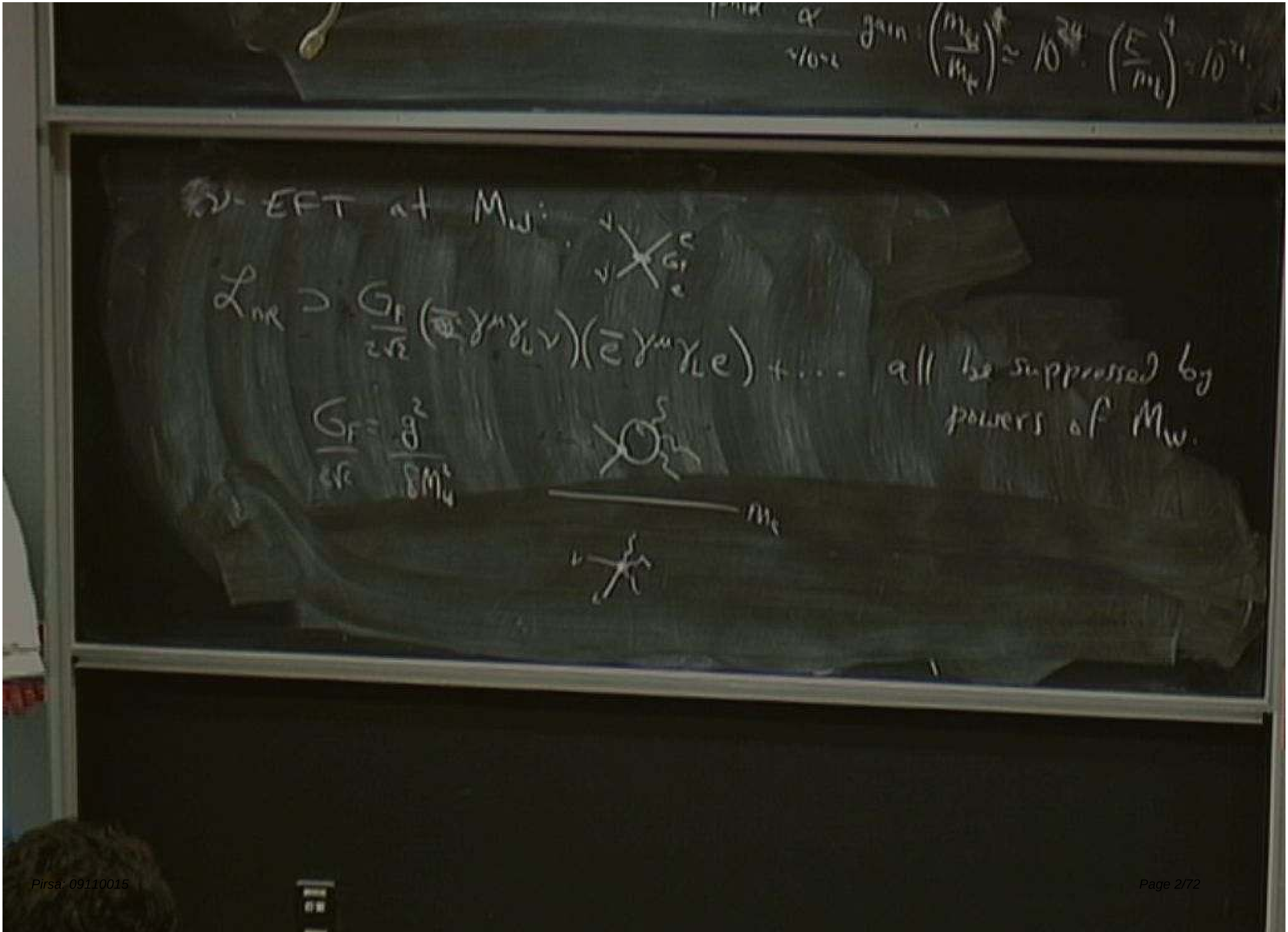
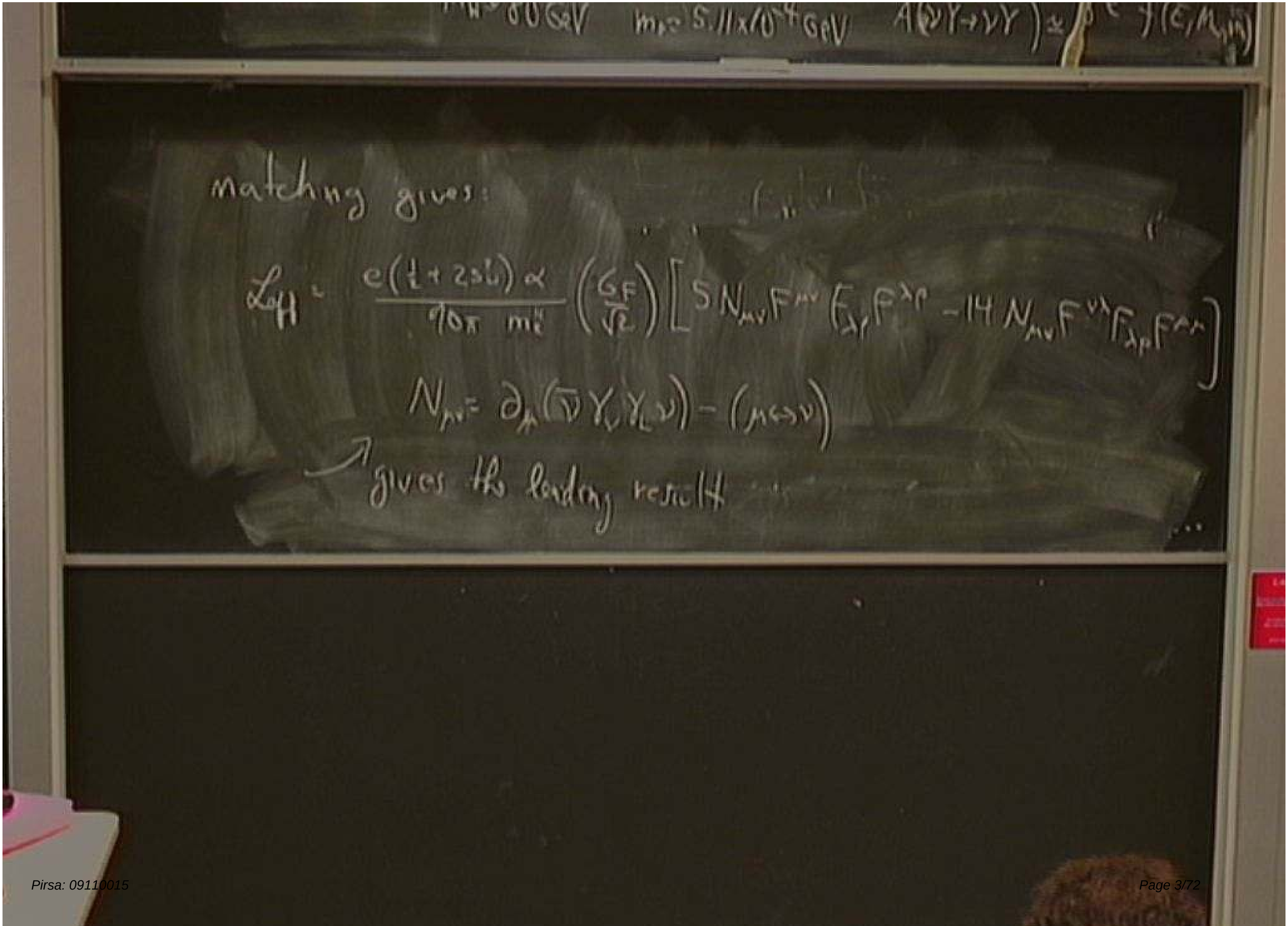
Title: Introduction to Effective Field Theory - Lecture 7B

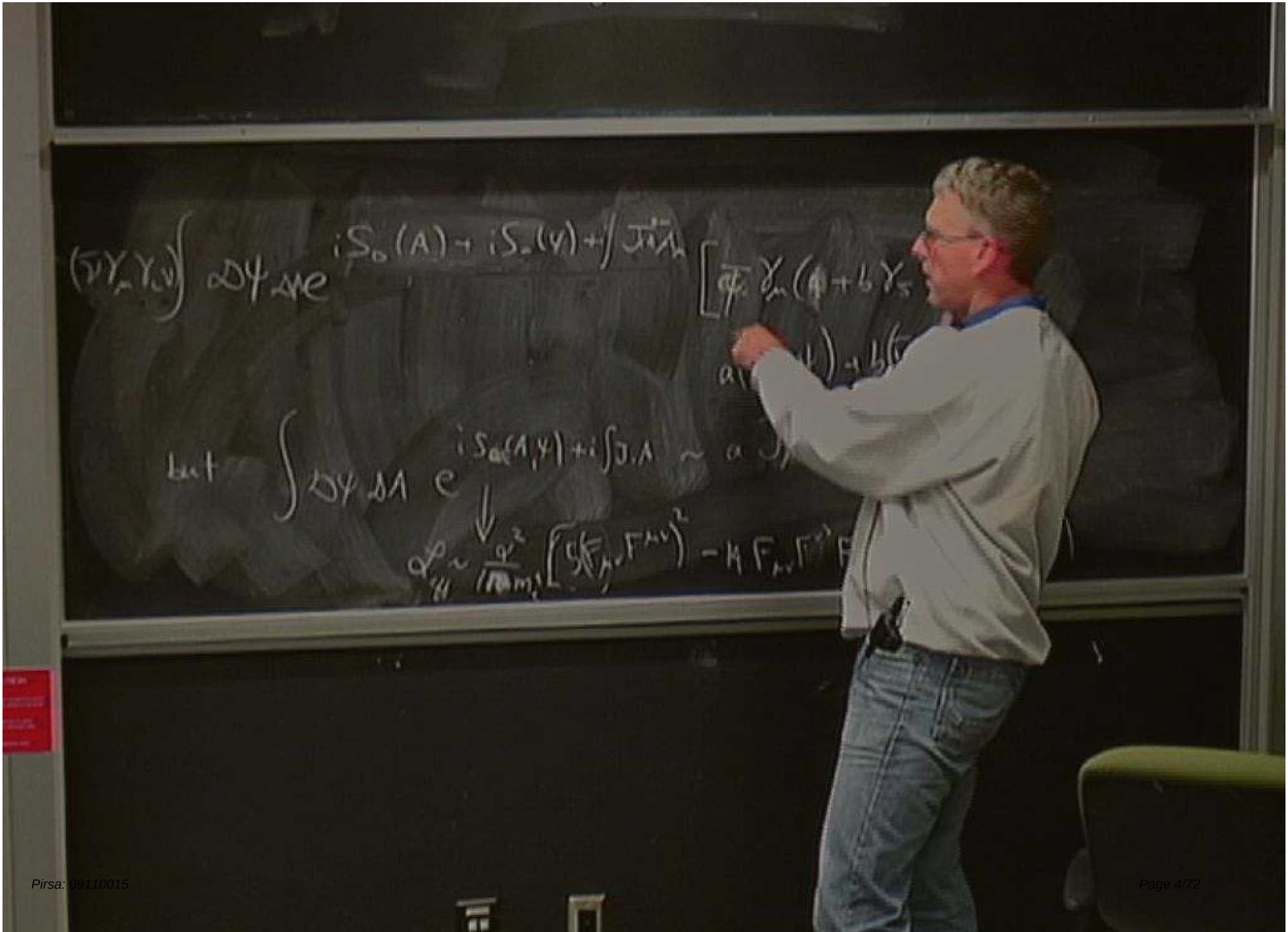
Date: Nov 04, 2009 03:30 PM

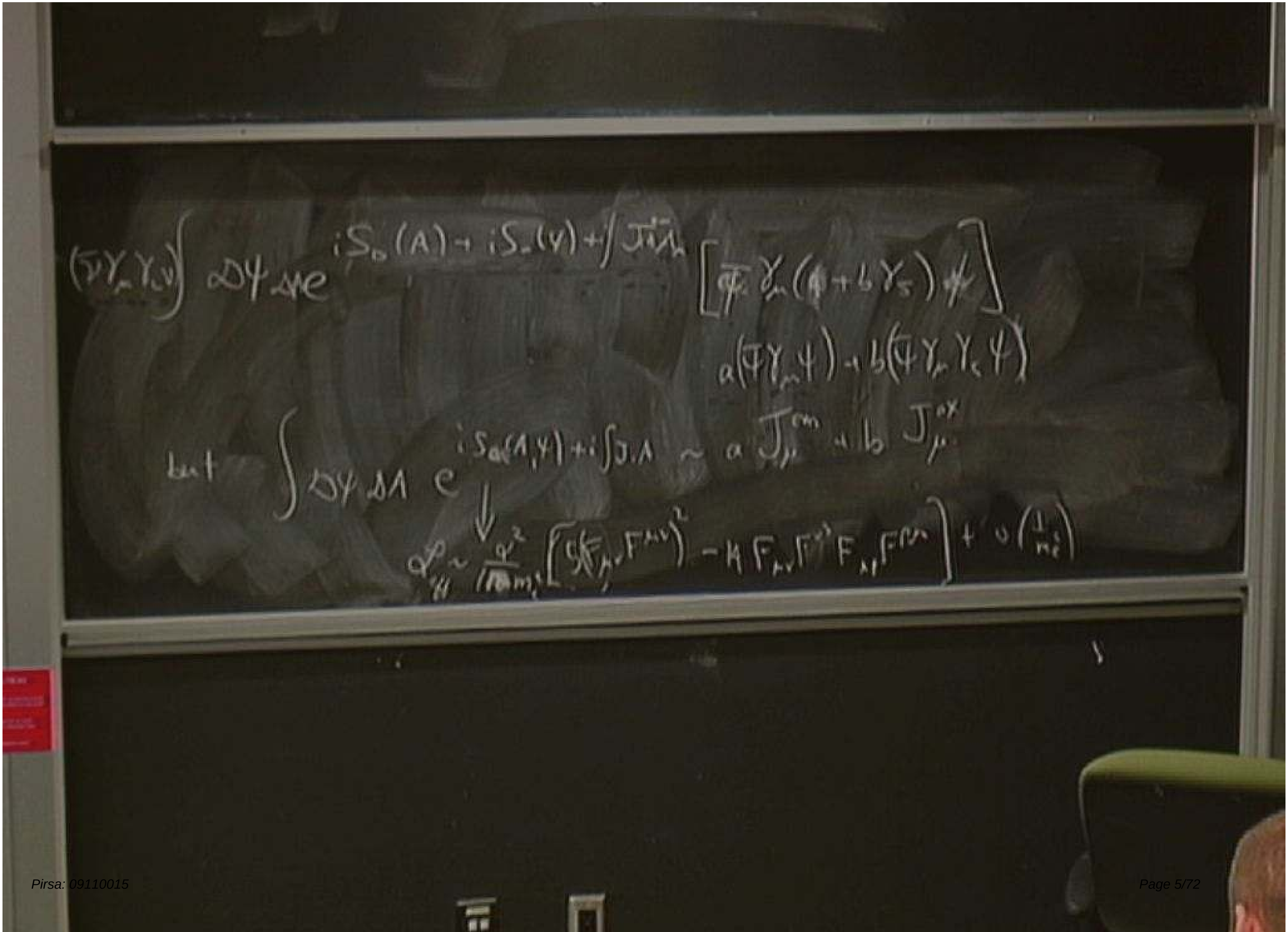
URL: http://pirsa.org/09110015

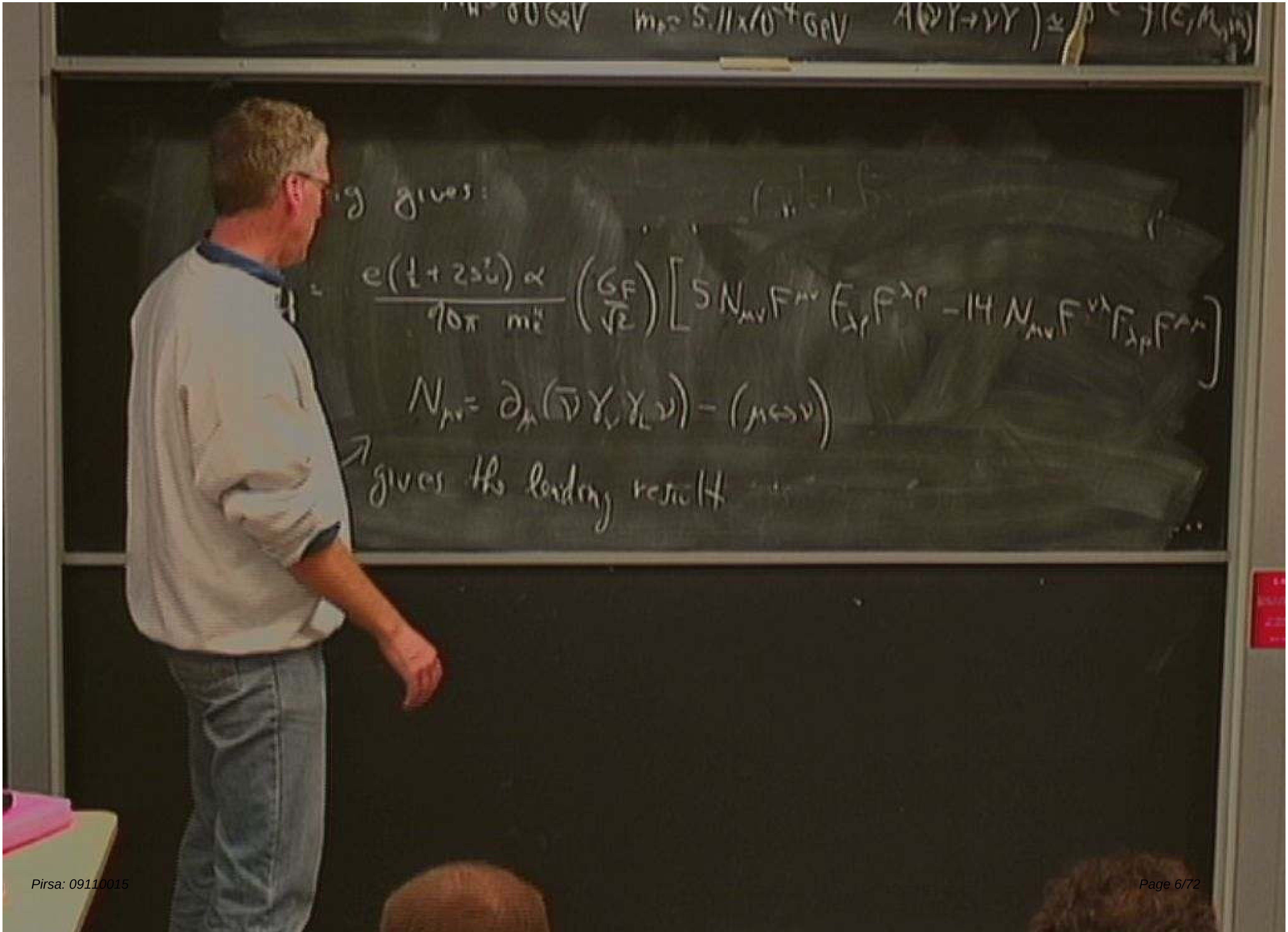
Abstract:


EFT at M_W:
L_NR ⊃ G_F/2√2 (ν̄ γ^μ γ_L ν)(ē γ^μ γ_L e) + . . . all be suppressed by powers of M_W.
G_F/√2 = g^2/8M_W^2
m_e


$m_e = 5.11\times10^{-4}$ GeV
matching gives:
$\mathcal{L}_H = \frac{e(\frac{1}{2}+2s_W^2)\alpha}{90\pi\, m_e^4}\left(\frac{G_F}{\sqrt{2}}\right)\left[5 N_{\mu\nu}F^{\mu\nu}F_{\lambda\rho}F^{\lambda\rho} - 14 N_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$
$N_{\mu\nu} = \partial_\mu(\bar{\nu}\gamma_\nu\gamma_L\nu) - (\mu\leftrightarrow\nu)$
gives the leading result ...




but



$m_\nu \simeq 5.11 \times 10^{-4}\,\mathrm{GeV}$
g gives:
$\frac{e(\frac{1}{2}+2s_w^2)\alpha}{90\pi\, m_e^4}\left(\frac{G_F}{\sqrt{2}}\right)\left[5 N_{\mu\nu}F^{\mu\nu}F_{\lambda\rho}F^{\lambda\rho} - 14 N_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu}\right]$
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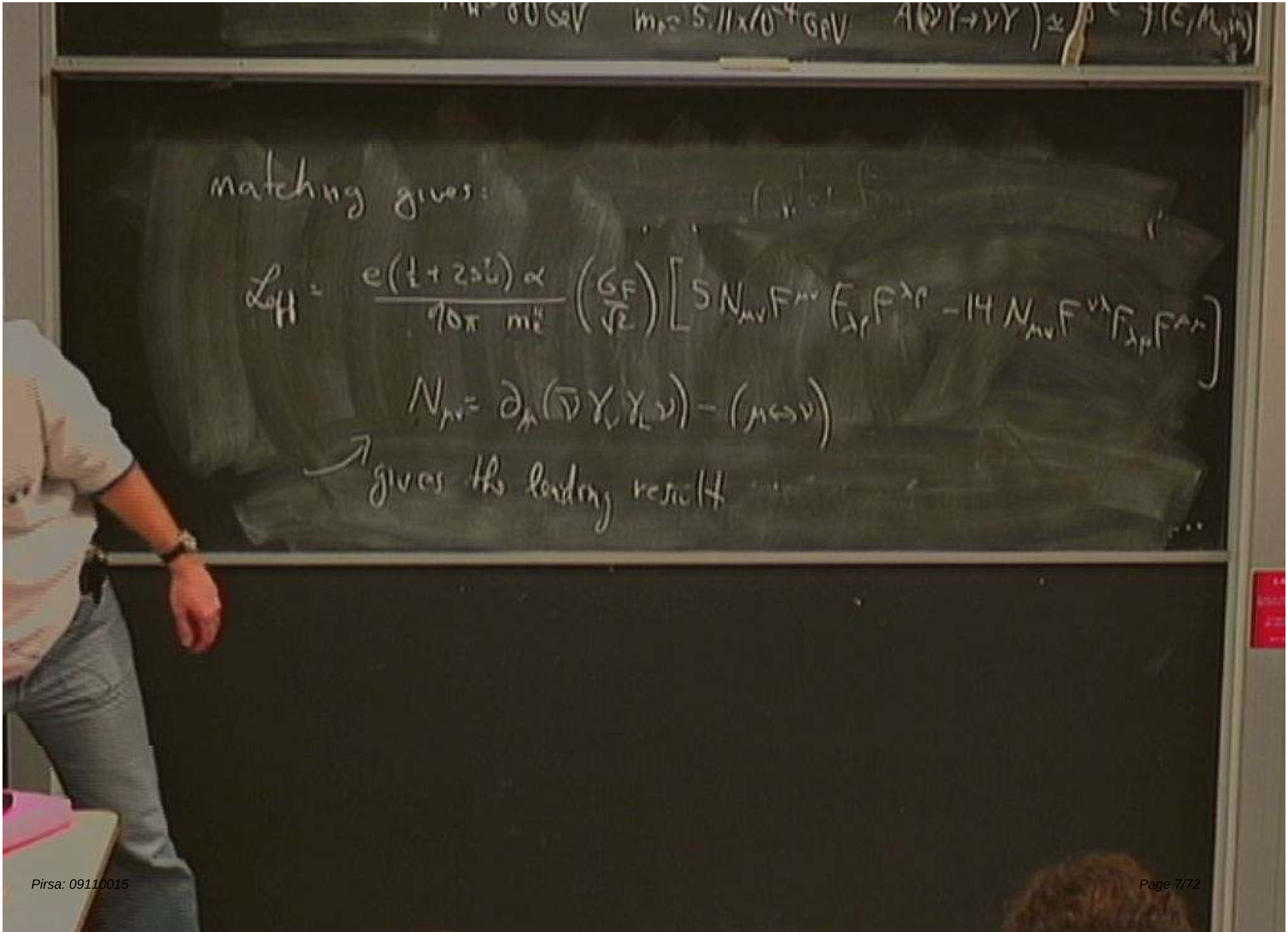
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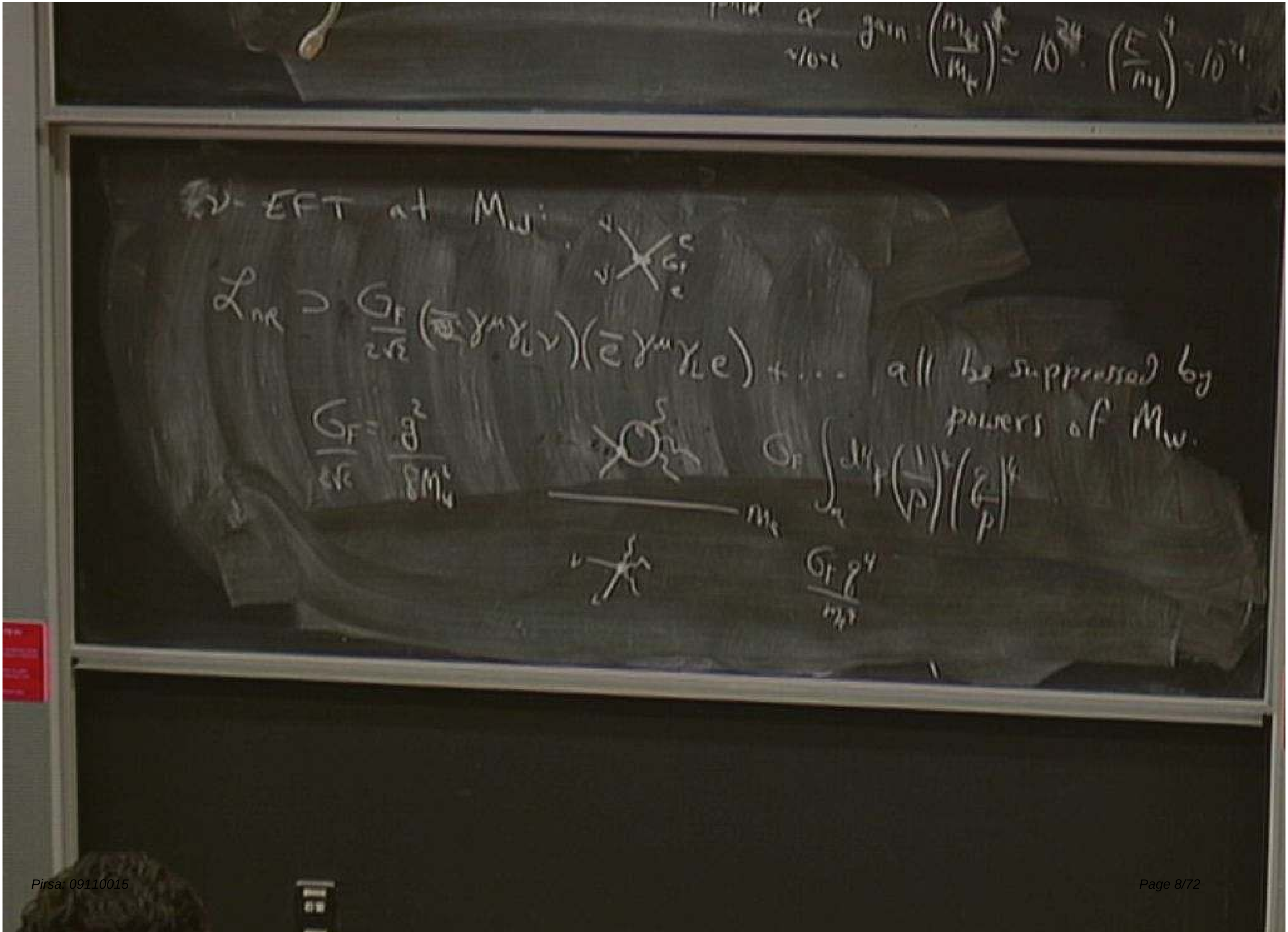
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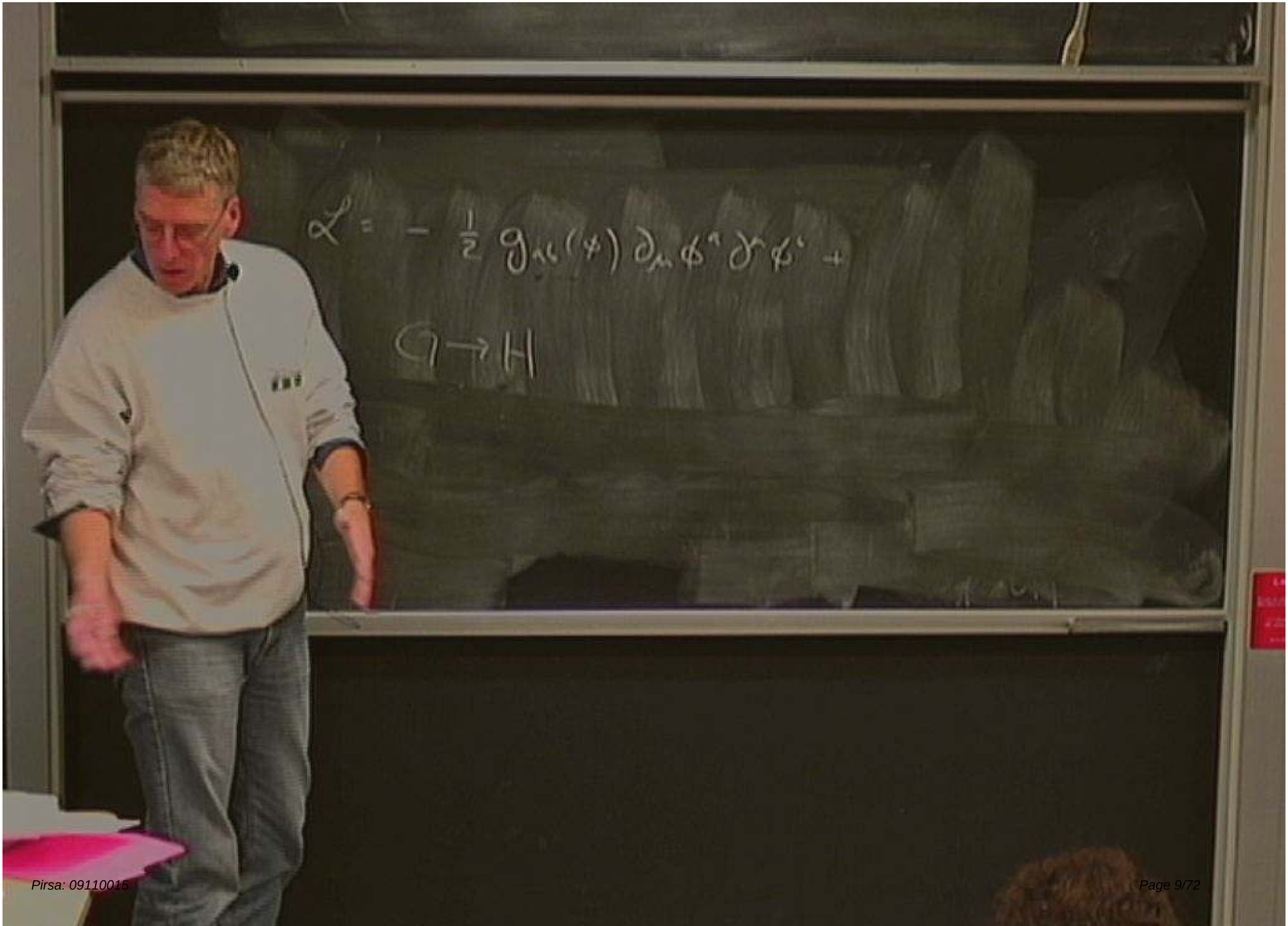
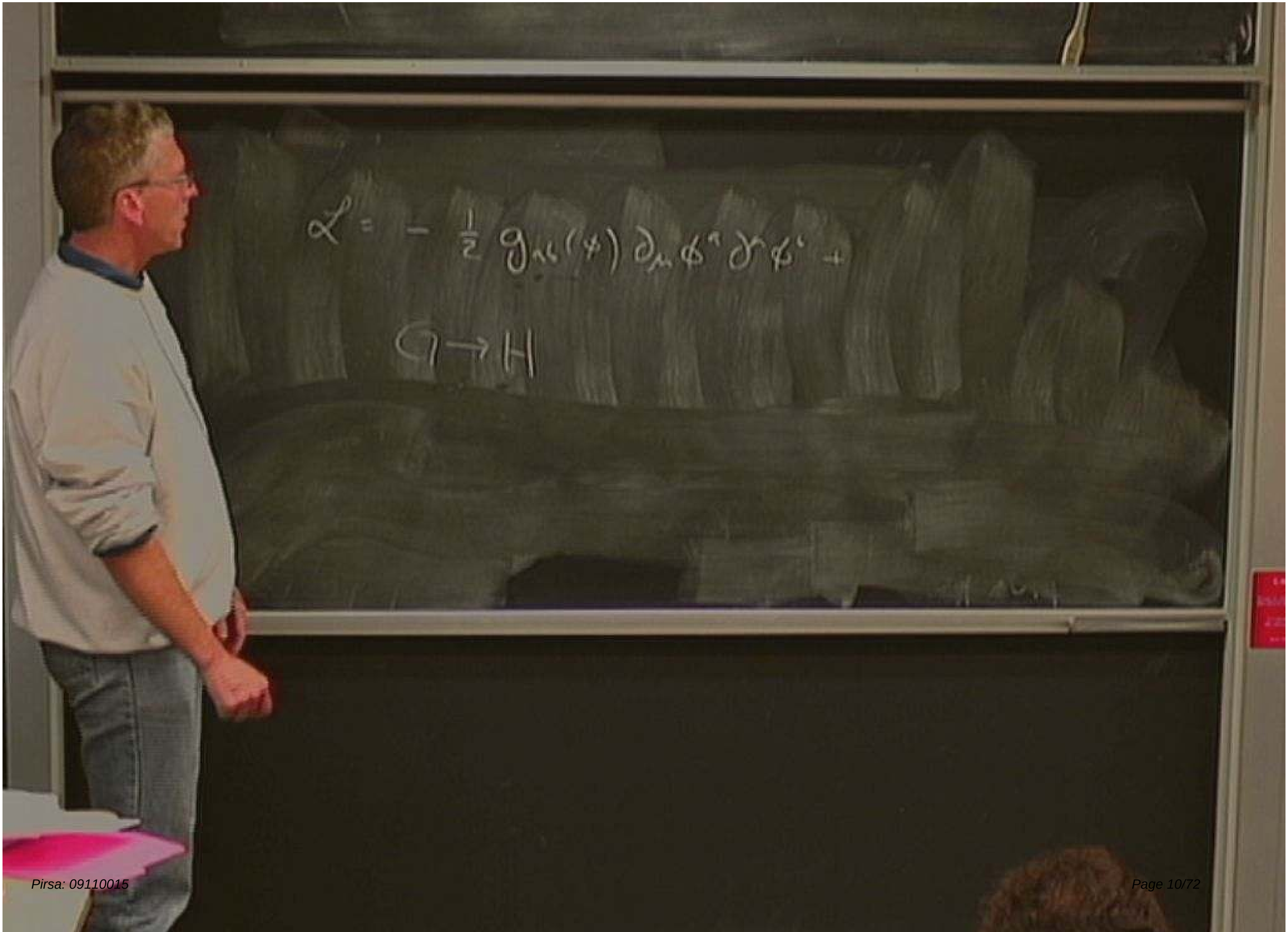
L_nR ⊃ G_F/2√2 (ν̄ γ^μ γ_L ν)(ē γ^μ γ_L e) + . . . all be suppressed by powers of M_W.
G_F/√2 = g²/8M_W²


