Title: Unified approach to classical and quantum dualities

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Abstract: Dualities appear in nearly all disciplines of physics and play a central role in statistical mechanics and field theory. I will discuss in a pedagogical way our recent findings motivated by a quest for a simple unifying framework for the detection and treatment of dualities.
I will explain how classical and quantum dualities, as well as duality relations that appear only in a sector of certain theories (i.e. emergent dualities), can be unveiled, and systematically established. Our method relies on the use of morphisms of the "bond algebra" of a quantum Hamiltonian. Dualities are characterized as unitary mappings implementing such morphisms, whose even powers become symmetries of the quantum problem. Dual variables (non-local mappings between the elementary degrees of freedom of the theory) which were guessed in the past can be derived
in our formalism. New self-dualities for four-dimensional Abelian gauge field theories will be discussed.

Unified Approach to Quantum and Classical Dualities

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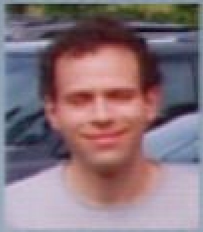


ohar Nussinov: Washington University - St. Louis

Perimeter Institute - November 2009

State of Affairs

The notion has been around for a very long time, and it has proven to be of great importance in Statistical Mechanics, Quantum Field Theory, and many other fields. Still, the best description available was:

"Dualities are certain mathematical transformations"

To be more precise some authors would add:

"Transformations like the ones considered by Kramers and Wannier in their study of the Ising model"

It is not clear how to find or derive dualities and whether classical and quantum versions are related or not

State of Affairs

State of Affairs

So, What are Dualities?

State of Affairs

So, What are Dualities?

Unitary Maps of Bond algebras

Why Dualities?

- Phase transitions and diagrams: Self-dual ↔ Critical Points
- Nature of (topological) excitations: Classification?
- Connect seemingly unrelated (dual) theories:
  Orbital ordered and superconducting theories
  AdS/CFT: Conjectured
- Allow exact solutions in special cases:
  Kitaev Toric Code model at finite T
  Kitaev honeycomb model in topological sectors

Why Dualities?

- Find simpler classical actions
- Numerical Simulations:
  Stochastic, Hierarchical Mean-fields or Renormalization Group Methods
- Unravel the power of Quantum Computation?

Old friends from the Zoo of Dualities

Classical Dualities

Wisdom: Low-temperature-to-High-temperature relations

Maxwell's equations in empty space (vacuum)

$$\nabla \times E = -\dot{B} \qquad \nabla \times B = \dot{E}$$

$$\nabla \cdot E = 0 \qquad \nabla \cdot B = 0$$

CLASSICAL ELECTROMAGNETIC DUALITY

$$E \rightarrow -B \qquad B \rightarrow E$$

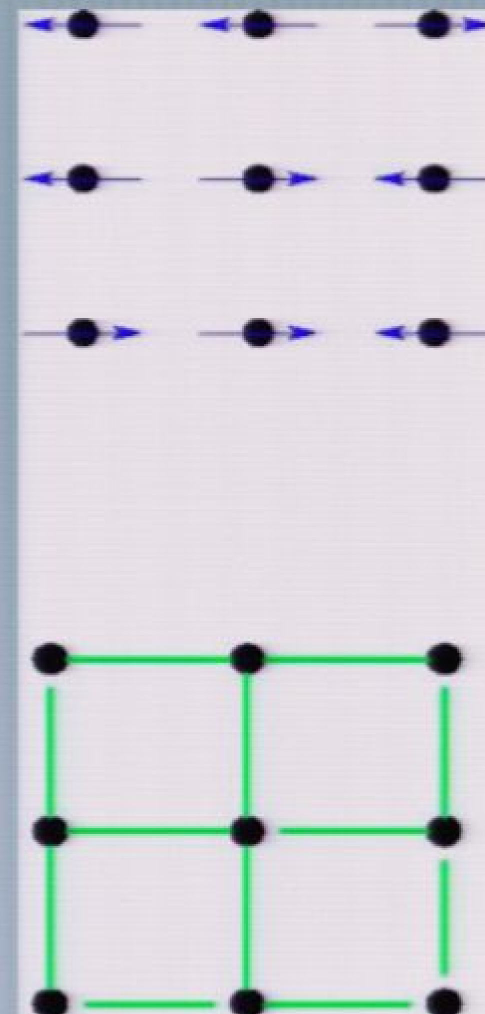
Things that look different are equivalent and interchangeable

Kramers–Wannier Self-Duality of the *D*=2 Ising Model

$$\mathcal{Z}(K) = \sum_{\sigma} \exp\left[K \sum_{\langle i,j \rangle} \sigma_i \sigma_j\right]$$

$$\sigma_i = 1 \text{ or } -1$$

$$K = \beta J \qquad \beta = 1/k_B T$$



has a remarkable property:

it is self-dual,

meaning...

KW SELF-DUALITY RELATION

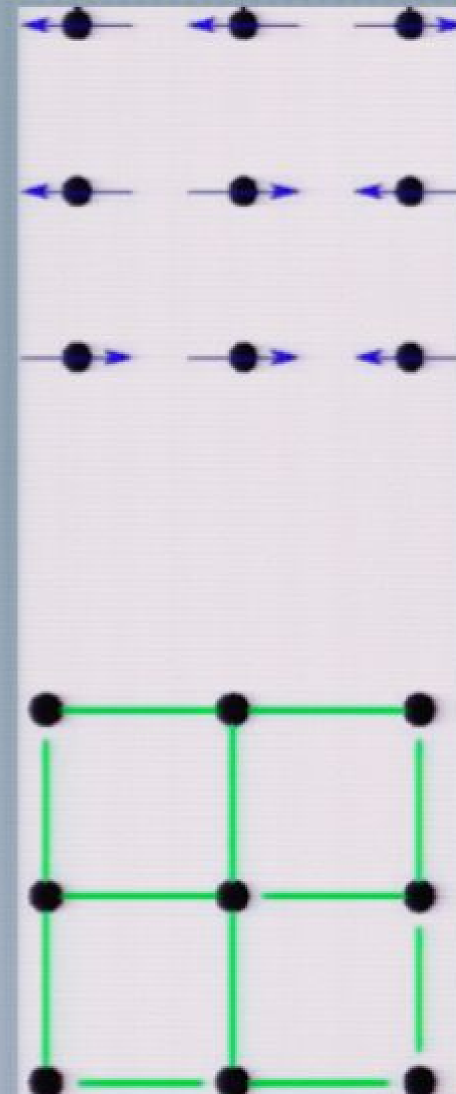
$$\frac{\mathcal{Z}(K)}{\sinh(2K)^{N^2/2}} = \frac{\mathcal{Z}(K^*)}{\sinh(2K^*)^{N^2/2}}$$

whenever $\sinh(2K)\sinh(2K^*) = 1$

CONCEPT:
High−T ⟷ Low−T relation

The critical point is located at the self-dual point: $K = K^* = K_c$

$$K_c = \frac{1}{2}\ln(1+\sqrt{2})$$



Quantum Dualities

Wisdom: Strong-coupling-to-Weak-coupling relations

Quantum dualities

$$H_{\text{IC}} = \sum_i j\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x = \sum_i h\mu_i^z\mu_{i+1}^z + j\mu_{i+1}^x$$

$$\mu_i^x = \sigma_{i-1}^z\sigma_i^z \qquad \mu_i^z = \sigma_i^x\sigma_{i+1}^x\sigma_{i+2}^x\sigma_{i+3}^x\cdots$$

