Title: Classification of topological insulators and superconductors

Date: Oct 07, 2009 12:00 PM

URL: http://pirsa.org/09100140

Abstract: Complete classification of topological insulators (including, e.g., the quantum Hall effect and the quantum spin Hall systems), and superconductors (including, e.g., chiral p-wave SC and the B-phase of 3He). An interacting bosonic model that realizes a topological superconducting phase in three spatial dimensions.

quantum condensed matter systems: classification of topological states and entanglement scaling

Shinsei Ryu
Univ. of California, Berkeley

phases and phase transitions in condensed matter systems

classical phases

Ginzberg-Laudau theory

Nambu-Goldstone modes

quantum phases

gapless phases

- Fermi liquid
- non Fermi liquid

gapped phases

- insulators
-topological insulators
-topological superconductors
-topological phase

quantum critical points

-relativistic conformal quantum critical point

quantum many-body physics beyond Landau-Ginzberg paradigm

- is it possible to have an exhausive classification of quantum phases in many-body systmes ?

- what is a good "order parameter" to distinguish all these phases ?

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- classification of topological insulators/superconductors

- example of interacting superconductor

- entanglement entropy in topological insulators/SCs

- holographic calculation of entanglement entropy

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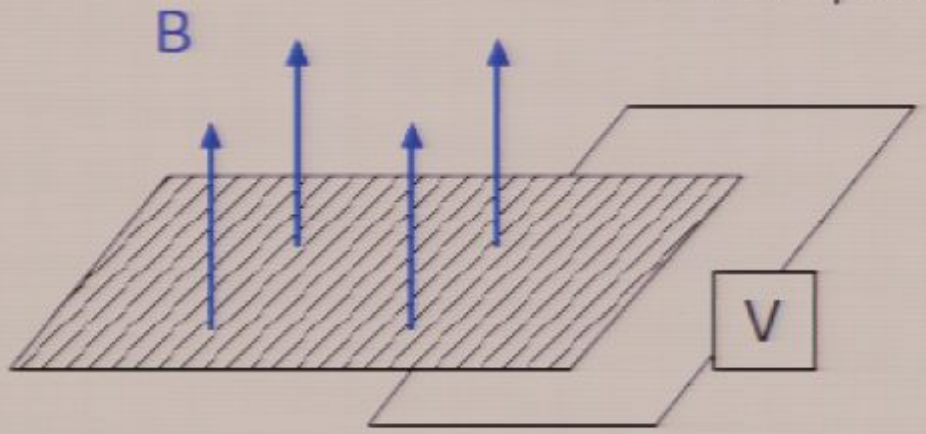
collaborators

integer quantum Hall effect (IQHE)

in d=2 spatial dimensions, with strong T breaking by B



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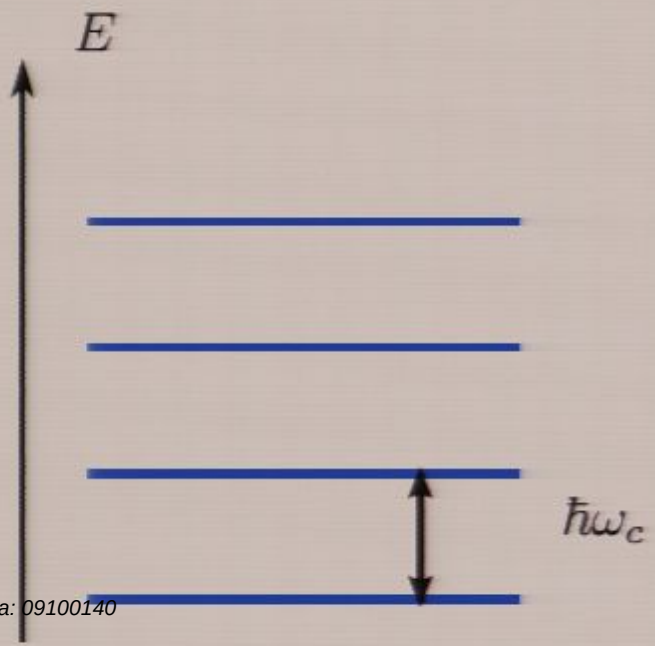

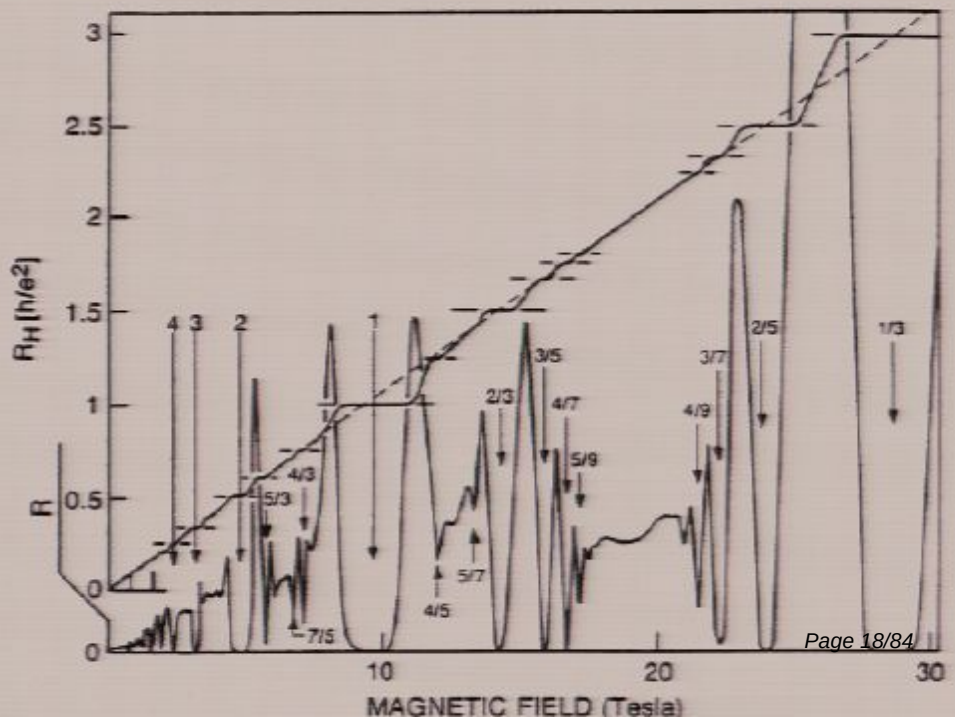

quantum spin Hall effect (QSHE)

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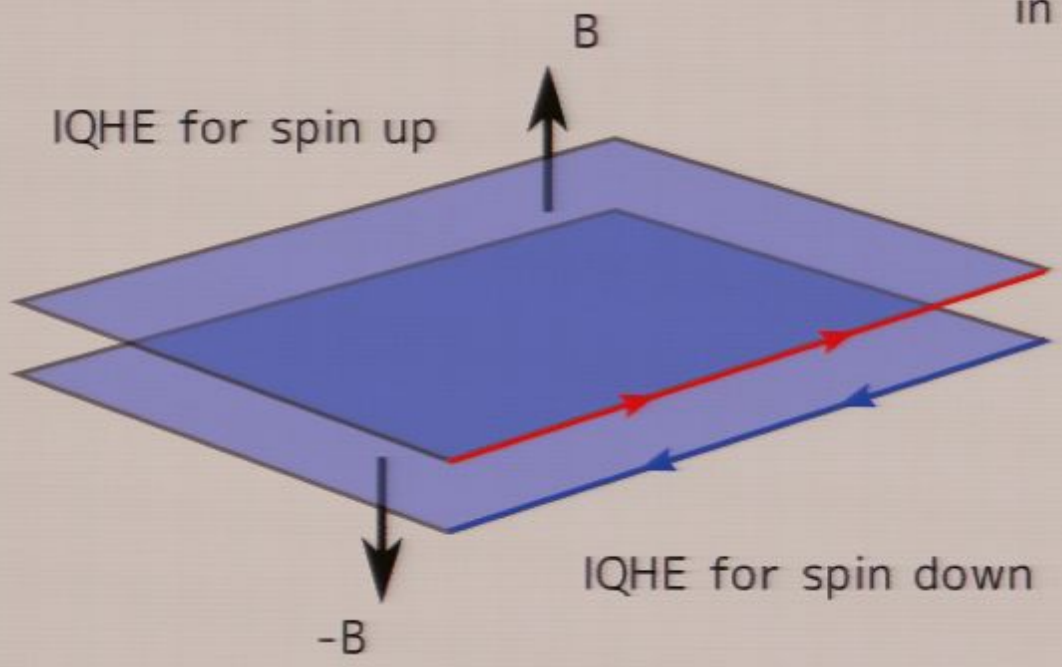

TRS

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when Sz is conserved, classification is Z $\quad \sigma_{xy,\uparrow} - \sigma_{xy,\downarrow}$ is quantized

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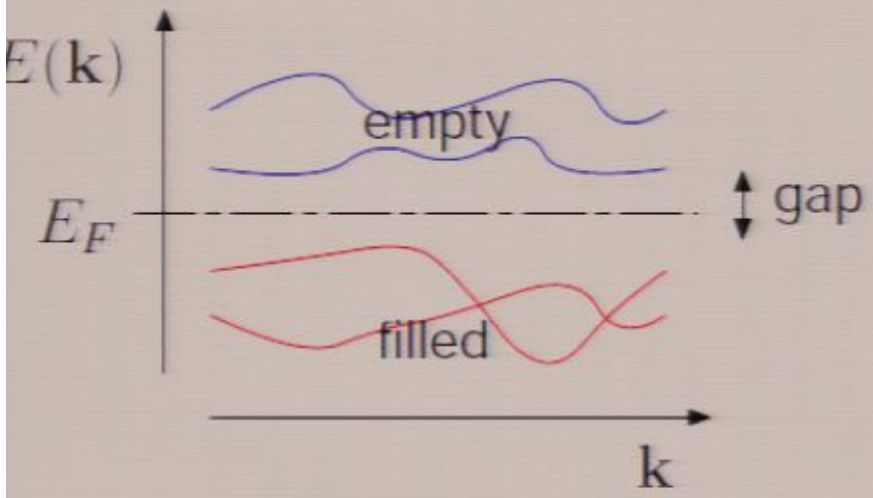
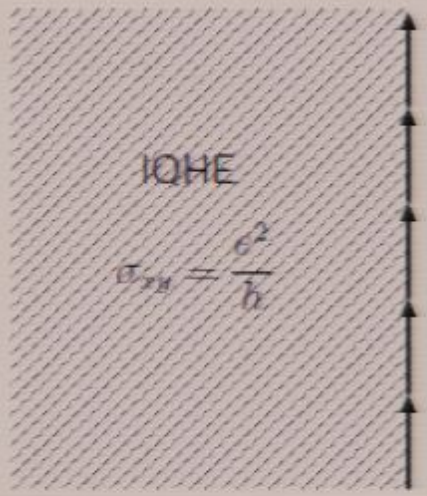
Kane-Mele, Roy, Moore-Balents (05-06)

IQHE as a topological insulator





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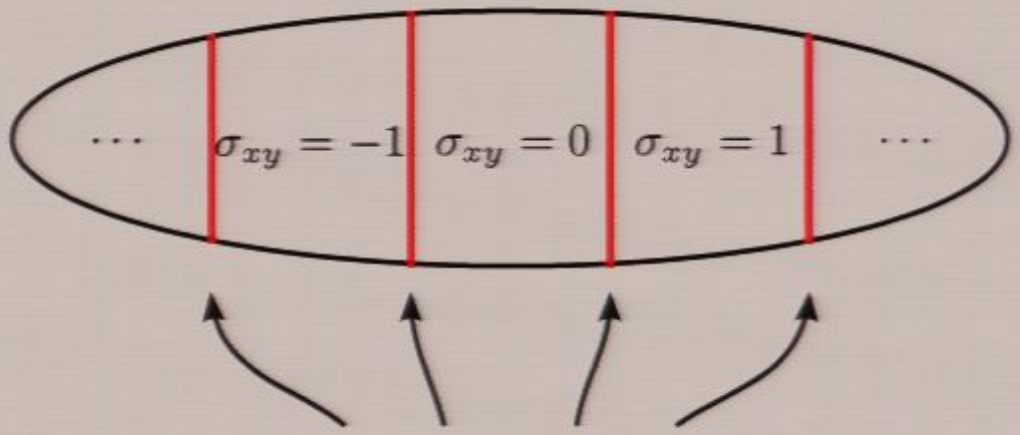

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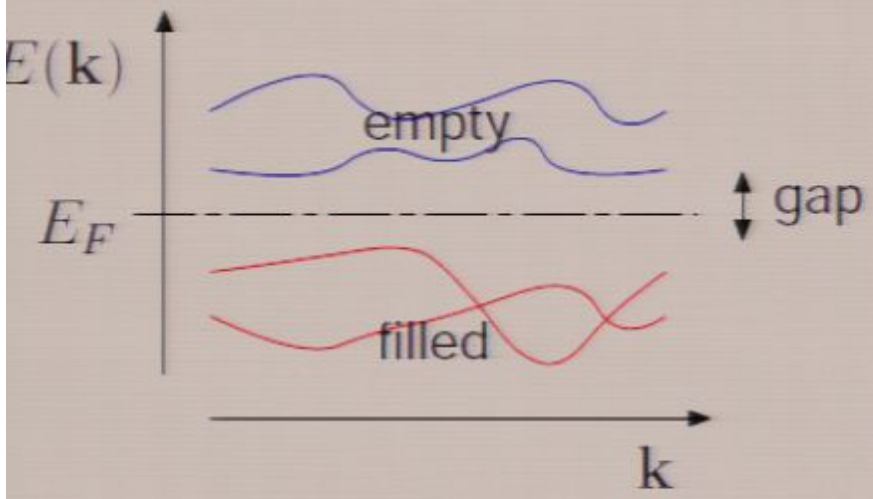

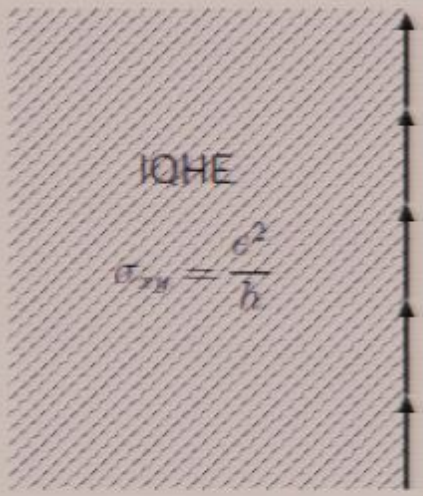

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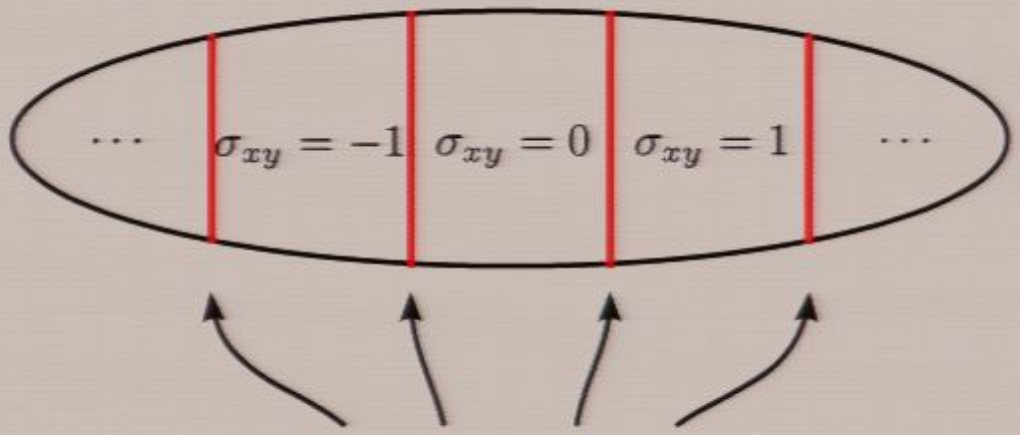

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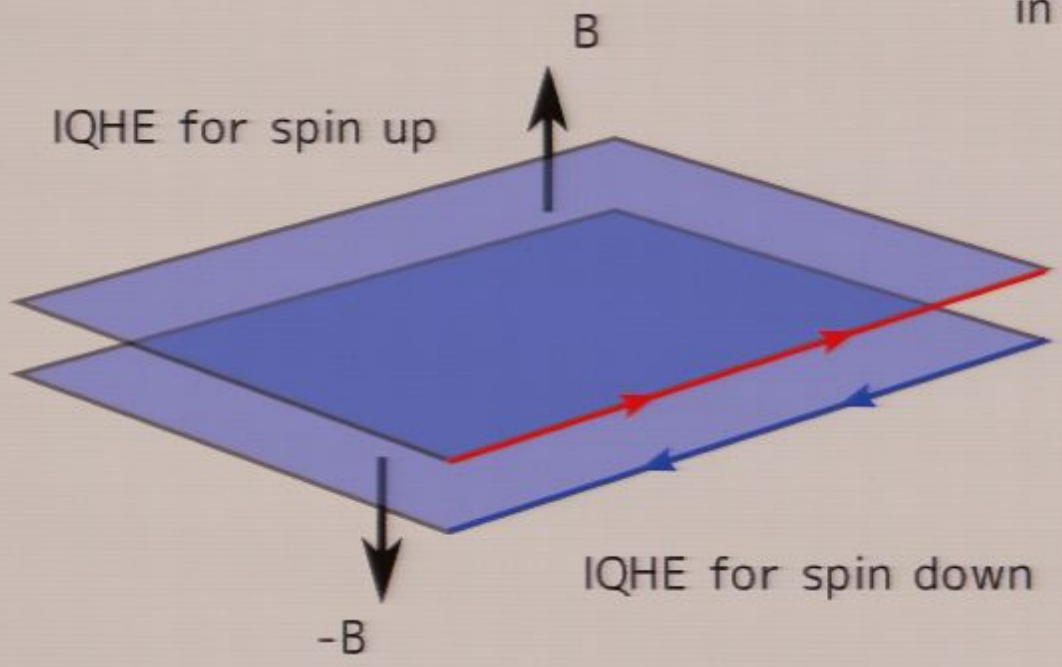