$\mathcal{L} = -\frac{1}{2} g_{ab}(\phi)\, \partial_\mu \phi^a \partial^\mu \phi^b +$
$G \to H$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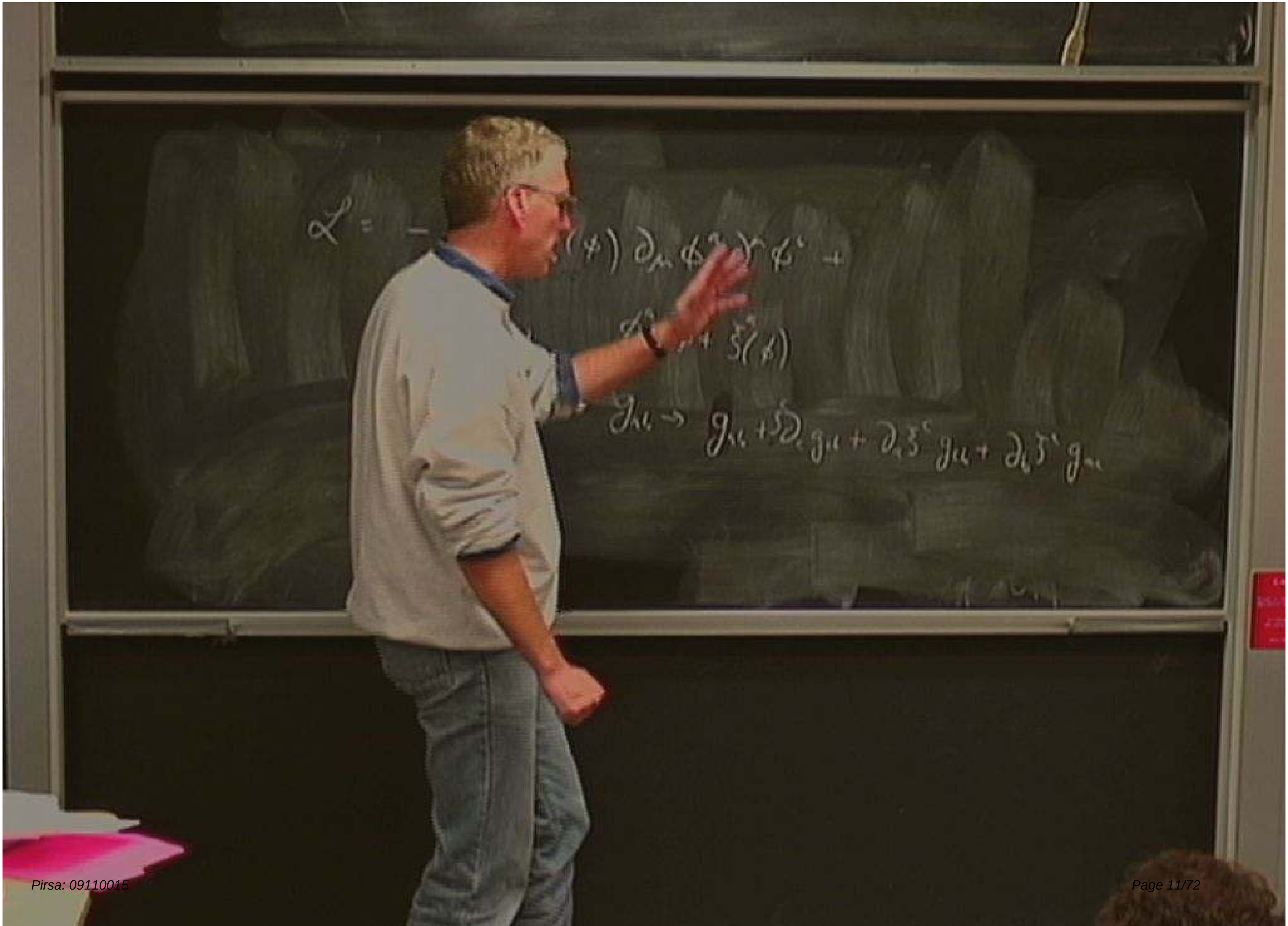
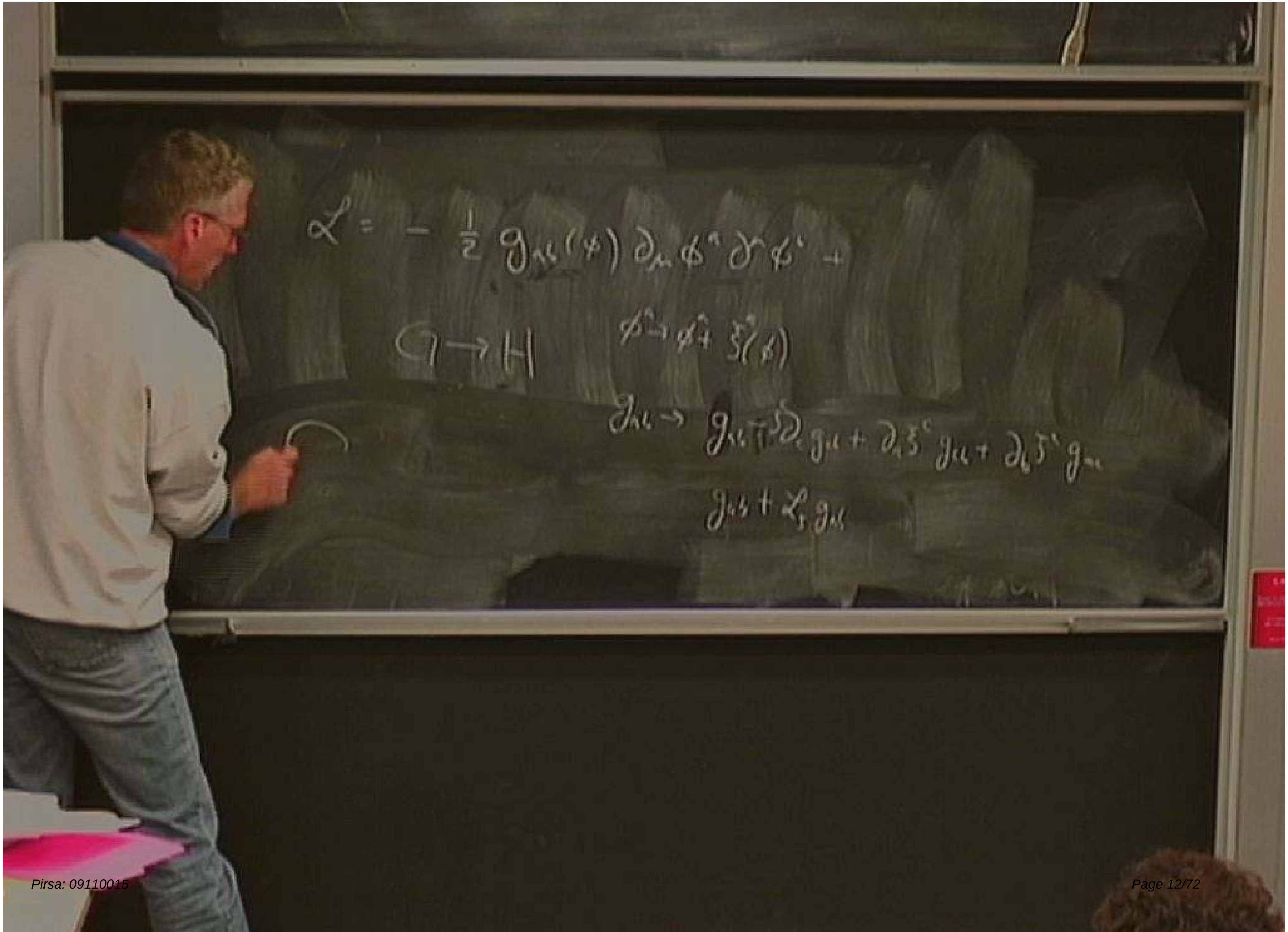
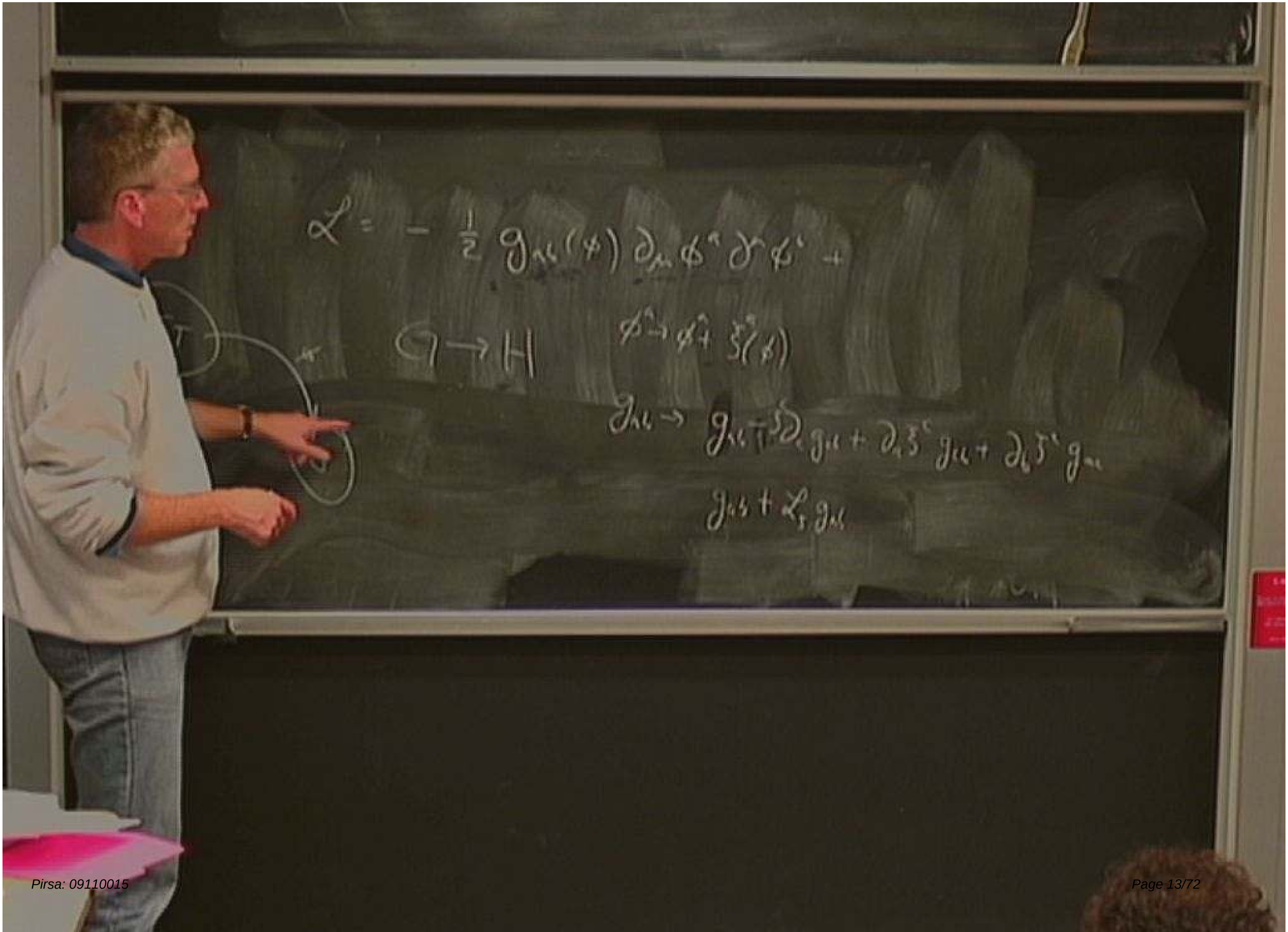
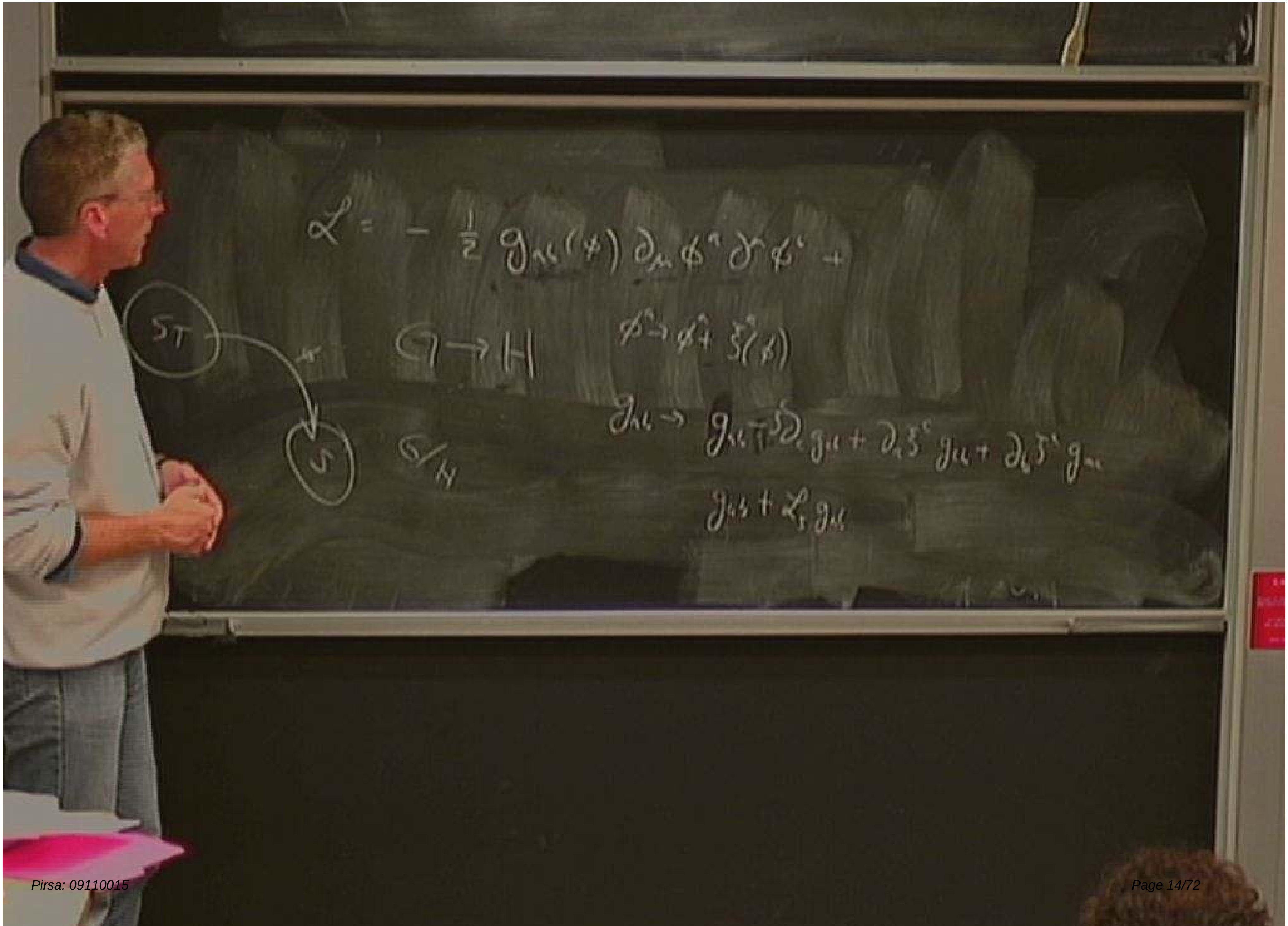




$\mathcal{L} = -\frac{1}{2} g_{ab}(\phi)\, \partial_\mu \phi^a \partial^\mu \phi^b +$
$G \to H$
$\phi^a \to \phi^a + \xi^a(\phi)$
$g_{ab} \to g_{ab} + \xi^c \partial_c g_{ab} + \partial_a \xi^c g_{cb} + \partial_b \xi^c g_{ac}$
$g_{ab} + \mathcal{L}_\xi g_{ab}$


L = − ½ g_ab(φ) ∂_μ φ^a ∂^μ φ^b +
G → H
φ^a → φ^a + ξ^a(φ)


L = − ½ g_ab(φ) ∂_μ φ^a ∂^μ φ^b +
ST
G → H
φ^a → φ^a + ξ^a(φ)
S
G/H
g_ab → g_ab + ξ^c ∂_c g_ab + ∂_a ξ^c g_cb + ∂_b ξ^c g_ac
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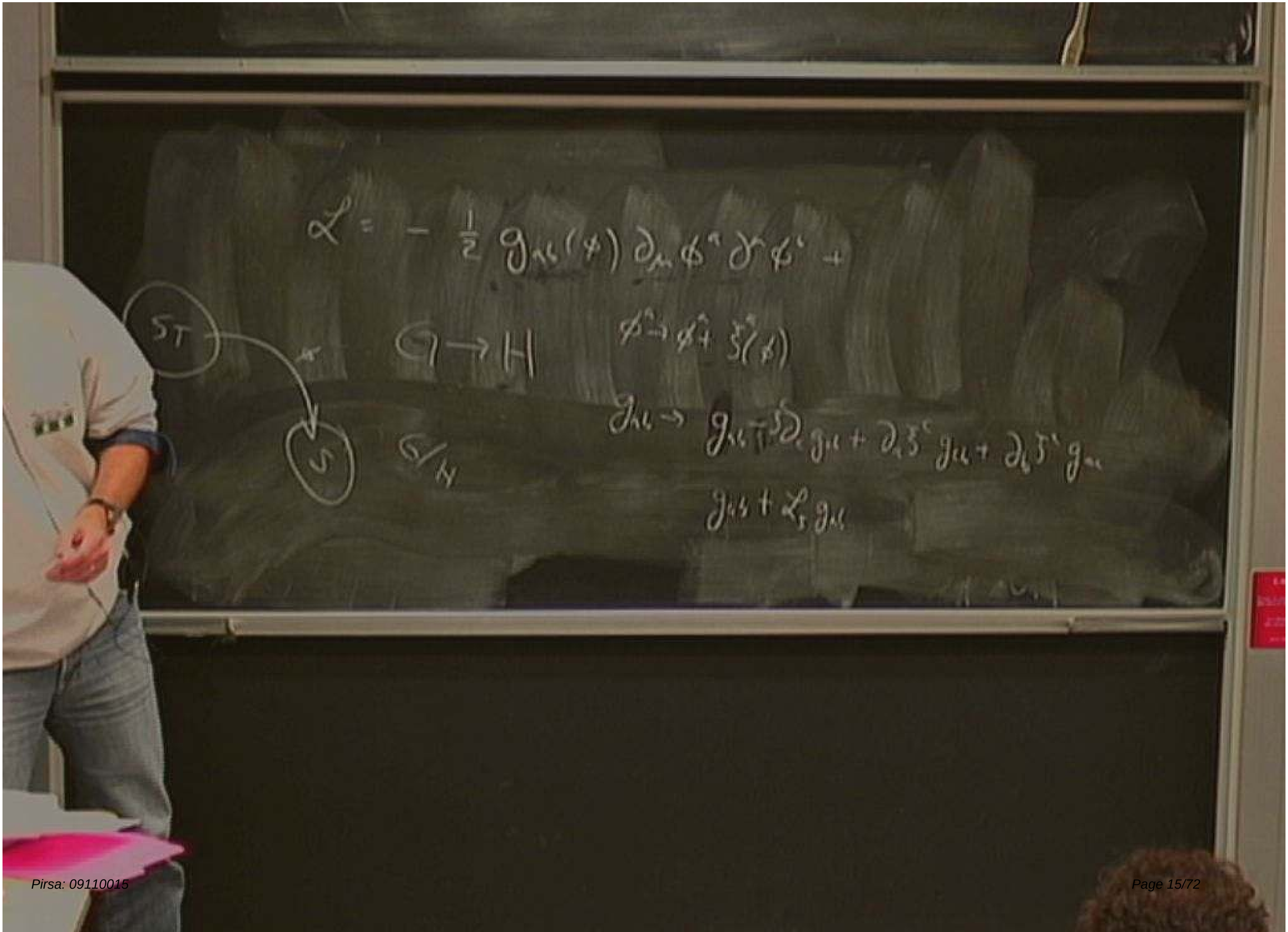

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S_T
$G \to H$
$\phi^a \to \phi^a + \xi^a(\phi)$
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G/H
$g_{ab} \to g_{ab} + \xi^c \partial_c g_{ab} + \partial_a \xi^c g_{cb} + \partial_b \xi^c g_{ac}$
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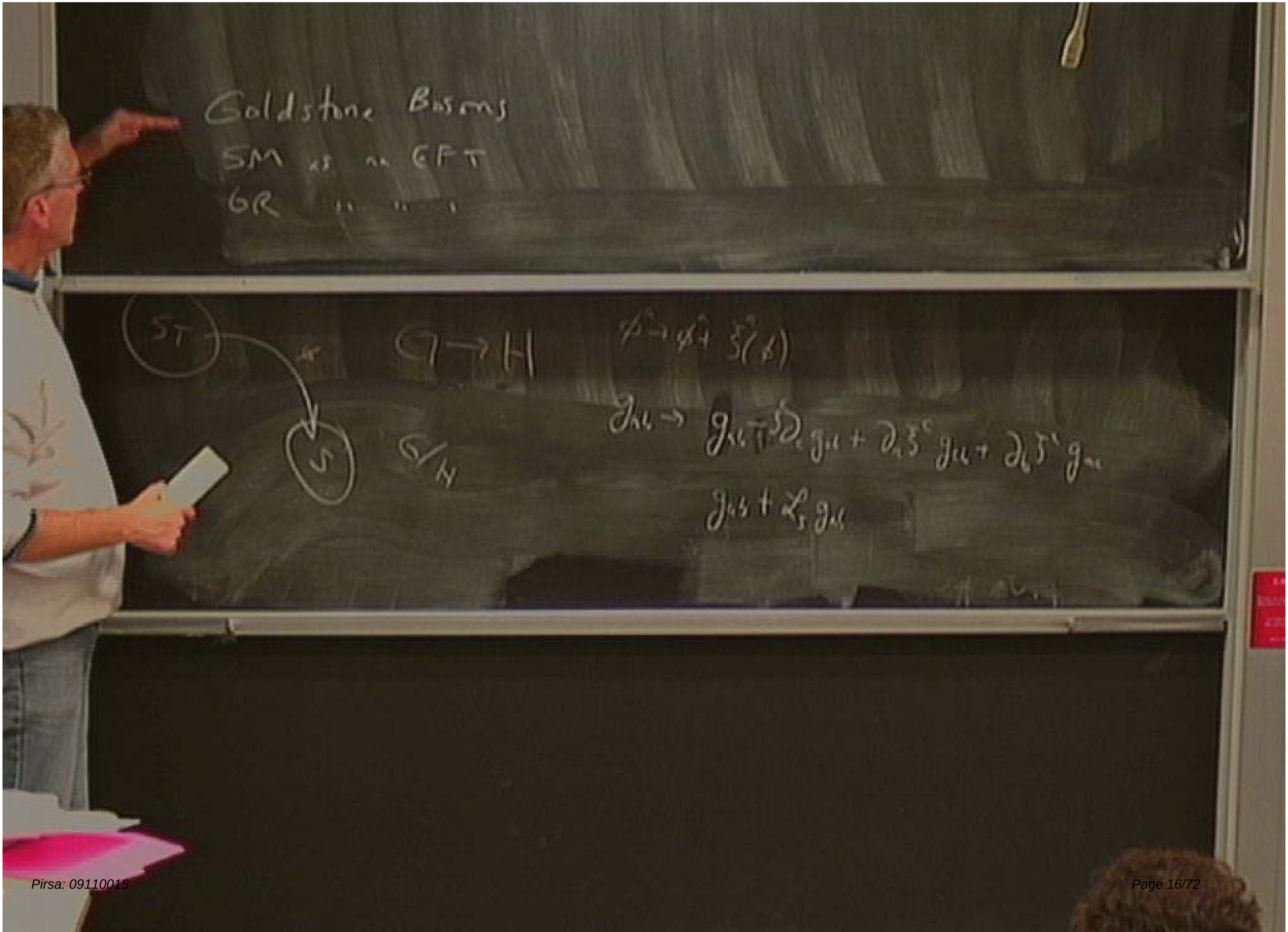
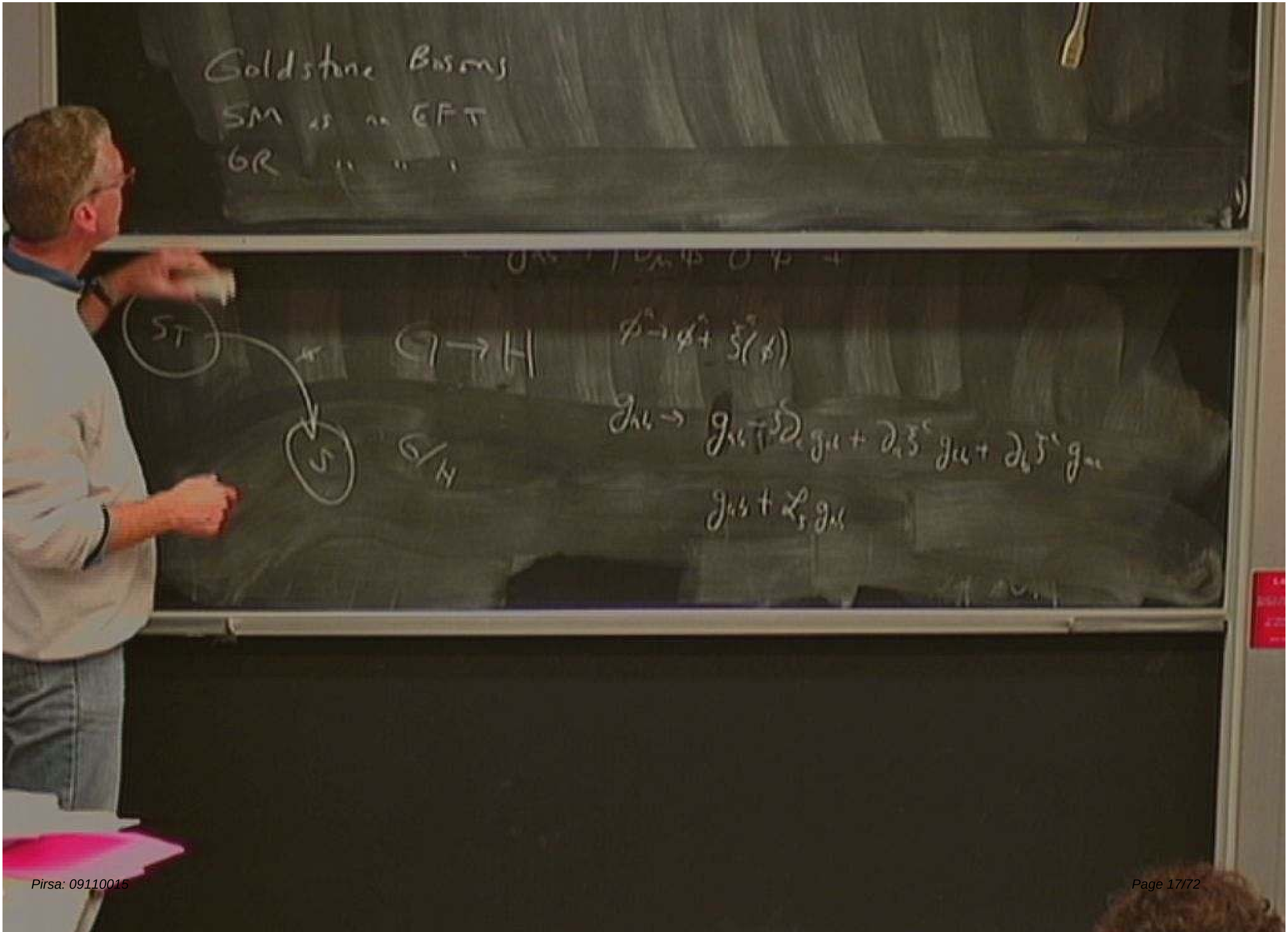

