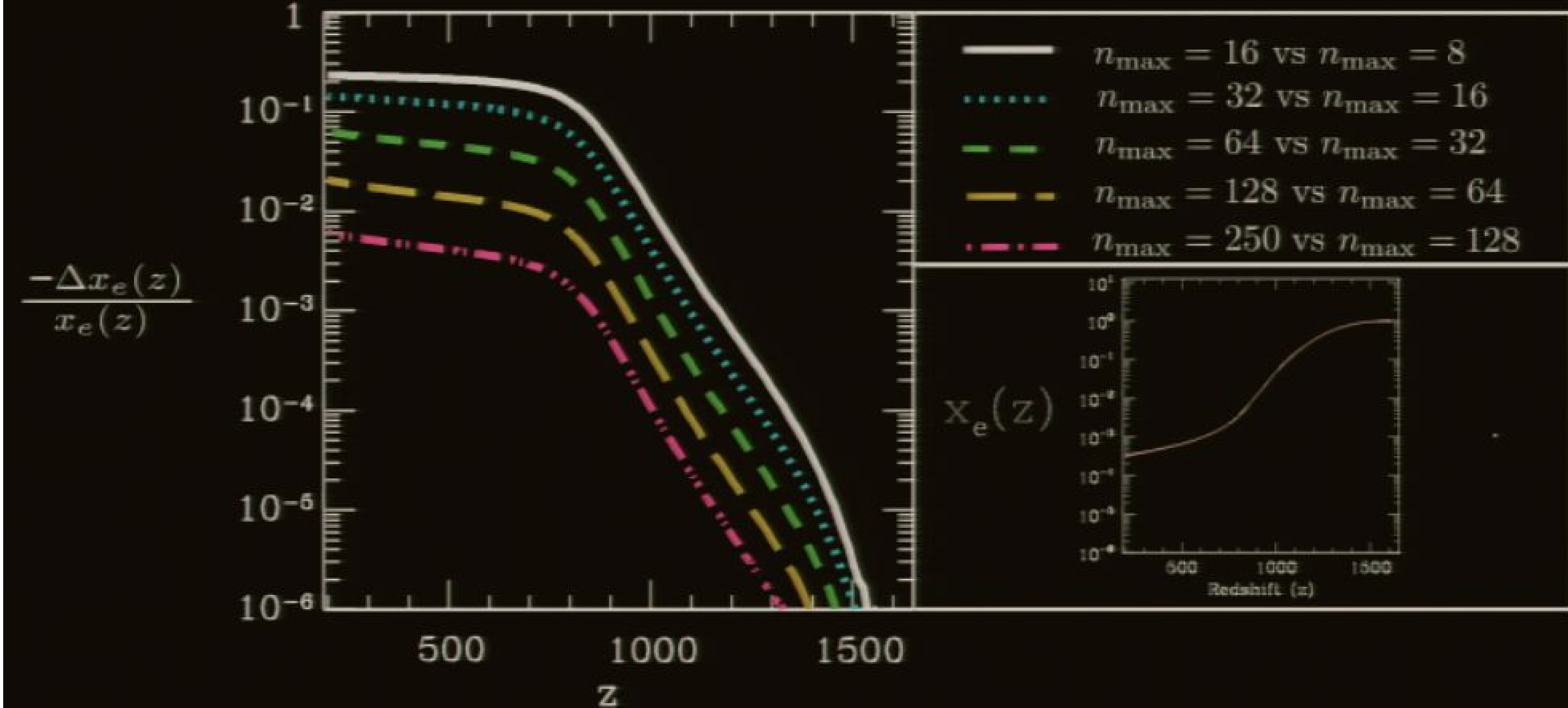
RESULTS: RECOMBINATION WITH HYDROGEN



$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

Negligible for Planck!

ESULTS: RECOMBINATION HISTORIES INCLUDING HIGH-n



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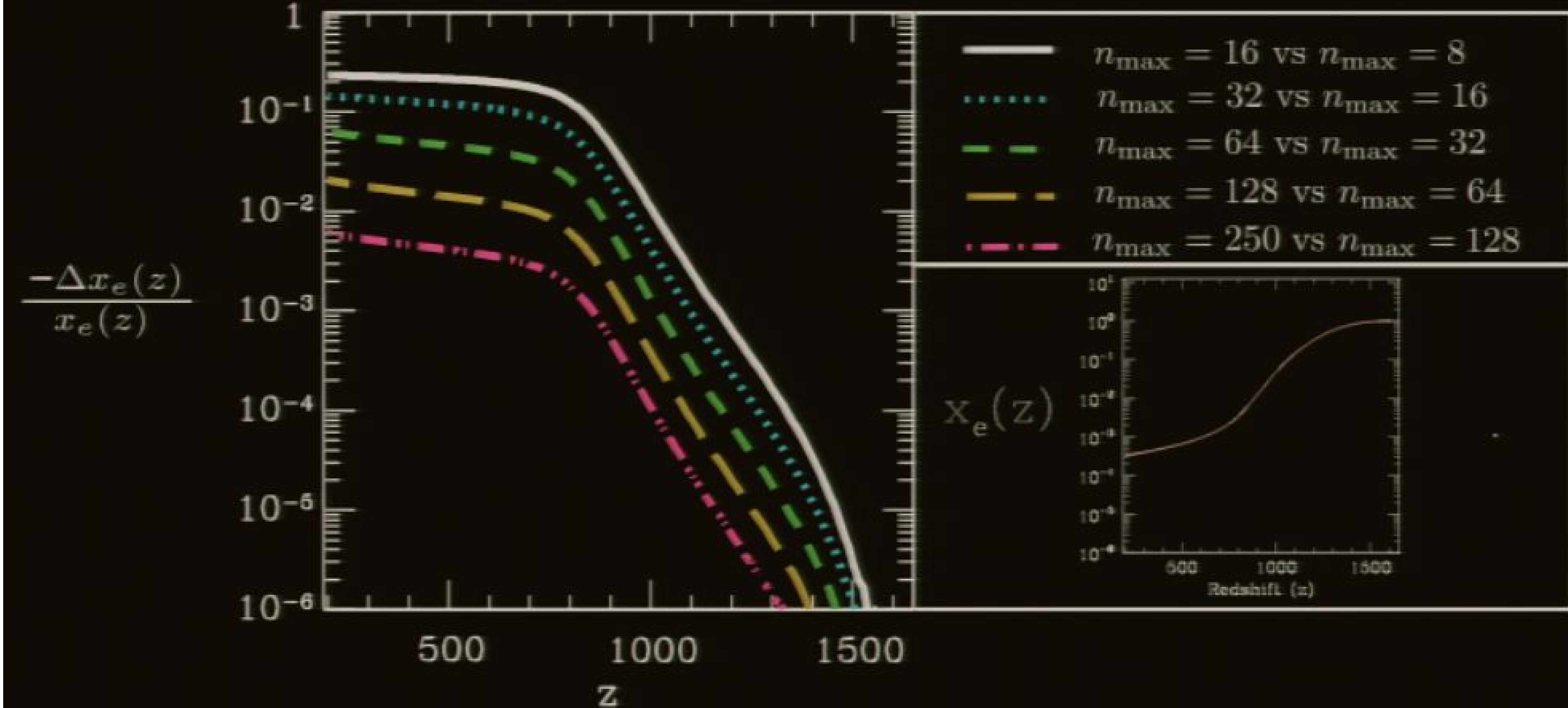

