Title: General Relativity for Cosmology - Lecture 16A

Date: Nov 12, 2009 04:00 PM

URL: http://pirsa.org/09110008

Abstract:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

GR for Cosmology, Achim Kempf, Fall 2009, Lecture 18

Causal Structure & "Singularities"

Recall:

- The "chronological future" of a set S is the set

$$I^+(S)$$

of events that can be reached from S on a future-directed timelike curve.

- The "causal future" of a set S is the set

$$J^+(S)$$

of events that can be reached from S on a future-directed causal curve.

recall: includes null curves.

- In Minkowski space: $J^+(S) = I^+(S) + \dot{I}^+(S)$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Recall: $\dot{I}^+(S)$ is the boundary of $I^+(S)$

Properties of $\dot{I}^+(S)$, in general?

Definition:

A subset $Q \subset M$ is called

"achronal"

if no two points in Q can be connected by a future-directed time-like curve, i.e., by a curve that, e.g., a clock (with mass) could travel. Thus, $Q \in M$ is achronal iff:

empty set

$$I^+(Q) \cap Q = \phi$$

(with mass) could travel. Thus, $Q \in \mathcal{M}$ is achronal iff:

$$I^+(Q) \cap Q = \phi$$

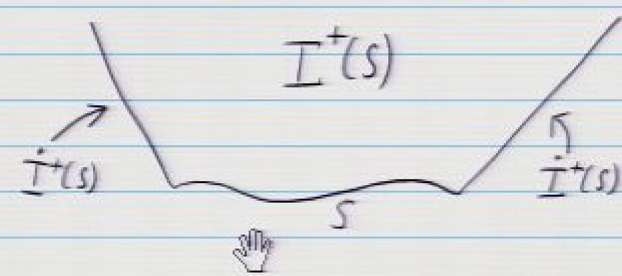
empty set

Theorem:

For all $S \subset \mathcal{M}$, the set

$$\dot{I}^+(S)$$

(if not empty) is an achronal 3-dimensional submanifold of $\mathcal{M}$.



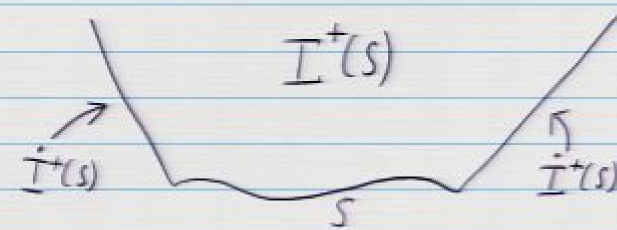
9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Theorem:

For all $S \subset \mathcal{M}$, the set

$$\dot{I}^+(S)$$



(if not empty) is an achronal 3-dimensional submanifold of $\mathcal{M}$.

Example: In Minkowski space, if S is a point p, then $\dot{I}^+(p)$ is the boundary of the light cone.

Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

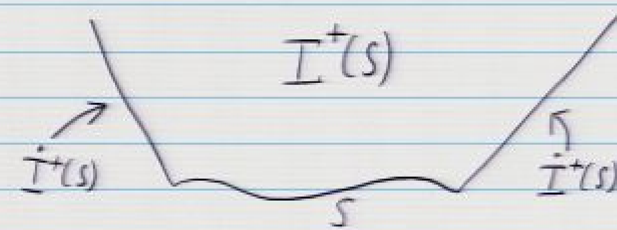
Page Width

Theorem:

For all $S \subset \mathcal{M}$, the set

$$\dot{I}^+(S)$$

(if not empty) is an achronal 3-dimensional submanifold of $\mathcal{M}$.



Example: In Minkowski space, if S is a point p, then $\dot{I}^+(p)$ is the boundary of the light cone.

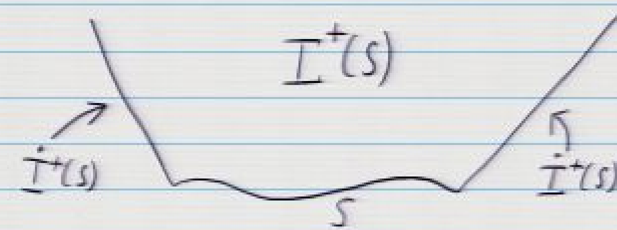
Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

Theorem:

For all $S \subset \mathcal{M}$, the set

$$\dot{I}^+(S)$$

(if not empty) is an achronal 3-dimensional submanifold of $\mathcal{M}$.



Example: In Minkowski space, if S is a point p, then $\dot{I}^+(p)$ is the boundary of the light cone.

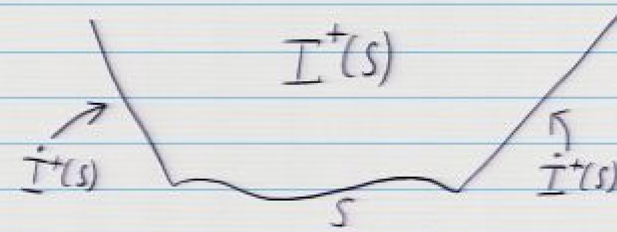
Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

Theorem:

For all $S \subset M$, the set

$$\dot{I}^+(S)$$

(if not empty) is an achronal 3-dimensional submanifold of M.



Example: In Minkowski space, if S is a point p, then $\dot{I}^+(p)$ is the boundary of the light cone.

Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

9gr18.jnt - Windows Journal

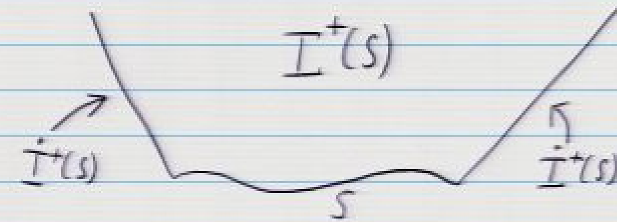
Edit View Insert Actions Tools Help

Page Width

For all $S \subset \mathcal{M}$, the set

$$\dot{I}^+(S)$$

(if not empty) is an achronal 3-dimensional submanifold of $\mathcal{M}$.



Example: In Minkowski space, if S is a point p, then $\dot{I}^+(p)$ is the boundary of the light cone.

Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

In general spacetimes, however:

3 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

$I^+(p)$ is the boundary of the light cone.

Indeed, no two points of the boundary of the lightcone are connected by time-like paths.

In general spacetimes, however:

It is clear that

$$\overline{J^+(S)} = \overline{I^+(S)}$$ ↙ bar denotes closure of the set

but we notice that, generally:

$$J^+(S) \subset \overline{I^+(S)}$$

In general spacetimes, however:

It is clear that

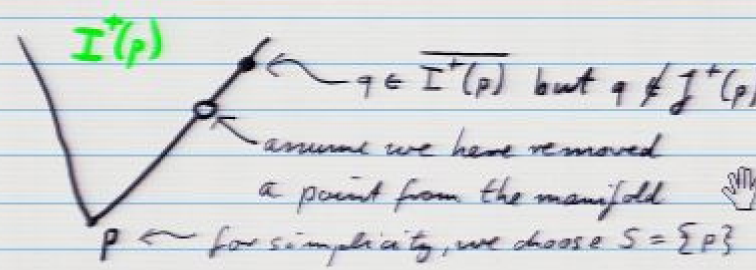
$$\overline{J^+(S)} = \overline{I^+(S)}$$

← bar denotes closure of the set

but we notice that, generally:

$$J^+(S) \subset \overline{I^+(S)}$$

Example:



⇒ here, $q \in \overline{I^+(p)}$, but $q \notin J^+(p)$ because there

It is clear that

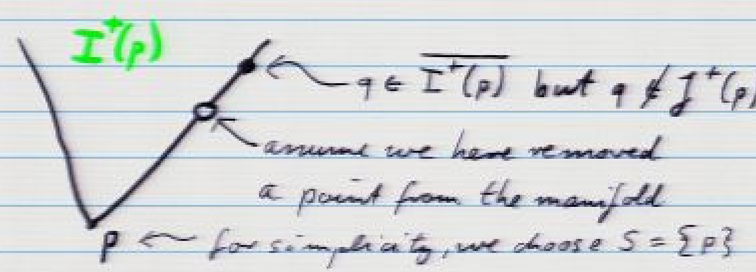
$$\overline{J^+(S)} = \overline{I^+(S)}$$

bar denotes closure of the set

but we notice that, generally:

$$J^+(S) \subset \overline{I^+(S)}$$

Example:



$\Rightarrow$ here, $q \in \overline{I^+(p)}$, but $q \notin J^+(p)$ because there is no nonspacelike curve between p and q.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

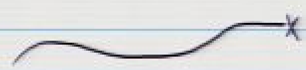
→ Idea:

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

$\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that



$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad \left(t < t_0 \text{ for past endpoint} \right)$$

- Note: p need not be reached by γ!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

⇝ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

$\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad \left(t < t_0 \text{ for past endpoint} \right)$$

- **Note:** p need not be reached by γ!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

⇝ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) end point of a curve γ if

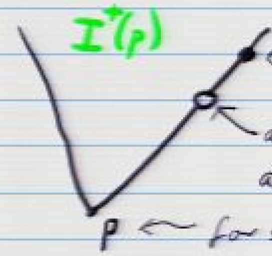
  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad \left(t < t_0 \text{ for past end point} \right)$$

- **Note:** p need not be reached by γ!

$J^+(S) \subset I^+(S)$

Example:



$\Rightarrow$ here, $q \in \overline{I^+(p)}$, but $q \notin J^+(p)$ because there is no nonspacelike curve between p and q.

$\leadsto$ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

is no nonspacelike curve between p and q.

⤳ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

We say that a point $p \in M$ is future (past) endpoint of a curve γ if

$\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that

$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0$ $\quad$ ($t < t_0$ for pas...

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

→ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

  $\forall$ neighborhoods U of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset U \quad \forall t > t_0 \qquad \left(\begin{array}{c} t < t_0 \text{ for past} \\ \text{endpoint} \end{array}\right)$$

- **Note:** p need not be reached by γ!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

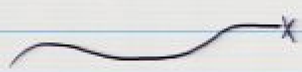
$\rightsquigarrow$ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that



$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad (t < t_0 \text{ for past endpoint})$$

- **Note:** p need not be reached by γ!

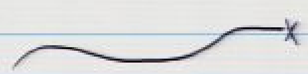
$\rightsquigarrow$ **Idea:**

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that



$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad \left(\begin{array}{c} t < t_0 \text{ for past} \\ \text{endpoint} \end{array}\right)$$

- **Note:** p need not be reached by γ!

9gr18.jnt - Windows Journal

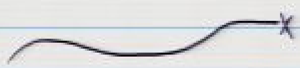
Edit View Insert Actions Tools Help

Let us use the extendibility or nonextendibility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in M$ is future (past) end point of a curve γ if

  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that



$$\gamma(t) \subset \mathcal{U} \quad \forall t > t_0 \qquad \left(t < t_0 \text{ for past end point} \right)$$

- **Note:** p need not be reached by γ!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

bility of curves (or especially of geodesics) as indicator for the absence or existence of a singularity.

Definition:

- We say that a point $p \in \mathcal{M}$ is future (past) endpoint of a curve γ if

  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset \mathcal{U} \quad \forall t > t_0 \qquad \left(t < t_0 \text{ for past endpoint} \right)$$

- **Note:** p need not be reached by γ!

- Since $p \in \mathcal{M}$, it is always possible to extend

5 / 25

Definition:

- We say that a point $p \in \mathcal{M}$ is future (past) endpoint of a curve γ if

  $\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0 \qquad \left(\begin{array}{c} t < t_0 \text{ for past} \\ \text{endpoint} \end{array}\right)$$

- **Note:** p need not be reached by γ!

- Since $p \in \mathcal{M}$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in \mathcal{M}$!

$\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we cho...

