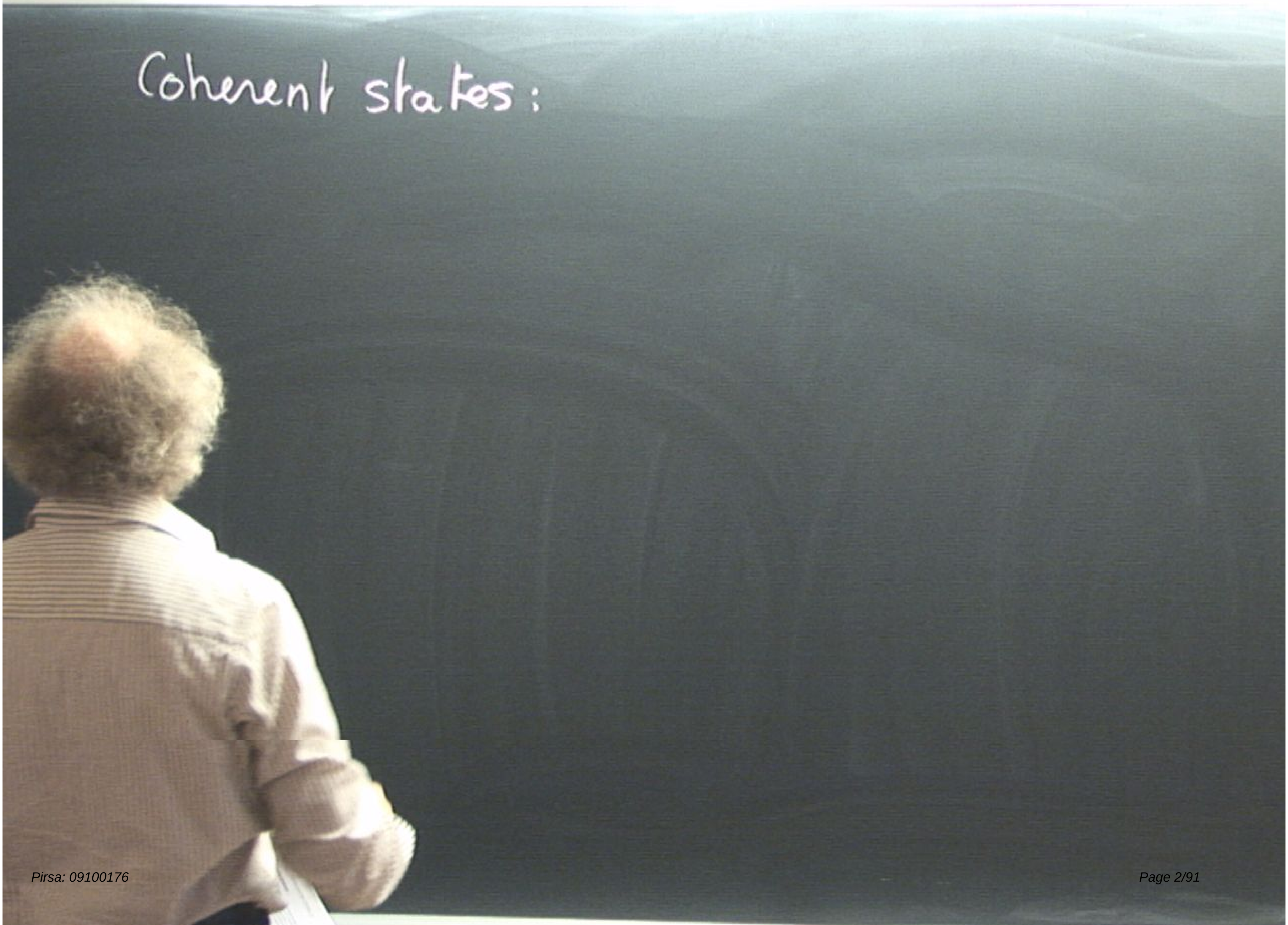
Title: Quantum Field Theory II (PHYS 603) - Lecture 5

Date: Oct 30, 2009 09:00 AM

URL: http://pirsa.org/09100176

Abstract:


Coherent states:

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Path Integral for anticommuting fields

Coherent states: (Bosons + Fermions)

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Bosons (N. Rel.)

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Path Integral for anticommuting fields

Bosons (N. Rel.)   1 state with 1 Boson $|1\rangle$

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Bosons (N. Rel.) $\quad$ 1 state with 1 Boson $|1\rangle$

energy $E_0 > 0$

$\mathcal{H}$ generated by $|n\rangle$ $\quad n \in \mathbb{N}$ number of bosons

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$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$a\,|n\rangle = \sqrt{n}\,|n-1\rangle$

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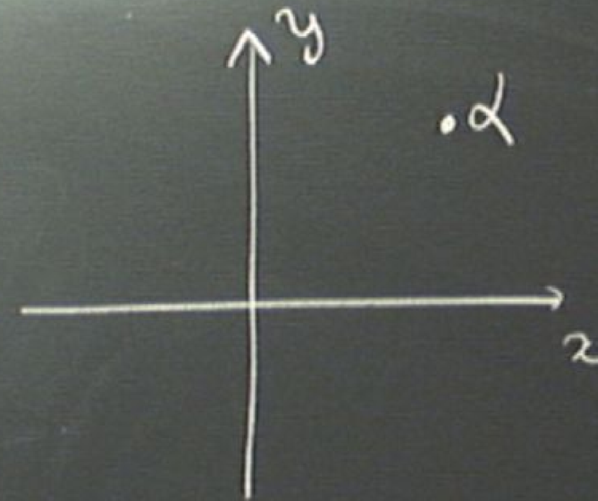
$[a, a^+] = 1$ $a|0> = 0$

$\alpha \in \mathbb{C}$

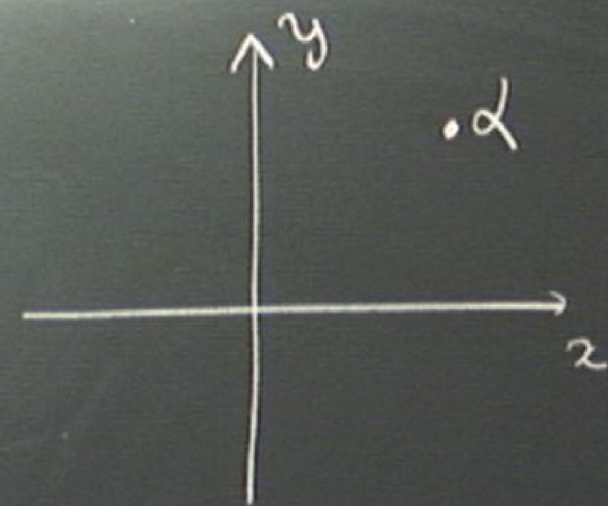
$|\alpha\rangle$



$\alpha \in \mathbb{C}$

$|\alpha\rangle = \quad e^{\alpha a^\dagger}|0\rangle$

coherent state : not an eigenstate of N
or H



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$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \cdot e^{\alpha a^\dagger} |0\rangle$$

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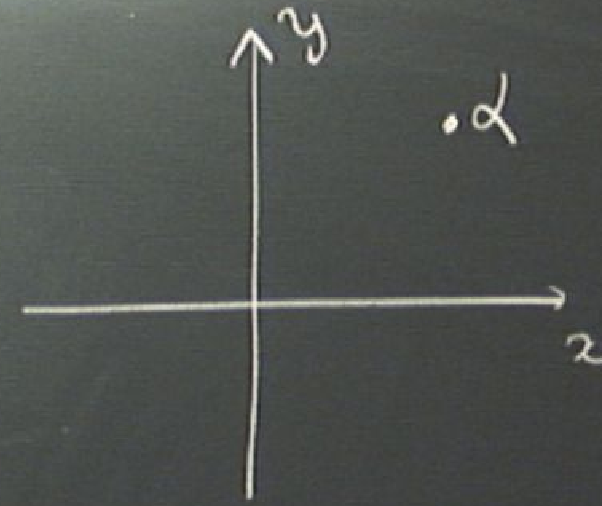
$$U(t) = e^{\frac{t}{i\hbar} H}$$



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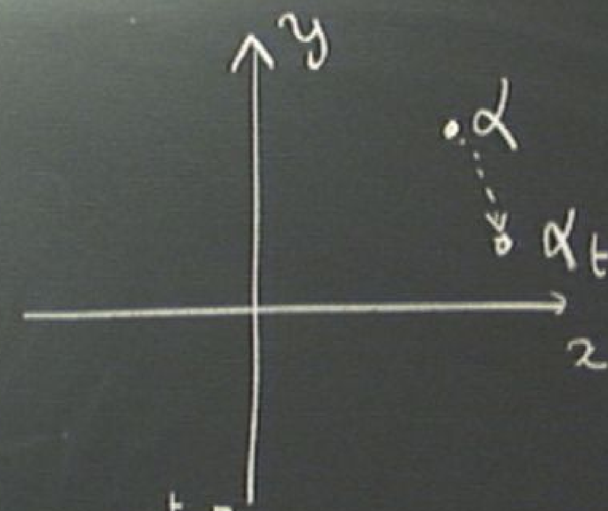

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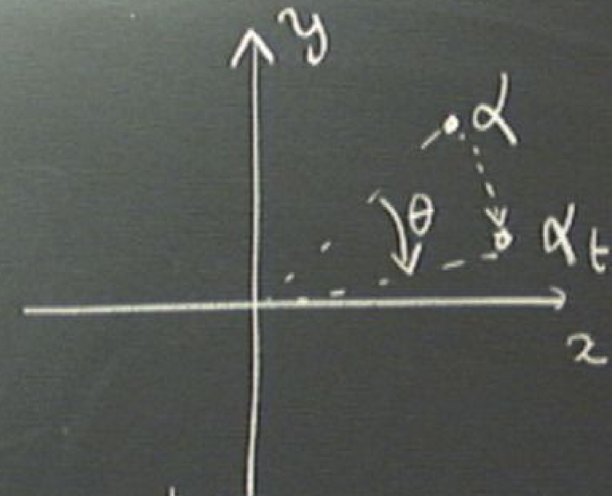

