Title: Introduction to Effective Field Theory - Lecture 8B

Date: Nov 11, 2009 11:30 AM

URL: http://pirsa.org/09110016

Abstract:

$\psi = U(x)\chi$

$\psi = e^{i\theta}\chi$

Goldstone fields

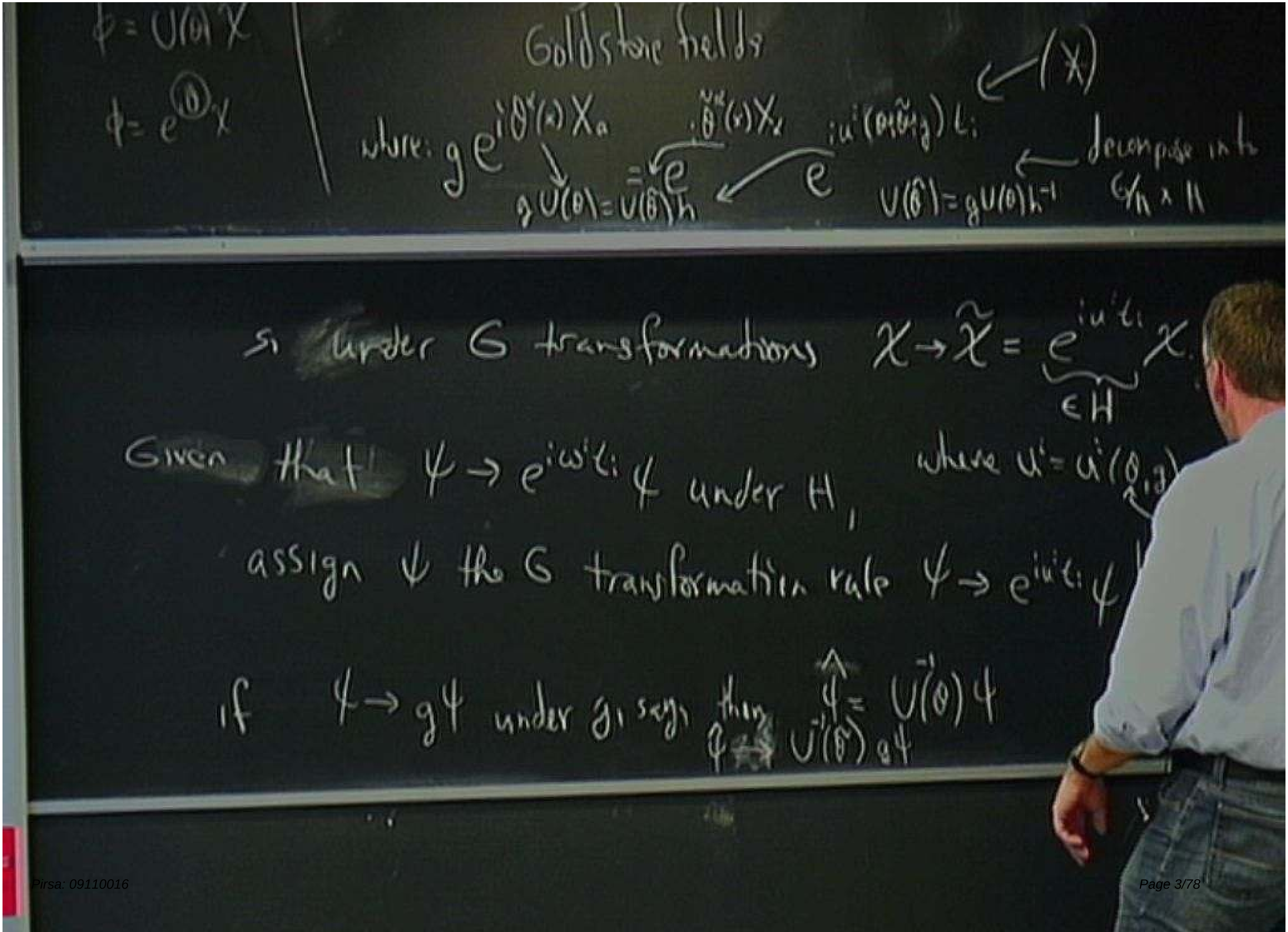
where: $g\, e^{i\theta^a(x) X_a} = e^{i\tilde\theta^a(x) X_a}\, e^{i u^i(\theta, \tilde\theta, g) t_i}$ ← decompose into $G/H \times H$ ← (χ)

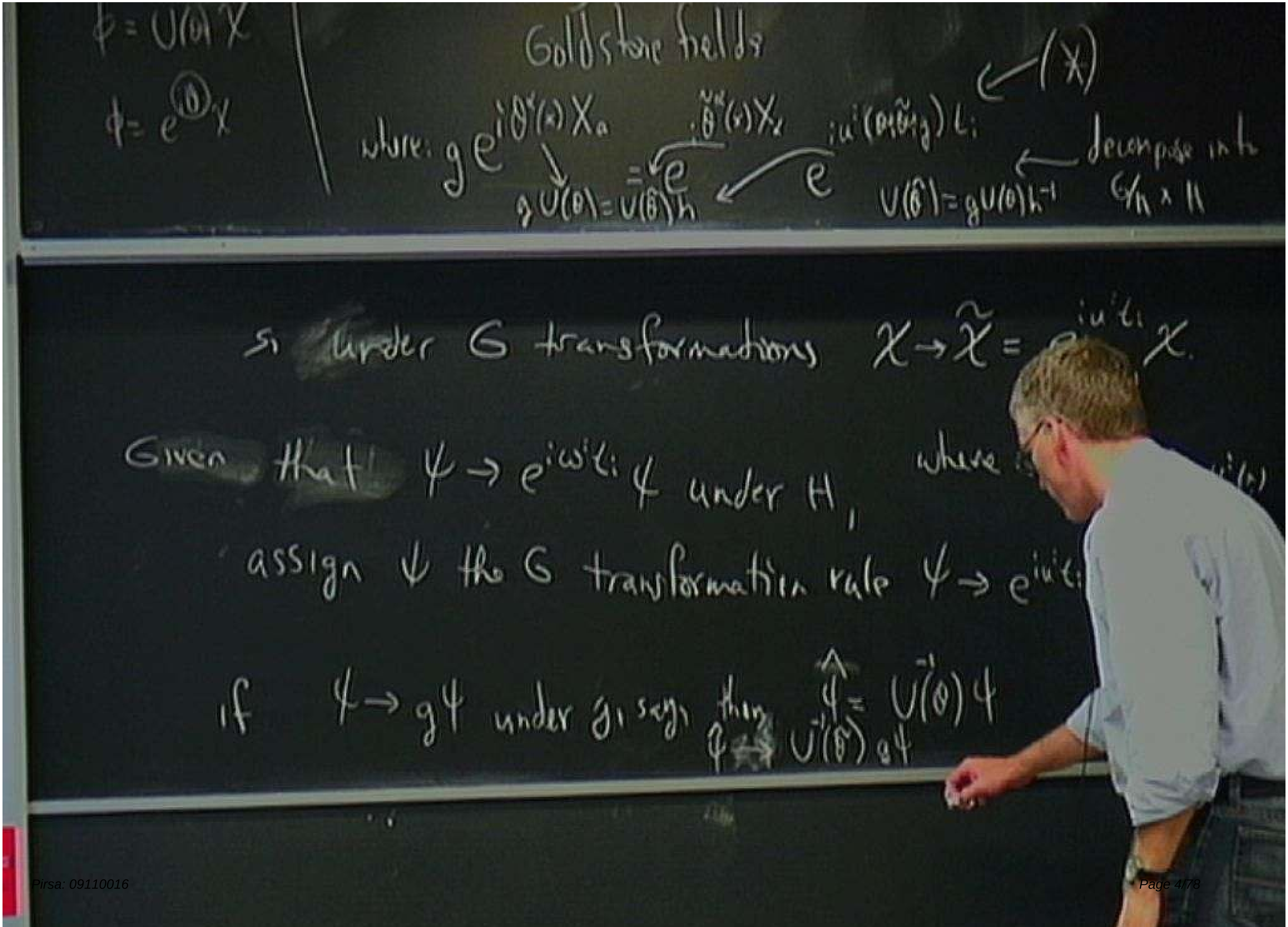
So under G transformations $\chi \to \tilde\chi = \underbrace{e^{i u^i t_i}}_{\in H}\chi$

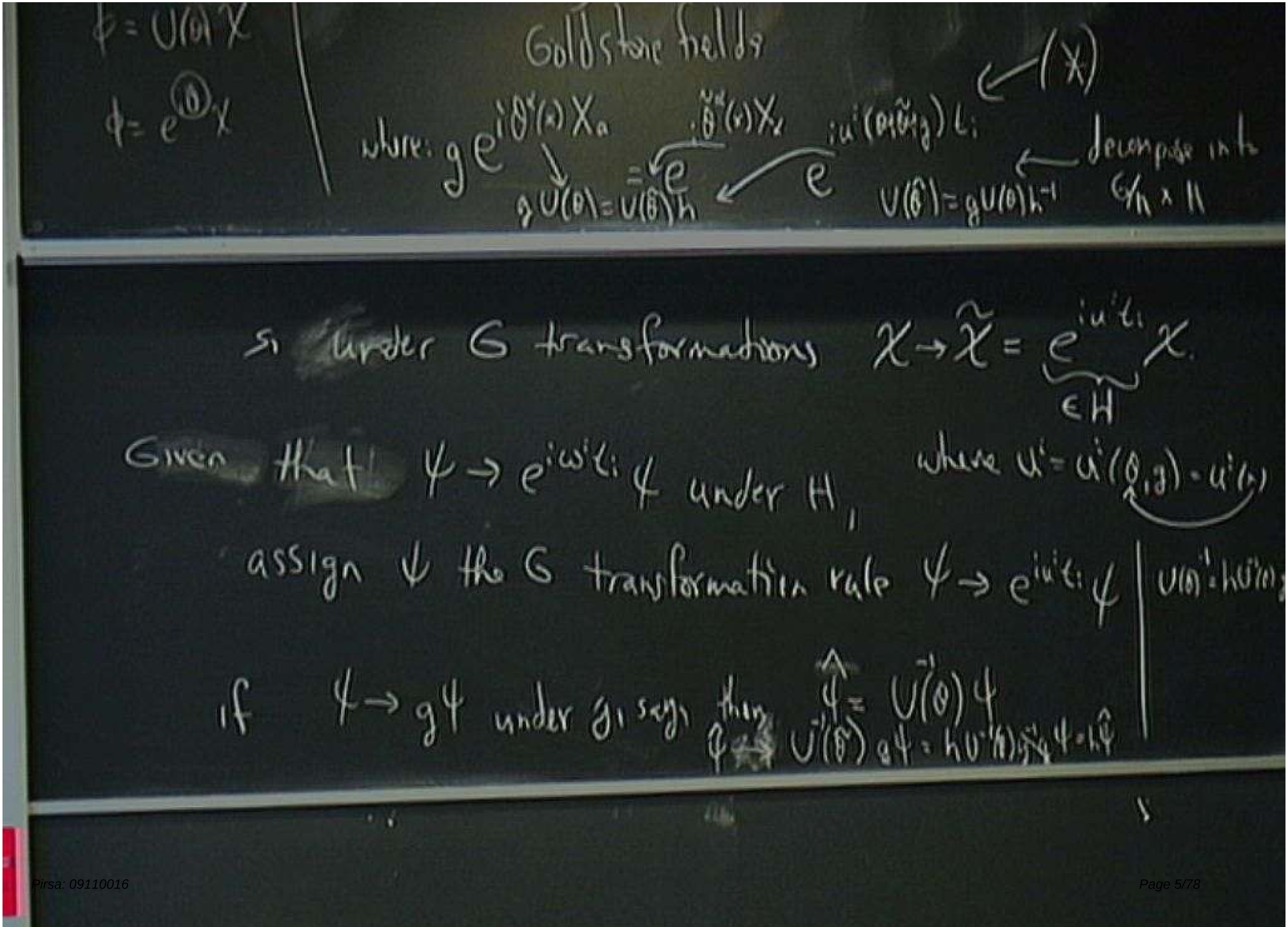
Given that $\psi \to e^{i\omega^i t_i}\psi$ under H, where $u^i = u^i(\theta, g) = u^i(x)$

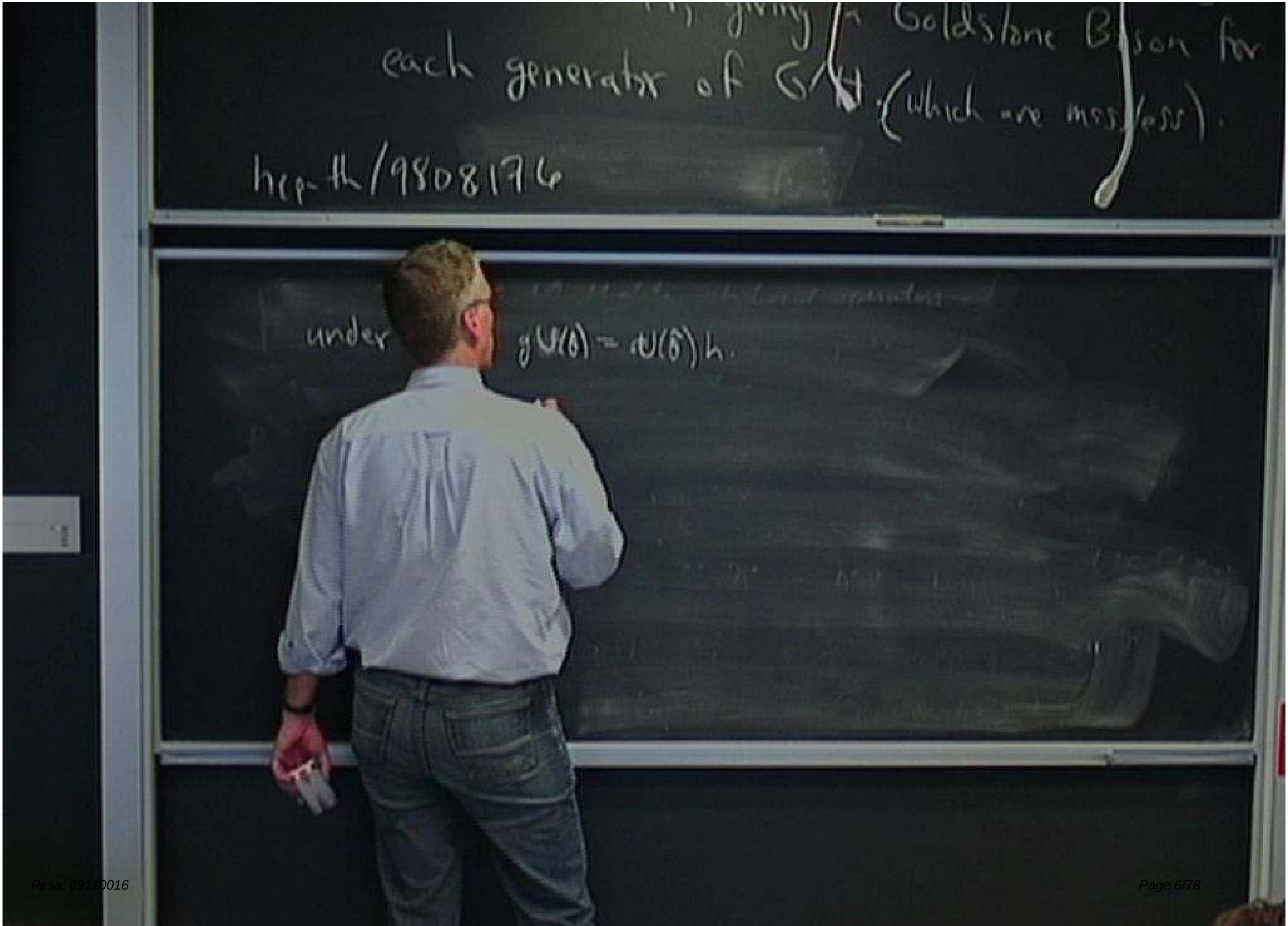
assign ψ the G transformation rule $\psi \to e^{i u^i t_i}\psi$

if $\psi \to g\psi$ under g, say, then


Goldstone fields
where:
decompose into G/H × H
under G transformations
∈H
Given that
under H,
where
assign ψ the G transformation rule
if ψ → gψ under g, says then


Goldstone fields
where
decompose into G/H × H
so under G transformations χ → χ̃ = e^{iu^i t_i} χ
Given that ψ → e^{iω^i t_i} ψ under H,
where
assign ψ the G transformation rule ψ → e^{iu^i t_i}
if ψ → gψ under G, say, then


ψ = U(θ) χ
ψ = e^{iθ} χ
Goldstone fields
where: g e^{iθ^a(x) X_a}
decompose into G/H × H
U(θ̃) = g U(θ) h^{-1}
So under G transformations χ → χ̃ = e^{iu^i t_i} χ
∈ H
Given that ψ → e^{iω^i t_i} ψ under H,
where u^i = u^i(θ, g)
assign ψ the G transformation rule ψ → e^{iu^i t_i} ψ
if ψ → gψ under g, say, then ψ̂ = U^{-1}(θ) ψ


giving a Goldstone Boson for
each generator of G/H (which are massless).
hep-th/9808176
under
gU(θ) = U(θ')h.

$\psi = U(\theta)\chi$

$\psi = e^{\textcircled{\theta}}\chi$

Goldstone fields

$\leftarrow (\ast)$

where: $g e^{i\theta^a(x) X_a} = e^{i\tilde{\theta}^a(x) X_a} e^{i u^i(\theta,\tilde{\theta},g) t_i}$ ← decompose into $G/H \times H$

$g U(\theta) = U(\tilde{\theta}) h$ $\quad U(\tilde{\theta}) = g U(\theta) h^{-1}$

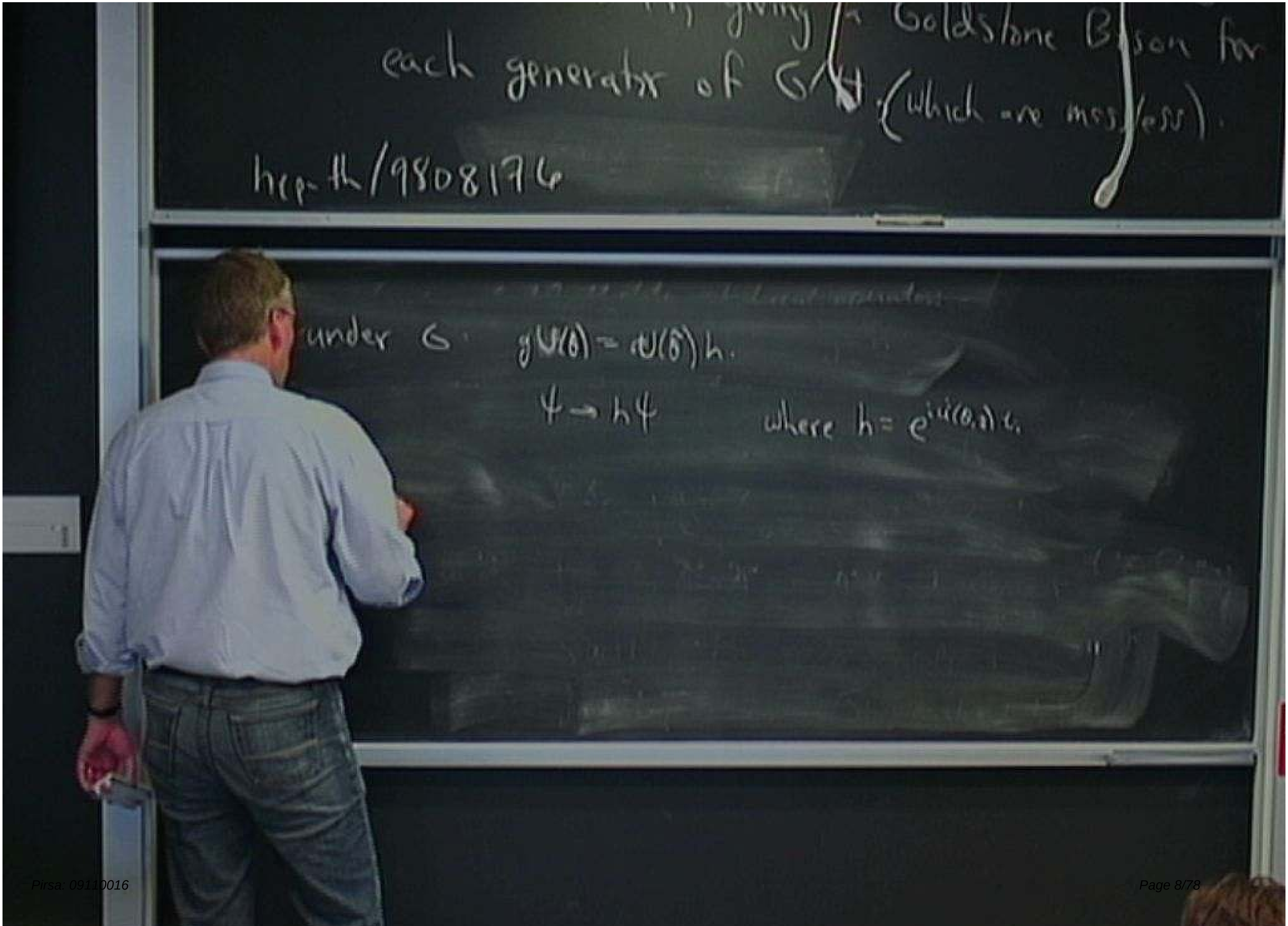
so under G transformations $\chi \to \tilde{\chi} = \underbrace{e^{i u^i t_i}}_{\in H} \chi$

Given that $\psi \to e^{i\omega^i t_i}\psi$ under H, where $u^i = u^i(\theta, g) = u^i(x)$

assign ψ the G transformation rule $\psi \to e^{i u^i t_i}\psi$ $\quad \big| \; U(\tilde{\theta})^{-1} = h U^{-1}(\theta) g^{-1}$

if $\psi \to g\psi$ under g, say, then $\hat{\psi} = U^{-1}(\theta)\psi$

$\hat{\psi} \to U^{-1}(\tilde{\theta}) g\psi = h U^{-1}(\theta) g^{-1} g \psi = h\hat{\psi}$


giving a Goldstone Boson for
each generator of G/H (which are massless).
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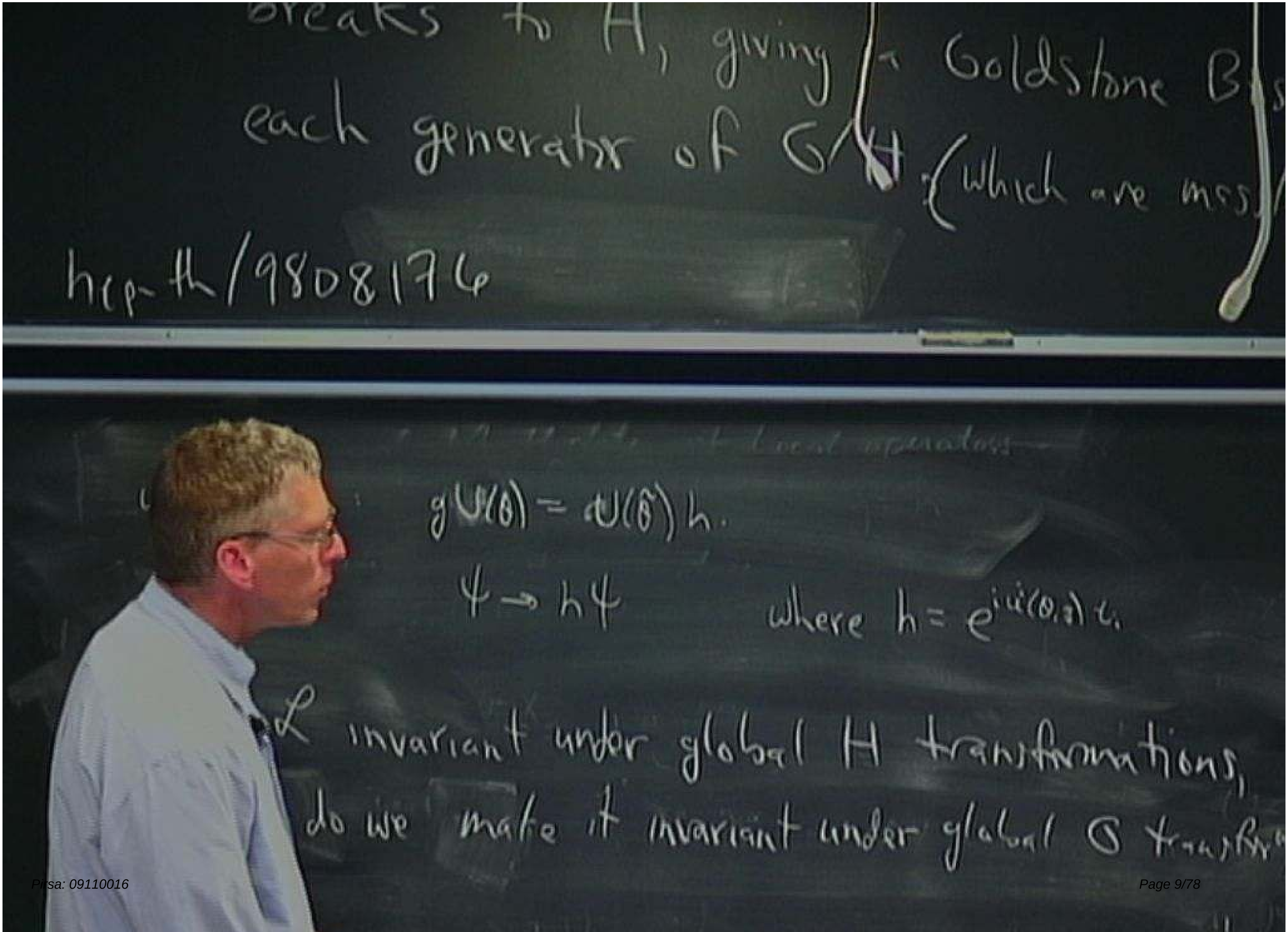
under G: gΨ(θ) = Ψ(θ')h.
ψ → hψ
where h =


