Title: The non-relativistic limit of AdS/CFT

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Abstract: Non-relativistic versions of the AdS/CFT conjecture have recently been investigated in some detail. These have primarily been in the context of the Schrodinger symmetry group. Here we talk of a study based on a different non-relativistic conformal symmetry: one obtained by a parametric contraction of the relativistic conformal group. The resulting Galilean conformal symmetry has the same number of generators as the relativistic symmetry group and thus is different from the Schrodinger group (which has fewer). One of the interesting features of the Galilean Conformal Algebra is that it admits an extension to an infinite dimensional symmetry algebra (which can potentially be dynamically realised). The latter contains a Virasoro-Kac-Moody subalgebra. We comment on realisations of this extended symmetry in a boundary field theory. We also propose a somewhat unusual geometric structure for the bulk gravity dual to any realisation of this symmetry. This involves taking a Newton-Cartan like limit of Einstein's equations in anti de Sitter space which singles out an AdS_2 comprising of the time and radial direction. The infinite dimensional Virasoro extension is identified with the asymptotic isometries of this AdS_2.

The Non-Relativistic Limit of AdS/CFT

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November 17, 2009

References

- **"Galilean Conformal Algebras and AdS/CFT"**
  A. Bagchi and R. Gopakumar
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- **"On Representations and Correlation Functions of GCA"**
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- **"Supersymmetric Extension of the GCA"**
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- **"Supersymmetric Extension of the GCA"**
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  Phys. Rev. D80:086011, 2009 [arXiv:0905.0540(hep-th)]

- **"GCA in 2D"**
  A. Bagchi, R. Gopakumar, I. Mandal, A. Miwa.
  (To appear)

Introduction and motivation

Why a non-relativistic limit?

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The usual route:

- NR symmetry group = Schrödinger symmetry group $\rightarrow$ the symmetries of free Schrödinger equations
- Relevant to study of cold atoms.

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The usual route:

- NR symmetry group = Schrödinger symmetry group $\rightarrow$ the symmetries of free Schrödinger equations
- Relevant to study of cold atoms.
- Gravity dual proposed with these symmetries in two higher dimensions. (Son 2008; Balasubramanian, McGreevy 2008.)

Introduction and motivation ....

We are interested in a *different* limit. $\rightarrow$ Why?

Plan of the talk

Galilean Conformal Algebra and Schrödinger Algebra

Introduction and motivation ....

We are interested in a *different* limit. → Why?

- Systematic construction of NR algebra from the parent Conformal symmetry: Inönu-Wigner contractions
- No reason to believe that all NR systems would have the symmetries of free Schrödinger equations.
- Wish to find a holographic description in the standard one higher dimension.

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From 2D Virasoro to GCA

Bulk Dual

Outline

Inonu-Wigner Contractions: A Simple Example

$SO(3)$ maps the surface of the sphere (S^2) embedded in R_3 to itself.

Inonu-Wigner Contractions: A Simple Example

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- Equation for S^3: $x_1^2 + x_2^2 + x_3^2 = R^2$.
- Infinitesimal generators: $X_{ij} = x_i \partial_j - x_j \partial_i$
- Algebra: $[X_{ij}, X_{rs}] = X_{is}\delta_{jr} + X_{jr}\delta_{is} - X_{ir}\delta_{js} - X_{js}\delta_{ir}$

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Let us look at the north pole: $x_{1,2} = 0$ and $x_3 = R$.

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Redefine generators:

$$Y_{12} = \lim_{R\to\infty} X_{12} = x_1\partial_2 - x_2\partial_1 \quad (1)$$

$$P_i = \lim_{R\to\infty} \frac{1}{R} X_{i,3} = \lim_{R\to\infty} \frac{1}{R}(x_i\partial_3 - x_3\partial_i) \to -\partial_i \quad (2)$$

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Redefined algebra: $[Y_{12}, P_i] = P_1\delta_{2i} - P_2\delta_{1i}, \quad [P_1, P_2] = 0 \quad (3)$

This is the $ISO(2)$ group.

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This is the $ISO(2)$ group. Expected! $\Rightarrow$ At North Pole, with $R \to \infty$, S^2 looks like R_2.
We will use this technique to investigate the non-relativistic limit of the conformal algebra.