- Since $p \in \mathcal{M}$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in \mathcal{M}$!

  $\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

Definition:

- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.
- Intuition: If γ future inextendible, then:

  a.) γ runs to ∞, or

  b.) γ runs around forever, or

  c.) γ hits a hole i.e. singularity, thus can't

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

- Since $p \in M$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in M$!

$\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

Definition:

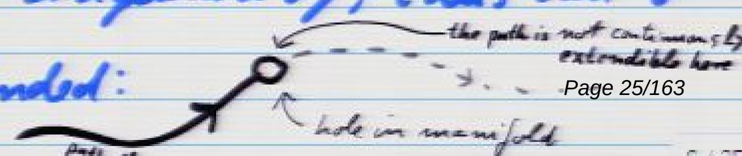
- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.

- Intuition: If γ future inextendible, then:

  a.) γ runs to ∞, or

  b.) γ runs around forever, or

  c.) γ hits a hole i.e. singularity, thus can't be continuously extended:



9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- Since $p \notin \mathcal{M}$, it is always possible to extend a curve γ continuously beyond p. Note: $p \notin \mathcal{M}$!

  $\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

<u>Definition:</u>

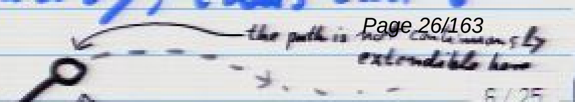
- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.

- Intuition: If γ future inextendible, then:

  a.) γ runs to ∞, or

  b.) γ runs around forever, or

  c.) γ hits a hole i.e. singularity, thus can't be continuously extended:



9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

- We say that a point $p \in M$ is future (past) endpoint of a curve γ if

$\forall$ neighborhoods U of p there exists a $t_0 \in \mathbb{R}$ so that

$$\gamma(t) \subset U \quad \forall\, t > t_0 \qquad \left(\begin{array}{c} t < t_0 \text{ for past} \\ \text{endpoint} \end{array} \right)$$

- **Note:** p need not be reached by γ!

- Since $p \in M$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in M$!

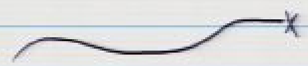
$\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

of a curve γ if

$\forall$ neighborhoods $\mathcal{U}$ of p there exists a $t_0 \in \mathbb{R}$ so that



$\gamma(t) \subset \mathcal{U} \quad \forall\, t > t_0$ ($t < t_0$ for past endpoint)

- **Note:** p need not be reached by γ!

- Since $p \in \mathcal{M}$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in \mathcal{M}$!

  $\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

Definition:

$\gamma(t) \in \mathcal{M} \quad \forall t > t_0$ ($t < t_0$ for past endpoint)

- **Note:** p need not be reached by γ!

- Since $p \notin \mathcal{M}$, it is always possible to extend a curve γ continuously beyond p. Note: $p \in \mathcal{M}$!

  $\Rightarrow$ The existence of an endpoint here indicates <u>not</u> a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

Definition:

- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.
- Intuition: If γ future inextendible, then:

  a) γ runs to ∞, or

a singularity, or 'hole' in the manifold, but merely that we chose to end the curve before it is necessary!

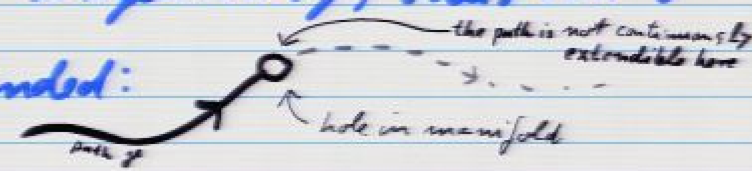
Definition:

- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.
- Intuition: If γ future inextendible, then:

  a.) γ runs to ∞, or

  b.) γ runs around forever, or

  c.) γ hits a hole i.e. singularity, thus can't be continuously extended:



Theorem:

Assume $\mathcal{C} \subset M$ is closed and assume:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

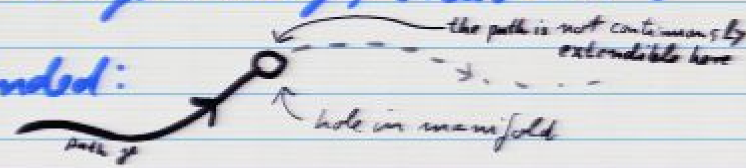
Definition:

- We say that a curve γ is future (past) inextendible if it does not possess a future (past) endpoint.
- Intuition: If γ future inextendible, then:

  a.) γ runs to ∞, or

  b.) γ runs around forever, or

  c.) γ hits a hole i.e. singularity, thus can't be continuously extended:



Theorem:

Assume $C \subset M$ is closed and assume:

$$q \in \dot{I}^{+}(C), \quad q \notin C$$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Theorem:

Assume $\mathcal{C} \subset M$ is closed and assume:

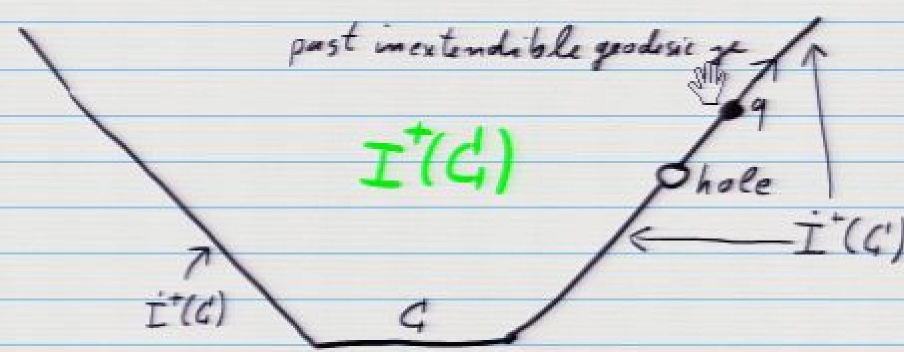
$$q \in \dot{I}^+(\mathcal{C}), \quad q \notin \mathcal{C}$$

Then, q lies on a null geodesic γ inside $\dot{I}^+(\mathcal{C})$ and the curve γ either:

a.) has past endpoint on $\mathcal{C}$

or b.) is past inextendible (because meets hole)

Example for b:



9gr18.jnt - Windows Journal

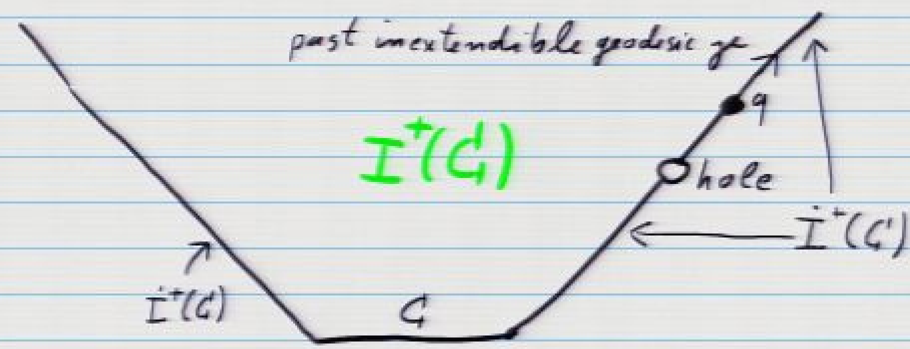
Edit View Insert Actions Tools Help

Page Width

a.) has past endpoint on C'

or b.) is past inextendible (because meets hole)

Example for b:



Strategy:

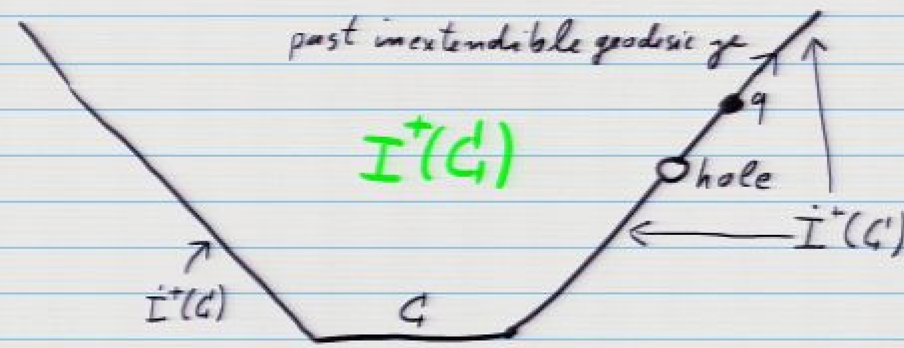
- Study inextendible curves!
- This includes these cases:
  - a.) γ hits singularity – will be main interest!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

or b.) is past inextendible (because meets hole)

Example for b:



Strategy:

- Study inextendible curves!
- This includes these cases:

a.) γ hits singularity - will be main interest!

b.) γ running off to ∞.

9gr18.jnt - Windows Journal

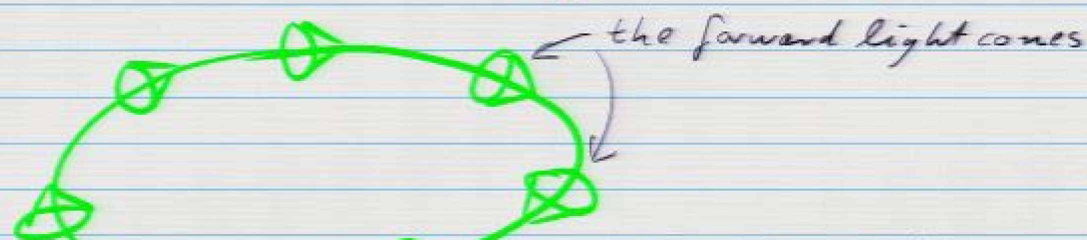
Edit View Insert Actions Tools Help

Strategy:

- Study inextendible curves!
- This includes these cases:
  - a.) γ hits singularity – will be main interest!
  - b.) γ running off to ∞.
  - c.) γ going round and round forever.

$\rightsquigarrow$ Must address potential causality problems of case c)

Example:



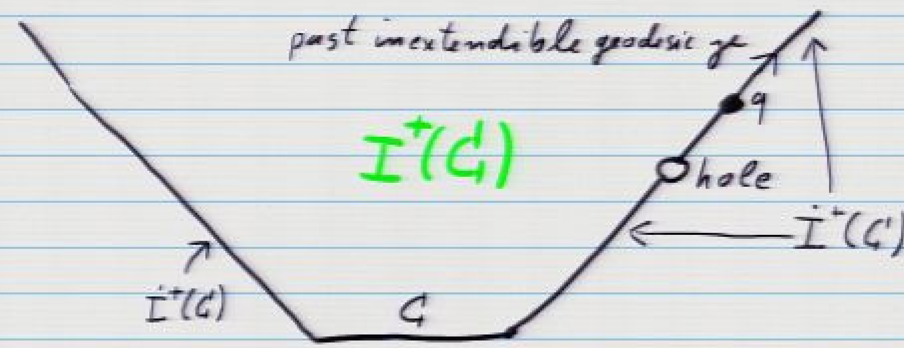
9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

or b.) is past inextendible (because meets hole)

Example for b:



Strategy:

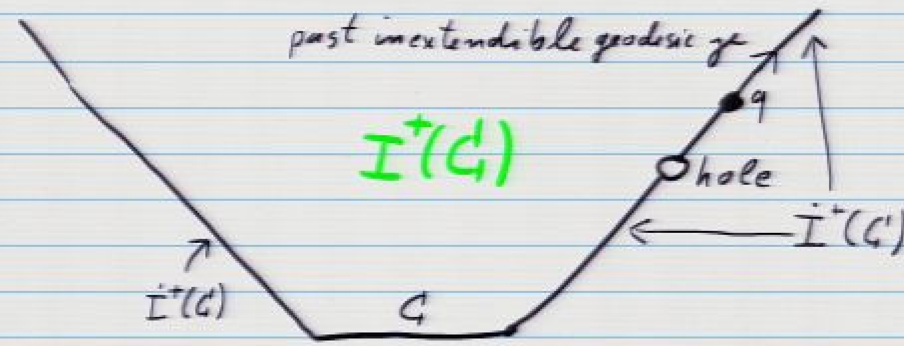
- Study inextendible curves!
- This includes these cases:

  a.) γ hits singularity - will be main interest!

  b.) γ running off to ∞.