The new operators are spin-1/2 operators as well, thus it has to be that

$$E_{\text{IC}}(j,h) = E_{\text{IC}}(h,j)$$

CONCEPT:
Strong-coupling ⟷ Weak-coupling relation

“Hand-waving” approach to quantum dualities

This is the traditional approach to Quantum Self-Dualities and Dualities

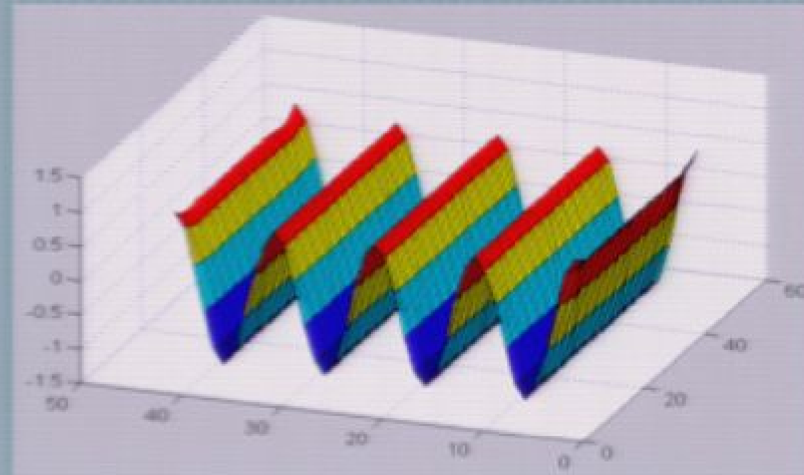
Idea: if you suspect a connection, try to prove it by GUESSING an OPERATOR (VERY NON-LOCAL) MAPPING, and good luck in finding it!!!!

Unconventional view on Particle-wave duality

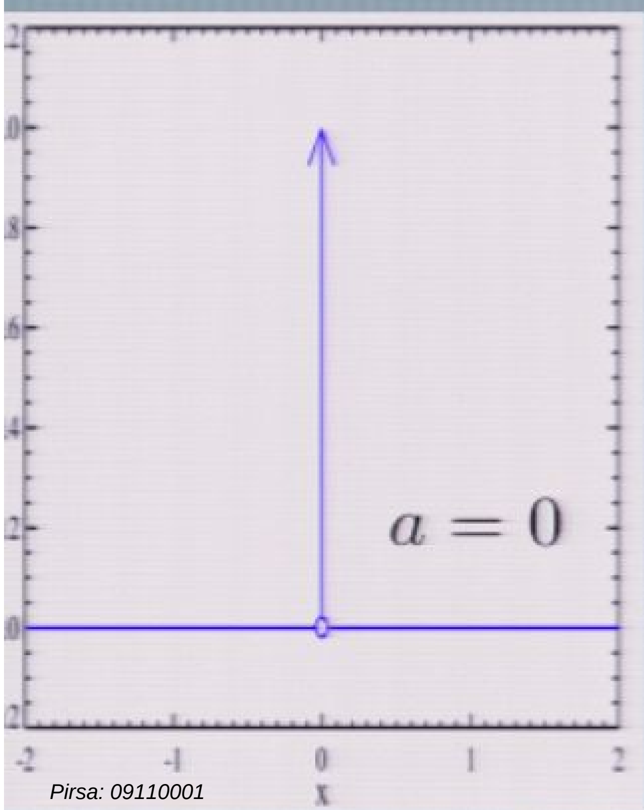
Our intuition about quantum motion has to deal with Heisenberg's Uncertainty Relation and its descendants and relatives

$$[x, p] = i$$

A particle with a definite momentum p is in a wavy state (wave)



$$\psi = \frac{1}{\sqrt{2\pi}} e^{ipx}$$



A particle with a definite position a is localized in space (particle)

$$\psi = \delta(x - a)$$

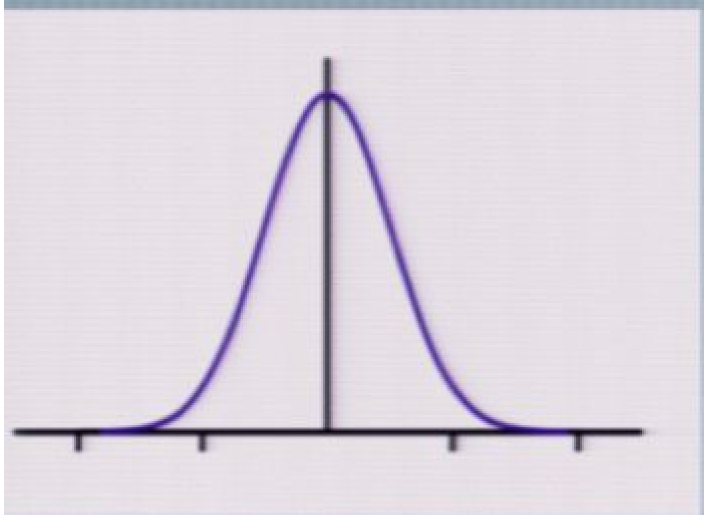
Both pictures are incompatible due to the Heisenberg uncertainty relation

nagine one introduces **new** position and momentum operators:

$$x' = -p \qquad\qquad p' = x$$

$$\boxed{[x', p'] = i}$$

his **unitary** transformation has surprising consequences:



A quantum state which is localized in momentum can be thought of as being localized in position, and viceversa.

Things that look very different seem equivalent

With only this info, one cannot distinguish between position and momentum

How can one distinguish position from momentum?

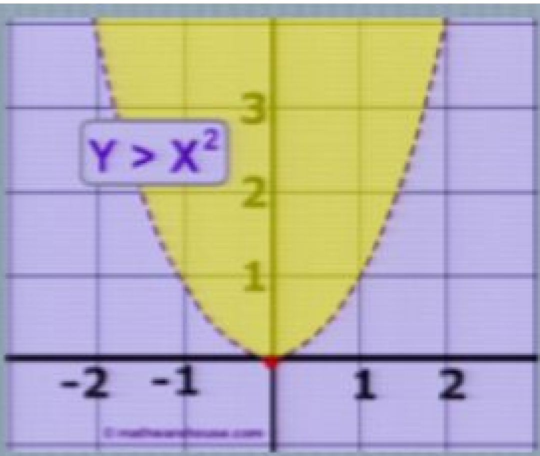
DYNAMICS BREAKS THE EQUIVALENCE

$$\boxed{H = \frac{1}{2m}p^2 + V(x)}$$

The Hamiltonian breaks the symmetry of the Heisenberg algebra

$$[x, p] = i$$

But for the Harmonic oscillator...



$$\frac{1}{2m}p^2 + \frac{1}{2}kx^2 \quad \leftrightarrow \quad \frac{1}{2}kp^2 + \frac{1}{2m}x^2$$

$$E_n = \sqrt{\frac{k}{m}}(n + \frac{1}{2}) \quad \longleftrightarrow \quad E_n = \sqrt{\frac{k}{m}}(n + \frac{1}{2})$$

Elementary SELF-DUALITY RELATION

The Harmonic Oscillator shares the symmetry of the Heisenberg algebra, but with new consequences

All these Dualities are examples of Unitary equivalence

So... What are Dualities?

