Title: Statistical Mechanics (PHYS 602) - Lecture 5

Date: Oct 02, 2009 10:30 AM

URL: http://pirsa.org/09100124

Abstract:

Correlation Length

$$<\sigma_j \sigma_{j+r}> = exp(-2r\tilde{K}) \qquad = \exp(-ar/\xi)$$

Here *ar* is distance between the sites of the two spins.

The result is that correlations fall off exponentially with distance, with the typical falloff distance, denoted as ξ, being the distance between lattice points (usually called *a*) times $1/(2D(K))=1/(2\tilde{K})$. This falloff distance is very important in field theory, particle physics, and phase transition theory. In the latter context it is called the coherence length. It is also called the Yukawa distance because it first came up in Hideki Yukjawa's description of mesons. Here, in the one dimensional Ising model, we have a very large coherence length for large K. Specifically

$$1/(2\tilde{K}) = \xi/a \to \exp(2K)/2 \text{ as } K \to \infty$$

while is very small in the opposite limit of small K.

$$\xi/a \to 1/(-\ln(2K)) \text{ as } K \to 0$$

Hideki Yukawa

Large correlation lengths, or equivalently small masses, play an important role in statistical and particle physics since they indicate a near-by phase transition or change in behavior.

ξ = correlation length= $a/[2D(K)] = a/(2\tilde{K})$

$$e^{k\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= \text{Const}\left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}}\right]$$

$W(\sigma,\sigma')$

$1 = d$

$$e^{k\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= \mathrm{Const}\left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}} \right]$$

$$\mathcal{L}_0 = \hat{K}_0 + \hat{K}_1 \hat{\tau}_1$$

$(d=0$

Bloch Walls in 1 d

In the Ising model at large values of the coupling, K, the spins tend to line up.

↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑↑

However, with a cost in probability exp(-2K) a whole region might flip its spins, producing a defect called a Bloch wall

↑↑↑↑↑↑↑↑↑↑↑↑↑↑↓↓↓↓↓↓↓↓↓↓↓↓↓↓

This kind of defect produces the decay of correlations in the Ising model at low temperatures. In any long Ising chain, many such defects will be randomly placed and ruin any possibility of correlations over infinitely long distances.

This is the simplest example of what is called a topological excitation, a defect which breaks the ordering in the system by separating two regions with different kinds of order. Since ordering is crucial in many situations, so are topological excitations.

Notice that, at low temperatures, this kind of excitation is much more likely than a simple flip of a single spin. The wall costs a factor of exp(-K); the flip costs exp(-2K).

Renormalization for 1D ising,

following ideas of Kenneth Wilson, this calculation is due to David Nelson and myself

$$Z = \sum_{\sigma_1,\sigma_2,\cdots} \exp(W_K\{\sigma\}) = \sum_{\sigma_1,\sigma_2,\cdots} \exp(K\sigma_1\sigma_2 + K\sigma_2\sigma_3 + \cdots)$$

Rearrange calculation: Rename spins separated by two lattice sites: let $\mu_1=\sigma_1$; $\mu_2=\sigma_3$, $\mu_3=\sigma_5$, ...; and sum over every other spin, σ_2, σ_4 .....

$$Z = \sum_{\mu_1,\mu_2,\cdots} \sum_{\sigma_2,\sigma_4,\cdots} \exp(K\mu_1\sigma_2 + K\sigma_2\mu_2 + \cdots) = \sum_{\mu_1,\mu_2,\cdots} \exp(w'\{\mu\})$$

Note that sum over σ_2, σ_4 ..... generates only nearest neighbor interactions for the μ's

$w'\{\mu\}$= const + $K' \mu_1 \mu_2 + K' \mu_2 \mu_3 + ...$

K' describes same system as before, with a new separation between lattice sites, which is twice as big as the old separation. Since the physical system is the same, physical quantities like the correlation length and the entropy are unchanged, but their description in terms of couplings and lattice constants has changed. In particular, the new lattice spacing is $a' = 2a$, but the correlation length is exactly the same $\xi' = \xi$. Since we know that the correlation length is given by

$\xi = a/[2D(K)]$, we know that the new coupling obeys $a/[2D(K)] = a'/[2D(K')]$ we find that the new coupling obeys $D(K') = 2\,D(K)$ before we do any detailed renormalization calculations. Since D is a decreasing function of K we know that the new, renormalized, coupling is smaller than the old one.

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$

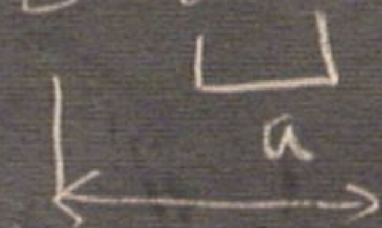
$e^{k\sigma_1\sigma_2} \qquad e^{k_2\sigma_3}$

a

$\mu_1 = \sigma_1$

$\mu_2 = \sigma_3$

$$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$$



$$\mu_1 = \sigma_1 \qquad e^{k\sigma_1\sigma_2} \, e^{k\sigma_2\sigma_3}$$

$$= \sum_{\sigma_2} e^{k\mu_1\sigma_2} \, e^{k\sigma_2\mu_2} \qquad \mu_2 = \sigma_3$$