breaks to H, giving a Goldstone B
each generator of G/H (which are
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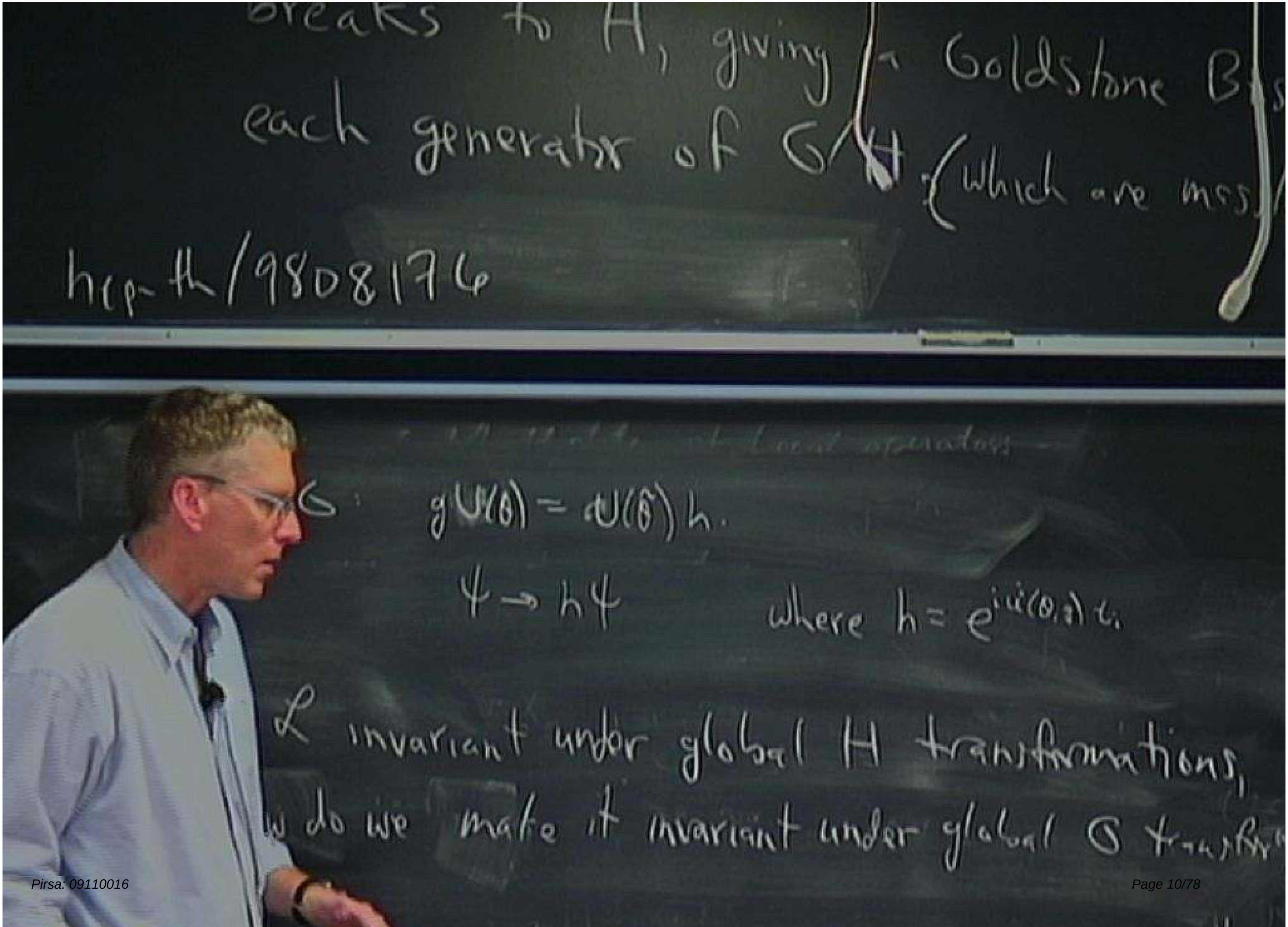
g U(θ) = U(θ') h.
ψ → hψ
where h =
invariant under global H transformations,
do we make it invariant under global G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hep-th/9808176
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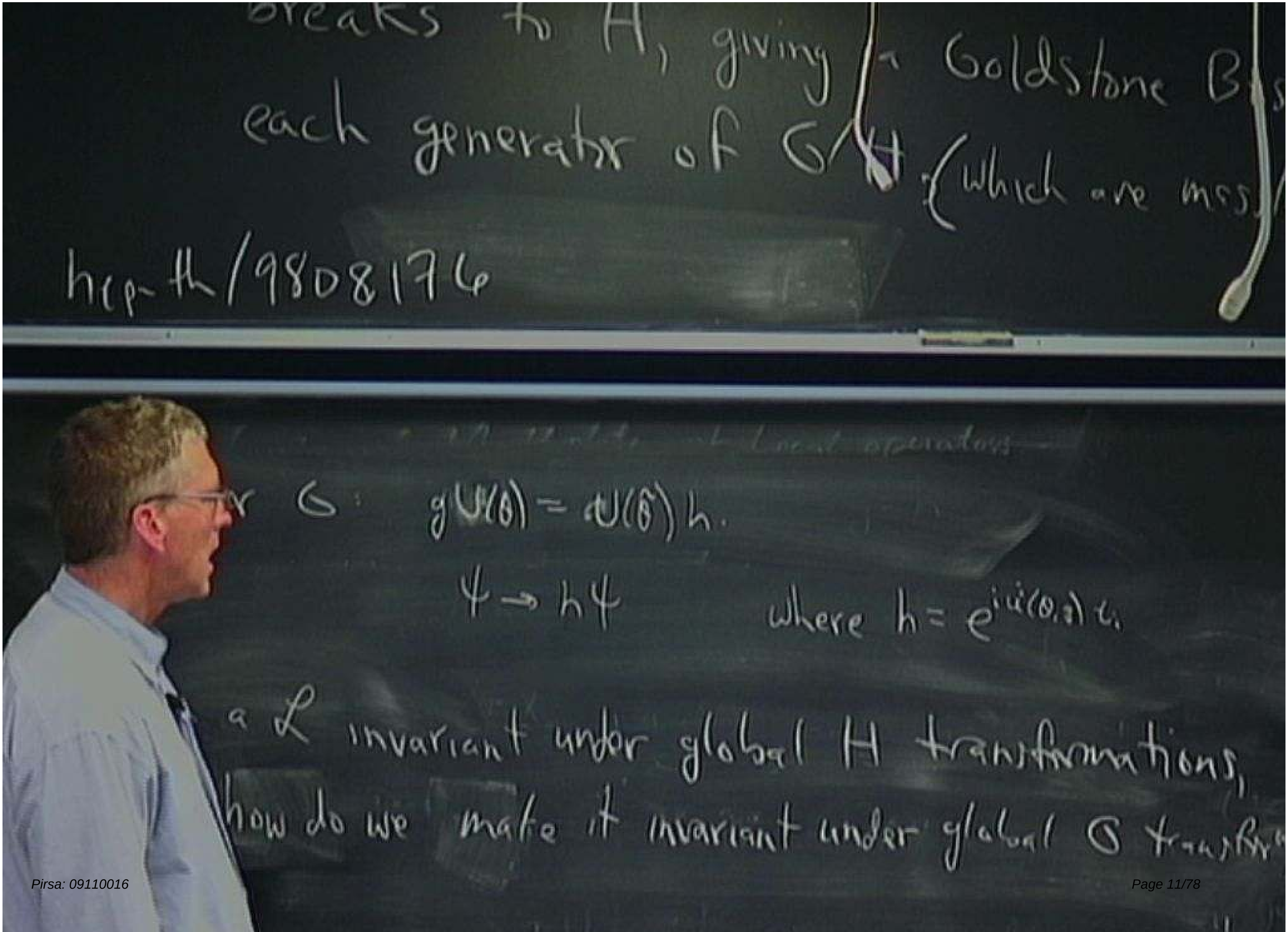
ψ → hψ
where h = e^{iu(θ,g) t_i}
L invariant under global H transformations,
w do we make it invariant under global G transf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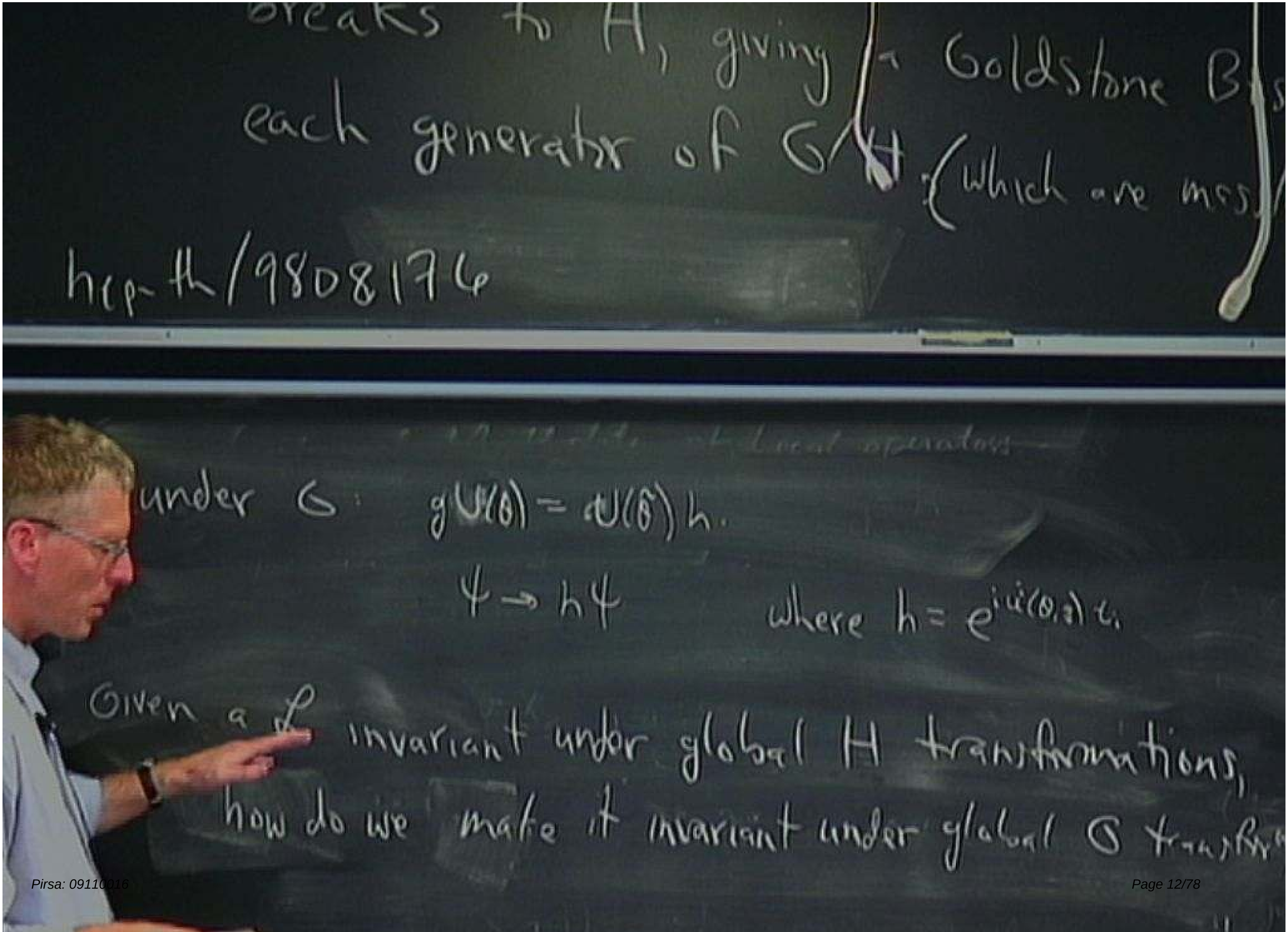
G: gU(θ) = U(θ') h.
ψ → hψ
where h = e
a ℒ invariant under global H transformations,
how do we make it invariant under global G


breaks to H, giving a Goldstone B
each generator of G/H (which are mas
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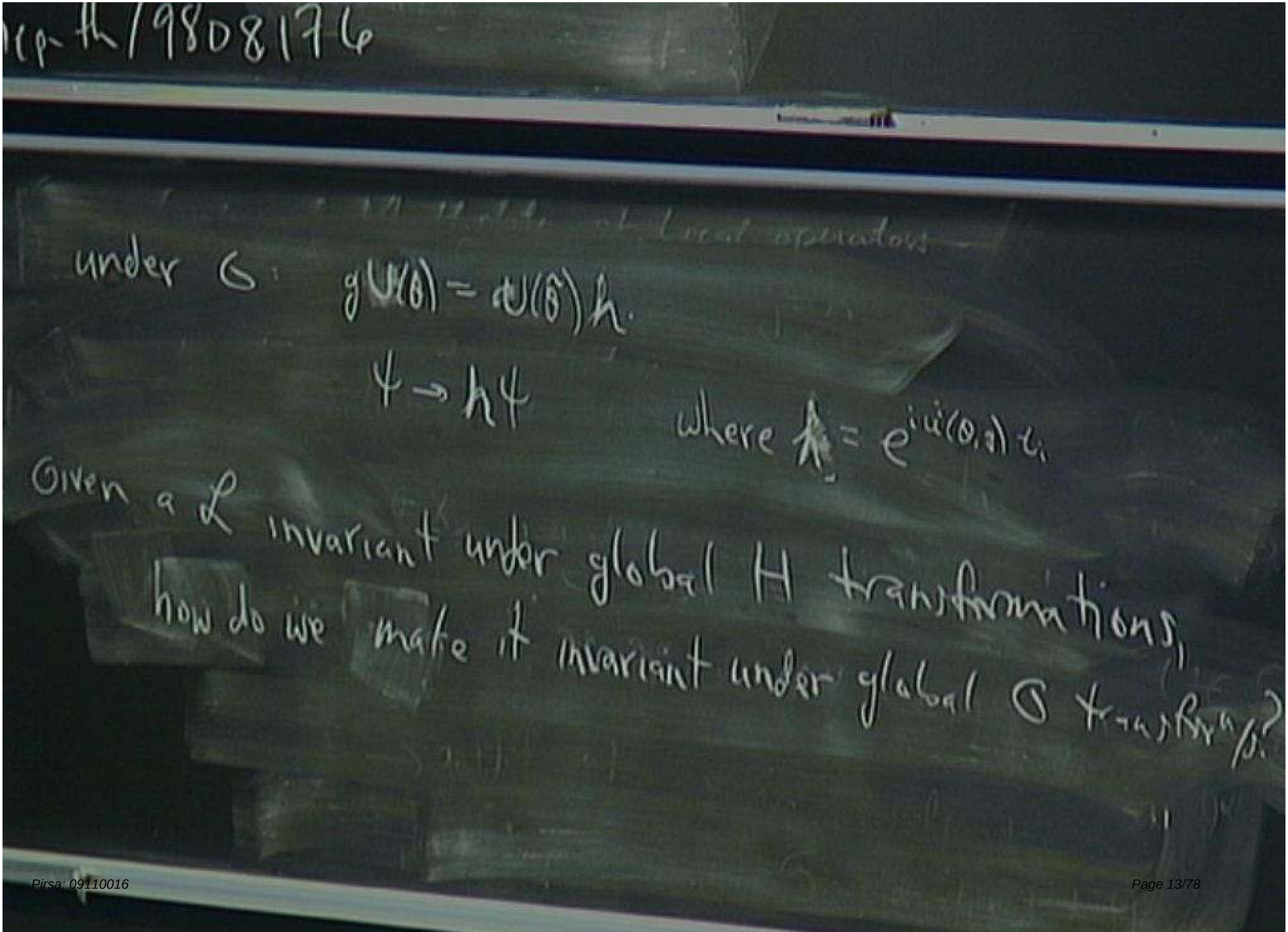
under G: g U(θ) = U(θ') h.
ψ → hψ
where h = e^{iu(θ,g) t}
Given a ℒ invariant under global H transformations,
how do we make it invariant under global G


hep-th/9808176
under G: gU(θ) = U(θ)h.
ψ → hψ
where h = e^{iu(θ,x) t_i}
Given a L invariant under global H transformations,
how do we make it invariant under global G transfms?

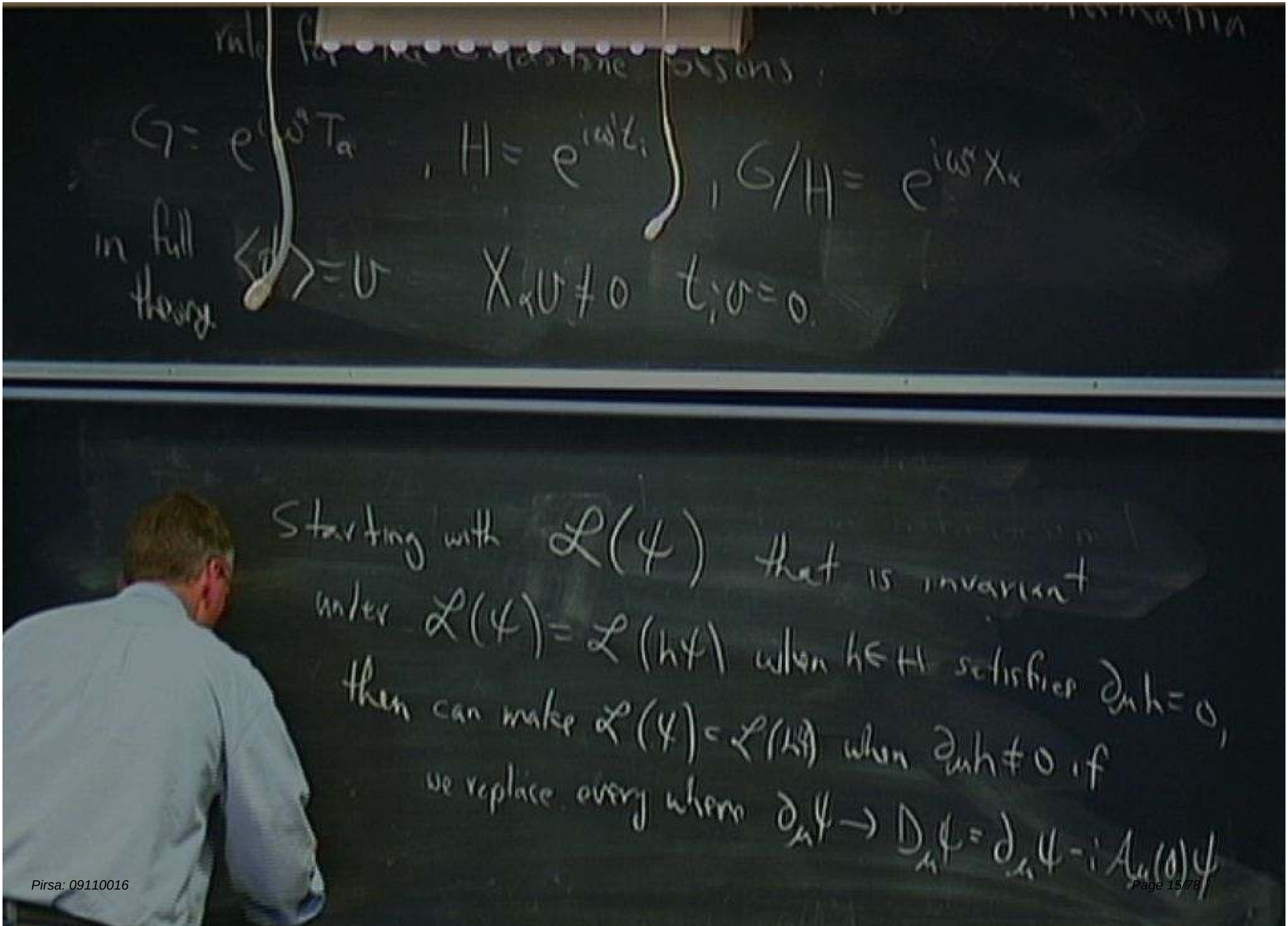
breaks to H, giving a Goldstone Bos
each generator of G/H (which are mas

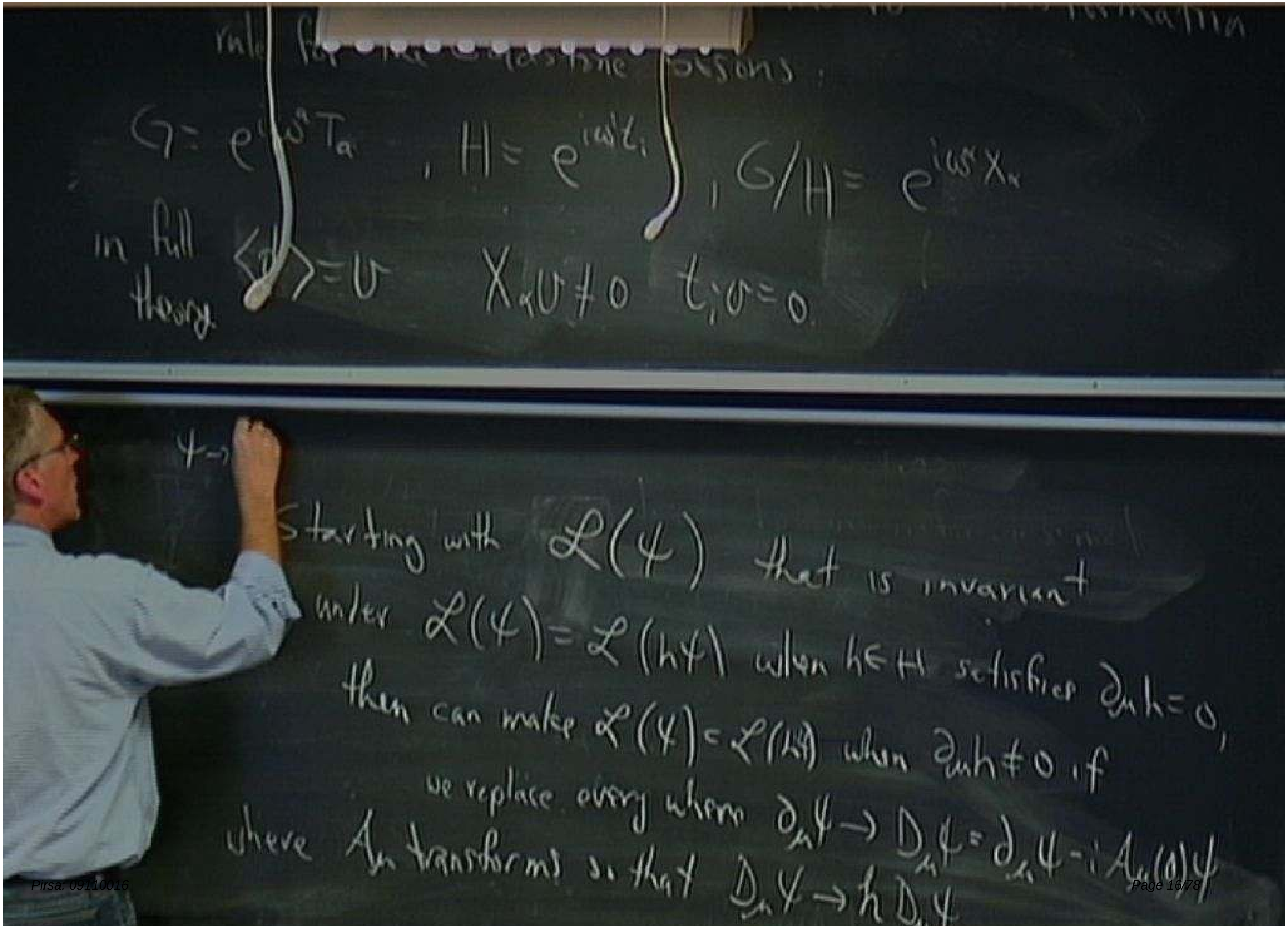
hep-th/9808176

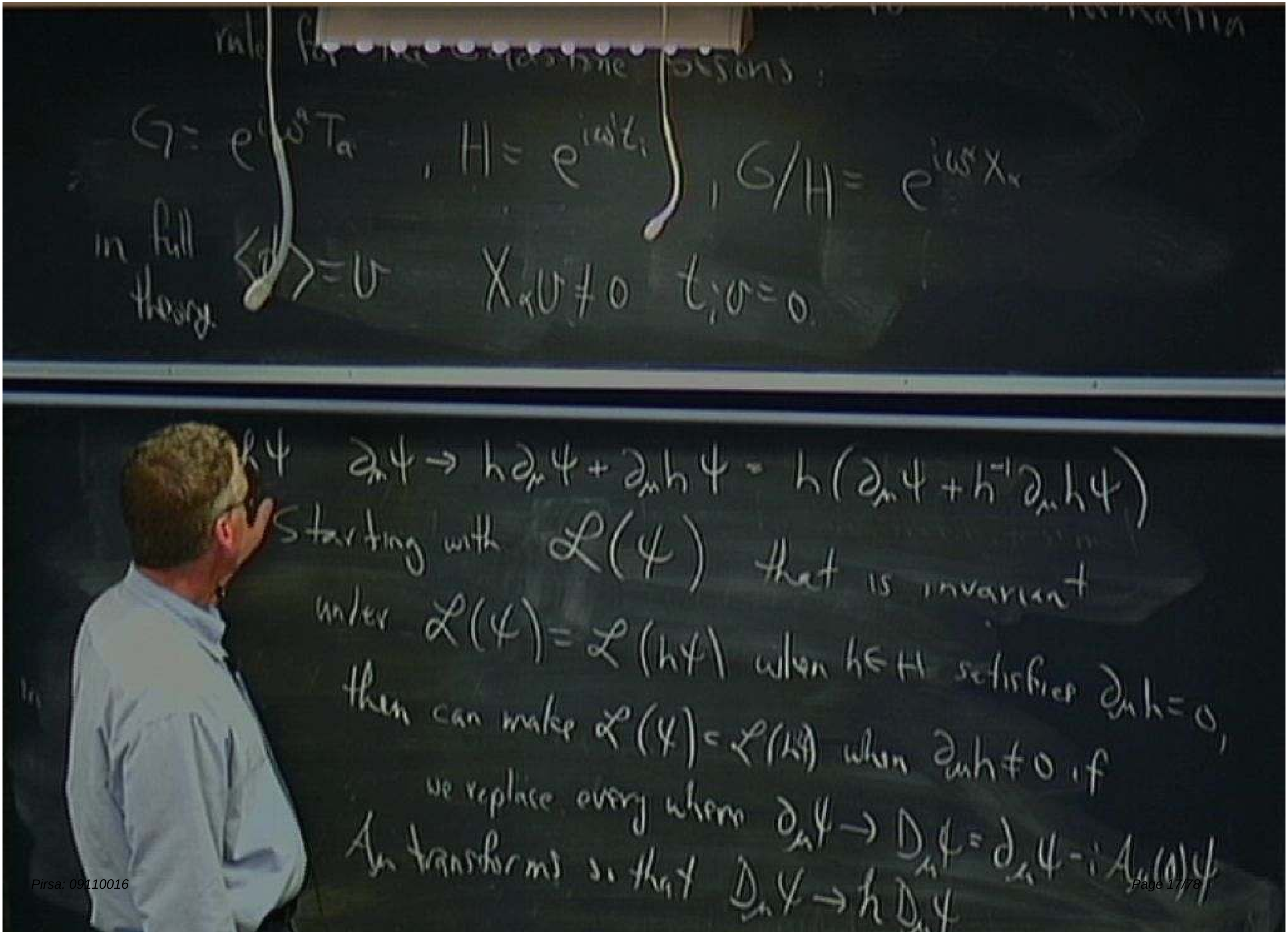
under G: $g\,U(\theta) = U(\theta')\,h$

$\psi \to h\psi$ where $h = e^{iu^i(\theta,g)\,t_i}$

Given a $\mathcal{L}$ invariant under global H transformations,
how do we make it invariant under global G transfor


$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$
in full theory $\langle\phi\rangle = v$ $X_\alpha v \neq 0$ $t_i v = 0$.
Starting with $\mathcal{L}(\psi)$ that is invariant
under $\mathcal{L}(\psi) = \mathcal{L}(h\psi)$ when $h \in H$ satisfies $\partial_\mu h = 0$,
then can make $\mathcal{L}(\psi) = \mathcal{L}(h\psi)$ when $\partial_\mu h \neq 0$ if
we replace every where $\partial_\mu \psi \to D_\mu \psi = \partial_\mu \psi - i A_\mu(\theta)\psi$


rule for ... Goldstone bosons
G = e^{iω^a T_a} , H = e^{iω^i t_i} , G/H = e^{iω^α X_α}
in full theory ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0
ψ →
Starting with L(ψ) that is invariant
under L(ψ) = L(hψ) when h ∈ H satisfies ∂_μ h = 0,
then can make L(ψ) = L(hψ) when ∂_μ h ≠ 0 if
we replace everywhere ∂_μ ψ → D_μ ψ = ∂_μ ψ − i A_μ(θ) ψ
where A_μ transforms so that D_μ ψ → h D_μ ψ



rule for ... bosons
G = e^{iω^a T_a} , H = e^{iω^i t_i} , G/H = e^{iω^α X_α}
in full theory ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0
∂_μψ → h∂_μψ + ∂_μh ψ = h(∂_μψ + h^{-1}∂_μh ψ)
Starting with ℒ(ψ) that is invariant
under ℒ(ψ) = ℒ(hψ) when h∈H satisfies ∂_μh = 0,
then can make ℒ(ψ) = ℒ(hψ) when ∂_μh ≠ 0 if
we replace everywhere ∂_μψ → D_μψ = ∂_μψ − iA_μ(x)ψ
A_μ transforms so that D_μψ → h D_μψ

... formation

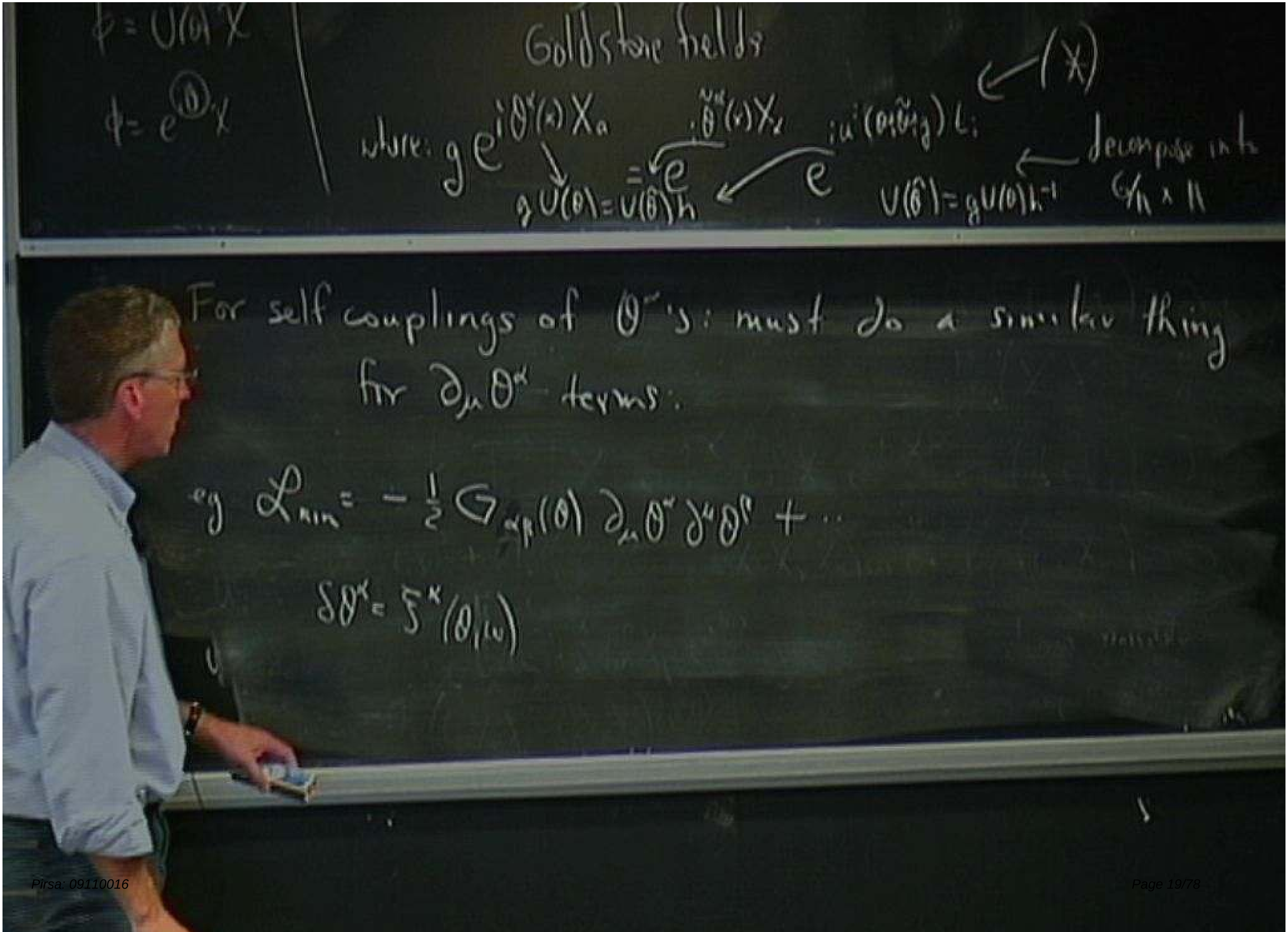
rule for the Goldstone bosons

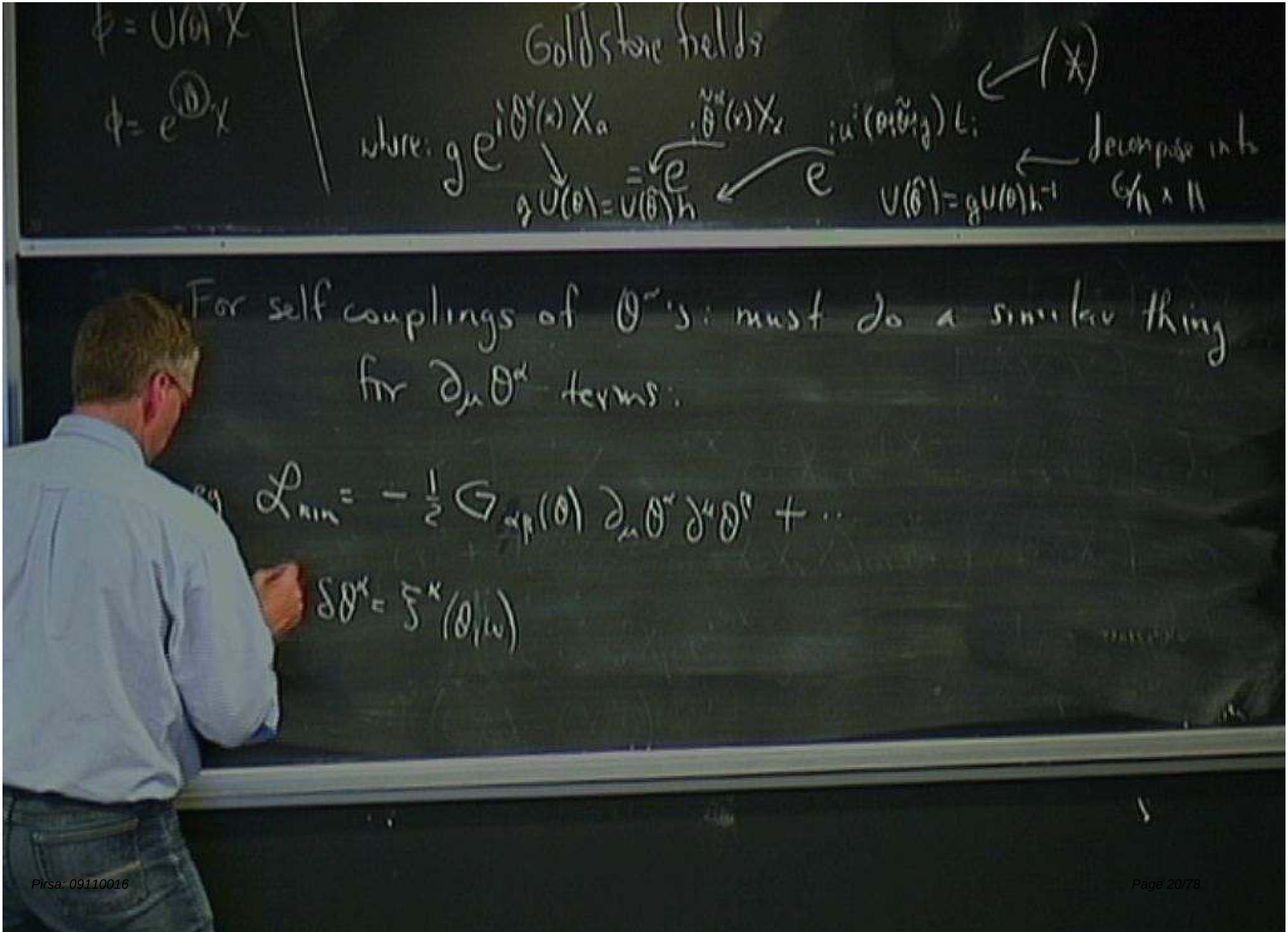
$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$