One would like to understand:

1)Their physical content and meaning in **classical** and **quantum** physics and their **connection if any**

2)A precise **mathematical characterization**

3)Methods to look for dualities systematically

4) New Applications

*** E. Cobanera, G. Ortiz, Z. Nussinov, "Unified approach to classical and quantum dualities", http://arxiv.org/abs/0907.0733**

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Our bond-algebraic approach

Bond algebras and their symmetries

Quantum Hamiltonians are built as a sum of quasi-local operators

We call these **BONDS**:

$$H = \sum_R J_R \mathcal{O}_R$$

A **bond algebra** for H is the set of all linear combinations of products of bonds

$$\mathcal{A}_H = \{1, \alpha\mathcal{O}_{\mathcal{R}}, \beta\mathcal{O}_R\mathcal{O}_{R'}, \mathcal{O}_R - \mathcal{O}_R\mathcal{O}_{R'}\mathcal{O}_{R''}, \cdots\}$$

It knows a lot about the Hamiltonian...

Exposing Quantum Dualities

When are two Hamiltonias Dual?

H_1 **and** H_2 **are dual if there is an**

isomorphism between their bond algebras

DUALITIES **are one to one, onto mappings between bond algebras that preserve every algebraic relation between bonds:**

$$\mathcal{O}^1_{R_1} \leftrightarrow \mathcal{O}^2_{R_2}$$

Self-Dualities are automorphims of bond algebras

that preserve the form of the Hamiltonian

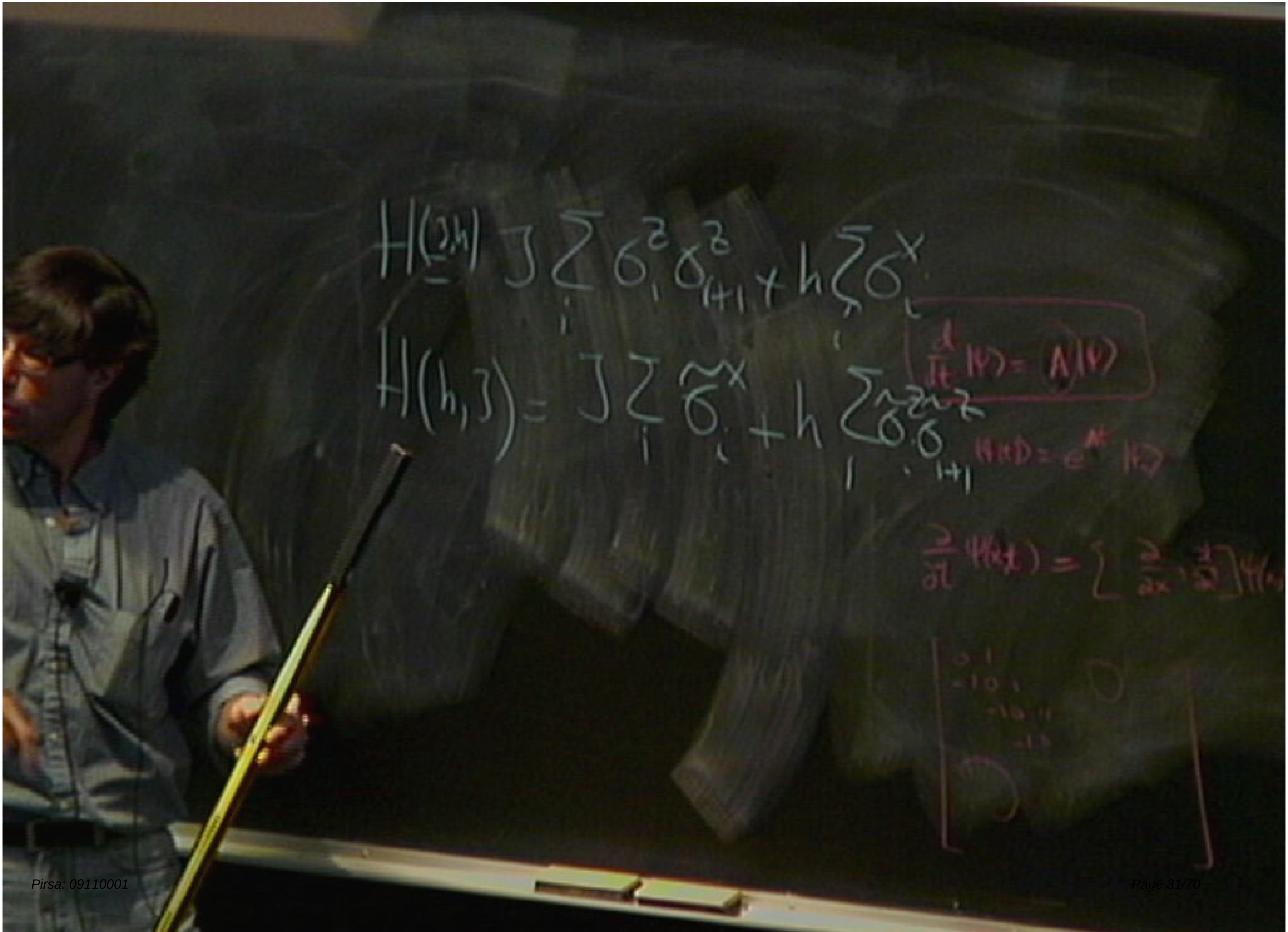
n other words:

A Self-Dualities is a symmetry of the bond algebra

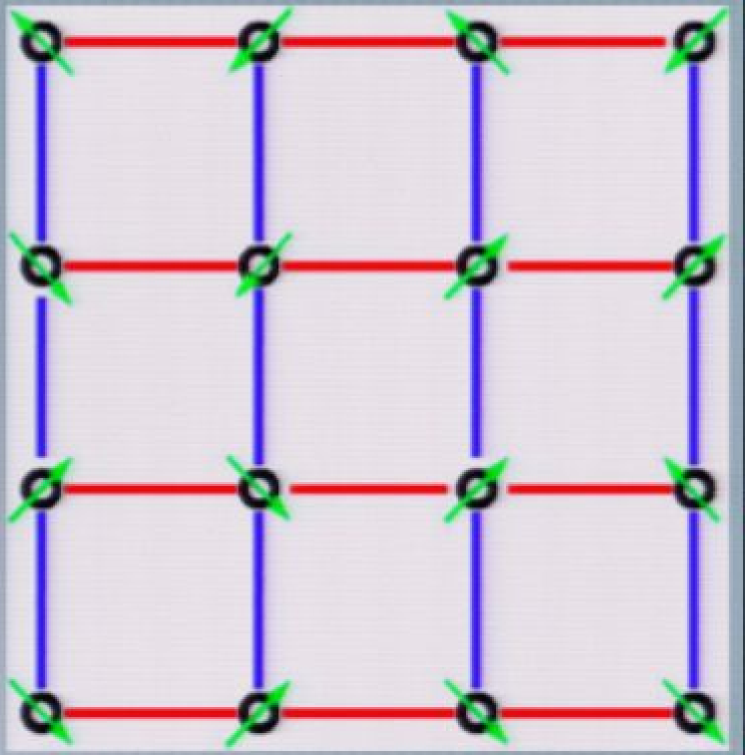
that preserves the form of the Hamiltonian

Quantum Mechanics requires these mappings to be

UNITARILY IMPLEMENTABLE


H(J,h) = J Σ σ^z_i σ^z_{i+1} + h Σ σ^x_i
H(h,J) = J Σ σ̃^x_i + h Σ σ̃^z_i σ̃^z_{i+1}

