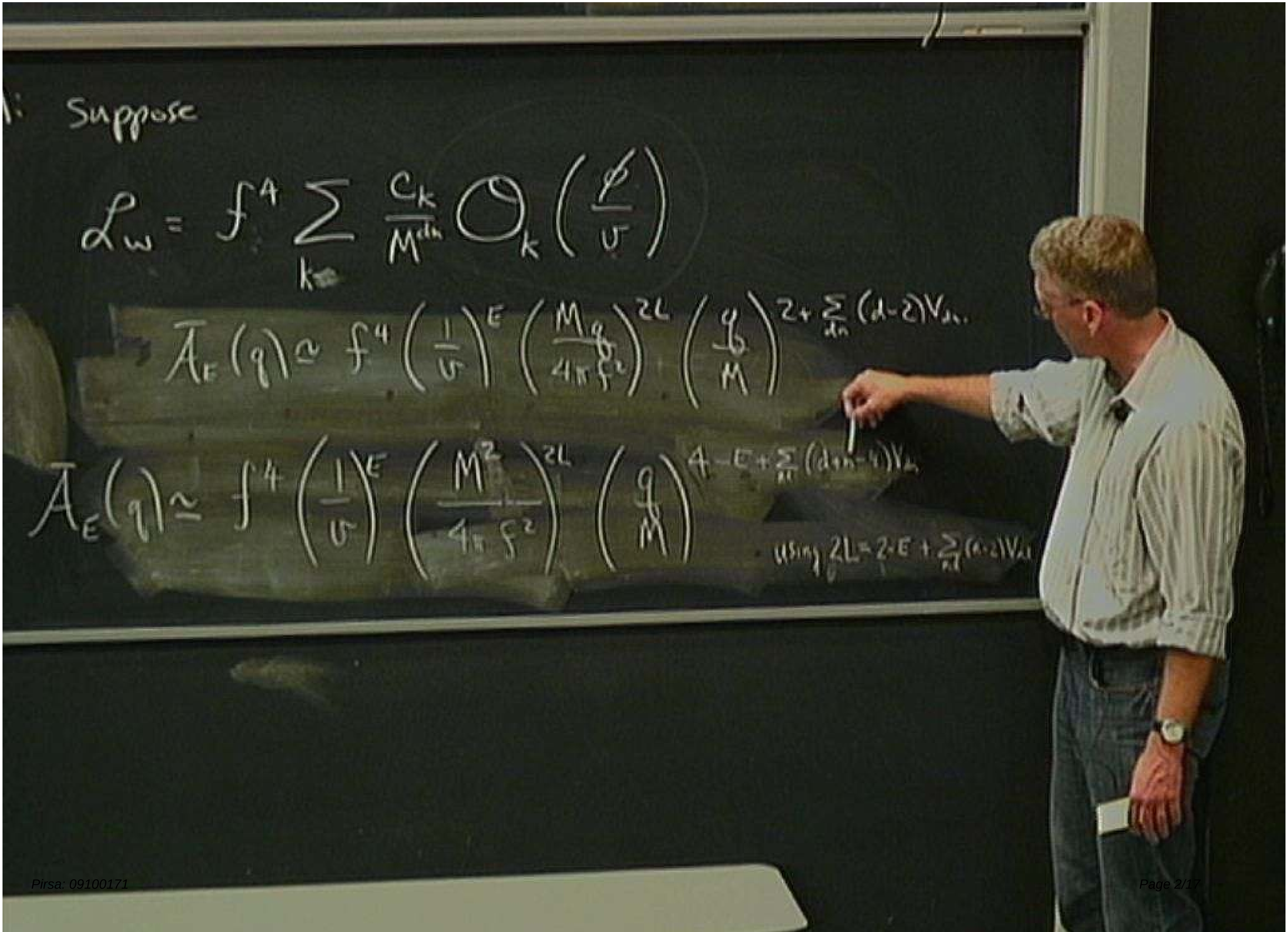
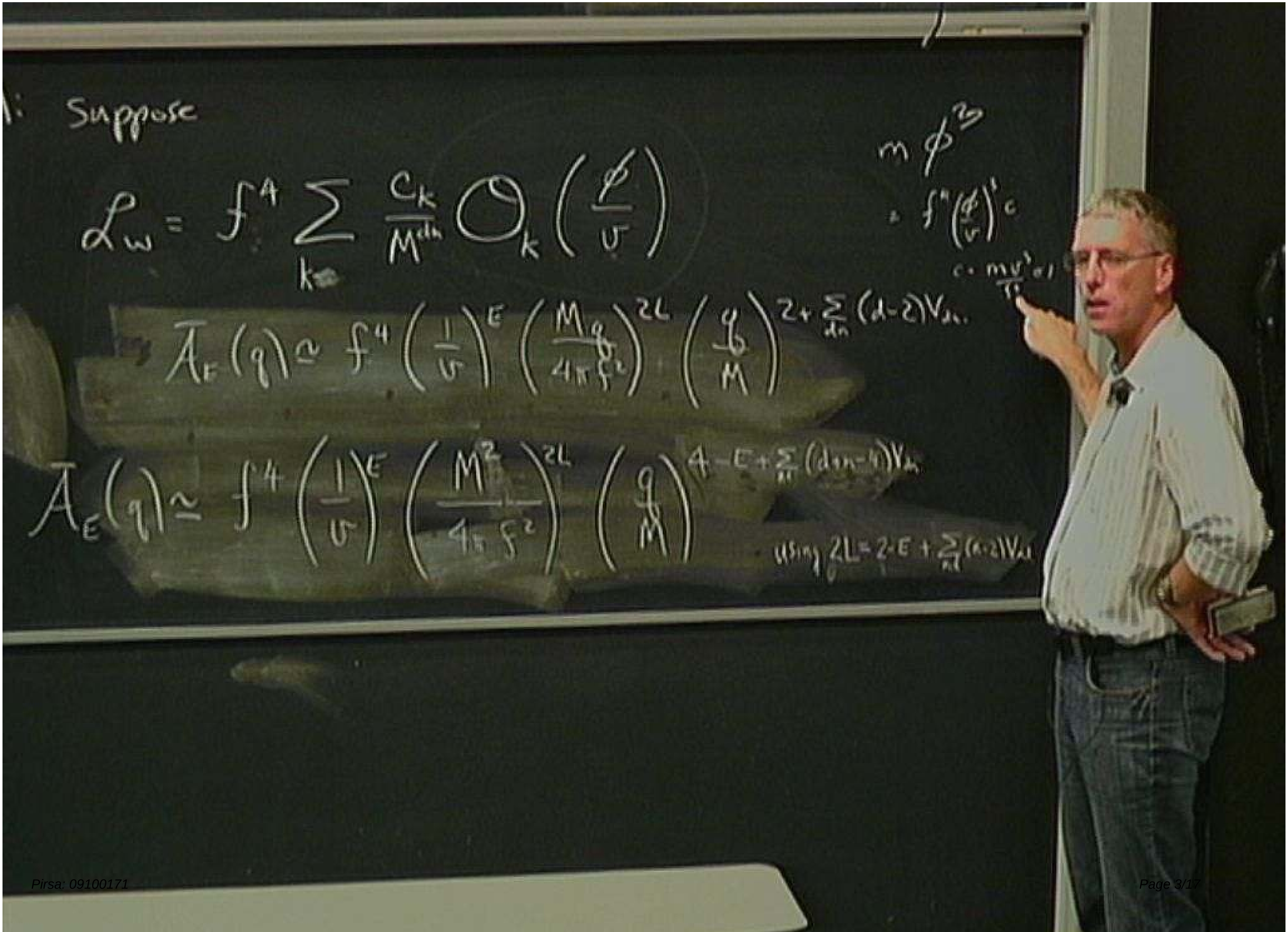
Title: Introduction to Effective Field Theory - Lecture 5B