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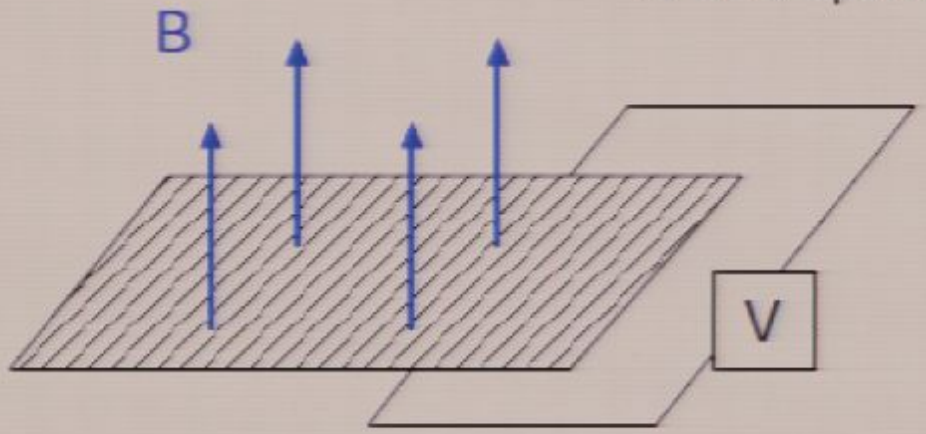

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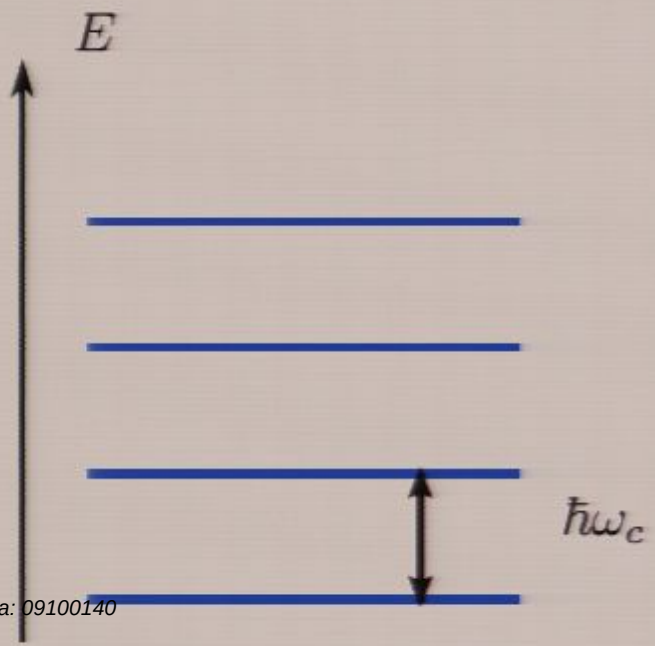

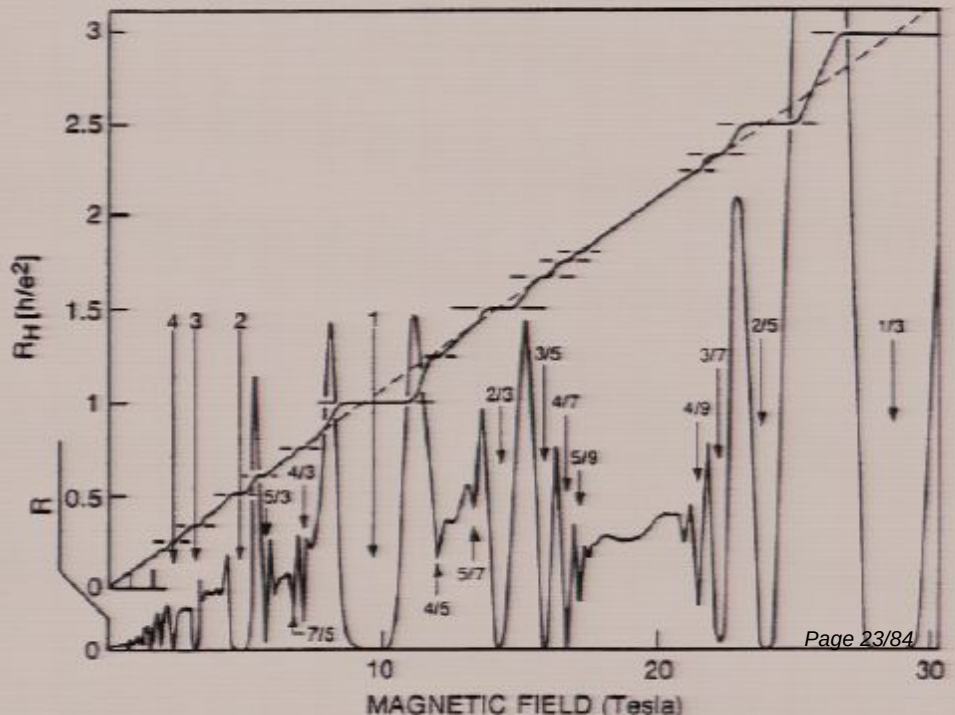

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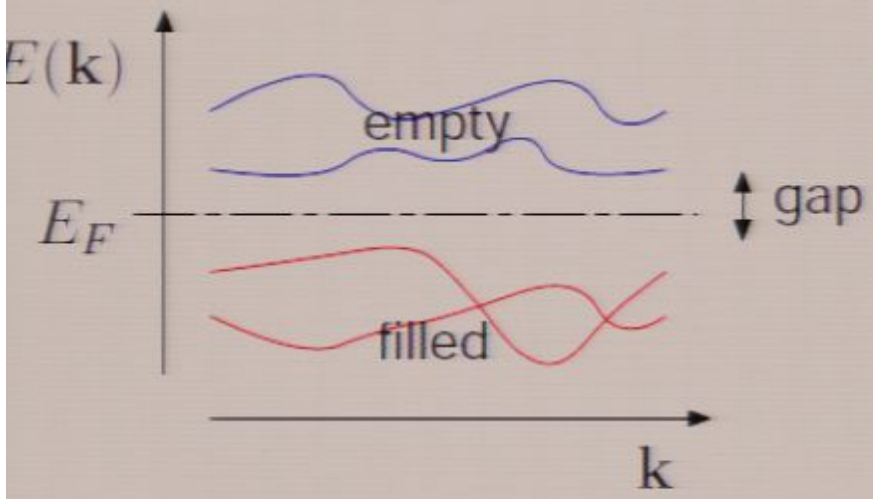

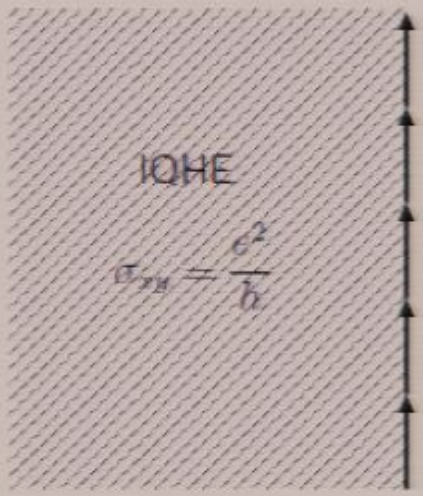

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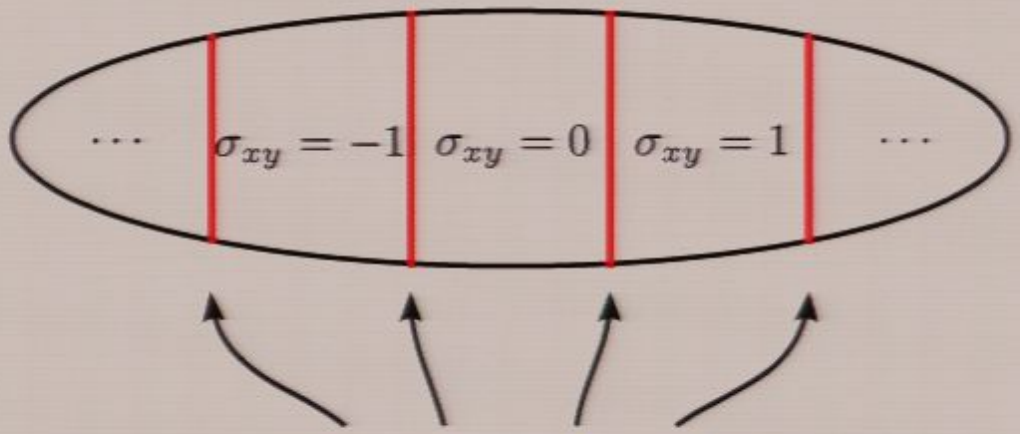

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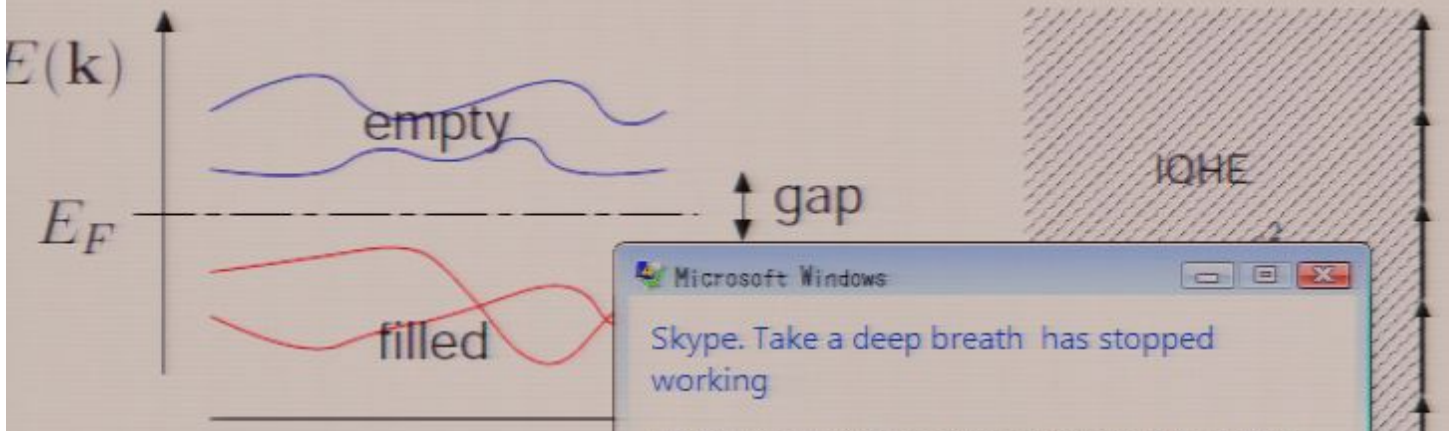

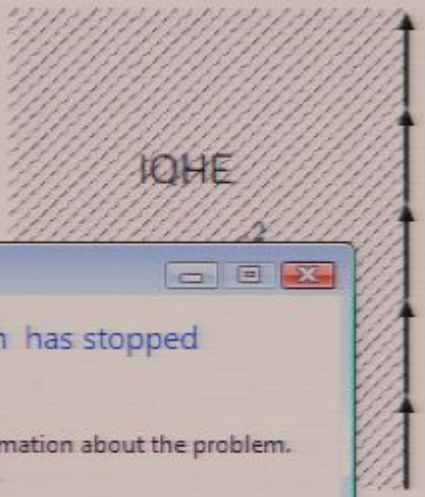

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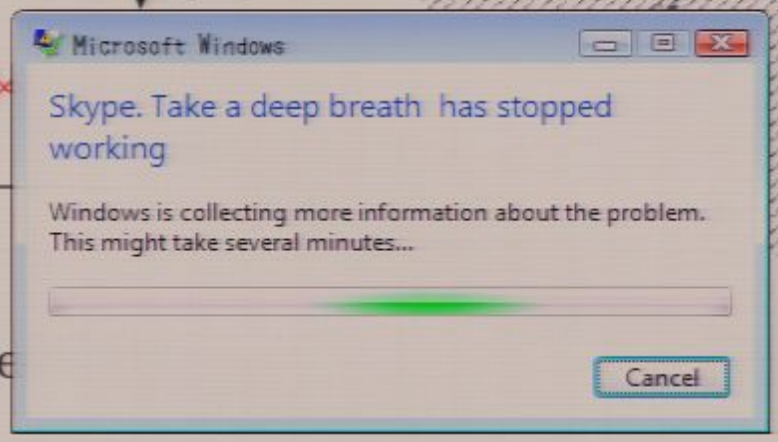

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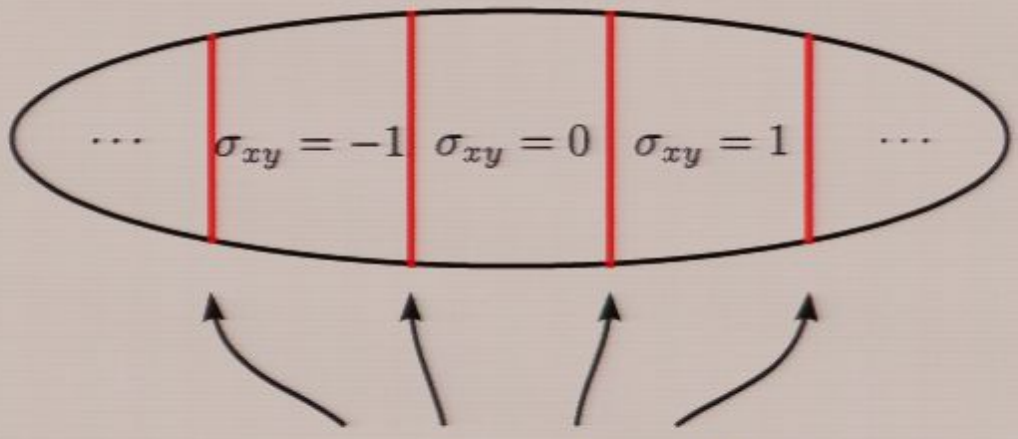

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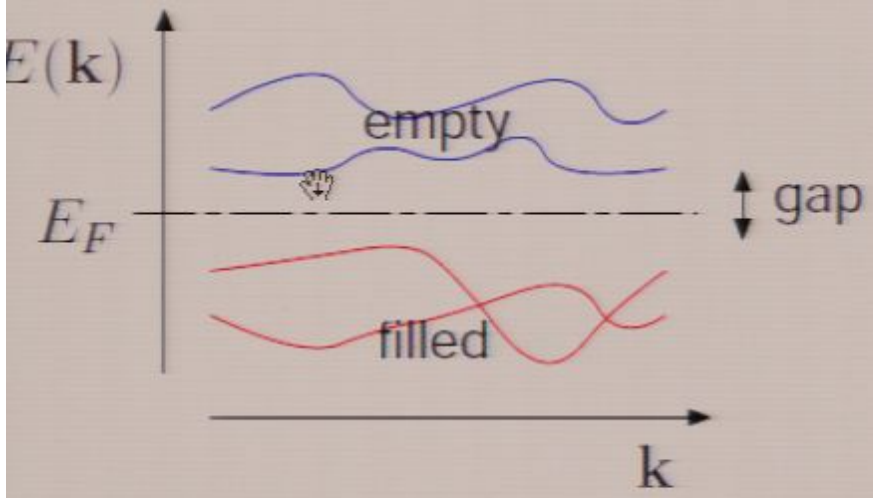

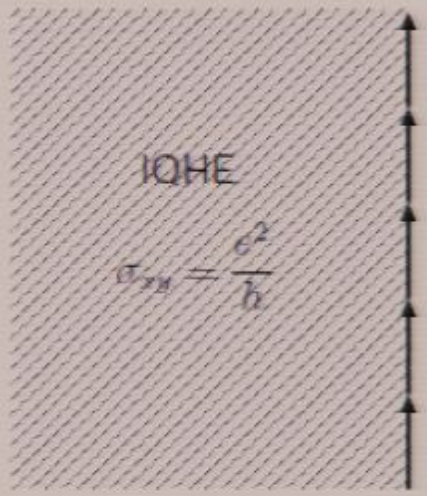

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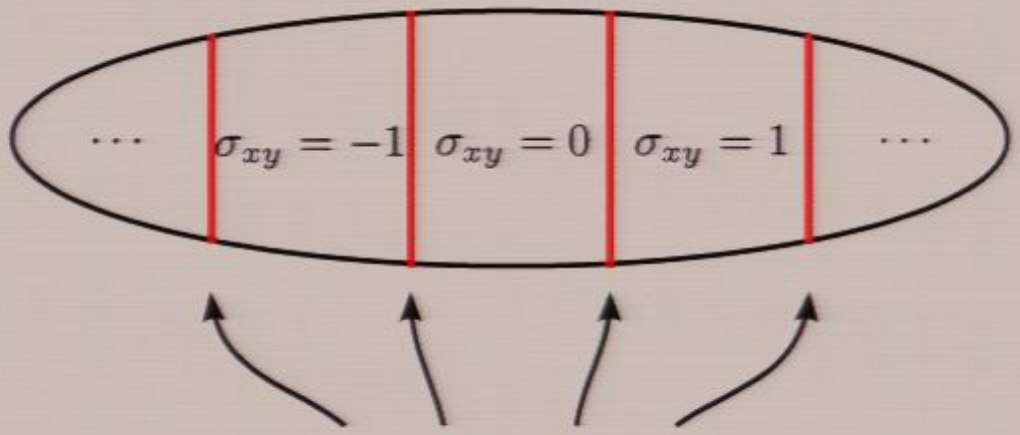