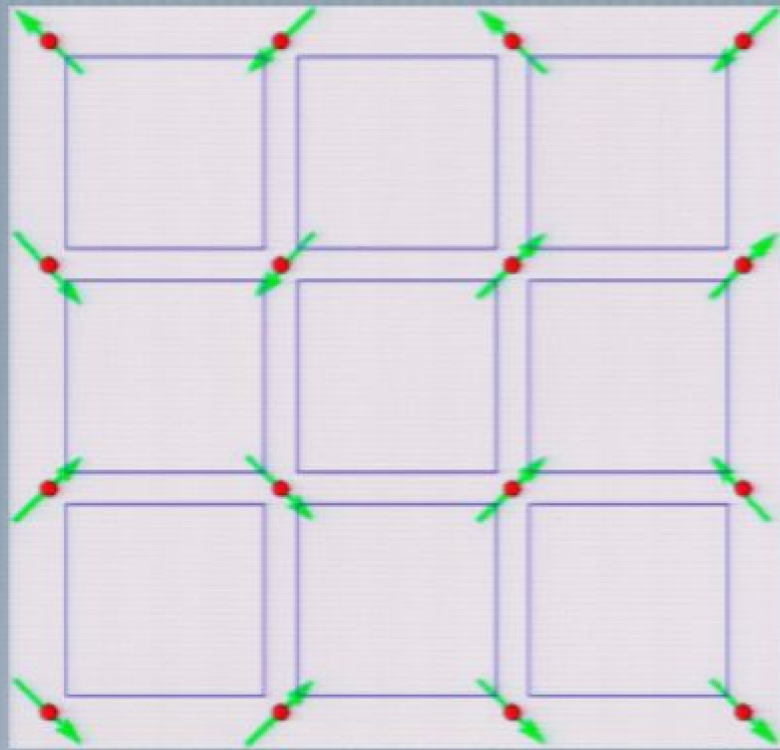
xample of Duality: Planar Orbital Compass (POC) and Xu-Moore Models (XM)



The POC model provides a simplified scenario to study orbital ordering in transition metal compounds

$$H_{POC} = \sum_{\vec{i}} j_x \sigma^x_{\vec{i}} \sigma^x_{\vec{i}+e_1} + j_y \sigma^y_{\vec{i}} \sigma^y_{\vec{i}+e_2}$$

The XM Hamiltonian was introduced as a simplified model of phase transitions in p+ip superconducting arrays



$$H_{XM} = \sum_{\vec{i}} j \sigma^z_{\vec{i}} \sigma^z_{\vec{i}+e_1} \sigma^z_{\vec{i}+e_1+e_2} \sigma^z_{\vec{i}+e_2} + h \sigma^x_{\vec{i}}$$

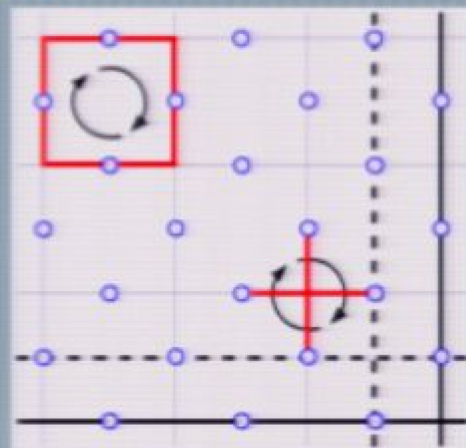
The two models are DUAL

$$\sigma^x_{\vec{i}}\sigma^x_{\vec{i}+\vec{e}_1} \mapsto \sigma^z{}_{\vec{i}}\sigma^z{}_{\vec{i}+e_1}\sigma^z{}_{\vec{i}+e_1+e_2}\sigma^z{}_{\vec{i}+e_2}$$

$$\sigma^y_{\vec{i}}\sigma^y_{\vec{i}+\vec{e}_2} \mapsto \sigma^x_{\vec{i}+\vec{e}_2}$$

THE TWO HAMILTONIANS ARE UNITARILY EQUIVALENT

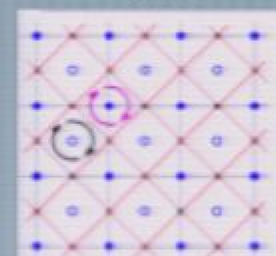
itaev's toric code model:



$$H_K = -\sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{ij \in star(s)} \sigma^x_{ij}$$

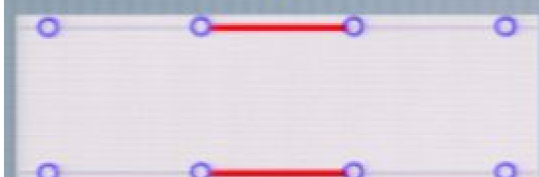
$$B_p = \prod_{ij \in boundary(p)} \sigma^z_{ij}$$



Duality mappings: Non-local

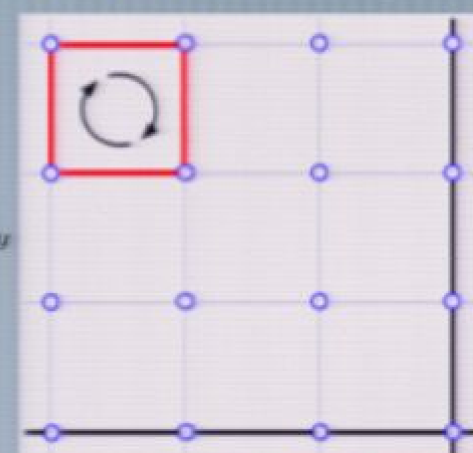
(Identical spectra)

2 Ising chains:



Wen's plaquette model:

$$H_W = -\sum_i \sigma^x_i \sigma^y_{i+\hat{e}_x} \sigma^x_{i+\hat{e}_x+\hat{e}_y} \sigma^y_{i+\hat{e}_y}$$

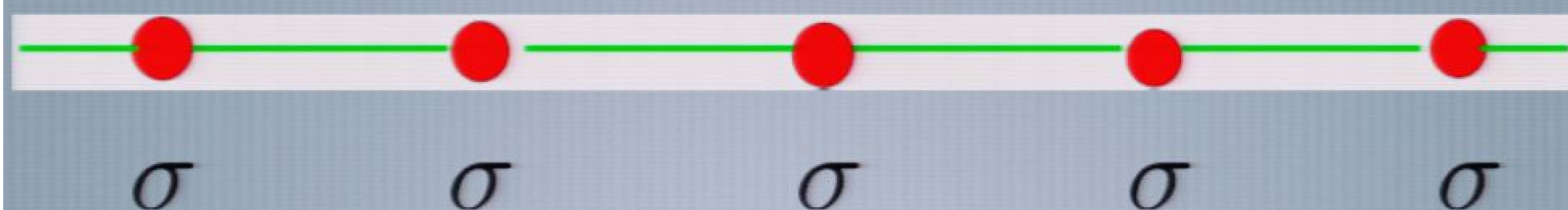


$$= -\sum_s \sigma^z_s \sigma^z_{s+1} - \sum_p \sigma^z_p \sigma^z_{p+1}$$

ussinov-Ortiz 2006)