Goldstone Bosons
SM as an EFT
GR
G → H
G/H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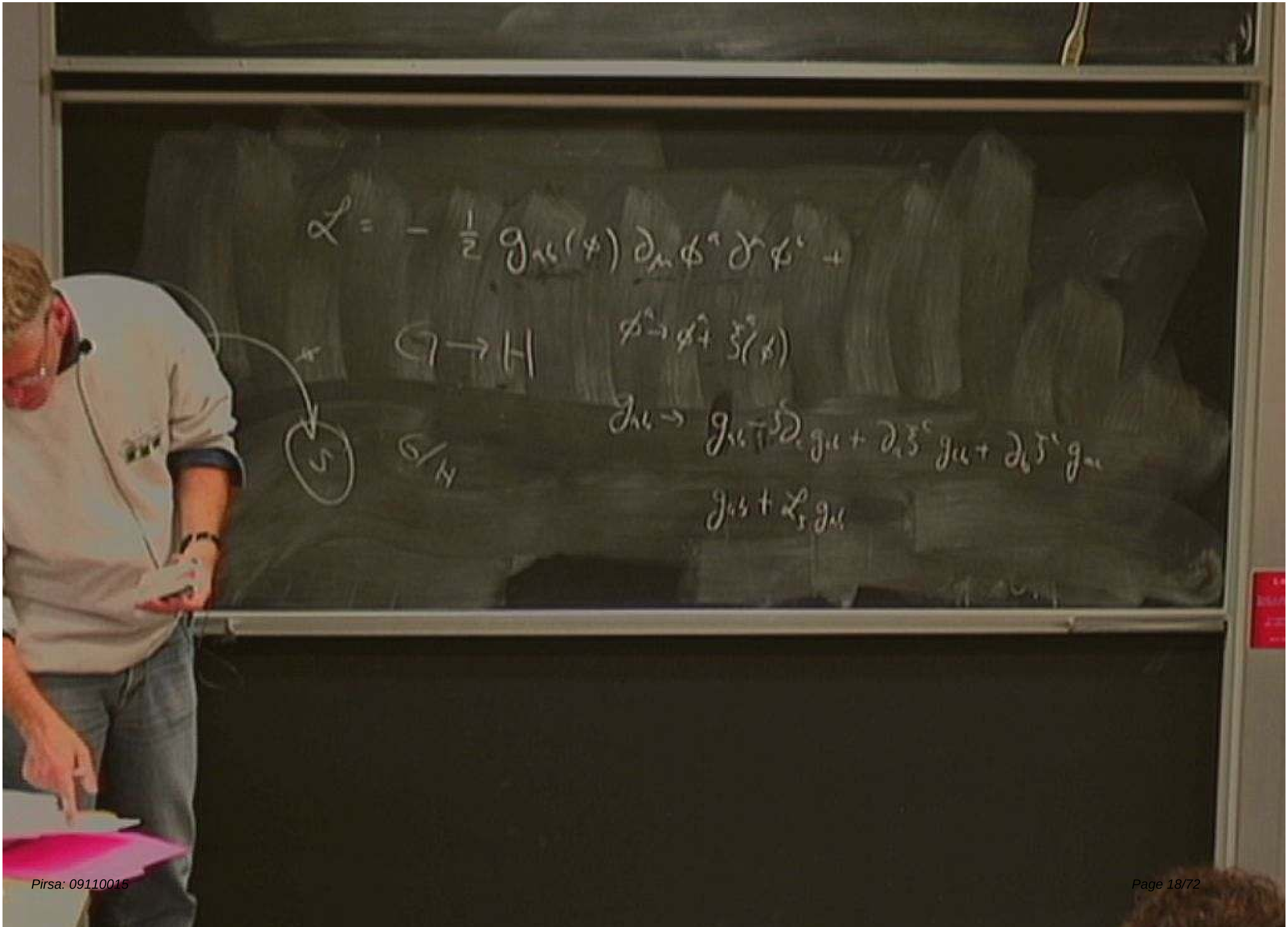
L = − 1/2 g_ab(φ) ∂_μ φ^a ∂^μ φ^b +
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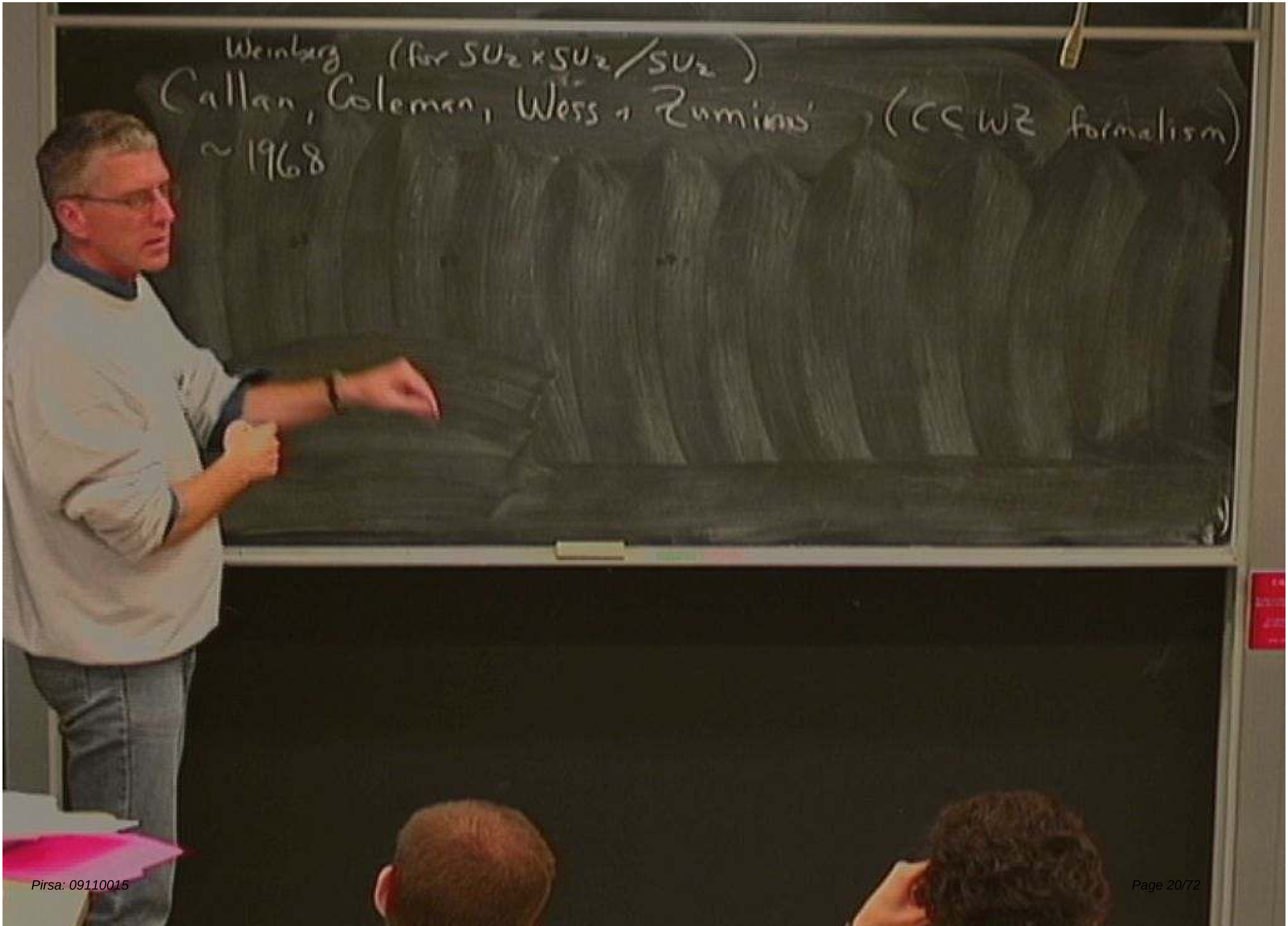
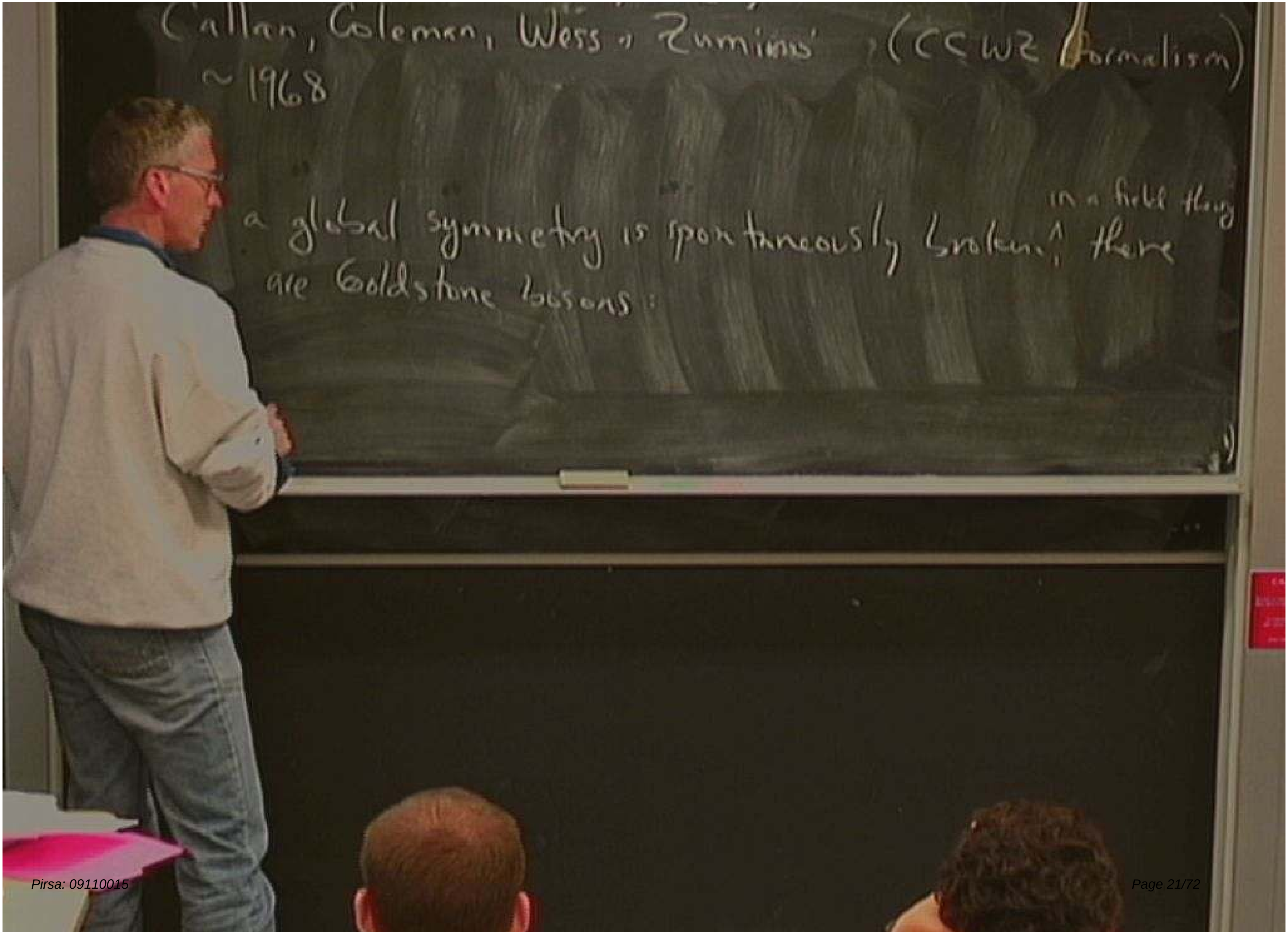
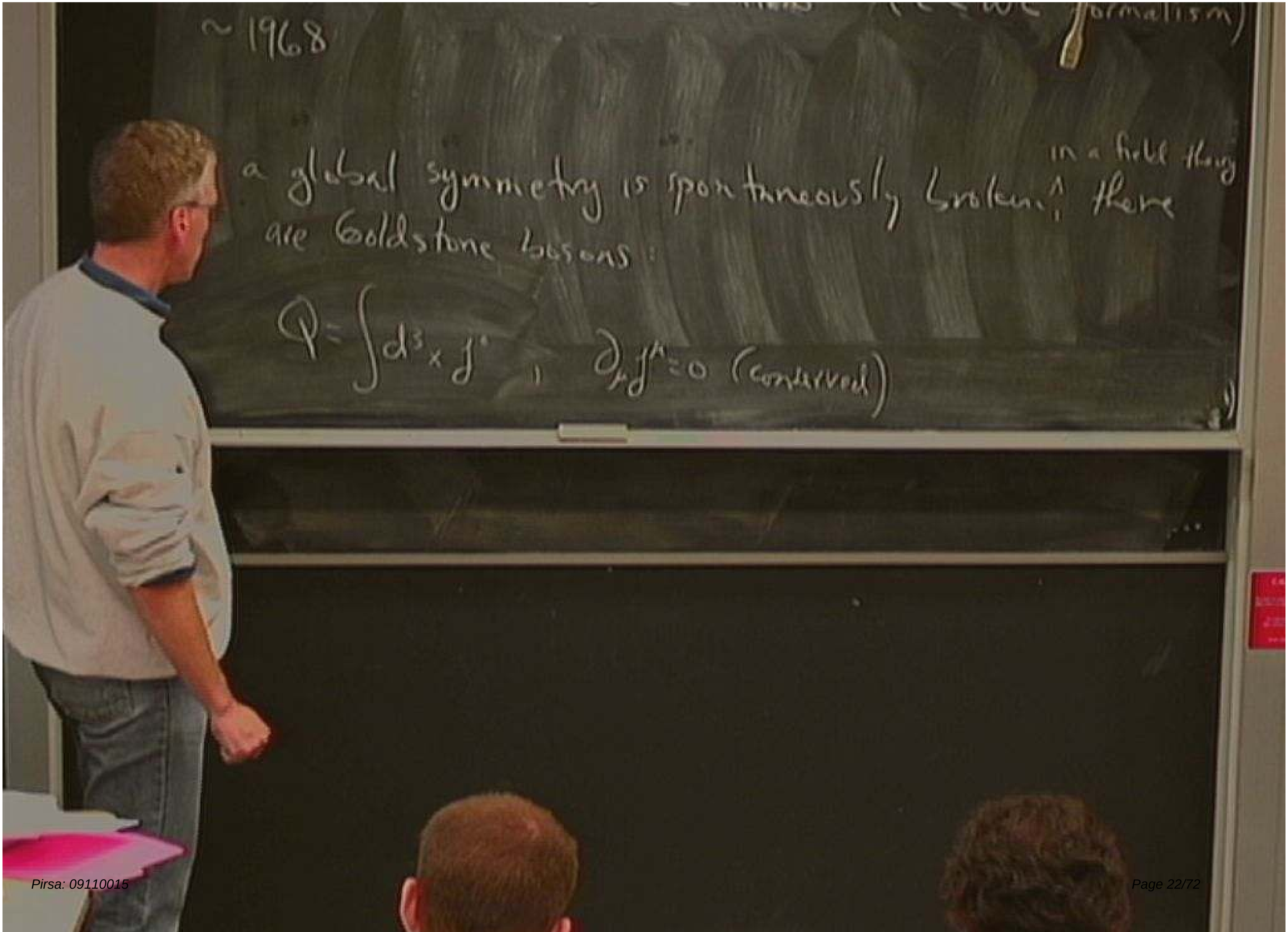
g_ab → g_ab + ξ^c ∂_c g_ab + ∂_a ξ^c g_cb + ∂_b ξ^c g_ac
g_ab + L_ξ g_ab
S
G/H


Weinberg (for SU2 x SU2/SU2)
Callan, Coleman, Wess + Zumino
(CCWZ formalism)
~1968


Callan, Coleman, Wess & Zumino (CCWZ formalism)
~1968
a global symmetry is spontaneously broken in a field theory there are Goldstone bosons: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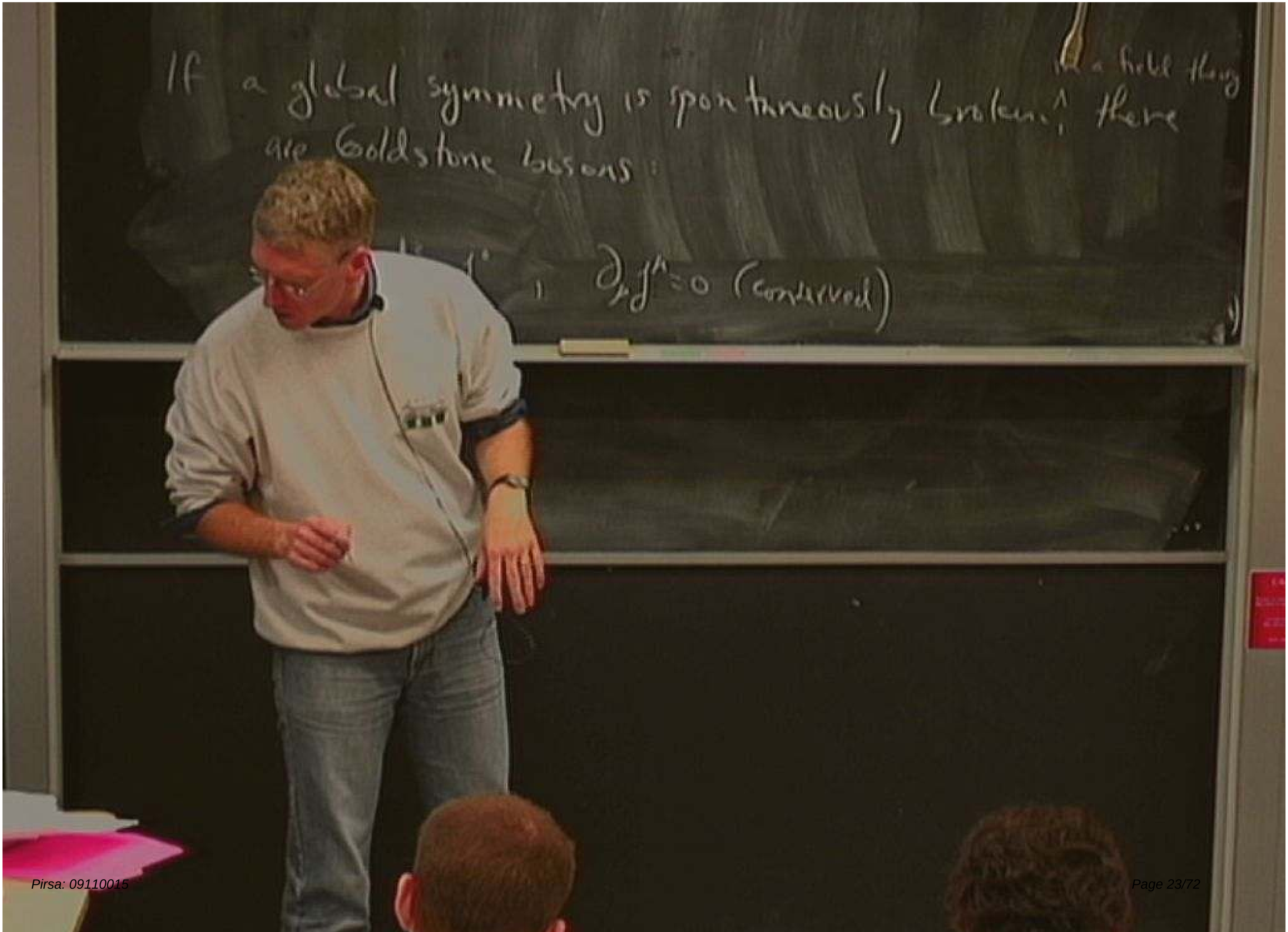
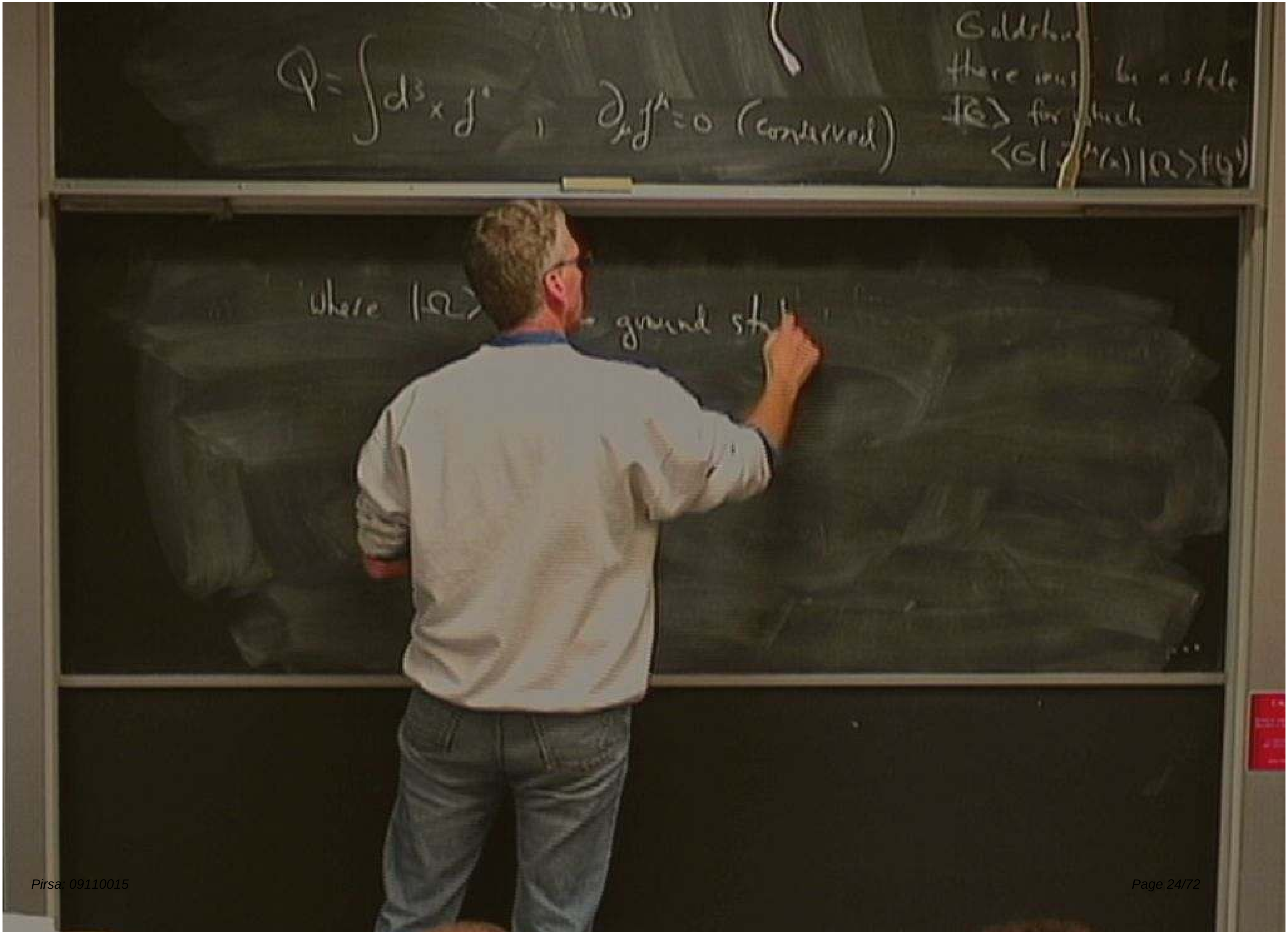
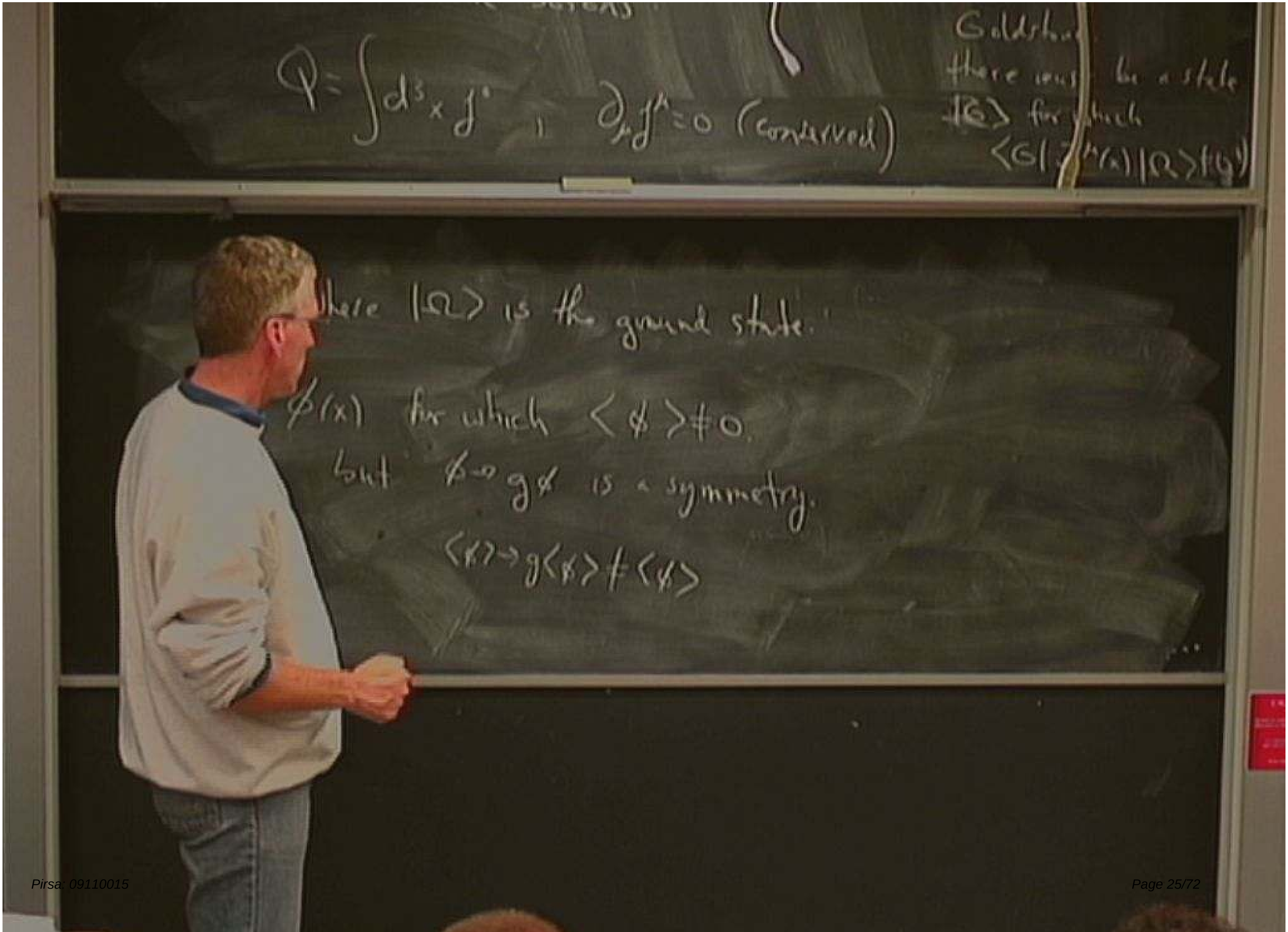
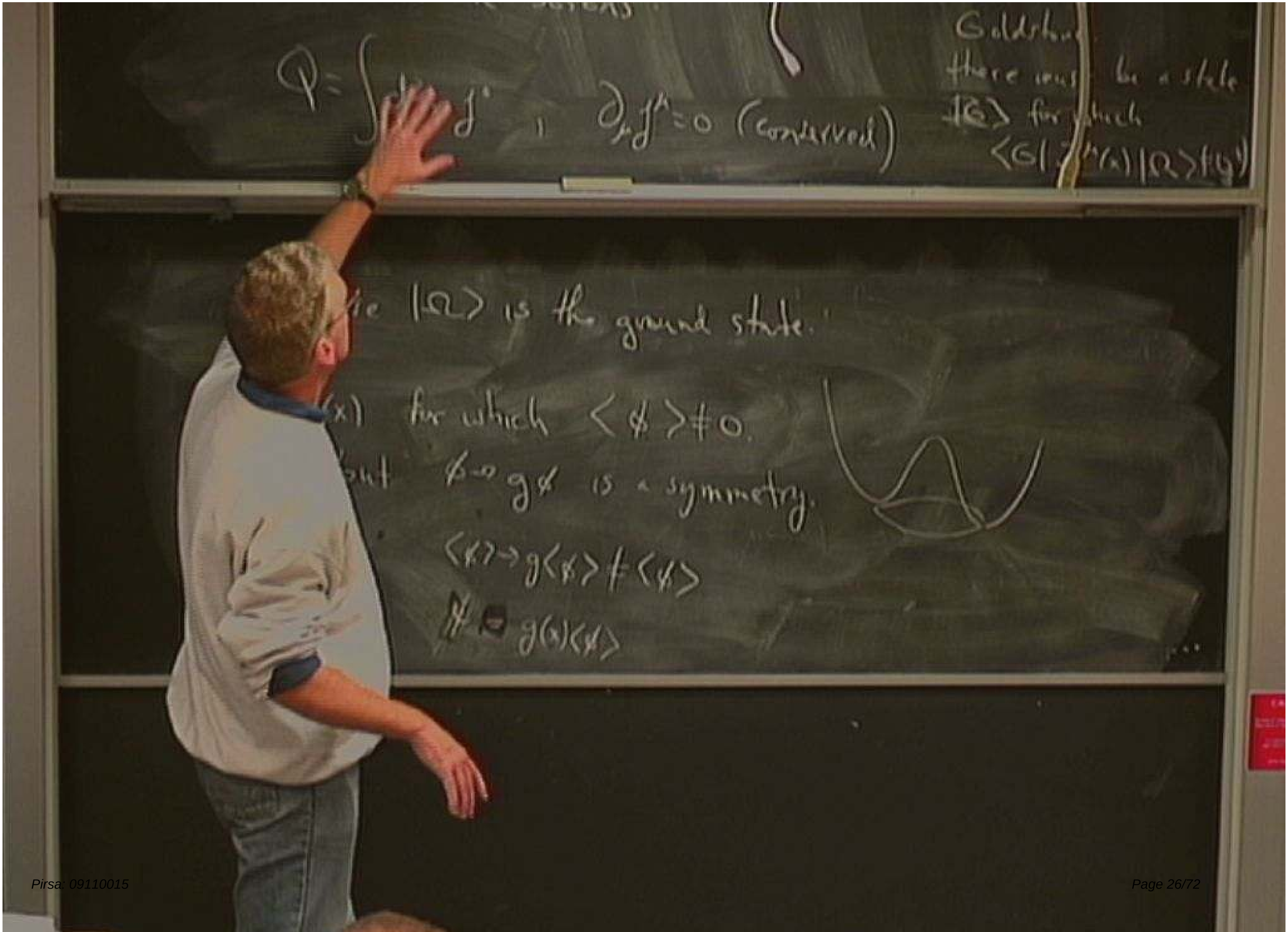
a global symmetry is spontaneously broken in a field theory there are Goldstone bosons:
$Q = \int d^3x\, j^0$, $\partial_\mu j^\mu = 0$ (conserved)