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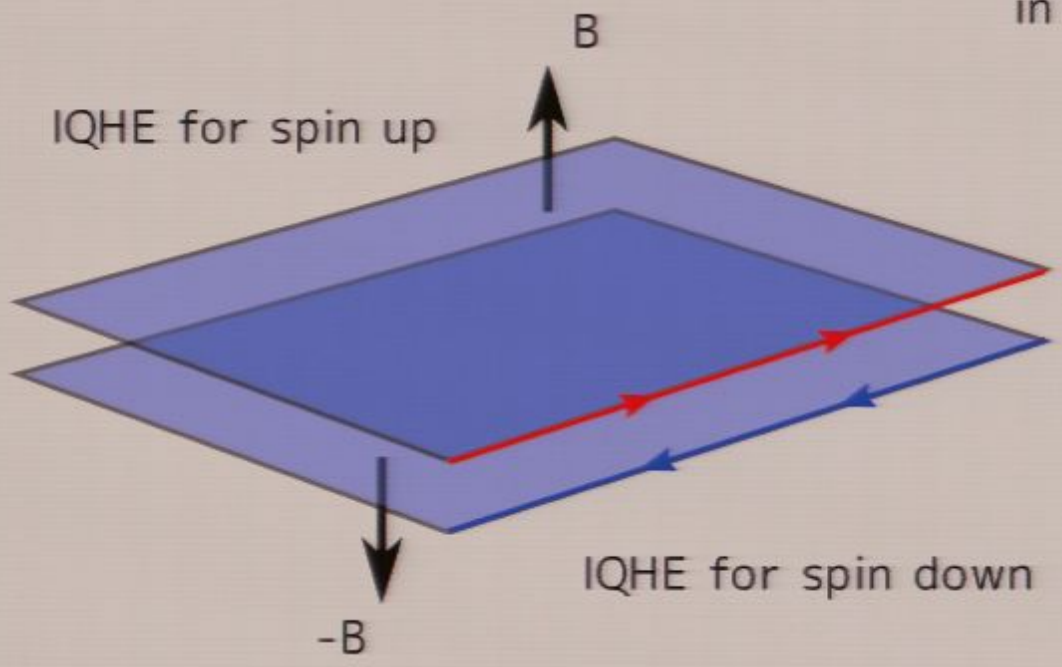
quantum spin Hall effect (QSHE)

in d=2 spatial dimensions, with good T



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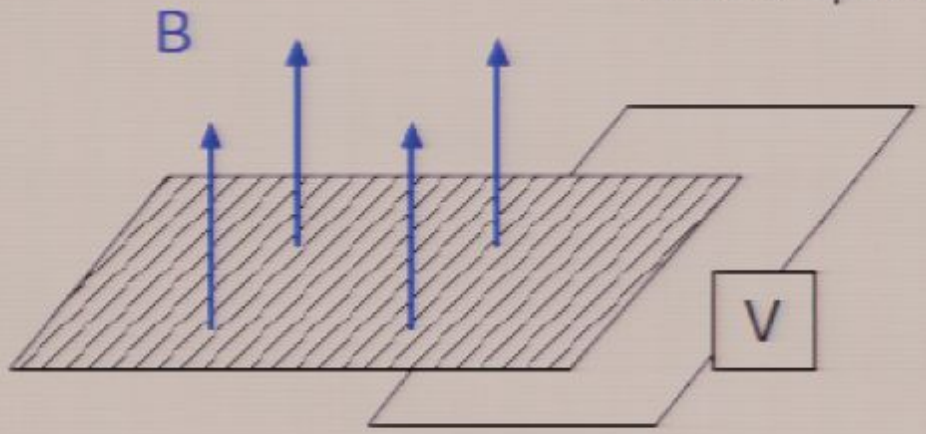

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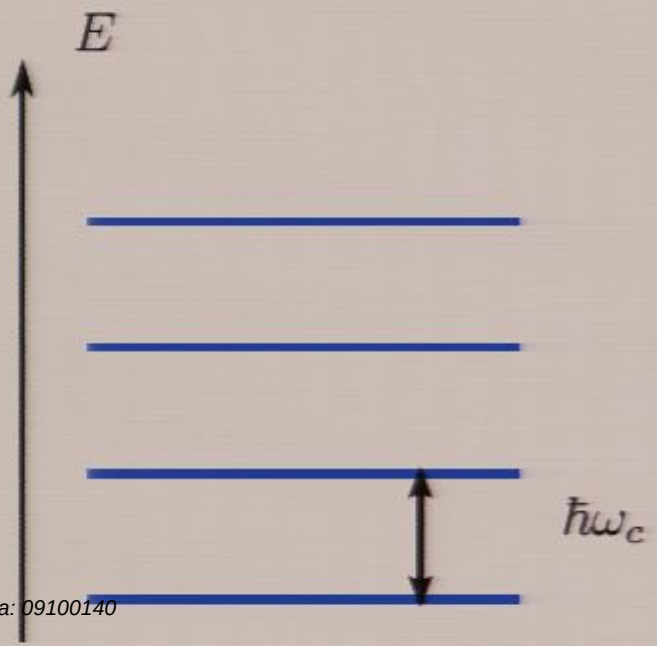

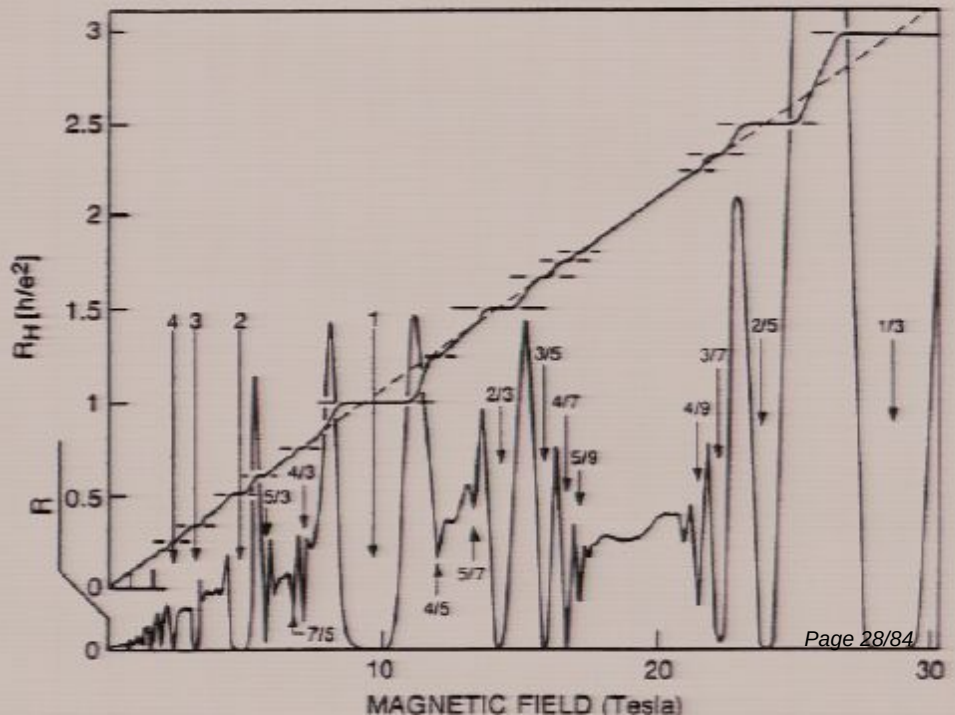

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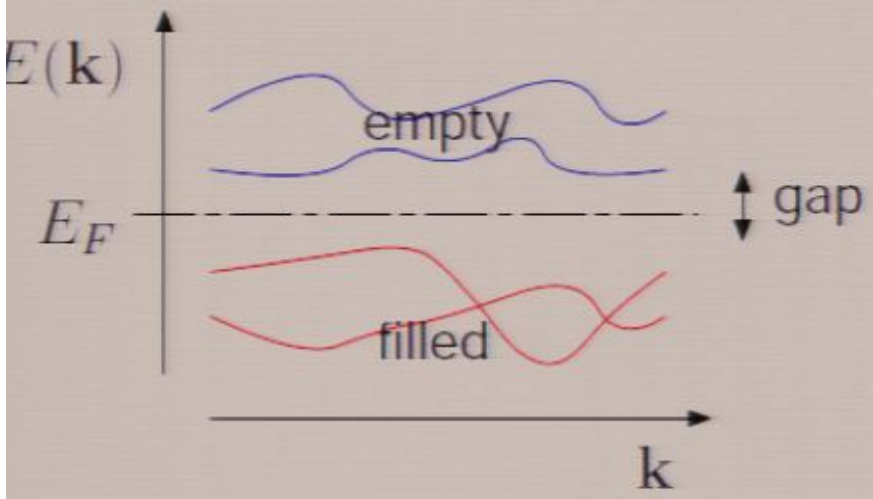

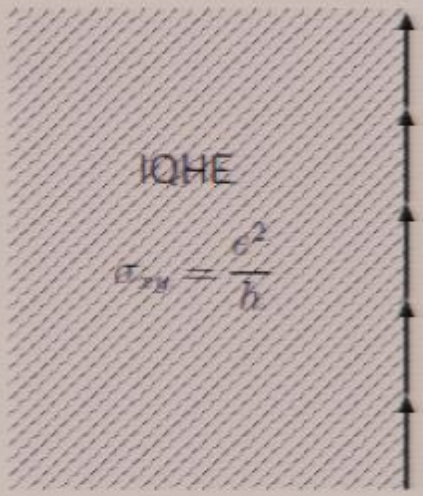

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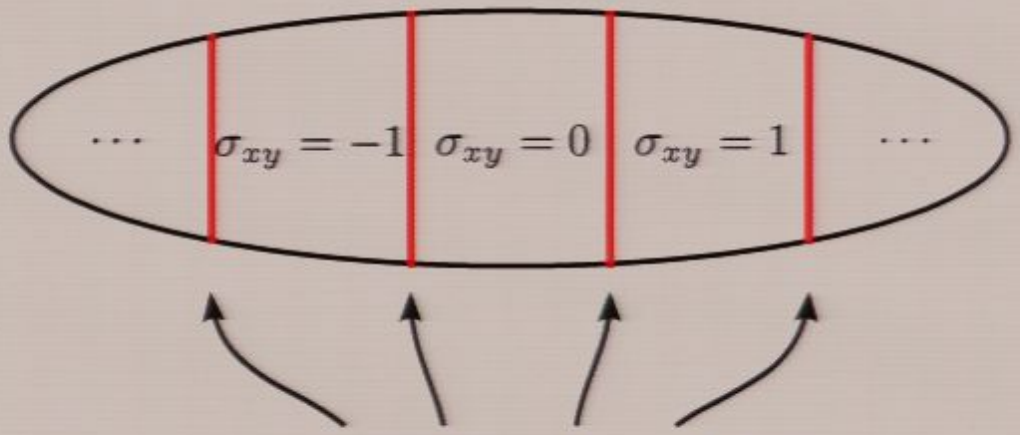

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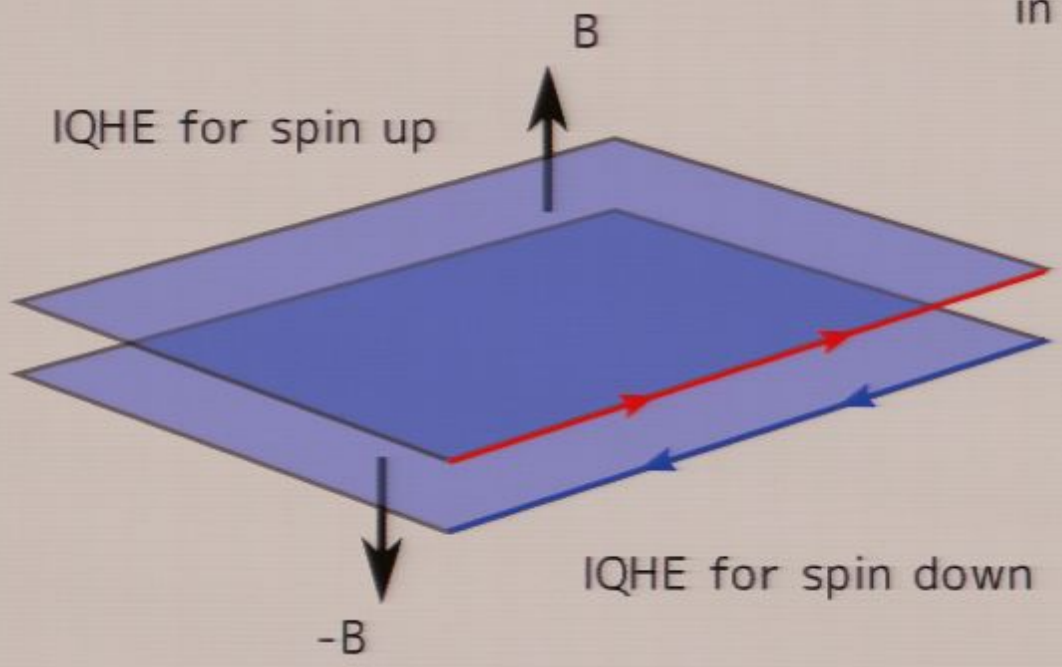

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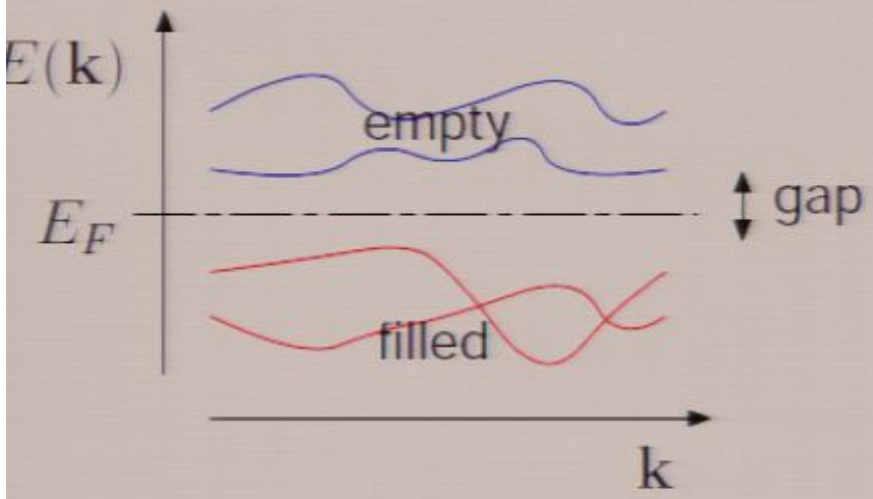

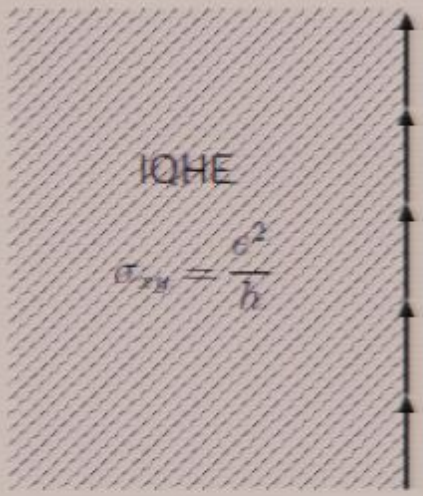

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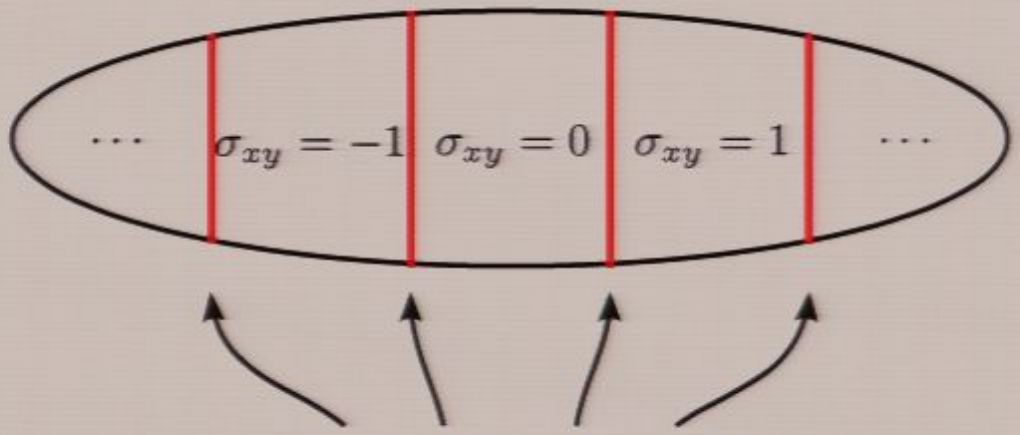

