Title: General Relativity for Cosmology - Lecture 13

Date: Nov 02, 2009 04:00 PM

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Abstract:

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GR for Cosmology, Achim Kempf, Fall 2009, Lecture 14

On $T^{\mu\nu}$, continued:

Recall: We defined $T^{\mu\nu}$ as that tensor which obeys for all $\delta g_{\mu\nu}(\lambda,x)$:

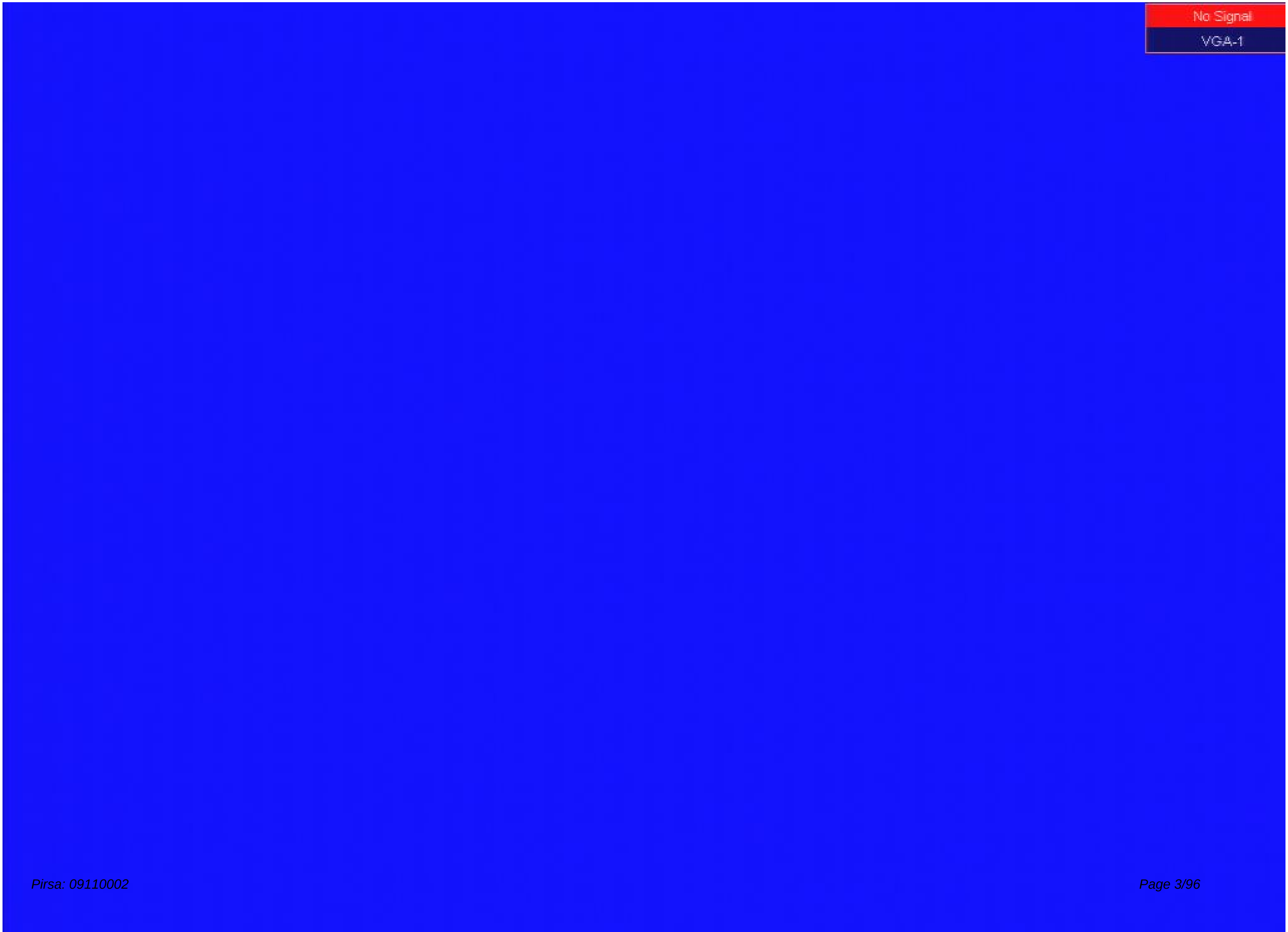
$$\left.\frac{dS}{d\lambda}\right|_{\lambda=0} = \frac{1}{2}\int_B T^{\mu\nu}\,\underbrace{\delta g_{\mu\nu}}_{=\left.\frac{d g_{\mu\nu}(\lambda,x)}{d\lambda}\right|_{\lambda=0}}\sqrt{g}\,d^4x$$