Relativistic Conformal Algebra

- **Poincare generators** $(\mu, \nu = 0, 1 \ldots d)$

$$J_{\mu\nu} = -(x_\mu \partial_\nu - x_\nu \partial_\mu), \quad B_i = J_{0i} \tag{4}$$
$$P_\mu = \partial_\mu, \quad P_0 = H$$

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- **Algebra**:

$$\begin{aligned} [J_{ij}, J_{rs}] &= so(d) \\ [J_{ij}, B_r] &= -(B_i \delta_{jr} - B_j \delta_{ir}) \\ [J_{ij}, P_r] &= -(P_i \delta_{jr} - P_j \delta_{ir}), \quad [J_{ij}, H] = 0 \\ [P_i, P_j] &= 0, \quad [H, P_i] = 0, \quad [H, B_i] = -P_i \end{aligned} \tag{5}$$

Relativistic Conformal Algebra ...

- **Other generators**: Dilatations(D) and SCT(K_μ):

$$D = -(x \cdot \partial) \qquad K_\mu = -(2x_\mu(x \cdot \partial) - (x \cdot x)\partial_\mu) \tag{7}$$

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- **Remaining algebra**:

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[K, K_i] &= 0, \quad [K, B_i] = K_i, \quad [K, P_i] = 2B_i \\
[J_{ij}, K_r] &= -(K_i\delta_{jr} - K_j\delta_{ir}), \quad [J_{ij}, K] = 0 \\
[J_{ij}, D] &= 0, \quad [K_i, K_j] = 0, \quad [H, K_i] = -2B_i, \\
[D, K_i] &= -K_i, \quad [D, B_i] = 0 \quad [D, P_i] = P_i \\
[D, H] &= H, \quad [H, K] = -2D, \quad [D, K] = -K.
\end{aligned} \tag{8}$$

$$[K_i, B_j] = \delta_{ij} K, \quad [K_i, P_j] = 2J_{ij} + 2\delta_{ij} D \tag{9}$$

Non-Relativistic Contraction: Finite GCA

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$$t \to \epsilon^r t \qquad x_i \to \epsilon^{r+1} x_i \tag{10}$$

Equivalent to $v_i \sim \epsilon$ $(c = 1)$. Set $r = 0$ for simplicity.

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- **Contracted Generators:**
  Galilean generators:

$$\begin{aligned} J_{ij} &= -(x_i\partial_j - x_j\partial_i), \quad H = -\partial_t \\ P_i &= \partial_i, \quad B_i = t\partial_i. \end{aligned} \tag{11}$$

  Galilean conformal generators:

$$\begin{aligned} D &= -(x_i\partial_i + t\partial_t), \quad K_i = t^2\partial_i, \\ K &= K_0 = -(2tx_i\partial_i + t^2\partial_t). \end{aligned} \tag{12}$$

Non-Relativistic Contraction: Finite GCA ..

Changes in Algebra: Galilean Conformal Algebra

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Changes in Algebra: Galilean Conformal Algebra

- RHS of Red Equations: (6), (9) $= 0$.
- Rest of algebra $\Rightarrow$ same.

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$$[J_{ij}, J_{rs}] = so(d)$$
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- For massive systems, consider rest energy > kinetic energy. Replace $\partial_0 \to -im_0 + \partial_t; m_0 \to \frac{m}{\epsilon^2}; x_i \to \epsilon x_i$.

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- For massive systems, consider rest energy > kinetic energy. Replace $\partial_0 \to -im_0 + \partial_t;\ m_0 \to \frac{m}{\epsilon^2};\ x_i \to \epsilon x_i$.
- Klein Gordon equation reduces to Schrodinger equation

$$(\partial_0^2 - \partial_i^2 + m_0^2)\phi = 0 \to (i\partial_t + \frac{1}{2m}\partial_i^2)\phi = 0. \qquad (13)$$

The parameter $\epsilon \sim \frac{v}{c}$ signifies taking the nonrelativistic limit

Schrödinger Algebra ...

Algebra:

- Galilean sub-group: $\{J_{ij}, B_i, P_i, H\}$

$$[B_i, P_j] = m\delta_{ij} \tag{14}$$

$m \rightarrow$ central extension, NR mass. Rest: same as in GCA.