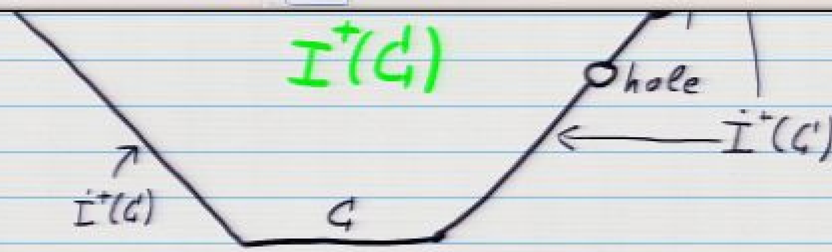
or b.) is past inextendible (because meets hole)

Example for b:



Strategy:

- Study inextendible curves!
- This includes these cases:
  - a.) γ hits singularity – will be main interest!
  - b.) γ running off to ∞.
  - c.) γ going round and round forever.



Strategy:

- Study inextendible curves!
- This includes these cases:
  - a.) γ hits singularity – will be main interest!
  - b.) γ running off to ∞.
  - c.) γ going round and round forever.

⇝ Must address potential causality problems of case c)

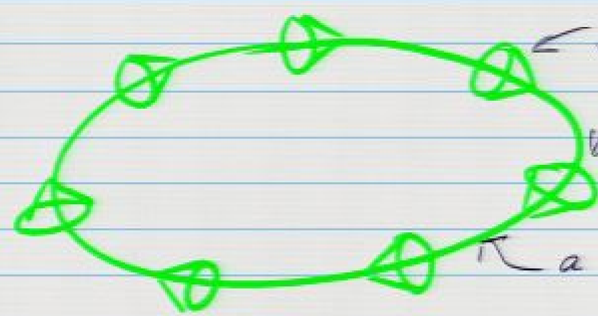
the forward light cones

Strategy:

- Study inextendible curves!
- This includes these cases:
  - a.) γ hits singularity – will be main interest!
  - b.) γ running off to ∞.
  - c.) γ going round and round forever.

⇝ Must address potential causality problems of case c)

Example:



the forward light cones

a closed timelike curve.

Note: No problem with time-orientability here!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

This includes these cases:

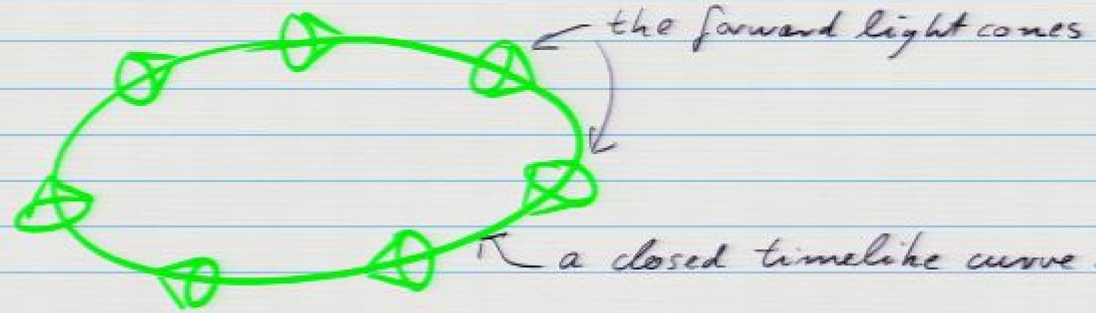
a.) γ hits singularity – will be main interest!

b.) γ running off to ∞.

c.) γ going round and round forever.

⇝ Must address potential causality problems of case c)

Example:



Note: No problem with time-orientability here!

Causality conditions:

9gr18.jnt - Windows Journal

a closed timelike curve.

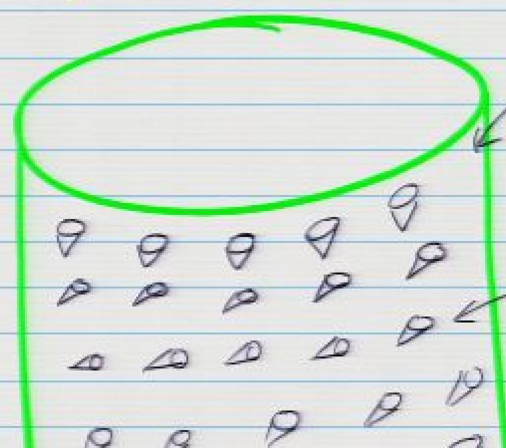
Note: No problem with time-orientability here!

Causality conditions:

- We say that (M, g) is "causal" if it does not contain closed causal (i.e. time or null) curves.

Problem: (M, g) may nevertheless be arbitrarily close to being acausal:

(M, g) is the boundary of a cylinder.



those light cones could be arbitrarily close to the picture above, to allowing a closed

9gr18.jnt - Windows Journal

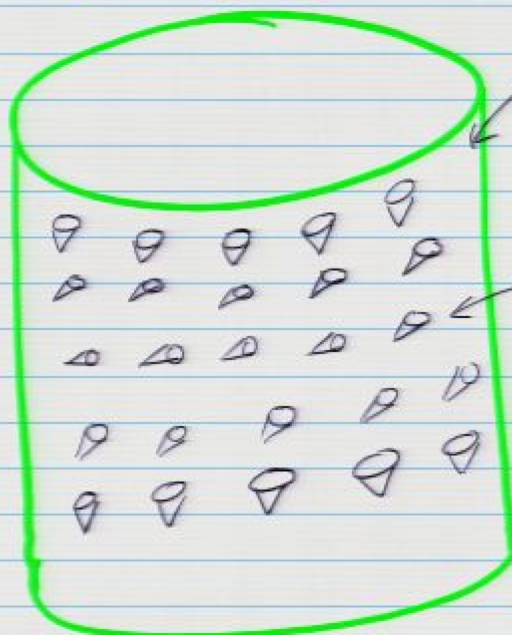
Edit View Insert Actions Tools Help

Causality conditions:

- We say that (M, g) is "causal" if it does not contain closed causal (i.e. time or null) curves.

Problem: (M, g) may nevertheless be arbitrarily close to being acausal:

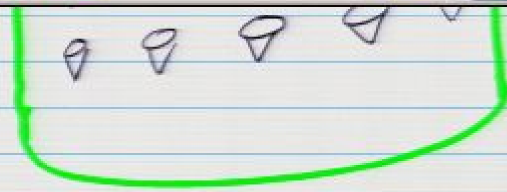
(M, g) is the boundary of a cylinder.



those light cones could be arbitrarily close to the picture above i.e. to allowing a closed timelike curve!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help



→ We say that a spacetime (M, g) is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods U of p there is a neighborhood $V \subset U$ so that:

No causal curve γ intersects V more than once.

Indeed:

If (M, g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

→ We say that a spacetime $(\mathcal{M}, g)$ is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods $\mathcal{U}$ of p there is a neighborhood $V \subset \mathcal{U}$ so that:

No causal curve γ intersects V more than once.

Indeed:

If $(\mathcal{M}, g)$ is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

→ We require strong causality to keep causal curves at

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

→ We say that a spacetime (M, g) is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods U of p there is a neighborhood $V \subset U$ so that:

No causal curve γ intersects V more than once.

Indeed:

If (M,g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

→ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

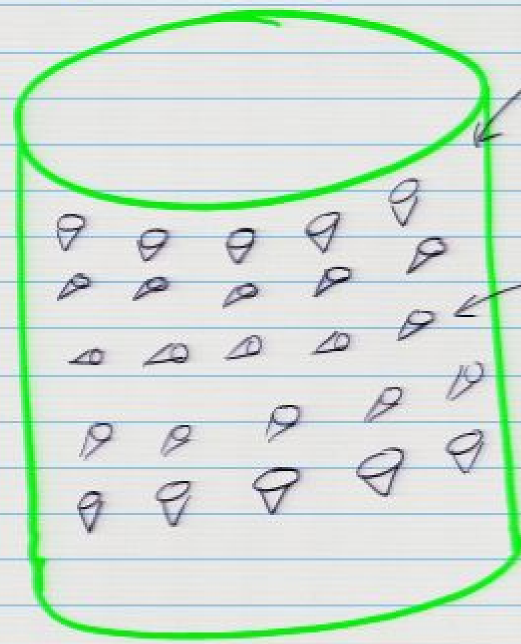
9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

contain closed causal (i.e. time or null) curves.

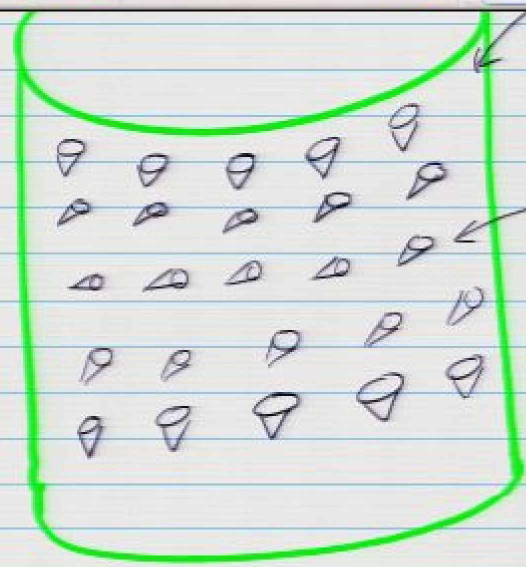
Problem: (M,g) may nevertheless be arbitrarily close to being acausal:

(M,g) is the boundary of a cylinder.



those light cones could be <u>arbitrarily close</u> to the picture above i.e. to allowing a closed timelike curve!

We say that a spacetime (M,g) is "strongly



those light cones could be arbitrarily close to the picture above i.e. to allowing a closed timelike curve!

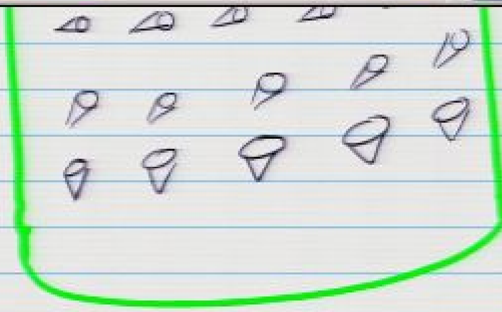
→ We say that a spacetime $(\mathcal{M}, g)$ is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods $\mathcal{U}$ of p there is a neighborhood $V \subset \mathcal{U}$ so that:

No causal curve γ intersects V more than once.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help



the picture above i.e. to allowing a closed timelike curve!

→ We say that a spacetime (M,g) is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods U of p there is a neighborhood $V \subset U$ so that:

No causal curve γ intersects V more than once.

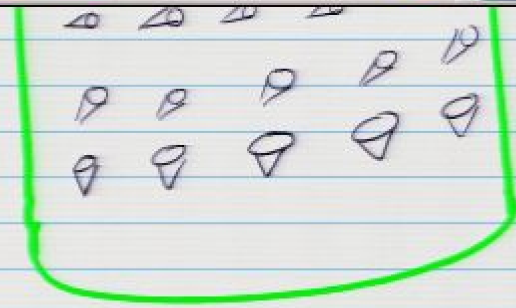
Indeed:

If (M,g) is not strongly causal $\Rightarrow$ there exists

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width



the picture above i.e. to allowing a closed timelike curve!