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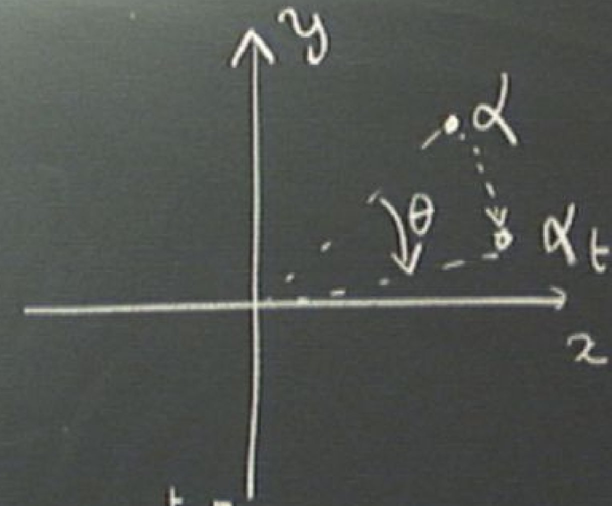

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Rotation (uniform) $\qquad \theta = -t \frac{E_0}{\hbar}$

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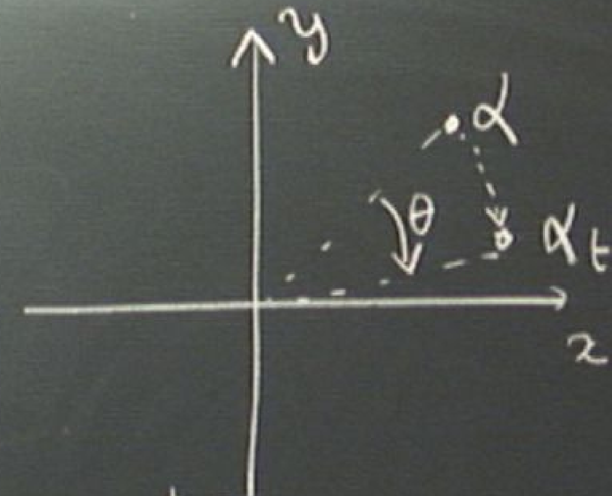

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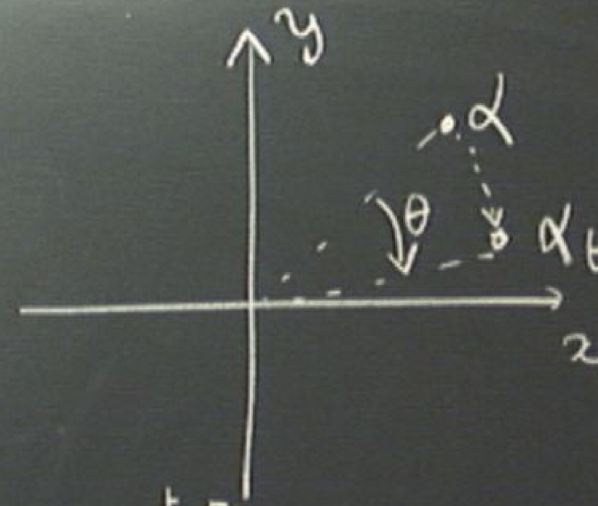

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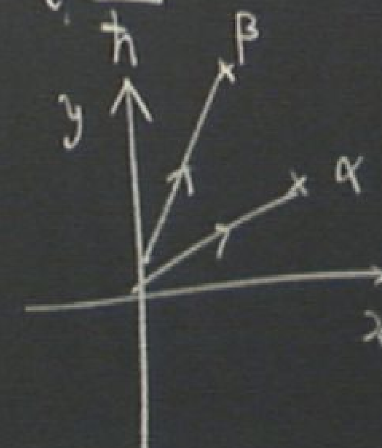

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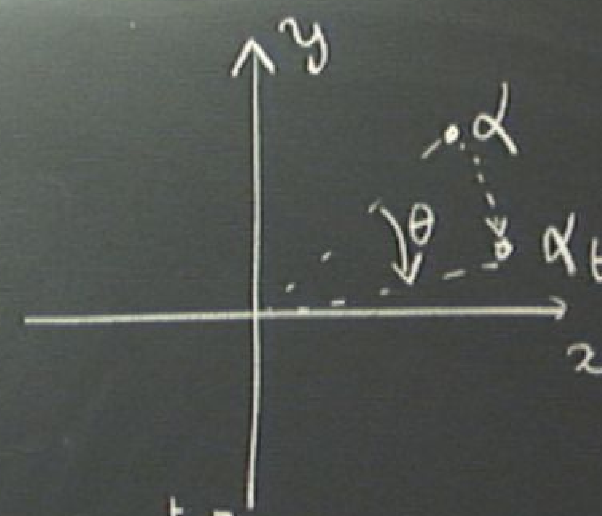

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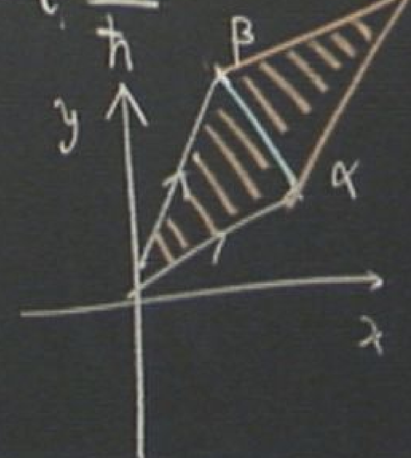

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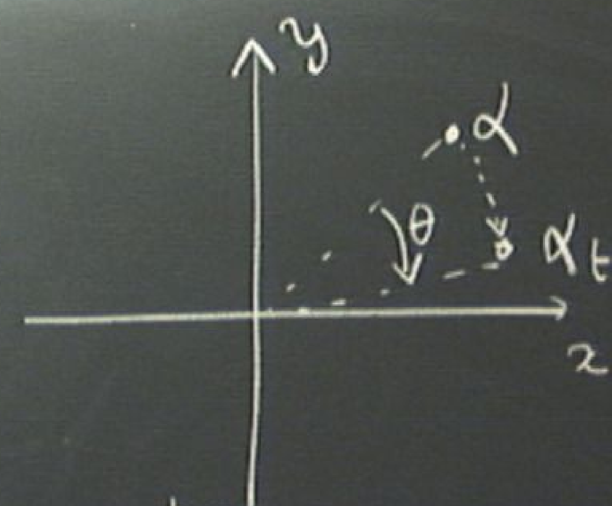

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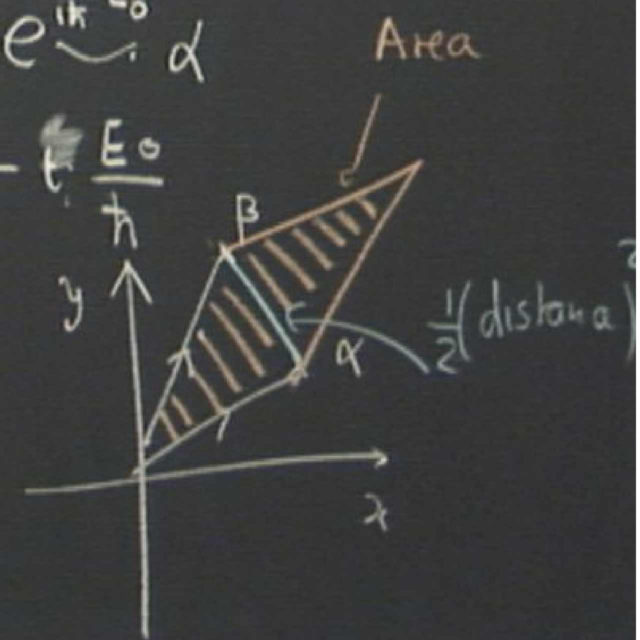

Integral over $\mathbb{C}$ $\quad \alpha = x + iy$

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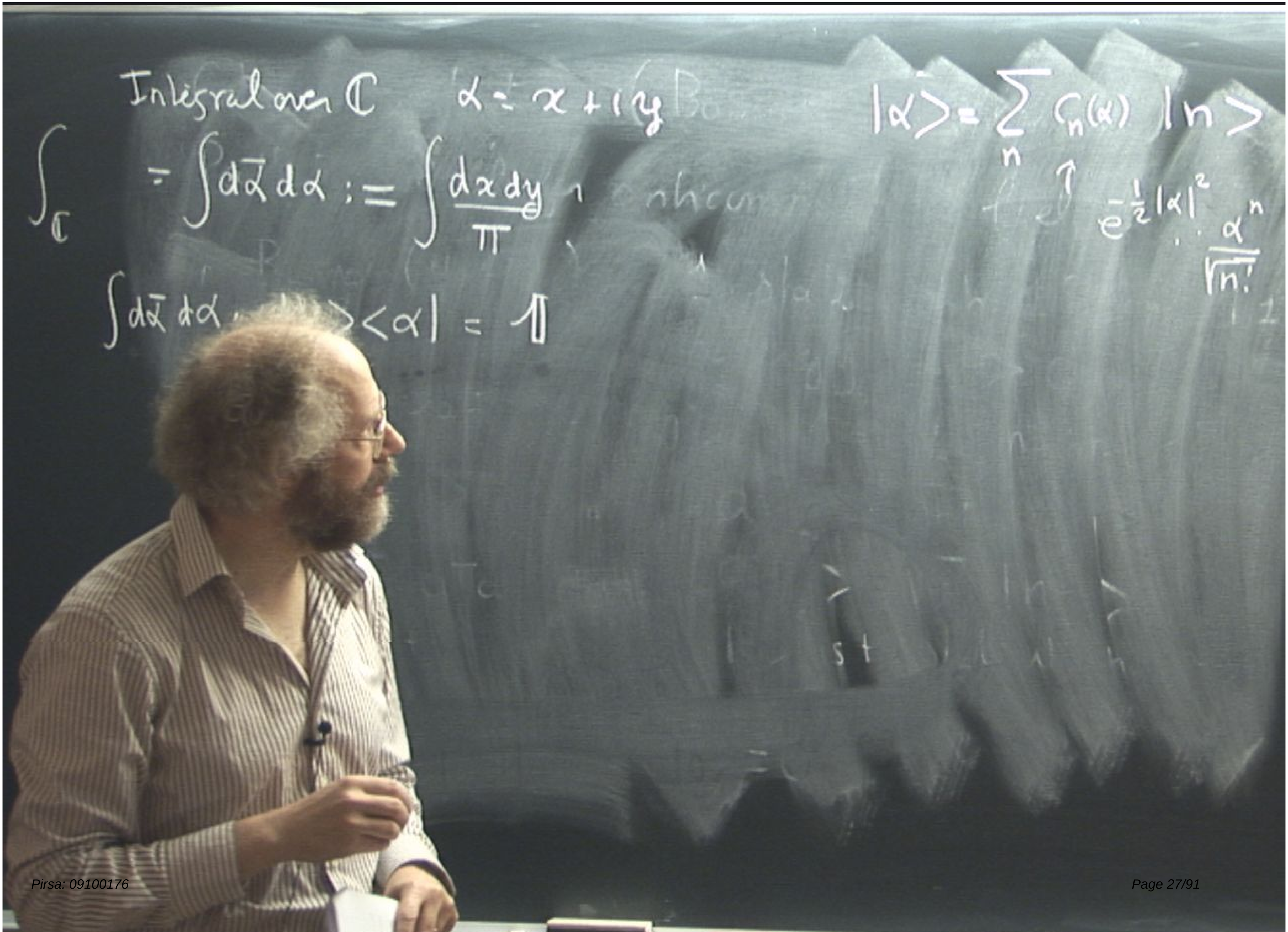
Integral over $\mathbb{C}$
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$e^{-\frac{1}{2}|\alpha|^2} \cdot \frac{\alpha^n}{\sqrt{n!}}$
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$$d\alpha \quad \alpha^p \bar{\alpha}^q\, |\alpha\rangle\langle\alpha|$$

p & q are ≥ 0 integers

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$$\bar{\alpha} \longrightarrow a^+$$

$$\alpha \longrightarrow a$$

$$\int d\bar{\alpha}\, d\alpha \;\; \alpha^p \bar{\alpha}^q \,|\alpha\rangle\langle\alpha| = (a)^p (a^+)^q$$

