Title: Holography & Heavy-Ion Collisions

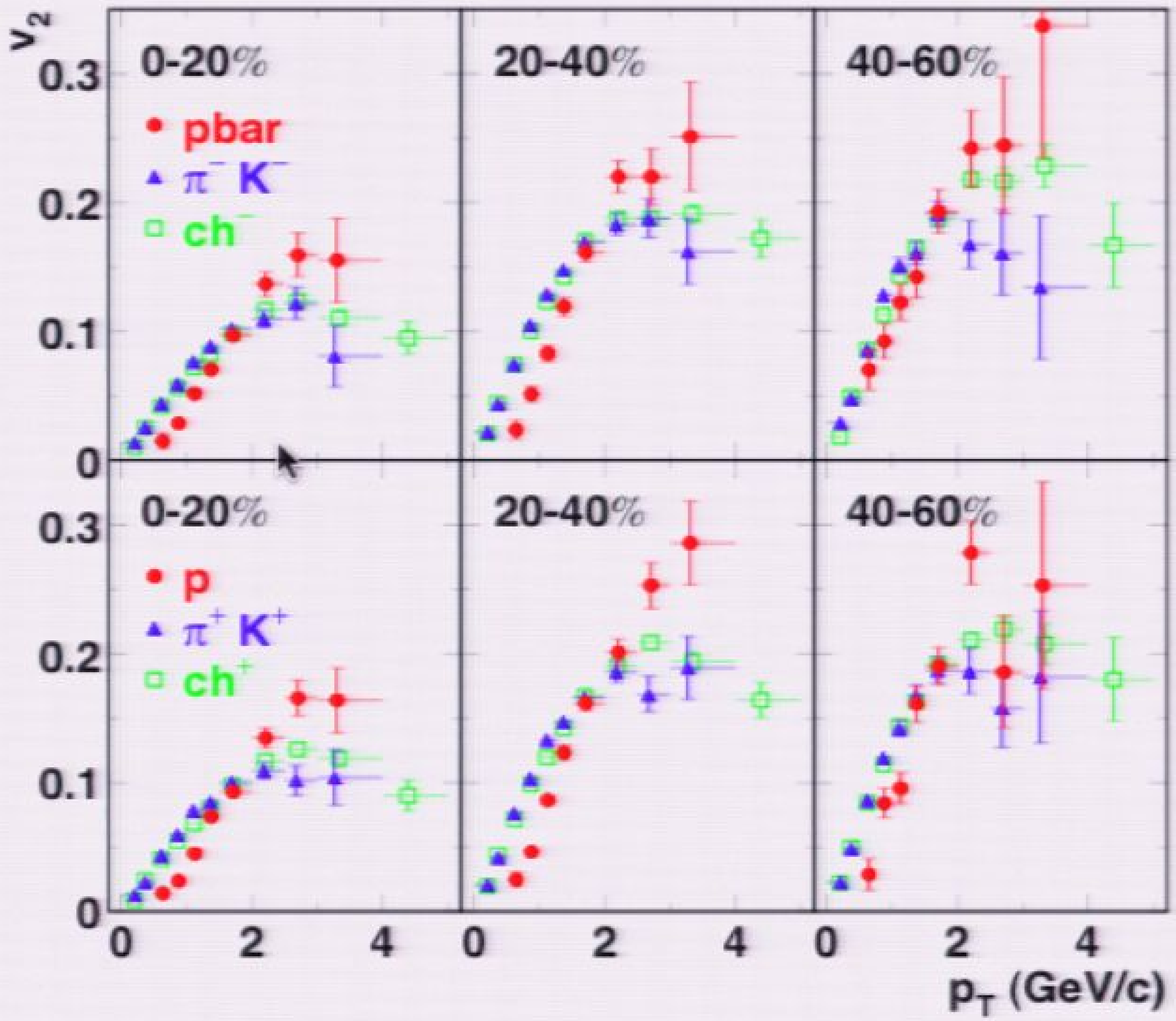
Date: Nov 03, 2009 11:00 AM

URL: http://pirsa.org/09110000

Abstract: Ultrarelativistic heavy-ion collisions are one of the most difficult problems for theoretical physicists: they probe non-abelian dynamics deep in the non-perturbative (strong coupling) regime in a many-body system, are highly dynamical (strong gradients), exhibit collective behavior, and involve phase transitions. Fluid dynamics with input from holography is surprisingly good at describing some aspects of experimental data in heavy-ion collisions. I will review some of this successful description, its limitations, and point out open problems that one may be able to understand using gauge/gravity duality.

Experimental Data

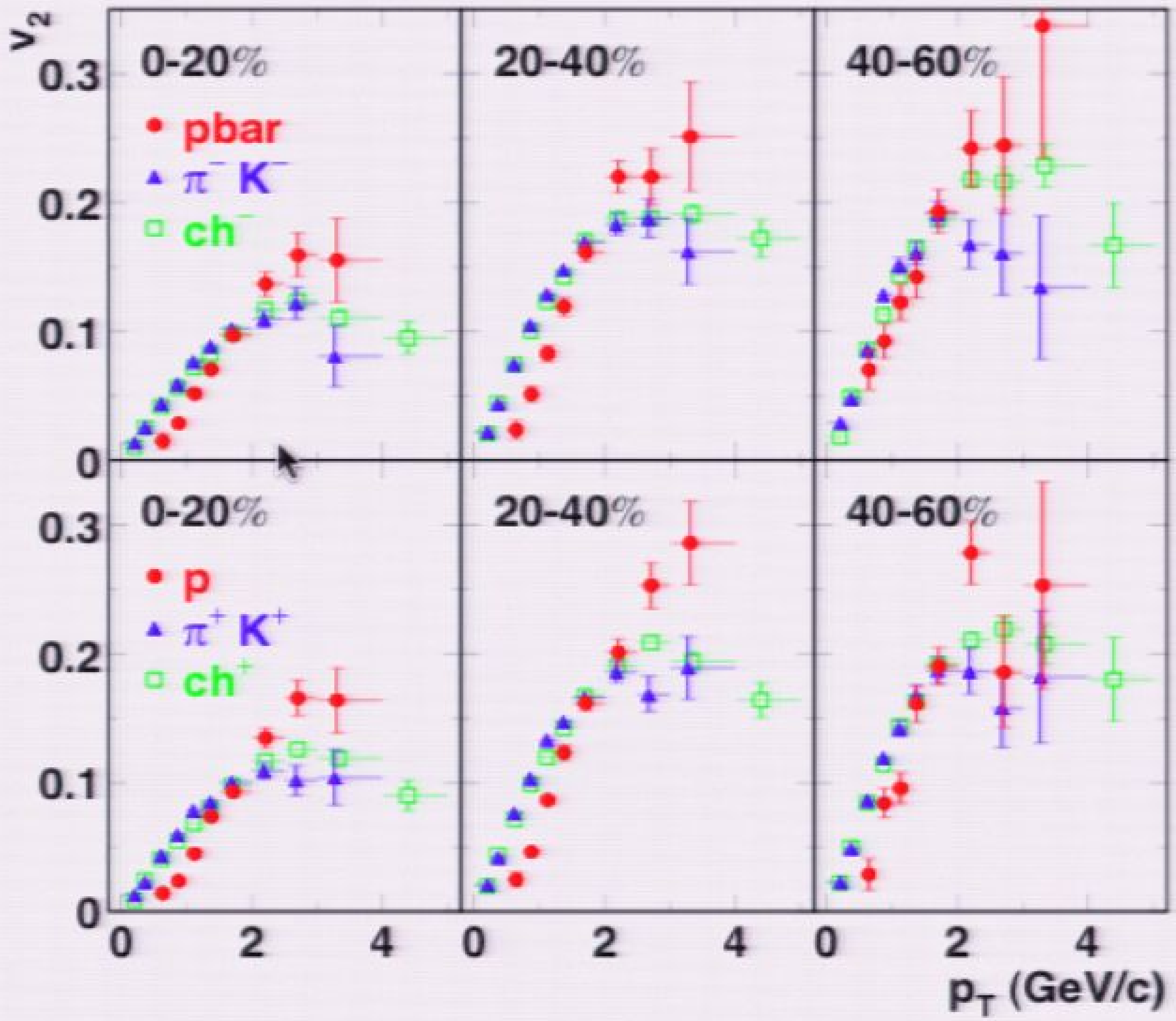
Elliptic Flow at RHIC $\sqrt{s} = 200$ GeV



[PHENIX, 2003]

Experimental Data

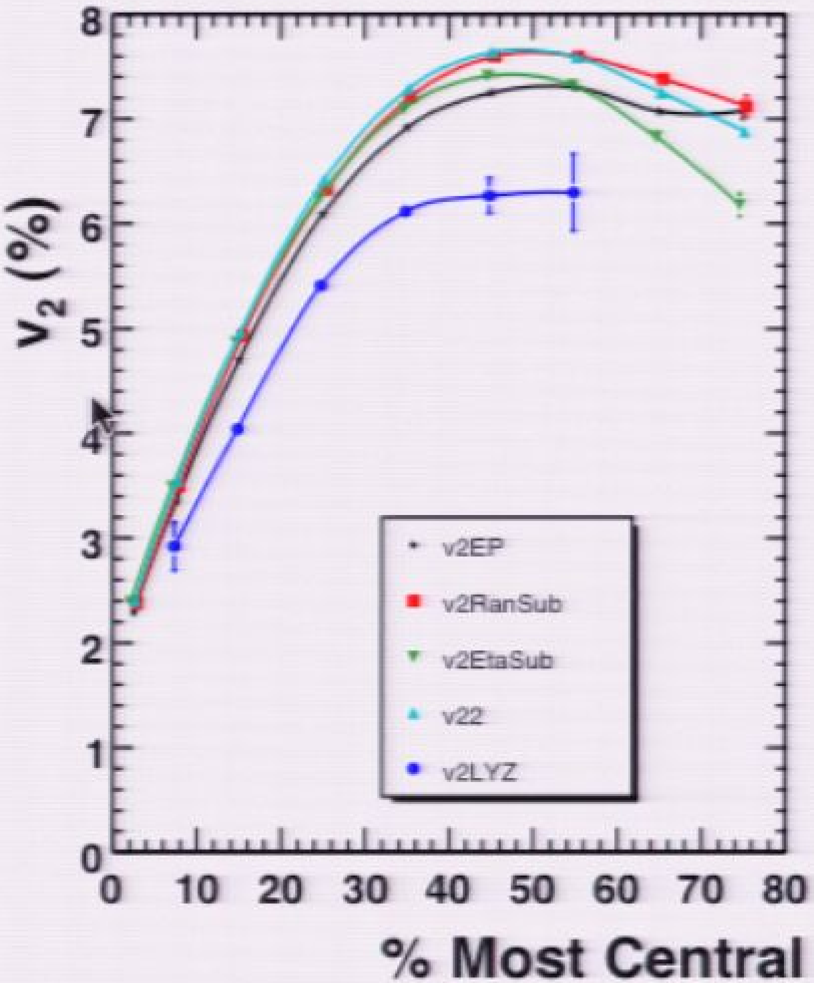
Elliptic Flow at RHIC $\sqrt{s} = 200$ GeV



[PHENIX, 2003]

Experimental Data

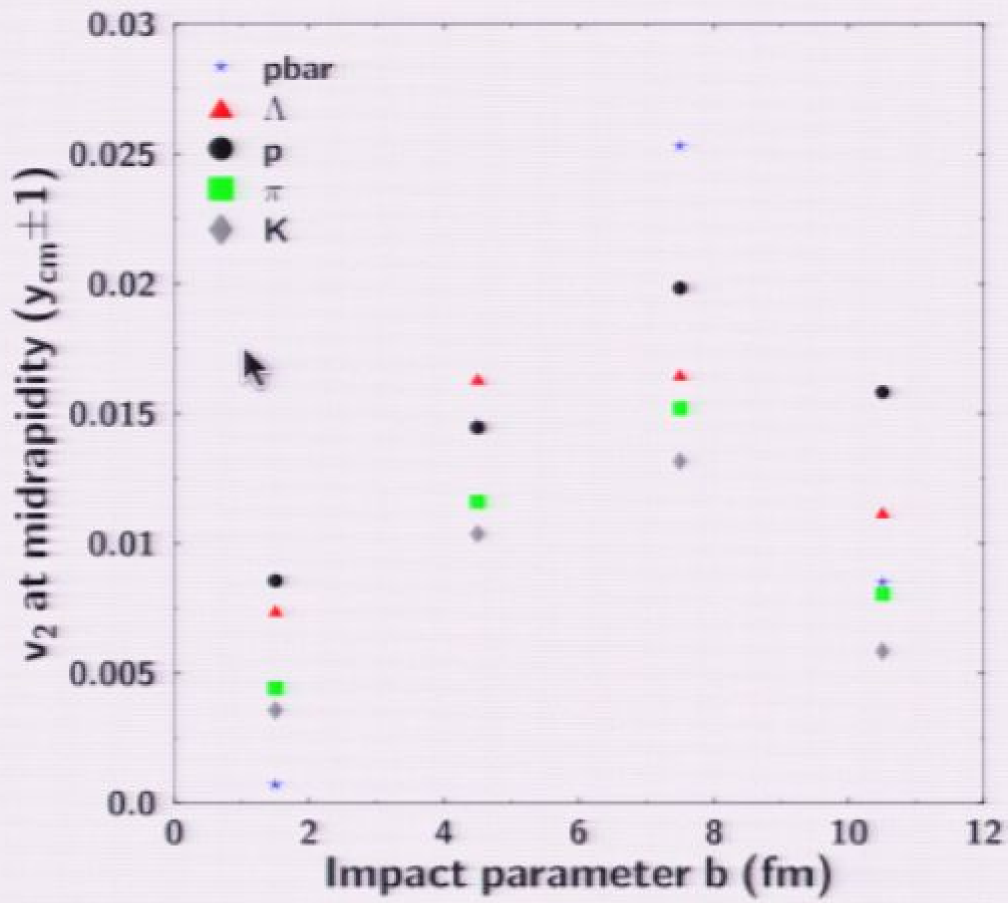
Elliptic Flow at RHIC: Not all measurements are equal



[Ollitrault, Poskanzer, Voloshin, 2009]

What kind of physics dominates at RHIC?