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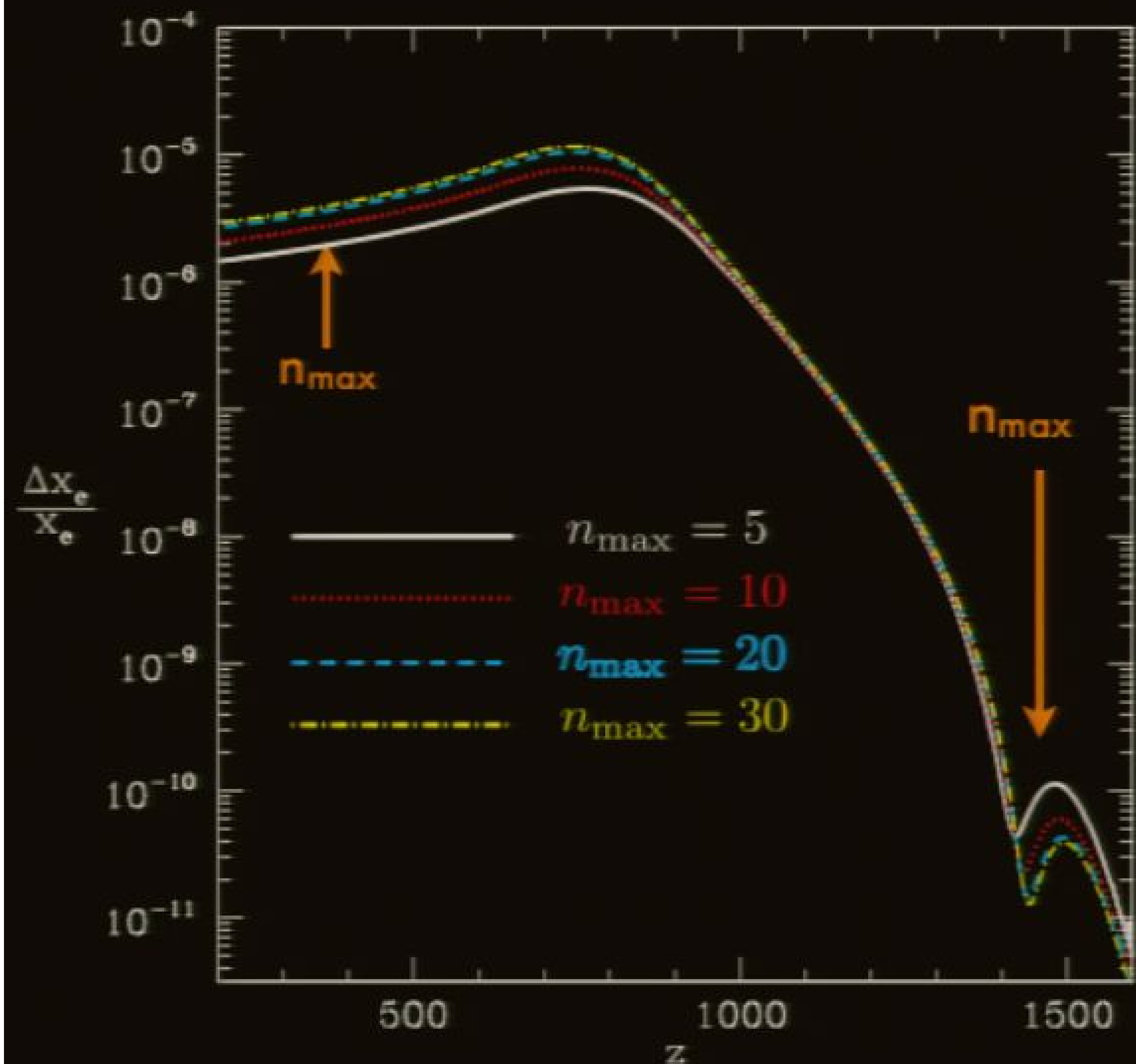

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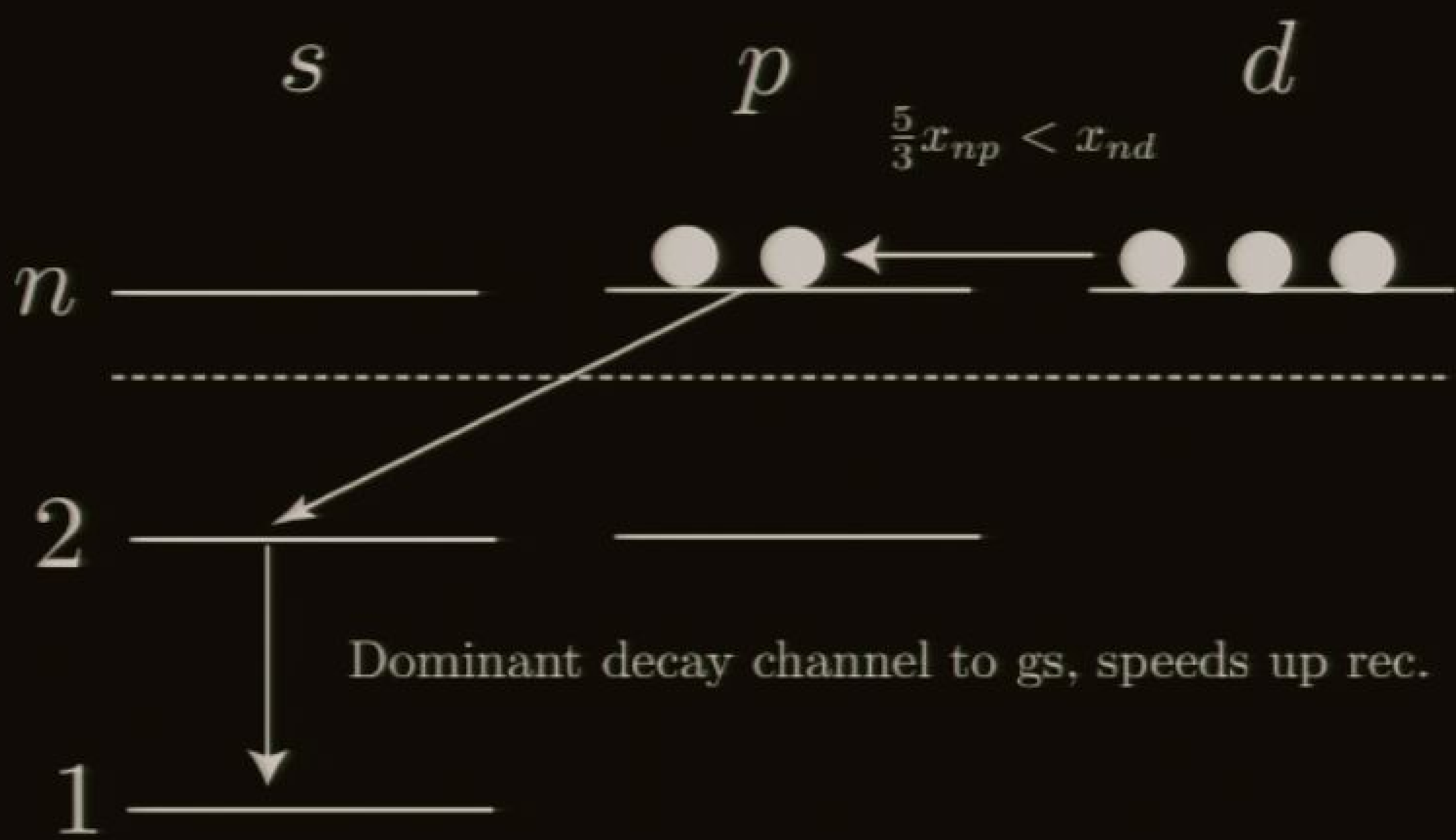
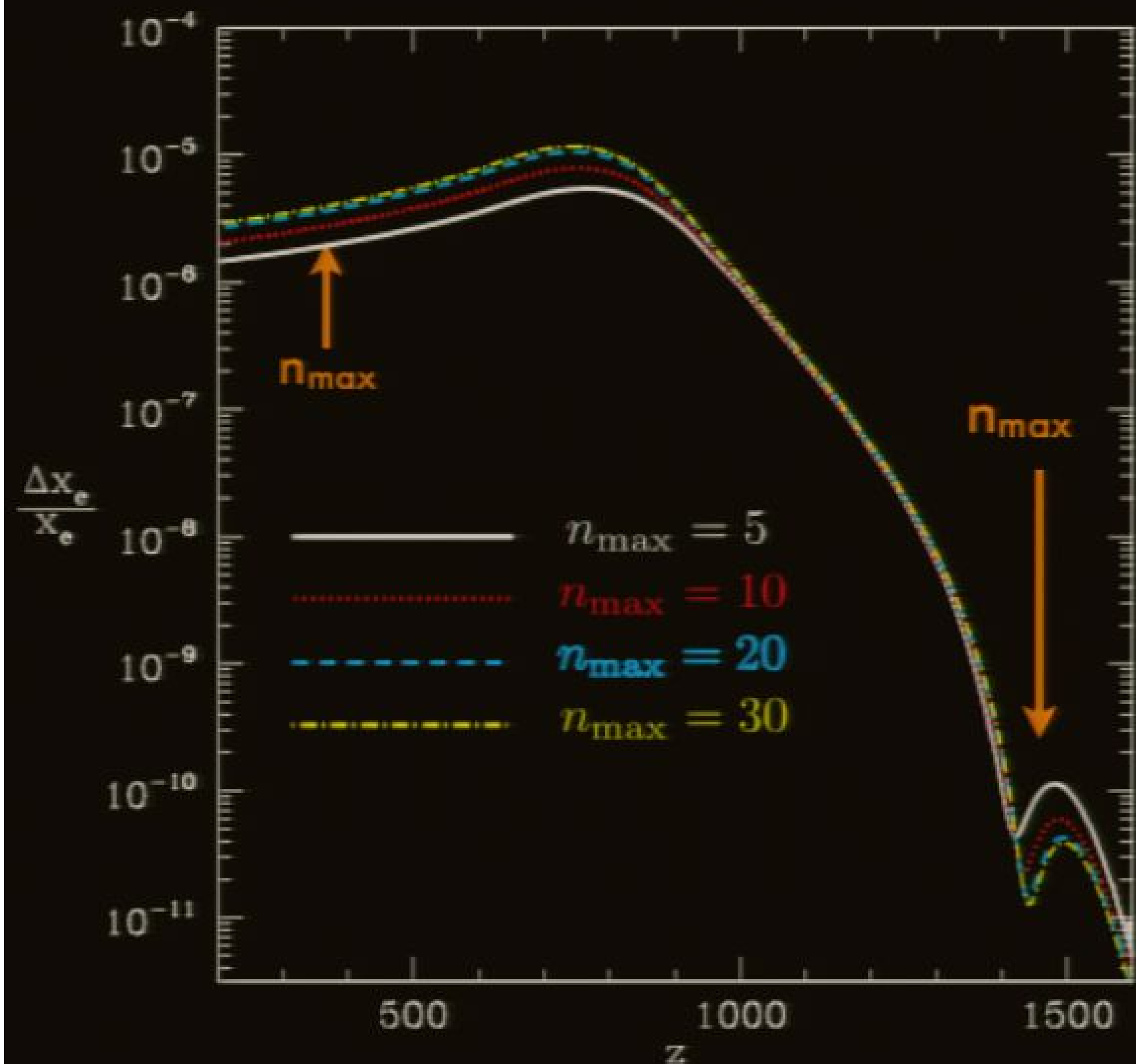