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- Other generators: $\{\tilde{D}, \tilde{K}\}$
  $\tilde{D} = -(2t\partial_t + x_i\partial_i) \rightarrow$ scales space and time differently.
  Action: $(x_i, t) \rightarrow (\lambda x_i, \lambda^2 t)$.
  $\tilde{K} = -(tx_i\partial_i + t^2\partial_t) \rightarrow$ like temporal SCT.
  Action: $(x_i, t) \rightarrow (\frac{x_i}{(1+\mu t)}, \frac{t}{(1+\mu t)})$.

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  Action: $(x_i, t) \rightarrow (\frac{x_i}{(1+\mu t)}, \frac{t}{(1+\mu t)})$.
- Non-zero commutators:

$$\begin{aligned} [\tilde{K}, P_i] &= B_i, \quad [\tilde{K}, B_i] = 0, \quad [\tilde{D}, B_i] = -B_i \\ [\tilde{D}, \tilde{K}] &= -2\tilde{K}, \quad [\tilde{K}, H] = -\tilde{D}, \quad [\tilde{D}, H] = 2H. \end{aligned} \quad (15)$$

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  Same number of generators as the parent theory (15 in D=4).

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- SA: Dilatation operator scales space and time differently.
  GCA: Dilatation scales space and time in the same way.
- SA: Allows mass, central extn between momentum and boosts.
  GCA: No mass (Jacobi id don't allow it).
  Symmetry group of massless/gapless NR system.

Outline

Redefining Finite algebra

- Redefine generators of finite algebra:

$$\begin{aligned} L^{(-1)} &= H, \quad L^{(0)} = D, \quad L^{(+1)} = K, \\ M_i^{(-1)} &= P_i, \quad M_i^{(0)} = B_i, \quad M_i^{(+1)} = K_i. \end{aligned} \tag{16}$$

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- The finite dimensional GCA (with $m, n = 0, \pm 1$):

$$
\begin{aligned}
[L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)}, \\
[L^{(m)}, M_i^{(n)}] &= (m-n)M_i^{(m+n)}, \quad [M_i^{(m)}, M_j^{(n)}] = 0, \\
[J_{ij}, L^{(n)}] &= 0, \quad [J_{ij}, M_k^{(m)}] = -(M_i^{(m)}\delta_{jk} - M_j^{(m)}\delta_{ik}).
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Infinite GCA

- Define for *arbitrary* integer n, vector fields:

$$\begin{aligned} L^{(n)} &= -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t, \quad M_i^{(n)} = t^{n+1}\partial_i \\ J_a^{(n)} &\equiv J_{ij}^{(n)} = -t^n(x_i\partial_j - x_j\partial_i) \end{aligned} \tag{17}$$

$n = 0, \pm 1.$ → vector fields that generate GCA

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$$\begin{aligned} [L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)}, \quad [L^{(m)}, J_a^{(n)}] = -nJ_a^{(m+n)} \\ [J_a^{(n)}, J_b^{(m)}] &= f_{abc}J_c^{(n+m)}, \quad [L^{(m)}, M_i^{(n)}] = (m-n)M_i^{(m+n)} \end{aligned}$$

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- Commuting generators $M_i^{(n)}$ function like generators of a global symmetry.
- Can consistently set these generators to zero and add usual Virasoro-Kac-Moody central terms.

Physical Significance of the GCA

- The $M_i^{(n)}$ act as generators of generalised time dependent but spatially homogeneous accelerations

$$x_i \rightarrow x_i + b_i(t). \tag{18}$$

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- These two together generate what is sometimes called the Coriolis group: the biggest group of "isometries" of "flat" Galilean spacetime.
- $L^{(n)}$ seem to be generators of a conformal "isometry" of Galilean spacetime.

$$t \rightarrow f(t), \qquad x_i \longrightarrow \frac{df}{dt}x_i \qquad (20)$$

Realisation of the GCA: NR Conformal Hydrodynamics

- NR limit of quantum field theories at finite temperature in the hydrodynamic limit $\longrightarrow$ recover Navier-Stokes eqns.

$$\partial_t v_i(x,t) + v_j \partial_j v_i(x,t) = -\partial_i p(x,t) + \nu_0 \partial_j \partial_j v_i(x,t) \qquad (21)$$

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- Has all symmetries of finite GCA, except D which is broken (by the viscous term) because of the choice of temperature.
- Has all $M_i^{(n)}$ as symmetries!
- Navier-Stokes eqn should describe the hydrodynamic limit of all NR field theories.
  A part of the infinitely extended GCA exists as its symmetries!