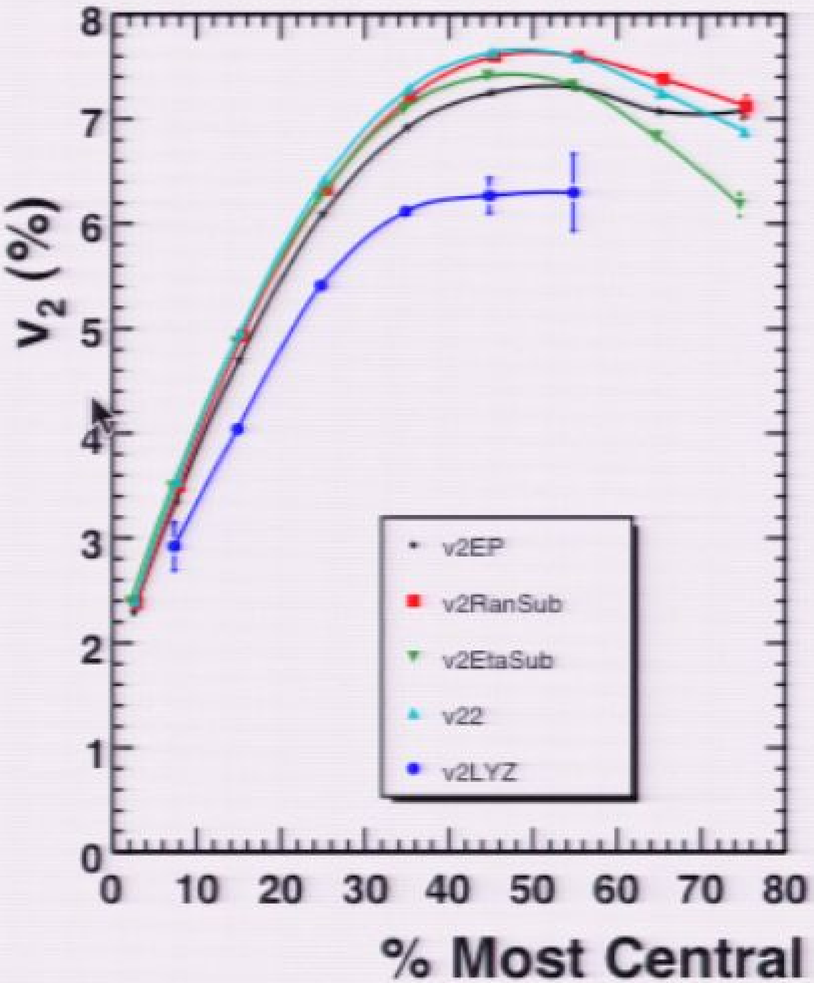
Is it Kinetic Theory?



[Bleicher & Stöcker, 2002]

Experimental Data

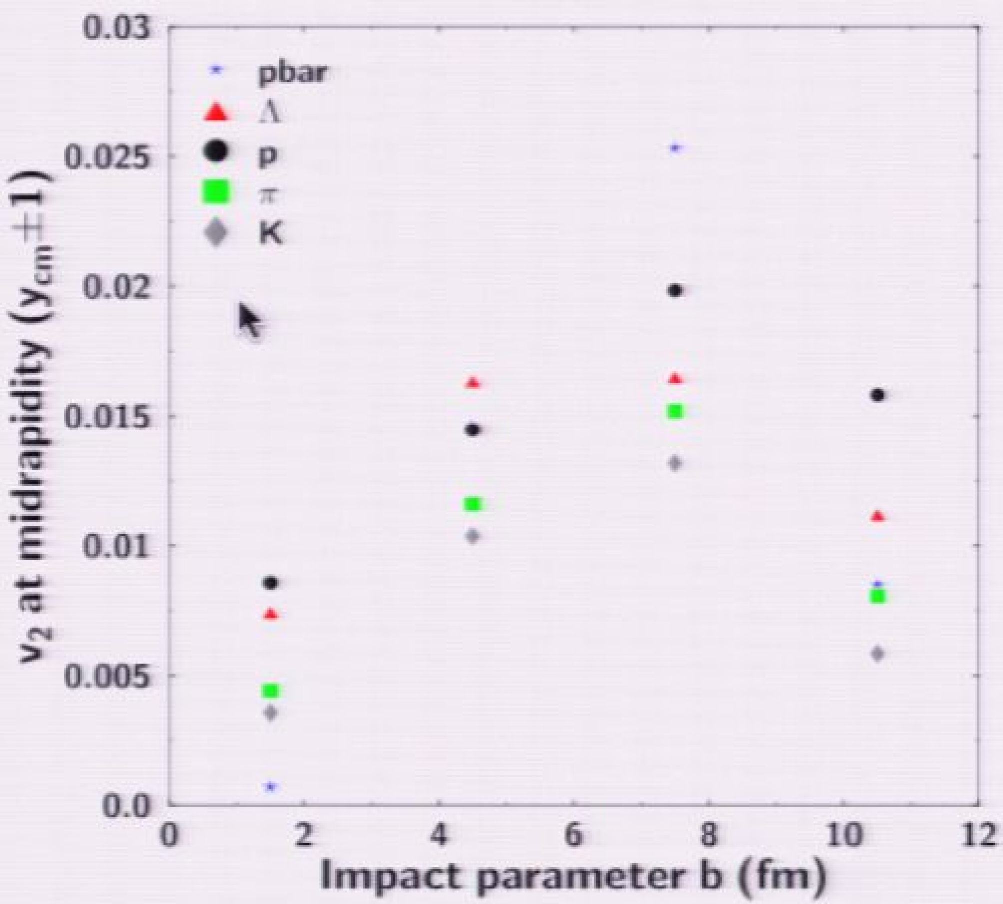
Elliptic Flow at RHIC: Not all measurements are equal



[Ollitrault, Poskanzer, Voloshin, 2009]

What kind of physics dominates at RHIC?

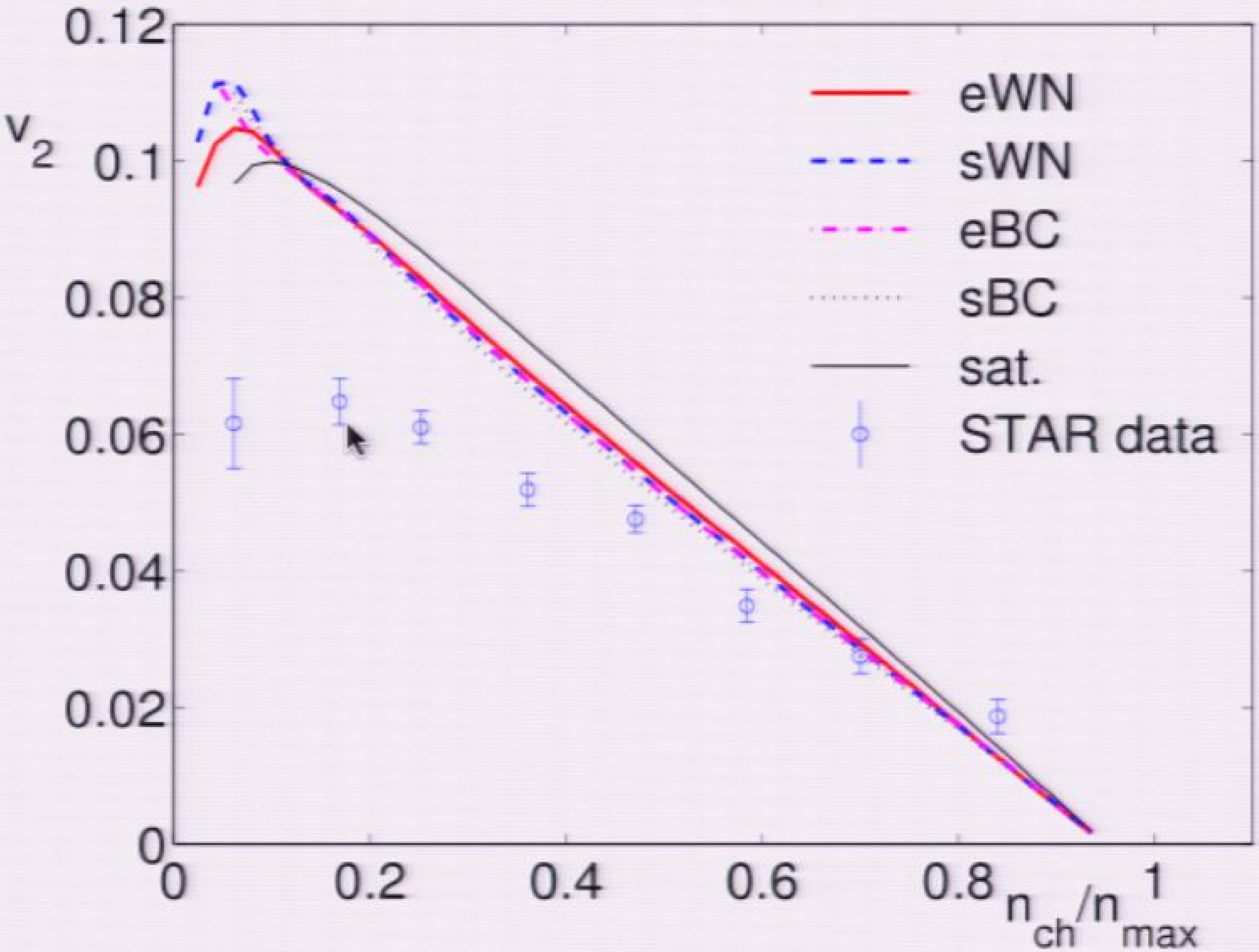
Is it Kinetic Theory? **No!**



[Bleicher & Stöcker, 2002]

What kind of physics dominates at RHIC?

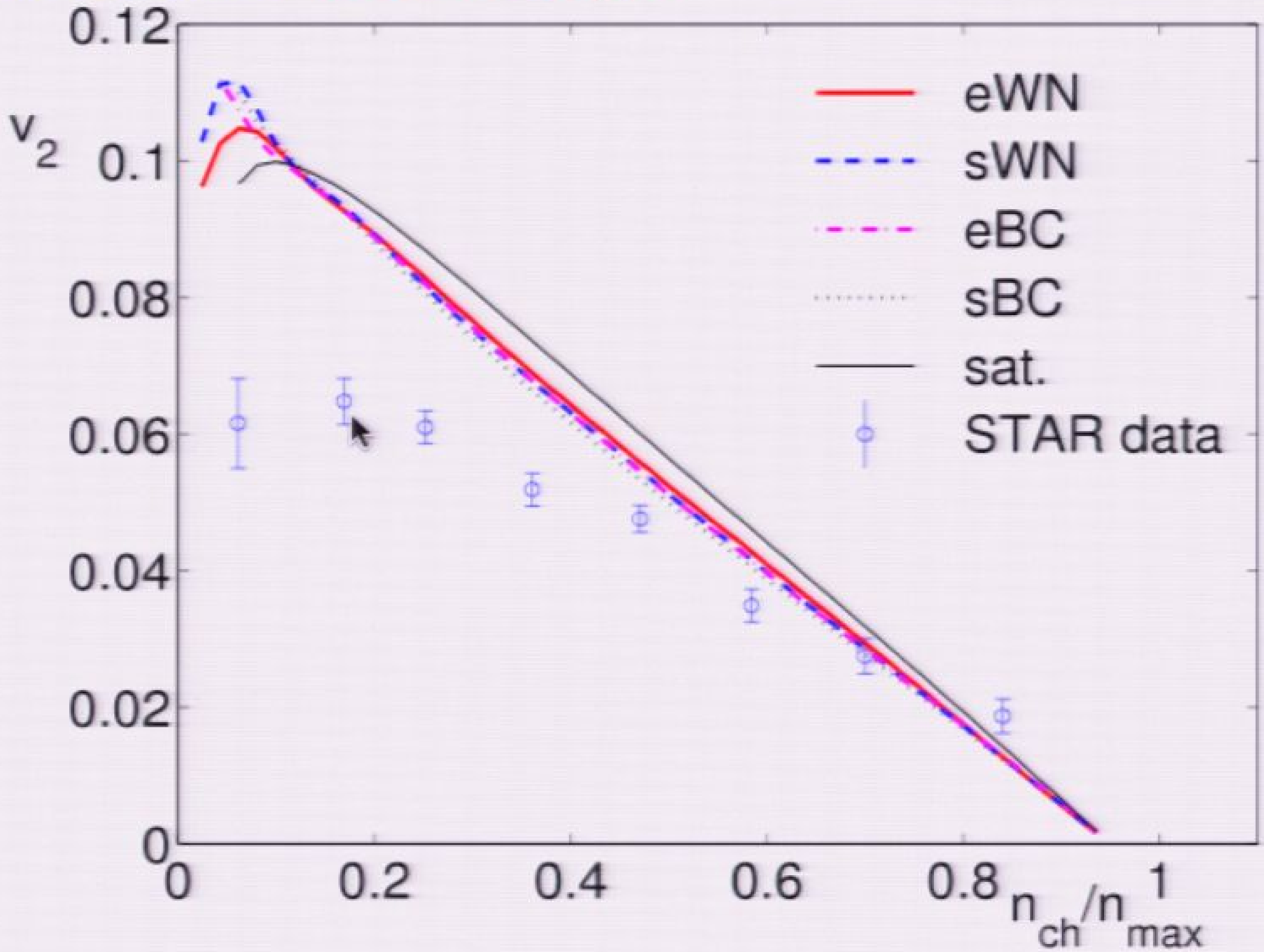
Is it Ideal Fluid Dynamics?



[P. Kolb et al., 2001]

What kind of physics dominates at RHIC?