→ We say that a spacetime (M, g) is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods U of p there is a neighborhood $V \subset U$ so that:

No causal curve γ intersects V more than once.

Indeed:

If (M, g) is not strongly causal $\Rightarrow$ there exists

9gr18.jnt - Windows Journal

→ We say that a spacetime (M,g) is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods U of p there is a neighborhood $V \subset U$ so that:

No causal curve γ intersects V more than once.

Indeed:

If (M,g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

→ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

→ We say that a spacetime $(\mathcal{M},g)$ is "strongly causal", if

$\forall p$ and $\forall$ neighborhoods $\mathcal{U}$ of p there is a neighborhood $V \subset \mathcal{U}$ so that:

No causal curve γ intersects V more than once.

Indeed:

If $(\mathcal{M},g)$ is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

→ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Indeed:

If (M,g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

$\rightsquigarrow$ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

Problem:

Still, arbitrarily small perturbations in the metric, somewhere, could allow causal curves to self-intersect!

10 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Indeed:

If (M,g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

$\rightsquigarrow$ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

Problem:

Still, arbitrarily small perturbations in the metric, somewhere, could allow causal curves to self-intersect!

Solution:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

If (M,g) is not strongly causal $\Rightarrow$ there exists a causal curve γ which comes arbitrarily close to intersecting itself.

$\rightsquigarrow$ We require strong causality to keep causal curves at least a finite distance from intersecting themselves.

Problem:

Still, arbitrarily small perturbations in the metric, somewhere, could allow causal curves to self-intersect!

Solution:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

least a finite distance from intersecting themselves.

Problem:

Still, arbitrarily small perturbations in the metric, somewhere, could allow causal curves to self-intersect!

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu} - w_\mu w_\nu$$

with a time-like cotangent vector field.

needed for theorem 2 below.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Problem:

Still, arbitrarily small perturbations in the metric, somewhere, could allow causal curves to self-intersect!

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu} - w_\mu w_\nu$$

with a time-like cotangent vector field.

needed for theorem 2 below.

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

self-intersect?

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu} - w_\mu w_\nu$$

with a time-like cotangent vector field.

needed for theorem 2 below.

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature

but light cones are now "wider":

If $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu w_\mu v^\nu w_\nu}_{} < 0$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu} - \omega_\mu \omega_\nu$$

with a time-like cotangent vector field.

↑ needed for theorem 2 below

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature

but light cones are now "wider":

If $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu \omega_\mu v^\nu \omega_\nu}_{\text{always } < 0} < 0$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

needed for theorem 2 below.

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature but light cones are now "wider":

If $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu w_\mu v^\nu w_\nu}_{\text{always} < 0} < 0$

Thus, it is easier for vectors v to be timelike or null for $\tilde{g}$ than for g.

$\Longrightarrow$

$(M, \tilde{g})$ has all the causal curves of (M, g), and more!

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu} - w_\mu w_\nu$$

with a time-like cotangent vector field.

needed for theorem 2 below.

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature

but light cones are now "wider":

If $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu w_\mu v^\nu w_\nu}_{\text{always} < 0} < 0$

11 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Solution:

a) Consider perturbing the metric g through

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = g_{\mu\nu} - \omega_\mu \omega_\nu$$

with a time-like cotangent vector field.

needed for theorem 2 below.

b) Notice: $\tilde{g}_{\mu\nu}$ still has same signature

but light cones are now "wider":

If $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu \omega_\mu v^\nu \omega_\nu}_{\text{always } < 0} < 0$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

if $v^\mu v^\nu g_{\mu\nu} < 0$

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu \omega_\mu v^\nu \omega_\nu}_{\text{always } < 0} < 0$

Thus, it is easier for vectors v to be timelike or null for $\tilde{g}$ than for g.

$\Longrightarrow$

$(M,\tilde{g})$ has all the causal curves of (M,g), and more!

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

then $v^\mu v^\nu \tilde{g}_{\mu\nu} = v^\mu v^\nu g_{\mu\nu} \underbrace{- v^\mu \omega_\mu v^\nu \omega_\nu}_{\text{always } < 0} < 0$

Thus, it is easier for vectors v to be timelike or null for $\tilde{g}$ than for g.

$\Longrightarrow$

$(M, \tilde{g})$ has all the causal curves of (M, g), and more!

c) Define:

(M, g) is called "stably causal", if there exists a ω so that even $(M, \tilde{g})$ is causal.

null for $\tilde{g}$ than for g.

$\Rightarrow$

$(M, \tilde{g})$ has all the causal curves of (M, g), and more!

c) Define:

(M, g) is called "stably causal", if there exists a ω so that even $(M, \tilde{g})$ is causal.

Theorem 1:

(M, g) stably causal $\Rightarrow$ (M, g) strongly causal.

Theorem 2:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

null for $\tilde{g}$ than for g.

$\Longrightarrow$

$(M, \tilde{g})$ has all the causal curves of (M,g), and more!

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth vector field but not that it is a gradient field

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$

c) Define:

(M, g) is called "stably causal", if there exists a ω so that even $(M, \tilde{g})$ is causal.

Theorem 1:

(M, g) stably causal $\Rightarrow$ (M, g) strongly causal.

Theorem 2:

(M, g) stably causal

$$\Updownarrow$$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

$(M,\tilde{g})$ has all the causal curves of (M,g), and more.

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

Theorem 1:

(M,g) stably causal $\Longrightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$$\Updownarrow$$

There exists a differentiable function $f \in \mathcal{F}(M)$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$

so that ∇f is a past-directed time-like vector fi...

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

c) Define:

(M,g) is called "stably causal", if there exists a ω so that even $(M,\tilde{g})$ is causal.

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$$\Updownarrow$$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Theorem 1:

(M,g) stably causal $\Rightarrow$ (M,g) strongly causal.

Theorem 2:

(M,g) stably causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth vector field but not that it is a gradient field.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Theorem 2:

(M, g) stably causal

$$\Updownarrow$$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth vector field but not that it is a gradient field.

We assume that spacetime is stably causal.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Chisel Marker (3.5 mm)
Medium Point
Fine Point
Marker (1 mm)
Very Fine Chisel
Pen Settings...

Theorem 2:

(M, g) ... y causal

$\Updownarrow$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth vector field but not that it is a gradient field.

We assume that spacetime is stably causal.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Theorem 2:

(M,g) stably causal

$$\Updownarrow$$

There exists a differentiable function $f \in \mathcal{F}(M)$ so that ∇f is a past-directed time-like vector field.

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth vector field but not that it is a gradient field.

We assume that spacetime is stably causal.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth timelike vector field but not that it is a gradient field.

→ We assume that spacetime is stably causal.

Intuition:

Therefore, inextendible paths either:

a.) go to ∞, or

b.) end in a singularity

→ Continue to study inextendible curves:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth timelike vector field but not that it is a gradient field.

⇝ We assume that spacetime is stably causal.

Intuition:

Therfore, inextendible paths either:

a.) go to ∞, or

b.) end in a singularity

⇝ Continue to study inextendible curves:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth timelike vector field but not that it is a gradient field.

We assume that spacetime is stably causal.

Intuition:

Therfore, inextendible paths either:

a.) go to ∞, or

b.) end in a singularity

Continue to study inextendible curves:

13 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Remark: This means that f can be viewed as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth timelike vector field but not that it is a gradient field.

⇝ We assume that spacetime is stably causal.

Intuition:

Therfore, inextendible paths either:

a.) go to ∞, or

b.) end in a singularity

⇝ Continue to study inextendible curves:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

as a cosmic "clock". (It is not unique, however)

Recall: Time-orientability means that there exists a past-pointing smooth timelike vector field but not that it is a gradient field.

⇝ We assume that spacetime is stably causal.

Intuition:

Therfore, inextendible paths either:

a.) go to ∞, or

b.) end in a singularity

⇝ Continue to study inextendible curves:

Recall:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

a.) go to ∞, or

b.) end in a singularity

⇝ Continue to study inextendible curves:

Recall:

- We considered the set of points $J^+(S)$ that can somehow be reached from a set S. (i.e. the set of points that are affected by S)
- Now consider set of points that can only be reached from S: (i.e. the set of events that depend on S and only S)

Definition:

i.e., a set of events among which no object could travel

Assume $S \subset \mathcal{M}$ is a closed achronal set.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

⇝ Continue to study inextendible curves:

Recall:

- We considered the set of points $J^+(S)$ that can somehow be reached from a set S. (i.e. the set of points that are affected by S)
- Now consider set of points that can only be reached from S: (i.e. the set of events that depend on S and only S)

Definition:

i.e., a set of events among which no object could travel

Assume $S \subset M$ is a closed achronal set. Then, the "future domain of dependence of S" is defined as:

$D^+(S) := \{ x \in M \mid$ Every past inextendible causal $\}$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

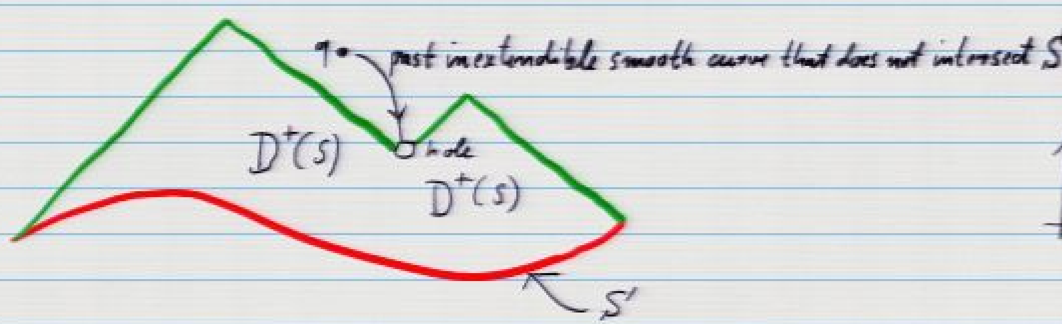
i.e., a set of events among which no object could travel

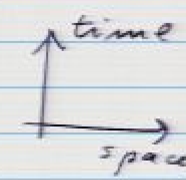
Assume $S \subset M$ is a closed achronal set.