If a global symmetry is spontaneously broken, a field theory there
are Goldstone bosons:
$\partial_\mu j^\mu = 0$ (conserved)


$Q = \int d^3x\, j^0$, $\partial_\mu j^\mu = 0$ (conserved)
there must be a state
where $|\Omega\rangle$
ground


Q = ∫d³x j⁰ , ∂μ j^μ = 0 (conserved)
here |Ω> is the ground state.
φ(x) for which <φ> ≠ 0.
but φ → gφ is a symmetry.
<φ> → g<φ> ≠ <φ>


(conserved)
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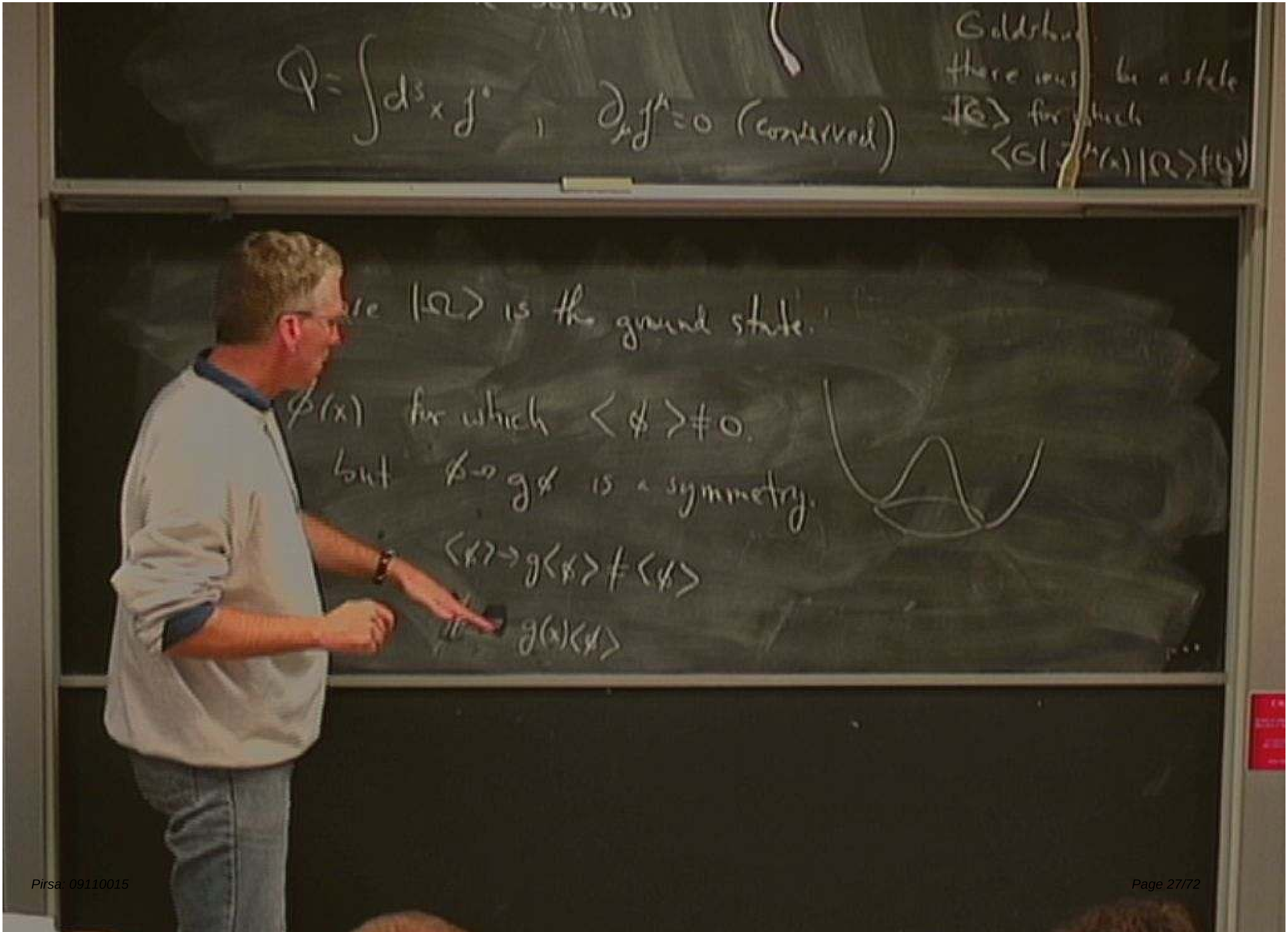
Q = ∫d³x j⁰ , ∂_μ j^μ = 0 (conserved)
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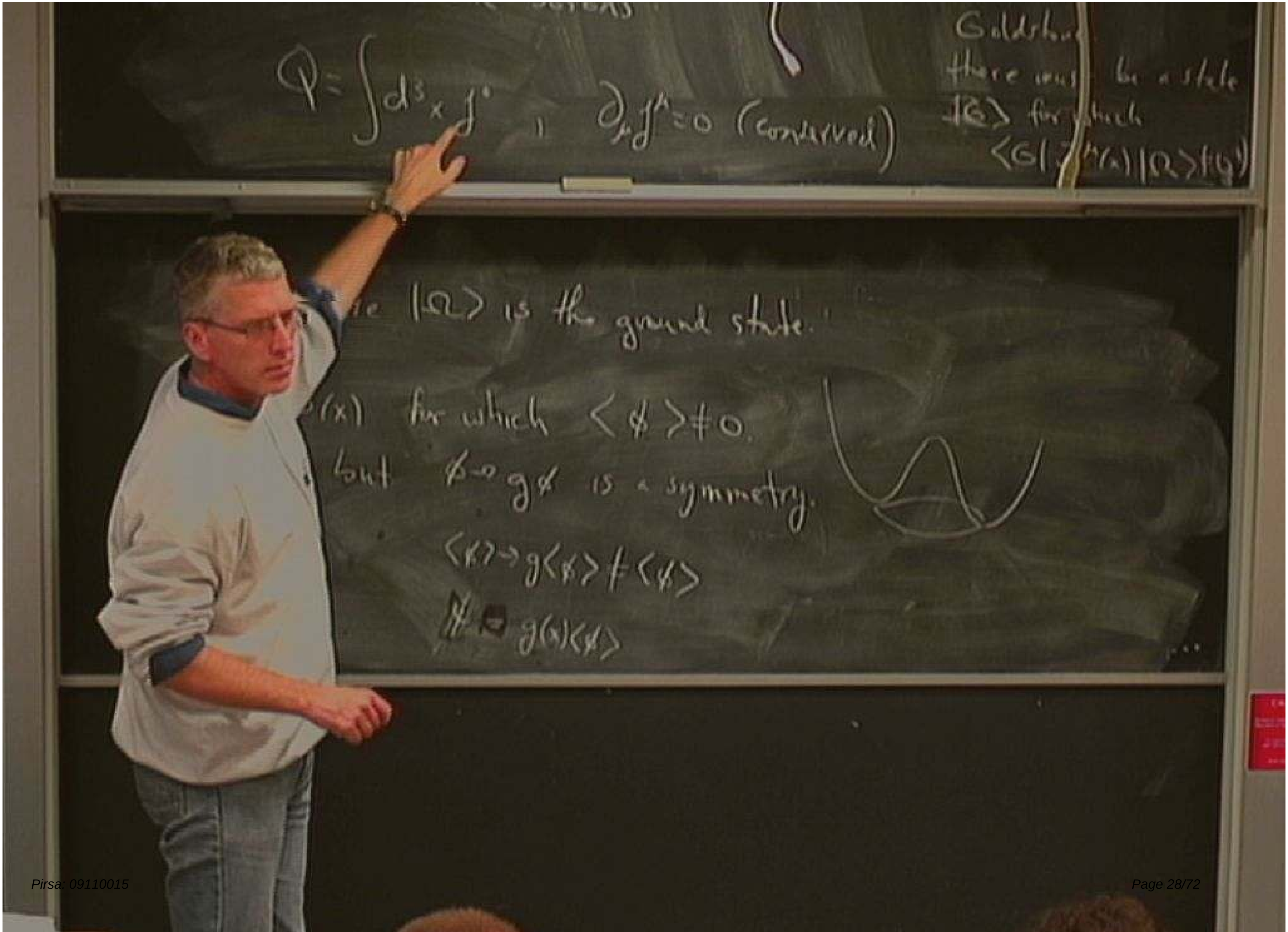
g(x)⟨φ⟩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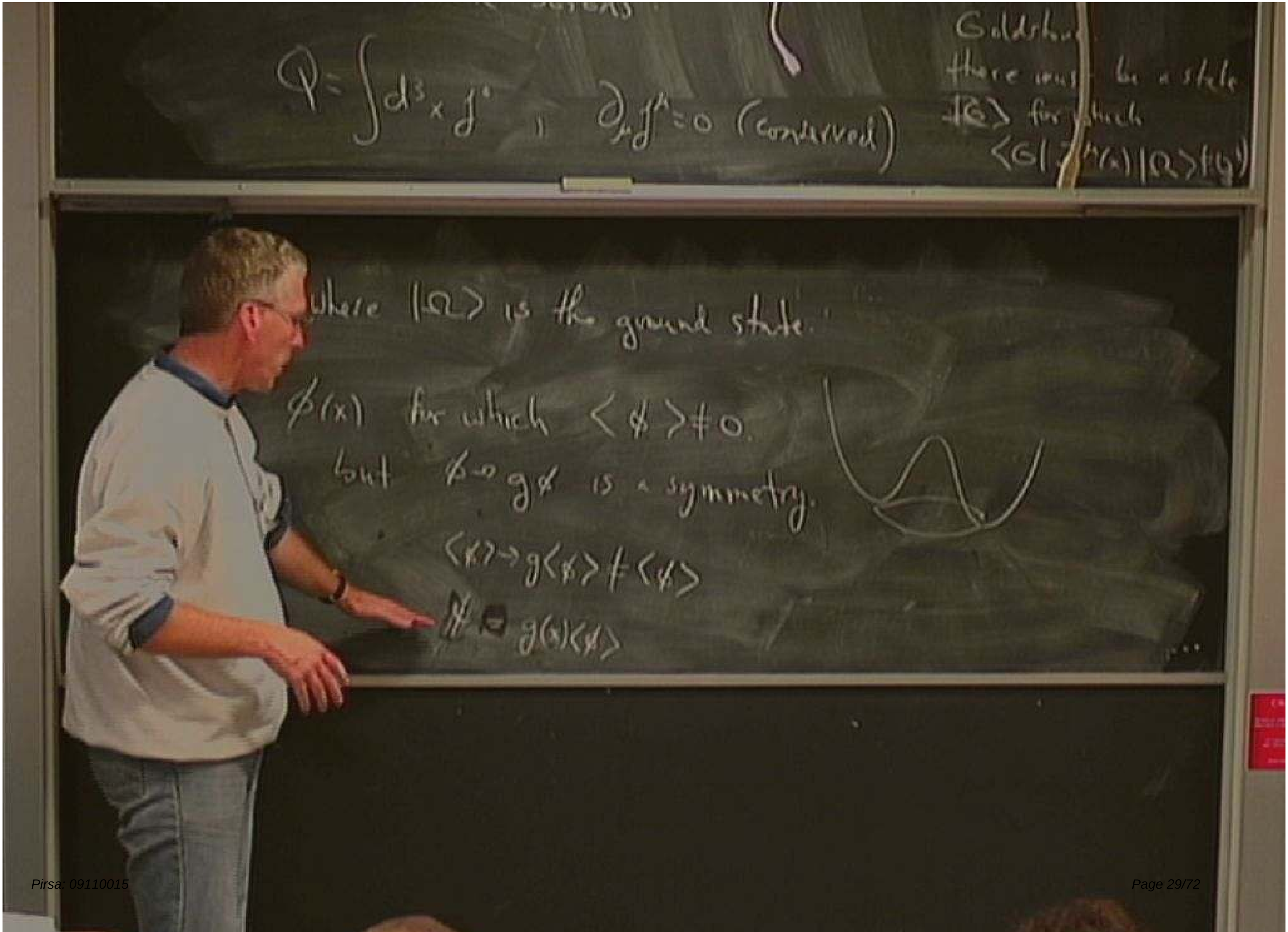
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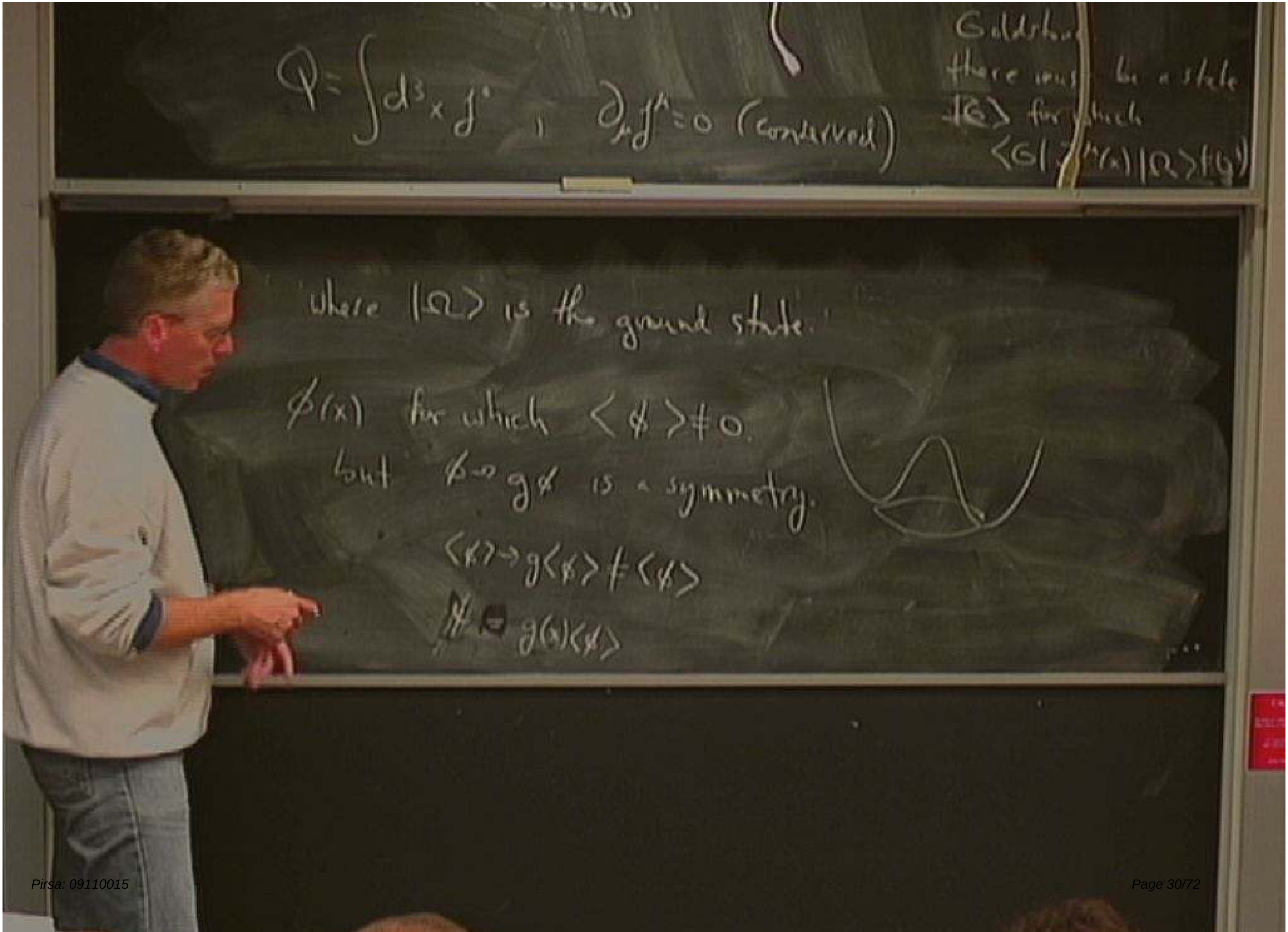
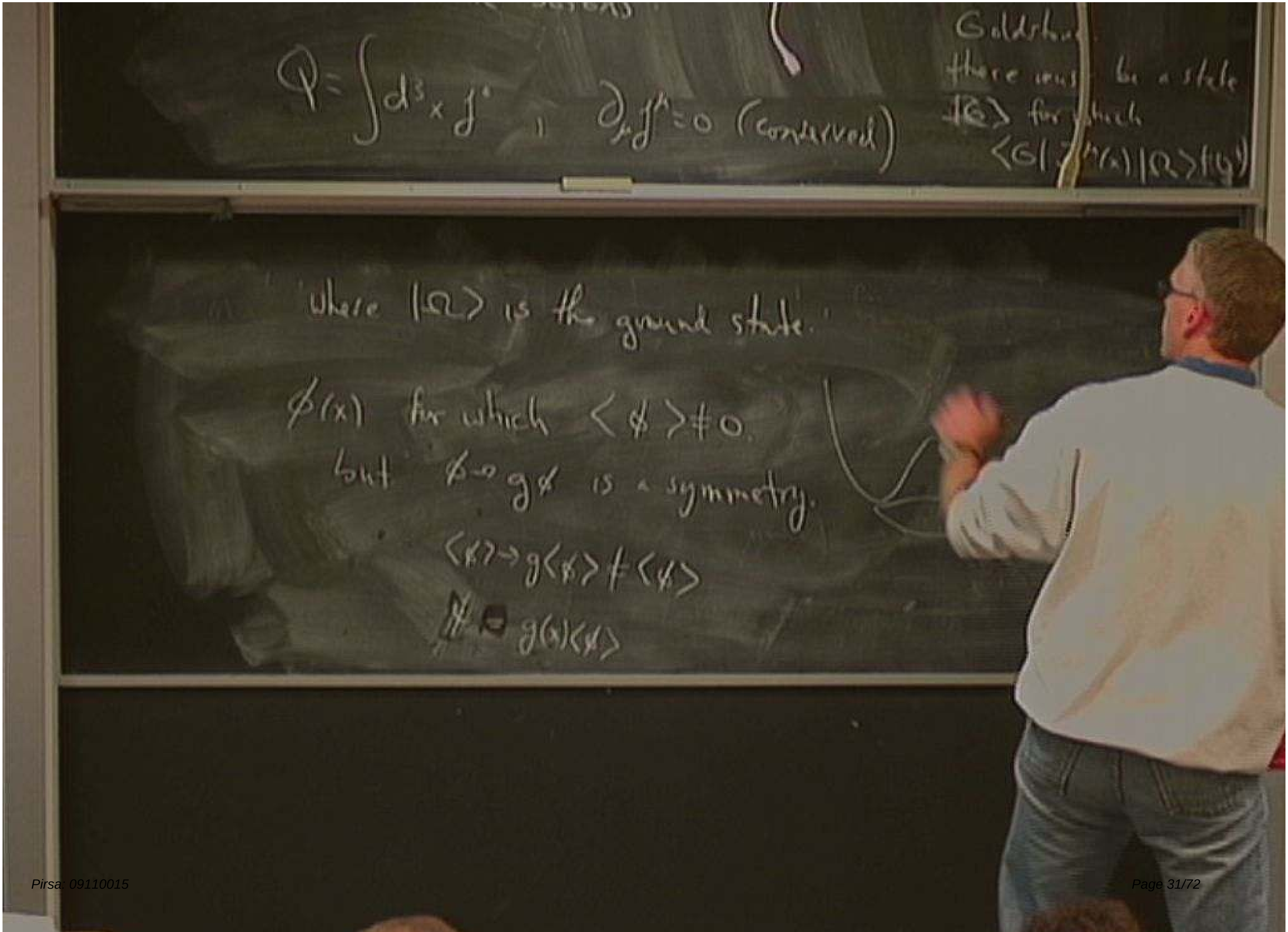

$Q = \int d^3x\, j^0$, $\partial_\mu j^\mu = 0$ (conserved)
there must be a state
$|G\rangle$ for which
where $|\Omega\rangle$ is the ground state.
$\phi(x)$ for which $\langle \phi \rangle \neq 0$.
but $\phi \to g\phi$ is a symmetry.
$\langle\phi\rangle \to g\langle\phi\rangle \neq \langle\phi\rangle$
$g(x)\langle\phi\rangle$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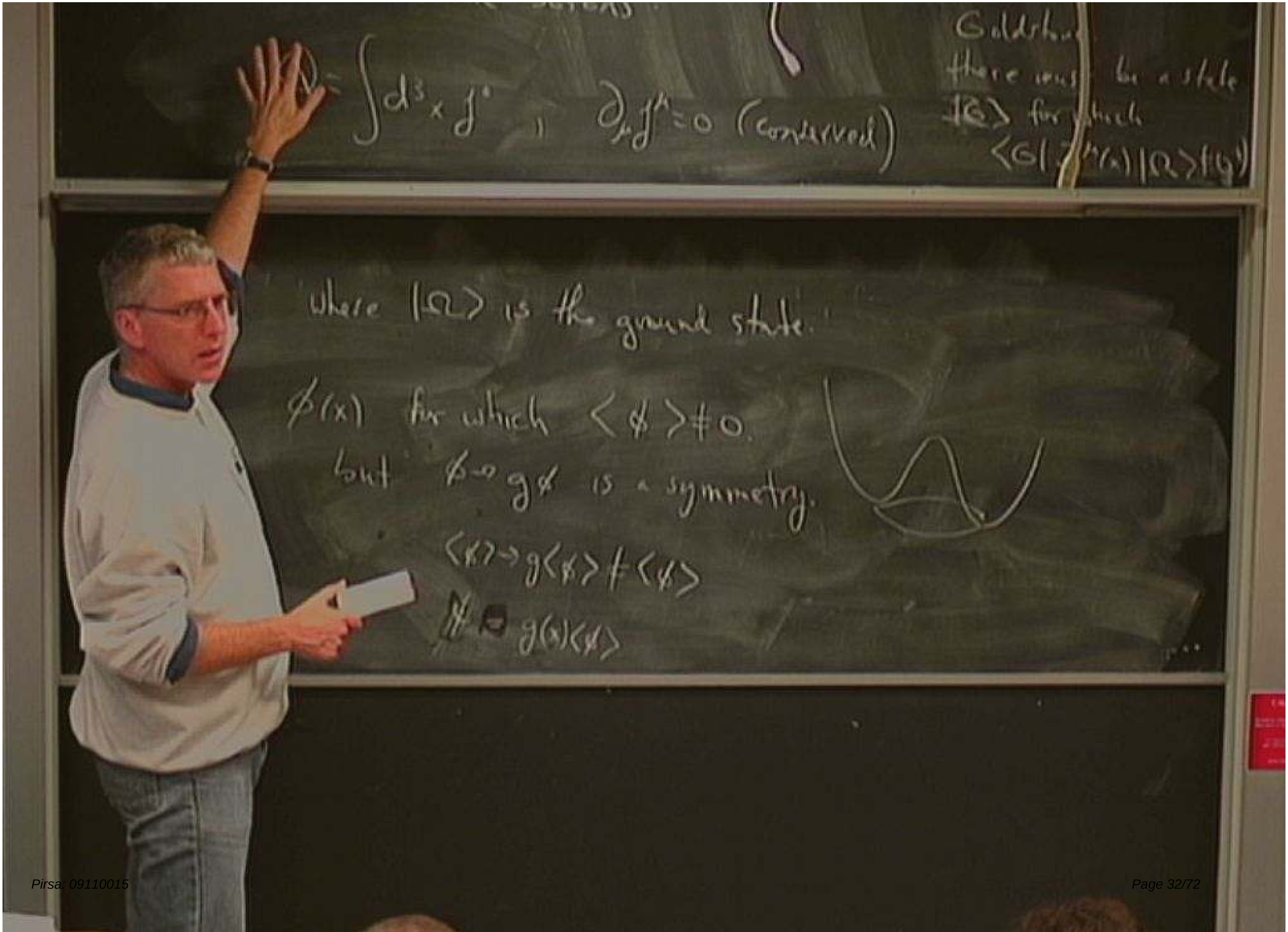

$Q = \int d^3x\, j^0$, $\partial_\mu j^\mu = 0$ (conserved)
Goldstone
there must be a state
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$\langle G | j^\mu(x) | \Omega \rangle \neq (\ldots)$
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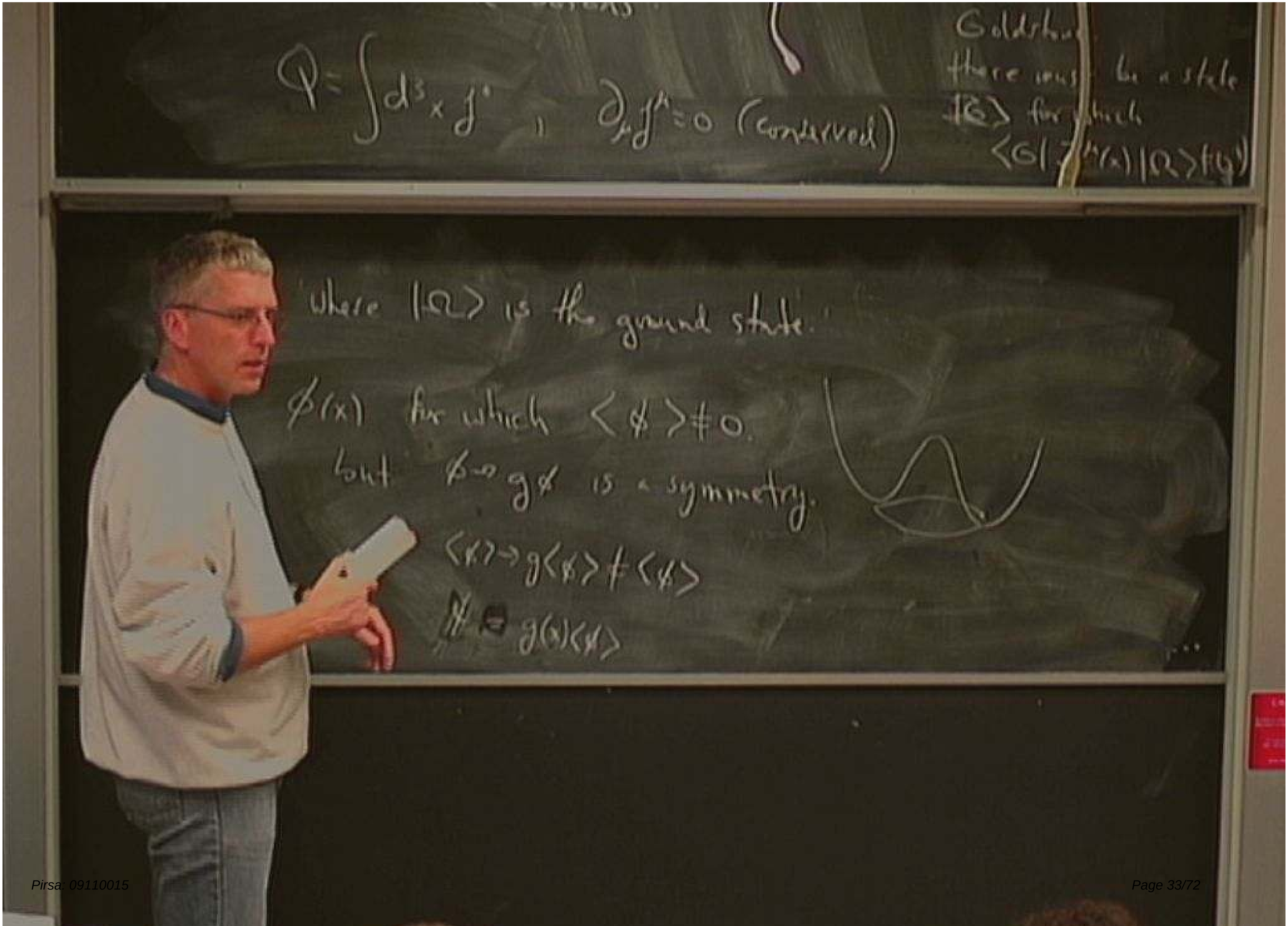
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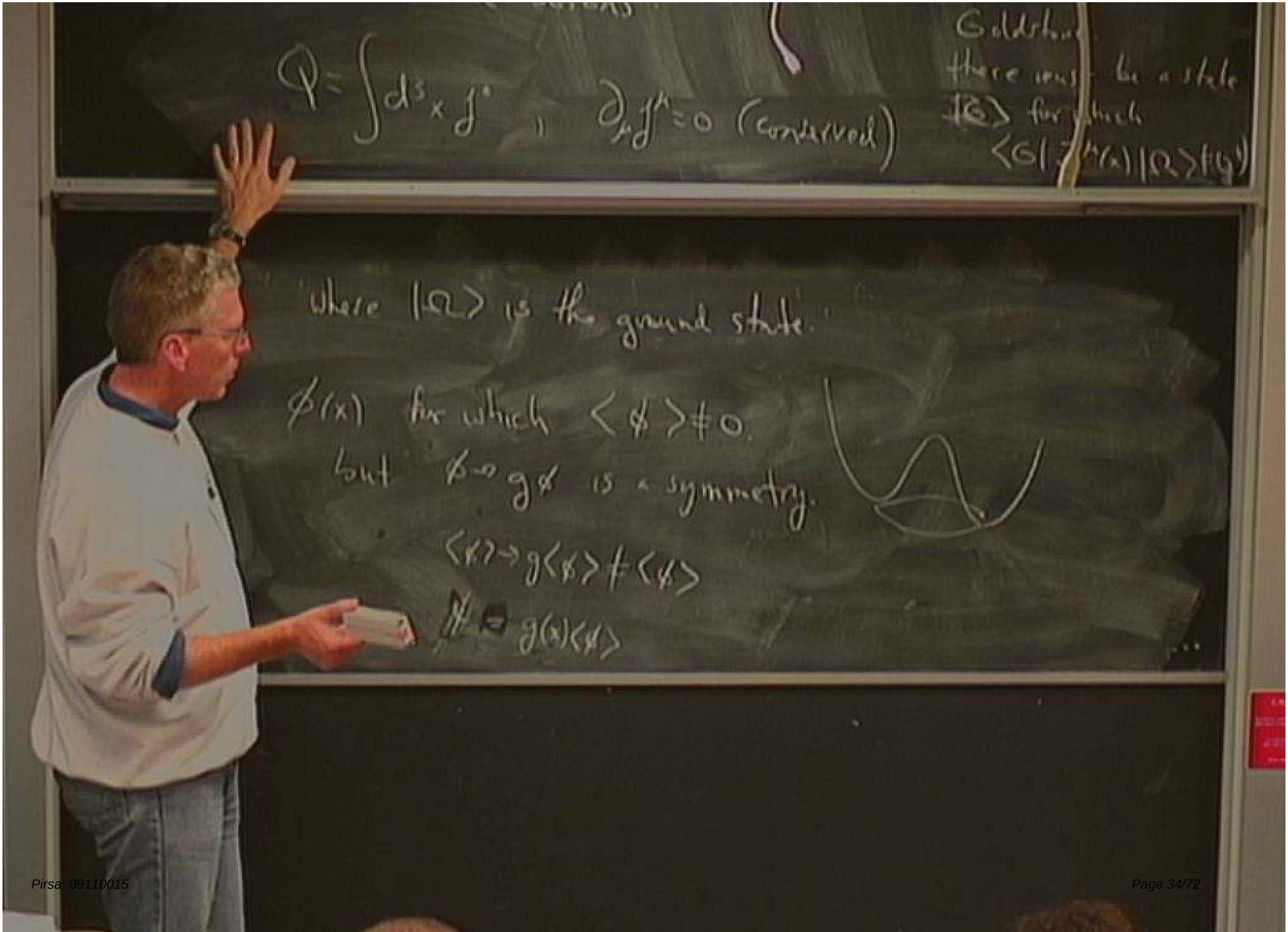
Q = ∫d³x j⁰ , ∂μ j^μ = 0 (conserved)
Goldstone
there must be a state
|G> for which
<G| j^μ(x)|Ω> ≠ 0
where |Ω> is the ground state.
φ(x) for which <φ> ≠ 0.
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<φ> → g<φ> ≠ <φ>
g(x)<φ>

