Title: Theoretical Constraints on the Higgs Effective Couplings.

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Abstract: We derive constraints on the sign of couplings in an effective Higgs Lagrangian using prime principles such as the naturalness principle, global symmetries, and unitarity. Specifically, we study four dimension-six operators, O_H, O_y, O_g, and O_gamma, which contribute to the production and decay of the Higgs boson at the Large Hadron Collider (LHC), among other things. Assuming the Higgs is a fundamental scalar, we find: 1) the coefficient of O_H is positive except when there are triplet scalars, resulting in a reduction in the Higgs on-shell coupling from their standard model (SM) expectations if no other operators contribute, 2) the linear combination of O_H and O_y controlling the overall Higgs coupling to fermion is always reduced, 3) the sign of O_g induced by a new colored fermion is such that it interferes destructively with the SM top contribution in the gluon fusion production of the Higgs, if the new fermion cancels the top quadratic divergence in the Higgs mass, and 4) the correlation between naturalness and the sign of O_gamma is similar to that of O_g, when there is a new set of heavy electroweak gauge bosons. Next considering a composite scalar for the Higgs, we find the reduction in the on-shell Higgs couplings persists. If further assuming a collective breaking mechanism as in little Higgs theories, the coefficient of O_H remains positive even in the presence of triplet scalars. In the end, we conclude that the gluon fusion production of the Higgs boson is reduced from the SM rate in all composite Higgs models. Our study suggests a wealth of information could be revealed by precise measurements of the Higgs couplings, providing strong motivations for both improving on measurements at the LHC and building a precision machine such as the linear collider.

Theoretical Constraints on the Higgs Effective Couplings

I.L., R. Rattazzi, and A. Vichi, arXiv:0907.5413 [hep-ph]

Ian Low
Argonne National Laboratory & Northwestern University

Outline

- Higgs boson search at the LHC
- Motivation and Overview
- Operators O_H and O_y
- Operators O_g and O_γ
- Discussions on size of effects and little Higgses.
- Summary/Outlook

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Higgs search at the LHC

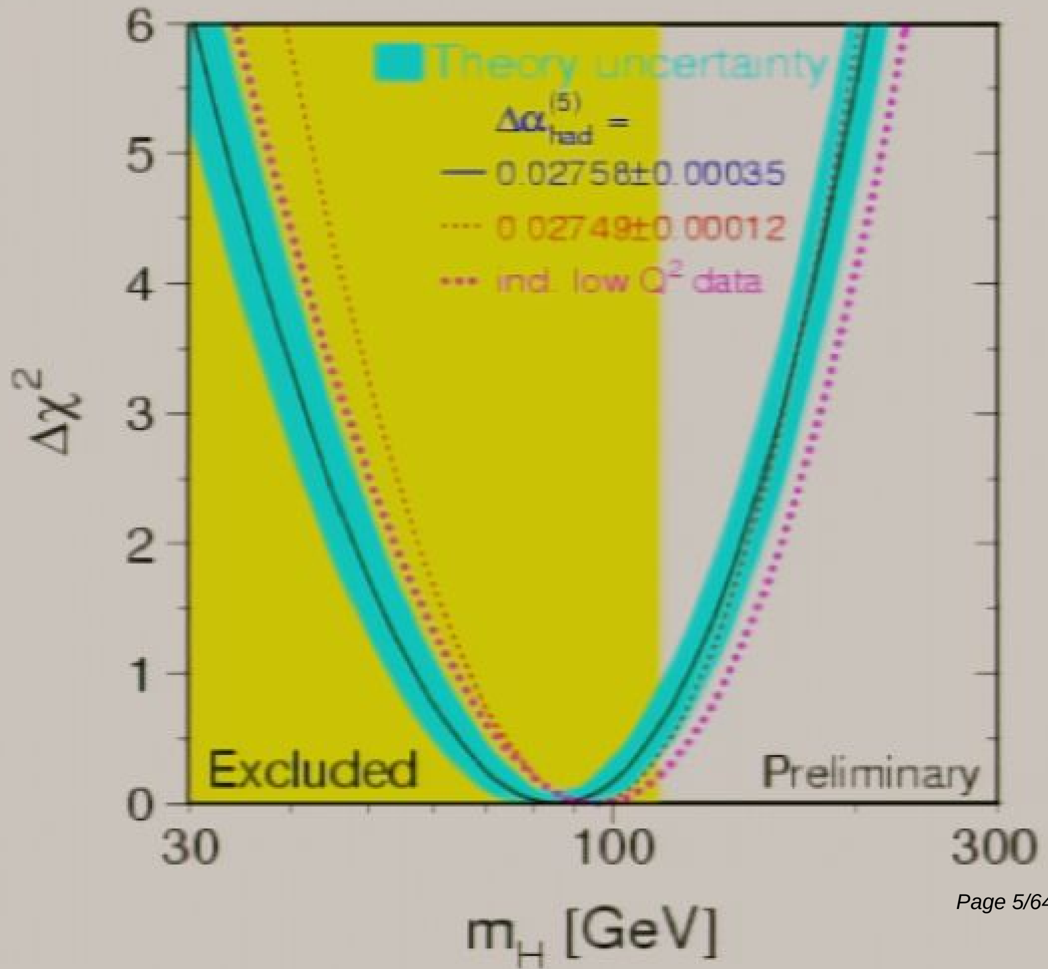
The Higgs boson is the last particle in the standard model that hasn't been observed directly!

The History:

Legacy of LEP -- precision electroweak measurements

LEPEWWG as of July 2006:

Minimal chi-square at Higgs mass = 85 GeV with an uncertainty of +39 GeV and -28 GeV



Unfortunately LEP did not see the standard model Higgs before it was shut down in 2000.

The combined four LEP experiments put a lower bound on the Higgs mass at 114.4 GeV at the 95% confidence Level. (hep-ex/0306033)

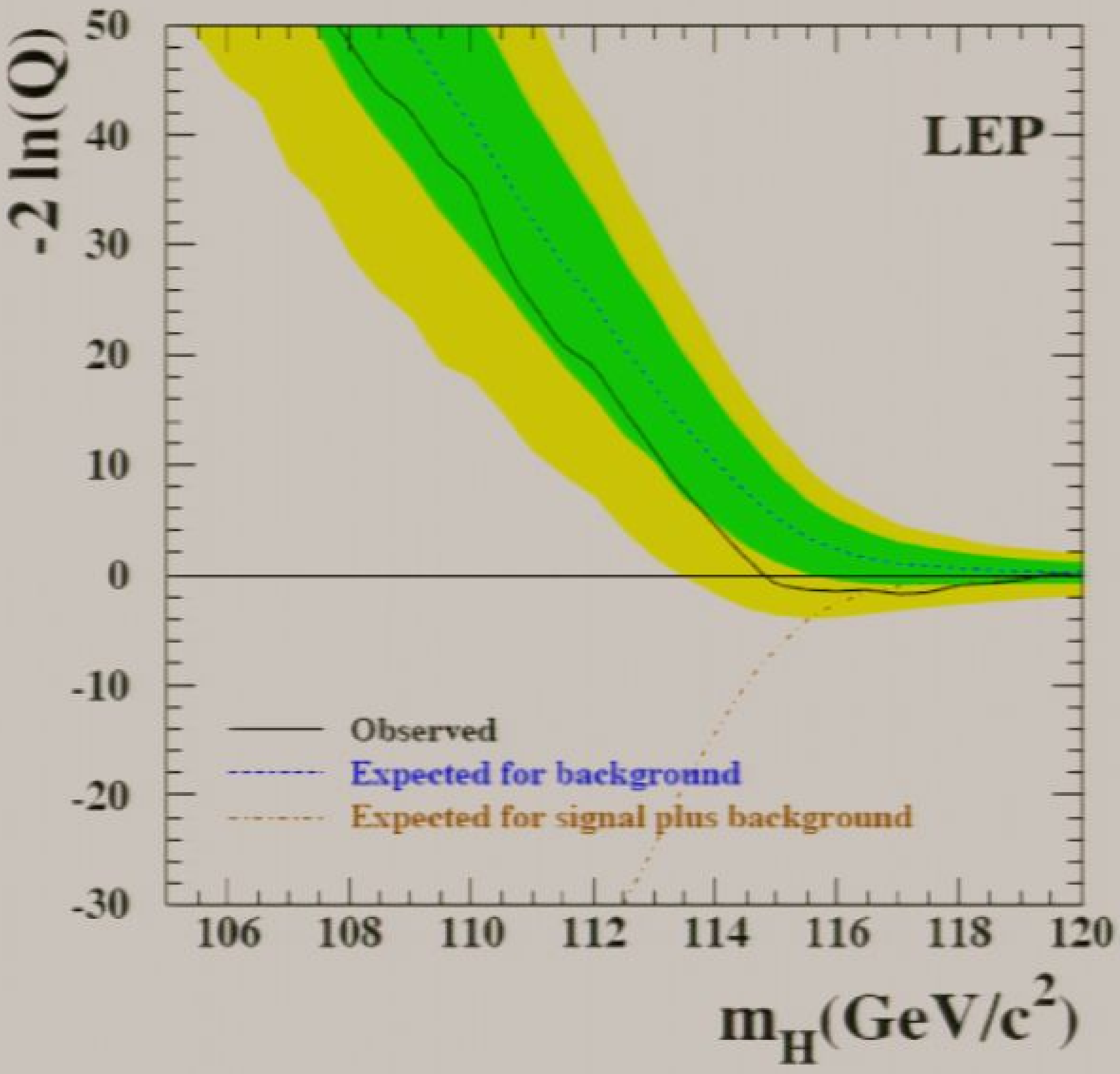


Figure 1: Observed and expected behaviour of the test statistic $-2\ln Q$ as a function of the test mass m_H, obtained by combining the data of the four LEP experiments. The full curve represents the observation; the dashed curve shows the median background expectation; the dark and light shaded bands represent the 68% and 95% probability bands about the median background expectation. The dash-dotted curve indicates the position of the minimum of the median expectation for the signal plus background hypothesis when the signal mass given on the abscissa is tested.

