Title: Reduction and Emergence in Bose-Einstein Condensates

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Abstract: A closer look at some proposed Gedanken-experiments on BECs promises to shed light on several aspects of reduction and emergence in physics. These include the relations between classical descriptions and different quantum treatments of macroscopic systems, and the emergence of new properties and even new objects as a result of spontaneous symmetry breaking.

Reduction and Emergence in Bose-Einstein Condensates

Richard Healey
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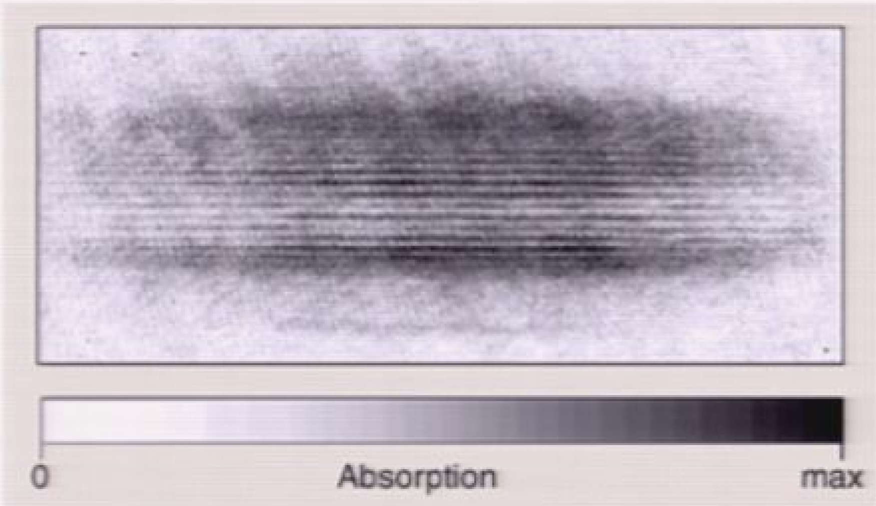
Plan of talk

1. Introduction
2. Interference between BECs
3. Emergence and reduction
4. Spontaneous symmetry breaking
5. Dynamic emergence of phase
6. Laloë’s analysis
7. Laloë’s “EPR” *gedankenexperiment*
8. A Bohrian anti-reductionist response
9. Leggett on relative phase between superfluids
10. Emergence of “vague objects”?

Interference between two BECs



Relative phase state

$$|\Lambda\rangle = \left[N_a^{\frac{1}{2}} \hat{a}\dagger_{u_a} e^{-i\Lambda/2} + N_b^{\frac{1}{2}} \hat{a}\dagger_{v_b} e^{i\Lambda/2}\right]^{N_a+N_b} |0\rangle \quad (1)$$

Emergence *vs.* Reduction?

- *Emergence* is a relation that may or may not hold between items in the world that scientists study---phenomena, behavior, properties, objects.
- *Reduction* is a relation applicable only to products of that study---theories, theoretical descriptions, sciences or laws (strictly, law statements).

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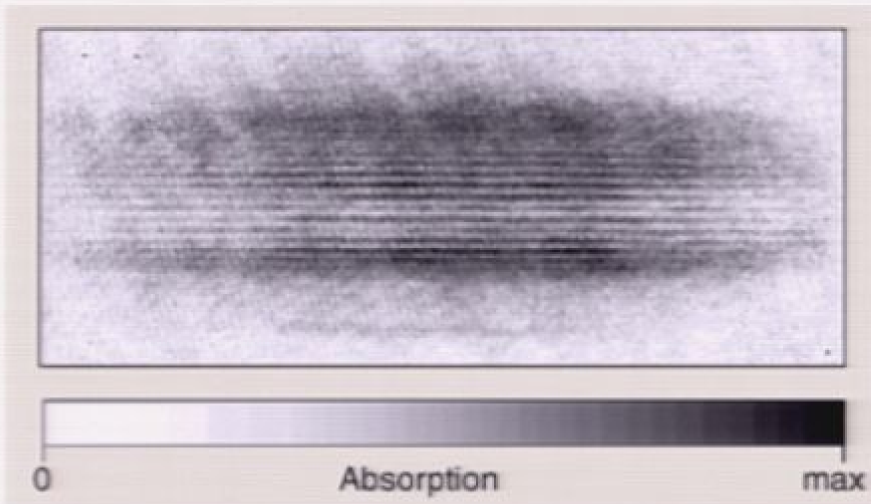