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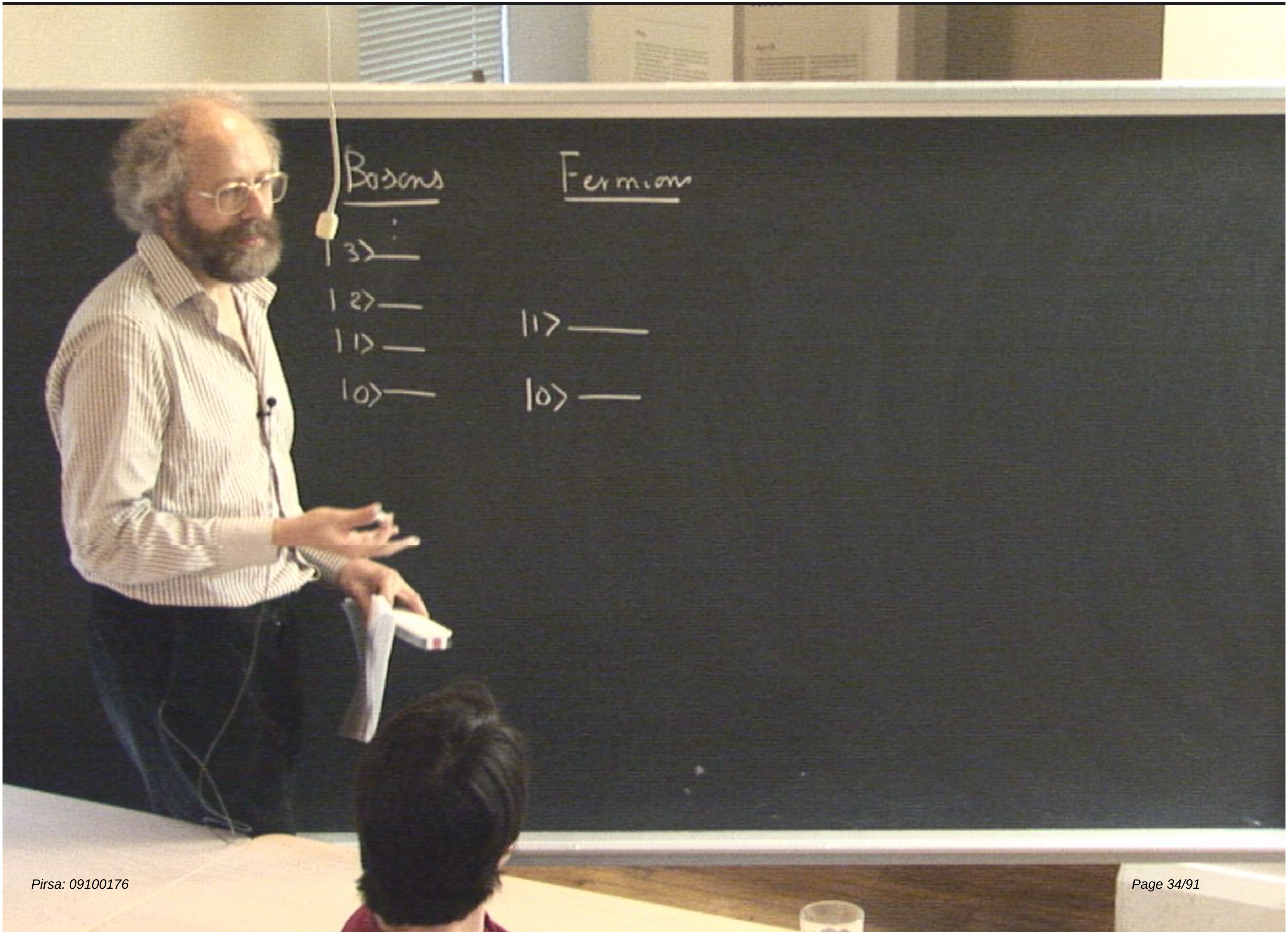
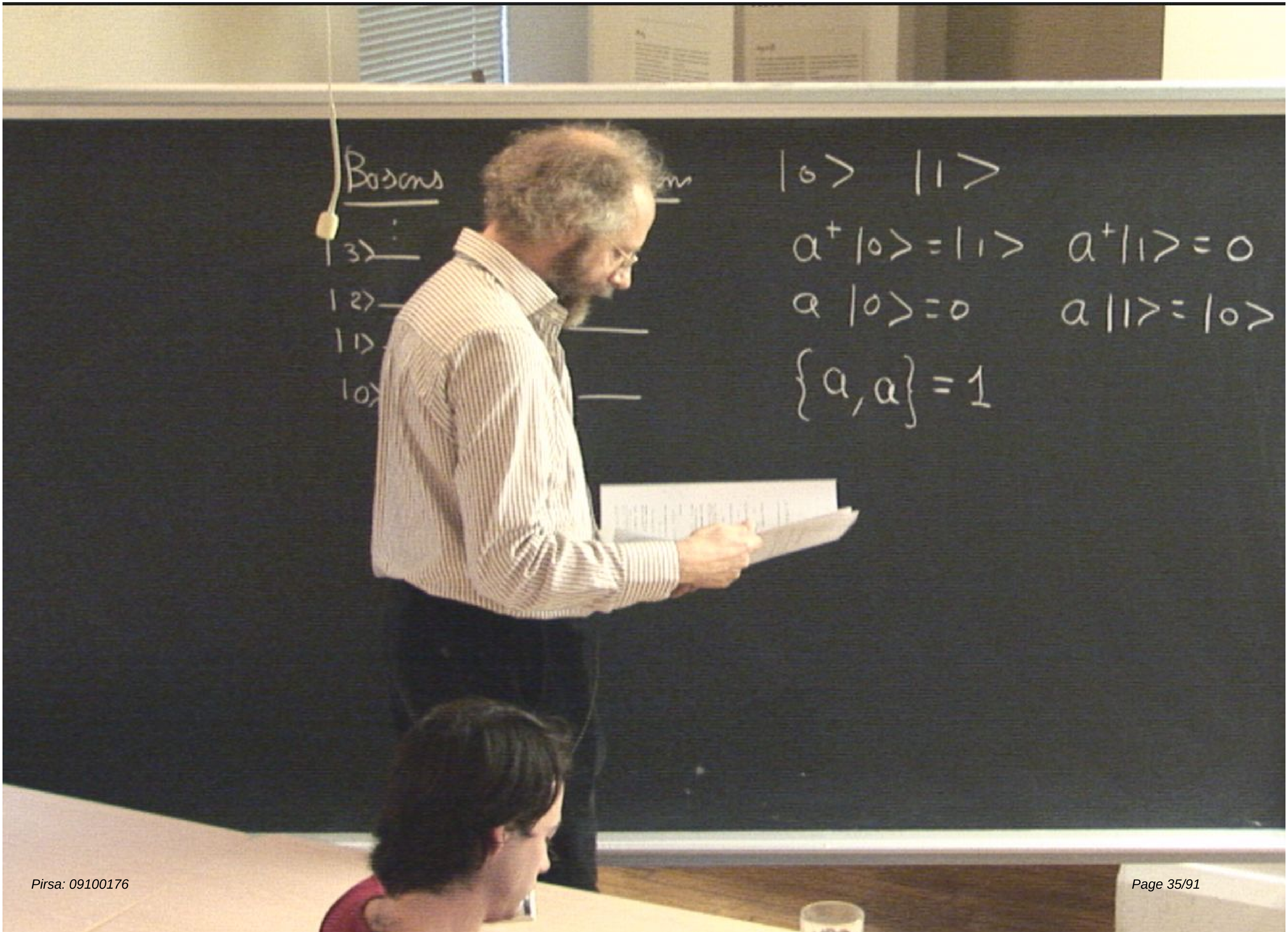
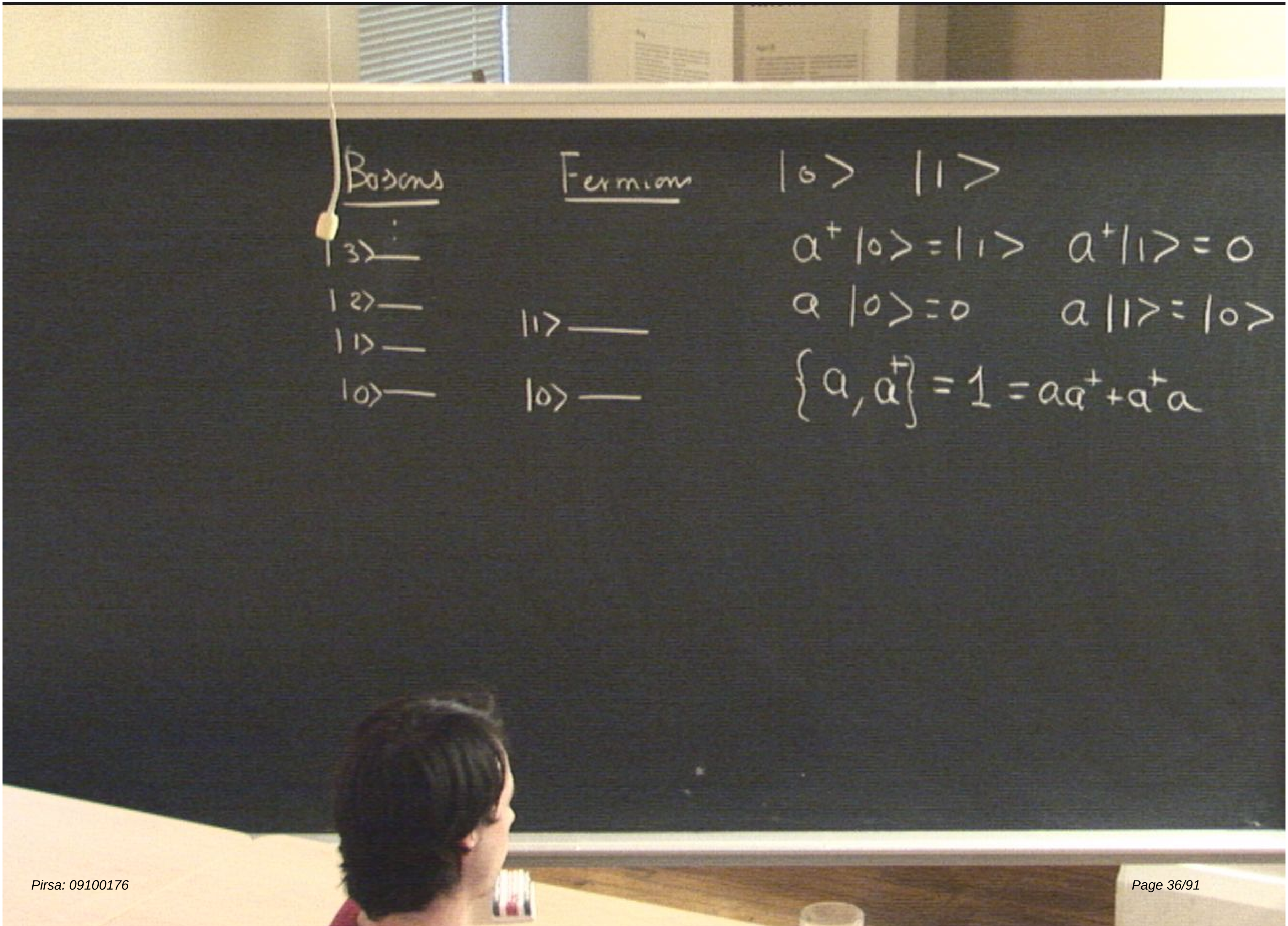
Operator O