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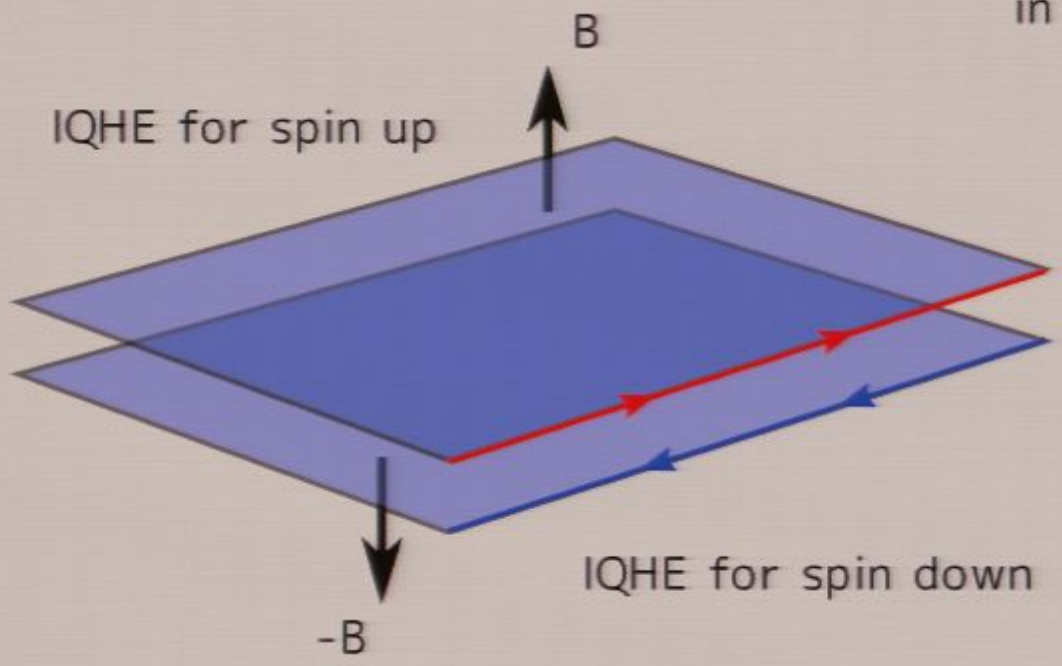

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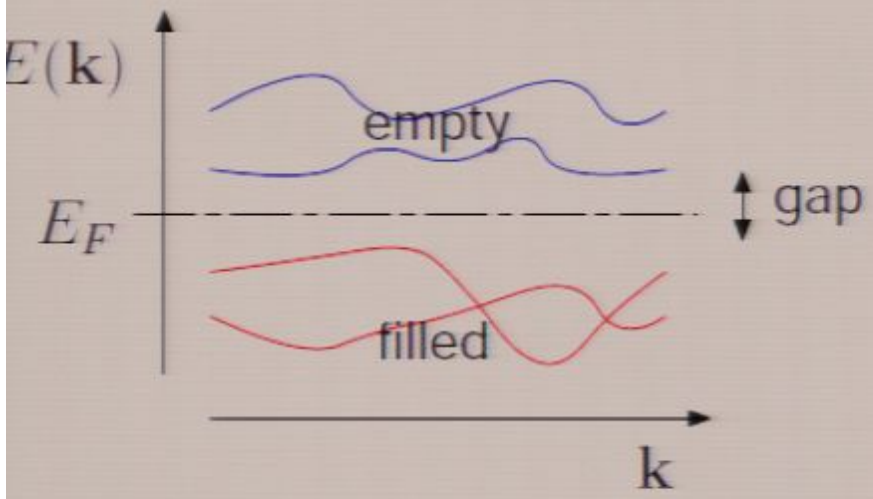

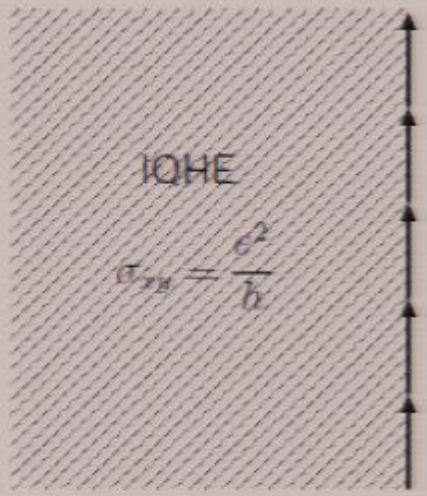

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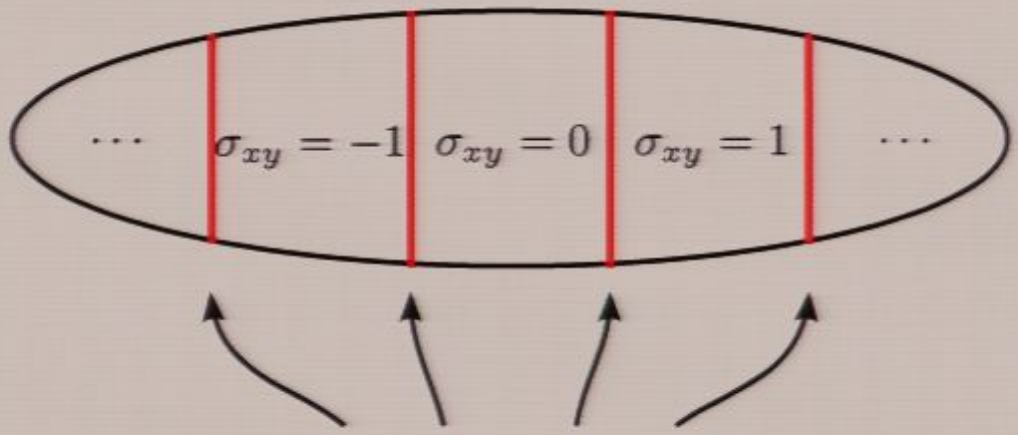

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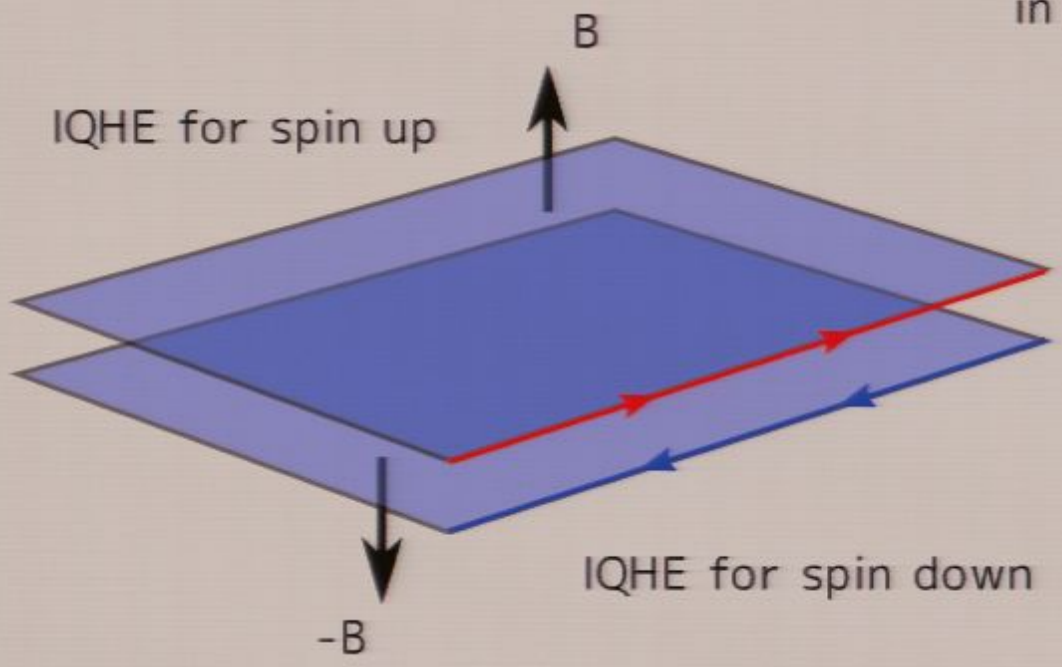

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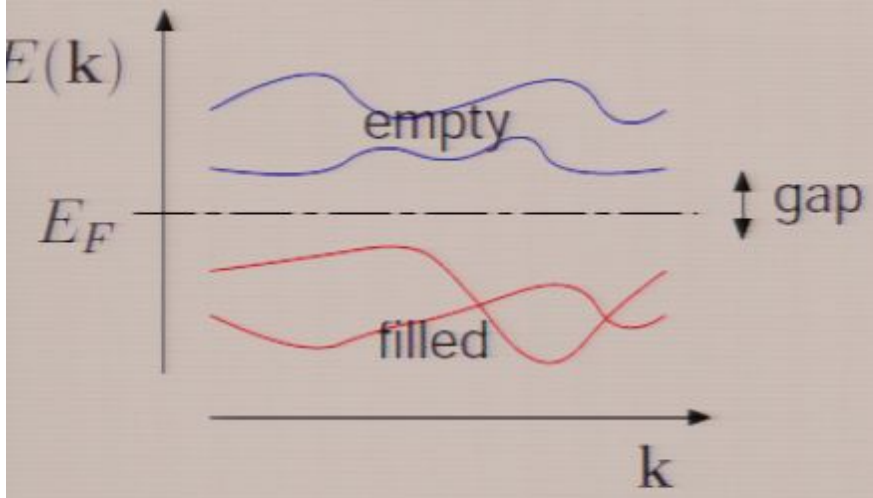

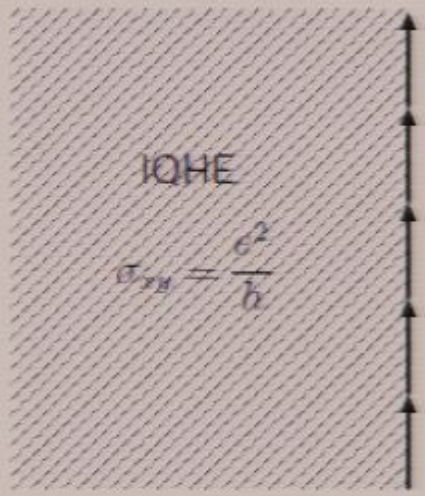

or

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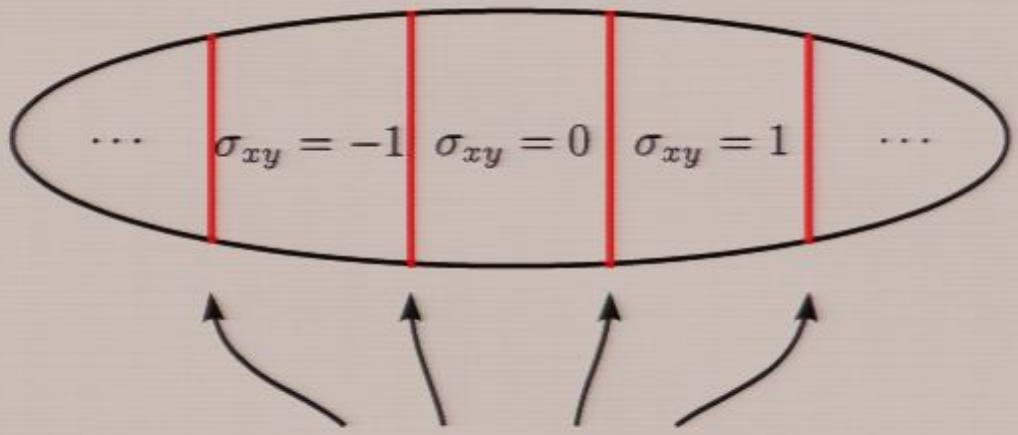
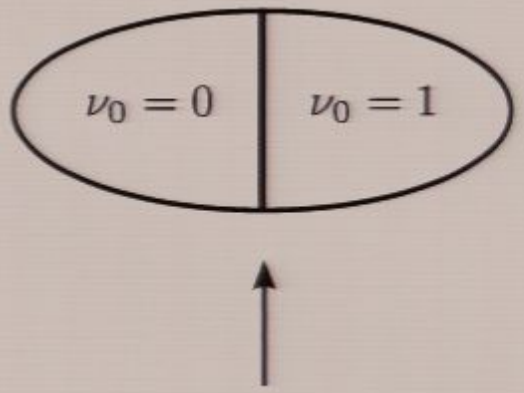


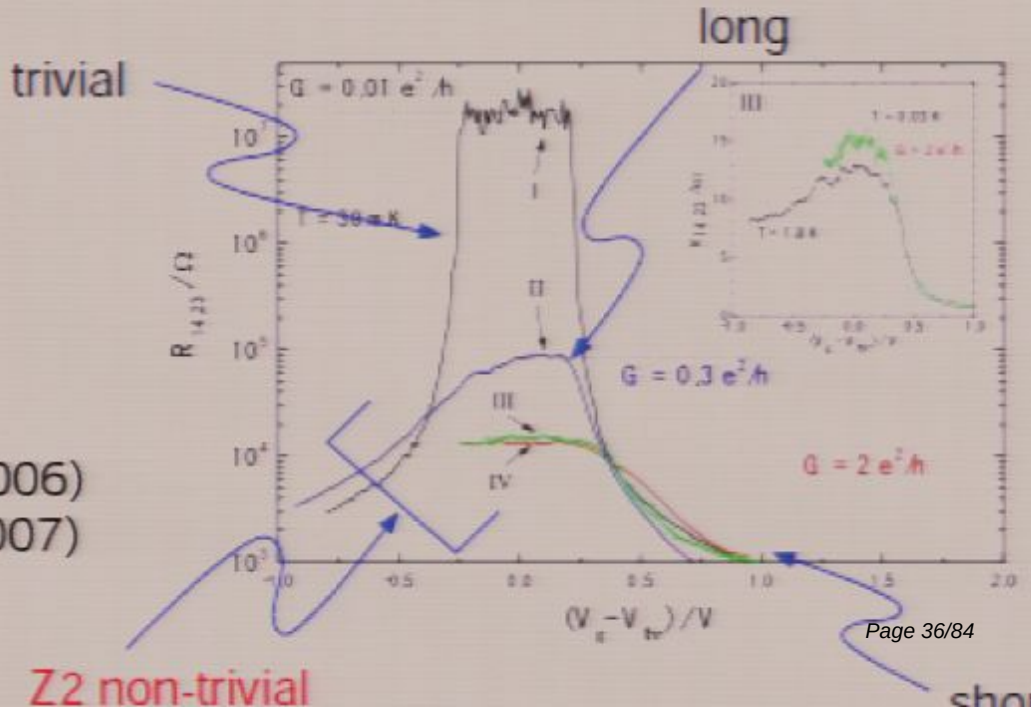
quantum phase transition

quantum spin Hall effect (QSHE)



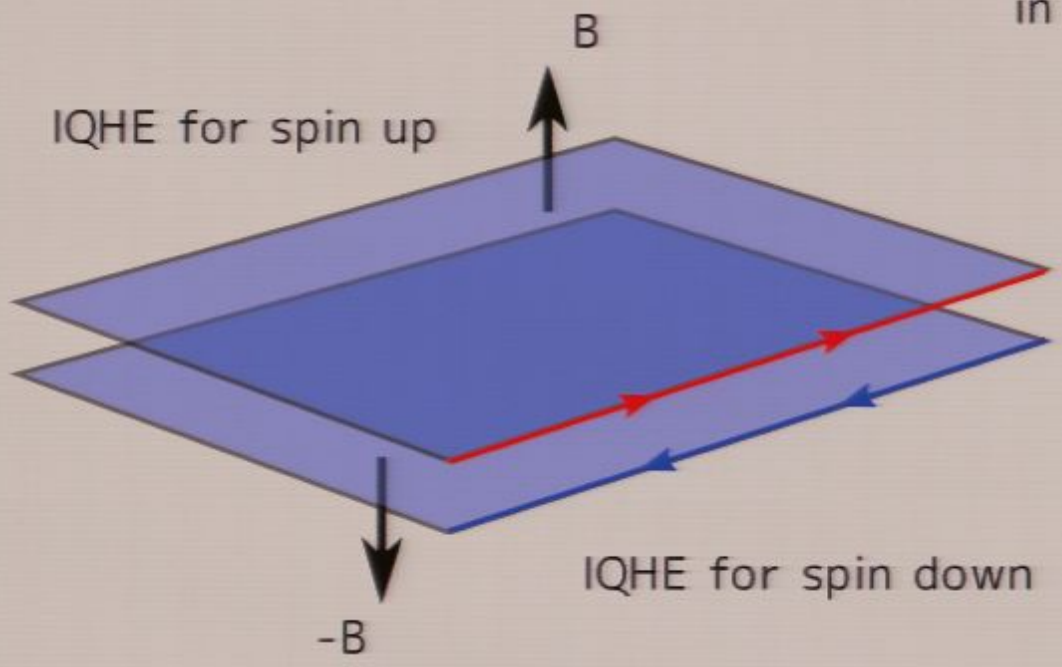
experimental realization:
HgTe quantum well