$$R^{\mathrm{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n < 5,$ early times

RESULTS: RECOMBINATION WITH HYDROGEN



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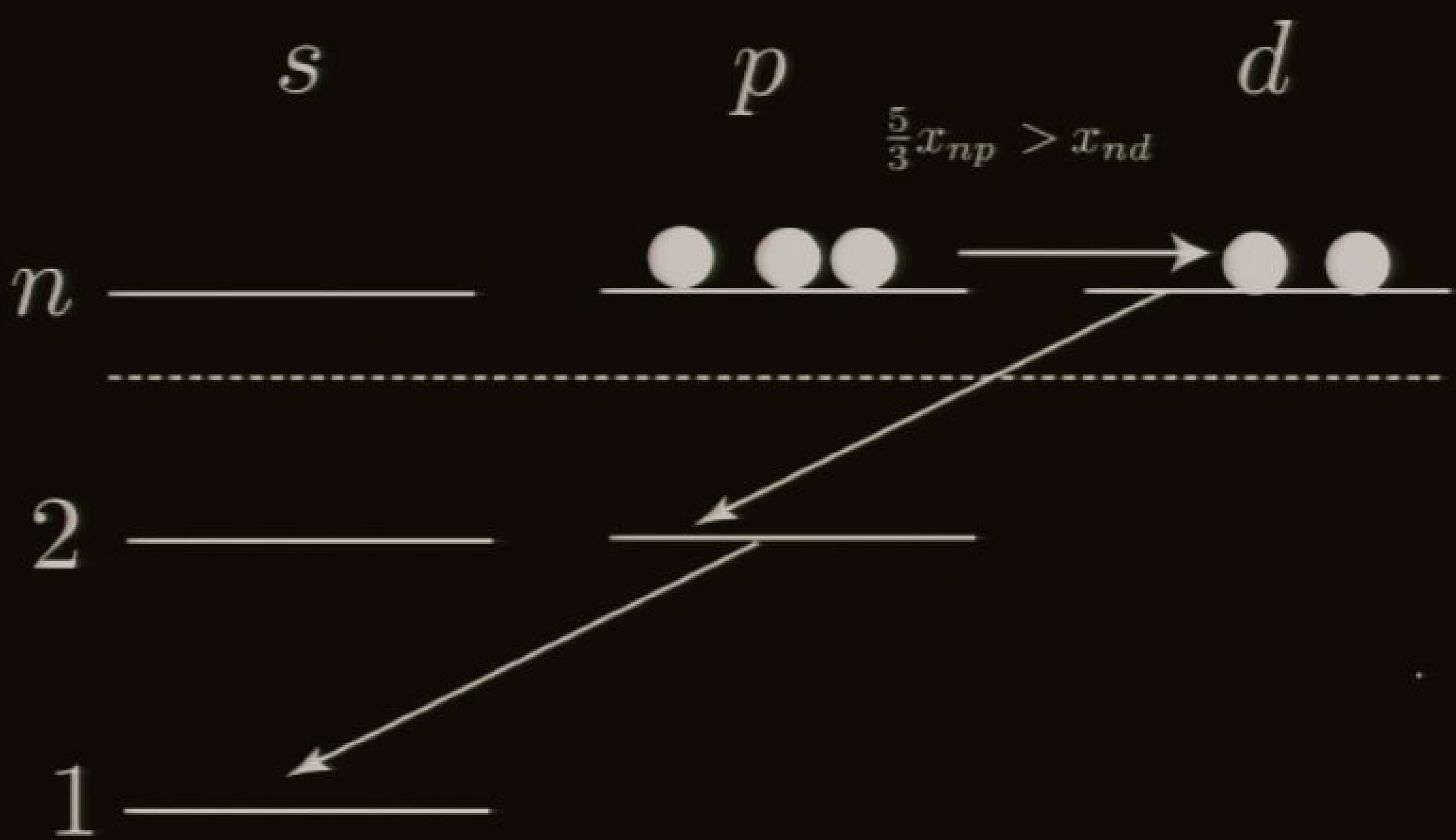
Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

