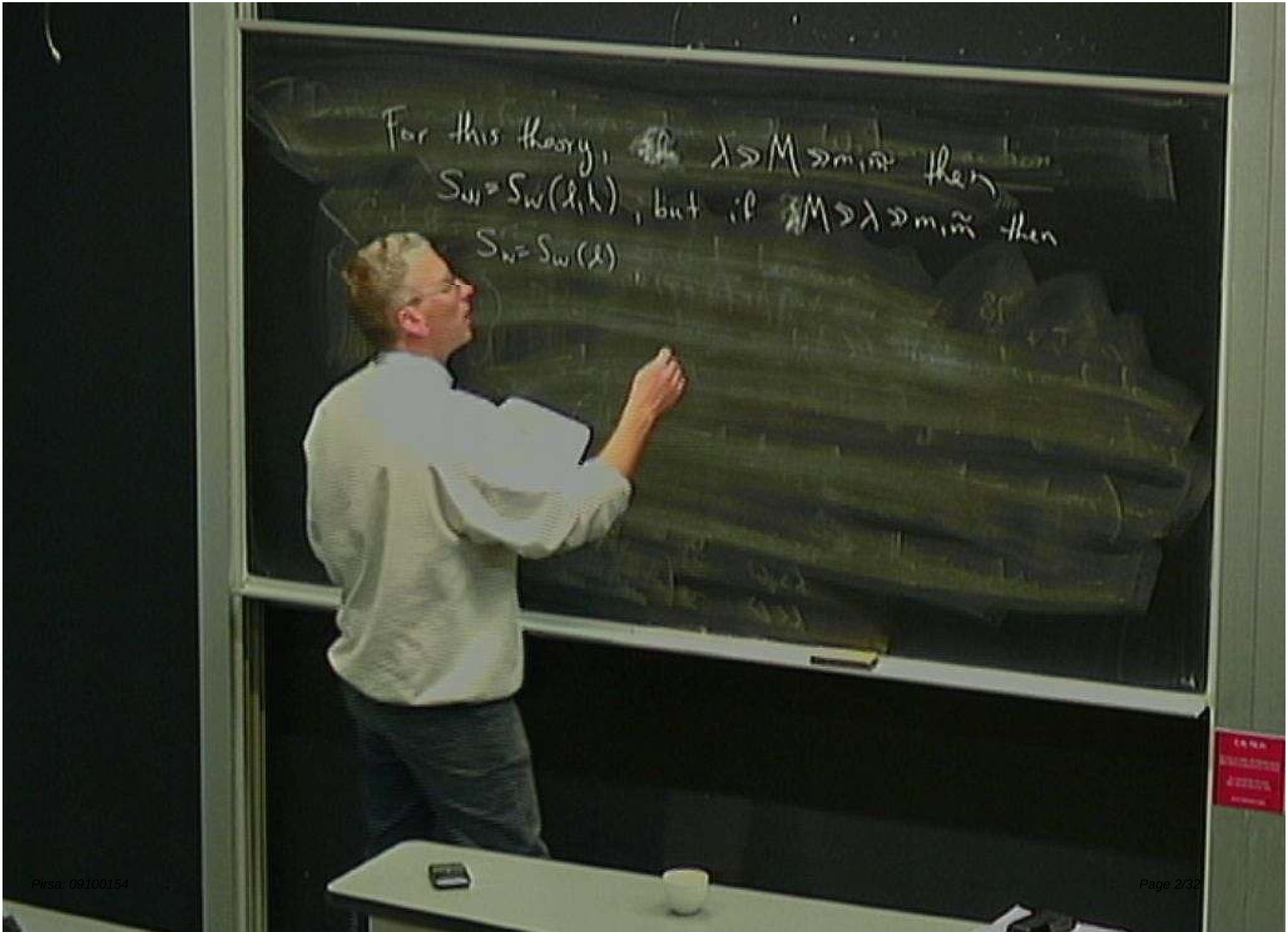
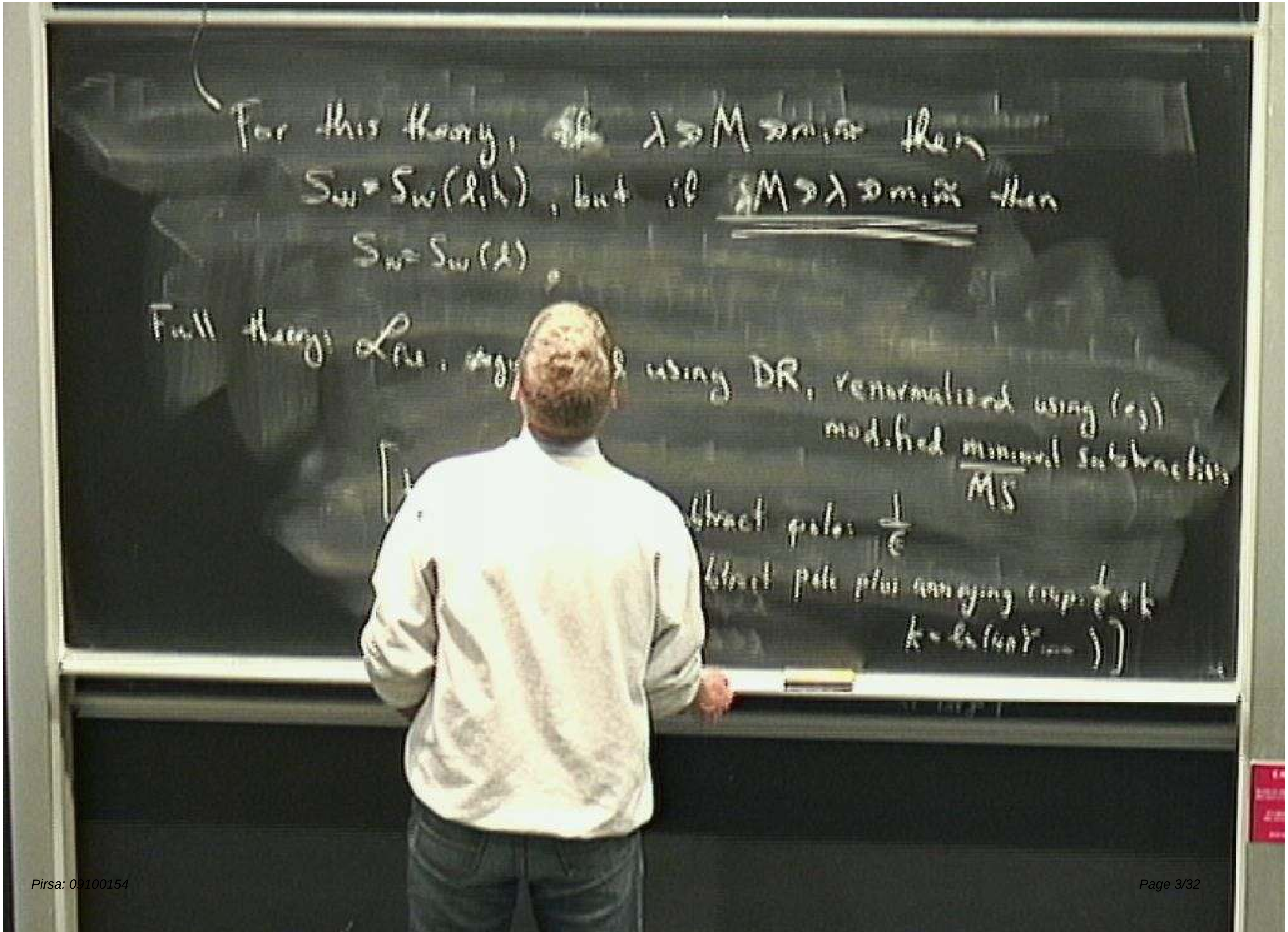
Title: Introduction to Effective Field Theory - Lecture 4B