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Emergence of properties

- *Synchronic* emergence: "at each level of complexity entirely new properties appear." (Philip Anderson, "More is Different")
- *Diachronic* (dynamic) emergence: as in cooking, brewing!
- *Illusive* emergence: "I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we place them is concerned, and that they reside only in the consciousness. Hence, if the living creature were removed, all these qualities would be wiped away and annihilated. But since we have imposed upon them special names, distinct from those of the other and real qualities mentioned previously, we wish to believe that they really exist as actually different from those." (Galileo, *The Assayer*)

Emergence of objects?

- *Dependent* emergence: any object with an emergent property may be called an emergent object.
- *Independent* emergence: an object composed wholly of basic parts, but not of any definite number of these.

Spontaneous symmetry breaking

- "The essential idea is that in the so-called $N \rightarrow \infty$ limit of large systems (on our own, macroscopic scale) it is not only convenient but essential to realize that matter will undergo sharp, singular phase transitions to states in which the microscopic symmetries, and even the microscopic equations of motion, are in a sense violated." (Anderson)
- "a superconductor is simply a material in which electromagnetic gauge invariance is spontaneously broken" (Stephen Weinberg)

Is the relative phase between two BECs a case of spontaneously broken **U(1)** symmetry?

- In the Heisenberg ferromagnet, *rotational* symmetry is spontaneously broken.
- This is modeled in the $N \rightarrow \infty$ limit as a quantum system with an infinite number of degrees of freedom.
- For such a system, the fundamental commutation relations have *unitarily inequivalent* Hilbert space representations.
- Spontaneous symmetry breaking is modeled as a selection of *one* of these (cf. Higgs mechanism).

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Problems with the analogy!

1. The **U(1)** phase of a BEC is specified only relative to *another similar BEC*: the orientation of a ferromagnet need not be specified relative to another *ferromagnet.*
2. The number of atoms in a BEC is large, but much smaller than the number of atoms in a typical ferromagnet, and certainly not infinite.

Dynamical emergence of relative phase between two BECs

As Javanainen and Yoo (1996) demonstrated numerically, and Castin and Dalibard (1997) showed analytically, an interference pattern will almost certainly build up from successive detections of individual atoms randomly selected from a pair of overlapping BECs, even if each BEC is initially described by a Fock state corresponding to a definite number of atoms.

Equivalence

- Castin and Dalibard showed that two different points of view on a system are available:
- Assuming an initial pair of coherent states with a definite relative phase Λ, successive measurements reveal that pre-existing phase in an interference phenomenon;
- Assuming each condensate is initially represented by a definite Fock state, with *no* well-defined relative phase, the same sequence of measurements progressively yields *exactly the same* interference pattern one would expect from some well-defined, but randomly chosen, relative phase Λ.

Laloë's analysis

Frank Laloë has devised an elegant analysis of how the statistics of detection of atoms from two overlapping Fock states can yield such an interference pattern.

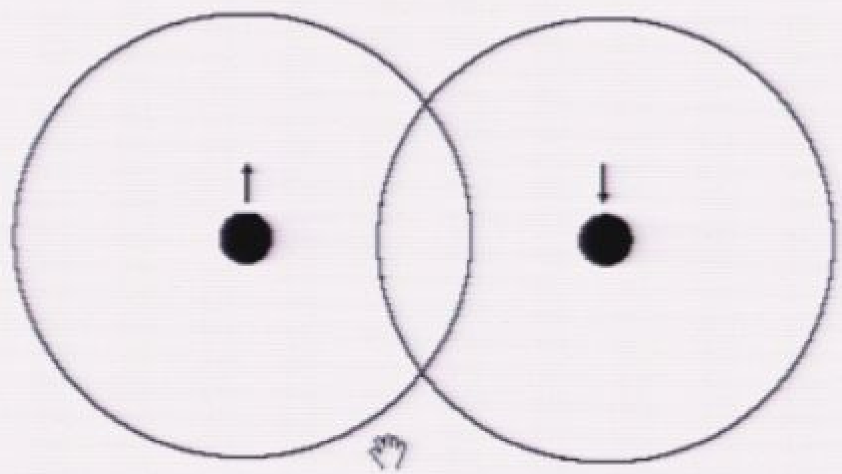
Double Fock state

$$|\Phi_0\rangle = \hat{a}\dagger_{u_a}^{N_a} \hat{a}\dagger_{v_b}^{N_b} |0\rangle \qquad (2)$$