Bernevig-Hughes-Zhang (2006)
M. Koenig et al. Science (2007)



quantum spin Hall effect (QSHE)

in d=2 spatial dimensions, with good T



TRS

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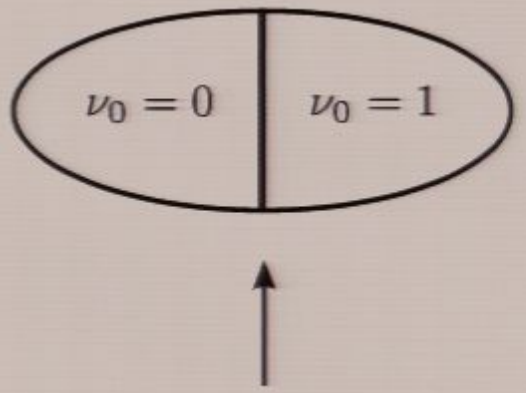

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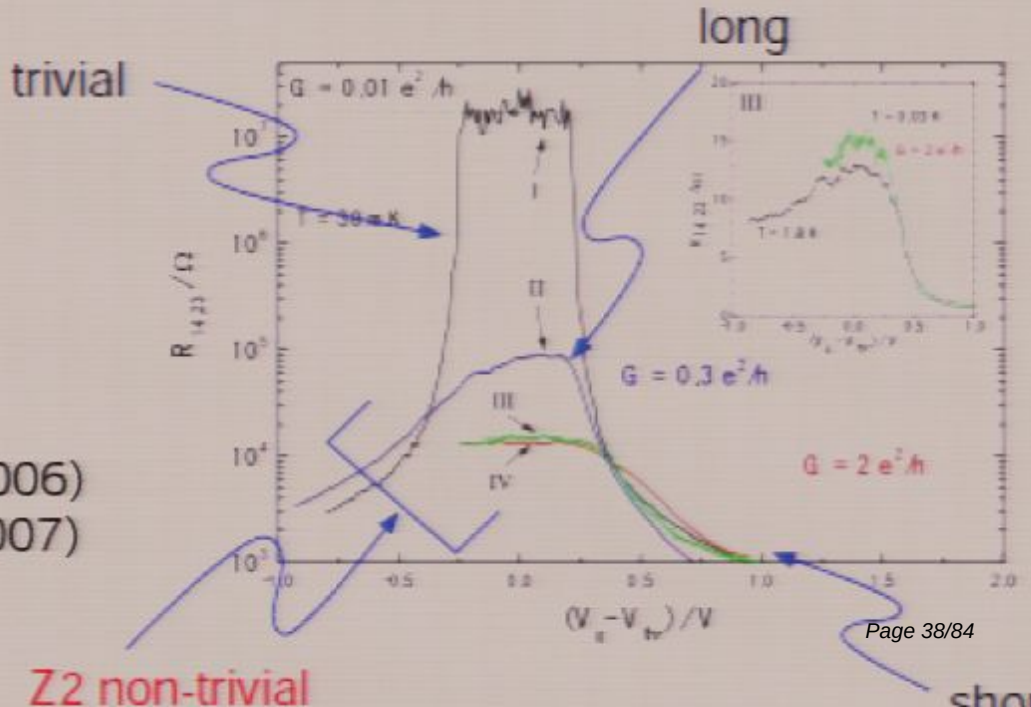

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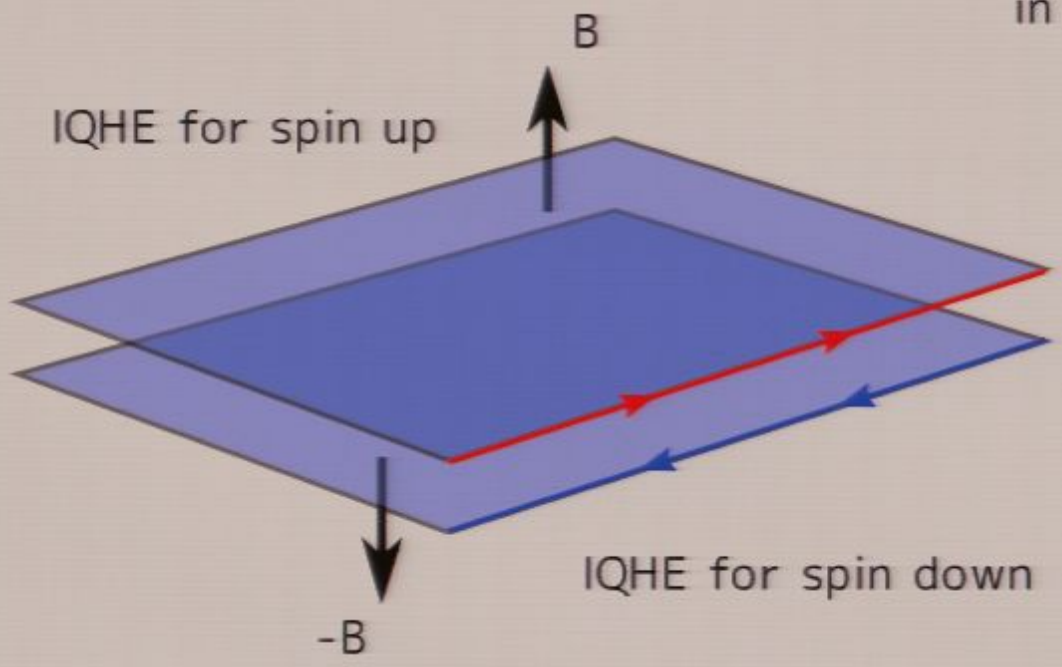

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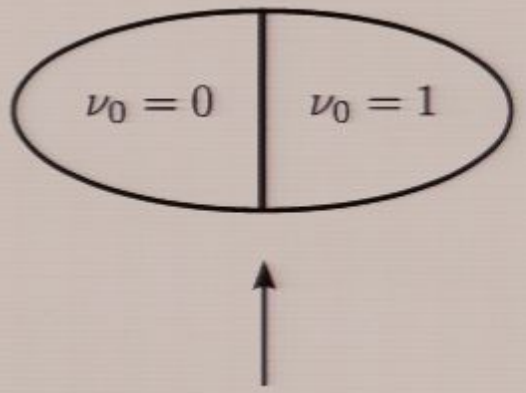

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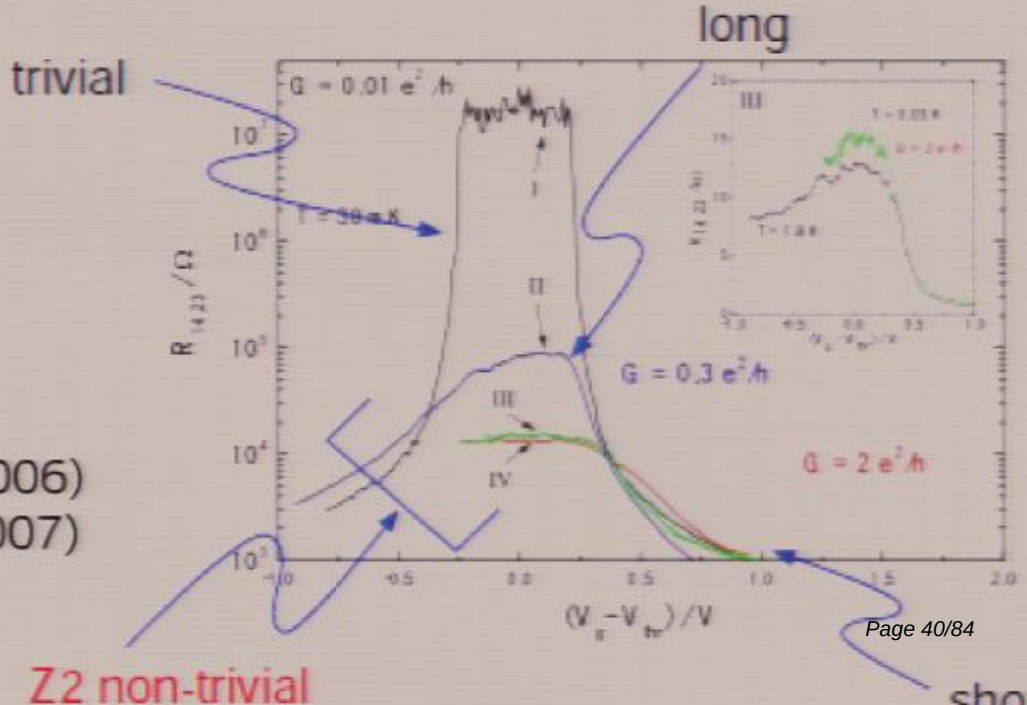

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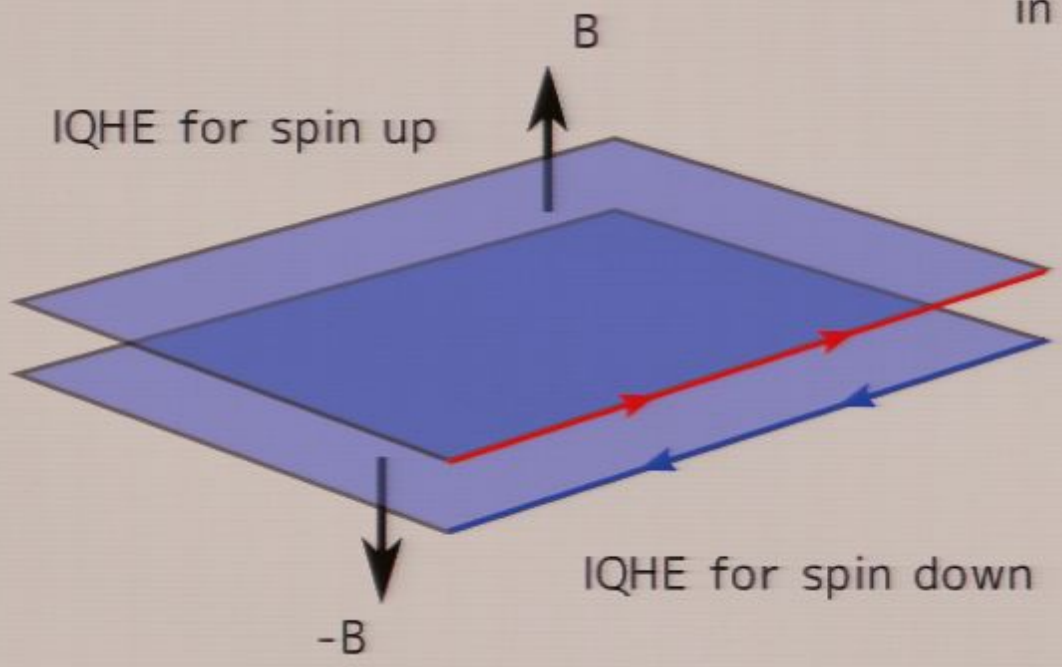

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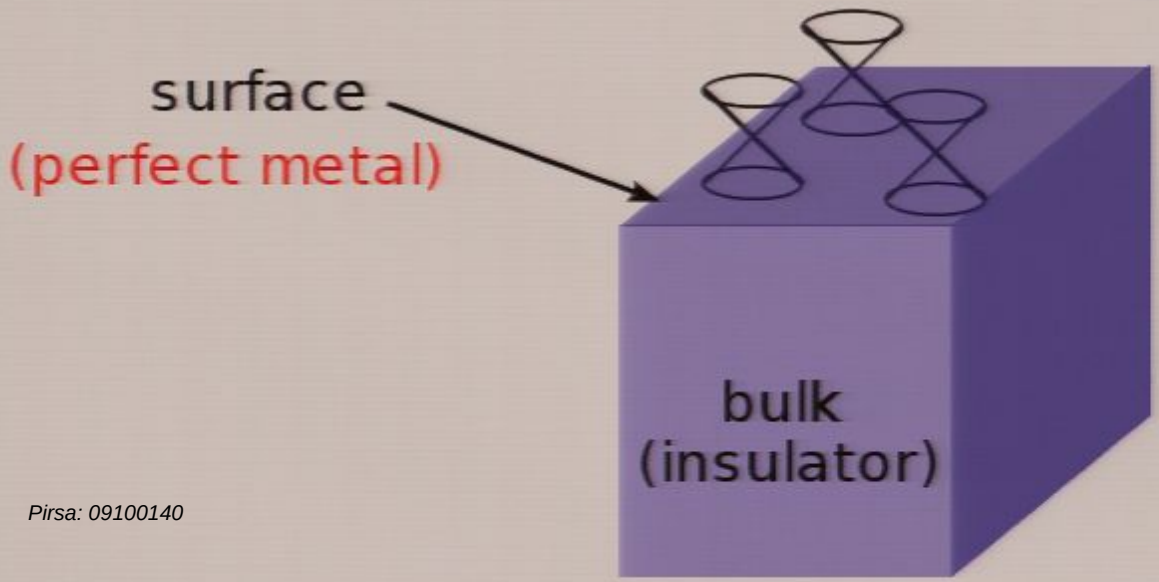
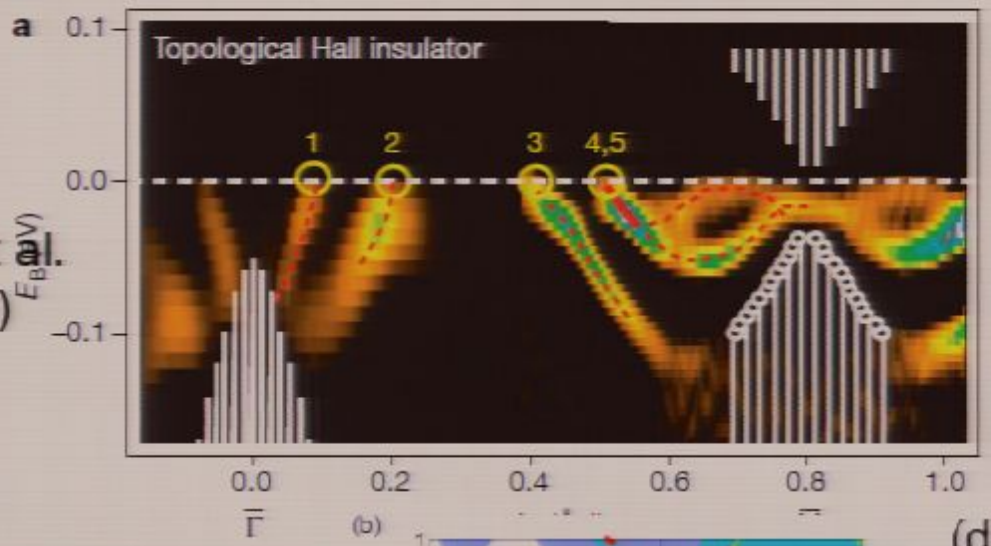


condensed matter realization of domain-wall fermion

BiSb

5 Dirac cones !



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ature (08)

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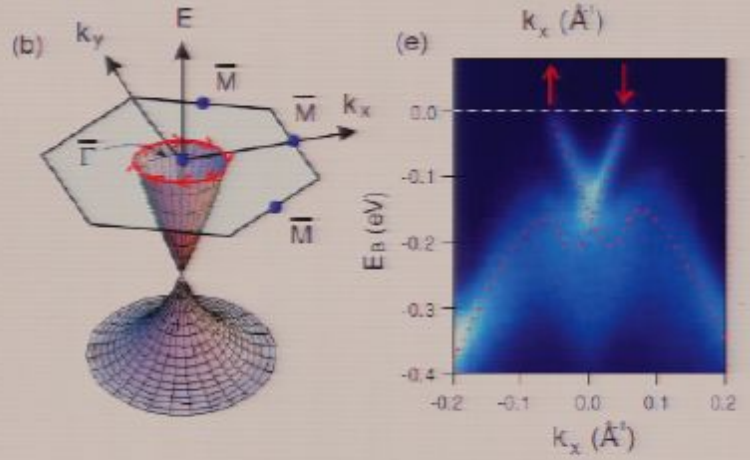

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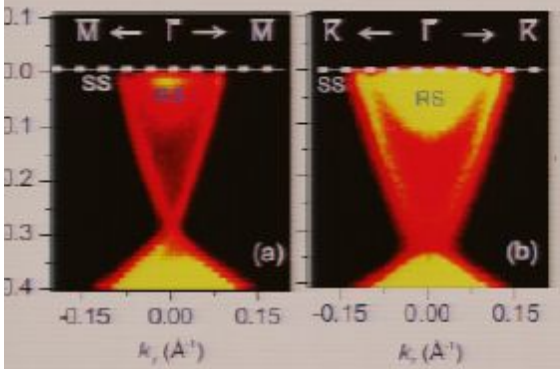
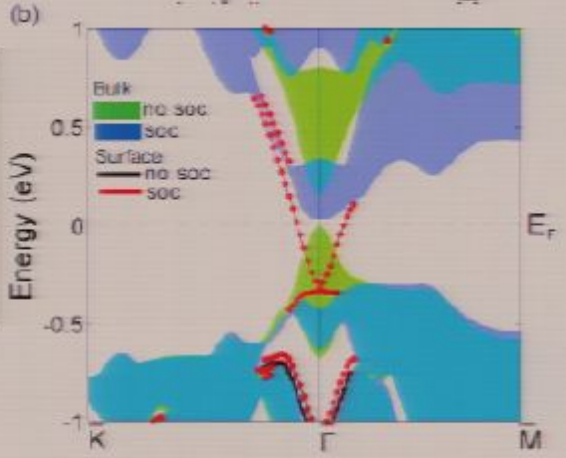
BiSe