$Q = \int d^3x \, j^0 \quad , \quad \partial_\mu j^\mu = 0$ (conserved)

Goldstone: there must be a state $|G\rangle$ for which $\langle G | j^\mu(x) | \Omega \rangle \neq 0$

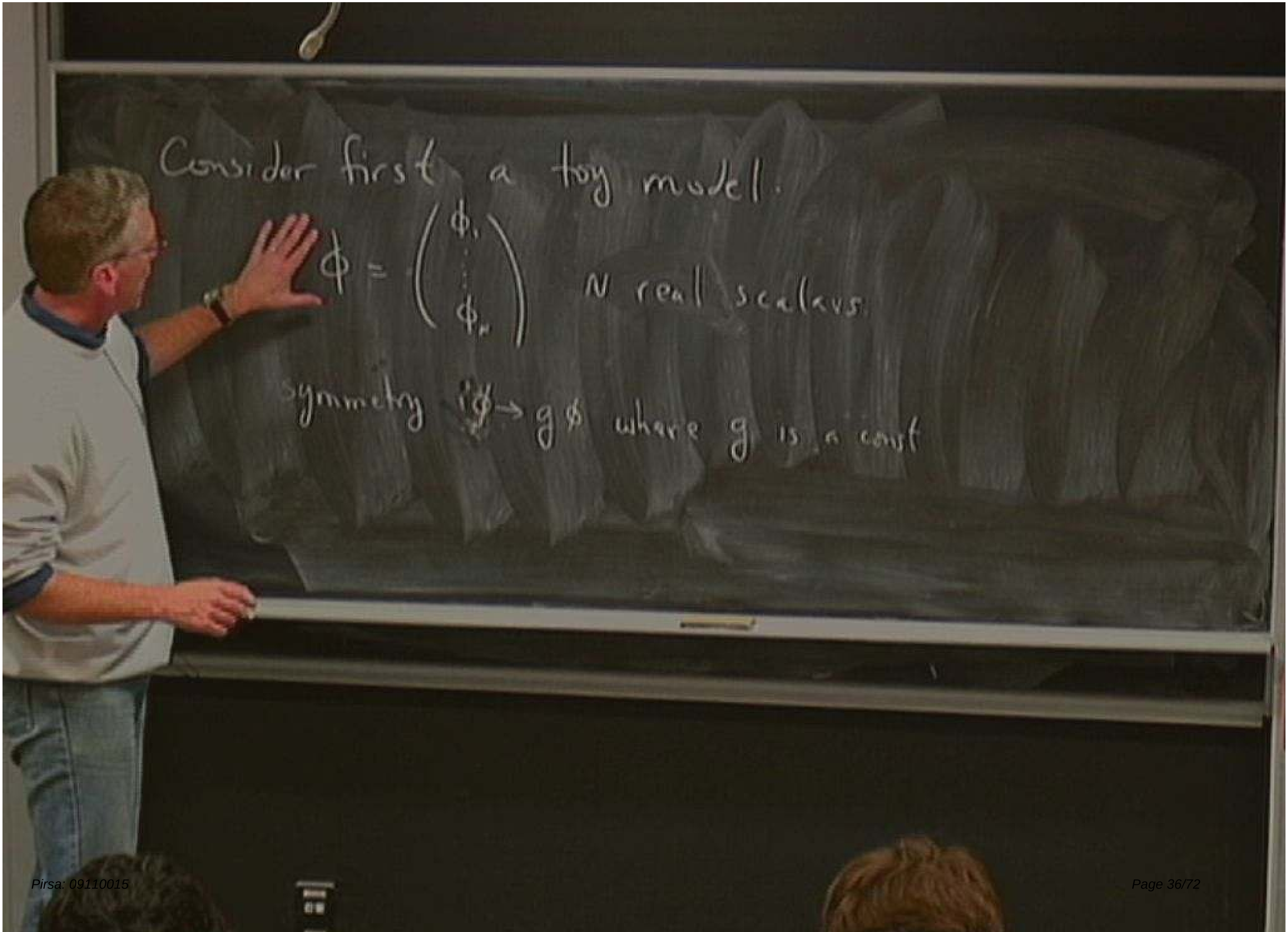
where $|\Omega\rangle$ is the ground state.

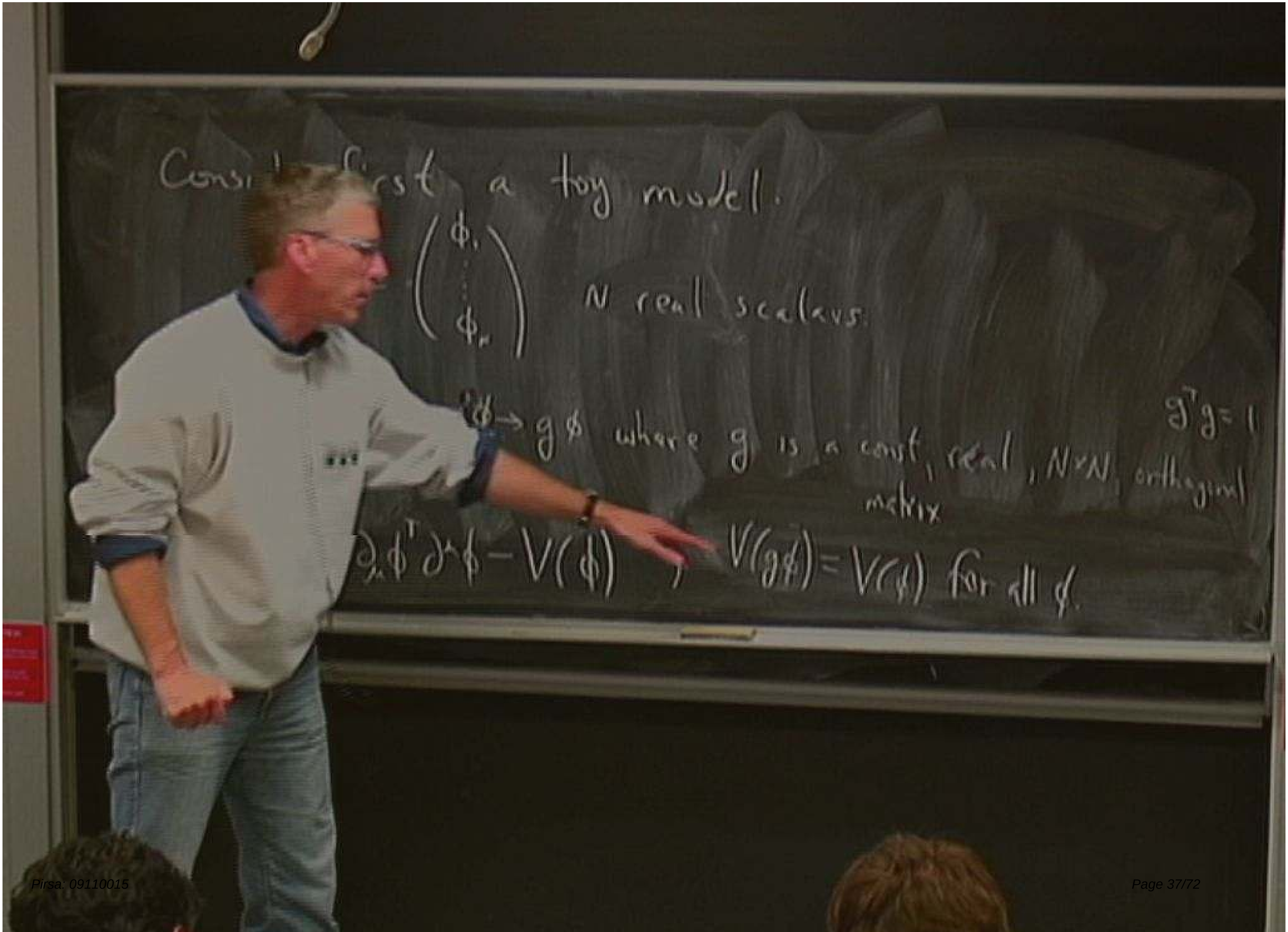
$\phi(x)$ for which $\langle \phi \rangle \neq 0$.

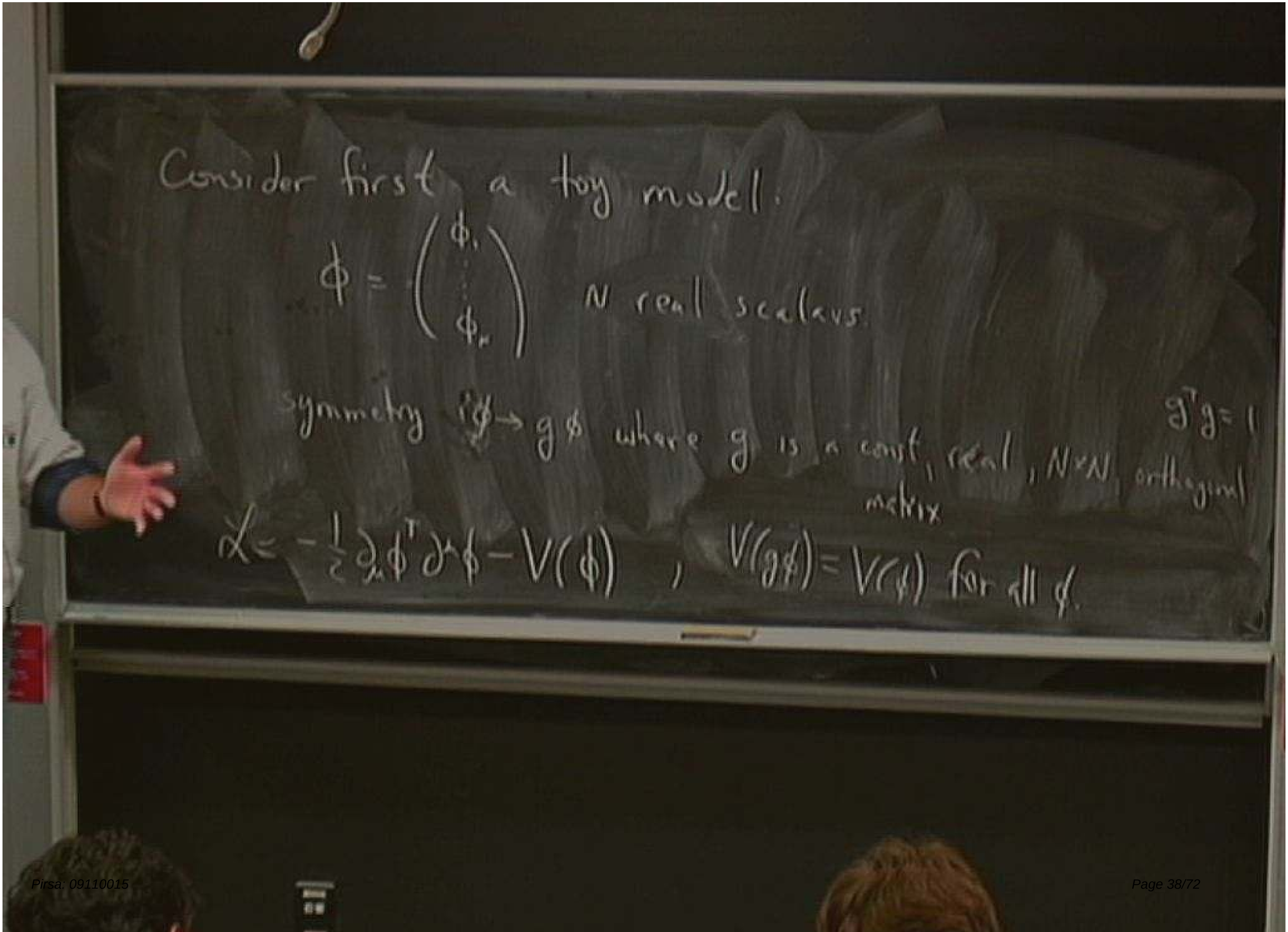
but $\phi \to g\phi$ is a symmetry.

$\langle \phi \rangle \to g \langle \phi \rangle \neq \langle \phi \rangle$

$\neq g(x) \langle \phi \rangle$


Consider first a toy model.
$\phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix}$ N real scalars
symmetry $\phi \to g\phi$ where g is a const


a toy model.
N real scalars.
where g is a const, real, N×N, orthogonal matrix
$g^T g = 1$
$V(g\phi) = V(\phi)$ for all ϕ.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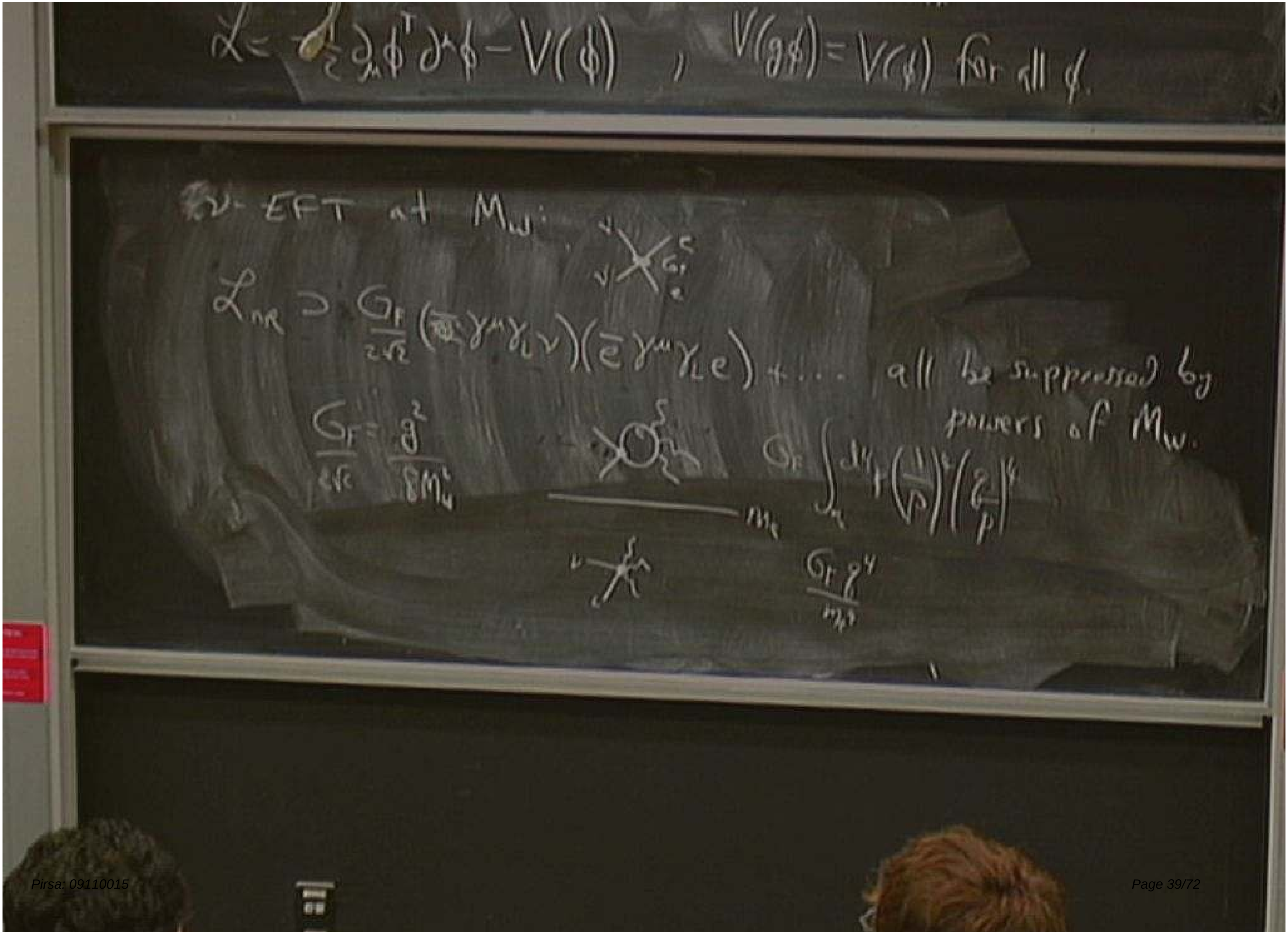
symmetry $\phi \to g\phi$ where g is a const, real, N×N, orthogonal matrix
$g^T g = 1$
$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - V(\phi)$, $V(g\phi) = V(\phi)$ for all ϕ.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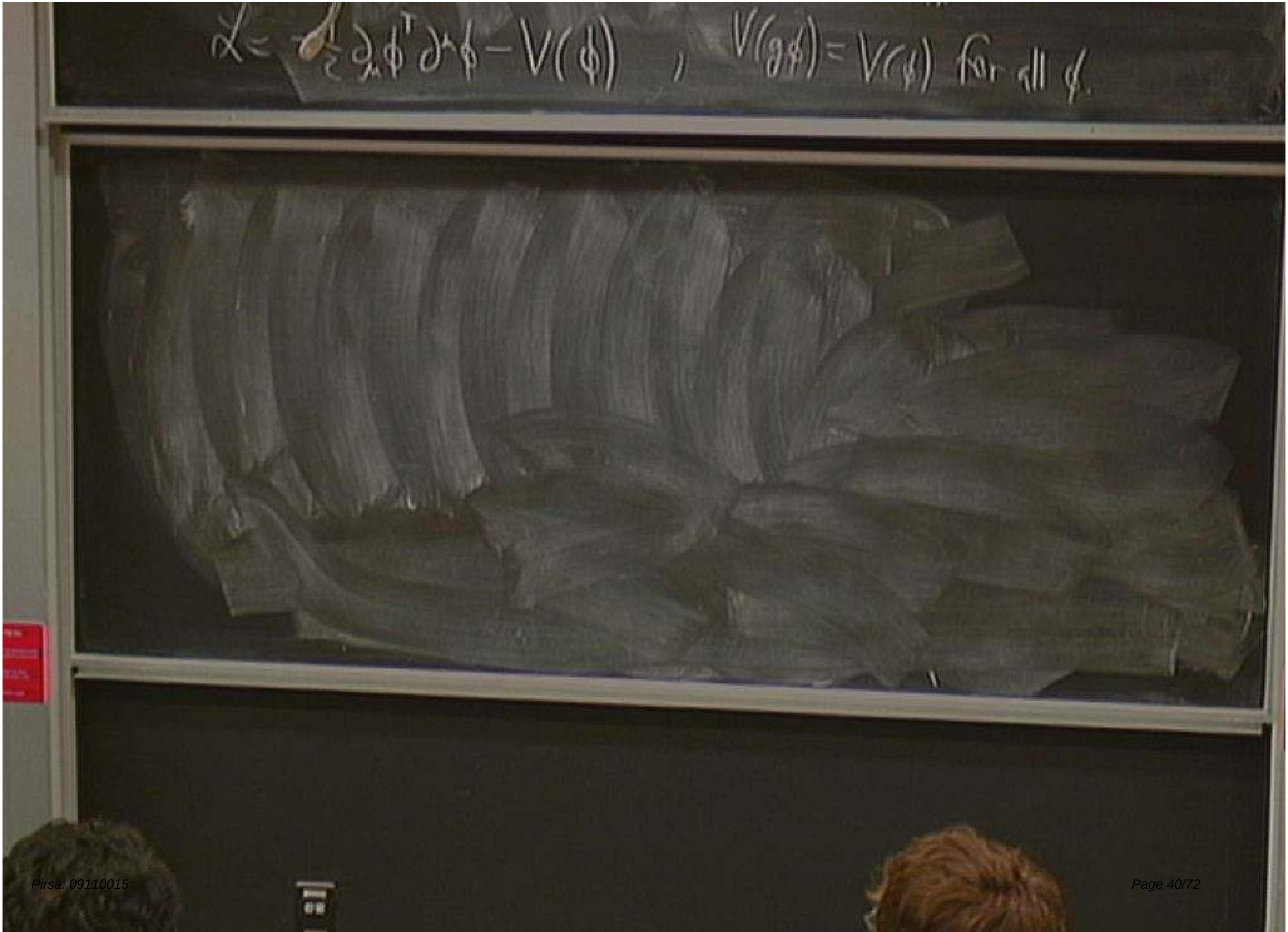
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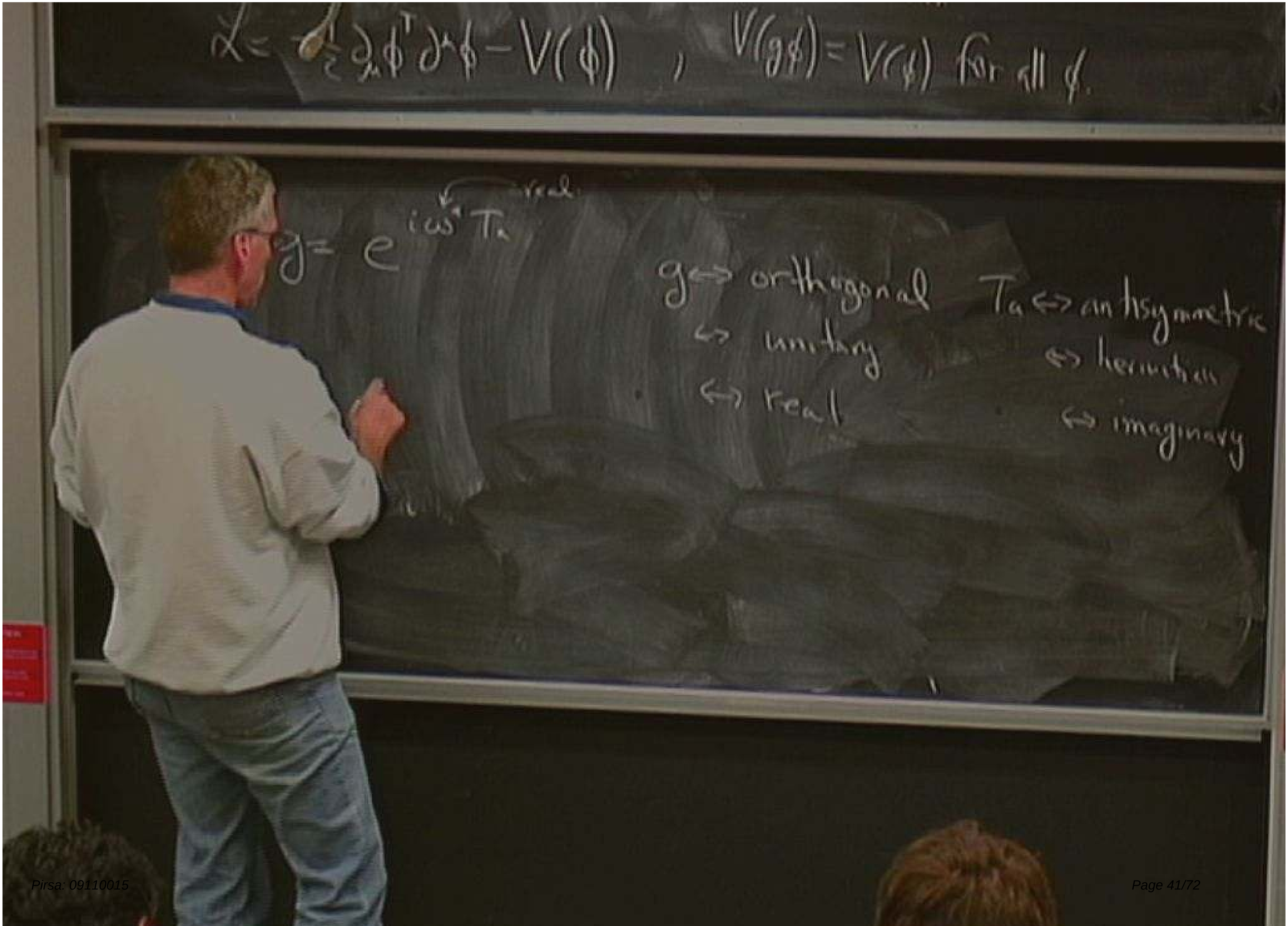
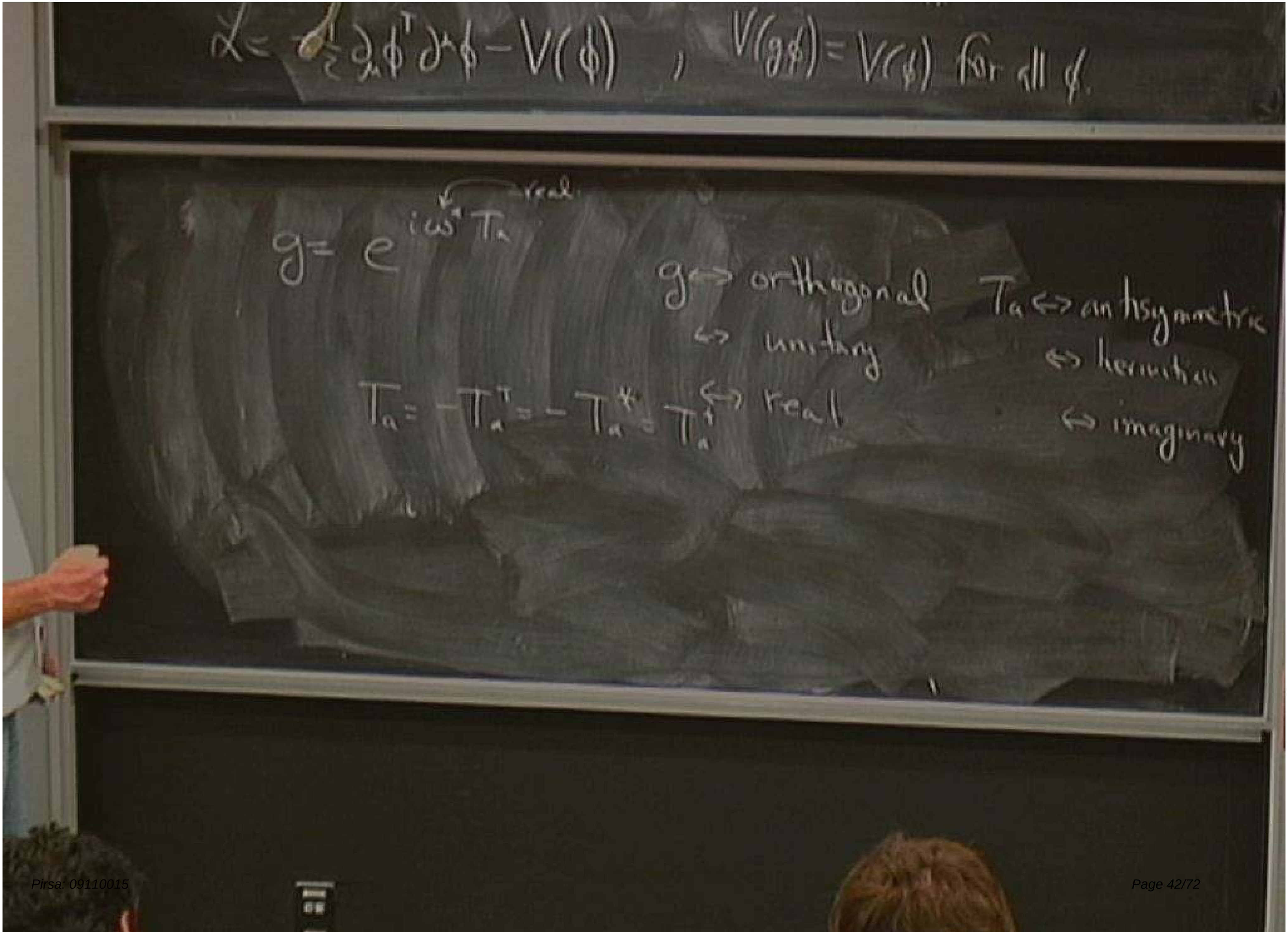


$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - V(\phi)$, $V(g\phi) = V(\phi)$ for all ϕ.
real
$g = e^{i\omega^a T_a}$
$g \Leftrightarrow$ orthogonal
$\Leftrightarrow$ unitary
$\Leftrightarrow$ real
$T_a \Leftrightarrow$ antisymmetric
$\Leftrightarrow$ hermitian
$\Leftrightarrow$ imaginary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real
$g = e^{i\omega^a T_a}$
$g \leftrightarrow$ orthogonal
$\leftrightarrow$ unitary
$T_a \leftrightarrow$ antisymmetric
$\leftrightarrow$ hermitian
$T_a = -T_a^T = -T_a^* = T_a^\dagger \leftrightarrow$ real
$\leftrightarrow$ imaginary

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - V(\phi) \quad , \quad V(g\phi) = V(\phi) \text{ for all } \phi.$$

$$g = e^{i\omega^a T_a} \qquad (\omega^a \text{ real})$$

$g \Leftrightarrow$ orthogonal $\quad T_a \Leftrightarrow$ antisymmetric

$\Leftrightarrow$ unitary $\quad \Leftrightarrow$ hermitian

$T_a = -T_a^T = -T_a^* = T_a^\dagger \Leftrightarrow$ real $\quad \Leftrightarrow$ imaginary

$$\mathcal{L} = -\tfrac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - V(\phi) \quad , \quad V(g\phi) = V(\phi) \text{ for all } \phi.$$

$$g = e^{i\omega^a T_a}$$

(ω^a real)

$g \leftrightarrow$ orthogonal $\qquad T_a \leftrightarrow$ antisymmetric

$\leftrightarrow$ unitary $\qquad \leftrightarrow$ hermitian

$$T_a = -T_a^T = -T_a^* = T_a^\dagger \leftrightarrow \text{real} \qquad \leftrightarrow \text{imaginary}$$

Define ϕ

$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - V(\phi)$, $V(g\phi) = V(\phi)$ for all ϕ.

