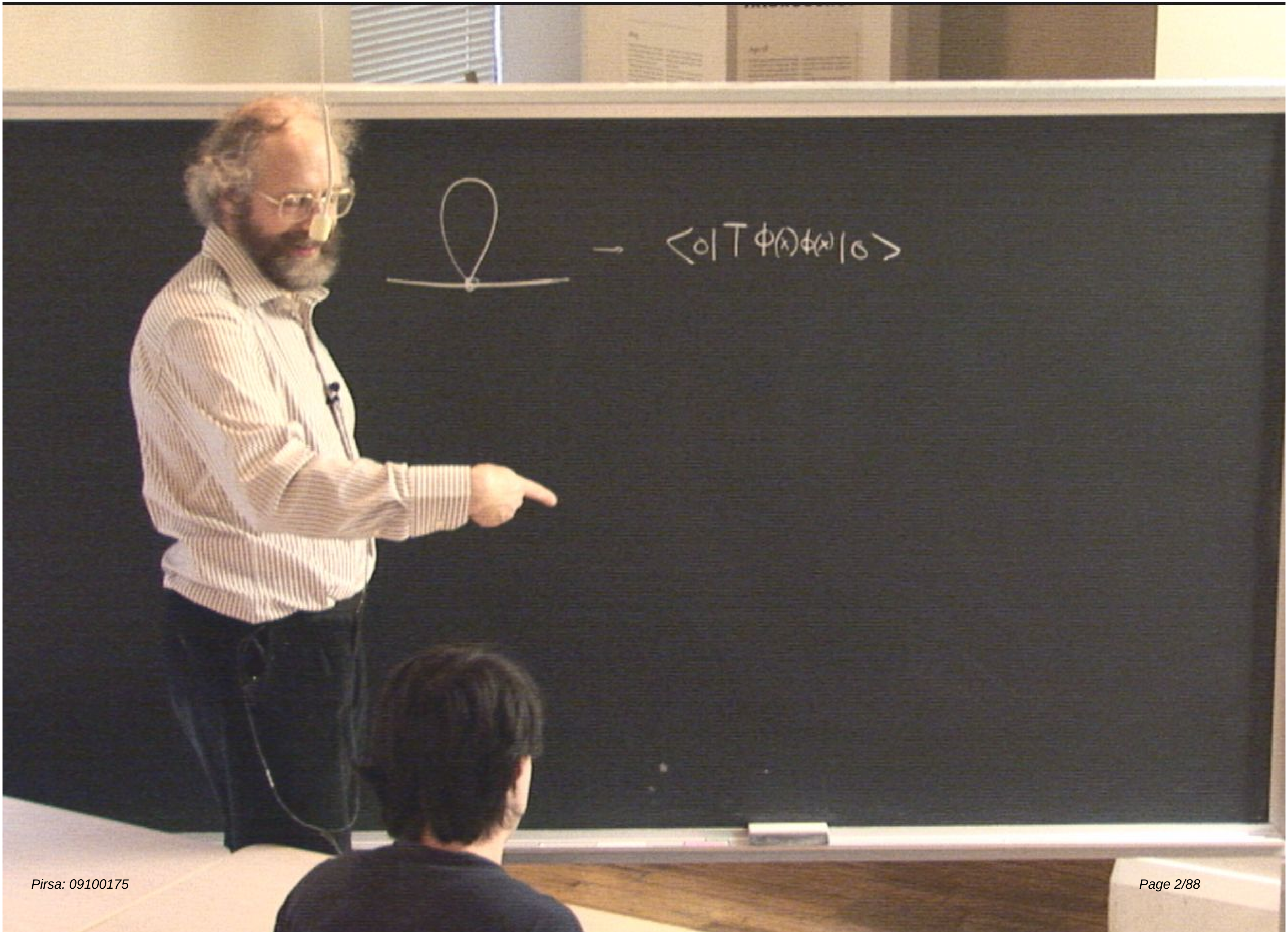
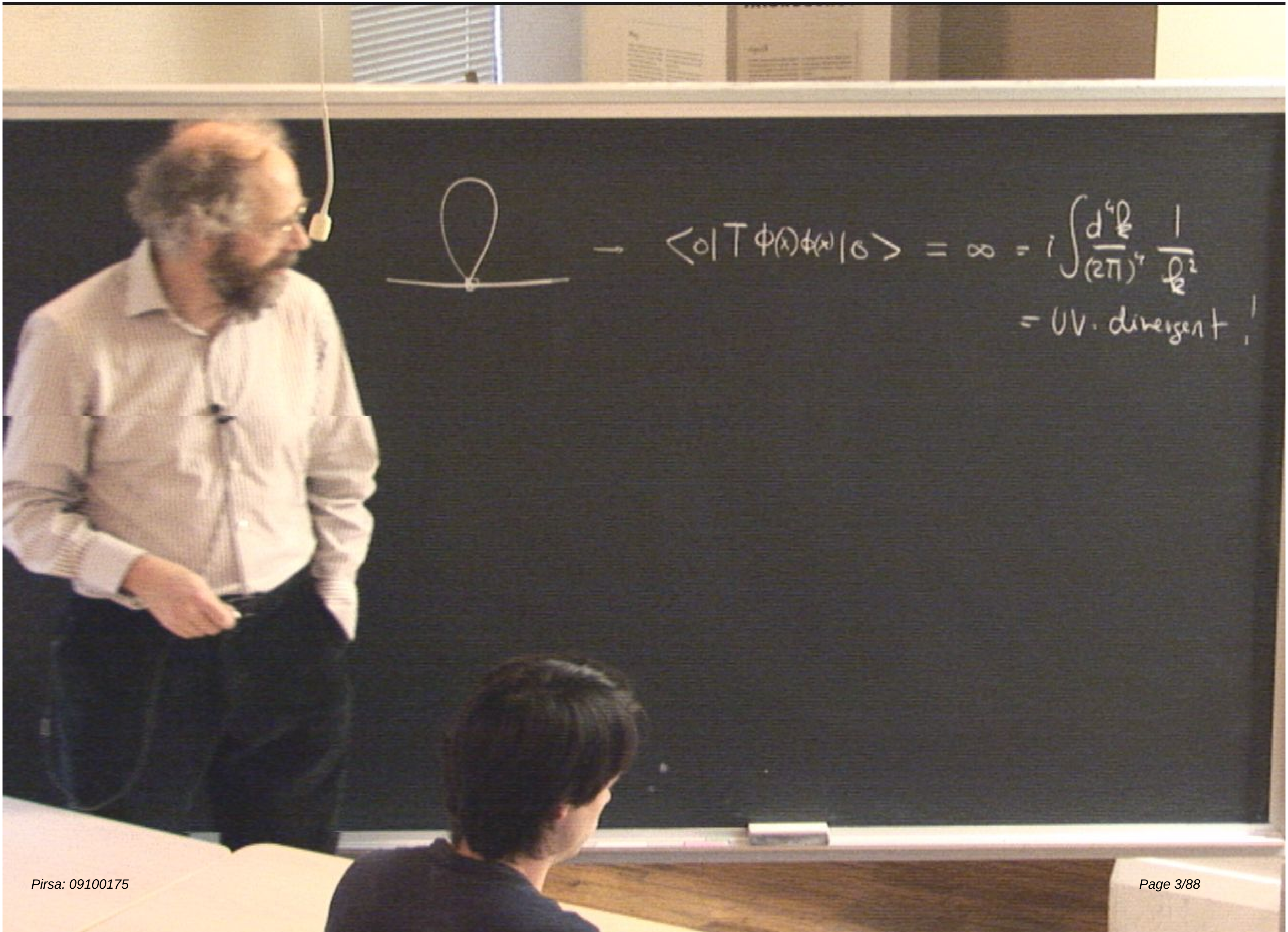
Title: Quantum Field Theory II (PHYS 603) - Lecture 4

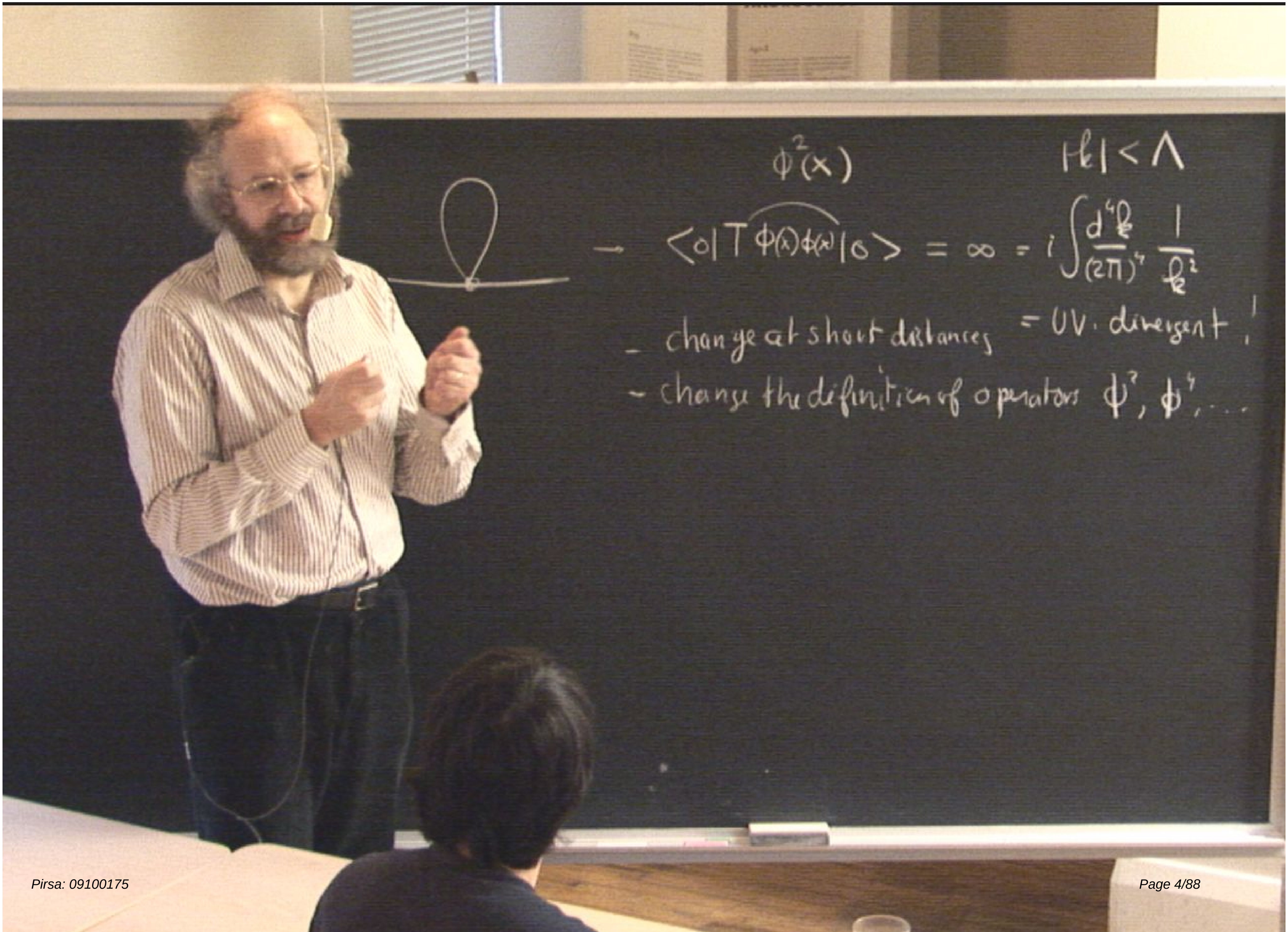
Date: Oct 29, 2009 09:00 AM

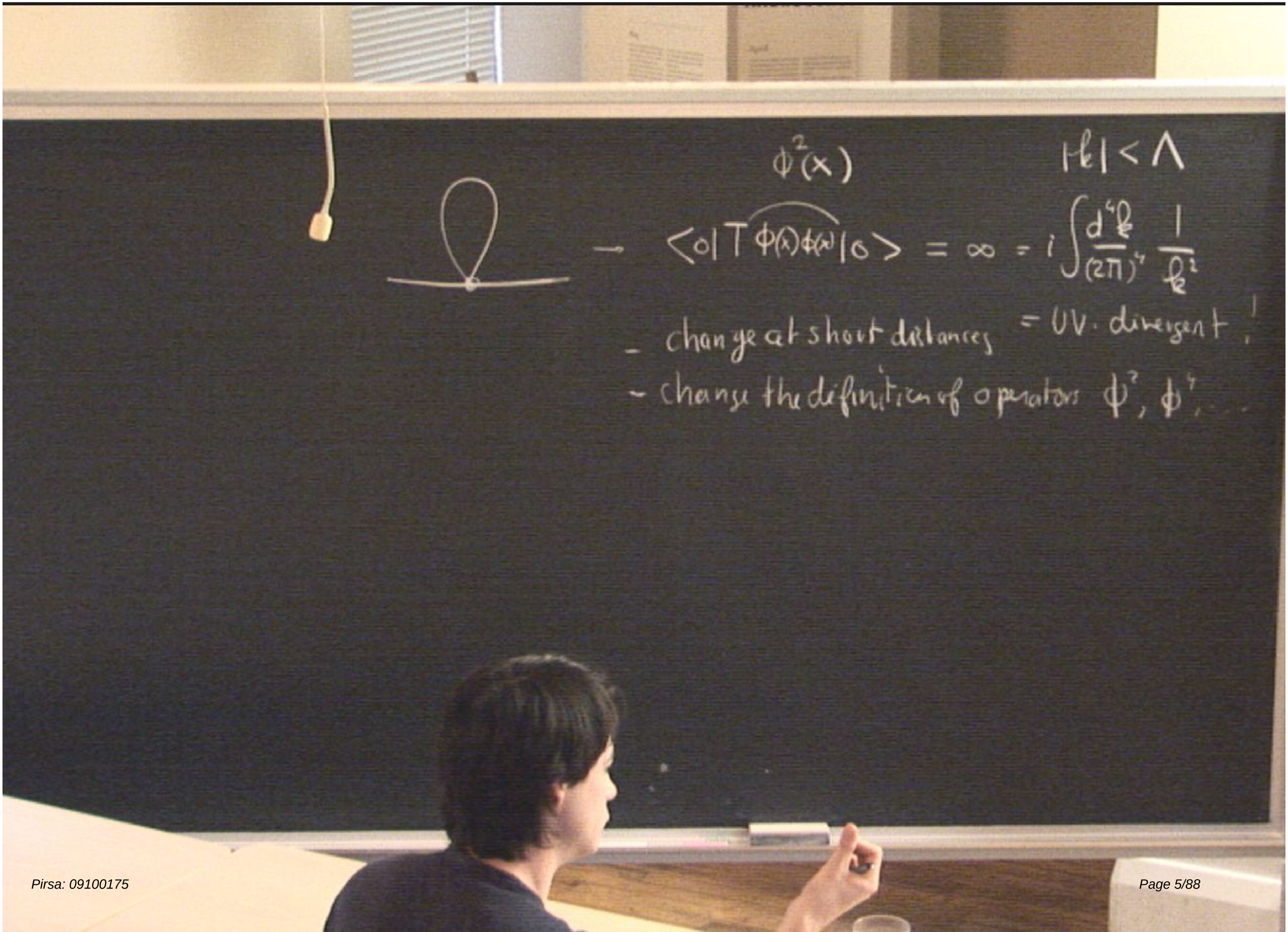
URL: http://pirsa.org/09100175

Abstract:


$\langle 0 | T\,\phi(x)\phi(x) | 0 \rangle$


$\langle 0 | T\, \phi(x)\phi(x) | 0 \rangle = \infty = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$
$=$ UV divergent !