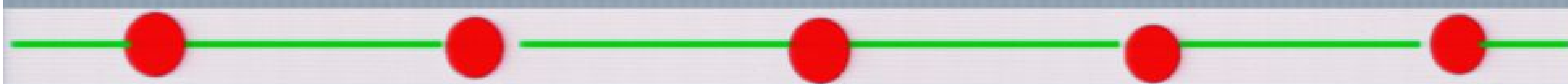
Example of Self-Duality:
Ising chain in a transverse field

$$H[j,h] = \sum_i j\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x$$



BOND ALGEBRA

$$\sigma^z_{i-1}\sigma^z_i \qquad \sigma^z_i\sigma^z_{i+1}$$



$$\sigma^x_{i-1} \qquad \sigma^x_i \qquad \sigma^x_{i+1}$$

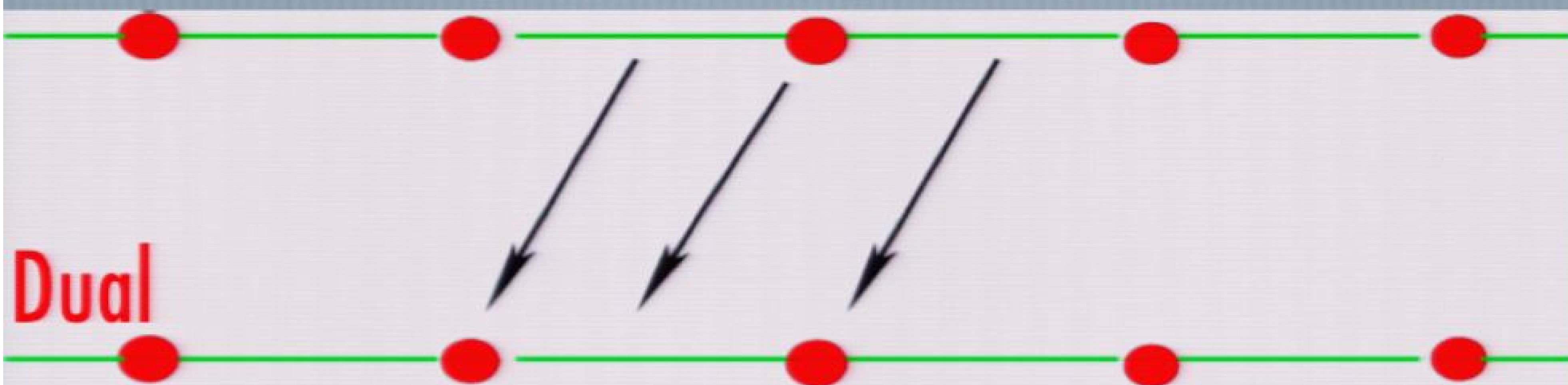
Every bond $\sigma^z\sigma^z$ anti-commutes with two bonds σ^x

Every bond σ^x anti-commutes with two bonds $\sigma^z\sigma^z$

SELF–DUALITY AUTOMORPHISM

$$\sigma_i^z \sigma_{i+1}^z \mapsto \sigma_i^x \qquad \sigma_i^x \mapsto \sigma_{i-1}^z \sigma_i^z$$

$\sigma_{i-1}^z \sigma_i^z \quad \sigma_i^x \quad \sigma_i^z \sigma_{i+1}^z$



$\sigma_{i-1}^x \quad \sigma_{i-1}^z \sigma_i^z \quad \sigma_i^x \quad \sigma_i^z \sigma_{i+1}^z$

Mapping is Unitarily implementable

$$\mathcal{U}_D \sigma_i^z \sigma_{i+1}^z \mathcal{U}_D^\dagger = \sigma_i^x \qquad \mathcal{U}_D \sigma_i^x \mathcal{U}_D^\dagger = \sigma_{i-1}^z \sigma_i^z$$

Ising chain in a transverse field is self-dual, meaning:

$$\mathcal{U}_D H[j,h] \mathcal{U}_D^\dagger = H[h,j]$$

Advantages:

- Better suited for **systematic** (ALGORITHMIC) **search** of (self-)dualities
- Allows us to **derive** the (in general) **non-local dual operator variables** - the ones that had to be guessed in the past

Dual operators, Disordered variables and Topological excitations

It is easy now to
COMPUTE DUAL OPERATOR VARIABLES:

bservation:

— Bond-algebraic mapping is local

— Mapping of microscopic degrees of freedom is non-local

Use bond-algebraic mapping to derive the dual variables

om bonds to dual variables: An example

$$\left.\begin{aligned} \mu_i^x &= \sigma_{i-1}^z \sigma_i^z \\ \mu_i^z \mu_{i+1}^z &= \sigma_i^x \end{aligned}\right\} \text{ Bond algebra map}$$

$$\mu_i^z \mu_{i+1}^z \mu_{i+1}^z \mu_{i+2}^z \cdots \mu_{i+4}^z \cdots = \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x \cdots$$

Dual variables

$$\mu_i^z = \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x \cdots$$

Generators of KINKS

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Bond algebra map

$$\mu_{i+1}^z \mu_{i+1}^z \mu_{i+2}^z \cdots \mu_{i+4}^z \cdots = \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x \cdots$$

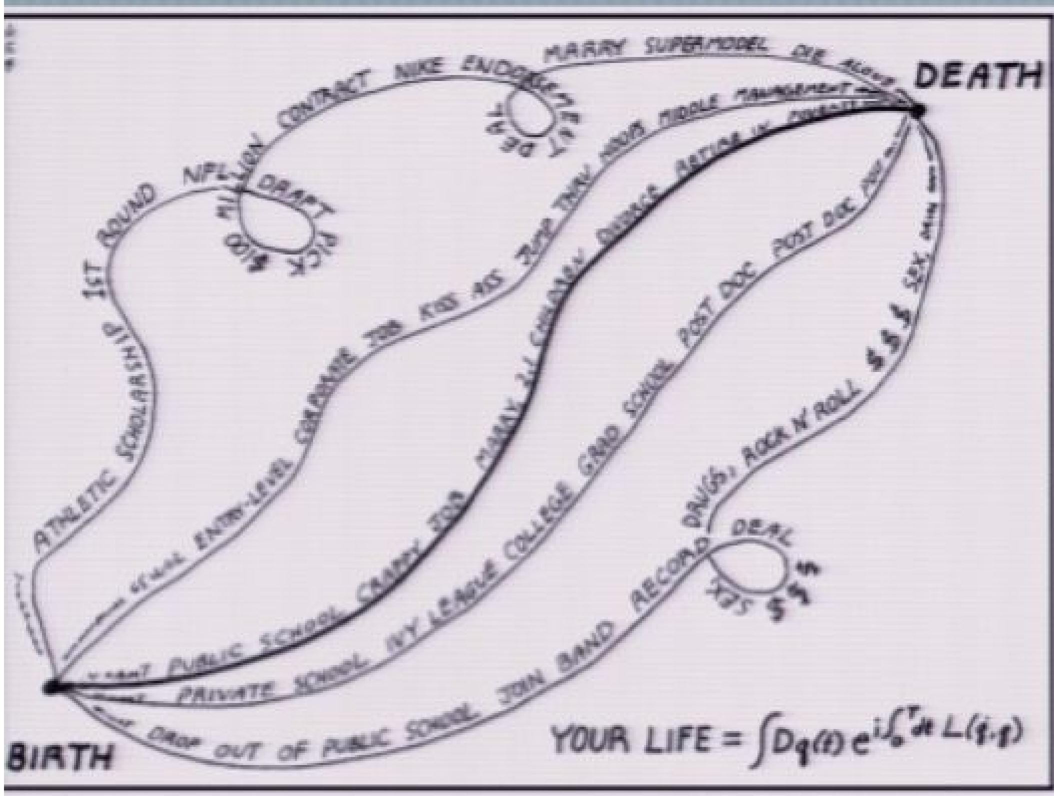
Dual variables

$$\mu_i^z = \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x \cdots$$

Generators of KINKS

onnecting Classical and Quantum Dualities

1) Classical System in $D+1$ ⟷ Quantum system in D



The Path Integral Formulation of Your Life

Feynman Path Integral

alias

Suzuki-Trotter decomposition

alias

World Line Monte Carlo

...