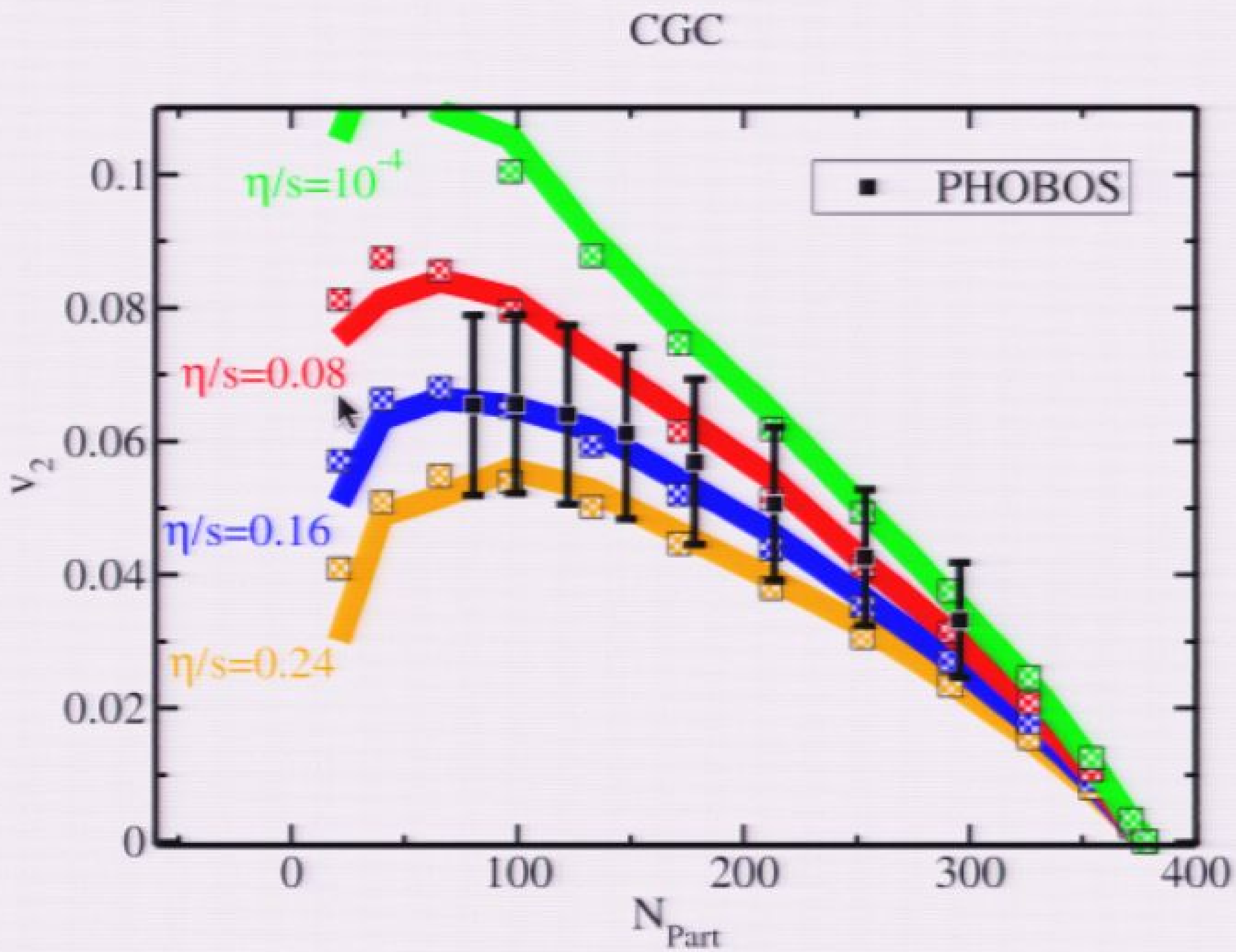
Is it Ideal Fluid Dynamics? **Not Really...**



[P. Kolb et al., 2001]

What kind of physics dominates at RHIC?

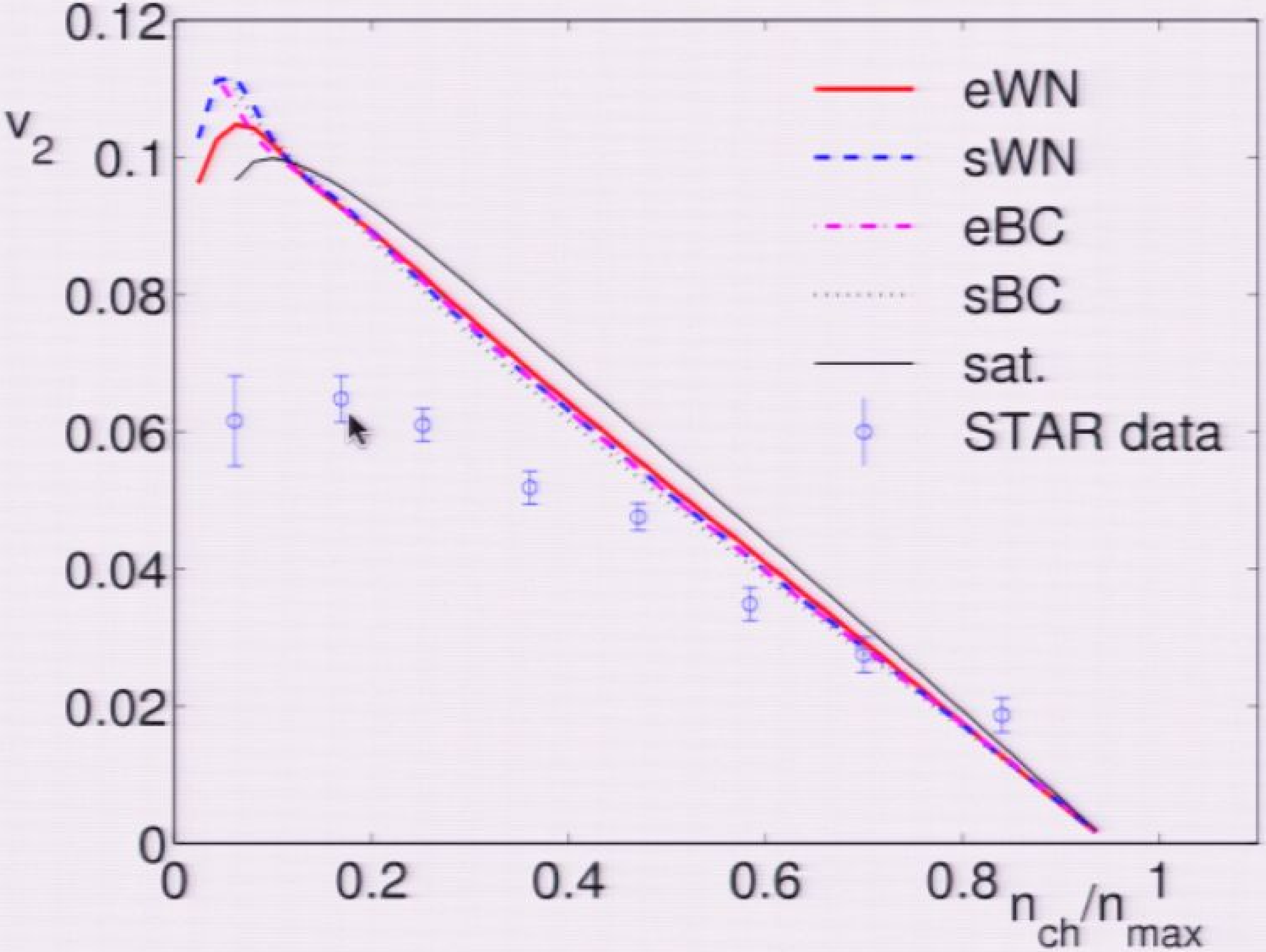
Something in between: Viscous Fluid Dynamics? Yes!



[M. Luzum & PR, 2008]

What kind of physics dominates at RHIC?

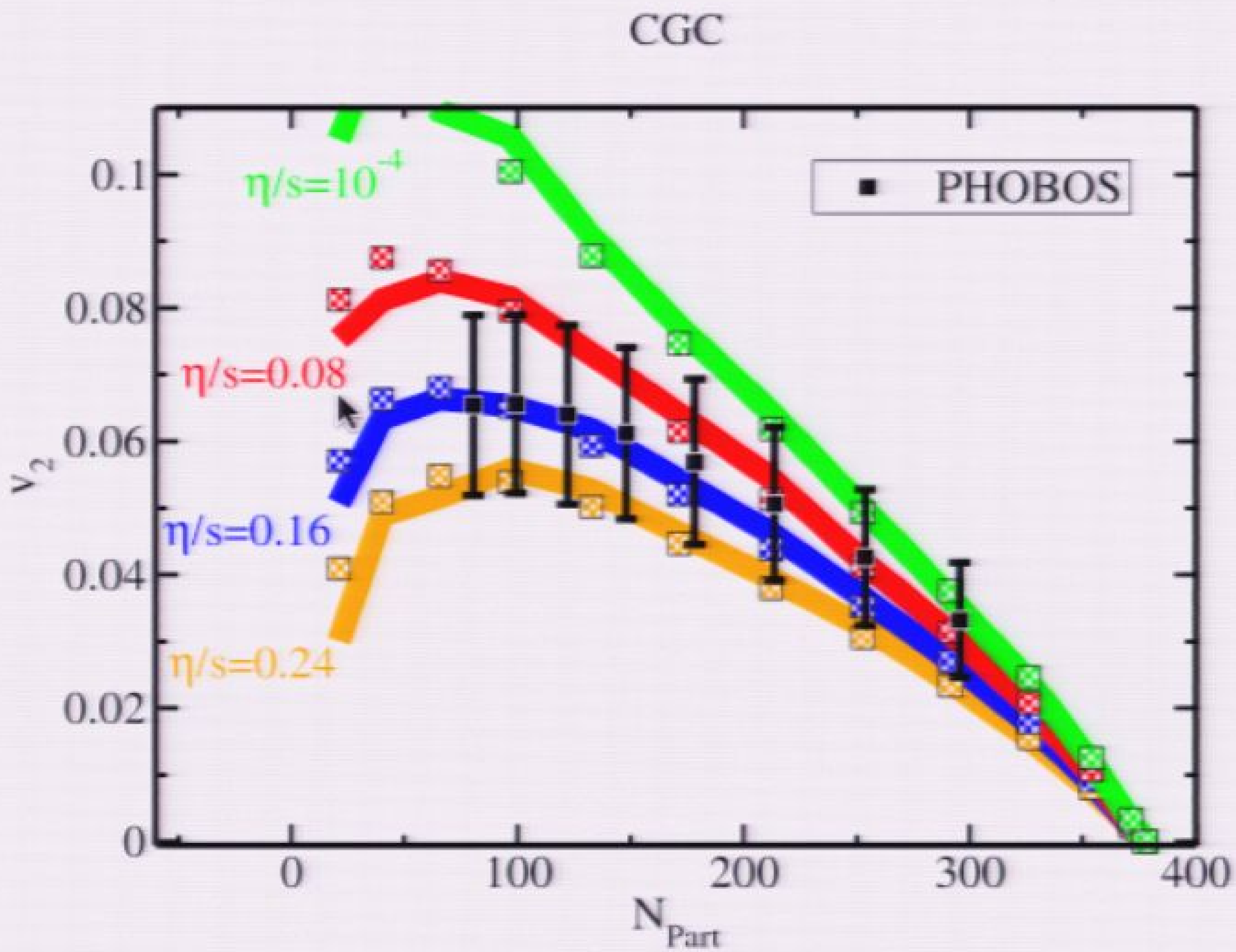
Is it Ideal Fluid Dynamics? Not Really...



[P. Kolb et al., 2001]

What kind of physics dominates at RHIC?

Something in between: Viscous Fluid Dynamics? Yes!



[M. Luzum & PR, 2008]

Connections to Holography

- Kinetic Theory for QCD ($\lambda \to 0$):

$$\frac{\eta}{s} \sim \mathcal{O}(1)$$

[Arnold, Moore, Yaffe, 2003]

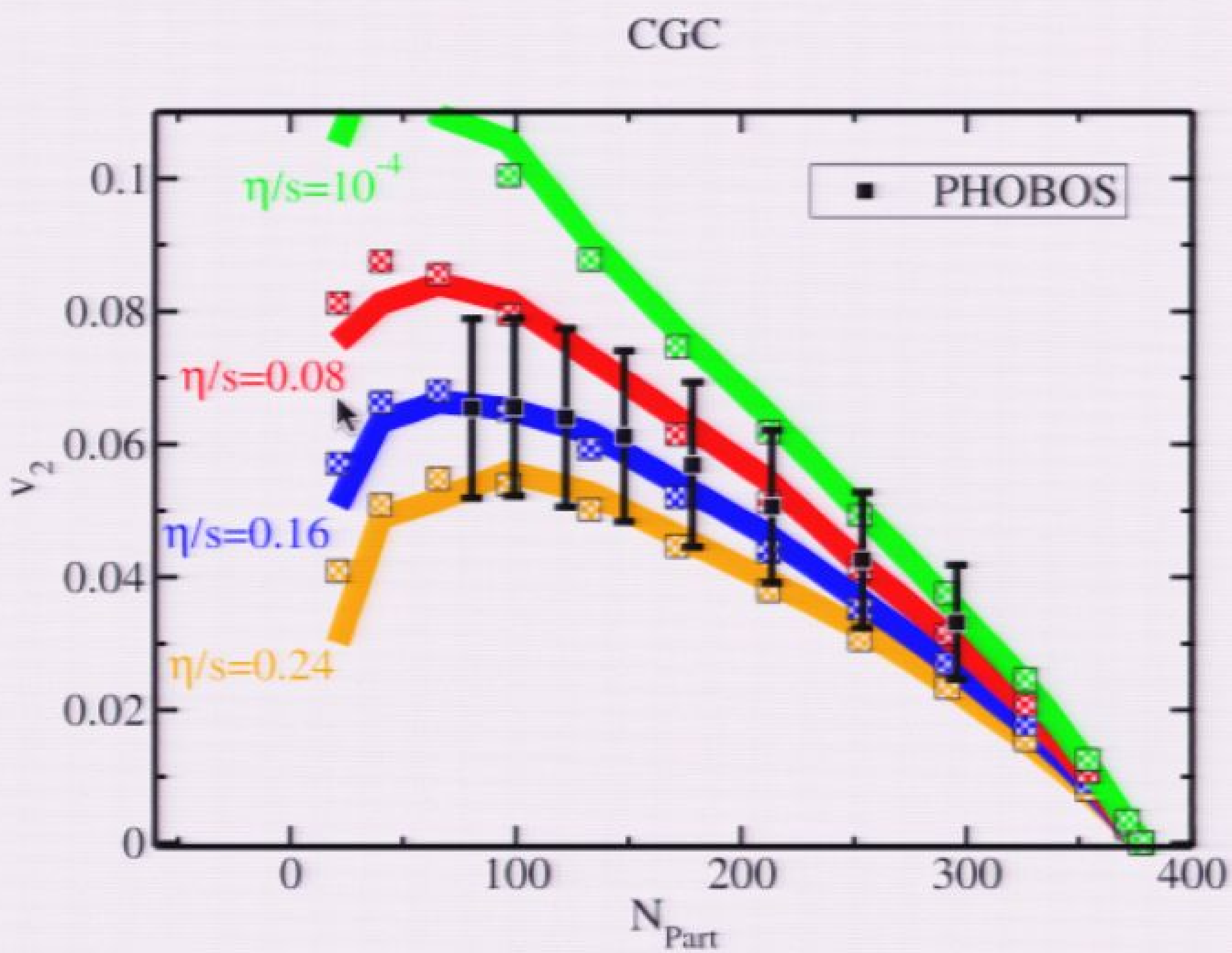
- Holography for $N_c \to \infty$ & $\lambda \to \infty$:

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

[Policastro, Son, Starinets, 2001]

What kind of physics dominates at RHIC?