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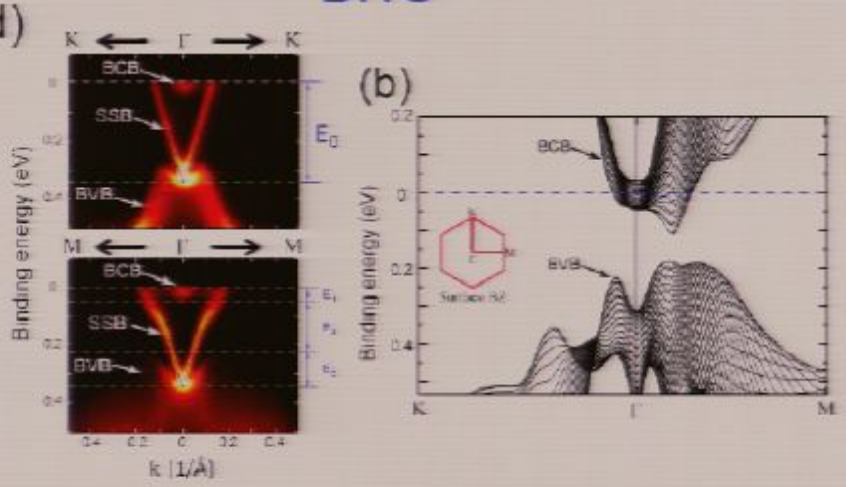

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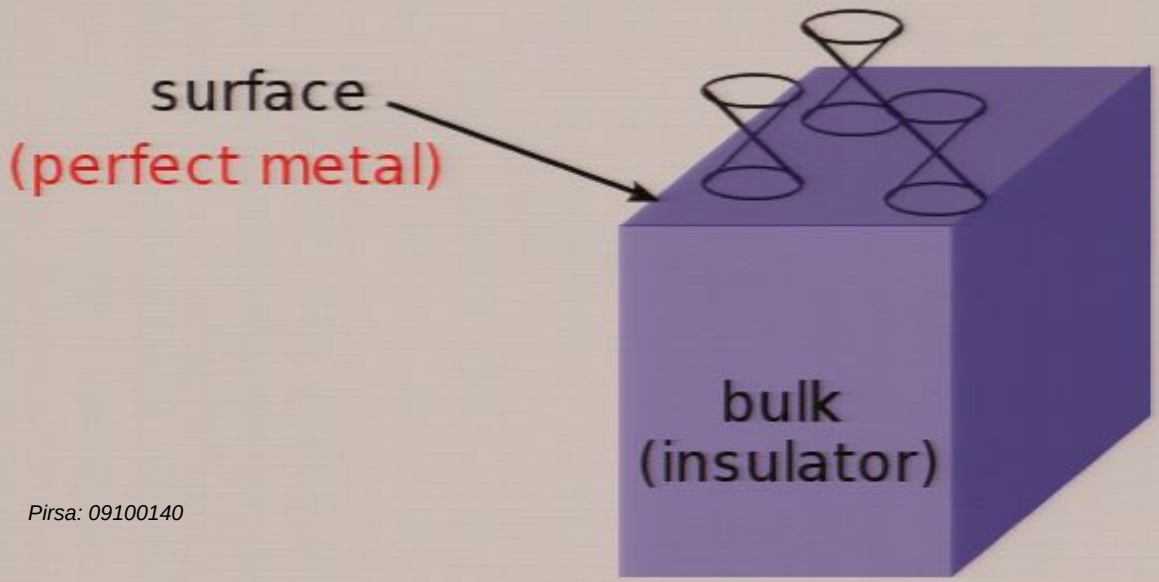

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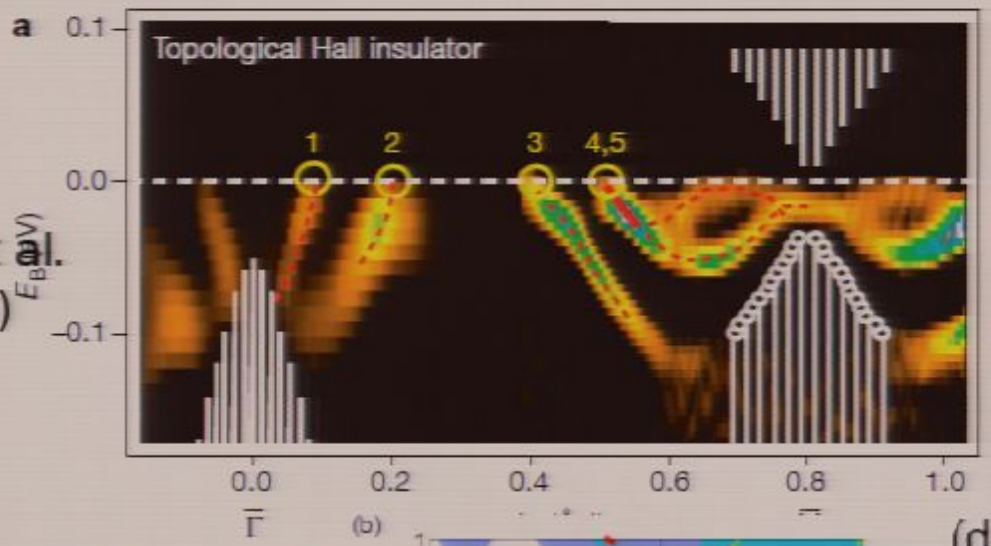

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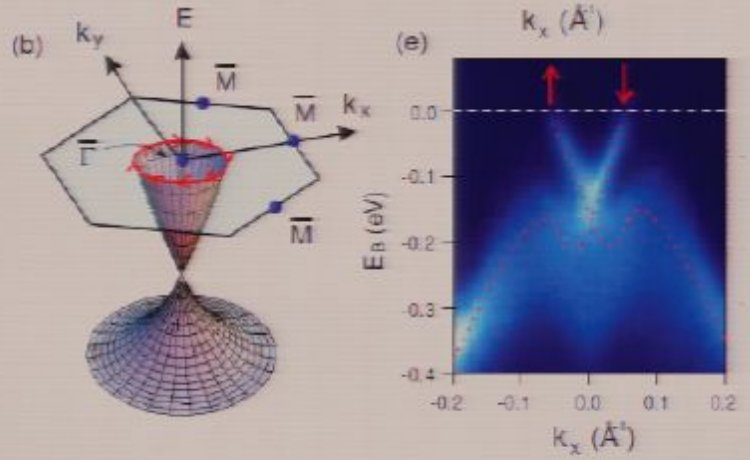

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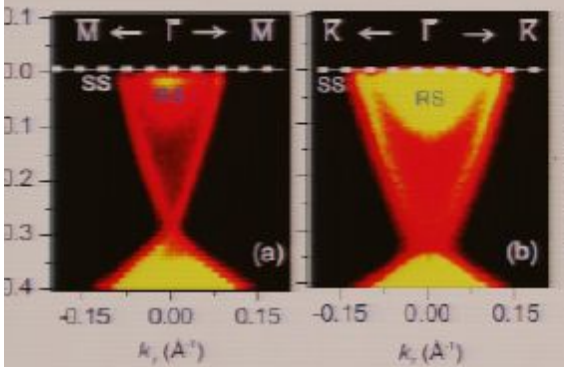

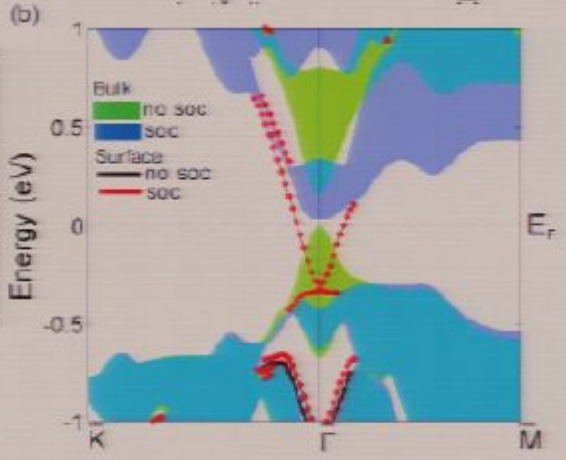

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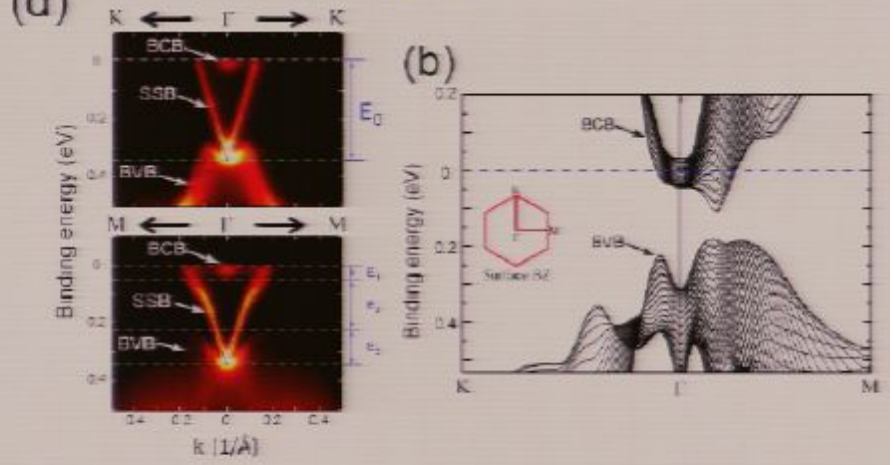

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chiral p-wave SC in d=2 - a topological SC

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$$H = \frac{1}{2}\int \Psi^\dagger \begin{pmatrix} \xi & \Delta \\ -\Delta^* & -\xi^T \end{pmatrix} \Psi$$

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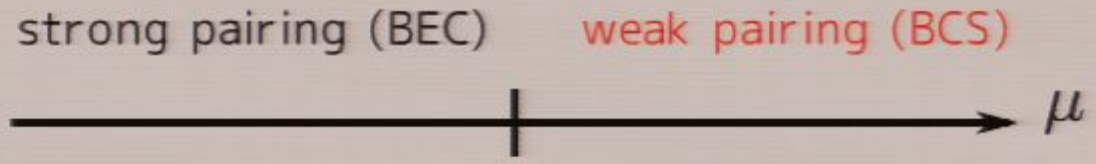

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Pirsa: 09100140

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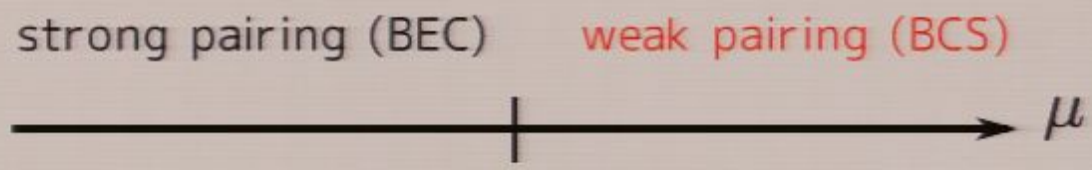
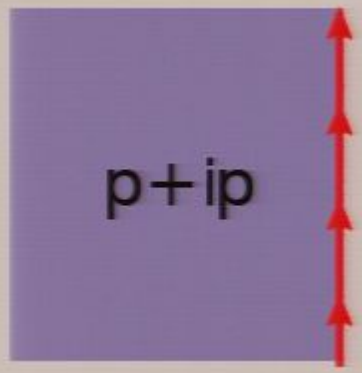


stable boundary Majorana-Weyl fermion in the weak pairing phase



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quantized thermal Hall conductivity

with inclusion of the dynamics of Cooper pair:

non-trivial ground state degeneracy — topologically protected q-bit

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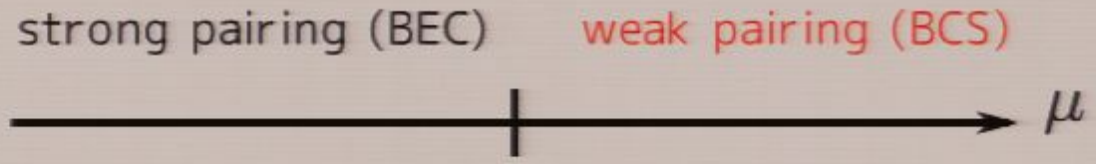

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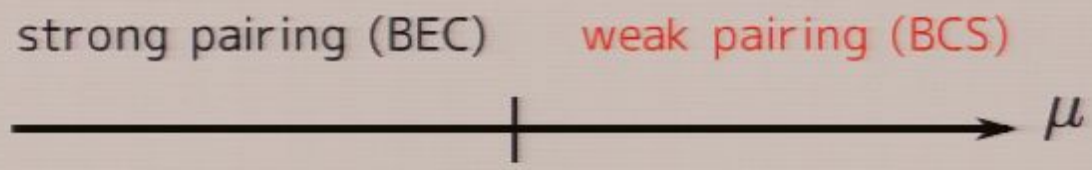

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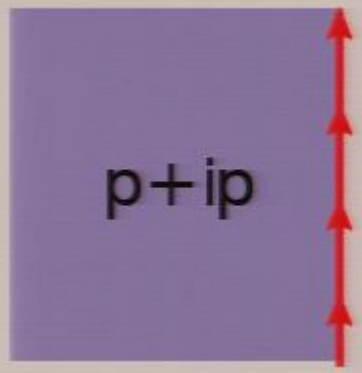

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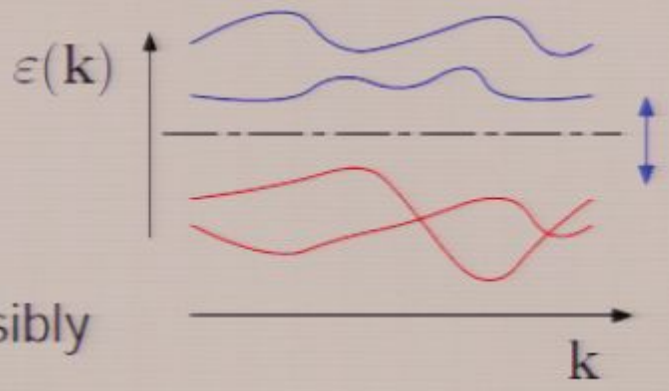


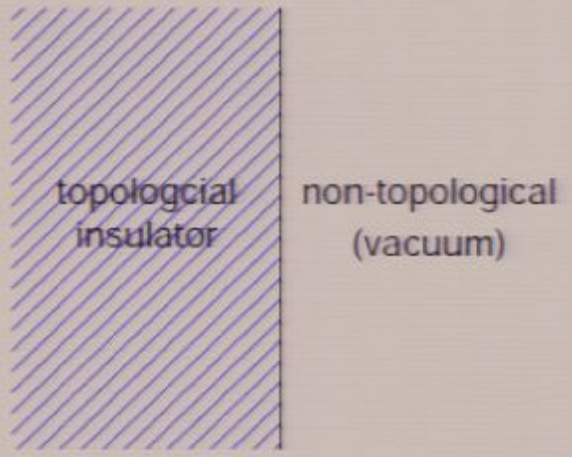
- gap

- support stable gapless modes at boundaries, possibly in the presence of general discrete symmetries

- states with and without boundary modes are not adiabatically connected

- may be characterized by a bulk topological invariant of some sort

(No interaction)



c.f. topological phase, topological field theory

How many different topoloigcal insulators and superconductors are possible in nature ?