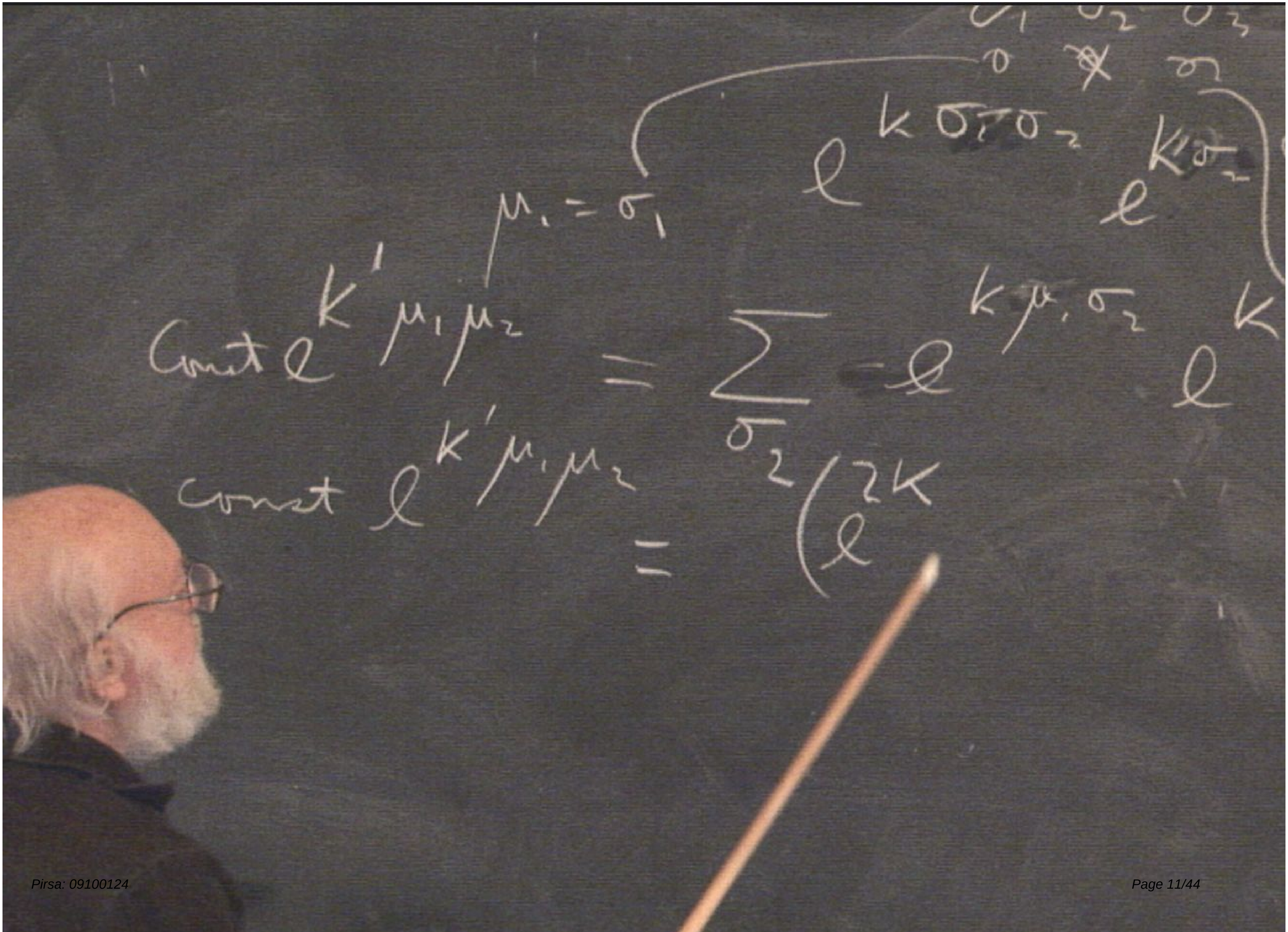
$\sigma_1 \quad \sigma_2 \quad \sigma_3$

$0 \quad \cancel{\sigma} \quad \sigma_2$

$e^{k\sigma_1\sigma_2} \quad e^{k\sigma_2}$

$\mu_1 = \sigma_1$

$$\text{Const}\; e^{K'\mu_1\mu_2} = \sum_{\sigma_2} e^{k\mu_1\sigma_2}\, e^{k}$$


σ1 σ2 σ3
0 σ2
μ1 = σ1
e^{k σ1 σ2} e^{k σ2}
Const e^{k' μ1 μ2} = Σ_{σ2} e^{k μ1 σ2} e^k
const e^{k' μ1 μ2} = (e^{2k}

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$

$0 \quad \cancel{\times} \quad \sigma_1 \quad \cancel{\times} \quad \sigma_1 \qquad 0 \quad 0$

$$e^{k\sigma_1\sigma_2} \; e^{k\sigma_2\sigma_3}$$

$\mu_1 = \sigma_1$ $\qquad$ $\mu_2 =$

$$\mu_1\mu_2 = \sum_{\sigma_2} e^{k\mu_1\sigma_2} \, e^{k\sigma_2\mu_2}$$

$$e^{k'\mu_1\mu_2} = \left(e^{2k} + e^{-2k}\right) \qquad \mu_1 = \mu$$

$$= 2$$

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4$

$e^{k\sigma_1\sigma_2} \qquad e^{k\sigma_2\sigma_3}$

$\mu_1 = \sigma_1 \qquad \mu_2 = \sigma_3$

a

$$\text{Const}\, e^{k'\mu_1\mu_2} = \sum_{\sigma_2} e^{k\mu_1\sigma_2}\, e^{k\sigma_2\mu_2}$$