Realisation of the GCA: NR Conformal Hydrodynamics ...

- Navier-Stokes equation with viscosity set to zero = Incompressible Euler equations

$$\partial_t v_i(x, t) + v_j \partial_j v_i(x, t) = -\partial_i p(x, t) \tag{22}$$

Entire finite GCA is a symmetry (since D is now also a symmetry).

This shows that one can readily realise "gapless" non-relativistic systems in which space and time scale in the same way!

Outline

Galilean Conformal Algebra and Schrödinger Algebra

Infinite extension of GCA

Correlation Functions of GCA

From 2D Virasoro to GCA

Bulk Dual

Representations

- States would be labeled under L_0 (dilatations) and M_0 (boosts).

$$L_0|\Delta, \xi^i\rangle = \Delta|\Delta, \xi^i\rangle, \quad M_0^i|\Delta, \xi^i\rangle = \xi^i|\Delta, \xi^i\rangle \tag{23}$$

- L_n, M_n^i lower the dilatation eigenvalue (Δ) and L_{-n}, M_{-n}^i raise it.
- Primary states: there must exist states with a lower bound on the Δ eigenvalue, so that for them Δ cannot be lowered further.

$$L_n|\Delta, \xi^i\rangle_p = 0, \quad M_n|\Delta, \xi^i\rangle_p = 0 \quad \forall n > 0 \tag{24}$$

- Can construct representations by acting on the primary state by raising operators.

Two and Three point functions of GCA

Look at correlation functions of quasi-primary operators.
(quasi-primary= primary wrt finite sub-algebra).

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Two pt function: $G^{(2)} = C_{(12)}\delta_{\Delta_1,\Delta_2}\delta_{\xi_1^i,\xi_2^i} t_{12}^{2\Delta_1} \exp\left(\frac{2\xi_i x_{12}^i}{t_{12}}\right)$

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Three pt function:
$G^{(3)} = C^{(3)} t_{13}^{\Delta^{132}} t_{23}^{\Delta^{231}} t_{12}^{\Delta^{123}} \exp\left(-\frac{\xi_i^{132} x_{13}^i}{t_{13}} - \frac{\xi_i^{231} x_{23}^i}{t_{23}} - \frac{\xi_i^{123} x_{12}^i}{t_{12}}\right)$

where $\Delta^{lmn} = -(\Delta^l + \Delta^m - \Delta^n)$ and similarly ξ_i^{lmn}.

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$$G^{(2)}(z_i, \bar{z_i}) = \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12} z_{12}^{-2h} \bar{z}_{12}^{-2\bar{h}}$$

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  - SA further constrained by mass selection rules. Not present in GCA: boosts act on co-ordinates unlike mass.

Outline

Mapping of the representations ...

- Representation of classical Virasoro algebra in 2d :

$$\mathcal{L}_n = -z^{n+1}\partial_z, \quad \bar{\mathcal{L}}_n = -\bar{z}^{n+1}\partial_{\bar{z}} \tag{25}$$

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$$\mathcal{L}_n + \bar{\mathcal{L}}_n = -t^{n+1}\partial_t - (n+1)t^n x\partial_x + O(\epsilon^2) \tag{26}$$

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- As $\epsilon \longrightarrow 0$, identification:

$$\mathcal{L}_n + \bar{\mathcal{L}}_n \longrightarrow L^{(n)}, \quad \epsilon(\mathcal{L}_n - \bar{\mathcal{L}}_n) \longrightarrow -M^{(n)} \tag{28}$$

Mapping of Infinite algebra

- 2d relativistic Virasoro Algebra:

$$
\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\mathcal{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\
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- The linear combinations before taking the limits:

$$[\mathcal{L}_m + \bar{\mathcal{L}}_m, \mathcal{L}_n + \bar{\mathcal{L}}_n] = (m-n)(\mathcal{L}_{m+n} + \bar{\mathcal{L}}_{m+n}) + \frac{c+\bar{c}}{12}m(m^2-1)\delta_{m+n,0}$$
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Mapping of Infinite algebra ...