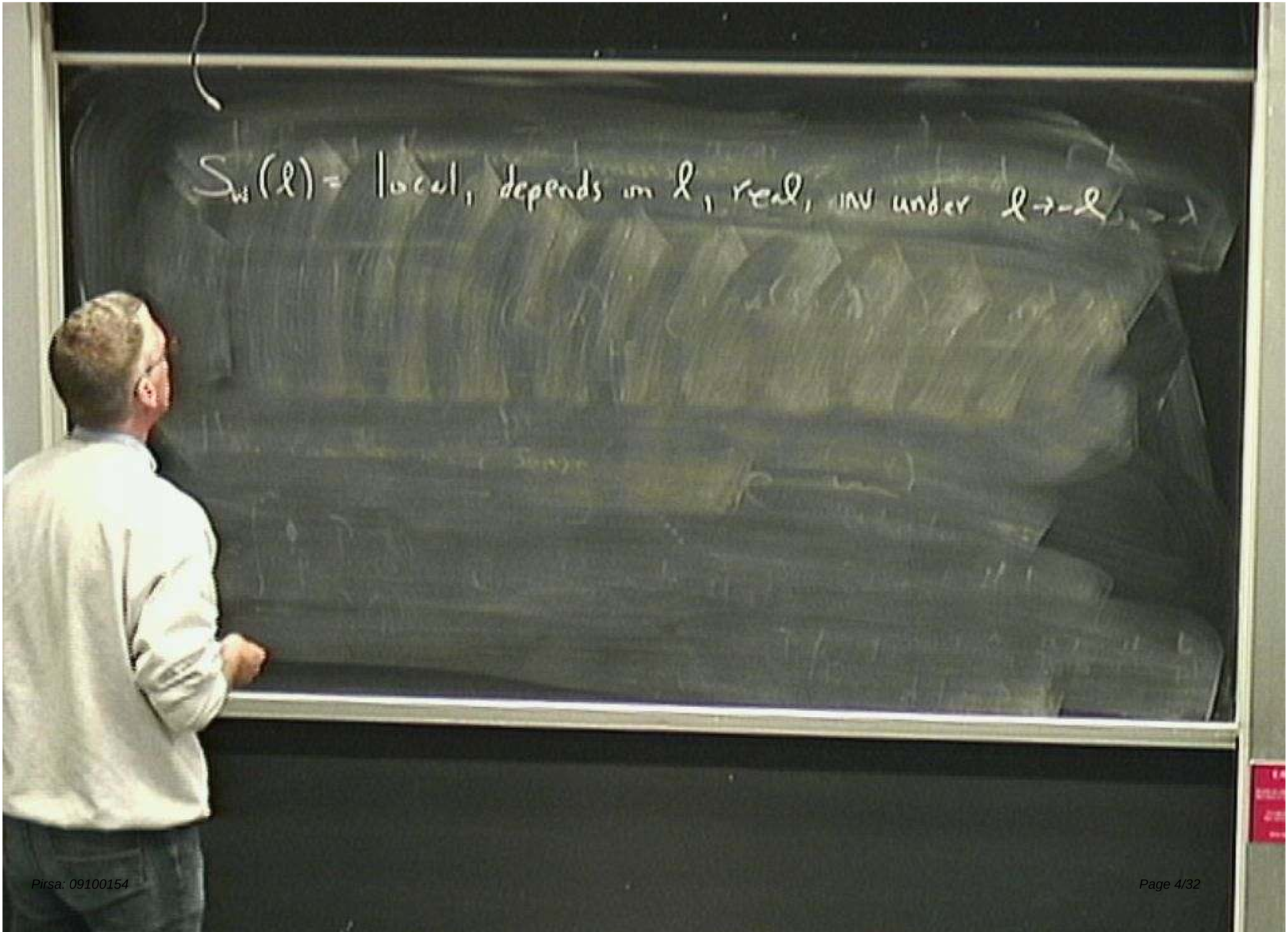
Date: Oct 14, 2009 11:00 AM

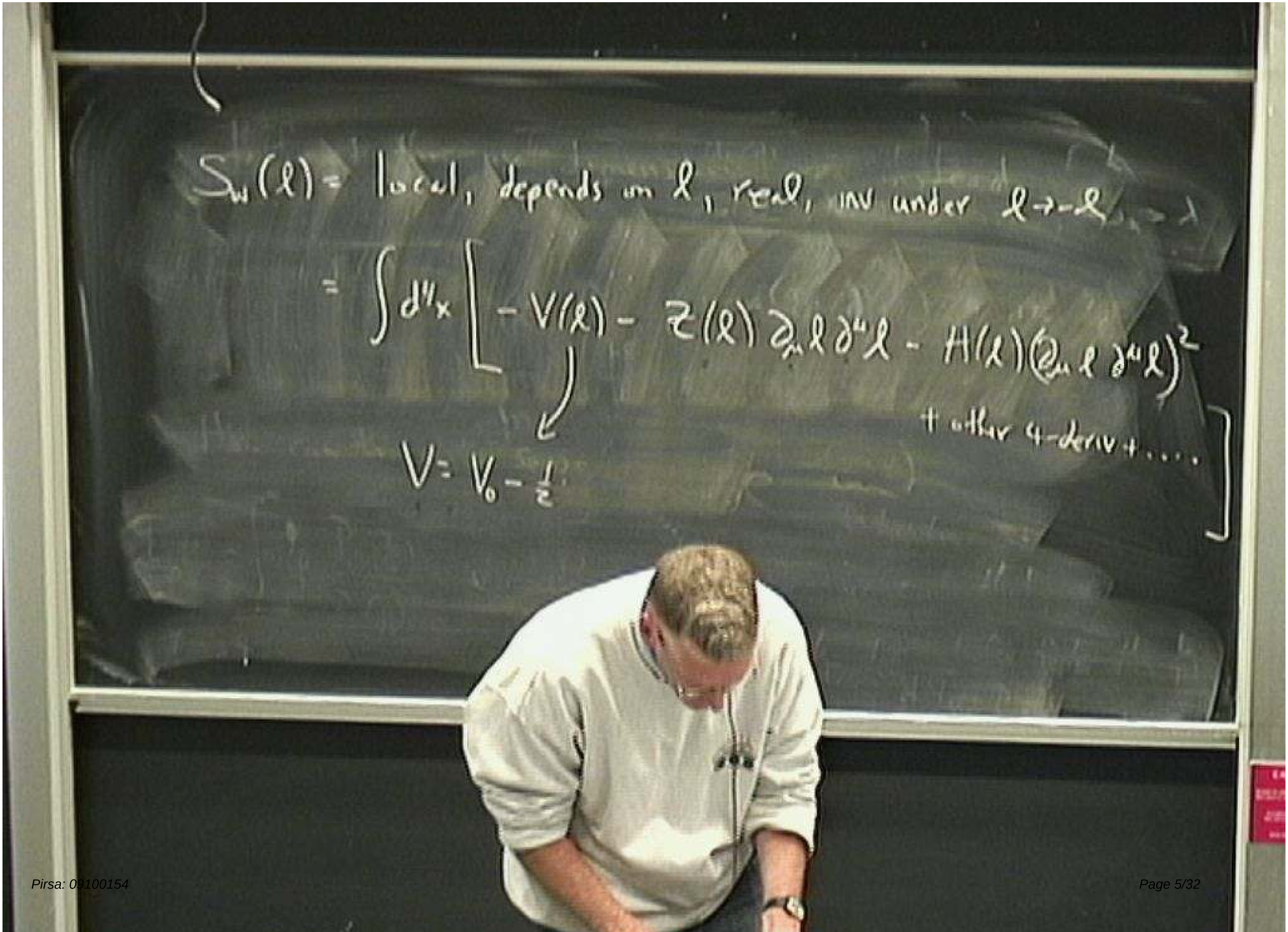
URL: http://pirsa.org/09100154

Abstract:


For this theory, λ≫M≫m,m̃ then
S_W = S_W(λ,Λ), but if M≫λ≫m,m̃ then
S_W = S_W(λ)


For this theory, if $\lambda \gg M \gg m_i$ then
$S_W = S_W(\lambda, \lambda)$, but if $M \gg \lambda \gg m_i$ then
$S_W = S_W(\lambda)$
Full theory: $\mathcal{L}_{FE}$ using DR, renormalized using (eg)
modified minimal subtraction
$\overline{MS}$
subtract poles $\frac{1}{\epsilon}$
subtract pole plus annoying crap $\frac{1}{\epsilon} + k$
$k = \ln(4\pi\ldots)$


S(ℓ) = local, depends on ℓ, real, inv under ℓ→−ℓ


$S_w(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$
$= \int d^4x \left[-V(\ell) - Z(\ell)\, \partial_\mu \ell\, \partial^\mu \ell - H(\ell) (\partial_\mu \ell\, \partial^\mu \ell)^2 \right.$
$\left. + \text{other 4-deriv} + \ldots \right]$
$V = V_0 - \frac{1}{2}$