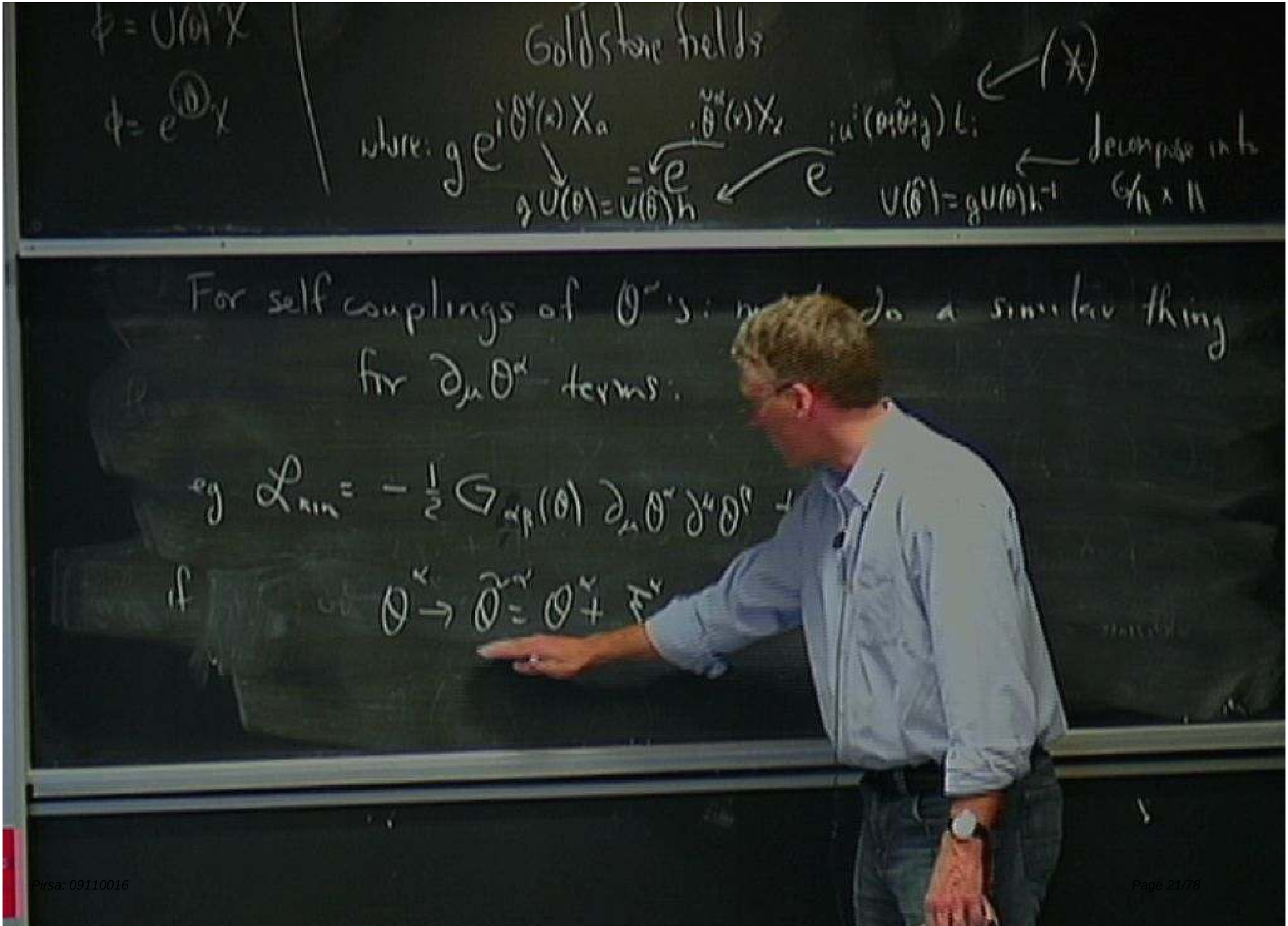
in full theory $\langle\phi\rangle = v$ $\quad X_\alpha v \neq 0$ $\quad t_i v = 0$.

$\psi \to h\psi \quad \partial_\mu\psi \to h\partial_\mu\psi + \partial_\mu h\,\psi = h(\partial_\mu\psi + h^{-1}\partial_\mu h\,\psi)$

Starting with $\mathcal{L}(\psi)$ that is invariant under $\mathcal{L}(\psi) = \mathcal{L}(h\psi)$ when $h \in H$ satisfies $\partial_\mu h = 0$, then can make $\mathcal{L}(\psi) = \mathcal{L}(h\psi)$ when $\partial_\mu h \neq 0$ if we replace everywhere $\partial_\mu\psi \to D_\mu\psi = \partial_\mu\psi - iA_\mu(x)\psi$ where A_μ transforms so that $D_\mu\psi \to h D_\mu\psi$


Goldstone fields
decompose into
For self couplings of θ's: must do a similar thing
for ∂μθα terms:
eg ℒkin = − ½ Gαβ(θ) ∂μθα ∂μθβ + ...
δθα = ξα(θ, ω)


Goldstone fields
where:
decompose into
G/H × H
For self couplings of θ^α's: must do a similar thing
for ∂_μ θ^α terms:
L_kin = − 1/2 G_αβ(θ) ∂_μ θ^α ∂^μ θ^β + ...
δθ^α = ξ^α(θ, ω)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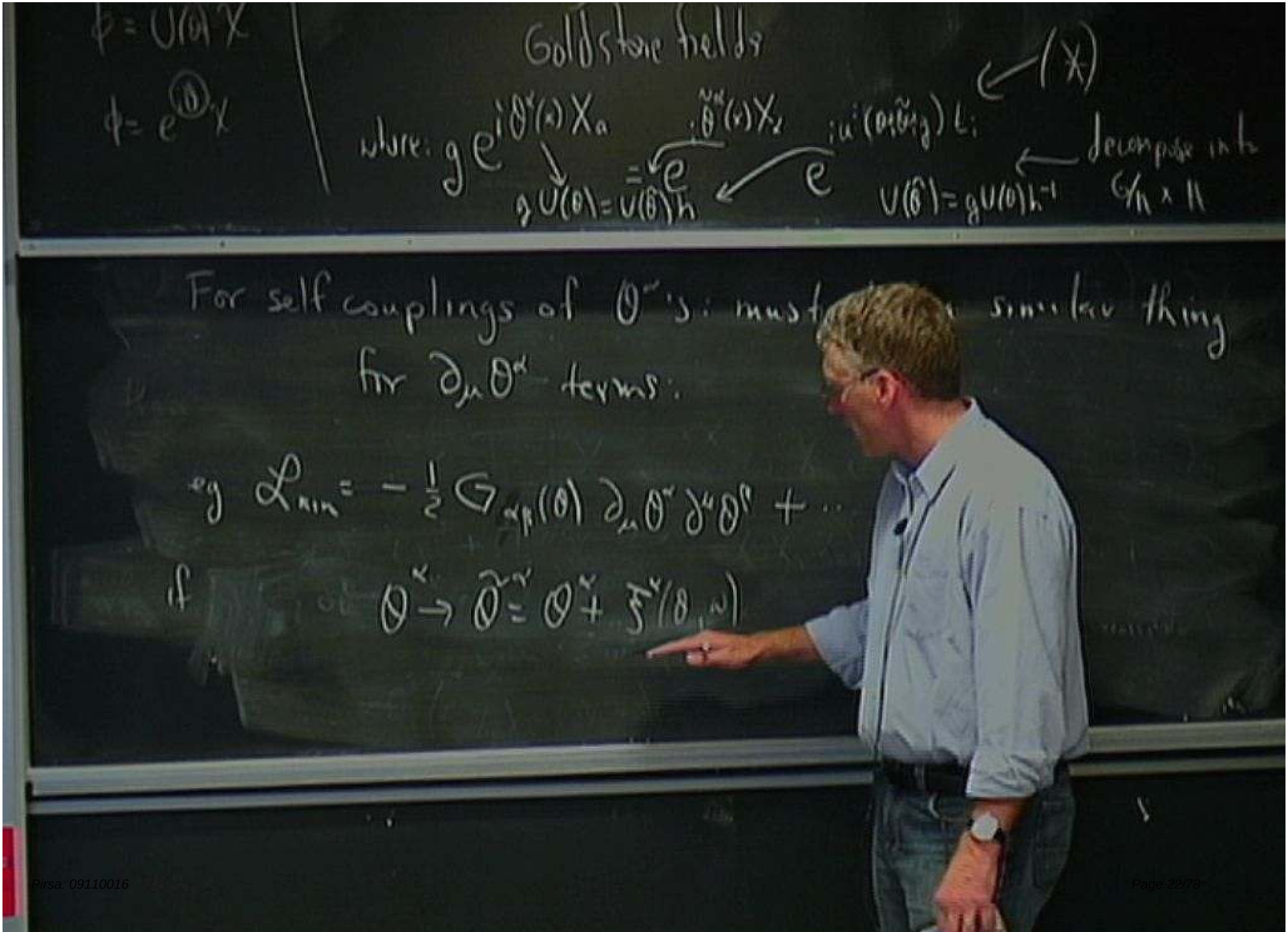

Goldstone fields
For self couplings of θ^α's: must ... similar thing
for ∂_μ θ^α terms:
eg ℒ_kin = − ½ G_αβ(θ) ∂_μ θ^α ∂^μ θ^β + ...
if θ^α → θ̃^α = θ^α + ξ^α(θ, ω)
decompose into G/H × H
V(θ̃) = g V(θ) h^{-1}

transformation rule: $\theta^\alpha(x) \to \tilde\theta^\alpha(x, \theta^\alpha(x), g)$ $g \in G$

↑ Goldstone fields

$\phi = U(\theta)\,\chi$

$\phi = e^{i\theta \cdot X}\chi$

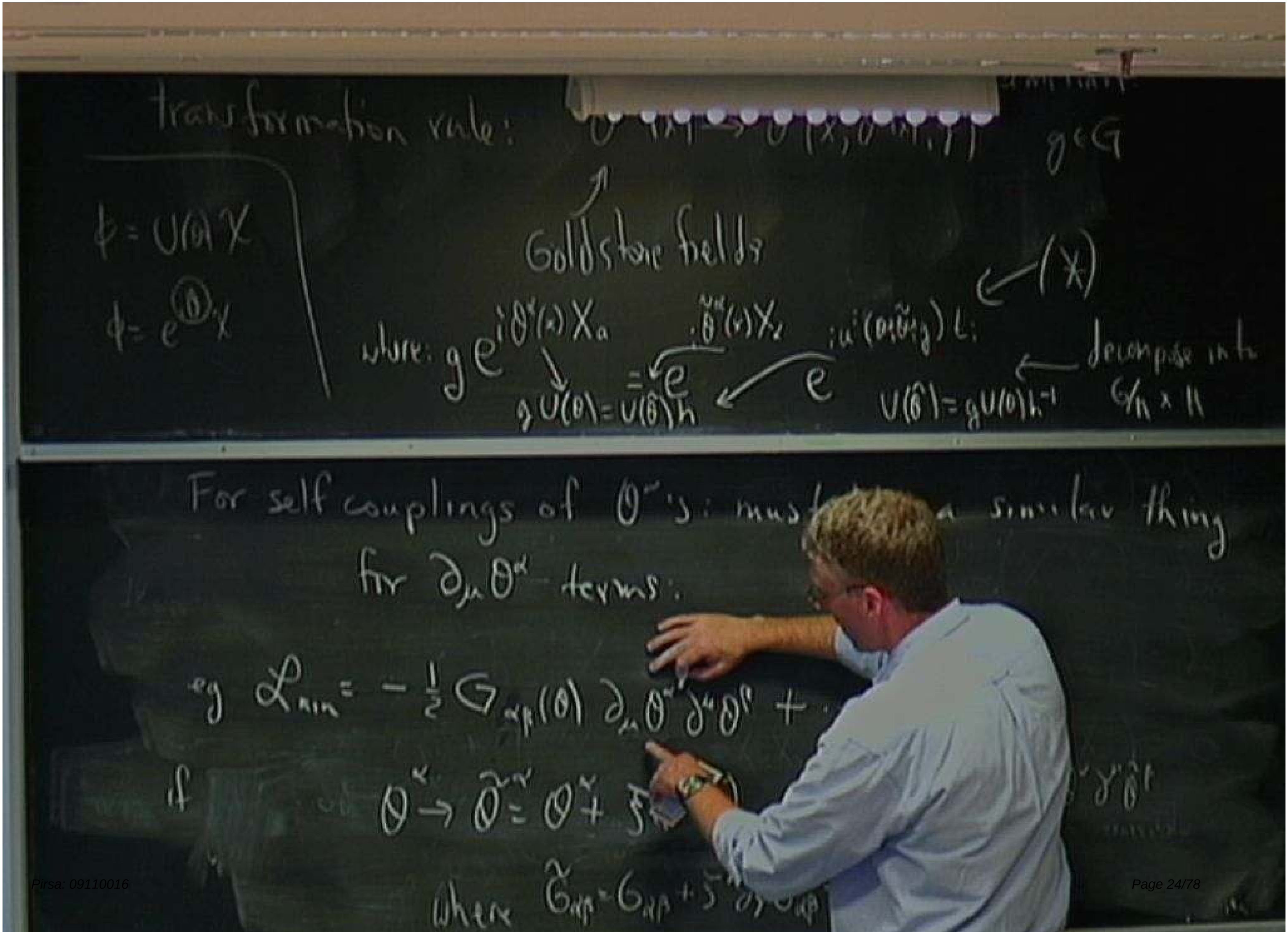
where: $g\, e^{i\theta^\alpha(x) X_a} = e^{i\tilde\theta^\alpha(x) X_a}\, e^{i u^i(\theta,\tilde\theta,g)\, t_i}$ ← decompose into $G/H \times H$ (*)

$g\,U(\theta) = U(\tilde\theta)\,h$ $U(\tilde\theta) = g\,U(\theta)\,h^{-1}$

For self couplings of θ^α's: must do a si[...] thing for $\partial_\mu \theta^\alpha$ terms:

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \dots$

if $\theta^\alpha \to \tilde\theta^\alpha = \theta^\alpha + \xi^\alpha(\theta, \omega)$ $\mathcal{L}_{kin} \to -\frac{1}{2}\tilde G \dots$


transformation rule:
g∈G
Goldstone fields
decompose into
For self couplings of θ^α's:
similar thing
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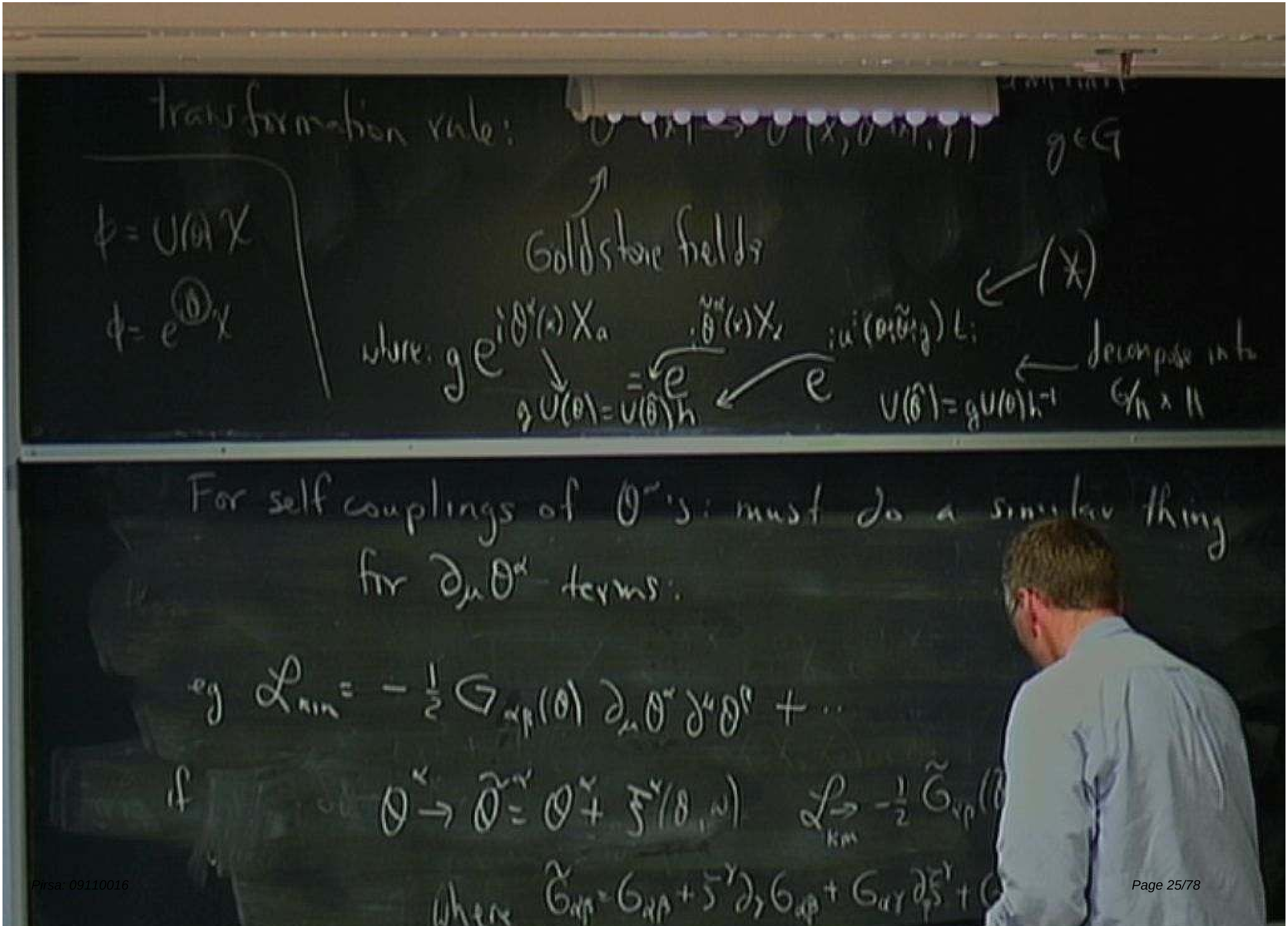
where


transformation rule:
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decompose into
G/H × H
For self couplings of θ's: must do a similar thing
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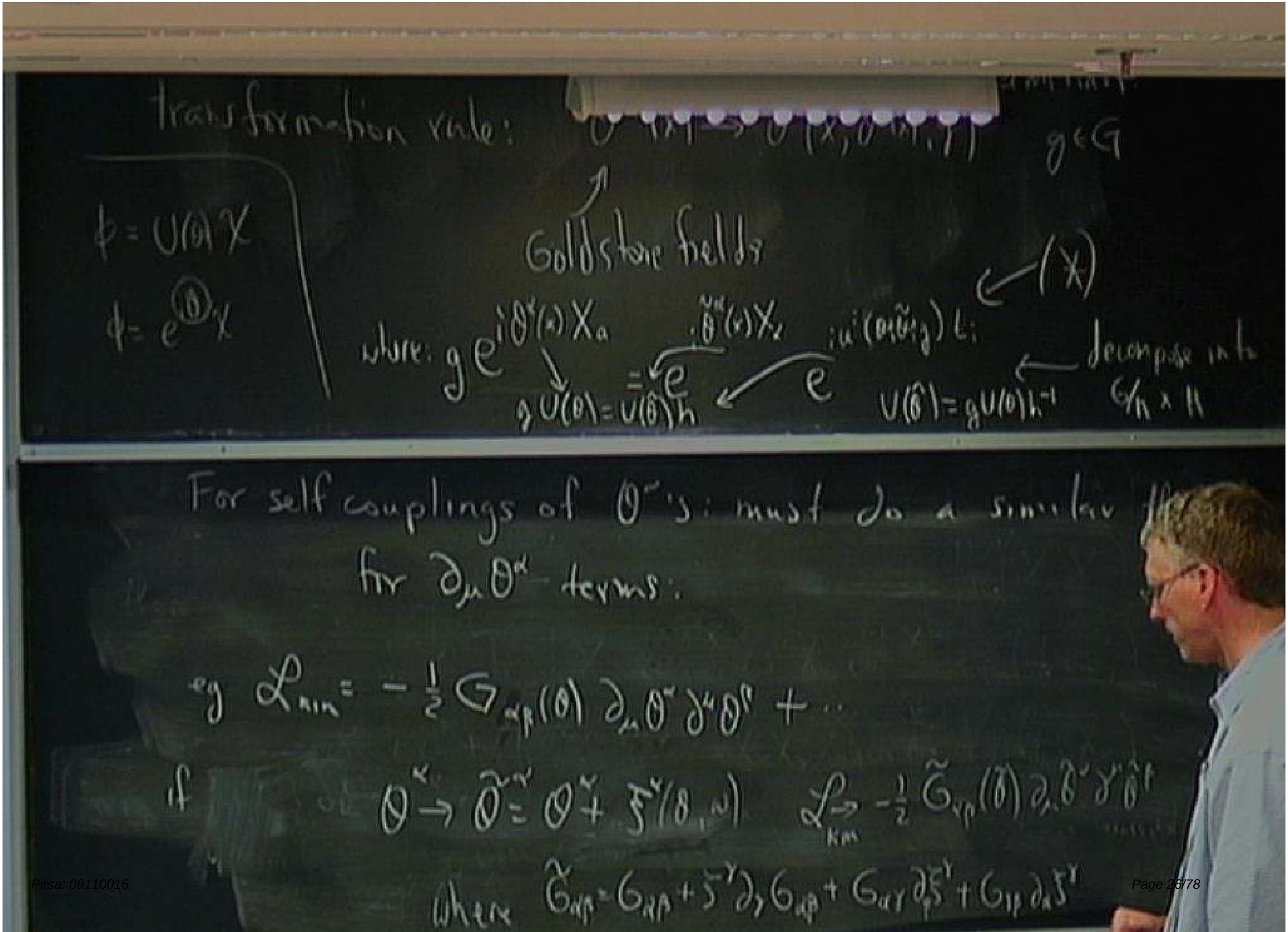
eg
if
where


transformation rule:
g∈G
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where
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For self couplings of θ's: must do a similar
for ∂μ θ^α terms:
eg ℒ_kin = −½ G_αβ(θ) ∂μθ^α ∂^μθ^β + ...
if
where

transformation rule: $\theta^\alpha(x) \to \tilde\theta^\alpha(x,\theta^\alpha(x),g)$ $\quad g\in G$

Goldstone fields

$\phi = U(\theta)\chi$

$\phi = e^{i\theta}\chi$

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\tilde\theta^\alpha(x) X_\alpha}\, e^{iu^i(\theta,\tilde\theta,g) t_i}$ $\leftarrow$ decompose into $G/H \times H$

$g\,U(\theta) = U(\tilde\theta)\,h$ $\qquad U(\tilde\theta) = g\,U(\theta)\,h^{-1}$

For self couplings of θ^α's: must do a similar ...

for $\partial_\mu \theta^\alpha$ terms:

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu\theta^\alpha \partial^\mu\theta^\beta + \ldots$

if $\theta^\alpha \to \tilde\theta^\alpha = \theta^\alpha + \xi^\alpha(\theta,\omega)$ $\quad \mathcal{L}_{kin} \to -\frac{1}{2}\tilde G_{\alpha\beta}(\theta)\,\partial_\mu\theta^\alpha \partial^\mu\theta^\beta$

where $\tilde G_{\alpha\beta} = G_{\alpha\beta} + \xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma}\partial_\beta \xi^\gamma + G_{\gamma\beta}\partial_\alpha \xi^\gamma$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta^\alpha(x), g)$ $\quad g \in G$

$\phi = U(\theta)\,\chi$

$\phi = e^{i\theta}\,\chi$

Goldstone fields

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\tilde{\theta}^\alpha(x) X_\alpha}\, e^{i u^i(\theta, g)\, t_i}$ ← decompose into $G/H \times H$

(X)

$g\, U(\theta) = U(\tilde{\theta})\, h$ $\qquad U(\tilde{\theta}) = g\, U(\theta)\, h^{-1}$

For self couplings of θ^α's: must do a similar thing for $\partial_\mu \theta^\alpha$ terms:

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \dots$

ST → X

$G_{\alpha\beta}$

if $\theta^\alpha \to \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha(\theta)$ $\qquad \mathcal{L}_{kin} \to -\frac{1}{2} \tilde{G}_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta$

where $\tilde{G}_{\alpha\beta} = G_{\alpha\beta} + \boxed{\xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma} \partial_\beta \xi^\gamma + G_{\gamma\beta} \partial_\alpha \xi^\gamma}$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta^\alpha(x), g)$ $\quad g \in G$

$\phi = U(\theta)\chi$

$\phi = e^{i\theta \cdot X}\chi$

Goldstone fields

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\tilde\theta^\alpha(x) X_\alpha}\, e^{i u^i(\theta,\tilde\theta,g)\, t_i}$ $\leftarrow$ decompose into $G/H \times H$

$g\, U(\theta) = U(\tilde\theta)\, h$ $\qquad U(\tilde\theta) = g\, U(\theta)\, h^{-1}$

For self couplings of θ^α's: must do a similar thing for $\partial_\mu \theta^\alpha$ terms:

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu\theta^\alpha \partial^\mu\theta^\beta + \dots$

ST $\to$ X

$G_{\alpha\beta}$ metric on X

if $\theta^\alpha \to \tilde\theta^\alpha = \theta^\alpha + \xi^\alpha(\theta)$ $\qquad \mathcal{L}_{kin} \to -\frac{1}{2} \tilde G_{\alpha\beta}(\theta)\, \partial_\mu\theta^\alpha \partial^\mu\theta^\beta$

where $\tilde G_{\alpha\beta} = G_{\alpha\beta} + \xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma}\partial_\beta \xi^\gamma + G_{\gamma\beta}\partial_\alpha \xi^\gamma$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta^\alpha(x), g)$ $g \in G$

Goldstone fields

$\phi = U(\theta)\chi$

$\phi = e^{i\theta}\chi$

where: $g\, e^{i\theta^a(x) X_a} = e^{i\tilde{\theta}^a(x) X_a}\, e^{i u^i(\theta,\tilde{\theta},g) t_i}$ ← decompose into $G/H \times H$

$g\, U(\theta) = U(\tilde{\theta})\, h$ $U(\tilde{\theta}) = g\, U(\theta)\, h^{-1}$

For self couplings of θ^α's: must do a similar thing for $\partial_\mu \theta^\alpha$ terms:

e.g. $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \dots$

if $\theta^\alpha \to \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha(\theta)$ $\mathcal{L}_{kin} \to -\frac{1}{2} \tilde{G}_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta$

where $\tilde{G}_{\alpha\beta} = G_{\alpha\beta} + \xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma}\partial_\beta \xi^\gamma + G_{\gamma\beta}\partial_\alpha \xi^\gamma$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta(x), g)$ $g \in G$

↑ Goldstone fields

$\phi = U(\theta)\chi$

$\phi = e^{i\theta \cdot X}\chi$

where: $g\, e^{i\theta^a(x) X_a} = e^{i\tilde{\theta}^a(x) X_a}\, e^{i u^i(\theta,\tilde{\theta},g)\, t_i}$ ← decompose into $G/H \times H$