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$$\mu_i^z \mu_{i+1}^z = \sigma_i^x$$

Bond algebra map

$$\mu_{i+1}^z \mu_{i+1}^z \mu_{i+2}^z \cdots \mu_{i+4}^z \cdots = \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x \cdots$$

Dual variables

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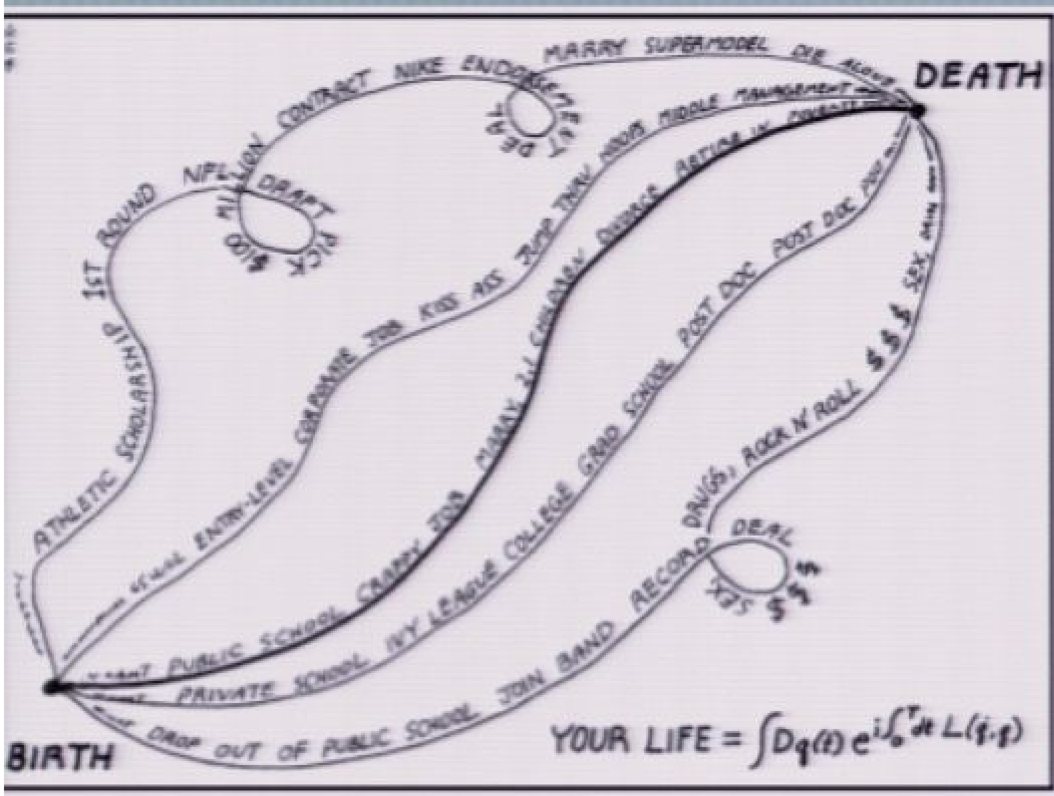

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Thus we propose that

2) **Classical Dualities ⟷ Quantum Dualities**

3) **Characterize Quantum Dualities as Hamiltonian dependent bond-algebraic equivalences**

And hope for Quantum Dualities to be easier to deal with than classical ones.
After all, the classical problem comes from exponentiating a quantum one and taking a trace...

Exposing Classical Dualities

Quantum-to-Classical: Suzuki-Trotter decomposition of the Ising chain

$$H[j,h] = -\sum_{i=1}^{N} j\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x$$

Quantum Ising Chain in a Transverse Field

$$\mathrm{Tr}\, e^{-\Delta\tau H[j,h]} \approx \left(\frac{1}{2}\sinh 2h_M\right)^{\frac{MN}{2}} \times$$

$$\sum_{\{\sigma\}} e^{\left[-\frac{1}{2}\ln\tanh(h_M)\sum\sigma_{m,n}\sigma_{m,n+1}+j_M\sum\sigma_{m,n}\sigma_{m+1,n}\right]}$$

$$h_M = \frac{\Delta\tau\ h}{M} \qquad j_M = \frac{\Delta\tau\ j}{M}$$

The larger M, the better it gets

Ve can fine-tune the couplings of the Quantum Model to get an sotropic **classical Ising magnet**

$$\text{Tr}\ e^{-\Delta\tau H[j,h]} \approx \left(\frac{1}{2}\sinh 2h_M\right)^{\frac{MN}{2}} \mathcal{Z}(K)$$

$$K \equiv -\frac{1}{2}\ln\tanh h_M = j_M$$

$$h_M = \frac{\Delta\tau\ h}{M} \qquad j_M = \frac{\Delta\tau\ j}{M}$$

On the other hand, exchanging couplings j and h gives

$$\text{Tr}\; e^{-\Delta\tau H[h,j]} \approx \left(\frac{1}{2}\sinh 2j_M\right)^{\frac{MN}{2}} \mathcal{Z}(\hat{K})$$

$$\hat{K} = -\frac{1}{2}\ln\tanh(j_M) = h_M$$

Any connection between the two?

YES!!!!!!

The QUANTUM Self-Duality guarantees that

$$\mathrm{Tr}\, e^{-\Delta\tau H[h,j]} = \mathrm{Tr}\, e^{-\Delta\tau H[j,h]}$$

OR BETTER, IN TERMS OF CLASSICAL PARTITION FUNCTIONS

$$\left(\frac{1}{2}\sinh 2\hat{K}\right)^{\frac{MN}{2}} \mathcal{Z}(K) = \left(\frac{1}{2}\sinh 2K\right)^{\frac{MN}{2}} \mathcal{Z}(\hat{K})$$

MOREOVER, FROM THE EXPLICIT FORMULAS FOR THE CLASSICAL COUPLINGS IN TERMS OF THE QUANTUM ONES, THIS RELATION FOLLOWS:

$$\sinh(2K)\sinh(2\hat{K}) = 1$$

These altogether are nothing but the classical self-duality relation of Kramers and Wannier!!!!

