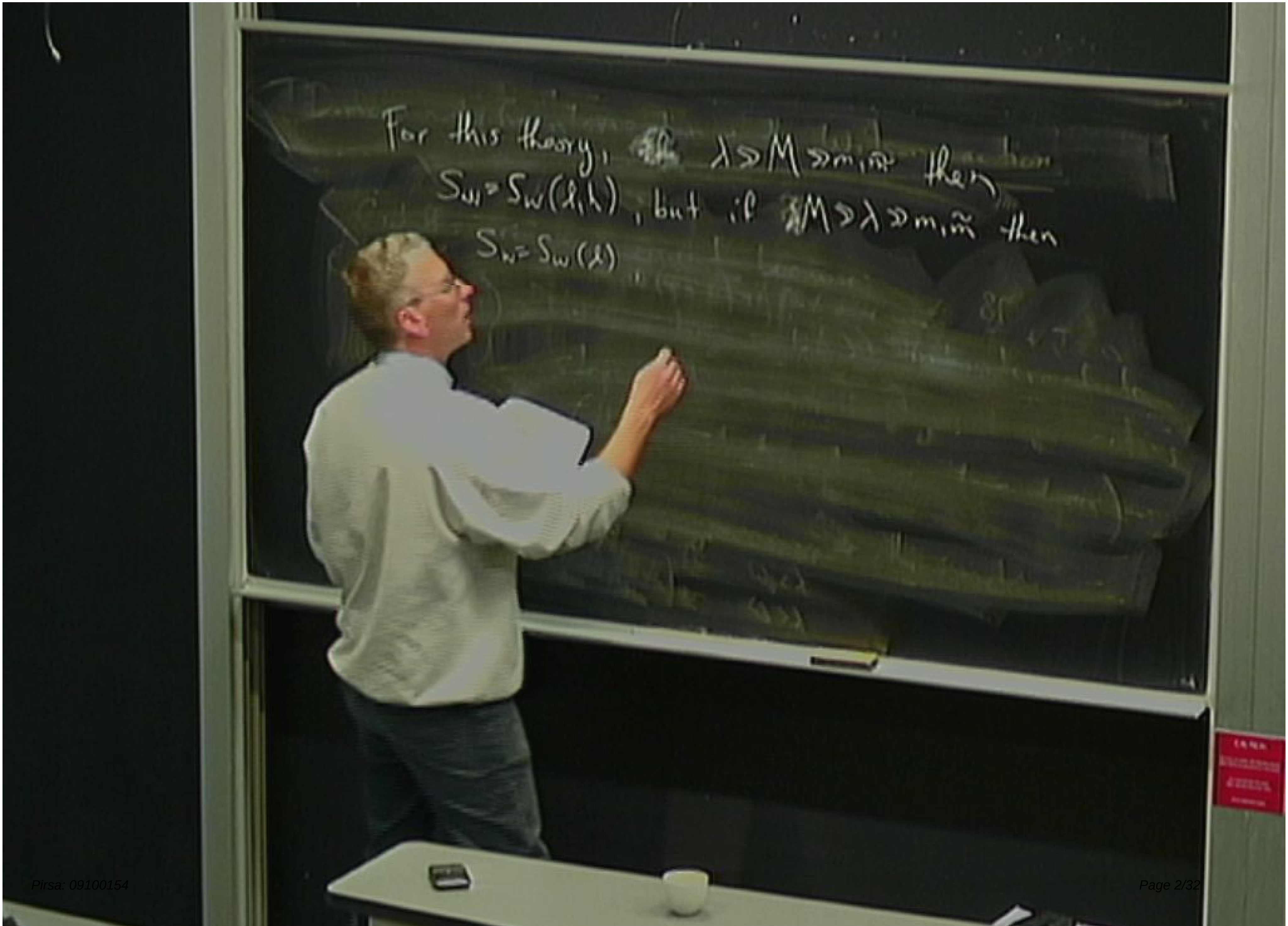
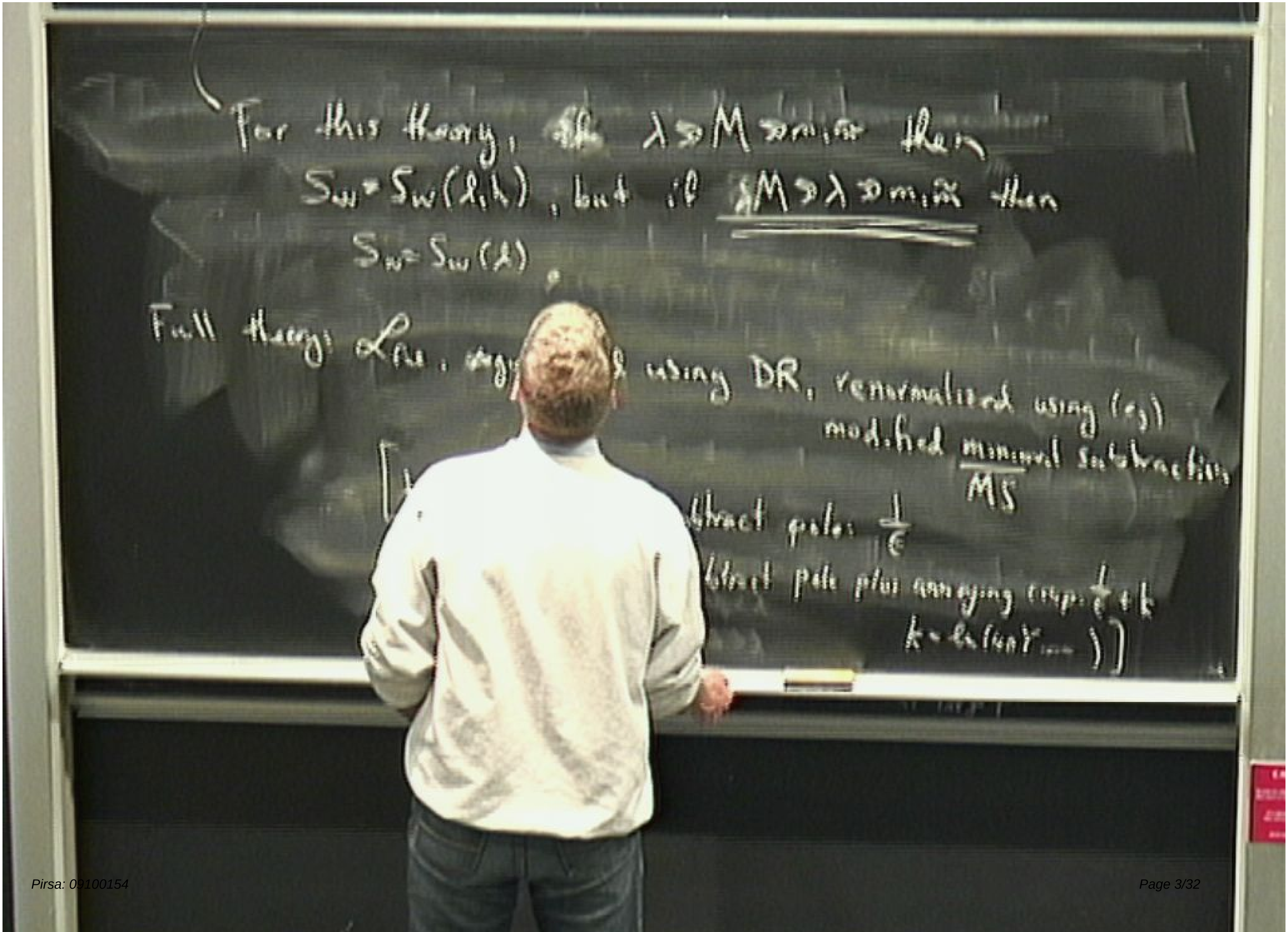
Title: Introduction to Effective Field Theory - Lecture 4B