- After taking the limit

$$
\begin{aligned}
[L^{(m)}, L^{(n)}] &= (m-n)L^{(m+n)} + C_1 m(m^2-1)\delta_{m+n,0} \\
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- Infinite extended GCA!

(Note that $C_1 = \frac{c+\bar{c}}{12}$ and $\frac{C_2}{\epsilon} = \frac{-c+\bar{c}}{12}$.)

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- Infinite GCA which was first written by observation has now been derived as a simple limit of the algebra of 2d CFTs.

GCA: The large spin sector of 2D Virasoro

- More subtlety in the above limit. Can be understood by looking at correlation functions.
- Would not get the important exponential pieces of correlation fns if one took the simple NR limit.
- Also need to take $h + \bar{h} = \Delta$ and $h - \bar{h} = \frac{\xi}{\epsilon}$.
- Two point correlator of $2d$ relativistic CFT in this limit:

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\begin{aligned}
G_R^{(2)}(z_i, \bar{z}_i) &= \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12}(t_{12} - x_{12})^{-2h}(t_{12} + x_{12})^{-2\bar{h}} \\
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&= \delta_{h_1,h_2}\delta_{\bar{h}_1,\bar{h}_2} C_{12}\ t_{12}^{\Delta}\ (1 - \frac{x_{12}}{t_{12}})^{2s}\ (1 - \frac{x_{12}^2}{t_{12}^2})^{-2\bar{h}}
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- Now, we want to take the limit of large spin along with the non-relativistic limit on the co-ordinates. Keeping in mind the identity $\lim_{N\to\infty}(1+\frac{x}{N})^N = e^x$, we find

$$G_R^{(2)} \to C_{12}\ \delta_{\Delta_1,\Delta_2}\ \delta_{\xi_1,\xi_2}\ t_{12}^{\Delta}\ \exp(\frac{2\xi x_{12}}{t_{12}}) = G_{GCA}^{(2)} \qquad (31)$$

A point to note here is that $\bar{h}$ goes at most like $\frac{1}{\epsilon}$ and $\frac{x_{12}^2}{t_{12}^2}$ goes like ϵ^2, so the last factor always scales to 1.

Outline

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$$ds^2 = R^2 \frac{dt^2 - dz^2 - dx_i^2}{z^2} \tag{32}$$

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- Boundary metric degenerates in the non-relativistic limit with the d spatial directions scaling as $x_i \propto \epsilon$ while $t \propto \epsilon^0$.
- Expect this feature to be shared by the bulk metric. Geometry on constant radial sections expected to have such a scaling.

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- $\Rightarrow$ In the bulk the time and radial directions of the metric *both* survive the scaling.
  $\Rightarrow$ AdS_2 sitting inside the original AdS_{d+2}.
- So expect the dual spacetime to have the structure $AdS_2 \times R^d$ with degenerate metric on the R^d.

Newton-Cartan Theory

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Newton-Cartan Theory

- Degenerate nature of the metric might seem to imply that the gravitational dynamics is singular.
- However, similar situation in asymptotically flat space in recovering Newtonian gravity from Einstein gravity in the non-relativistic limit.
- The answer: there is a well-defined geometric theory of Newtonian gravitation - Newton-Cartan theory.

Newton-Cartan Theory ...

- The ingredients: A space time endowed with absolute time function t, a non-dynamical *spatial* (Euclidean) metric, a dynamical *non-metric* connection $\Gamma^i_{00} \propto \partial_i \Phi$.
- Einstein's equations reduce to $R_{00} \propto \rho$ which is Poisson's equation for Φ.

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- One can write all this in a more covariant form: A d dimensional fibre R^d over a base R which is parametrised by the time t.
- Nothing singular about this classical geometric description, only unusual.

Proposed Modifications for *AdS*

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- Thus the bulk boundary relation is some kind of an AdS_2/CFT_1 duality.
- The correlators on the boundary theory also point to the same thing. "Conformal" pieces in t. But non-trivial pieces from the fibre-bundle structure.