Date: Oct 21, 2009 11:30 AM

URL: http://pirsa.org/09100171

Abstract:


Suppose
$\mathcal{L}_W = f^4 \sum_k \frac{c_k}{M^{d_k}} O_k\left(\frac{\phi}{v}\right)$
$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^E \left(\frac{M q}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{2+\sum_{d_n}(d-2)V_{d_n}}$
$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_{n}(d+n-4)V_{dn}}$
using $2L = 2-E+\sum_{n}(n-2)V_{dn}$


Suppose
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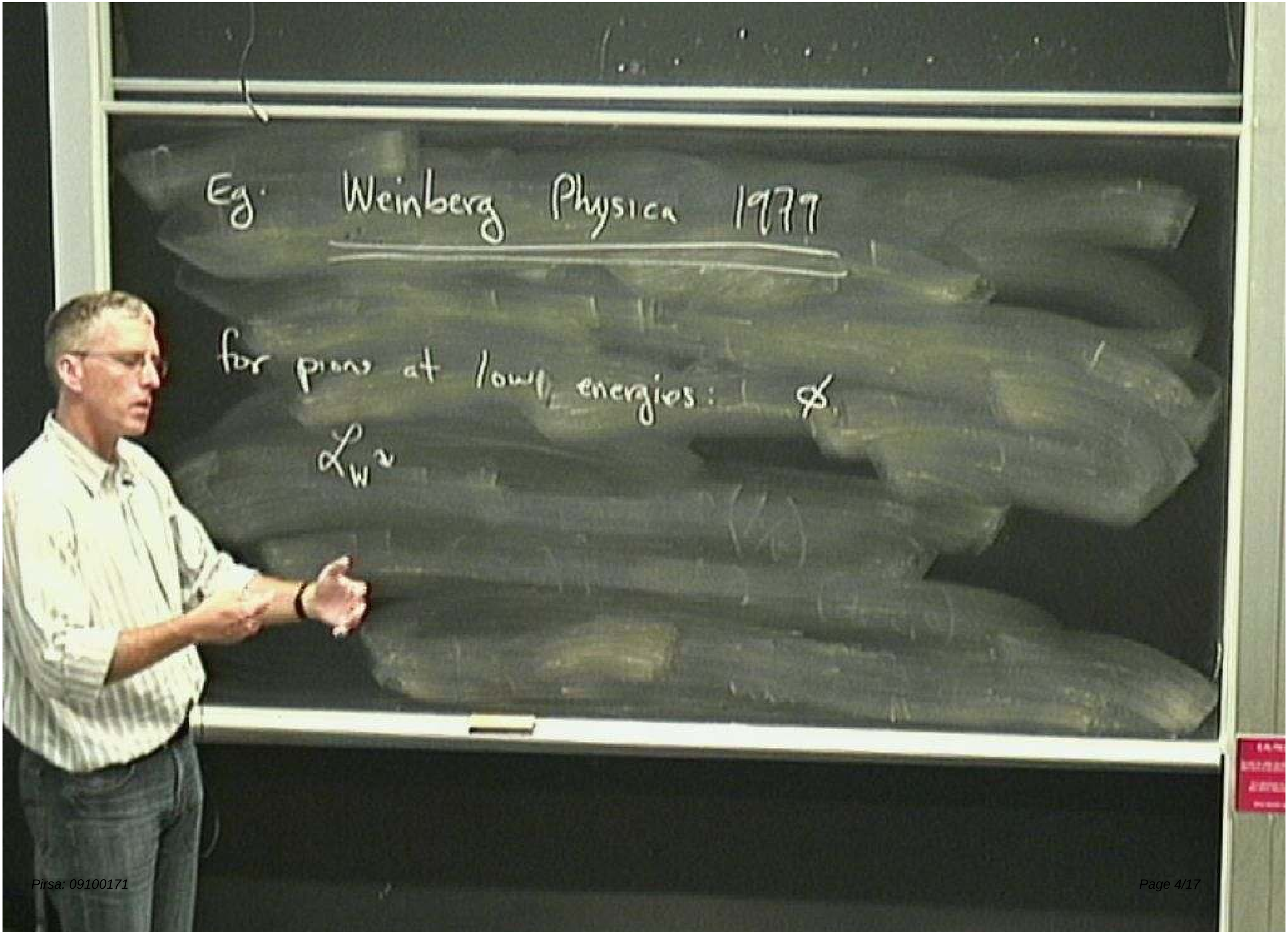
$m\phi^3$
$\bar{A}_E(q) \simeq f^4 \left(\frac{1}{v}\right)^E \left(\frac{M^2}{4\pi f^2}\right)^{2L} \left(\frac{q}{M}\right)^{4-E+\sum_n (d_n-4)V_n}$