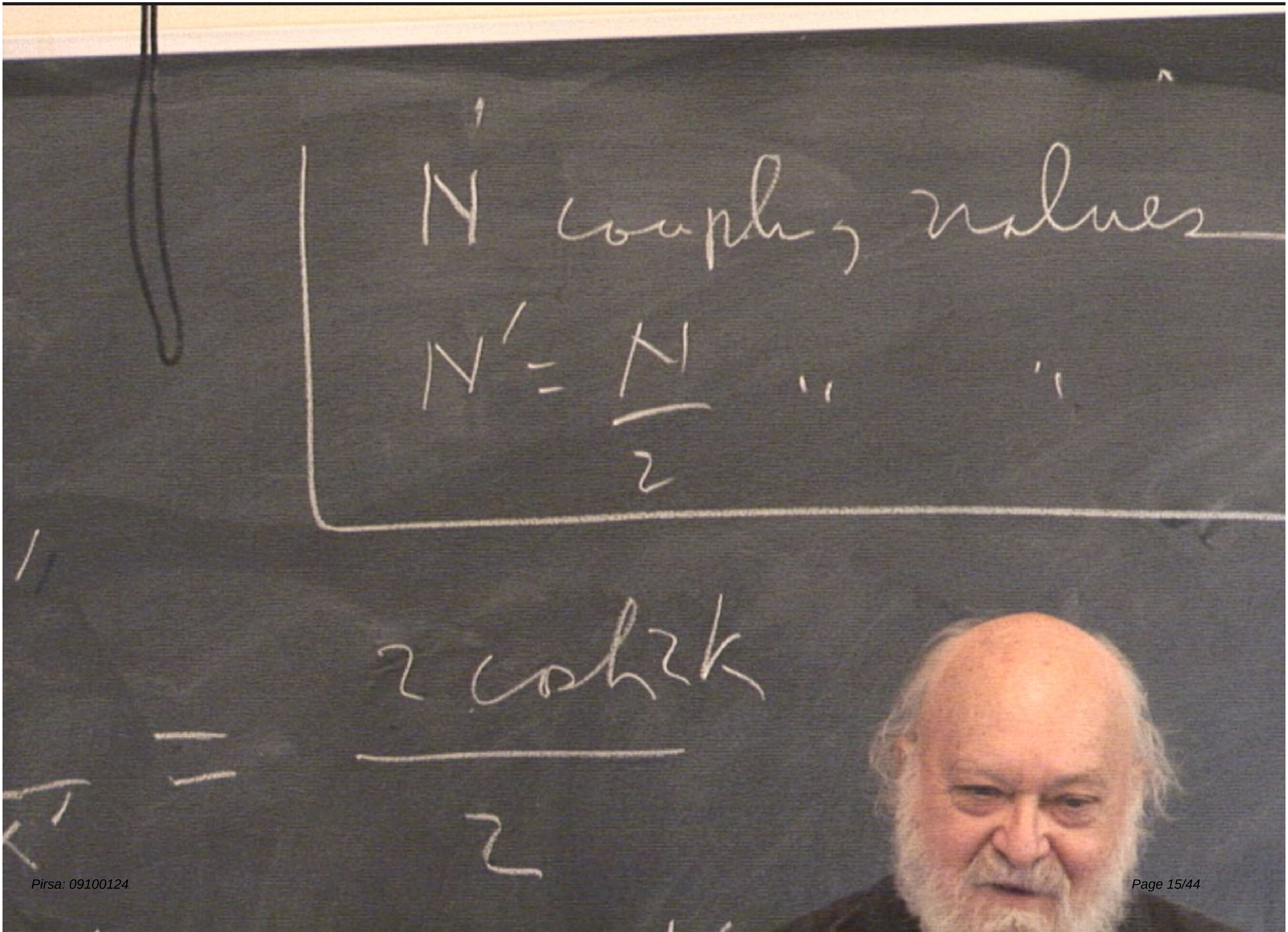
$$\text{Const}\, e^{k'\mu_1\mu_2} = \left(e^{2k} + e^{-2k}\right) \quad \text{if } \mu_1 = \mu_2$$

$$= 2 \quad \text{if } \mu_1 \neq \mu_2$$

$= \sigma_3$

$$\mu_2 \quad \frac{e^{K'}}{e^{-K'}} = \frac{2\cosh 2K}{2}$$

$$\mu_2 \quad e^{2K'} = \cosh 2K$$


N coupling values
N' = N/2
2 cosh 2K
2

$$\text{Const}\, \ell^{k\mu_1\mu_2} = \sum_{\sigma_2} \ell^{k\mu}$$

$$\text{const}\, \ell^{k'\mu_1\mu_2} = \left(\ell^{2k} + \right.$$

$$= 2$$

renormalization group

semi-

2

$2\cosh 2k$

$\{k'\} = R(\{k\})$

2

$k' = R(k)$

$\cosh 2K$

(σ, σ')

$\ell = d$

$$e^{k\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= \text{Const}\left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}}\right]$$

$$\mathcal{L}_0 = \tilde{K}_0 + \tilde{k}_1 \tilde{\tau}_1$$

$(d = 0 \qquad Z =$

$$\text{const}\left(e^{\tilde{k}} + e^{-\tilde{k}} \sigma\sigma'\right)$$

$(\sigma, \sigma') \quad = d$

$$e^{K\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= \text{const} \left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}} \right]$$

$$\mathcal{L}_0 = \tilde{K}_0 + \tilde{k}_1 \tau_1$$

$$(d = 0 \qquad Z = \int$$

$$\text{const} \left(e^{\tilde{k}} + e^{-\tilde{k}} \sigma\sigma' \right)$$

$$(\text{const})^2 \sum_{\sigma_2} \left(e^{\tilde{k}} + \mu_1 \sigma_2 e^{-\tilde{k}} \right) \left(e^{\tilde{k}} + \sigma_2 \mu_1 e^{-\tilde{k}} \right)$$

$$(\sigma, \sigma') \qquad e^{k\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= d$$

$$\mathcal{L} = \hat{k}_0 + \hat{k}_1 \hat{\tau}_1 = \text{const}\left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}}\right]$$

$$(d = 0 \qquad Z =$$

$$\text{const}\left(e^{\tilde{k}} + e^{-\tilde{k}}\sigma\sigma'\right)$$

$$(\text{const})^2 \sum \left(e^{\tilde{k}} + \mu_1 \sigma_2 e^{-\tilde{k}}\right)\left(e^{\tilde{k}} + \sigma_2 \mu_1 e^{-\tilde{k}}\right)$$

$$2\,(\text{const})^2 \left[\sigma_2^2\, e^{2\tilde{k}}\right.$$

(σ, σ') $\quad e^{K\sigma\sigma'} = \cosh K + \sigma\sigma' \sinh K$

$= \mathrm{const}\left[e^{\tilde{K}} + \sigma\sigma' e^{-\tilde{K}}\right]$

$\mathcal{L}_0 = \tilde{K}_0 + \tilde{K}_1 \hat{T}_1$

$\mathcal{L}_0 = 0 \qquad Z =$

$\mathrm{const}\left(e^{\tilde{K}} + e^{-\tilde{K}} \sigma\sigma'\right)$

$(\mathrm{const})^2 \sum \left(e^{\tilde{K}} + \mu_1 \sigma_2 e^{-\tilde{K}}\right)\left(e^{\tilde{K}} + \sigma_2 \mu_2 e^{-\tilde{K}}\right)$