The focus has now shifted to Tevatron and the LHC:

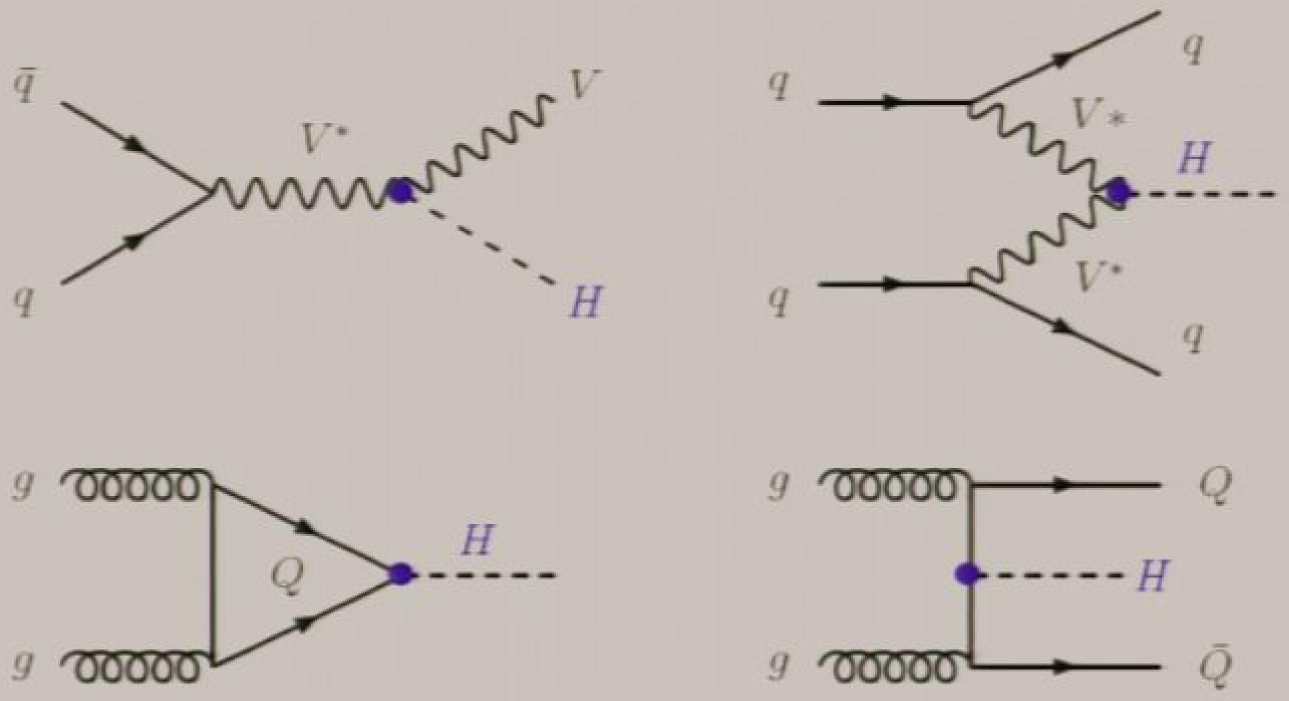
Main production mechanisms of the Higgs at hadron colliders:

associated production with W/Z : $q\bar{q} \longrightarrow V + H$

vector boson fusion : $qq \longrightarrow V^*V^* \longrightarrow qq + H$

gluon – gluon fusion : $gg \longrightarrow H$

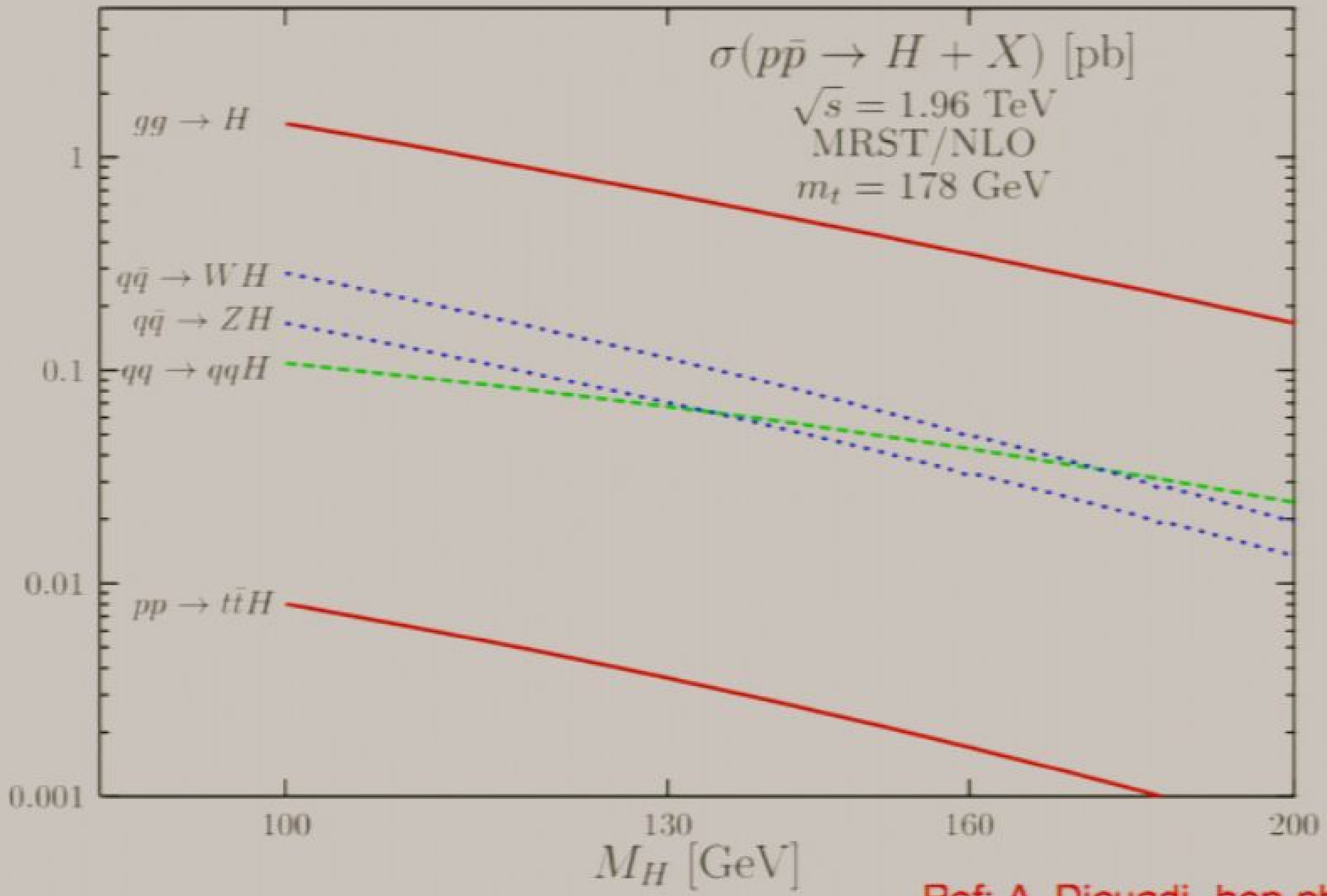
associated production with heavy quarks : $gg, q\bar{q} \longrightarrow Q\bar{Q} + H$



Ref: A. Djouadi, hep-ph/0503172

Among them gluon fusion is the dominant mechanism!

At Tevatron:



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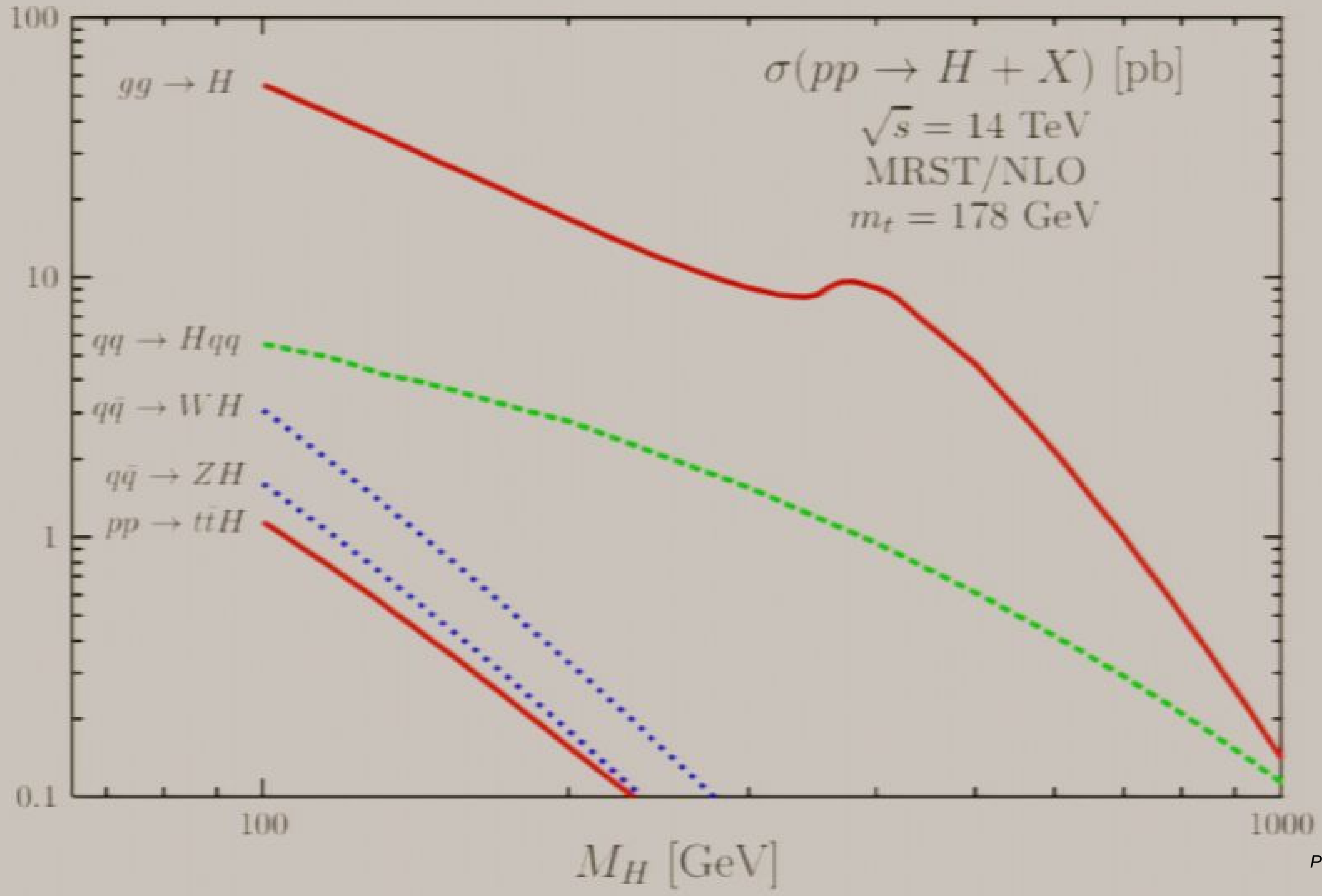
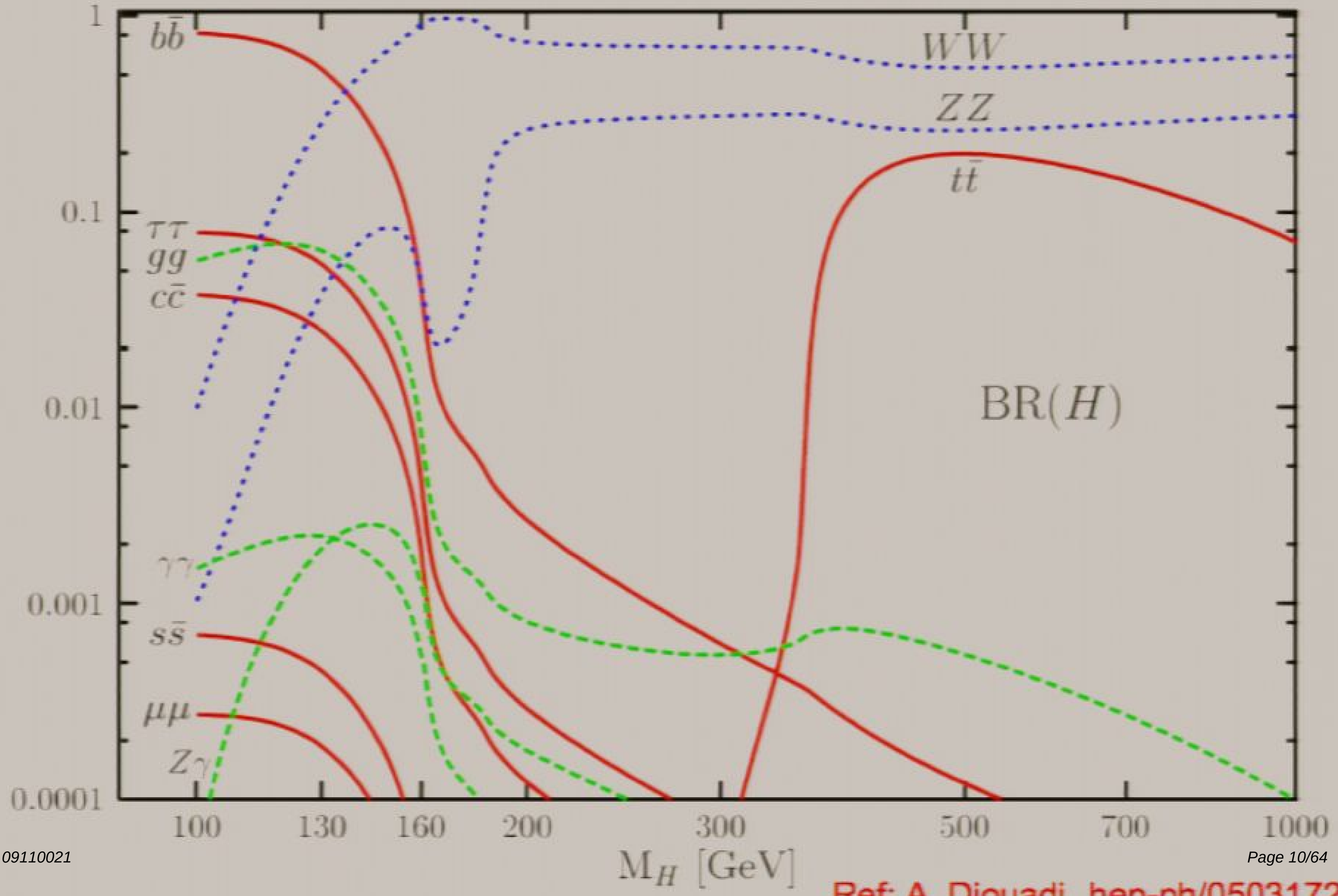
At LHC:



$\sigma(pp \to H + X)$ [pb]
$\sqrt{s} = 14$ TeV
MRST/NLO
$m_t = 178$ GeV
$gg \to H$
$qq \to Hqq$
$q\bar{q} \to WH$
$q\bar{q} \to ZH$
$pp \to t\bar{t}H$
100
10
1
0.1
100
1000
M_H [GeV]

Decay channels depend on the Higgs mass:



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Very recently Fermilab announced exclusion of a SM Higgs in the mass range 160 - 170 GeV from an inclusive search at CDF and