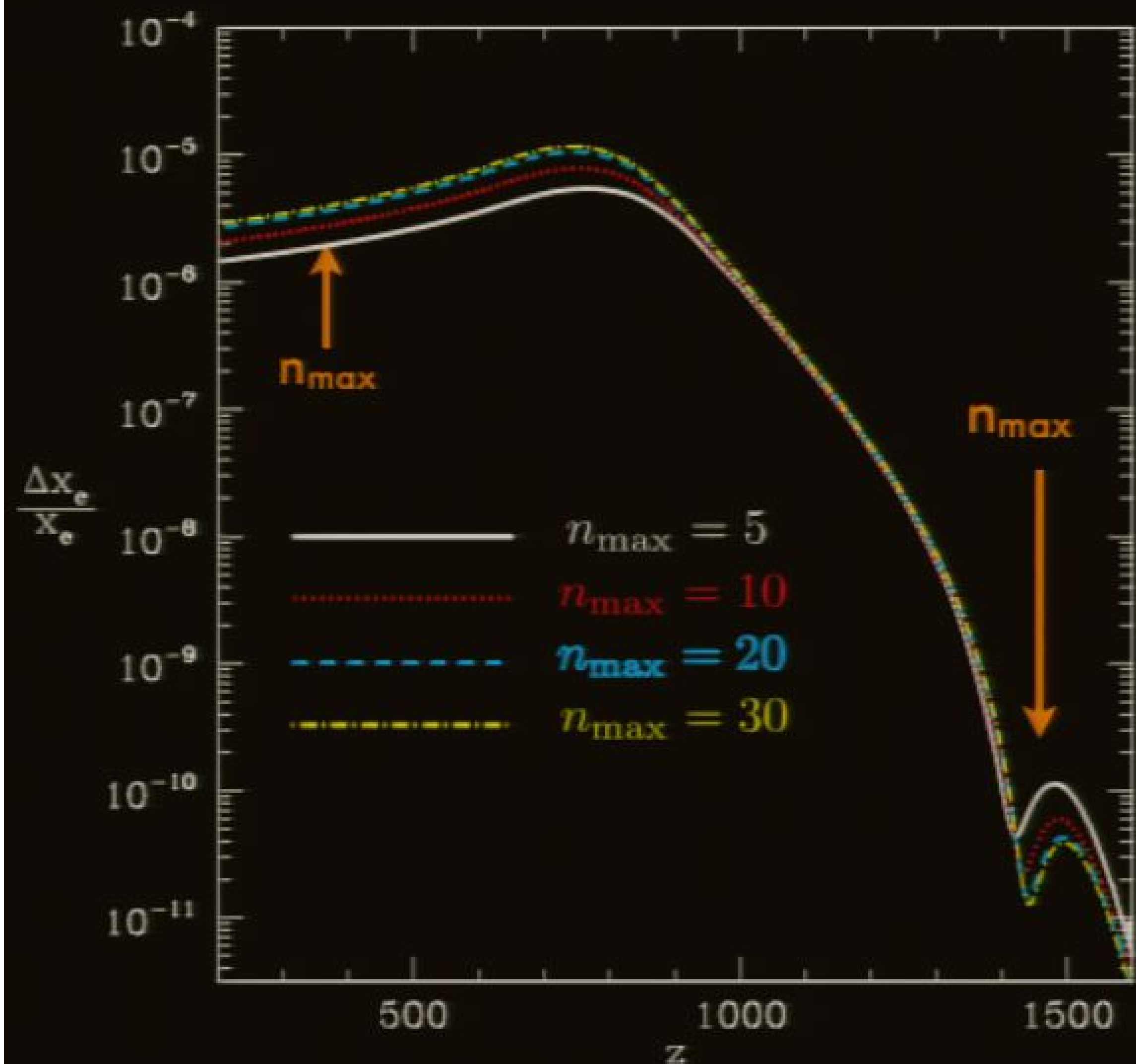


Sub-Dominant decay channel to gs, slows rec down rel. to $n < 5$

$$R^{\text{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

$n \geq 5$, early times

RESULTS: RECOMBINATION WITH HYDROGEN



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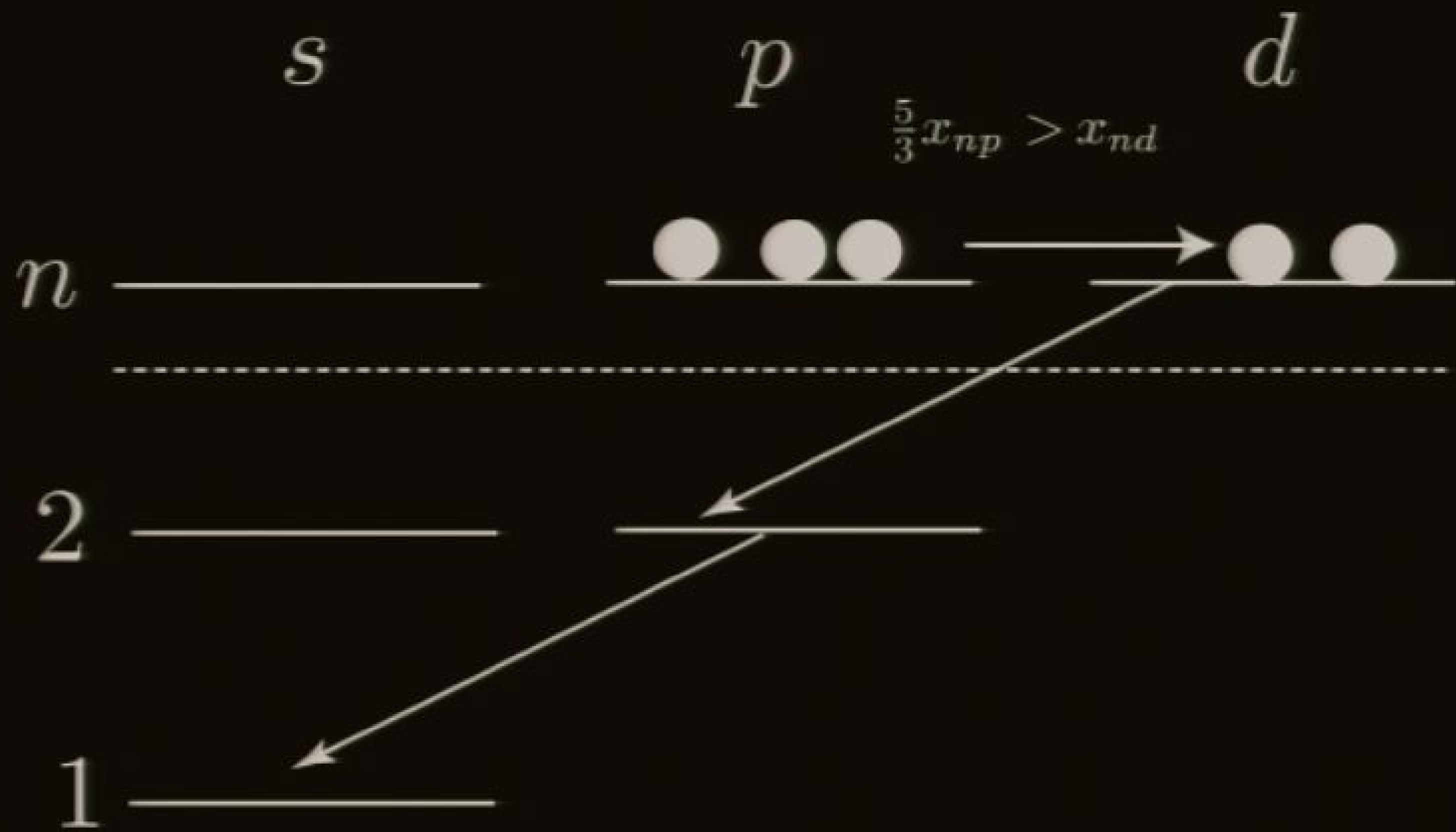
Negligible for Planck!