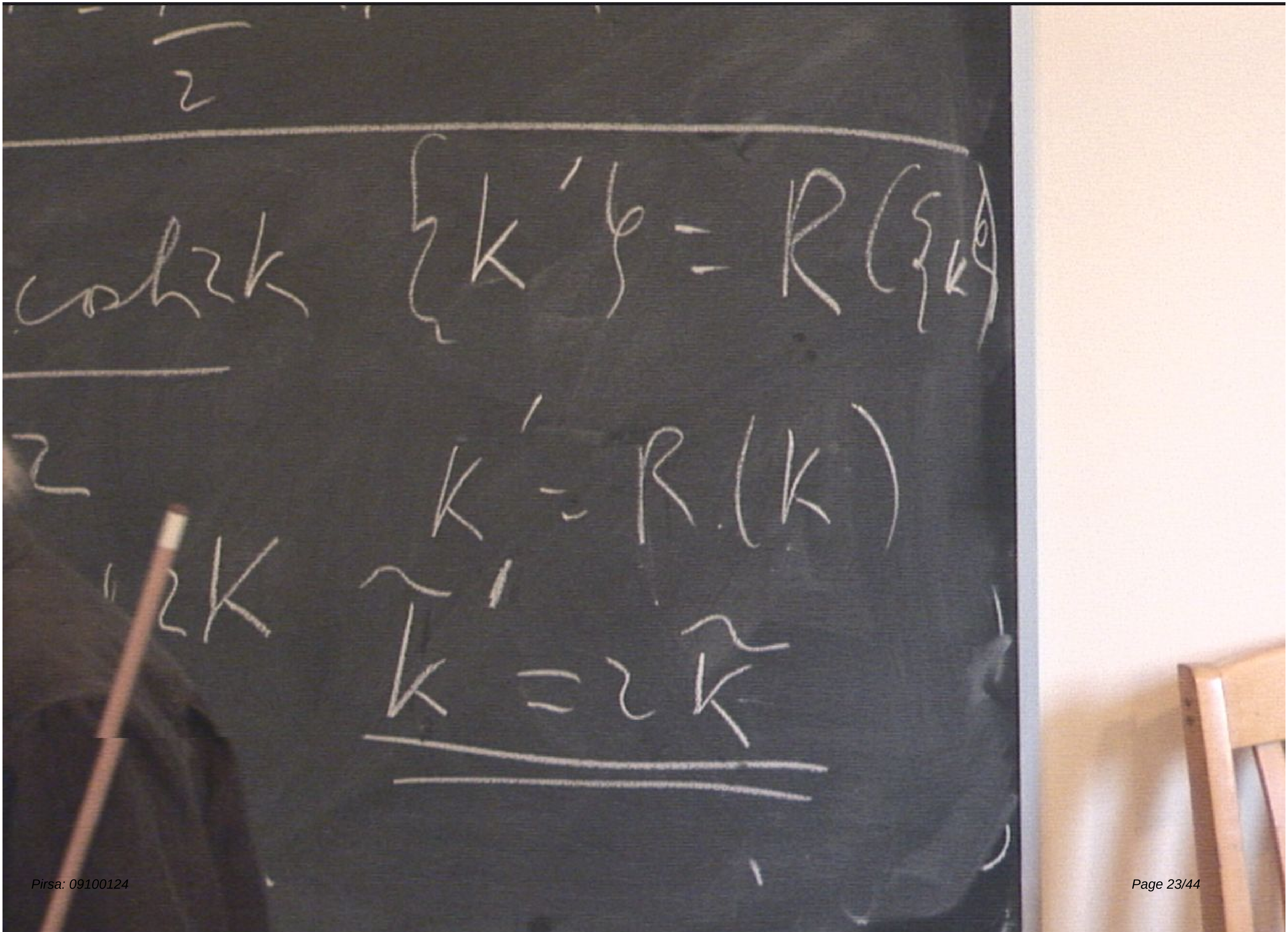
$2 (\mathrm{const})^2 \left[e^{2\tilde{K}} + 0 + 0 + \mu_1 \mu_2 e^{-2\tilde{K}} \right]$

$$\boxed{N' = \frac{N}{2} \quad " \quad "}$$

$$= \frac{2\cosh 2k}{2} \qquad \{k'\} = R(\{k\})$$

$$\cosh 2K \qquad K' = R(k)$$

$$\tilde{k}'$$


cosh2k
{k'} = R({k})
k' = R(k)
k = 2k

const ℓ $= \sum \ell$

const $\ell^{k' \mu_1 \mu_2}$ $= \sum_{\sigma_2} \left(\ell^{2k} + \ell \right.$

$\simeq 2$

renormalization group

$a' \to m\, a$ semi-

$\hat{k}' = m\, \hat{k}$

(σ, σ')