$g\,U(\theta) = U(\tilde{\theta})\,h$ $U(\tilde{\theta}) = g\,U(\theta)\,h^{-1}$

(*)

For self couplings of θ^α's: must do a similar thing for $\partial_\mu \theta^\alpha$ terms:

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \ldots$

ST → X ; $G_{\alpha\beta}$ metric on X

if $\theta^\alpha \to \tilde{\theta}^\alpha = \theta^\alpha + \xi^\alpha(\theta)\,\gamma$ $\mathcal{L}_{kin} \to -\frac{1}{2}\tilde{G}_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta$

where $\tilde{G}_{\alpha\beta} = G_{\alpha\beta} + \left[\xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma}\partial_\beta \xi^\gamma + G_{\gamma\beta}\partial_\alpha \xi^\gamma\right]$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta(x), g)$ $\quad g \in G$

Goldstone fields

$\phi = U(\theta)\,\chi$

$\phi = e^{i\theta}\chi$

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\theta'^\alpha(x) X_\alpha}\, e^{i u^i(\theta, g) t_i}$ ← decompose into $G/H \times H$

$g\,U(\theta) = U(\theta')\,h$ $\qquad U(\theta') = g\,U(\theta)\,h^{-1}$

For self couplings of θ^α's: must do a similar thing for $\partial_\mu \theta^\alpha$ terms:

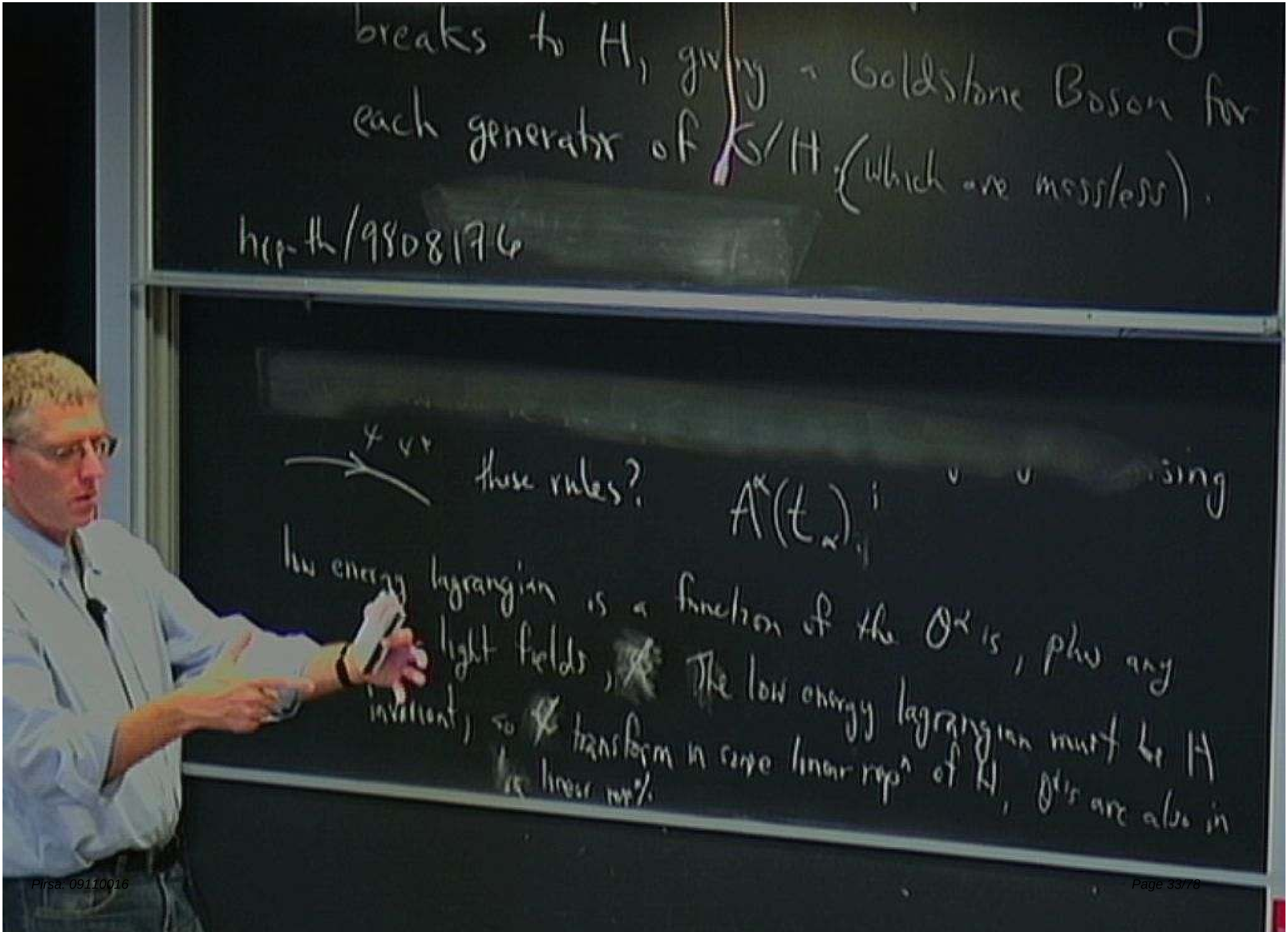
ST → X ← G/H

$G_{\alpha\beta}$ metric on X

eg $\mathcal{L}_{kin} = -\frac{1}{2} G_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta + \dots$

if $\theta^\alpha \to \theta'^\alpha = \theta^\alpha + \xi^\alpha(\theta)\,\gamma$ $\qquad \mathcal{L}_{kin} \to -\frac{1}{2} \tilde{G}_{\alpha\beta}(\theta)\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta$

where $\tilde{G}_{\alpha\beta} = G_{\alpha\beta} + \left[\xi^\gamma \partial_\gamma G_{\alpha\beta} + G_{\alpha\gamma} \partial_\beta \xi^\gamma + G_{\gamma\beta} \partial_\alpha \xi^\gamma\right]$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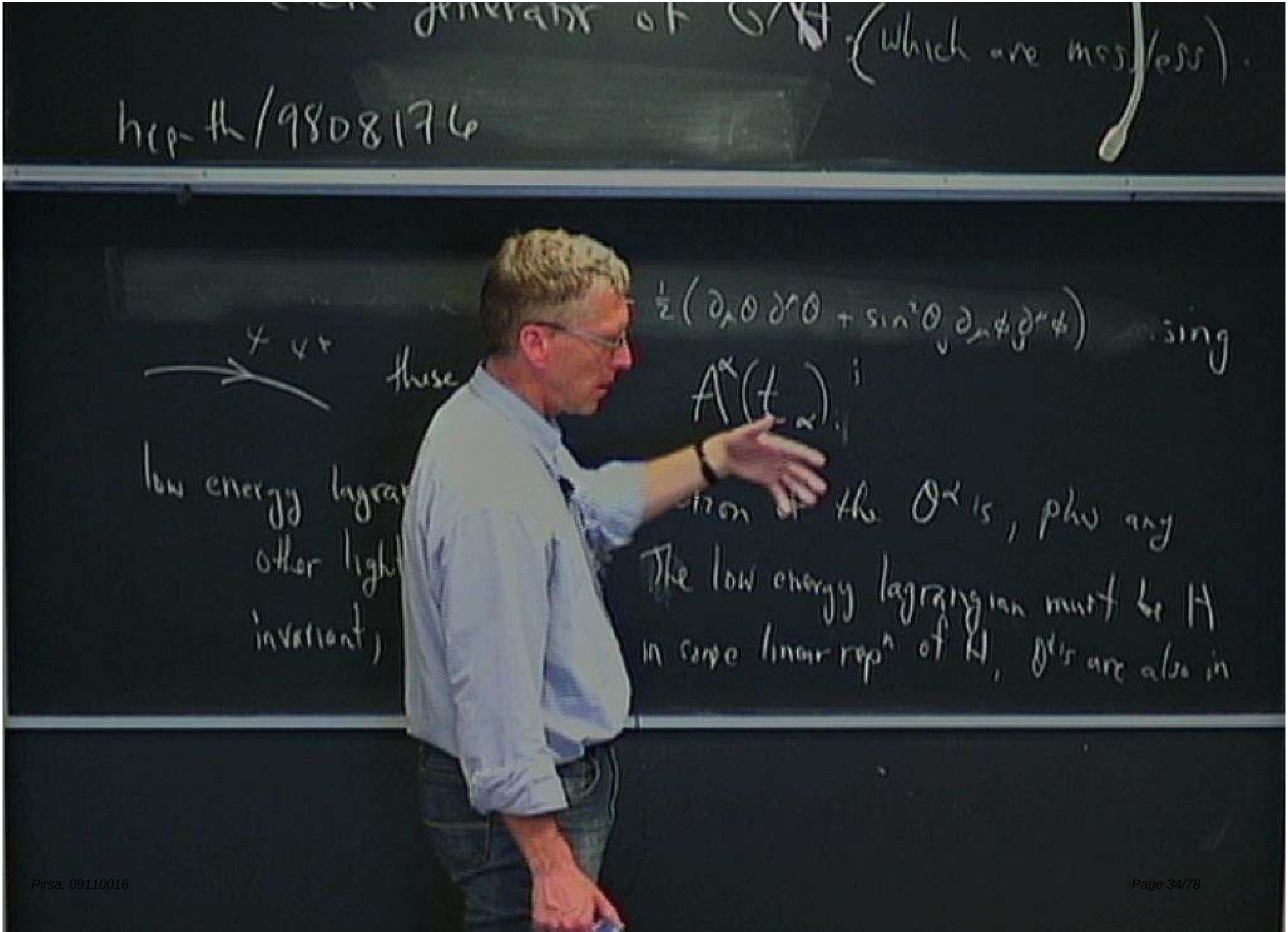
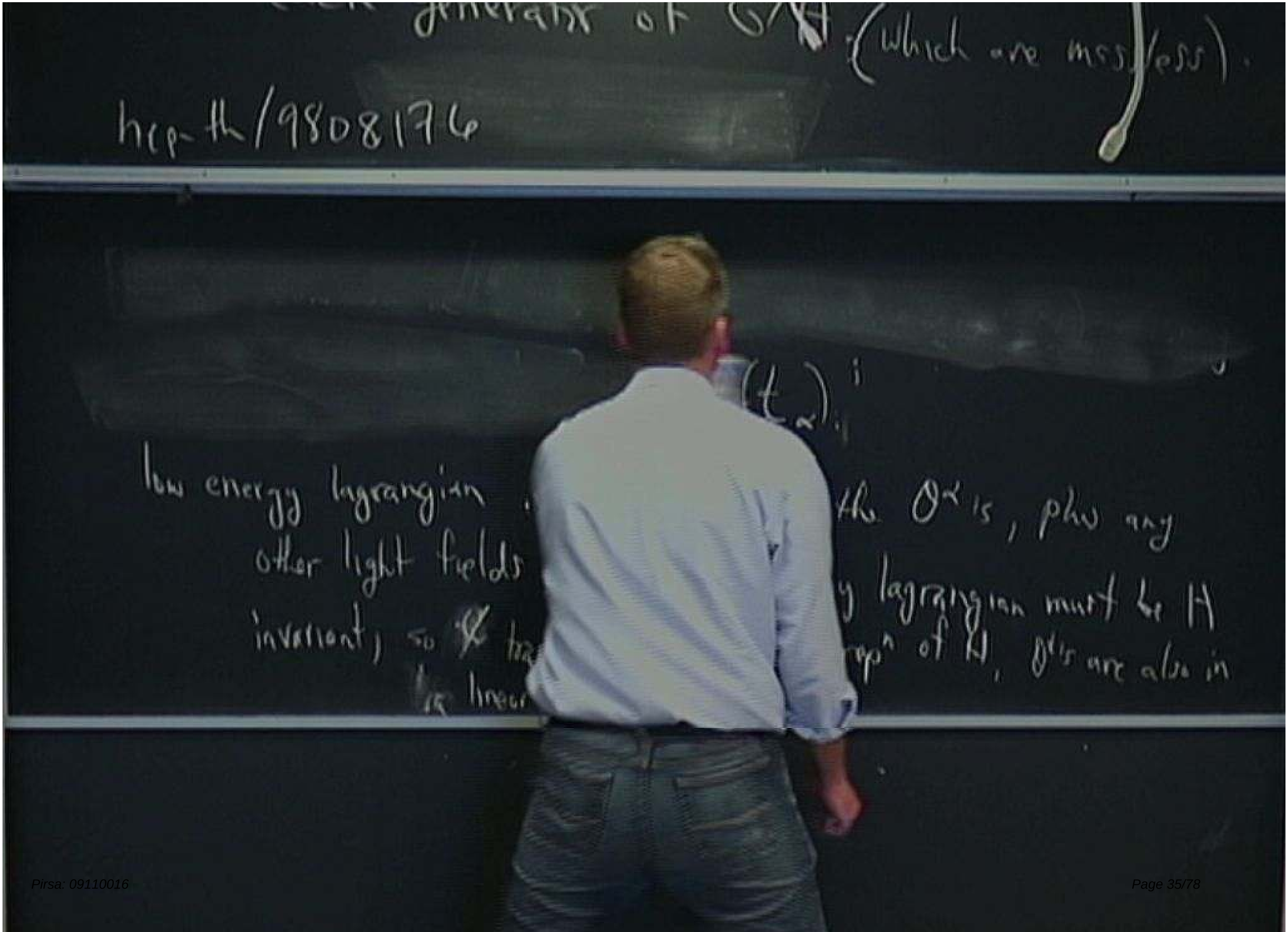
each generator of G/H (which are massless).
hep-th/9808176
these rules?
low energy lagrangian is a function of the


generators of G/H (which are massless).
hep-th/9808176
$\frac{1}{2}(\partial_\mu\theta\,\partial^\mu\theta + \sin^2\theta\,\partial_\mu\phi\,\partial^\mu\phi)$
these
$A^\alpha(t_\alpha)$
low energy lagra
the θ^α's, plus any
other light
The low energy lagrangian must be H
invariant,
in some linear rep^n of H, θ's are also in


generators of G/H (which are massless).
hep-th/9808176
low energy lagrangian
the θ^α's, plus any
other light fields
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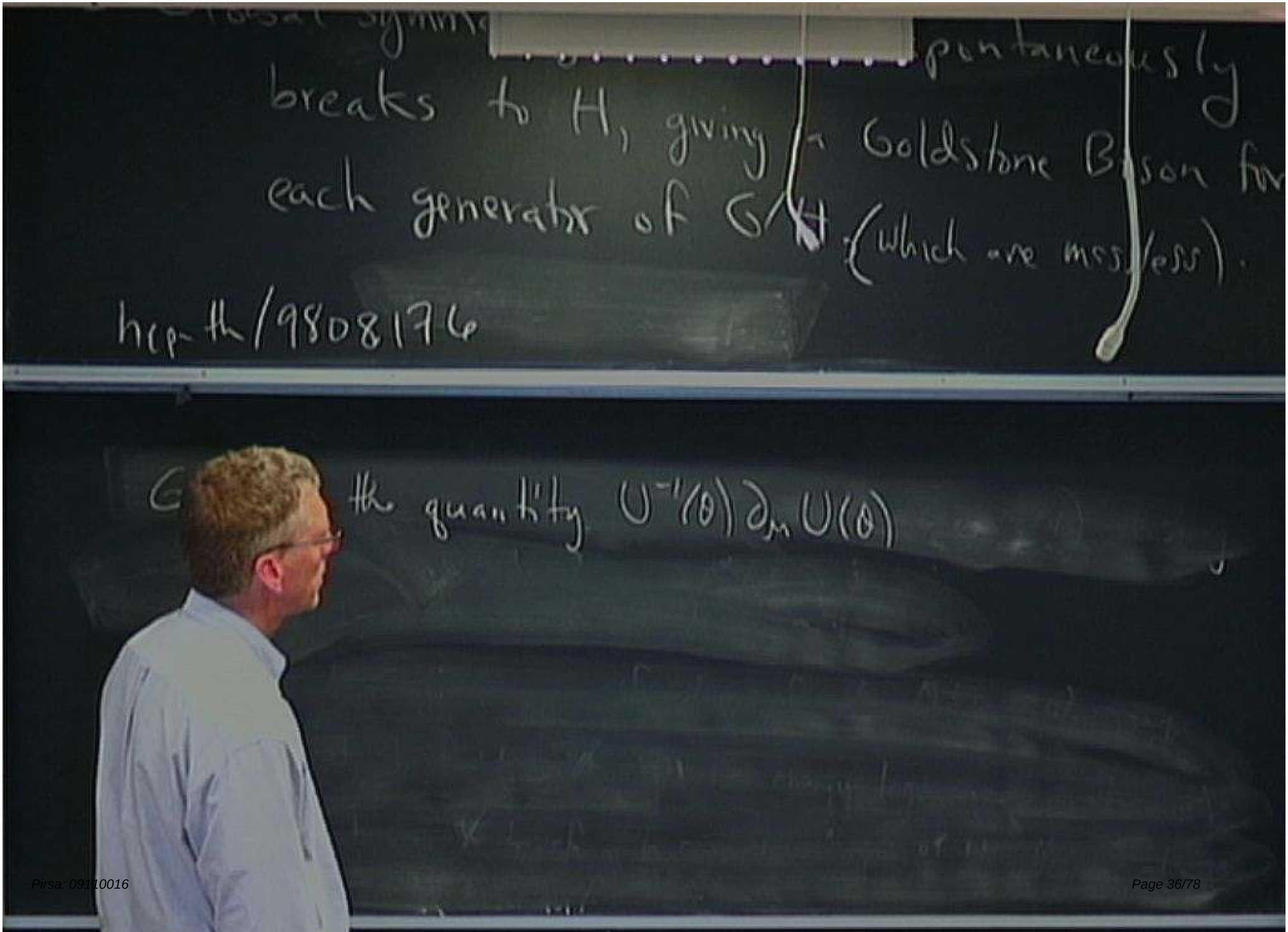
breaks to H, giving a Goldstone Boson for
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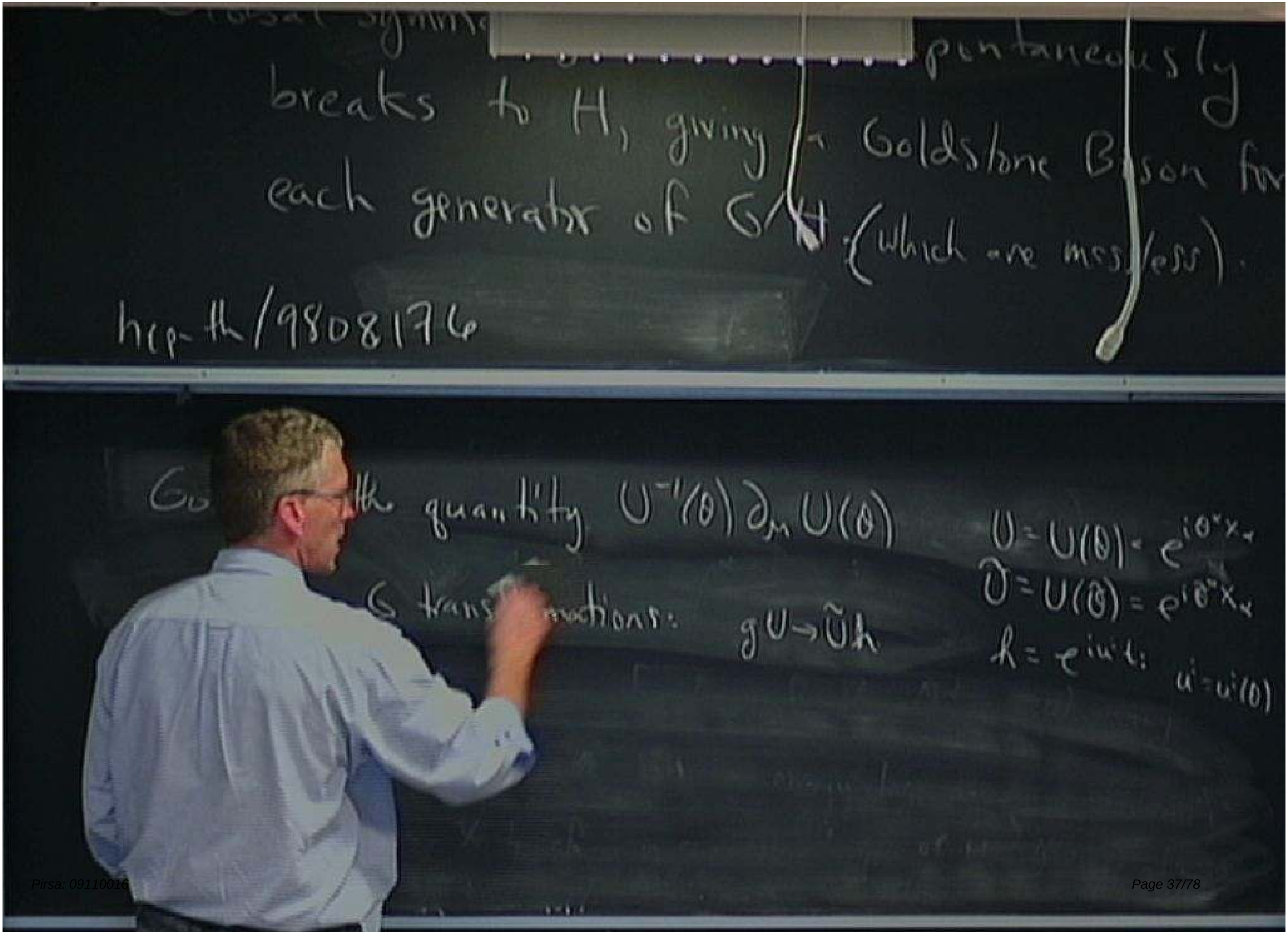
hep-th/9808176
the quantity U^-1(θ) ∂μ U(θ)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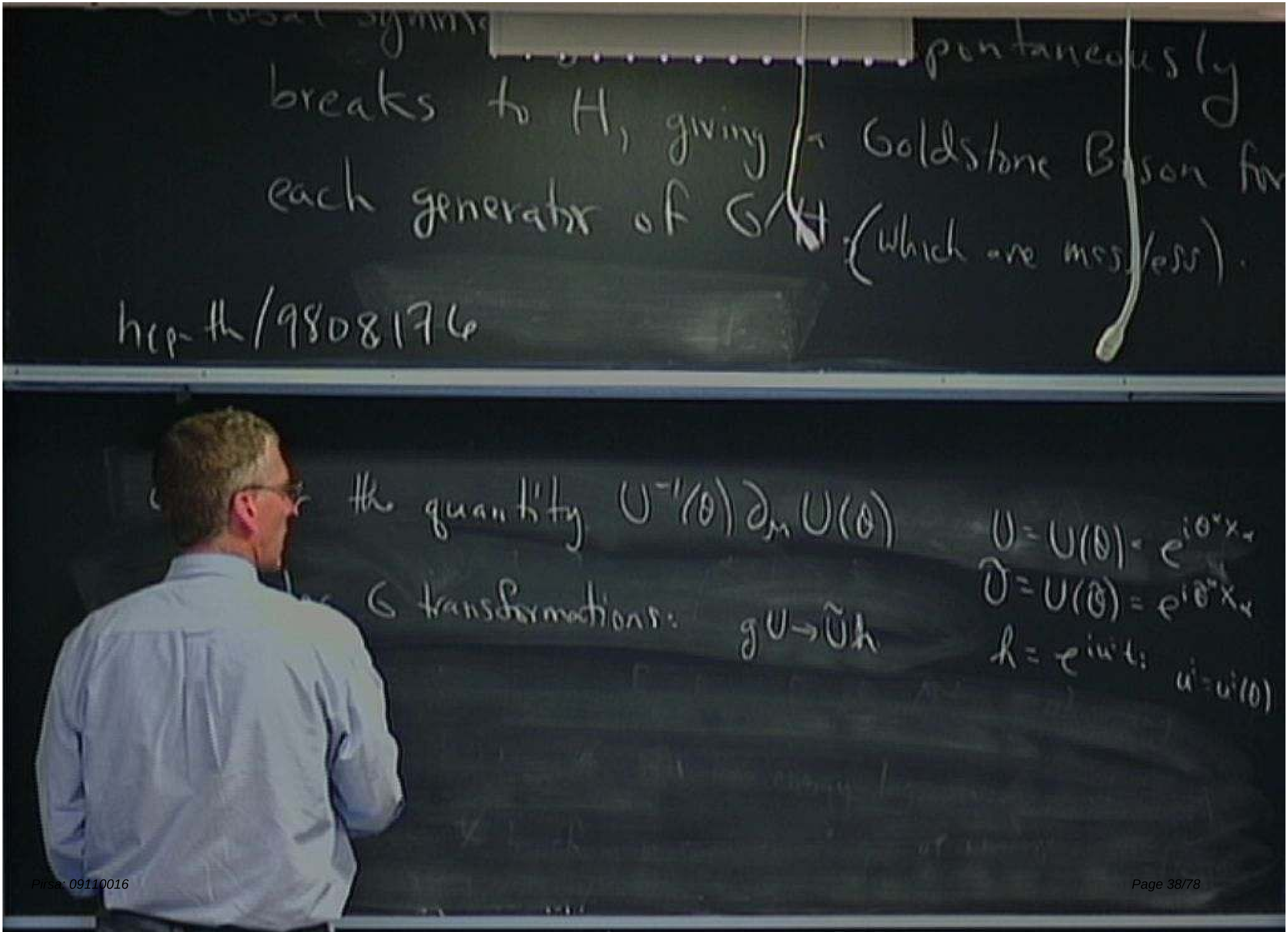
quantity $U^{-1}(\theta)\,\partial_\mu U(\theta)$
G transformations: $gU \to \tilde{U}h$
$U = U(\theta) = e^{i\theta^\alpha X_\alpha}$
$\tilde{U} = U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$
$h = e^{iu^i t_i}$
$u^i = u^i(\theta)$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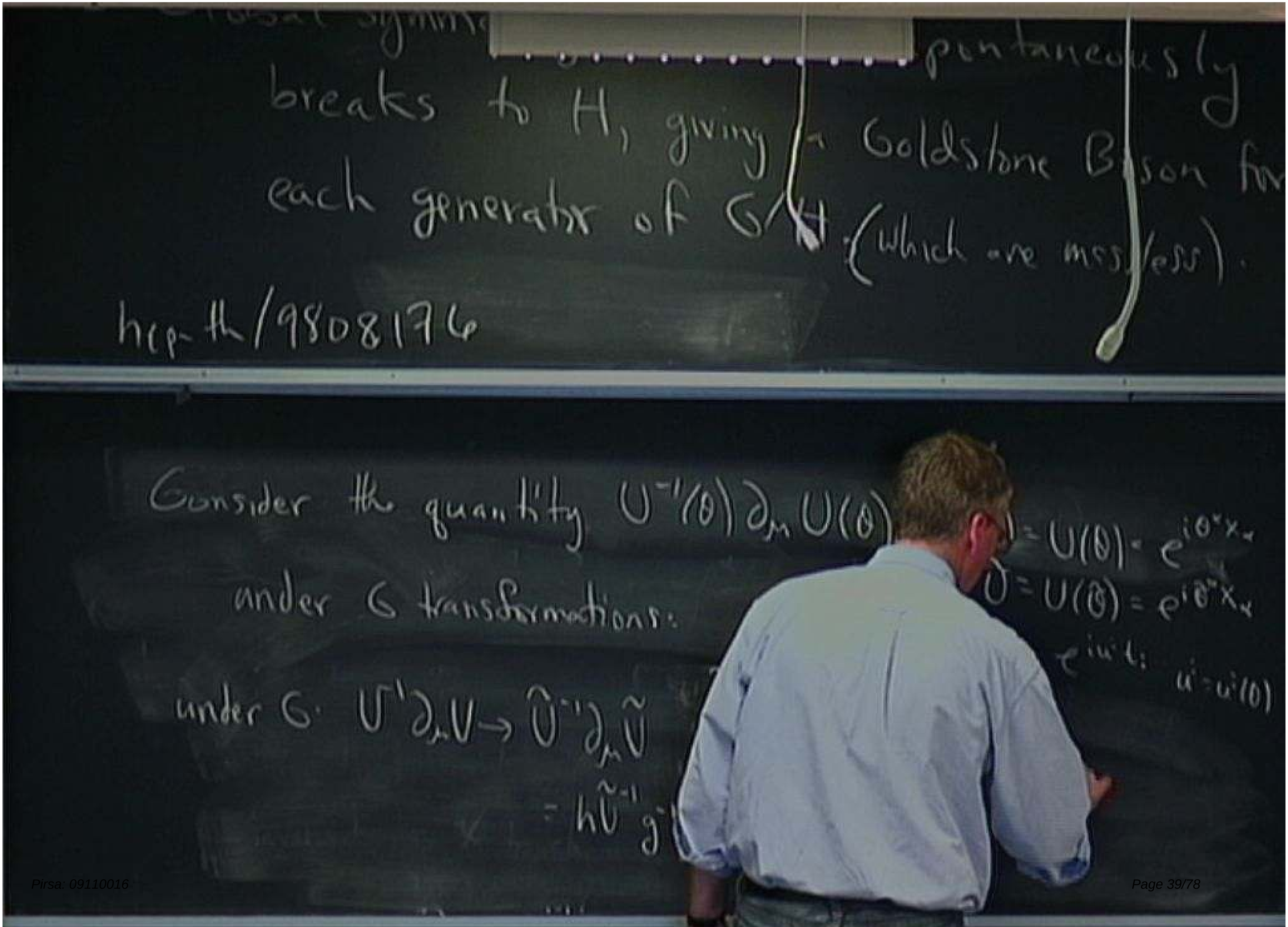
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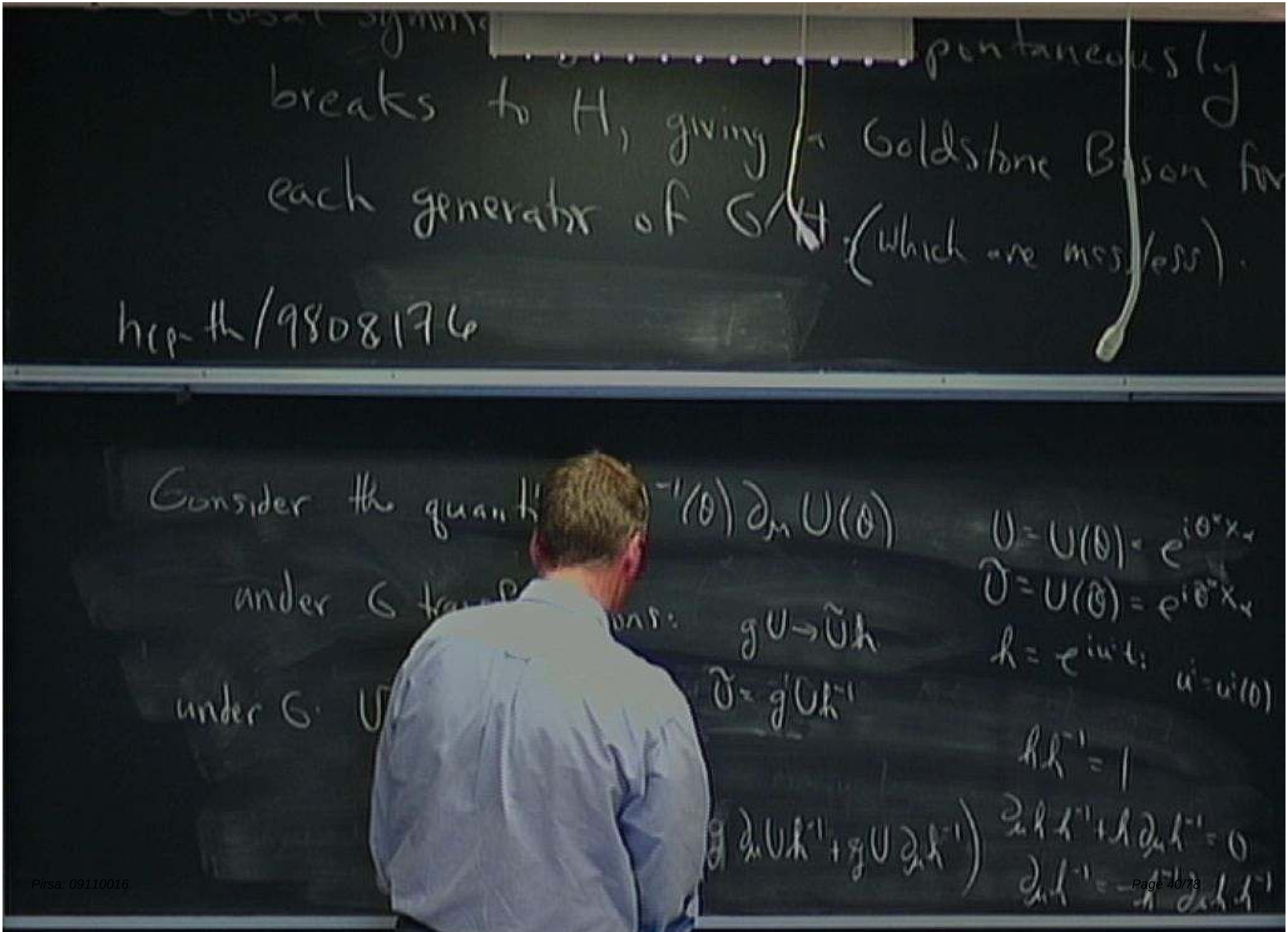
each generator of G/H (which are massless).
hep-th/9808176
Consider the quantity $U^{-1}(\theta)\partial_\mu U(\theta)$
$= U(\theta) = e^{i\theta^\alpha X_\alpha}$
under G transformations:
$\tilde{U} = U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$
$e^{iu^i t_i}$
$u^i = u^i(\theta)$
under G: $U^{-1}\partial_\mu U \to \tilde{U}^{-1}\partial_\mu \tilde{U}$
$= h\tilde{U}^{-1} g^{-1}$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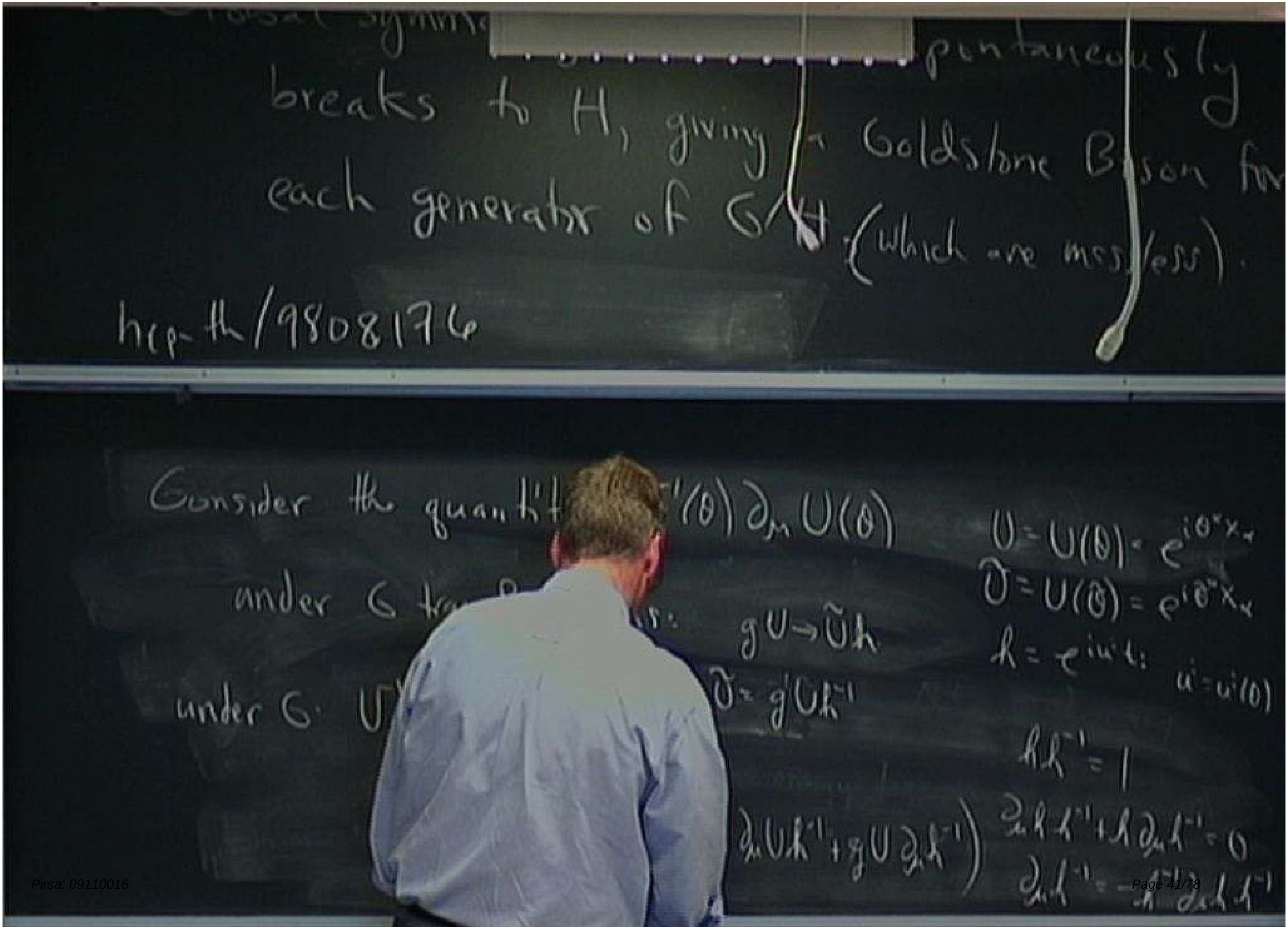
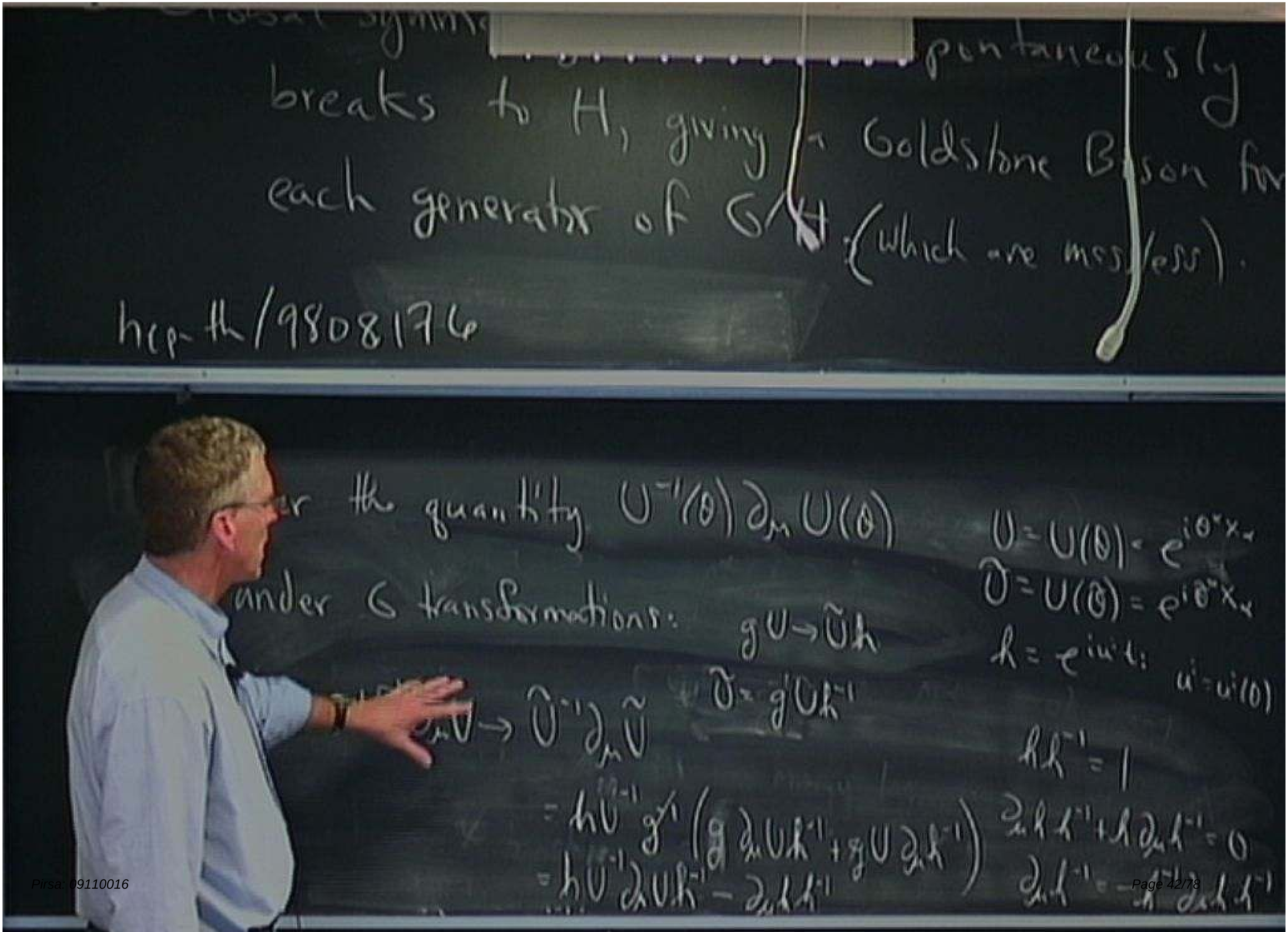
under G transformations:
under G