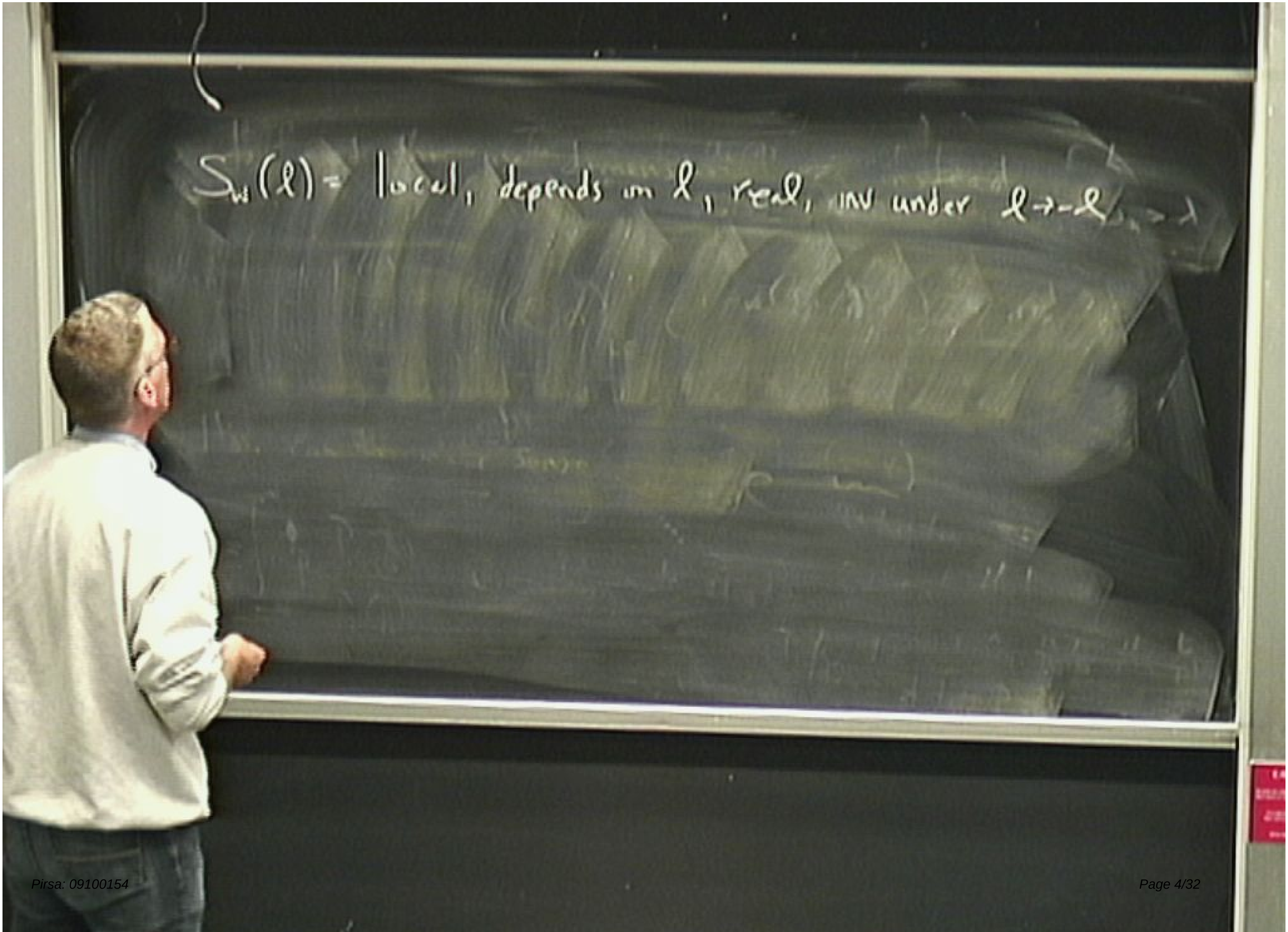
Date: Oct 14, 2009 11:00 AM

URL: http://pirsa.org/09100154

Abstract:


For this theory, λ ≫ M ≫ m, m̃ then
S_W = S_W(λ, Λ), but if M ≫ λ ≫ m, m̃ then
S_W = S_W(λ)


For this theory, if λ ≫ M ≫ m_i then
S_W = S_W(λ, λ), but if M ≫ λ ≫ m_i then
S_W = S_W(λ)
Full theory: L_FULL. using DR, renormalized using (eg)
modified minimal subtraction
MS
subtract poles 1/ε
subtract pole plus annoying crap 1/ε + k
k = ln(4π γ ...)]

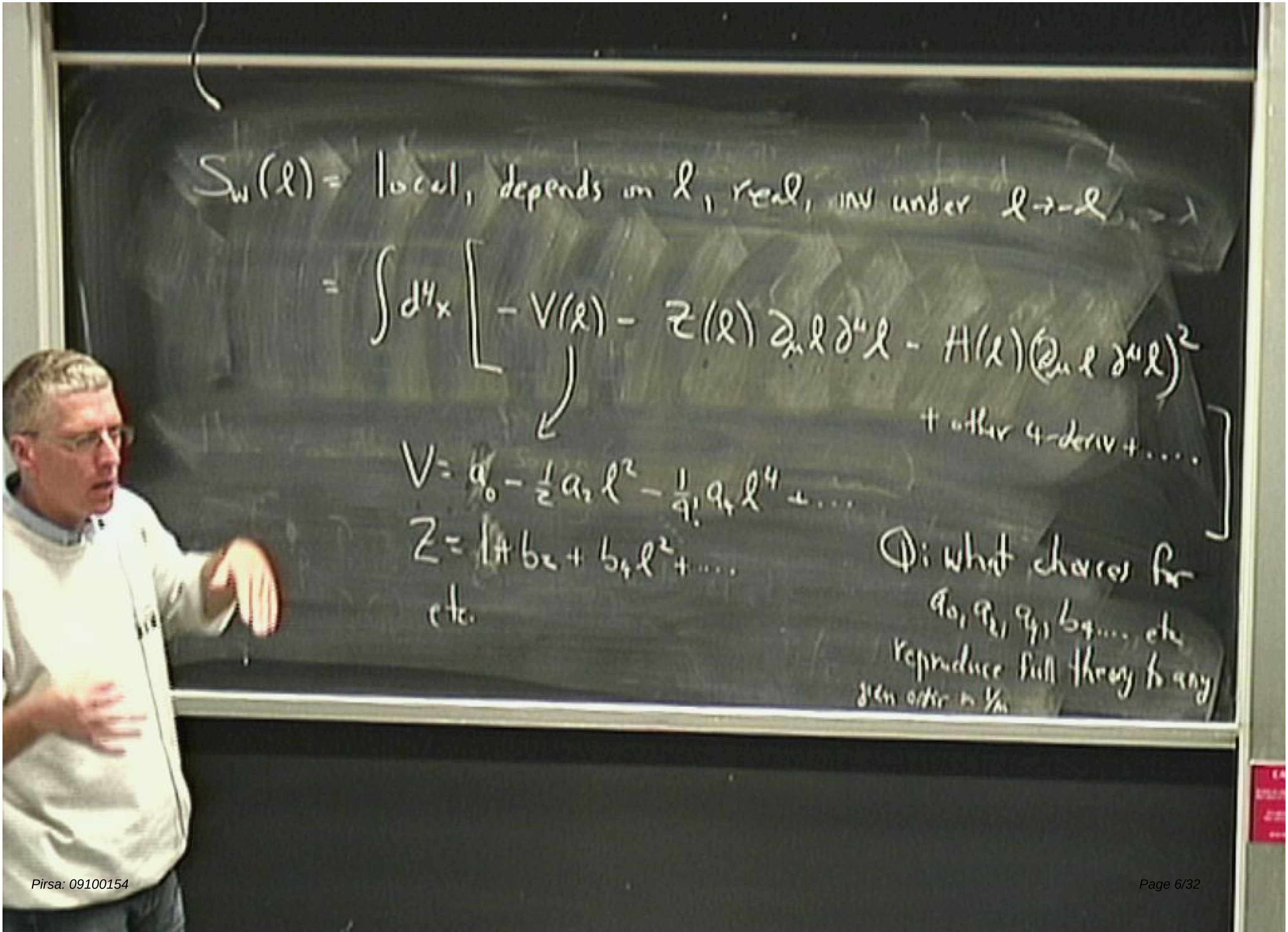

$S_w(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$

$S_w(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$, $\lambda \to -\lambda$

$$= \int d^4x \left[-V(\ell) - Z(\ell)\, \partial_\mu \ell\, \partial^\mu \ell - H(\ell) (\partial_\mu \ell\, \partial^\mu \ell)^2 \right.$$