$\phi^2(x)$
$|k| < \Lambda$
$\langle 0|T\,\phi(x)\phi(x)|0\rangle = \infty = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$
- change at short distances = UV. divergent !
- change the definition of operators $\phi^2, \phi^4, \ldots$


$\phi^2(x)$
$|k| < \Lambda$
$\rightarrow \langle 0|T\,\phi(x)\phi(x)|0\rangle = \infty = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$
= UV. divergent !
- change at short distances
- change the definition of operators $\phi^2, \phi^4, \ldots$

Generating functionals & functional calculus

Green Functions

$$\langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle$$

Generating functionals & functional calculus

een Functions

$$\langle 0|\, T\, \phi(z_1)\cdots\phi(z_N)\,|0\rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1)\cdots\phi(z_N)}{\int D[\phi]\, e^{iS[\phi]}}$$

Generating functionals & functional calculus

een Functions

$$\langle 0|\, T\, \phi(z_1)\cdots\phi(z_N)\,|0\rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1)\cdots\phi(z_N)}{\boxed{\int D[\phi]\, e^{iS[\phi]}}}$$

$$\rangle = 1$$

cuum of the interacting theory

vacuum diagrams 8, ⊖, ...

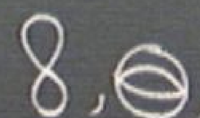
Generating functionals & functional calculus

een Functions

$${}_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) \,|0\rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z_N)}{\boxed{\int D[\phi]\, e^{iS[\phi]}}}$$

$$\langle 0 | 0 \rangle = 1$$

vacuum of the interacting theory

vacuum diagrams  , . . .

Generating functionals & functional calculus

Green Functions

$$\ldots z_N) = \langle 0|\, T\, \phi(z_1)\cdots\phi(z_N)\,|0\rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1)\cdots\phi(\ldots}{\boxed{\int D[\phi]\, e^{iS[\phi]}}}$$

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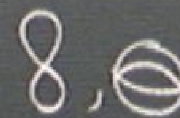
vacuum of the interacting theory

vacuum diagrams  , . . .

$$S = \int d^4x \left[-\frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{m^2}{2}\phi^2\right]$$

Generating functionals & functional calculus

Green Functions

$$G^{(N)}(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\boxed{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z_N)}}{\boxed{\int D[\phi]\, e^{iS[\phi]}}}$$

$$\langle 0 | 0 \rangle = 1$$

vacuum of the interacting theory

vacuum diagrams  , . . .

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Generating functionals & functional calculus

Green Functions

$$(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi}{\boxed{\int D[\phi]\, e^{iS[\phi]}}}$$

$$\langle 0 | 0 \rangle = 1$$

↑

vacuum of the interacting theory

vacuum diagrams , . . .

$$S = \int d^4x \left[-\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

Functionnal

$$j\cdot\phi = \int d^4x\, j(x)\, \phi(x)$$

classical "source term" function.

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}\left[S[\phi] + j\cdot\phi\right]}$$

$$j\cdot\phi = \int d^4x\, j(x)\, \phi(x)$$

↑ Functional

$j(x)$ ↑ "classical source term" function.

Expand this in $j(x)$

$Z[j=0]$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

$$j\cdot\phi = \int d^4x\, j(x)\,\phi(x)$$

Functional

classical "source term" function.

Expand this in $j(x)$

$$Z[j=0] \qquad Z[j] = \sum_{K=0}^{\infty} \frac{1}{K!}\left(\frac{i}{\hbar}\right)^K \int dx_1 \cdots dx_K \int D[\phi]\, e^{\frac{i}{\hbar}S[\phi]}\, \phi(x_1)\cdots\phi(x_K)$$

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Functional derivatives

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

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Functional derivatives

$$\frac{\delta \cdots}{\delta j(x)}$$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

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Functional derivatives

$$\frac{\delta \cdots}{\delta j(x)} = \text{term containing } j(x) \text{ in this integral}$$

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$$j(x_1) \cdots j \ldots$$

Functional derivatives

$$\frac{\delta \cdots}{\delta j(x)} = \text{term containing } j(x) \text{ in this integral}$$

$$\frac{\delta^{K}}{\delta j(x_1) \cdots \delta(x_K)} Z[j] =$$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

↑ Functional

$$j\cdot\phi = \int d^4x\, j(x)\,\phi(x)$$

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$$j(x_1)\cdots j(x_K)$$

Functional derivatives

$$\frac{\delta \cdots}{\delta j(x)} = \text{term containing } j(x) \text{ in this integral}$$

$$\frac{\delta^k}{\delta j(y_1)\cdots\delta(x_K)} Z[j] =$$

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Functional derivatives

$$\frac{\delta \cdots}{\delta j(x)} = \text{term containing } j(x) \text{ in this integral dropping the } j \text{ and the integral}$$

$$\frac{\delta^k}{\delta j(x_1)\cdots\delta(x_K)} Z[j] =$$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]} \qquad j\cdot\phi = \int d^4x\, j(x)\,\phi(x)$$

Functional

"classical source term" function.

Expand this in $j(x)$

$$Z[j=0]\,,\quad Z[j] = \sum_{K=0}^{\infty} \boxed{\frac{1}{K!}\left(\frac{i}{\hbar}\right)^K \int dx_1 \cdots dx_K \int D[\phi]\, e^{\frac{i}{\hbar}S[\phi]}\, \phi(x_1)\cdots\phi(x_K)\, j(x_1)\cdots j(x_K)}$$

Functional derivatives

$\frac{\delta \cdots}{\delta j(x)}$ = term containing $j(x)$ in this integral dropping the j and the integr

$$\boxed{\frac{\delta^K}{\delta j(y_1)\cdots\delta j(y_K)} Z[j]\Big|_{J=0} := \left(\frac{i}{\hbar}\right)^K \int D[\phi]\, e^{\frac{i}{\hbar}S[\phi]}\, \phi(y_1)\cdots\phi(y_K)}$$

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}[S[\phi] + j\cdot\phi]}$$

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Functional derivatives

$\frac{\delta \cdots}{\delta j(x)}$ = term containing $j(x)$ in this integral dropping the j and the integral

$$\boxed{\frac{\delta^k}{\delta j(x_1)\cdots\delta j(x_M)} Z[j]\Big|_{J=0} := \left(\frac{i}{\hbar}\right)^M \int D[\phi]\, e^{\frac{i}{\hbar}S[\phi]}\, \phi(x_1)\cdots\phi(x_M)}$$

Generating functionals & functional calculus

Green Functions

$$G^{(N)}(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z_N)}{\int D[\phi]\, e^{iS[\phi]}}$$

$$\boxed{G^{N}(z_1, z_N) = \prod_{a=1}^{N} \left(\frac{i}{\hbar} \frac{\delta}{\delta J(z_a)} \right) Z[J] \Big|_{J=0} \Big/ Z[J=0]}$$

Generating functionals & functional calculus : $\boxed{\text{real time}}$

Green Functions

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Generating functionals & functional calculus : $\boxed{\text{real time}}$

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$Z[$

Euclidean time:

$$Z[J] = \int D[\phi]\, e^{-\frac{1}{\hbar}(S_E[\phi] - j \cdot \phi)}$$

$$G_E^N(z_1 \ldots z_N)$$

Generating functionals & functional calculus: $\boxed{\text{real time}}$

Green Functions

$$G^{(N)}(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z_N)}{\int D[\phi]\, e^{iS[\phi]}}$$

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Generating functionals & functional calculus : $\boxed{\text{real time}}$

Green Functions

$$G^{(N)}(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z}{\int D[\phi]\, e^{iS[\phi]}}$$

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Z[

Euclidean time: $Z[J] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left(S_E[\phi] - j \cdot \phi\right)}$

$$G_E^{N}(z_1 .. z_N) = \langle \phi(z_1) .. \phi(z_N) \rangle = \prod_a \left(\hbar \frac{\delta}{\delta j(z_a)} \right) Z[J]$$

Generating functionals & functional calculus : $\boxed{\text{real time}}$

Green Functions

$$G^{(N)}(z_1 \cdots z_N) = \langle 0 | T\, \phi(z_1) \cdots \phi(z_N) | 0 \rangle = \frac{\int D[\phi]\, e^{iS[\phi]}\, \phi(z_1) \cdots \phi(z_N)}{\int D[\phi]\, e^{iS[\phi]}}$$

$$\boxed{G^{N}(z_1, z_N) = \prod_{a=1}^{N} \left(\frac{\hbar}{i} \frac{\delta}{\delta j(z_a)} \right) Z[j] \Big|_{J=0} \Big/ Z[J=0]}$$

Euclidean time:

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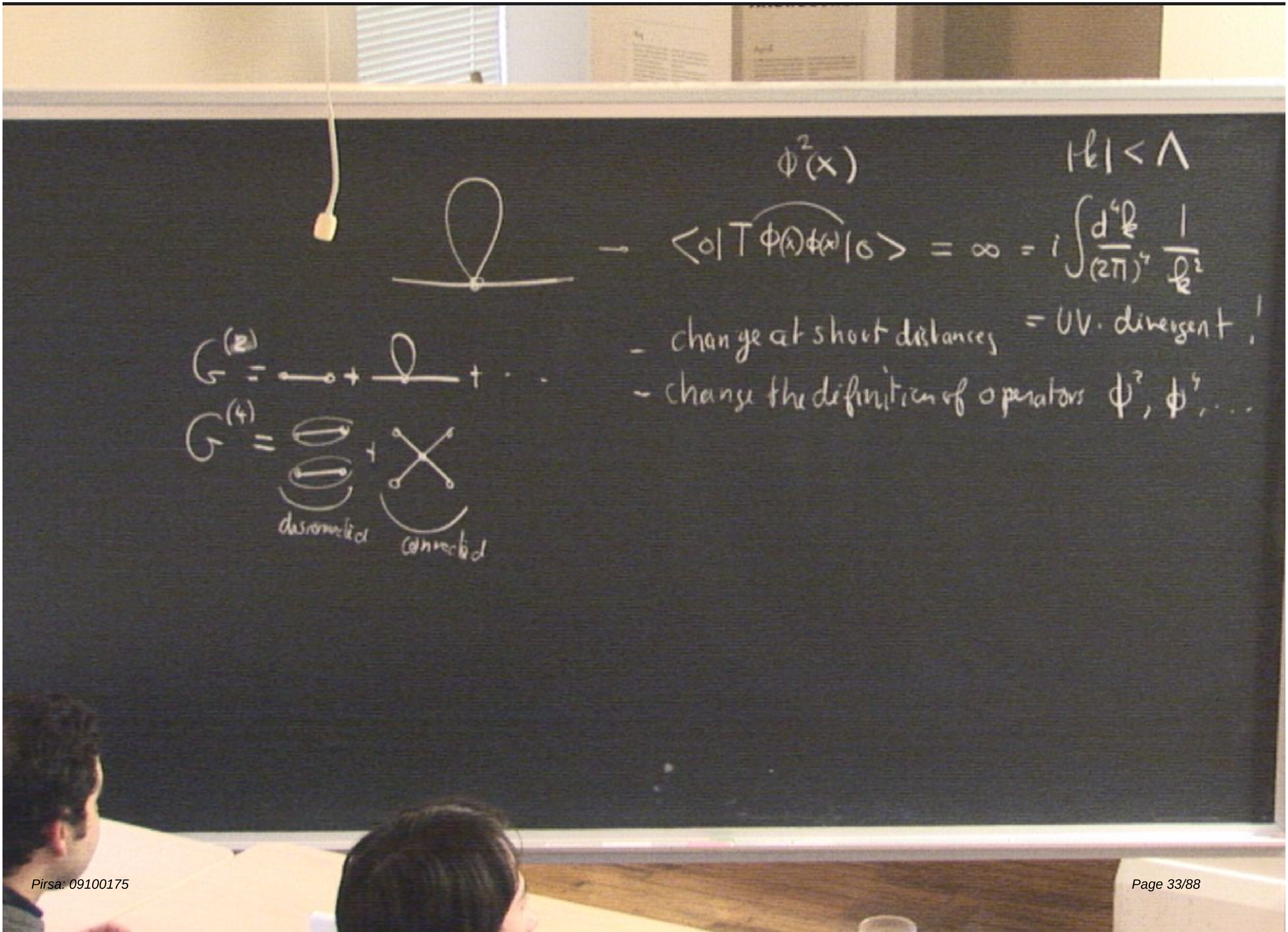
$$\boxed{G_E^N(z_1 \cdots z_N) = \langle \phi(z_1) \cdots \phi(z_N) \rangle = \prod_a \left(\hbar \frac{\delta}{\delta j(z_a)} \right) Z[J] \Big|_{J=0} \Big/ Z[0]}$$