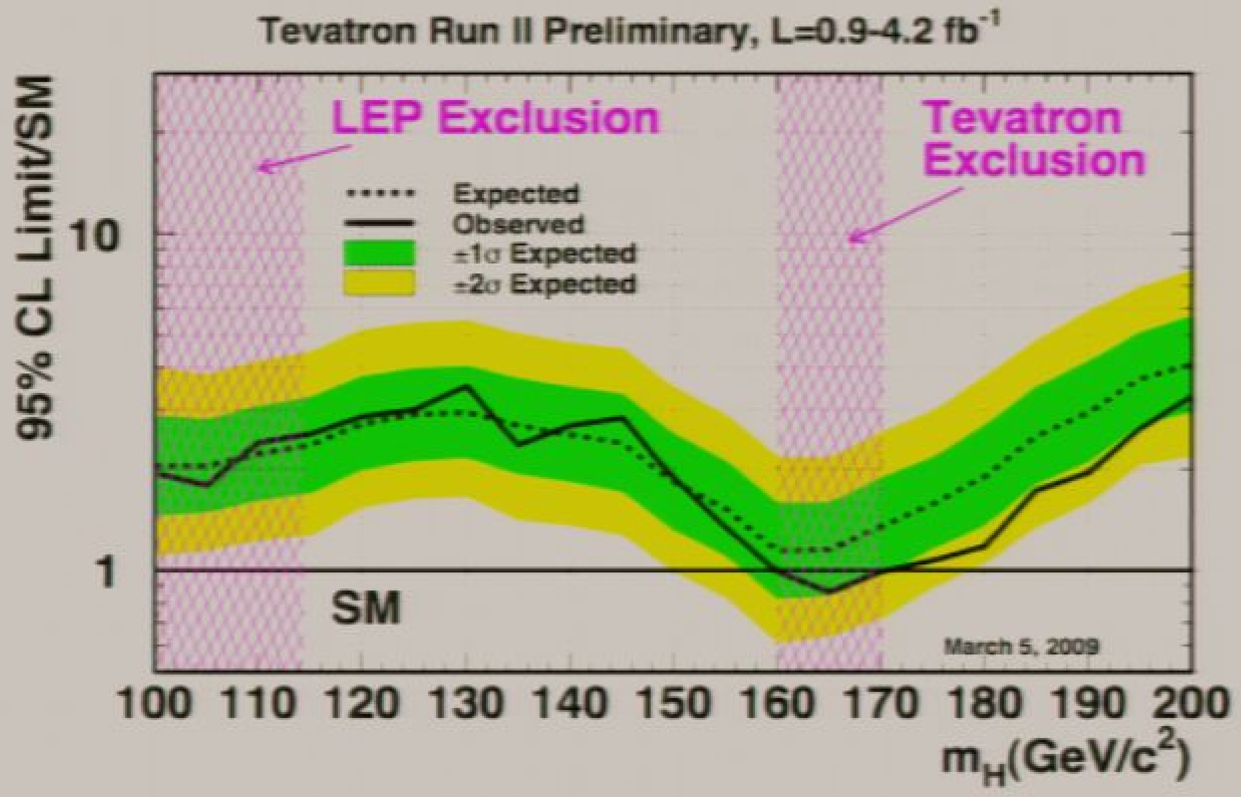


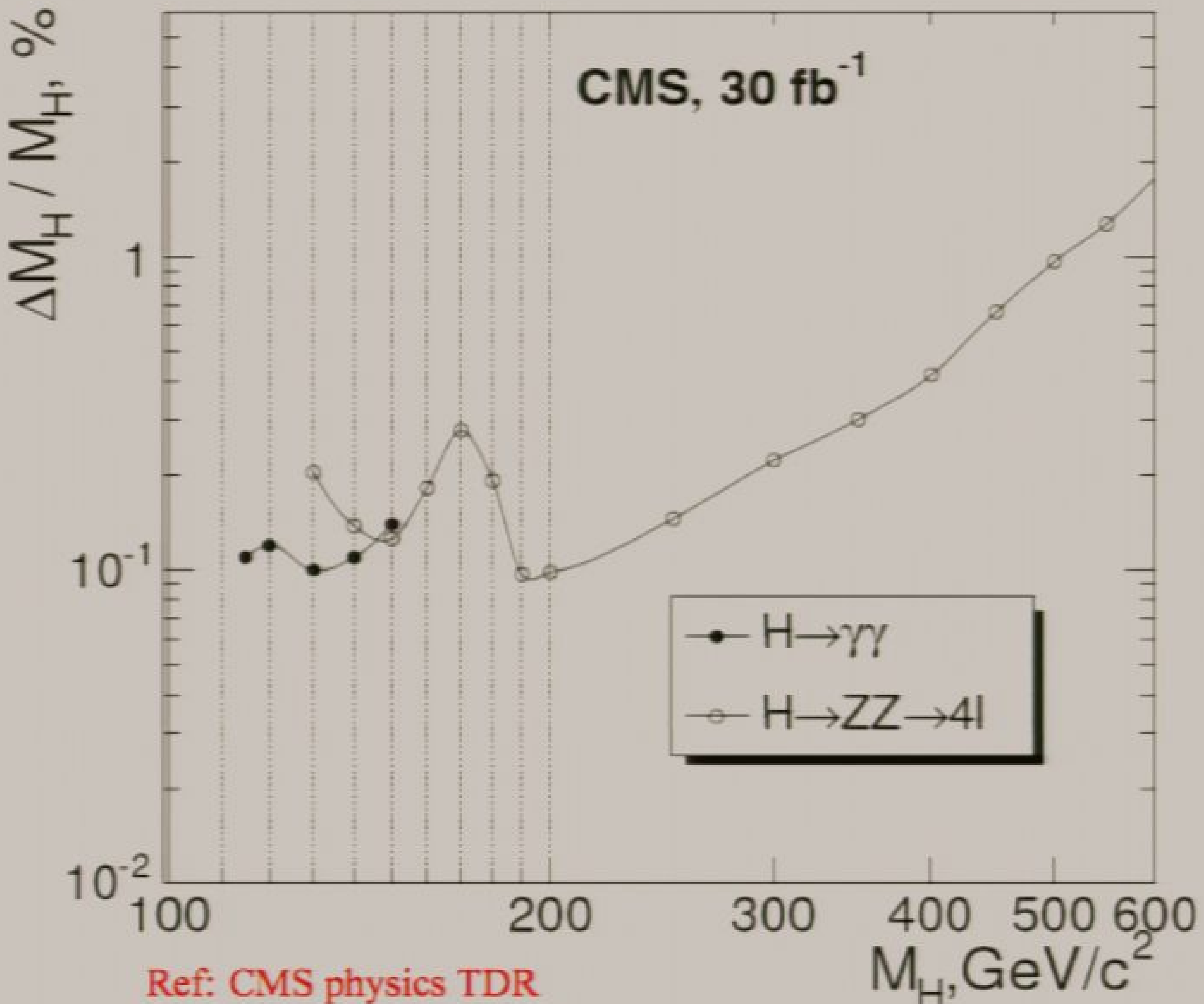
FIG. 4: Observed and expected (median, for the background-only hypothesis) 95% C.L. upper limits on the ratios to the SM cross section, as functions of the Higgs boson mass for the combined CDF and DØ analyses. The limits are expressed as a multiple of the SM prediction for test masses (every 5 GeV/c^2) for which both experiments have performed dedicated searches in different channels. The points are joined by straight lines for better readability. The bands indicate the 68% and 95% probability regions where the limits can fluctuate, in the absence of signal. The limits displayed in this figure are obtained with the Bayesian calculation.

- For low Higgs mass $m_h \leq 150$ GeV, the Higgs mostly decays to two b-quarks, two tau leptons, two gluons and etc.

- In hadron colliders these modes are difficult to extract because of the large QCD jet background.

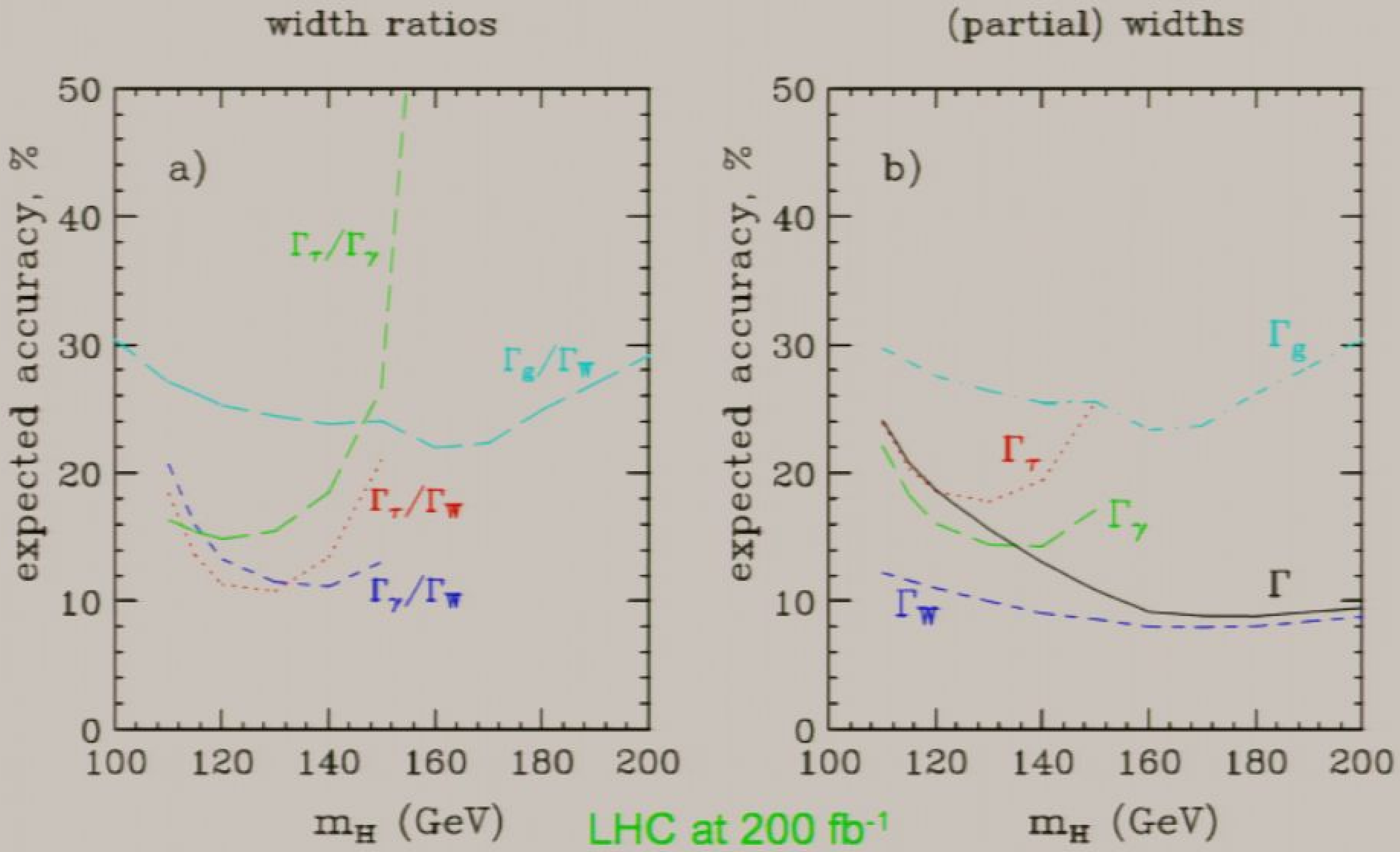
- The silver detection mode in this mass range is the two photons mode: $h \to \gamma\gamma$, which like the gluon fusion is a loop-induced process.

Higgs mass can be measured very precisely by, for example, looking at the invariant mass of the di-photon.



Ref: CMS physics TDR

Furthermore, it is possible to extract individual partial widths of the Higgs boson.



LHC at 200 fb^{-1}

D. Zeppenfeld, hep-ph/0203123

Motivation and Overview

How do we study theoretical properties of the Higgs?

A useful tool is the effective Lagrangian approach, when new states are heavier than the Higgs:

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- Pick a model, compute the effective coefficients, and scan through allowed parameter space.

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A useful tool is the effective Lagrangian approach, when new states are heavier than the Higgs:

- Write down all possible operators in a large mass expansion, without any dynamical assumptions.
- Pick a model, compute the effective coefficients, and scan through allowed parameter space.
- Making some simple dynamical assumptions to encompass a large class of models:

  The Strongly Interacting Light Higgs Lagrangian (SILH), which assumes the Higgs is a light composite Pseudo-Nambu-Goldstone boson (PNGB). Giudice, Grojean, Pomarol, Rattazzi, hep-ph/0703164

Which approach to use depends on what questions one wants to ask. We are interested in structural and global questions such as:

- Is physics at the electroweak scale natural?
- Is the Higgs boson fundamental or composite?
- If new particles are discovered, who ordered them? Are they responsible for canceling the Higgs quadratic divergences?

In this regard, neither of the first two approaches seems useful.

Our goal is to seek answers to the above structural and global questions.

We adopt a top-down approach by studying possible constraints on coefficients of Higgs effective couplings from prime principles such as: naturalness principle, global symmetry patterns, and unitarity.

Our methodology is quite general and apply to theories even with a fundamental Higgs scalar (e.g. supersymmetry).

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An Overview of SILH:

- The Higgs is a composite particle arising from new strong dynamics characterized by g_ρ and m_ρ.
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- The standard model (SM) gauge group is a weakly-gauged subgroup of G, the global symmetry group of the strong dynamics.

The advantage of using SILH is power-counting:

- Allow us to understand which operators are genuinely sensitive to the underlying strong dynamics.
- Relative importance of different operators in collider phenomenology.

At leading order there are (only) 10 dim-6 operators. We study four of them that control the Higgs production and decay at the LHC:

$$\mathcal{O}_H = \partial^\mu (H^\dagger H)\partial_\mu (H^\dagger H) \qquad \mathcal{O}_y = H^\dagger H\, \bar{f}_L H f_R + \text{h.c.}$$
$$\mathcal{O}_g = H^\dagger H G^a_{\mu\nu} G^{a\,\mu\nu} \qquad \mathcal{O}_\gamma = H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$$

The sizes of these four operators in SILH power-counting are:

$$\frac{c_H}{2f^2}\mathcal{O}_H, \quad \frac{c_y y_f}{f^2}\mathcal{O}_y, \quad \frac{c_g g_s^2}{16\pi^2 f^2}\frac{y_t^2}{g_\rho^2}\mathcal{O}_g, \quad \frac{c_\gamma g'^2}{16\pi^2 f^2}\frac{g^2}{g_\rho^2}\mathcal{O}_\gamma$$

The first two are tree-level effects while the latter two one-loop induced.

It is worth mentioning another operator

$$\frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\left(H^\dagger \overleftrightarrow{D}_\mu H\right)$$

which breaks custodial symmetry and is severely constrained by the ρ parameter.