Contrast: Quantum vs Classical

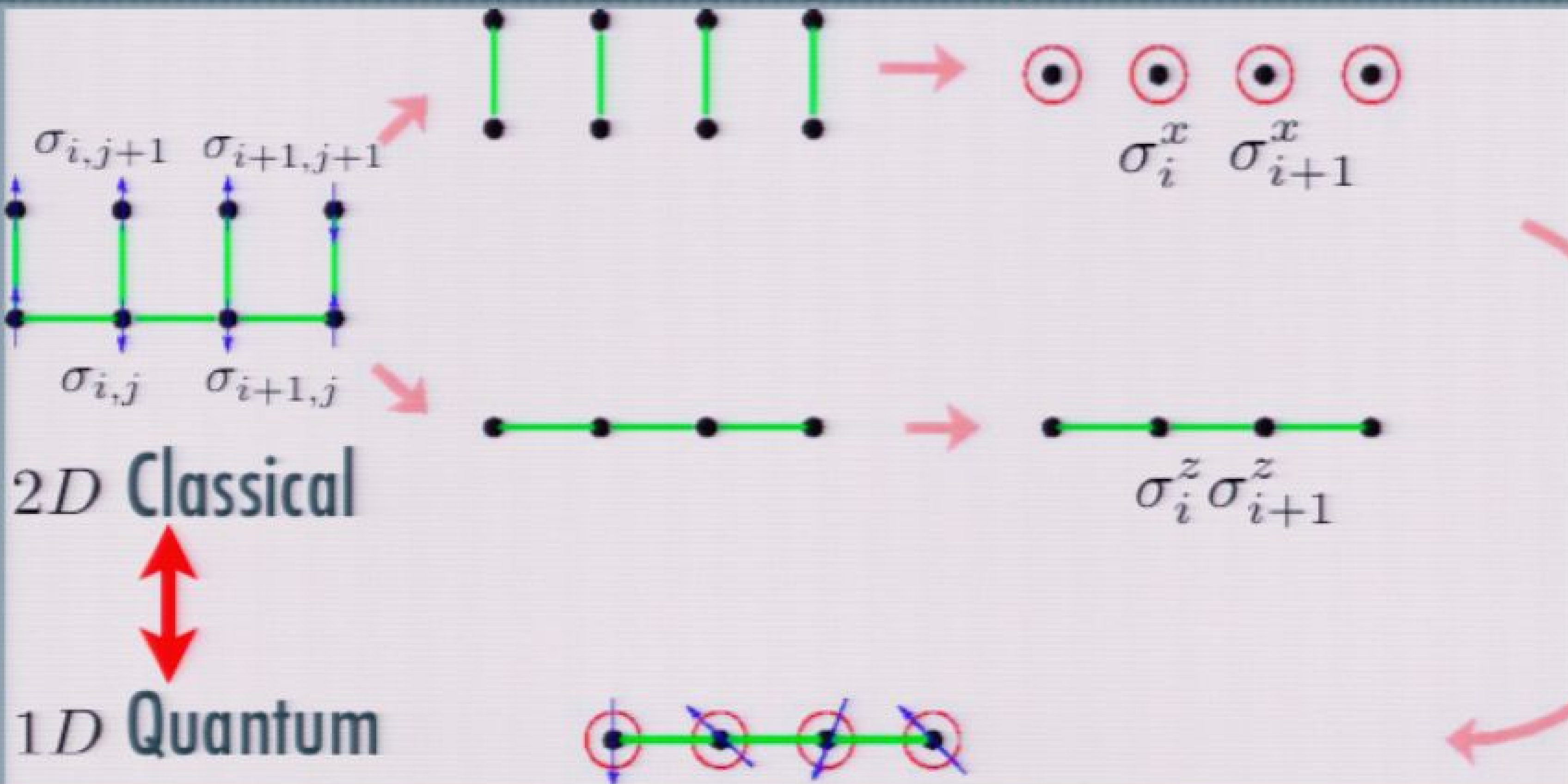
uantum Self-duality relation

$$h = j$$

lassical Self-duality relation

$$\sinh(2K)\,\sinh(2\hat{K}) = 1$$

lassical-to-Quantum:
rom the transfer matrix to a quantum Hamilt



$$\sigma' | \exp(\lambda \sigma^x) | \sigma \rangle = \sqrt{\frac{1}{2}\sinh(2\lambda)}\, \exp\left[-\frac{1}{2} \ln \tanh(\lambda) \sigma' \sigma \right]$$

Classical and Quantum (Self-)Dualities are equivalent and in correspondence:

We have managed to UNIFY them.

Dualities and New Symmetries

$$\mathcal{U}_D H[j,h] \mathcal{U}_D^\dagger = H[h,j]$$

A **self-duality** is **not a symmetry** in general, but

$$\mathcal{U}_D^2 H[j,h] \mathcal{U}_D^{\dagger 2} = H[j,h]$$

$$\boxed{\text{Self-duality} \leftrightarrow \sqrt{\text{Quantum Symmetry}}}$$

A **self-duality** is **an emergent symmetry** at the self-dual point

New (Self-)Dualities
Enlarging the Zoo of Dualities

Dualities and New Symmetries

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Enlarging the Zoo of Dualities

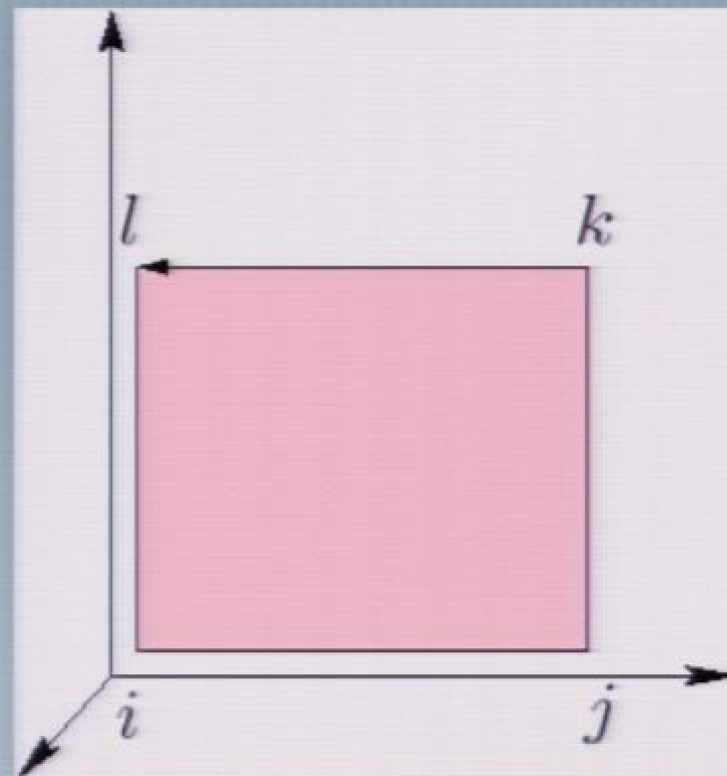
Gauge Field Theories

Four-Dimensional (D=4) Euclidean Lattice

't Hooft idea: the most important degrees of freedom in a **confinement-deconfinement phase transition** should be the field configurations taking values on $\mathbb{Z}_N$, the center of SU(N)

With 't Hooft ideas in mind, several authors attempted rigorous studies of Wilson's action for Lattice Gauge Field Theories

$$S = \frac{1}{g^2} \sum_{\square} \text{Re Tr } U_{ij} U_{jk} U_{kl}^{\dagger} U_{li}^{\dagger}$$



restricting however the fields to taking alues on a unitary representation of $\mathbb{Z}_N$, that is, on Nth roots of unity

From Euclidean to Quantum Hamiltonian Formulation

The reverse of the Suzuki-Trotter decomposition gives a quantum problem in 3 dimensions

$$H_{LG} = \sum_n \sum_{i=1}^{3} V_n^i + \lambda\left(\frac{1}{g^2}\right)\Theta_n^i + h.c.$$

$$\Theta_n^1 = U_n^2 U_{n+e_2}^3 U_{n+e_3}^{2\dagger} U_n^{3\dagger}$$ and cyclic permutations

$$VU = \omega UV, \quad \omega = \exp i\frac{2\pi}{N}$$

The **Weyl Algebra**

With ‘t Hooft ideas in mind, several authors attempted rigorous studies of Wilson’s action for Lattice Gauge Field Theories