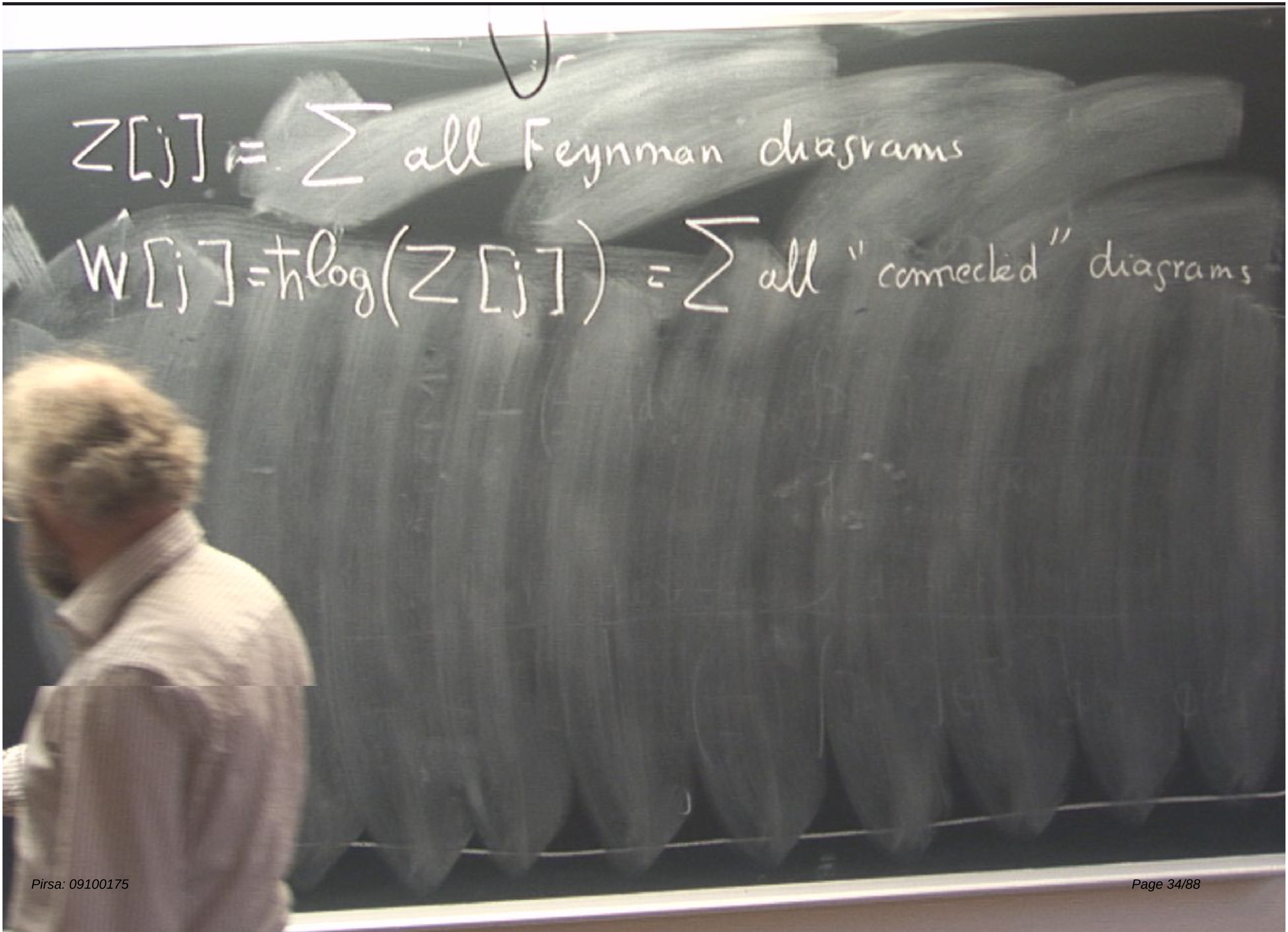
$$Z[j] = \sum \text{all Feynman diagrams}$$

$$W[j] = \log(Z[j])$$

$$Z[j] = \sum \text{all Feynman diagrams}$$

$$W[j] = \hbar \log(Z[j])$$


$\phi^2(x)$
$|k| < \Lambda$
$\langle 0| T \phi(x)\phi(x) |0\rangle = \infty = i\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2}$
= UV. divergent !
- change at short distances
- change the definition of operators $\phi^2, \phi^4, \ldots$
$G^{(2)} =$ + + ...
$G^{(4)} =$ +
disconnected
connected


$Z[j] = \sum$ all Feynman diagrams
$W[j] = \hbar \log(Z[j]) = \sum$ all "connected" diagrams

$$Z[j] = \sum \text{all Feynman diagrams}$$

$$W[j] = \hbar \log(Z[j]) = \sum \text{all ``connected'' diagrams}$$

$$Z[0] = \sum \text{vaccum diagrams}$$

$$= 1$$

$$Z[j] = \sum \text{all Feynman diagrams}$$

$$W[j] = \hbar \log(Z[j]) = \sum \text{all ``connected'' diagrams}$$

$$Z[0] = \sum \text{vaccum diagrams}$$

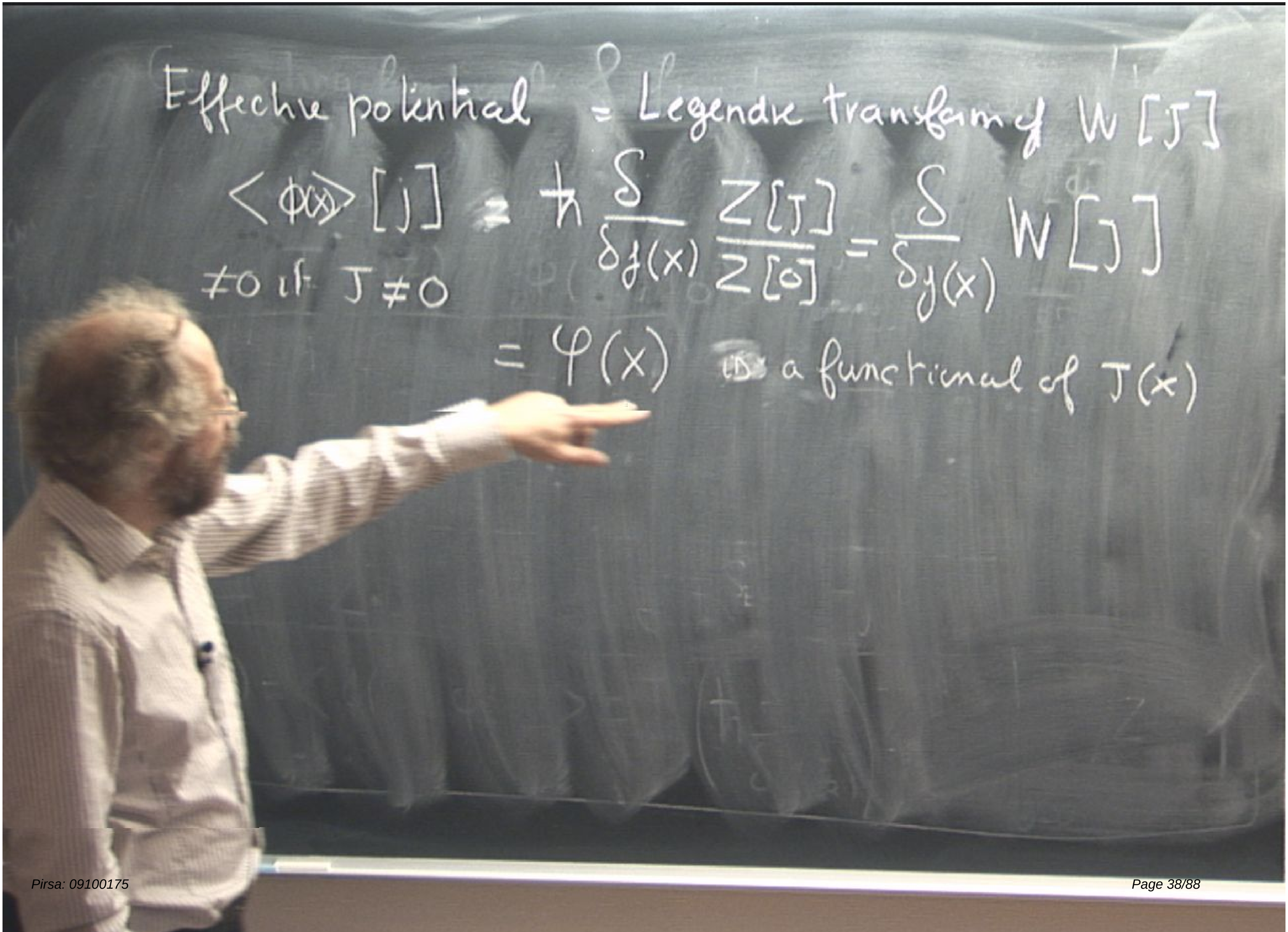
$= 1 +$  $+$  $+$  $+ \cdots$

$$Z[j] = \sum \text{all Feynman diagrams}$$

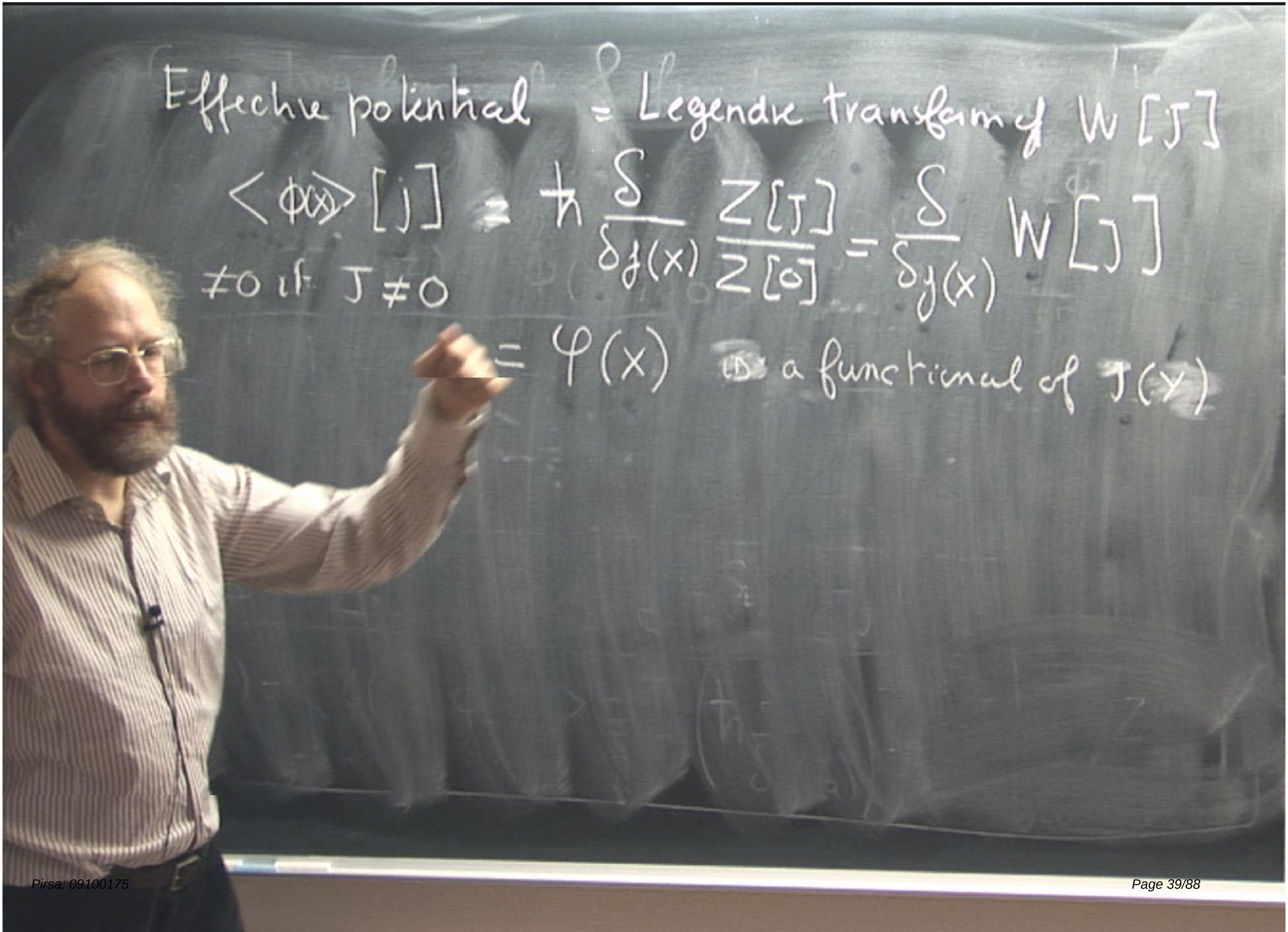
$$W[j] = \hbar \log(Z[j]) = \sum \text{all "connected" diagrams}$$

$$Z[0] = \sum \text{vaccum diagrams}$$

$$= 1 + \frac{1}{8}\, 8 + \frac{1}{48}\, \ominus + 8\,8\, \underset{\substack{\shortparallel \\ \left(\frac{1}{8}\right)^2 \cdot \frac{1}{2}}}{\frac{1}{128}} + \ldots = \exp\left(\frac{1}{2}\, 8\right)$$