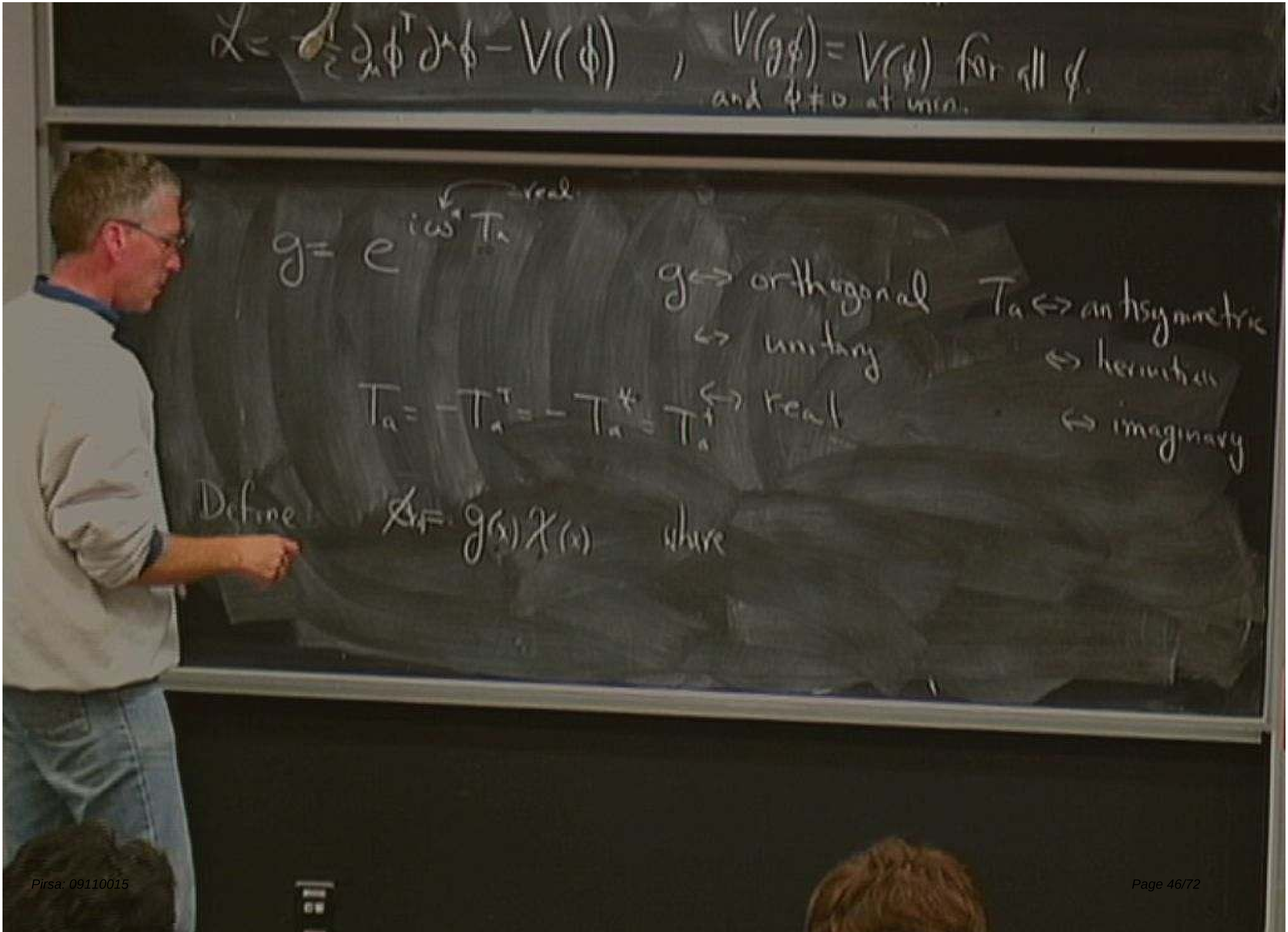
$g = e^{i\omega^a T_a}$ (ω^a real)

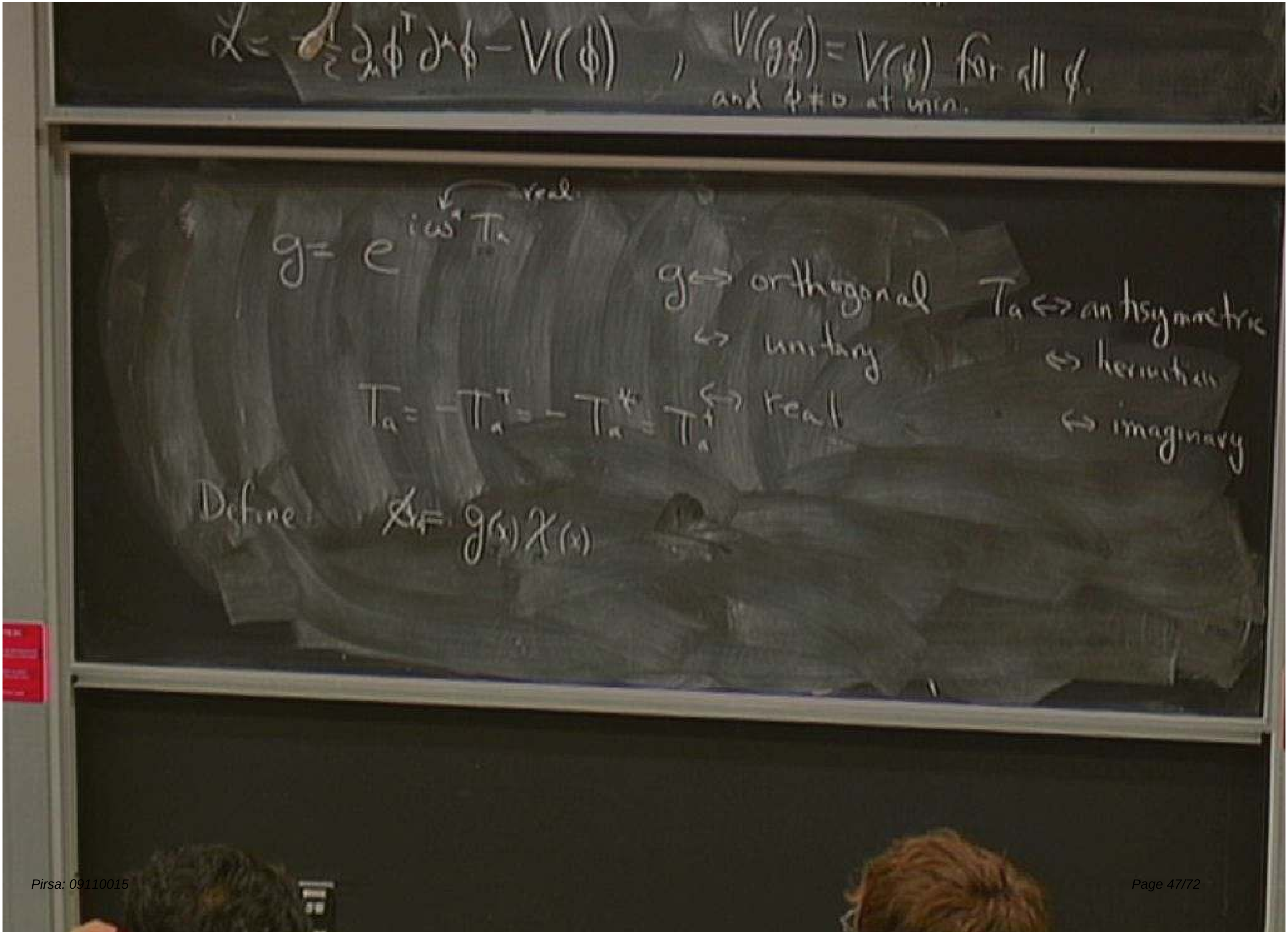
$g \leftrightarrow$ orthogonal $\qquad T_a \leftrightarrow$ antisymmetric

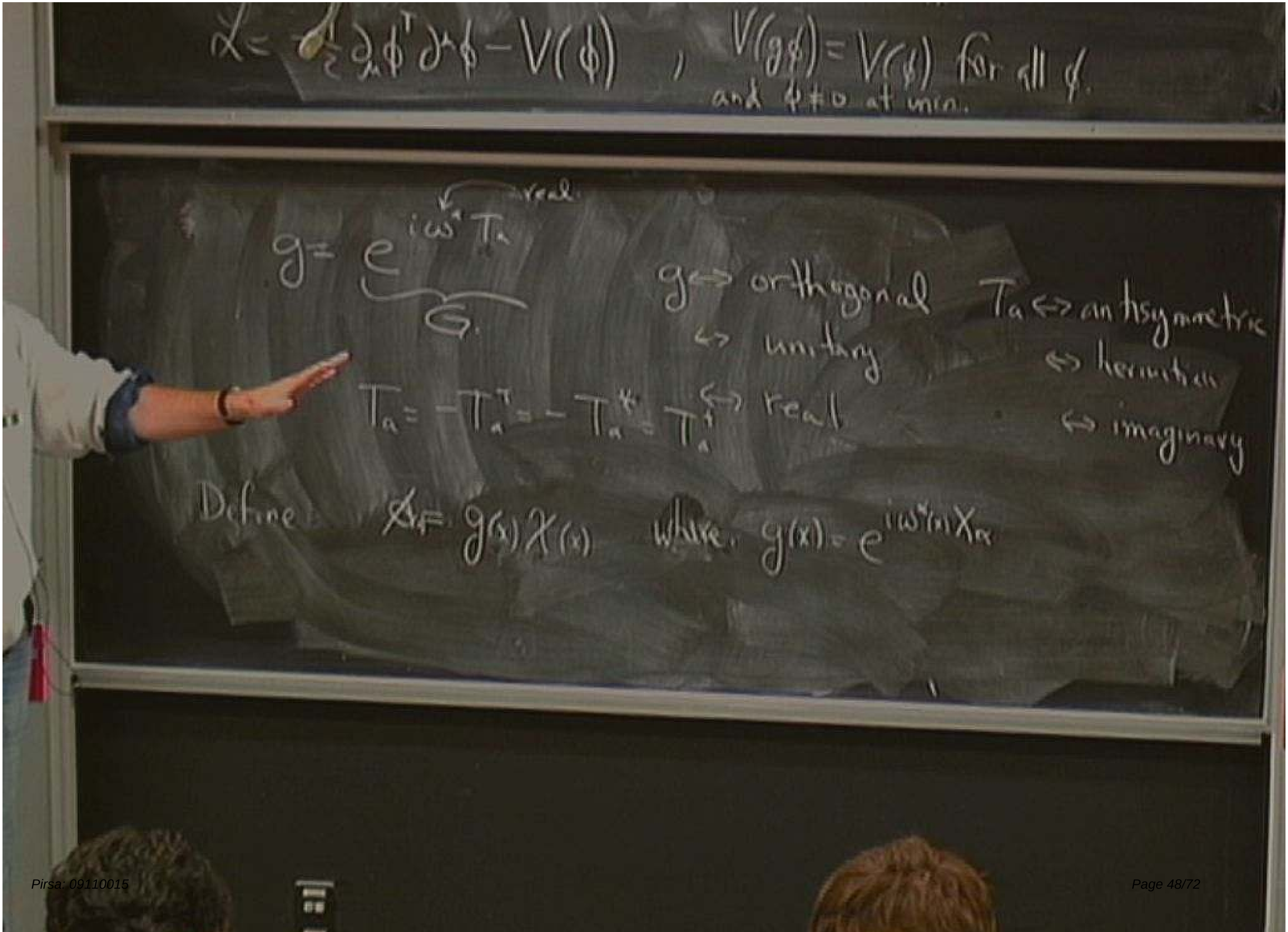
$\leftrightarrow$ unitary $\qquad \leftrightarrow$ hermitian

$T_a = -T_a^T = -T_a^* = T_a^\dagger$ $\leftrightarrow$ real $\qquad \leftrightarrow$ imaginary

Define $\phi =$


L = -1/2 ∂μφT ∂μφ - V(φ) , V(gφ) = V(φ) for all φ.
and φ ≠ 0 at min.
real
g = e^(iω^a Ta)
g ⇔ orthogonal
⇔ unitary
Ta ⇔ antisymmetric
⇔ hermitian
Ta = -Ta^T = -Ta^* = Ta^† ⇔ real
⇔ imaginary
Define: χF = g(x) χ(x) where


$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^T\partial^\mu\phi - V(\phi)$, $V(g\phi) = V(\phi)$ for all ϕ.
and $\phi \neq 0$ at min.
real
$g = e^{i\omega^a T_a}$
$g \leftrightarrow$ orthogonal
$\leftrightarrow$ unitary
$T_a \leftrightarrow$ antisymmetric
$\leftrightarrow$ hermitian
$T_a = -T_a^T = -T_a^* = T_a^\dagger \leftrightarrow$ real
$\leftrightarrow$ imaginary
Define $\phi = g(x)\chi(x)$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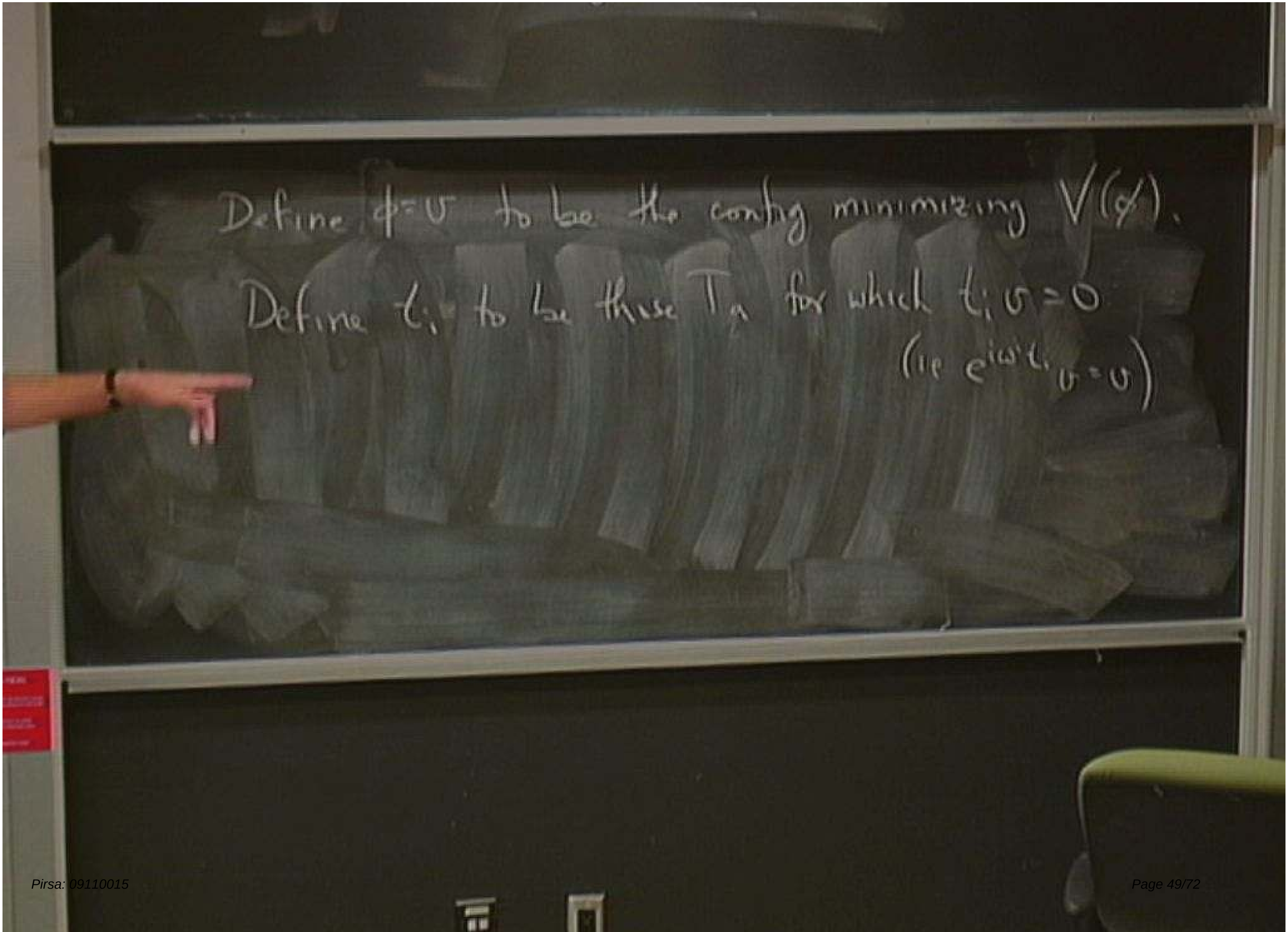
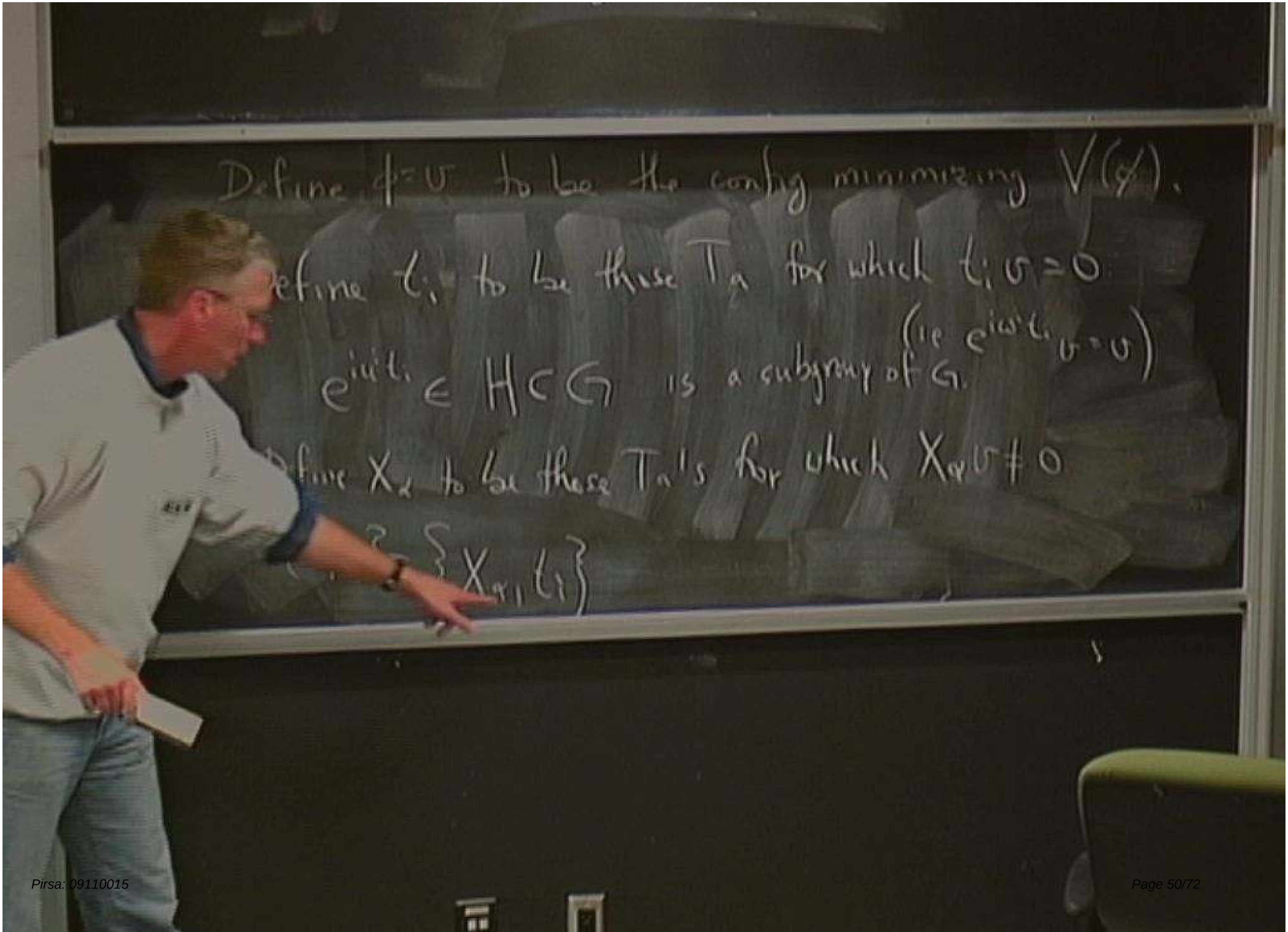
$T_a = -T_a^T = -T_a^* = T_a^\dagger$ $\leftrightarrow$ real
$\leftrightarrow$ imaginary
Define $\phi \equiv g(x)\chi(x)$ where $g(x) = e^{i\omega^a(x) X_a}$


Define $\phi = v$ to be the config minimizing $V(\phi)$.
Define t_i to be those T_a for which $t_i v = 0$
(ie $e^{i\omega^i t_i} v = v$)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(ie $e^{i\omega^i t_i} v = v$)
$e^{i\omega^i t_i} \in H \subset G$ is a subgroup of G.
Define X_α to be those T_a's for which $X_\alpha v \neq 0$
$\{X_\alpha, t_i\}$

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(i.e. $e^{i\omega^i t_i} v = v$)

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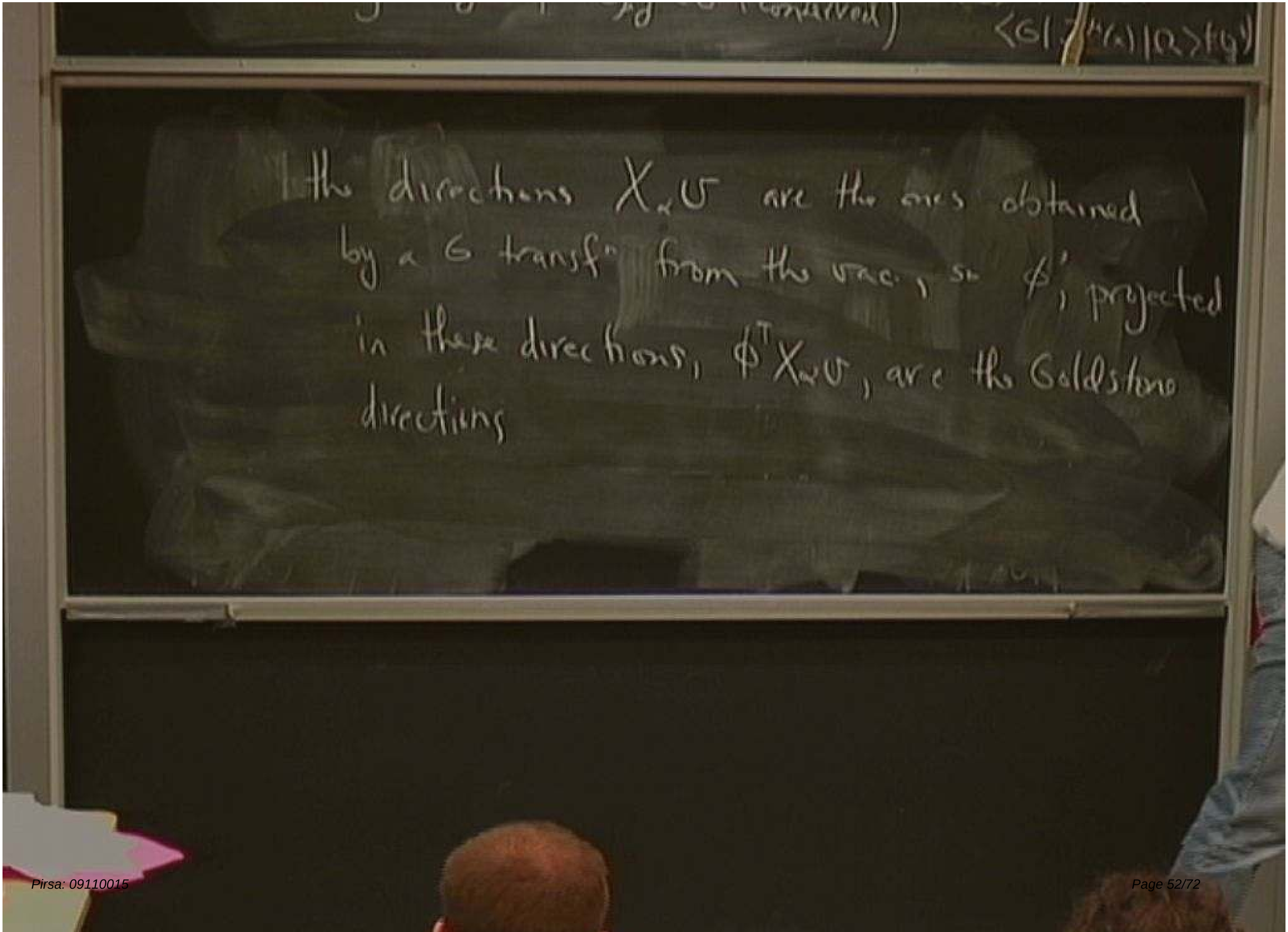
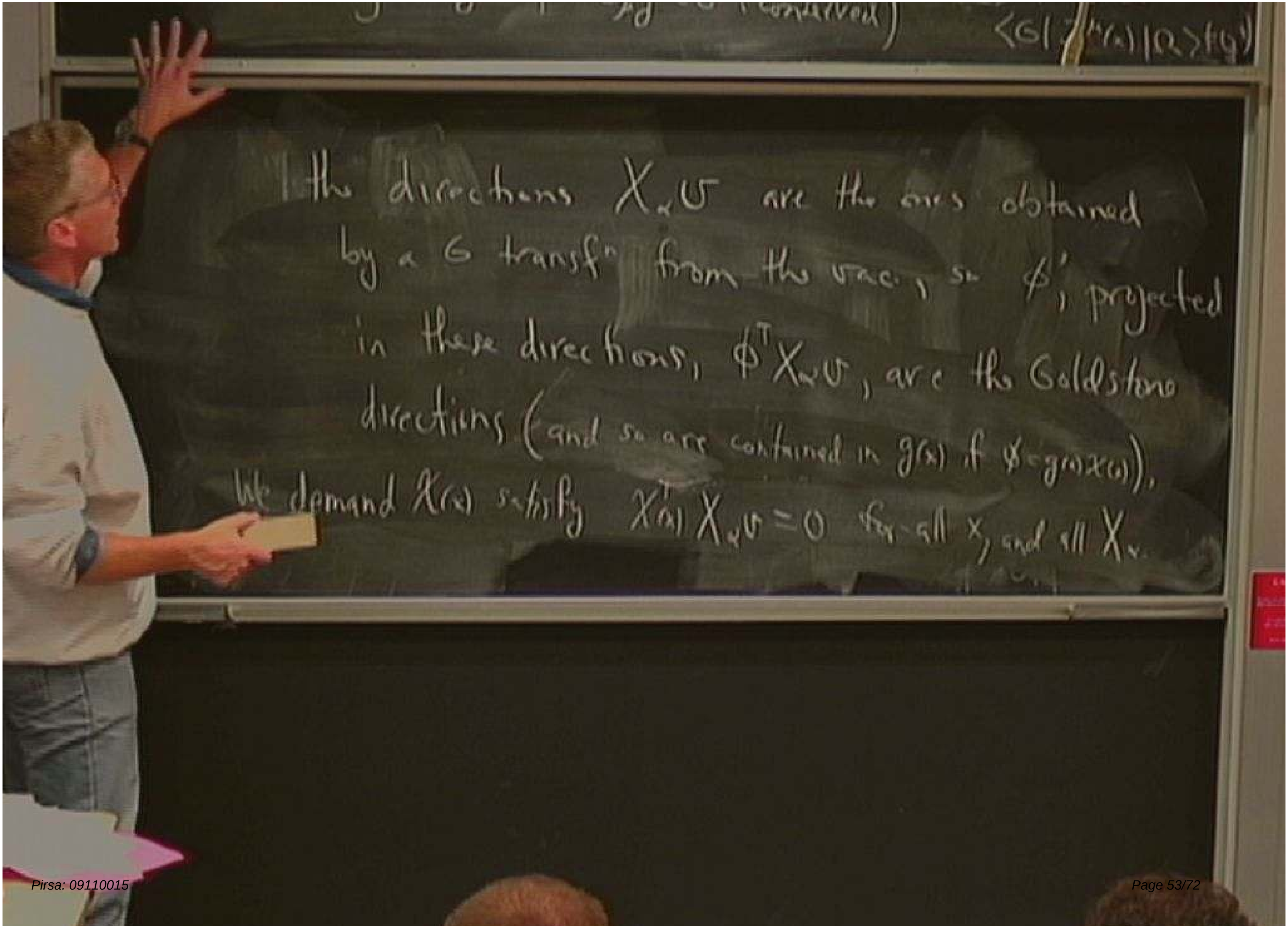
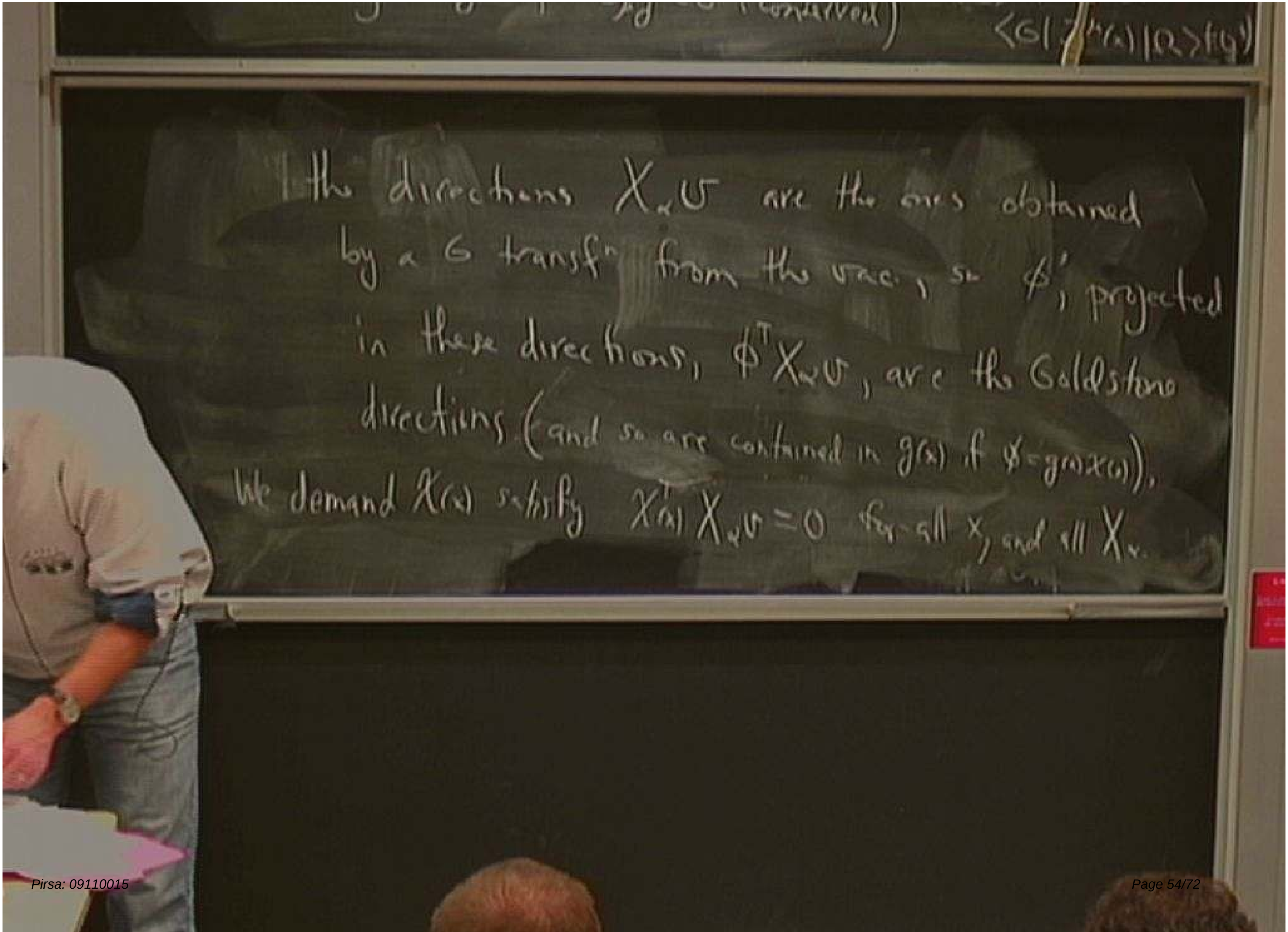
Define X_α to be those T_a's for which $X_\alpha v \neq 0$

$\{T_a\} = \{X_\alpha, t_i\}$ X_α's label the coset G/H


the directions $X_\alpha v$ are the ones obtained
by a G transf^n from the vac., so ϕ', projected
in these directions, $\phi^T X_\alpha v$, are the Goldstone
directions


the directions Xα v are the ones obtained
by a G transfn from the vac., so φ', projected
in these directions, φᵀ Xα v, are the Goldstone
directions (and so are contained in g(x) if φ = g(x)χ(x)).
We demand χ(x) satisfy χᵀ(x) Xα v = 0 for all x, and all Xα.