Something in between: Viscous Fluid Dynamics? Yes!



[M. Luzum & PR, 2008]

Connections to Holography

- Kinetic Theory for QCD ($\lambda \to 0$):

$$\frac{\eta}{s} \sim \mathcal{O}(1)$$

[Arnold, Moore, Yaffe, 2003]

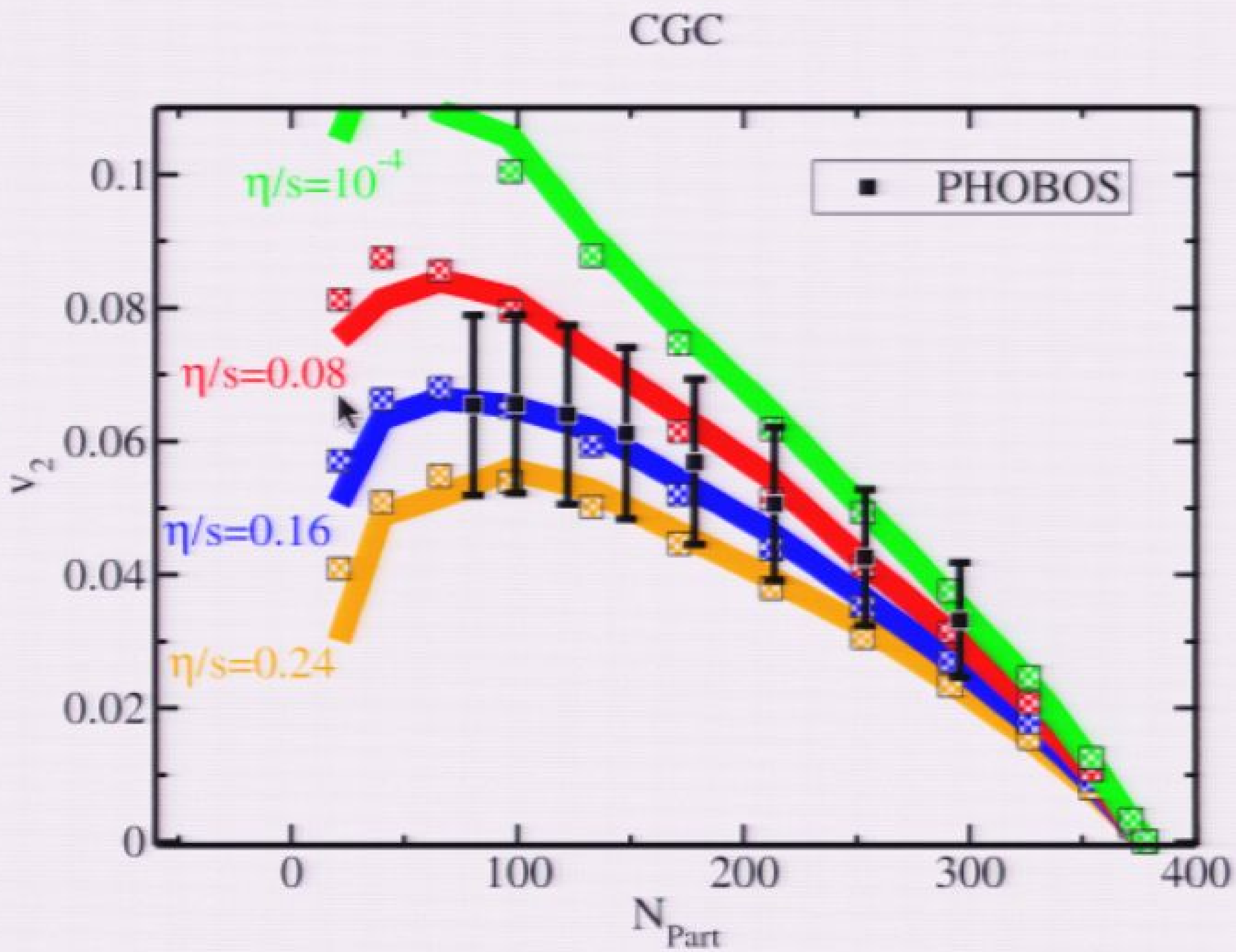
- Holography for $N_c \to \infty$ & $\lambda \to \infty$:

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

[Policastro, Son, Starinets, 2001]

What kind of physics dominates at RHIC?

Something in between: Viscous Fluid Dynamics? Yes!



[M. Luzum & PR, 2008]

Connections to Holography

- Kinetic Theory for QCD ($\lambda \to 0$):

$$\frac{\eta}{s} \sim \mathcal{O}(1)$$

[Arnold, Moore, Yaffe, 2003]

- Holography for $N_c \to \infty$ & $\lambda \to \infty$:

$$\frac{\eta}{s} = \frac{1}{4\pi} \sim 0.08$$

[Policastro, Son, Starinets, 2001]

Viscous Fluid Dynamics

Fluid Dynamics = Conservation of Energy+Momentum for long wavelength modes

$$T^{\mu\nu}_{id} = \epsilon u^\mu u^\nu - p(g^{\mu\nu} - u^\mu u^\nu)$$ (Fluid EMT, no gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

=

Ideal Fluid Dynamics

Non-relativistic Ideal Fluid Dynamics

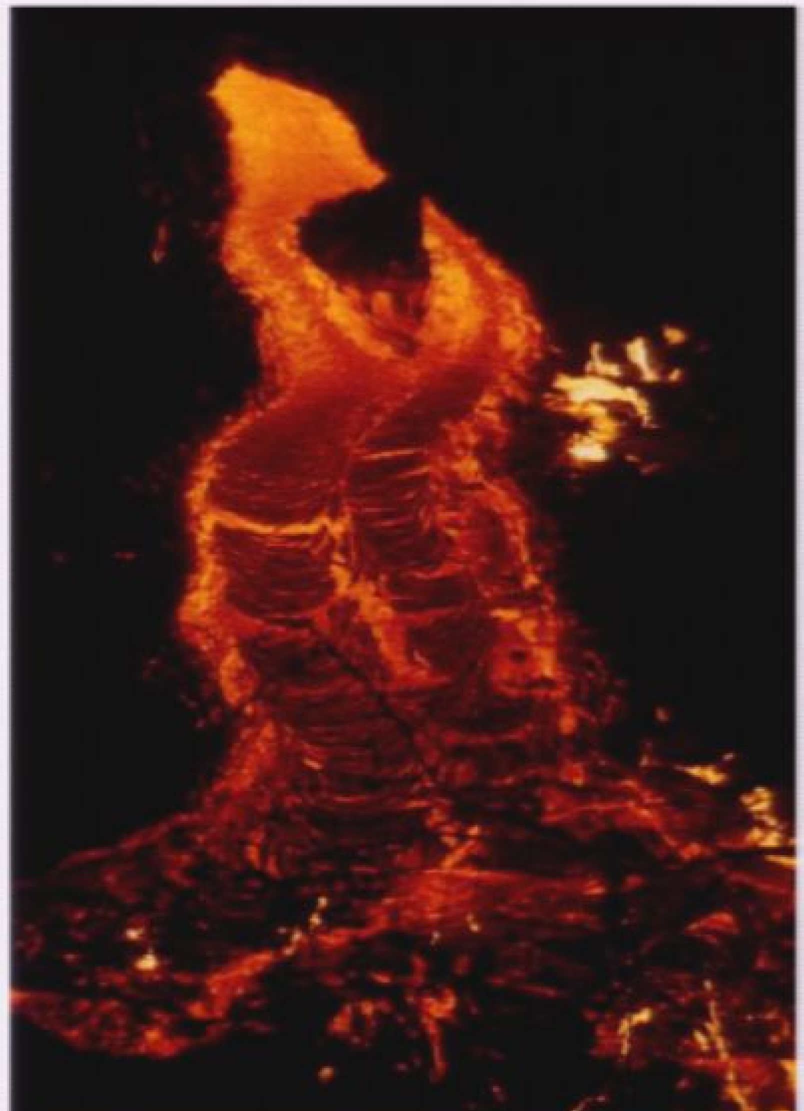
$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \delta^{ij} p$$

[L. Euler, 1755]

- Non-linear
- Non-dissipative: “Ideal Fluid Dynamics”

Non-linear & Non-dissipative: Turbulence

Non-linear & Dissipative: Laminar



Non-linear & Dissipative: Laminar

Viscosity dampens turbulent instability!

Relativistic Ideal Fluid Dynamics

$$T^{\mu\nu} = T^{\mu\nu}_{id}$$ (Fluid EMT, no gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

=

Ideal Fluid Dynamics

Relativistic Viscous Fluid Dynamics

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id}} + \Pi^{\mu\nu}$$ (Fluid EMT, 1^{st} o. gradients)

+

$$\partial_\mu T^{\mu\nu} = 0$$ ("EMT Conservation")

=

Relativistic Navier-Stokes Equation

Relativistic Viscous Fluid Dynamics

- L. Euler, 1755:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \delta^{ij} p$$

- C. Navier, 1822; G. Stokes 1845:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \left[\delta^{ij} p + \Pi^{ij}\right] ,$$

$$\Pi^{ij} = -\eta \left(\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i} - \frac{2}{3}\delta^{ij}\frac{\partial v^l}{\partial x^l} \right) - \zeta \delta^{ij} \frac{\partial v^l}{\partial x^l} ,$$

- $\eta, \zeta \ldots$ transport coefficients (“viscosities”)

Relativistic Navier-Stokes Equation

- Good enough for non-relativistic systems
- NOT good enough for relativistic systems

Relativistic Viscous Fluid Dynamics

- L. Euler, 1755:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \delta^{ij} p$$

- C. Navier, 1822; G. Stokes 1845:

$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \left[\delta^{ij} p + \Pi^{ij}\right] ,$$

$$\Pi^{ij} = -\eta \left(\frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i} - \frac{2}{3}\delta^{ij}\frac{\partial v^l}{\partial x^l} \right) - \zeta \delta^{ij} \frac{\partial v^l}{\partial x^l} ,$$

- $\eta, \zeta \ldots$ transport coefficients ("viscosities")

Relativistic Navier-Stokes Equation

- Good enough for non-relativistic systems
- NOT good enough for relativistic systems

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \; (k \gg 1)$$

- Know how to regulate: "second-order" theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Second Order Fluid Dynamics

- Limiting speed is finite

$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon+p)} + \frac{\zeta}{\tau_\Pi(\epsilon+p)}}$$

[PR, 2009]

- $\tau_\pi, \tau_\Pi \ldots$...: "2$^{\rm nd}$ order" regulators for "1$^{\rm st}$ order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \ (k \gg 1)$$

- Know how to regulate: "second-order" theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \ (k \gg 1)$$

- Know how to regulate: "second-order" theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Second Order Fluid Dynamics

- Limiting speed is finite

$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon + p)} + \frac{\zeta}{\tau_\Pi(\epsilon + p)}}$$

[PR, 2009]

- $\tau_\pi, \tau_\Pi \ldots$...: "$2^{nd}$ order" regulators for "1^{st} order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \; (k \gg 1)$$

- Know how to regulate: “second-order” theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Second Order Fluid Dynamics

- Limiting speed is finite

$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon+p)} + \frac{\zeta}{\tau_\Pi(\epsilon+p)}}$$

[PR, 2009]

- $\tau_\pi, \tau_\Pi \ldots\ldots$: "2nd order" regulators for "1st order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Navier-Stokes: Problems with Causality

Consider small perturbations around equilibrium

- Transverse velocity perturbations obey

$$\partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

- Diffusion speed of wavemode k:

$$v_T(k) = 2k \frac{\eta}{\epsilon + p} \to \infty \ (k \gg 1)$$

- Know how to regulate: "second-order" theories:

$$\tau_\pi \partial_t^2 \delta u^y + \partial_t \delta u^y - \frac{\eta}{\epsilon + p} \partial_x^2 \delta u^y = 0$$

[Maxwell (1867), Cattaneo (1948)]