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In terms of physical amplitudes and partial decay width:

$$\Gamma(h \to f\bar{f}) \;=\; \Gamma(h \to f\bar{f})_{SM}[1 - \xi(\overbrace{c_H + 2c_y})],$$

$$\Gamma(h \to gg) \;=\; \Gamma(h \to gg)_{SM}\left[1 - \xi\,\mathrm{Re}\left(\overbrace{c_H + 2c_y} + \frac{\overbrace{4y_t^2 c_g}}{g_\rho^2 I_g}\right)\right],$$

$$\Gamma(h \to \gamma\gamma) \;=\; \Gamma(h \to \gamma\gamma)_{SM}\left[\underset{g_\rho}{1} - \xi\,\mathrm{Re}\left(\frac{\overbrace{c_H + 2c_y}}{1 + J_\gamma/I_\gamma} + \frac{c_H - (g^2/g_\rho^2)c_W}{1 + I_\gamma/J_\gamma} + \frac{\overbrace{(4g^2/g_\rho^2)c_\gamma}}{I_\gamma + J_\gamma}\right)\right].$$

$$\boxed{\xi \equiv v^2/f^2}$$

In addition, O_H also contributes to the high energy limit of the scattering amplitude for longitudinal gauge bosons.

An important technical remark is in order:

Coefficients of these operators are not physical and depend on the operator basis one chooses.

Under reparametrization of field variables:

$$H \to H + a(H^\dagger H)H/f^2$$

the Higgs kinetic term transform as

$$(D_\mu H)^\dagger(D^\mu H) \to (D_\mu H)^\dagger(D^\mu H) + \frac{a}{f^2}\partial^\mu(H^\dagger H)\partial_\mu(H^\dagger H) + \frac{2a}{f^2}H^\dagger H(D_\mu H)^\dagger(D^\mu H)$$

We have used this degree of freedom to set

$$\mathcal{O}_r = H^\dagger H(D_\mu H)^\dagger(D^\mu H)$$

to zero in the SILH Lagrangian.

Also the Yukawa coupling transforms:

$$-y_f f_L H f_R \to -y_f \left(1 + a\frac{H^\dagger H}{f^2}\right) f_L H f_R$$

We will always stay in the $O_r = 0$ basis.

The physical amplitude is, of course, invariant under the field reparametrization.

The field reparametrization is equivalent to using the equation of motion, and can be used to move all new physics effects into so-called oblique operators.

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Operators O_H and O_y

Here we prove a positivity constraint on c_H and c_H+2c_y. The latter controls the overall Higgs-fermion-fermion coupling. There are three sources:

- From a non-linear sigma model. (-->Higgs is composite):
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  $c_H+2c_y > 0$ in all cases
- From integrating out heavy vectors. (--> Higgs is fundamental or composite)

  Both c_H and $c_H+2c_y > 0$

I will not go into the technical details except the following remarks:

- It's possible to derive a dispersion relation

$$c_H = \frac{f^2}{\pi}\int_0^\infty \left(\sigma_{+-}(x) - \sigma_{-+}(x)\right)\frac{dx}{x}$$

$$\mathcal{A}_{++}(s) \equiv \mathcal{A}\left(\pi^+\pi^+ \to \pi^+\pi^+\right)(s, t=0)$$
$$\mathcal{A}_{+-}(s) \equiv \mathcal{A}\left(\pi^+\pi^- \to \pi^+\pi^-\right)(s, t=0)$$

The second term is non-vanishing only in the presence of a doubly-charged state.

We showed through explicit computation that c_H is negative only when there are triplet scalars, which fits nicely with the dispersion relation result.

However in this case $c_H + 2c_y$ is still positive!

- Using group theory it is possible to show that the only scalars that have cubic interactions with the Higgs doublet are a singlet, a real triplet, and a complex triplet:

$$\phi_s \sim \vec{h} \cdot \vec{h} = H^\dagger H,$$
$$\phi_r^a \sim \vec{h}^T \; T_L^a T_R^3 \; \vec{h} = H^\dagger \frac{\sigma^a}{2} H,$$
$$\phi_c^a \sim \vec{h}^T \; T_L^a (T_R^1 - i T_R^2) \; \vec{h} = H^T \epsilon \frac{\sigma^a}{2} H.$$

from here one can write down the effective interaction and integrate out the heavy mode explicitly.

- One popular model with a complex triplet scalar is the SU(5)/SO(5) littlest Higgs model. The scalar potential is

$$V(\phi,\phi^*) = g_L^2 f^2 \left|\phi_{ij} + \frac{i}{2f}(H_i H_j + H_j H_i)\right|^2 + g_R^2 f^2 \left|\phi_{ij} - \frac{i}{2f}(H_i H_j + H_j H_i)\right|^2$$

and c_H is

$$c_H^{(s)} = -\frac{(g_R^2 - g_L^2)^2}{(g_R^2 + g_L^2)^2}$$

The important observation here is the collective breaking structure and that c_H is maximally negative when one coupling is much larger than the other.

We will use this observation to prove that c_H in all little Higgs models is still positive, despite the negative contribution from the triplet.

- The Higgs couples to a vector only through the current, whose quantum number is again completely specified by group theory:

$$(\mathbf{3}_L, \mathbf{1}_R) \ : \ J^a_{HL\,\mu} \equiv \frac{1}{2} i\, \vec{h}^T\, T^a_L \overleftrightarrow{D}_\mu\, \vec{h} = i H^\dagger\, \frac{\sigma^a}{2}\, \overleftrightarrow{D}_\mu\, H$$

$$(\mathbf{1}_L, \mathbf{3}_R) \supset J^3_{HR\,\mu} = i\, \vec{h}^T\, T^3_R \overleftrightarrow{D}_\mu\, \vec{h} = i H^\dagger\, \overleftrightarrow{D}_\mu\, H$$

$$(\mathbf{1}_L, \mathbf{3}_R) \supset J^-_{HR\,\mu} = i\, \vec{h}^T\, (T^1_R - i T^2_R) \overleftrightarrow{D}_\mu\, \vec{h} = i\, H^T\, \epsilon \overleftrightarrow{D}_\mu\, H$$

A subtlety here is one can also build vector currents out of SM gauge fields and fermions:

$$(\mathbf{3}_L, \mathbf{1}_R) \ : \ J^a_{F\,\mu} \equiv (D^\nu W_{\nu\mu})^a \qquad \text{and} \qquad J^a_{\psi\,\mu} \equiv \bar{\psi}\, \gamma_\mu \frac{\sigma^a}{2}\, \psi ,$$

$$(\mathbf{1}_L, \mathbf{3}_R) \supset J^Y_{F\,\mu} \equiv D^\nu B_{\nu\mu} \qquad \text{and} \qquad J^Y_{\psi\,\mu} \equiv \bar{\psi}\, \gamma_\mu Y\, \psi .$$

In this case integrating out heavy vectors could generate, say, four-fermi operators, which is non-oblique.

One could either use a specific choice of low-energy interpolating field, or equivalently the equation of motion

$$\begin{aligned}(D^\nu W_{\nu\mu})^a + gJ^a_{HL\mu} + gJ^a_\mu &= 0 \\ D^\nu B_{\nu\mu} + g'J^3_{HR\mu} + g'J^Y_\mu &= 0\end{aligned}$$

to go back to the basis where all new physics effects are oblique.

Given these currents, it's possible again to explicitly compute effects of integrating out heavy vectors. We find both c_H and $c_H+2c_y > 0$.

- When integrating out heavy vectors within a nlsm, it's possible to prove a stronger result, that c_H is not only positive, but also independent of gauge couplings.

  (It's just a group-theoretic factor.)

  This statement is again used in proving that $c_H > 0$ in all little Higgs theories despite the negative contribution from triplet scalars.

Our convention of sign is such that the positivity constraint implies the on-shell Higgs coupling is reduced from the SM expectation, if no other operators contribute!

Operators O_g and O_γ

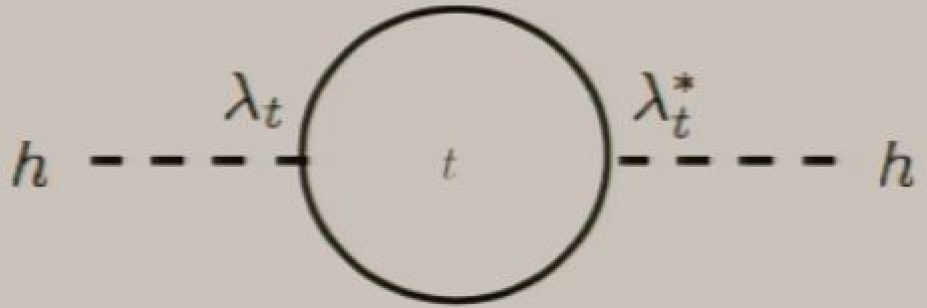
Here we establish a correlation between cancellation of Higgs quadratic divergence in the top sector and the sign of O_g (and O_γ) induced by new fermionic top partners that are colored:

If the top quadratic divergence is cancelled, the interference is destructive.

Conversely, if the quadratic divergence is enhanced, the interference is constructive.

- First a diagrammatic argument:

 The interaction of the Higgs with the top quark induces a quadratically divergent contribution in the Higgs mass:



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Q: How do we use another fermion to cancel the above divergence?

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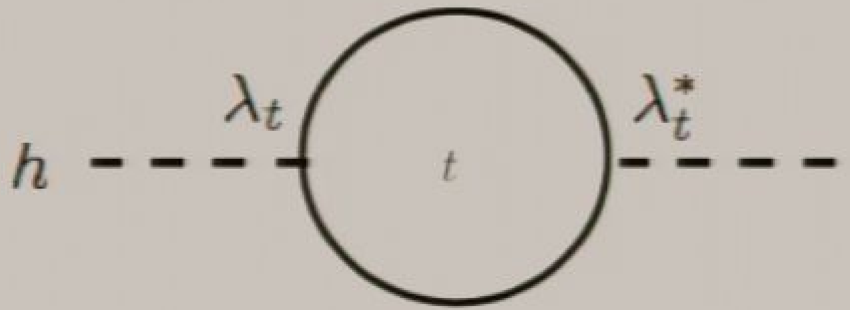
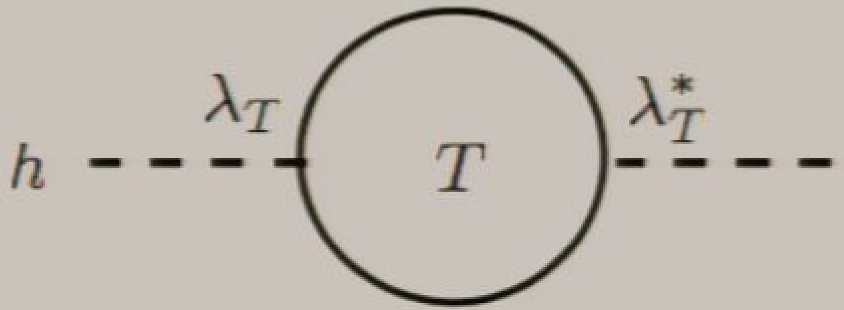


$$= -\frac{3}{8\pi^2}|\lambda_t|^2\Lambda_{\rm NP}^2$$

Q: How do we use another fermion to cancel the above divergence?