(conserved)
the directions $X_\alpha v$ are the ones obtained
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in these directions, $\phi^T X_\alpha v$, are the Goldstone
directions (and so are contained in $g(x)$ if $\phi = g(x)\chi(x)$).
We demand $\chi(x)$ satisfy $\chi^T(x) X_\alpha v = 0$ for all x, and all X_α.

(conserved) $\langle G| J^{\mu}(x) |\Omega\rangle f(y')$

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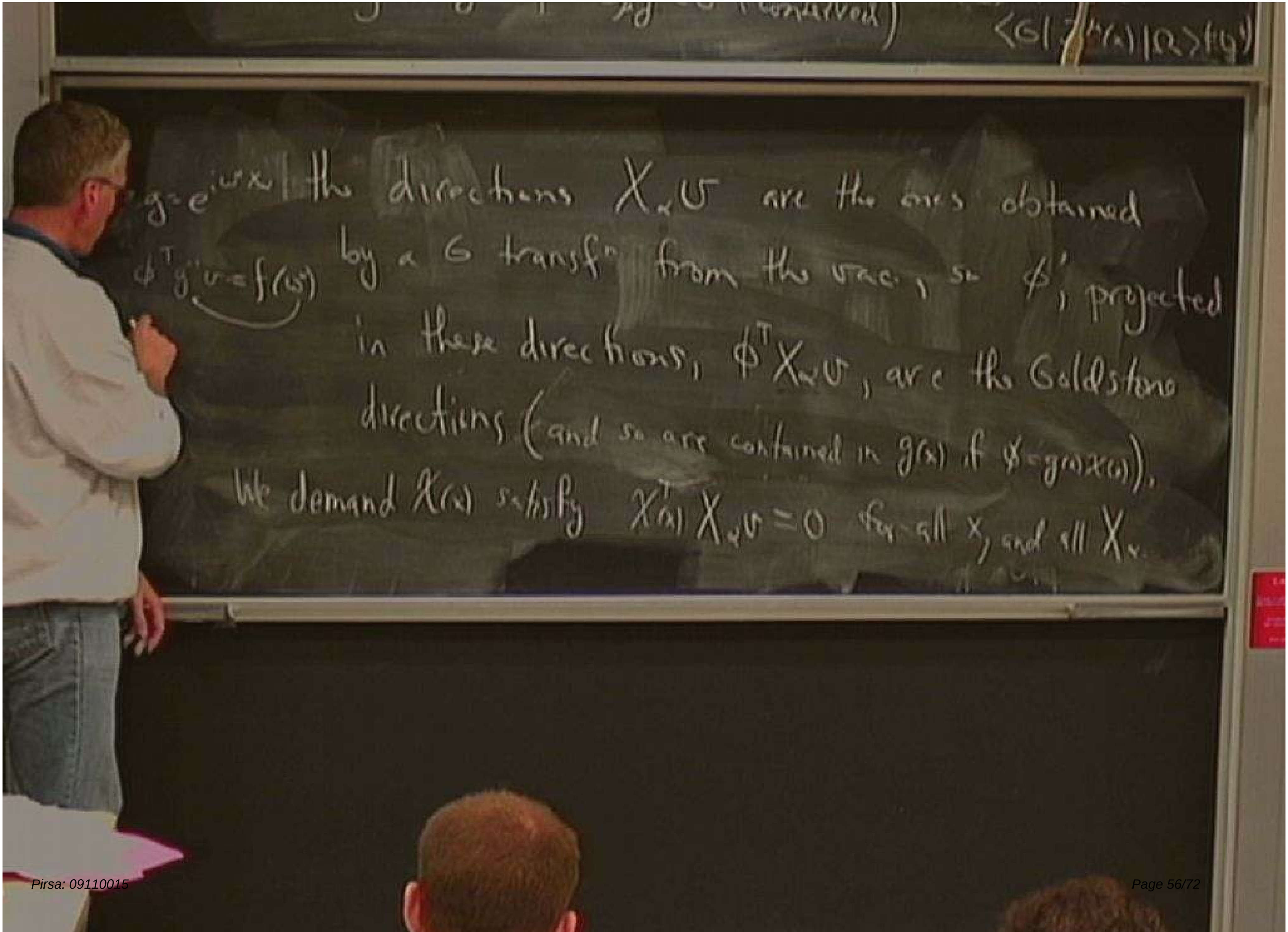
(conserved)
⟨G| J^μ(x)|Ω⟩f(g')
g = e^{iω·X} | the directions X_α v are the ones obtained
φ^T g' v = f(ω') by a G transf^n from the vac., so φ', projected
in these directions, φ^T X_α v, are the Goldstone
directions (and so are contained in g(x) if φ = g(x)χ(x)).
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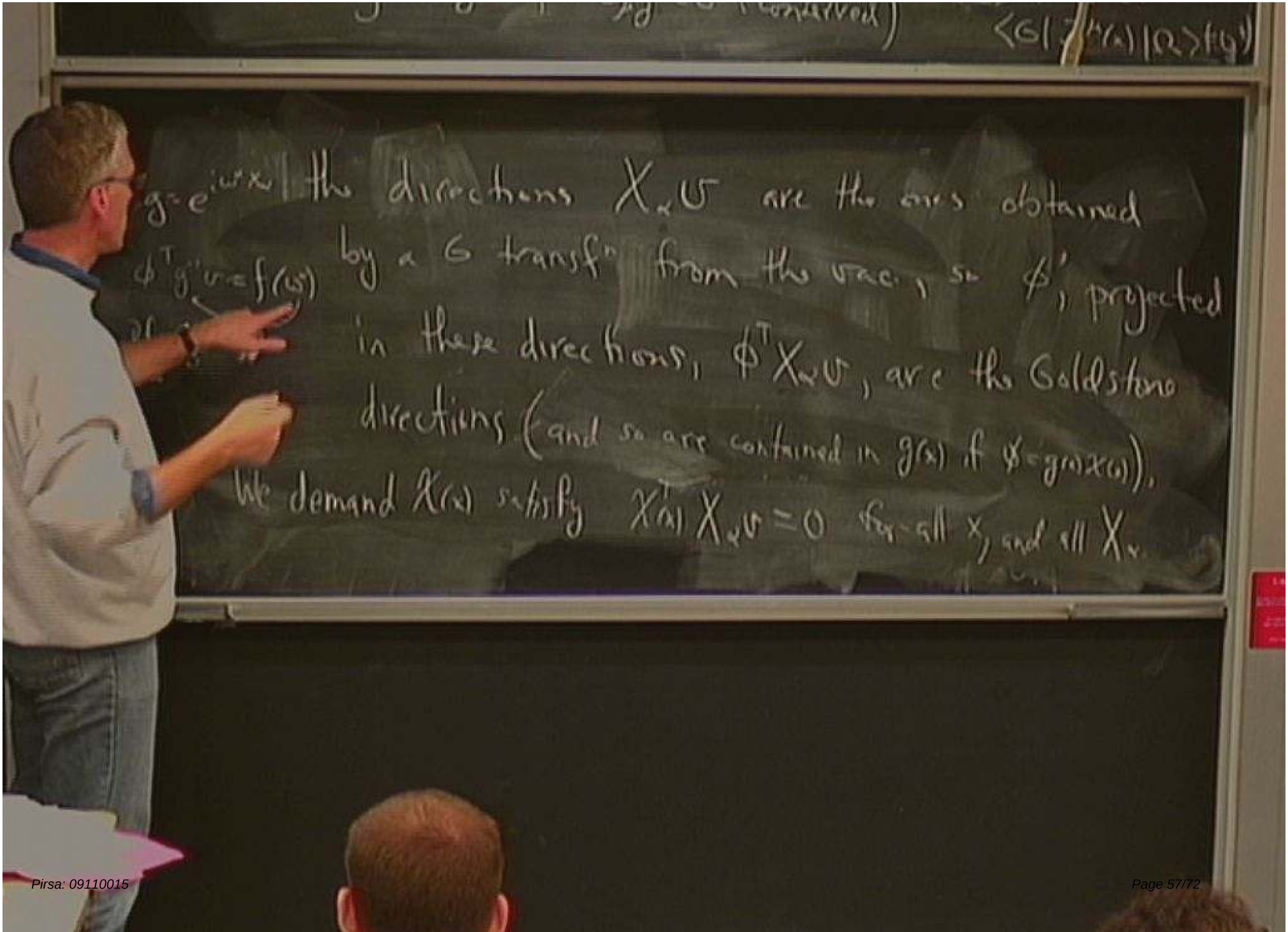
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in these directions, φ^T X_α v, are the Goldstone
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We demand χ(x) satisfy χ^T(x) X_α v = 0 for all x, and all X_α.

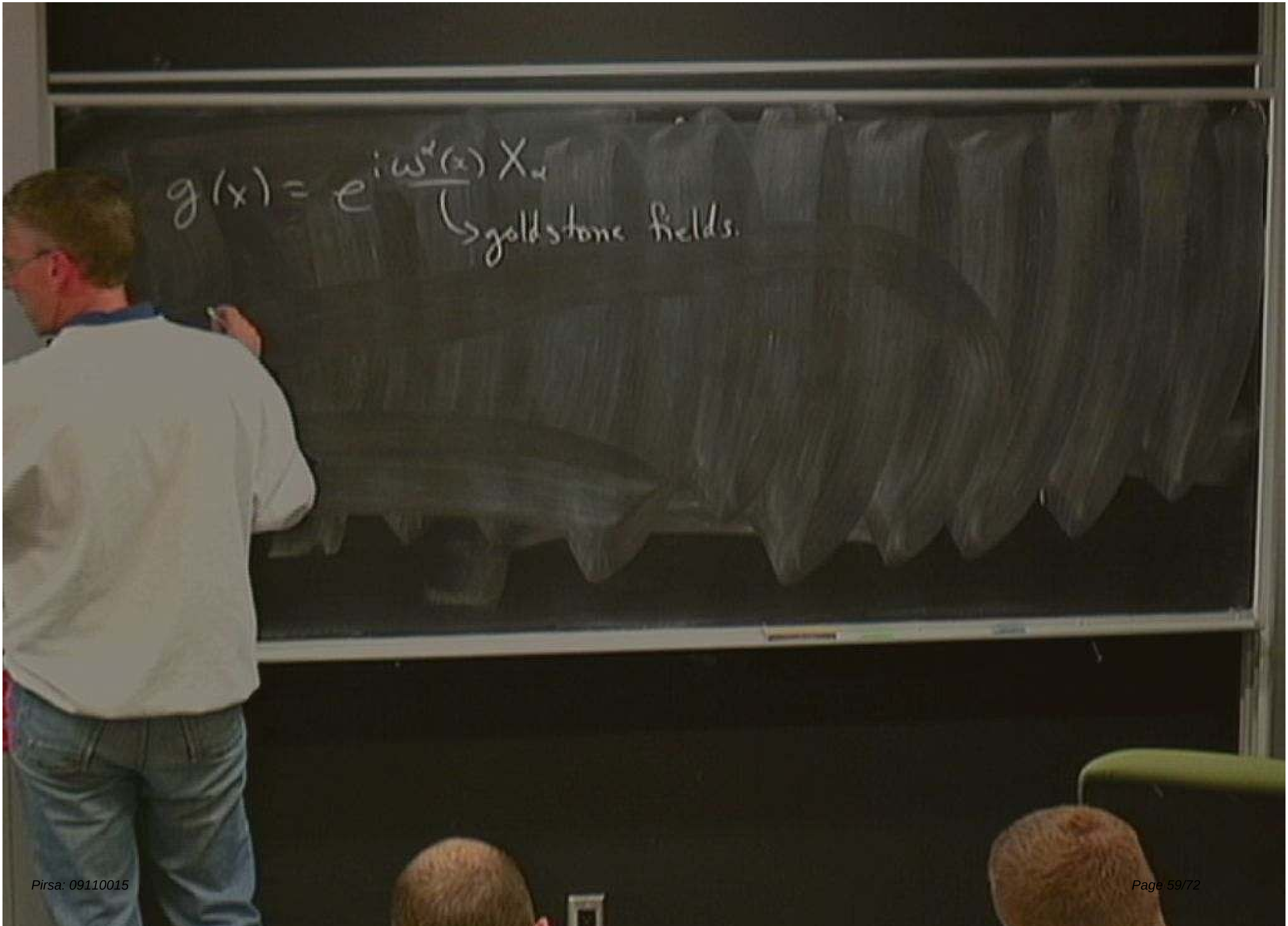
... conserved) $\langle G | J^\mu(x) | \Omega \rangle f(g')$

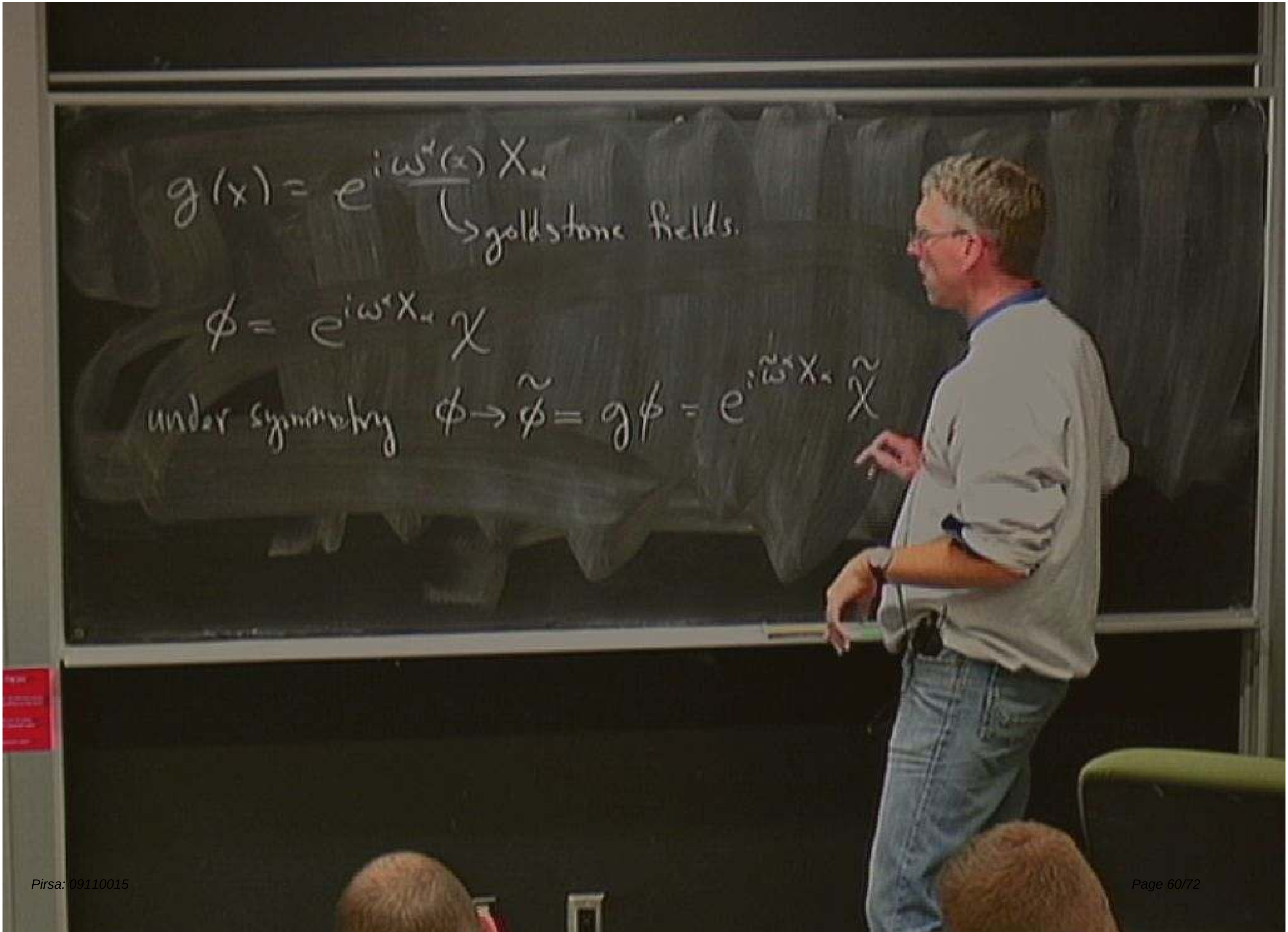
$g = e^{i\omega X_\alpha}$ the directions $X_\alpha v$ are the ones obtained by a G transf^n from the vac., so ϕ^T, projected in these directions, $\phi^T X_\alpha v$, are the Goldstone directions (and so are contained in $g(x)$ if $\phi = g(x)\chi(x)$).

$\phi^T g' v = f(\omega)$

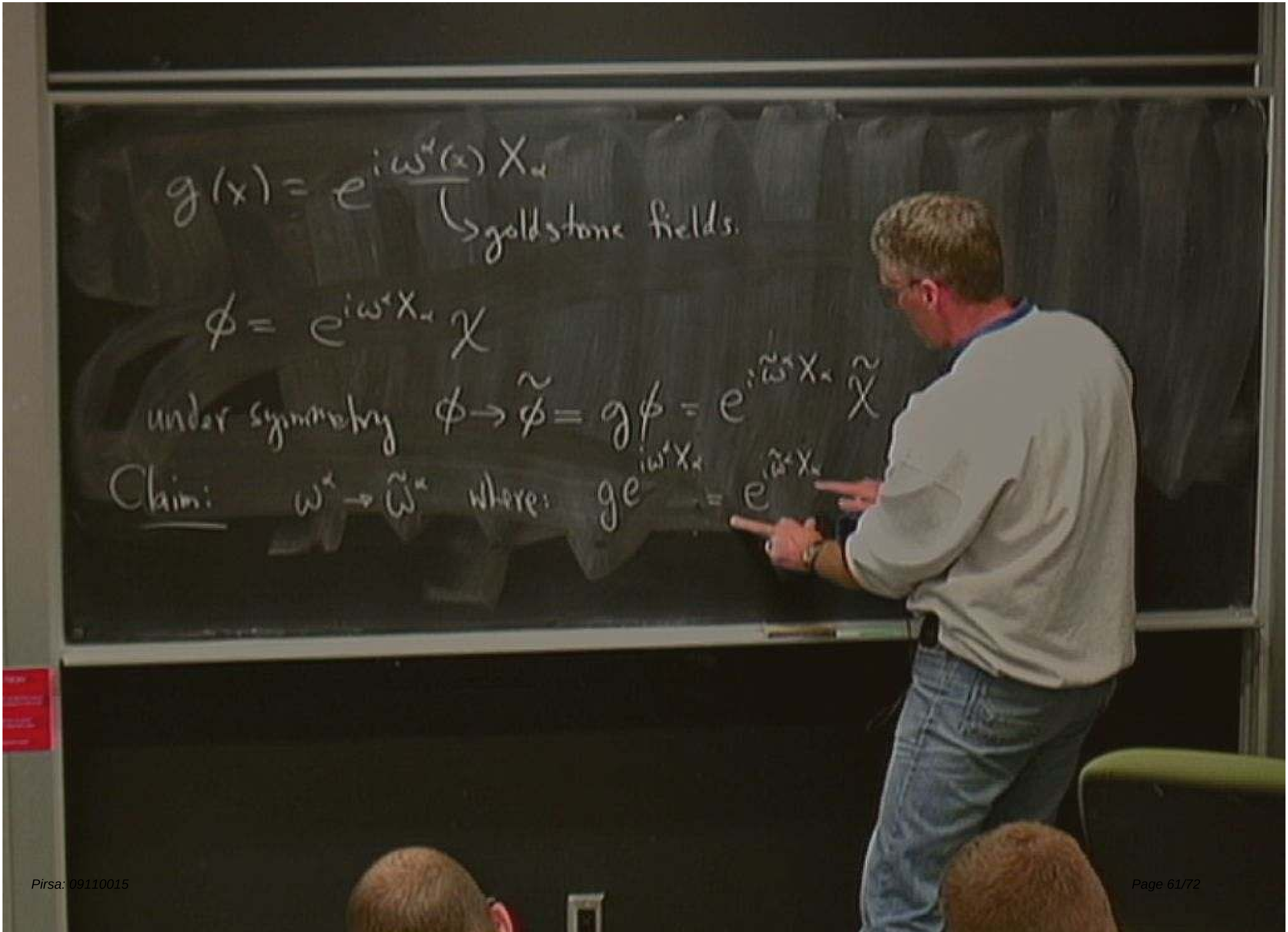
$\frac{\partial f}{\partial \omega} = 0$

We demand $\chi(x)$ satisfy $\chi^T(x) X_\alpha v = 0$ for all x, and all X_α


$g(x) = e^{i\omega^\alpha(x) X_\alpha}$
↳ goldstone fields.



$g(x) = e^{i\omega^\alpha(x) X_\alpha}$
↳ goldstone fields.
$\phi = e^{i\omega^\alpha X_\alpha} \chi$
under symmetry $\phi \to \tilde{\phi} = g\phi = e^{i\tilde{\omega}^\alpha X_\alpha} \tilde{\chi}$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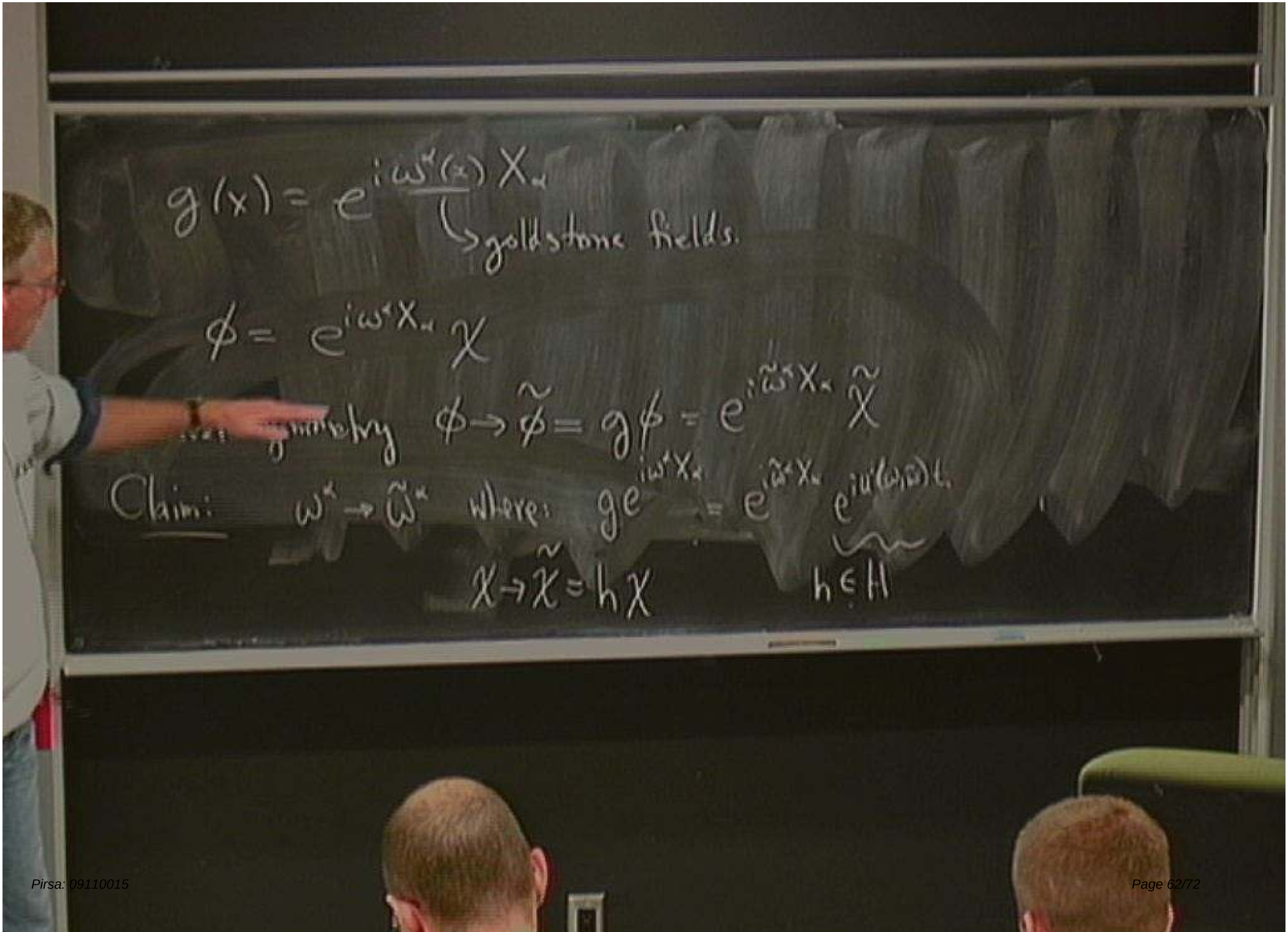
goldstone fields.
$\phi = e^{i\omega^\alpha X_\alpha} \chi$
under symmetry $\phi \to \tilde{\phi} = g\phi = e^{i\tilde{\omega}^\alpha X_\alpha} \tilde{\chi}$
Claim: $\omega^\alpha \to \tilde{\omega}^\alpha$ where: $g e^{i\omega^\alpha X_\alpha} = e^{i\tilde{\omega}^\alpha X_\alpha}$



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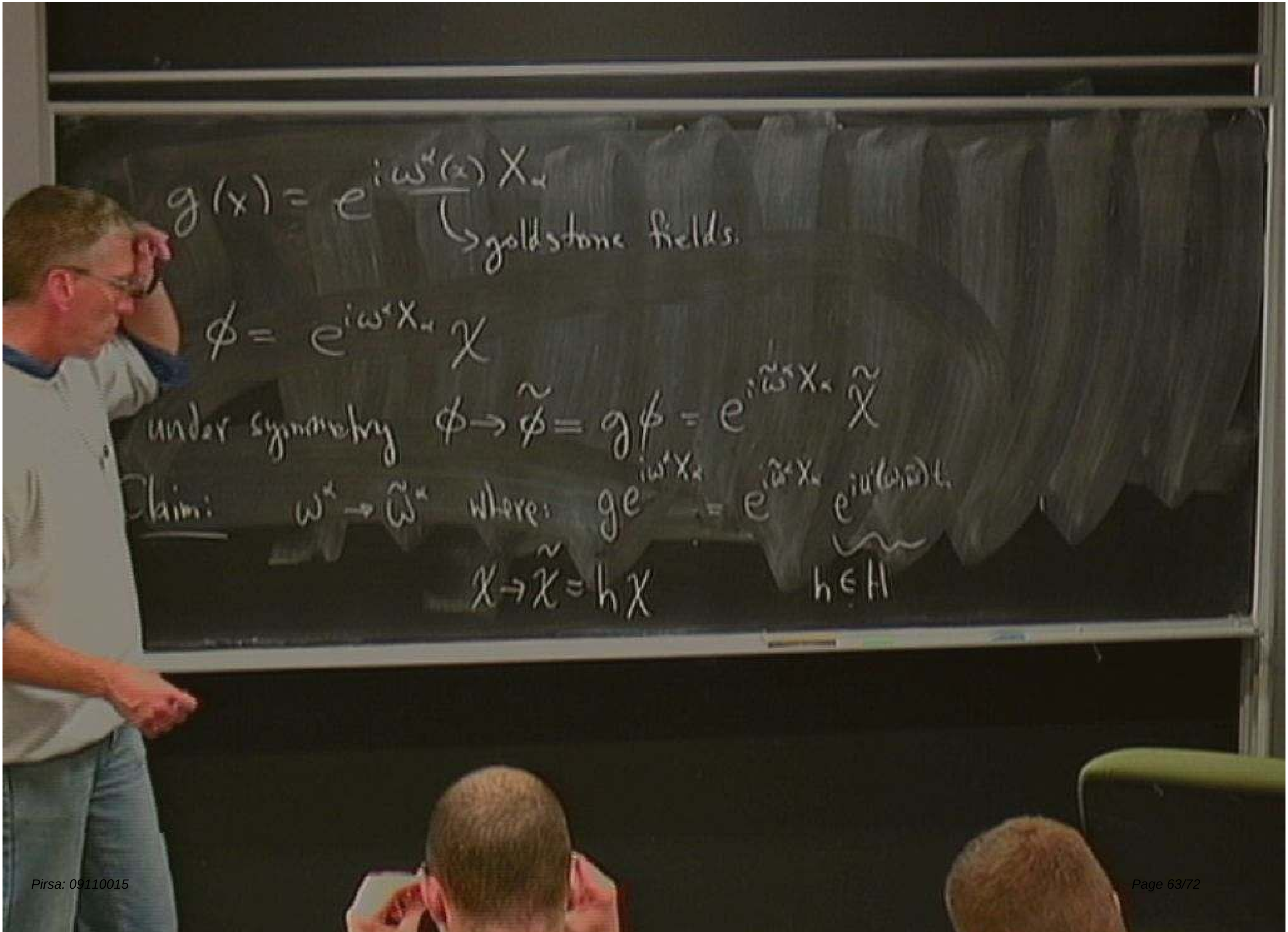
Claim: $\omega^\alpha \to \tilde{\omega}^\alpha$ where: $g e^{i\omega^\alpha X_\alpha} = e^{i\tilde{\omega}^\alpha X_\alpha} e^{iu(\omega,\tilde{\omega})t}$
$\chi \to \tilde{\chi} = h\chi$
$h \in H$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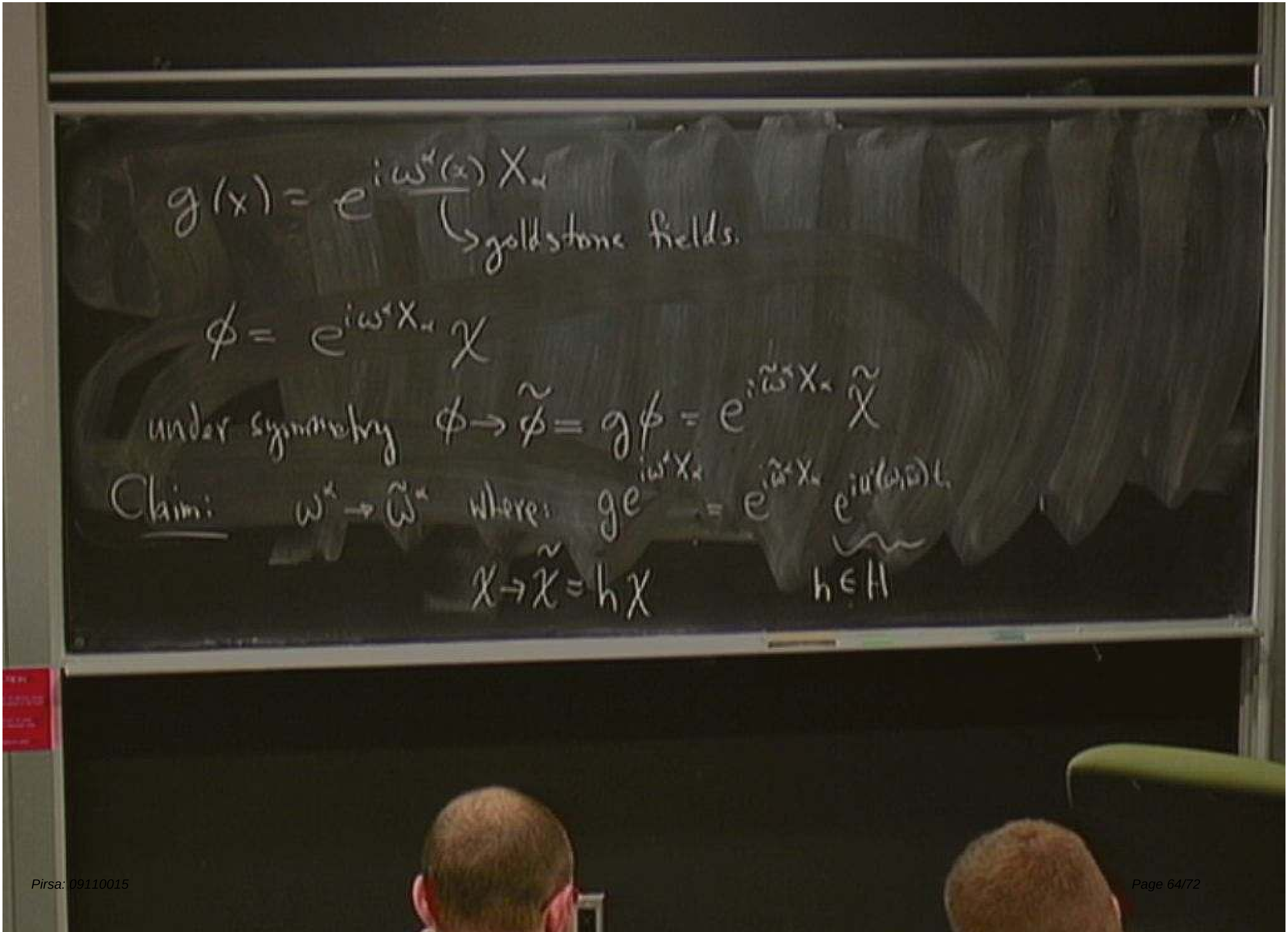
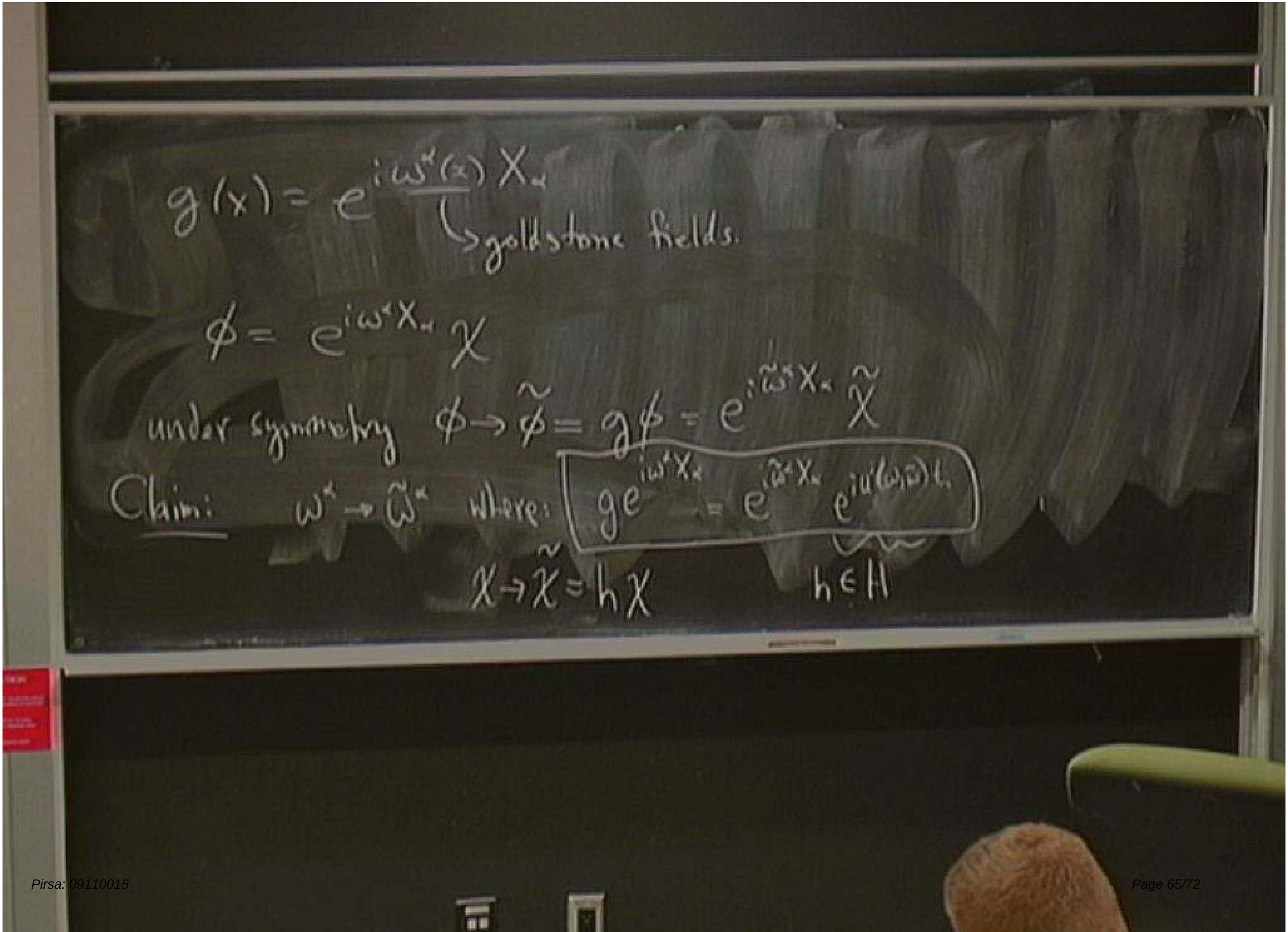