Second Order Fluid Dynamics

- Limiting speed is finite

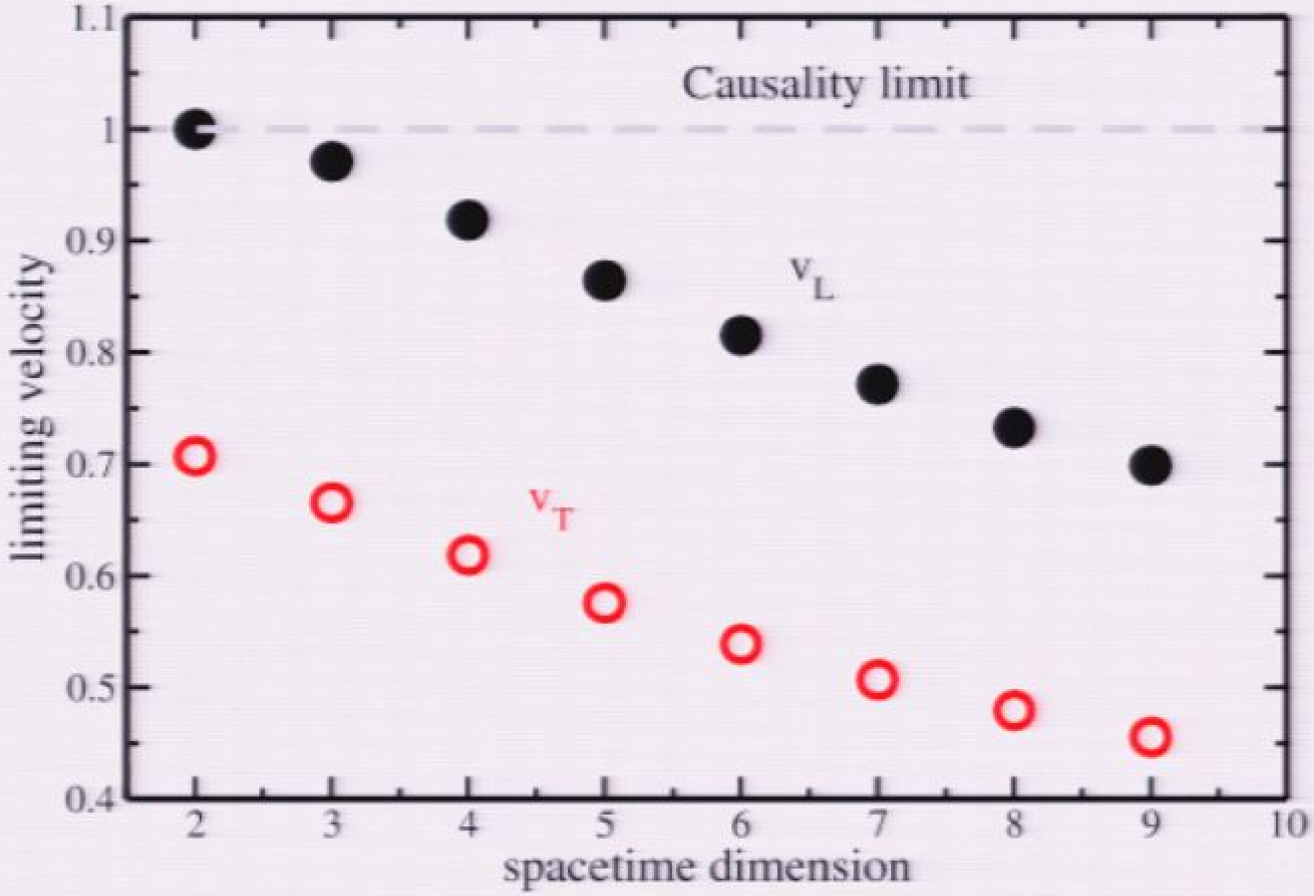
$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon+p)} + \frac{\zeta}{\tau_\Pi(\epsilon+p)}}$$

[PR, 2009]

- $\tau_\pi, \tau_\Pi \ldots$...: "2nd order" regulators for "1st order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Holography: Results for limiting velocities

$\lambda \to \infty$, AdS_{D+1}



[PR, 2009; based on Bhattacharyya et al., 2008]

Second Order Fluid Dynamics

- Limiting speed is finite

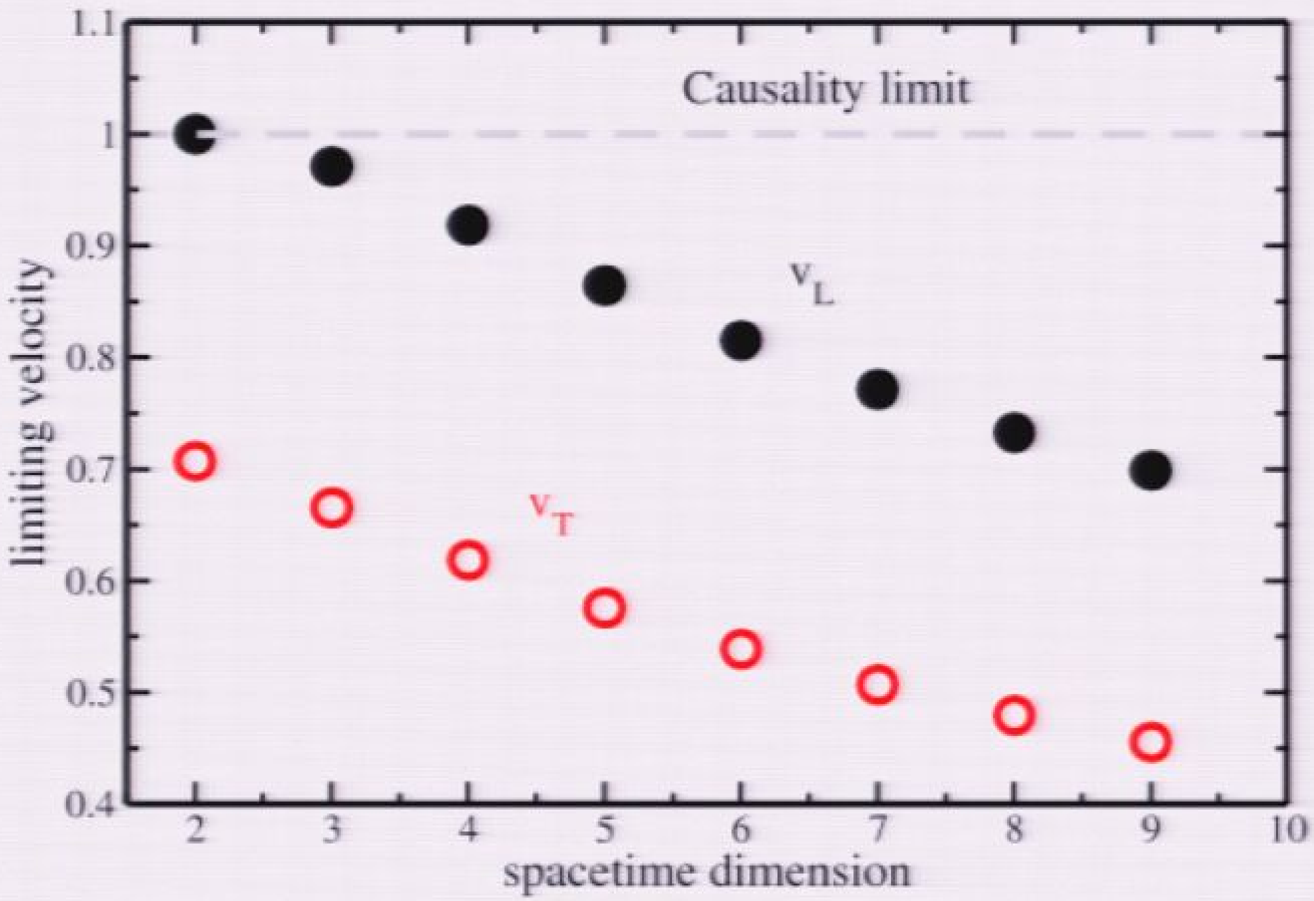
$$\lim_{k\to\infty} v_L(k) = \sqrt{c_s^2 + \frac{4\eta}{3\tau_\pi(\epsilon+p)} + \frac{\zeta}{\tau_\Pi(\epsilon+p)}}$$

[PR, 2009]

- $\tau_\pi, \tau_\Pi \ldots\ldots$: "2$^{\text{nd}}$ order" regulators for "1$^{\text{st}}$ order" fluid dynamics
- Regulators acts in UV, low momentum (fluid dynamics) regime is still Navier-Stokes

Holography: Results for limiting velocities

$\lambda \to \infty$, AdS_{D+1}



[PR, 2009; based on Bhattacharyya et al., 2008]

Holography: Results for limiting velocities

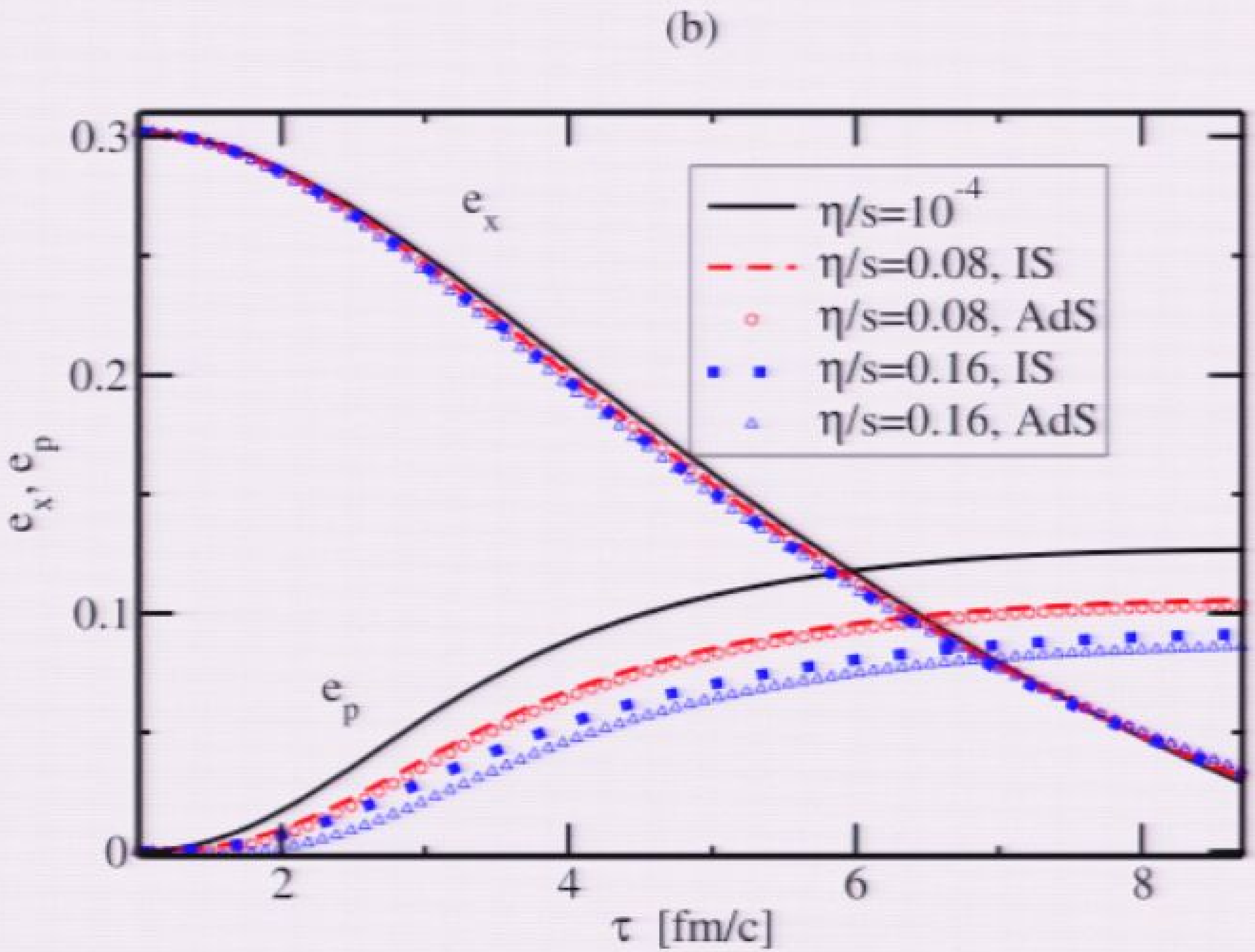
- Shear sector τ_π: $v_{T,L}$ decrease with decreasing λ

  [A. Buchel & M. Paulos, 2009]
- $v_{T,L} < 1$ as long as QFT is causal in GB gravity

  [A. Buchel & R.C. Myers, 2009]

τ_π from experiment

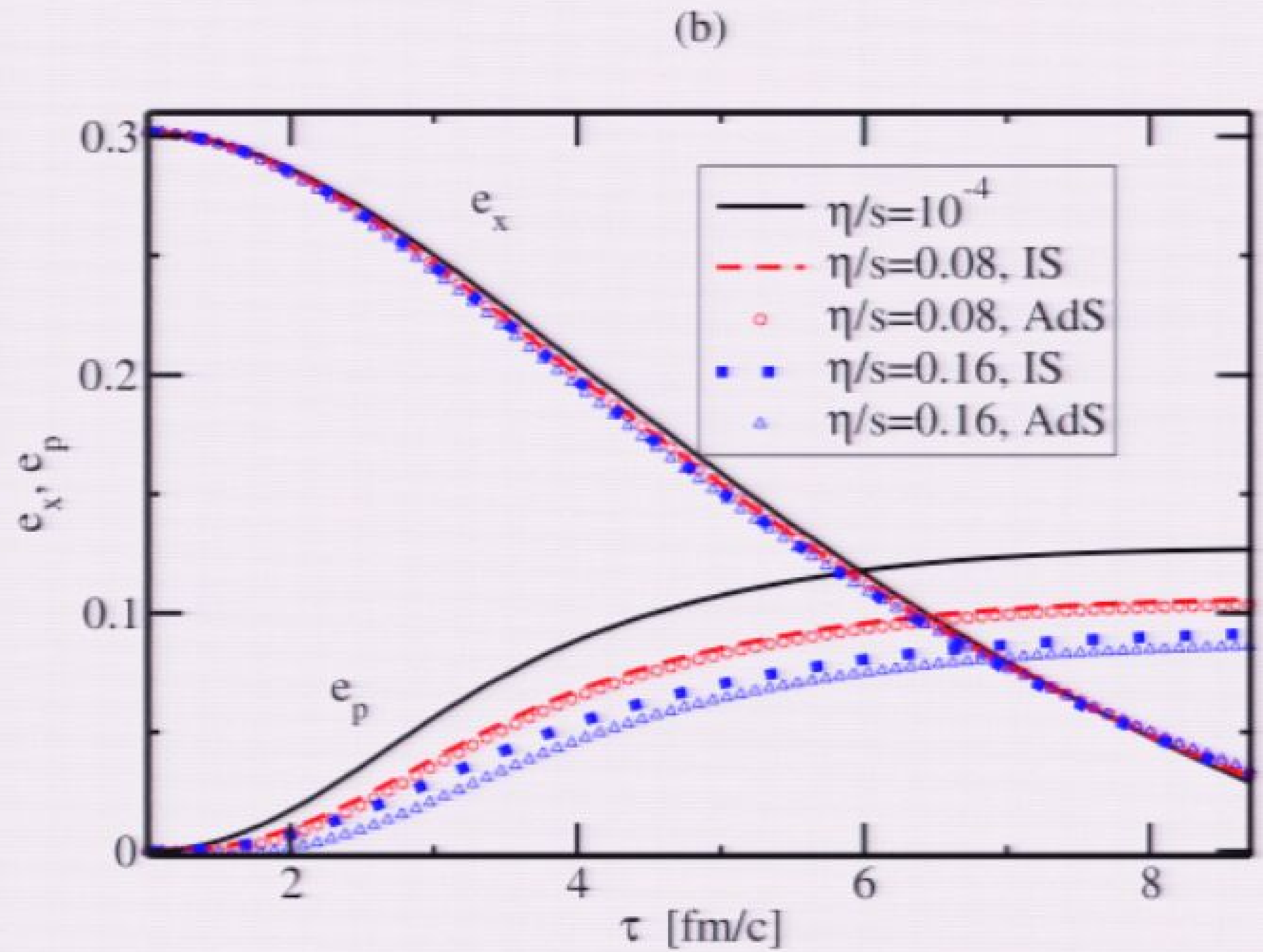
Is it possible to constrain value of τ_π?