$$\left. + \text{other 4-deriv} + \ldots \right]$$

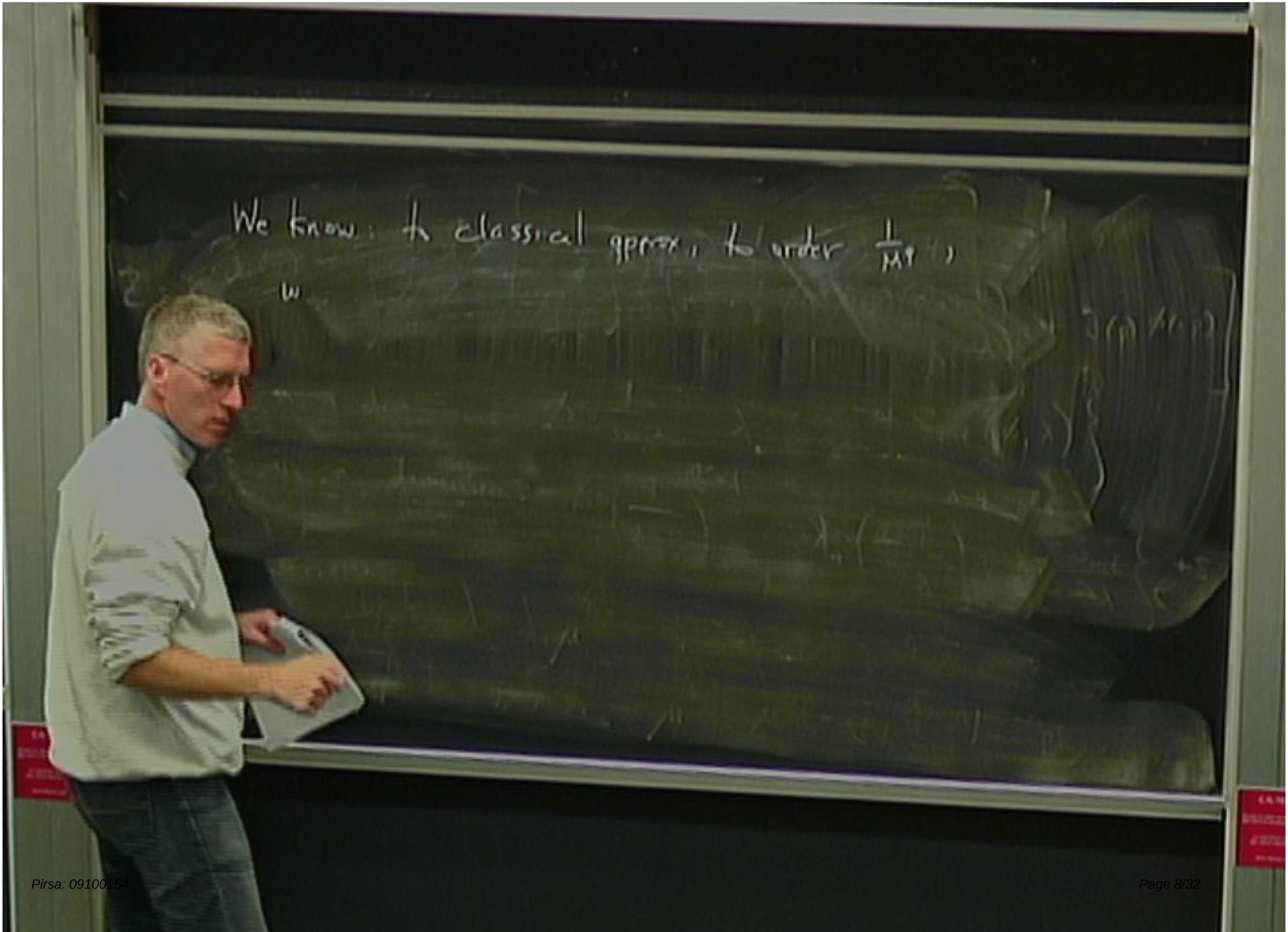
$$V = V_0 - \frac{1}{2}$$

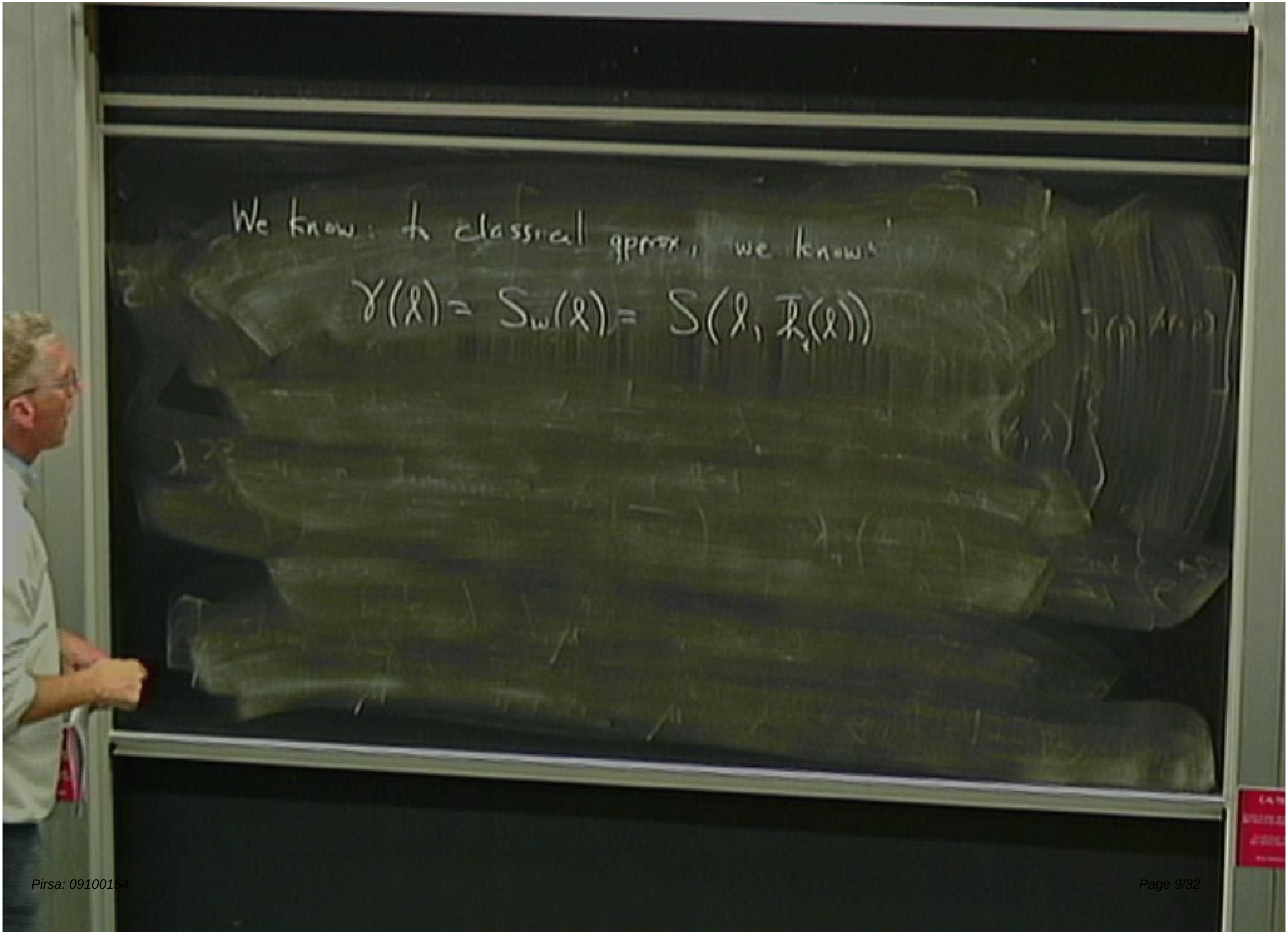


$S_w(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$
$= \int d^4x \left[-V(\ell) - Z(\ell)\, \partial_\mu \ell\, \partial^\mu \ell - A(\ell) (\partial_\mu \ell\, \partial^\mu \ell)^2 \right.$
$\left. + \text{other 4-deriv} + \dots \right]$
$V = a_0 - \frac{1}{2} a_2 \ell^2 - \frac{1}{4!} a_4 \ell^4 + \dots$
$Z = 1 + b_2 + b_4 \ell^2 + \dots$
etc.
Q: what choices for
$a_0, a_2, a_4, b_4, \dots$ etc
reproduce full theory to any
given order in $1/m$

2 parts: 1) Which couplings are required at all to a fixed order in $1/M$? ('Power counting')

2) What value is required of the necessary couplings. ('matching')