GCA from Bulk Killing Vectors

- AdS_{d+2} in radially infalling coordinates for null geodesics $(t' = t + z, z' = z)$ (tranforming from Poincare co-ordinates)

$$ds^2 = \frac{R^2}{z'^2}(-dt'(2dz' - dt') - dx_i^2). \tag{34}$$

- Take the generators of the AdS_{d+2} isometries and perform the contraction by taking $t', z' \to \epsilon^r, x_i' \to \epsilon^{r+1} x_i$.

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- Contracted Killing vectors given by

$$\begin{aligned} P_i &= -\partial_i, \quad B_i = -(t' - z')\partial_i, \quad K_i = -(t'^2 - 2t'z')\partial_i \\ H &= \partial_{t'}, \quad D = t'\partial_{t'} + z'\partial_{z'} + x_i\partial_i, \\ K &= t'^2\partial_{t'} + 2(t' - z')(z'\partial_{z'} + x_i\partial_i). \end{aligned} \tag{35}$$

GCA from Bulk Killing Vectors ...

- More compactly (for $m, n = 0, \pm 1, l = 0$).

$$\begin{aligned} L^{(n)} &= t'^{n+1}\partial_{t'} + (n+1)(t'^n - nzt'^{n-1})(x_i\partial_i + z'\partial_{z'}) \\ M_i^{(m)} &= -(t'^{m+1} - (m+1)zt'^m)\partial_i \\ J_{ij}^{(l)} &= -t'^n(x_i\partial_j - x_j\partial_i) \end{aligned} \quad (36)$$

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- Reduces at the boundary ($z = 0$) to the generators of the contracted conformal algebra. And satisfies the *same* algebra.
- In fact, these bulk vector fields (for arbitrary m, n, l) reduce to that of the extended Kac-Moody algebra at the boundary.

Interpretation of Bulk Vector Fields

- The $M_i^{(n)}$ and $J_a^{(n)}$ act only on the R^d. Rotation and translation of the spatial slices which depend on t, z.
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Isometries of the spatial metric. Act trivially on the AdS_2.
- The Virasoro generators act as the generators of asymptotic symmetries of the AdS_2.
- Under its action (with infinitesimal parameter a_n)

$$\begin{aligned} z \to \tilde{z} &= z[1 + a_n(n+1)(t^n - nzt^{n-1})] \\ t \to \tilde{t} &= t[1 + a_n t^n] \\ x_i \to \tilde{x}_i &= x_i[1 + a_n(n+1)(t^n - nzt^{n-1})]. \end{aligned} \tag{37}$$

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  1. Consider action on Poincare metric on AdS_5 (EF co-ords).
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- On taking the scaling limit:

$$ds^2 = \frac{1}{z^2}(-2dtdz+dt^2) \to \frac{1}{z^2}(-2dtdz+dt^2+2n(n^2-1)a_n z^2 t^{n-2} dt^2).$$

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- Action of the $L^{(n)}$ on the spatial metric on the slices of constant t, z is again an isometry.
- $L^{(n)}, J_a^{(n)}, M_i^{(N)}$ together generate (asymptotic) isometries of the spatial and AdS_2 metrics γ^{ij} and g_{ab}.
- Therefore it seems natural to consider the action of these generators on the Newton-Cartan like geometry.

Summary

- GCA is obtained by a parametric group contraction of the relativistic conformal algebra.
- It can be given an infinite lift for any spacetime dimensions.
- Finite algebra realised as invariant symmetry algebra of the Euler equations.
- Novel structures for the 2 and 3 pt functions
- For d=2, a mapping exists from the Virasoro to the GCA.
- Bulk dual is in standard one higher dimension and is a Newton-Cartan like $AdS_2 \times R^d$.
- GCA is obtained by contracting bulk killing vectors and can be interpretted as the asympototic isometry of the NC structure.

Future Directions

- Interpreting and using the infinite symmetry in conformal theories with $d > 2$.
- Infinite symmetry $=$ integrability ?
- Understanding the bulk better: bulk-bdy dictionary, N-C structure.
- Embeddings in string theory.
- Other cond mat systems with GCA. (e.g. aging systems, quantum hall systems.)
- A host of other questions!

Thank You!!