breaks to H, giving a Goldstone Boson for
each generator of G/H (which are massless).
hep-th/9808176
Consider the quantit
$\partial_\mu U(\theta)$
under G
$gU \to \tilde{U}h$
under G
$\tilde{U} = gUh^{-1}$
$U = U(\theta) = e^{i\theta^\alpha X_\alpha}$
$\tilde{U} = U(\tilde{\theta}) = e^{i\tilde{\theta}^\alpha X_\alpha}$
$hh^{-1} = 1$
$\partial_\mu h h^{-1} + h \partial_\mu h^{-1} = 0$



pontaneously
breaks to H, giving a Goldstone Boson for
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hep-th/9808176
the quantity U⁻¹(θ) ∂μ U(θ)
under G transformations:
gU → Ũh
U = U(θ) = e^{iθ^a X_a}
Ũ = U(θ̃) = e^{iθ̃^a X_a}
h = e^{iu^i t_i}
u^i = u^i(θ)
Ũ = gUh⁻¹
hh⁻¹ = 1

Global symmetry ... spontaneously breaks to H, giving a Goldstone Boson for each generator of G/H (which are massless).

hep-th/9808176

Consider the quantity $U^{-1}(\theta)\partial_\mu U(\theta)$

under G transformations: $gU \to \tilde{U}h$

$U = U(\theta) = e^{i\theta^a X_a}$

$\tilde{U} = U(\tilde{\theta}) = e^{i\tilde{\theta}^a X_a}$

$h = e^{iu^i t_i}$ $\quad u^i = u^i(\theta)$

$\tilde{U} = gUh^{-1}$

under G: $U^{-1}\partial_\mu U \to \tilde{U}^{-1}\partial_\mu \tilde{U}$

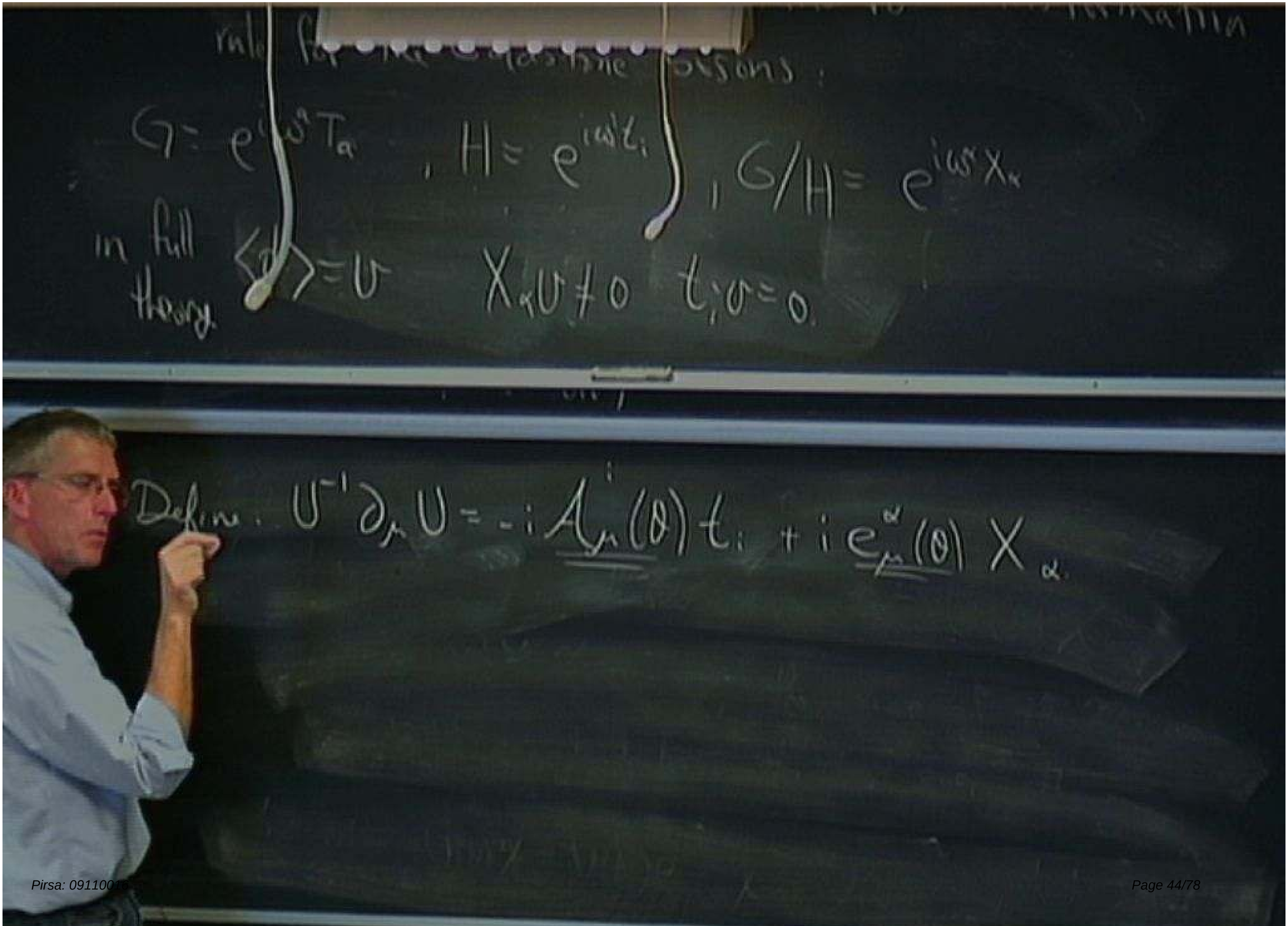
$hh^{-1} = 1$

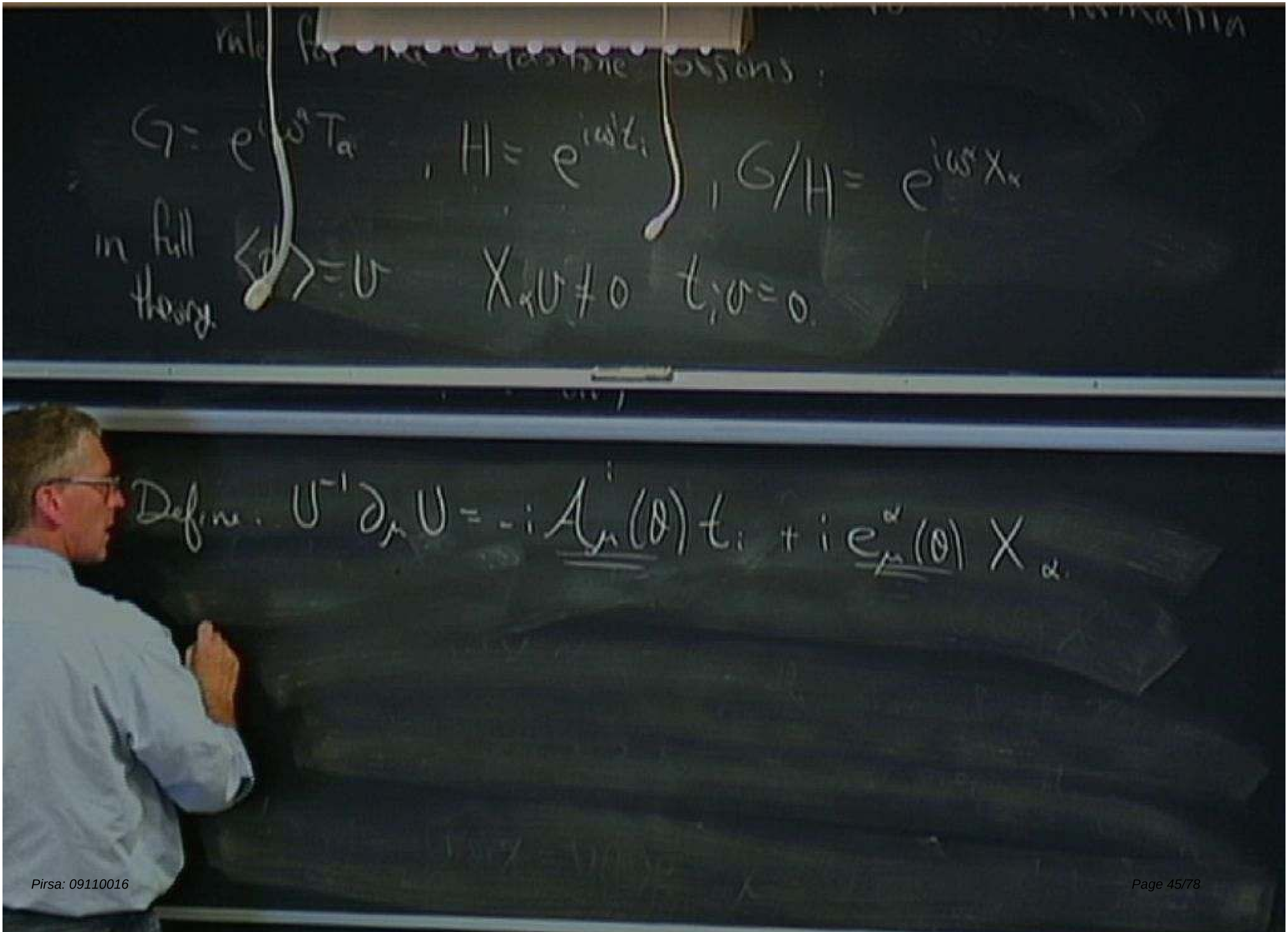
$= hU^{-1}g^{-1}(g\,\partial_\mu U h^{-1} + gU\,\partial_\mu h^{-1})$

$\partial_\mu h\, h^{-1} + h\,\partial_\mu h^{-1} = 0$

$= hU^{-1}\partial_\mu U h^{-1} - \partial_\mu h\, h^{-1}$

$\partial_\mu h^{-1} = -h^{-1}\partial_\mu h\, h^{-1}$


$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$
in full theory $\langle\phi\rangle = v$ $X_\alpha v \neq 0$ $t_i v = 0$.
Define: $U^{-1}\partial_\mu U = -i A_\mu^i(\theta) t_i + i e_\mu^\alpha(\theta) X_\alpha$


bosons :
$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$
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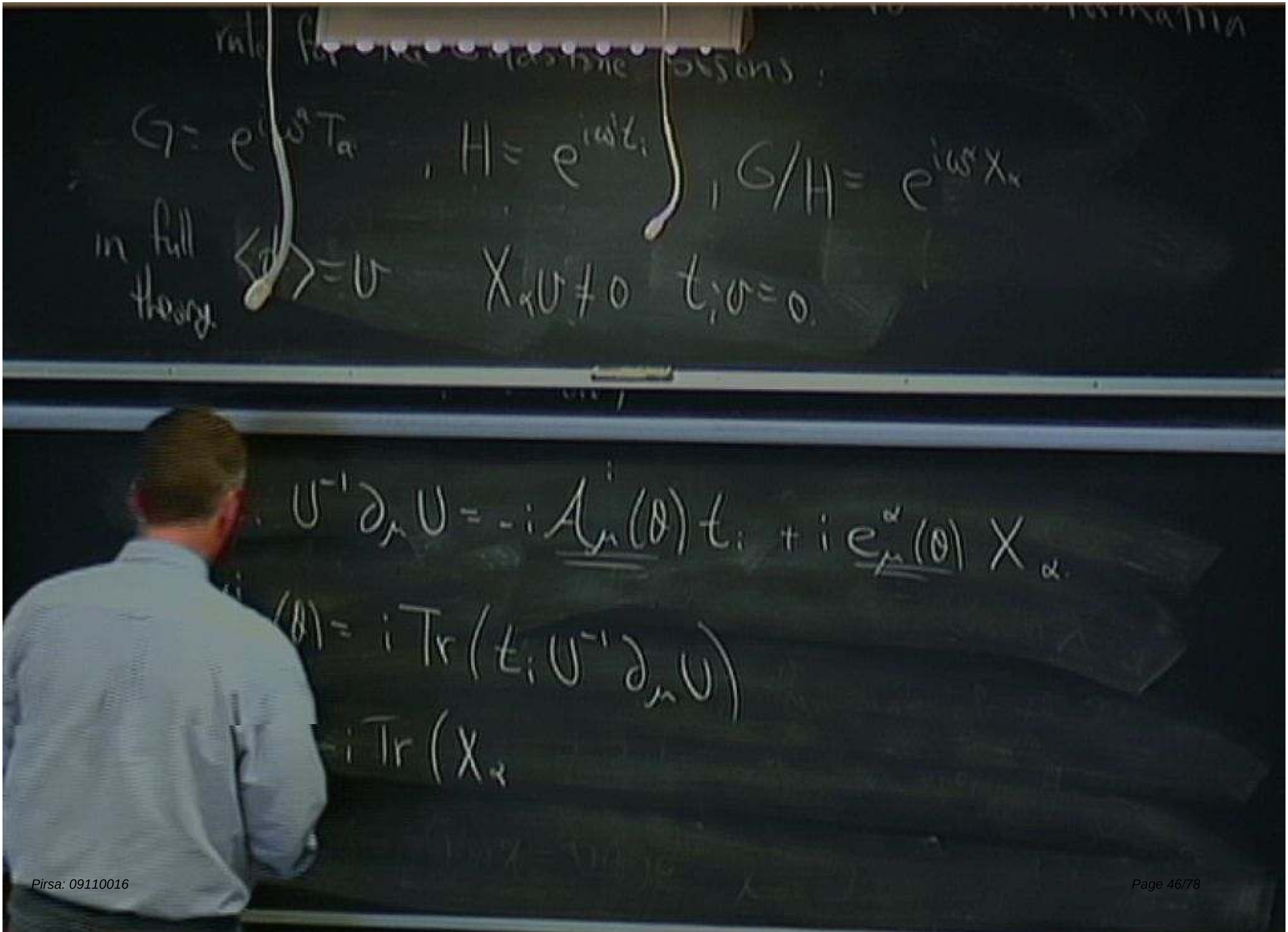
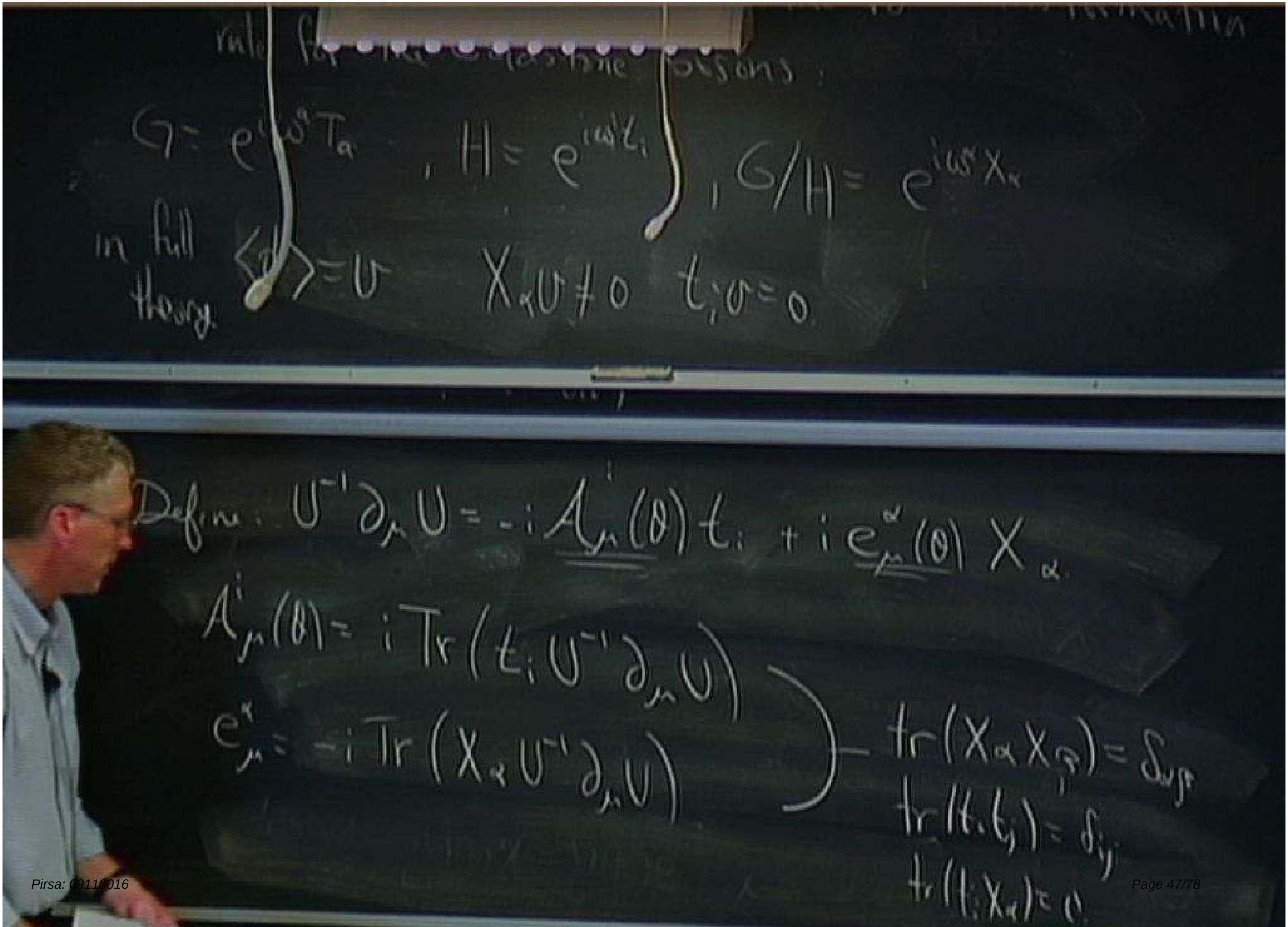
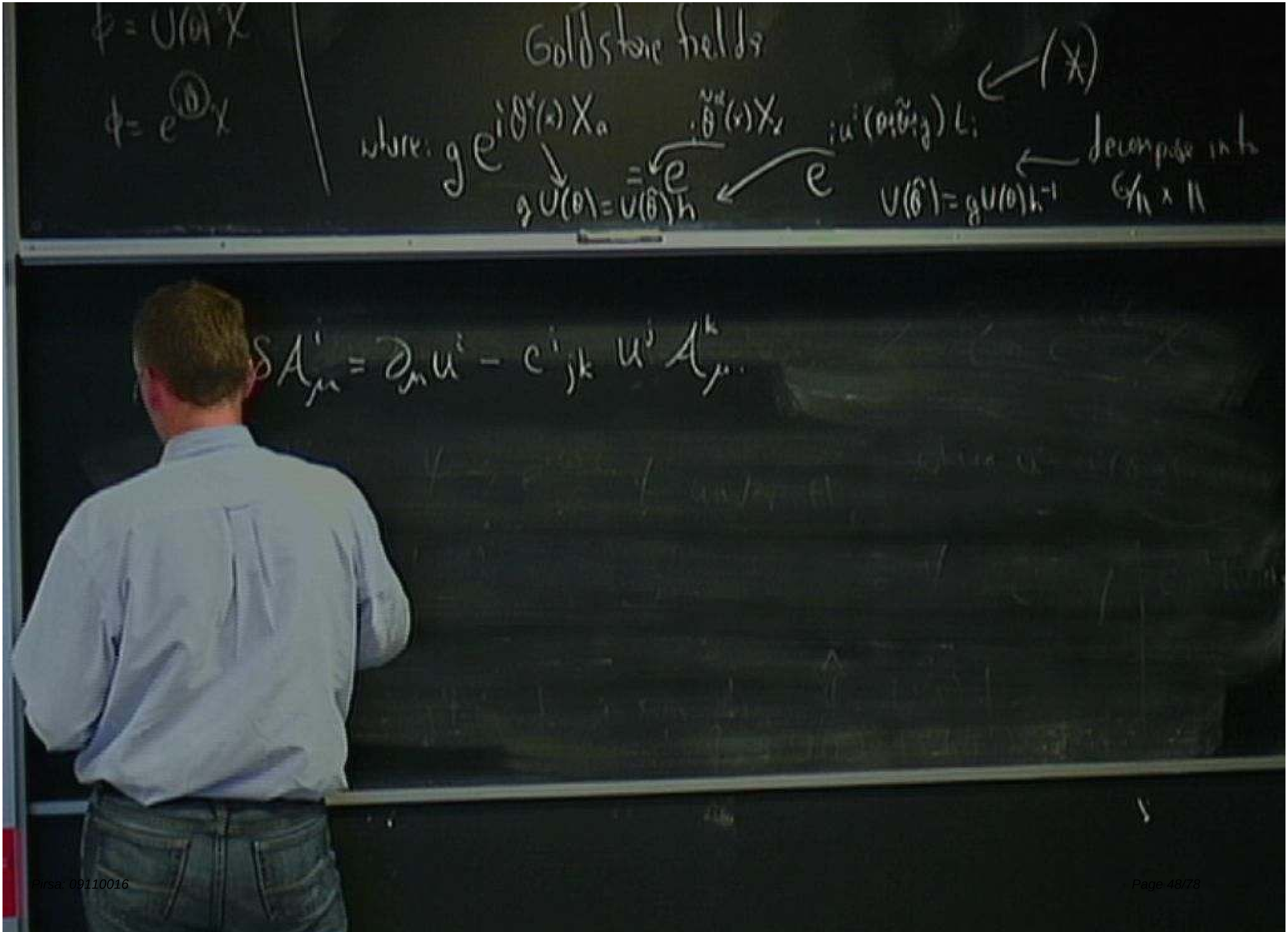
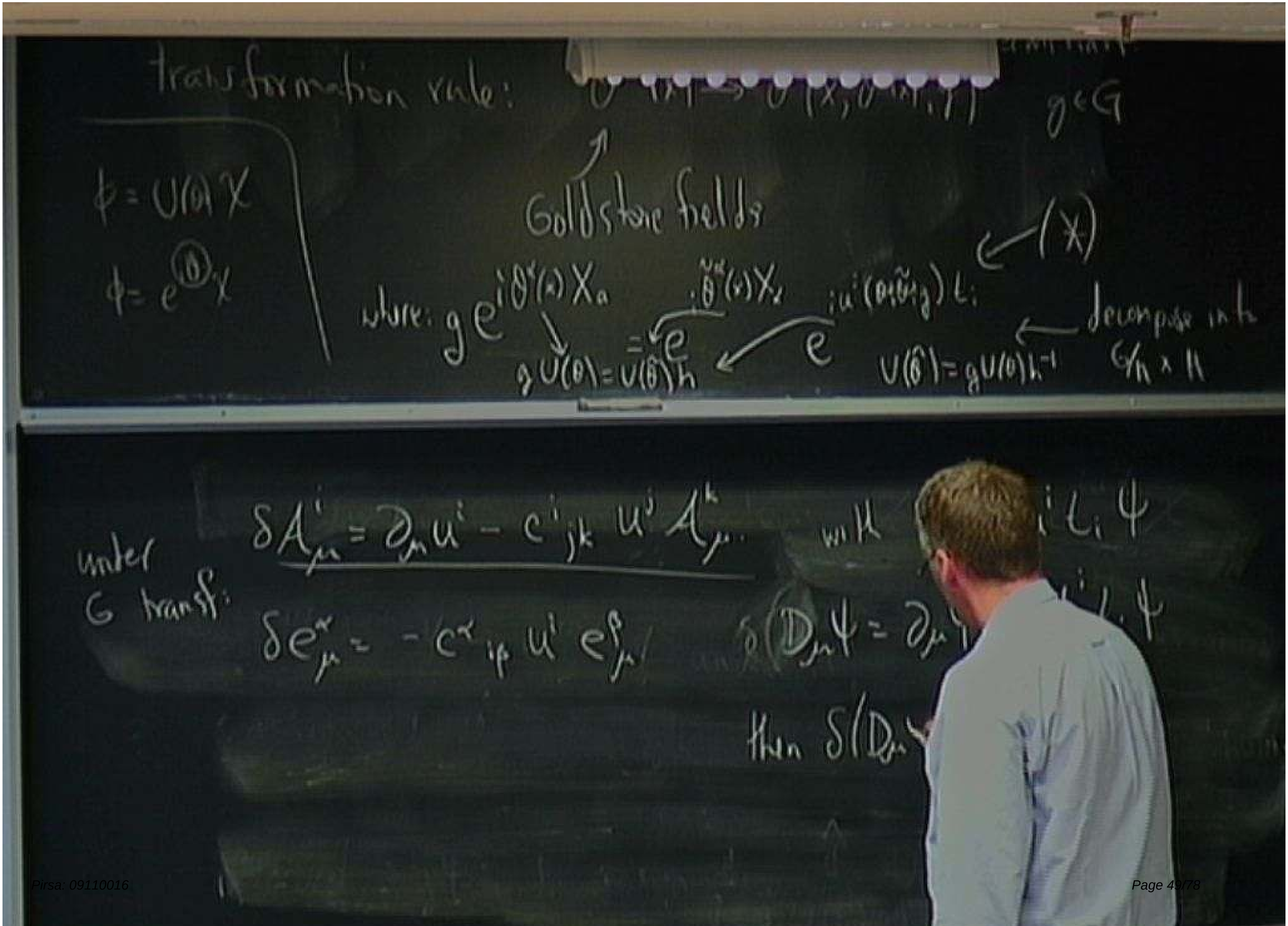
Define. $U^{-1}\partial_\mu U = -i A^i_\mu(\theta) t_i + i e^\alpha_\mu(\theta) X_\alpha$