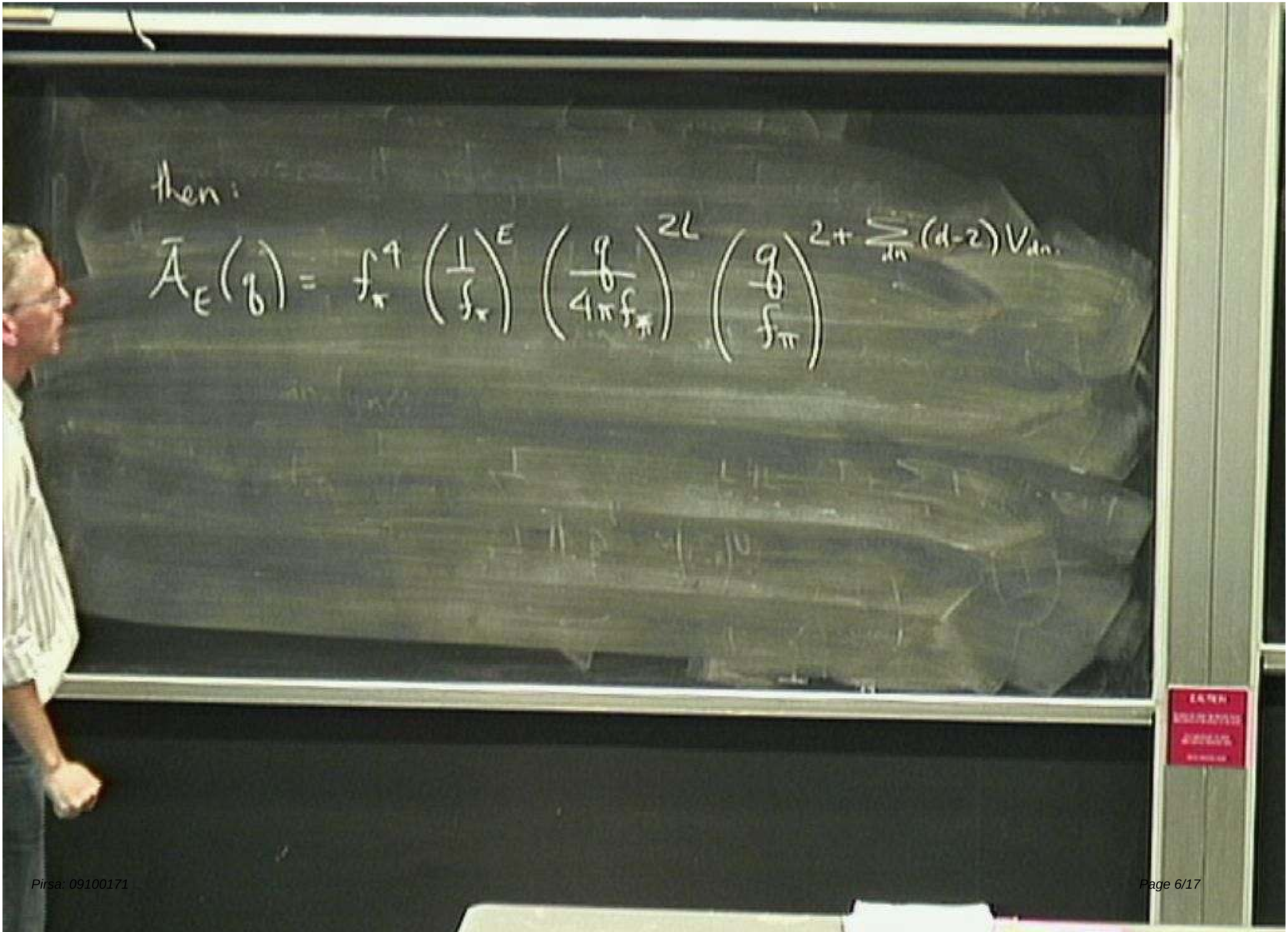

Eg: Weinberg Physica 1979
for pions at low energies: ϕ,
L_W ≈

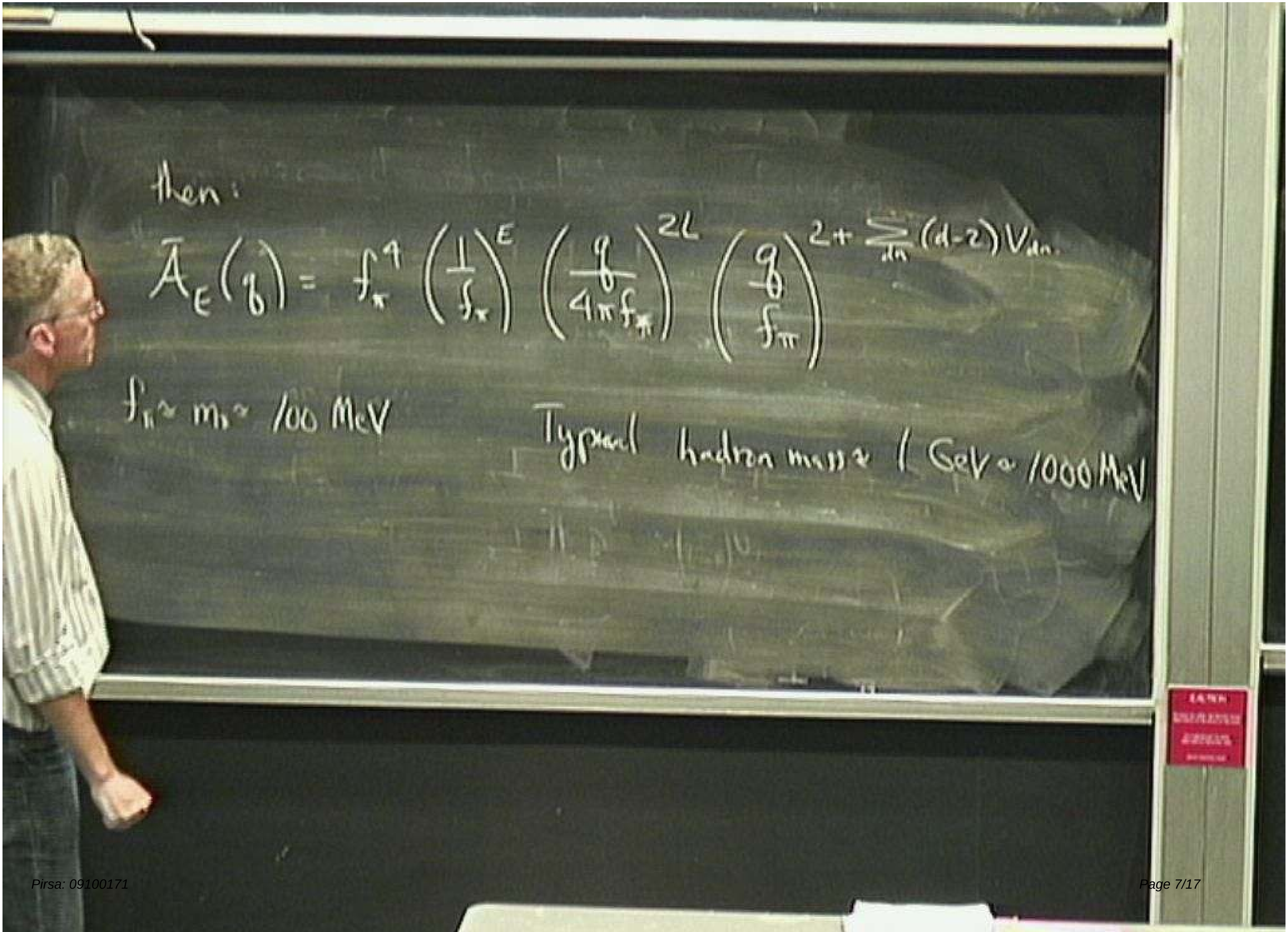
Eg. Weinberg … 1979

for pions at low energies: ϕ,

$$\mathcal{L}_W \approx f_\pi^2\, G_2\!\left(\phi/f_\pi\right) \partial_\mu\!\left(\phi/f_\pi\right) \partial^\mu\!\left(\phi/f_\pi\right) + G_4\!\left(\phi/f_\pi\right) \partial_\mu\!\left(\phi/f_\pi\right) \partial^\mu\!\left(\phi/f_\pi\right) \cdot \partial_\nu\!\left(\phi/f_\pi\right) \partial^\nu\!\left(\phi/f_\pi\right) + \dots$$

Has our form with $d \geq 2$ $\quad f = M = v = f_\pi$


then:
$\bar{A}_E(g) = f_\pi^4 \left(\frac{1}{f_\pi}\right)^E \left(\frac{g}{4\pi f_\pi}\right)^{2L} \left(\frac{g}{f_\pi}\right)^{2+\sum_{d,n}(d-2)V_{d,n}}$


then:
$\bar{A}_E(q) = f_\pi^4 \left(\frac{1}{f_\pi}\right)^E \left(\frac{q}{4\pi f_\pi}\right)^{2L} \left(\frac{q}{f_\pi}\right)^{2+\sum_{dn}(d-2)V_{dn}}$
$f_\pi \simeq m_\pi \simeq 100$ MeV
Typical hadron mass $\simeq$ 1 GeV $\simeq$ 1000 MeV