We know: to classical approx, to order $\frac{1}{M^4}$,
w



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1LPI $\longrightarrow \Gamma(\ell) = S_w(\ell) = S(\ell, \bar{\Phi}_\ell(\ell))$

$$e^{i\Gamma(\ell)} = \int \mathcal{D}\phi \, e^{iS(\ell+\phi) + i\int J\phi}$$

$$S_w = S_w(\varphi) + S_w''\phi^2 + S_{int}$$

$$= e^{iS_w(\varphi)} \int \mathcal{D}\phi \, e^{iS_1\phi + iS_2\phi^2} \left[1 + \text{int. terms} \right]$$

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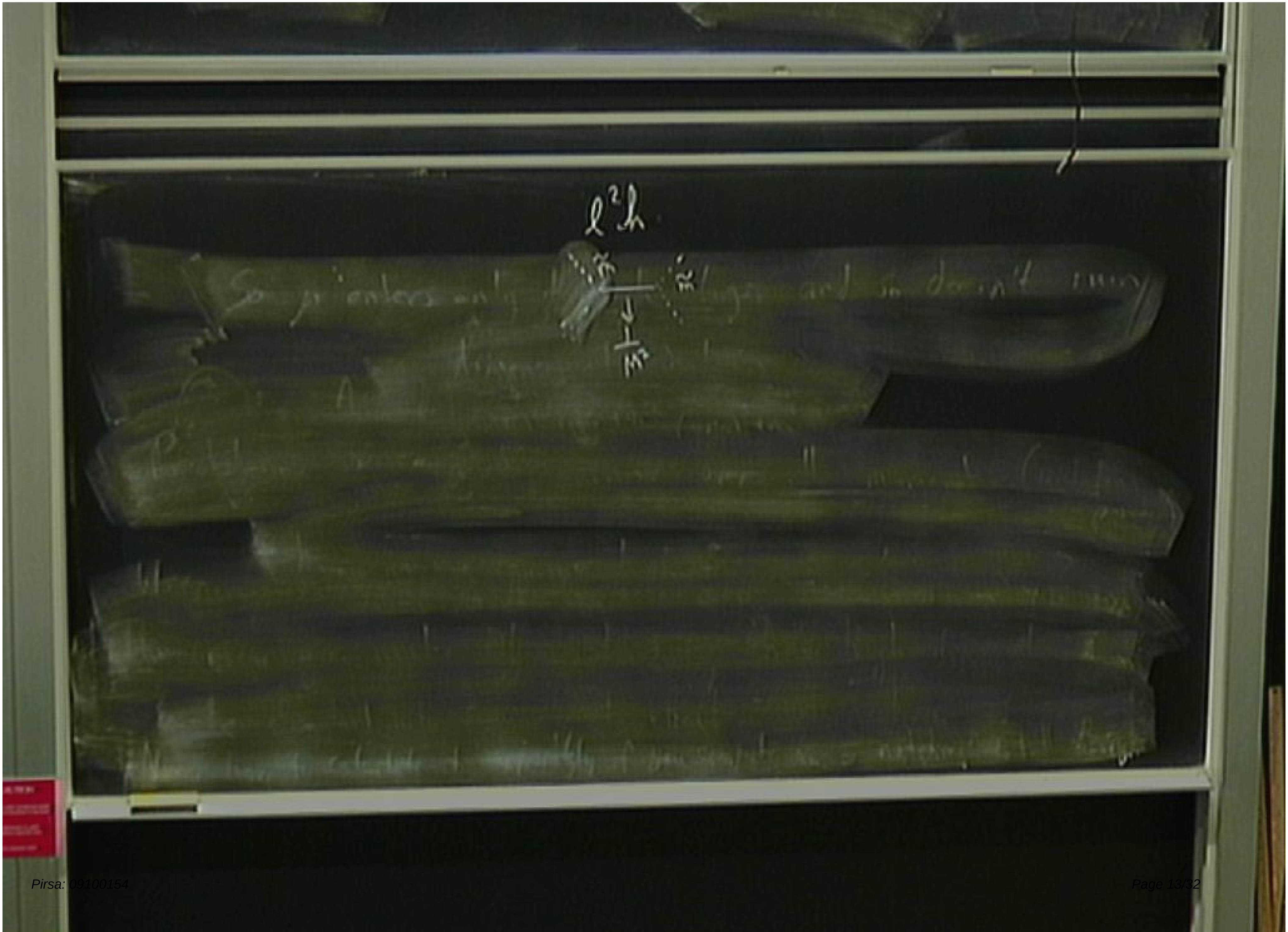
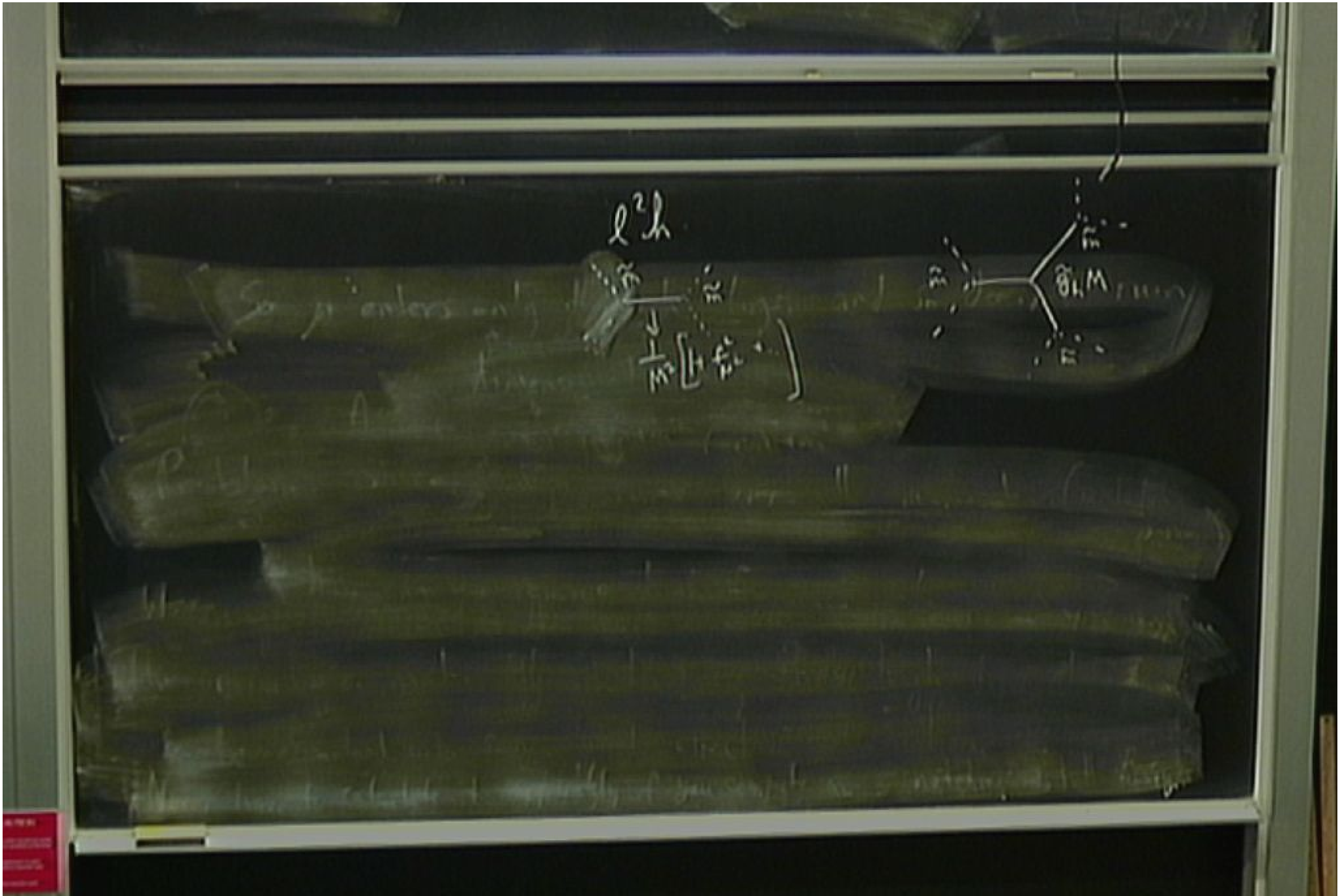
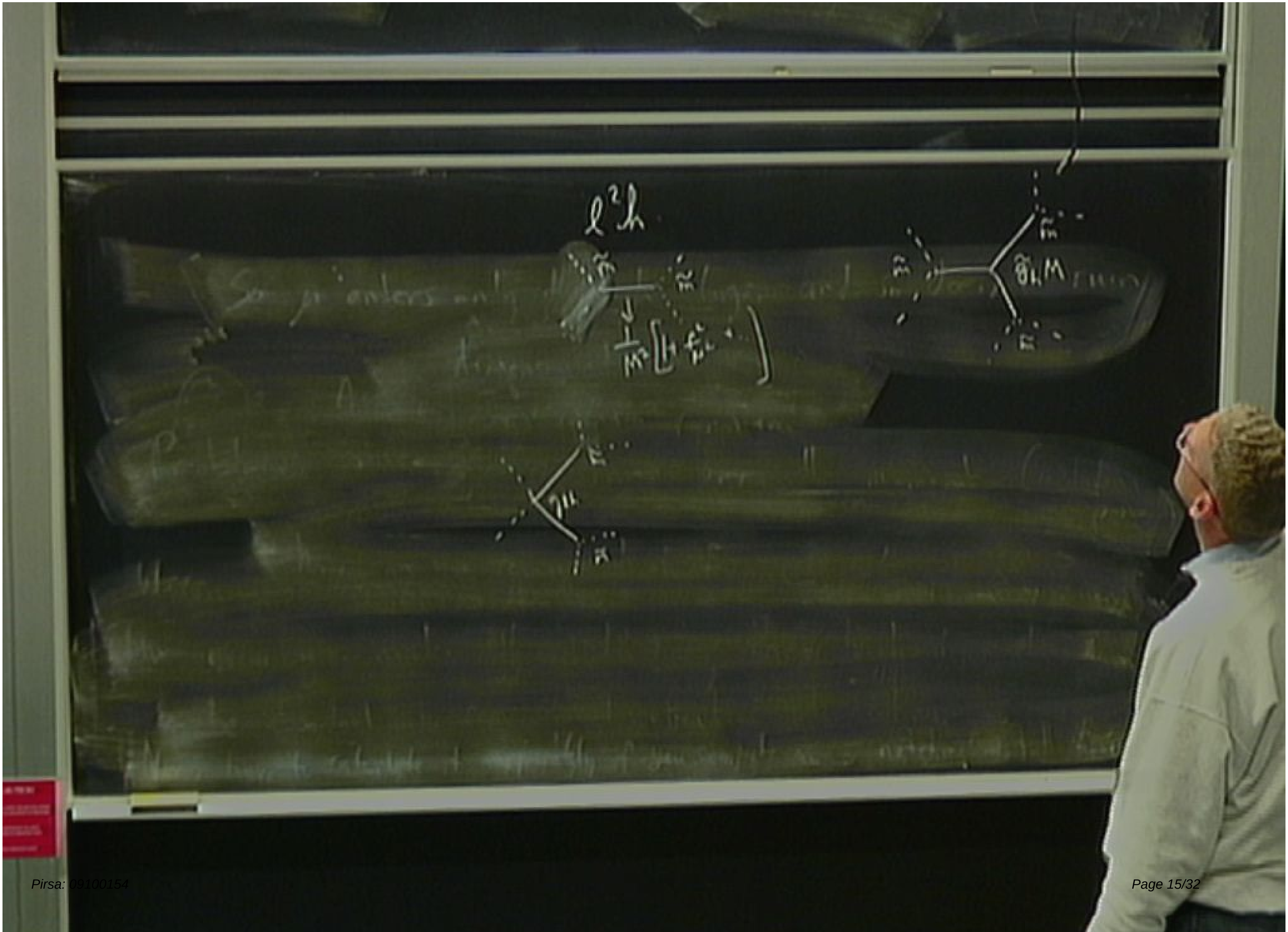
$$= e^{iS_w(\varphi)} \int \mathcal{D}\phi \, e^{i S_1\phi + i S_2'\phi^2} \left[1 + \text{int. terms} \right]$$