(we choose $T^{\mu\nu}$ symmetric because $g_{\mu\nu}$ is symmetric)

The above is meant when writing:

$$T^{\mu\nu} = \frac{2}{\sqrt{g}}\frac{\delta}{\delta g_{\mu\nu}} S$$

We found that $T^{\mu\nu}{}_{;\nu} = 0$ always holds.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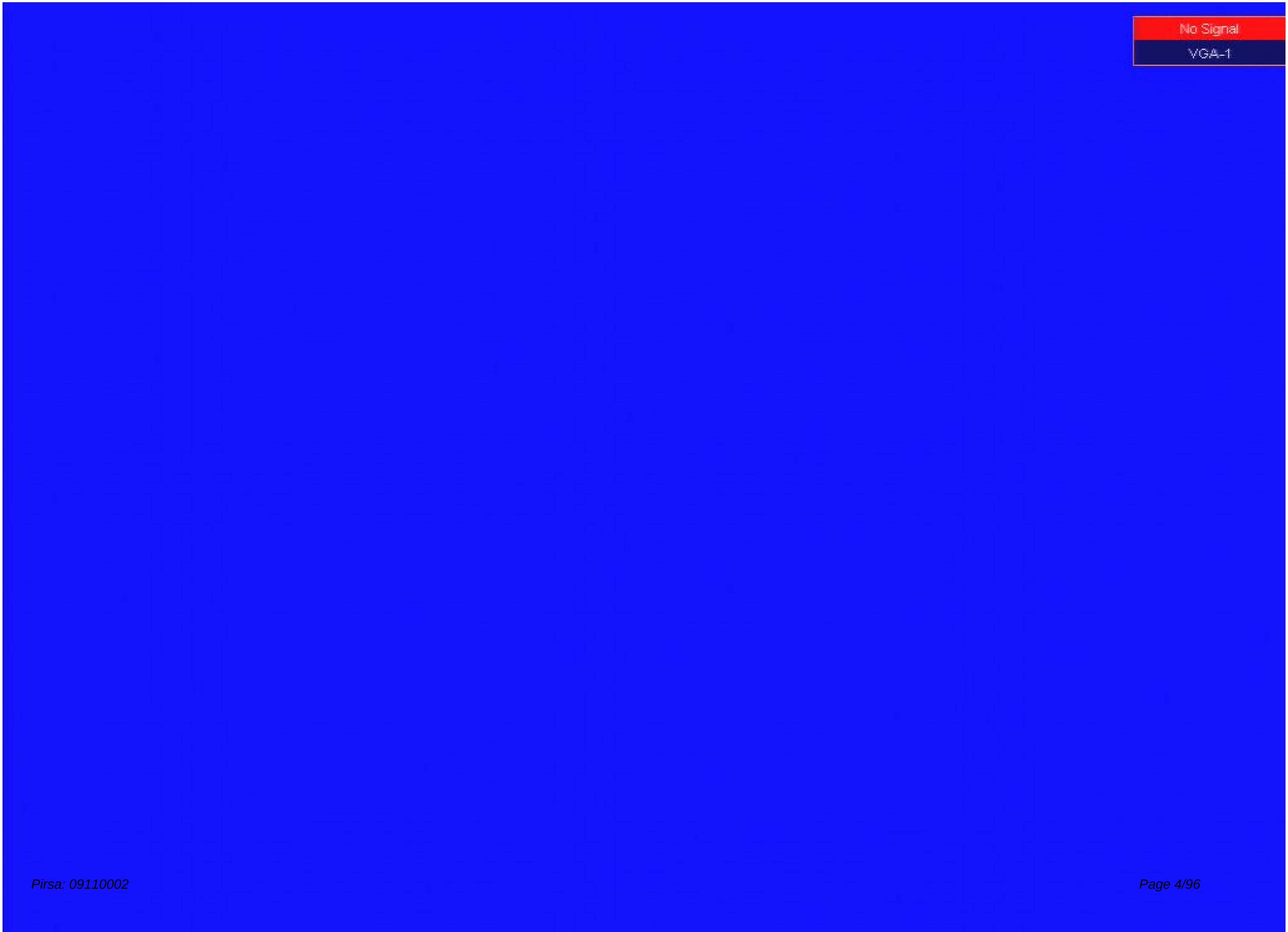
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- We found that $T^{\mu\nu}{}_{;\nu} = 0$ always holds.
  (Since is consequence of diffeomorphism invariance)