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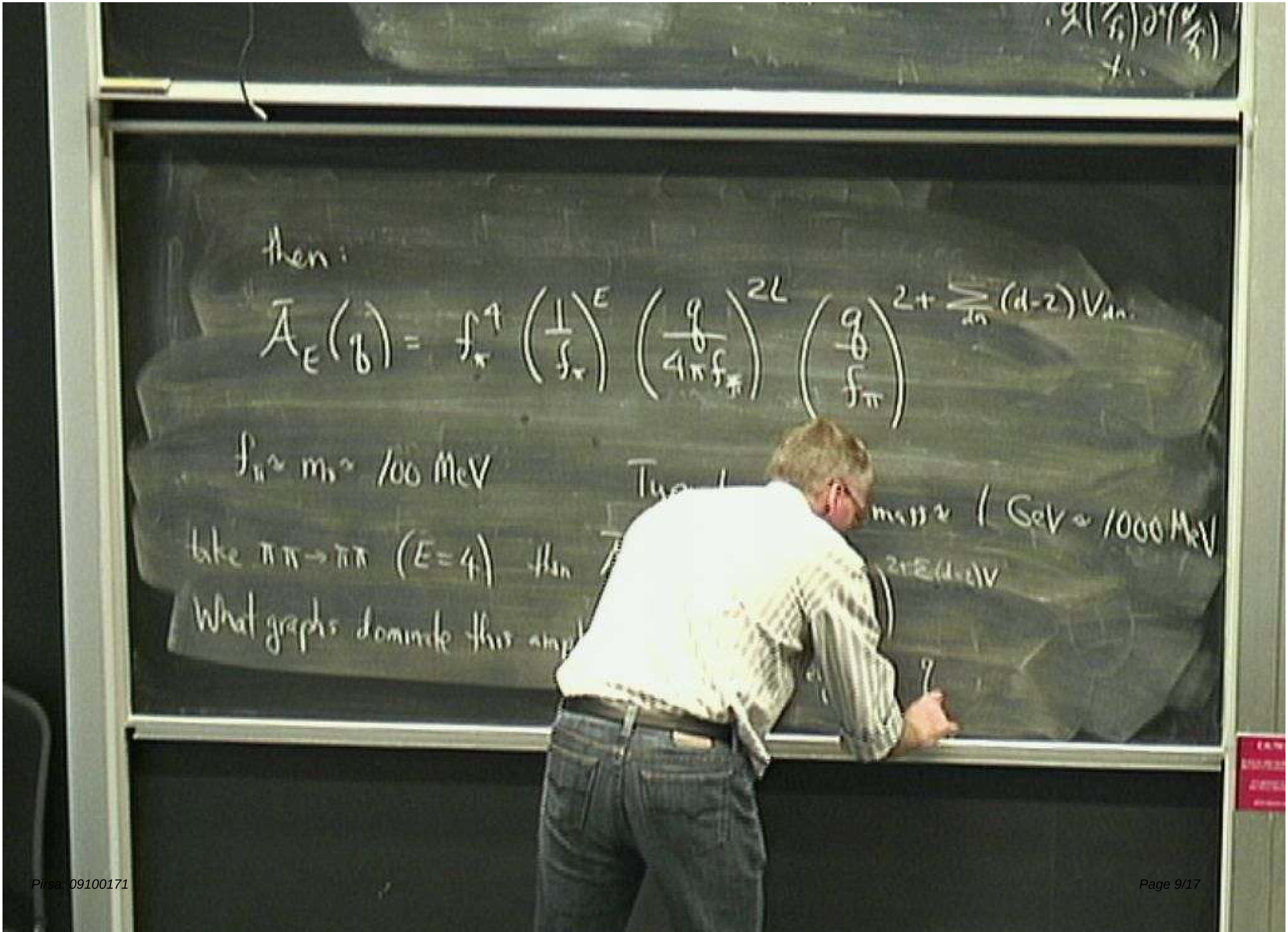
$f_\pi \approx m_\pi \approx 100$ MeV

Typical hadron mass $\approx$ 1 GeV $\approx$ 1000 MeV

take $\pi\pi \to \pi\pi$ $(E=4)$ then $\bar{A}_4(q) \approx \left(\frac{q}{4\pi f_\pi}\right)^{2L} \left(\frac{q}{f_\pi}\right)^{2+\sum(d-2)V}$