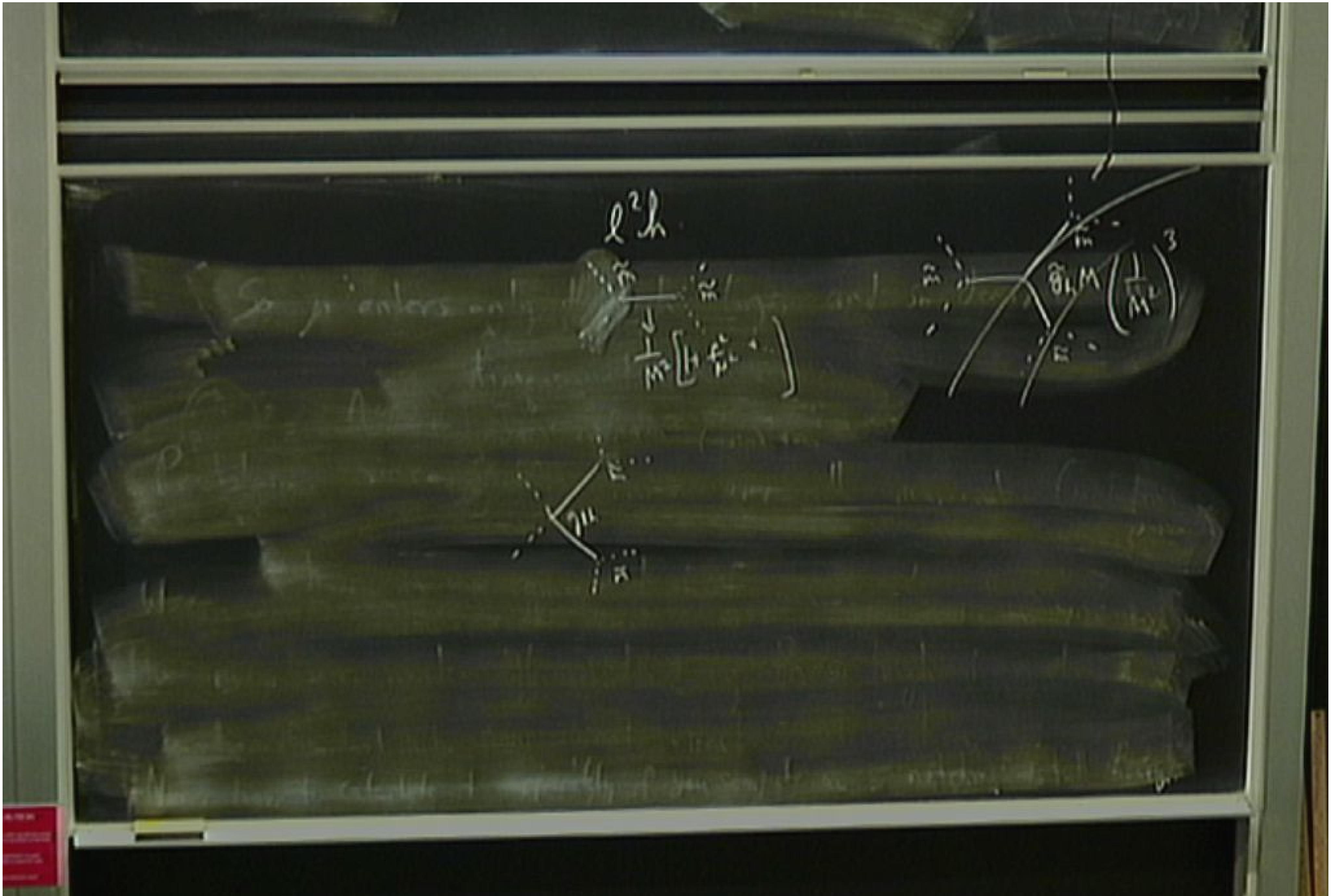
$$\rightarrow \mathcal{L}_{\ell}^{\text{eff}} = -\frac{1}{2}\partial_\mu \ell \partial^\mu \ell - \frac{m^2}{2}\ell^2 - \left(\frac{g}{4!}\ell^4 - \frac{m^2}{8M^2}\right)\ell^4 - \left(\frac{m^2}{2M^2}\right)\ell^2 \partial_\mu \ell \partial^\mu \ell - \left(\frac{g_{\ell\ell} m^2}{16 M^4}\right)\ell^6 + O\left(\frac{1}{M^6}, g_\ell g_\ell\right)$$