Then, the "future domain of dependence of S" is defined as:

$$D^+(S') := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

Example:



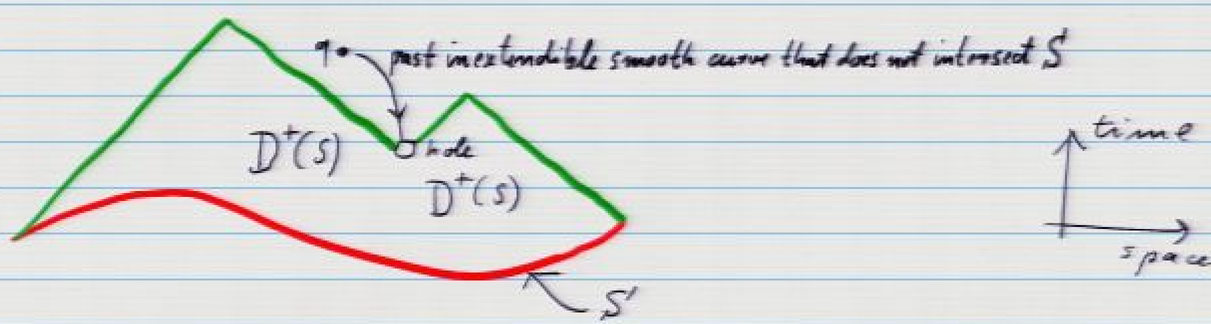


Why $q \notin D^+(S)$? Some of its past inextendible

Then, the future domain of dependence of S is defined as:

$$D^+(S) := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

Example:



Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

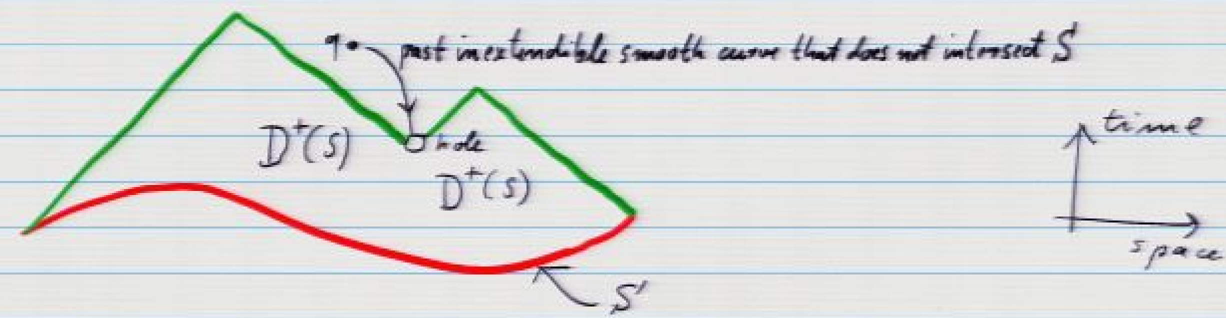
Definition:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

$$D^+(S') := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

Example:



Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

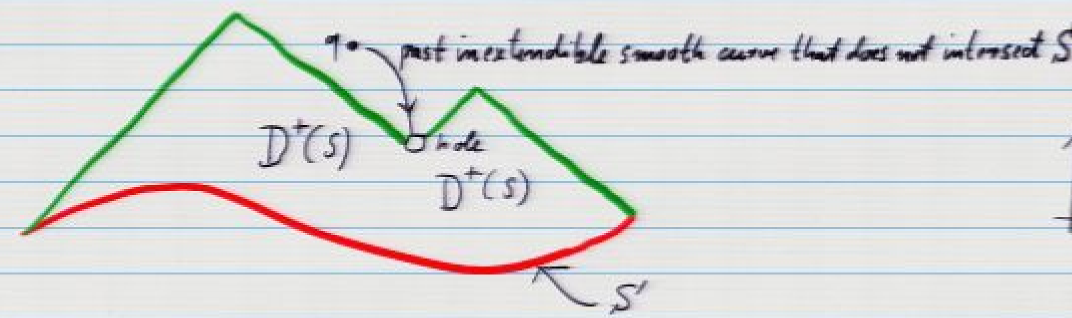
Analogously, the "past domain of dependence of S'" is:

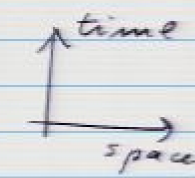
9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Example:





Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

Analogously, the "past domain of dependence of S'" is:

$$D^-(S') := \left\{ p \in M \;\middle|\; \text{Every future inextendible causal curve through } p \text{ intersects } S \right\}$$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

Analogously, the "past domain of dependence of S" is:

$$D^-(S) := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S" is:

$$D(S) := D^+(S) \cup D^-(S)$$

9gr18.jnt - Windows Journal

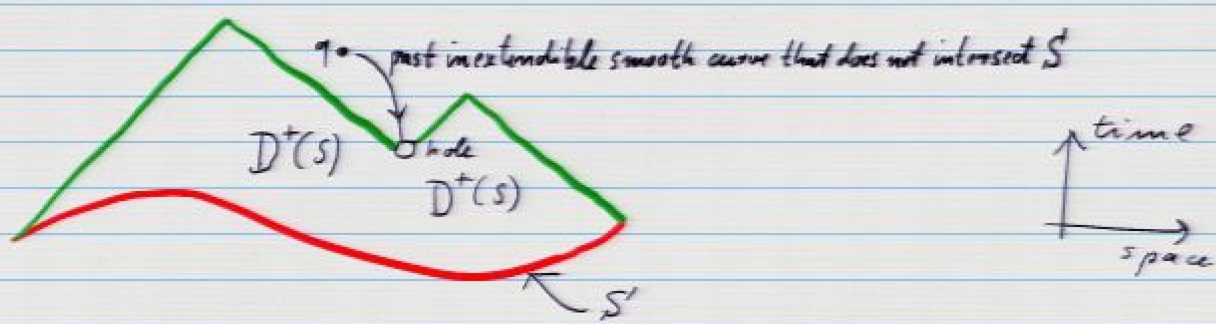
Edit View Insert Actions Tools Help

Page Width

is defined as:

$$D^+(S') := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

Example:



Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

Analogously, the "past domain of dependence of S'"

15 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

Analogously, the "past domain of dependence of S'" is:

$$D^-(S') := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S'" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?)

The "future Cauchy horizon of S'", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

16 / 25

9gr18.jnt - Windows Journal

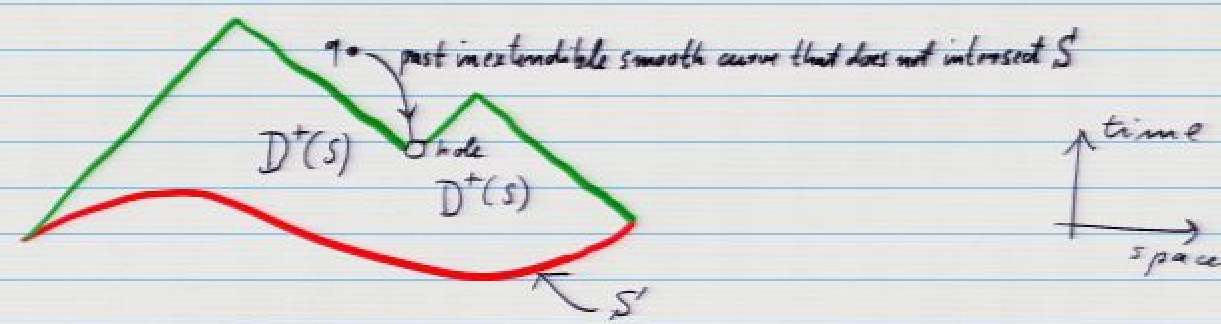
Edit View Insert Actions Tools Help

Page Width

is defined as:

$$D^+(S') := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

Example:



Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

Analogously, the "past domain of dependence of S'"

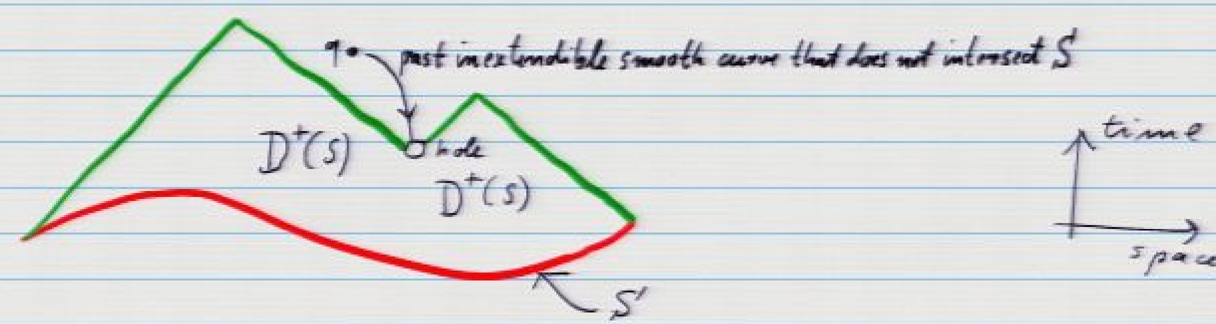
15 / 25

Then, the future domain of dependence of S

is defined as:

$$D^+(S') := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every past inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

Example:



Why $q \notin D^+(S)$? Some of its past inextendible causal curves do not intersect S because they get stuck at the hole!

(q is affected by events in the "shadow" of the hole)

Definition:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

Analogously, the "past domain of dependence of S'" is:

$$D^-(S') := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S'" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?

The "future Cauchy horizon of S'", denoted $H^+(S)$

9gr18.jnt - Windows Journal

Analogously, the "past domain of dependence of S′" is:

$$D^-(S') := \left\{ p \in M \;\middle|\; \begin{array}{l} \text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S′" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?

The "future Cauchy horizon of S′", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: ⇒ $H^+(S)$ is achronal. Why!)

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Analogously, the "past domain of dependence of S'" is:

$$D^-(S') := \left\{ p \in \mathcal{M} \,\middle|\, \begin{array}{l} \text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S \end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S'" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?)

The "future Cauchy horizon of S'", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example: $H^+(S')$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

$$D^-(S') := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S'" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?)

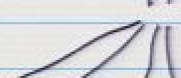
The "future Cauchy horizon of S'", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:

$H^+(S')$



$$D^-(S') := \left\{ p \in M \;\middle|\; \begin{array}{l}\text{Every future inextendible causal} \\ \text{curve through } p \text{ intersects } S\end{array} \right\}$$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S'" is:

$$D(S') := D^+(S') \cup D^-(S')$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?

The "future Cauchy horizon of S'", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:

$D^-(S) := \{ p \in M \mid$ ... curve through p intersects S $\}$

(the set of events p that affect only S)

Definition:

The "full domain of dependence of S" is:

$$D(S) := D^+(S) \cup D^-(S)$$

Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?)

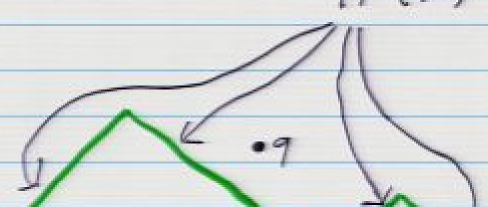
The "future Cauchy horizon of S", denoted $H^+(S)$ is:

$$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:

$H^+(S)$



9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

The "full domain of dependence of S" is:

$$D(S) := D^+(S) \cup D^-(S)$$

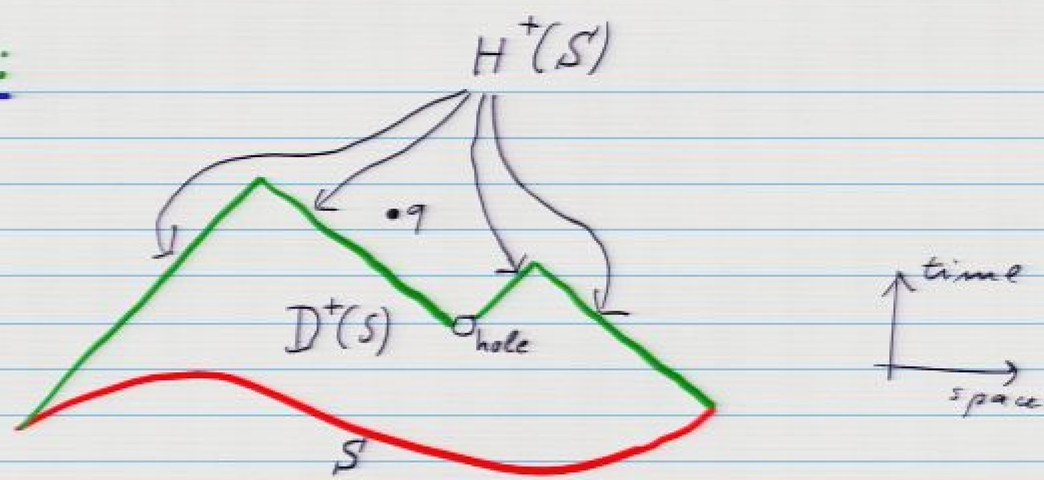
Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?