g(x) = e^{iω^α(x) X_α}
↳ goldstone fields.
φ = e^{iω^α X_α} χ
under symmetry φ → φ̃ = gφ = e^{iω̃^α X_α} χ̃
Claim: ω^α → ω̃^α where: g e^{iω^α X_α} = e^{iω̃^α X_α} e^{iu(ω,ω̃)t}
χ → χ̃ = hχ
h∈H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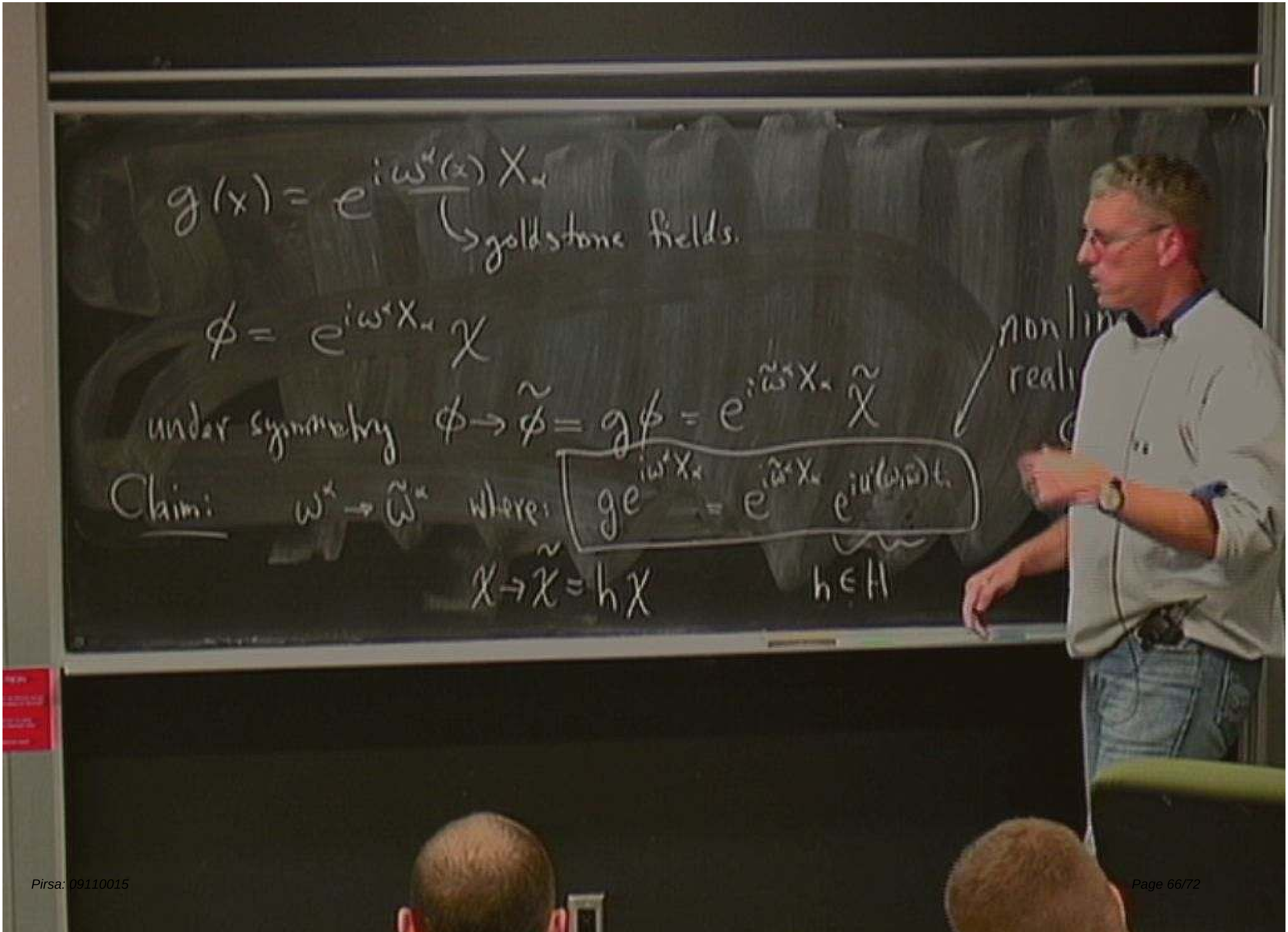
Claim: $\omega^\alpha \to \tilde{\omega}^\alpha$ where: $g e^{i\omega^\alpha X_\alpha} = e^{i\tilde{\omega}^\alpha X_\alpha} e^{iu^l(\omega,\tilde{\omega}) t_l}$
$\chi \to \tilde{\chi} = h\chi$ $h \in H$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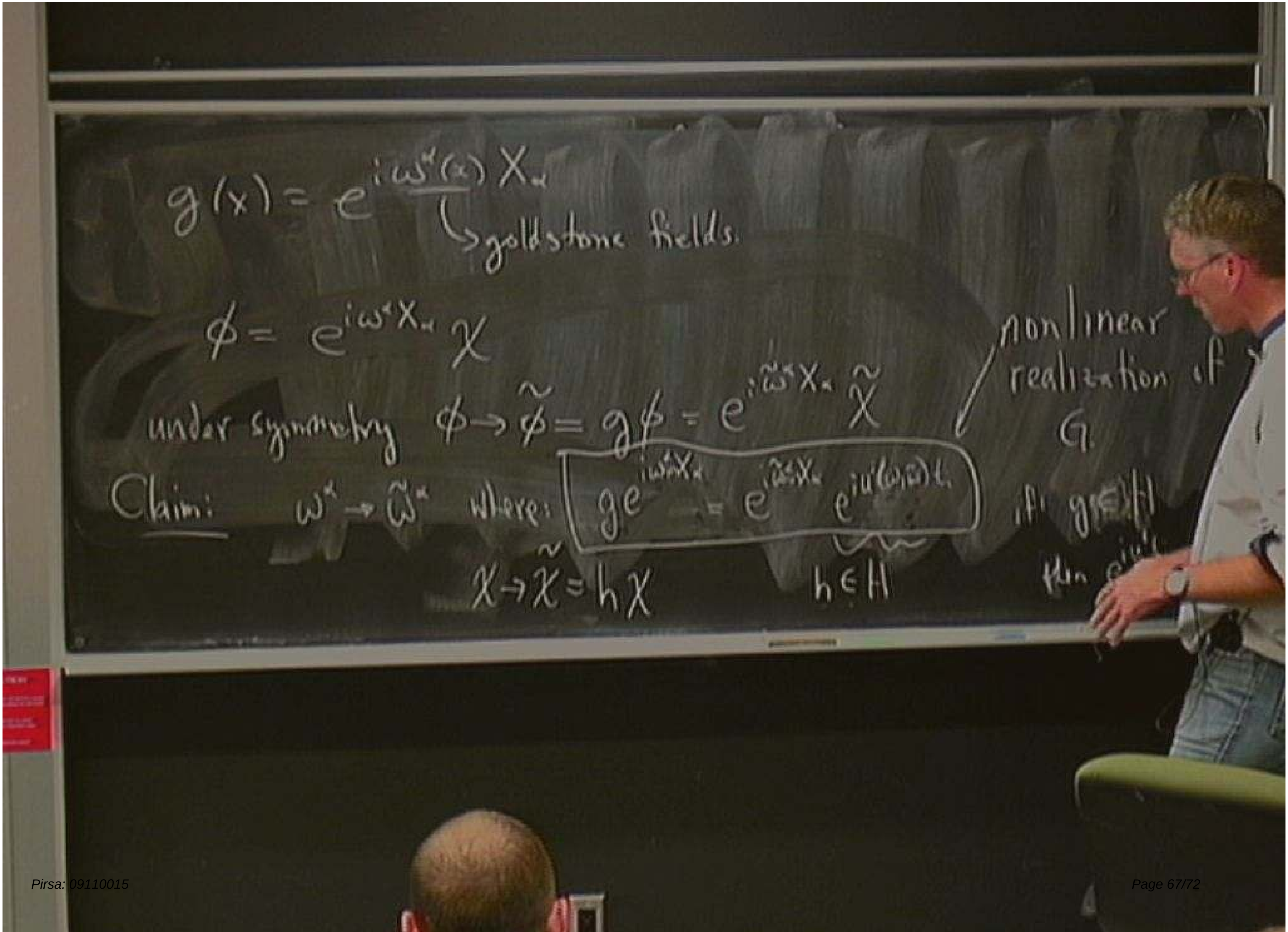
nonlin
reali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φ = e^{iω^α X_α} χ
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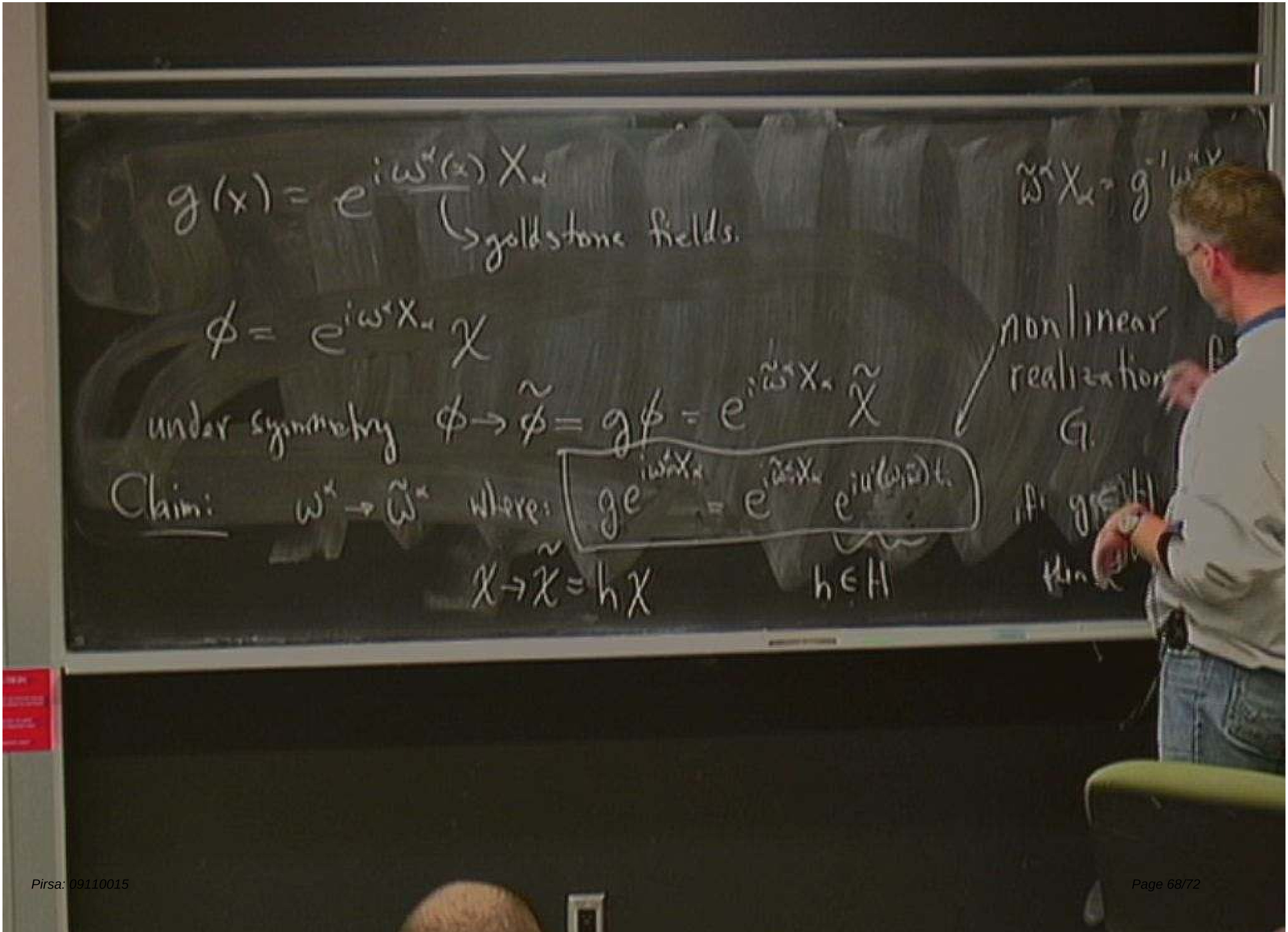
nonlinear realization of G.
Claim: ω^α → ω̃^α where: g e^{iω^α X_α} = e^{iω̃^α X_α} e^{iu(ω,ω̃)t}
χ → χ̃ = hχ
h∈H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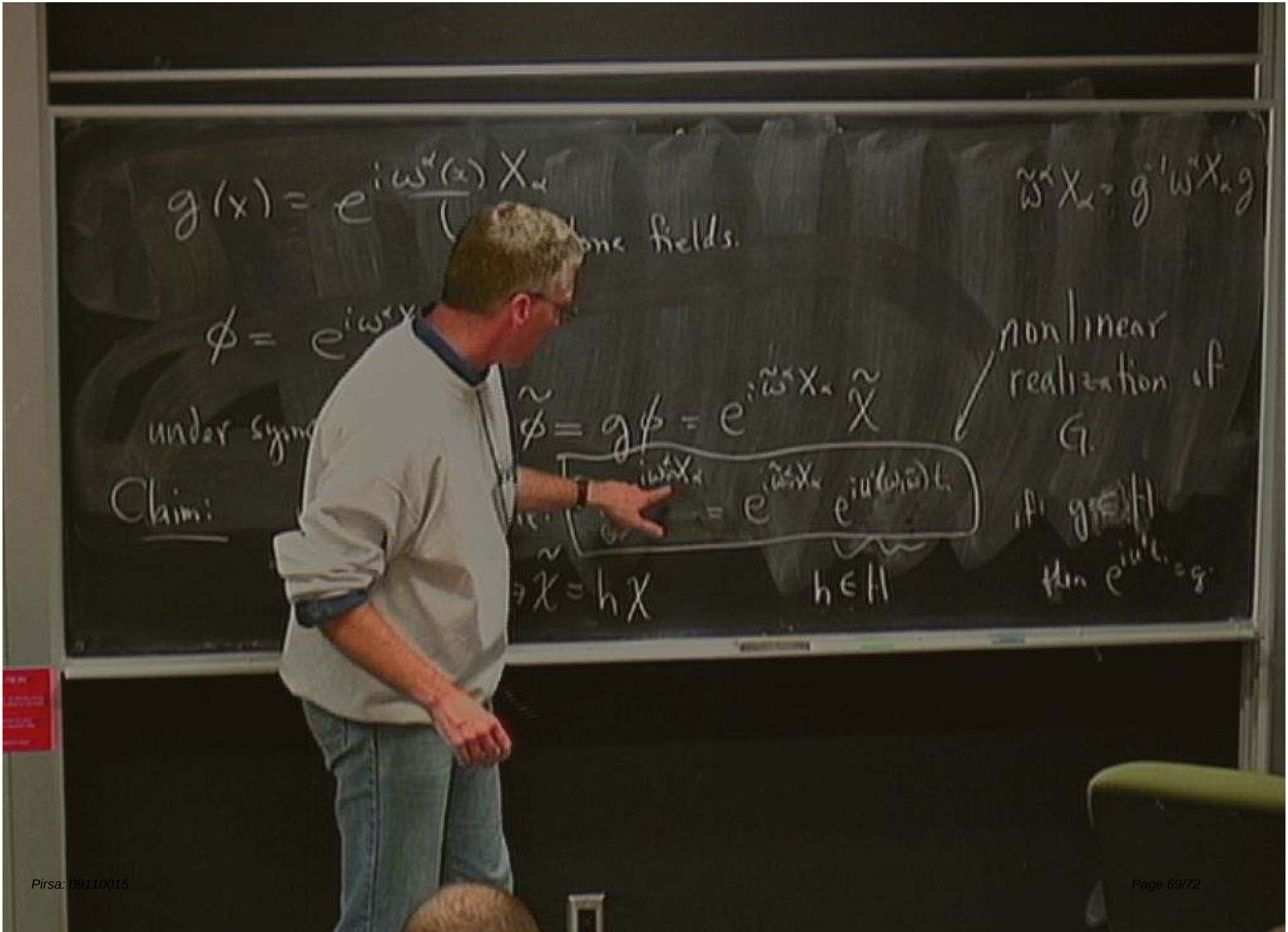
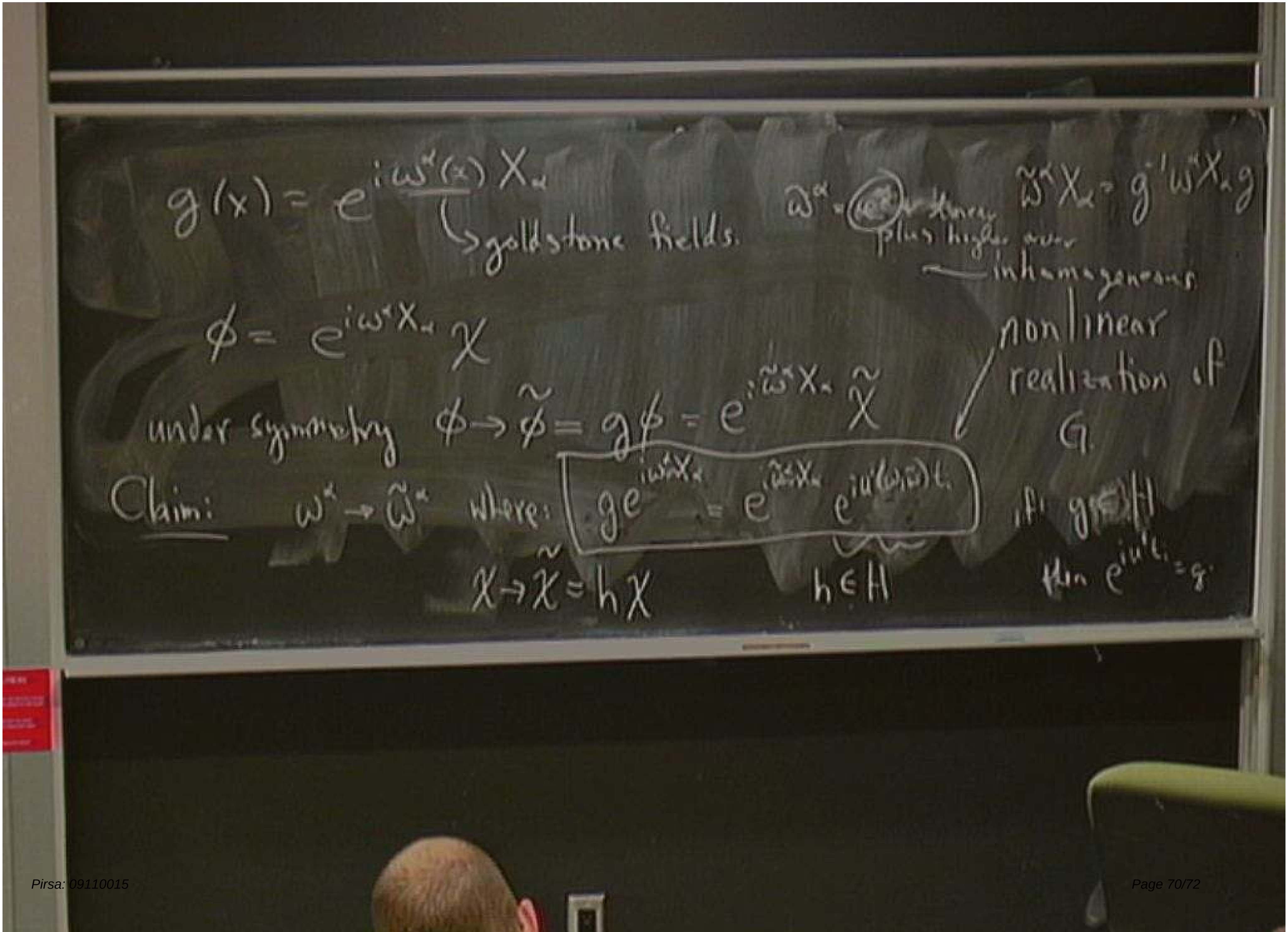
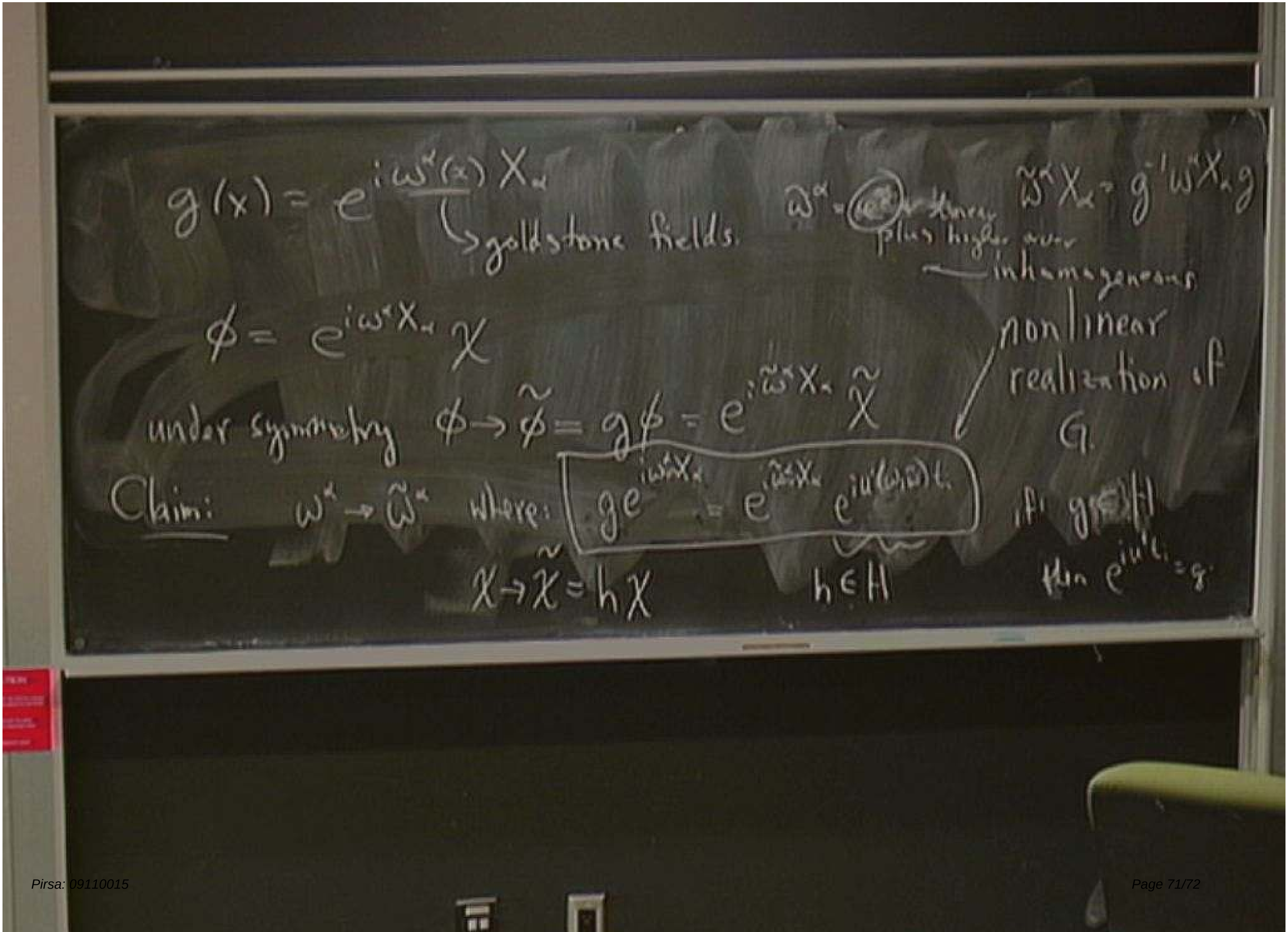
χ → χ̃ = hχ
h ∈ H
nonlinear realization
G


g(x) = e
fields.
nonlinear
realization of
G.
under
Claim:
h ∈ H


$g(x) = e^{i\omega^\alpha(x) X_\alpha}$
goldstone fields.
$\tilde{\omega}^\alpha X_\alpha = g^{-1}\omega^\alpha X_\alpha g$
plus higher order
inhomogeneous
$\phi = e^{i\omega^\alpha X_\alpha}\chi$
nonlinear realization of G.
under symmetry $\phi \to \tilde{\phi} = g\phi = e^{i\tilde{\omega}^\alpha X_\alpha}\tilde{\chi}$
Claim: $\omega^\alpha \to \tilde{\omega}^\alpha$ where: $g e^{i\omega^\alpha X_\alpha} = e^{i\tilde{\omega}^\alpha X_\alpha} e^{iu^i(\omega,\tilde{\omega})t_i}$
$\chi \to \tilde{\chi} = h\chi$
$h \in H$
if $g \in H$ then $e^{iu^i t_i} = g$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Claim: ω^α → ω̃^α where: g e^{iω^α X_α} = e^{iω̃^α X_α} e^{iu(ω, g) t}
χ → χ̃ = hχ
h ∈ H

matrix

$\mathcal{L} = -\frac{1}{2}\partial_\mu \phi^T \partial^\mu \phi - V(\phi)$, $V(g\phi) = V(\phi)$ for all ϕ.

and $\phi \neq 0$ at min.

$g = \underbrace{e^{i\omega^a T_a}}_{G.}$ (ω^a real)

$g \leftrightarrow$ orthogonal $\quad T_a \leftrightarrow$ antisymmetric

$\leftrightarrow$ unitary $\quad \leftrightarrow$ hermitian

$\leftrightarrow$ real $\quad \leftrightarrow$ imaginary

$T_a = -T_a^T = -T_a^* = T_a^\dagger$

Define $\phi(x) = g(x)\chi(x)$ where $g(x) = e^{i\omega^a(x) X_a}$