Effective potential = Legendre transform of $W[J]$
$\langle \phi(x) \rangle [J] = \hbar \frac{\delta}{\delta J(x)} \frac{Z[J]}{Z[0]} = \frac{\delta}{\delta J(x)} W[J]$
$\neq 0$ if $J \neq 0$
$= \varphi(x)$ is a functional of $J(x)$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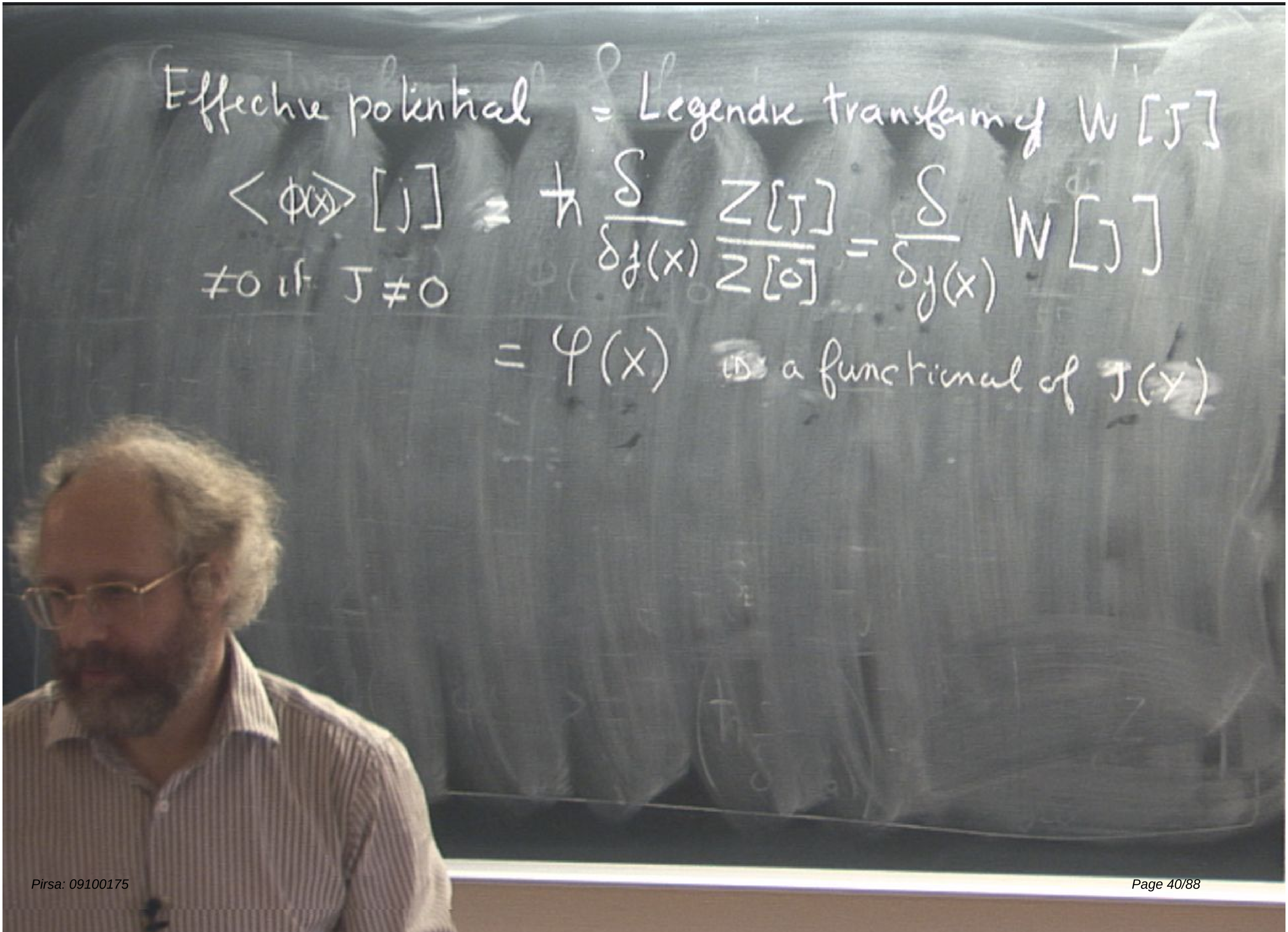
Effective potential = Legendre transform of W[J]
$\langle \phi(x) \rangle [j] = \hbar \frac{\delta}{\delta j(x)} \frac{Z[J]}{Z[0]} = \frac{\delta}{\delta j(x)} W[j]$
$\neq 0$ if $J \neq 0$
$= \varphi(x)$ is a functional of $J(y)$

Effective potential = Legendre transform of $W[J]$

$$\langle \phi(x) \rangle [j] = \hbar \frac{\delta Z[j]}{\delta j(x)} \Big/ Z[j] = \frac{\delta}{\delta j(x)} W[j]$$

$\neq 0$ if $J \neq 0$

$$= \varphi(x)$$ is a functional of $J(y)$

Effective potential = Legendre transform of $W[J]$

$$\langle \phi(x) \rangle [j] = \hbar \frac{\delta Z[j]}{\delta j(x)} \Big/ Z[j] = \frac{\delta}{\delta j(x)} W[j]$$

$\neq 0$ if $J \neq 0$

$$= \varphi(x)$$ is a functional of $J(y)$

change of variable from $J(y) \to \varphi(x)$

$$\boxed{\Gamma[\varphi] = j \cdot \varphi - W[j] = \int d^4x\, j(x)\varphi(x) - W[j]}$$

$$\varphi(x) = \frac{\delta}{\delta j(x)} W[j]$$ Background Field.

action

Effective potential = Legendre transform of $W[J]$

$$\langle \phi(x) \rangle [j] = \hbar \frac{\delta Z[j]}{\delta j(x)} \Big/ Z[j] = \frac{\delta}{\delta j(x)} W[j]$$

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$$\boxed{\Gamma[\varphi] = j\cdot\varphi - W[J] = \int d^4x\, j(x)\varphi(x) - W[j]}$$

$$\boxed{\varphi(x) = \frac{\delta}{\delta j(x)} W[j]}$$

Background Field.

What does this generates?

$$Z[j] = \int D[\phi]\, e^{\frac{i}{\hbar}\left[S[\phi] - j\phi\right]}$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: ϕ_c

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}[S[\phi] - j\cdot\phi]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$Z[j] = \int D[\phi]\, e^{\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow$ Integrate over $\tilde{\phi}$

Function(nal) of j

$S[$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\quad \phi_c(y) \Leftarrow \dfrac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

Function(nal) of j

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\, S''(\phi_c)\, \tilde{\phi}$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \dfrac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

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Function(nal) of j

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi}$$

$$\int dx\,dy\, \tilde{\phi}(x)\tilde{\phi}(y)\, \frac{\delta^2}{\delta\phi(x)\delta\phi(y)} S[\phi]$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}[S[\phi] - j\cdot\phi]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

$\uparrow$

Function(nal) of j

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi}$$

Integral Kernel of $S''(\phi_c)$

$$\int dx\,dy\, \tilde{\phi}(x)\tilde{\phi}(y) \boxed{\frac{\delta^2}{\delta\phi(x)\delta\phi(y)} S[\phi]}$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

Function(nal) of j

Integral Kernel of $S''(\phi_c)$

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi} + O(\tilde{\phi}^3)$$

Semiclassical expansion

$$\int dx\,dy\,\tilde{\phi}(x)\tilde{\phi}(y)\left(\frac{\delta^2}{\delta\phi(x)\delta\phi(y)} S[\phi]\right)$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

$\uparrow$ Function(nal) of j

Integral Kernel of $S''(\phi_c)$

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi} + \cancel{O(\tilde{\phi}^3)}$$

Semiclassical expansion

$$\int dx\,dy\, \tilde{\phi}(x)\tilde{\phi}(y) \boxed{\frac{\delta^2}{\delta\phi(x)\delta\phi(y)} S[\phi]}$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\cdot\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

Integral Kernel of $S''(\phi_c)$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

Function(nal) of j

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\,\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi} + \cancel{O(\tilde{\phi}^3)}$$

Semiclassical expansion

$$\boxed{\int dx\,dy\,\tilde{\phi}(x)\tilde{\phi}(y)\left(\frac{\delta^2}{\delta\phi(x)\,\delta\phi(y)} S[\phi]\right)}$$

$$\boxed{Z[j] = \exp\left(-\frac{1}{\hbar}\left(S[\phi_c] - j\phi_c\right)\right)\mathrm{Det}\left(S''(\phi_c)\right)^{-1/2}}$$

$$Z[j] = \int D[\phi]\, e^{-\frac{1}{\hbar}\left[S[\phi] - j\phi\right]}$$

Saddle point Approximation

minimum of $S[\phi] - j\cdot\phi$: $\phi_c(y) \Leftarrow \frac{\delta S[\phi]}{\delta\phi(x)} - j(x) = 0$

$$\phi(x) = \phi_c(x) + \tilde{\phi}(x) \leftarrow \text{Integrate over } \tilde{\phi}$$

$\uparrow$ Function(nal) of j

Integral Kernel of $S''(\phi_c)$

$$S[\phi] - j\phi = \left(S[\phi_c] - j\phi_c\right) + \frac{1}{2}\tilde{\phi}\cdot S''(\phi_c)\cdot\tilde{\phi} + \cancel{O(\tilde{\phi}^3)}$$

$$\int dx\,dy\, \tilde{\phi}(x)\tilde{\phi}(y) \left(\frac{\delta^2}{\delta\phi(x)\delta\phi(y)} S[\phi]\right)$$

Semiclassical expansion

$$\boxed{Z[j] = \exp\left(-\frac{1}{\hbar}\left(S[\phi_c] - j\phi_c\right)\right) \mathrm{Det}\left(S''(\phi_c)\right)^{-1/2}}$$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2} \operatorname{Tr} \log\left(S''[\phi_c]\right) + O(\hbar^2)$$

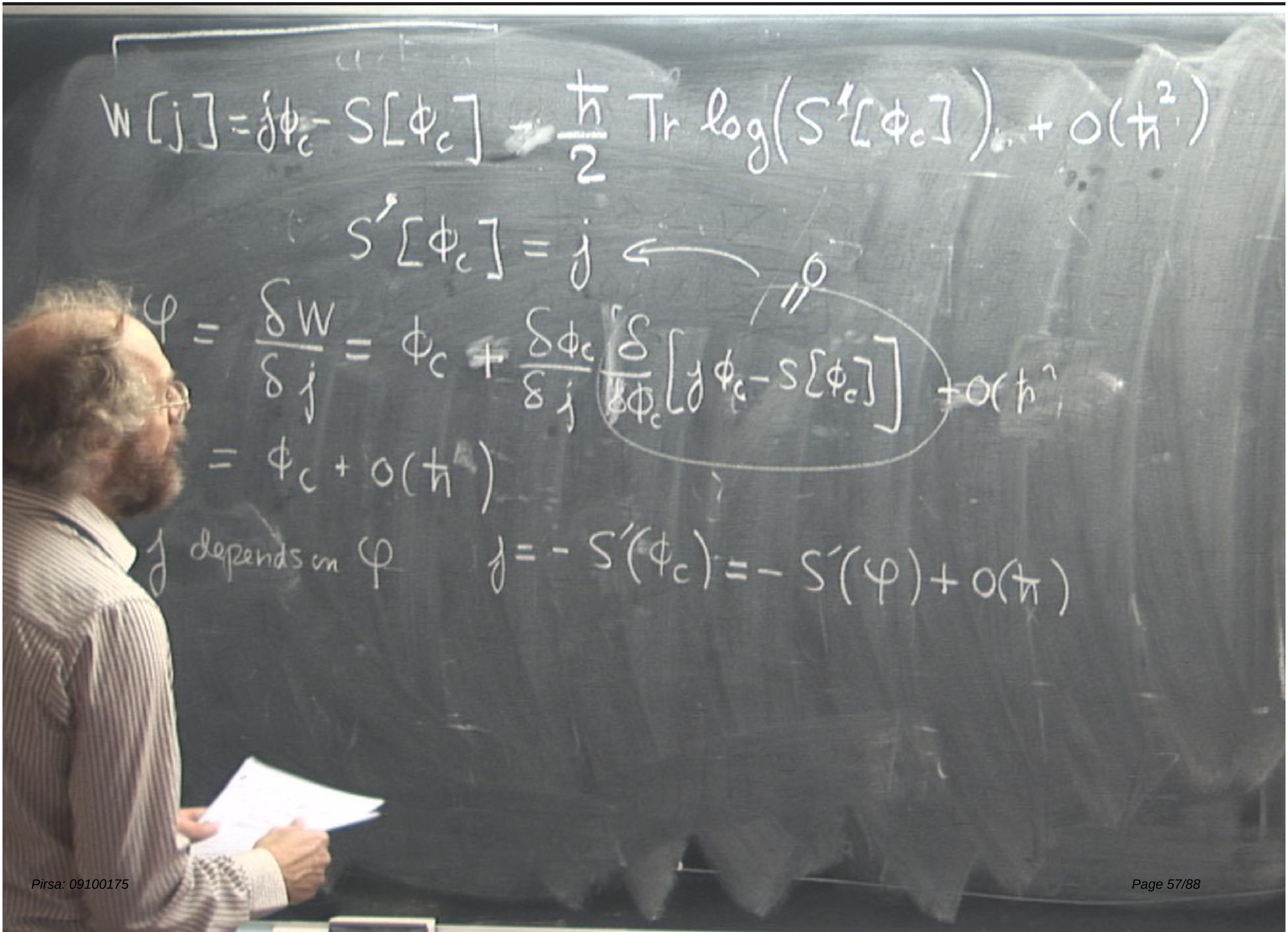
$$S'[\phi_c] = j$$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2} \operatorname{Tr} \log\left(S''[\phi_c]\right) + O(\hbar^2)$$

$$S'[\phi_c] = j$$

$$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j} \underbrace{\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right]}_{=0} + O(\hbar^2)$$