RESULTS: RECOMBINATION WITH HYDROGEN

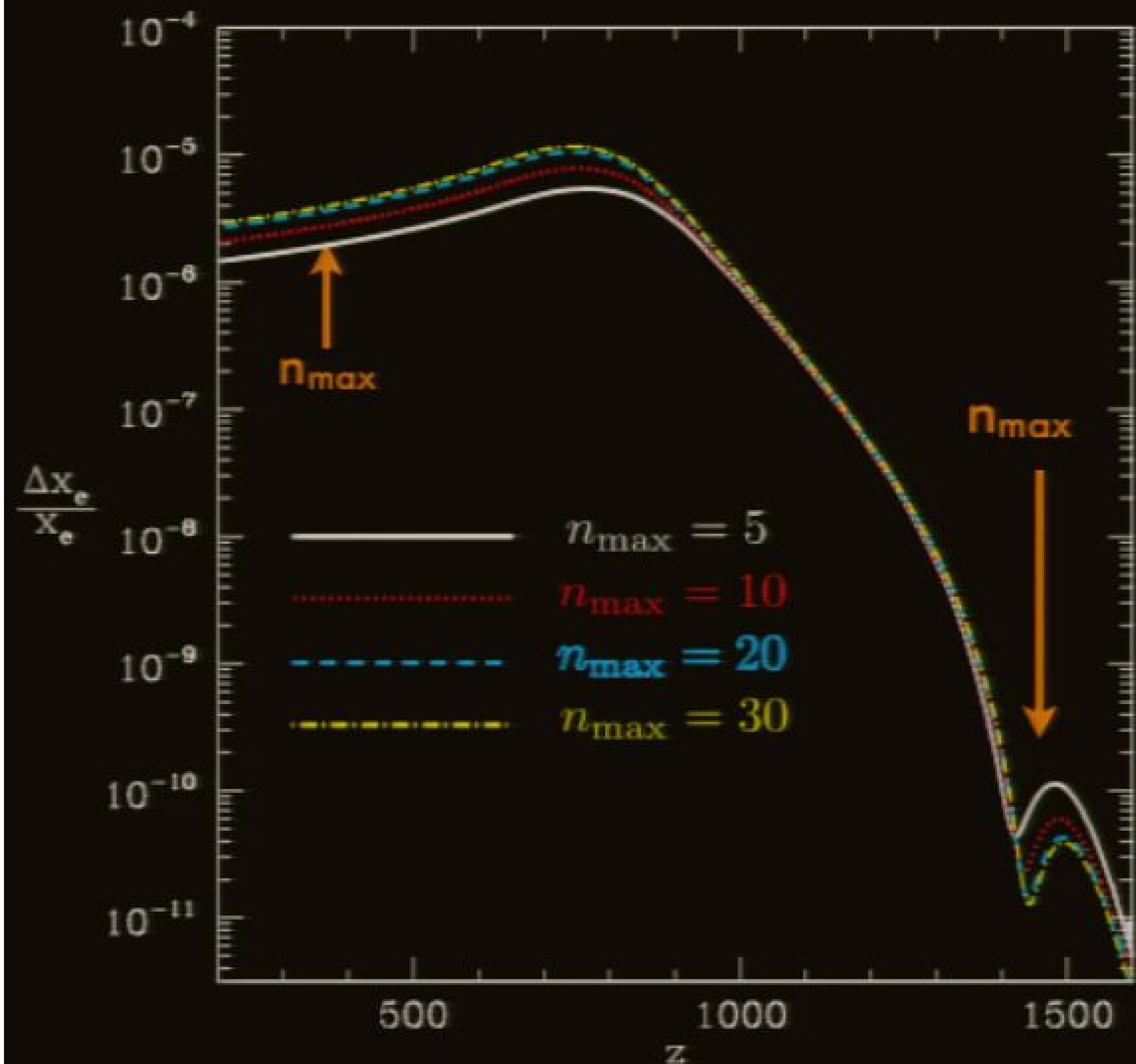


Dominant decay channel to gs, speeds up rec

$$R^{\mathrm{quad}}_{nd\to np} = A_{nd\to 1s}\left(x_{nd} - \frac{5}{3}x_{np}\right)$$

All n, late times

RESULTS: RECOMBINATION WITH HYDROGEN

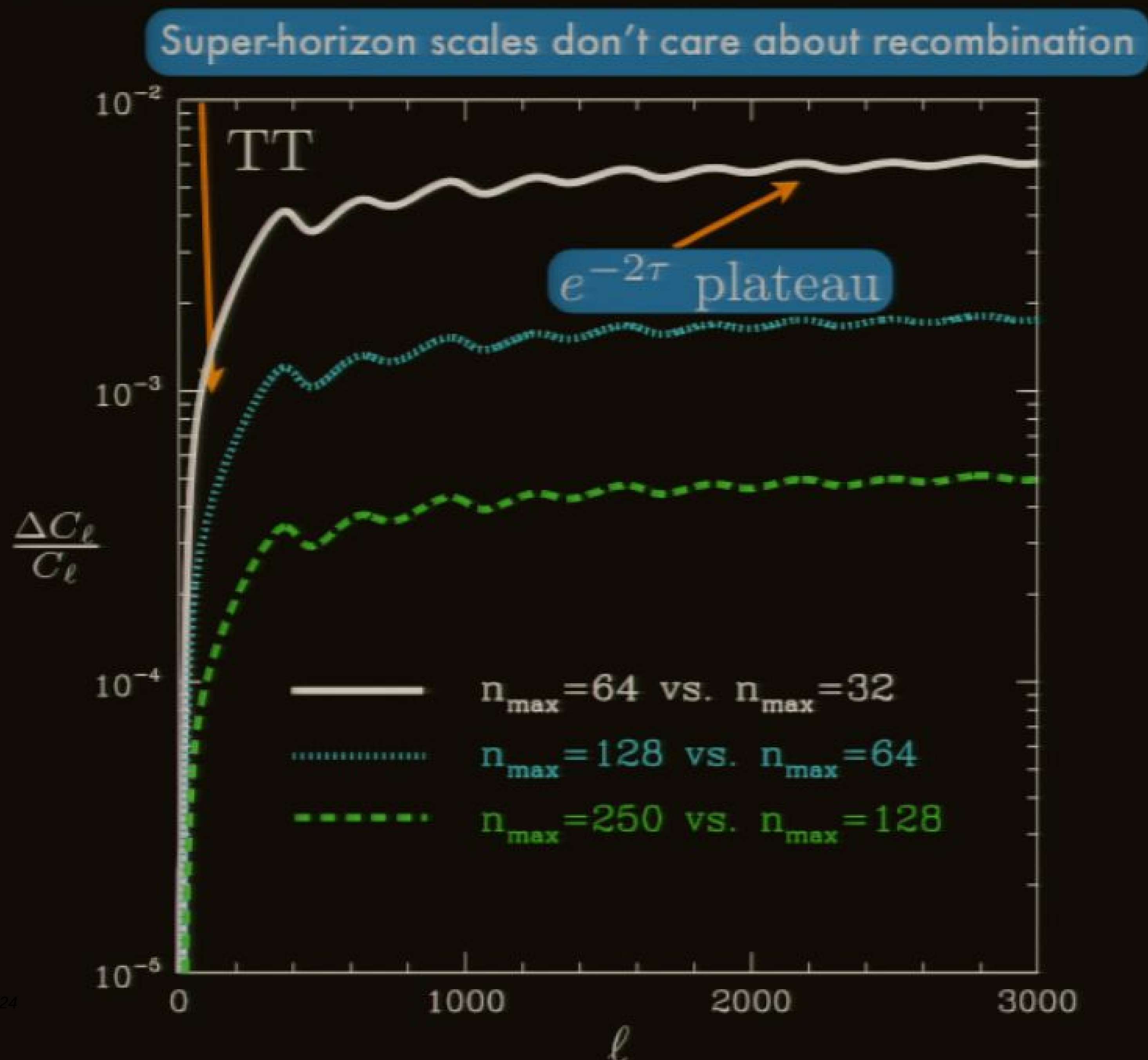


$$\Delta x_e \equiv x_e\big|_{\text{no } E2 \text{ transitions}} - x_e\big|_{\text{with } E2 \text{ transitions}}$$