However:

- $T^{\mu\nu}{}_{;\nu} = 0$ is not conservation law!

Why? $T^{\mu\nu}{}_{;\nu}\sqrt{g}$ is not a divergence, unlike $K^{\mu}{}_{;\mu}\sqrt{g} = \mathrm{div}_{\Omega} K$

Except: if space-time possesses isometries, i.e., covariant so-called Killing fields, ξ, i.e., fields obeying:

$$L_\xi g = 0, \quad \text{i.e.,} \quad \xi_{\mu;\nu} = -\xi_{\nu;\mu}$$

Because then: $P^\mu := T^{\mu\nu}\xi_\nu$ obeys $P^\mu{}_{;\mu} = 0$

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Thus:

$$\int_B \underbrace{P^\mu{}_{;\mu}\sqrt{g}}_{\mathrm{div}_\Omega P \,=\, d\, i_P \Omega} \, d^4x \overset{\text{Stokes}}{\underset{\text{(Gauß)}}{=}} \underbrace{\int_{\partial B} i_P \Omega}_{\text{a conservation law}} = 0$$

Proposition: maximal number of indep. Killing vector fields on spacetime: 10

Actual spacetime has no Killing vector fields, but realistic simplified

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Actual spacetime has no Killing vector fields, but realistic simplified models of parts or all of spacetime often do:

Definition: A space-time (M, g) is called **"stationary"** if it possesses a **time-like** Killing vector field ξ, i.e., which obeys:

$$L_\xi g = 0 \ ; \quad g(\xi, \xi) < 0$$

($=0$ would be called "null" or light-like; >0 would be called space-like)

i.e., if: $\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0, \quad \xi^\mu \xi_\mu < 0$

→ In this case, observers can travel along the integral curves of ξ and they will see no change in the prevailing local curvature, i.e., following the observer, one can find a (so-called comoving) coordinate system, in which:

$$\frac{\partial}{\partial x^0} g_{\mu\nu}(x^0, x^1, x^2, x^3) = 0$$

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□ **But:** This does not imply that there is a cds in which

$$g = \begin{pmatrix} g_{00}, 0, 0, 0 \\ 0, & \\ 0, & \times \\ \cdots \end{pmatrix} \quad (\times) \quad !$$

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Definition: A space-time is called "static", if the time-like Killing field ξ, viewed as a 1-form,

$$\xi = \xi_\mu \, dx^\mu$$

obeys the "Frobenius condition":

$$\xi \wedge d\xi = 0 \qquad (F)$$

Exercise: write it out in coordinates

Significance? (F) holds $\Leftrightarrow$ $g = \begin{pmatrix} g_{00}, & 0,0,0 \\ 0 & \\ 0 & (*) \\ 0 & \end{pmatrix}$ in suitable cds.

- If, in a suitable coordinate system, g is of the form $(*)$ and the time-like ξ is $\xi = g_{00}\, dt$, then $\xi \wedge d\xi = 0$ trivially.