The "future Cauchy horizon of S", denoted $H^+(S)$ is:

$$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:



analogously:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

The full domain of dependence of S' is:

$$D(S') := D^+(S') \cup D^-(S')$$

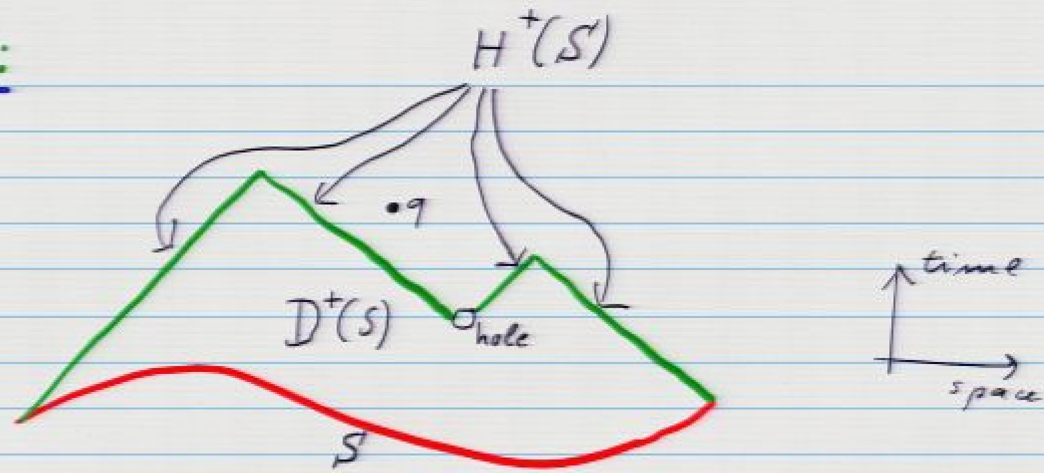
Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?

The "future Cauchy horizon of S'", denoted $H^+(S)$ is:

$$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: ⇒ $H^+(S)$ is achronal. Why?)

Example:



analogously:

Definition:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

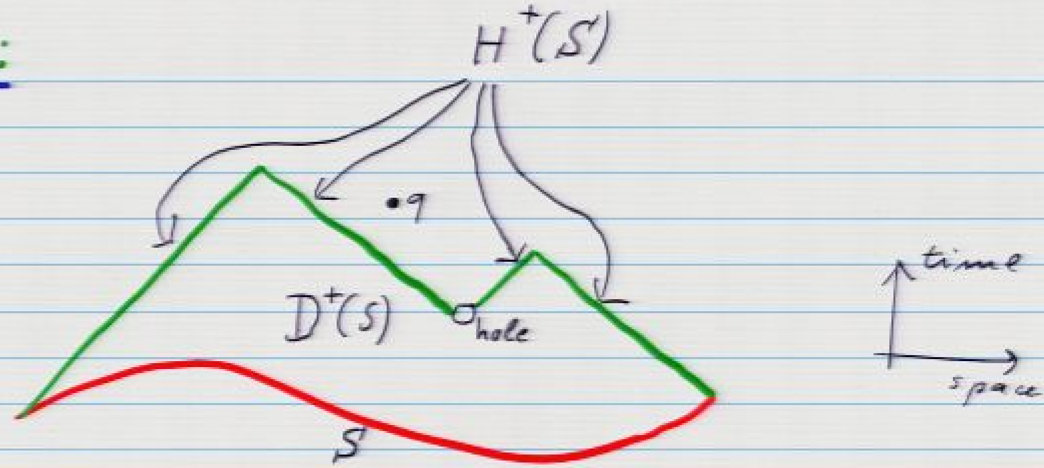
Definition: (set of latest events that are affected only by S? How far have initial conditions on S full predictive power?)

The "future Cauchy horizon of S", denoted $H^+(S)$ is:

$$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$$

(Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:



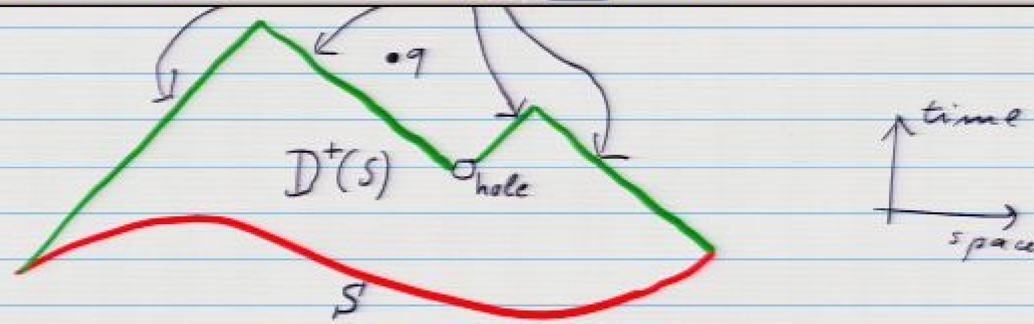
analogously

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

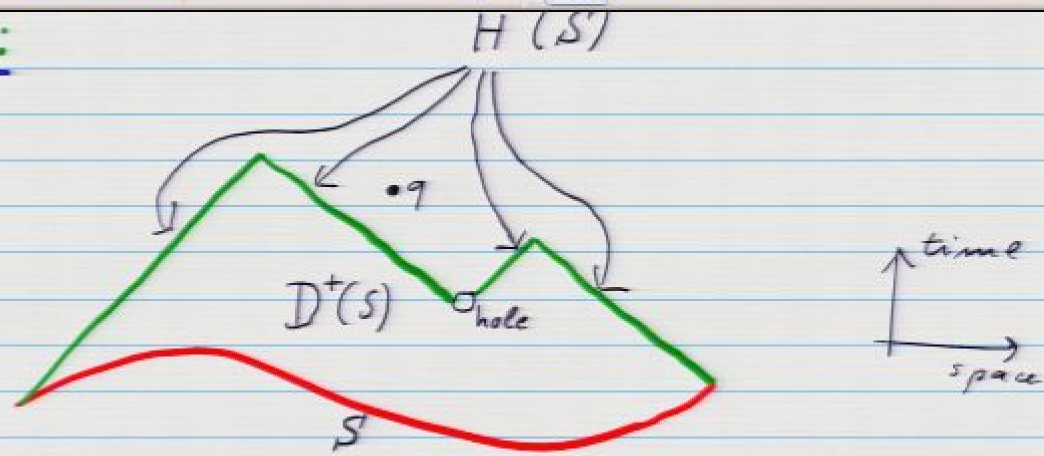
$$H(S) := H^+(S) \cup H^-(S)$$

Proposition:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Example:



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

analogously:

Definition:

The "past Cauchy horizon of S'", denoted $H^-(S)$

is: $H^-(S') := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S'" is defined as:

$$H(S') := H^+(S) \cup H^-(S)$$

Proposition:

$$H(S') = \dot{D}(S')$$

Definition:

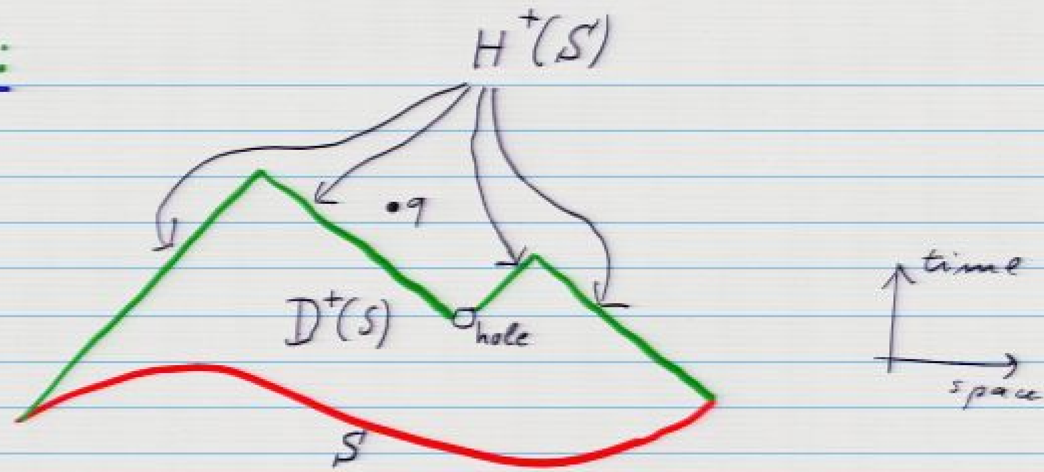
A closed achronal set S' is called a

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

is: $$H^+(S') := \overline{D^+(S)} - I^-(D^+(S))$$ (Note: $\Rightarrow H^+(S)$ is achronal. Why!)

Example:



analogously:

Definition:

The "past Cauchy horizon of S'", denoted $H^-(S)$

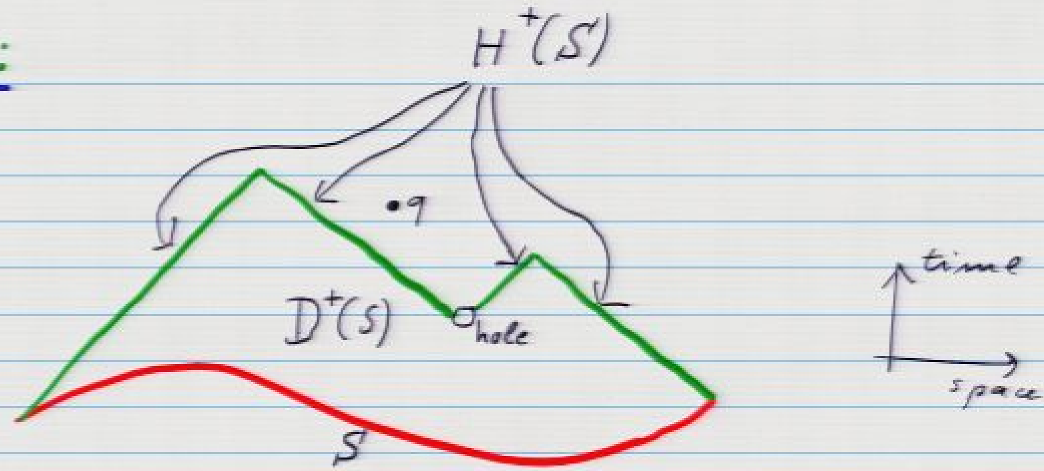
is: $$H^-(S') := \overline{D^-(S)} - I^+(D^-(S))$$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S'" is defined as

$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$ (achronal. Why?)