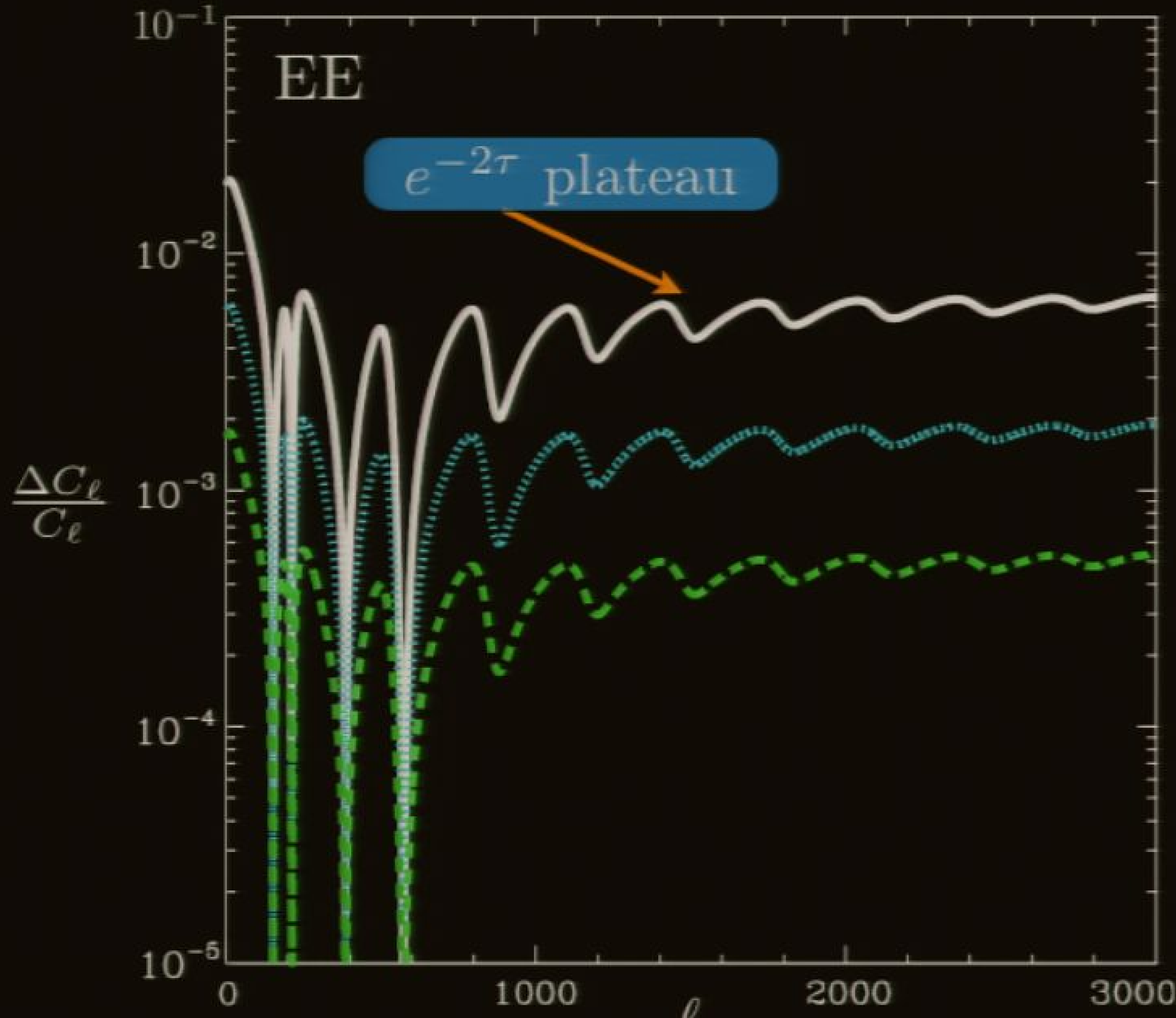
Negligible for Planck!

RESULTS: CMB ANISOTROPIES

RESULTS: TT $C_l s$ WITH HIGH-N STATES

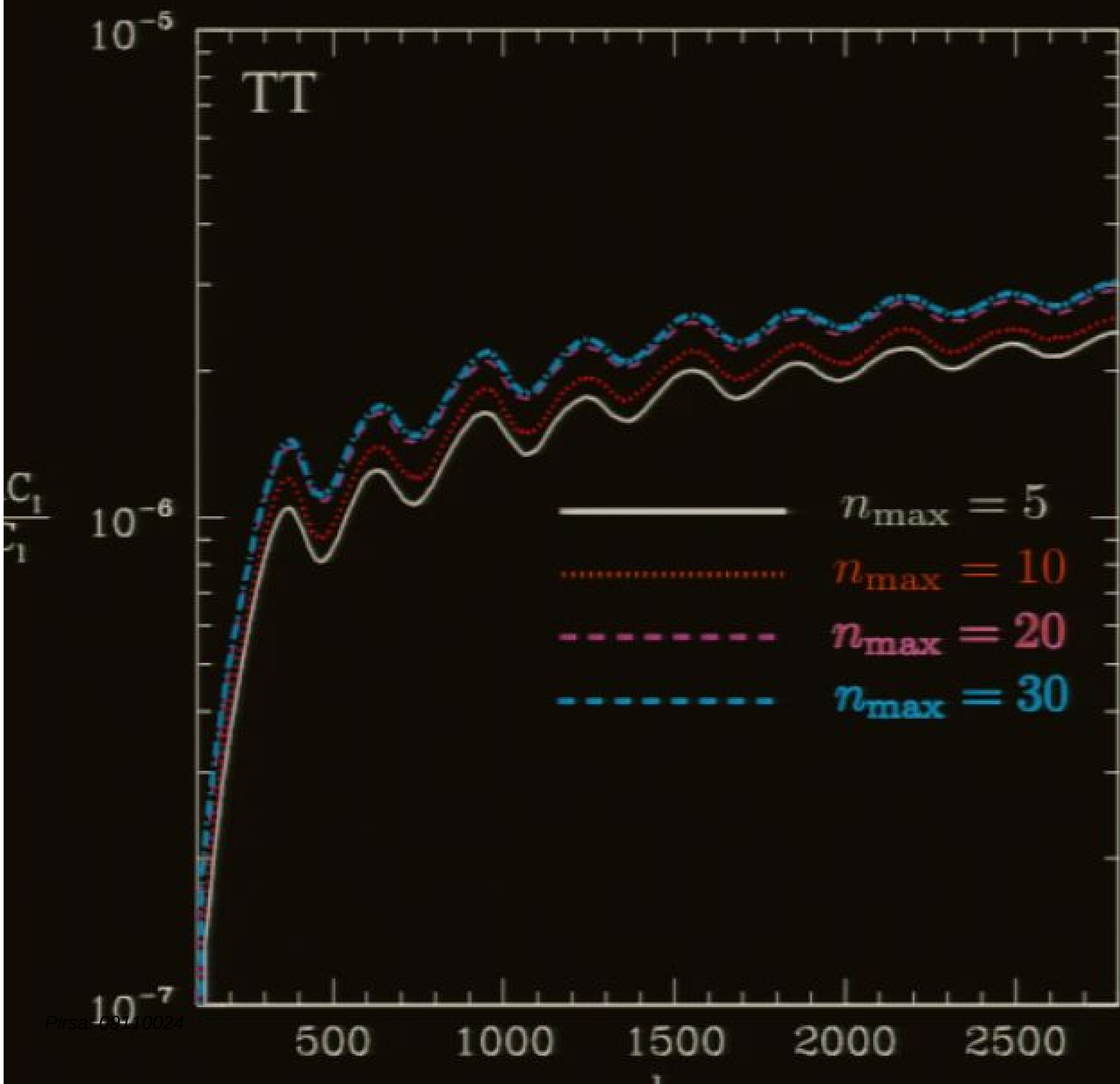


RESULTS: EE $C_l s$ WITH HIGH-N STATES



RESULTS: TEMPERATURE (TT) $C_l s$ WITH HYDROGEN QUADRUPOLES,

Bulk of integral from late times, higher n_{max} → lower x_e → lower τ → higher $e^{-2\tau}$ → higher C_l