$S_W(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$

$$= \int d^4x \Big[-V(\ell) - Z(\ell)\, \partial_\mu \ell\, \partial^\mu \ell - H(\ell)\left(\partial_\mu \ell\, \partial^\mu \ell\right)^2 + \text{other 4-deriv} + \dots \Big]$$

$$V = a_0 - \tfrac{1}{2} a_2 \ell^2 - \tfrac{1}{4!} a_4 \ell^4 + \dots$$

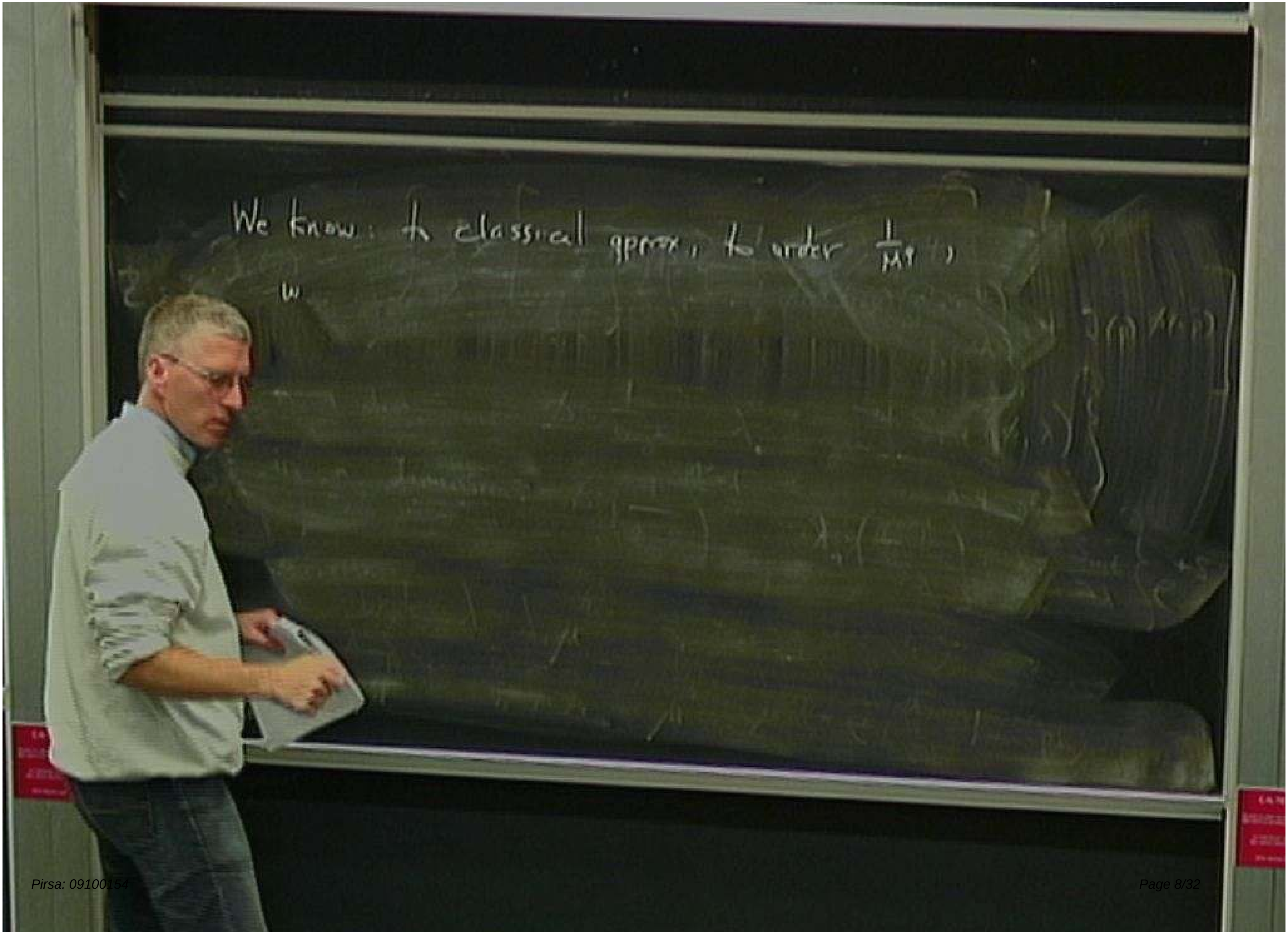
$$Z = 1 + b_2 + b_4 \ell^2 + \dots$$

etc.

Q: what choices for $a_0, a_2, a_4, b_4, \dots$ etc reproduce full theory to any given order in $1/m$

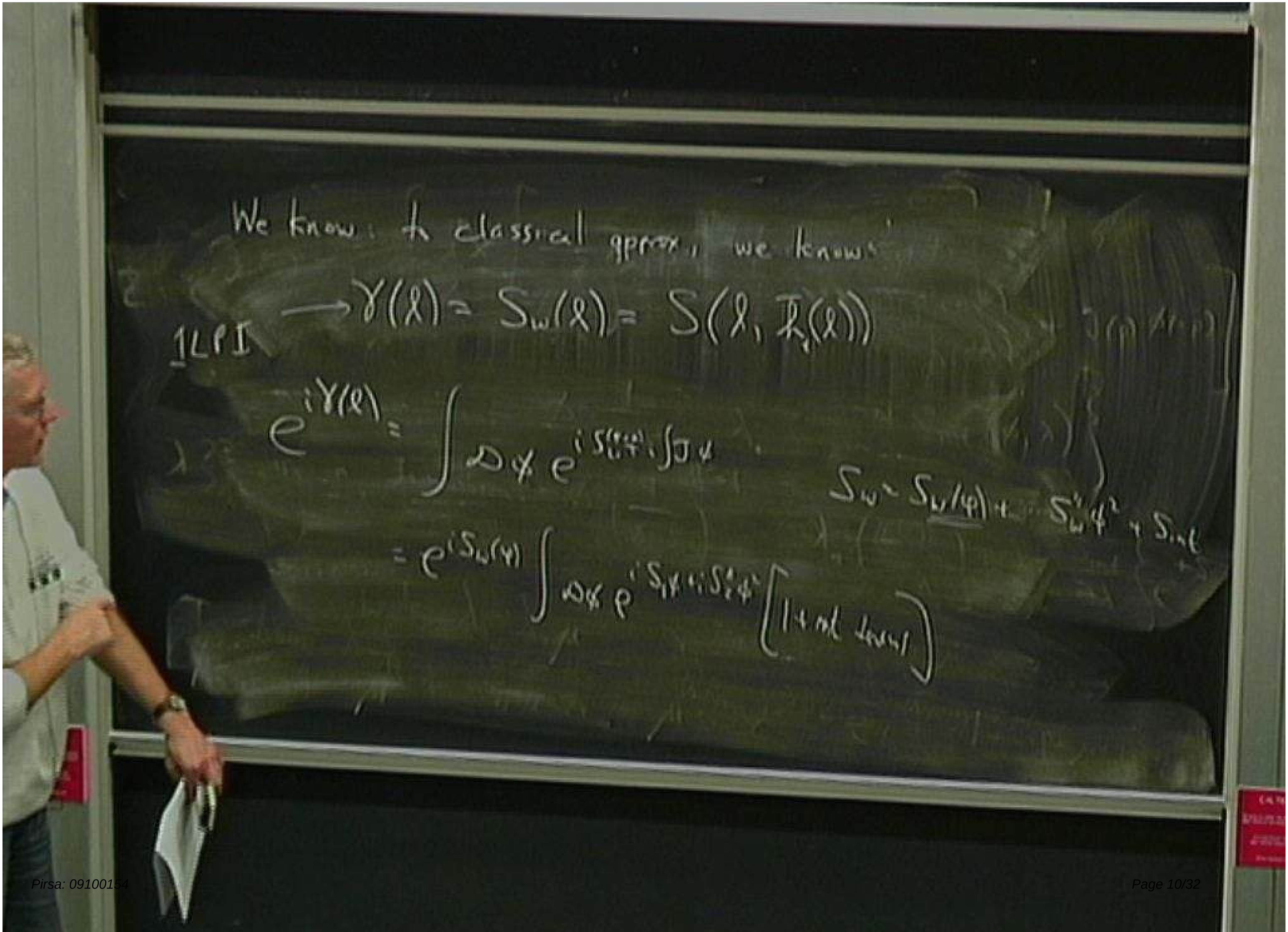
2 parts: 1) Which couplings are required at all to a fixed order in $1/M$? ('Power counting')

2) What value is required of the necessary couplings. ('matching')


We know: th classical approx, to order

We know: to classical approx., we know:

$$\gamma(\ell) = S_w(\ell) = S(\ell, \bar{h}_c(\ell))$$


We know: to classical approx, we know:
1LPI
Γ(φ) = S_W(φ) = S(φ, Φ_φ(φ))
e^{iΓ(φ)} = ∫ Dϕ e^{iS(φ+ϕ) + i∫Jϕ}
S_W ≈ S_W(φ) + S_W'' ϕ^2 + S_int
= e^{iS_W(φ)} ∫ Dϕ e^{iS_1ϕ + iS_2ϕ^2} [1 + int terms]

We know: to classical approx, we know:

1LPI $\longrightarrow \gamma(\ell) = S_W(\ell) = S(\ell, \bar{\phi}_\ell(\ell))$

$$e^{i\gamma(\ell)} = \int \mathcal{D}\phi \, e^{iS_W(\phi) + i\int J\phi}$$

$$S_W = S_W(\varphi) + S''_W \phi^2 + S_{int}$$

$$= e^{iS_W(\varphi)} \int \mathcal{D}\phi \, e^{iS_1\phi + iS_2\phi^2} \left[1 + \text{int terms} \right]$$