Example:



analogously:

Definition:

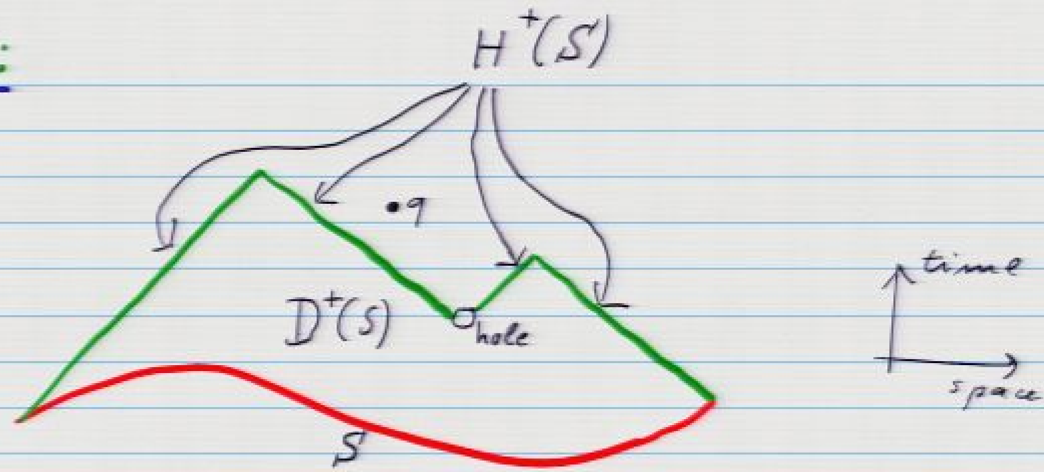
The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

Example:



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

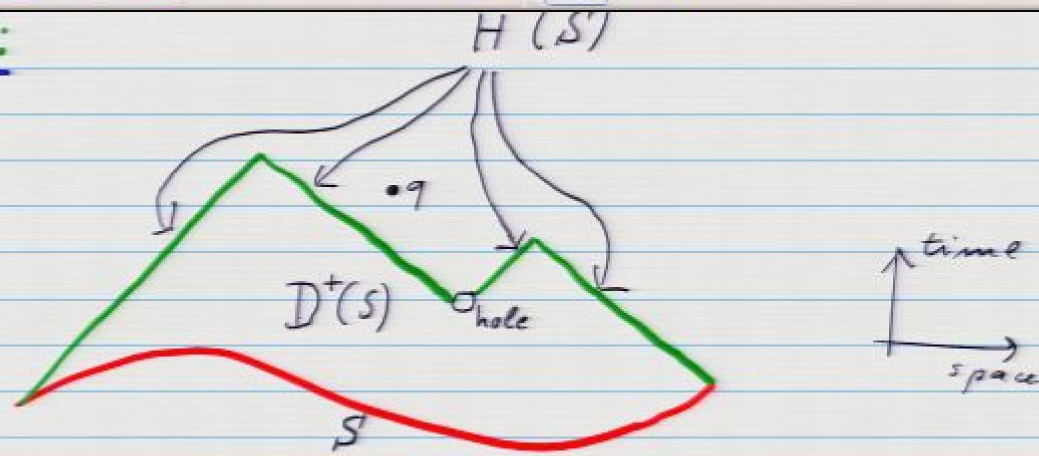
is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

Example:



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

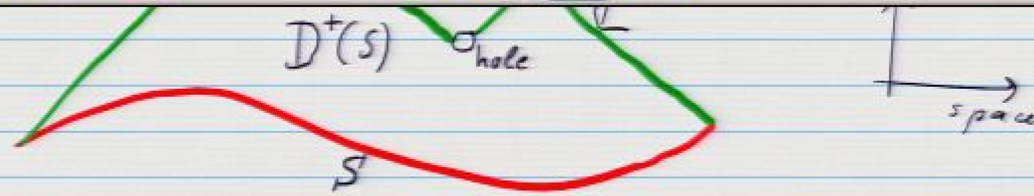
The "full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

Proposition:

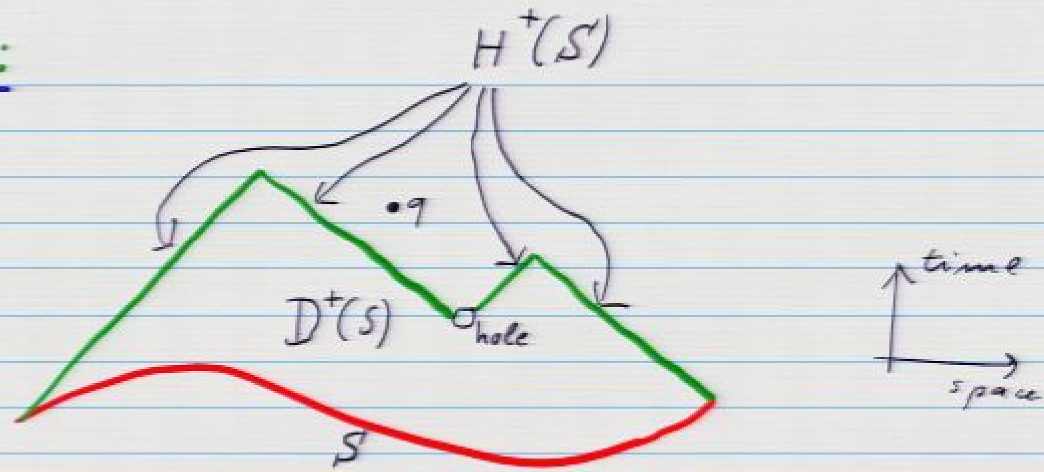
$$H(S) = \dot{D}(S)$$

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

is: $H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$ (Note: $\Rightarrow H^+(S)$ is achronal. Why?)

Example:



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

Proposition:

$$H(S) = \dot{D}(S)$$

Definition:

A closed, achronal set S is called a

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

The full Cauchy horizon of S" is defined as:

$$H(S) := H^+(S) \cup H^-(S)$$

Proposition:

$$H(S') = \dot{D}(S')$$

Definition:

A closed, achronal set S is called a "Cauchy surface", if its full Cauchy horizon vanishes, i.e. if

a) $H(S) = \phi$ ← empty set or equivalently if

b) $\dot{D}(S) = \phi$ ... equivalently if

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Proposition:

$$H(S') = \dot{D}(S')$$

Definition:

A closed, achronal set S' is called a "**Cauchy surface**", if its full Cauchy horizon vanishes, i.e. if

a) $H(S) = \emptyset$ ← empty set or equivalently if

b) $\dot{D}(S) = \emptyset$ or equivalently if

c) $D(S') = \mathcal{M}$

Hawking, Ellis, Geroch et al.

Note: This follows Wald. The definitions by others are equivalent

9gr18.jnt - Windows Journal

Proposition:

$$H(S') = \dot{D}(S')$$

Definition:

A closed, achronal set S' is called a "Cauchy surface", if its full Cauchy horizon vanishes, i.e. if

a.) $H(S) = \phi$ ← empty set   or equivalently if

b.) $\dot{D}(S) = \phi$ or equivalently if

c.) $D(S') = M$

Hawking, Ellis, Geroch et al.

Note: This follows Wald. The definitions by others are equivalent.

but more tedious

b.) $\dot{D}(S) = \phi$ or equivalently if

c.) $D(S') = M$

Hawking, Ellis, Geroch et al.

Note: This follows Wald. The definitions by others are equivalent.

but more tedious

Remarks:

- Cauchy surfaces are important because if the conditions on a Cauchy surface are known, then everything on M can be predicted and retrodicted.
- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".
- The term "surface" is motivated by a **theorem:**

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

c.) $D(S') = M$

Hawking, Ellis, Geroch et al.

Note: This follows Wald. The definitions by others are equivalent.

but more tedious

Remarks:

- Cauchy surfaces are important because if the conditions on a Cauchy surface are known, then everything on M can be predicted and retrodicted.
- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".
- The term "surface" is motivated by a **theorem:**

Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Remarks:

- Cauchy surfaces are important because if the conditions on a Cauchy surface are known, then everything on $\mathcal{M}$ can be predicted and retrodicted.

- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".

- The term "surface" is motivated by a theorem:

Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of $\mathcal{M}$.

Definition:

If $(\mathcal{M}, g)$ possesses a Cauchy surface then

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Remarks:

- Cauchy surfaces are important because if the conditions on a Cauchy surface are known, then everything on M can be predicted and retrodicted.
- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".
- The term "surface" is motivated by a theorem:

Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

Definition:

If (M, g) possesses a Cauchy surface then it is called "globally hyperbolic".

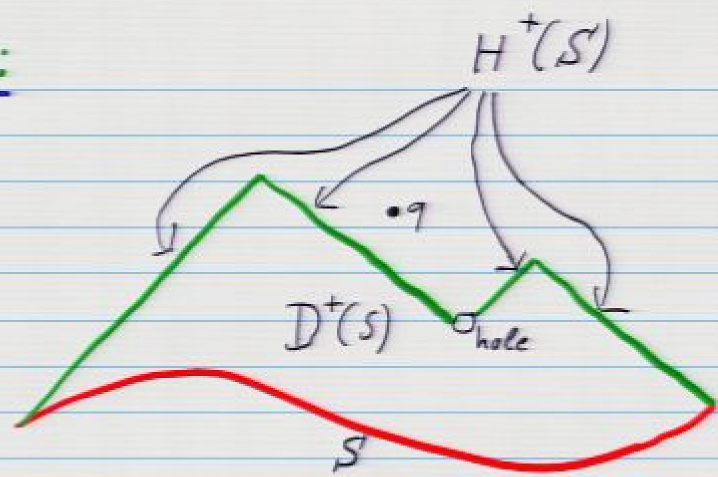
9gr18.jnt - Windows Journal

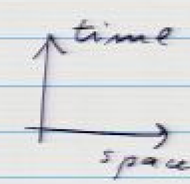
Edit View Insert Actions Tools Help

Page Width

$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$ (achronal. Why?)

Example:





analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$ (set of earliest events that affect only S)

Definition:

The "full Cauchy horizon of S" is defined as

9gr18.jnt - Windows Journal

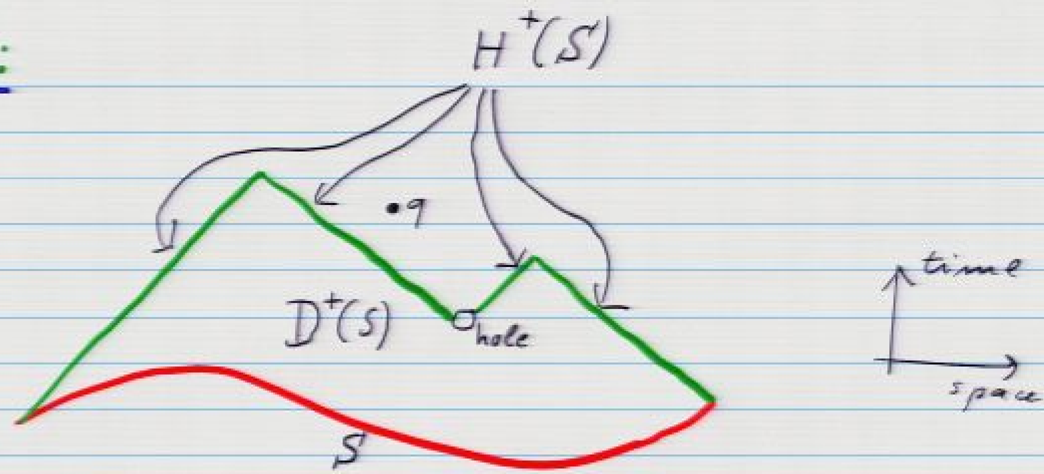
Edit View Insert Actions Tools Help

Page Width

The future Cauchy horizon of S, denoted $H^+(S)$

is: $$H^+(S) := \overline{D^+(S)} - I^-(D^+(S))$$ (Note: $\Rightarrow H^+(S)$ is achronal. Why!)

Example:



analogously:

Definition:

The "past Cauchy horizon of S", denoted $H^-(S)$

is: $$H^-(S) := \overline{D^-(S)} - I^+(D^-(S))$$ (set of earliest events that affect only S)

Definition:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Proposition:

$$H(S') = \dot{D}(S')$$

Definition:

A closed, achronal set S' is called a "Cauchy surface", if its full Cauchy horizon vanishes, i.e. if

a.) $H(S) = \phi$ ← empty set or equivalently if

b.) $\dot{D}(S) = \phi$ or equivalently if

c.) $D(S') = M$

Hawking, Ellis, Geroch et al.

Note: This follows Wald. The definitions by others are equivalent. but more tedious

Remarks:

- Cauchy surfaces are important because if the conditions on a Cauchy surface are known, then everything on M can be predicted and retrodicted.

- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".

- The term "surface" is motivated by a **theorem:**

  Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

Definition:

If (M, g) possesses a Cauchy surface then

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

everything on M can be predicted and retrodicted.

- Since a Cauchy surface is achronal, it can be viewed as an "instant in time".

- The term "surface" is motivated by a theorem:

Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

Definition:

If (M, g) possesses a Cauchy surface then it is called "globally hyperbolic".

Proposition:

9gr18.jnt - Windows Journal

- The term "surface" is motivated by a **theorem:**

  Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

Definition:

If (M, g) possesses a Cauchy surface then it is called "globally hyperbolic".