Wrong answer: another fermion *T* with only Yukawa coupling to the Higgs wouldn't work. The divergences always add up!



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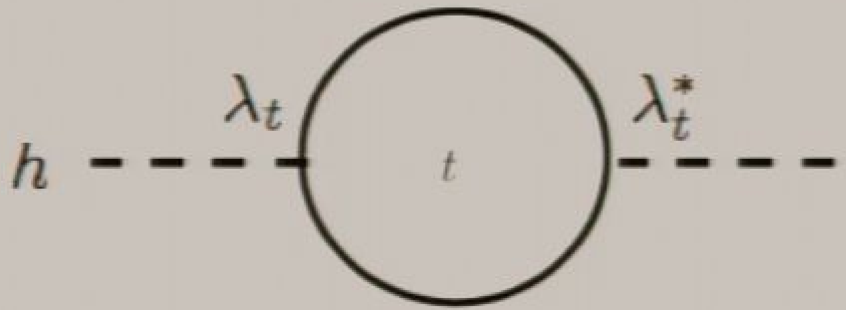
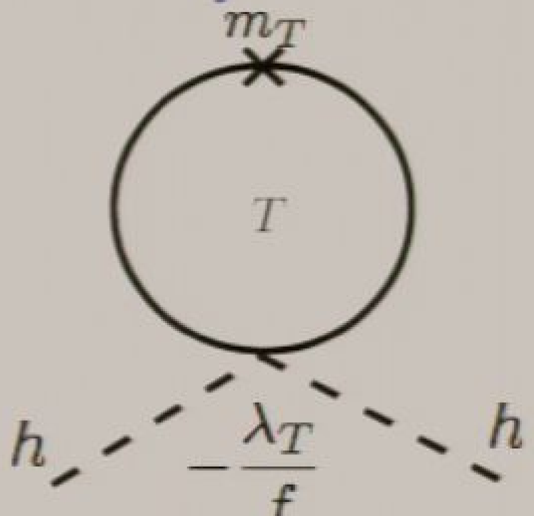


$$= -\frac{3}{8\pi^2}|\lambda_t|^2\Lambda_{\rm NP}^2$$

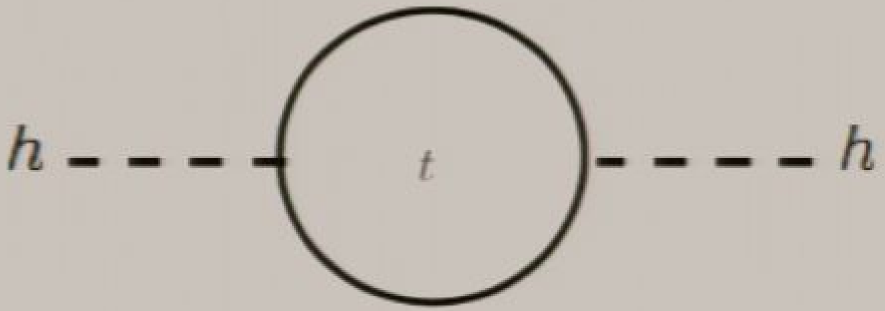
Q: How do we use another fermion to cancel the above divergence?

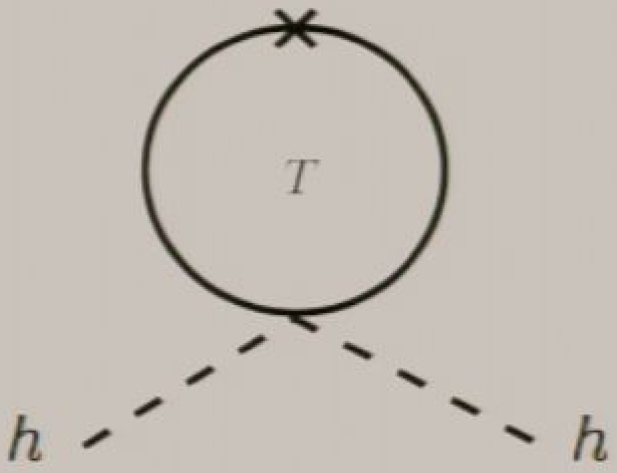
Correct answer: always need a dimension-five coupling with the Higgs!



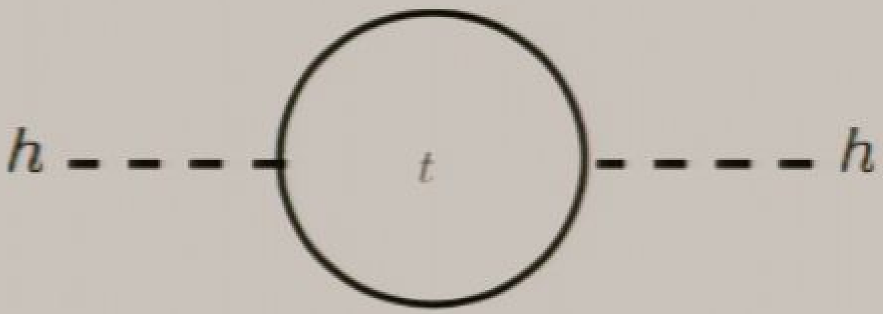
$$= -\frac{3}{8\pi^2}\left(-m_T\frac{\lambda_T}{f}\right)\Lambda_{\rm NP}^2$$

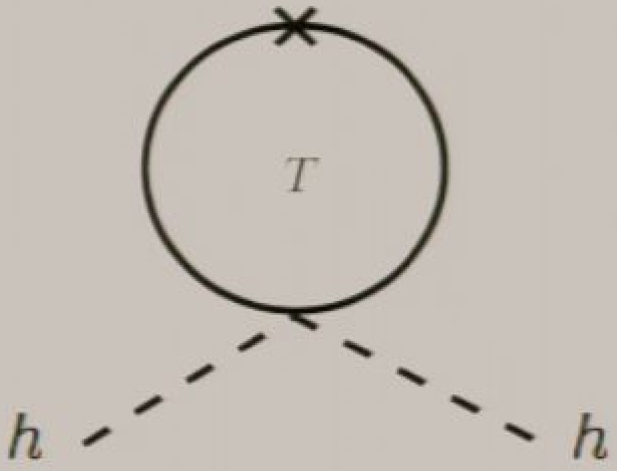
- If the following two diagrams have a relative minus sign, then Higgs quadratic divergence is cancelled. Otherwise, the divergences add up.





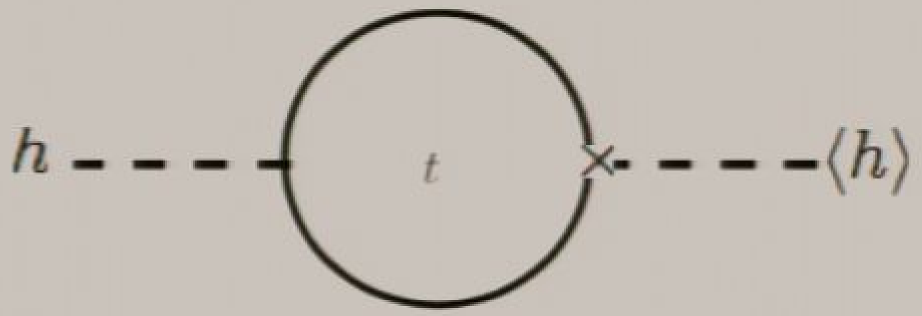
Now let’s massage the diagrams a little bit:

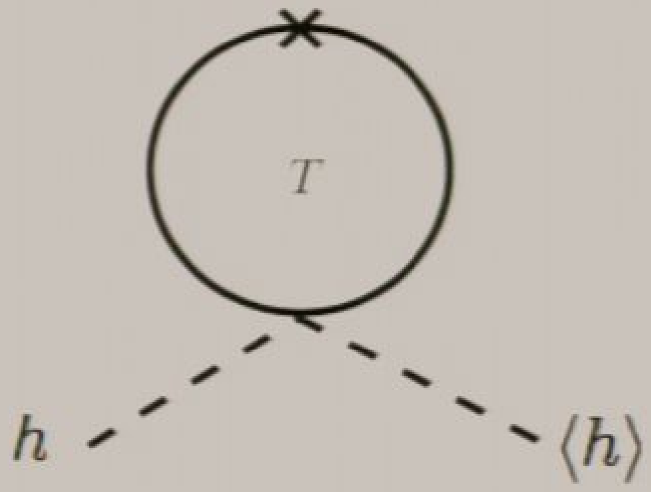




Now let's massage the diagrams a little bit:

-- First putting one of the Higgs field in its VEV.