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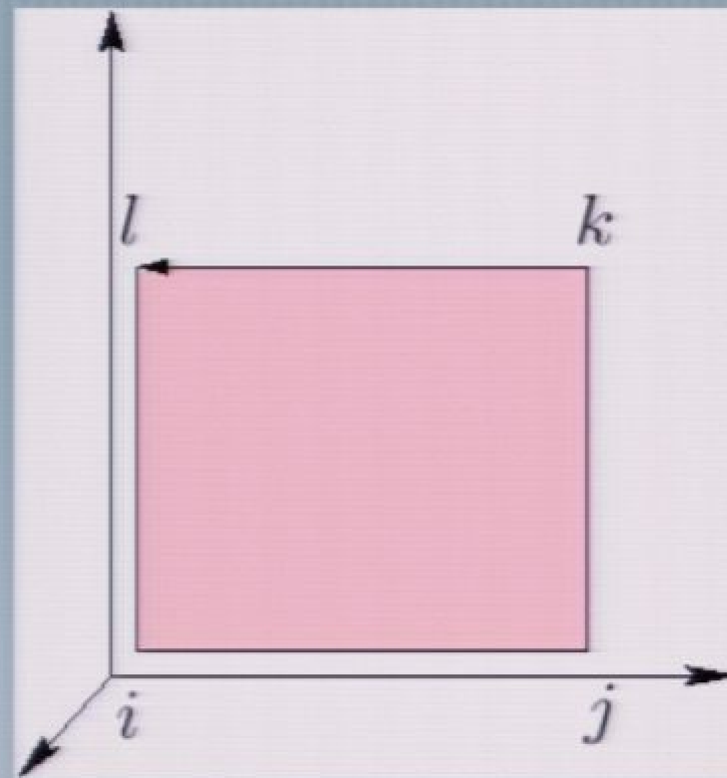

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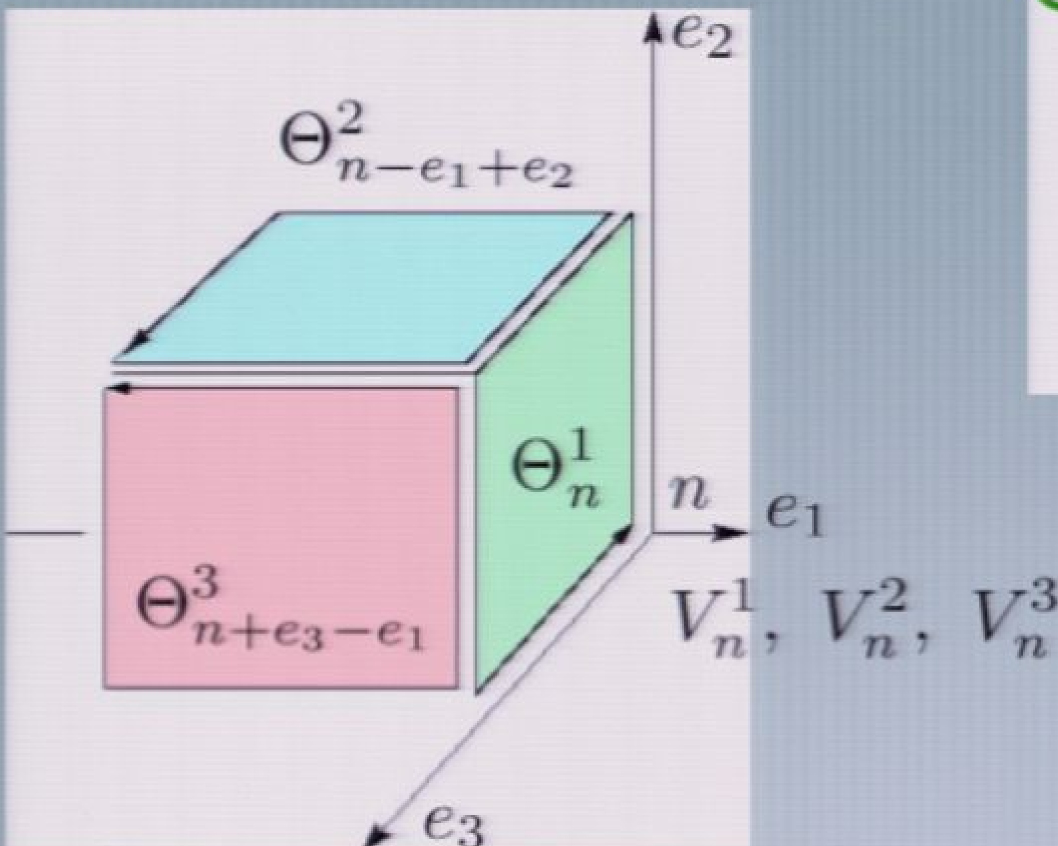
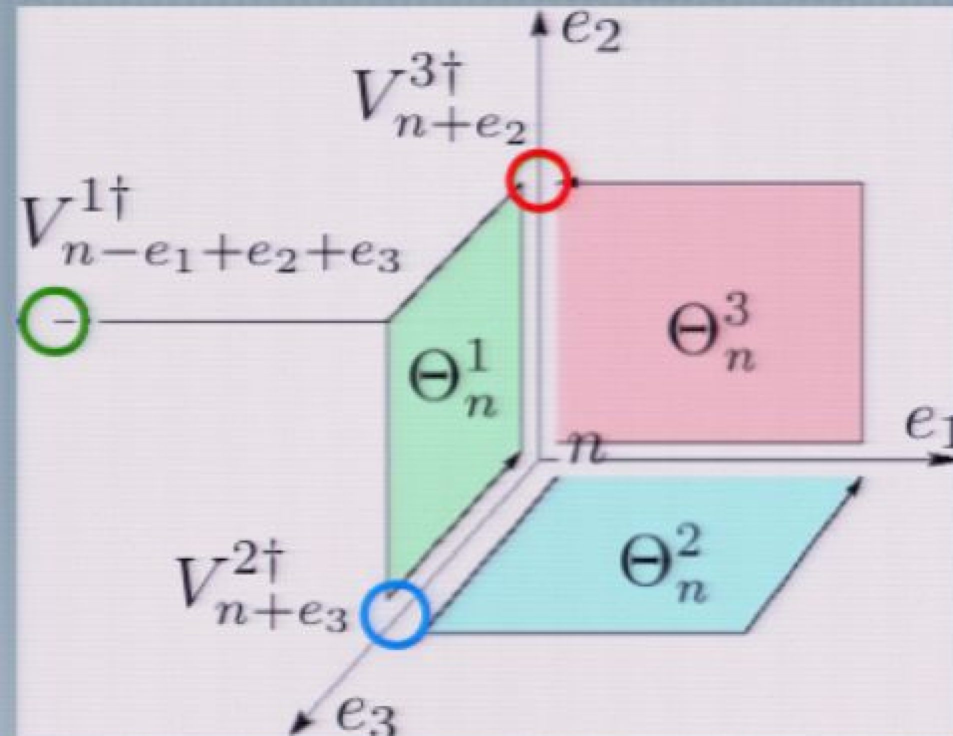
The Weyl Algebra

The Self-Duality Mapping





Some features of the Self-Duality

- The critical couplings have to distribute symmetrically relative to the self dual point
- The self-duality unitary has period four, thus it reveals a **new** discrete symmetry of these theories
- An explicit **analytic** formula to compute the self-dual coupling can be obtained
- **Prove** connection between these GFTs and the vector Potts model
- We do not use Villain's trick. It is not necessary!

Summary of Main Results

- Quantum (self-)dualities can now be looked for **systematically** as **bond algebra (unitary)** mappings
- An **algebraic approach** to quantum (self-)dualities **explains classical dualities** as well, in any space dimension d
- **Dual Variables can be computed** and carry information on the topological excitations of the system
- **New (self-)dualities** can be discovered with this new algebraic approach. We showed the case of Abelian GFTs with a confinement-deconfinement phase transition

Summary of Main Results

- Dualities may emerge is certain sectors (**emergent dualities**)
- Self-dualities are **square roots of symmetries**
- Bond-algebra mappings allow **exact** solution of several many-body models in high space dimensions
- Other Self-dualities: Potts, p-clock, etc. models
- Other Dualities: Extended Kitaev, Blume-Emery-Griffiths, etc. models