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Generic properties of $T^{\mu\nu}$:

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- $T^{\mu\nu}$ has contributions from known and also from as yet unknown matter fields (e.g. from dark matter).
- Thus, in order to draw generic conclusions about, e.g.

  a.) the occurrence of singularities, or (Note: black hole formation stops energy dropping)

  b.) the overall positivity of the energy (despite universal attraction!)

  one needs plausible conjectures about the full $T_{\mu\nu}$:

The "weak energy condition" (assumption):

$$T_{\mu\nu} v^\mu v^\nu \geq 0 \quad \text{for all timelike } v: \; g(v,v) < 0$$

(by continuity it then also holds for lightlike v)

Why assume it?

(Note: Negative energy would be anti-gravitating) ⇝ All observers travel with a time-like tangent v.

[illegible]

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All observers travel with a time-like tangent v.

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Why assume it? → The local energy-momentum flow vector K ... 5 / 23

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Why assume it?

- The local energy-momentum flow vector, K, may not be conserved but should be non-space-like: "All flow should be into the future."

- In an orthonormal basis, the dominant energy condition takes the form:

$$T^{00} \geq |T^{ab}|$$

Note: This is all intuition from fluid mechanics analogy. Quantum fields may or may not

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The dynamics of space-time!

- Consider the full matter action:

$$S[g,\psi] = \int_M L(g,\psi)\sqrt{g}\, d^4x$$

all matter fields: e^-, photons, quarks, gluons etc.

- The equations of motion of matter fields are

$$\frac{\delta S}{\delta \psi_{(i)}{}^{a..b}{}_{c..d}} = 0$$

i.e.:

$$\frac{\partial L}{\partial \psi^{a..b}} = \left(\frac{\partial L}{\partial \psi^{a..b}} \right)$$

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- Do we obtain suitable equations of motion for $g_{\mu\nu}$ by setting

$$\frac{\delta S'}{\delta g_{\mu\nu}} = 0 \quad ?$$

Apparently not, because it would mean:

$$\frac{\delta S'}{\delta g_{\mu\nu}} = \frac{1}{2} T^{\mu\nu} \sqrt{g} = 0 \;!$$

Thus, the universe would have to be empty of matter (assuming all matter has positive energy).

- Andrei Sakharov (1968):

The quantum effects of matter **induce** suitable extra terms in the action!

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Sakharov's reasoning: (modernized version)

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$$\text{prob.ampl.}\left[\psi_{(i)}\right] = N\, e^{\frac{iS'[g,\psi]}{\hbar}}$$

- As usual in quantum theory, the actual or "effective" matter evolution $\langle \psi_{(i)}(x,t) \rangle$ is close

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- As usual in quantum theory, the actual or "effective" matter evolution $\langle\psi_{(i)}(x,t)\rangle$ is close to but not identical to the classical matter evolution $\psi_{(i)}(x,t)$.

Why? Path integral picture: The field evolutions with close-to-extremal actions have very similar values $e^{\frac{iS}{\hbar}}$ because for them $\frac{\delta S'}{\delta\psi} \simeq 0$ i.e. their prob. amplitudes add up. Other matter evolutions $\psi_{(i)}(x,t)$ have widely different $e^{\frac{iS}{\hbar}}$, so their probabilities average another away.

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- Thus, the effective quantum fields obey equations of motion that are somewhat modified!

→ Aim: Calculate the "effective action"

$$S_{eff}[g,\psi]$$

which yields the effective evolution of matter fields when matter quantum effects are taken into account.

- Problems:

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  ⇒ Use perturbative methods.

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- Problems:

  - These calculations are very difficult.

    $\Rightarrow$ Use perturbative methods.

  - There occur integrals that are divergent at short distances.

    $\Rightarrow$ Introduce a cutoff at some minimum length l_c (or maximum momentum $\frac{\hbar}{l_c}$).

- But it is clear that: as always, the effective action will contain terms of all possible forms that are consistent with the symmetries of the theory i.e.

The only question is which prefactors these terms will have.

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The only question is which prefactors these terms will have. →

quantum "vacuum energy" of matter

$$S_{eff}[g,\psi] = \int_M \left(L^{\text{matter}} + L^{\text{matter}}_{\text{quantum effects}} + c_1 + c_2 R + c_3\, \mathcal{O}(R^2) \right) \sqrt{g}\, d^4x$$

this is the local change of the vacuum energy due to curvature deforming t...

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this is the local change of the vacuum energy due to curvature deforming the quantum harmonic oscillators of the field modes.