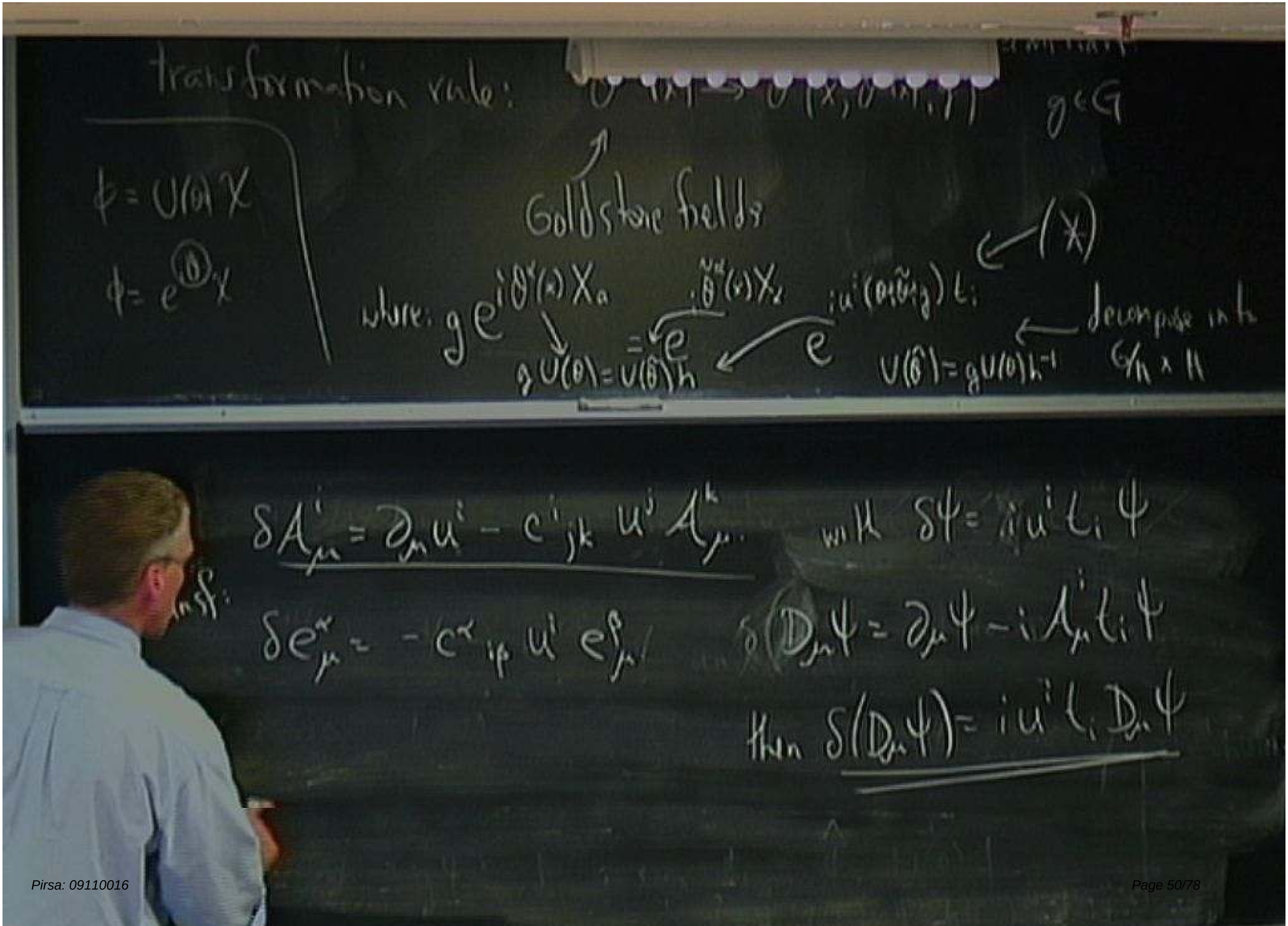


$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$
in full theory $\langle\phi\rangle = v$ $X_\alpha v \neq 0$ $t_i v = 0$.
$U^{-1}\partial_\mu U = -i A^i_\mu(\theta) t_i + i e^\alpha_\mu(\theta) X_\alpha$.
$(\theta) = i \mathrm{Tr}(t_i U^{-1}\partial_\mu U)$
$= i \mathrm{Tr}(X_\alpha$


rule for the Goldstone bosons
G = e^{iω^a T_a} , H = e^{iω^i t_i} , G/H = e^{iω^α X_α}
in full theory ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0
Define: U^{-1} ∂_μ U = -i A^i_μ(θ) t_i + i e^α_μ(θ) X_α
A^i_μ(θ) = i Tr(t_i U^{-1} ∂_μ U)
e^α_μ = -i Tr(X_α U^{-1} ∂_μ U)
tr(X_α X_β) = δ_αβ
tr(t_i t_j) = δ_ij
tr(t_i X_α) = 0


Goldstone fields
where:
decompose into


transformation rule:
g∈G
Goldstone fields
φ = U(θ) χ
φ = e^{iθ} χ
where: g e^{iθ^a(x) X_a}
decompose into
G/H × H
U(θ̃) = g U(θ) h^{-1}
g U(θ) = U(θ̃) h
under
G transf:
δA^i_μ = ∂_μ u^i − c^i_{jk} u^j A^k_μ
δe^α_μ = − c^α_{iβ} u^i e^β_μ
with
then δ(D_μ


transformation rule:
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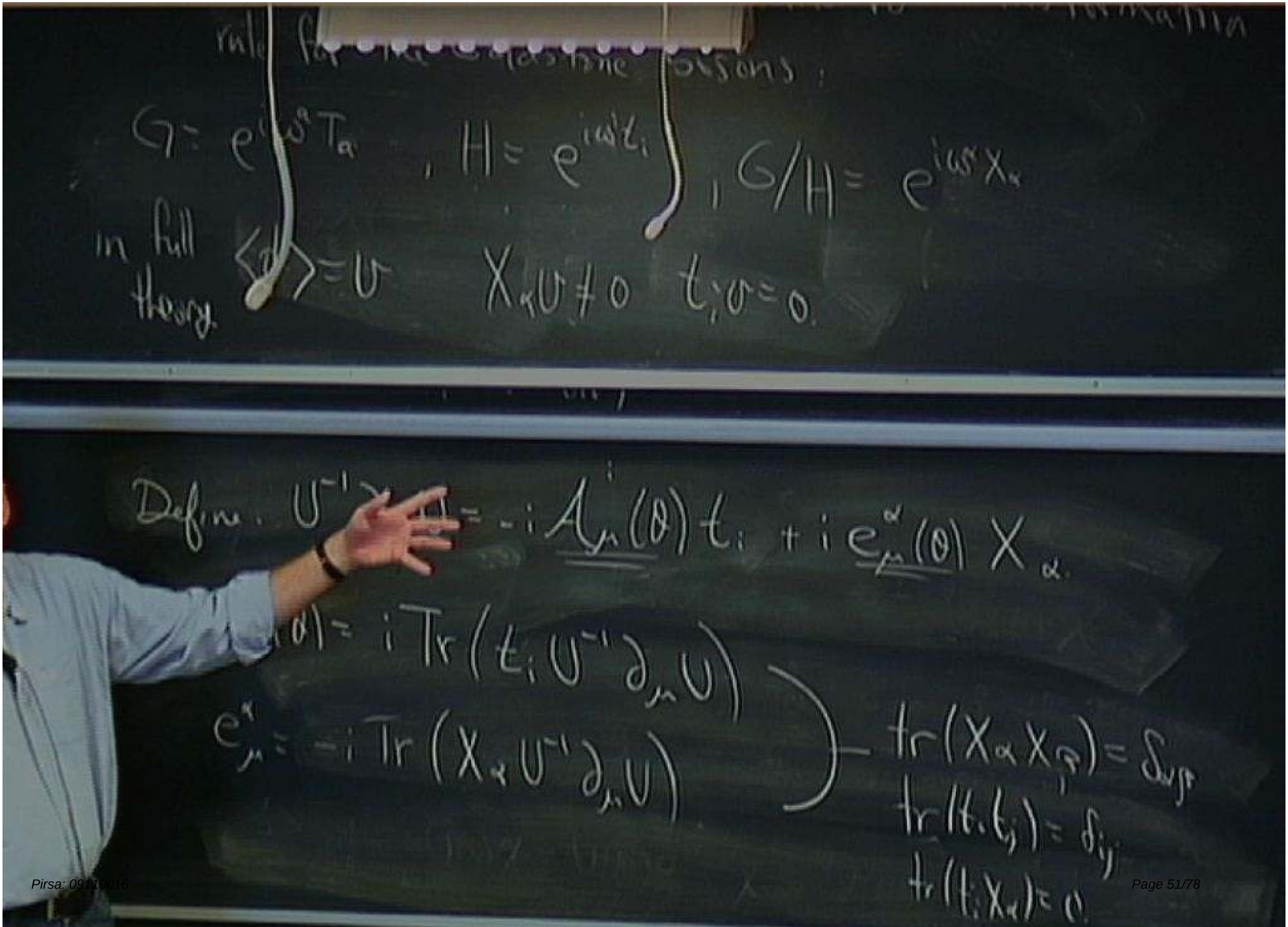
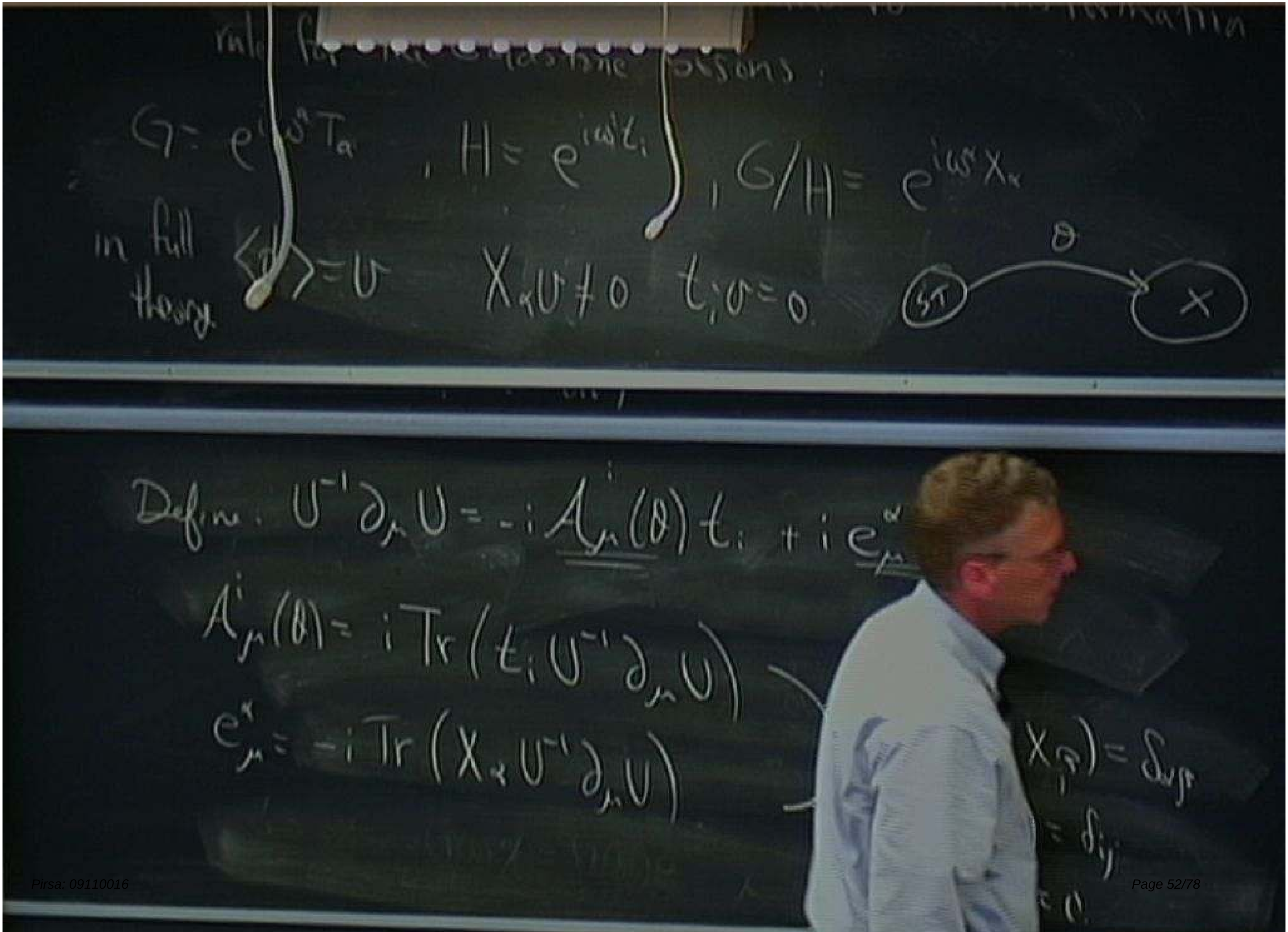
Goldstone fields
φ = U(θ) χ
φ = e^θ χ
where: g e^{iθ^a(x) X_a}
g U(θ) = U(θ̃) h
U(θ̃) = g U(θ) h^{-1}
decompose into G/H × H
δA^i_μ = ∂_μ u^i − c^i_{jk} u^j A^k_μ
with δψ = i u^i t_i ψ
δe^α_μ = − c^α_{iβ} u^i e^β_μ
D_μ ψ = ∂_μ ψ − i A^i_μ t_i ψ
then δ(D_μ ψ) = i u^i t_i D_μ ψ



G = e^{iω^a T_a} , H = e^{iω^i t_i} , G/H = e^{iω^α X_α}
in full theory. ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0.
Define. U^{-1} … = -i A^i_μ(θ) t_i + i e^α_μ(θ) X_α.
(θ) = i Tr(t_i U^{-1} ∂_μ U)
e^α_μ = -i Tr(X_α U^{-1} ∂_μ U)
tr(X_α X_β) = δ_αβ
tr(t_i t_j) = δ_ij
tr(t_i X_α) = 0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θ
ST
X
Define: U^{-1} ∂_μ U = -i A^i_μ(θ) t_i + i e^α_μ
A^i_μ(θ) = i Tr(t_i U^{-1} ∂_μ U)
e^α_μ = -i Tr(X_α U^{-1} ∂_μ U)
X_β) = δ_αβ

rule for the Goldstone bosons

$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$

in full theory $\langle\phi\rangle = v$ $X_\alpha v \neq 0$ $t_i v = 0$.

Define: $U^{-1}\partial_\mu U = -i A^i_\mu(\theta)\, t_i + i\, e^\alpha_\mu(\theta)\, X_\alpha$.

$A^i_\mu(\theta) = i\,\mathrm{Tr}(t_i U^{-1}\partial_\mu U)$

$e^\alpha_\mu = -i\,\mathrm{Tr}(X_\alpha U^{-1}\partial_\mu U)$

$\mathrm{tr}(X_\alpha X_\beta) = \delta_{\alpha\beta}$

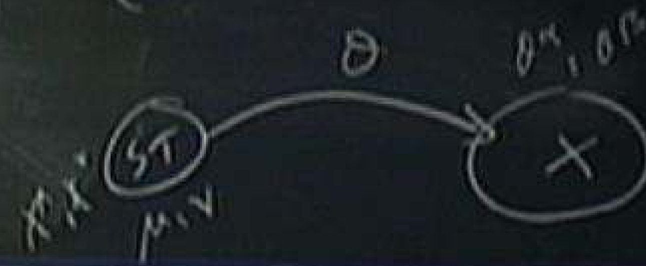
$\mathrm{tr}(t_i t_j) = \delta_{ij}$

$\mathrm{tr}(t_i X_\alpha) = 0$

rule for the Goldstone bosons:

$$G = e^{i\omega^a T_a}, \quad H = e^{i\omega^i t_i}, \quad G/H = e^{i\omega^\alpha X_\alpha}$$

in full theory $\langle \phi \rangle = v \qquad X_\alpha v \neq 0 \qquad t_i v = 0.$



Define: $U^{-1}\partial_\mu U =: -i A^i_\mu(\theta)\, t_i + i e^\alpha_\mu(\theta)\, X_\alpha$.

$$A^i_\mu(\theta) = i \, \mathrm{Tr}\left(t_i U^{-1} \partial_\mu U\right)$$

$$e^\alpha_\mu = -i \, \mathrm{Tr}\left(X_\alpha U^{-1} \partial_\mu U\right)$$

$\mathrm{tr}(X_\alpha X_\beta) = \delta_{\alpha\beta}$

$\mathrm{tr}(t_i t_j) = \delta_{ij}$

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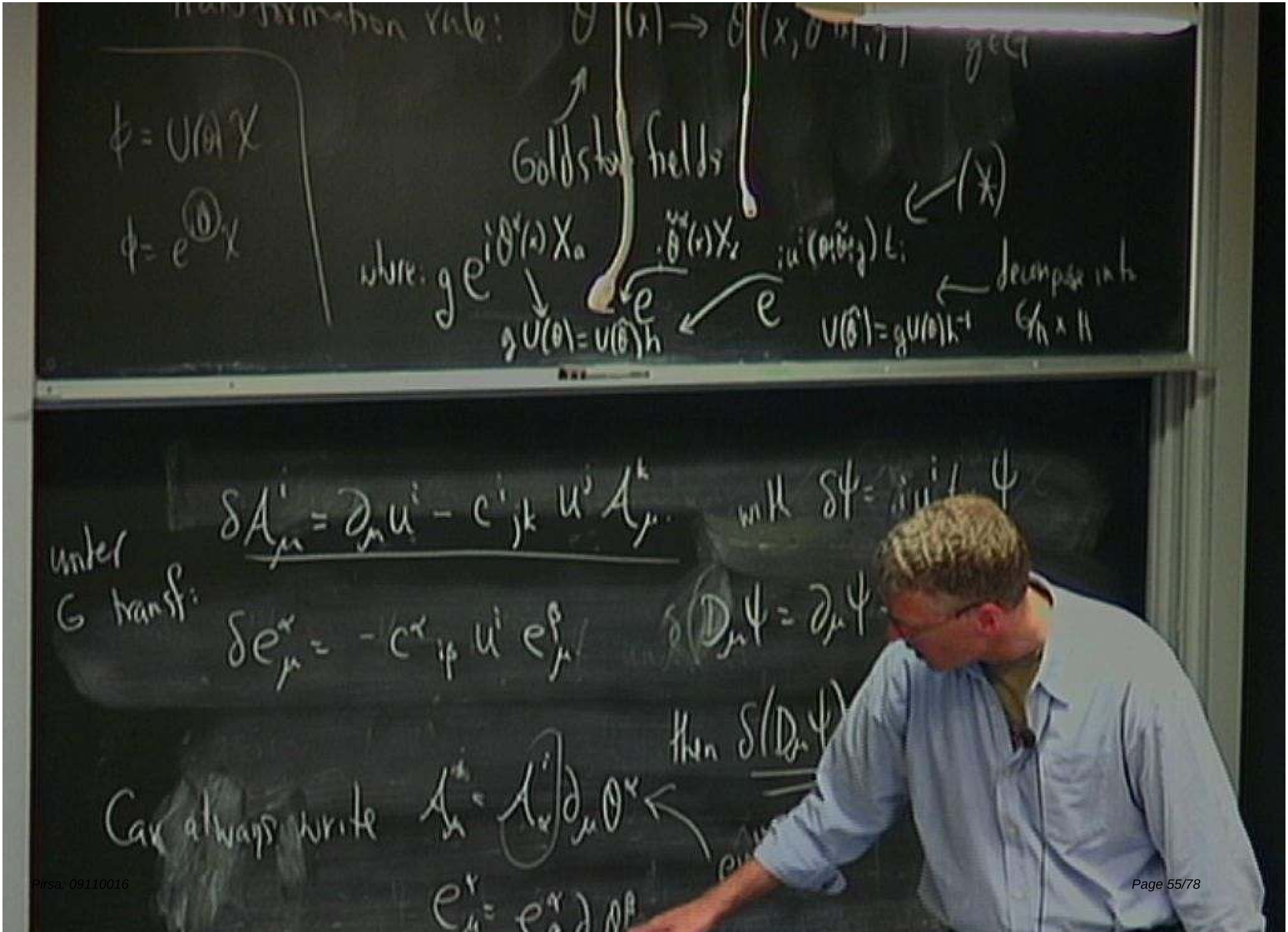
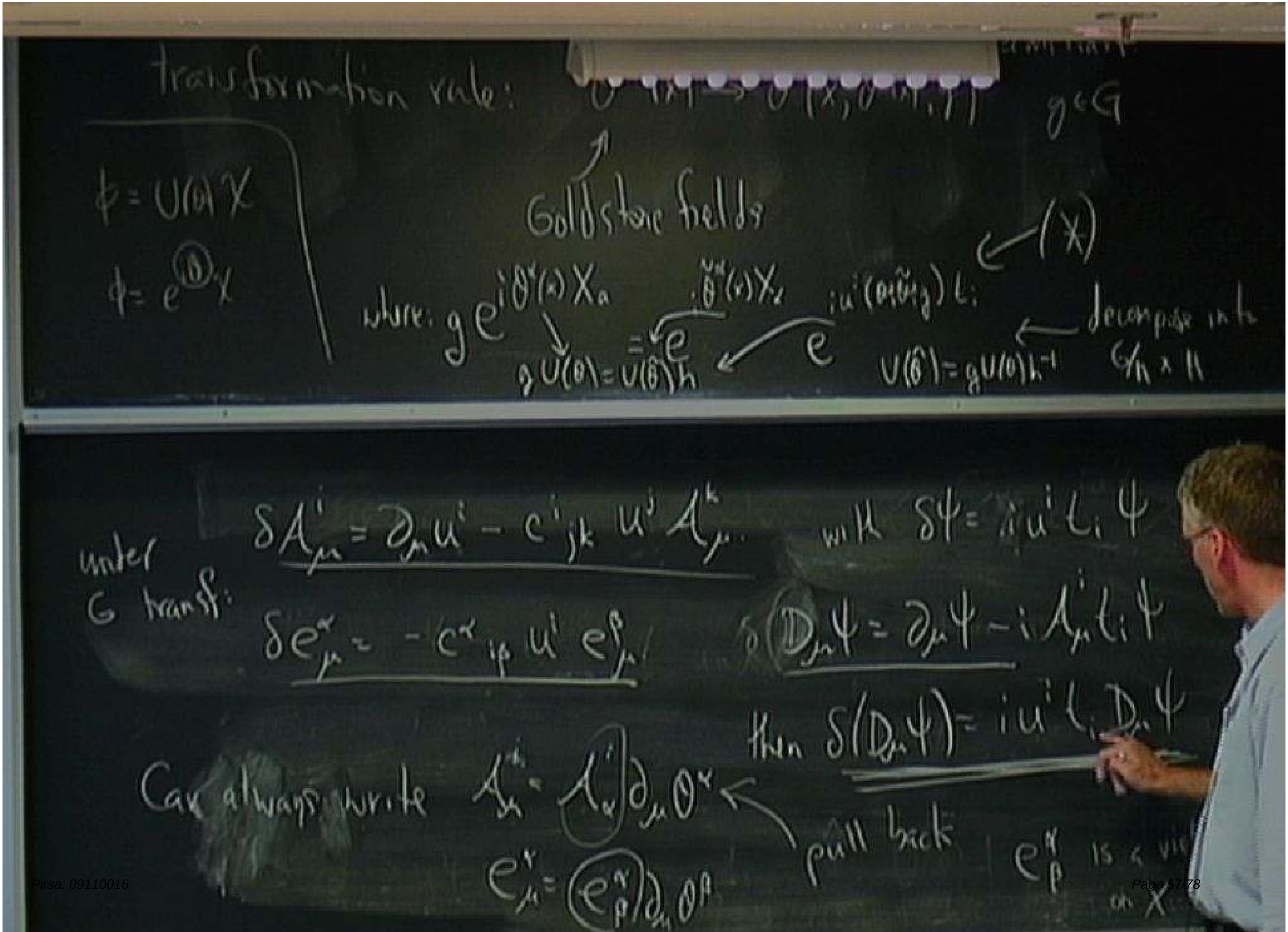