$$\mathrm{Tr}(O) = \int d\bar{\alpha}\, d\alpha \,\langle\alpha| O |\alpha\rangle$$


Bosons
Fermions
|3⟩
|2⟩
|1⟩
|0⟩
|1⟩
|0⟩


Bosons
|3>
|2>
|1>
|0>
|0> |1>
a⁺|0> = |1>   a⁺|1> = 0
a |0> = 0   a|1> = |0>
{a, a} = 1


Bosons
Fermions
$|0\rangle \quad |1\rangle$
$|3\rangle$
$|2\rangle$
$|1\rangle$
$|0\rangle$
$|1\rangle$
$|0\rangle$
$a^+|0\rangle = |1\rangle \quad a^+|1\rangle = 0$
$a|0\rangle = 0 \quad a|1\rangle = |0\rangle$
$\{a, a^+\} = 1 = aa^+ + a^+a$

Calculus & Integration with anticommuting "numbers"

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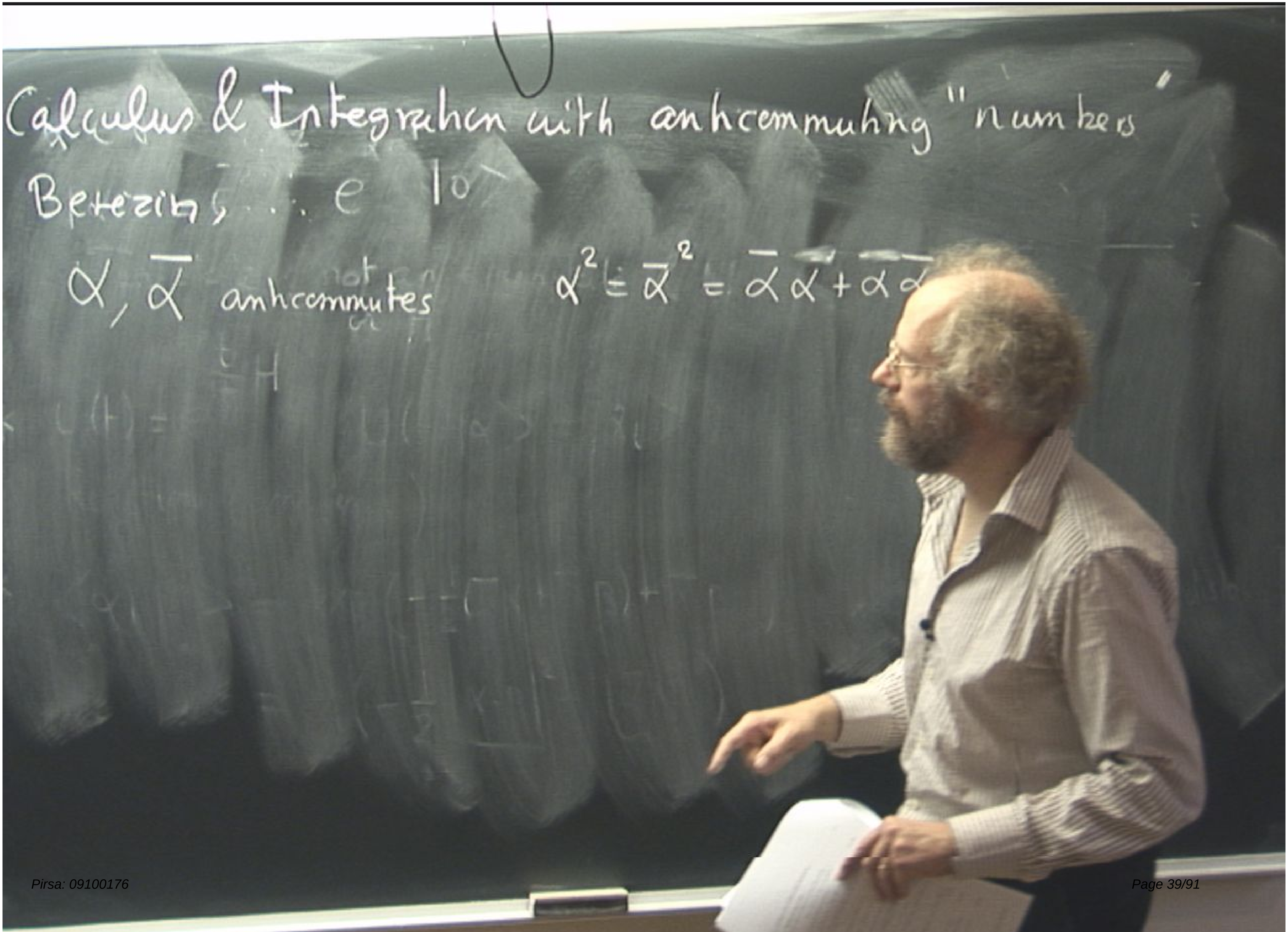
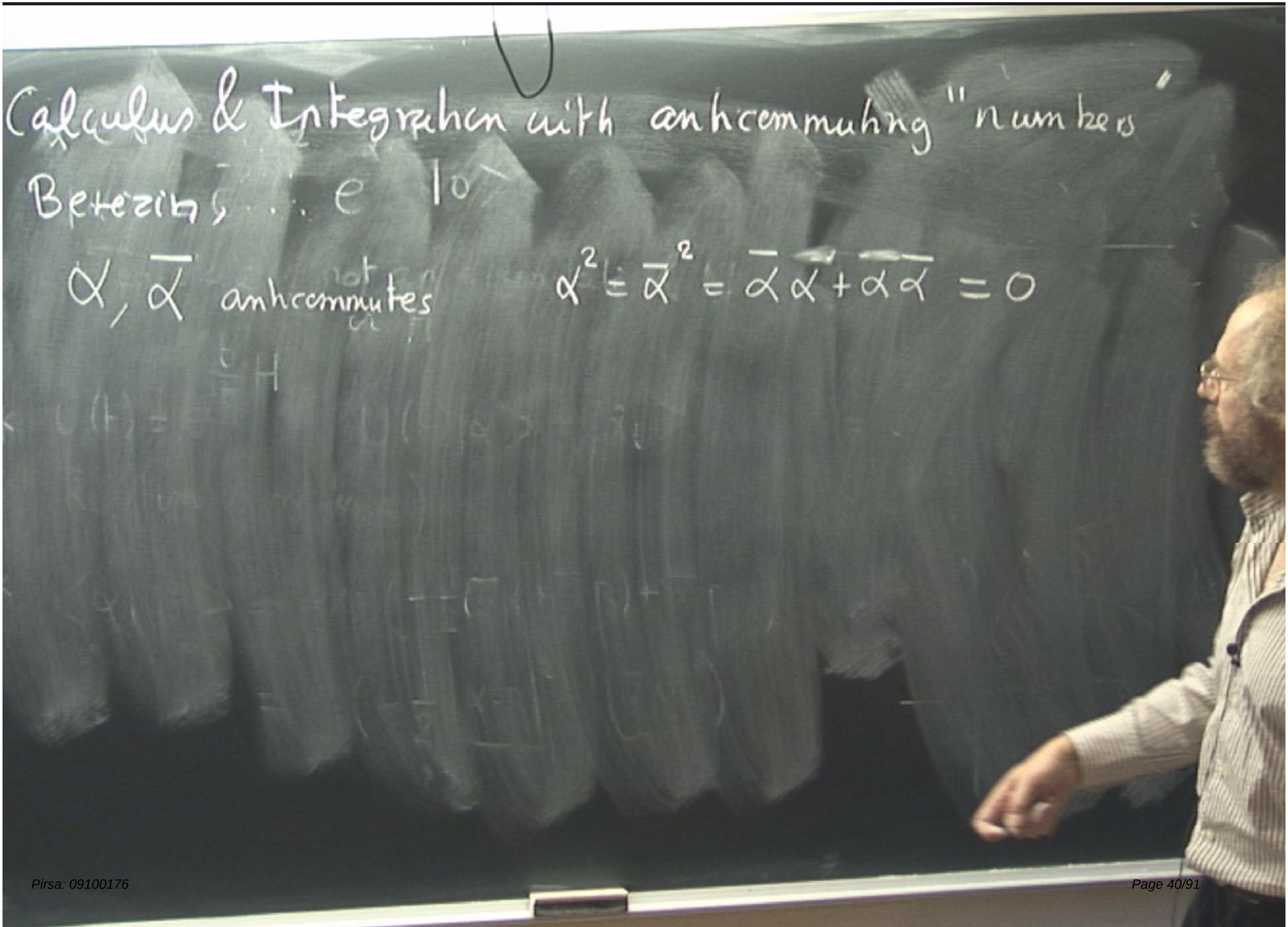
Berezin