[M. Luzum & PR, 2008]

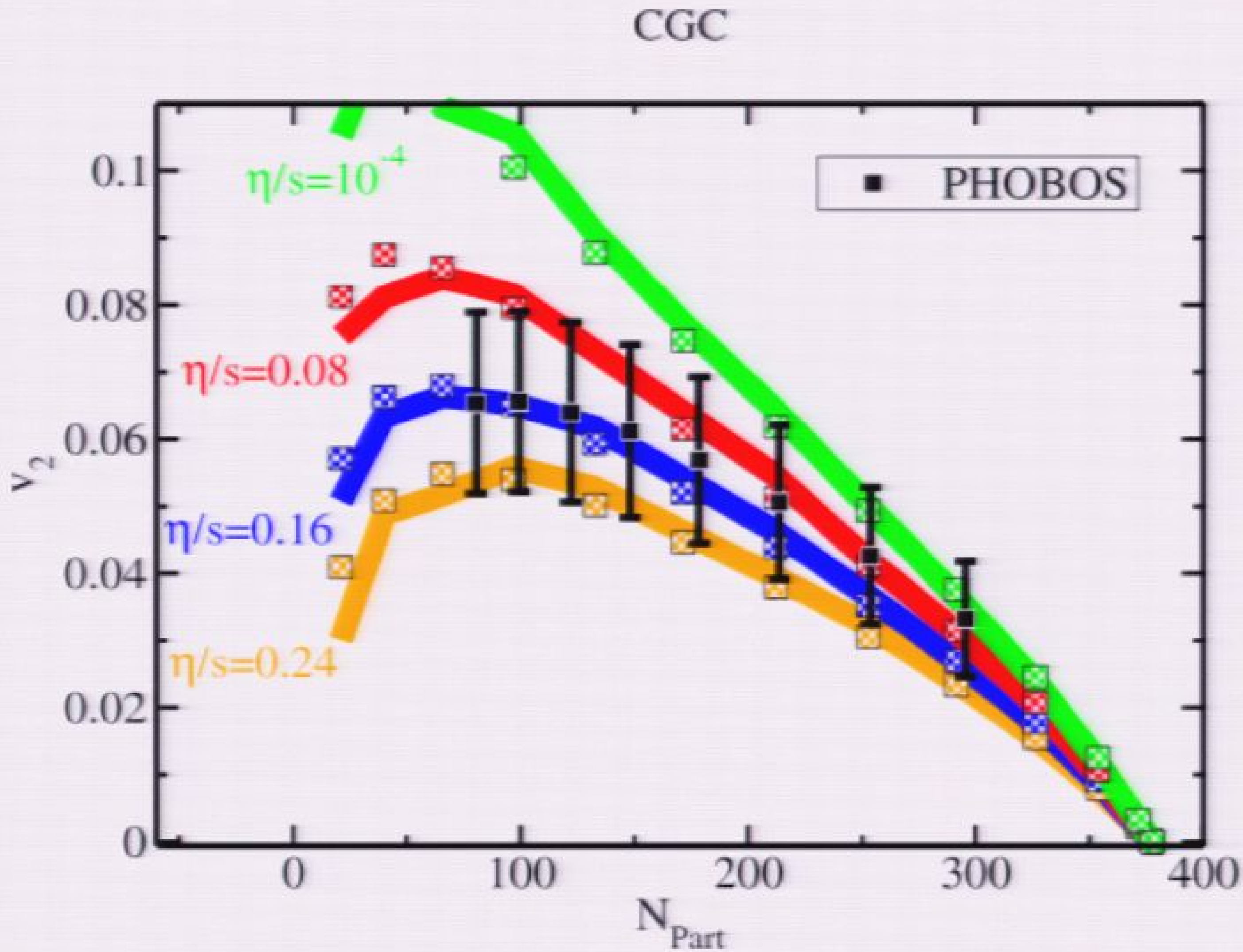
τ_π from experiment

Is it possible to constrain value of τ_π? Not Really...



[M. Luzum & PR, 2008]

Good prospects for extracting η/s!



[M. Luzum & PR, 2008]

Open Problems

The effect from bulk viscosity

- Viscous Stress Tensor

$$\Pi^{\mu\nu} = -\eta\nabla^{<\mu}u^{\nu>} - \zeta\nabla\cdot u + \mathcal{O}(\nabla^2)$$

- Full second-order structure known

[PR, 2009]

- ζ/s known in wQCD and for some gauge-gravity duals

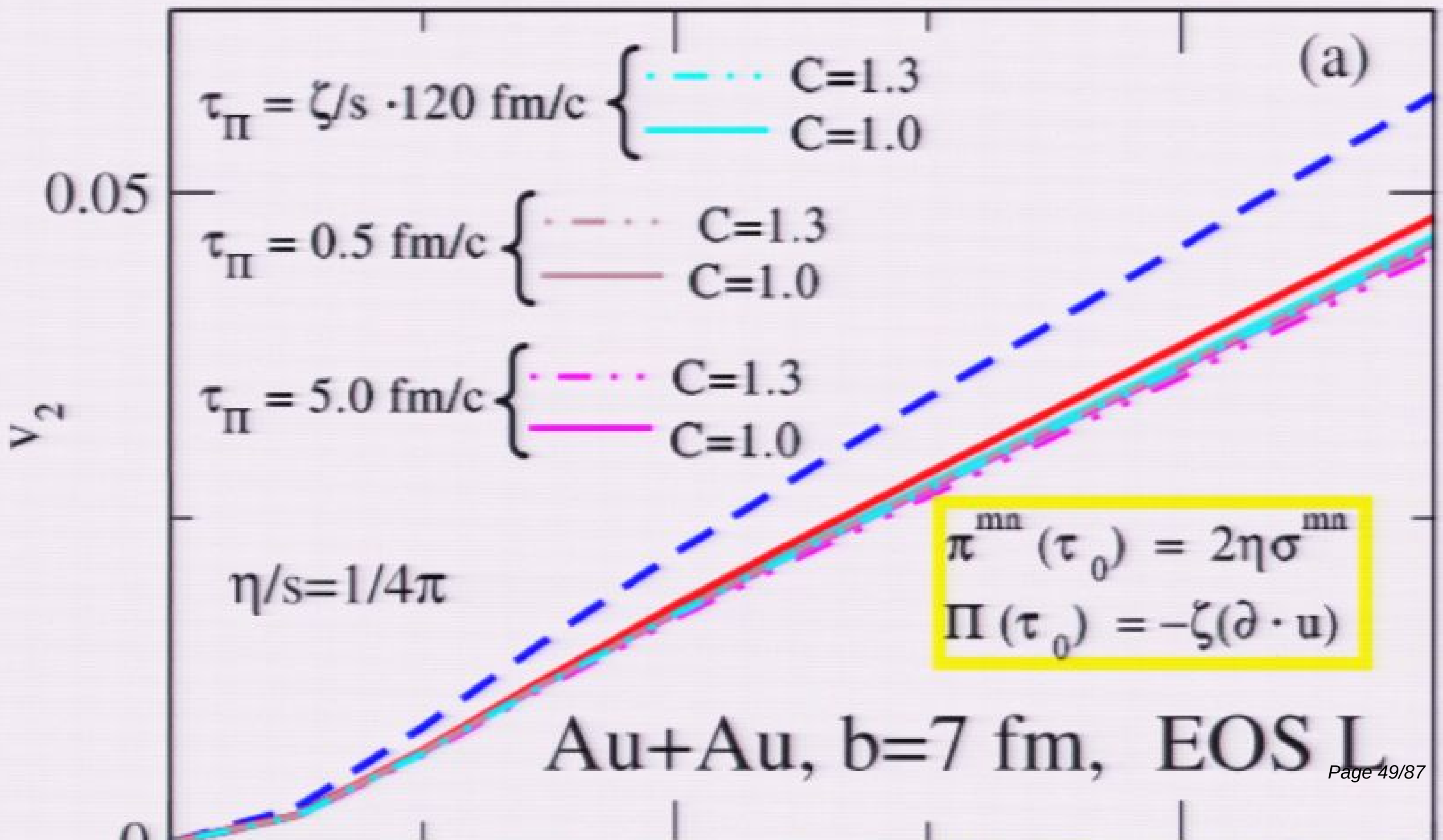
[Arnold, Dogan & Moore, 2006; Buchel et al., 2007,2008]

- Value of some 2nd order coefficients known

[Betz, Henkel & Rischke, 2009; Buchel, 2009]

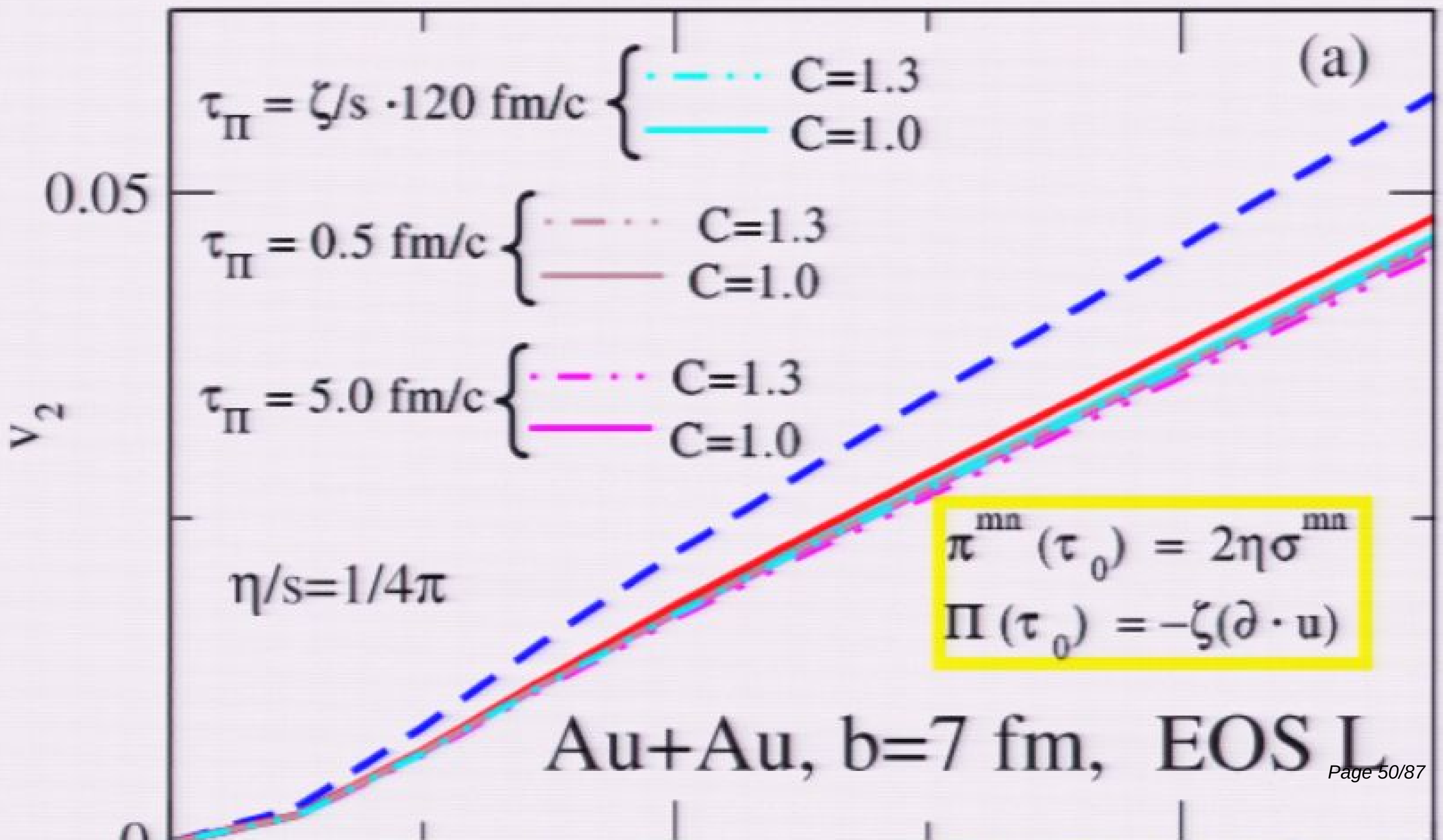
Effects of Bulk viscosity

[H. Song and U. Heinz, 2009]



Effects of Bulk viscosity: Negligible...

[H. Song and U. Heinz, 2009]



Cavitation

- Viscous Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu + (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

where

$$\Pi = -\zeta \nabla \cdot u + \mathcal{O}(\nabla^2) \qquad \pi^{\mu\nu} = -\eta \nabla^{<\mu} u^{\nu>} + \mathcal{O}(\nabla^2)$$

- If $\zeta/P \gg 1/T$ then

$$P + \Pi < 0$$

- "Negative Effective Pressure"
- Cavitation = Formation of Vacuum Bubbles

[see e.g. K. Rajagopal & N. Tripuraneni, 2009]

Cavitation

- Viscous Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^{\mu}u^{\nu} + (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

where

$$\Pi = -\zeta\nabla\cdot u + \mathcal{O}(\nabla^2) \qquad \pi^{\mu\nu} = -\eta\nabla^{<\mu}u^{\nu>} + \mathcal{O}(\nabla^2)$$

- If $\zeta/P \gg 1/T$ then

$$P + \Pi < 0$$

- “Negative Effective Pressure”
- Cavitation = Formation of Vacuum Bubbles

[see e.g. K. Rajagopal & N. Tripuraneni, 2009]

Cavitation

- Viscous Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu + (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

where

$$\Pi = -\zeta \nabla \cdot u + \mathcal{O}(\nabla^2) \qquad \pi^{\mu\nu} = -\eta \nabla^{<\mu} u^{\nu>} + \mathcal{O}(\nabla^2)$$

- If $\zeta/P \gg 1/T$ then

$$P + \Pi < 0$$

- "Negative Effective Pressure"
- Cavitation = Formation of Vacuum Bubbles

[see e.g. K. Rajagopal & N. Tripuraneni, 2009]

Cavitation

Does Cavitation Happen in QCD plasmas?