We know: to classical approx, we know:

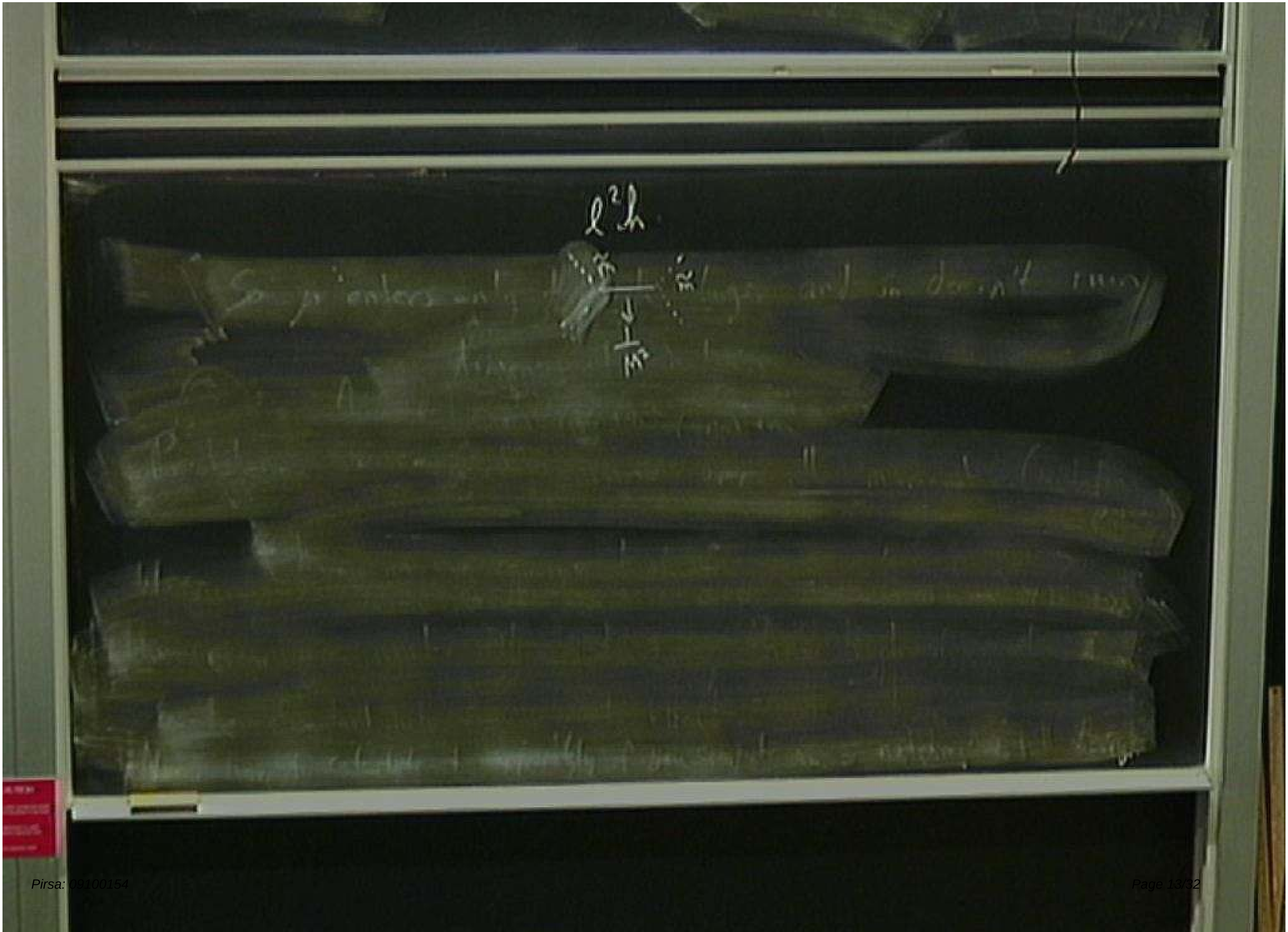
$$\xrightarrow{\text{1LPI}} \underline{\gamma(\ell)} = S_w(\ell) = S(\ell, \bar{\Phi}_1(\ell))$$

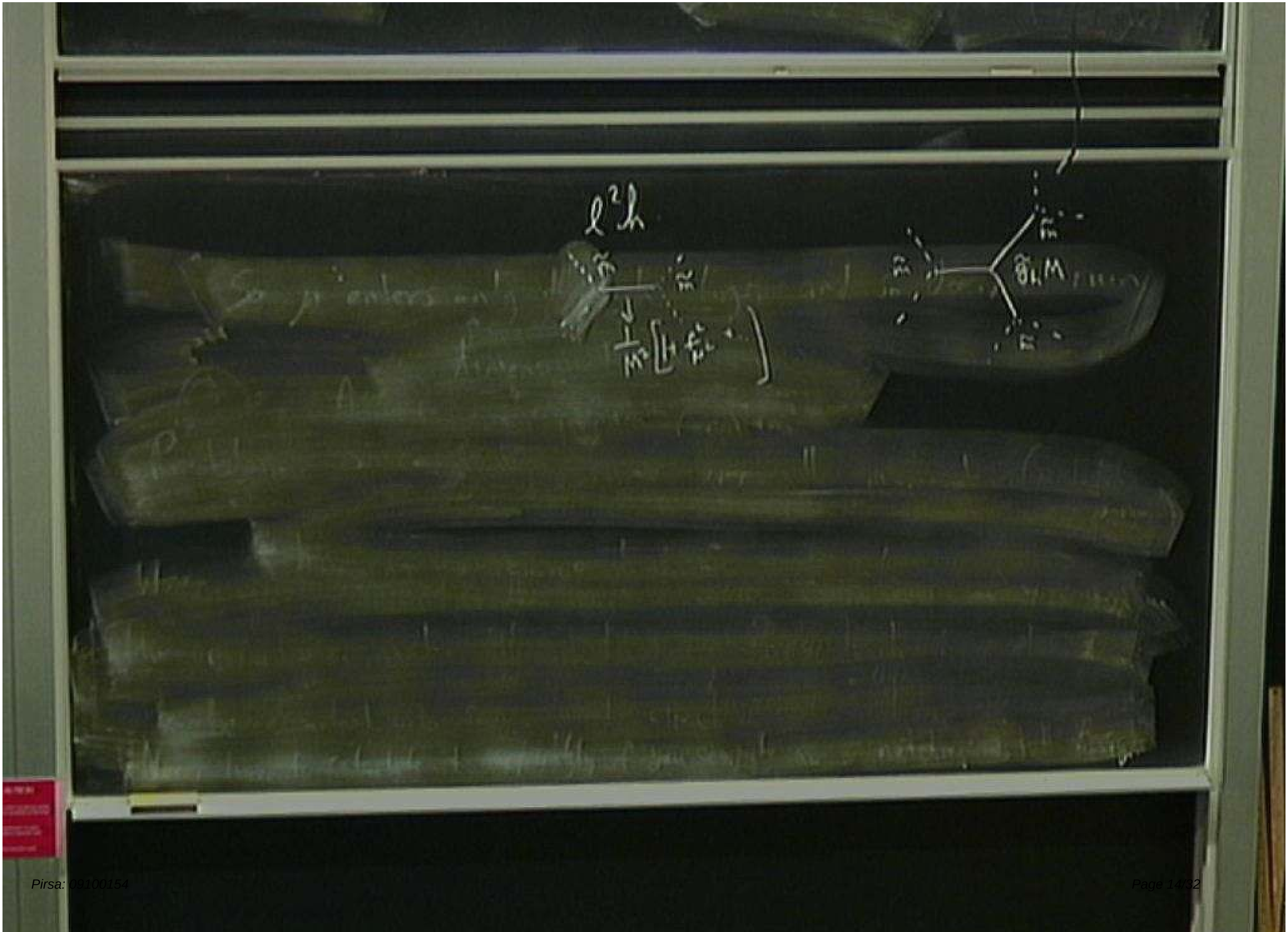
$$e^{i\gamma(\ell)} = \int \mathcal{D}\phi \, e^{iS(\ell,\phi) + i\int J\phi}$$