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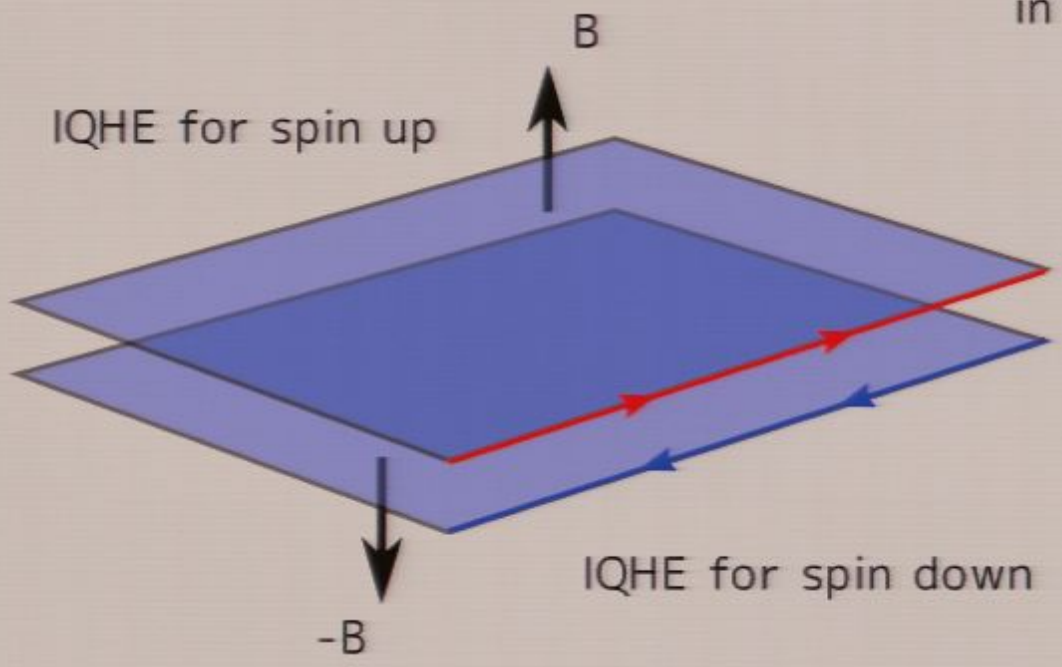

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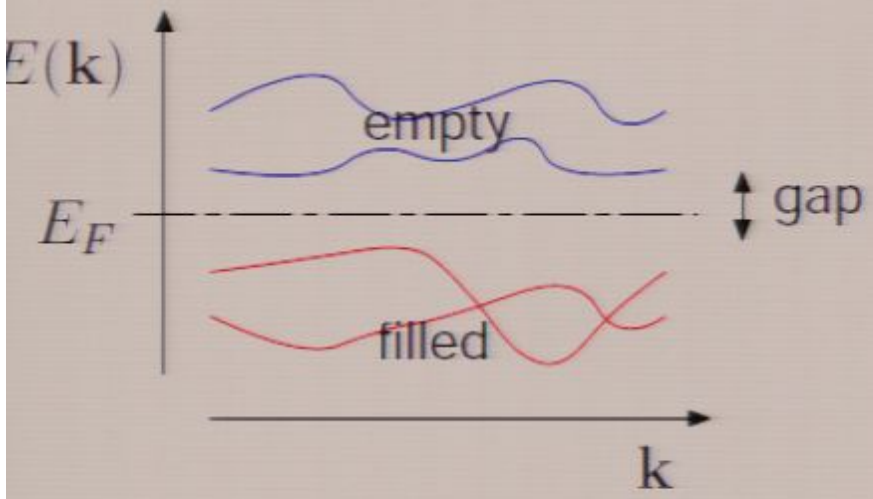

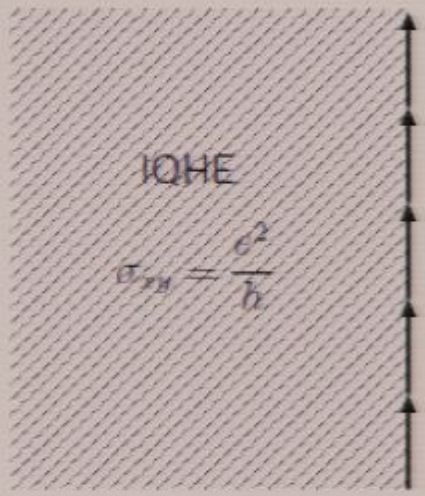

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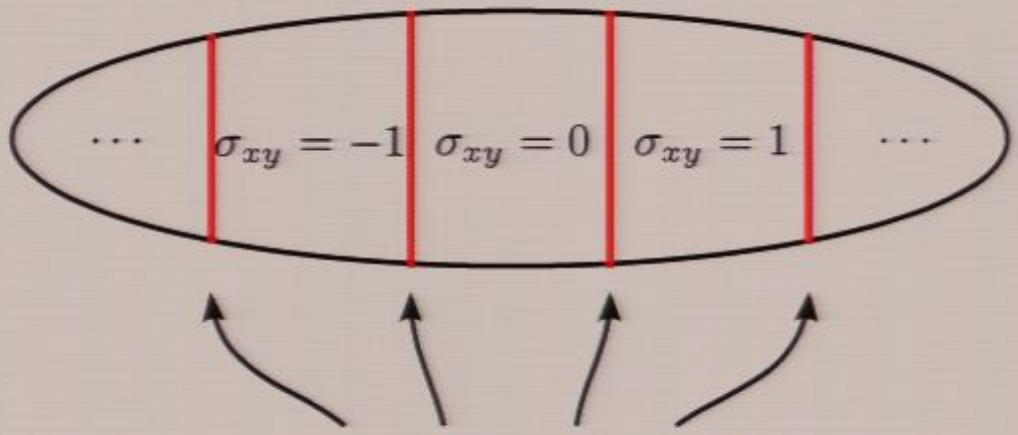

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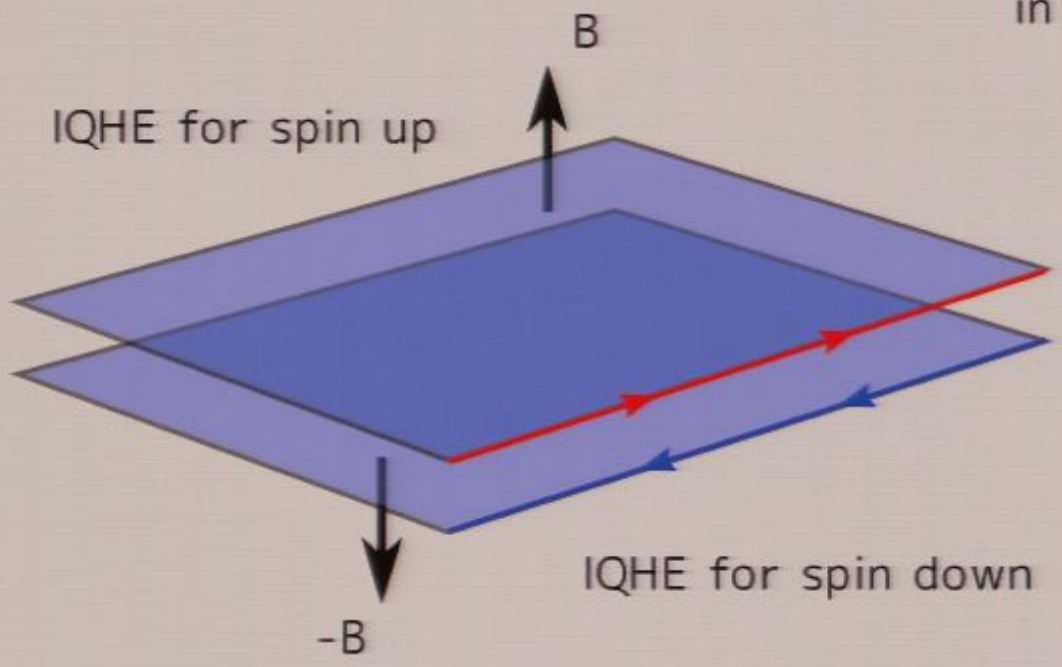

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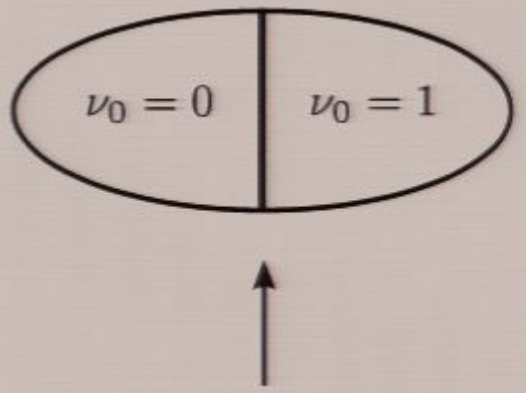

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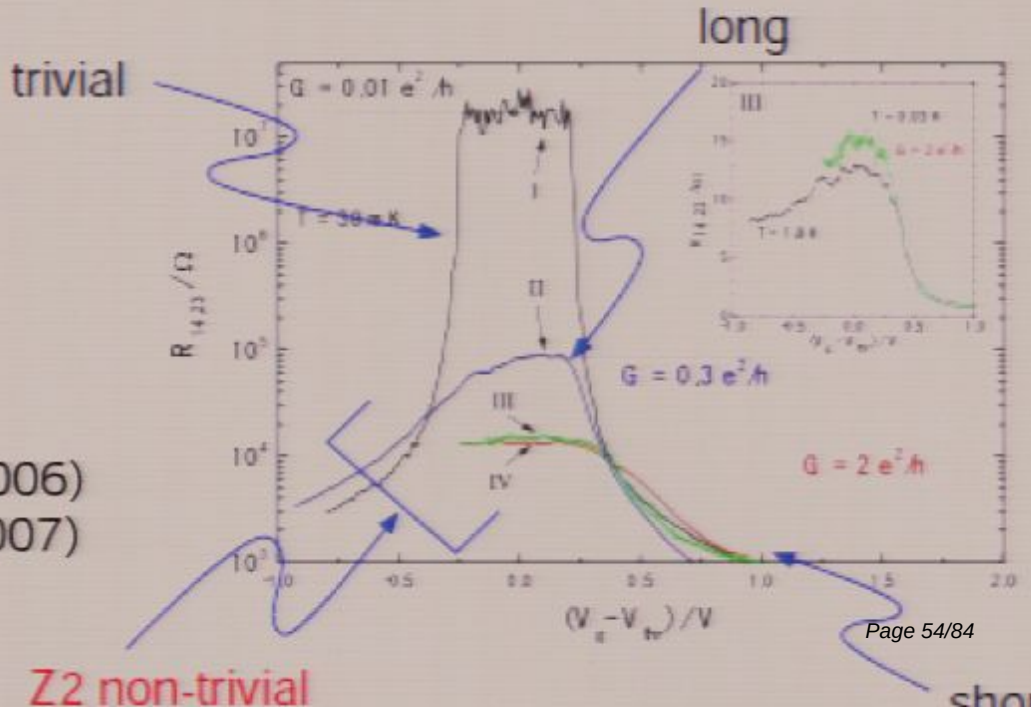

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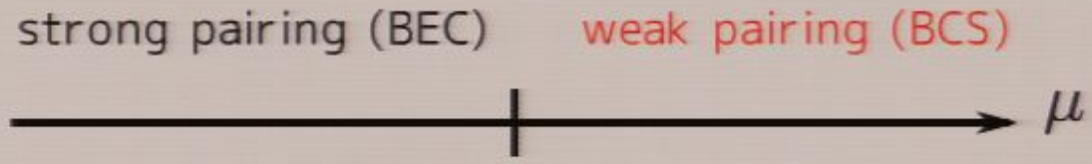
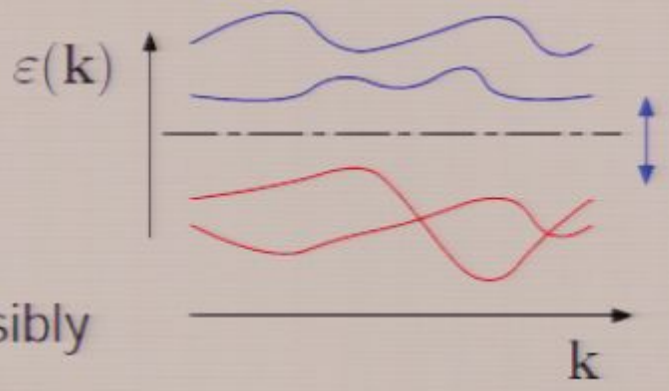
Read-Green (2000)



$$\Psi(\{\boldsymbol{r}_i\}) \sim \text{short ranged} \qquad \Psi(\{\boldsymbol{r}_i\}) \sim \text{Pf}\left(\frac{1}{z_i - z_j}\right)$$

Pirsa: 09100140

topological:



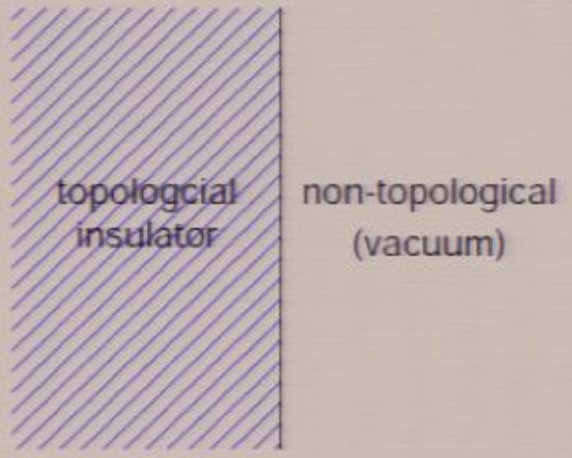
- gap

- support stable gapless modes at boundaries, possibly in the presence of general discrete symmetries

- states with and without boundary modes are not adiabatically connected

- may be characterized by a bulk topological invariant of some sort

(No interaction)



c.f. topological phase, topological field theory

How many different topoloigcal insulators and superconductors are possible in nature ?

Discrete symmetries

two types of anti-unitary symmetries

Time-Reversal Symmetry (TRS)

$$\mathcal{T}\mathcal{H}^*\mathcal{T}^{-1} = \mathcal{H}$$

integer spin particle

$$\text{TRS} = \begin{cases} 0 & \text{no TRS} \\ +1 & \text{TRS with } \mathcal{T}^T = +\mathcal{T} \\ -1 & \text{TRS with } \mathcal{T}^T = -\mathcal{T} \end{cases}$$

half-odd integer spin particle

Particle-Hole Symmetry (PHS)

$$C\mathcal{H}^T C^{-1} = -\mathcal{H}$$

$$\text{PHS} = \begin{cases} 0 & \text{no PHS} \\ +1 & \text{PHS with } C^T = +C \\ -1 & \text{PHS with } C^T = -C \end{cases}$$

PHS + TRS = chiral symmetry

$$\left.\begin{aligned} \mathcal{T}\mathcal{H}^*\mathcal{T}^{-1} &= \mathcal{H} \\ C\mathcal{H}^*C^{-1} &= -\mathcal{H} \end{aligned}\right\} \longrightarrow \quad TC\mathcal{H}(TC)^{-1} = -\mathcal{H}$$

random matrix ensembles

Altland-Zirnbauer (1997)

		TRS	PHS	SLS	description	RM ensembles
Wigner-Dyson (standard)	A	0	0	0	unitary	$U(N)$
	AI	+1	0	0	orthogonal	$U(N)/O(N)$
	AII	-1	0	0	symplectic (spin-orbit)	$U(2N)/Sp(N)$
chiral (sublattice)	AIII	0	0	1	chiral unitary	$U(2N)/U(N) \times U(N)$
	BDI	+1	+1	1	chiral orthogonal	$O(2N)/O(2N) \times O(2N)$
	CII	-1	-1	1	chiral symplectic	$Sp(4N)/Sp(2N) \times Sp(2N)$
BdG	D	0	+1	0	singlet/triplet SC	$O(N)$
	C	0	-1	0	singlet SC	$Sp(N)$
	DIII	-1	+1	1	singlet/triplet SC with TRS	$O(2N)/U(N)$
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classification of topological insulators and superconductors

result:

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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
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AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

SR, Schnyder, Furusaki, Ludwig (for d=1,2,3, 2008)

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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\cdots$
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\cdots$
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\cdots$
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	$\cdots$
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	$\cdots$

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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

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classification of topological insulators and superconductors

spatial dimensions

presence/absence
of topological band structure

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AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	...
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

ymmetry classes of quadratic fermionic
lamiltonians (Altland-Zirnbauer)

$\mathbb{Z}$ integer classification

$\mathbb{Z}_2$ Z2 classification

0 no top. ins./SC

classification of topological insulators and superconductors

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AZ\d	0	1	2	3	4	5	6	7	8	9
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D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
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CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
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ymmetry classes of quadratic fermionic
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$\mathbb{Z}$ integer classification

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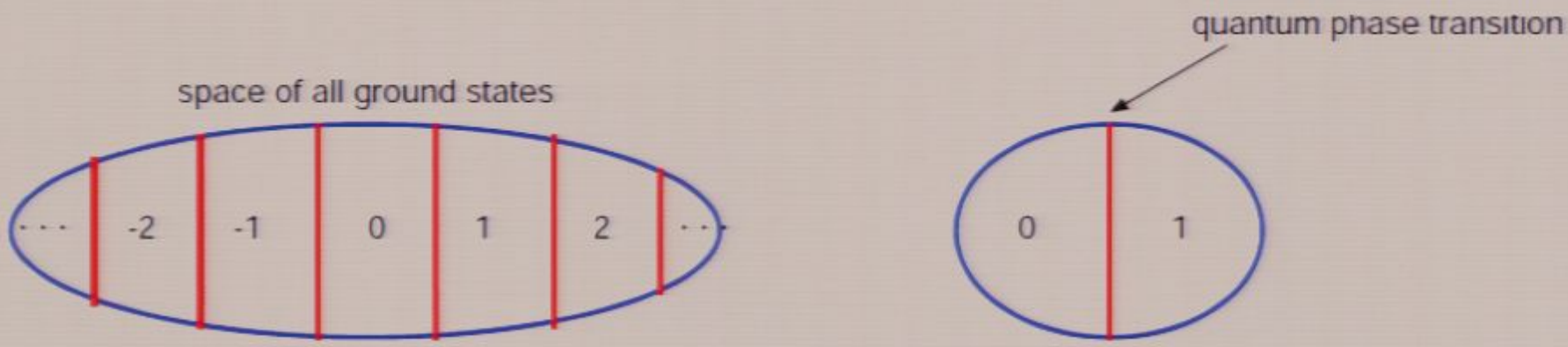
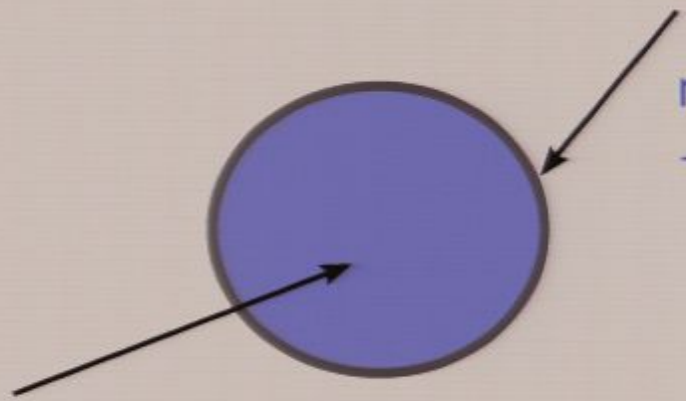
underlying strategies for classification

- discover a topological invariant



- bulk-boundary correspondence



classification of topological insulators and superconductors

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AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	...
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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