- The constants c_1, c_2 etc depend on:
  - the details of the matter action L^{matter}.

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total effective matter Lagrangian (for L)

and postulate now that the equations of motion for the metric follow from the action principle:

$$\frac{\delta S_{\psi}[g,\psi]}{\delta g_{\mu\nu}} = 0$$

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δS

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in principle, it is the effective quantum expectation value

Evaluate the left hand side:

a.) $$\delta \int_B c_1 \sqrt{g}\, d^4x = \int_B c_1 \underbrace{\frac{\delta \sqrt{g}}{\delta g_{\mu\nu}}}_{\text{recall: } = \frac{1}{2} g^{\mu\nu} \sqrt{g}} \delta g_{\mu\nu}\, d^4x$$

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b.) $$\delta \int_B c_2 R_{\mu\nu} g^{\mu\nu} \sqrt{g}\, d^4x$$

$$= \underbrace{\int_B c_2 (\delta R_{\mu\nu}) g^{\mu\nu} \sqrt{g}\, d^4x}_{\text{Term I}} + \underbrace{\int_B c_2 R_{\mu\nu} \delta(g^{\mu\nu}\sqrt{g})\, d^4x}_{\text{Term II}}$$

Proposition: Term I = 0

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Proposition: Term I $= 0$

Proof: Choose origin of geodesic coordinate system

$$R_{\mu\nu} = \Gamma^{\alpha}{}_{\mu\nu,\alpha} - \Gamma^{\alpha}{}_{\mu\alpha,\nu} + \underbrace{\Gamma^{\cdot}_{\cdot\cdot}\Gamma^{\cdot}_{\cdot\cdot} + \Gamma^{\cdot}_{\cdot\cdot}\Gamma^{\cdot}_{\cdot\cdot}}_{\text{vanish at origin because } \Gamma = 0}$$

Thus:

$$\delta R_{\mu\nu} = (\delta \Gamma^{\alpha}{}_{\mu\nu})_{,\alpha} - (\delta \Gamma^{\alpha}{}_{\mu\alpha})_{,\nu}$$

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Thus:
$$\delta R_{\mu\nu} = (\delta \Gamma^{\alpha}_{\mu\nu})_{,\alpha} - (\delta \Gamma^{\alpha}_{\mu\alpha})_{,\nu}$$

$$\Rightarrow \quad g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}(\delta \Gamma^{\alpha}_{\mu\nu})_{,\alpha} - g^{\mu\nu}(\delta \Gamma^{\alpha}_{\mu\alpha})_{,\nu}$$

$$= w^{\alpha}{}_{;\alpha} \text{ for } w^{\alpha} = g^{\mu\nu}\delta\Gamma^{\alpha}_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^{\nu}_{\mu\nu}$$

recall: $g^{\mu\nu}{}_{;\alpha} = 0$ here.

Thus, in arbitrary coordinate system:

$$g^{\mu\nu}\delta R_{\mu\nu} = w^{\alpha}{}_{;\alpha}$$

$$\Rightarrow \int_B c_2\, g^{\mu\nu}\delta R_{\mu\nu}\sqrt{g}\, d^4x = \int_B w^{\alpha}{}_{;\alpha}\sqrt{g}\, d^4x$$

$$= \int_B \mathrm{div}_w \Omega$$

= 0 on ∂B, ass

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$i_w = 0$ on ∂B, assuming $\delta g_{\mu\nu}$ and $\delta g_{\mu\nu,s} = 0$ on ∂B

$$= 0$$

c.) Evaluate term II:

$$\int_B c_2 R_{\mu\nu}\,\delta(g^{\mu\nu}\sqrt{g})\, d^4x = ?$$

We have:

$$\delta(g^{\mu\nu}\sqrt{g}) = (\delta g^{\mu\nu})\sqrt{g} + g^{\mu\nu}\frac{\partial\sqrt{g}}{\partial g_{ab}}\,\delta g_{ab}$$

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"Einstein tensor"