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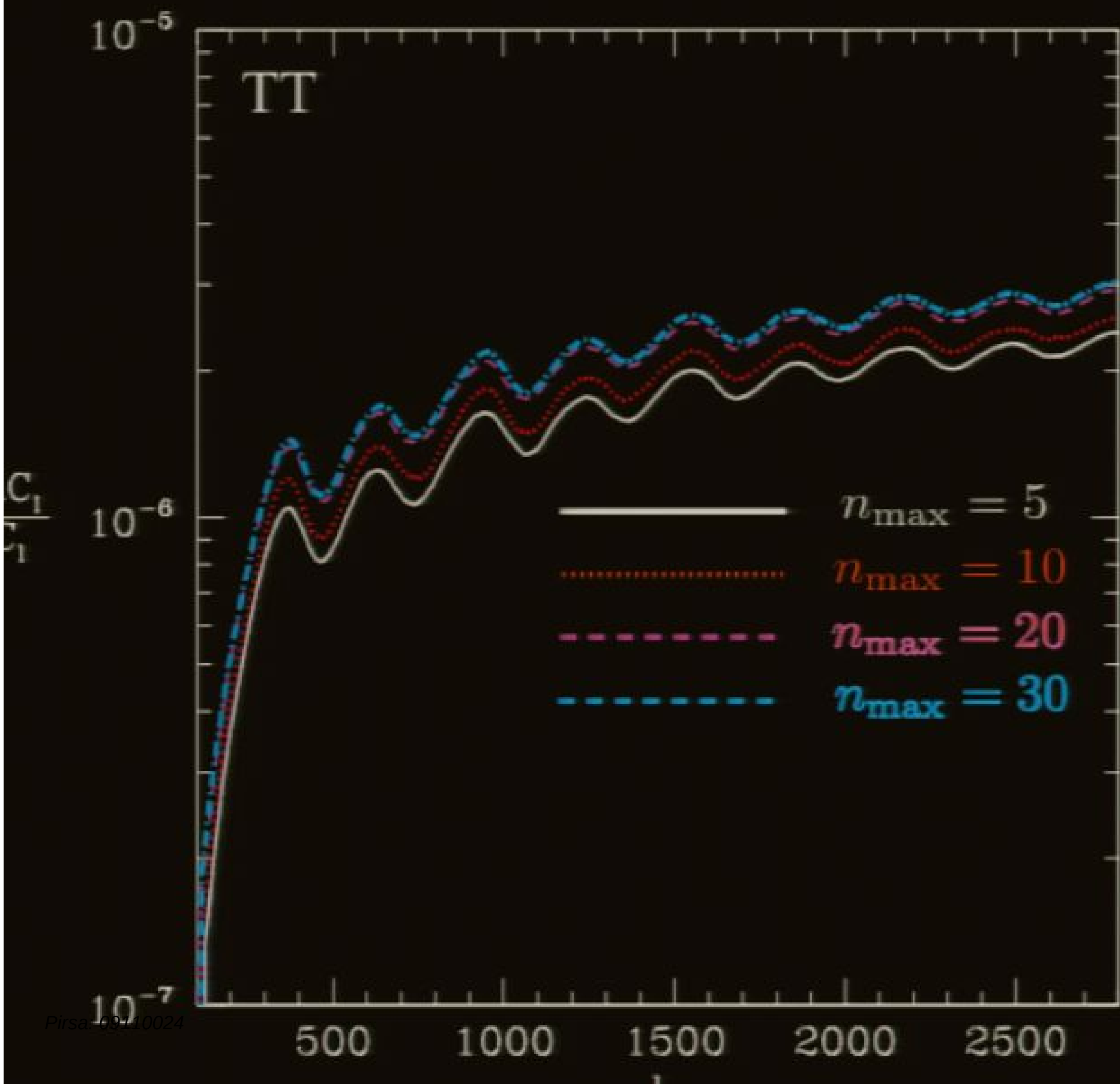
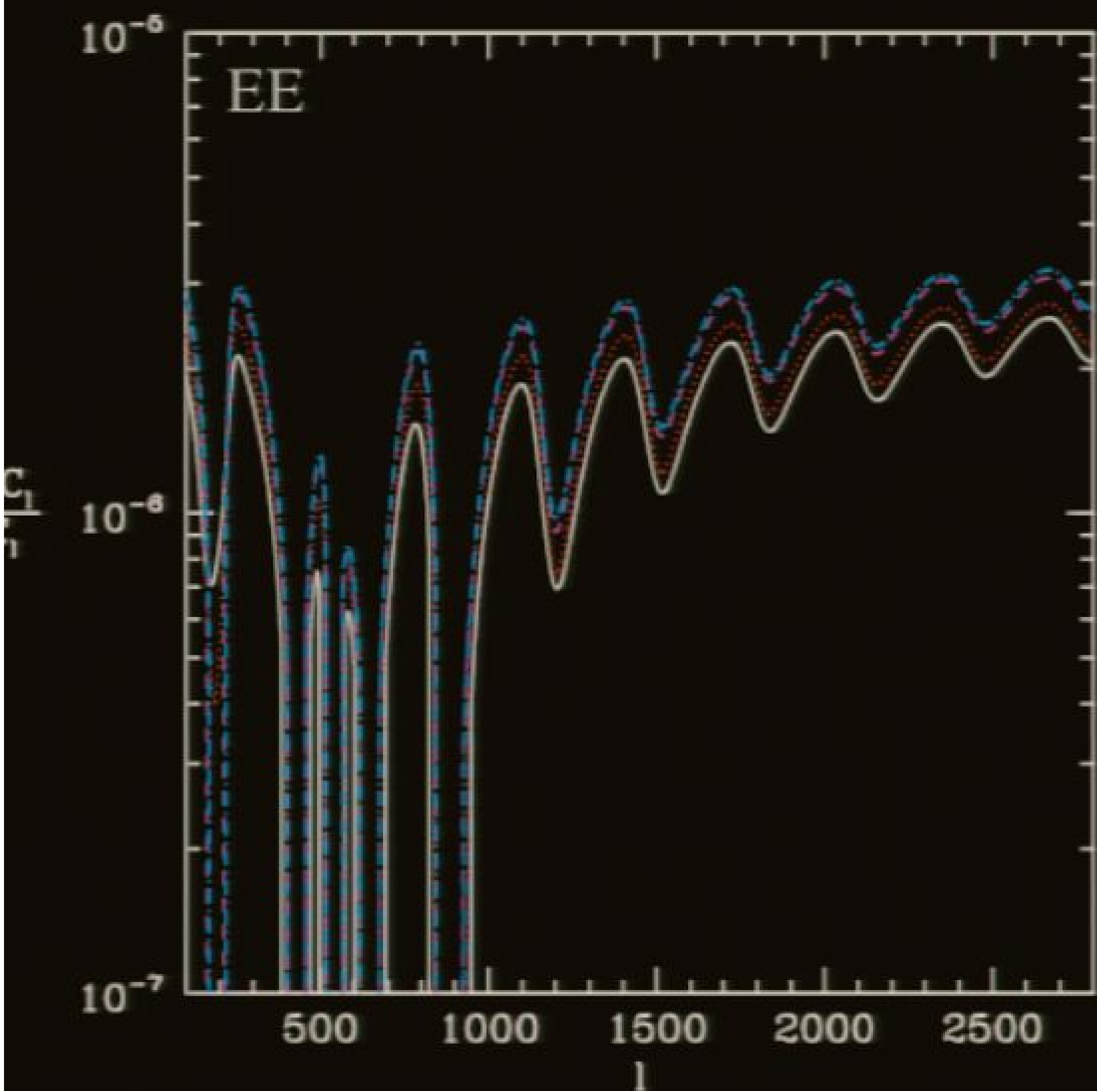


Overall effect is negligible for CMB experiments!