where:
under
G transf:
then
Can always write

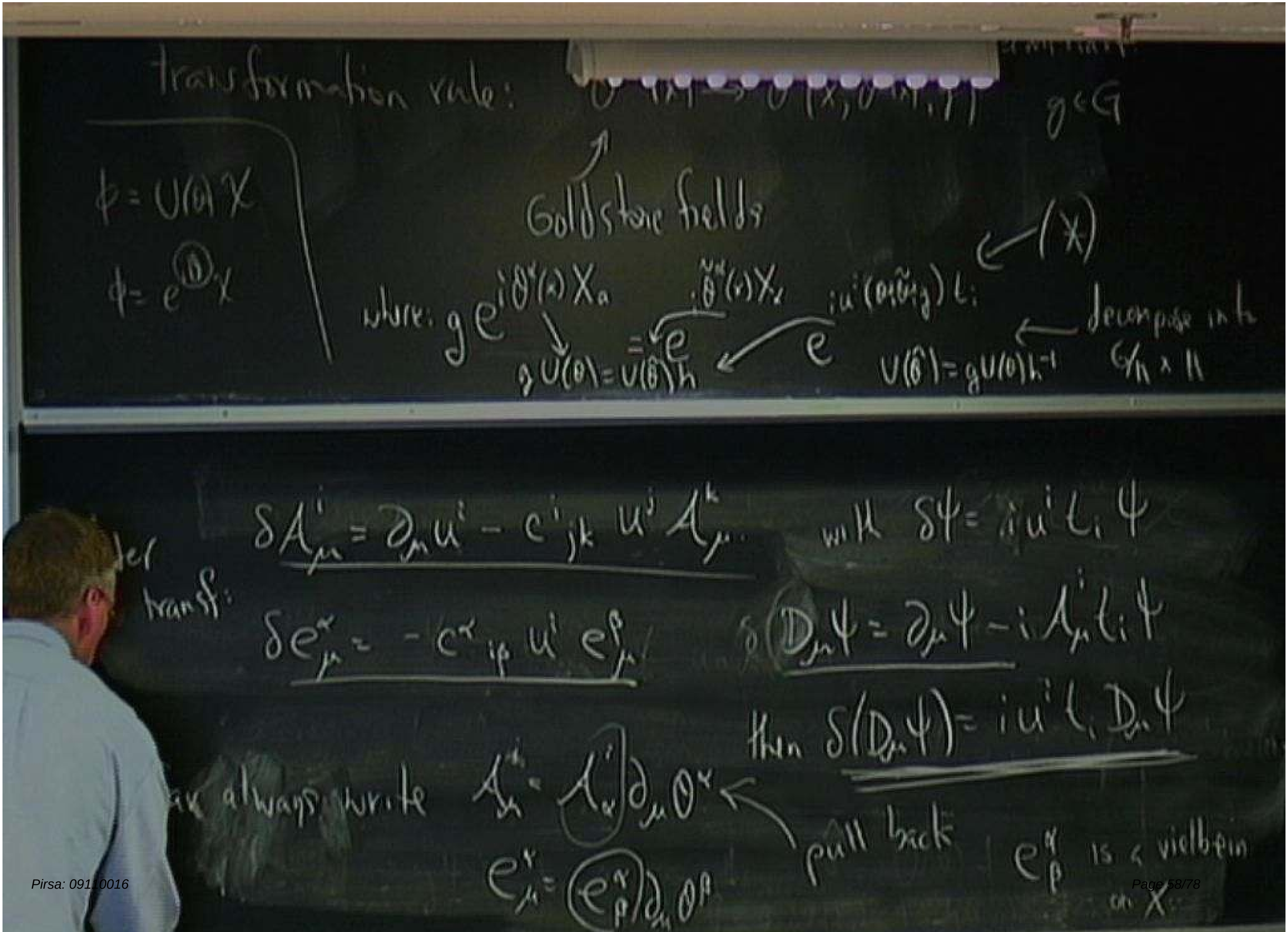
... symm... spontaneously breaks to H, giving a Goldstone Boson for each generator of G/H (which are massless).

hep-th/9808176

Define $G_{\alpha\beta} = e_\alpha^{\gamma} e_\beta^{\delta} \frac{\delta_{\gamma\delta}}{Tr(X_\gamma X_\delta)}$

Claim: $G_{\alpha\beta}$


transformation rule:
g∈G
Goldstone fields
ϕ = U(θ) χ
ϕ = e^{iθ} χ
where: g e^{iθ^α(x) X_α}
g U(θ) = U(θ̃) h
U(θ̃) = g U(θ) h^{-1}
decompose into G/H × H
under G transf:
δA^i_μ = ∂_μ u^i − c^i_{jk} u^j A^k_μ
with δψ = i u^i t_i ψ
δe^α_μ = − c^α_{iβ} u^i e^β_μ
D_μ ψ = ∂_μ ψ − i A^i_μ t_i ψ
then δ(D_μ ψ) = i u^i t_i D_μ ψ
Can always write A^i_μ = A^i_α ∂_μ θ^α
pull back
e^α_μ = e^α_β ∂_μ θ^β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Goldstone fields
(X)
where: g e^{iθ^α(x) X_α}
= e^{iθ̃^α(x) X_α}
e^{iu^i(θ, θ̃, g) t_i}
g U(θ) = U(θ̃) h
U(θ̃) = g U(θ) h^{-1}
decompose into
G/H × H
δA^i_μ = ∂_μ u^i − c^i_{jk} u^j A^k_μ
with δψ = i u^i t_i ψ
transf:
δe^α_μ = − c^α_{iβ} u^i e^β_μ
D_μ ψ = ∂_μ ψ − i A^i_μ t_i ψ
then δ(D_μ ψ) = i u^i t_i D_μ ψ
always write A^i_μ = A^i_α ∂_μ θ^α
pull back
e^α_μ = e^α_β ∂_μ θ^β
e^α_β is a vielbein on X

transformation rule: $U(x) \to U(x, \theta'(x), g)$ $\quad g \in G$

Goldstone fields

$\phi = U(\theta)\chi$

$\phi = e^{i\theta}\chi$

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\theta'^\alpha(x) X_\alpha}\, e^{i u^i(\theta, \theta', g) t_i}$ $\leftarrow$ decompose into $G/H \times H$ $\quad (*)$

$g\, U(\theta) = U(\theta')\, h$ $\qquad U(\theta') = g\, U(\theta)\, h^{-1}$

under G transf:

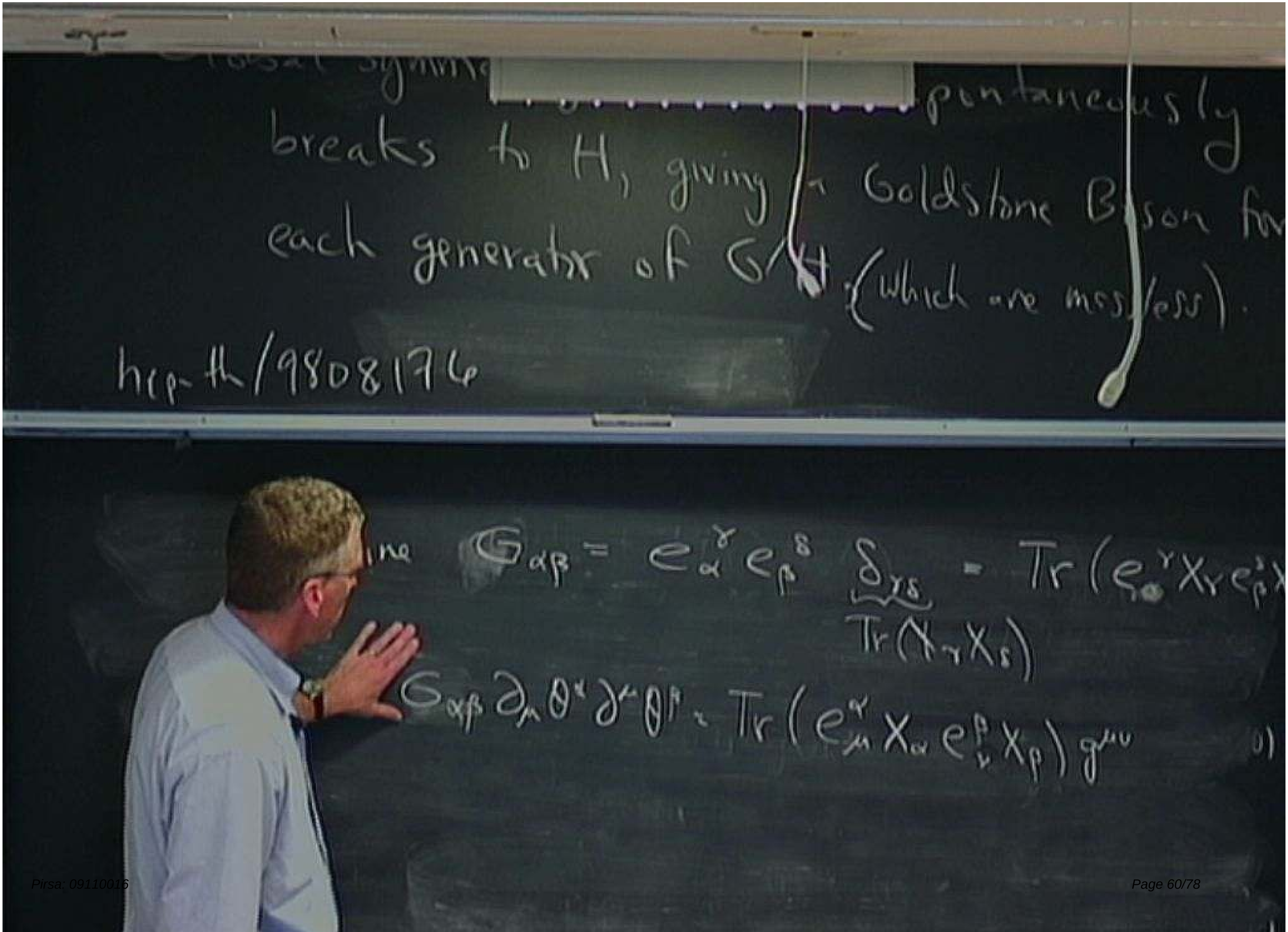
$\delta A^i_\mu = \partial_\mu u^i - c^i{}_{jk}\, u^j A^k_\mu$ $\quad$ with $\delta\psi = i u^i t_i \psi$

$\delta e^\alpha_\mu = -c^\alpha{}_{i\beta}\, u^i e^\beta_\mu$ $\qquad \mathcal{D}_\mu\psi = \partial_\mu\psi - i A^i_\mu t_i \psi$

then $\delta(\mathcal{D}_\mu\psi) = i u^i t_i \mathcal{D}_\mu\psi$

Can always write $A^k_\mu = A^i_\alpha\, \partial_\mu \theta^\alpha$ $\leftarrow$ pull back

$e^\alpha_\mu X_\alpha \to h\, e^\alpha_\mu X_\alpha h^{-1}$ $\qquad e^\alpha_\mu = e^\alpha_\beta\, \partial_\mu \theta^\beta$ $\qquad e^\alpha_\beta$ is a vielbein on X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hep-th/9808176
$G_{\alpha\beta} = e_\alpha^{\gamma} e_\beta^{\delta} \delta_{\gamma\delta} = Tr(e_\alpha^{\gamma} X_\gamma e_\beta^{\delta} X$
$Tr(X_\gamma X_\delta)$
$G_{\alpha\beta} \partial_\mu \theta^\alpha \partial^\mu \theta^\beta = Tr(e_\mu^\alpha X_\alpha e_\nu^\beta X_\beta) g^{\mu\nu}$

... spontaneously
breaks to H, giving a Goldstone Boson for
each generator of G/H (which are massless).

hep-th/9808176

Define $G_{\alpha\beta} = e_\alpha^{\ \gamma} e_\beta^{\ \delta} \underbrace{\delta_{\gamma\delta}}_{Tr(X_\gamma X_\delta)} = Tr(e_\alpha^{\ \gamma} X_\gamma e_\beta^{\ \delta} X$

Claim: $G_{\alpha\beta}\, \partial_\mu \theta^\alpha \partial^\mu \theta^\beta = Tr(e_\mu^\alpha X_\alpha e_\nu^\beta X_\beta)\, g^{\mu\nu}$

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta(x), g)$ $g \in G$

$\phi = U(\theta)\,\chi$

$\phi = e^{i\theta \cdot X}\chi$

Goldstone fields

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\theta'^\alpha(x) X_\alpha}\, e^{i u^i(\theta, \theta', g)\, t_i}$ (*) ← decompose into $G/H \times H$

$g\,U(\theta) = U(\theta')\,h$

$U(\theta') = g\,U(\theta)\,h^{-1}$

under G transf:

$$\delta A^i_\mu = \partial_\mu u^i - c^i{}_{jk}\, u^j A^k_\mu \qquad \text{with } \delta\psi = i u^i t_i \psi$$

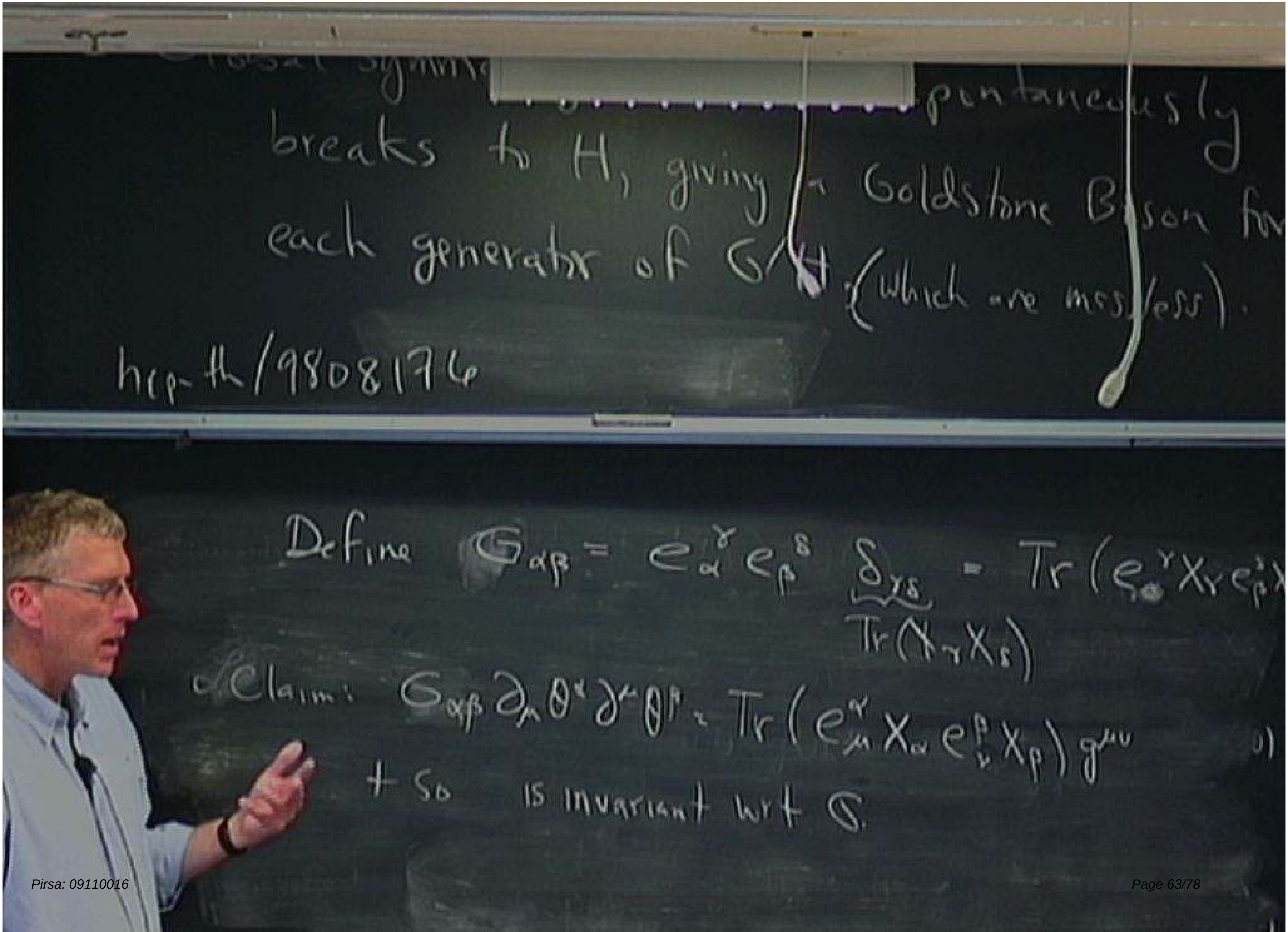
$$\delta e^\alpha_\mu = - c^\alpha{}_{i\beta}\, u^i e^\beta_\mu \qquad D_\mu\psi = \partial_\mu\psi - i A^i_\mu t_i \psi$$

then $\delta(D_\mu\psi) = i u^i t_i D_\mu\psi$

Can always write $A^i_\mu = A^i_\alpha\, \partial_\mu \theta^\alpha$ ← pull back

$e^\alpha_\mu = e^\alpha_\beta\, \partial_\mu \theta^\beta$ e^α_β is a vielbein on X

$e^\alpha_\mu X_\alpha \to h\, e^\alpha_\mu X_\alpha\, h^{-1}$


pontaneously
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hep-th/9808176
Define Gαβ = eα^γ eβ^δ δγδ = Tr(eα^γ Xγ eβ^δ X
Tr(Xγ Xδ)
Claim: Gαβ ∂μ θ^α ∂^μ θ^β = Tr(eμ^α Xα eν^β Xβ) g^μν
+ S0 is invariant wrt G.

transformation rule: $\theta^\alpha(x) \to \theta'^\alpha(x, \theta(x), g)$ $g \in G$

↑ Goldstone fields

$\psi = U(\theta)\chi$

$\psi = e^{(\theta)}\chi$

where: $g\, e^{i\theta^\alpha(x) X_\alpha} = e^{i\theta'^\alpha(x) X_\alpha}\, e^{i u^i(\theta, \tilde{\theta}, g) t_i}$ ← decompose into $G/H \times H$

$g\,U(\theta) = U(\theta')h$ $U(\theta') = g\,U(\theta)\,h^{-1}$

under G transf:

$\delta A^i_\mu = \partial_\mu u^i - c^i{}_{jk}\, u^j A^k_\mu$ with $\delta\psi = i u^i t_i \psi$

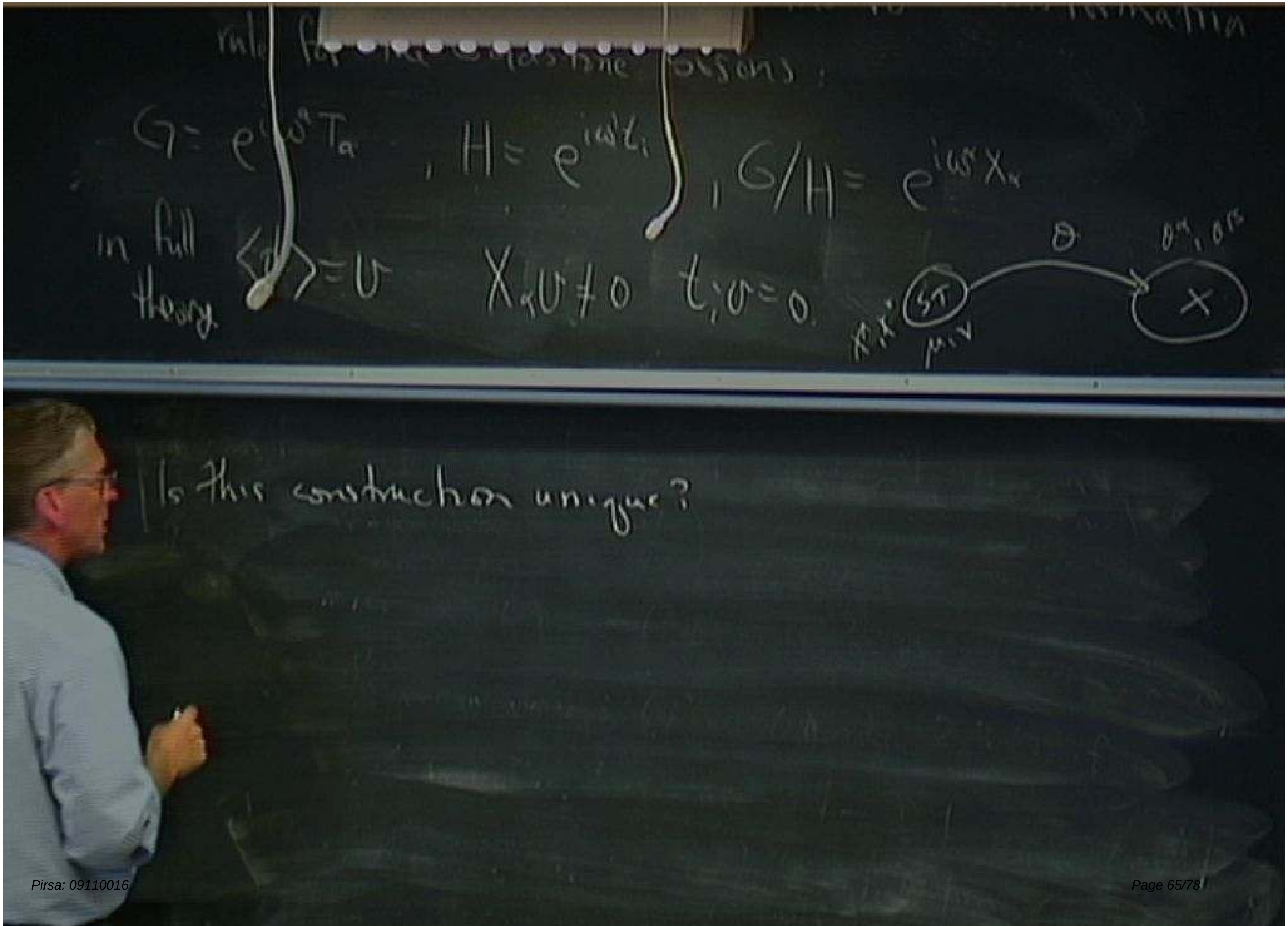
$\delta e^\alpha_\mu = -c^\alpha{}_{i\beta}\, u^i e^\beta_\mu$ $D_\mu\psi = \partial_\mu\psi - i A^i_\mu t_i \psi$

then $\delta(D_\mu\psi) = i u^i t_i D_\mu\psi$

Can always write $A^i_\mu = A^i_\alpha \partial_\mu \theta^\alpha$ ← pull back

$e^\alpha_\mu = e^\alpha_\beta \partial_\mu \theta^\beta$ e^α_β is a vielbein on X

$e^\alpha_\mu X_\alpha \to h\, e^\alpha_\mu X_\alpha h^{-1}$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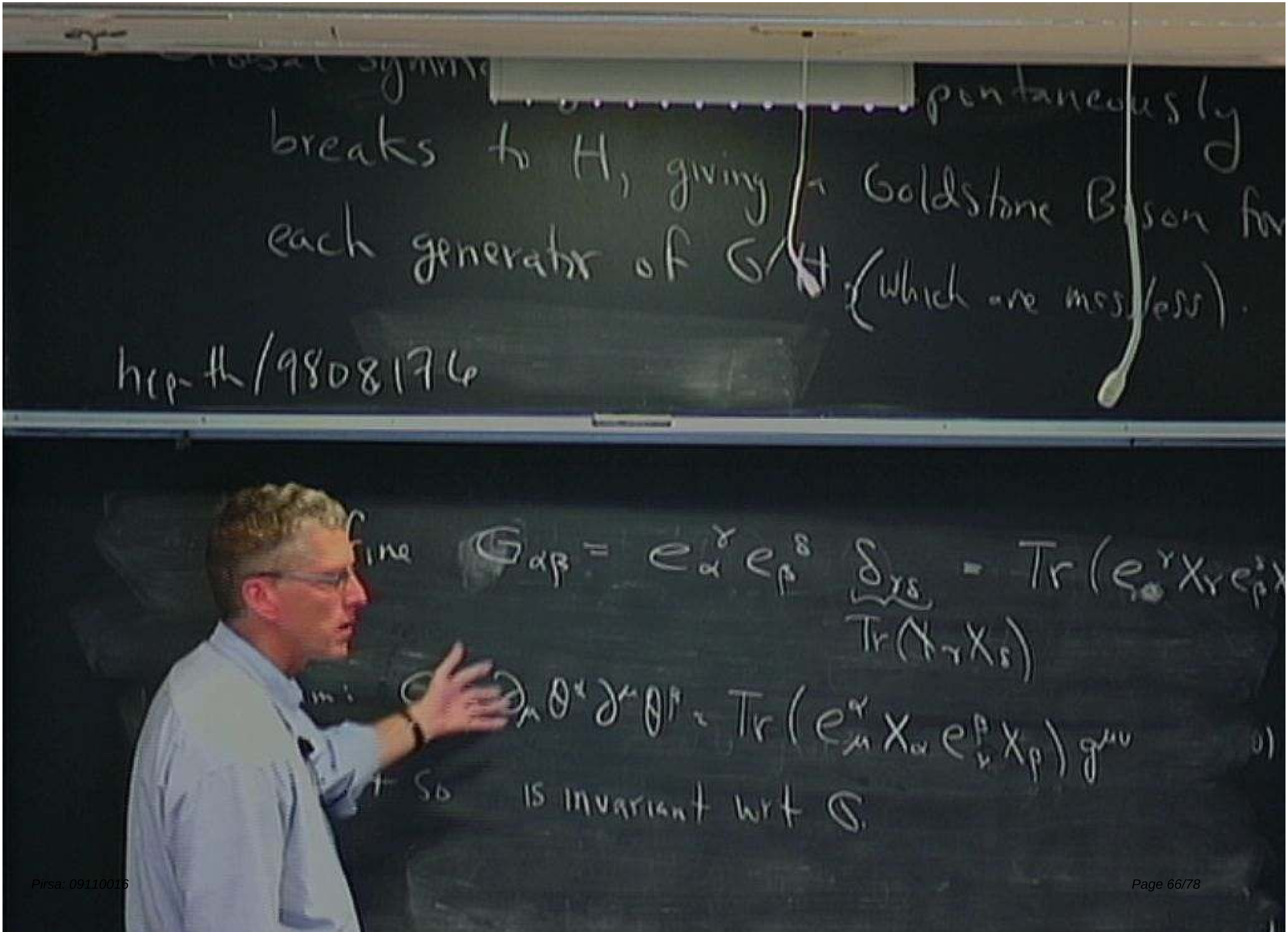
in full theory ⟨φ⟩ = v
X_α v ≠ 0 t_i v = 0
Is this construction unique?


pontaneously
breaks to H, giving a Goldstone Boson for
each generator of G/H (which are massless).
hep-th/9808176
ine Gαβ = eα^γ eβ^δ δγδ = Tr(eα^γ Xγ eβ^δ X
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μ θ^α ∂^μ θ^β = Tr(eμ^α Xα eν^β Xβ) g^μν
S0 is invariant wrt G.

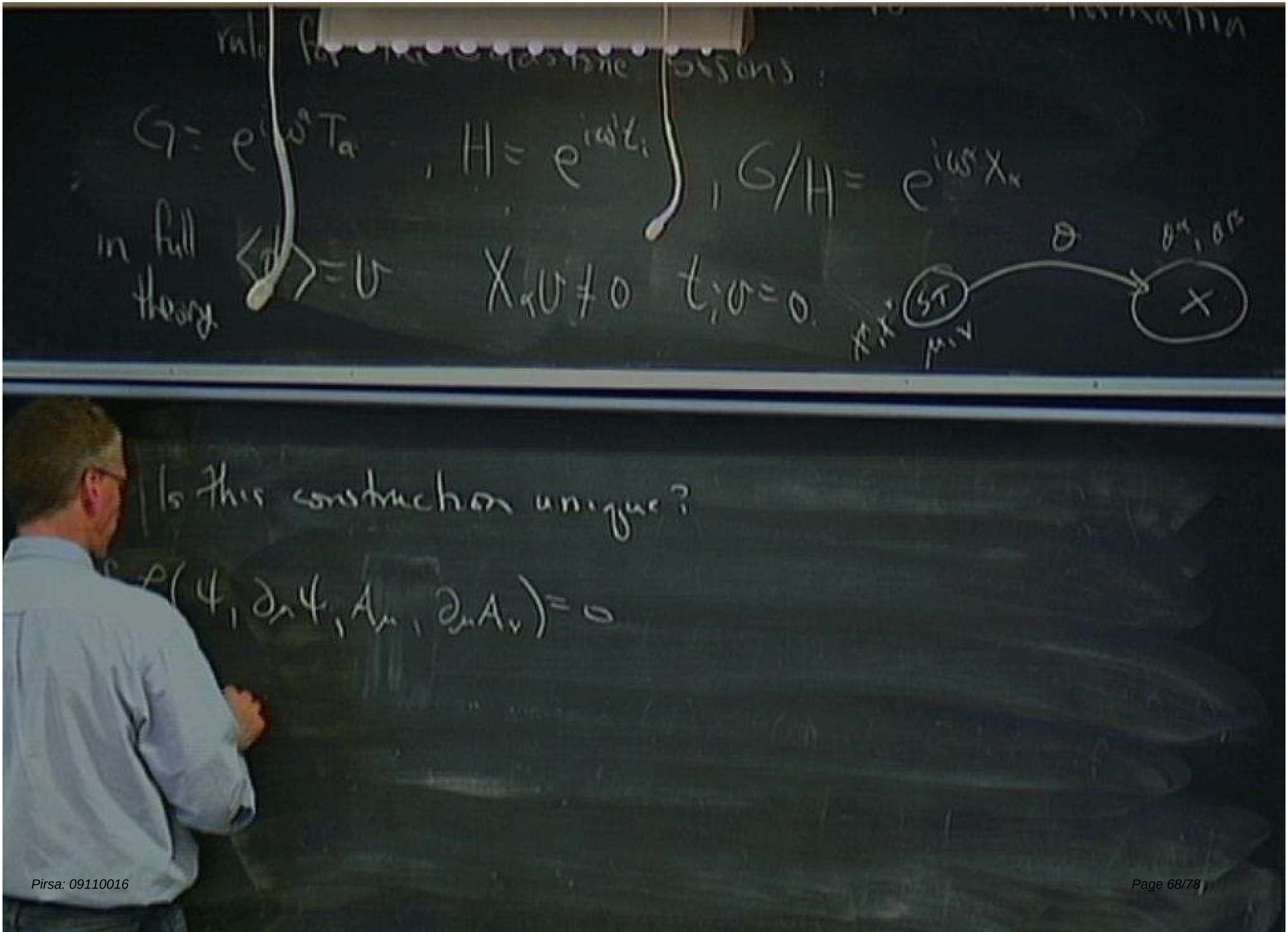
... symm... spontaneously breaks to H, giving a Goldstone Boson for each generator of G/H (which are massless).

hep-th/9808176

Define $G_{\alpha\beta} = e_\alpha^{\ \gamma} e_\beta^{\ \delta} \underbrace{\delta_{\gamma\delta}}_{Tr(X_\gamma X_\delta)} = Tr(e_\alpha^{\ \gamma} X_\gamma e_\beta^{\ \delta} X$

Claim: $G_{\alpha\beta}\partial_\mu\theta^\alpha\partial^\mu\theta^\beta = Tr(e_\mu^{\ \alpha} X_\alpha e_\nu^{\ \beta} X_\beta)\, g^{\mu\nu}$

$+ S_0$ is invariant wrt G.