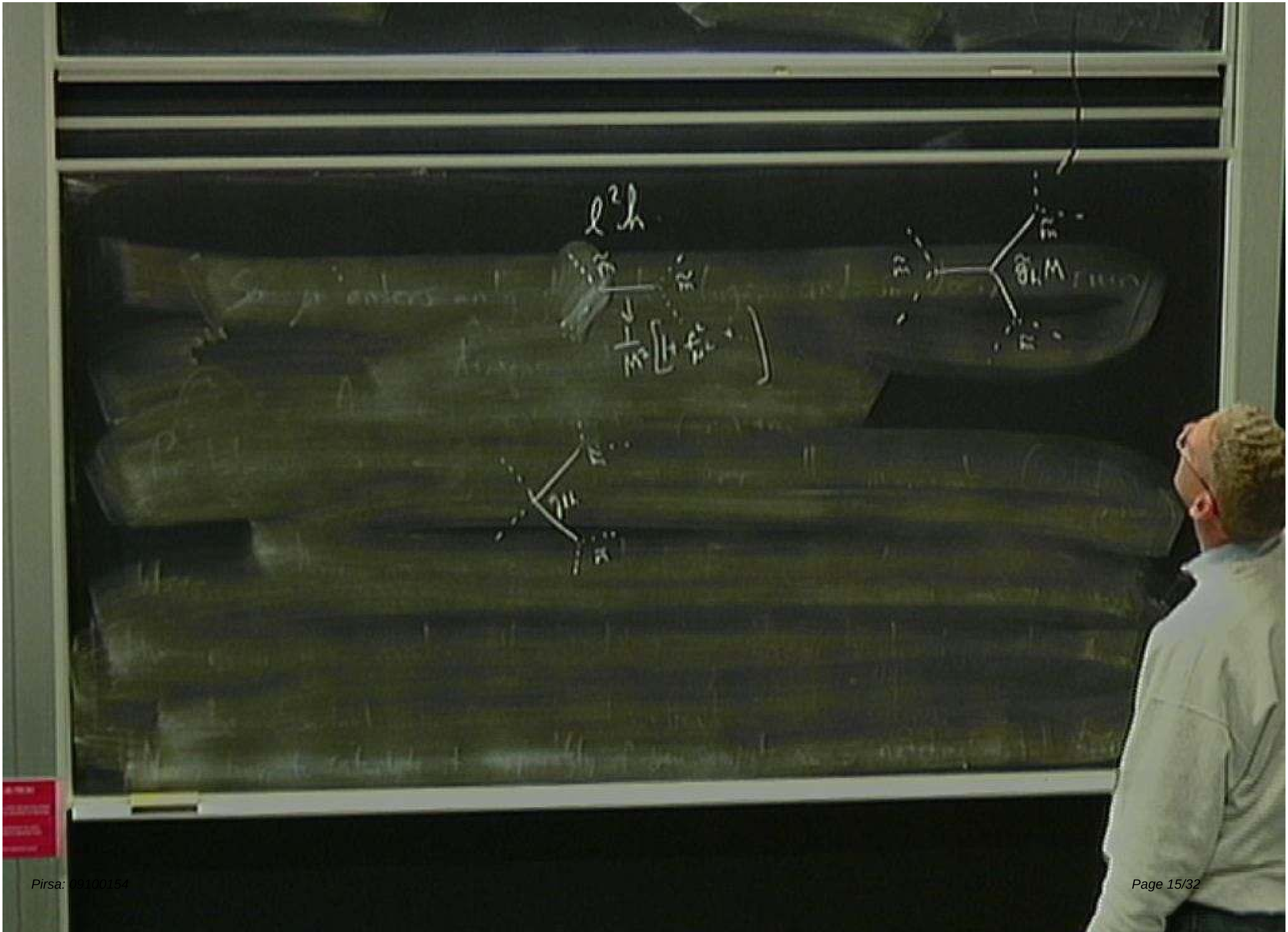
$$S_w = \underline{S_w(\varphi)} + \quad S_w''\phi^2 + S_{int}$$

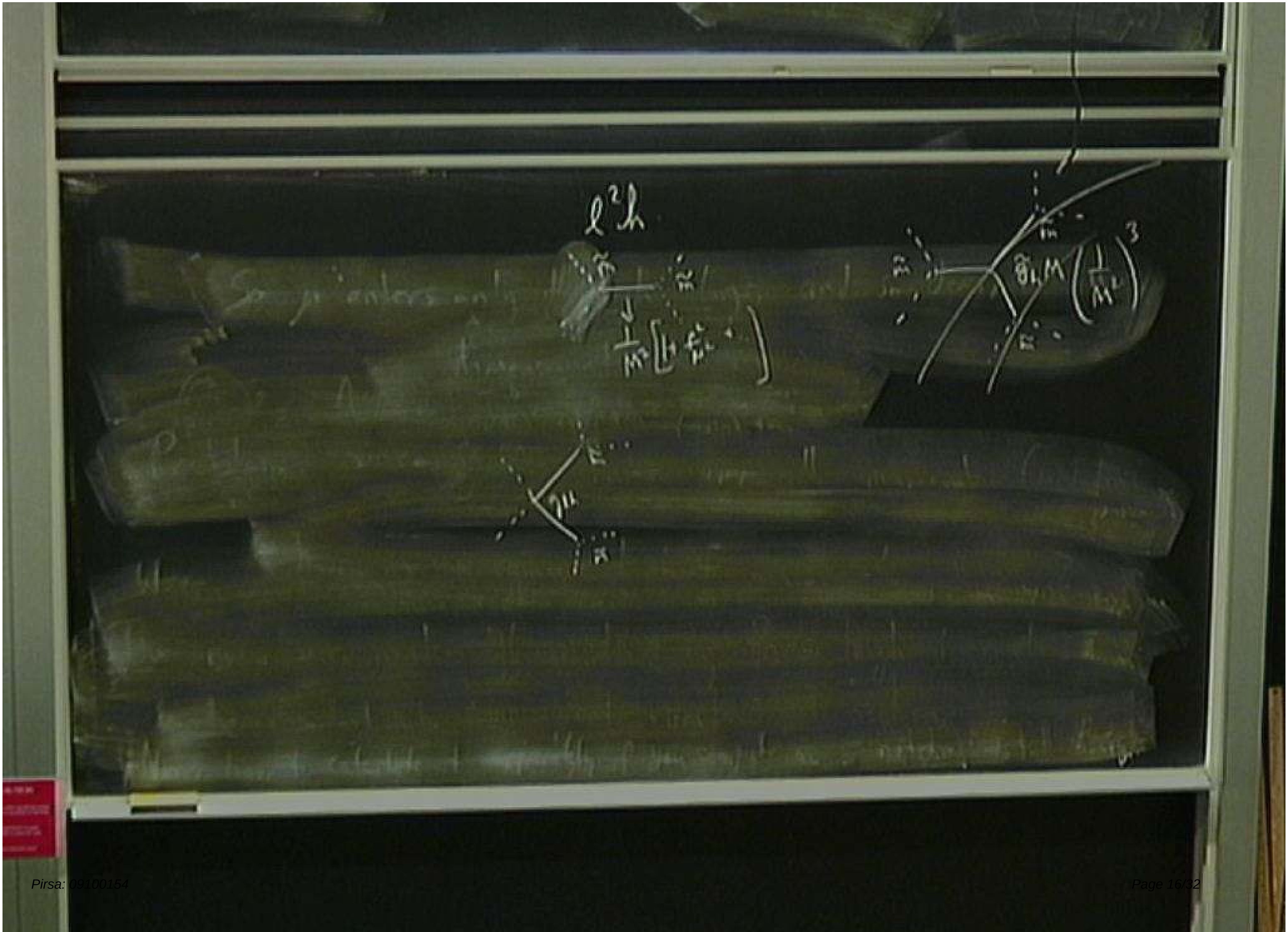
$$= e^{iS_w(\varphi)} \int \mathcal{D}\phi \, e^{iS_1\phi + iS_2\phi^2} \left[1 + \text{int. terms} \right]$$

$$\Rightarrow \mathcal{L}^{\ell}_{W} = -\tfrac{1}{2}\partial_\mu \ell \partial^\mu \ell - \frac{m^2}{2}\ell^2 - \left(\frac{g}{4!}\ell^4 - \frac{m^2}{8 M^2}\right)\ell^4 - \left(\frac{m^2}{2M^2}\right)\ell^2 \partial_\mu \ell \partial^\mu \ell - \left(\frac{g_{H} m^2}{16 M^2}\right)\ell^6 + O\left(\frac{1}{M^4}, g_{H} g_{\ell}\right)$$