Bringing together a) + b) + c) $\Rightarrow$

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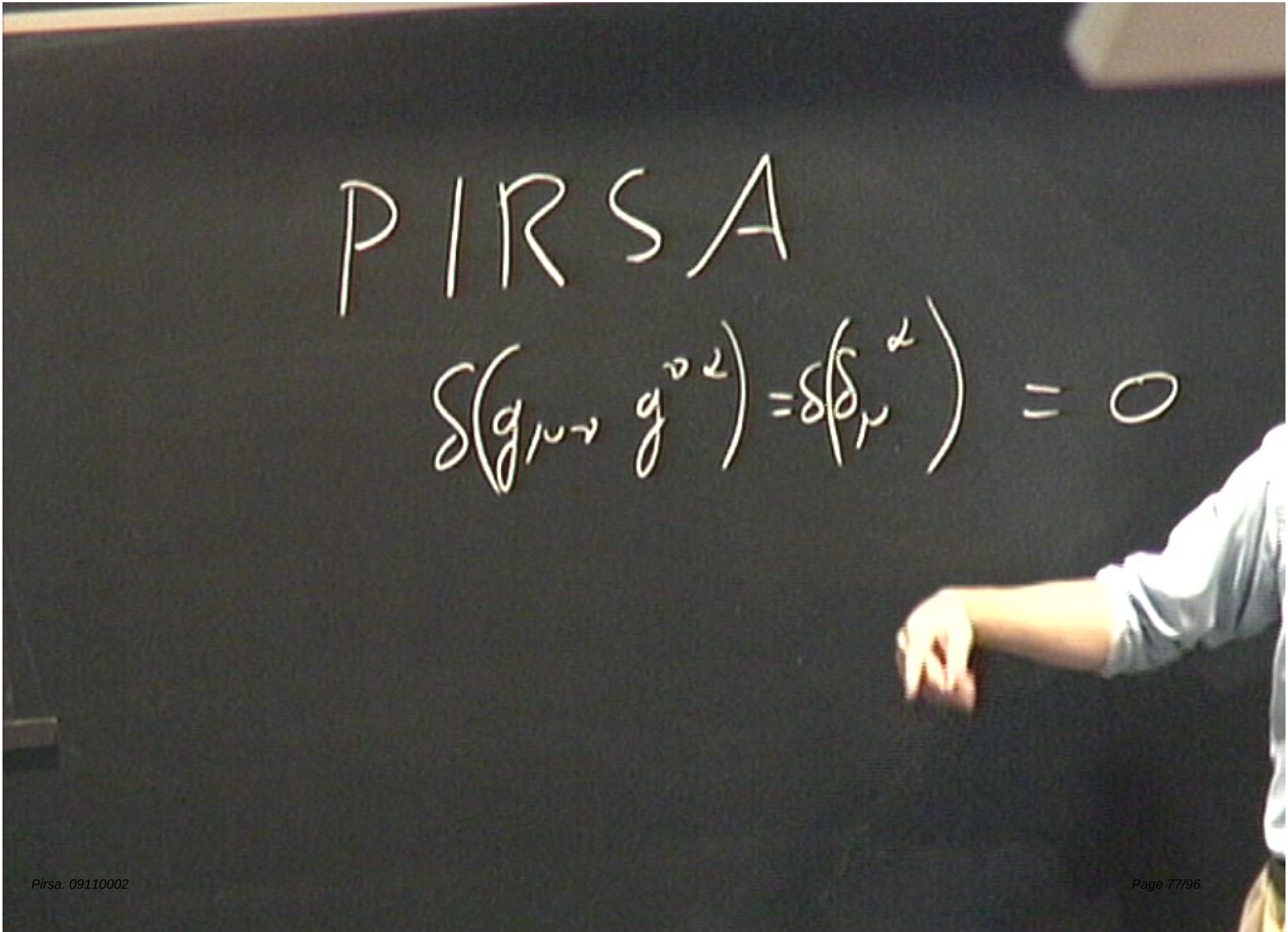
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$$\delta \int_B (c_1 + c_2 R_{\mu\nu} g^{\mu\nu}) \sqrt{g}\, d^4x$$

$$= \int_B \underbrace{\left(c_1 \frac{1}{2} g^{\mu\nu} - c_2 G^{\mu\nu}\right)}_{} \sqrt{g}\, \underbrace{\delta g_{ab}}_{\text{symmetric}} d^4x$$

as in the case of the $T^{\mu\nu}$ calculation, one could add an antisymmetric part here and it would drop from the integrand.