If yes, what are it's effects?

Cavitation

Does Cavitation Happen in QCD plasmas?

If yes, what are it's effects?

Cavitation in a different context

Cosmologists call

$$P + \Pi < 0$$

"Inflation"

Science Project Proposal

- Take holographic dual to theory with $\zeta \neq 0$ (e.g. $\mathcal{N} = 2^*$)
- Consider isotropically expanding boundary space (e.g. FRW)

[c.f. Kajantie, Louko, Tahkokallio, 2008]

- Study regime where fluid dynamics would predict cavitation/inflation & compare to full QFT result

Equilibration

Fluid dynamics seems to describe RHIC data

We do not know why fluid dynamics works

Equilibration

Fluid dynamics seems to describe RHIC data

We do not know why fluid dynamics works

Equilibration

- Energy Momentum Tensor of a Fluid = Gradient Expansion of Quantum Field Theory

$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} + \Pi^{\mu\nu}$$

- Fluid Dynamics works well if gradients are small
- If Gradients are small, T^{ij} should nearly isotropic in l.r.f

$$T^{ij} = P\delta^{ij} + \mathcal{O}(\nabla), \qquad \text{for } u^{\mu} = (1, \mathbf{0})$$

Equilibration

- Energy Momentum Tensor of a Fluid = Gradient Expansion of Quantum Field Theory

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \Pi^{\mu\nu}$$

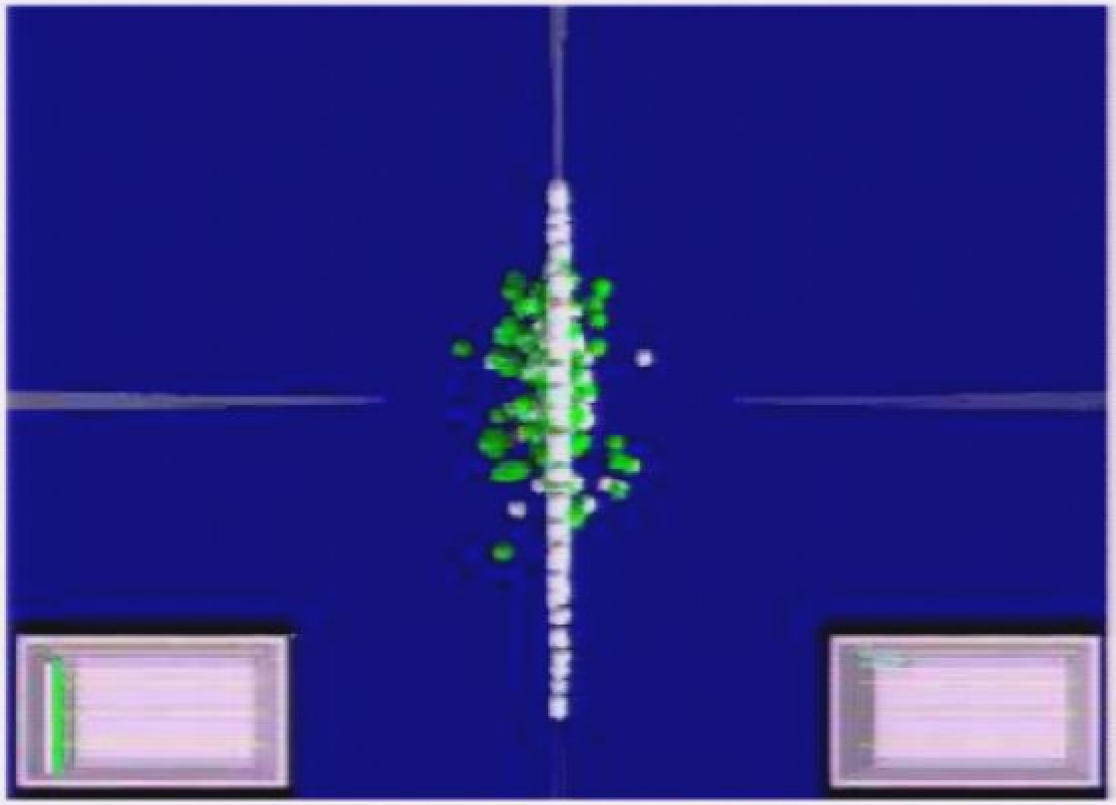
- Fluid Dynamics works well if gradients are small
- If Gradients are small, T^{ij} should nearly isotropic in l.r.f

$$T^{ij} = P\delta^{ij} + \mathcal{O}(\nabla), \qquad \text{for } u^\mu = (1, \mathbf{0})$$

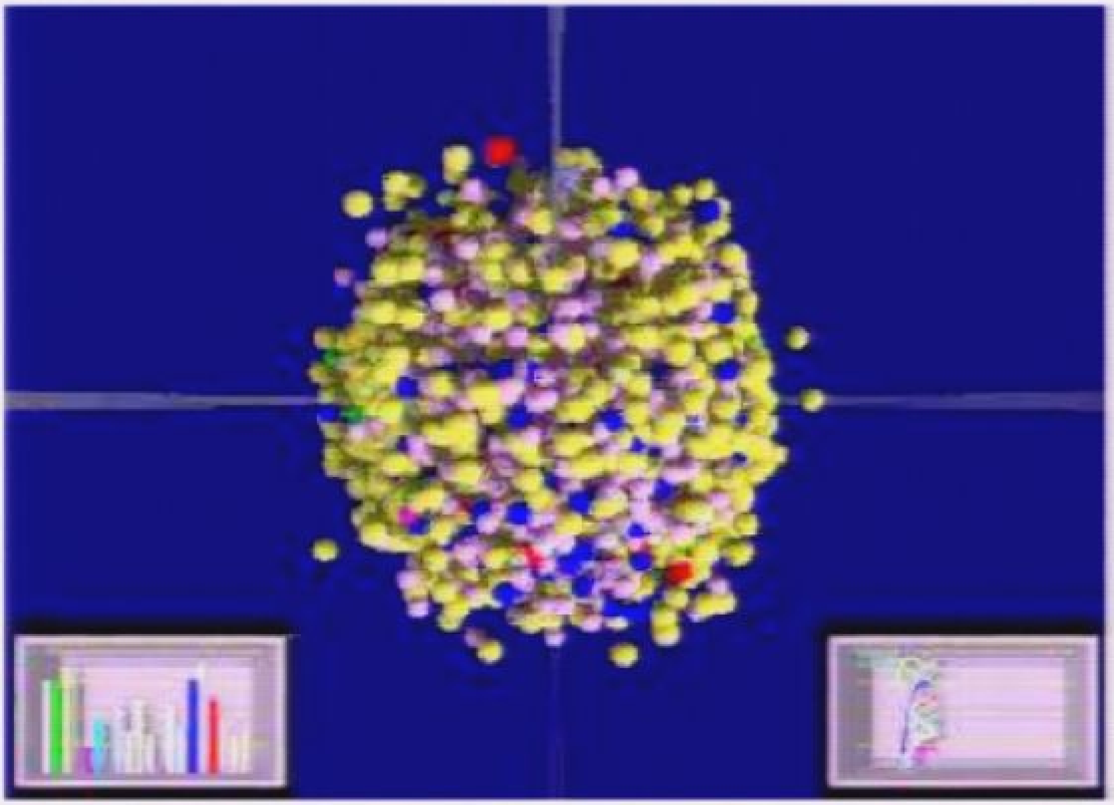
Fluid Dynamics

If we understand how T^{ij} becomes isotropic, we understand why fluid dynamics works at RHIC

Au+Au Collisions at RHIC



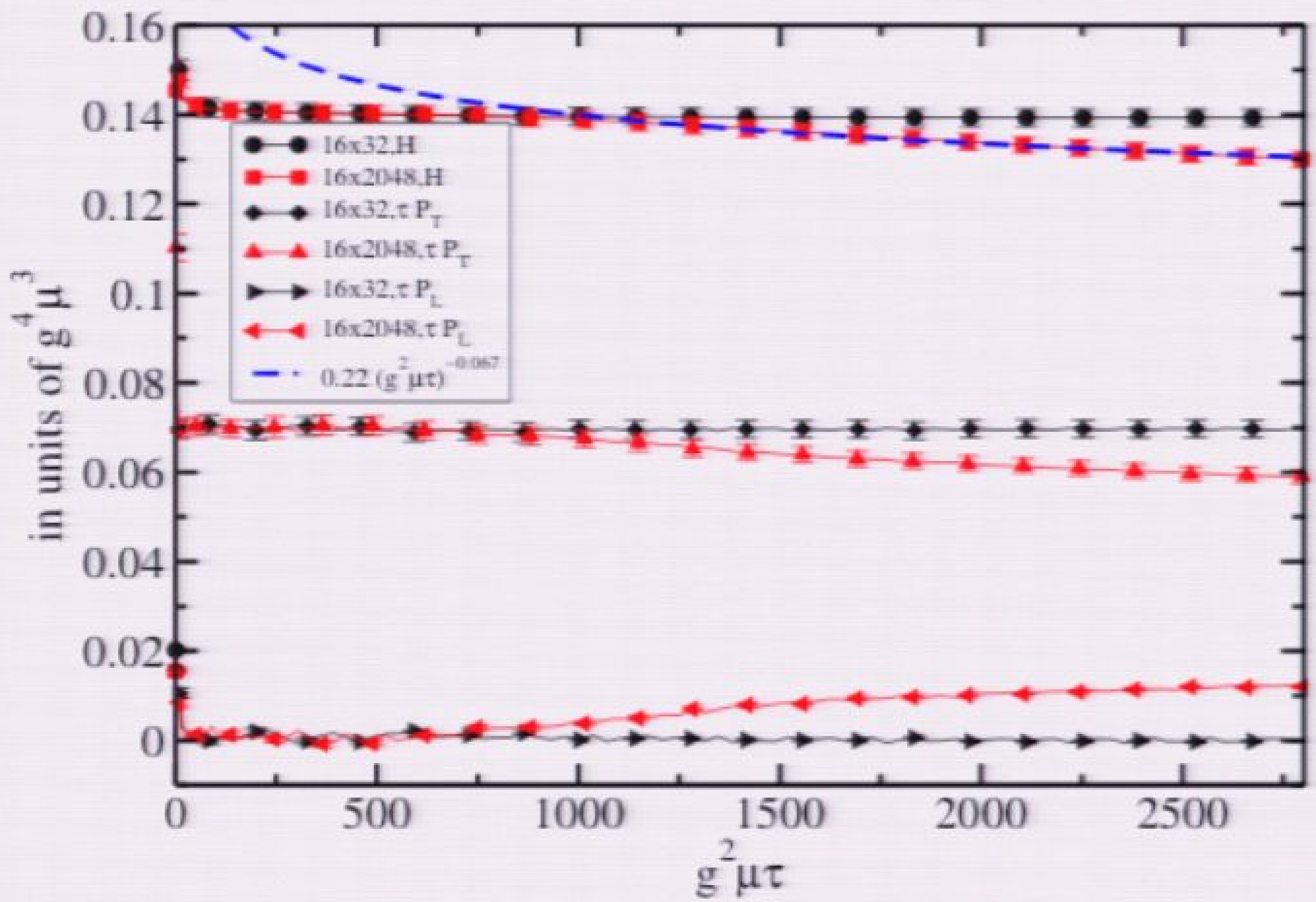
Au+Au Collisions at RHIC



Weak Coupling

Equilibration at Weak Coupling

Isotropization in Expanding CGC Medium (3+1d)



Classical Yang-Mills simulations

[PR and R. Venugopalan 2006]

Weak Coupling

Equilibration at Weak Coupling: Incomplete!?