What graphs dominate this amplitude?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$f_\pi \sim m_\rho \sim 100$ MeV
mass ~ 1 GeV ~ 1000 MeV
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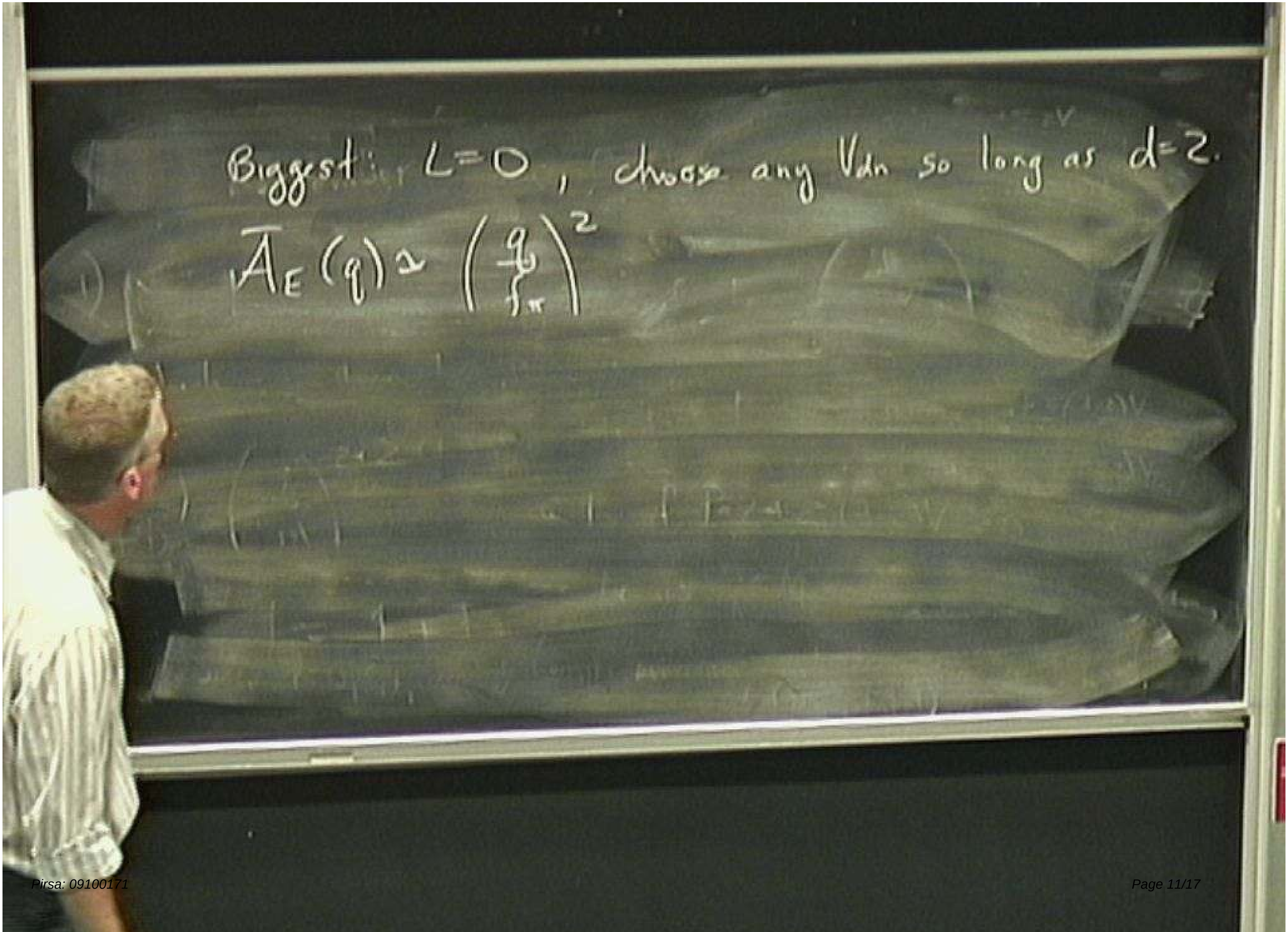
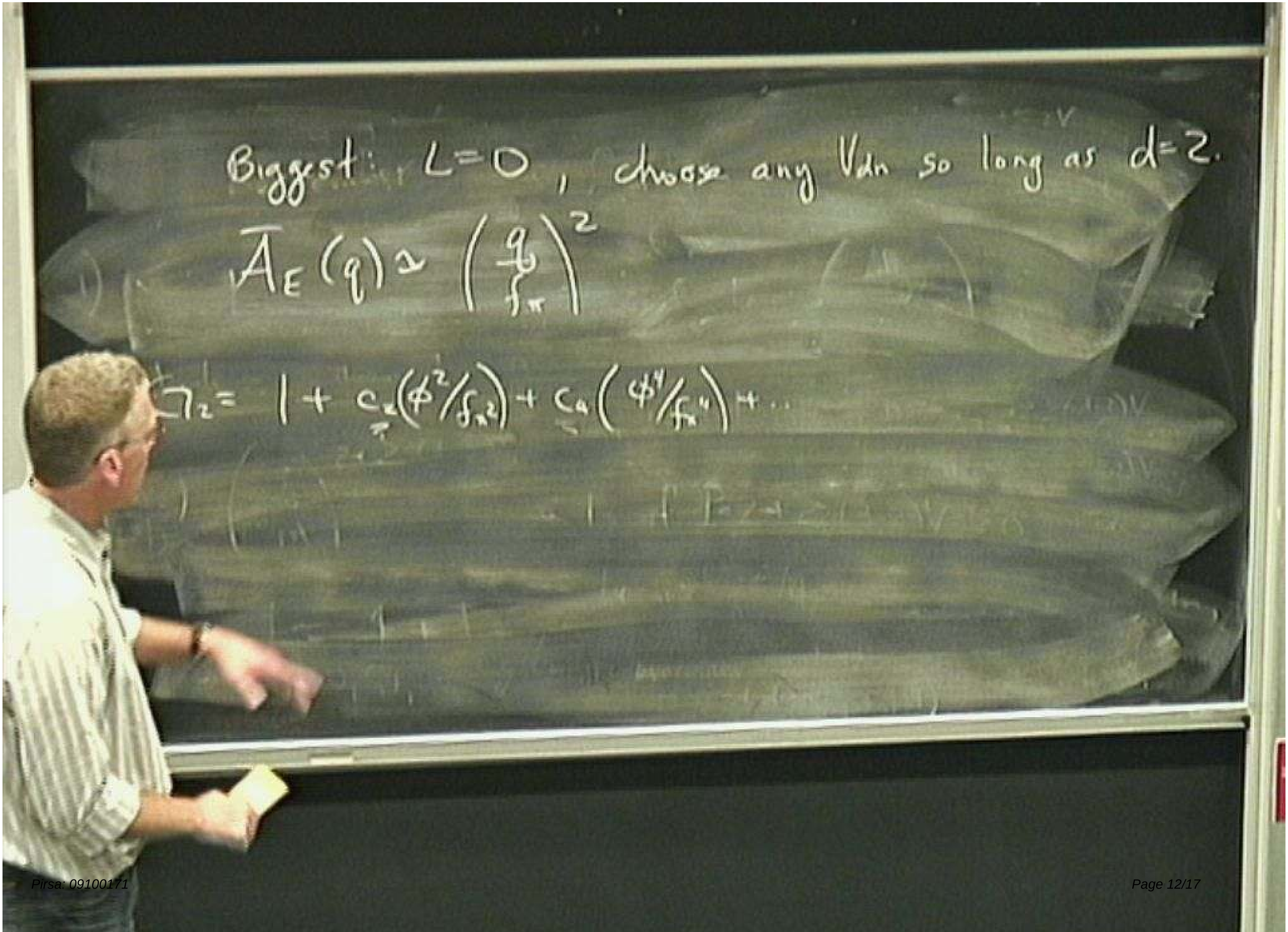
What graphs dominate this amplitude at low energies: $(q \ll 4\pi f_\pi)$