Proposition:

If (M, g) is globally hyperbolic, then:

- There exists a "global time function f" so that every

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

▯ The term "surface" is motivated by a **theorem:**

Every Cauchy surface, Σ, is a 3-dimensional C^0 submanifold of M.

Definition:

If (M, g) possesses a Cauchy surface then it is called "globally hyperbolic".

Proposition:

If (M, g) is globally hyperbolic, then:

▯ There exists a "global time function f" so that every surface of constant f is a Cauchy surface.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

If (M,g) possesses a Cauchy surface then it is called "globally hyperbolic".

Proposition:

If (M,g) is globally hyperbolic, then:

- There exists a "global time function f" so that every surface of constant f is a Cauchy surface.
- (M,g) is stably (and therefore also strongly) causal.

Remark: It is clear that any globally hyperbolic (M,g) is (not the converse though) causal:

19 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Proposition:

If (M,g) is globally hyperbolic, then:

- There exists a "global time function f" so that every surface of constant f is a Cauchy surface.

- (M,g) is stably (and therefore also strongly) causal.

Remark: It is clear that any globally hyperbolic (M,g) is causal: (not the converse though)

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.

- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Proposition:

If (M,g) is globally hyperbolic, then:

- There exists a "global time function f" so that every surface of constant f is a Cauchy surface.

- (M,g) is stably (and therefore also strongly) causal.

(not the converse though)

Remark: It is clear that any globally hyperbolic (M,g) is causal:

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.

- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

If (M,g) is globally hyperbolic, then:

- There exists a "global time function f" so that every surface of constant f is a Cauchy surface.

- (M,g) is stably (and therefore also strongly) causal.

Remark: It is clear that any globally hyperbolic (M,g) is causal: (not the converse though)

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.

- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

Recall:

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Remark: It is clear that any globally hyperbolic (M,g) is causal: (not the converse though)

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.
- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

Recall:

Plan is to study inextendible geodesics in order to detect singularities.

Now: How to identify these geodesics which are inextendible because they end at a hole in the manifold?

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

(not the converse though)

Remark: It is clear that any globally hyperbolic (M,g) is causal:

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.
- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

Recall:

Plan is to study inextendible geodesics in order to detect singularities.

Now: How to identify these geodesics which are inextendible because they end at a hole in the manifold?

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- There exists a "global time function f" so that every surface of constant f is a Cauchy surface.

- (M, g) is stably (and therefore also strongly) causal.

(not the converse though)

Remark: It is clear that any globally hyperbolic (M, g) is causal:

- Each of its Cauchy surfaces Σ is intersected by every causal curve γ.

- Any causal curve intersects Σ but if γ is closed this contradicts the achronicity of Σ.

Recall:

Plan is to study inextendible geodesics in order to

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Recall:

Plan is to study inextendible geodesics in order to detect singularities.

Now: How to identify these geodesics which are inextendible because they end at a hole in the manifold?

First: Avoid trivial cases

Definition:

We say that (M, g) is inextendible, if it is not isometric to a proper subset of another spacetime (M', g').

$\rightarrow$ We will always assume that (M, g) is inextendible.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

are inextendible because they end at a hole in the manifold?

First: Avoid trivial cases

Definition:

We say that (M, g) is inextendible, if it is not isometric to a proper subset of another spacetime (M', g').

$\rightsquigarrow$ We will always assume that (M, g) is inextendible.

Definition:

A geodesic which is inextendible but possesses

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

First: Avoid trivial cases

Definition:

We say that (M,g) is inextendible, if it is not isometric to a proper subset of another spacetime (M',g').

$\rightsquigarrow$ We will always assume that (M,g) is inextendible.

Definition:

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Definition:

We say that (M,g) is inextendible, if it is not isometric to a proper subset of another spacetime (M',g').

$\rightsquigarrow$ We will always assume that (M,g) is inextendible.

Definition:

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

→ We will always assume that (M, g) is inextendible.

Definition:

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance or proper time respectively.

Definition:

We say that (M, g) possesses a singularity

21 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance or proper time respectively.

Definition:

We say that (M, g) possesses a singularity if it possesses an incomplete geodesic.

(I.e. there are time-like, null and spacelike singularities)

Edit View Insert Actions Tools Help

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance or proper time respectively.

Definition:

We say that (M, g) possesses a singularity if it possesses an incomplete geodesic.

(I.e. there are time-like, null and spacelike singularities)

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

parameter can be chosen to be the proper distance or proper time respectively.

Definition:

We say that (M, g) possesses a **singularity** if it possesses an incomplete geodesic.
(I.e. there are time-like, null and spacelike singularities)

a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\beta\delta}$,

e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

or proper time respectively.

Definition:

- We say that (M, g) possesses a **singularity** if it possesses an incomplete geodesic.
(I.e. there are time-like, null and spacelike singularities)

- When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\rho s}$,

e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

A geodesic which is inextendible but possesses a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance or proper time respectively.

Definition:

- We say that (M, g) possesses a singularity if it possesses an incomplete geodesic.
  (I.e. there are time-like, null and spacelike singularities)

- When going along an incomplete geodesic

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

a finite range of its affine parameter is called "incomplete".

Recall: For space- or timelike geodesics the affine parameter can be chosen to be the proper distance or proper time respectively.

Definition:

We say that (M, g) possesses a singularity if it possesses an incomplete geodesic.
(I.e. there are time-like, null and spacelike singularities)

When going along an incomplete geodesic towards

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Definition:

- We say that (M, g) possesses a **singularity** if it possesses an incomplete geodesic.
  (I.e. there are time-like, null and spacelike singularities)

- When going along an incomplete geodesic towards a "singularity", 3 things can happen:

  I) A scalar constructed from $R^{\mu\nu}{}_{\rho\sigma}$,

  e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

  $\rightsquigarrow$ We say it is a "scalar curvature singularity"

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

We say that $(\mathcal{M}, g)$ possesses a singularity if it possesses an incomplete geodesic.
(I.e. there are time-like, null and spacelike singularities)

When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\sigma\delta}$,
e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.
$\rightsquigarrow$ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame,

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\beta\delta}$, e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

→ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}_{\beta\delta}$ or its covariant derivatives diverge.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\rho\sigma}$, e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

$\rightsquigarrow$ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}_{\rho\sigma}$ or its covariant derivatives diverge.

$\rightsquigarrow$ We say it is a "parallel-propagated curvature

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

- When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\rho\sigma}$,

e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

⇝ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}{}_{\rho\sigma}$ or its covariant derivatives diverge.

⇝ We say it is a "parallel-propagated curvature singularity"

23 / 25

When going along an incomplete geodesic towards a "singularity", 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\rho\sigma}$, e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

$\rightsquigarrow$ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}{}_{\rho\sigma}$ or its covariant derivatives diverge.

$\rightsquigarrow$ We say it is a "parallel-propagated curvature singularity"

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

a singularity, 3 things can happen:

I) A scalar constructed from $R^{\mu\nu}{}_{\rho\sigma}$,

e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

→ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}{}_{\rho\sigma}$ or its covariant derivatives diverge.

→ We say it is a "parallel-propagated curvature singularity"

III) N...

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

II) In a parallel transported tetrad frame, a scalar component of R_{ss} or its covariant derivatives diverge.

→ We say it is a "parallel-propagated curvature singularity"

III) None of the above. Example: "Conical singularity". (cut out a suitable piece and identify the boundaries of the cut)

(This way mfld can be diffable while some paths cannot.)

→ We say it is a non-curvature singularity

Fundamental problem:

In concrete solutions such as Schwarzsch...

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

II) In a parallel transported tetrad frame, a scalar component of $\widetilde{R}_{ss}$ or its covariant derivatives diverge.

→ We say it is a "parallel-propagated curvature singularity"

III) None of the above. Example: "Conical singularity". (cut out a suitable piece and identify the boundaries of the cut)

(This way mfld can be diffable while some paths cannot.)

→ We say it is a non-curvature singularity

Fundamental problem:

- In concrete solutions, such as Schwarzschild

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

I) A scalar constructed from $R^{\cdots}{}_{ss}$,

e.g. R, $R^{\mu\nu}R_{\mu\nu}$, etc diverges.

$\rightsquigarrow$ We say it is a "scalar curvature singularity".

II) In a parallel transported tetrad frame, a scalar component of $R^{\mu\nu}_{ss}$ or its covariant derivatives diverge.

$\rightsquigarrow$ We say it is a "parallel-propagated curvature singularity"

III) None of the above. Example: "Conical singularity".

(cut out a suitable piece and identify the boundaries of the cut)

(The ...ay mfld can be diffable while some ...)

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

a scalar component of R_{ss} or its covariant derivatives diverge.

→ We say it is a "parallel-propagated curvature singularity"

III) None of the above. Example: "Conical singularity". (cut out a suitable piece and identify the boundaries of the cut)

(This way mfld can be diffable while some paths cannot.)

→ We say it is a non-curvature singularity

Fundamental problem:

- In concrete solutions, such as Schwarzschild or FRW cosmologies, curvature singularities are

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

singularity

III) None of the above. Example: "Conical singularity".
(cut out a suitable piece and identify the boundaries of the cut)

(This way mfld can be diffable while some paths cannot.)

⇝ We say it is a non-curvature singularity

Fundamental problem:

- In concrete solutions, such as Schwarzschild or FRW cosmologies, curvature singularities are obviously present.

- But these spacetimes are highly symmetric.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

singularity"

III) None of the above. Example: "Conical singularity".
(cut out a suitable piece and identify the boundaries of the cut)

(This way mfld can be diffable while some paths cannot.)

→ We say it is a non-curvature singularity

Fundamental problem:

- In concrete solutions, such as Schwarzschild or FRW cosmologies, curvature singularities are obviously present.

- But these spacetimes are highly symmetric.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Fundamental problem:

- In concrete solutions, such as Schwarzschild or FRW cosmologies, curvature singularities are obviously present.
- But these spacetimes are highly symmetric.

  Do more realistic, i.e. perturbed spacetimes also show these singularities?

- Example:

Spherically symmetric dust shell infall.

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Fundamental problem:

- In concrete solutions, such as Schwarzschild or FRW cosmologies, curvature singularities are obviously present.

- But these spacetimes are highly symmetric.

  Do more realistic, i.e. perturbed spacetimes also show these singularities?

- Example:

Spherically symmetric dust shell infall.

24 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

show these singularities?

Example:

Spherically symmetric dust shell infall.

In Newton gravity:

→ predict ∞ mass density to occur,
but not if symmetry perturbed!

In Einstein gravity:

→ predict black hole singularity to occur,
even if symmetry is perturbed,

Remark:
Black holes provide finite energy endpoint of grav. collapse, thus stabilizing GR energetically
Note: In QM, charge driven

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

Example:

Spherically symmetric dust shell infall.

In Newton gravity:

→ predict ∞ mass density to occur,
but not if symmetry perturbed!

In Einstein gravity:

→ predict black hole singularity to occur,
even if symmetry is perturbed,
(if assuming e.g. dominant energy cond. etc.)

Remark:
Black holes provide finite energy endpoint of grav. collapse, thus stabilizing GR energetically.
Note: In QM, charge driven collapse is bounded at finite energy by uncertainty principle.

25 / 25

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

9gr18.jnt - Windows Journal

Edit View Insert Actions Tools Help

Page Width

spherically symmetric dust shell infall.

In Newton gravity:

→ predict ∞ mass density to occur, but not if symmetry perturbed!

In Einstein gravity:

Remark: Black holes provide finite energy endpoint of grav. collapse, thus stabilizing GR energetically.
Note: In QM, charge driven collapse is bounded at finite energy by uncertainty principle.

→ predict black hole singularity to occur, even if symmetry is perturbed, (if assuming e.g. dominant energy cond. etc.)

⇝ Next: "Singularity theorems"