Now add *spin*

$$|\Phi\rangle = \hat{a}\dagger_{u_a,\alpha}^{N_a} \hat{a}\dagger_{v_b,\beta}^{N_b} |0\rangle \qquad (3)$$

Here is one possibility



'igure 1: Two condensates, corresponding to two different internal spin states, expand and overlap ı a region of space. In this region, particle spin measurements are performed in transverse directions perpendicular to the spin directions of the two initial condensates).

If $\hat{\Psi}_\alpha(\mathbf{r})$ is the field operator for z-spin α, $\hat{\Psi}_\beta(\mathbf{r})$ for z-spin β, and † indicates the adjoint operation, then the number density operator of the BECs is

$$\hat{n}(\mathbf{r}) = \hat{\Psi}^\dagger_\alpha(\mathbf{r})\hat{\Psi}_\alpha(\mathbf{r}) + \hat{\Psi}^\dagger_\beta(\mathbf{r})\hat{\Psi}_\beta(\mathbf{r}) \tag{4}$$

and the density operator for their spin component in a direction in the $x-y$ plane at an angle φ from the x-axis is

$$\hat{\sigma}_\varphi(\mathbf{r}) = e^{-i\varphi}\hat{\Psi}^\dagger_\alpha(\mathbf{r})\hat{\Psi}_\beta(\mathbf{r}) + e^{+i\varphi}\hat{\Psi}^\dagger_\beta(\mathbf{r})\hat{\Psi}_\alpha(\mathbf{r}) \tag{5}$$

Suppose that one measurement is made of the φ-component of particle spin in a small region of space Δr centered around point $\mathbf{r}$. The corresponding spin operator is

$$\hat{S}(\mathbf{r},\varphi) = \int_{\Delta r} d^3\mathbf{r}'\hat{\sigma}_\varphi(\mathbf{r}') \tag{6}$$

For sufficiently small Δr, this has only three eigenvalues $\eta = 0, \pm 1$ since no more than one particle would be found in Δr. The single-particle eigenstates for finding a particle there with $\eta = \pm 1$ are

$$|\Delta r, \eta\rangle = |\Delta r\rangle \otimes \frac{1}{\sqrt{2}} \left[e^{-i\varphi/2} \, |\alpha\rangle + e^{+i\varphi/2} \, |\beta\rangle \right] \tag{7}$$

where $|\Delta r\rangle$ is a single-particle spatial state whose wave-function equals 1 inside Δr but 0 everywhere outside Δr. The corresponding N-particle projector is

$$\hat{P}_{\eta=\pm 1}(\mathbf{r},\varphi) = \frac{1}{2} \int_{\Delta r} d^3\mathbf{r}' \left[\hat{n}(\mathbf{r}') + \eta \hat{\sigma}_{\varphi}(\mathbf{r}') \right] \tag{8}$$

and the projector for finding no particle there is

$$\hat{P}_{\eta=0}(\mathbf{r}) = \left(\mathbf{1} - \int_{\Delta r} d^3\mathbf{r}' \hat{n}(\mathbf{r}') \right) \tag{9}$$

As $\Delta r \to 0$, the corresponding eigenstates (for variable $\mathbf{r}$) form a quasi-complete basis for the N-particle space.

Now consider a sequence of m measurements of transverse spin-components φ_j in very small non-overlapping regions Δr_j, each of volume Δ, centered around points $\mathbf{r}_j$ $(1 \leq j \leq m)$. Since the projectors for non-overlapping regions commute, the joint probability for detecting m particles with spins η_j in regions Δr_j is

$$\left\langle \Phi \left| \hat{P}_{\eta_1}(\mathbf{r}_1,\varphi_1) \times \hat{P}_{\eta_2}(\mathbf{r}_2,\varphi_2) \times \ldots \times \hat{P}_{\eta_m}(\mathbf{r}_m,\varphi_m) \times \right| \Phi \right\rangle \quad (10)$$