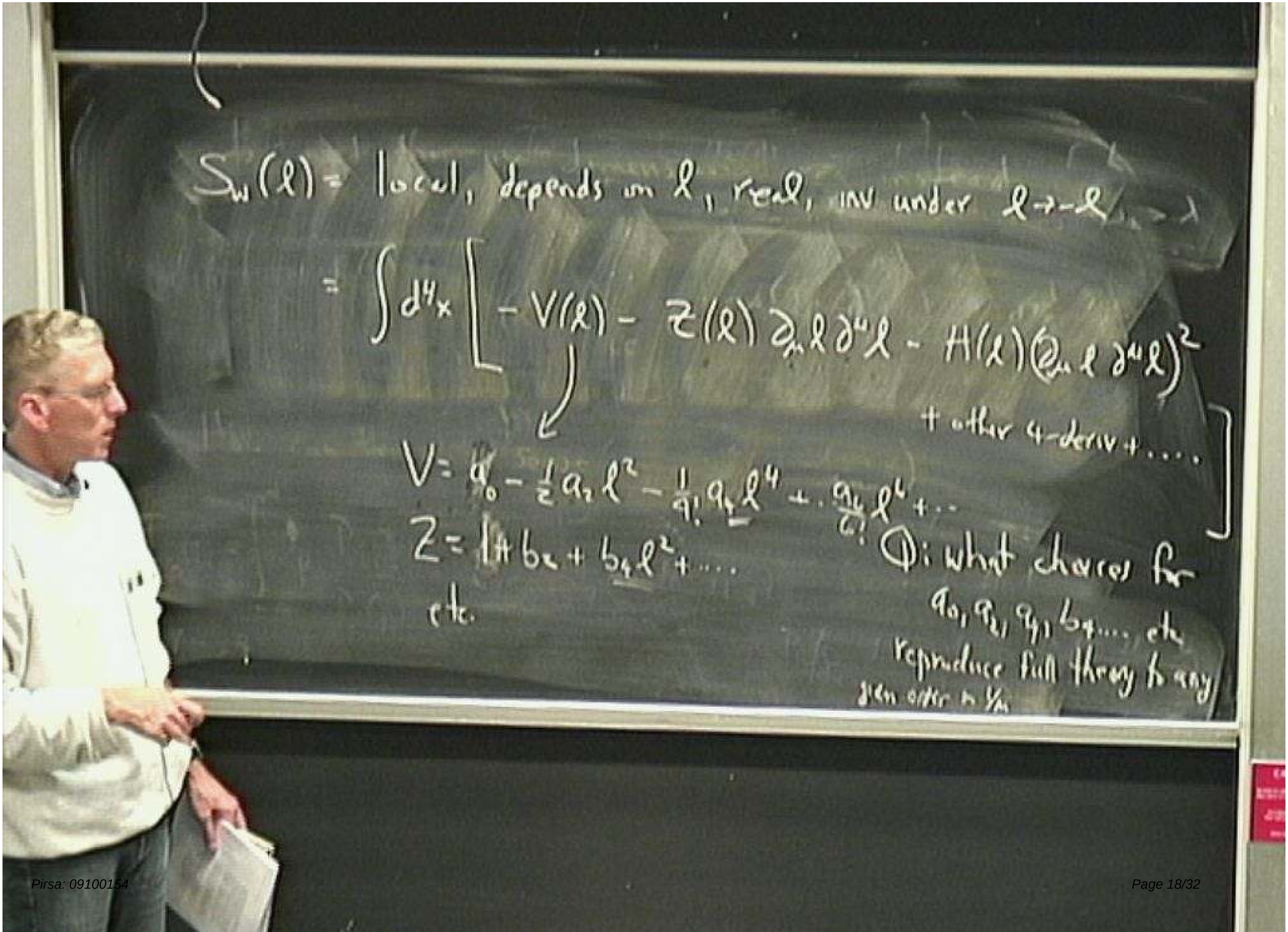


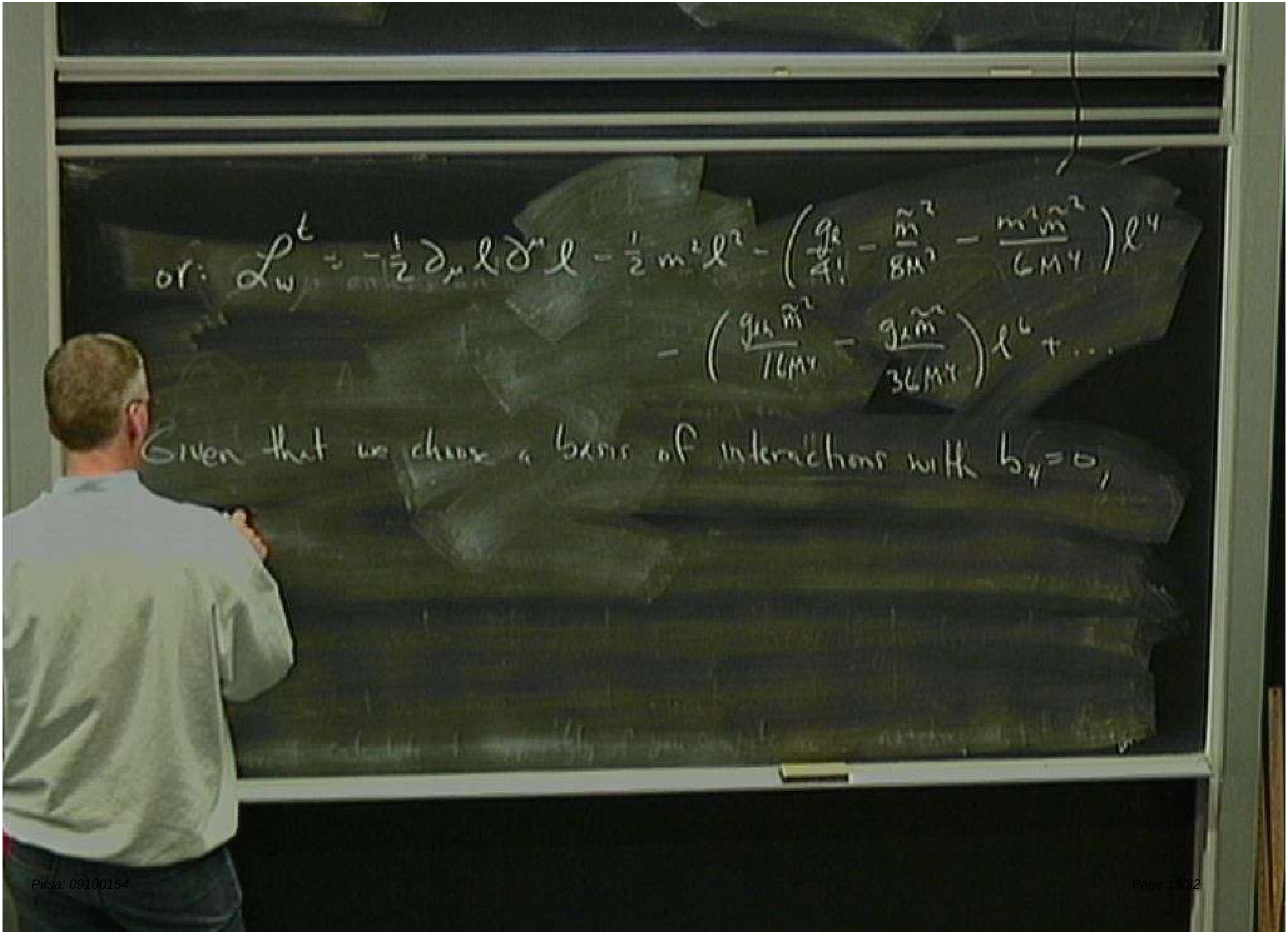

$\ell^2 h$
$g_H M$





or: $$\mathcal{L}_w^t = -\frac{1}{2}\partial_\mu \ell \, \partial^\mu \ell - \frac{1}{2} m^2 \ell^2 - \left(\frac{g_4}{4!} - \frac{\tilde{m}^2}{8M^2} - \frac{m^2 \tilde{m}^2}{6M^4} \right) \ell^4 - \left(\frac{g_4 \tilde{m}^2}{16M^4} - \frac{g_4 \tilde{m}^2}{36M^4} \right) \ell^6 + \ldots$$


S_w(ℓ) = local, depends on ℓ, real, inv under ℓ → −ℓ
= ∫d⁴x [−V(ℓ) − Z(ℓ) ∂_μℓ ∂^μℓ − H(ℓ)(∂_μℓ ∂^μℓ)²
+ other 4-deriv + ...
V = a₀ − ½ a₂ ℓ² − (1/4!) a₄ ℓ⁴ + ...
Z = 1 + b₂ + b₄ ℓ² + ...
etc.
Q: what choices for
a₀, a₂, a₄, b₄, ... etc
reproduce full theory to any
given order in 1/M


or: $\mathcal{L}^t_W = -\frac{1}{2}\partial_\mu \ell \partial^\mu \ell - \frac{1}{2} m^2 \ell^2 - \left(\frac{g_4}{4!} - \frac{\tilde{m}^2}{8M^2} - \frac{m^2\tilde{m}^2}{6M^4}\right)\ell^4$
$- \left(\frac{g_4 \tilde{m}^2}{16M^4} - \frac{g_4 \tilde{m}^2}{36M^4}\right)\ell^6 + \dots$
Given that we choose a basis of interactions with $b_4 = 0$,

At one loop level, ask what matching implies for $a_{d_i}, \ldots$ to $o(M^0)$.