$\tilde{d}$

$$e^{k\sigma\sigma'} = \cosh k + \sigma\sigma' \sinh k$$

$$= \text{const}\left[e^{\tilde{k}} + \sigma\sigma' e^{-\tilde{k}}\right]$$

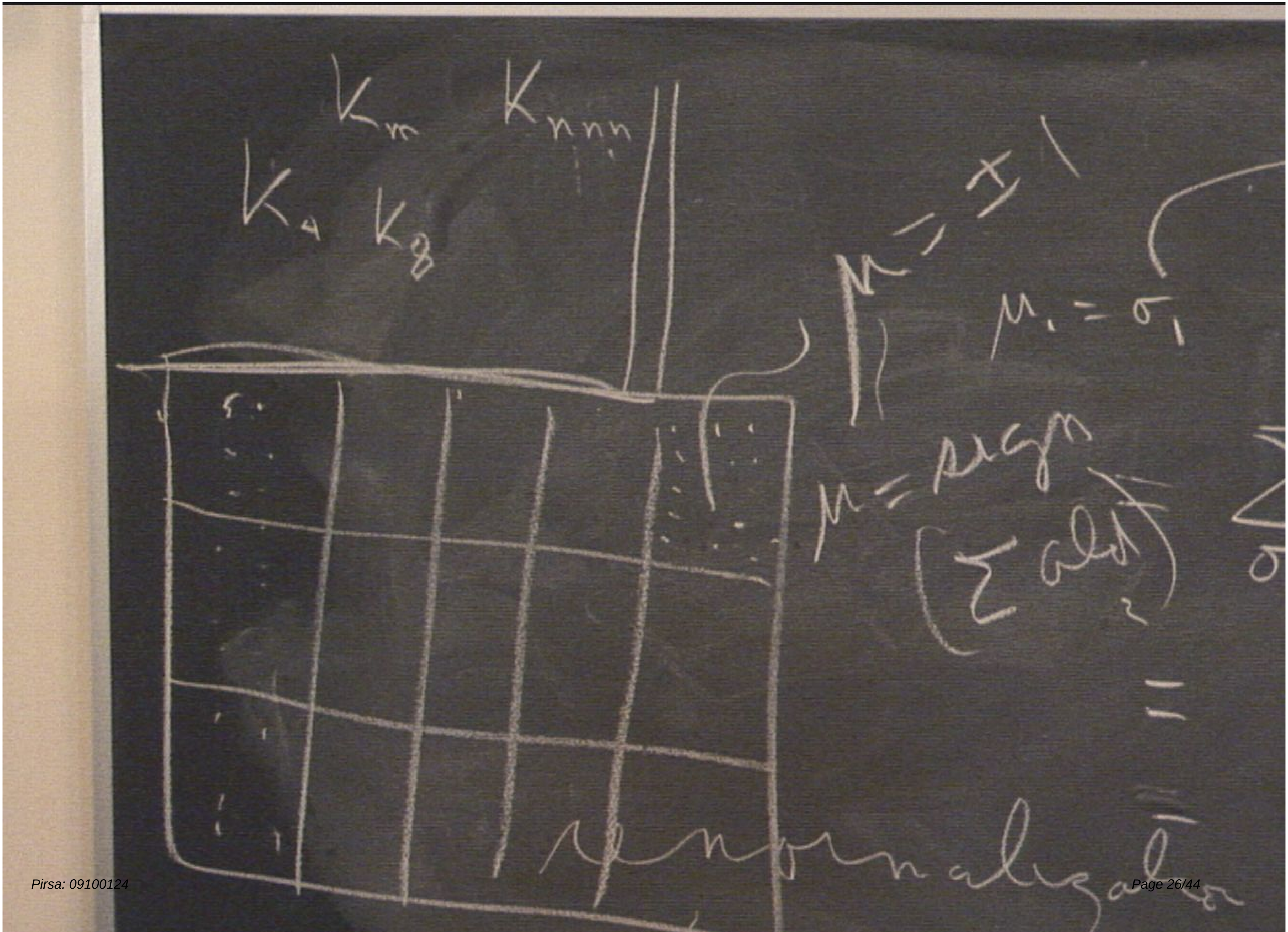
$$\ell_0 = \tilde{k}_0 + \tilde{k}_1 \tilde{\tau}_1$$