$$= \phi_c + O(\hbar^2)$$


$W[j] = j\phi_c - S[\phi_c] + \frac{\hbar}{2} \mathrm{Tr} \log(S''[\phi_c]) + O(\hbar^2)$
$S'[\phi_c] = j$
0
$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j} \frac{\delta}{\delta \phi_c}[j\phi_c - S[\phi_c]] + O(\hbar)$
$= \phi_c + O(\hbar)$
j depends on φ
$j = -S'(\phi_c) = -S'(\varphi) + O(\hbar)$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\phi_c]\right) + O(\hbar^2)$$

$$S'[\phi_c] = j$$

$$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j}\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right] + O(\hbar)$$

$$\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right] = 0$$

$$\boxed{\varphi = \phi_c + O(\hbar)}$$

j depends on φ

$$\boxed{j = +S'(\phi_c) = +S'(\varphi) + O(\hbar)}$$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2} \operatorname{Tr} \log\left(S''[\phi_c]\right) + O(\hbar^2)$$

$$S'[\phi_c] = j$$

$$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j} \frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right] + O(\hbar)$$

$$\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right] = 0$$

$$\boxed{\varphi = \phi_c + O(\hbar)}$$

j depends on φ

$$\boxed{j = +S'(\phi_c) = +S'(\varphi) + O(\hbar)}$$

$$\Gamma[\varphi] = j\varphi - W[j] = \cancel{S'(\varphi)\varphi - S'(\varphi)\phi_c} + S[\varphi] + \frac{\hbar}{2} \operatorname{Tr} \log S''($$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\phi_c]\right) + O(\hbar^2)$$

$$S'[\phi_c] = j$$

$$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j}\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right] + O(\hbar)$$

$$\boxed{\varphi = \phi_c + O(\hbar)}$$

j depends on φ

$$\boxed{j = + S'(\phi_c) = + S'(\varphi) + O(\hbar)}$$

$$\Gamma[\varphi] = j\varphi - W[j] = \cancel{S'(\varphi)\varphi - S'(\varphi)\phi_c} + S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\log[S''(\varphi)]$$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

$$W[j] = j\phi_c - S[\phi_c] - \frac{\hbar}{2} \operatorname{Tr} \log\left(S''[\phi_c]\right) + O(\hbar^2)$$

$$S'[\phi_c] = j$$

$$\varphi = \frac{\delta W}{\delta j} = \phi_c + \frac{\delta \phi_c}{\delta j} \underbrace{\frac{\delta}{\delta \phi_c}\left[j\phi_c - S[\phi_c]\right]}_{0} + O(\hbar^2)$$

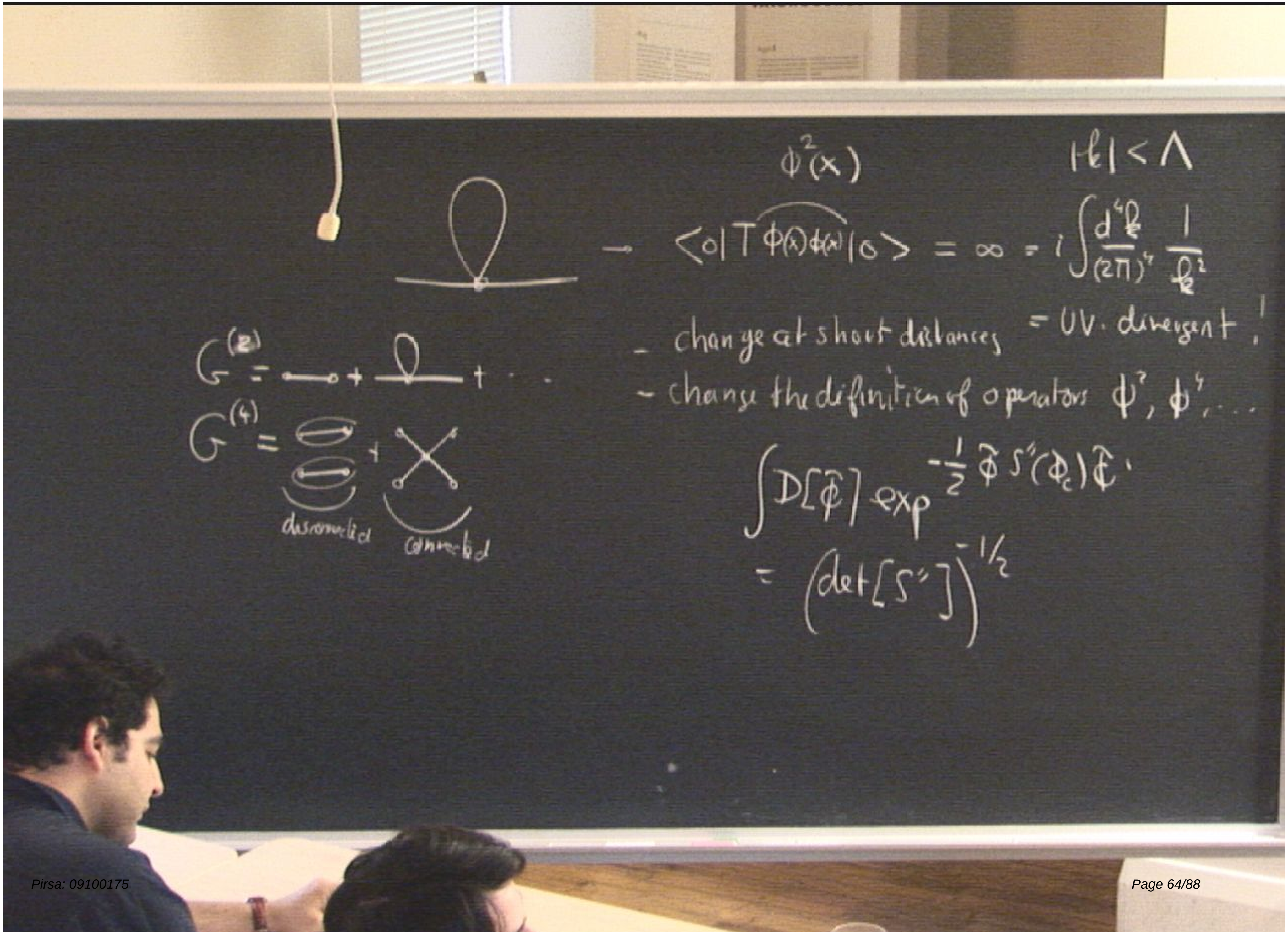
$$\boxed{\varphi = \phi_c + O(\hbar^2)}$$

j depends on φ $\quad \boxed{j = +S'(\phi_c) = +S'(\varphi) + O(\hbar)}$

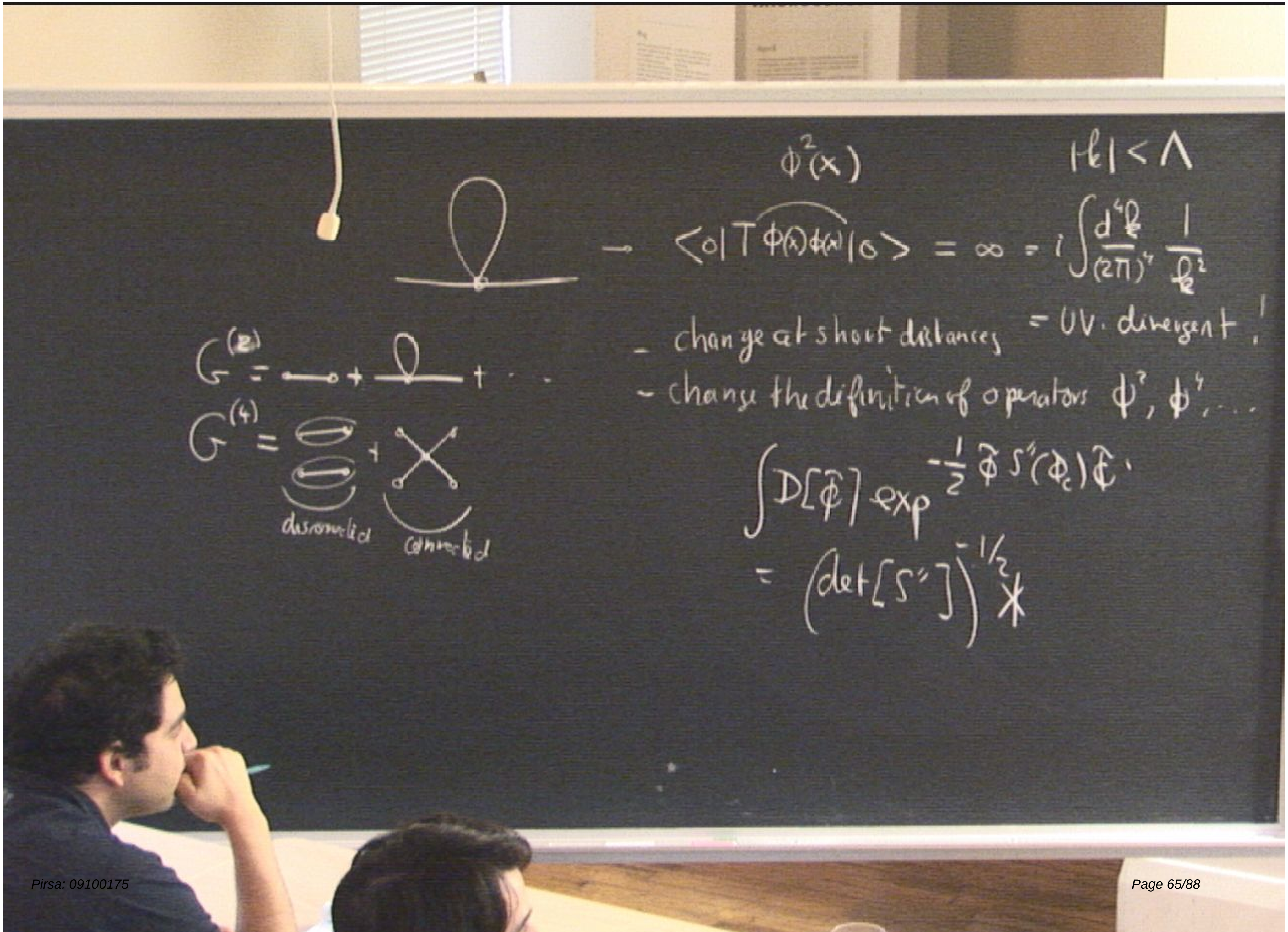
$$\Gamma[\varphi] = j\varphi - W[j] = \underbrace{S'(\varphi)\varphi - \cancel{S'(\varphi)\phi_c}}_{O(\hbar^2)} + S[\varphi] + \frac{\hbar}{2}\operatorname{Tr}\log S''($$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

Any action and any QFT (Bosonic Field . ϕ, A_μ, . . .



$\phi^2(x)$
$|k| < \Lambda$
$\rightarrow \langle 0| T \phi(x)\phi(x) |0\rangle = \infty = i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}$
– change at short distances
= UV. divergent !
– change the definition of operators $\phi^2, \phi^4, \ldots$
$G^{(2)} = \; + \; + \cdots$
$G^{(4)} = \; + \;$
disconnected
connected
$\int D[\tilde{\phi}] \exp -\frac{1}{2}\tilde{\phi} S''(\phi_c)\tilde{\phi}$
$= \left(\det[S'']\right)^{-1/2}$


$\phi^2(x)$
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$\int D[\tilde{\phi}] \exp^{-\frac{1}{2}\tilde{\phi} S''(\phi_c)\tilde{\phi}}$
$= (\det[S''])^{-1/2}$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

Any action and any QFT (Bosonic Field . ϕ, A_μ, . .)

$$S[\varphi] = \int d^4x \left[\varphi \frac{1}{2}(-\Delta + m^2)\varphi + \frac{g}{4!}\varphi^4\right]$$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

Any action and any QFT (Bosonic Field. ϕ, A_μ, . .)

$$S[\varphi] = \int d^4x \left[\frac{1}{2}\varphi(-\Delta + m^2)\varphi + \frac{g}{4!}\varphi^4\right]$$

Euclidean time

$S''[\varphi]$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + o(\hbar^2)}$$

Any action and any QFT (Bosonic Field. ϕ, A_μ, . .)

$$S[\varphi] = \int d^4x \left[\varphi \frac{1}{2}(-\Delta + m^2)\varphi + \frac{g}{4!}\varphi^4\right]$$

Euclidean time

$$S''[\varphi]_{xy} = \left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)_{xy}$$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + o(\hbar^2)}$$

Any action and any QFT (Bosonic Field. ϕ, A_μ, . .)

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Any action and any QFT (Bosonic Field. $\phi, A_\mu, \dots$)

$$S[\varphi] = \int d^4x \left[\varphi \frac{1}{2}(-\Delta + m^2)\varphi + \frac{g}{4!}\varphi^4\right]$$

Euclidean time

$$S''[\varphi]_{xy} = \left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)_{xy} = (-\Delta + m^2)\left(1 + \frac{g}{2}(-\Delta + m^2)^{-1}\cdot\varphi^2\right)$$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

Any action and any QFT (Bosonic Field. ϕ, A_μ, . .)