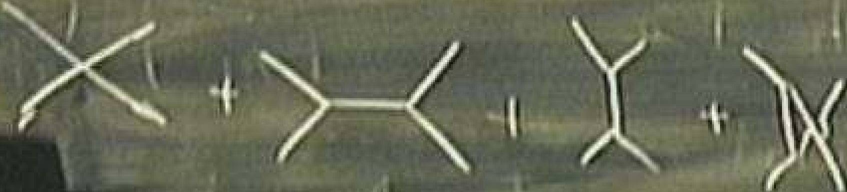

Biggest: $L=0$, choose any V_{dn} so long as $d=2$.
$\bar{A}_E(q) \propto \left(\frac{q}{f_\pi}\right)^2$


Biggest: L=0, choose any V_dn so long as d=2.
Ā_E(q) ~ (q/f_π)^2
η_2 = 1 + c_2(φ^2/f_π^2) + c_4(φ^4/f_π^4) + ...

Biggest: $L = 0$, choose any V_{dn} so long as $d = 2$.

$$\bar{A}_E(q) \simeq \left(\frac{q}{f_\pi}\right)^2$$

if: $G_2 = 1 + c_1 \frac{\phi}{f_\pi} + c_2 \frac{\phi^2}{f_\pi^2} +$



1st subleading terms:

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