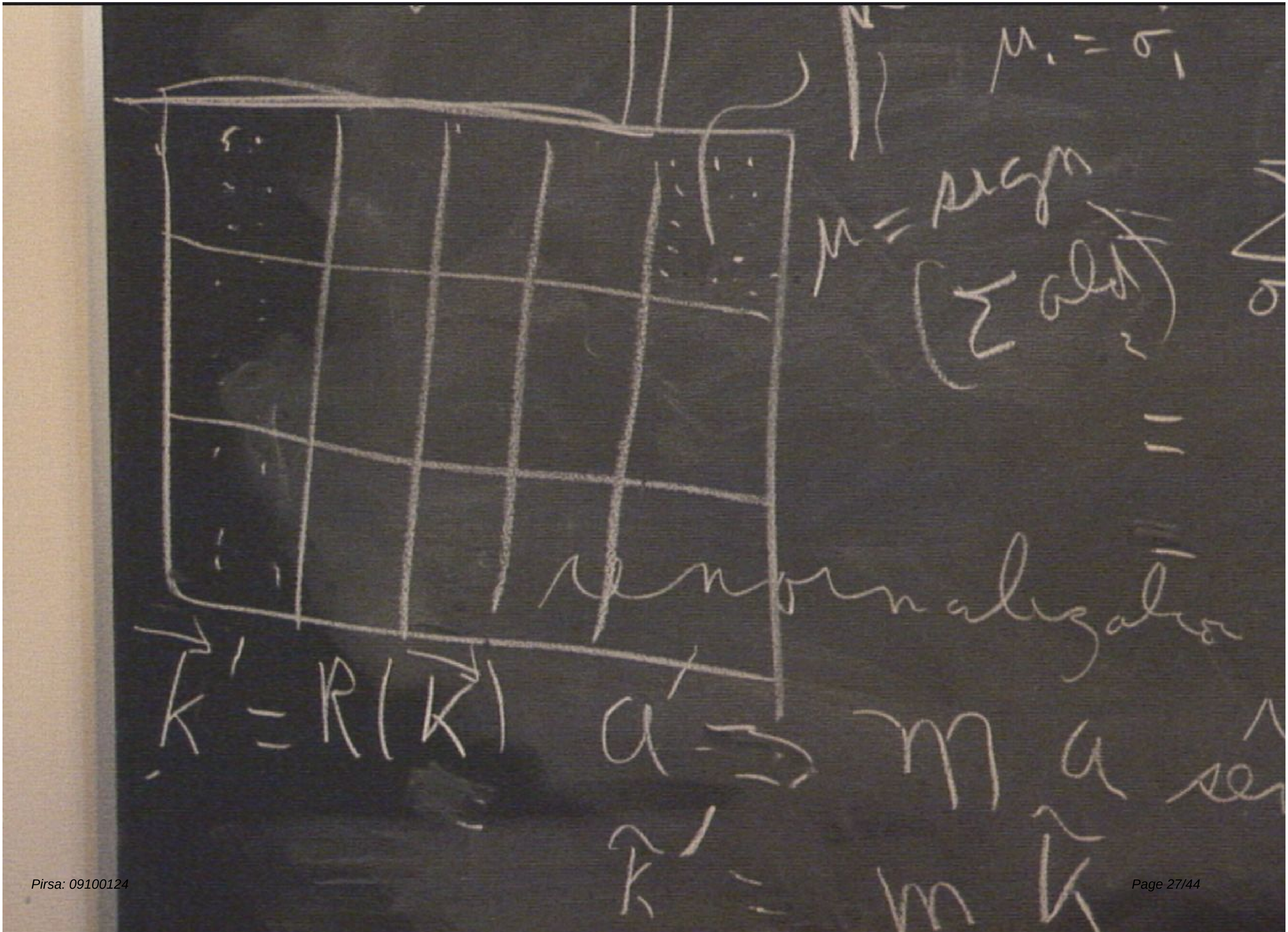
$d = 0 \qquad Z =$

$$\text{const}\left(e^{\tilde{k}} + e^{-\tilde{k}}\sigma\sigma'\right)$$

$$(\text{const})^2 \sum \left(e^{\tilde{k}} + \mu_1\sigma_2 e^{-\tilde{k}}\right)\left(e^{\tilde{k}} + \sigma_2\mu_2 e^{-\tilde{k}}\right)$$

$$2(\text{const})^2 \left[e^{2\tilde{k}} + 0 + 0 + \mu_2\mu_1 e^{-2\tilde{k}}\right]$$


K_m K_{nnn}
K_4 K_8
$\mu = \pm 1$
$\mu_i = \sigma_i$
$\mu = \mathrm{sign}$


renormalization
K' = R(K)

$$\mu = \operatorname{sign}\left(\sum a_{l_2}\right) \sum_{\sigma_2} e^{k\mu\sigma_2} e^{k\sigma}$$

$$= \left(e^{2K} + e^{-2K}\right)$$

$$= 2$$

renormalization group

$a' = m\, a$ semi-

$\hat{k}' = m \tilde{k}$

$a' = m\, a$

$a'' = m^2 a$

N coupling values

$N' = \frac{N}{2}$ " "

$$-\beta H\{\sigma\} = K_0 \sum_r 1 + K_{nn} \sum_{\langle r\,r'\rangle} \sigma_r \sigma_{r'}$$

$$+ K_{nnn} \sum_{nnn} \sigma_r \sigma_{r'}$$

$$e^{2K'} =$$

$$+ K_4 \left(\sigma_1 \sigma_2 \sigma_3 \sigma_4 + \text{translations} \right)$$

$\mu_1 \sigma_2 \quad K \sigma_2 \mu_2$

$\mu_2 = \sigma_3$

ℓ

$-\beta H\{\sigma\} =$

e^{-2K}

if $\mu_1 = \mu_2$

if $\mu_1 \neq \mu_2$

$e^{2K'} =$

$a' = m\, a$

$a = m^2 a$

$K_0' = f(\vec{K}_0)$

Other Variables

Ising variable takes on 2 values coupling between variables depends on whether they are the same or different. Z_2. Ferromaqgnetic, antiferromagnetic coupling, even mixed. Phase transition in dimensions greater than one.

q-state Potts model, variable takes on q-values, simplectic coupling.

interaction ***s.s'*** ***s*** is a q-component vector. q=2 XY equivalent to U1
q=3 called (classical) heisenberg model, higher q's as well

SU_2 and any symmetry group you can think of
phase transitions explored in all dimensions

Renormalization Calculation

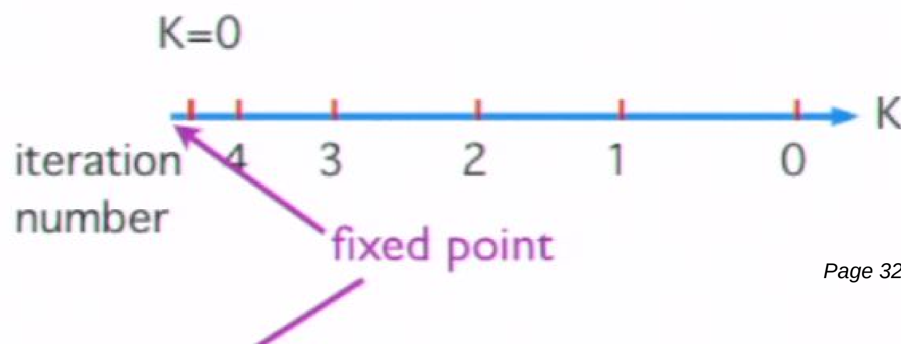
$$Z = \sum_{\mu_1,\mu_2,\cdots}\ \sum_{\sigma_2,\sigma_4,\cdots} \exp(K\mu_1\sigma_2 + K\sigma_2\mu_2 + \cdots) = \sum_{\mu_1,\mu_2,\cdots} \exp(w'\{\mu\})$$

So the new nearest neighbor coupling term is given by

$$\exp(K_0' + K'\mu_1\mu_2) = \sum_{\sigma_2} \exp(K\mu_1\sigma_2 + K\sigma_2\mu_2)$$

which then gives us $\exp(K_0' + K') = e^{2K} + e^{-2K}$ and $\exp(K_0' - K') = 2$
so that $e^{2K'} = \cosh 2K$.

The renormalization calculation tells us what we know already, namely that the one-dimensional model has no phase transition. A phase transition is a change in the long-ranged structure of correlations in a system. Here the couplings gradually weaken as you renormalize to longer and longer distances. All possible values of the coupling reduce to weak couplings at long distances. The system is always in the weak coupling phase. So there is no phase transition.