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To do this, regularize <u>both</u> the Full theory <u>and</u> the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

At one loop level, ask what matching implies for ald... to $o(M^0)$.

To do this, regularize both the Full theory and the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

recall, if $A_R = A_0 - \frac{1}{64\pi^2}(M^4+m^4)L$ $\quad L = \frac{1}{\epsilon}+k$ $\quad V=$

$B_R = B_0$

Matching: Use the toy model:

$$-A_0 - B_0 h$$

$$\mathcal{L}_{full}(\ell, h) = -\frac{1}{2}\partial_\mu \ell\, \partial^\mu \ell - \frac{1}{2}\partial_\mu h\, \partial^\mu h - \frac{1}{2} m_0^2 \ell^2 - \frac{1}{2} M_0^2 h^2$$

$$-\frac{\tilde{g}_h}{3!} M h^3 - \frac{1}{2}\tilde{m}\, \ell^2 h - \frac{1}{4!} g_h h^4 - \frac{1}{4!} g_\ell \ell^4 - \frac{1}{4} g_{\ell h} \ell^2 h^2$$

(inv under $\ell \to -\ell$)

$$m, \tilde{m} \ll M$$

At one loop level, ask what matching implies for ald...
to $o(M^0)$.

To do this, regularize <u>both</u> the Full theory <u>and</u>
the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

recall, if $A_R = A_0 - \frac{1}{64\pi^2}(M^4 + m^4)L$ $\qquad L = \frac{1}{\epsilon} + k$ $\qquad V =$

$B_R = B_0 - \frac{1}{32\pi^2}(m^2\tilde{m} + \hat{g}_h M^3)L$

$m_R^2 = m_0^2 - \frac{1}{32\pi^2}(g_0 m^2 + g_h M^2 + 2\tilde{m}^2)L$

; etc

At one loop level, ask what matching implies for $a, d_i, \ldots$ to $o(M^0)$.

To do this, regularize **both** the Full theory **and** the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

recall, if $A_R = A_0 - \frac{1}{64\pi^2}(M^4 + m^4)L$ $\qquad L = \frac{1}{\epsilon} + k$

$B_R = B_0 - \frac{1}{32\pi^2}(m^2\tilde{m} + \hat{g}_h M^3)L$

$m_R^2 = m_0^2 - \frac{1}{32\pi^2}(g_l m^2 + g_{lh} M^2 + 2\tilde{m}^2)L$ $\quad$ s.t. $A_R, B_R \ldots$ etc are finite as $n \to 4$

; etc

Similarly renormalize $\mathcal{L}_{W}$ in dim reg with $\overline{MS}$:

Similarly renormalize $\mathcal{L}_{(4)}$ in dim reg with $\overline{MS}$:

so eg: $a_{1_R} = a_{1_0} - \frac{1}{32\pi^2}$ [illegible] L

$a_{4_R} = a_{4_0} - \frac{3}{32\pi^2} a_4^2 L$

etc

Similarly renormalize $\mathcal{L}_{W}$ in dim reg with $\overline{MS}$:

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$$a_{4_R} = a_{4_0} - \frac{3}{32\pi^2} a_4^2 L$$

etc

Matching: ask how $a_{2_R}, a_{4_R}, \dots$ etc depend on A_R, g_R, m_R, etc in order to make observables agree

Similarly renormalize $\mathcal{L}_W$ in dim reg with $\overline{MS}$:

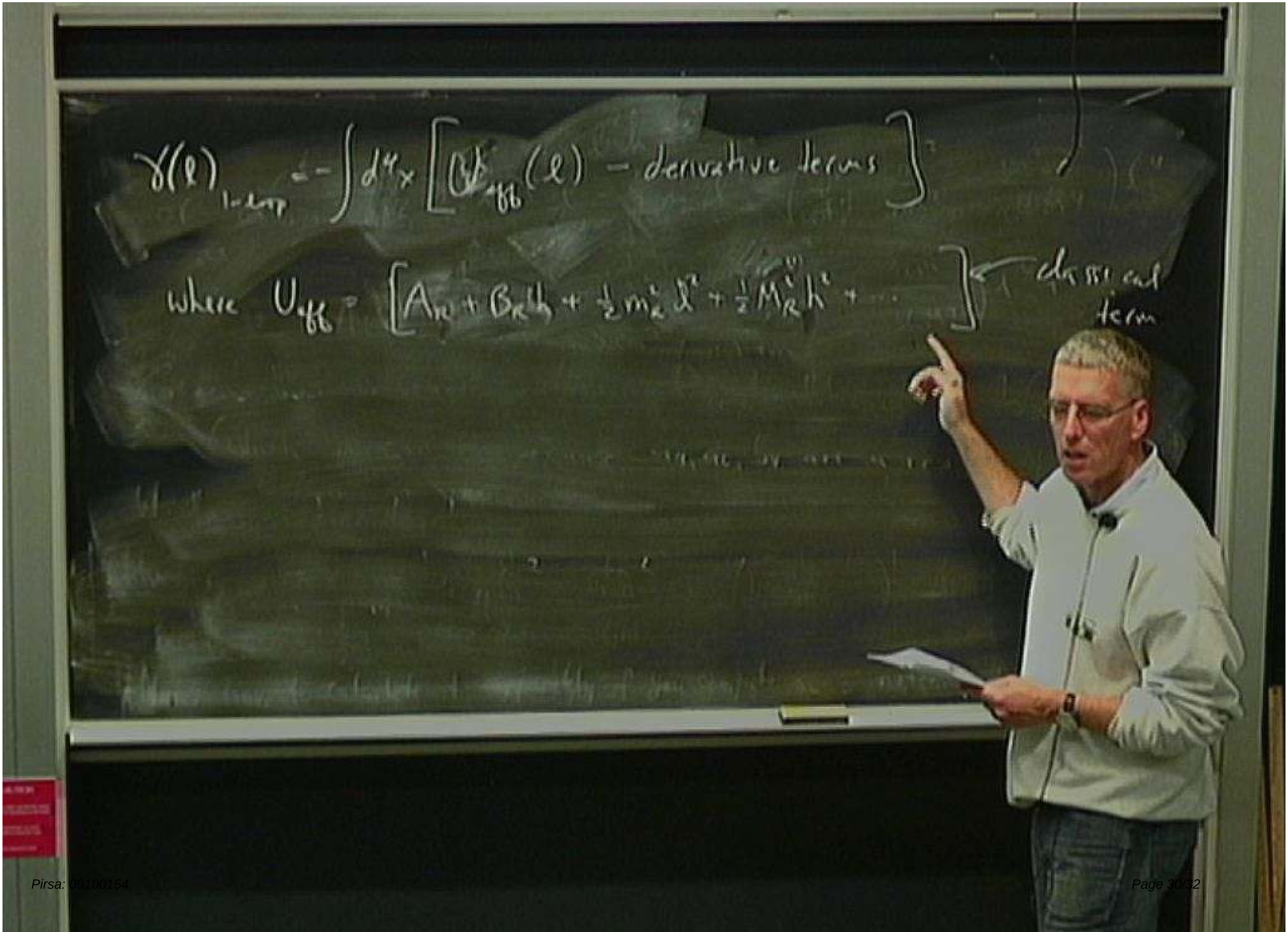
so eg: $a_{2_R} = a_{2_0} - \frac{1}{32\pi^2}$ [illegible] L