Strong Coupling

Equilibration at Strong Coupling

Energy-Momentum Tensor of a Boosted Charge

- Charge at rest

$$E^i = -\frac{ex^i}{|\mathbf{x}|^3}$$

- Associated Energy-Momentum Tensor

$$T^{00} = \frac{1}{2}E^2 = \frac{e^2}{2|\mathbf{x}|^4}$$

A Gravity Dual to Heavy-Ion Collisions

- EMT of boosted charge:

$$T^{++} \propto e^2 \delta(x^+)$$

- Line element for gravity dual:

$$ds^2 = \frac{-dt^2 + d\mathbf{x}^2 + dz^2 + z^4 T^{\mu\nu} dx_\mu dx_\nu + \mathcal{O}(z^6)}{z^2}$$

- Aichelburg-Sexl shock waves,

$$ds^2 = -2dx^+ dx^- + d\mathbf{x}_\perp{}^2 - 4\mu \ln(\mathbf{x}_\perp{}^2)\delta(x^+) dx^{+2}$$

[P. Aichelburg and R. Sexl, 1971]

Gravity Duals for Heavy-Ion Collisions

Collisions of Shock Waves in $AdS_5 \leftrightarrow$ Heavy-Ion Collisions

[H. Nastase, 2004; 2006]

Equilibration in QFT $\leftrightarrow$ Black Hole formation in AdS

Gravity Duals for Heavy-Ion Collisions

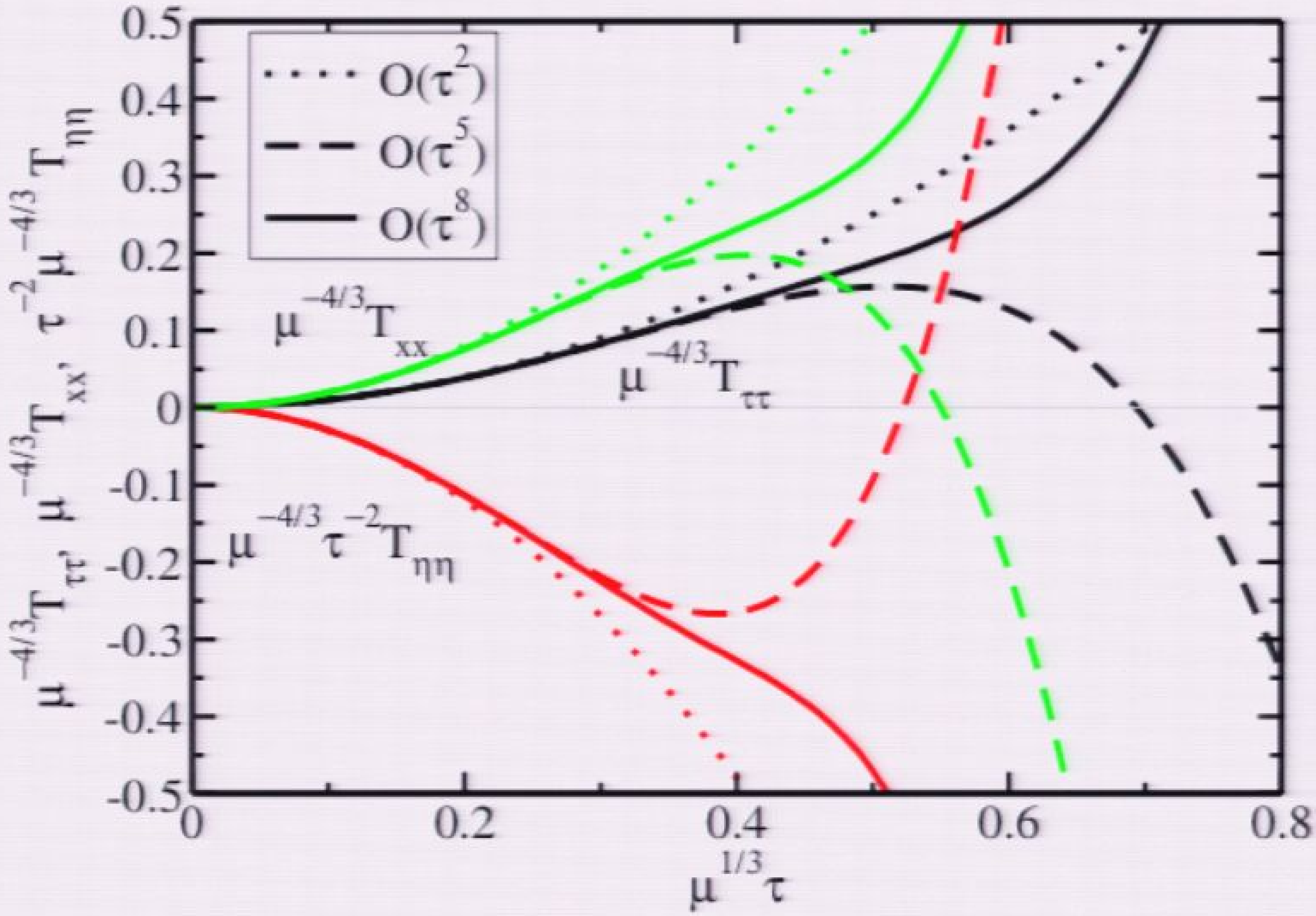
Collisions of Shock Waves in $AdS_5 \leftrightarrow$ Heavy-Ion Collisions

[H. Nastase, 2004; 2006]

Equilibration in QFT $\leftrightarrow$ Black Hole formation in AdS

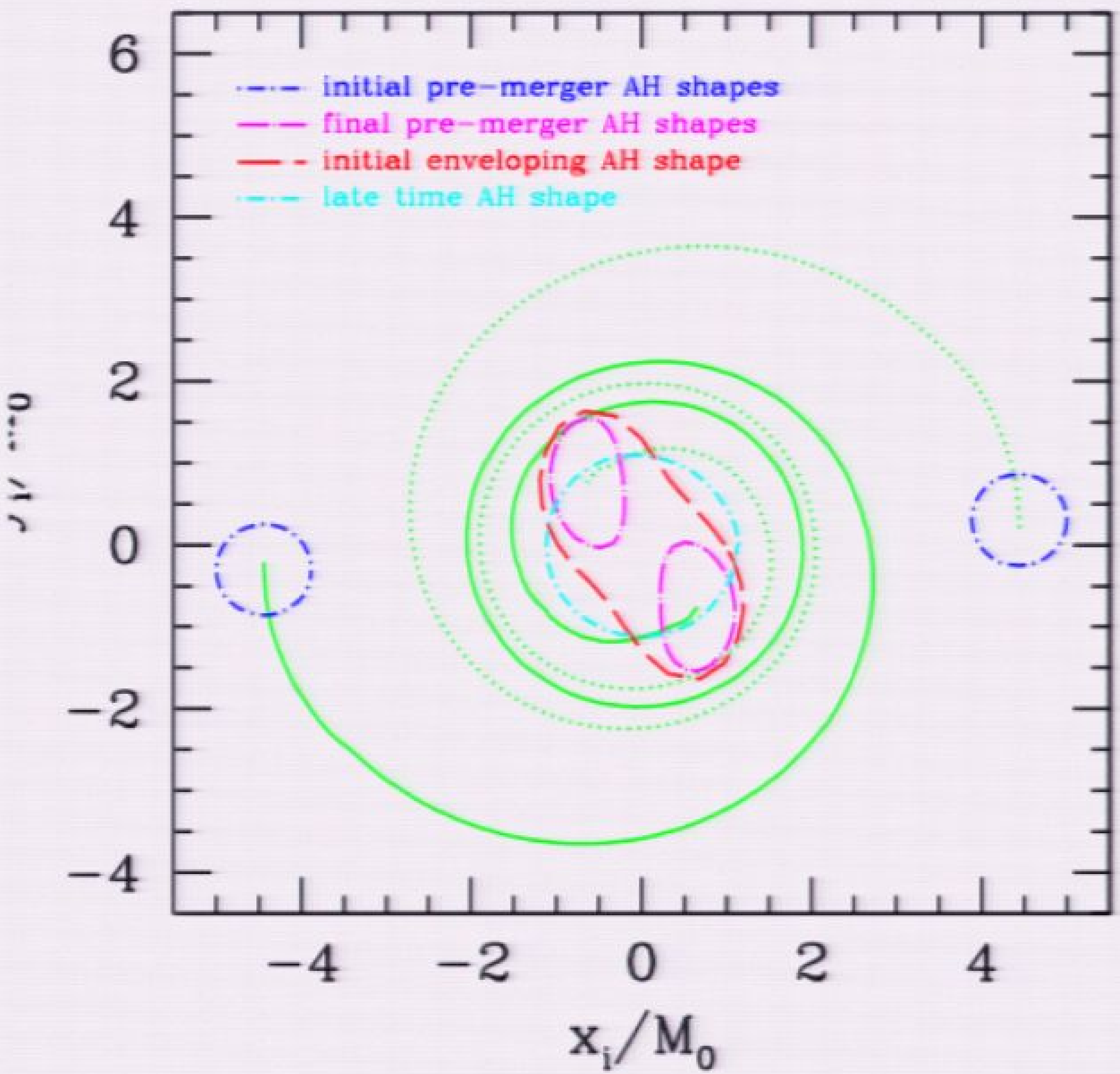
Collisions of Shock Waves in AdS_5

Analytical Results



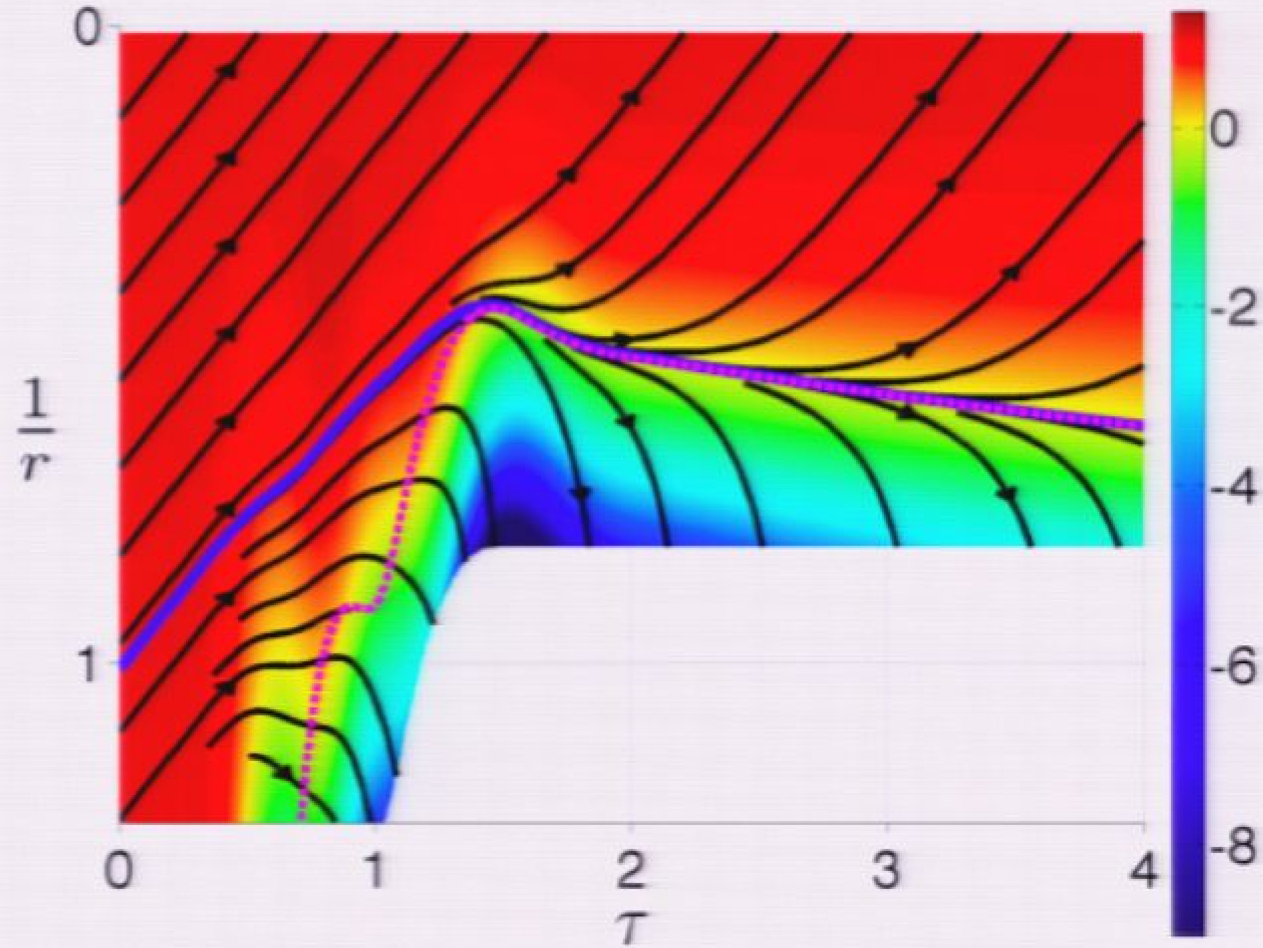
[D. Grumiller & PR 2008]

Black Hole Mergers in 4d Minkowski



[F. Pretorius, 2006]

Numerical Gravity in AdS_5



[P. Chesler & L. Yaffe, 2009]

Conclusions

- Holography is a vital tool to understand Heavy-Ion Collisions qualitatively
- Many open problems to be solved...may need some numerics

Conclusions

- Holography is a vital tool to understand Heavy-Ion Collisions qualitatively
- Many open problems to be solved...may need some numerics

Conclusions

- Holography is a vital tool to understand Heavy-Ion Collisions qualitatively
- Many open problems to be solved...may need some numerics

Weak Coupling

Equilibration at Weak Coupling: Incomplete!?

Equilibration

- Energy Momentum Tensor of a Fluid = Gradient Expansion of Quantum Field Theory

$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} + \Pi^{\mu\nu}$$

- Fluid Dynamics works well if gradients are small
- If Gradients are small, T^{ij} should nearly isotropic in l.r.f

$$T^{ij} = P\delta^{ij} + \mathcal{O}(\nabla), \qquad \text{for } u^{\mu} = (1, \mathbf{0})$$

Science Project Proposal

- Take holographic dual to theory with $\zeta \neq 0$ (e.g. $\mathcal{N} = 2^*$)
- Consider isotropically expanding boundary space (e.g. FRW)

[c.f. Kajantie, Louko, Tahkokallio, 2008]

- Study regime where fluid dynamics would predict cavitation/inflation & compare to full QFT result

Cavitation in a different context

Cosmologists call

$$P + \Pi < 0$$

"Inflation"

Cavitation

Does Cavitation Happen in QCD plasmas?

If yes, what are it's effects?

Cavitation

- Viscous Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu + (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

where

$$\Pi = -\zeta \nabla \cdot u + \mathcal{O}(\nabla^2) \qquad \pi^{\mu\nu} = -\eta \nabla^{<\mu} u^{\nu>} + \mathcal{O}(\nabla^2)$$

- If $\zeta / P \gg 1/T$ then

$$P + \Pi < 0$$

- "Negative Effective Pressure"
- Cavitation = Formation of Vacuum Bubbles

[see e.g. K. Rajagopal & N. Tripuraneni, 2009]

Cavitation in a different context

Cosmologists call

$$P + \Pi < 0$$

"Inflation"

Science Project Proposal

- Take holographic dual to theory with $\zeta \neq 0$ (e.g. $\mathcal{N} = 2^*$)
- Consider isotropically expanding boundary space (e.g. FRW)

  [c.f. Kajantie, Louko, Tahkokallio, 2008]
- Study regime where fluid dynamics would predict cavitation/inflation & compare to full QFT result

Cavitation

- Viscous Stress-Energy Tensor:

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu + (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

where

$$\Pi = -\zeta \nabla \cdot u + \mathcal{O}(\nabla^2) \qquad \pi^{\mu\nu} = -\eta \nabla^{<\mu} u^{\nu>} + \mathcal{O}(\nabla^2)$$

- If $\zeta / P \gg 1/T$ then

$$P + \Pi < 0$$

- "Negative Effective Pressure"
- Cavitation = Formation of Vacuum Bubbles

[see e.g. K. Rajagopal & N. Tripuraneni, 2009]