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Euclidean time

$$S''[\varphi]_{XY} = \left(-\Delta + m^2 + \frac{g}{2}\varphi^2\right)_{XY} = (-\Delta+m^2)\left(1 + \frac{g}{2}\underbrace{(-\Delta+m^2)^{-1}}_{\text{Propagator of the free field}}\cdot\,\varphi^2\right)$$

$$\log[S''[\varphi]] = \log(-\Delta+m^2) + \frac{g}{2}G_0\varphi^2 - \frac{g^2}{8}G_0\varphi^2G_0\varphi^2 + \frac{g^3}{3}\frac{1}{8}G_0\varphi^2G_0\varphi^2G_0\varphi^2 \ldots$$

$G_0\cdot\varphi^2$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + o(\hbar^2)}$$

Any action and any QFT (Bosonic Field . ϕ, A_μ, . .)

$$S[\varphi] = \int d^4x \left[\varphi \frac{1}{2}(-\Delta + m^2)\varphi + \frac{g}{4!}\varphi^4\right]$$

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Propagator of the free field

$G_0 \cdot \varphi^2$

$$\frac{\hbar}{2} \mathrm{Tr} \log\left(S''[\varphi]\right) = \frac{\hbar}{2} \mathrm{Tr} \log\left(-\Delta + m^2\right)$$

$+$

$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

$$+\frac{\hbar}{4}\int dx\; G_0(x-x)\,\varphi^2(x)$$

$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

$$+\frac{\hbar}{4}\int dx\, G_0(x-x)\,\varphi^2(x) - \frac{\hbar}{2}\,\frac{1}{2}\left(\frac{g}{2}\right)^2\int dx\,dy\; G_0(x-y)\,\varphi^2(y)\, G_0(y-x)\,\varphi^2(x)$$

$$+\frac{\hbar}{2}\,\frac{1}{3}\left(\frac{g}{2}\right)^3\int dx\,dy\,dz\; G_0(x-y)\,\varphi^2(y)\, G_0(y-z)\,\varphi^2(z)\, G_0(z-x)\,\varphi^2(x)$$

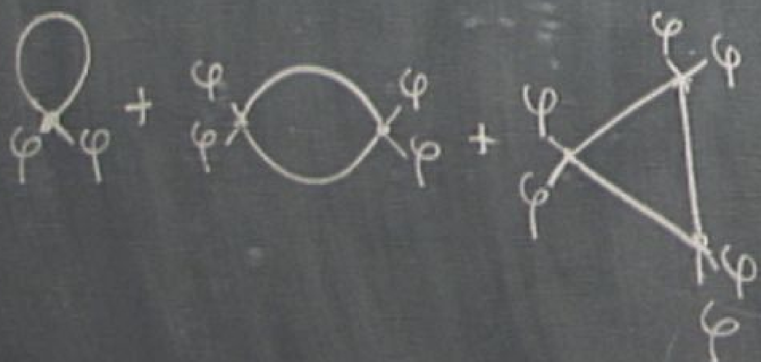
$$+\cdots$$

$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right)=\frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

$$+\frac{\hbar}{2}\frac{g}{2}\int dx\; G_0(x-x)\,\varphi^2(x)-\frac{\hbar}{2}\,\frac{1}{2}\left(\frac{g}{2}\right)^2\int dx\,dy\; G_0(x-y)\,\varphi^2(y)\,G_0(y-x)\,\varphi^2(x)$$

$$+\frac{\hbar}{2}\,\frac{1}{3}\left(\frac{g}{2}\right)^3\int dx\,dy\,dz\; G_0(x-y)\,\varphi^2(y)\,G_0(y-z)\,\varphi^2(z)\,G_0(z-x)\,\varphi^2(x)$$

$$+\;\ldots$$



$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

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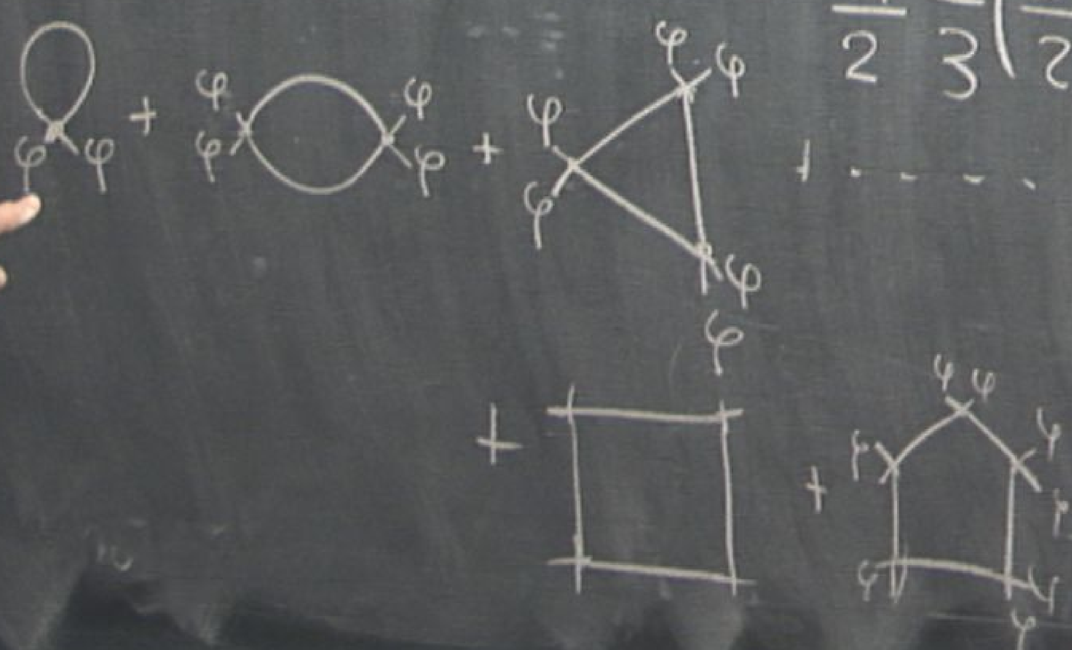
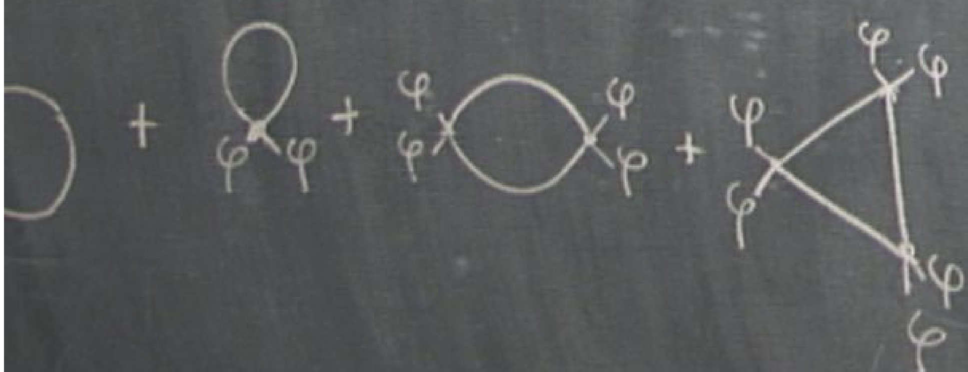
$$+\;\cdots\cdots$$

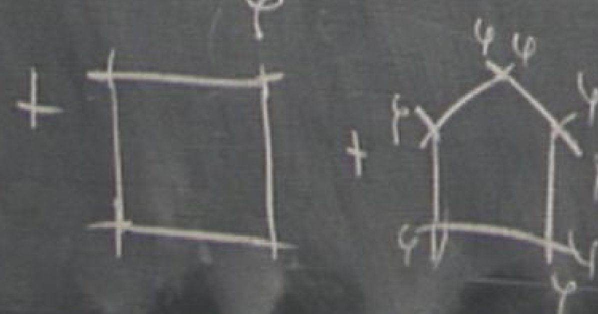


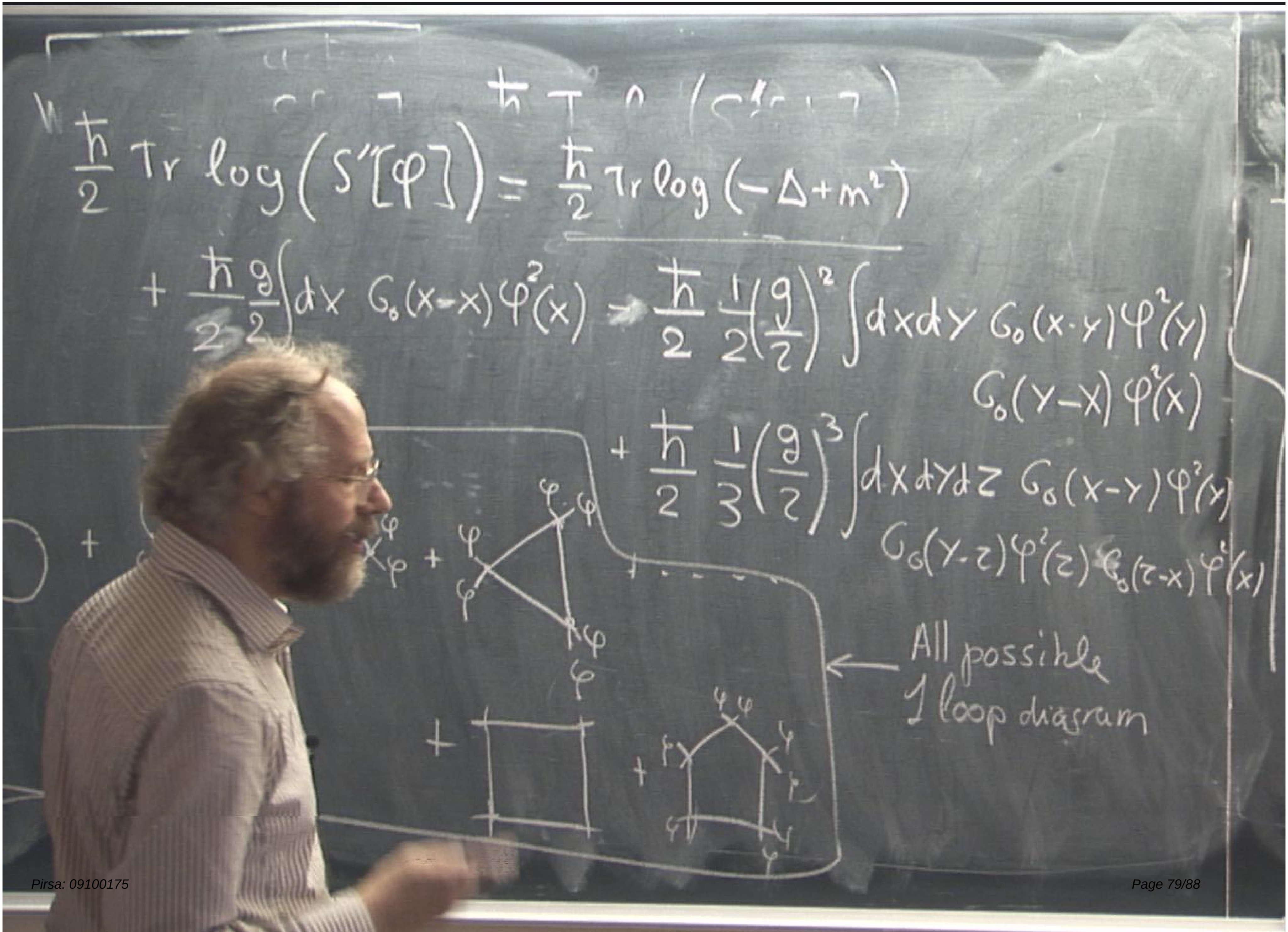
$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

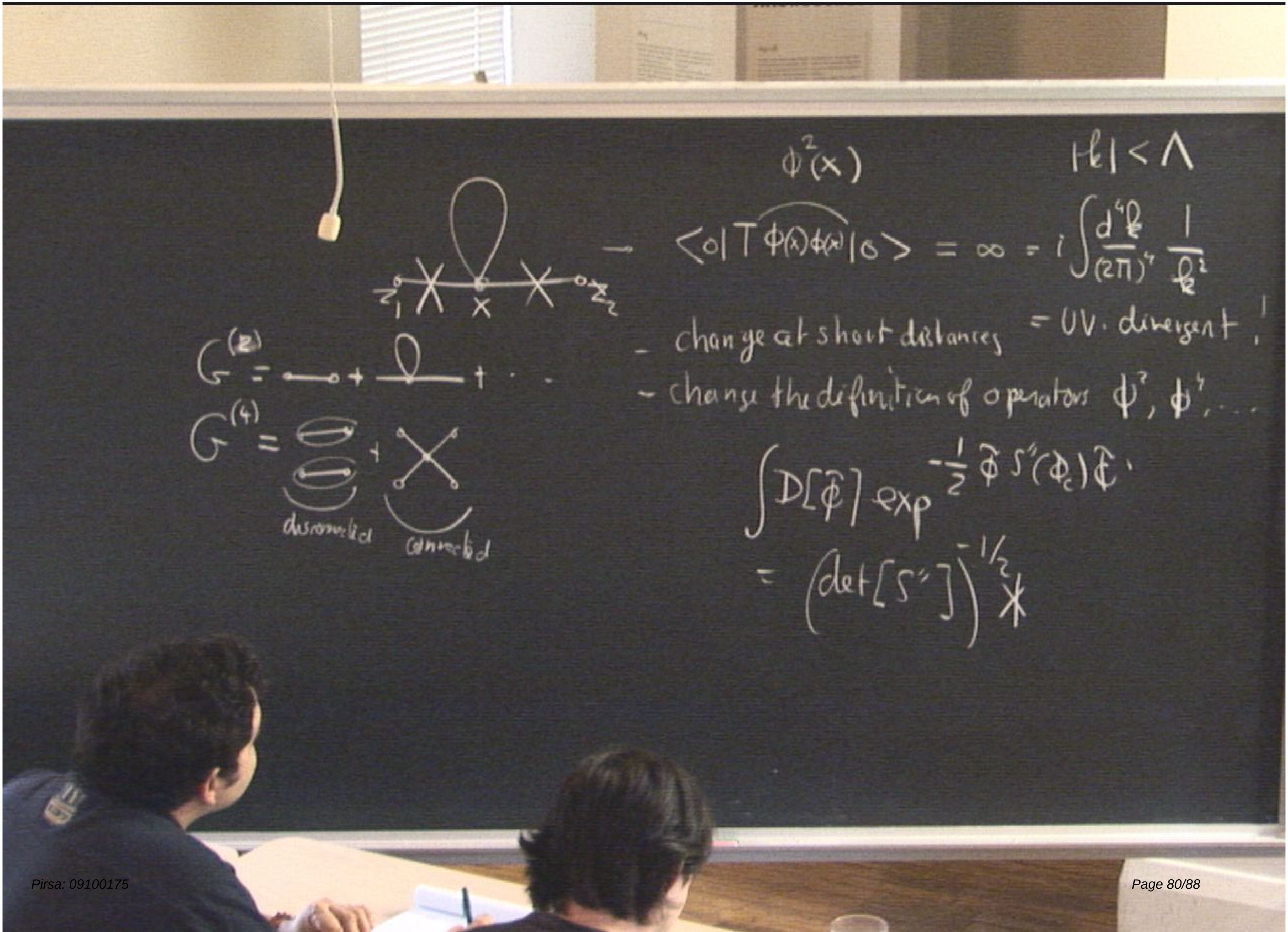
$$+\frac{\hbar}{2}\frac{g}{2}\int dx\; G_0(x-x)\,\varphi^2(x) - \frac{\hbar}{2}\frac{1}{2}\left(\frac{g}{2}\right)^2\int dx\,dy\; G_0(x-y)\,\varphi^2(y)\; G_0(y-x)\,\varphi^2(x)$$

$$+\frac{\hbar}{2}\frac{1}{3}\left(\frac{g}{2}\right)^3\int dx\,dy\,dz\; G_0(x-y)\,\varphi^2(y)\; G_0(y-z)\,\varphi^2(z)\; G_0(z-x)\,\varphi^2(x) + \ldots$$