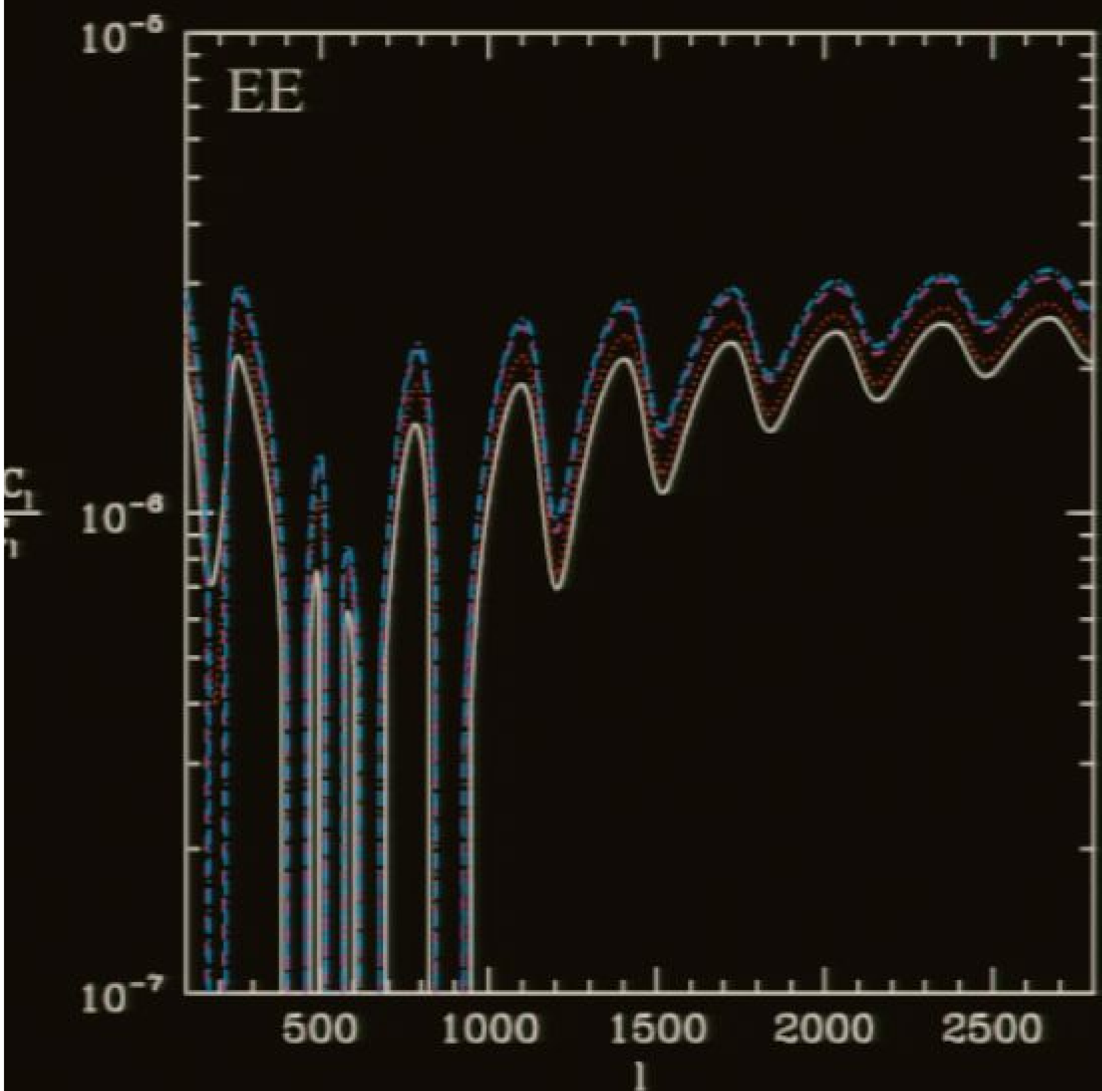
RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES



$$\Delta C_l \equiv C_l\big|_{\text{with } E2 \text{ transitions}} - x_e\big|_{\text{no } E2 \text{ transitions}}.$$

Bulk of integral from late times, higher $n_{\max}$ $\rightarrow$ lower x_e $\rightarrow$ lower τ $\rightarrow$ higher $e^{-2\tau}$ $\rightarrow$ higher C_l

RESULTS: POLARIZATION (EE) C_ls WITH HYDROGEN QUADRUPOLES

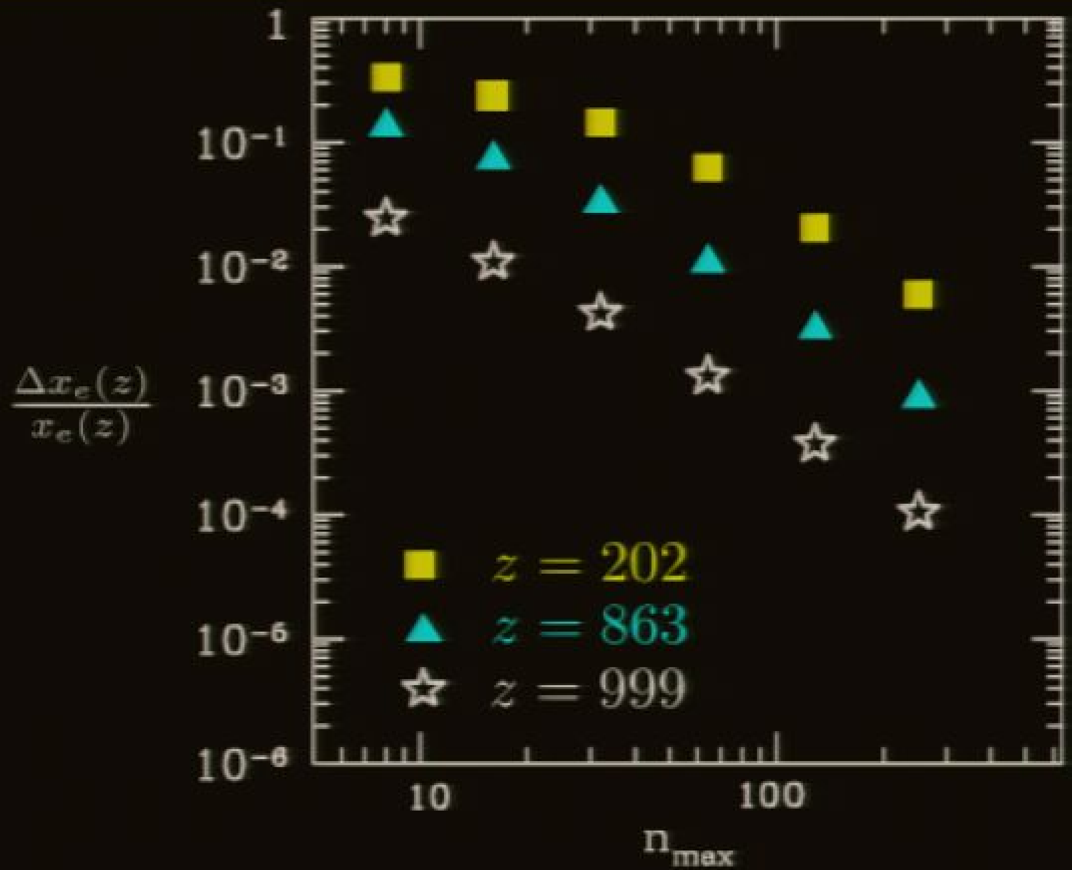


$$\Delta C_l \equiv C_l\big|_{\text{with } E2 \text{ transitions}} - x_e\big|_{\text{no } E2 \text{ transitions}}.$$

Overall effect is negligible for upcoming CMB experiments!

Bulk of integral from late times, higher $n_{max} \rightarrow$ lower x_e $\rightarrow$ lower $\tau \rightarrow$ higher $e^{-2\tau} \rightarrow$ higher C_l

CONVERGENCE



- Relative error well described by power law at high $n_{\max}$

$$\Delta x_e / x_e \propto n_{\max}^{-1.9}$$

- Can extrapolate to absolute error

THE UPSHOT FOR COSMOLOGY

- Can explore effect on overall Planck likelihood analysis

$$Z^2 = \sum_{ll',X,Y} F_{ll'} \Delta C_l^{\mathrm{X}} \Delta C_l^{\mathrm{Y}}$$

$$Z = 1.8 \text{ if } n_{\max} = 64,$$
$$Z = 0.50 \text{ if } n_{\max} = 128,$$
$$Z = 0.14 \text{ if } n_{\max} = 250.$$

CONCLUSIONS

- RecSparse: a new tool for MLA recombination calculations (*arXiv:0911.1359*)
- Highly excited levels (n~64 and higher) are relevant for Planck CMB data analysis
- E2 transitions in H are not relevant for Planck CMB data analysis

FUTURE WORK

- Include line-overlap
- Develop cutoff method for excluded levels
- Generalize RecSparse to calc. rec. line. spectra
- Compute and include collisional rates
- Monte-Carlo analyses
- Cosmological masers