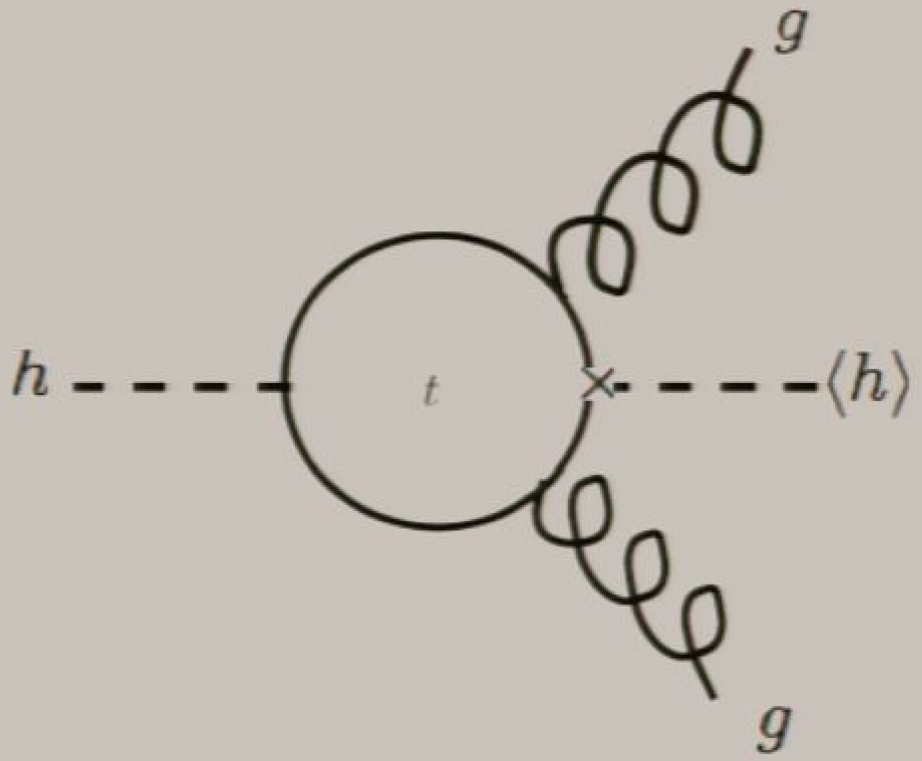


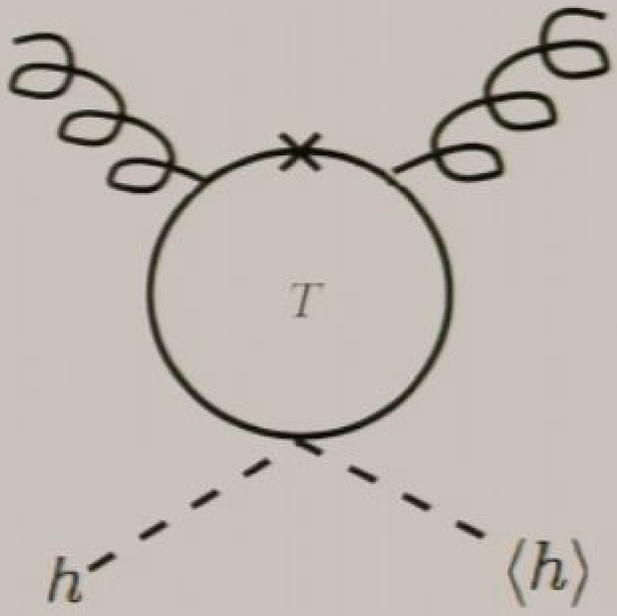


Now let's massage the diagrams a little bit:

-- First putting one of the Higgs field in its VEV.

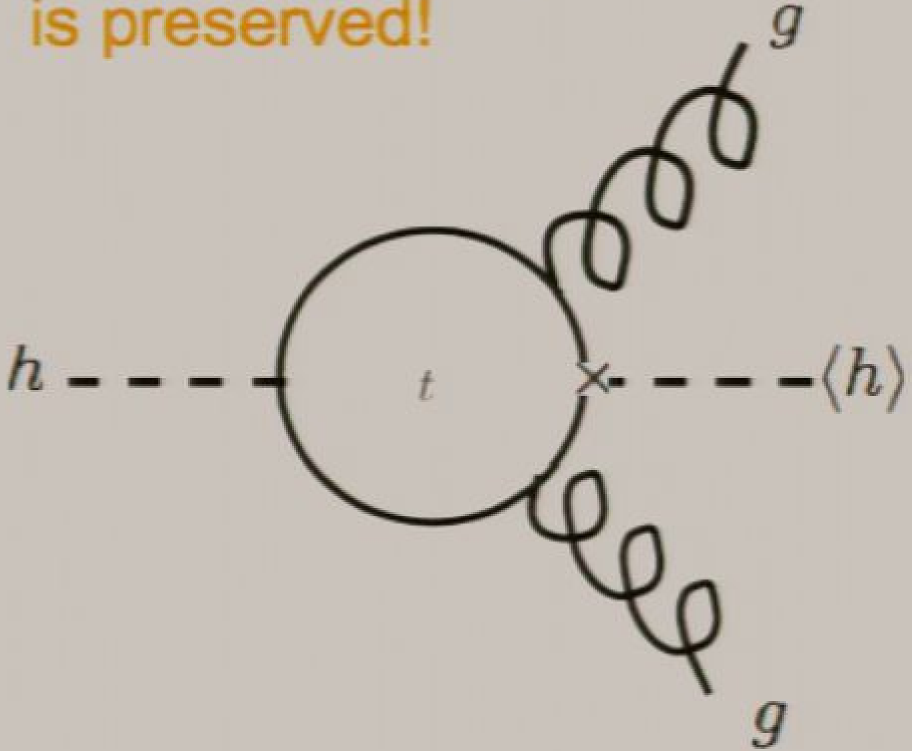
-- Next let's insert two gluons into the fermion line.

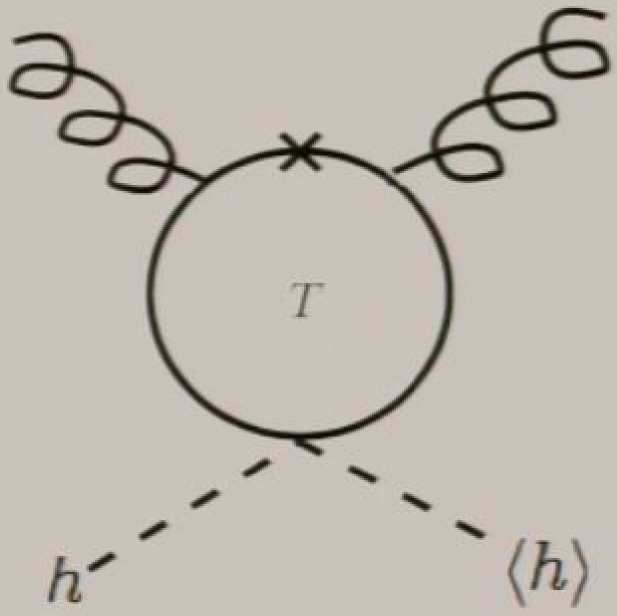




These are exactly the two diagrams contributing to gluon fusion from the top quark and the new state!

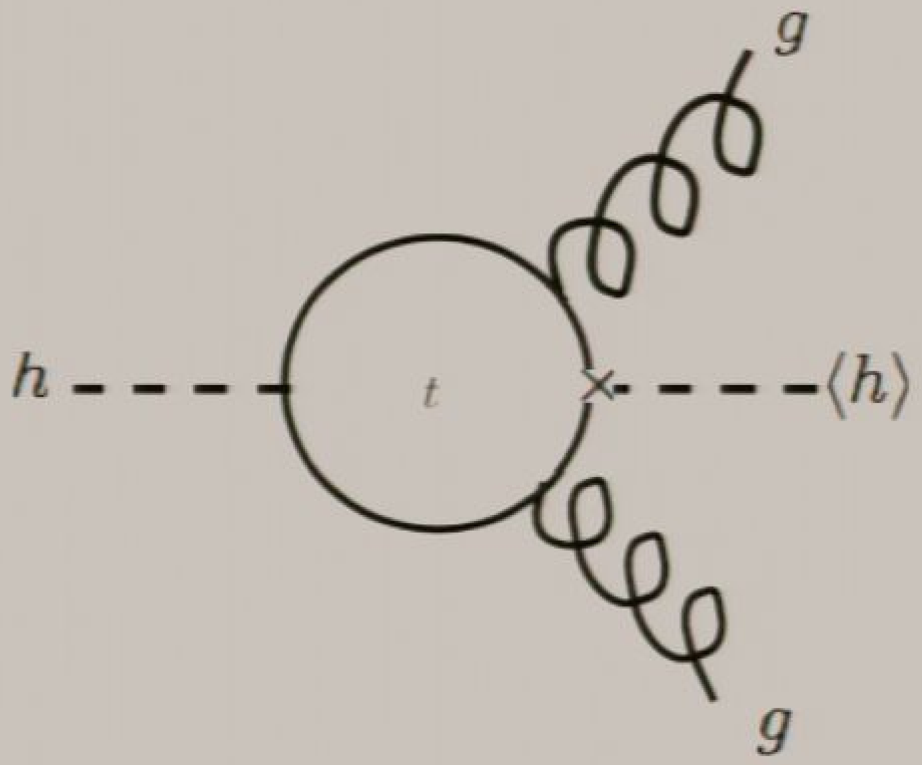
Because we have the same number of insertions along the fermion line, the relative sign between the diagrams is preserved!