$\ell^2 h$
$\frac{1}{M^2}$



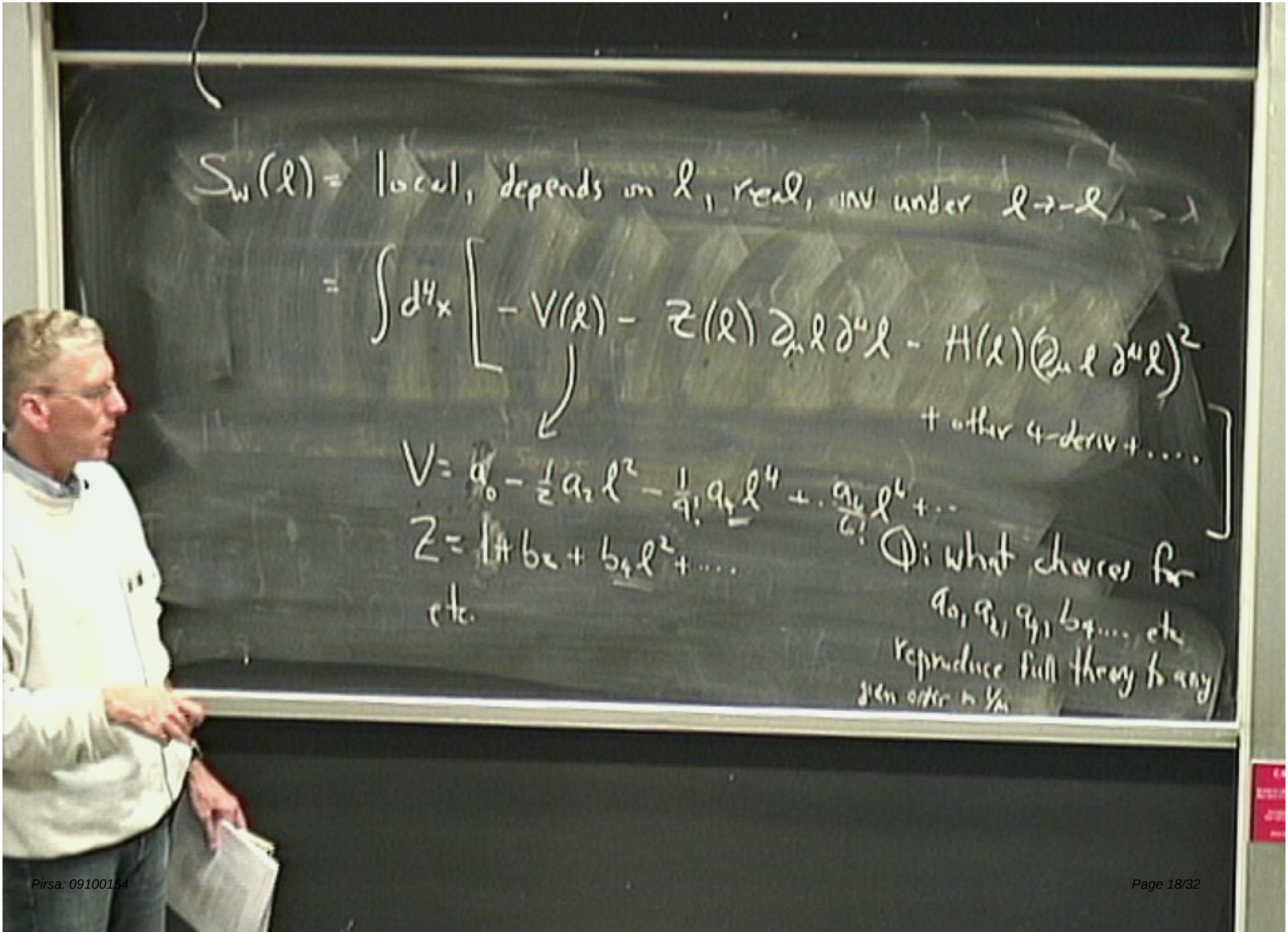
$\ell^2 h$
$g_H M$

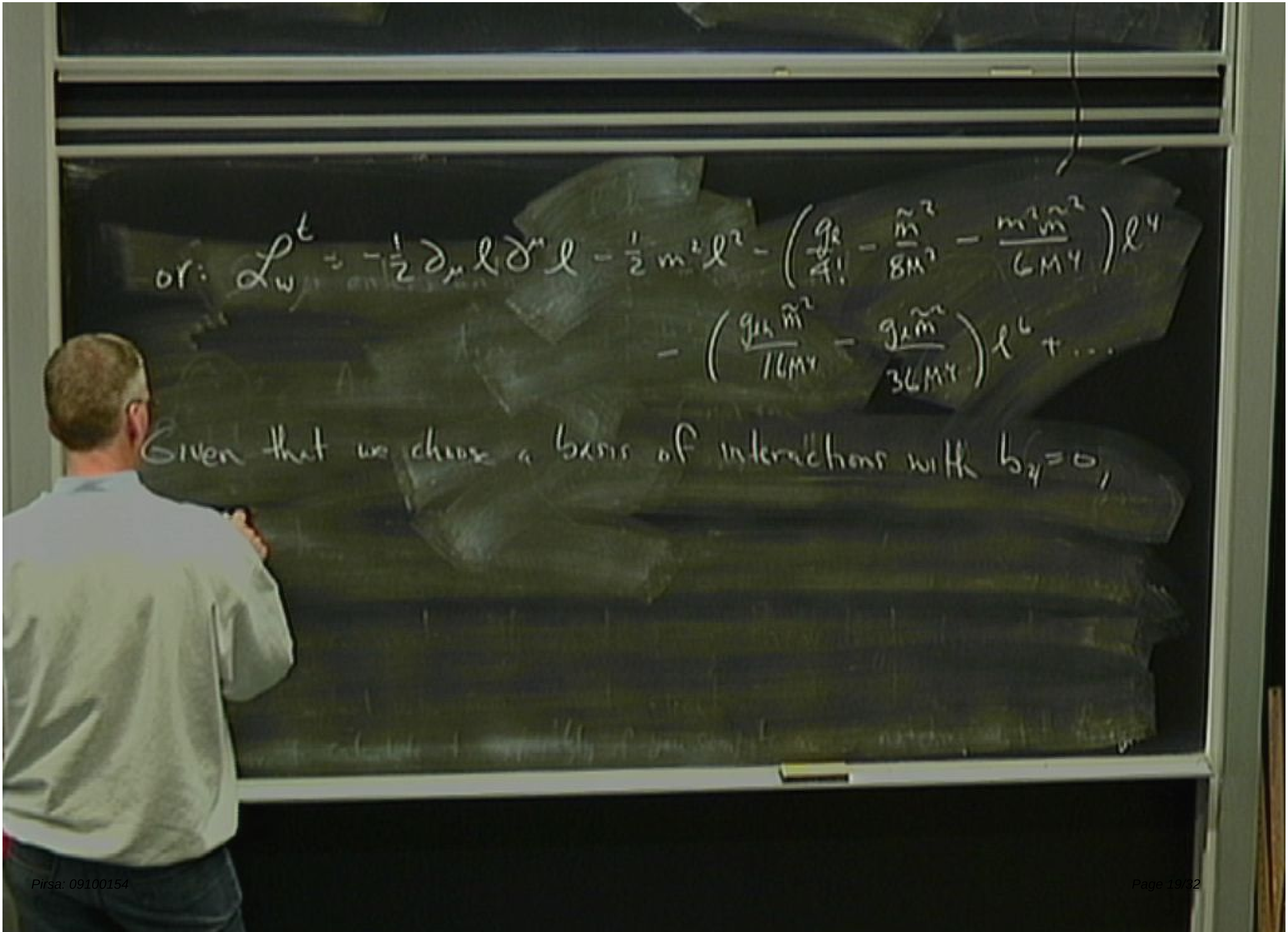



l²h

or: $\mathcal{L}_W^t = -\frac{1}{2}\partial_\mu \ell \partial^\mu \ell - \frac{1}{2} m^2 \ell^2 - \left(\frac{g_R}{4!} - \frac{\tilde{m}^2}{8M^2} - \frac{m^2\tilde{m}^2}{6M^4}\right)\ell^4$

$$- \left(\frac{g_R \tilde{m}^2}{16M^4} - \frac{g_R \tilde{m}^2}{36M^4}\right)\ell^6 + \ldots$$


$S_w(\ell) =$ local, depends on ℓ, real, inv under $\ell \to -\ell$
$= \int d^4x \left[-V(\ell) - Z(\ell)\, \partial_\mu \ell\, \partial^\mu \ell - H(\ell) (\partial_\mu \ell\, \partial^\mu \ell)^2 + \text{other 4-deriv} + \dots \right]$
$V = a_0 - \frac{1}{2} a_2 \ell^2 - \frac{1}{4!} a_4 \ell^4 + \frac{a_6}{6!} \ell^6 + \dots$
$Z = 1 + b_2 + b_4 \ell^2 + \dots$
etc.
Q: what choices for $a_0, a_2, a_4, b_4, \dots$ etc reproduce full theory to any given order in $1/m$


or: $\mathcal{L}_W^t = -\frac{1}{2}\partial_\mu \ell \partial^\mu \ell - \frac{1}{2} m^2 \ell^2 - \left(\frac{g_4}{4!} - \frac{\tilde{m}^2}{8M^2} - \frac{m^2 \tilde{m}^2}{6M^4}\right)\ell^4$
$- \left(\frac{g_4 \tilde{m}^2}{16M^4} - \frac{g_4 \tilde{m}^2}{36M^4}\right)\ell^6 + \dots$
Given that we choose a basis of interactions with $b_4 = 0$,

At one loop level, ask what matching implies for a_{d_i} … to $o(M^0)$.