Using (8) together with (4) and (5) this gives a product of several terms, each containing various products of field operators. Since these commute, we can push all the creation operators to the left and all the annihilation operators to the right. Expanding the field operators in terms of a basis $|u_a, \alpha\rangle$, $|v_b, \beta\rangle$ of single particle states

$$\hat{\Psi}_\alpha(\mathbf{r}) = u_a(\mathbf{r}) \times \hat{a}_{u_a,\alpha} + \ldots \quad ; \quad \hat{\Psi}_\beta(\mathbf{r}) = v_b(\mathbf{r}) \times \hat{a}_{v_b,\beta} + \ldots \quad (11)$$

But none of the "dotted" terms will contribute to (10), since $|\Phi\rangle$ contains no particles in states other than $|u_a, \alpha\rangle$, $|v_b, \beta\rangle$.

Since Δ is very small, the spatial wave-functions are each approximately constant over each region Δr_j.The joint probability for detection of m particles with spins η_j in regions Δr_j $(1 < j < m)$, each of volume Δ, is then proportional to

$$\int_0^{2\pi} \frac{d\Lambda}{2\pi} \prod_{j=1}^{m} \left\{ \begin{array}{c} N_a \left|u_a(\mathbf{r}_j)\right|^2 + N_b \left|v_b(\mathbf{r}_j)\right|^2 + \\ \eta_j \sqrt{N_a N_b} \left(e^{i\left(\Lambda - \varphi_j\right)} u_a(\mathbf{r}_j) v_b^*(\mathbf{r}_j) + c.c\right) \end{array} \right\} \tag{13}$$

or, with $\xi(\mathbf{r}) = \arg\left[u_a(\mathbf{r})/v_b(\mathbf{r})\right]$,

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There is a clever trick to take account of particle number conservation in each sequence, using the mathematical identity

$$\int_0^{2\pi} \frac{d\Lambda}{2\pi} e^{in\Lambda} = \delta_{n,0} \tag{12}$$

By multiplying each $\hat{\Psi}_\alpha(\mathbf{r})$ (or rather $\sqrt{N_a} u_a(\mathbf{r})$) by $e^{i\Lambda}$, and each $\hat{\Psi}^\dagger_\alpha(\mathbf{r})$ (or rather $\sqrt{N_a} u^*_a(\mathbf{r})$) by $e^{-i\Lambda}$, and integrating Λ over 2π (and similarly for the b particles), we automatically take account of particle number conservation!

Each term now contains between $\langle\Phi|$ and $|\Phi\rangle$ a string of creation operators ollowed by a string of annihilation operators. If a state $|u_a, \alpha\rangle$ or $|v_b, \beta\rangle$ does ıot appear exactly the same number of times in each of these, it will not con- ribute to (10): if it does appear exactly the same number of times in each of hese, every creation or annihilation operator will introduce a factor $\sqrt{N_{a,b} - q}$ vhere q depends on the term but $q < m$. If $m \ll N_a, N_b$, these factors can e approximated by $\sqrt{N_{a,b}}$ respectively. So now each field operator has been eplaced in (10) by a factor $\sqrt{N_{a,b}}$ multiplying a position wave-function u_a or v_b or its complex conjugate). But we still have to take account of particle number

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This may well already give rise to a slight correlation between the results η_1, η_2: f η_1 is $+1$ and φ_1 and φ_2 are close, then η_2 is more likely than not also to equal $+1$.

But as one considers additional transverse spin measurements, strong correlations become apparent. The probability distribution for the transverse spin of the $(m+1)$st particle *conditional on outcomes η_j for the other m measurements* becomes strongly peaked as m increases. So for large m the probability listribution from the double Fock state (3) coincides with that of the relative phase state (15)!

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Laloë's "EPR" *gedankenexperiment*

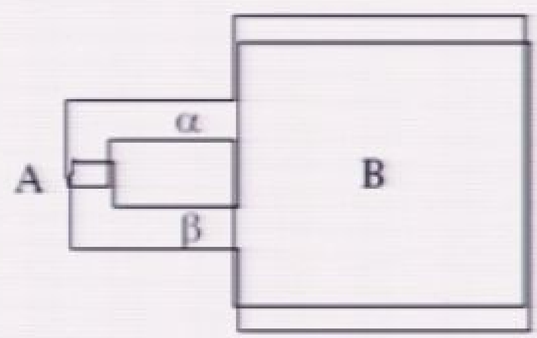


Figure 2: Two different spin states ($\alpha = +$ and $\beta = -$) are associated with two orbital wave functions that overlap mostly in a large region B, but also have "fingers" that overlap in a much smaller region A. The two states are macroscopically populated. Under the influence of a few measurements of the spin of particles performed in region A, a macroscopic transverse spin polarization appears in region B.