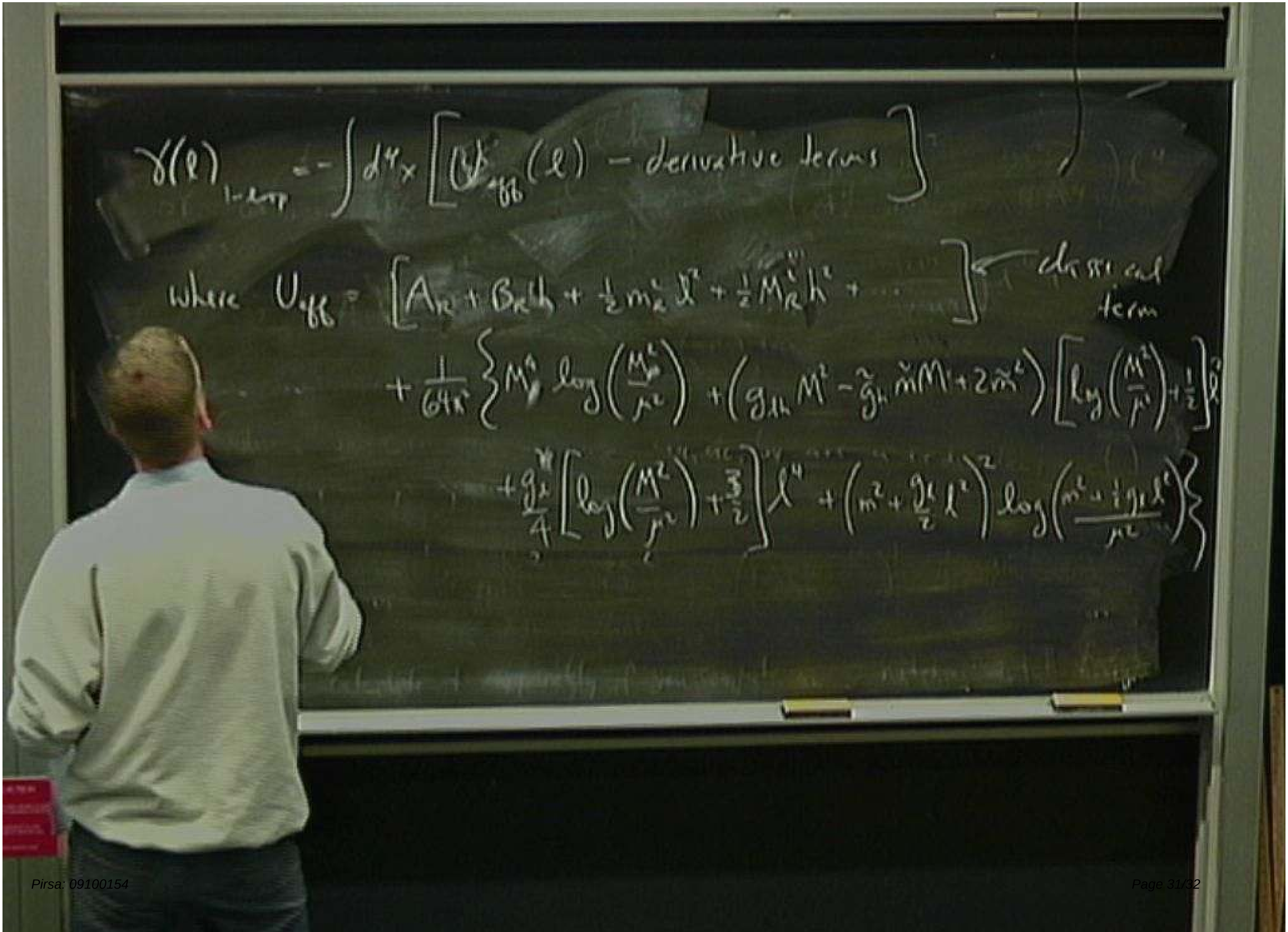
$$a_{4_R} = a_{4_0} - \frac{3}{32\pi^2} a_4^2 L$$

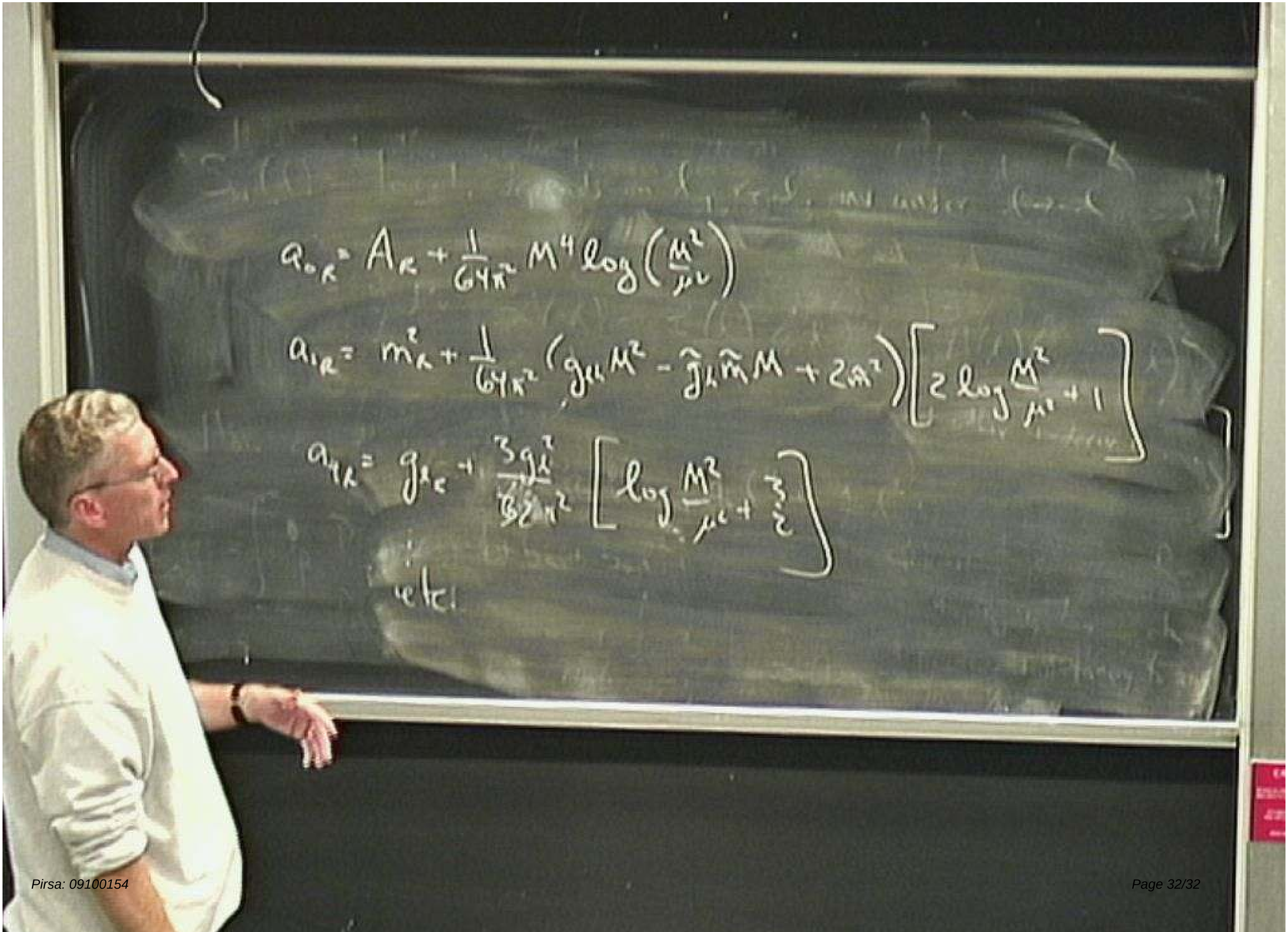
etc

Matching: ask how $a_{2_R}, a_{4_R}, \ldots$ etc depend on $A_R, g_R, m_R, \tilde{g}_R$ in order to make observables agree

If we ask $\mathcal{L}_W$ to reproduce $\gamma(\ell)$ to 1 loop + to order M^4, must


$\gamma(\ell)_{1\text{-loop}} = -\int d^4x \left[U_{eff}(\ell) - \text{derivative terms} \right]$
where $U_{eff} = \left[A_R + B_R h + \frac{1}{2} m_R^2 \ell^2 + \frac{1}{2} M_R^2 h^2 + \cdots \right]$
classical term


$\gamma(\ell)_{1\text{-loop}} = -\int d^4x \left[U_{eff}(\ell) - \text{derivative terms} \right]$
where $U_{eff} = \left[A_R + B_R h + \frac{1}{2} m_R^2 \ell^2 + \frac{1}{2} M_R^2 h^2 + \dots \right]$
classical term
$+ \frac{1}{64\pi^2} \Big\{ M_\phi^4 \log\left(\frac{M_\phi^2}{\mu^2}\right) + \left(g_{4h} M^2 - \tilde{g}_3 \tilde{m} M + 2\tilde{m}^2\right) \left[\log\left(\frac{M^2}{\mu^2}\right) + \frac{1}{2} \right] \ell^2$
$+ \frac{g_4^2}{4} \left[\log\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} \right] \ell^4 + \left(m^2 + \frac{g_4}{2} \ell^2\right)^2 \log\left(\frac{m^2 + \frac{1}{2} g_4 \ell^2}{\mu^2}\right) \Big\}$



$a_{0R} = A_R + \frac{1}{64\pi^2} M^4 \log\left(\frac{M^2}{\mu^2}\right)$
$a_{1R} = m_R^2 + \frac{1}{64\pi^2}\left(g_{11} M^2 - \tilde{g}_{1}\tilde{m} M + 2\tilde{m}^2\right)\left[2 \log \frac{M^2}{\mu^2} + 1\right]$
$a_{4R} = g_{4R} + \frac{3 g_4^2}{32\pi^2}\left[\log \frac{M^2}{\mu^2} + \frac{3}{2}\right]$
etc.