After many iterations coupling approaches fixed point at $K=0$

Ising Model in d=2

$$-H/(kT) = K\sum_{nn}\sigma_r\sigma_s + h\sum_r \sigma_r$$

$\sigma_r = \pm 1$

nn indicates a sum over nearest neighboirs

square lattice

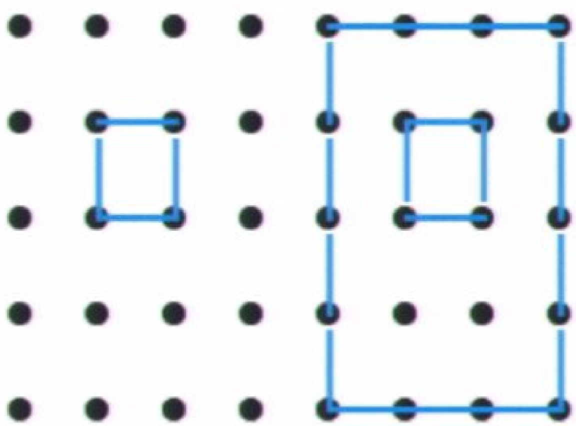
Onsager calculated partition function and phase transition for this situation

Nearest neighbor structure
s's are nearest neighbors to r
Bonds=exp($K\sigma\sigma'$) connect nearest neighbors

High Temperature Expansion

Nearest neighbor structure
Bonds=exp($K\sigma\sigma'$) connect nearest neighbors
Bond=cosh K +$\sigma\sigma'$ sinh K =cosh K[1+ $\sigma\sigma'$ tanh K]

Z=(2 cosh K cosh K)^N < products of [1+ $\sigma\sigma'$ tanh K] >
= (2 cosh K cosh K)^N sum < products of (tanh K)^M >
for nonzero terms, when there are N sites



To get a non-zero value each spin must appear on a even number of bonds. You then get the lattice covered by closed polygons.

With a lot of hard work one can calculate a series up to ten or even twenty terms long and estimate behavior of thermodynamic functions from these seres

$$e^{K\sigma\sigma'} = \cosh K\left[1 + \sigma\sigma' \tanh K\right]$$

$$\text{const}\left(e^{k} + e^{-k}\sigma\sigma'\right)$$

$$(\text{const})^2 \sum \left(e^{\tilde{k}} + \mu_1\sigma_2 e^{-\tilde{k}}\right)\left(e^{\tilde{k}} + \sigma_2\mu_2 e^{-\tilde{k}}\right)$$

$$2(\text{const})^2\left[e^{2\tilde{k}} + 0 + 0 + \mu_1\mu_2 e^{-2\tilde{k}}\right]$$

$$e^{K\sigma\sigma'} = \cosh K\left[1 + \sigma\sigma' \tanh K\right]$$

$$Z = \left[\cosh K\right]^{N}$$

$$\text{const}\left(e^{K} + e^{-K}\sigma\sigma'\right)$$

$$\left(\text{const}\right)^{2} \sum \left(e^{\hat{k}} + \mu_1 \sigma_2 e^{-\hat{k}}\right)\left(e^{\hat{k}} + \sigma_2 \mu_2 e^{-\hat{k}}\right)$$

$$2\left(\text{const}\right)^{2} \left[e^{2\hat{k}} + 0 + 0 + \mu_2 \mu_1 e^{-2\hat{k}}\right]$$

$$e^{K\sigma\sigma'} = \cosh K \left[1 + \sigma\sigma' \tanh K \right]$$

$$Z = 2^{L^2} \left[\cosh K \right]^{2L^2}$$

$$\text{const} \left(e^{k} + e^{-k} \sigma\sigma' \right)$$

$$(\text{const})^2 \sum \left(e^{\tilde{k}} + \mu_1 \sigma_2 e^{-\tilde{k}} \right) \left(e^{\tilde{k}} + \sigma_2 \mu_2 e^{-\tilde{k}} \right)$$

$$2 (\text{const})^2 \left[\sigma_2^2 e^{2\tilde{k}} + 0 + 0 + \mu_2 \mu_1 e^{-2\tilde{k}} \right]$$

$$e^{K\sigma\sigma'} = \cosh k\left[1+\sigma\sigma'\tanh k\right]$$

$$Z = 2^{L^2}\left[\cosh k\right]^{2L^2}\left[\quad\right]_2$$

$$1 + \tanh k\,\sigma\sigma + \tanh\,\sigma\sigma\sigma\sigma$$

$$\text{const}\left(e^{k} + e^{-k}\sigma\sigma'\right)$$

$$(\text{const})^2\sum\left(e^{\tilde{k}} + \mu_1\sigma_2 e^{-\tilde{k}}\right)\left(e^{\tilde{k}} + \sigma_2\mu_2 e^{-\tilde{k}}\right)$$

$$(\text{const})^2\sum_{\sigma_2}\left[e^{2\tilde{k}} + 0 + 0 + \mu_1\mu_2 e^{-2\tilde{k}}\right]$$

$\tanh^4 K \quad \sigma_1 \sigma_2 \quad e^{K\sigma\sigma'} =$

$\sigma_2 \sigma_3 \sigma_3 \sigma_4 \sigma_4 \sigma_1 \quad Z = 2^{L^2} [$

$= \tanh^4 K \qquad 1 + \tan$

$\cosh (e$

$(\cosh)^2 \sum \left(e^{\hat{k}} + \mu_1 \sigma_2\right.$

$2 (\cosh)^2 \left[e^{2\hat{k}} + O + O\right.$

$$\sigma_1 \sigma_2 \qquad e^{K\sigma\sigma'} = \cosh K \left[1 + \sigma\sigma' \tanh K \right]$$

$$\sigma_4 \sigma_4 \sigma_1 \qquad Z = 2^{L^2} \left[\cosh K\right]^{2L^2} \left[\quad \right]_2$$

$$\sim \underline{1} + \tanh K \, \sigma\sigma + \tanh \underline{\sigma\sigma\sigma\sigma}$$

$$\text{const}\left(e^{k} + e^{-k}\sigma\sigma'\right)$$

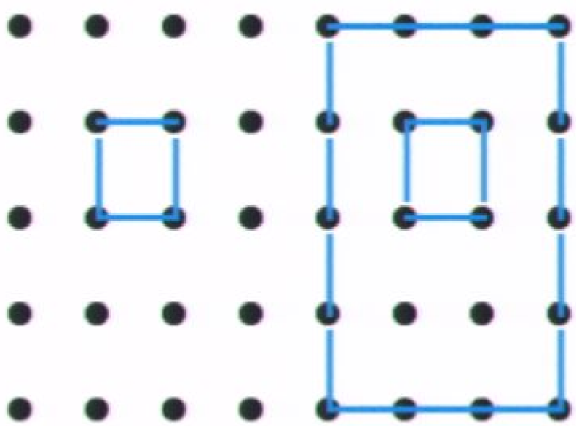
$$(\text{const})^2 \sum \left(e^{\hat{k}} + \mu_1 \sigma_2 e^{-k}\right)\left(e^{\hat{k}} + \sigma_2 \mu_2 e^{-\hat{k}}\right)$$

$$2 (\text{const})^2 \left[\sigma_2^2 e^{2\hat{k}} + 0 + 0 + \mu_1 \mu_2 e^{-2\hat{k}} \right]$$