Laloë's claim

"the EPR argument can be transposed to this case, and ... the argument becomes stronger, mostly because the measured systems themselves are now macroscopic."

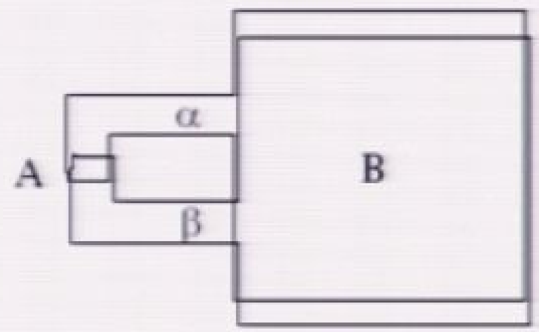
Application of his analysis

To simplify, suppose u_a, v_b have the same relative *phase* everywhere, even though their relative amplitudes very spatially (so ξ=0 in (14),(16)).

Then after, say *m=100* measurements of transverse spin on atoms in *A*, there will be a certain angle φ for which *every* measurement of transverse φ-spin on an atom in *B* is almost certain to yield φ-spin up!

(The angle φ depends on the "emerging" relative phase Λ between the two condensates.)

Laloë's "EPR" *gedankenexperiment*



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Laloë's conclusion

We have a situation that is similar to the usual EPR situation: measurements erformed in A can determine the direction of spins in both regions A and B. If re rephrase the EPR argument to adapt it to this case, we just have to replace he words 'before the measurement in A' by 'before the series of measurements n A', but all the rest of the reasoning remains exactly the same: since the lements of reality in B cannot appear under the effect of what is done at an rbitrary distance in region A, these elements of reality must exist even before he measurements performed in A. Since the double Fock state (3) of quantum nechanics does not contain any information on the direction of spins in B, this heory is incomplete."

His challenge to Bohr

'What is new here is that the EPR elements of reality in B correspond to a ystem that is macroscopic. One can no longer invoke its microscopic character o deprive the system contained in B of any physical reality! The system can ven be at our scale, correspond to a macroscopic magnetization that can be lirectly observable with a hand compass; is it then still possible to state that it as no intrinsic physical reality? When the EPR argument is transposed to the nacroscopic world, it is clear that Bohr's refutation does not apply in the form vritten in his article; it has to be at least modified in some way."

How Bohr would (should?) respond

- Quantum mechanics requires no microscopic/macroscopic distinction.
- Nor does it presuppose a division in the world between quantum and classical objects.
- But in order to apply quantum mechanics to a system *S* one must *describe* the experimental arrangement surrounding that system classically.
- That is true irrespective of whether *S* is microscopic or macroscopic.

as he put it…

"The necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical descriptions of physical phenomena.*"

Bohr's radical pragmatism

- All ascriptions of physical reality to properties of systems are contextual: taken out of context they lack significance.
- This is a pragmatic, not a verificationist view of meaning: Bohr was no positivist.
- In applying quantum mechanics to the BECs in Laloë's "EPR" *gedankenexperiment*, an ascription of a macroscopic magnetization to the contents of region *B* lacks significance in the absence of classically described conditions external to the system of BECs.
- It is placing a hand compass in the vicinity of *B* that renders that ascription meaningful, and indeed true.

Implications for reduction

- Can classical physics be reduced to quantum physics?
- According to classical physics, the behavior of a hand compass near region *B* following 100 or so measurements of transverse spin on particles in region *A* would warrant ascribing a macroscopic magnetization to (the contents of) region *B*.
- But if one endorses (what I take to be) Bohr's response, the statement that *B* contains macroscopic magnetization is not true, or even significant, outside of an appropriate context,
- so there can be no universal reduction of classical to quantum physics.