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To do this, regularize both the Full theory and
the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

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recall, if $A_R = A_0 - \frac{1}{64\pi^2}(M^4 + m^4)L$ $\qquad L = \frac{1}{\epsilon} + k$ $\qquad V =$

$B_R = B_0$

Matching: Use the toy model

$$-A_0 - B_0 h$$

$$\mathcal{L}_{\text{full}}(l,h) = -\frac{1}{2}\partial_\mu l\,\partial^\mu l - \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}m^2 l^2 - \frac{1}{2}M^2 h^2$$

$$-\frac{\tilde{g}_h}{3!}M h^3 - \frac{1}{2}\tilde{m}\,l^2 h - \frac{1}{4!}g_h h^4 - \frac{1}{4!}g_l l^4 - \frac{1}{4}g_{lh}\,l^2 h^2$$

(inv under $l \to -l$)

$$m, \tilde{m} \ll M$$

At one loop level, ask what matching implies for g_4 ... to $O(M^0)$.

To do this, regularize both the Full theory and the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

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$B_R = B_0 - \frac{1}{32\pi^2}(m^2\tilde{m} + \hat{g}_{11} M^3)L$

$m_R^2 = m_0^2 - \frac{1}{32\pi^2}(g_0 m^2 + g_{11} M^2 + 2\tilde{m}^2)L$

; etc

At one loop level, ask what matching implies for a_{k}... to $O(M^0)$.

To do this, regularize both the Full theory and the Wilsonian theory using dim reg with $\overline{MS}$ renorm.

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$m_R^2 = m_0^2 - \frac{1}{32\pi^2}(g_l m^2 + g_{lh} M^2 + 2\tilde{m}^2)L$ $\quad$ s.t. $A_R, B_R, \ldots$ etc are finite as $n \to 4$

; etc

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$$a_{4_R} = a_{4_0} - \frac{3}{32\pi^2} a_2^2 L$$

etc

Similarly renormalize $\mathcal{L}_W$ in dim reg with $\overline{MS}$:

so eg: $a_{1_R} = a_{1_0} - \frac{1}{32\pi^2}$ [illegible] L

$$a_{4_R} = a_{4_0} - \frac{3}{32\pi^2} a_4^2 L$$

etc

Matching: ask how $a_{2_R}, a_{4_R}, \ldots$ etc depend on $A_R, g_R, m_R, \bar{g}_R$, etc in order to make observables agree

Similarly renormalize $\mathcal{L}_W$ in dim reg with $\overline{MS}$:

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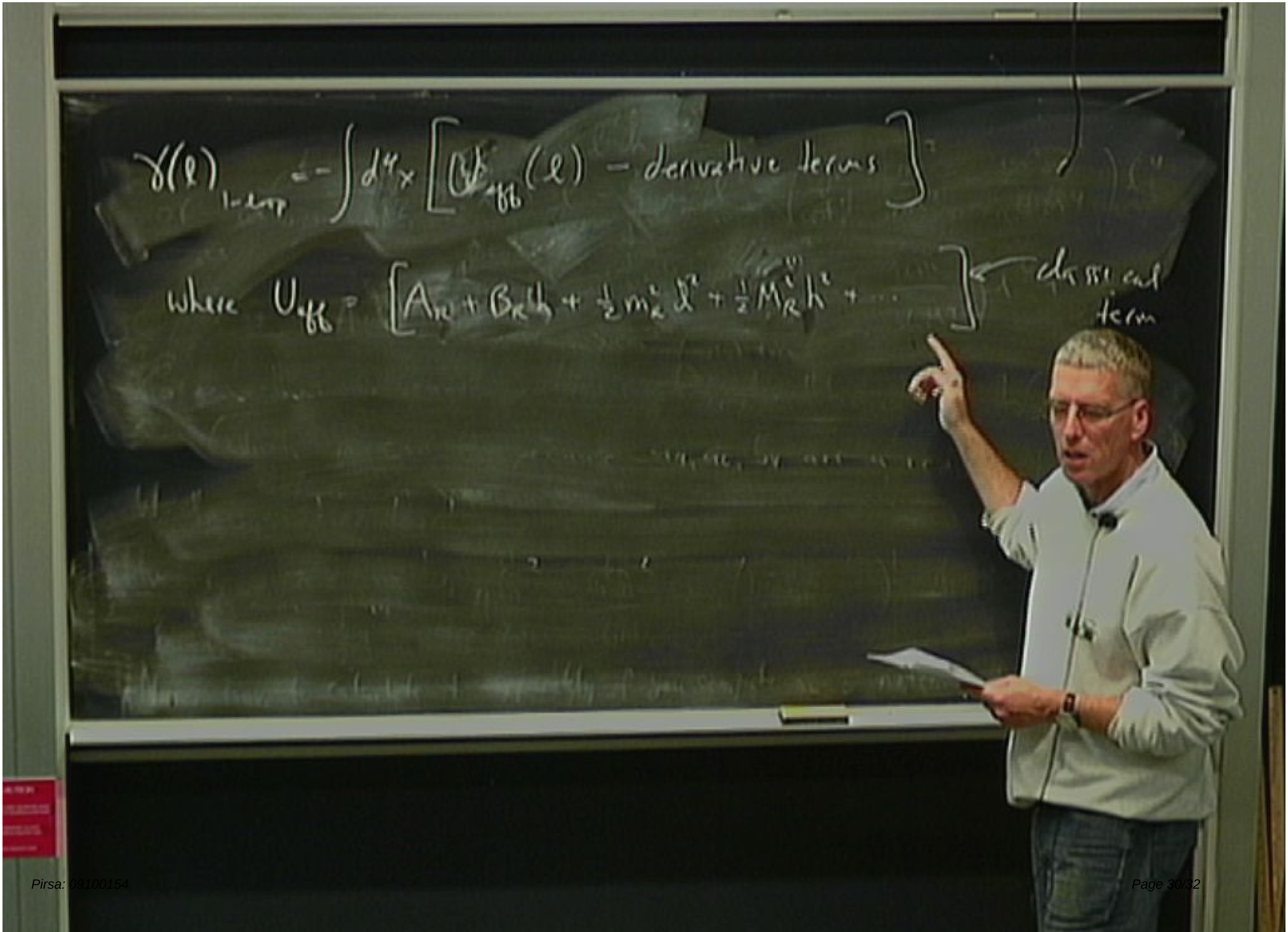
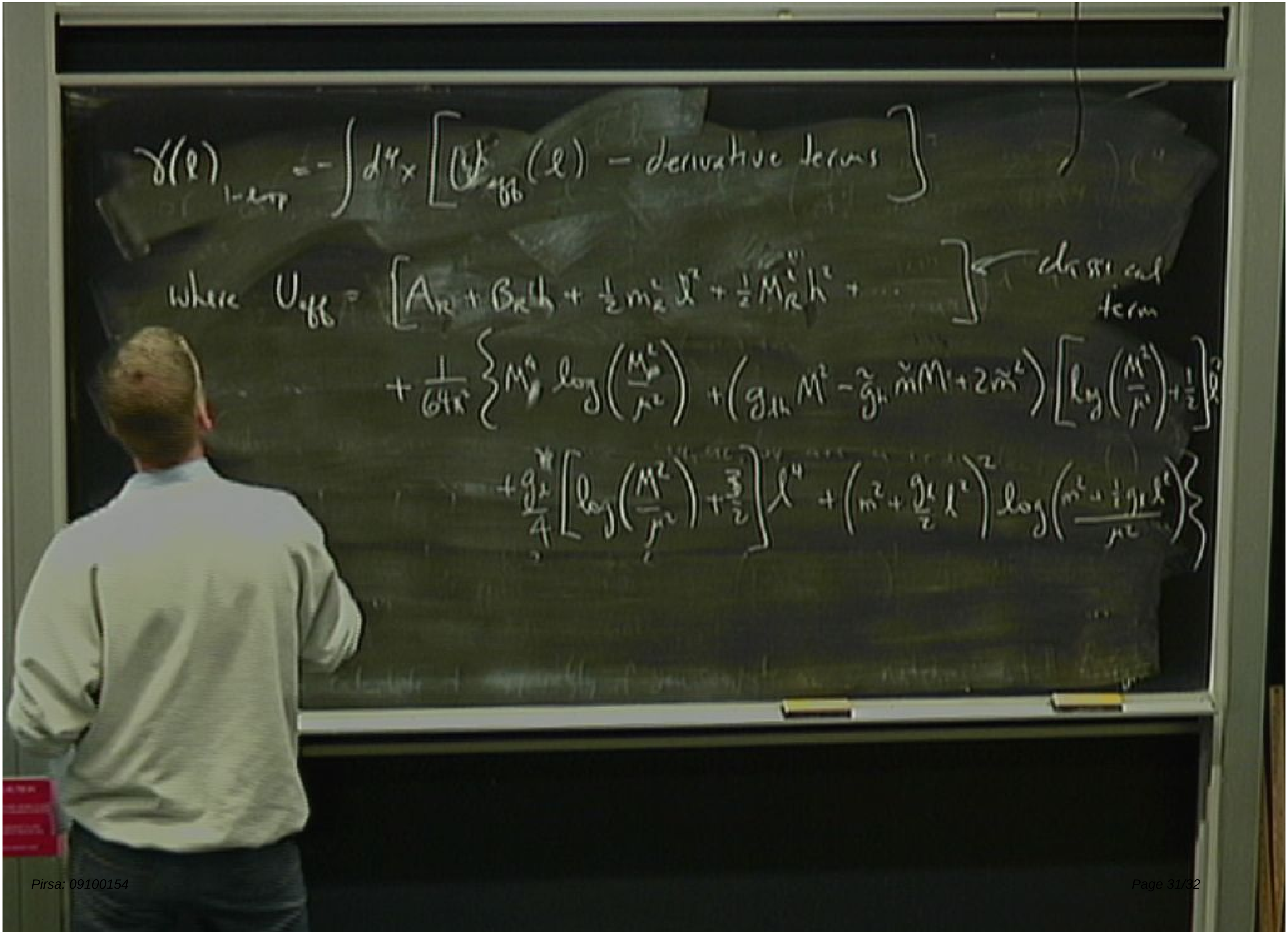
If we ask $\mathcal{L}_W$ to reproduce $\gamma(\ell)$ to 1 loop + to order M^0, must