High Temperature Expansion

Nearest neighbor structure
Bonds=exp($K\sigma\sigma'$) connect nearest neighbors
Bond=cosh K +$\sigma\sigma'$ sinh K =cosh K[1+ $\sigma\sigma'$ tanh K]

Z=(2 cosh K cosh K)^N < products of [1+ $\sigma\sigma'$ tanh K] >
= (2 cosh K cosh K)^N sum < products of (tanh K)^M >
for nonzero terms, when there are N sites



To get a non-zero value each spin must appear on a even number of bonds. You then get the lattice covered by closed polygons.

With a lot of hard work one can calculate a series up to ten or even twenty terms long and estimate behavior of thermodynamic functions from these seres

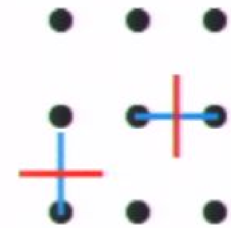
Low Temperature Expansion

Nearest neighbor structure

Bonds=exp($K\sigma\sigma'$) =$e^{K}\delta_{\sigma,\sigma'}$ +$e^{-K}\delta_{\sigma,-\sigma'}$

Bond =$e^{K}[\delta_{\sigma,\sigma'}$ +$e^{-2K}\delta_{\sigma,-\sigma'}]$

We draw these bonds differently from the high T bonds. We draw them rotated 90 degrees in comparison to the other bonds.

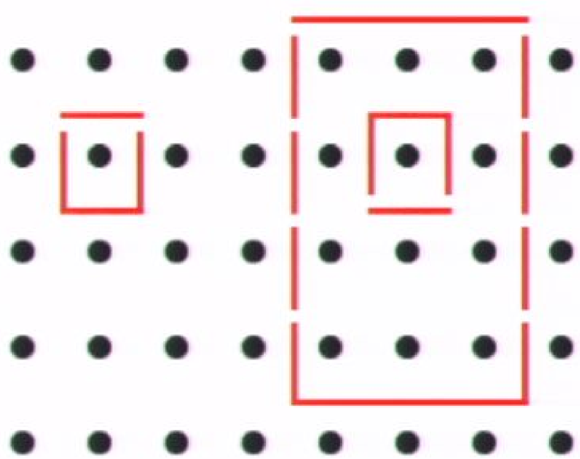


note e^{-2K} = tanh $\tilde{K}$

$Z=2(e^{K})^{N}$ < products of $[\delta_{\sigma,\sigma'}$ +$e^{-2K}\delta_{\sigma,-\sigma'}]$ >

= $2e^{NK}$ sum < products of $(e^{-2K})^{M}$ >

for nonzero terms



To get a non-zero term, assign a value to one spin. Then every time you cross a red line, change the spin-value to the opposite. Your valid pictures become a series of closed red polygons.

With a lot of hard work one can calculate a series up to ten or even twenty terms long and estimate behavior of thermodynamic functions from these seres

High Temperature Expansion

Nearest neighbor structure
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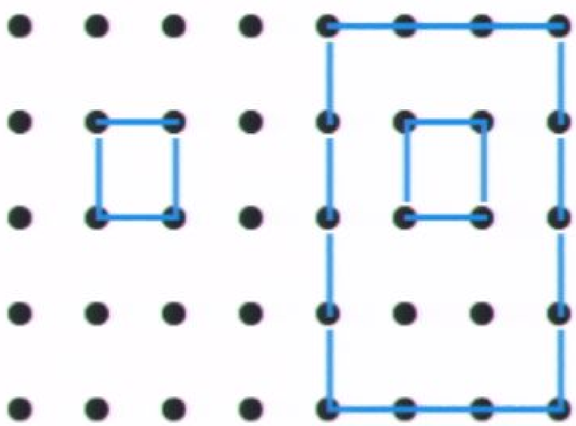

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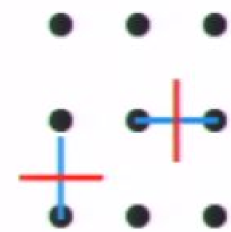

note e^{-2K} = tanh $\tilde{K}$

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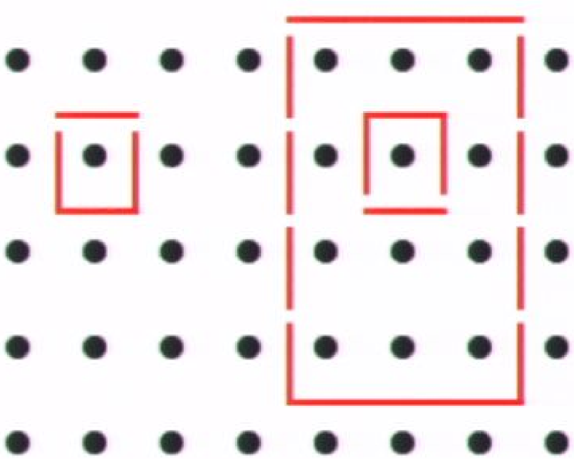
= $2e^{NK}$ sum < products of $(e^{-2K})^{M}$ >

for nonzero terms



To get a non-zero term, assign a value to one spin. Then every time you cross a red line, change the spin-value to the opposite. Your valid pictures become a series of closed red polygons.

With a lot of hard work one can calculate a series up to ten or even twenty terms long and estimate behavior of thermodynamic functions from these seres