$$\Rightarrow \quad \frac{\delta}{\delta g_{\mu\nu}} \int_B \left(c_1 + c_2 R_{\mu\nu} g^{\mu\nu}\right) \sqrt{g}\, d^4x$$

(up to anti-sym. components, which we set to zero)

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Remark: Choosing to add an antisymmetric part would make $G^{\mu\nu}$ and $R^{\mu\nu}$ non-symmetric, in violation of $g_{\mu\nu;\rho} = 0$.

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Finally, we conclude:

$$\frac{\delta S'_{eff}}{\delta g_{\mu\nu}} = 0$$

leads to this equation of motion for g:

$$\left(\frac{1}{2} c_1 g^{\mu\nu} - c_2 G^{\mu\nu}\right)\sqrt{g} = -\frac{1}{2}\sqrt{g}\, T^{\mu\nu}$$

i.e.:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \frac{c_1}{2c_2} g^{\mu\nu} = \frac{1}{2c_2} T^{\mu\nu}$$

As is well-known, comparison with experiment requires:

$$c_2 = \frac{1}{16\pi G}$$

Newton's constant.

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(Notice: the "reduced Bianchi identity" $(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R)_{;\nu}=0$ now is equivalent to the statement $T^{\mu\nu}{}_{;\nu}=0$)

Notice: The symmetry of $R^{\mu\nu}$ (due to $g_{\mu\nu;\rho}=0$) enforces the symmetry of $T^{\mu\nu}$.

⇒ Einstein equation:

$$\boxed{R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\underbrace{-8\pi G c_1}_{=\Lambda\ \text{"cosmol. constant"}}g^{\mu\nu}=8\pi G T^{\mu\nu}}$$

What is Λ?

Recall: The λ_i are constants prefactor of order one which depends on the matter Lagrangian.

First find ℓ_c:

(I.e. also: $G=\ell_c^2$, which we'll need on the next slide)

Given that $\frac{1}{16\pi G}=c_2=\lambda_2\ell_c^{-2}$

undoing $\hbar=1$ and $c=1$ so that get l_g m, s:

we obtain: $\ell_c=\lambda_2\sqrt{16\pi G}=\sqrt{16\pi\frac{\hbar G}{c}}$

$=\lambda_2\cdot 4\sqrt{\pi}\cdot 1.616\times 10^{-35}\,m$

Recall: The λ_i are some numbers of order $O(1)$,

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identity "$(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R)_{;\nu} = 0$" now is equivalent to the statement $T^{\mu\nu}{}_{;\nu} = 0$

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Recall: The λ_i are some numbers of order $O(1)$, depending on details which sort of particles are in the matter action and their quantum effects

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We note, therefore: When quantum field theories are cut off at about the Planck length they induce gravity with the correct eqns of motion and coupling strength!

Now find c_1:

Given that $\ell_c \approx 10^{-35}$ m, quantum field theories generate a value of c_1 i.e. a cosmological constant of about:

$$c_1 = \lambda_1 \ell_c^{-4}$$

of $\mathcal{O}(1)$

Finally, find Λ:

$$\Lambda_{theory} = -8\pi G c_1 = -8\pi G \lambda_1 \ell_c^{-4}$$

$$\approx G \ell_c^{-4} \approx \ell_c^{-2}$$ (using $\lambda_1 \approx 1$ and $G \approx \ell_c^2$)

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This is the worst physical prediction ever:

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Experiment:

Based on cosmic microwave background data and on supernova brightness versus redshift data:

$$\Lambda_{experiment} \simeq 10^{-52}\ m^{-2} \qquad \text{i.e. } \frac{\Lambda_{th}}{\Lambda_{exp}} \simeq 10^{122}$$

→ For some unknown reason, the constant part, c_1, of the vacuum energy of quantum field theories does essentially not gravitate - while its disturbance through curvature $c_1 R$ is real: it induces regular gravit...

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