Calculus & Integration with anticommuting "numbers"
Berezin
$\alpha, \bar{\alpha}$ anticommutes
$\alpha^2 = \bar{\alpha}^2 = \bar{\alpha}\alpha + \alpha\bar{\alpha}$


Calculus & Integration with anticommuting "numbers"
Berezin
α, ᾱ anticommutes
α² = ᾱ² = ᾱα + αᾱ = 0

Calculus & Integration with anticommuting "numbers"

Berezin ... e 10

$\alpha, \bar{\alpha}$ anticommutes $\qquad \alpha^2 = \bar{\alpha}^2 = \bar{\alpha}\alpha + \alpha\bar{\alpha} = 0$

Algebra of complex combination generated by these numbers

Calculus & Integration with anticommuting "numbers"

Berezin ... e 10

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Algebra of complex combination generated by these numbers

$G = \{ g = a_0 + a_1 \alpha + a_2 \bar{\alpha}$

ordinary complex number

Calculus & Integration with anticommuting "numbers"

Berezin ... e 10

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Algebra of complex combination generated by these numbers

$$G = \{ g = a_0 + a_1 \alpha + a_2 \bar{\alpha} + a_3 \bar{\alpha}\alpha \}$$

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Berezin, ... e 10

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Berezin ... e 10

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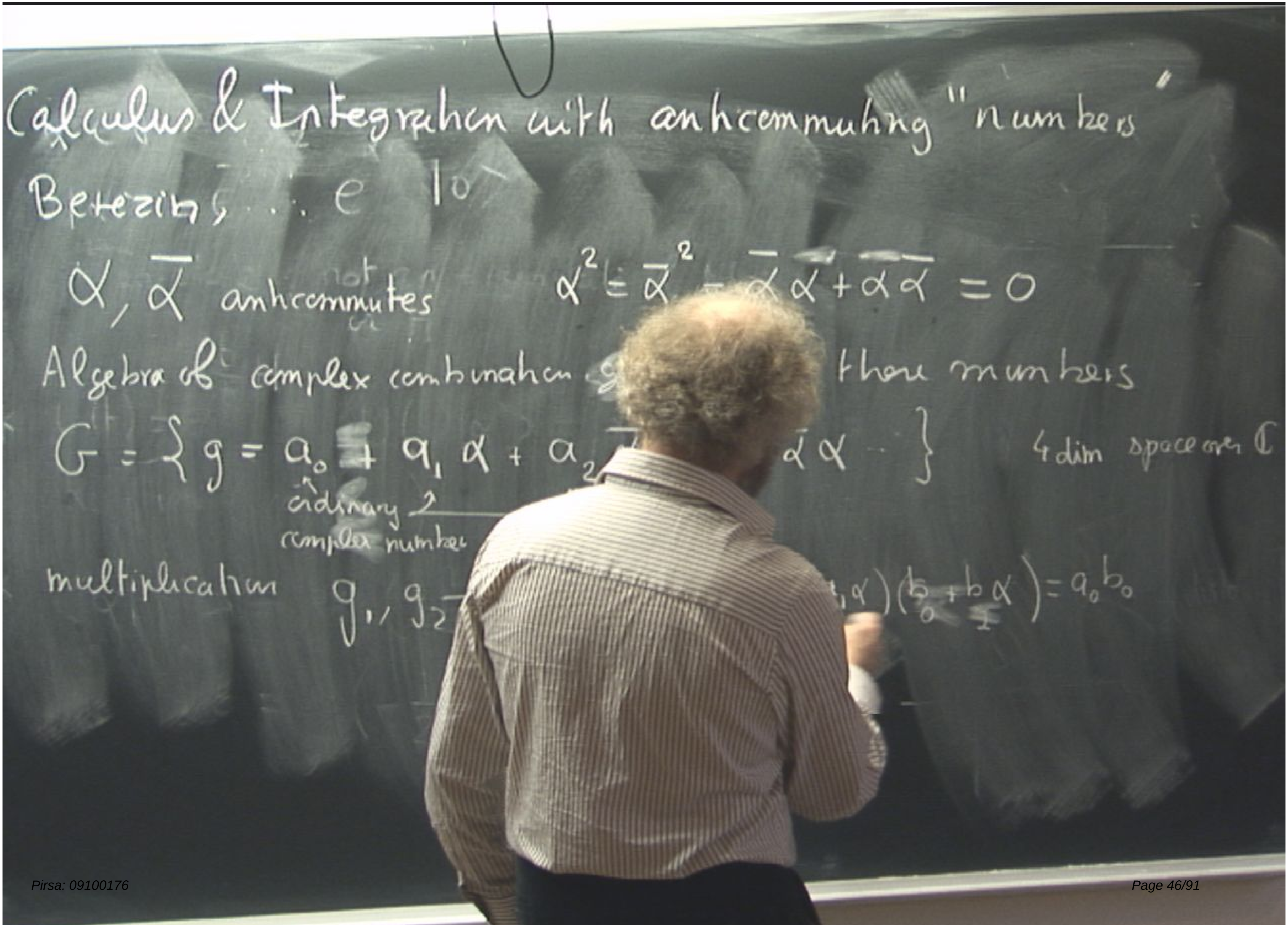
Al... complex combination generated by these numbers

G ... $a_0 + a_1 \alpha + a_2 \bar{\alpha} + a_3 \bar{\alpha}\alpha \,\}$ 4 dim space over $\mathbb{C}$

ordinary complex number

...tion $\quad g_1, g_2 \mapsto g_1 \cdot g_2$


Calculus & Integration with anticommuting "numbers"
Berezin
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4 dim space over $\mathbb{C}$
ordinary complex number
multiplication
g_1, g_2
$)(b_0 + b\alpha) = a_0 b_0$

Calculus & Integration with anticommuting "numbers"

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ordinary complex number

multiplication $\quad g_1, g_2 \mapsto g_1 \cdot g_2$

$(a_0 + a_1 \alpha)(b_0 + b_1 \alpha) = a_0 b_0 + a_0 b_1 \alpha$
$+ a_1 b_0 \alpha +$

Calculus & Integration with anticommuting "numbers"

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ordinary complex number

multiplication $\quad g_1, g_2 \to g_1 \cdot g_2 \qquad (a_0 + a_1\alpha)(b_0 + b_1\alpha) = a_0 b_0 + a_0 b_1 \alpha + a_1 b_0 \alpha +$

addition $\quad g_1, g_2 \to g_1 + g_2 \qquad (a_0 + a_2\bar{\alpha})(b_0 + b_1\alpha)$

Algebra not commutable $\quad g_1 \cdot g_2 \neq g_2 \cdot g_1$

associative $(g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$

Calculus & Integration with anticommuting "numbers"

Berezin . . . (Grassmann Algebra)

$\alpha, \bar{\alpha}$ anticommutes $\qquad \alpha^2 = \bar{\alpha}^2 = \bar{\alpha}\alpha + \alpha\bar{\alpha} = 0$

Algebra of complex combination generated by these numbers

$$G = \{ g = a_0 + a_1 \alpha + a_2 \bar{\alpha} + a_3 \bar{\alpha}\alpha \}$$ 4 dim space over $\mathbb{C}$