Relative phase in the Josephson effect

When a pair of conductors separated by a thin metal oxide junction is cooled to become superconducting, a current flows across the junction even in the absence of an applied voltage. This DC Josephson effect may be explained quantum mechanically by appeal to a well-defined phase difference ϕ across the junction in the wave-function representing the state of the system: the DC current is proportional to $\sin\phi$.

Relative phase *vs*. Fock states

Leggett and Sols write the wave-function as follows,

$$\Phi \sim \left(|a|\, e^{i\phi/2}\psi_L + |b|\, e^{-i\phi/2}\psi_R\right)^N \tag{17}$$

where the system consists of N "bosons" (Cooper pairs) and ψ_L (ψ_R) is the Schrödinger amplitude for a boson to be on the left (right) of the junction.

This state (17) may be expanded in a basis of double Fock states $|N_a, N_b\rangle$ as

$$\Phi \sim \sum_{M=-N/2}^{+N/2} |C_M|\, e^{iM\phi}\, |N_a, N_b\rangle \tag{18}$$

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What if two superfluids never met?

Leggett proposed his own *gedankenexperiment*. Suppose one condensate is prepared in Peking, while the other is made in Toronto.

The "experiment" simply consists in weighing them at separate times ... that can be arbitrarily far separated, so as to determine the number difference $[N_a - N_b]$ at these times, without ever making Josephson contact between them.

If (17) correctly represents their total state, then there is no reason to expect the results to agree: indeed, one would expect them to differ by an amount of the order of $N^{\frac{1}{2}}$. If, on the other hand, the correct representation is a double Fock state (or mixture of these), then the results would be expected to agree (within the margin of error of the experiment).

Ontological emergence?

Leggett says he “can see no reason whatever to doubt that it is the latter conclusion which would be found experimentally”.

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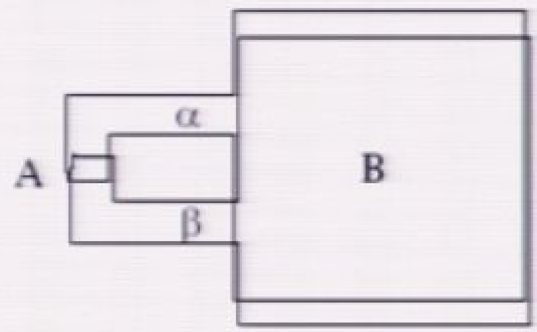

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Now consider the case $m = 2$: joint measurement of φ_1-spin and φ_2-spin with results η_1, η_2 respectively on two particles. The Λ-probability distribution for result η_2 *conditional on outcome* η_1 is now weighted by a factor that depends both on the angle φ_1 of the measurement on particle 1 and on its outcome and location $(\eta_1, \mathbf{r}_1)$ and is proportional to

$$N_a \left|u_a(\mathbf{r}_1)\right|^2 + N_b \left|v_b(\mathbf{r}_1)\right|^2 + 2\eta_1 \sqrt{N_a N_b} \left|u_a(\mathbf{r}_1)\right| \left|v_b(\mathbf{r}_1)\right| \cos(\Lambda + \xi(\mathbf{r}_1) - \varphi_1) \quad (16)$$

This may well already give rise to a slight correlation between the results η_1, η_2: if η_1 is $+1$ and φ_1 and φ_2 are close, then η_2 is more likely than not also to equal $+1$.

But as one considers additional transverse spin measurements, strong correlations become apparent. The probability distribution for the transverse spin of the $(m+1)$st particle *conditional on outcomes* η_j *for the other* m *measurements* becomes strongly peaked as m increases. So for large m the probability distribution from the double Fock state (3) coincides with that of the relative phase state (15)!

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There is a clever trick to take account of particle number conservation in each sequence, using the mathematical identity

$$\int_0^{2\pi} \frac{d\Lambda}{2\pi} e^{in\Lambda} = \delta_{n,0} \tag{12}$$

By multiplying each $\hat{\Psi}_\alpha(\mathbf{r})$ (or rather $\sqrt{N_a}u_a(\mathbf{r})$) by $e^{i\Lambda}$, and each $\hat{\Psi}^\dagger_\alpha(\mathbf{r})$ (or rather $\sqrt{N_a}u^*_a(\mathbf{r})$) by $e^{-i\Lambda}$, and integrating Λ over 2π (and similarly for the b particles), we automatically take account of particle number conservation!

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