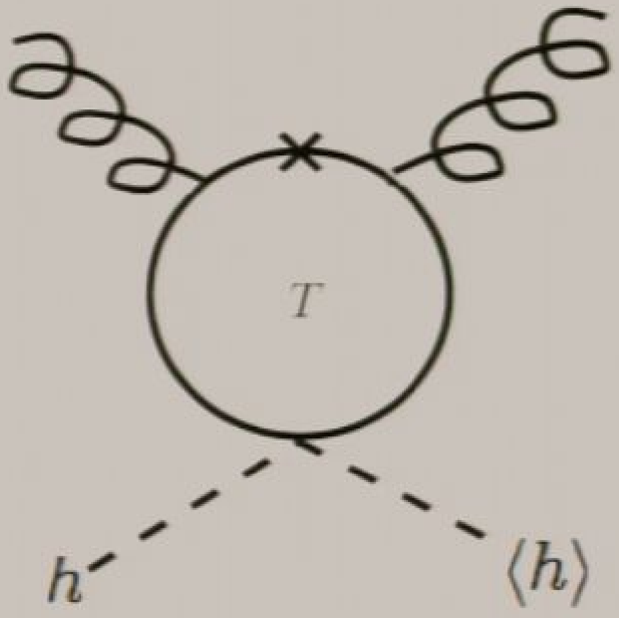




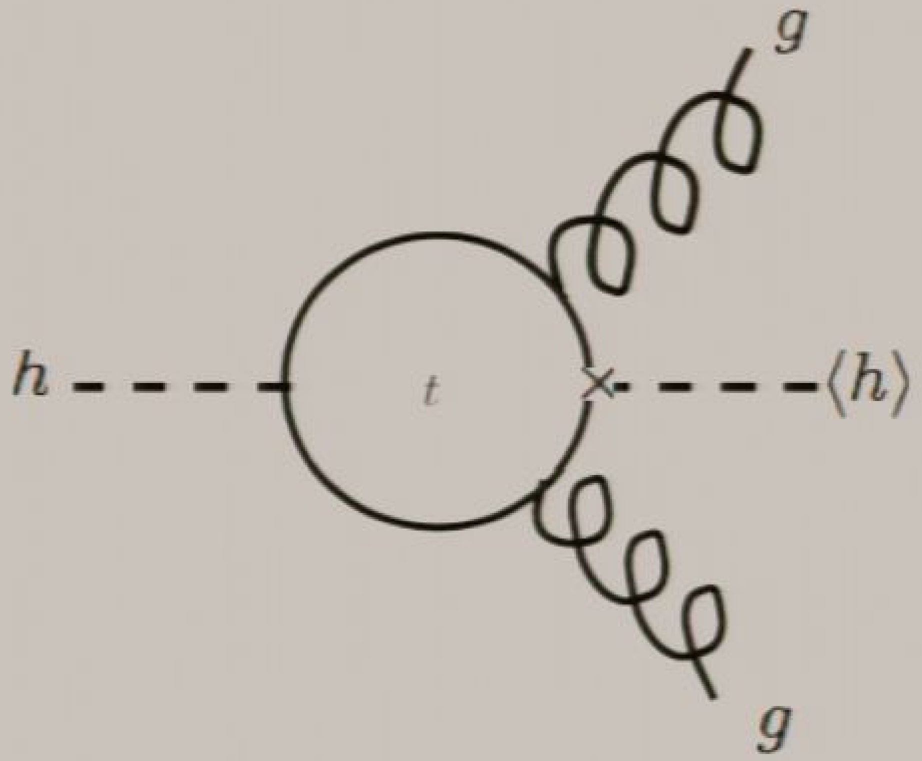
In other words, if the Higgs divergence is cancelled, the new state would interfere destructively with the top quark.

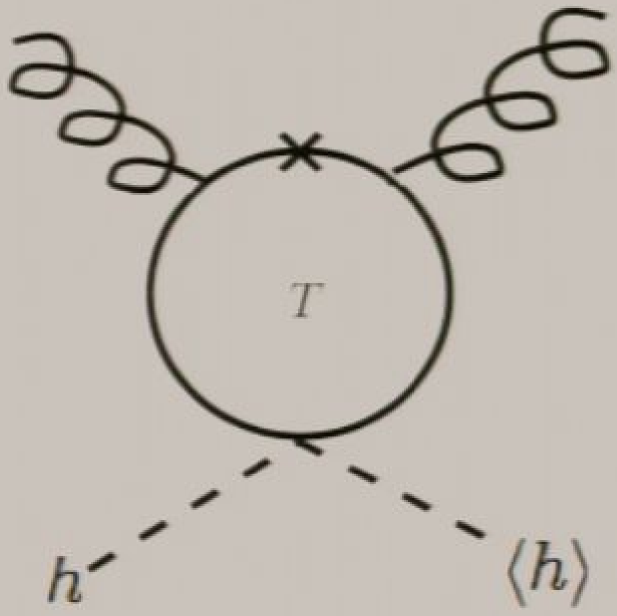
But if the divergence is NOT canceled, the new state would interfere constructively with the top quark.





The only assumption here is there is a new degree of freedom that is colored and has a significant coupling to the Higgs. Otherwise, our statement is completely general, model independent, and applies to any non-supersymmetric theories.





- The diagrammatic arguments can be formalized using the low-energy Higgs theorem, where the coefficient c_g is related to the one-loop QCD beta function:

$$\sum_{r_F} t_{r_F} \frac{\partial}{\partial \log v} \log\left(\mathcal{M}^\dagger_{r_F}(h)\mathcal{M}_{r_F}(h)\right) + \frac{1}{4}\sum_{r_S} t_{r_S} \frac{\partial}{\partial \log v} \log\left(\mathcal{M}^\dagger_{r_S}(h)\mathcal{M}_{r_S}(h)\right)$$

On the other hand, the one-loop Higgs quadratic divergence can be computed using Coleman-Weinberg potential:

$$\frac{1}{16\pi^2}\Lambda^2 \,\mathrm{Str}\, \mathcal{M}^\dagger(h)\mathcal{M}(h)$$

- Assuming a vector-like top partner, one can diagonalize the femionic mass matrix to get the mass eigenvalues:

$$m_t(h) = \frac{\lambda_t}{\sqrt{2}} h \left(1 - c_y^{(t)} \frac{h^2}{2f^2}\right) + \mathcal{O}(h^5), \qquad m_T(h) = \lambda_T f \left(1 - \frac{\lambda_t^2}{4\lambda_T^2} \frac{h^2}{f^2}\right) + \mathcal{O}(h^4).$$

Absence of one-loop quadratic divergence in Coleman-Weinberg requires

$$m_t(h)^2 + m_T(h)^2 = \text{constant} + \mathcal{O}(h^4)$$

The coefficient of the dim-5 operator $(1/v)\; hG_a^{\mu\nu}G^{a\,\mu\nu}$ is

$$\frac{g_s^2}{48\pi^2}\frac{h}{v} \times \frac{1}{2}\frac{\partial}{\partial \log h} \log \frac{\det \mathcal{M}^\dagger \mathcal{M}}{\mu^2}\bigg|_{h=v} = \frac{g_s^2}{48\pi^2}\frac{h}{v}\left(1 - c_y^{(t)}\frac{v^2}{f^2} - \frac{m_t^2}{m_T^2}\right)$$

- In the end $c_g = -1/6$, where the minus sign is fixed by naturalness and 1/6 by QCD beta function.

- There is a new contribution to c_y in the h^3 term in the light mass eigenvalue, which is due to the non-linearity in the implementation of top Yukawa coupling.

  We surveyed a large number of composite Higgs models and found it to be positive (again!)

- Taking into account all the effects discussed so far

$$\mathcal{L}_{hgg} = \frac{g_s^2}{48\pi^2}\frac{h}{v}\left[1-\left(\frac{c_H}{2}+c_y-\frac{3c_g\lambda_t^2}{\lambda_T^2}\right)\frac{v^2}{f^2}\right]G'^a_{\mu\nu}G'^{a\,\mu\nu}$$

We conclude the gluon fusion production rate is reduced in composite Higgs models!

- The situation with O_γ is more complicated, because all charged particles contribute.

- In the SM the dominant contribution is in fact the W boson loop, followed by the top quark.

- If we focus on O_γ induced by a new set of heavy electroweak gauge bosons, it is easy to see that the arguments for O_g apply as well and there is a connection between the interference effect and the cancellation of Higgs quadratic divergences in the electroweak sector.

Discussions

- Size of effects:

  There are two classes of effects,

  $\mathcal{O}(v^2/f^2)$ for c_H and c_y

  $\mathcal{O}(g_{SM}^2 v^2/g_\rho^2 f^2)$ for c_g and c_γ

  The latter one includes both $(m_t/m_T)^2$ and $m_W^2/m_{W'}^2$

  Constraints from the *S* parameter requires

  $$\frac{v^2}{f^2} \lesssim \frac{g_\rho^2}{g_{SM}^2} \times \frac{g_{SM}^2}{16\pi^2} = \frac{g_\rho^2}{16\pi^2}$$

  which is enhanced over the SM one-loop effect by

  $$\frac{g_\rho^2}{g_{SM}^2}$$

In composite Higgs models without collective breaking mechanism, such as the holographic Higgs, fine-tuning is required to obtain a large quartic coupling for the Higgs:

$$g_\rho \gg g_{SM} \qquad v^2/f^2 < 0.1 - 0.3$$

while in little Higgs models (without T-parity), precision electroweak constraints also prefer g_ρ to be slightly larger than g_{SM}.

On the other hand, naturalness prefers

$$\frac{m_t^2}{m_T^2} = \frac{\lambda_t^2}{\lambda_T^2} \times \frac{v^2}{f^2}$$

to be large to cut off the top quadratic divergence.

- Sign of c_H in little Higgs theories:

  There are three ingredients in the proof:

  1) The G/H nlsm flows to an infrared G_{IR}/H_{IR} when $g_{SM} = 0$.

  2) The contribution from the triplet is maximally negative when $g_{SM} = 0$.

  3) The contribution from vectors is independent of g_{SM}.

  Then it is simple to argue that

$$\lim_{g_\rho \sim g_{SM}} c_H > \lim_{g_\rho \gg g_{SM}} c_H > 0$$

Summary and Outlook

First, if we assume the Higgs is a fundamental scalar,

- the contribution from integrating out heavy states at tree level satisfies $c_H > 0$ except when there are the triplet scalars coupling to the Higgs, implying the on-shell coupling of the Higgs boson is reduced from the SM expectations if effects from other operators can be neglected.

- $c_H + 2c_y > 0$ when integrating out heavy scalars (including the triplet scalars) and vectors, implying the overall Higgs coupling to fermions are reduced from the SM expectations.

- $c_g < 0$ if there is a new colored fermion canceling the top quadratic divergence, implying the interference with the SM top is destructive, while $c_g > 0$ if the new colored fermion add to the top quadratic divergence. Similarly for c_γ.

Next assuming the Higgs sector is composite and belongs to a nlσm.

- $c_H > 0$ and $c_H + 2c_y > 0$ for the contribution from the non-linearity in the Higgs kinetic term.
- $c_y > 0$ for the contribution from the non-linearity in the Yukawa interaction, among the models we surveyed.

Finally assuming the underlying model has a collective breaking mechanism such as in little Higgs theories.

- $c_H > 0$ even in the presence of triplet scalars.

In the end we conclude that the gluon fusion production of the Higgs is reduced from the SM rate for all composite Higgs models.

The size of effects depends on the underlying models, ranging from modest, in the order of 10-30 %, to as small as SM one-loop effect.

Given the wealth of information that could be revealed, there's strong motivation to improve on measurements at the LHC.

If the LHC sees the Higgs and nothing else, in my mind there is still a strong scientific case for building a precision machine such as the linear collider!