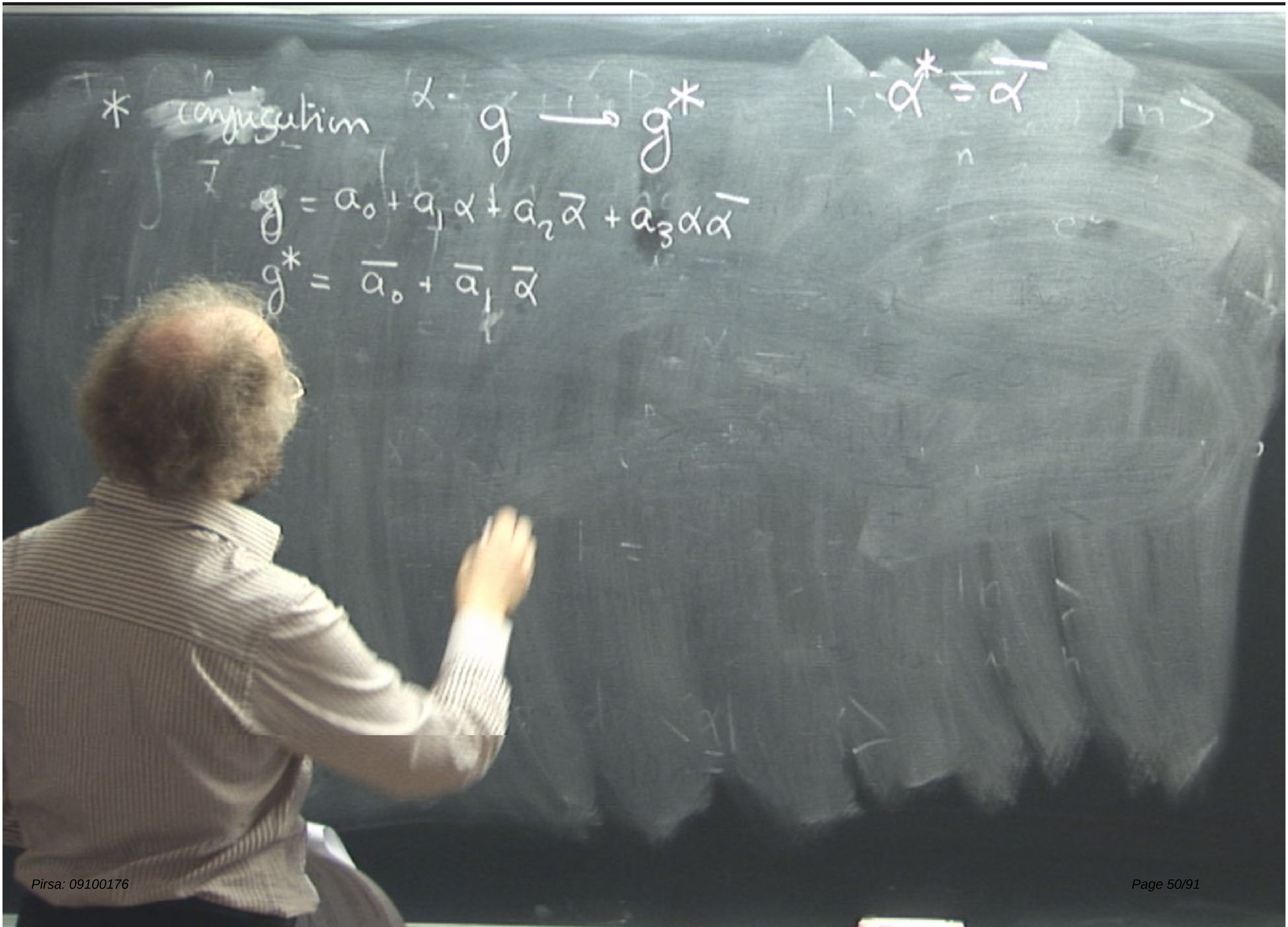
ordinary complex number

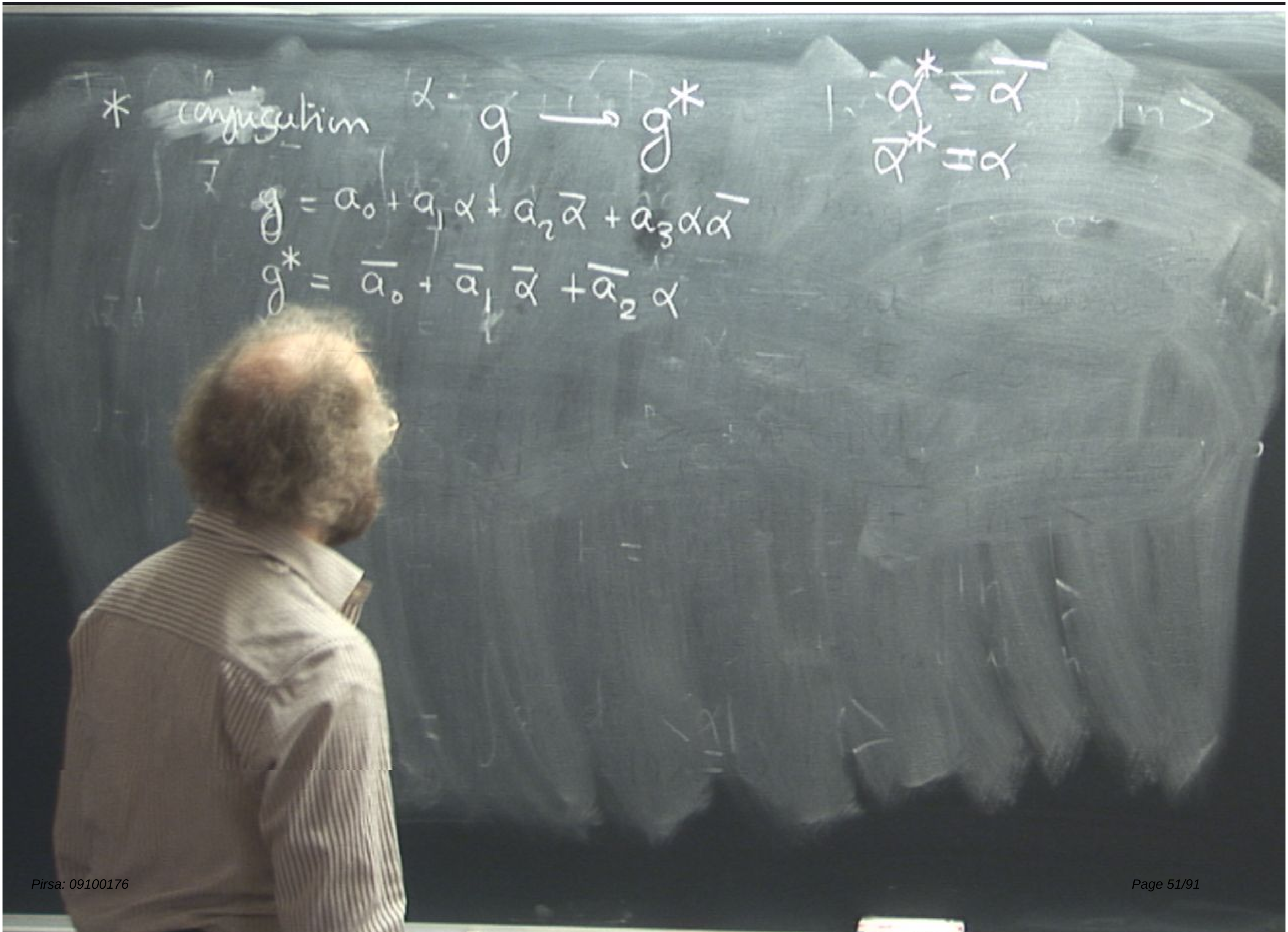
multiplication $\quad g_1, g_2 \to g_1 \cdot g_2 \qquad (a_0 + a_1\alpha)(b_0 + b_1\alpha) = a_0 b_0 + a_0 b_1 \alpha + a_1 b_0 \alpha +$

addition $\quad g_1, g_2 \to g_1 + g_2 \qquad (a_0 + a_2\bar{\alpha})(b_0 + b_1\alpha)$

Algebra not commutable $\quad g_1 \cdot g_2 \neq g_2 \cdot g_1$

associative $\quad (g_1 \cdot g_2) \cdot g_3 = g_1 \cdot (g_2 \cdot g_3)$


* conjugation $g \longrightarrow g^*$
$\alpha^* = \bar{\alpha}$
$g = a_0 + a_1\alpha + a_2\bar{\alpha} + a_3\alpha\bar{\alpha}$
$g^* = \bar{a}_0 + \bar{a}_1\bar{\alpha}$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$*$ conjugation $\quad g \longrightarrow g^*$

$\alpha^* = \bar{\alpha} \qquad (\alpha\bar{\alpha})^* = \alpha\bar{\alpha}$

$\bar{\alpha}^* = \alpha$

$g = a_0 + a_1\alpha + a_2\bar{\alpha} + a_3\alpha\bar{\alpha}$

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$(g_1 g_2)^* = g_2^* g_1^*$

$*$ conjugation $\quad g \longrightarrow g^*$

$$g = a_0 + a_1 \alpha + a_2 \bar{\alpha} + a_3 \alpha\bar{\alpha}$$

$$g^* = \overline{a_0} + \overline{a_1}\, \bar{\alpha} + \overline{a_2}\, \alpha + \overline{a_3}\, \alpha\bar{\alpha}$$

$\alpha^* = \bar{\alpha} \qquad (\alpha\bar{\alpha})^* = \alpha\bar{\alpha}$

$\bar{\alpha}^* = \alpha$

$$(g_1 g_2)^* = g_2^*\, g_1^*$$

$$(g^*)^* = g$$

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* Integration.

* conjugation $\quad g \longrightarrow g^*$

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* Integration. (Rules)

$$\int d\alpha\, 1 = 0 \; ; \quad \int d\alpha\, \alpha = 1 \qquad \int d\alpha\, \bar{\alpha} = 0$$

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* Integration (Rules) everything anticommutes

$$\int d\alpha\, 1 = 0 ; \quad \int d\alpha\, \alpha = 1 \quad \int d\alpha\, \bar{\alpha} = 0$$

$$\int d\alpha \cdot (\alpha\bar{\alpha}) = \bar{\alpha}$$

* conjugation $\alpha \cdot \quad g \longrightarrow g^*$

1. $\alpha^* = \overline{\alpha} \qquad (\alpha\overline{\alpha})^* = \alpha\overline{\alpha}$

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$$g = a_0 + a_1\alpha + a_2\overline{\alpha} + a_3\alpha\overline{\alpha}$$

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E LAW

$g^* = \bar a_0 + \bar a_1 \bar\alpha + \bar a_2 \alpha + \bar a_3 \alpha\bar\alpha$

$(g_1 g_2)^* = g_2^* g_1^*$

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* "Integration" (Rules) everything anticommute,

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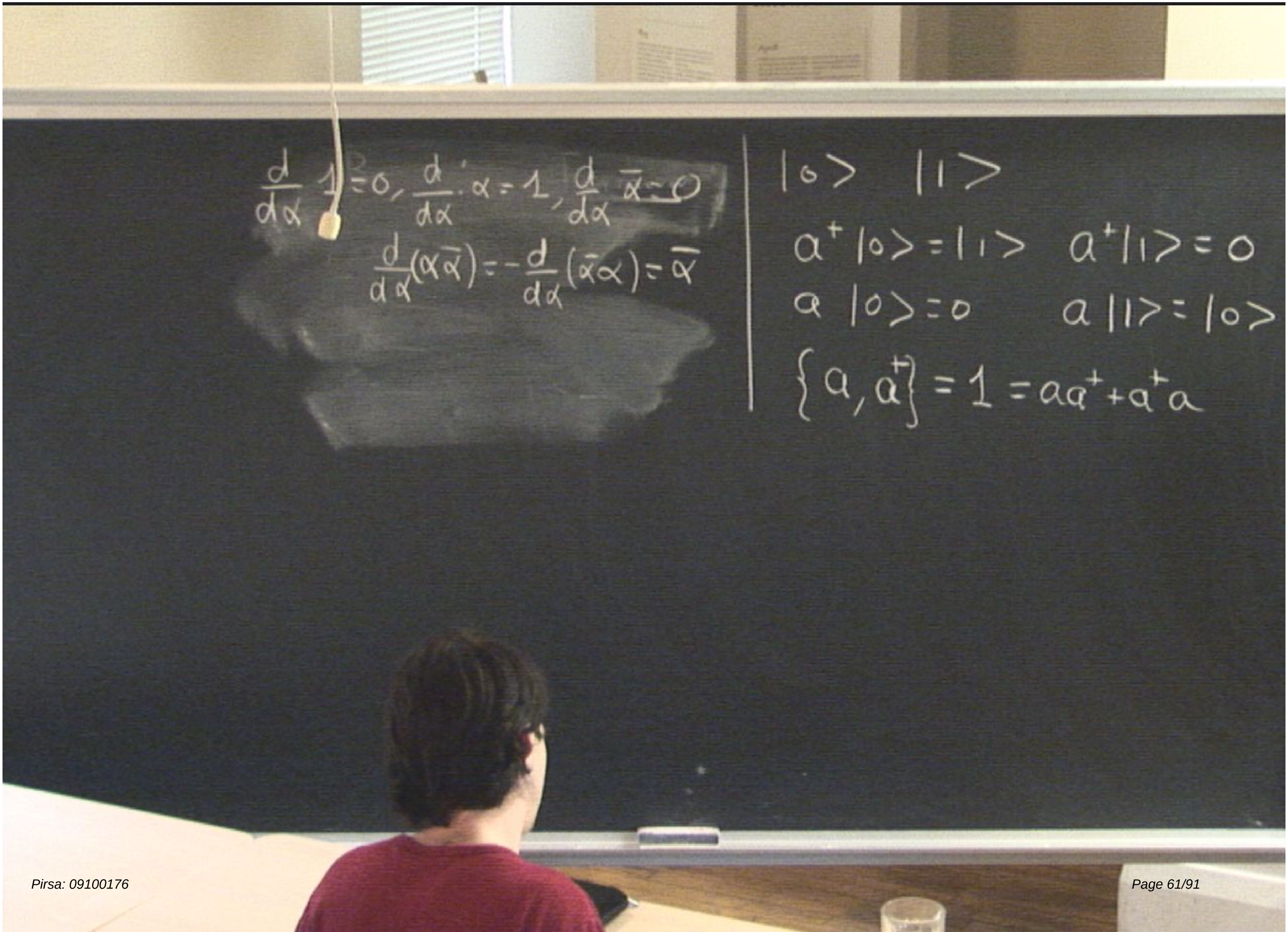
$$\int d\bar{\alpha}\, 1 = 0, \int d\bar{\alpha}\, \alpha = 0, \int d\bar{\alpha}\, \bar{\alpha} = 1, \int d\bar{\alpha}(\alpha\bar{\alpha}) = -\alpha$$