$\frac{\hbar}{2} \mathrm{Tr} \log\left(S''[\varphi]\right) = \frac{\hbar}{2} \mathrm{Tr} \log(-\Delta + m^2)$
$+ \frac{\hbar}{2} \frac{g}{2} \int dx \, G_0(x-x) \varphi^2(x) - \frac{\hbar}{2} \frac{1}{2} \left(\frac{g}{2}\right)^2 \int dx dy \, G_0(x-y) \varphi^2(y) G_0(y-x) \varphi^2(x)$
$+ \frac{\hbar}{2} \frac{1}{3} \left(\frac{g}{2}\right)^3 \int dx dy dz \, G_0(x-y) \varphi^2(y) G_0(y-z) \varphi^2(z) G_0(z-x) \varphi^2(x)$
All possible 1 loop diagram


$\phi^2(x)$
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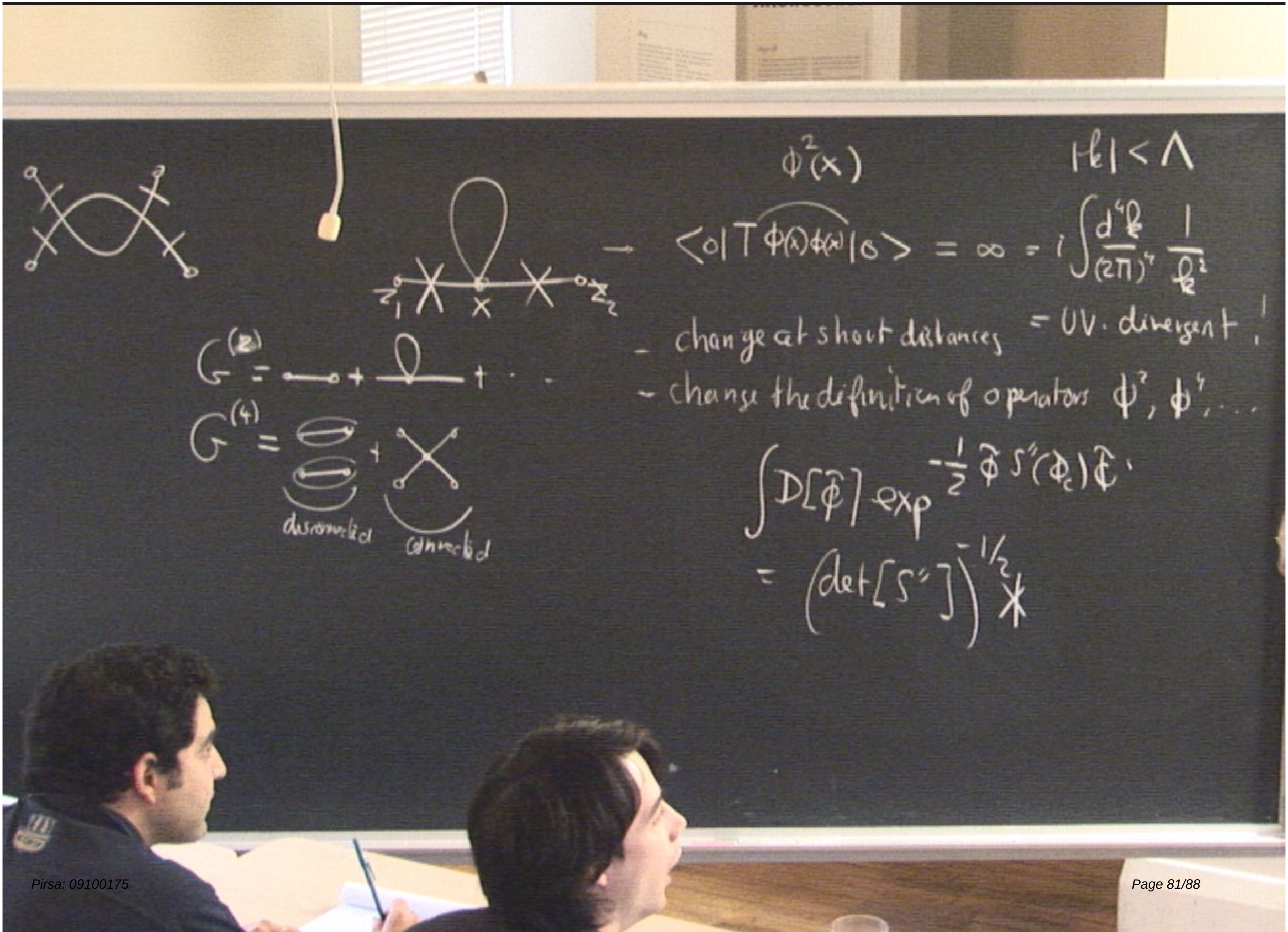
$\phi^2(x)$
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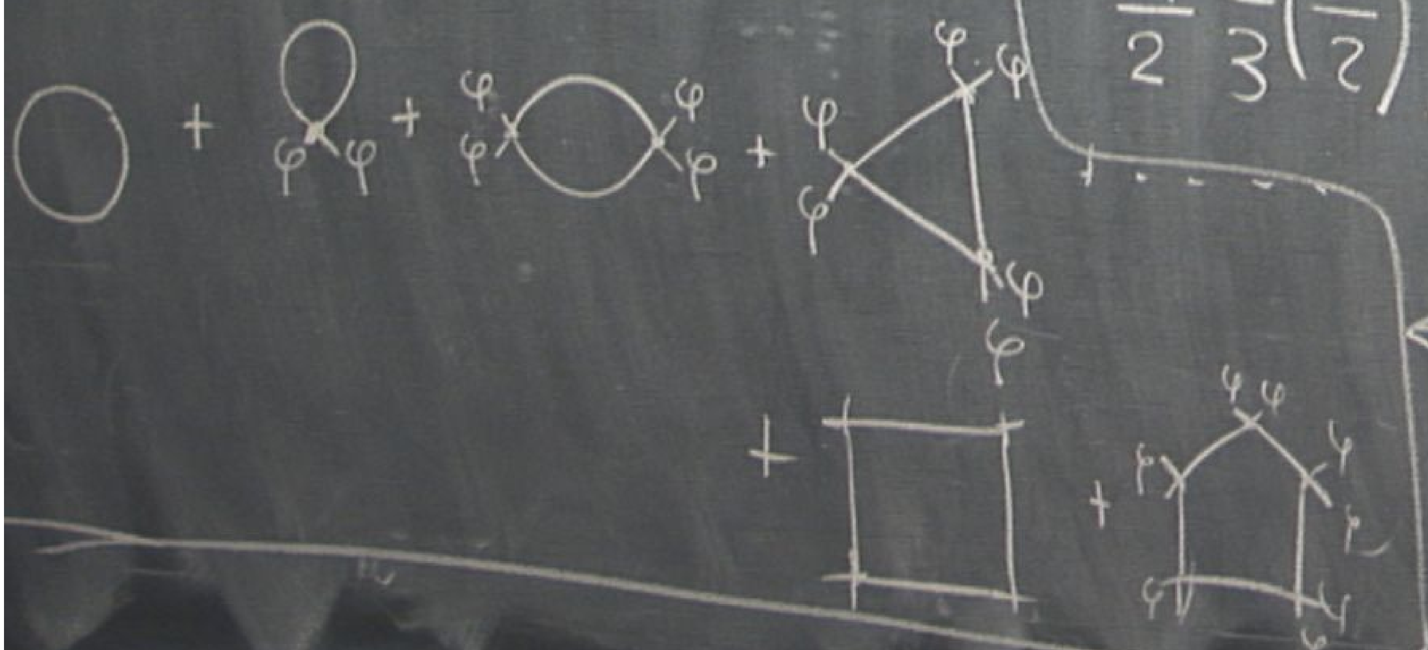
$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \underline{\frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)}$$

$$+\frac{\hbar}{2}\frac{g}{2}\int dx\; G_0(x-x)\,\varphi^2(x) - \frac{\hbar}{2}\,\frac{1}{2}\left(\frac{g}{2}\right)^2\int dx\,dy\; G_0(x-y)\,\varphi^2(y)\, G_0(y-x)\,\varphi^2(x)$$

$$+\frac{\hbar}{2}\,\frac{1}{3}\left(\frac{g}{2}\right)^3\int dx\,dy\,dz\; G_0(x-y)\,\varphi^2(\ldots\, G_0(y-z)\,\varphi^2(z)\, G_0(z-x)\,\ldots$$

$$+ \ldots$$



All possible
1 loop diagram
amputated
1 line irreducible

$$\frac{\hbar}{2}\,\mathrm{Tr}\log\left(S''[\varphi]\right) = \frac{\hbar}{2}\,\mathrm{Tr}\log\left(-\Delta+m^2\right)$$

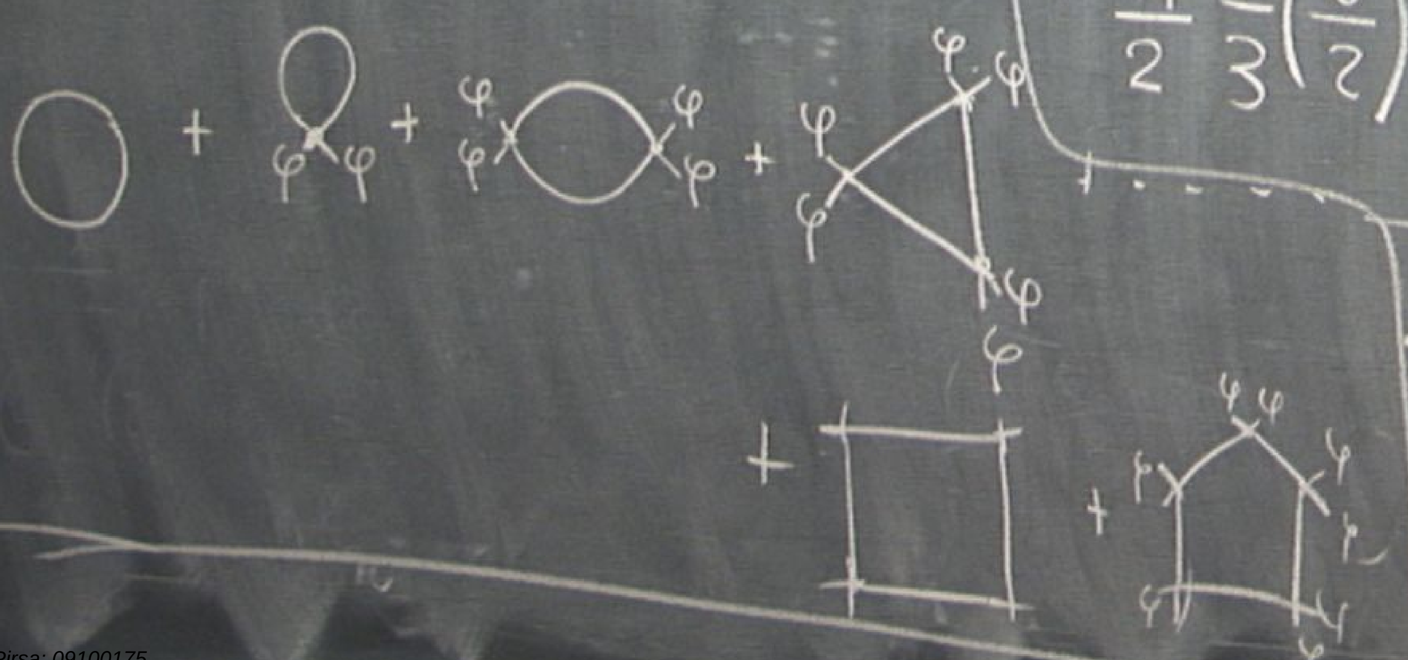
$$+\ \frac{\hbar}{2}\frac{g}{2}\int dx\; G_0(x-x)\,\varphi^2(x)\ -\ \frac{\hbar}{2}\,\frac{1}{2}\left(\frac{g}{2}\right)^2\int dx\,dy\; G_0(x-y)\,\varphi^2(\ldots$$