G = e^{iω^a T_a} , H = e^{iω^i t_i} , G/H = e^{iω^α X_α}
in full theory ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0
θ
ST
X
Is this construction unique?
ℒ(ψ, ∂_μψ, A_μ, ∂_μA_ν) = 0

rule for the Goldstone bosons :

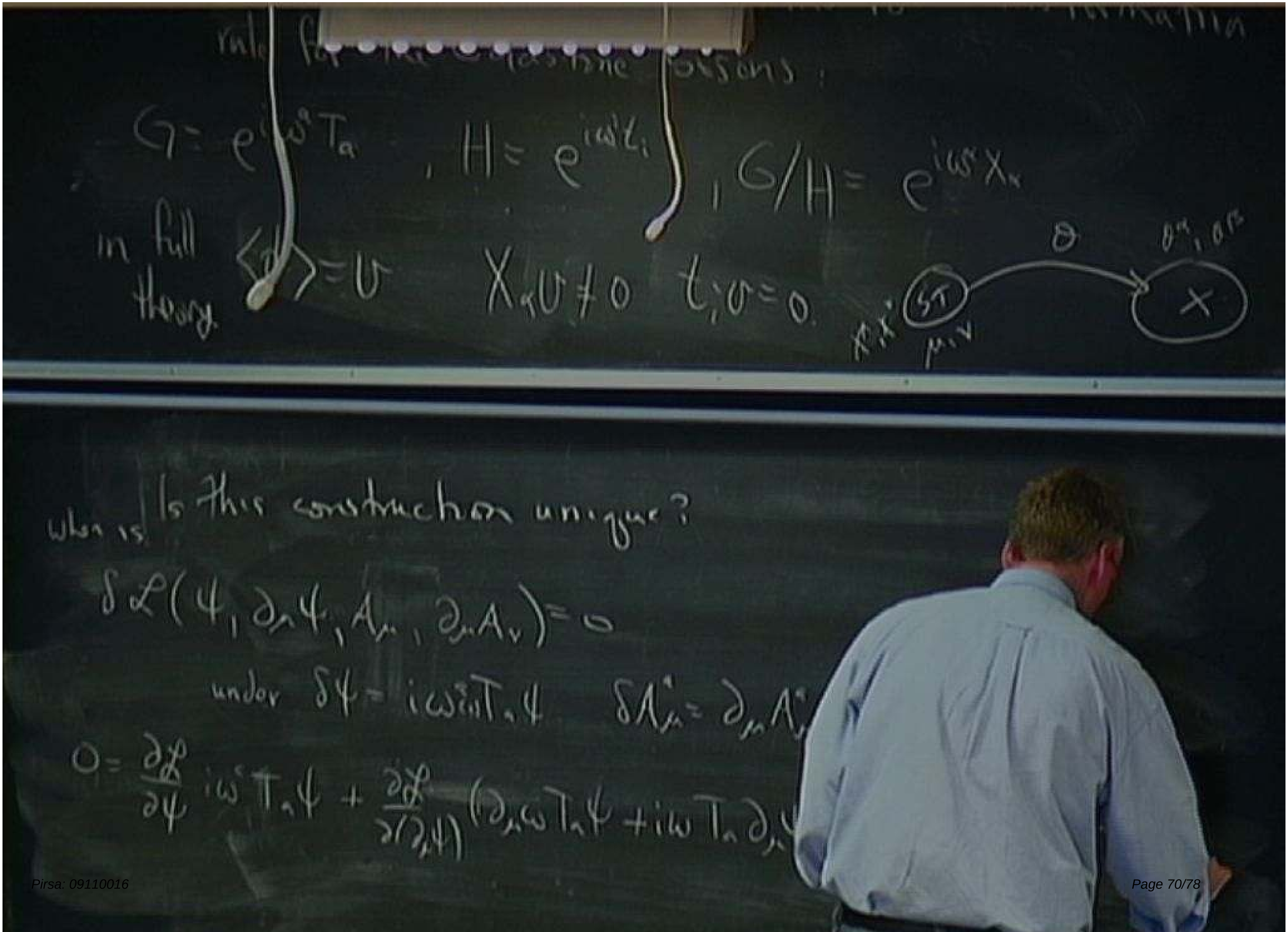
$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$

in full theory $\langle \phi \rangle = v$ $\quad X_\alpha v \neq 0 \quad t_i v = 0.$

When is / Is this construction unique?

$$\delta \mathcal{L}(\psi, \partial_\mu \psi, A_\mu, \partial_\mu A_\nu) = 0$$

under $\delta\psi = i\omega^a(x) T_a \psi \quad \delta A^a_\mu = \partial_\mu \Lambda^a_\mu + c^a{}_{bc}\,\omega^b A^c_\mu$?


rules for the Goldstone bosons
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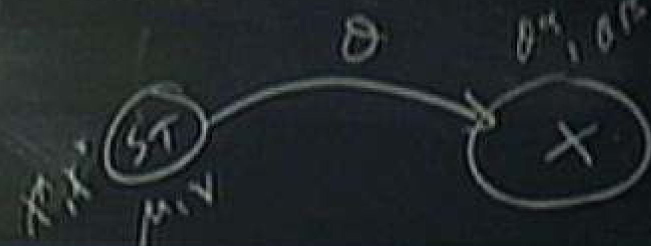
in full theory ⟨φ⟩ = v   X_α v ≠ 0   t_i v = 0
Is this construction unique?
δ𝓛(ψ, ∂_μψ, A_μ, ∂_μA_ν) = 0
under δψ = iω^a T_a ψ   δA_μ = ∂_μ Λ
0 = ∂𝓛/∂ψ iω T_a ψ + ∂𝓛/∂(∂_μψ) (∂_μω T_a ψ + iω T_a ∂_μψ

rule for [illegible] Goldstone bosons:

$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$

in full theory $\langle \phi \rangle = v$ $\quad X_\alpha v \neq 0 \quad t_i v = 0$



when is / Is this construction unique?

$$\delta \mathcal{L}(\psi, \partial_\mu \psi, A_\mu, \partial_\mu A_\nu) = 0$$

under $\delta\psi = i\omega^a(x) T_a \psi \quad \delta A^a_\mu = \partial_\mu A^a_\mu + c^a{}_{bc}\, \omega^b A^c_\mu$?

$$0 = \frac{\partial \mathcal{L}}{\partial \psi} i\omega^a T_a \psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\left(\partial_\mu \omega T_a \psi + i\omega T_a \partial_\mu \psi\right) + \frac{\partial \mathcal{L}}{\partial A^a_\mu}\left(\partial_\mu \omega^a + c^a{}_{bc}\, \omega^b A^c_\mu\right)$$

$G = e^{i\omega^a T_a}$, $H = e^{i\omega^i t_i}$, $G/H = e^{i\omega^\alpha X_\alpha}$

in full theory $\langle \phi \rangle = v$ $X_\alpha v \neq 0$ $t_i v = 0$

$x^\mu x^\nu$ (ST) μ,ν $\xrightarrow{\theta}$ (X) $\theta^\alpha, \theta^\beta$

when is Is this construction unique? $\partial_\mu A^a \to F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + c \cdots$

$\delta \mathcal{L}(\psi, \partial_\mu \psi, A_\mu, \partial_\mu A_\nu) = 0$

under $\delta\psi = i\omega^a(x) T_a \psi$ $\delta A^a_\mu = \partial_\mu A^a_\mu + c^a{}_{bc}\omega^b A^c_\mu$?

$$0 = \frac{\partial \mathcal{L}}{\partial \psi} i\omega^a T_a \psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\left(\partial_\mu \omega T_a \psi + i\omega T_a \partial_\mu \psi\right) + \frac{\partial \mathcal{L}}{\partial A^a_\lambda}\left(\partial_\lambda \omega^a + c^a{}_{bc}\omega^b A^c_\lambda\right)$$

$$= (\quad)\,\omega^a + (\quad)\,\partial_\mu \omega^a + (\quad)\,\partial_\mu \partial_\nu \omega^a + \frac{\partial \mathcal{L}}{\partial \partial_\mu A^a_\nu}\left(\partial_\mu \partial_\nu \omega^a + c^a{}_{bc}(\partial_\mu \omega^b A^c_\nu + \omega^b \partial_\mu A^c_\nu)\right)$$

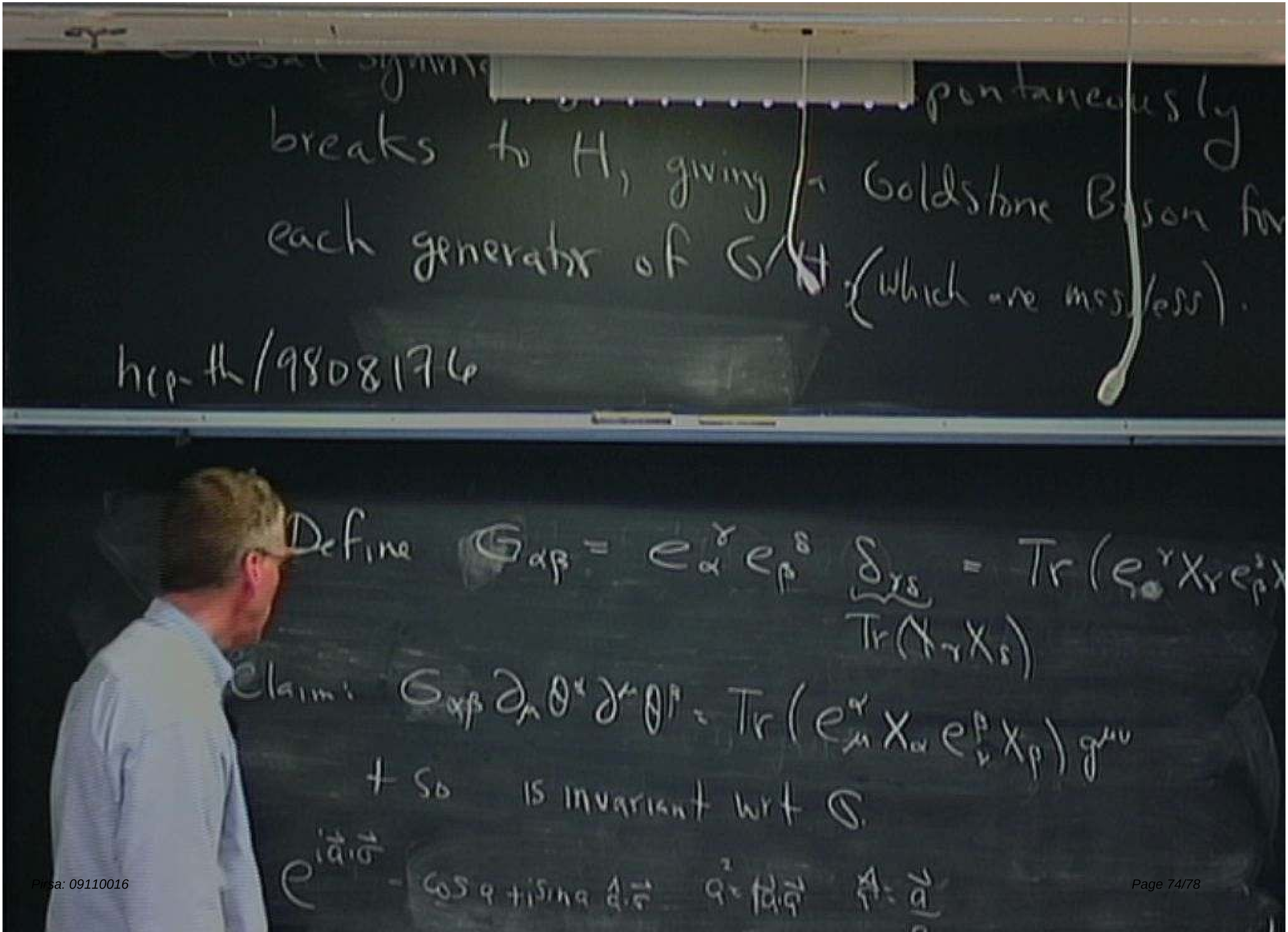
which spontaneously breaks to H, giving a Goldstone Boson for each generator of G/H. (which are massless).

hep-th/9808176

Define $G_{\alpha\beta} = e_\alpha^{\gamma} e_\beta^{\delta} \underbrace{\delta_{\gamma\delta}}_{Tr(X_\gamma X_\delta)} = Tr(e_\alpha^{\gamma} X_\gamma e_\beta^{\delta} X_\delta)$

Claim: $G_{\alpha\beta}\partial_\mu\theta^\alpha\partial^\mu\theta^\beta = Tr(e_\mu^{\alpha} X_\alpha e_\nu^{\beta} X_\beta) g^{\mu\nu}$

$+ S_0$ is invariant wrt G.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Claim: G_αβ ∂_μ θ^α ∂^μ θ^β = Tr(e_μ^α X_α e_ν^β X_β) g^μν
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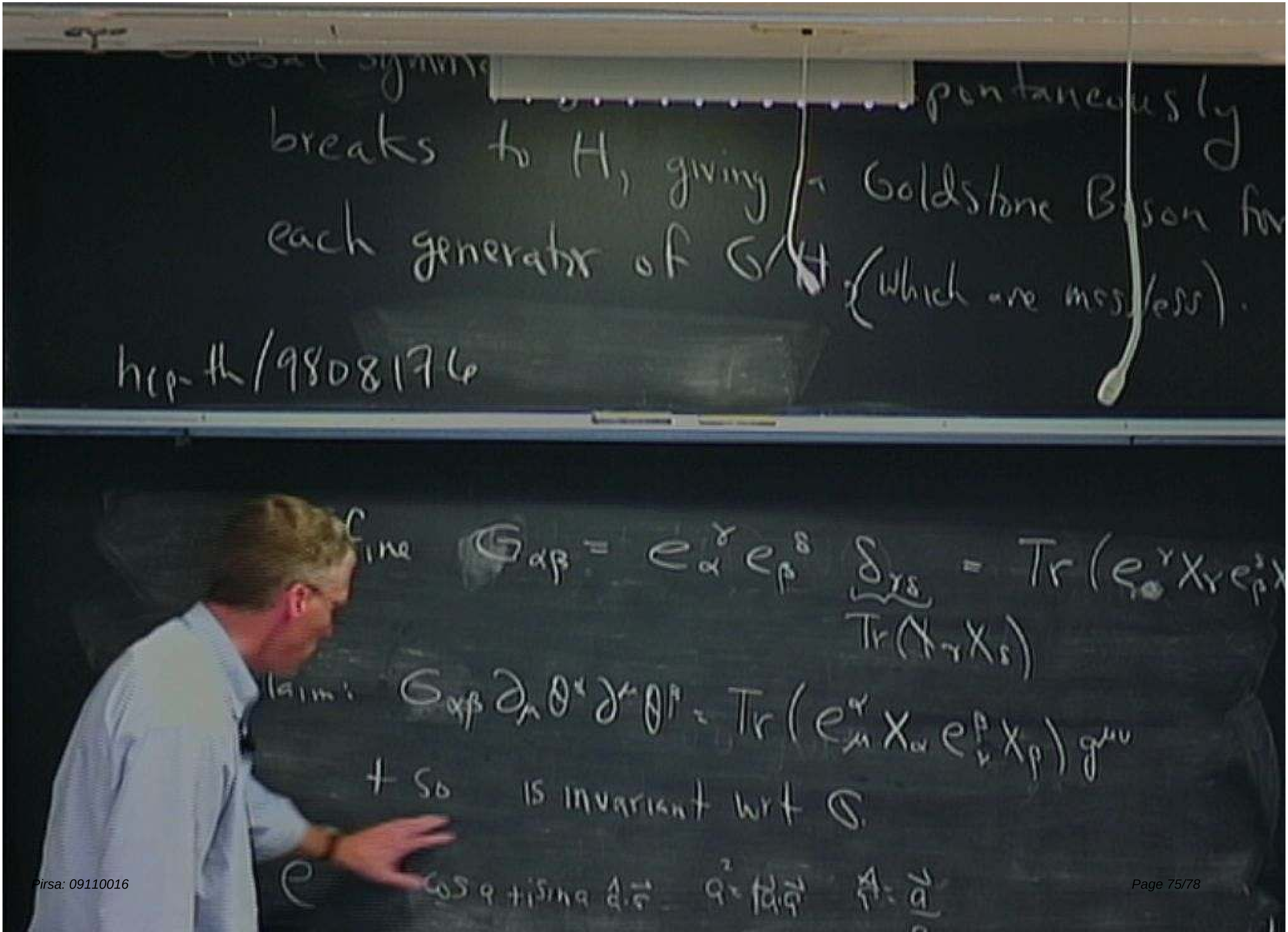
e^{i a·σ} = cos a + i sin a â·σ a² = a·a â = a/a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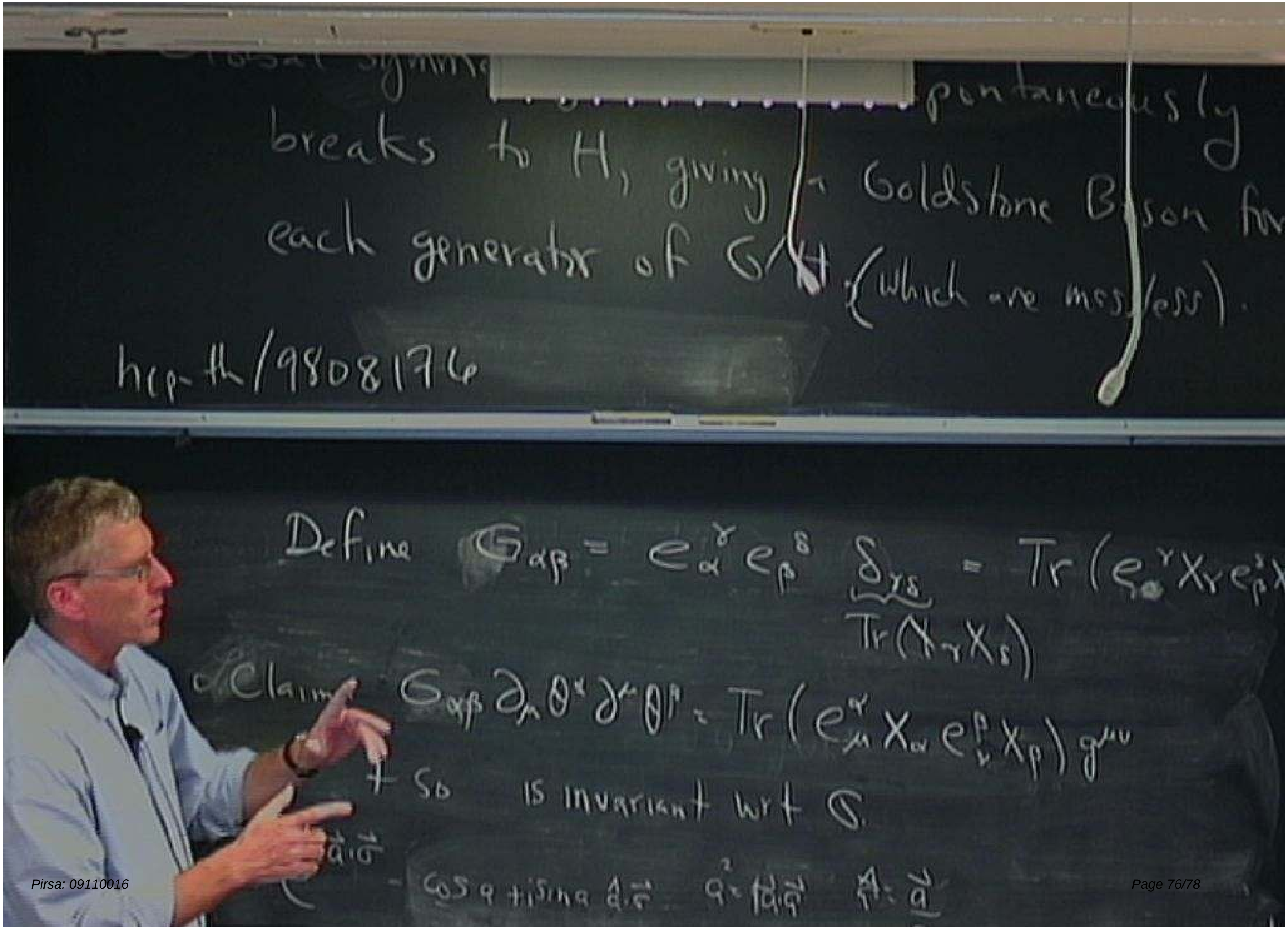
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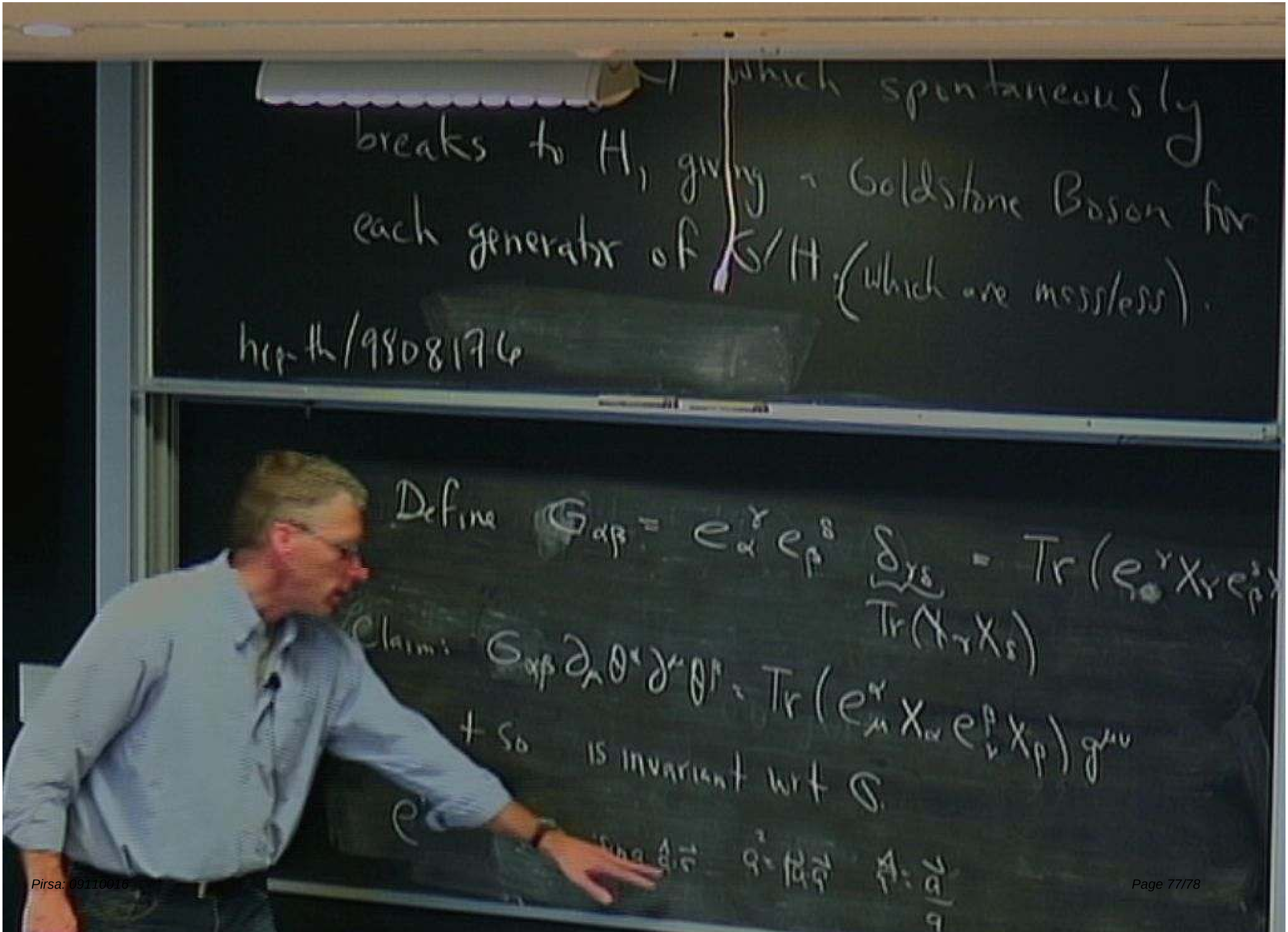
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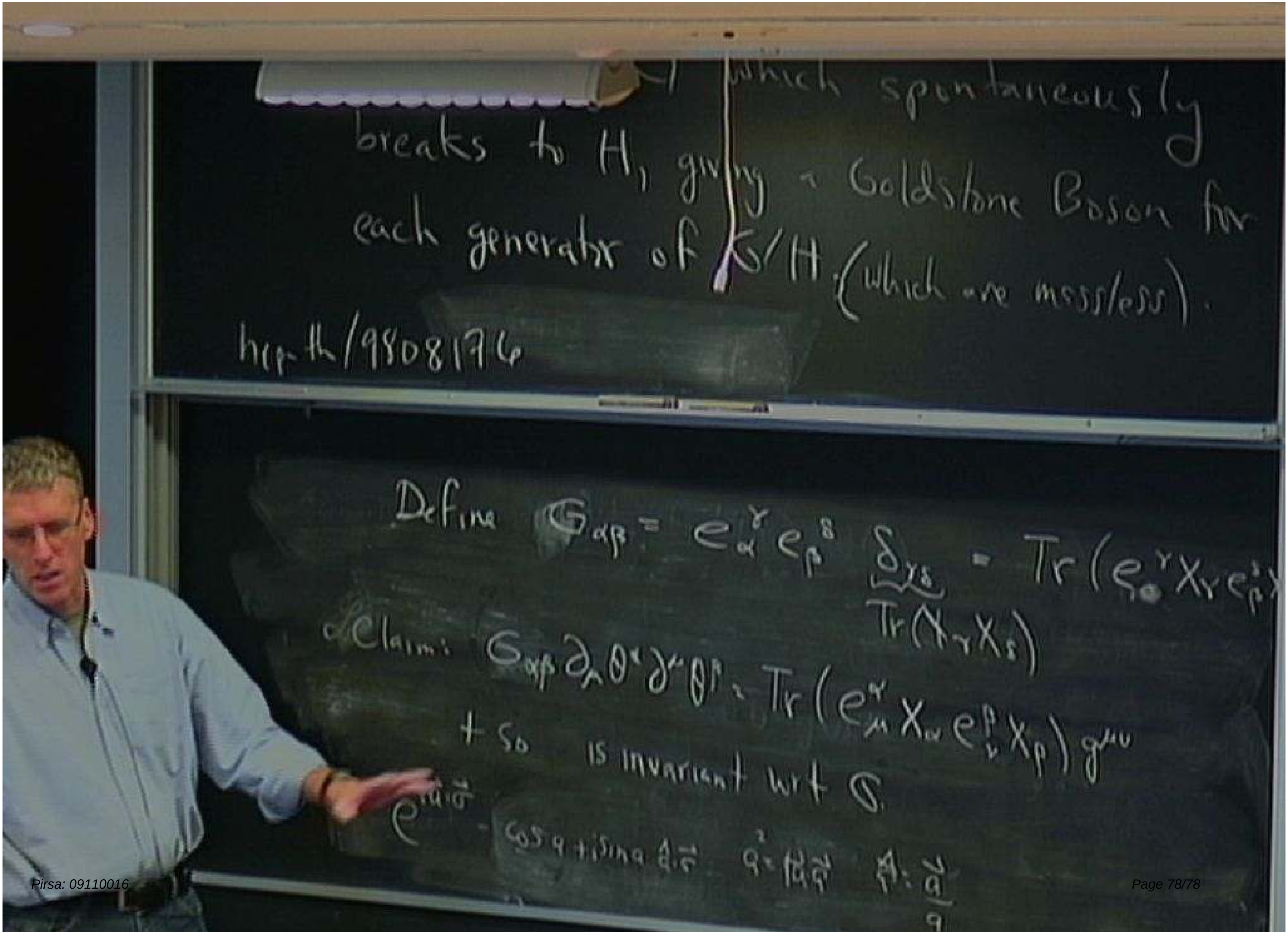
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