$$\int d\bar{\alpha}(\bar{\alpha}\alpha) = +\alpha$$



$\frac{d}{d\alpha} 1 = 0, \frac{d}{d\alpha} \alpha = 1, \frac{d}{d\alpha} \bar{\alpha} = 0$
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$|0\rangle \quad |1\rangle$
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$a|0\rangle = 0 \quad a|1\rangle = |0\rangle$
$\{a, a^+\} = 1 = aa^+ + a^+a$

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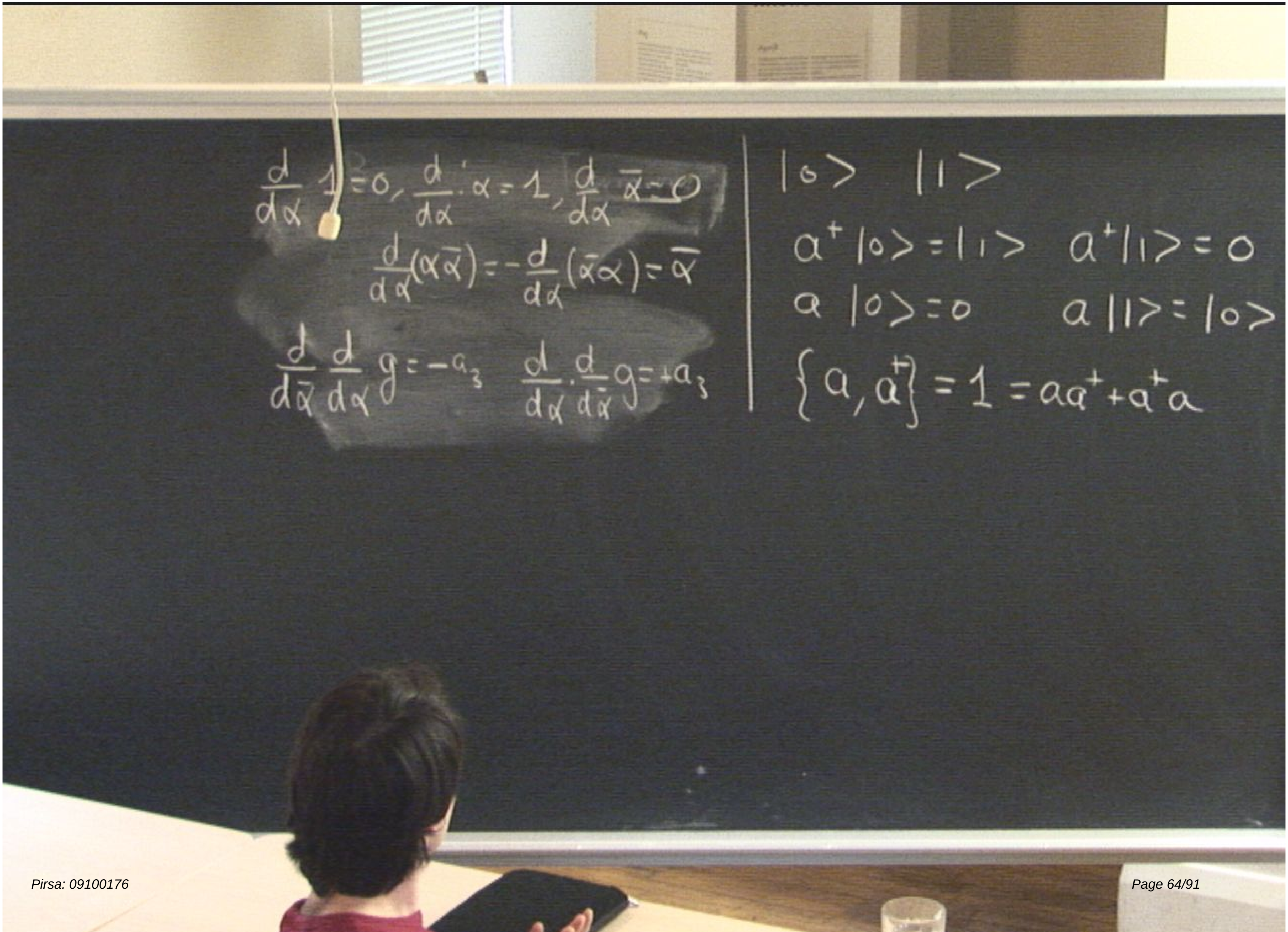
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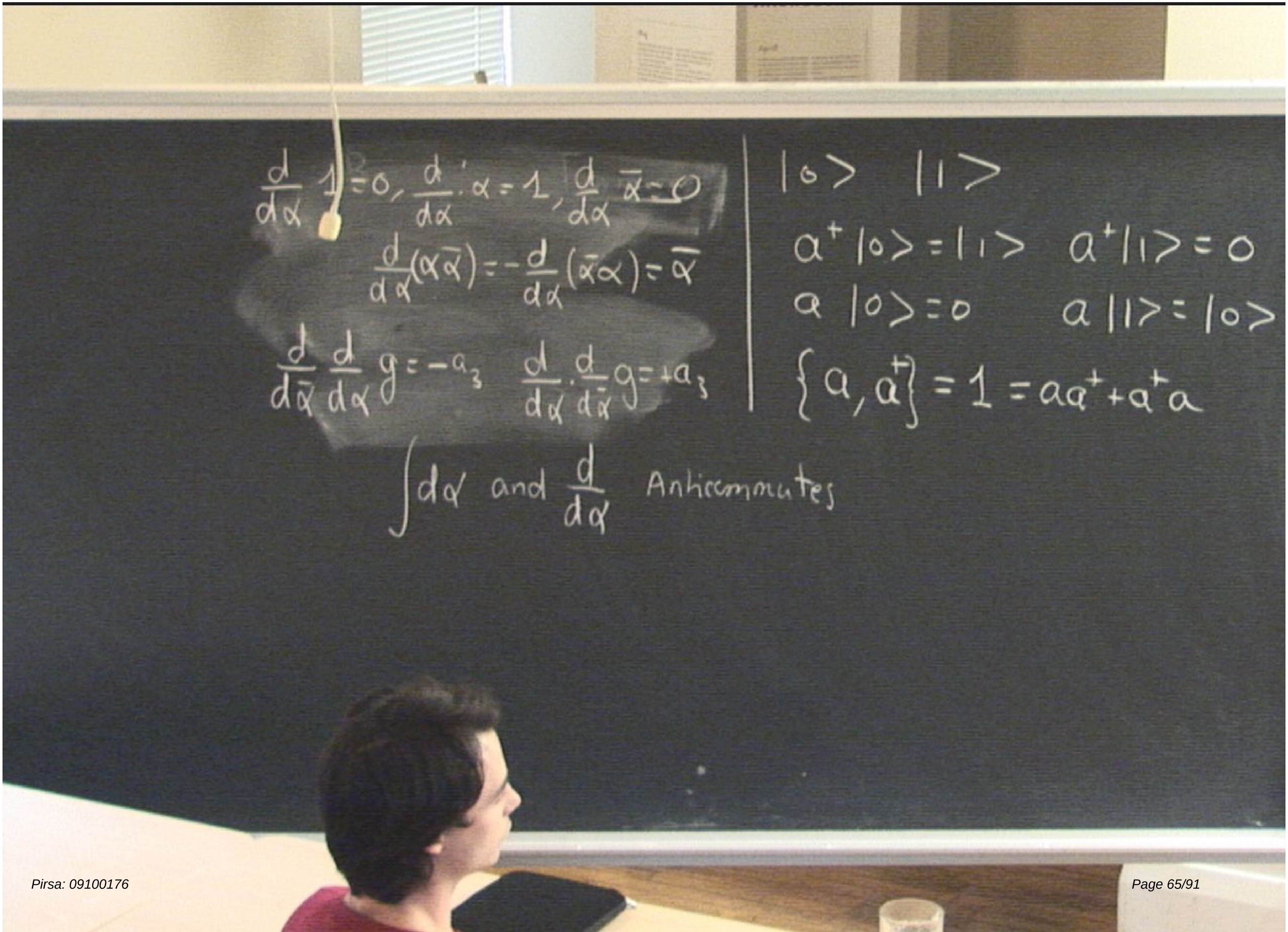

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$$\int d\bar{\alpha}\, d\alpha\, \exp(-A\bar{\alpha}\alpha)$$

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$$\boxed{\exp(-A\bar{\alpha}\alpha) = 1 - A\bar{\alpha}\alpha} + \cancel{\frac{1}{2}A^2(\bar{\alpha}\alpha)^2} + \cdots$$

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Many "independent" numbers $\alpha_a, \bar{\alpha}_a \quad a = 1, N$

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Many "independent" numbers $\alpha_a, \bar\alpha_a$ $a = 1, N$

"N variable" Grassman algebra

$$G = 1 + \cdot\alpha_a + \cdot\bar\alpha_a + \cdot\, \bar\alpha_a \alpha_b + \bar\alpha_a \alpha_b \alpha_c \alpha_d$$