$\gamma(\ell)_{1\text{-loop}} = -\int d^4x \left[U_{eff}(\ell) - \text{derivative terms} \right]$
where $U_{eff} = \left[A_R + B_R h + \frac{1}{2} m_R^2 \ell^2 + \frac{1}{2} M_R^2 h^2 + \dots \right]$
classical term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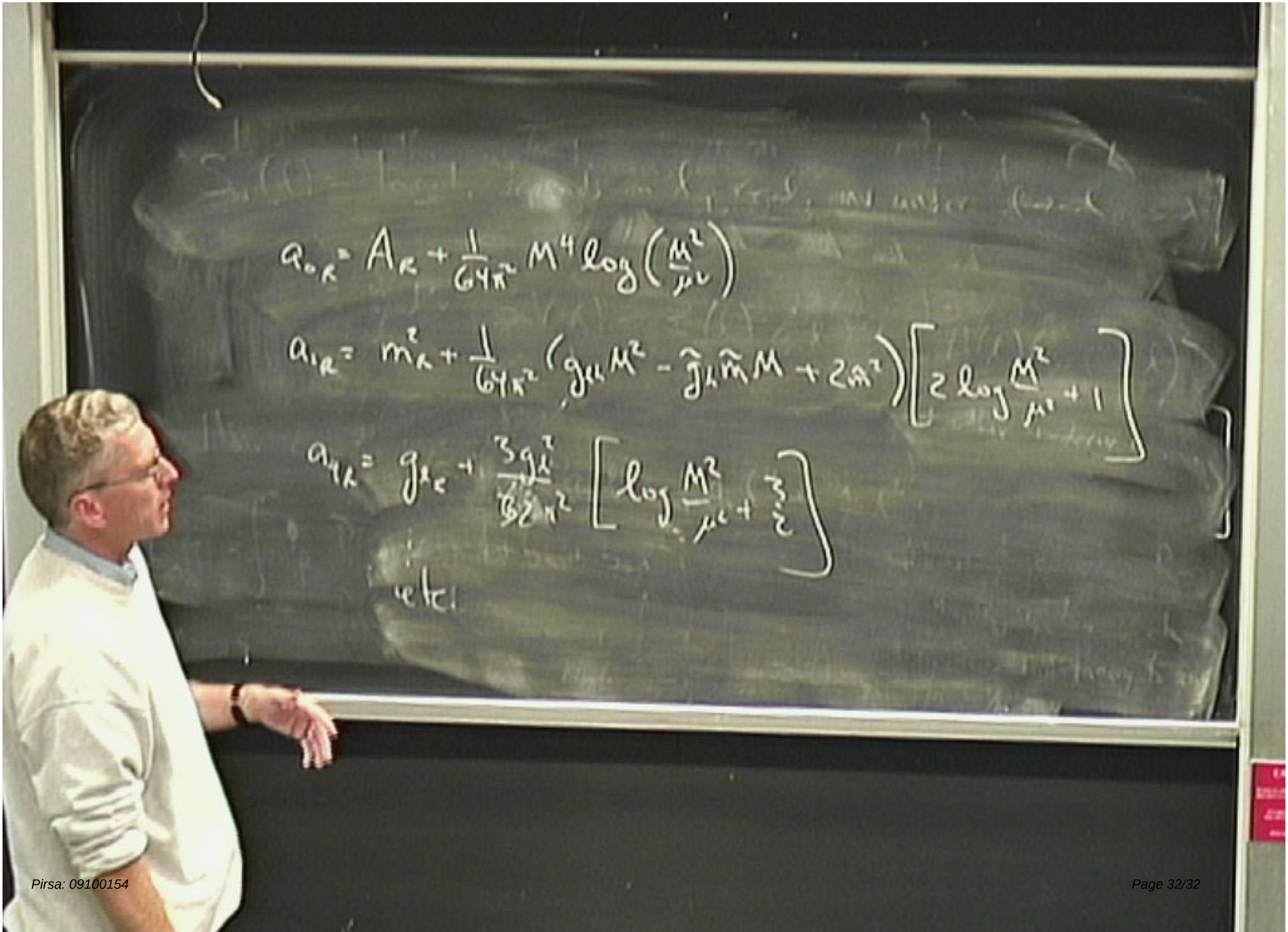
classical term
$+ \frac{1}{64\pi^2} \Big\{ M_\phi^4 \log\left(\frac{M_\phi^2}{\mu^2}\right) + \left(g_{\ell h} M^2 - \tilde{g}_h \tilde{m} M + 2\tilde{m}^2\right)\left[\log\left(\frac{M^2}{\mu^2}\right) + \frac{1}{2}\right]\ell^2$
$+ \frac{g_\ell^2}{4}\left[\log\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2}\right]\ell^4 + \left(m^2 + \frac{g_\ell}{2}\ell^2\right)^2 \log\left(\frac{m^2 + \frac{1}{2} g_\ell \ell^2}{\mu^2}\right) \Big\}$


$a_{0R} = A_R + \frac{1}{64\pi^2} M^4 \log\left(\frac{M^2}{\mu^2}\right)$
$a_{1R} = m_R^2 + \frac{1}{64\pi^2}\left(g_\lambda M^2 - \tilde{g}_\lambda \tilde{m} M + 2\tilde{m}^2\right)\left[2\log\frac{M^2}{\mu^2} + 1\right]$
$a_{4R} = g_{\lambda R} + \frac{3g_\lambda^2}{32\pi^2}\left[\log\frac{M^2}{\mu^2} + \frac{3}{2}\right]$
etc.