$$G_0(y-x)\,\varphi^2(x\ldots$$

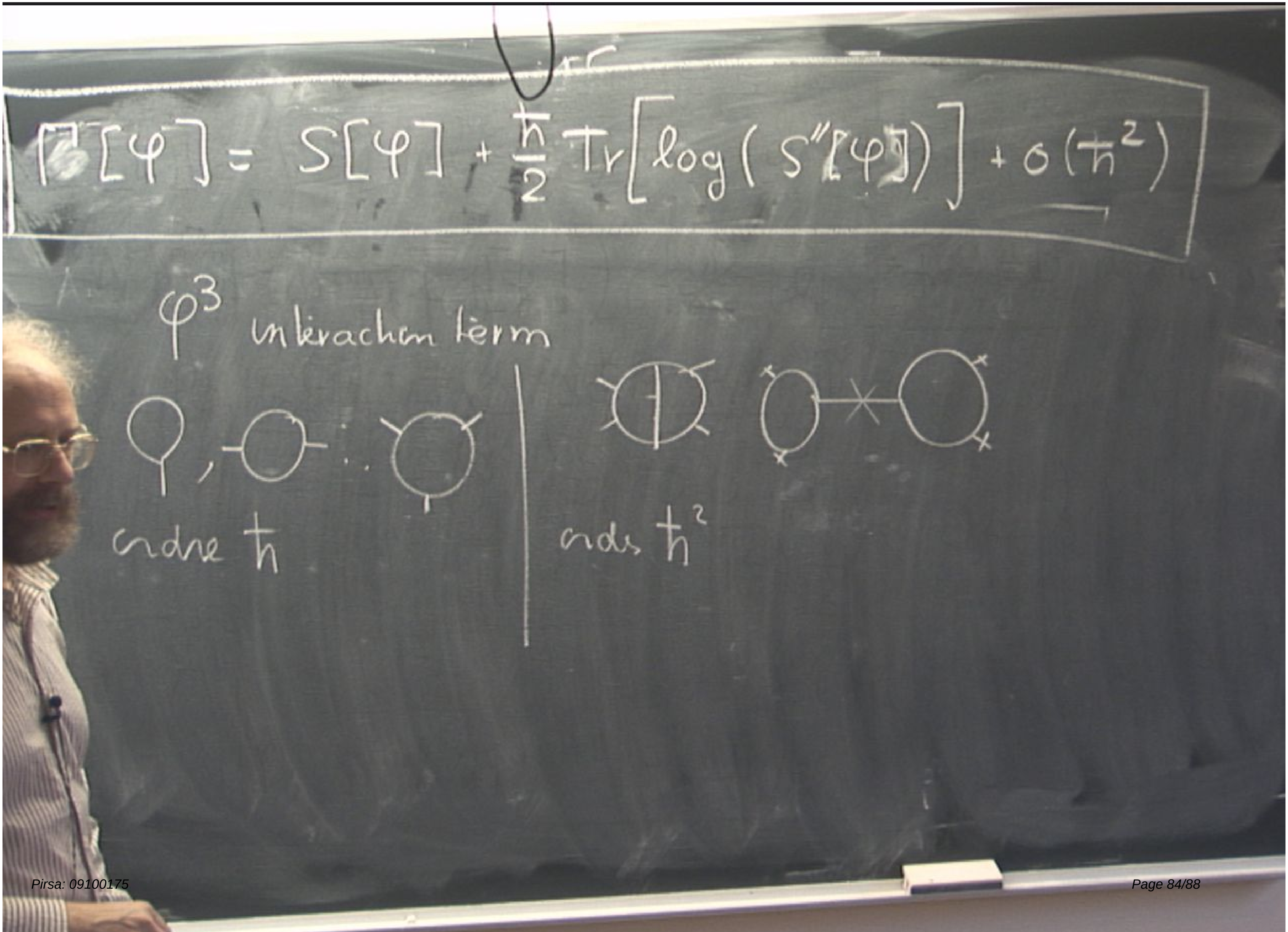
$$+\ \frac{\hbar}{2}\,\frac{1}{3}\left(\frac{g}{2}\right)^3\int dx\,dy\,dz\; G_0(x-y)\,\varphi\ldots$$

$$G_0(y-z)\,\varphi^2(z)\,G_0(z-x\ldots$$

$+ \ldots$



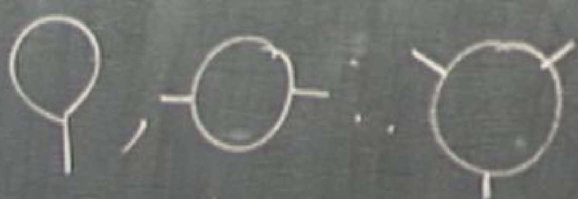
All possible
1 loop diagram
amputated
1 line irreducable



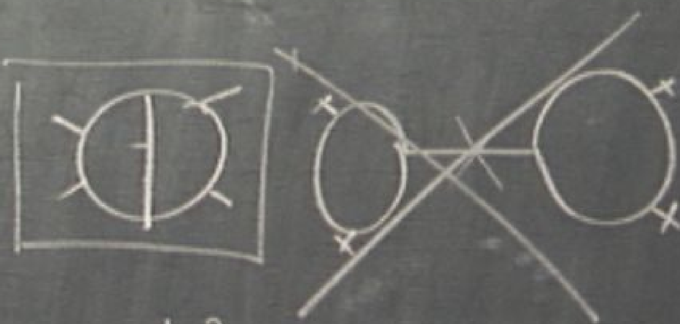
$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\log(S''[\varphi])\right] + O(\hbar^2)$
φ^3 interaction term
ordre $\hbar$
ordre $\hbar^2$

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\Big[\log\big(S''[\varphi]\big)\Big] + o(\hbar^2)}$$

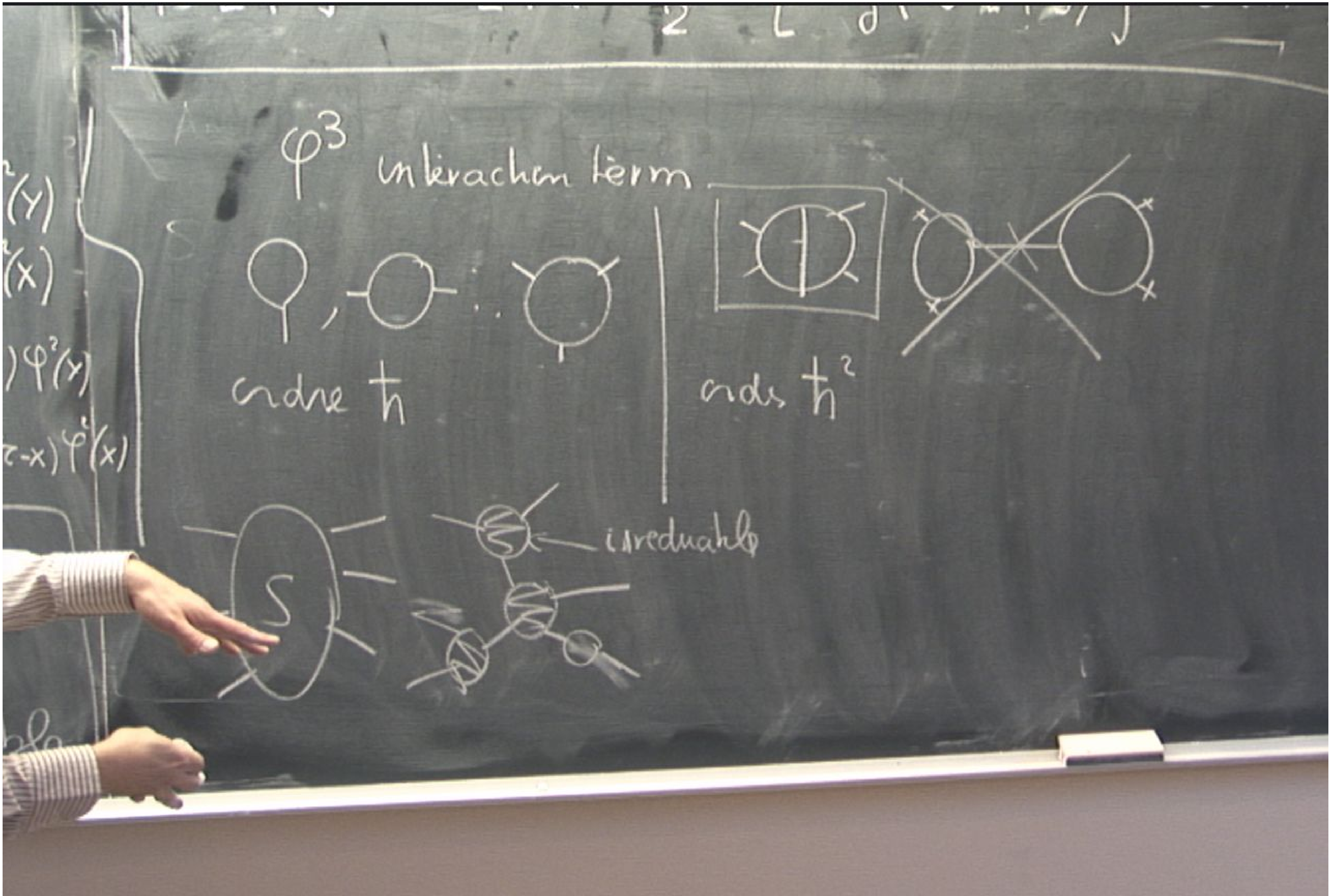
φ^3 interaction term

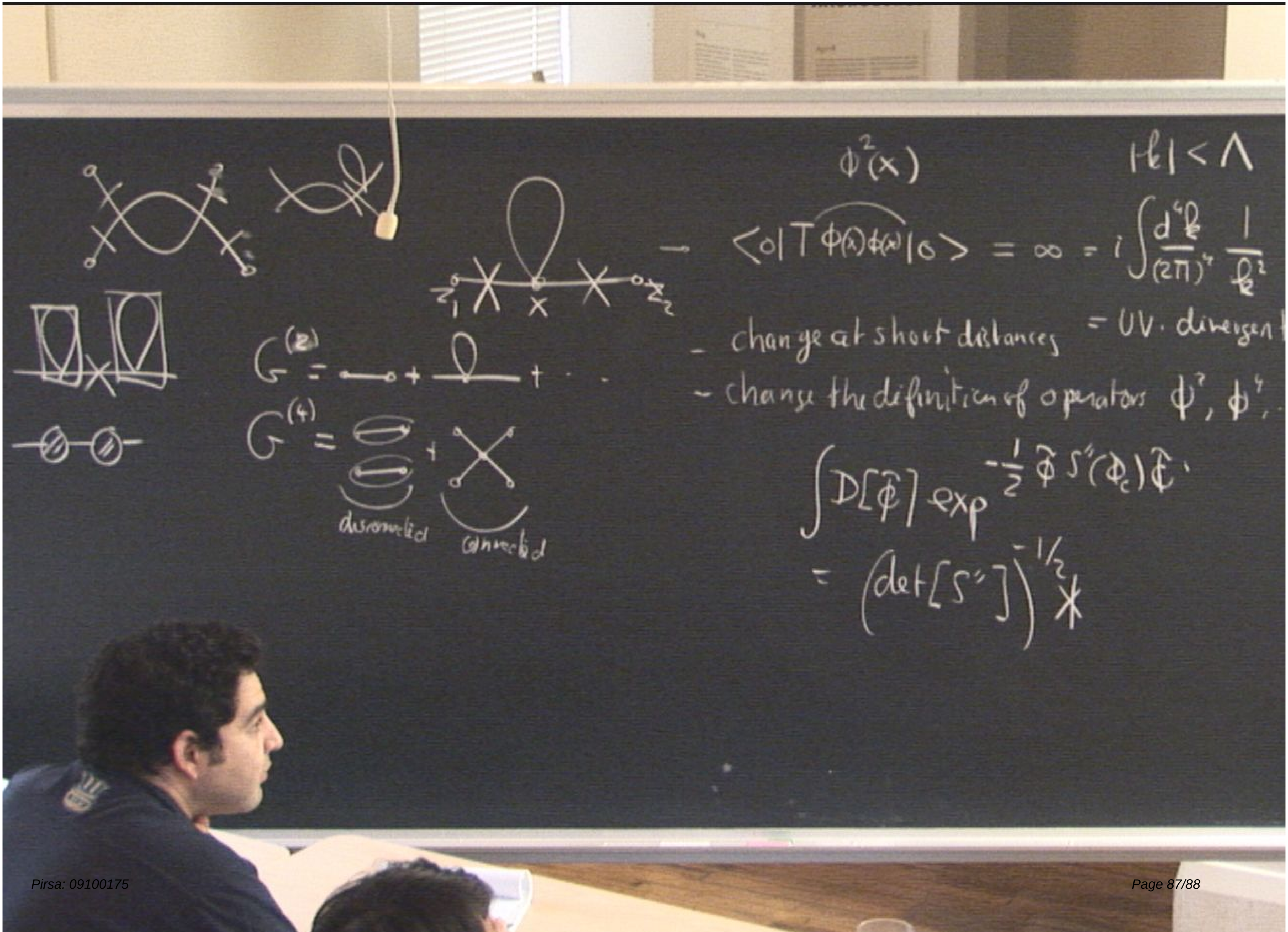


ordre $\hbar$



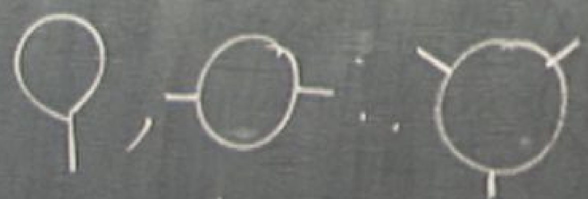
ordre $\hbar^2$


φ^3 interaction term
order $\hbar$
order $\hbar^2$
S
irreducible


φ²(x)
|k| < Λ
⟨0|T φ(x)φ(x)|0⟩ = ∞ = i ∫ d⁴k/(2π)⁴ 1/k²
z₁
z₂
– change at short distances = UV. divergen
– change the definition of operators φ², φ⁴,
G⁽²⁾ = + + . . .
G⁽⁴⁾ = +
disconnected
connected
∫D[φ̃] exp −½ φ̃ S''(φ_c) φ̃
= (det[S''])^(−1/2)

$$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2}\,\mathrm{Tr}\left[\log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$$

φ^3 interaction term



order $\hbar$

order $\hbar^2$

$\hbar^K$

$\downarrow$

diagrams with

K loops