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"N ~~variable~~ Grassman algebra

$$\prod_{a=1}^{N} d\bar{\alpha}_a\, d\alpha_a$$

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"N variable Grassman algebra

$$\int \prod_{a=1}^{N} d\bar{\alpha}_a\, d\alpha_a \exp\left(-A_{ab} \bar{\alpha}_a \alpha_b\right)$$

$$g = a_0 + a_1 \bar{\alpha} + a_2 \alpha + a_3 \bar{\alpha}\alpha$$

$$\int d\alpha\, g = a_1 + a_3 \bar{\alpha}$$

$$\int d\bar{\alpha}\, g = a_2 + a_3 \alpha$$

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"N Grassman algebra

$$\int \prod_{a=1}^{N} d\bar\alpha_a \, d\alpha_a \exp\left(-\sum_{cd} A_{cd} \bar\alpha_c \alpha_d\right) =$$

$$g = a_0 + a_1\bar{\alpha} + a_2\bar{\alpha} + a_3\,\bar{\alpha}\alpha$$

ordre N

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"N variable Grassman algebra"

$$\int \prod_{a=1}^{N} d\bar{\alpha}_a\, d\alpha_a\; \exp\Big(-\sum_{cd} A_{cd}\,\bar{\alpha}_c\,\alpha_d\Big) = \det(A_{ab})$$

$$g = a_0 + a_1 \bar\alpha + a_2 \bar\alpha + a_3 \bar\alpha \alpha$$

$$\int d\alpha\, g = a_1 + a_3 \bar\alpha$$

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order N

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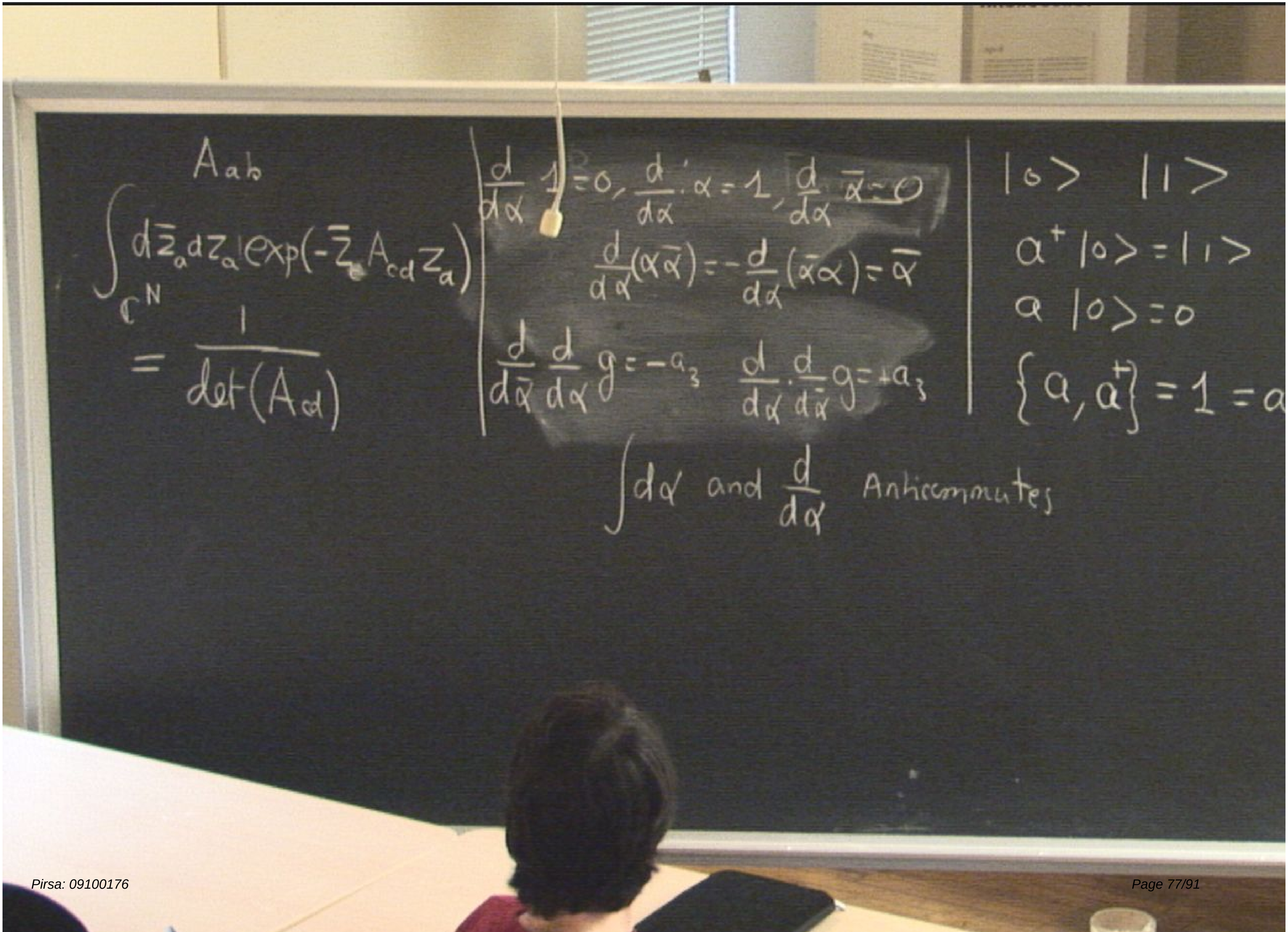
$$\boxed{\int \prod_{a=1}^{N} d\bar\alpha_a\, d\alpha_a \exp\Big(-\sum_{cd} A_{cd}\, \bar\alpha_c\, \alpha_d\Big) = \det(A_{ab})}$$


A_{ab}
$\int_{\mathbb{C}^N} d\bar{z}_a dz_a \exp(-\bar{z}_c A_{cd} z_d)$
$= \frac{1}{\det(A_{cd})}$
$\frac{d}{d\alpha} 1 = 0, \frac{d}{d\alpha} \alpha = 1, \frac{d}{d\alpha} \bar{\alpha} = 0$
$\frac{d}{d\alpha}(\alpha\bar{\alpha}) = -\frac{d}{d\alpha}(\bar{\alpha}\alpha) = \bar{\alpha}$
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$\int d\alpha$ and $\frac{d}{d\alpha}$ Anticommutes
$|0\rangle \quad |1\rangle$
$a^+ |0\rangle = |1\rangle$
$a |0\rangle = 0$
$\{a, a^+\} = 1 = a$

Fermion

$|1\rangle$ ___

$a \downarrow \quad \uparrow a^{+}$

$|0\rangle$ ___

Fermion

$|1\rangle$ ——— $a \downarrow$ $\uparrow a^{+}$ $|0\rangle$ ———

α element of G generated by α & $\bar{\alpha}$

$$|\alpha\rangle = e^{-\frac{1}{2}\bar{\alpha}\alpha}\, e^{\alpha a^{+}}|0\rangle$$

Fermion

$|1\rangle$ ——— $a \downarrow \quad \uparrow a^{+}$

$|0\rangle$ ———

α element of G generated by α & $\bar{\alpha}$

$$|\alpha\rangle = e^{-\frac{1}{2}\bar{\alpha}\alpha}\, e^{\alpha a^{+}} |0\rangle$$

$$= \left(1 - \frac{1}{2}\bar{\alpha}\alpha\right)\left(|0\rangle + \alpha|1\rangle\right)$$

$$= \left(1 - \frac{1}{2}\bar{\alpha}\alpha\right)|0\rangle + \alpha|1\rangle$$

coherent states does not leave in $\mathcal{H}$

but in $\mathcal{H} \otimes G$

Hilbert space

Fermion

$|1\rangle$ ———

$a \downarrow \quad \uparrow a^+$

$|0\rangle$ ———

α_1 α element of G generated by α & $\bar{\alpha}$

$$|\alpha\rangle = e^{-\frac{1}{2}\bar{\alpha}\alpha} \, e^{\alpha a^+} |0\rangle \leftarrow \text{standard operators}$$

KET

$$= \left(1 - \frac{1}{2}\bar{\alpha}\alpha\right)\left(|0\rangle + \alpha|1\rangle\right)$$

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coherent states does not leave in $\mathcal{H}$

but in $\mathcal{H} \otimes G = \mathcal{H}_{\text{enlarged}}$

Hilbert space

BRA

$$\langle\alpha| = \left(1 - \frac{1}{2}\bar{\alpha}\alpha\right)\langle 0| + \bar{\alpha}\langle 1|$$

$$\langle\alpha|\alpha\rangle = 1$$

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$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle\langle\alpha|$$

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$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle\langle\alpha| = \mathbb{1}$$

$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle \, \alpha^p \bar{\alpha}^q \langle\alpha|$$

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$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle \alpha^p \bar{\alpha}^q \langle\alpha| = a^p a^{+q}$$

p & $q = 0$ or 1

$$p+q=1$$

$$\langle\alpha|\alpha\rangle = 1$$

$$|\alpha\rangle\,\alpha^{p}\bar{\alpha}^{q} = \alpha^{p}\bar{\alpha}^{q}\,|-\alpha\rangle$$

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$$a \longrightarrow \alpha$$

$$a^+ \longrightarrow \bar{\alpha}$$

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$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle \alpha^p \bar{\alpha}^q \langle\alpha| \mp a^p a^{+q}$$

anh normal ordered

$p \,\&\, q = 0$ or 1

$$\mathrm{Tr}(O) = \int d\bar{\alpha}\, d\alpha \, \langle\alpha| O |-\alpha\rangle$$

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anti normal ordered

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Antiperiodic boundary condition

$$p + q = 1$$

$$\langle \alpha | \alpha \rangle = 1$$

$$|\alpha\rangle \alpha^p \bar{\alpha}^q = \alpha^p \bar{\alpha}^q |-\alpha\rangle$$

$$\int d\bar{\alpha}\, d\alpha \, |\alpha\rangle\langle\alpha| = \mathbb{1}$$

$$a \longrightarrow \alpha$$

$$a^+ \longrightarrow \bar{\alpha}$$

$$\int d\bar{\alpha}\, d\alpha \; |\alpha\rangle \alpha^p \bar{\alpha}^q \langle\alpha| = \mp\, a^p a^{+q}$$

anti normal ordered

p & $q = 0$ or 1

$$\mathrm{Tr}(O) = \int d\bar{\alpha}\, d\alpha \, \langle\alpha| O |-\alpha\rangle$$

"Anti periodic" boundary condition ← anticommuting