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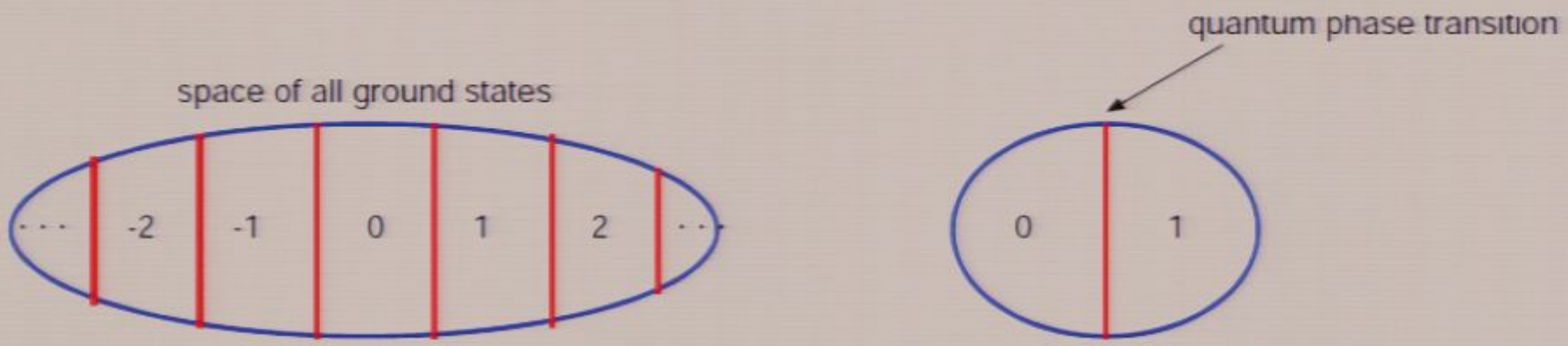

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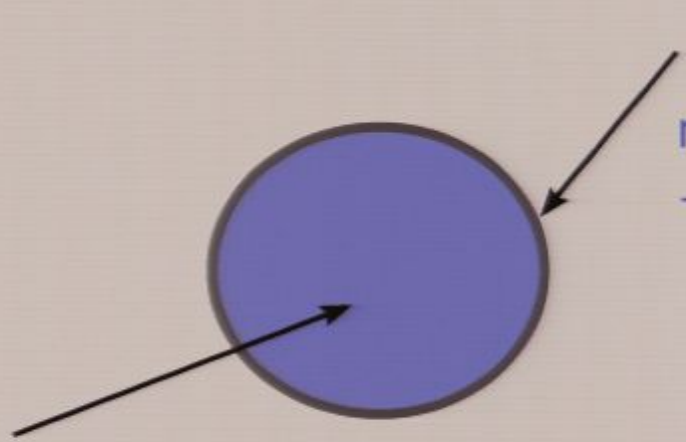


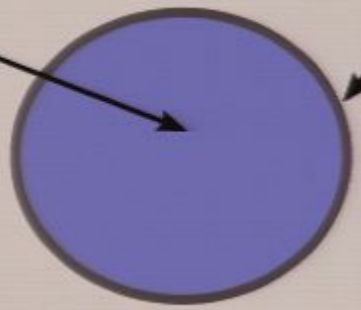
bulk-boundary correspondence

topological insulators/SC

fully gapped,
no excitations

Anderson delocalization

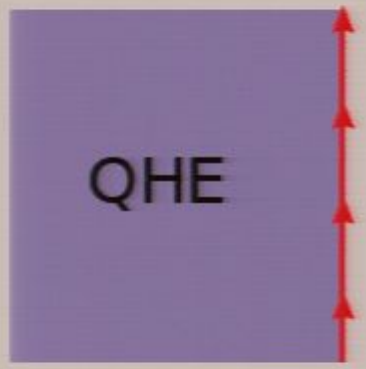
non-linear sigma model on G/H
+ (discrete) topological term



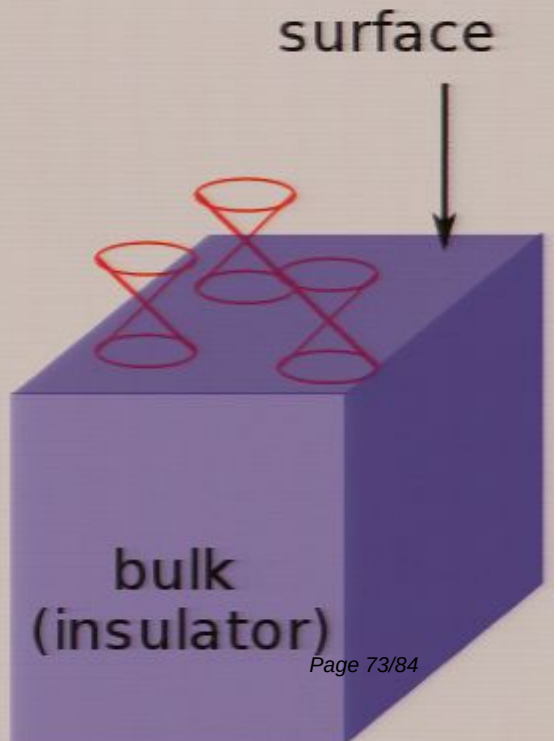
IQHE

QSHE

chiral p+ip wave SC



vac



case study: Z2 topological insulator in d=3

bulk-boundary correspondence

topological insulators/SC

fully gapped,
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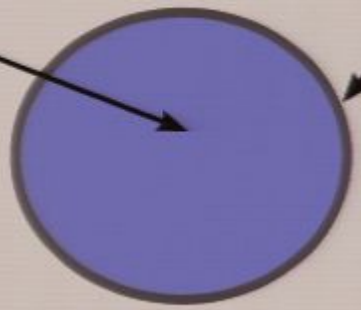
Anderson delocalization

non-linear sigma model on G/H
+ (discrete) topological term



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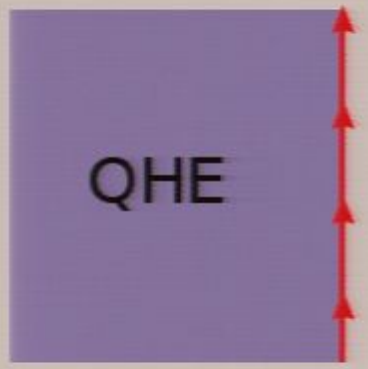
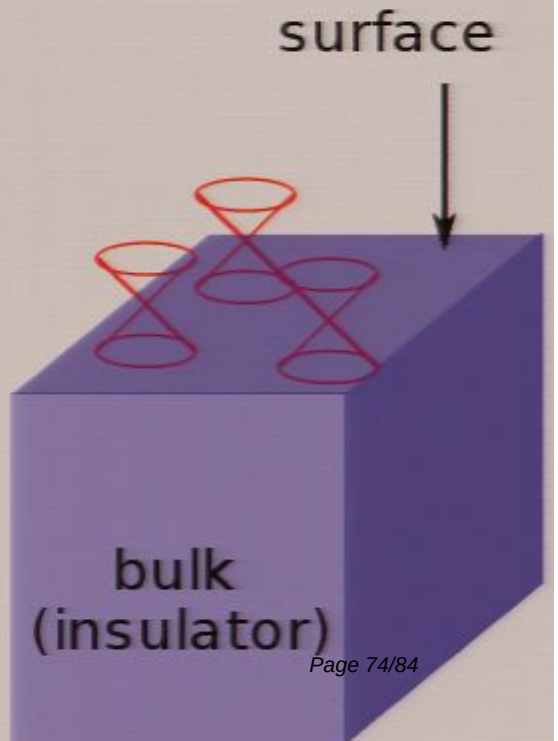
QSHE

chiral p+ip wave SC



vac



case study: Z2 topological insulator in d=3

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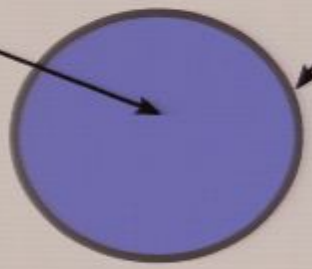
bulk-boundary correspondence

topological insulators/SC

fully gapped,
no excitations

Anderson delocalization

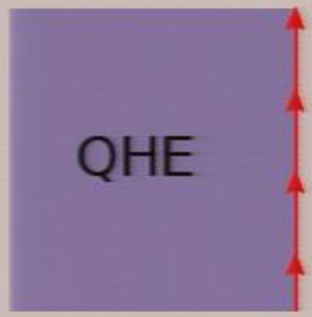
non-linear sigma model on G/H
+ (discrete) topological term



IQHE

QSHE

chiral p+ip wave SC



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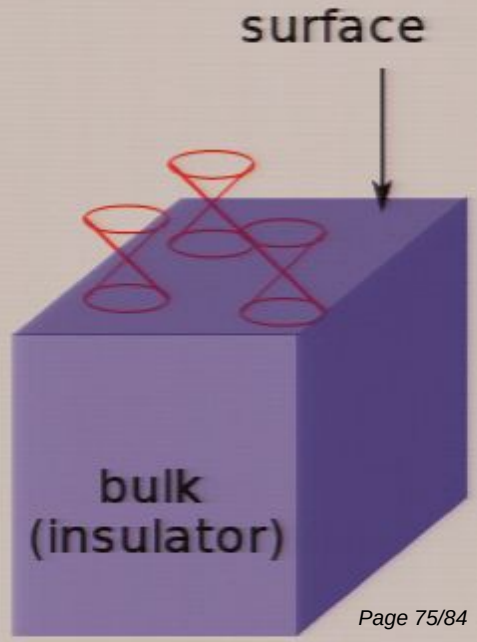

case study: Z2 topological insulator in d=3

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Z2 topological term in symplectic symmetry class

SR, Obuse, Mudry, Furusaki (07)

microscopic model:

$$\mathcal{H} = v_F\left(\sigma_x p_x + \sigma_y p_y\right) + V(\mathbf{r}) \qquad i\sigma_y \mathcal{H}^*(-i\sigma_y) = \mathcal{H}$$

effective field theory: non-linear sigma model

$Q(\mathbf{r}) \in \mathrm{O}(4N)/[\mathrm{O}(2N) \times \mathrm{O}(2N)]$ (diffusive motion of electrons)

$$S = \sigma_{xx} \int d^2 r \, \mathrm{tr}\left[\partial_\mu Q \partial_\mu Q\right]$$

even number of Dirac

$$Z = \int \mathcal{D}[Q] e^{-S}$$

odd number of Dirac
-> Z2 topological term

$$Z = \int \mathcal{D}[Q] (-1)^{n[Q]} e^{-S} \qquad n[Q] = 0, 1$$

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classification of topological insulators and superconductors

- integer quantum Hall effect
- quantum spin Hall effect
- topological superconductor
- classification of topological insulators and SCs

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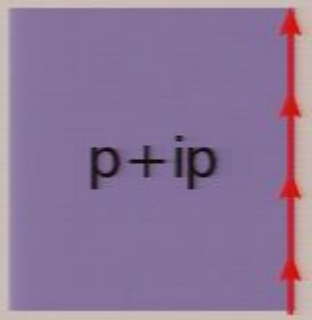
chiral p-wave SC in d=2 - a topological SC

strong pairing (BEC) weak pairing (BCS)

μ

stable boundary Majorana-Weyl fermion in the weak pairing phase



vac.

$$S = \int dx d\tau \bar{\psi} \bar{\partial} \psi$$

quantized thermal Hall conductivity

with inclusion of the dynamics of Cooper pair:

non-trivial ground state degeneracy topologically protected q-bit

non-Abelian statistics of vortices

vortex supports an isolated Majorana mode

$\nu = 1/2$ FQHE Moore-Read state

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bulk-boundary correspondence

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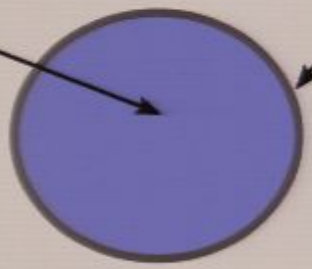

IQHE

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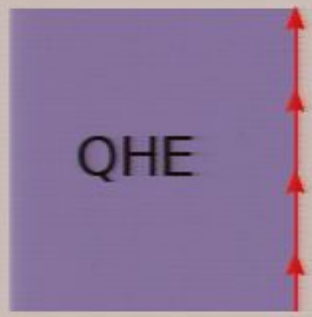

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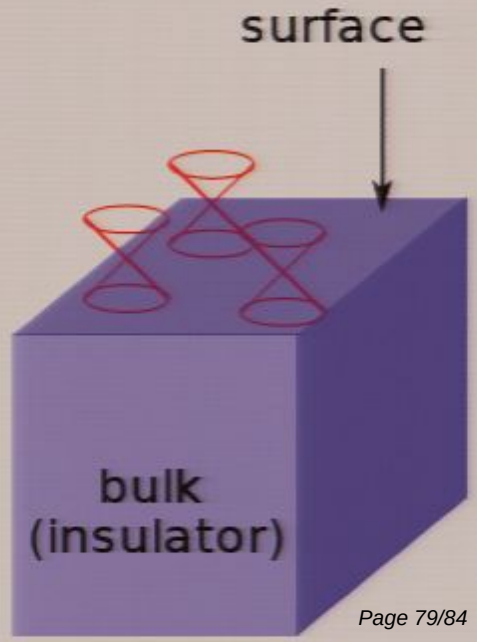

case study: Z2 topological insulator in d=3

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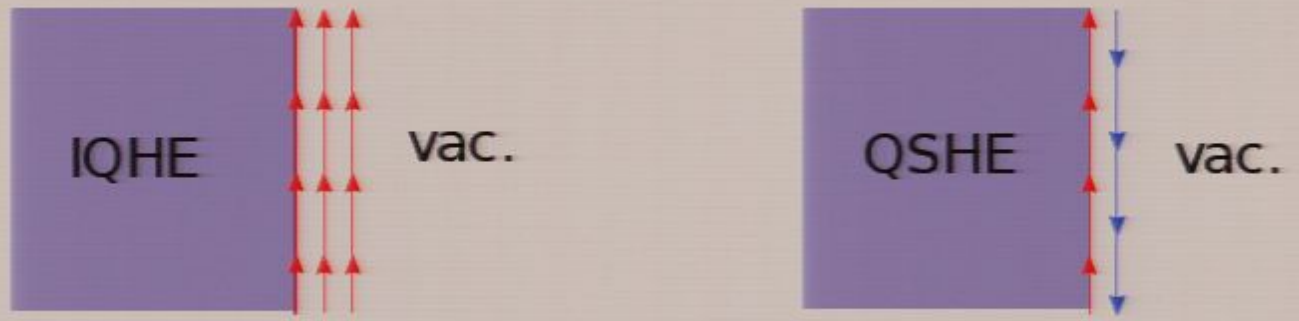
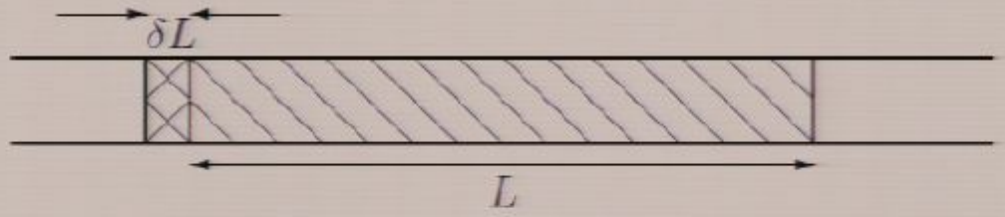
classification in (2+1)-dimensions

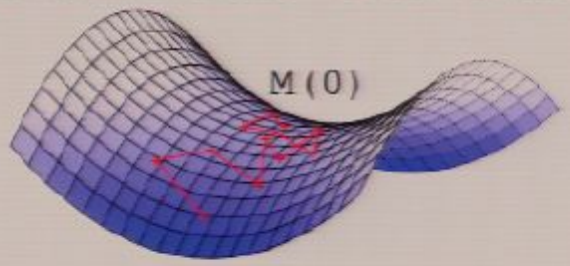


classification of (1+1)-dimensional Anderson delocalization

$$\mathcal{M}_E(L+\delta L) = \mathcal{M}_E(\delta L)\mathcal{M}_E(L)$$



$\Rightarrow$ "Brownian motion" of the transfer matrix



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AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	...
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	...
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	...
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

SR, Schnyder, Furusaki, Ludwig (for d=1,2,3, 2008)

Kitaev (all d and periodicity, 2009)

Qi, Hughes, Zhang (cases with one discrete symmetry, 2008)

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spatial dimensions

presence/absence
of topological band structure

AZ\d	0	1	2	3	4	5	6	7	8	9
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	...
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	...
AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	...
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

symmetry classes of quadratic fermionic Hamiltonians (Altland-Zirnbauer)

$\mathbb{Z}$ integer classification

$\mathbb{Z}_2$ Z2 classification

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IQHE

p+ip wave SC

AZ\d	0	1	2	3	4	5	6	7	8	9
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AI	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	...
BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

polyacetylene

3He B

TMTSF

Z2 topological insulator

QSHE

d+id wave SC

some outcomes of classification:

- 3He B is newly identified as a topological SC (superfluid) in d=3.
- topological singlet SC in d=3 is predicted.

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IQHE

p+ip wave SC

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BDI	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	...
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	...
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	...
AII	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	...
CII	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	...
C	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	...
CI	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	...

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